x-dependent Baryon Light-cone DA from Lattice QCD

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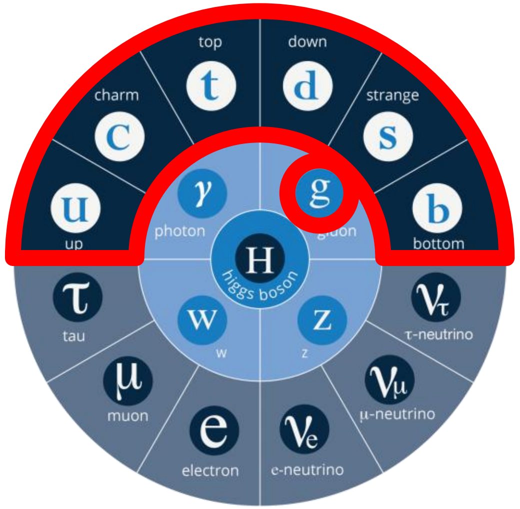
(Lattice Parton Collaboration)

Sep 23rd @ SPIN2025, Qingdao

OUT LINE

Why Baryon ?

Baryon — cornerstone particles

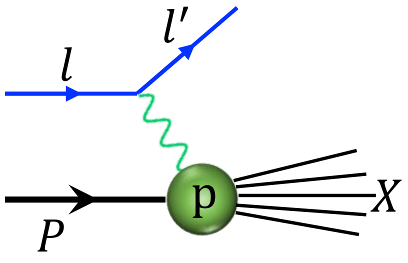


- Matter genesis in the Universe
- Tools to probe micro world
- Astronomy & Cosmology
- …………

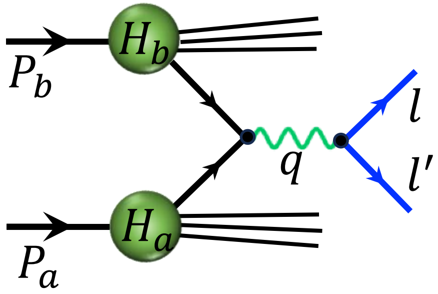
Why LCDA ?

- Probe internal structure of nucleons in **Inclusive processes**
- **Defined PDFs**

DIS

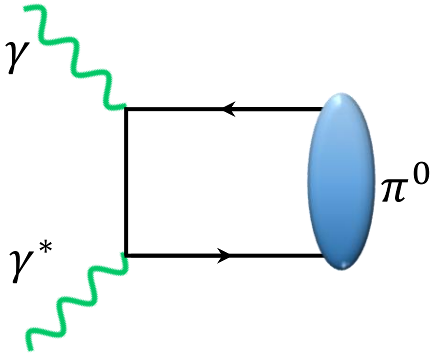


Drell-Yan

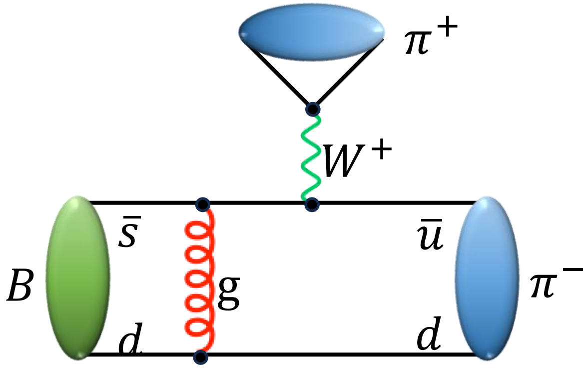


- Obtain richer QCD dynamical information in **Exclusive scatterings**
- **Defined LCDAs**

$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$



Why Baryon LCDA ?

Establish CP Violation in b-Baryon Decays

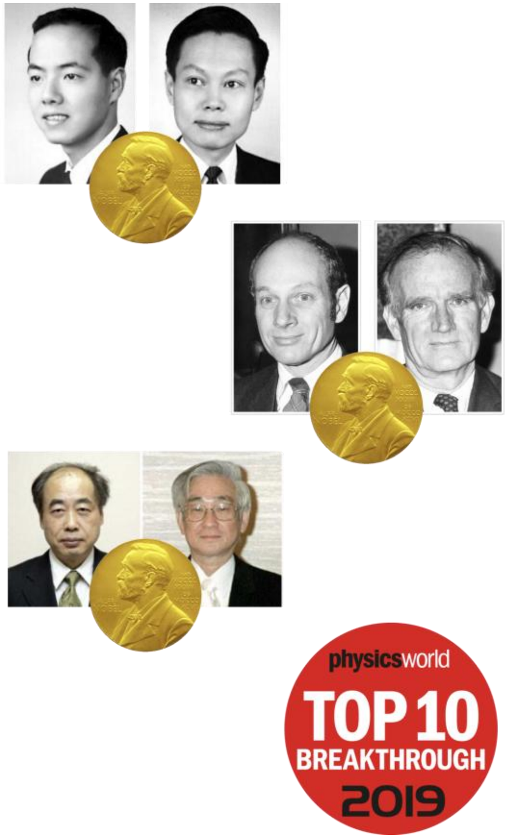
- 1956, Parity violation in weak interaction;
- 1973, Kobayashi-Maskawa mechanism;
- CPV well established in meson since 1964:

  1964 Kaon; 2004 B meson; 2019 D meson

....

- 2025, first bservation of direct CPV in **Baryon**:

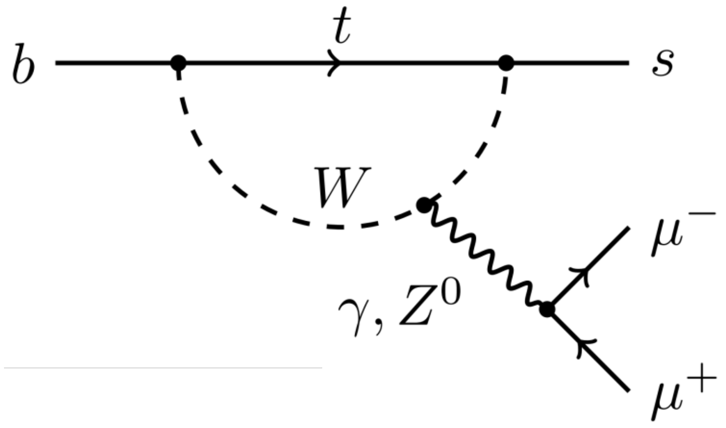
$$\Lambda_b^0 \to p$$

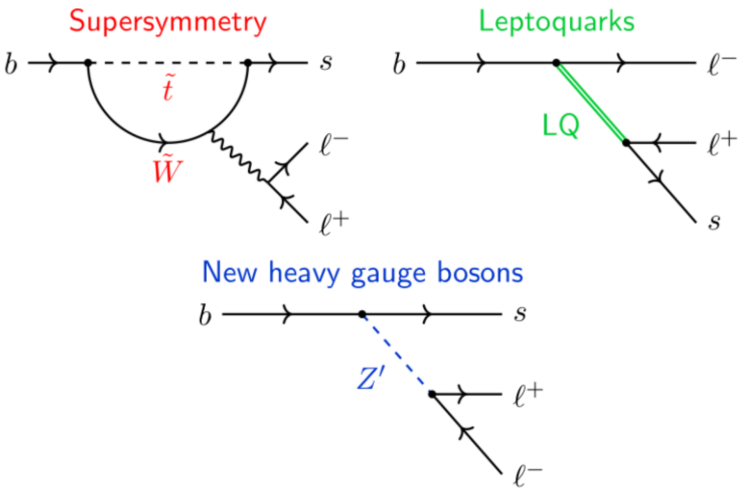


LHCb, Nature (2025);
J.J.Han, et.al. PRL 134, 221801 (2025)

Why Baryon LCDA ?

Searching New Physics beyond the SM





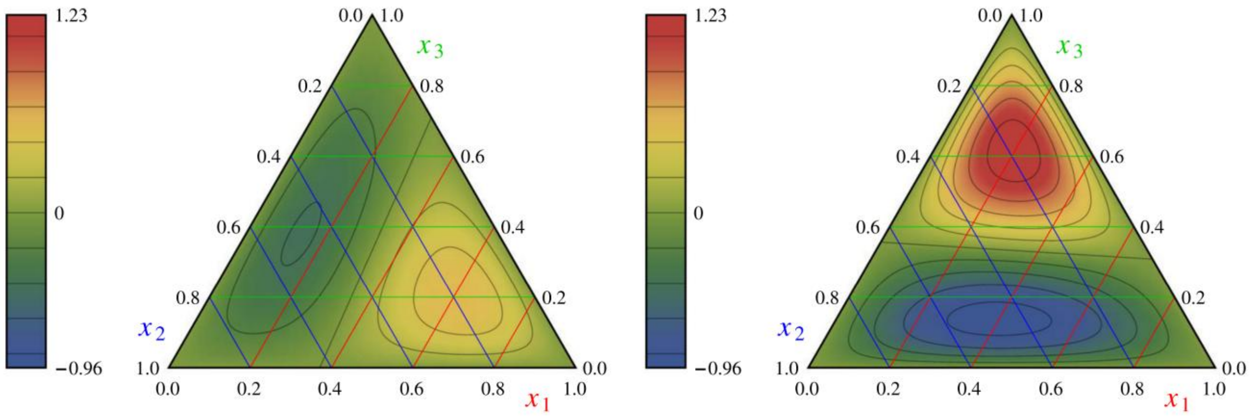
- FCNC is highly suppressed in SM;
- Thus b-baryon decays, such as $\Lambda_b \to \Lambda$ are highly sensitive to NP;
- Leading power FF can factorize into convolution of hard kernel and LCDAs

W.Wang, PLB 708, (2012) 119-126

How to study LCDA ?

Light Baryon LCDAs: (1983 - now)

- **Asymptotic form**
  Chernyak, Zhitnitsky, 1983; ……
- **QCD Sum Rules**
  King, Sachrajda,1987;
  Stefanis, Bergmann, 1993;
  Braun, et·al, 2000, 2006; ……
- **Models parametrization**
  Bell, et·al, 2013; ……



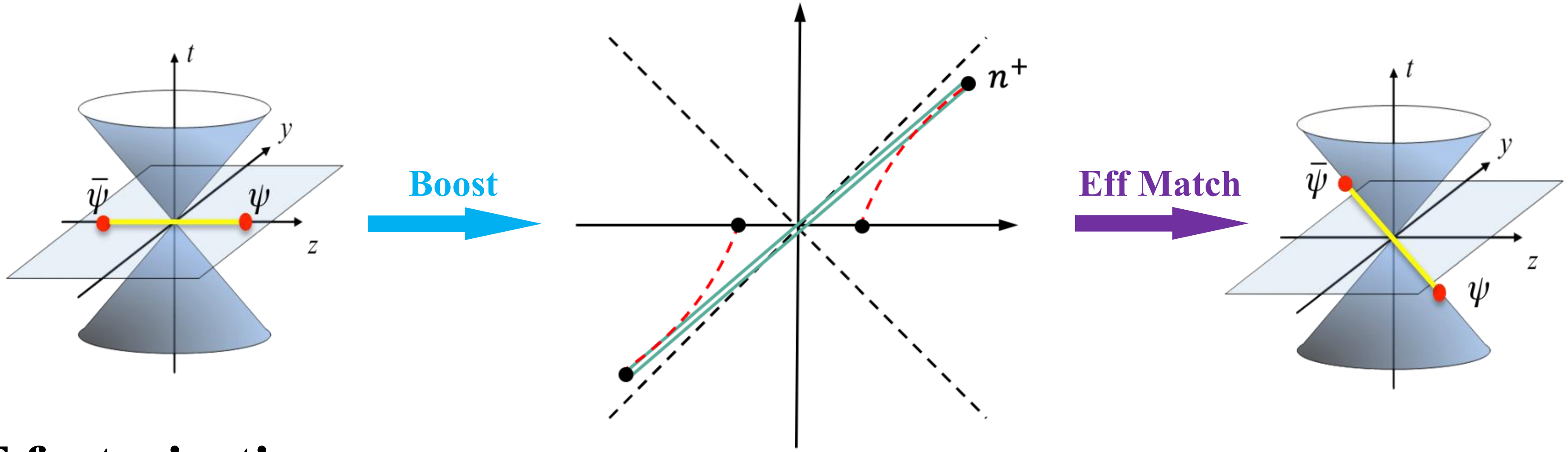
- **Lattice QCD with OPE**
  QCDSF, 2008, 2009;
  RQCD, 2016, 2019, 2025; ……
- **Lattice QCD with LaMET**
  LPC, PRD 111, 034510 (2025); ……

But LQCD in Euclidean space has NO light-cone correlation!

RQCD, Eur. Phys. J. A (2019) 55;
LPC, PRD 111, 034510 (2025)

How to study LCDA ?

Large-momentum Effective Theory



Ji, PRL 110, 262002 2013

LaMET factorization:

Quasi-DA — Matching kernel — LCDA

$$\tilde{\phi}(x_1, x_2, P^z, \mu) = \int \mathrm{d}y_1 \mathrm{d}y_2 \, \mathcal{C}(x_1, x_2; y_1, y_2; P^z, \mu) \, \phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(x_1 P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x_2 P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{[(1-x_1-x_2)P^z]^2}\right)$$

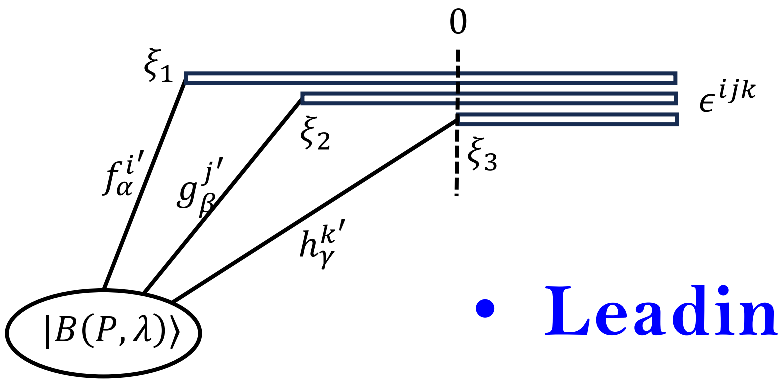
Higher power correction

Ji, PRL 110, 262002 2013;
LPC, PRD 111, 034510 (2025)

Definition of LCDA

- **Operator definition in coordinate space:**

$$H_{\alpha\beta\gamma}(\xi_1,\xi_2,\xi_3) = \epsilon^{ijk}\langle 0|f_\alpha^{i'}(\xi_1 n)W^{i'i}(\xi_1 n,\xi_0 n)\times g_\beta^{j'}(\xi_2 n)W^{j'j}(\xi_2 n,\xi_0 n) \times h_\gamma^{k'}(\xi_3 n)W^{k'k}(\xi_3 n,\xi_0 n)|B(P,\lambda)\rangle$$



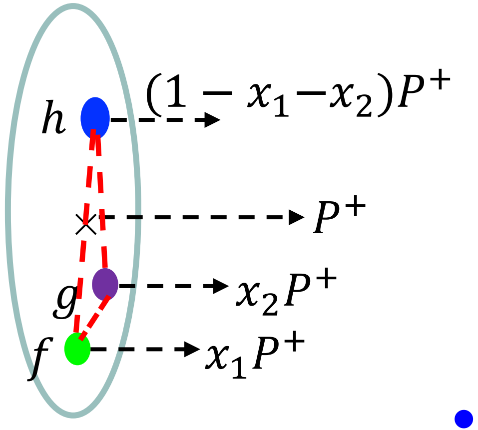
- **Leading twist for octet baryon:**

$$\langle 0|f_\alpha(\xi_1 n)g_\beta(\xi_2 n)h_\gamma(\xi_3 n)|B(P,\lambda)\rangle = \frac{1}{4}f_V\left[(\not{P}C)_{\alpha\beta}(\gamma_5 u_B)_\gamma \boldsymbol{\Phi_V}(\xi_i n\cdot P) + (\not{P}\gamma_5 C)_{\alpha\beta}(u_B)_\gamma \boldsymbol{\Phi_A}(\xi_i n\cdot P)\right] + \frac{1}{4}f_T(\mathrm{i}\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_B)_\gamma \boldsymbol{\Phi_T}(\xi_i n\cdot P)$$

Octet	n	p	Λ
fgh	d d u	u u d	u d s

C.Han et.al. JHEP 12 044 (2023); LPC, PRD 111, 034510 (2025)

Definition of LCDA



- **Definition of baryon LCDA:**

$$\phi(x_1,x_2,\mu)=\int\frac{P^+\mathrm{d}\xi_1^-}{2\pi}\frac{P^+\mathrm{d}\xi_2^-}{2\pi}\,\mathrm{e}^{\mathrm{i}(x_1\xi_1^-+x_2\xi_2^-)P^+}\,\Phi(\xi_1,\xi_2,\mu)$$

- **Leading twist for octet baryon LCDA**

$$-f_V\Phi_V(\xi_i n\cdot P)P^+\gamma_5 u_B=\langle 0|f^T(\xi_1 n)(C\not{n})g(\xi_2 n)\ h(0)|B\rangle$$

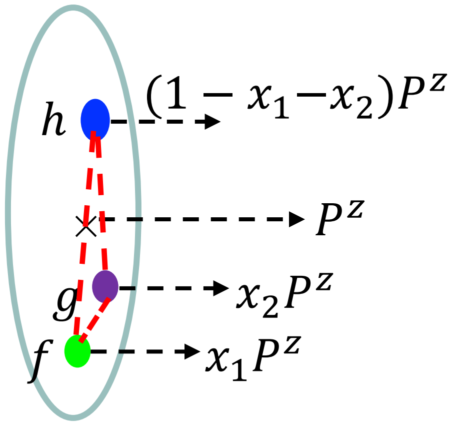
PRD 111, 034510 (2025) —— Λ: $$f_V\Phi_A(\xi_i n\cdot P)P^+u_B=\langle 0|f^T(\xi_1 n)(C\gamma_5\not{n})g(\xi_2 n)\ h(0)|B\rangle$$

$$2f_T\Phi_T(\xi_i n\cdot P)P^+\gamma_5 u_B=\langle 0|f^T(\xi_1 n)(\mathrm{i}C\sigma_{\mu\nu}n^\nu)g(\xi_2 n)\ \gamma^\mu h(0)|B\rangle$$

C.Han et.al. JHEP 07 019 (2024);
V.L.C & I.R.Z NPB 24652(1984);
G.R.Farrar et.al. NPB 311585(1989)

Definition of LCDA

- **Definition of baryon quasi-DA:**



$$\tilde{\phi}(x_1,x_2,\mu)=\int\frac{P^z\mathrm{d}z_1}{2\pi}\frac{P^z\mathrm{d}z_2}{2\pi}\,\mathrm{e}^{-\mathrm{i}(x_1z_1+x_2z_2)P^z}\,\tilde{\Phi}(z_1,z_2,\mu)$$

- **Leading twist for octet baryon quasi-DA:**

$$-f_V\tilde{\Phi}_V(z_i,P^z)P^\nu\gamma_5u_B=\langle 0|f^T(z_1)(C\gamma^\nu)g(z_2)\;h(0)|B\rangle$$

PRD 111, 034510 (2025) — Λ: $$f_V\tilde{\Phi}_A(z_i,P^z)P^\nu u_B=\langle 0|f^T(z_1)(C\gamma_5\gamma^\nu)g(z_2)\;h(0)|B\rangle$$

$$2f_T\tilde{\Phi}_T(z_i,P^z)P^\nu\gamma_5u_B=\langle 0|f^T(z_1)\left(\frac{1}{2}C[\gamma^\nu,\gamma^\mu]\right)g(z_2)\;\gamma^\mu h(0)|B\rangle$$

C.Han et.al. JHEP 07 019 (2024);

Lattice simulation

Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1, z_2; t, P^z) = \int d^3x\, e^{-i\vec{x}\cdot\vec{P}} \langle 0 | \mathcal{O}^{\gamma'}_{\rm Sink}(\vec{x}, t; z_1, z_2) \overline{\mathcal{O}}^{\gamma}_{\rm Src}(0, 0; 0, 0) T_{\gamma'\gamma} | 0 \rangle$$

- **Extract & norm quasi-DA from 2-point function**

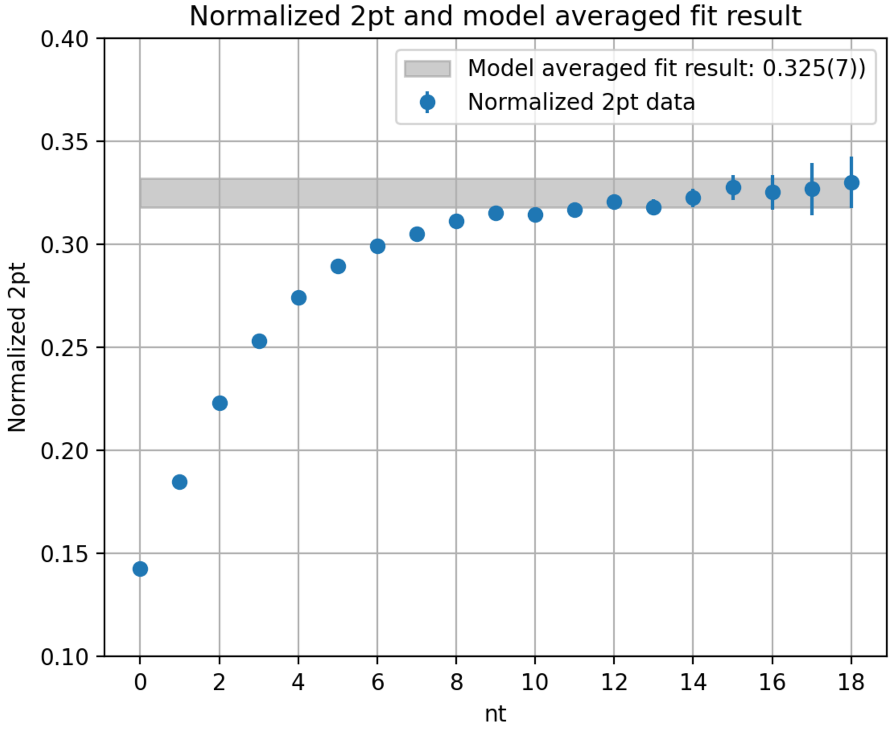
$$\frac{C_2(z_1, z_2; P^z, t)}{C_2(0, 0; P^z, t)} \xrightarrow{t\to\infty} \frac{\widetilde{\Phi}(z_1, z_2; P^z)}{\widetilde{\Phi}(0, 0; P^z)}$$

$$C_2^{\rm Norm}(z_1, z_2; P^z, t) = \underbrace{\widetilde{\Phi}(z_1, z_2; P^z)}_{\text{ground-state}} \Big(1 + \underbrace{\Delta A(z_1, z_2; P^z) e^{-\Delta E t}}_{\text{excited-states}}\Big)$$

ground-state

excited-states

~10^3 points!



Lattice simulation

Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1, z_2; t, P^z) = \int d^3x \; e^{-i\vec{x}\cdot\vec{P}} \langle 0| \underbrace{\mathcal{O}^{\gamma'}_{\text{Sink}}(\vec{x}, t; z_1, z_2)}_{\text{Determined by DA}} \underbrace{\overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0, 0; 0, 0) T_{\gamma'\gamma}}_{\text{Up to choice ?}} |0\rangle$$

- **Interpolators and projection operators**

  - Sink-side Interpolators

$$\mathcal{O}_{\text{Sink},V} = f^T(z_1)(C\gamma^t)g(z_2)\; h(0)$$
$$\mathcal{O}_{\text{Sink},A} = f^T(z_1)(C\gamma_5\gamma^t)g(z_2)\; h(0)$$
$$\mathcal{O}_{\text{Sink},T} = f^T(z_1)(C[\gamma^t, \gamma^y]/2)g(z_2)\; \gamma_y\; h(0)$$

  - Projection operator ?

$$T = I, \gamma^t, \gamma^z$$

  - Source-side Interpolators ?

$$\mathcal{O}^P_{\text{Src}} = (u^T\; C\gamma_5\; d)u$$
$$\mathcal{O}^\Lambda_{\text{Src}} = \frac{1}{\sqrt{6}}\Big(2(u^T\; C\gamma_5\; d)s + (u^T\; C\gamma_5\; s)d + (s^T\; C\gamma_5\; d)u\Big)$$

Lattice simulation

Improvement from interpolator: Kinematically-enhancement

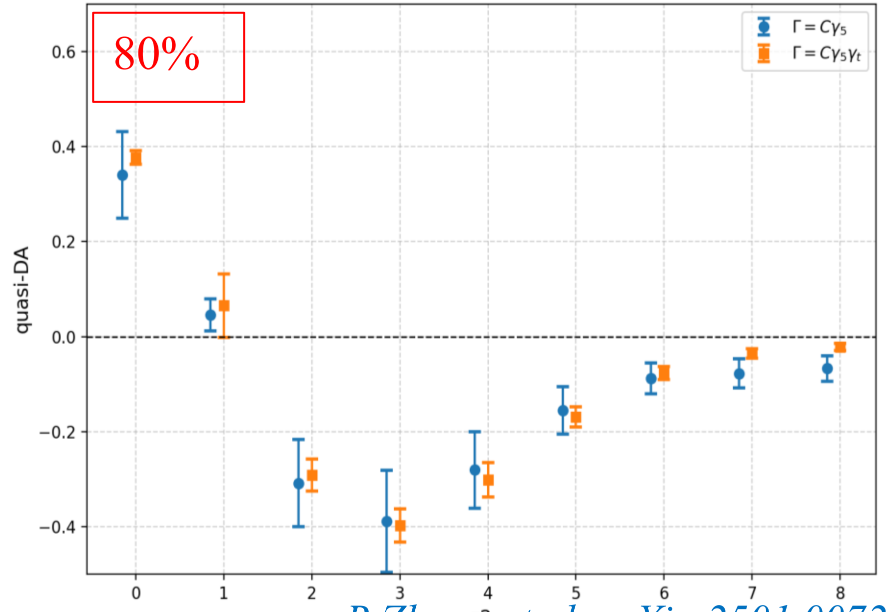
- Standard interpolator: $N_{\gamma_5} = \epsilon^{ijk}(f_i^T\ C\gamma_5\ g_j)h_k$ for a static baryon states

$$\langle 0|N_{\gamma_5}|N(P)\rangle = \lambda u(P)$$

- Enhanced interpolator : $N_{\gamma_5\gamma_\mu} = \epsilon^{ijk}(f_i^T\ C\gamma_5\gamma_\mu\ g_j)h_k$ better overlap with boosted states

$$\langle 0|N_{\gamma_5\gamma_\mu}|N(P)\rangle = \alpha P_\mu u(P) + \beta\gamma_\mu u(P)$$

$$\Rightarrow \quad \mathrm{SNR}(C_{2\mathrm{pt}}(t\to\infty)) \propto \frac{P_\mu}{M_0} e^{-E't}$$



R.Zhang et.al, arXiv:2501.00729

Lattice simulation

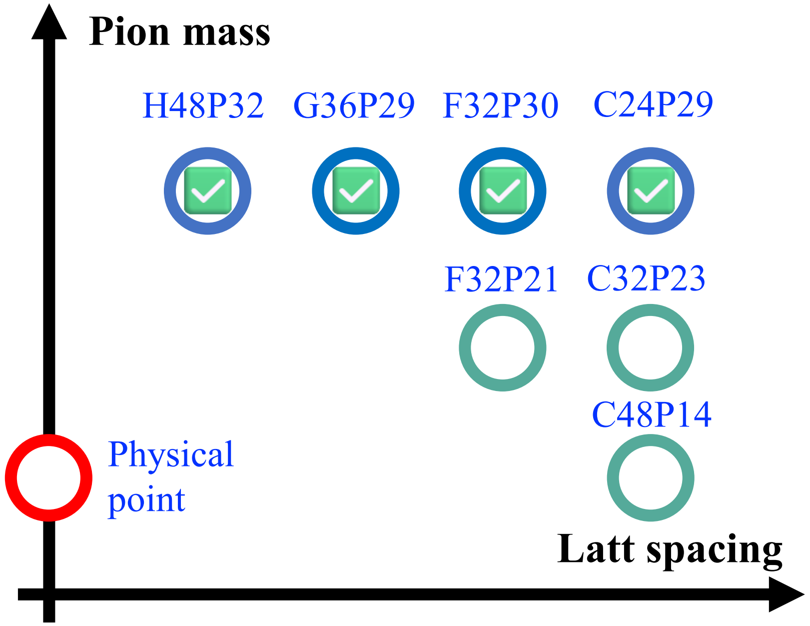
CLQCD Ensembles

- 4 lattice spacings for $a \to 0$ limit & Self-renormalization
- 4 momentums for $P_z \to \infty$ limit

Ensemble	Volume	Lattice spacing	π mass	measurements	P^z
C24P29	$24^3 \times 72$	0.1052 fm	292.3 MeV	864 *4src *9it0	1.96, 2.45, 2.94, 3.43 GeV
F32P30	$32^3 \times 96$	0.0775 fm	300.4 MeV	777 *4src *8it0	2.00, 2.50, 3.00, 3.49 GeV
G36P29	$36^3 \times 108$	0.0689 fm	297.2 MeV	656 *6src *9it0	2.01, 2.52, 3.02, 3.52 GeV
H48P32	$48^3 \times 144$	0.0520 fm	316.6 MeV	550 *6src *8it0	1.99, 2.48, 2.98, 3.48 GeV

CLQCD Ensembles

- 4 lattice spacings for $a \rightarrow 0$ limit & Self-renormalization
- 4 momentums for $P_z \rightarrow \infty$ limit
- More pion masses to physical mass limit



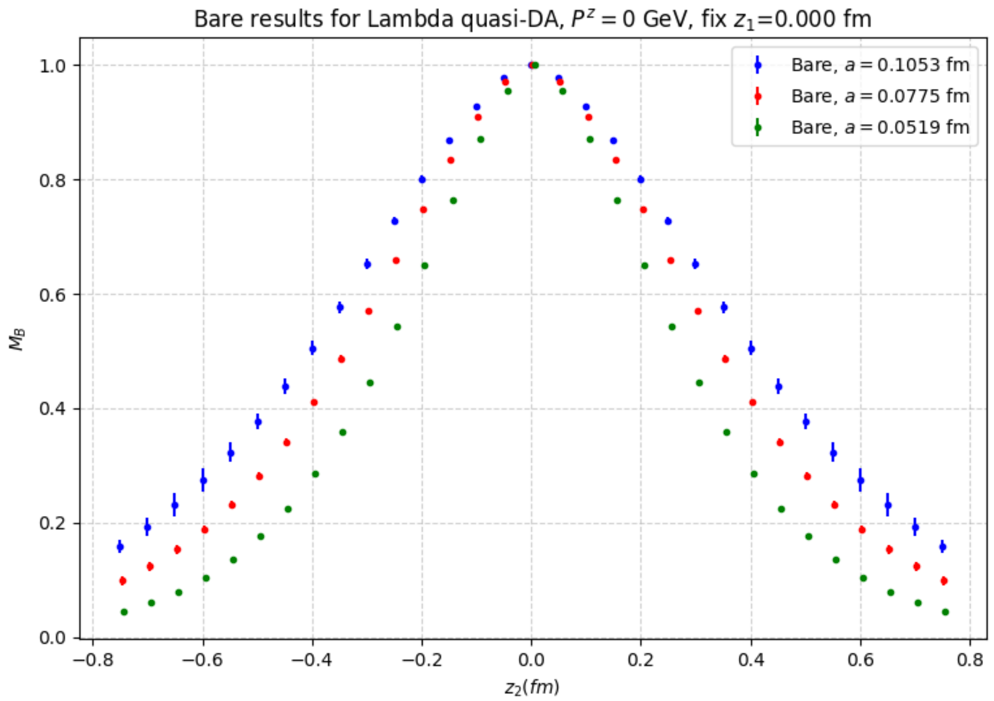
Ensemble	Lattice spacing	π mass	measurements
C24P29	0.1052 fm	292 MeV	864 *4 src *9 it0
F32P30	0.0775 fm	300 MeV	777 *4 src *8 it0
H48P32	0.0520 fm	317 MeV	550 *6 src *9 it0
G36P29	0.0689 fm	297 MeV	656 *6 src *8 it0
Other ensembles with different pion masses			

Bare quasi-DA results

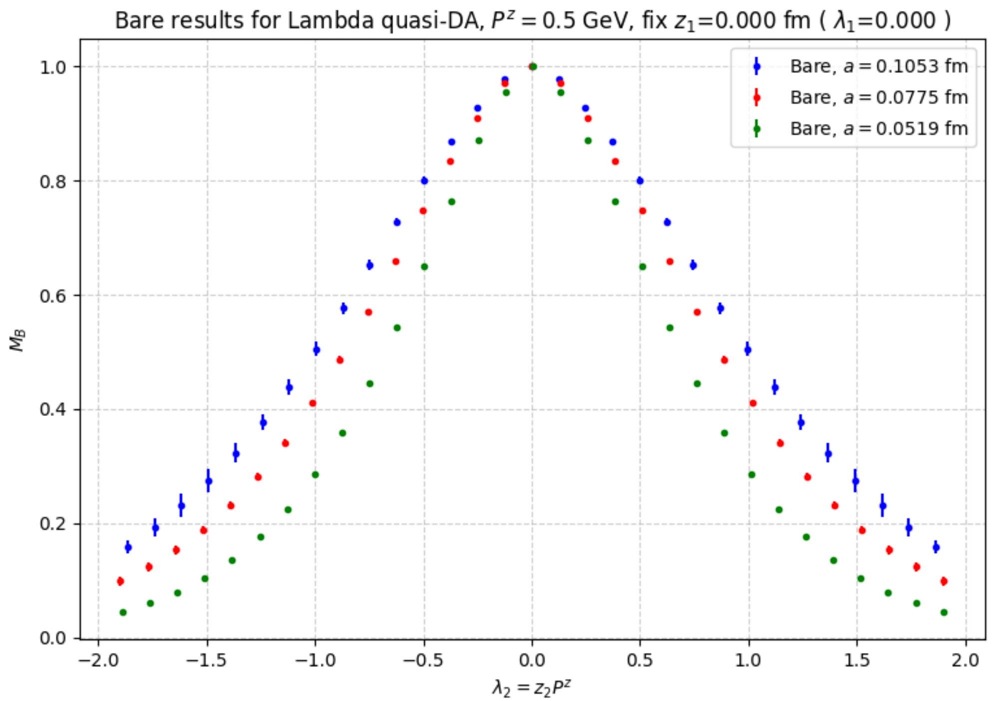
z-dependence of normed bare quasi-DA on different lattice spacing

Lambda A term

P^z=0 GeV



P^z=0.5 GeV



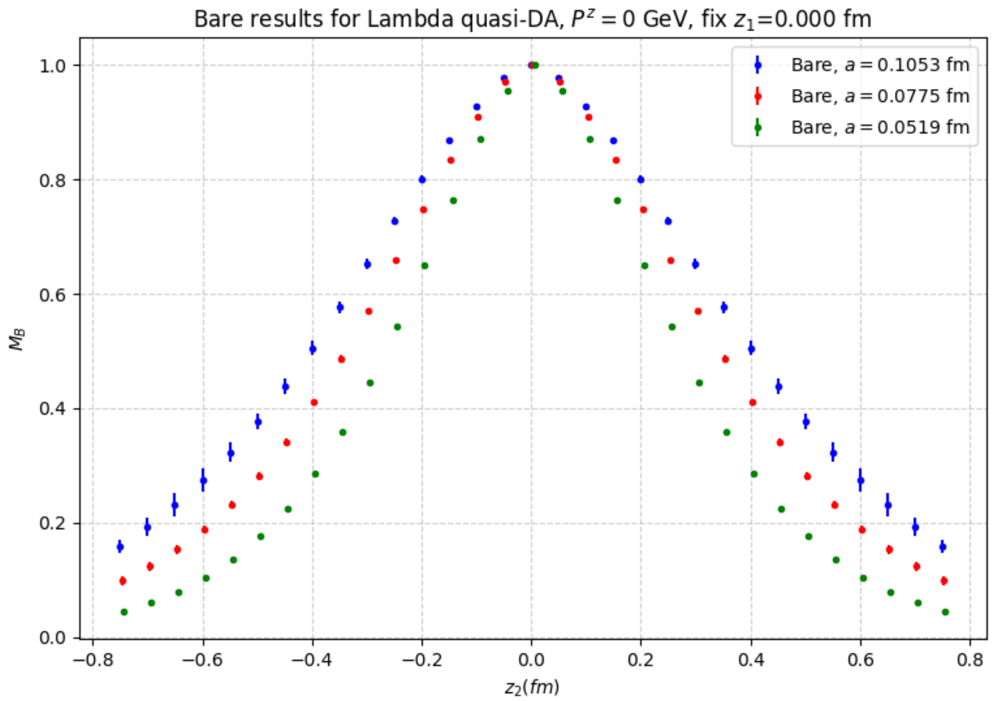
Bare quasi-DA results: Linear divergence

Linear divergence of Normed Bare quasi-DA

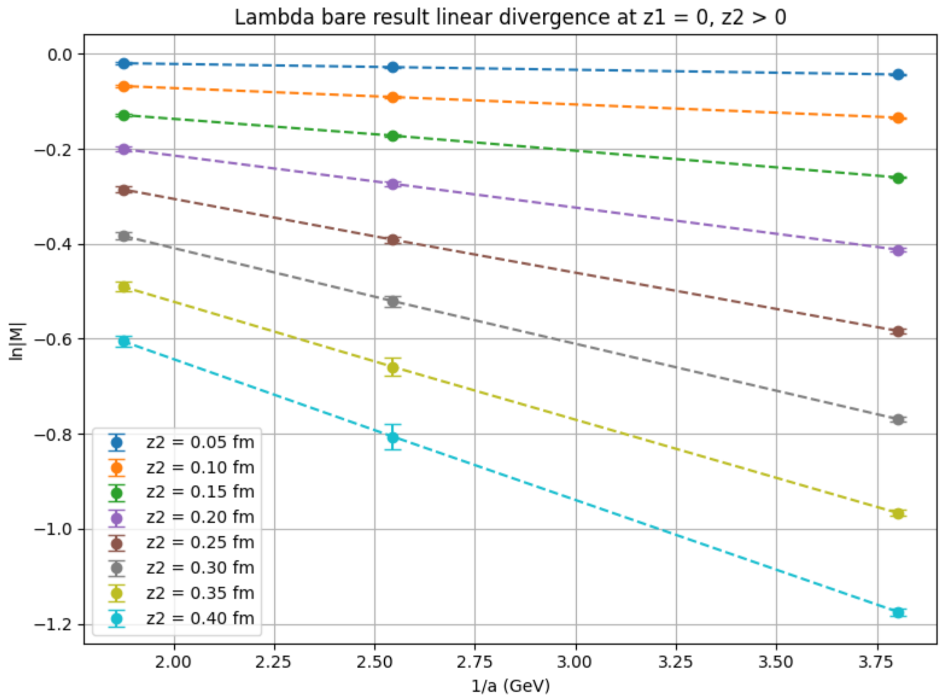
$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

$$M(z_1, z_2; P^z, a) \propto \exp\left[\left(\boxed{\frac{k}{a\ln(a\Lambda_{\rm QCD})}} + m_0\right)\tilde{z}\right] m(z_1, z_2, P^z)$$

P^z=0 GeV



log scale



Bare quasi-DA results: Linear divergence

Linear divergence of Normed Bare quasi-DA

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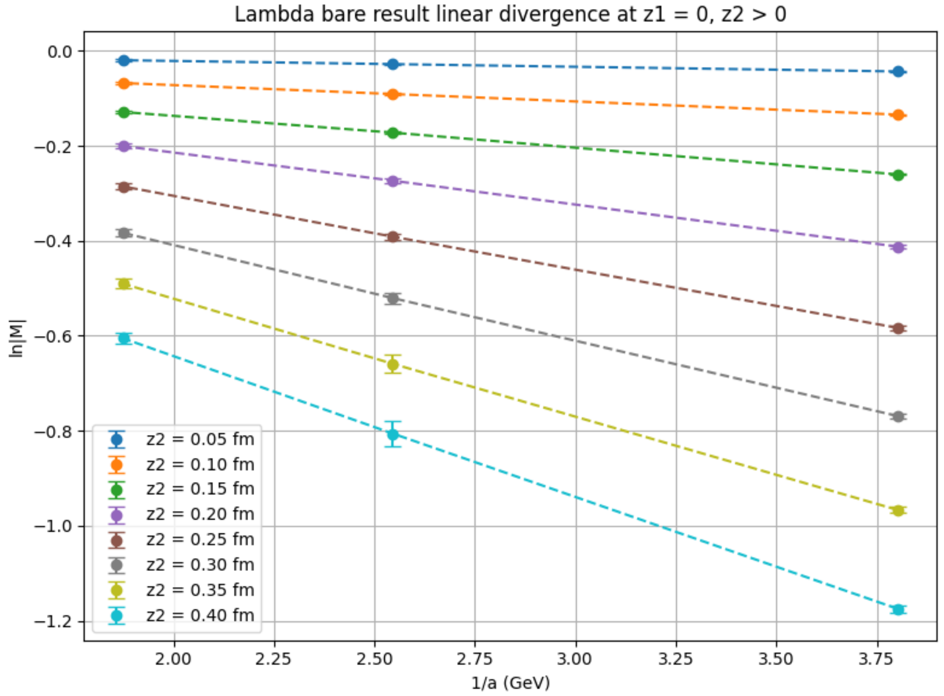
$$M(z_1, z_2; P^z, a) \propto \exp\left[\left(\boxed{\frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})}} + m_0\right)\tilde{z}\right] m(z_1, z_2, P^z)$$

log scale

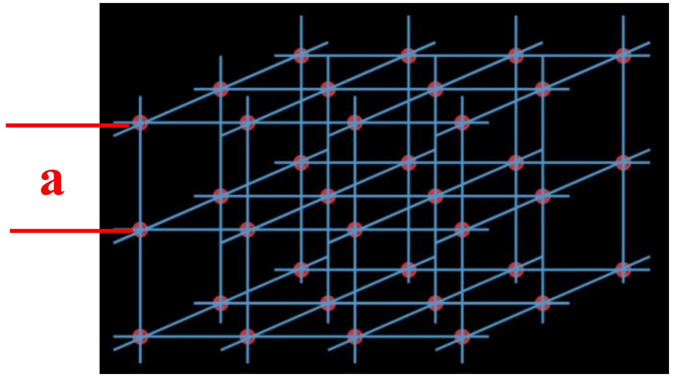


We need proper scheme
to eliminate the linear divergences !

Hybrid Renormalization

Renormalization on Lattice

Scheme conversion: $\frac{1}{a}$ cutoff $\rightarrow$ $\overline{\mathrm{MS}}$



Schemes on Lattice

- **RI/MOM Scheme**

  Alexandrou et.al, NPB 2017; Stewart, Zhao, PRD 2018

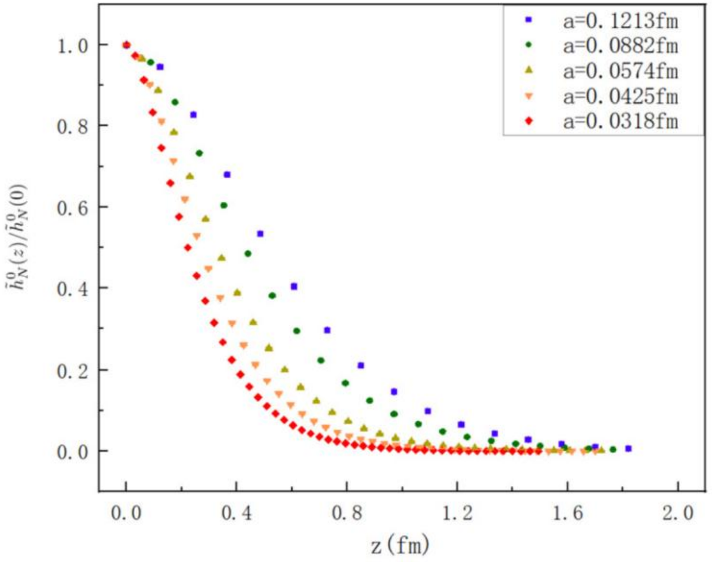
- **Ratio Scheme**

  Radyushkin et.al, PRD 2017

- **Hybrid Scheme with Self Renormalization**

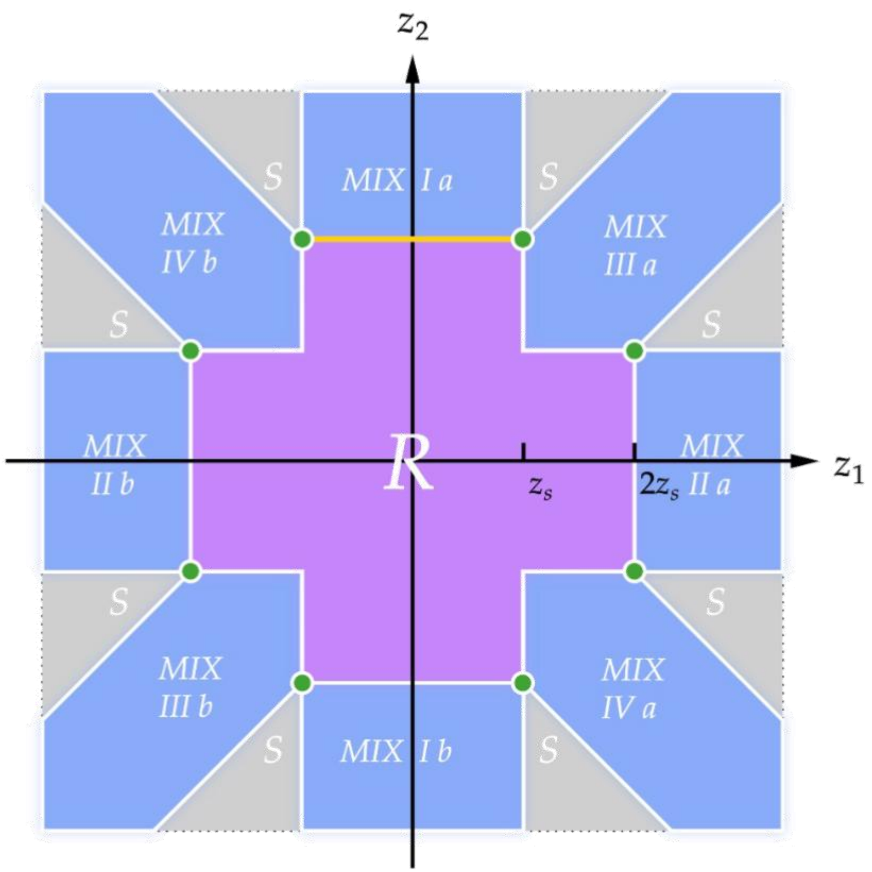
  Ji et.al, NPB 2021; Huo et.al, NPB 2021

—— The only solution by now to correctly eliminate linear divergence in PDFs and DAs



Hybrid Renormalization

2D renormalization in Hybrid scheme

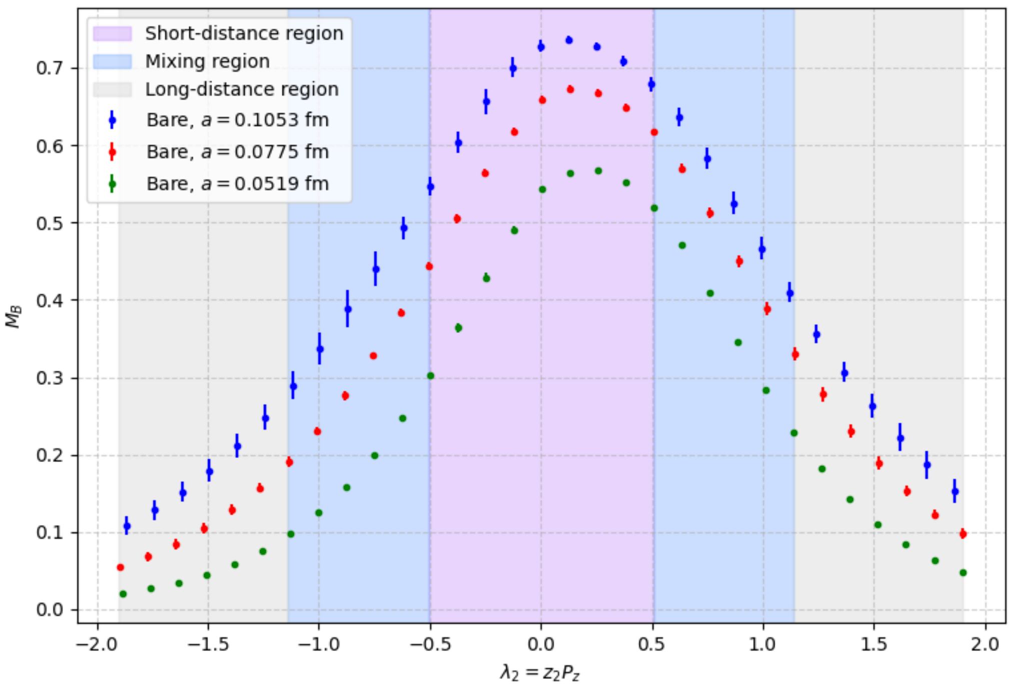


- **Different schemes** on different scales
- Extract divergences from different spacings

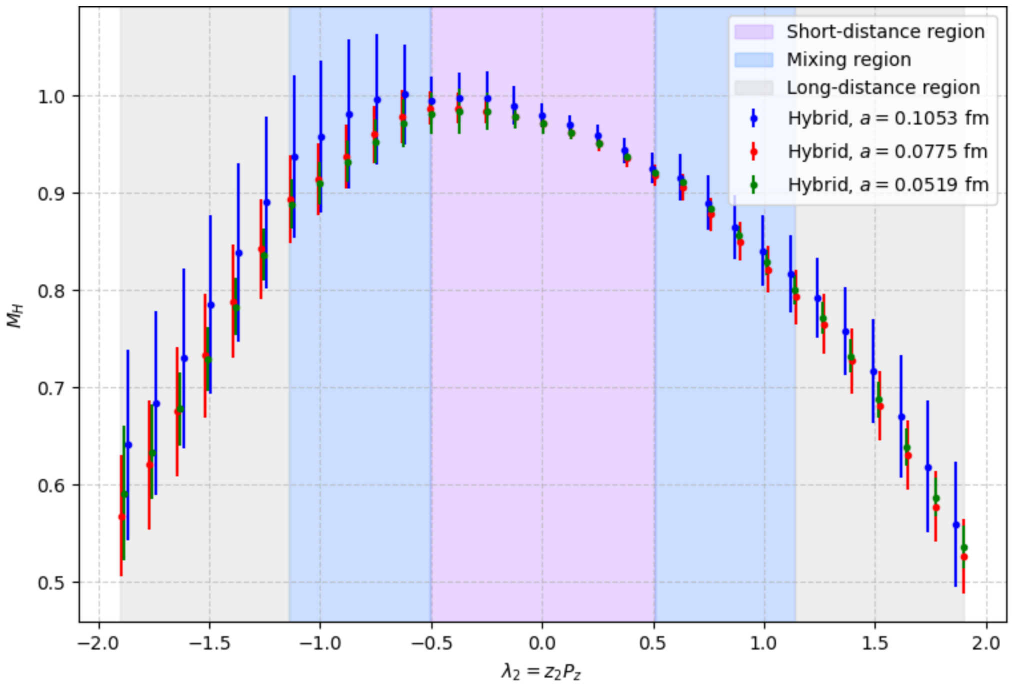
Very complicated in applying to Baryon DAs
—— **LPC, arXiv: 2508.08971**

Hybrid Renormalization

Bare Matrix Element



Hybrid scheme result



LPC, arXiv: 2508.08971

Renormalization & Matching

2D matching in Hybrid scheme

$$\widetilde{\phi}_H(x_1, x_2, P^z, \mu) = \int \mathrm{d}y_1 \mathrm{d}y_2\, \mathcal{C}_H(x_1, x_2; y_1, y_2; P^z, \mu)\, \phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(x_i P^z)^2}\right)$$

- Inverse matching:

  $C(x_1, x_2, y_1, y_2) \to$ 4 Dimensional tensor $\to$ Reduce to 2D matrix $\to$ inverse

- Iterative matching:

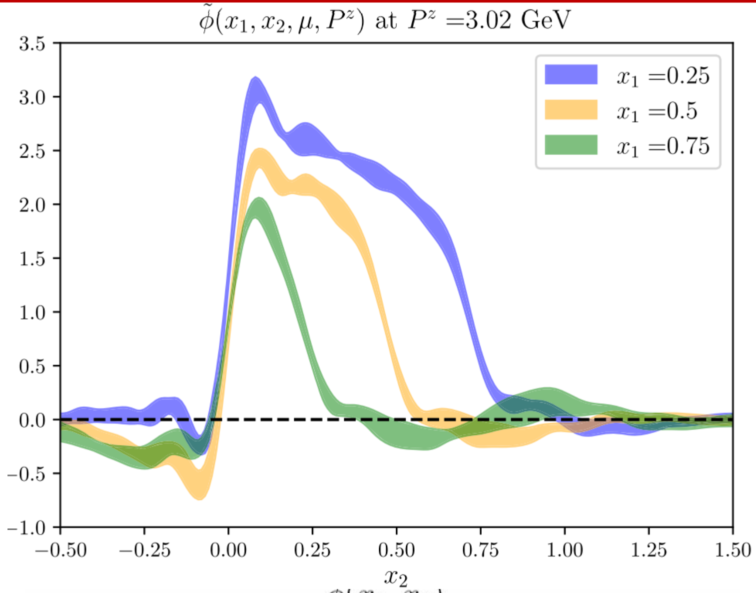
$$\widetilde{\phi}(x_1, x_2) = \phi(x_1, x_2) + \frac{\alpha_s C_F}{2\pi} \int \mathrm{d}y_1 \mathrm{d}y_2\, \mathcal{C}^{(1)}(x_1, x_2; y_1, y_2)\, \phi(y_1, y_2) + \mathcal{O}\left(\alpha_s^2\right)$$

The difference between $\tilde{\phi}(x_1, x_2)$ and $\phi(x_1, x_2)$ introduces error only at higher order

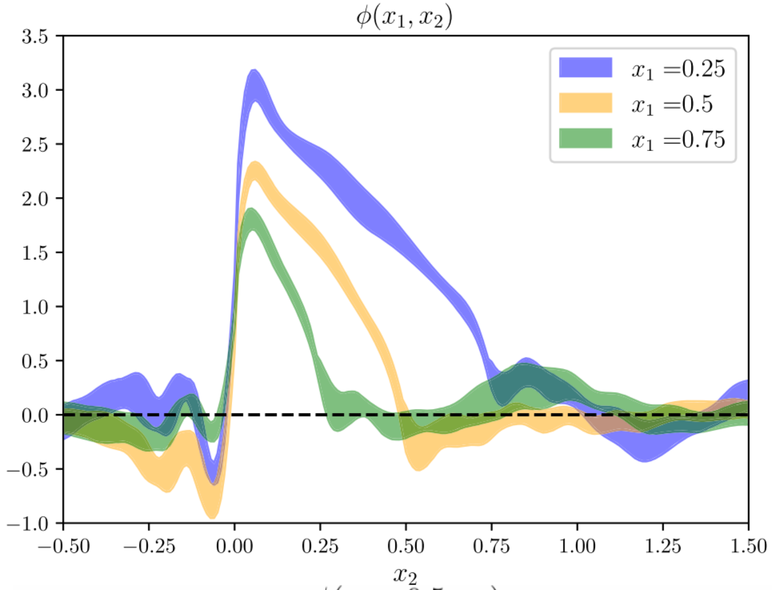
$$\phi(x_1, x_2) = \widetilde{\phi}(x_1, x_2) - \frac{\alpha_s C_F}{2\pi} \int \mathrm{d}y_1 \mathrm{d}y_2\, \mathcal{C}^{(1)}(x_1, x_2; y_1, y_2)\, \widetilde{\phi}(y_1, y_2) + \mathcal{O}\left(\alpha_s^2\right)$$

C.Han et.al. JHEP 12 044 (2023)
LPC, PRD 111, 034510 (2025)

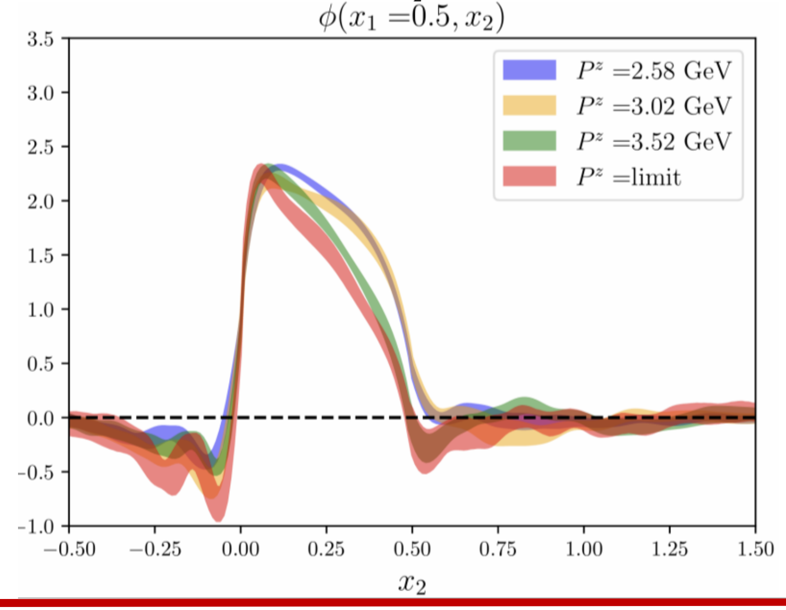
Renormalization & Matching



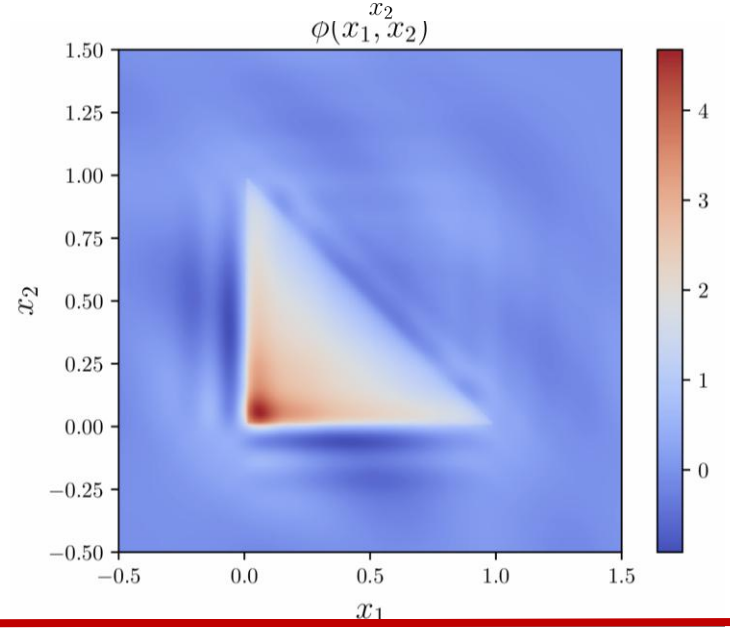
Matching →



↓ **Large Pz limit**



← **LCDA**



Summary & Outlook

- We have done:
  - ☐ *LPC, PRD 111, 034510*, validated the concept of lattice calculation of baryon LCDA from LaMET
  - ☐ *arXiv: 2508.08971*, successfully adopted the Hybrid renormalization on baryon matrix element

- We now proceeding:
  - ☐ Continuum & physical mass extrapolation
  - ☐ More strategies to enhance lattice simulation
  - ☐ Methods for limited FT, inverse & matching schemes

- Following:
  - ☐ Please stay tuned our results for all leading twists LCDAs of Proton and Lambda !

Thanks for attention!

Backup

Lattice simulation

Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1, z_2; t, P^z) = \int \mathrm{d}^3x\ \mathrm{e}^{-i\vec{x}\cdot\vec{P}} \langle 0 | \mathcal{O}^{\gamma'}_{\text{Sink}}(\vec{x}, t; z_1, z_2) \overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0, 0; 0, 0) T_{\gamma'\gamma} | 0 \rangle$$

- **Extract & normalize quasi-DA from 2-point function**

$$C_2(z_1, z_2; P^z, t) = \sum_k \langle 0 | \mathcal{O}^{\gamma'}_{\text{Sink}}(z_1, z_2; t) | B(P^z)_k \rangle \langle B(P^z)_k | \overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0, 0; t) | 0 \rangle T_{\gamma'\gamma}$$

$$\overset{t\to\infty}{\sim} \widetilde{\Phi}(z_1, z_2; P^z) \widetilde{\Phi}^*(0, 0; P^z) |f_\Lambda|^2 \left[u_\Lambda(P^z) u^\dagger_\Lambda(P^z) \right]^{\gamma'\gamma} T_{\gamma'\gamma}$$

$$\Longrightarrow \quad \frac{C_2(z_1, z_2; P^z, t)}{C_2(0, 0; P^z, t)} = f(t) \frac{\widetilde{\Phi}(z_1, z_2; P^z) \widetilde{\Phi}^*(0, 0; P^z)}{\widetilde{\Phi}(0, 0; P^z) \widetilde{\Phi}^*(0, 0; P^z)} \frac{\cdots}{\cdots} \overset{t\to\infty}{\longrightarrow} \frac{\widetilde{\Phi}(z_1, z_2; P^z)}{\widetilde{\Phi}(0, 0; P^z)}$$

Lattice simulation

Symmetries of quasi-DA

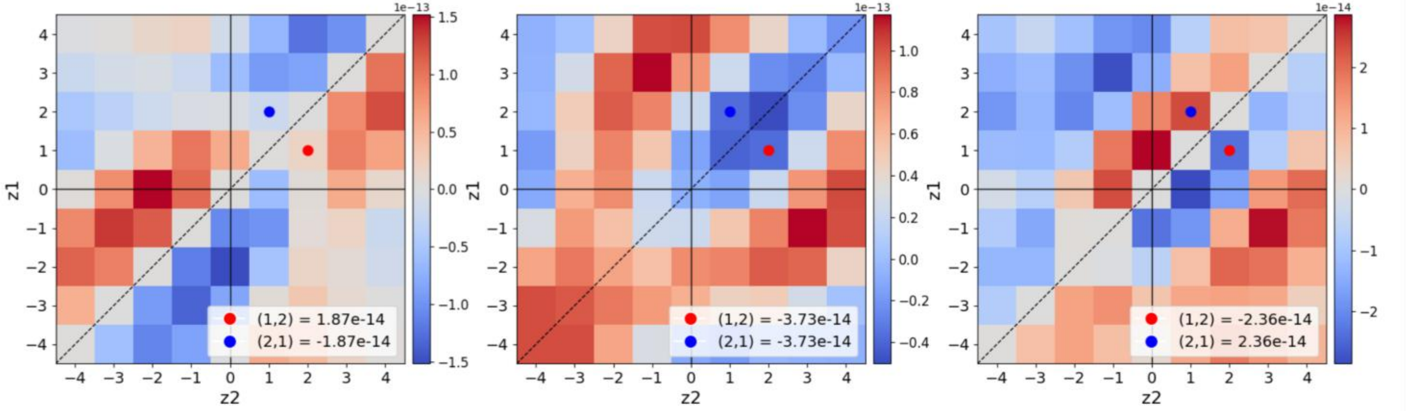
- isospin symmetry for “u, d” quarks (on a single configuration);

symmetry of z_1, z_2 exchanging:

$$V^{\Lambda}(x_2,x_1,x_3) = -V^{\Lambda}(x_1,x_2,x_3)\,,$$
$$A^{\Lambda}(x_2,x_1,x_3) = +A^{\Lambda}(x_1,x_2,x_3)\,,$$
$$T^{\Lambda}(x_2,x_1,x_3) = -T^{\Lambda}(x_1,x_2,x_3)\,.$$

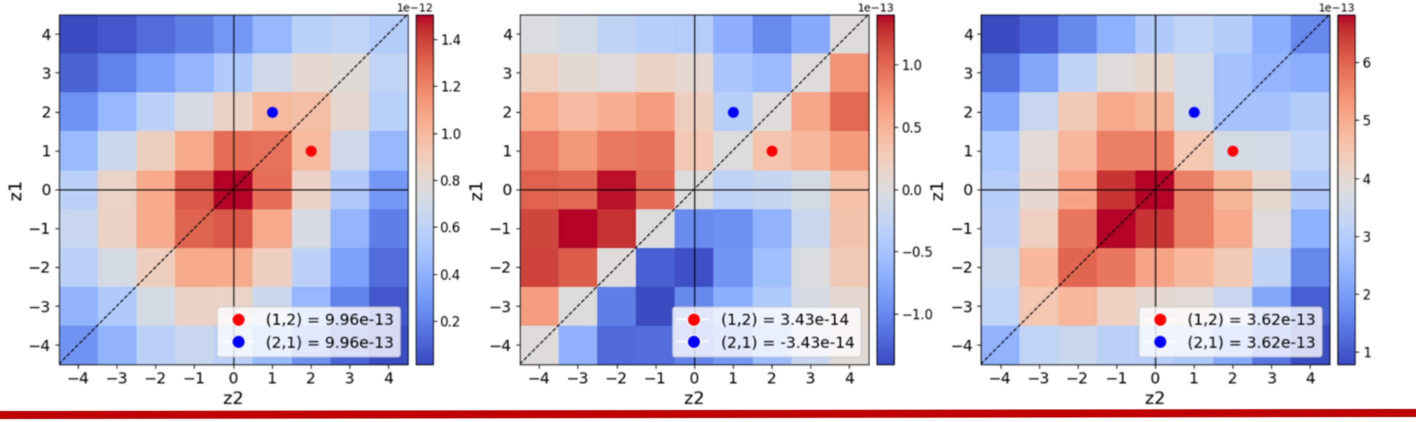
Lambda

$\widetilde{\Phi}_V$ $\widetilde{\Phi}_A$ $\widetilde{\Phi}_T$



$$V^{B\neq\Lambda}(x_2,x_1,x_3) = +V^{B}(x_1,x_2,x_3)\,,$$
$$A^{B\neq\Lambda}(x_2,x_1,x_3) = -A^{B}(x_1,x_2,x_3)\,,$$
$$T^{B\neq\Lambda}(x_2,x_1,x_3) = +T^{B}(x_1,x_2,x_3)\,,$$

Proton



Lattice simulation

Symmetries of quasi-DA

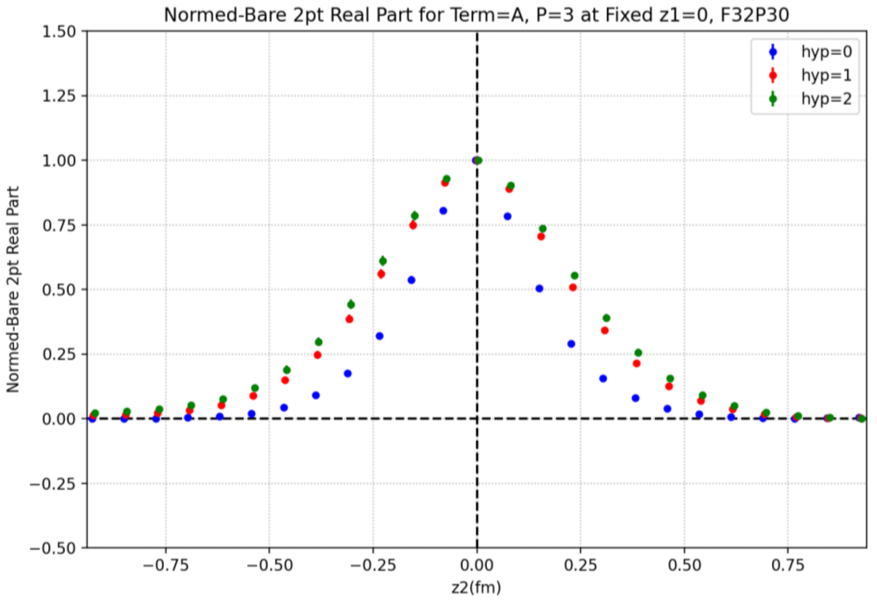
$$\tilde{\Phi}(z_1, z_2, \mu) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, e^{i(x_1 z_1 + x_2 z_2)P^z} \times \tilde{\phi}(x_1, x_2, \mu)$$

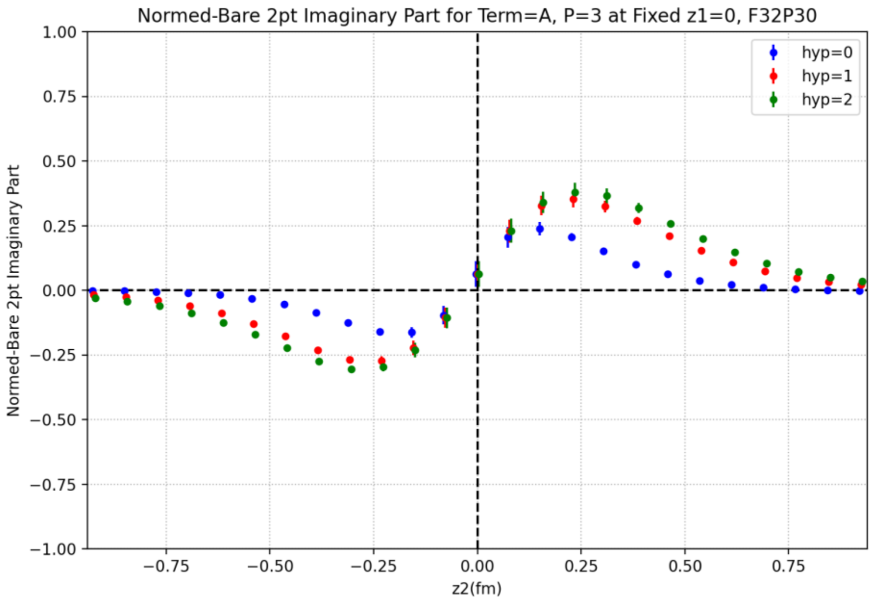
Pure real

- spatial parity symmetry (at statistical level);

the reality in momentum space $\tilde{\Phi}(x_1, x_2)$ leads to: $\tilde{\Phi}(z_1, z_2) = \tilde{\Phi}^*(-z_1, -z_2)$

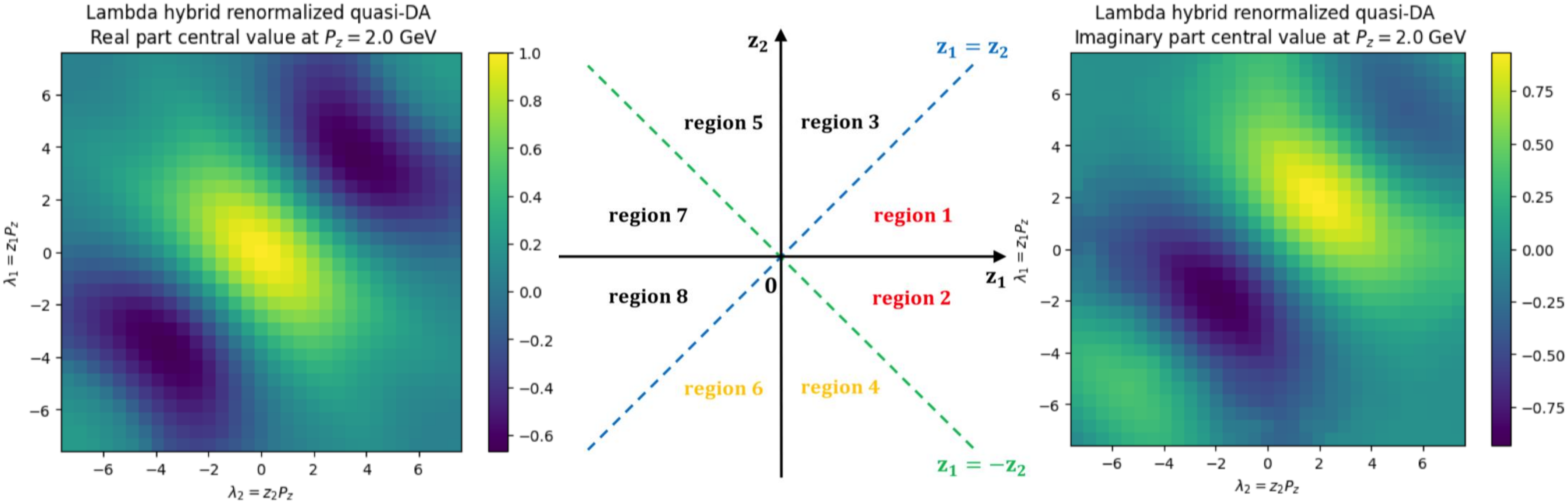
$$\mathrm{Re}\ \tilde{\Phi}(-z_1, -z_2) = \mathrm{Re}\ \tilde{\Phi}(z_1, z_2) \qquad \mathrm{Im}\ \tilde{\Phi}(-z_1, -z_2) = -\mathrm{Im}\ \tilde{\Phi}(z_1, z_2)$$





Lattice simulation

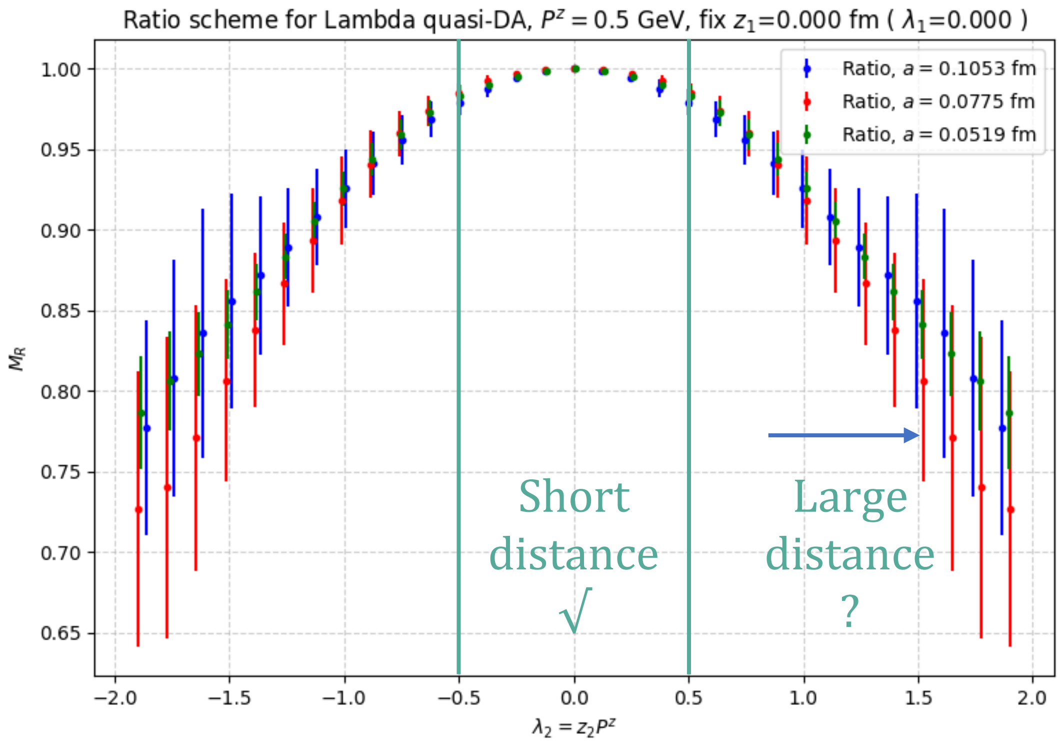
Symmetries of quasi-DA



Ratio renormalization scheme

$$\tilde{\psi}^{\text{ratio}}(z_1, z_2, P^z) = \frac{\tilde{\psi}^{\text{bare}}(z_1, z_2, P^z)}{\tilde{\psi}^{\text{bare}}(z_1, z_2, P^z = 0)}$$

Lambda A term $\quad P^z$=0.5 GeV



Uncontrollable IR structures on large z region !

Self renormalization scheme

Step-1 (in all regions with linear div.)

b0

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{k}{a \ln(a\Lambda_{\rm QCD})}\tilde{z} + g(z_1, z_2) + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\overline{\rm MS}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})}\right]$$

Step-2 (in perturbative region)

$$g(z_1, z_2) - \ln Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = m_0\tilde{z} + b_0$$

$$Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

Self renormalization scheme

Renormalization factor

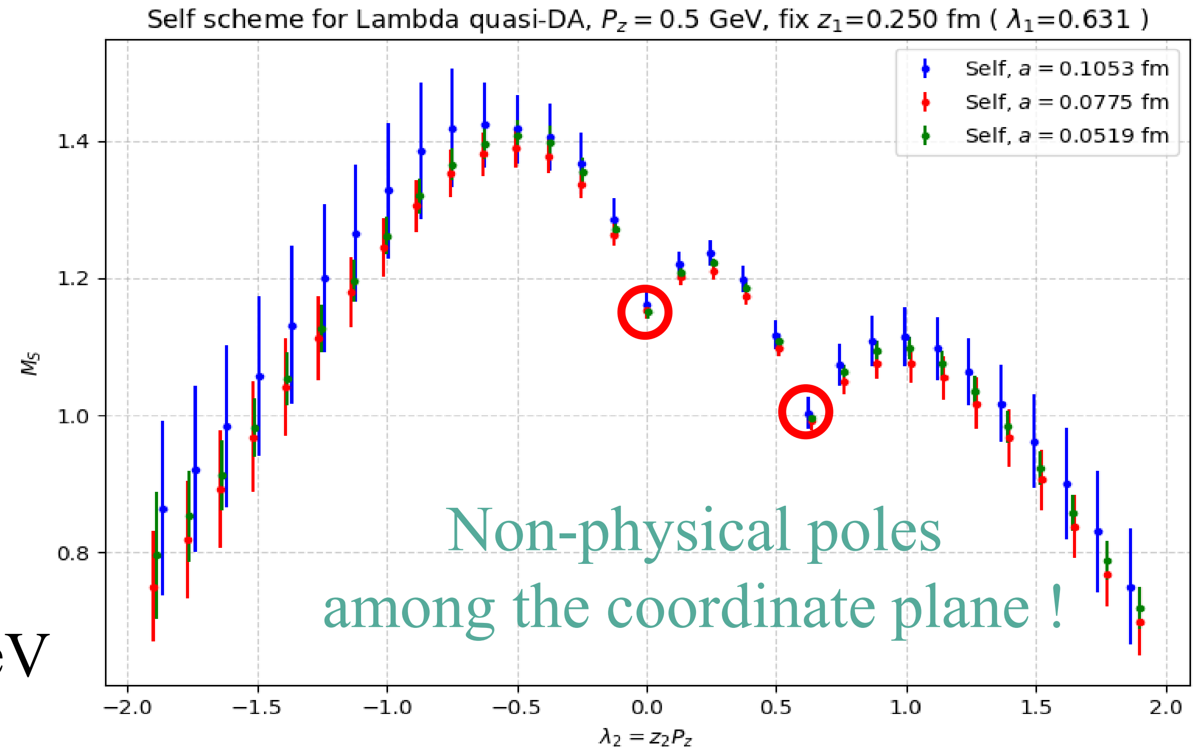
$$Z_R(z_1, z_2; a) = \exp\left[\frac{k}{a\ln(a\Lambda_{\text{QCD}})}\tilde{z} + m_0\tilde{z} + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\overline{\text{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\text{QCD}})}\right]\right]$$

Renormalized 0 & large-momentum matrix element

$$M_R(z_1, z_2; P_z = 0) = \exp\left(g(z_1, z_2) - m_0\tilde{z} - b_0\right)$$

$$M_R(z_1, z_2; P_z) = \frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$

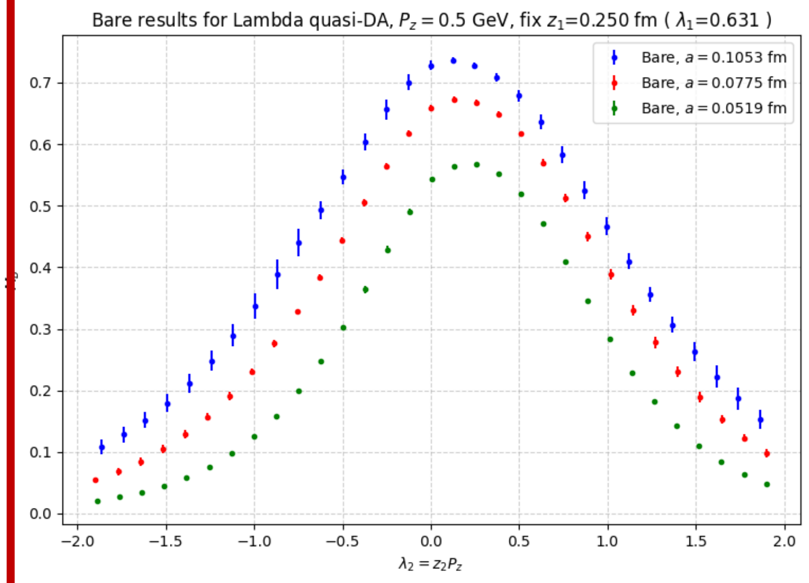
P^z=0.5 GeV



Self renormalization scheme

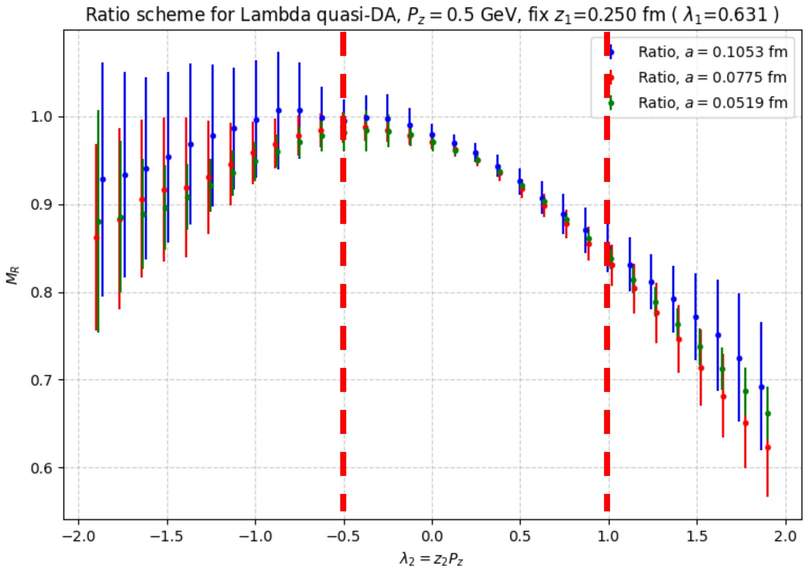
Bare Matrix Element

$$M(z_1, z_2; P_z; a)$$



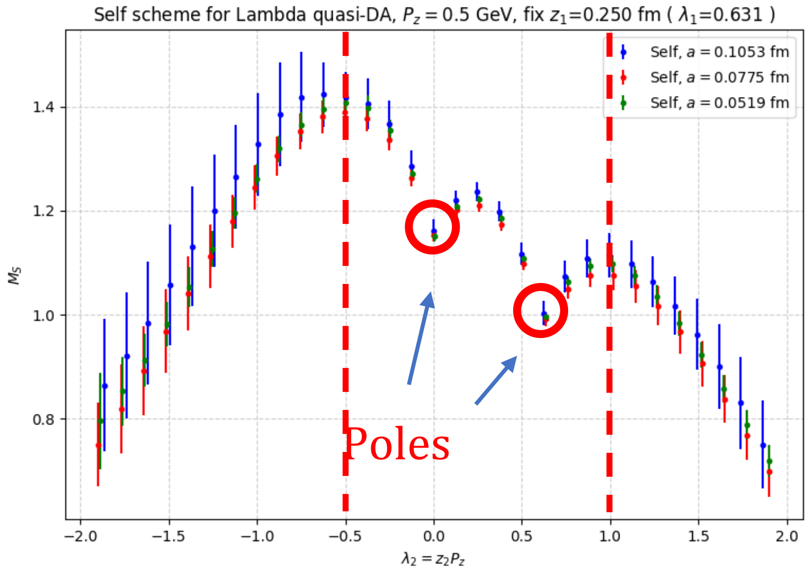
Ratio scheme

$$\frac{M(z_1, z_2; P_z; a)}{M(z_1, z_2; 0; a)}$$



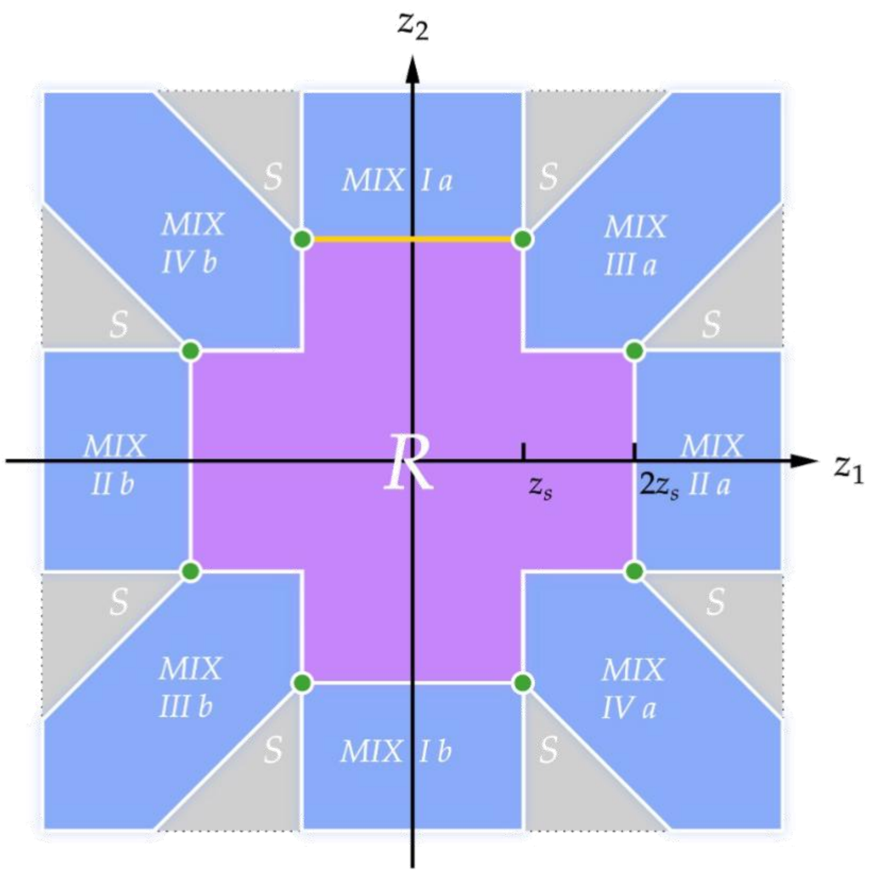
Self scheme

$$\frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$



Hybrid Renormalization

2D renormalization in Hybrid scheme



- 1) **Short-distance region**

$$\frac{\hat{M}_{\overline{\rm MS}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\rm MS}}(z_1, z_2, 0, 0, \mu)}$$

- 2) **Long-distance region**

$$\frac{\hat{M}_{\overline{\rm MS}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\rm MS}}(\text{sign}(z_1) z_s, \text{sign}(z_2) 2z_s, 0, 0, \mu)}$$

- 3) **Mixing regions**

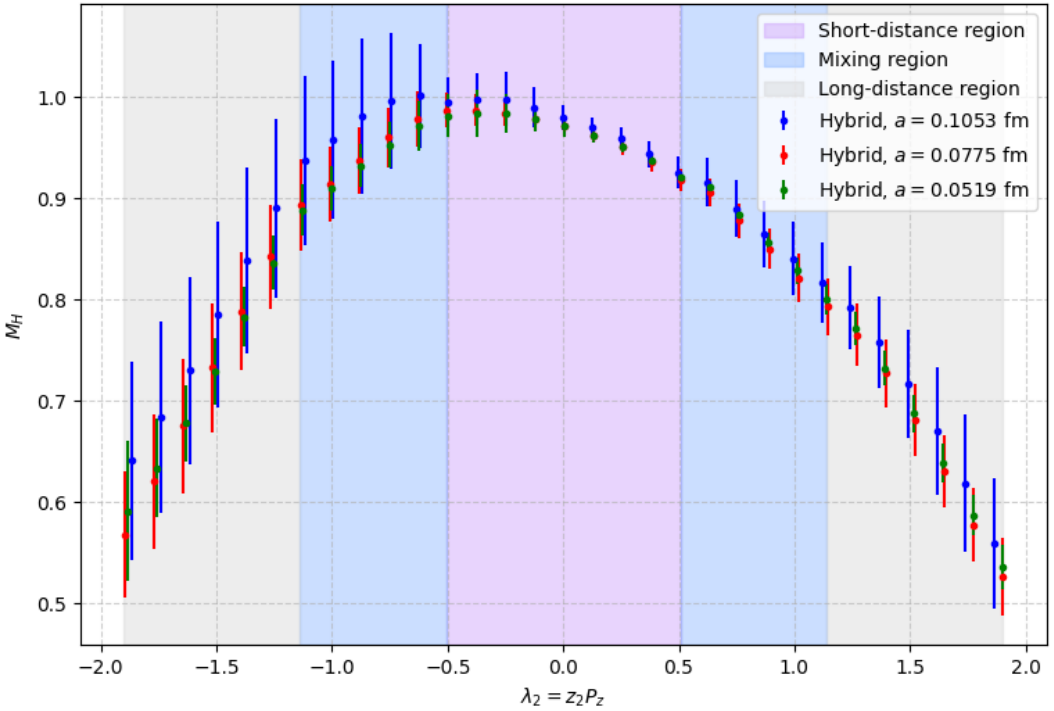
$$\frac{\hat{M}_{\overline{\rm MS}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\rm MS}}(z_1, \text{sign}(z_2) 2z_s, 0, 0, \mu)} \cdot \theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$

Hybrid Renormalization results

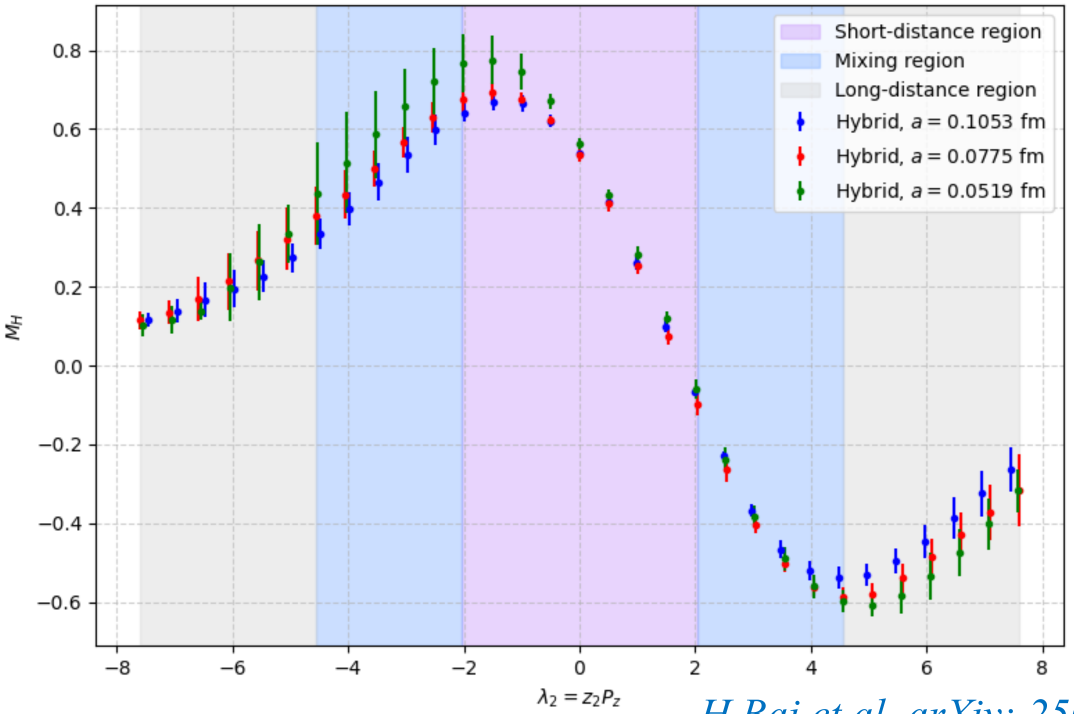
z-dependence of Hybrid renormalized quasi-DA on different lattice spacing

Lambda A term z_1=0.250 fm

P^z=0.5 GeV



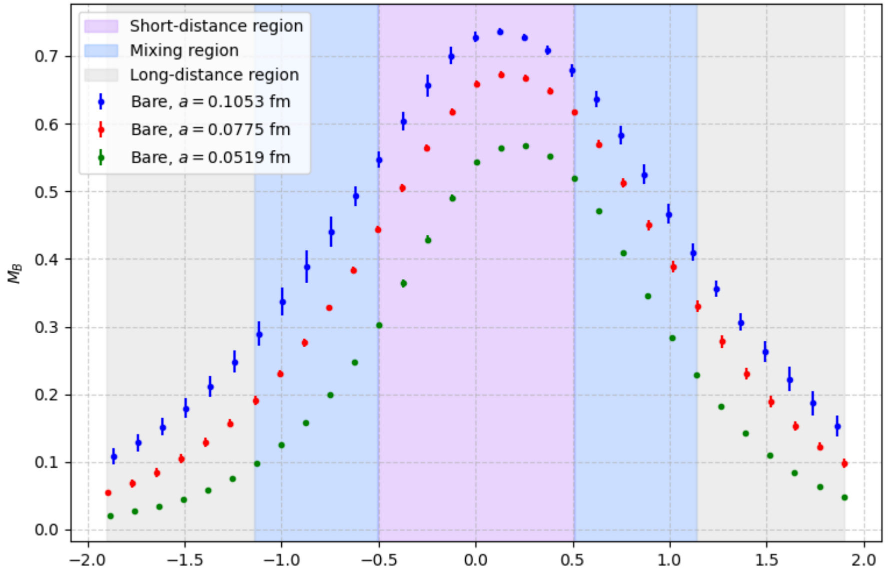
P^z=2.0 GeV



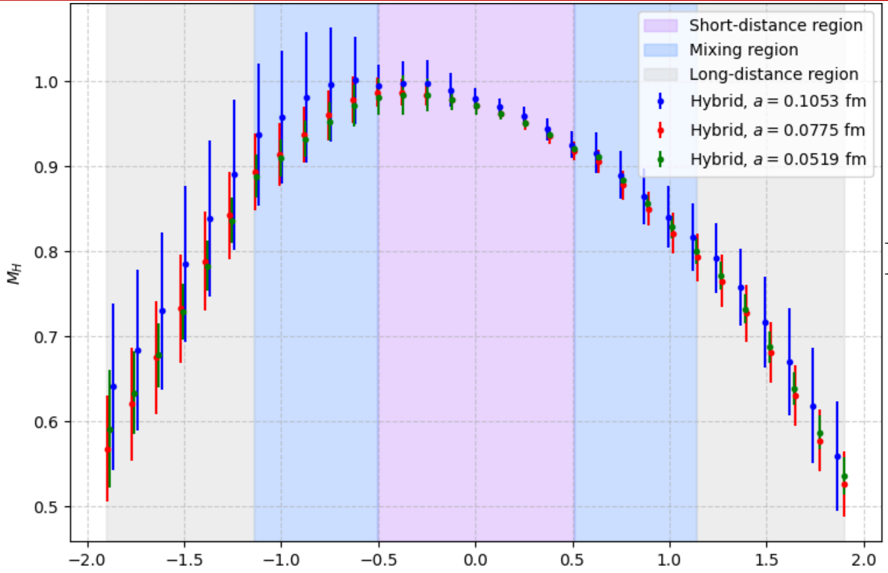
H.Bai et.al. arXiv: 2508.08971

Hybrid Renormalization

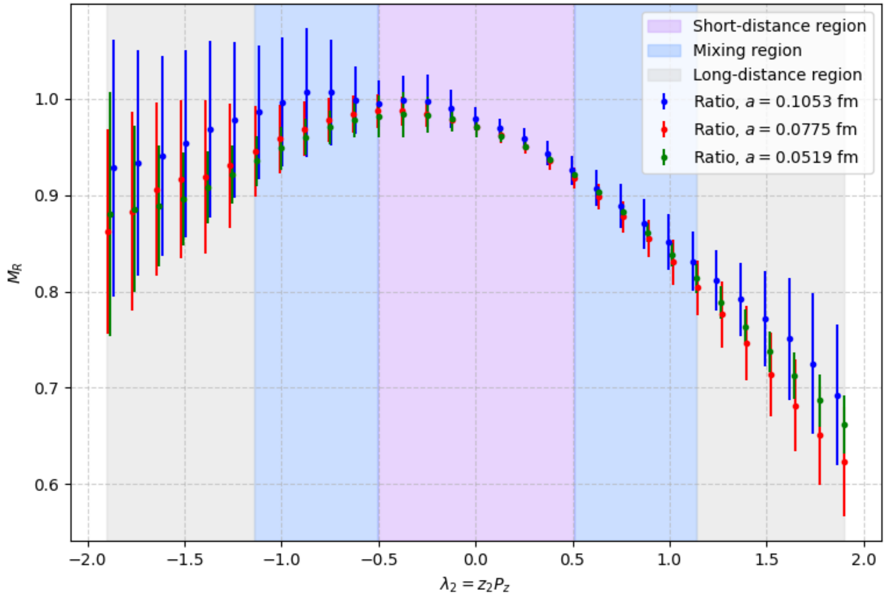
Bare Matrix Element



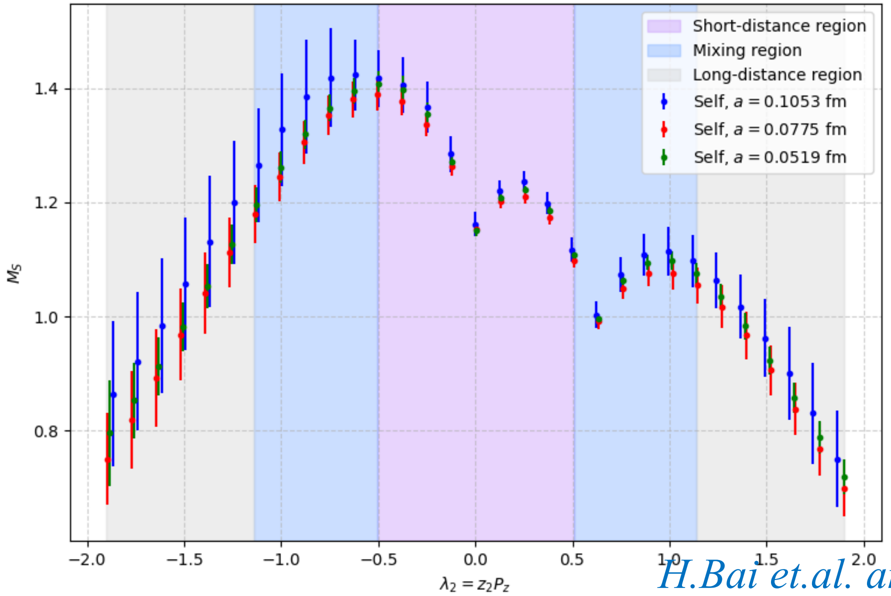
Hybrid scheme result



Ratio scheme result



Self scheme result



H.Bai et.al. arXiv: 2508.08971

How to study LCDA ?

2 Lattice methods for LCDA

Operator Product Expansion (OPE)

the local moments are consistent between light-cone coordinate and Euclidean space

- Focus on several lowest moments
- Inverse problem from moments to LCDA

Light baryon:

QCDSF, 2008, 2009;

RQCD, 2016, 2019, 2025 ……

Large momentum effective theory (LaMET)

the light-cone correlator relates to an equal-time correlator with a large momentum boost

- x-dependent LCDA directly
- Power correction at both endpoints

Light baryon:

LPC PRD 111, 034510 (2025)