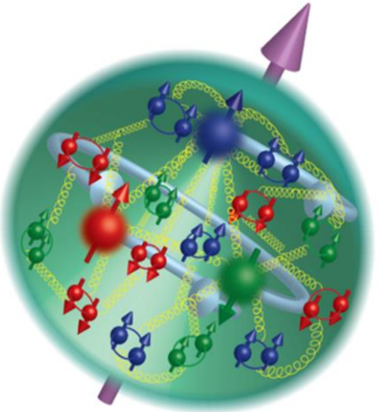
100 YEARS SPIN 2025

26th International Symposium on Spin Physics

A Century of Spin



Nucleon 3D intrinsic spin structure from the weak-neutral axial-vector form factors

Yi Chen

Tsinghua University

Based on:
[PRD 110, L091503 (2024)]
[JHEP 04, 132 (2025)]
[arXiv: 25xx.xxxx (to appear)]

Thanks to:

Bing-Song Zou, Qun Wang, Cédric Lorcé, Yang Li, Feng-Kun Guo, Qing Chen

Sep. 23, 2025 @ SPIN2025, Qingdao, China

Outline

1. Introduction and motivations

2. Weak-neutral axial-vector form factors (FFs) and hadronic matrix elements

3. Relativistic 3D intrinsic spin structure of a spin-1/2 hadron (e.g. the proton)

4. Summary and outlook

Acknowledgements:

Special thanks to the International Spin Physics Committee and Local Organizing Committee for this excellent Symposium - SPIN2025.

Based on:

[YC, Yang Li, Cédric Lorcé, & Qun Wang. PRD 110, L091503 (2024)]

[YC. JHEP 04, 132 (2025)]

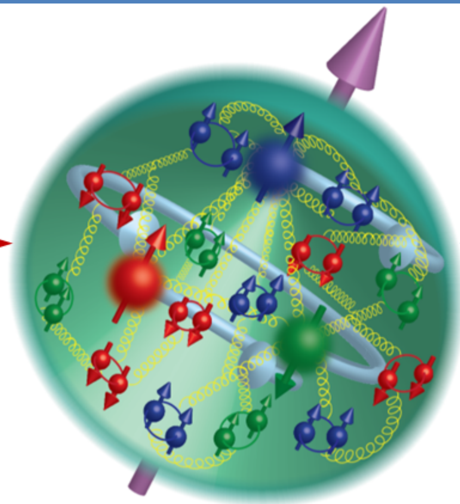
[YC, Qing Chen, Feng-Kun Guo, Qun Wang, & Bing-Song Zou. arXiv: 25xx.xxxx (to appear)]

Probing the internal structures of a hadron

EM probes

Virtual photon γ*

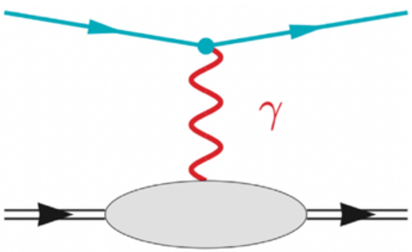
Vector



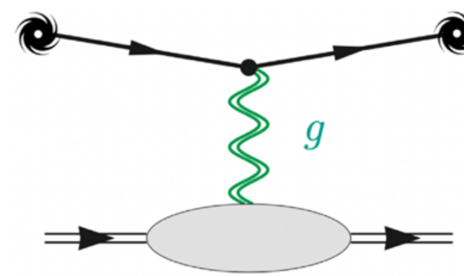
Tensor

Gravity-like probes

Virtual gluons g*



Electric charge: $Q=G_E(0)$
Magnetic moment: $\mu=G_M(0)$

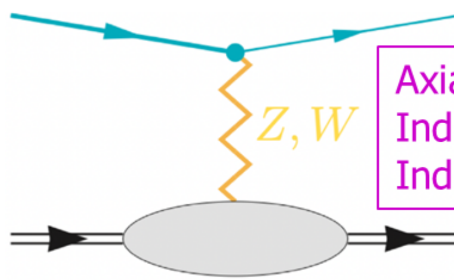


Mass: $M,\ A(0)=1$
Spin: $J,\ J(0)=1/2$
D-term: $D(0)$

Axial-vector

Weak probes

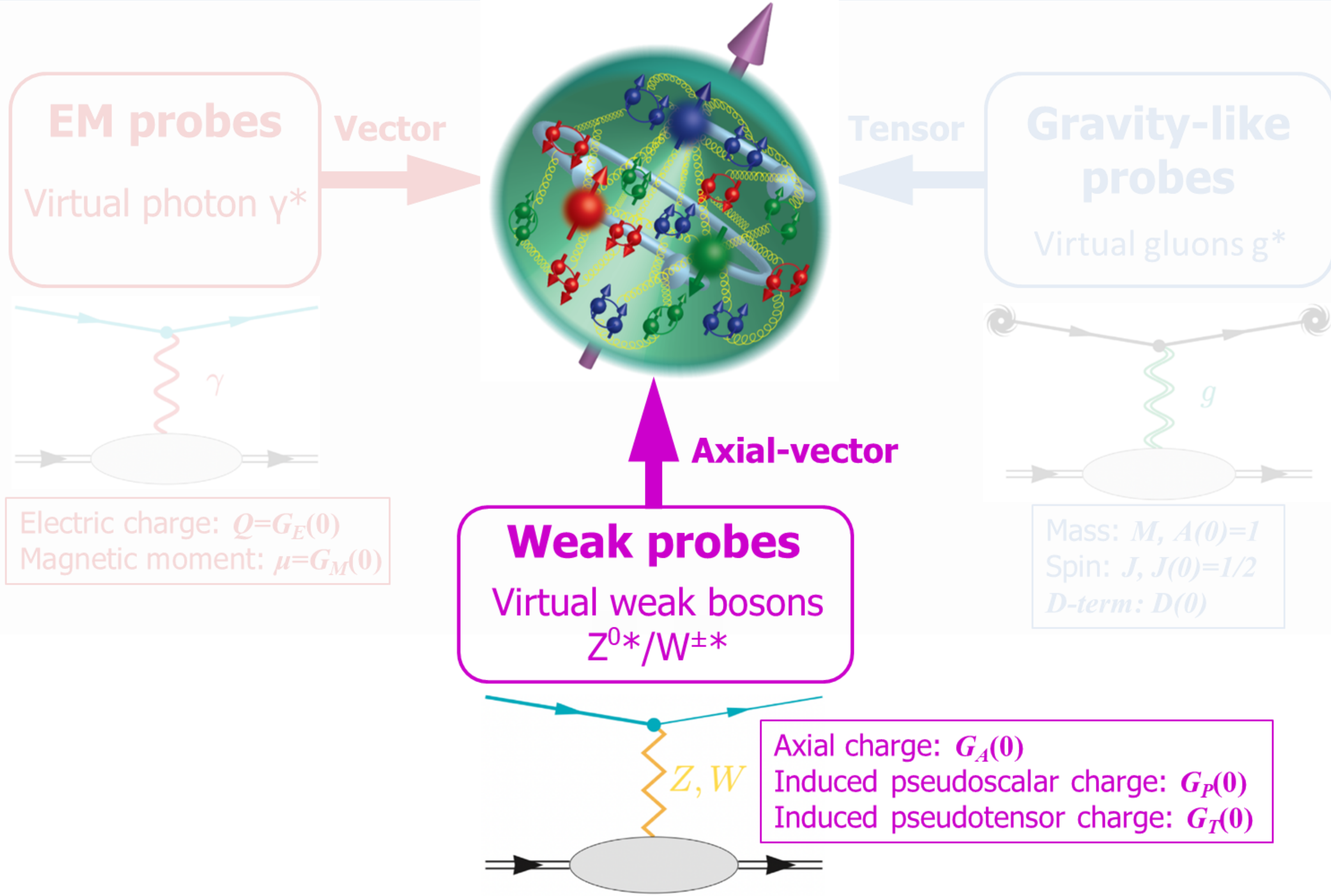
Virtual weak bosons
$Z^0*/W^{\pm}*$



Axial charge: $G_A(0)$
Induced pseudoscalar charge: $G_P(0)$
Induced pseudotensor charge: $G_T(0)$

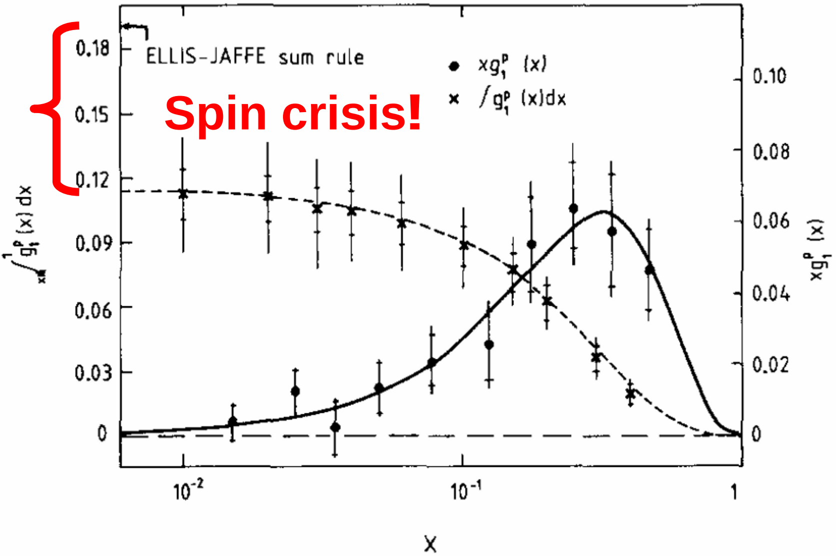
[M. Polyakov & P. Schweitzer, IJMPA 33(2018)1830025]
[Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan. RMP 95, 041002 (2023)] …

Probing the internal structures of a hadron



[M. Polyakov & P. Schweitzer, IJMPA 33(2018)1830025]
[V. Bernard, L. Elouadrhiri, & U. Meissner. J. Phys. G 28, R1 (2002)] ...

The proton spin crisis



EMC experiment [PLB 206 (1988) 364]

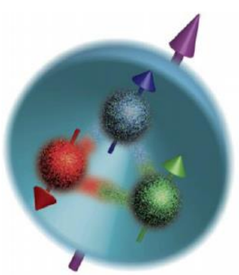
Total quark spin contribution:

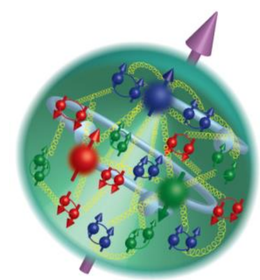
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s + \cdots$$
$$= G_A^u(0) + G_A^d(0) + G_A^s(0) + \cdots$$

$$\boxed{G_A^Z(0) = \frac{1}{2}\left[\Delta u - \Delta d - \Delta s + \Delta c + \cdots\right]}$$

→ **u and d quarks contribute only (14±9±21)% of the proton spin!**

$$\Delta\Sigma(\overline{Q^2} = 10.7\ \mathrm{GeV}^2) = 0.060 \pm 0.047 \pm 0.069$$
$$\int_0^1 \mathrm{d}x\, g_1^p(x) = 0.114 \pm 0.012 \pm 0.026$$



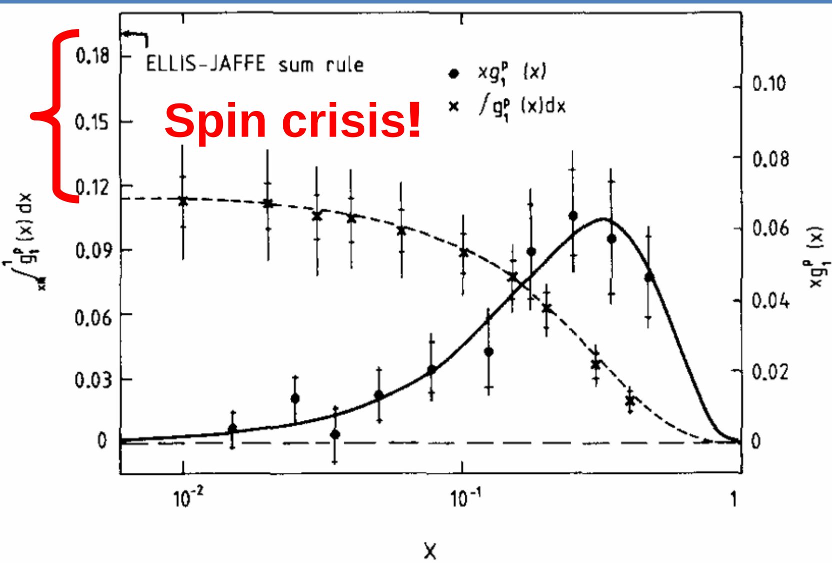


$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}(\Delta u + \Delta d)$$

Ellis-Jaffe sum rule

$$\langle J_z\rangle^p = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z^q + L_z^g$$

Jaffe-Manohar IMF sum rule



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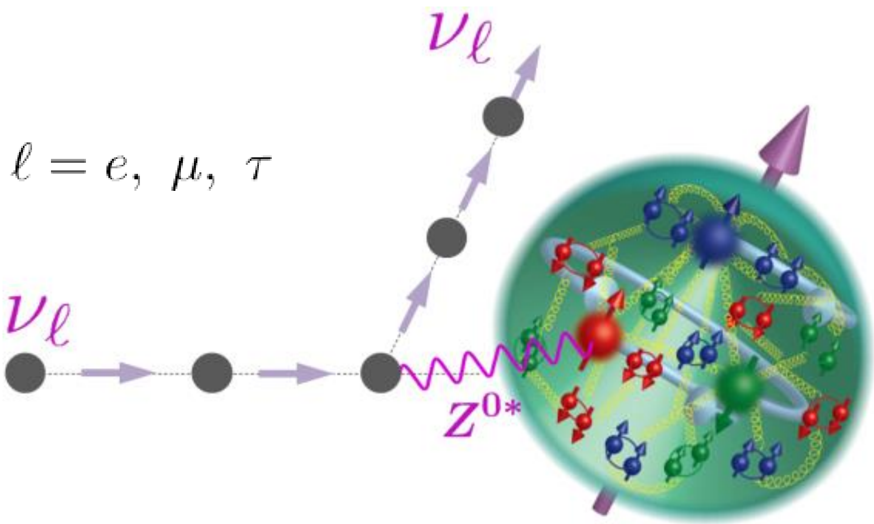
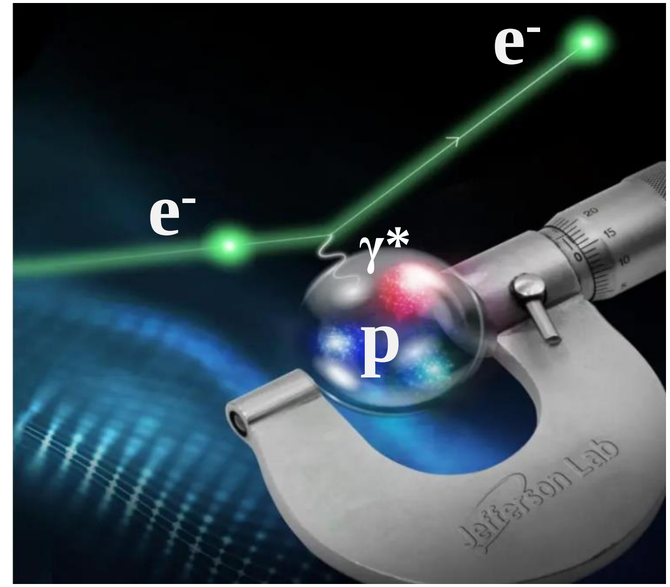
See also: Stephen Pate's talk (Time: Sep 23rd, Room 7)

08:55 - 09:15	Progress on constraining the strange quark contribution to the nucleon spin	Stephen Pate

$$\Delta s = G_A^s(0)$$

Elastic (anti)neutrino-proton scattering and why it?





$\nu_\ell + N \to \nu_\ell + N$ ← $\ell^- + N \to \ell^- + N$

(1). By a simple analogy with the elastic electron-nucleon scattering.

(2). Very clean, since (anti)neutrinos participate only* in weak interactions.

(3). Key QCD bound-states (nucleons) in the weak sector: the weak content of the most important baryonic matter in Nature.

(4). Strangeness contributions to the nucleon spin → spin crisis!

(5). $G_A^s(0)$ is only* accessible in weak neutral-current elastic scattering!

(6). Constraining uncertainties in neutrino oscillation or P-violating exps.

Relation between spin tensor and axial-vector four-current

- Using the QCD equations of motion, one can explicitly show that

$$\hat{S}^{\mu\alpha\beta}(x) = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\hat{\bar{\psi}}(x)\gamma_\lambda\gamma^5\hat{\psi}(x) = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\hat{j}_{5\lambda}$$

with the axial-vector four-current operator given by $\hat{j}_5^\mu(x) \equiv \hat{\bar{\psi}}(x)\gamma^\mu\gamma^5\hat{\psi}(x)$

[E. Leader & C. Lorcé. Physics Reports 541 (2014) 163]
[Lorcé et al. PLB 776(2018)38]
...

- Most generic matrix elements of the weak-neutral axial-vector four-current operator for a spin-1/2 hadron (e.g. the nucleon N=p, n):

$${}_N\langle p', s'|\hat{j}_5^\mu(0)|p, s\rangle_N = \bar{u}(p', s')\left[\gamma^\mu \underbrace{G_A}_{\text{Axial}} + \frac{\Delta^\mu}{2M}\underbrace{G_P}_{\text{Induced pseudoscalar}} - \frac{\sigma^{\mu\nu}\Delta_\nu}{2M}\underbrace{G_T}_{\text{Induced pseudo-tensor}}\right]\gamma^5 u(p, s)$$

$$\Delta = p' - p$$

$$\Delta^2 = t \equiv -Q^2 \leq 0$$

* Assuming G-parity invariance (or exact isospin symmetry) eliminates $G_T^Z(Q^2)$.

[Ohlsson & Snellman. EPJC 6, 285 (1999)]...
[V. Bernard, L. Elouadrhiri, & U. Meissner. J. Phys. G 28, R1 (2002)]
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP 04, 132 (2025)]

Exact relation between spin density and axial-vector current

- **Spin density operator and spin (vector) density**

$$\boxed{\hat{S}^i(x) = \frac{1}{2}\epsilon^{ijk}\hat{S}^{0jk} = \frac{1}{2}\hat{j}_5^i(x)} \quad \longrightarrow \quad \boxed{\boldsymbol{S} = \frac{1}{2}\boldsymbol{J}_5}$$

→ **Spin vector density S** is so closely related to the **axial current density J_5.**

[Lorcé & Mantovani, & Pasquini. PLB 776 (2018) 38]
[Chen, Li, Lorcé, Wang. PRD 110, L091503 (2024)]

G_A G_P G_T

- **Weak-neutral axial-vector FFs :**

1). Elastic (anti)neutrino-nucleon scatterings.

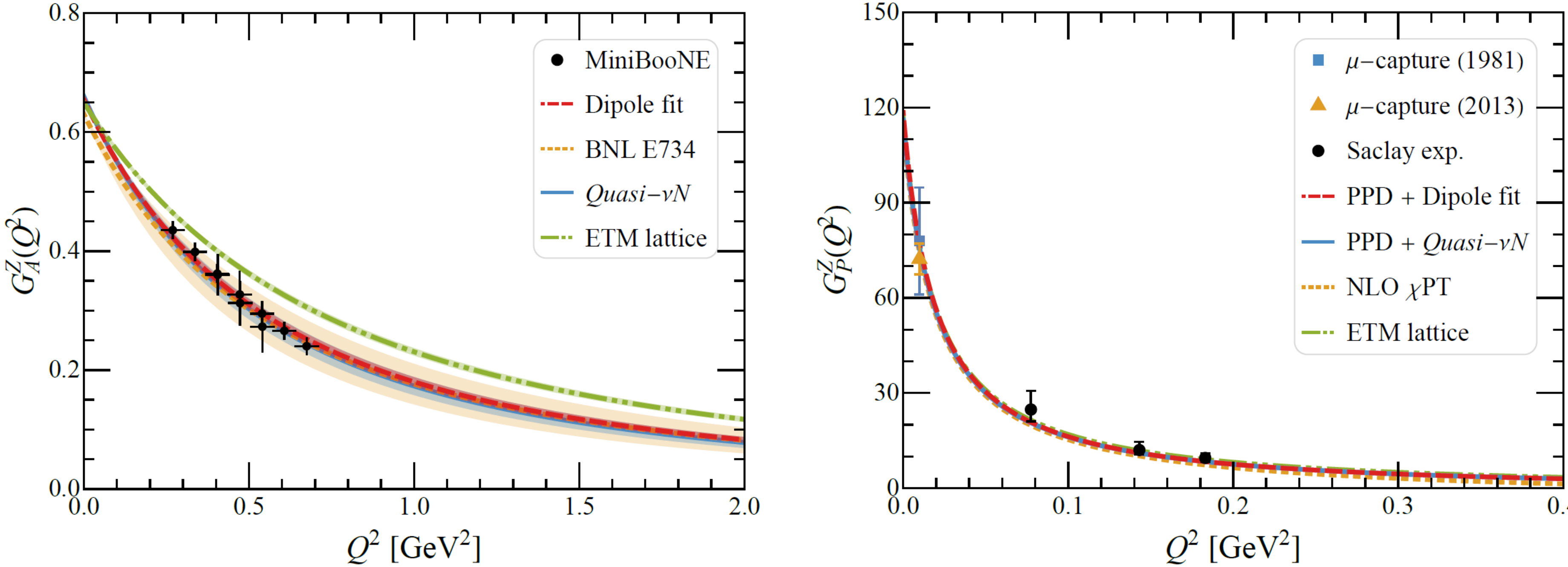
[1]. Horstkotte, et al. Phys. Rev. D 25, 2743 (1982);
[2]. Ahrens, et al. Phys. Rev. D 35, 785 (1987);
[3]. Aguilar-Arevalo, et al. Phys. Rev. D 82, 092005 (2010);
[4]. Aguilar-Arevalo, et al. Phys. Rev. D 91, 012004 (2015);

2). Recent lattice QCD calculations.

[1]. Alexandrou, et al. Phys. Rev. D 103, 034509 (2021);
[2]. Alexandrou, et al. Phys. Rev. D 104, 074503 (2021);
[3]. Djukanovic, et al. Phys. Rev. D 106, 074503 (2022);
[4]. Jang, et al. Phys. Rev. D 109, 014503 (2024);…

See also: Weizhi Xiong's talk for future measurements of the nucleon axial form factor at JLab. (Time: Sep 23rd, Room 9)

Weak-neutral axial-vector FFs of the nucleon: $G^Z_{A,P,T}(Q^2)$



BNL E734 { **[1]. Horstkotte, et al. Phys. Rev. D 25, 2743 (1982);**
[2]. Ahrens, et al. Phys. Rev. D 35, 785 (1987); }

MiniBooNE { **[3]. Aguilar-Arevalo, et al. Phys. Rev. D 82, 092005 (2010);**
[4]. Aguilar-Arevalo, et al. Phys. Rev. D 91, 012004 (2015); }

ETM lattice { **[5]. Alexandrou, et al. Phys. Rev. D 103, 034509 (2021);**
[6]. Alexandrou, et al. Phys. Rev. D 104, 074503 (2021);… }

μ-capture { **[7]. Bardin, et al. Phys. Lett. B 104, 320 (1981);**
[8]. Andreev, et al. Phys. Rev. Lett. 110, 012504 (2013); }

Saclay exp. { **[9]. Choi, et al. Phys. Rev. Lett. 71, 3927 (1993);**

[Bernard, Kaiser & Meissner, PRD 50, 6899 (1994)]
[R. Sufian, K. Liu & D. Richards. JHEP 01, 136 (2020)]
[YC. JHEP 04, 132 (2025)]

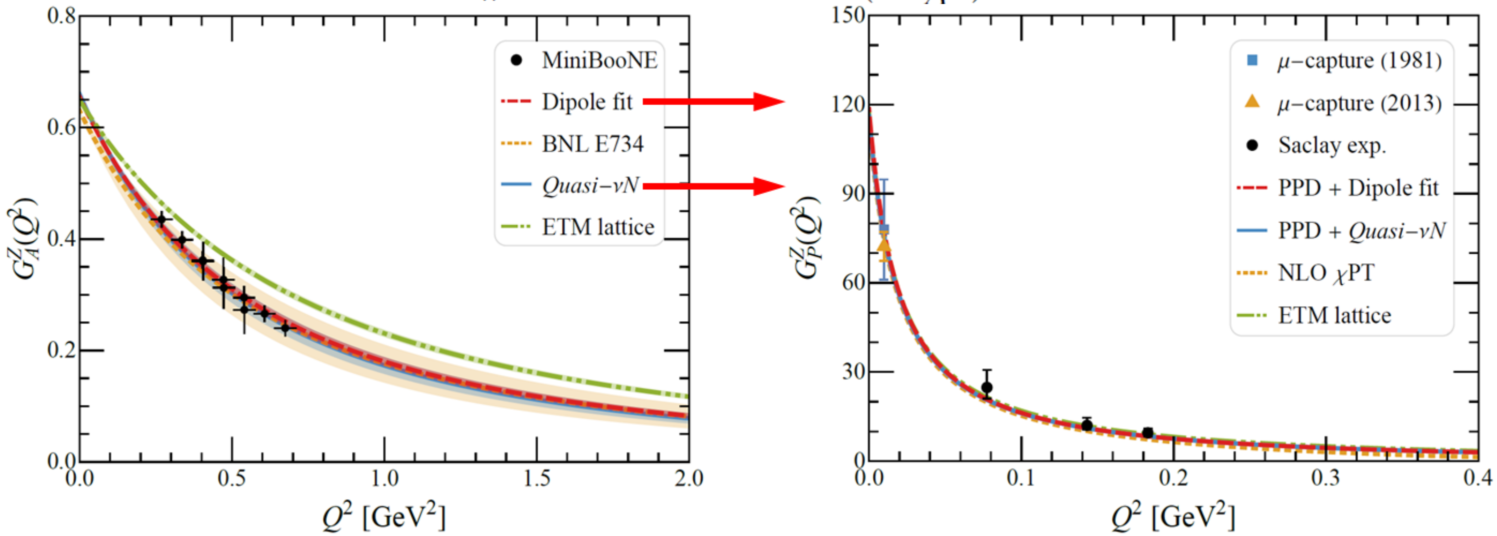
Pion-pole dominance (PPD) hypothesis and scaling ansätz

- **PPD:** $$G_P^Z(Q^2) = \underbrace{\frac{4M^2}{M_\pi^2 + Q^2}}_{\text{pion pole}} G_A^Z(Q^2)$$ ← **PCAC and Goldberger-Treiman relations**

- **NLO ChPT (χPT):**

$$g_{\pi^\pm pn} = \frac{\sqrt{4\pi}(M_p + M_n)}{M_\pi} f_{\pi^\pm pn} \approx (13.22613 \pm 0.04369)$$

$$G_P^W(Q^2) = g_{\pi^\pm pn} \frac{2(M_p + M_n)F_\pi}{Q^2 + M_\pi^2} - 2G_A^W(0)\frac{(M_p + M_n)^2}{(M_A^W)^2} + \mathcal{O}(Q^2; M_\pi^2)$$



- **Induced pseudotensor FF (scaling ansätz):** $G_T^Z(Q^2) = \kappa_T \cdot G_A^Z(Q^2), \quad \kappa_T \approx 0.1$

[Y. Jang, R. Gupta, B. Yoon & T. Bhattacharya, PRL 124, 072002 (2020)]
[C. Alexandrou [ETM], et al. PRD 103, 034509 (2021)]
[Bernard, Kaiser & Meissner, PRD 50, 6899 (1994); Reinert et al. PRL 126, 092501 (2021)]
[M. Day & K. McFarland, PRD 86, 053003 (2012)]
[C. Chen, C. Fischer, C. Roberts & J. Segovia, PRD 105, 094022 (2022); EPJA 58, 206 (2022)]

Nucleon 3D mean-square axial radius $\langle r_A^2 \rangle \neq R_A^2$

- **Standard definition of 3D mean-square axial (charge) radius $\langle r_A^2 \rangle$:**

$$\langle r_A^2 \rangle \equiv \frac{\int \mathrm{d}^3 r \, r^2 \, J^0_{5,B}(\boldsymbol{r})}{\int \mathrm{d}^3 r \, J^0_{5,B}(\boldsymbol{r})}$$

$$\langle r_E^2 \rangle \equiv -\frac{6}{G_E(0)} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q=0}$$

- **Naïve traditional definition of axial (charge) radius R_A^2, by a simple analogy with the traditional definition of the mean-square proton charge radius:**

$$R_A^2 \equiv -\frac{6}{G_A(0)} \frac{\mathrm{d}G_A(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Justification of the naïve 3D mean-square nucleon axial (charge) radius R_A^2 has never been rigorously discussed since 1986!

(for almost 40 years!!!)

[Meissner & Kaiser, PLB 180, 129 (1986)]
[Meissner & Kaiser & Weise, NPA 466, 129 (1986)]
[A1 Collaboration, PLB 468, 20 (1999)]
[Hill, et al. Rept. Prog. Phys. 81, 096301 (2018)]
[MINERvA Collaboration, Nature 614, 48 (2023)]
…

3D mean-square axial radius $\langle r_A^2 \rangle$ in the Breit frame (BF)

(1). Assuming G-parity invariance,

$$_N\langle p', s'|\hat{j}_5^\mu(0)|p, s\rangle_N = \bar{u}(p', s')\left[\gamma^\mu \underbrace{G_A}_{\text{Axial}} + \frac{\Delta^\mu}{2M}\underbrace{G_P}_{\text{Induced pseudoscalar}}\right]\gamma^5 u(p, s)$$

In the 3D BF: $_N\langle p', s'|\hat{j}_5^0(0)|p, s\rangle_N = 0 \longrightarrow J_{5,B}^0(\boldsymbol{r}) = 0$

$\longrightarrow$ **Totally vanishing 3D axial charge distribution, thus no 3D axial radius!**

(2). Without assuming G-parity invariance,

$$_N\langle p', s'|\hat{j}_5^\mu(0)|p, s\rangle_N = \bar{u}(p', s')\left[\gamma^\mu \underbrace{G_A^Z}_{\text{Axial}} + \frac{\Delta^\mu}{2M}\underbrace{G_P^Z}_{\text{Induced pseudoscalar}} - \frac{\sigma^{\mu\nu}\Delta_\nu}{2M}\underbrace{G_T^Z}_{\text{Induced pseudo-tensor}}\right]\gamma^5 u(p, s)$$

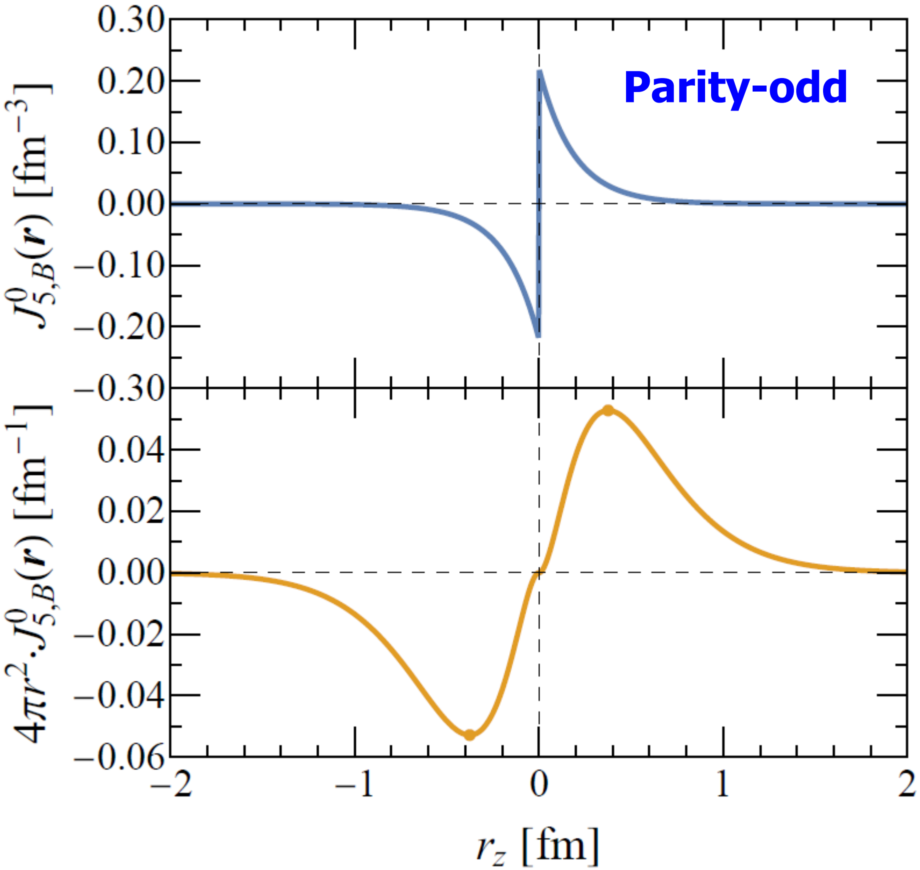
In the 3D BF: $_N\langle p', s'|\hat{j}_5^0(0)|p, s\rangle_N = \sqrt{1+\tau}\,\underbrace{(\boldsymbol{\sigma}\cdot i\boldsymbol{\Delta})}_{\text{P-odd}}G_T^Z(\boldsymbol{\Delta}^2)$

$\longrightarrow$ **1). 3D axial charge distribution is related to $G_T^Z(Q^2)$ rather than $G_A^Z(Q^2)$;**
2). It is parity-odd, thus there is no 3D mean-square axial radius!

Nucleon 3D axial charge distribution & axial radius $\langle r_A^2 \rangle$

In either cases, i.e. $G_T^Z(Q^2) = 0$ or $G_T^Z(Q^2) \neq 0$, 3D mean-square axial radius for a spin-1/2 hadron does not exist. This is dictated by the parity symmetry.

$$J^0_{5,B}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \frac{(\boldsymbol{\sigma}\cdot i\boldsymbol{\Delta})}{2M} G_T^Z(\boldsymbol{\Delta}^2) \qquad \int \mathrm{d}^3 r\, J^0_{5,B}(\boldsymbol{r}) = 0$$

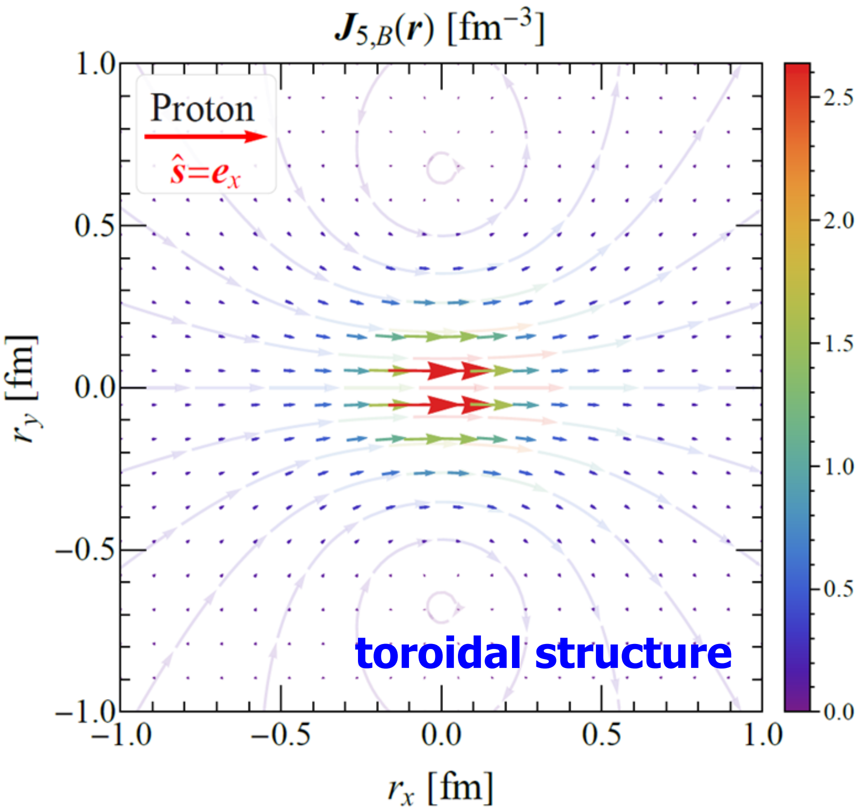


[YC, Y. Li, C. Lorcé, and Q. Wang. PRD 110, L091503 (2024)]
[YC. JHEP 04, 132 (2025)]

Relativistic 3D intrinsic spin structure of the nucleon

◆ **3D BF spin vector density** $\boldsymbol{S}_B(\boldsymbol{r}) = \frac{1}{2}\boldsymbol{J}_{5,B}(\boldsymbol{r})$

$$\boldsymbol{J}_{5,B}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{ \left[\boldsymbol{\sigma} - \frac{\boldsymbol{\Delta}(\boldsymbol{\Delta}\cdot\boldsymbol{\sigma})}{4P_B^0(P_B^0+M)}\right] G_A^Z(\boldsymbol{\Delta}^2) - \frac{\boldsymbol{\Delta}(\boldsymbol{\Delta}\cdot\boldsymbol{\sigma})}{4MP_B^0} G_P^Z(\boldsymbol{\Delta}^2) \right\}$$



■ **Input from MiniBooNE @ Fermilab (anti)neutrino data:**

$G_A^Z(Q^2)$: **MiniBooNE data;**
$G_P^Z(Q^2)$: **MiniBooNE data of G_A^Z (Q^2) + PPD (pion pole dominance);**

■ **Multipole decomposition:**

$$\boldsymbol{S}_B(\boldsymbol{r}) = \boldsymbol{S}_B^{(M)}(\boldsymbol{r}) + \boldsymbol{S}_B^{(Q)}(\boldsymbol{r})$$

Up-down: mirror-symmetric

Left-right: mirror-antisymmetric

[MiniBooNE: PRD 82, 092005 (2010); PRD 91, 012004 (2015)]
[YC, Y. Li, C. Lorcé, Q. Wang. PRD 110, L091503 (2024); YC, JHEP 04, 132 (2025)]
[Sufian et al. JHEP 01, 136 (2020)]
[P. Neumann-Cosel et al. PRL 133, 233502 (2024)]

Toroidal structure of ^{208}Pb & proton

PHYSICAL REVIEW LETTERS **133,** 232502 (2024)

Editors' Suggestion   Featured in Physics

Candidate Toroidal Electric Dipole Mode in the Spherical Nucleus ^{58}Ni

P. von Neumann-Cosel,[1,*] V. O. Nesterenko,[2,3,†] I. Brandherm,[1] P. I. Vishnevskiy,[2,4] P.-G. Reinhard,[5] J. Kvasil,[6] H. Matsubara,[7,8] A. Repko,[9] A. Richter,[1] M. Scheck,[10,11] and A. Tamii[7]

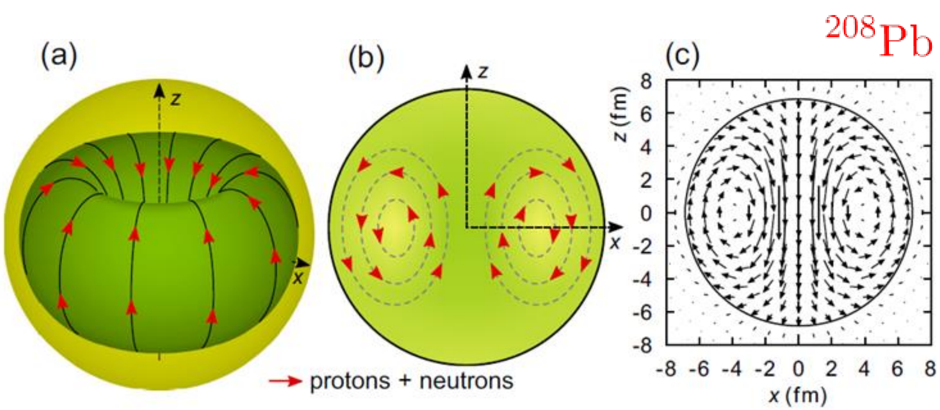
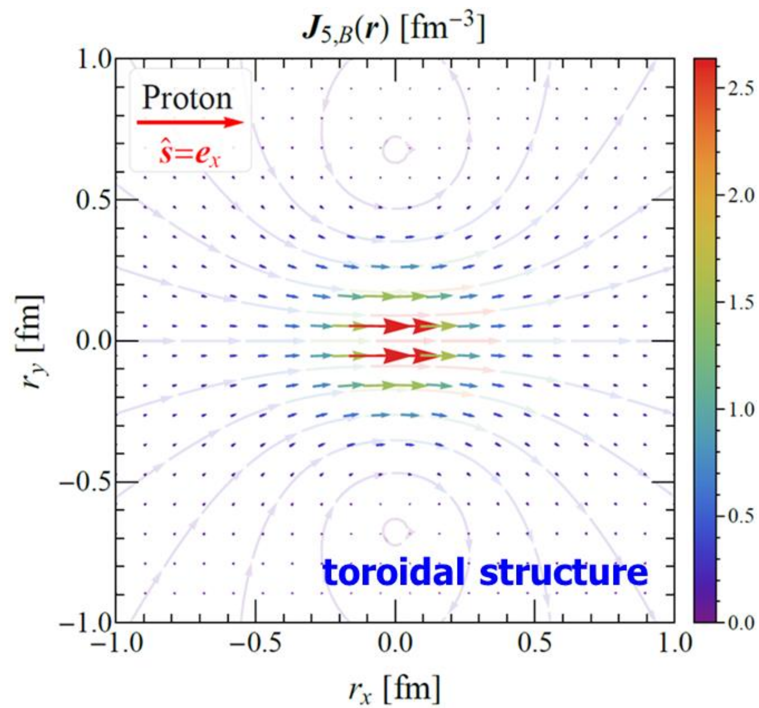


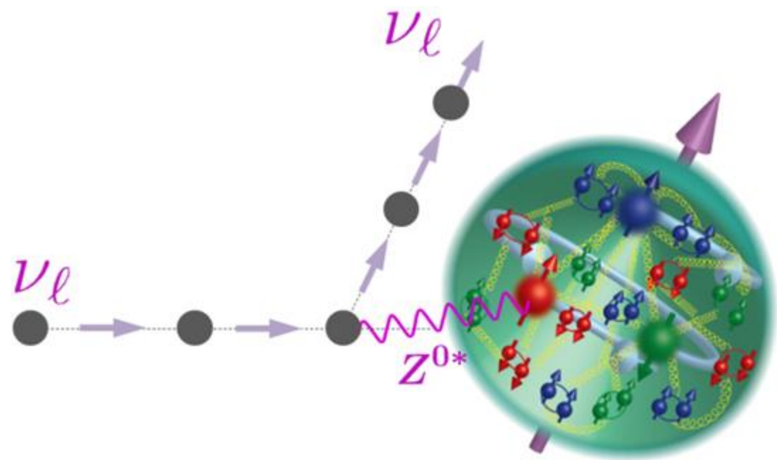
FIG. 1. Nuclear toroidal excitations. (a) Schematic view and (b) its cut in the x-z plane. (c) Same as (b) for the toroidal mode predicted in the nucleus ^{208}Pb [14]. The arrows mark the current along stream lines and their length is a measure of the current density.



➡ **Nucleon intrinsic spin structure is naturally of toroidal structure!**

[P. Neumann-Cosel et al. PRL 133, 233502 (2024)]
[YC, Y. Li, C. Lorcé, Q. Wang. PRD 110, L091503 (2024); YC, JHEP 04, 132 (2025)]

Different nature of axial-vector and vector four-currents

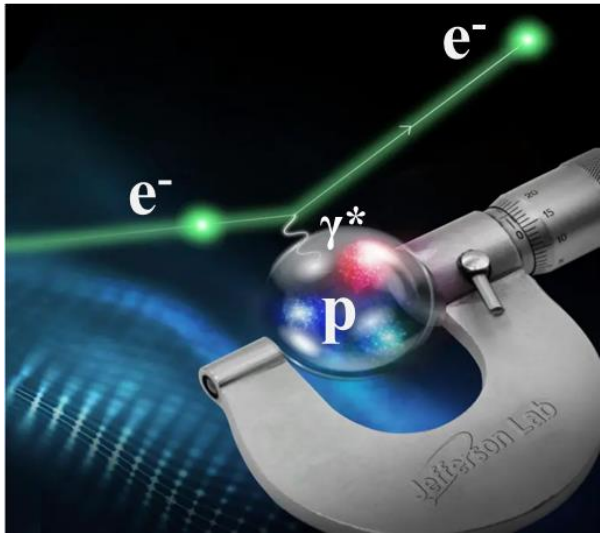


$P_\mu \Gamma_5^\mu = 0$ $\hat{j}_5^\mu = \hat{\bar{\psi}}\gamma^\mu\gamma^5\hat{\psi}$

spacelike four-current J_5^μ

intrinsic part

$\boldsymbol{J}_5 = 2\boldsymbol{S}$ **twice of spin distribution**



$\Delta_\mu \Gamma^\mu = 0$ $\hat{j}^\mu = \hat{\bar{\psi}}\gamma^\mu\hat{\psi}$

timelike four-current J^μ

intrinsic part

$J^0 = \rho_{\mathrm{ch}}$ **electric charge distribution**

$P = (p' + p)/2$

$\Delta = p' - p$

[YC, Y. Li, C. Lorcé, and Q. Wang. PRD 110, L091503 (2024); YC, JHEP 04, 132 (2025)]
[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC, and Cédric Lorcé, PRD 106, 116024 (2022)]
[YC, and Cédric Lorcé, PRD 107, 096003 (2023)] ...

Nucleon 3D mean-square spin radius $\langle r^2_{\mathrm{spin}} \rangle$

- Physically meaningful **3D mean-square spin radius**:

$$\langle r^2_{\mathrm{spin}} \rangle \equiv \frac{\int \mathrm{d}^3 r\, r^2\, \hat{\boldsymbol{s}} \cdot \boldsymbol{S}_B(\boldsymbol{r})}{\int \mathrm{d}^3 r\, \hat{\boldsymbol{s}} \cdot \boldsymbol{S}_B(\boldsymbol{r})} = R_A^2 + \frac{1}{4M^2} \left[1 + \frac{2G_P^Z(0)}{G_A^Z(0)} \right]$$

Model-independent!

Based on **recent lattice QCD results** and **PPD hypothesis**, one can show that

$$\frac{G_P(0)}{G_A(0)} \gg 1$$

the last term $\frac{1}{2M^2} \frac{G_P(0)}{G_A(0)}$ **actually plays an dominant role!**

- R_A^2 has recently been measured with high-precision by the **MINERvA Collaboration** at Fermilab.

  [MINERvA Collaboration, Nature 614, 48 (2023)]

- However, $\boldsymbol{R_A^2}$ **alone is not enough** to fix the meaningful 3D mean-square spin radius $\langle r^2_{\mathrm{spin}} \rangle$. **One still needs to determine the ratio $\boldsymbol{G_P^Z(0)/G_A^Z(0)}$.**

- This **provides an additional key motivation for ongoing lattice QCD and model calculations or future experimental measurements** of the nucleon induced pseudoscalar form factor $\boldsymbol{G_P^Z(Q^2)}$.

Covariant Lorentz transformations (based on Poincaré symmetry)

◆ **Axial-vector four-current case:**

$$\langle p', s'|\hat{j}_5^{\mu}(0)|p, s\rangle = \sum_{s'_B, s_B} \underbrace{D^{\dagger(j)}_{s's'_B}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^{\mu}{}_{\nu} \langle p'_B, s'_B|\hat{j}_5^{\nu}(0)|p_B, s_B\rangle}_{\text{Lorentz mixing}}$$

◆ **Exactly the same angular conditions as before* for the Wigner rotation angle θ:**

$$\cos\theta = \frac{P^0 + M(1+\tau)}{(P^0 + M)\sqrt{1+\tau}}, \qquad \sin\theta = -\frac{\sqrt{\tau}P_z}{(P^0 + M)\sqrt{1+\tau}}$$

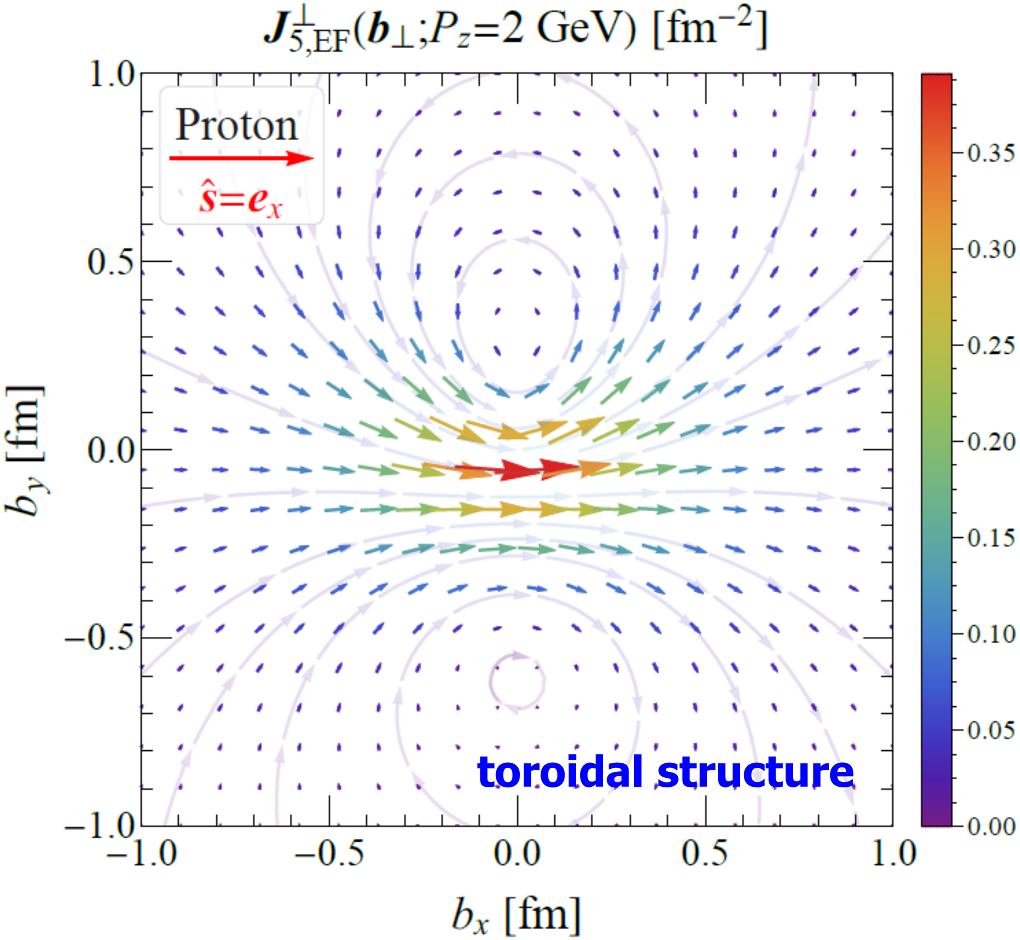
◆ **Elastic frame (EF) axial-vector four-current distributions:**

$$\left.\begin{aligned}
J^0_{5,\text{EF}}(\boldsymbol{b}_\perp; P_z) &= \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{s's}}{2M} G^Z_T(\boldsymbol{\Delta}^2_\perp) + \left(\frac{P_z}{P^0}\right)(\sigma_z)_{s's} G^Z_A(\boldsymbol{\Delta}^2_\perp)\right] \\
J^z_{5,\text{EF}}(\boldsymbol{b}_\perp; P_z) &= \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \left[\left(\frac{P_z}{P^0}\right)\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{s's}}{2M} G^Z_T(\boldsymbol{\Delta}^2_\perp) + (\sigma_z)_{s's} G^Z_A(\boldsymbol{\Delta}^2_\perp)\right]
\end{aligned}\right\} \text{free from Winger spin rotation}$$

$$\left.\begin{aligned}
\boldsymbol{J}^\perp_{5,\text{EF}}(\boldsymbol{b}_\perp; P_z) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \bigg\{ &-\frac{\boldsymbol{\Delta}_\perp(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{s's}}{4P^0}\left[\frac{G^Z_A(\boldsymbol{\Delta}^2_\perp)}{P^0 + M} + \frac{G^Z_P(\boldsymbol{\Delta}^2_\perp)}{M}\right] \\
&+ \frac{\sqrt{P^2}}{P^0}\left[\cos\theta\,(\boldsymbol{\sigma}_\perp)_{s's} - \frac{(\boldsymbol{e}_z \times i\boldsymbol{\Delta}_\perp)_\perp}{2M\sqrt{\tau}} \sin\theta\,\delta_{s's}\right] G^Z_A(\boldsymbol{\Delta}^2_\perp)\bigg\}
\end{aligned}\right\} \text{suffer from Winger spin rotation}$$

[Durand, De Celles, Marr, PR 126, 1882 (1962)]
[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC, and Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]*
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]
[YC. JHEP 04, 132 (2025)]

Relativistic EF distributions of a moving proton



$$S^{\perp}_{\mathrm{EF}} = \frac{1}{2} J^{\perp}_{5,\mathrm{EF}}$$

$$J^{\perp}_{5,\mathrm{EF}} = J^{\perp(M)}_{5,\mathrm{EF}} + J^{\perp(D)}_{5,\mathrm{EF}} + J^{\perp(Q)}_{5,\mathrm{EF}}$$

← Dipole contribution breaks explicitly the up-down symmetry!

Up-down: mirror-asymmetric

Left-right: mirror-antisymmetric

- **Both $S_{\perp,\mathrm{EF}}$ and $J^{\perp}_{5,\mathrm{EF}}$ are free from Lorentz mixing effect but suffer from the spin Wigner rotation!**

[YC, Y. Li, C. Lorcé, and Q. Wang. PRD 110, L091503 (2024)]
[YC. JHEP 04, 132 (2025)]
[P. Neumann-Cosel et al. PRL 133, 233502 (2024)]

Proton's 2D transverse mean-square axial and spin radii

- **Axial radius** (for a longitudinally polarized, moving spin-1/2 hadron).

$$\langle b_A^2 \rangle_{\mathrm{EF}}(P_z) = \frac{1}{2E_P^2} + \frac{2}{3} R_A^2$$

- **Longitudinal spin radius** (for a longitudinally polarized spin-1/2 hadron).

$$\langle b_{\mathrm{spin},L}^2 \rangle_{\mathrm{EF}}(P_z) \equiv \frac{\int \mathrm{d}^2 b_\perp \, b^2 \, S_{\mathrm{EF}}^z(\boldsymbol{b}_\perp; P_z)}{\int \mathrm{d}^2 b_\perp \, S_{\mathrm{EF}}^z(\boldsymbol{b}_\perp; P_z)} = \frac{2}{3} R_A^2$$

- **Transverse spin radius** (for a transversely polarized spin-1/2 hadron).

$$\langle b_{\mathrm{spin},T}^2 \rangle_{\mathrm{EF}}(P_z) = \frac{2}{3} R_A^2 + \frac{1}{2M^2} \frac{G_P(0)}{G_A(0)} - \frac{1}{2M(E_P + M)} + \frac{1}{2E_P^2}$$

Conclusion: **the second-class current contribution [associated with $G_T^Z(Q^2)$], although explicitly included, does not contribute in fact to the mean-square axial and spin radii.**

[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024)]
[YC. JHEP 04, 132 (2025)]

Relativistic light-front (LF) distributions

- **In the IMF limit ($P_z \to \infty$):** $J^{+}_{5,\mathrm{LF}}(\boldsymbol{b}_\perp;P^+) = J^{0}_{5,\mathrm{EF}}(\boldsymbol{b}_\perp;\infty) = J^{z}_{5,\mathrm{EF}}(\boldsymbol{b}_\perp;\infty)$

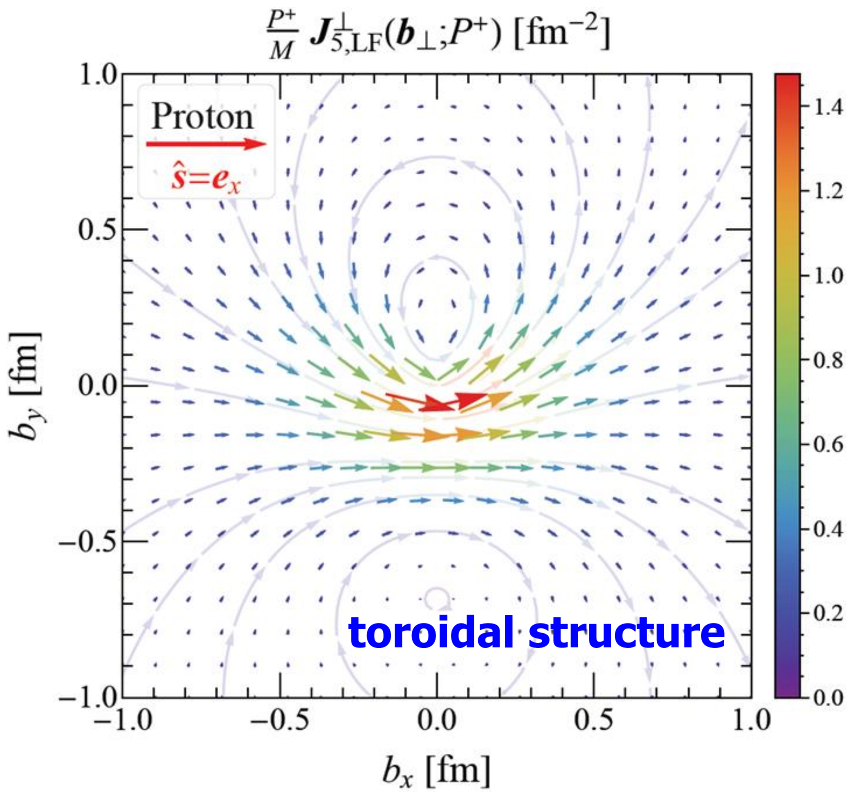
- **Mean-square LF radii:**

$$\langle b_A^2\rangle_{\mathrm{LF}}(P^+) = \langle b^2_{\mathrm{spin},L}\rangle_{\mathrm{LF}}(P^+) = \frac{2}{3}R_A^2$$

$$\langle b^2_{\mathrm{spin},T}\rangle_{\mathrm{LF}}(P^+) = \frac{2}{3}R_A^2 + \frac{1}{2M^2}\frac{G_P^Z(0)}{G_A^Z(0)}$$

~ independent of G_T^Z
(second-class current)

- **Scaled transverse LF axial current distribution:**



$$\boldsymbol{S}^{\perp}_{\mathrm{LF}} = \frac{1}{2}\boldsymbol{J}^{\perp}_{5,\mathrm{LF}}$$

Transversely polarized proton

Up-down: mirror-asymmetric

Left-right: mirror-antisymmetric

[YC, Y. Li, C. Lorcé, and Q. Wang. PRD 110, L091503 (2024); YC. JHEP 04, 132 (2025)]
[P. Neumann-Cosel et al. PRL 133, 233502 (2024)]

LF amplitudes via EF amplitudes at proper IMF limit

Conjecture. *Any light-front (LF) amplitudes for well-defined LF distributions in principle can be explicitly reproduced from the corresponding elastic frame (EF) amplitudes in the proper infinite-momentum frame (IMF) limit.*

◆ EF amplitudes

$$\mathcal{A}^0_{\mathrm{EF}} = 2P^0\left[\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)}{2M}G_T^Z(\boldsymbol{\Delta}_\perp^2)+\frac{P_z}{P^0}\sigma_z G_A^Z(\boldsymbol{\Delta}_\perp^2)\right],$$

$$\mathcal{A}^z_{\mathrm{EF}} = 2P^0\left[\frac{P_z}{P^0}\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)}{2M}G_T^Z(\boldsymbol{\Delta}_\perp^2)+\sigma_z G_A^Z(\boldsymbol{\Delta}_\perp^2)\right],$$

$$\boldsymbol{\mathcal{A}}^\perp_{\mathrm{EF}} = 2\sqrt{P^2}\left[\frac{P^0+M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}\boldsymbol{\sigma}_\perp+\frac{(\boldsymbol{e}_z\times i\boldsymbol{\Delta}_\perp)_\perp}{2M}\frac{P_z}{(P^0+M)\sqrt{1+\tau}}\right]G_A^Z(\boldsymbol{\Delta}_\perp^2)$$
$$-\frac{\boldsymbol{\Delta}_\perp(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)}{2}\left[\frac{G_A^Z(\boldsymbol{\Delta}_\perp^2)}{P^0+M}+\frac{G_P^Z(\boldsymbol{\Delta}_\perp^2)}{M}\right],$$

◆ LF amplitudes

$$\mathcal{A}^+_{\mathrm{LF}} = 2P^+\left[\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{\lambda'\lambda}}{2M}G_T^Z(\boldsymbol{\Delta}_\perp^2)+(\sigma_z)_{\lambda'\lambda}G_A^Z(\boldsymbol{\Delta}_\perp^2)\right],$$

$$\mathcal{A}^-_{\mathrm{LF}} = 2P^-\left[\frac{(i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{\lambda'\lambda}}{2M}G_T^Z(\boldsymbol{\Delta}_\perp^2)-(\sigma_z)_{\lambda'\lambda}G_A^Z(\boldsymbol{\Delta}_\perp^2)\right],$$

$$\boldsymbol{\mathcal{A}}^\perp_{\mathrm{EF}} = 2M\left\{\left[(\boldsymbol{\sigma}_\perp)_{\lambda'\lambda}+\frac{(\boldsymbol{e}_z\times i\boldsymbol{\Delta}_\perp)_\perp}{2M}\delta_{\lambda'\lambda}\right]G_A^Z(\boldsymbol{\Delta}_\perp^2)-\frac{\boldsymbol{\Delta}_\perp(\boldsymbol{\Delta}_\perp\cdot\boldsymbol{\sigma}_\perp)_{\lambda'\lambda}}{4M^2}G_P^Z(\boldsymbol{\Delta}_\perp^2)\right\}$$

This conjecture has been verified independently in:

(1). Electromagnetic four-current case;

(2). Polarization-magnetization tensor case;

(3). Axial-vector four-current case (with/without G_T^Z);

...

supporting LaMET?

[Cédric Lorcé, PRL 125, 232002 (2020)]
[YC & Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]
[YC, Y. Li, C. Lorcé, Q. Wang. PRD 110, L091503 (2024)]
[YC. JHEP 04, 132 (2025)]

Summary & Outlook

1. **Relativistic 3D and 2D intrinsic spin structures and axial-vector structures of the nucleon in position space are studied for the first time by using the nucleon weak-neutral axial-vector form factors.**

2. **Physically meaningful 3D axial (charge) radius $\langle r_A^2 \rangle$ does not exist for any spin-1/2 hadrons due to the parity symmetry. Relative to the naïve axial (charge) radius R_A^2, the mean-square nucleon spin radius $\langle r_{\rm spin}^2 \rangle$ is a physically more meaningful quantity that better characterizes the spatial extension of the weak content size of the nucleon.**

3. **Measurement of (naïve) axial radius R_A^2 alone is not enough to fix the nucleon spin radius $\langle r_{\rm spin}^2 \rangle$. On top of R_A^2, one also needs to determine the ratio $G_P(0)/G_A(0)$, which provides an additional key motivation for ongoing lattice QCD and model calculations and future experimental measurements of the nucleon induced pseudoscalar form factor $G_P^Z(Q^2)$.**

4. **Nucleon axial-vector FFs provide additional constraints on GPDs $\widetilde{H}_q$ and $\widetilde{E}_q$ extracted from data at e.g. JLab, NICA, LHC, EIC, EicC, etc.**

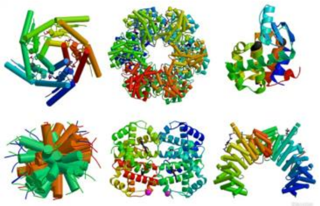
$$\begin{bmatrix} G_A(t) \\ G_P(t) \end{bmatrix} = \sum_f g_A^f \int_{-1}^{1} \mathrm{d}x \begin{bmatrix} \tilde{H}_f(x,\xi,t) \\ \tilde{E}_f(x,\xi,t) \end{bmatrix} \qquad f = u, d, s, \ldots$$

Take-home message: Structure dictates properties

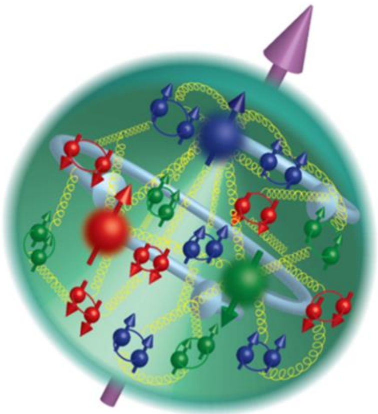
- **Analogy:**

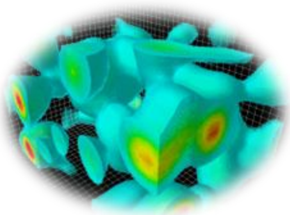
"Symmetry dictates interactions"

"Structure dictates properties"



functional proteins





QCD vacuum

- **Hadron structures are highly non-trivial and complicated!**

(1). Hadron structures are closely associated with the nonperturbative QCD dynamics between the internal quark and gluon degrees of freedom.

(2). The QCD vacuum itself is also highly non-trivial, due to quantum fluctuations (loop effects, pair creations and annihilations, instanton/sphaleron/renormelon transitions), non-trivial topologies, etc.

(3). On top of the QCD dynamics and non-trivial QCD vacuum structure, hadron structures are also affected by electromagnetic and weak interactions.

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Guang-Peng Zhang,
Bo-Wen Xiao,
Stephen Pate,
...

Thank you very much!

Backup: First- and second-class currents

$$ {}_N\langle p', s'|\hat{j}_5^\mu(0)|p,s\rangle_N = \bar{u}(p',s')\left[\gamma^\mu G_A^Z + \frac{\Delta^\mu}{2M} G_P^Z - \frac{\sigma^{\mu\nu}\Delta_\nu}{2M} G_T^Z\right]\gamma^5 u(p,s) $$

Axial

Induced pseudoscalar

Induced pseudo-tensor

- **Weinberg's classification via G-parity transformation**

$$ \mathrm{G} \equiv \mathrm{C}\exp(i\pi I_y) \qquad I_y = \sigma_y/2 $$

- **First-class currents, e.g.**

$$ \mathrm{G}j^\mu\mathrm{G}^{-1} = +j^\mu \qquad \mathrm{G}j_5^\mu\mathrm{G}^{-1} = -j_5^\mu $$

$$ j^\mu = \bar{\psi}\gamma^\mu\psi $$
$$ j_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi $$

$G_A^Z(Q^2)$ and $G_P^Z(Q^2)$ are FFs associated with the first-class currents.

- **Second-class currents, e.g.**

$$ \mathrm{G}j^\mu\mathrm{G}^{-1} = -j^\mu \qquad \mathrm{G}j_5^\mu\mathrm{G}^{-1} = +j_5^\mu $$

$G_T^Z(Q^2)$ is the FF associated with the second-class current.

In strong interactions (e.g. strong decays of mesons), G-parity is exact. However, due to different electric charges and masses of u, d, s … quarks, G-parity invariance is in general not conserved in electromagnetic or weak interactions.

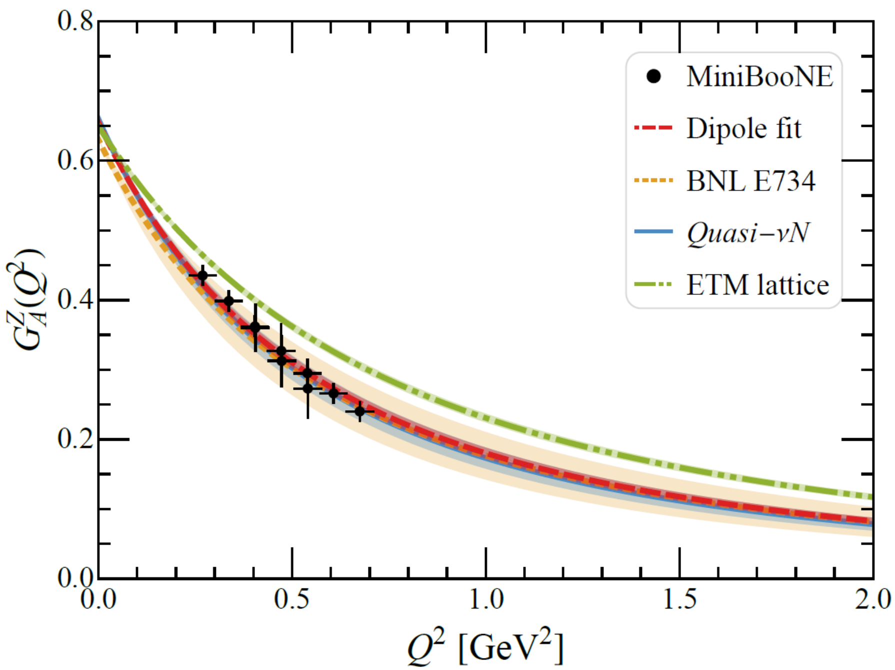
[Weinberg, PR 112, 1375 (1958)]
[H. Shiomi, NPA 603, 281 (1996)]

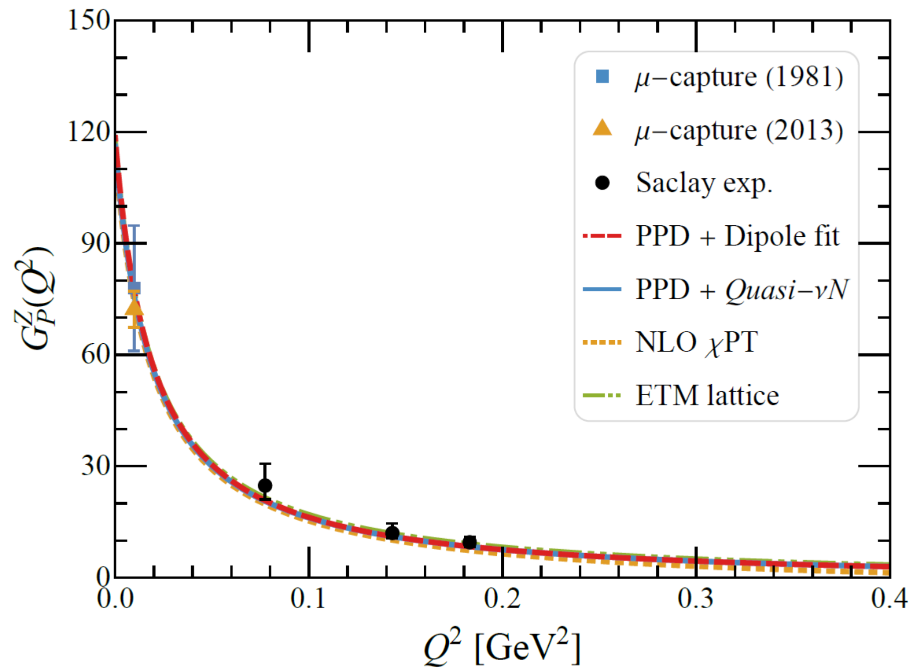
Backup: Quark flavor decomposition of axial-vector FFs

- **Weak-neutral axial-vector FFs:**

$$G_X^Z(Q^2) = \sum_f g_A^f\, G_X^f(Q^2)$$
$$\simeq \underline{\frac{1}{2}\left[G_X^W(Q^2) - G_X^s(Q^2)\right.} + G_X^c(Q^2) - G_X^b(Q^2) + \left.G_X^t(Q^2)\right]$$

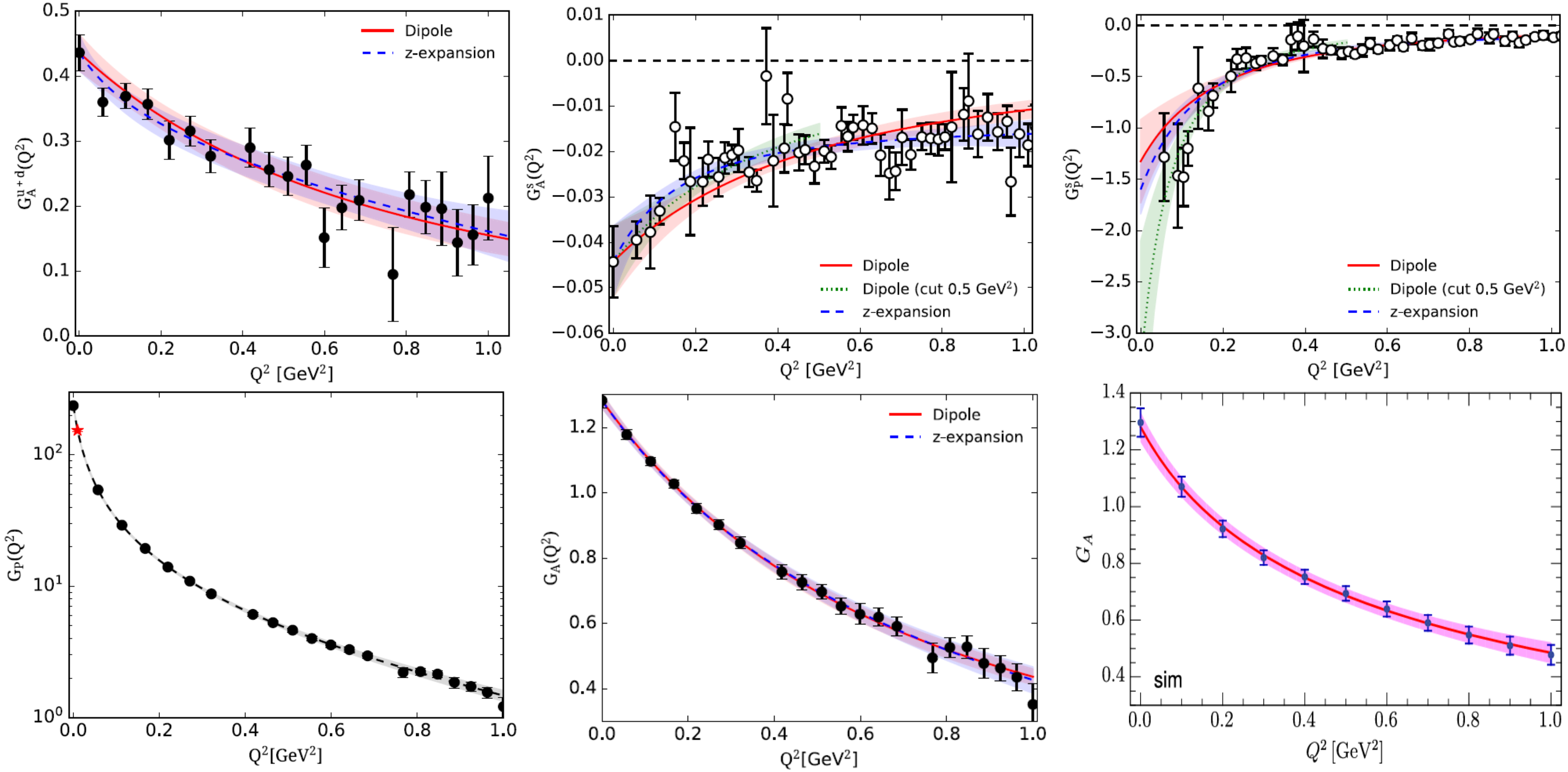
$$G_X^W \simeq G_X^u - G_X^d \equiv G_X^{(u-d)} \qquad X = A, P, T \qquad f = u, d, s, c, b, t \qquad g_A^{u,c,t} = +\frac{1}{2}$$
$$g_A^{d,s,b} = -\frac{1}{2}$$





[YC. JHEP 04, 132 (2025)]

Backup: Recent lattice QCD results



Recent lattice QCD results:

[1]. H.-W. Lin. PRL 127, 182001 (2021);
[2]. Alexandrou, et al. Phys. Rev. D 103, 034509 (2021);
[3]. Alexandrou, et al. Phys. Rev. D 104, 074503 (2021);
[4]. Djukanovic, et al. Phys. Rev. D 106, 074503 (2022);
[5]. Jang et al. Phys. Rev. D 109, 014503 (2024);
…

Backup: Explicit Wigner spin rotation in the generic EF

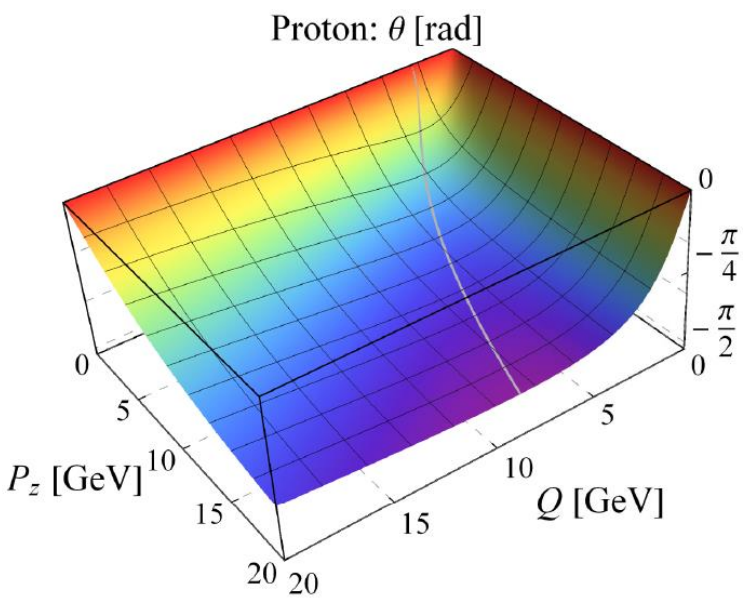
- **2D EF Wigner spin rotation matrices:**

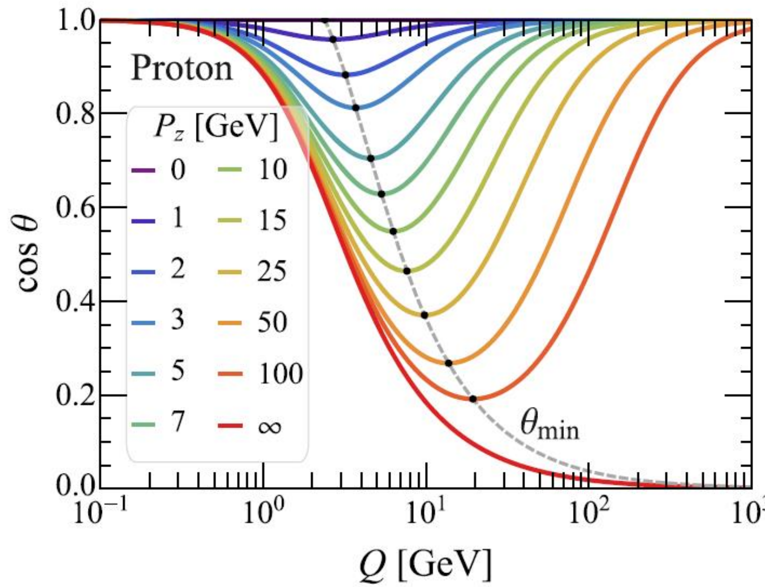
$$D^{(1/2)}(p_B,\Lambda) = D^{\dagger(1/2)}(p'_B,\Lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi_\Delta}\sin\frac{\theta}{2} \\ e^{i\phi_\Delta}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

with

$$\cos\theta = \frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}, \qquad \sin\theta = -\frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}}$$

satisfying $\cos^2\theta + \sin^2\theta = 1.$





[YC, & Cédric Lorcé. PRD 106, 116024 (2022); PRD 107, 096003 (2023)]
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); YC. JHEP 04, 132 (2025)]