

Recent Developments in perturbative calculations on DVCS

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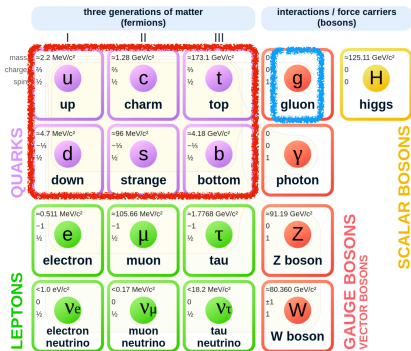
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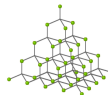
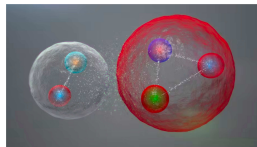


Standard model of particle physics

Standard Model of Elementary Particles

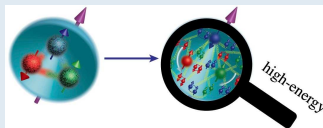


theory for strong interaction:
Quantum Chromodynamics (QCD)

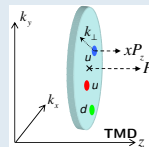
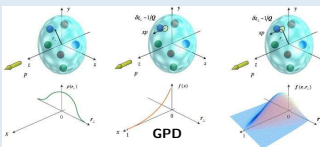


Internal structures of hadrons

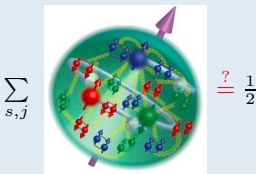
- Hadrons are composite particles with complicated internal structures



- PDFs only encode longitudinal information; GPDs and TMDs include transverse information



- GPDs are crucial for understanding the spin structure of hadrons

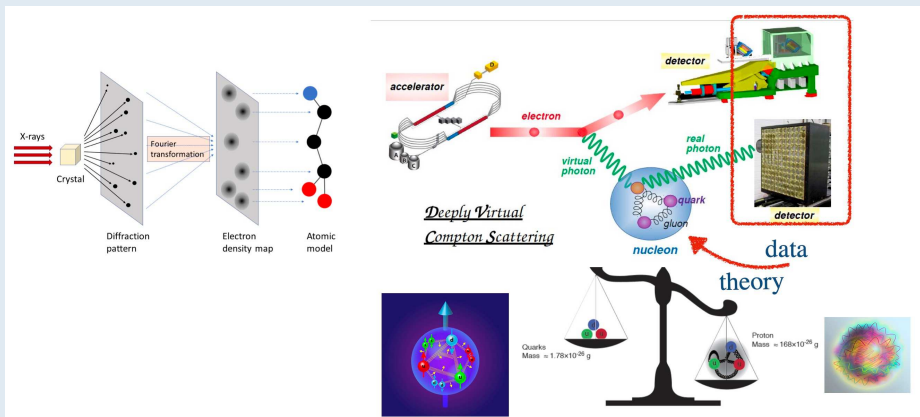


Unsolved problem in physics:

? How do the quarks and gluons carry the spin of protons?



Structure information from experiment

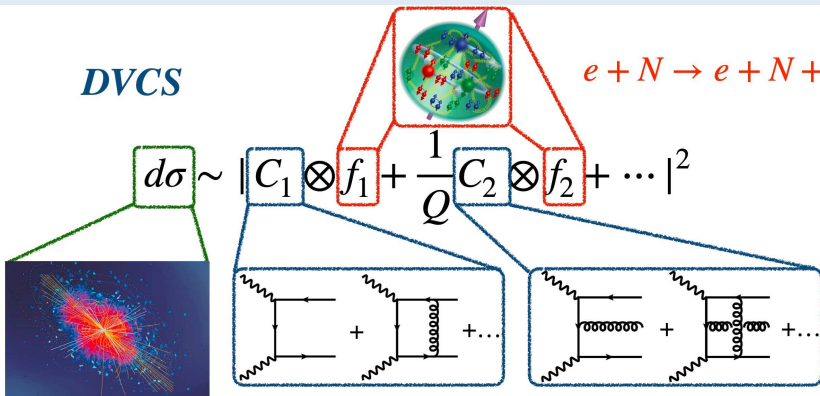


Extracting structure information of hadrons from high-energy experiment

- Theoretical foundation for accessing structural information of hadrons: factorization

DVCS

$e + N \rightarrow e + N + \gamma$



- One of the main scientific goal of JLAB 12 and EIC (EicC)



Deeply Virtual Compton Scattering (DVCS)

- Golden channel for extracting the Generalized Parton Distributions (GPDs)

$$\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p') \quad \gamma^*(q') \text{ for DDVCS process}$$

Kinematics: $q'^2 = 0$, $q^2 = -Q^2$, $t = \Delta^2 = (p' - p)^2$, $p^2 = p'^2 = m^2$, $P_\mu = (p_\mu + p'_\mu)/2$

- DVCS amplitude defined as (hadronic)

$$\delta(p + q - p' - q') \mathcal{A}_{\mu\nu}(q, q', p) = i \int \frac{d^4x d^4y}{(2\pi)^4} e^{-iq \cdot x + iq' \cdot y} \langle p' | T \{ j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(y) \} | p \rangle$$

decomposable into helicity amplitudes

[V. Braun, A. Manashov, B. Pirnay, (2012)]

$$\begin{aligned} \mathcal{A}_{\mu\nu}(q, q', p) = & \varepsilon_\mu^+ \varepsilon_\nu^{*+} \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^{*-} \mathcal{A}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^{*+} \mathcal{A}^{0+} + \varepsilon_\mu^0 \varepsilon_\nu^{*-} \mathcal{A}^{0-} \\ & + \varepsilon_\mu^+ \varepsilon_\nu^{*-} \mathcal{A}^{+-} + \varepsilon_\mu^- \varepsilon_\nu^{*+} \mathcal{A}^{-+} + q'_\nu \mathcal{A}_\mu^{(3)} \end{aligned}$$

Parity conservation dictates:

$$\mathcal{A}^{++} = \mathcal{A}^{--} \equiv \mathcal{A}^{(0)}, \quad \mathcal{A}^{0\pm} \equiv (\varepsilon_\mu^\pm P^\mu) \mathcal{A}^{(1)}, \quad \mathcal{A}^{\mp\pm} \equiv (\varepsilon_\mu^\pm P^\mu)^2 \mathcal{A}^{(2)},$$

- Other parameterizations available, exact relations between them known

[A. Belitsky, D. Müller, YJ, (2014)], [V. Braun, A. Manashov, D. Müller, B. Pirnay, (2014)]



Higher α_s corrections at leading-power

- The helicity amplitudes $\mathcal{A}^{(0)}$ contributes at the leading-power in $1/Q$

$$\mathcal{A}^{(0)} = \mathcal{A}_{\text{LP}}^{(0)} + \mathcal{O}(1/Q), \quad \mathcal{A}^{(1)} = \mathcal{O}(1/Q), \quad \mathcal{A}^{(2)} = \mathcal{O}(1/Q^2)$$

- $\mathcal{A}_{\text{LP}}^{(0)}$ factorizes into a convolution of perturbative (Wilson) coefficient function and GPD

$$\mathcal{A}_{\text{LP}}^{(0)}(\xi, Q^2) \sim \mathcal{H}_{V/A,i}^{\text{LP}} = \int_{-1}^1 \frac{dx}{\xi} C_{V/A,i} \left(\alpha_s, \frac{x}{\xi}, \frac{\mu^2}{Q^2} \right) F_{V/A,i}(x, \xi, \mu), \quad t\text{-dependence} \sim \frac{t}{Q^2}$$

- $\mathcal{A}_{\text{LP}}^{(0)}$ is the main contribution in JLAB-12 and EIC DVCS process (BH as noise)
- $C_{V/A,i}$ captures dynamics at $\sim Q$ scale, whereas $F_{V/A}$ describes physics at $\sim \Lambda_{\text{QCD}}$
- Both $C_{V/A}$ and $\sim \Lambda_{\text{QCD}}$ are factorization scale μ -dependent

$$\mu \frac{d\mathcal{H}}{d\mu} = 0, \quad \text{resummation/RGE evolution} \implies \mu \frac{d}{d\mu} \mathcal{H} \Big|_{\alpha_s^n} = \mathcal{O}(\alpha_s^{n+1})$$



Scale dependence of GPDs

- The GPD satisfies the RGE
 - Flavor nonsinglet case

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \alpha_s} + \mathbb{H}_{\text{NS}} \right) F_q(x, \xi, \mu) = 0,$$

$\mathbb{H}$ is called evolution kernel. In position space $\mathbb{H}_{\text{NS}} \Leftrightarrow \mathbb{H}_{\text{ERBL}}$, partially known to three-loops

[S. Mikhailov and A. Radyushkin (1985), G. Katz (1985)], [V. Braun, A. Manashov, S. Moch, M. Strohmaier (2017)]

- Flavor singlet case known to two-loops

$$\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \alpha_s} + \begin{pmatrix} \mathbb{H}_S^{qq} & \mathbb{H}_S^{qg} \\ \mathbb{H}_S^{gq} & \mathbb{H}_S^{gg} \end{pmatrix} \right] \begin{pmatrix} F_q(x, \xi, \mu) \\ F_g(x, \xi, \mu) \end{pmatrix} = 0$$

[A. Belitsky, D. Müller (2000)], [V. Braun, A. Manashov, S. Moch, M. Strohmaier (2019)]

- $\mathbb{H}$ obtainable by computing conformal anomaly Δ of one-loop less, a general quantity
- $\mathbb{H}_{\text{pos.}} \xrightarrow{\text{F.T.}} \mathbb{H}_{\text{mom.}}$ is easy, but NOT vice versa
- Evolution kernels for higher-twist GPDs known to one-loop

[V. Braun, A. Manashov, J. Rohrwild (2009)], [A. Belitsky, YJ (2014)]



Perturbative Wilson coefficient function in DVCS

- The complete Wilson coefficients $C_{V/A,i}$ for DVCS at LP are known to $\mathcal{O}(\alpha_s^2)$

- Flavor nonsinglet vector case computed via two-loop conformal anomaly

[V. Braun, A. Manashov, S. Moch, J. Schoenleber (2021)]

- Flavor nonsinglet axial-vector case computed
require evanescent operators $\Leftrightarrow$ renormalization scheme

[V. Braun, A. Manashov, S. Moch, J. Schoenleber (2021)] (by $\Delta^{(2)}$), [J. Gao, T. Huber, YJ, Y.-M. Wang (2021)] (IBP, MI, evan. operator)

- Flavor singlet vector contribution computed using IBP, MI technique

[V. Braun, YJ, J. Schoenleber (2022)]

- Flavor singlet axial-vector and transversity contribution known

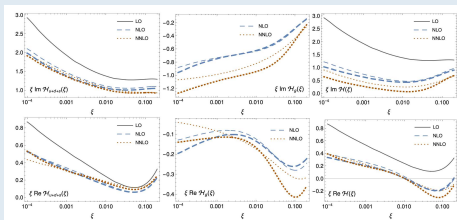
[YJ, J. Schoenleber (2023)]

- All coefficient functions are expressible using HPL functions of weight 0 and 1



Numerical impact of NNLL' corrections

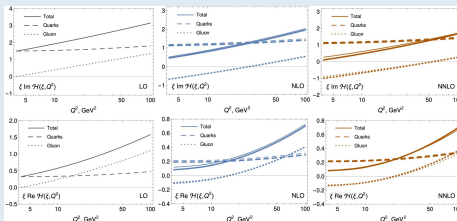
- The gluonic contribution at two-loop is significant compared to one-loop



CFF $\mathcal{H}$ as a function of ξ at $\mu^2 = Q^2 = 4 \text{ GeV}^2$.

- In total, the two-loop correction is smaller, but non-negligible

[V. Braun, Y.J. J. Schoenleber (2022)]



CFF $\mathcal{H}$ as a function of Q^2 at $\xi = 0.005$. The growth of $\mathcal{H}$ is almost entirely driven by gluon contribution.



Full helicity amplitudes in DVCS

- Helicity amplitudes $\mathcal{A}^{(k)}$ can be expanded in $1/Q$,

$$\begin{aligned}\mathcal{A}^{(0)} &= \mathcal{A}_{0,1}^{(0)} + \mathcal{A}_{2,t}^{(0)} + \mathcal{A}_{2,m^2}^{(0)} + \mathcal{A}_{4,t^2}^{(0)} + \mathcal{A}_{4,m^2 t}^{(0)} + \mathcal{A}_{4,m^4}^{(0)} + \dots \\ \mathcal{A}^{(1)} &= \mathcal{A}_{1,1}^{(1)} + \mathcal{A}_{3,t}^{(1)} + \mathcal{A}_{3,m^2}^{(1)} + \mathcal{A}_{5,t^2}^{(1)} + \mathcal{A}_{5,m^2 t}^{(1)} + \mathcal{A}_{5,m^4}^{(1)} + \dots \\ \mathcal{A}^{(2)} &= \mathcal{A}_{2,1}^{(2)} + \mathcal{A}_{4,t}^{(2)} + \mathcal{A}_{4,m^2}^{(2)} + \mathcal{A}_{6,t^2}^{(2)} + \mathcal{A}_{6,m^2 t}^{(2)} + \mathcal{A}_{6,m^4}^{(2)} + \dots\end{aligned}$$

with

$$\mathcal{A}_{j,z}^{(i)} \sim \frac{z}{Q^j},$$

- subleading corrections are generated from two sources:

$$\mathcal{A}_{j>0,z}^{(i)} = \mathcal{A}_{j,z}^{(i),\text{kin}} + \mathcal{A}_{j,z}^{(i),\text{dyn}}$$

- dynamical corrections carries **genuine higher-twist** information of the hadron, while kinematic ones are induced by **twist-two GPDs**
- Each term is a convolution of coefficient function $C_{j,z}^{(i)}$ and GPD H_k , schematically*:

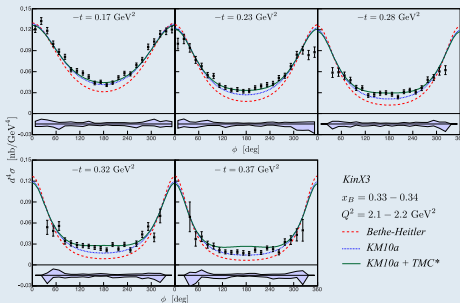
$$\mathbb{A}_{j,z}^{(i),\text{kin}} = C_{j,z}^{(i)} \otimes H_2, \quad \mathcal{A}_{j,z}^{(i),\text{dyn}} = C_{j,z}^{(i,k)} \otimes H_k, k > 2, \quad \text{*if factorization holds}$$

- DIS case: $t = 0 \mapsto$ Nachtmann correction, (1973)



Motivation

- Theory side:
 - necessary to remove frame dependence, restore **EM gauge and translation symmetry**
 - a systematic framework for higher-power kinematic contribution
 - verify factorization at **NNLP** order
 - application of conformal symmetry to higher-power corrections
 - Phenomenological side:
 - provide theory support for nuclei DVCS measurements (significant m^2 corrections)
- [M. Hattawy et al. [CLAS], Phys. Rev. Lett. 119, no.20, 202004 (2017)]
- mismatch between theory and experiment, resolvable by higher-power corrections?



Kinematic corrections: local form

- OPE expansion of current correlator (schematically, scalar current for simplicity):

[V. Braun, YJ, A. Manashov (2021)]

$$\begin{aligned}
 T\{j(x_1)j(x_2)\} &= \sum_N \left\{ A_N^{\mu_1 \cdots \mu_N} \underbrace{\mathcal{O}_{\mu_1 \cdots \mu_N}^N}_{\text{twist-2 operators}} + B_N^{\mu_1 \cdots \mu_N} \underbrace{\partial^\mu \mathcal{O}_{\mu, \mu_1 \cdots \mu_N}^N}_{\text{descendants of twist-2}} \right. \\
 &\quad \left. + C_N^{\mu_1 \cdots \mu_N} \partial^2 \underbrace{\mathcal{O}_{\mu_1 \cdots \mu_N}^N}_{\text{twist-2 operators}} + D_N^{\mu_1 \cdots \mu_N} \underbrace{\partial^\mu \partial^\nu \mathcal{O}_{\mu, \nu, \mu_1 \cdots \mu_N}^N}_{\text{descendants}} + \cdots \right\} \\
 &\quad + \text{genuine higher-twist contributions} \\
 &\equiv \sum_N C_N^{\mu_1 \cdots \mu_N}(x, \partial) \mathcal{O}_{\mu_1 \cdots \mu_N}^N + \text{genuine higher-twist contributions}
 \end{aligned}$$

- employ conformally covariant OPE:

[S. Ferrara, A. F. Grillo, and Gatto, 1971-1973]

CFs of descendants are related to the CFs of the twist-2 operators by conformal symmetry

$$A_N^{\mu_1 \cdots \mu_N} \xrightarrow{O(4,2)} C_N^{\mu_1 \cdots \mu_N}(x, \partial)$$



CFs for kinematic corrections: local OPE of leading twist and descendants

- A glimpse of local coefficient functions ($\otimes$ with $\mathcal{O}_{N,V}^{(k)}(x_{12}^u), \mathcal{O}_{N,A}^{(k)}(x_{12}^u), k = 0, 1, 2$)

[V. Braun, Y.J. A. Manashov (2021)]

- vector case:

$$\begin{aligned} \mathbb{C}_{N,V}^{\mu\nu,(0)} = & \frac{r_{N,V}}{(-x_{12}^2 + i0)^2} \left[(N+1)g_{\mu\nu} \left(1 - \frac{1}{4} \frac{u\bar{u}}{N+1} x_{12}^2 \partial^2 \right) \right. \\ & + \frac{1}{2N} x_{12}^2 (\partial_1^\mu \partial_2^\nu - \partial_1^\nu \partial_2^\mu) + \left(1 - \frac{1}{4} \frac{u\bar{u}}{N} x_{12}^2 \partial^2 \right) \left(\frac{\bar{u}}{u} x_{21}^\mu \partial_1^\nu + \frac{u}{\bar{u}} x_{12}^\nu \partial_2^\mu \right) \\ & - \frac{1}{4} \frac{u\bar{u}}{N(N+1)} x_{12}^2 \partial^2 (x_{21}^\nu \partial_1^\mu + x_{12}^\mu \partial_2^\nu) \\ & \left. - \frac{x_{12}^\mu x_{12}^\nu}{N+1} u\bar{u} \partial^2 \left(1 - \frac{1}{4} \frac{u\bar{u}}{N+2} x_{12}^2 \partial^2 \right) \right] + \dots \end{aligned}$$

- axial-vector case:

$$\begin{aligned} \mathbb{C}_{N,A}^{\mu\nu,(0)} = & \frac{r_{N,A}}{(-x_{12}^2 + i0)^2} \left\{ \epsilon^{\mu\nu}{}_{\beta\gamma} x_{12}^\beta \left[N \left(\frac{u}{\bar{u}} \partial_2^\gamma - \frac{\bar{u}}{u} \partial_1^\gamma \right) \right. \right. \\ & - \frac{1}{4} \frac{u\bar{u} x_{12}^2 \partial^2}{(N+1)} \left(\partial_2^\gamma - \partial_1^\gamma + (N+1) \left(\frac{u}{\bar{u}} \partial_2^\gamma - \frac{\bar{u}}{u} \partial_1^\gamma \right) \right) \\ & \left. \left. - (x_{12}^\nu \epsilon^\mu{}_{\alpha\beta\gamma} + x_{12}^\mu \epsilon^\nu{}_{\alpha\beta\gamma}) x_{12}^\alpha \left(1 - \frac{1}{4} \frac{u\bar{u} x_{12}^2 \partial^2}{N+1} \right) \partial_1^\beta \partial_2^\gamma \right] \right\} + \dots \end{aligned}$$



Kinematic corrections: nonlocal form

- resumming back into nonlocal operator $\mapsto$ GPD/DD

[V. Braun, Y.J. A. Manashov (2022)]

$$\begin{aligned}
 \mathbb{A}_V^{\mu\nu} &= \text{T}\{j^\mu(x)j^\nu(0)\} \\
 &= \frac{1}{i\pi^2} \left\{ \frac{1}{(-x^2 + i0)^2} \int_0^1 dv \left[[g^{\mu\nu}(x\partial) - x^\mu\partial^\nu] O(v, 0) - x^\nu(\partial^\mu - i\Delta^\mu) O(1, v) \right] \right. \\
 &\quad - \frac{1}{(-x^2 + i0)} \int_0^1 du \int_0^u dv \left[\frac{i}{2} (\Delta^\nu\partial^\mu - \Delta^\mu\partial^\nu) O(u, v) - \frac{\Delta^2 \bar{u}}{4} x^\mu\partial^\nu O(u, v) \right] \\
 &\quad \left. + \frac{\Delta^2}{2} \frac{x^\mu x^\nu}{(-x^2 + i0)^2} \int_0^1 du u \int_0^u dv O(u, v) + \dots \right\}, \\
 O(z_1, z_2) &\text{ non-local twist-2 light-ray operators } \mapsto \text{GPD/DD}, \text{ also } \Delta \cdot \partial_x O(z_1, z_2), \dots
 \end{aligned}$$

- many highly nontrivial cancellations must happen to resum local $\mapsto$ nonlocal
 - need “intertwining operator” to relate operators of different conformal spin
- similar for axial-vector case



Phenomenology: nucleon DVCS

- Fourier transform to momentum space: final prediction to NNLP*

$$\begin{aligned} \mathcal{A}_{\mu\nu} = & -g_{\mu\nu}^\perp \mathcal{A}^{(0)} + \frac{1}{Q} \left(q_\mu + q'_\mu \frac{Q^2}{q \cdot q'} \right) g_{\nu\rho} P^\rho \mathcal{A}^{(1)} + \frac{1}{2} \left(g_{\mu\rho}^\perp g_{\nu\rho}^\perp - \epsilon_{\mu\rho} \epsilon_{\nu\sigma} \right) P^\rho P^\sigma \mathcal{A}^{(2)} \\ & + q'_\nu A_\mu^{(3)}, \\ g_{\mu\nu}^\perp = & g_{\mu\nu} - \frac{q_\mu q'_\nu}{(qq')} - q'_\mu q'_\nu \frac{Q^2}{(qq')^2}, \quad \epsilon_{\mu\nu}^\perp = \frac{1}{(qq')} \epsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta, \quad P^\mu = \frac{1}{2}(p + p')_\mu, \end{aligned}$$

- Fourier transform to momentum space: final prediction to NNLP*

$$\begin{aligned} \mathcal{A}^{(0)} & \sim 1 + \frac{1}{Q} + \frac{1}{Q^2} + \dots, \\ \mathcal{A}^{(1)} & \sim \frac{1}{Q} + \frac{1}{Q^3} + \dots, \\ \mathcal{A}^{(2)} & \sim \frac{1}{Q^2} + \frac{1}{Q^4} + \dots, \end{aligned}$$

- Further contributions can be calculated if necessary



A glimpse to the final analytic results

- structure of $\mathbb{A}_{j,k}^{(i)}$, an example:

[V. Braun, Y.J, A. Manashov (2024)]

$$\mathcal{A}_{1/Q^4}^{(2)} = -\frac{\varkappa}{(q \cdot q')^2} \left(4|P_\perp|^2 D_\xi^2 + \frac{3}{\xi} t D_\xi - 4t \right) \\ \times D_\xi^2 \int_{-1}^1 \frac{dx}{\xi} \mathcal{H}(x, \xi, t) \left(\frac{1}{\bar{x}_\xi} (\text{Li}_2(x_\xi) - \zeta_2) - \ln \bar{x}_\xi \right) + \dots$$

$$D_\xi = \xi^2 \partial_\xi, \quad x_\xi = \frac{x + \xi}{2\xi} + i0, \quad \bar{x}_\xi = 1 - x_\xi, \quad |P_\perp|^2 = -m^2 - t(1 - \xi^2)/(4\xi^2).$$

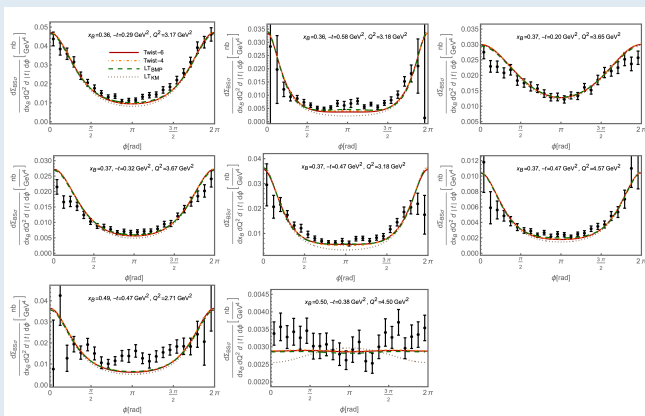
- $\mathbb{A}^{(1)}$ and $\mathbb{A}^{(2)}$ relatively simple, $\mathbb{A}_{4,k}^{(0)}$ complicated; factorization holds!
 - factorization violating $1/q'^2, \ln q'^2$ cancel in final result, highly nontrivial for $\mathcal{A}^{(0)}$
 - only $\text{Li}_2, \ln^2$ appears in final result
 - agree with previous lower-order result

[V. Braun, A. Manashov, B. Pirnay, Phys. Rev. D 86 (2012) 014003]



Numerics

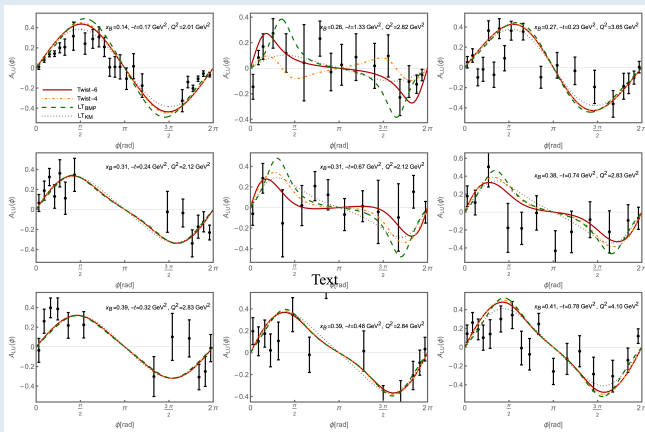
- Expansion parameter $1/Q^2 \rightarrow 1/(qq') = 2/(Q^2 + t)!$



GPD model: GK12; angular dependence of the spin-averaged total cross section



Numerics



GPD model: GK12; angular dependence of beam spin asymmetry



Conclusions and outlooks

• Conclusions

- complete one-loop Wilson coefficient functions available
- indispensable for analyzing JLab12 and EIC data
- complete LO NNLP kinematic corrections are obtained, tree-level factorization holds
- kinematic corrections can become large in certain kinematic regions
- result respects all symmetries to $1/Q^5$ order

• Outlooks

- three-loop GPD evolution kernels
- three-loop DVCS coefficient functions: convergence of gluon contribution
- two DDVCS and DVMP coefficient functions: accessible in JLAB and EIC data
- more extensive numerical analysis for JLAB/EIC data
- embedding into public codes
- α_s corrections, kinematic corrections from gluon GPD
- apply to other exclusive processes: target mass correction
- . . .

[P. Ball, V. Braun, NPB543 (1999)]

