LQCD Determination of Quark Spin in Octet Baryons and SU(3)-Flavor Symmetry

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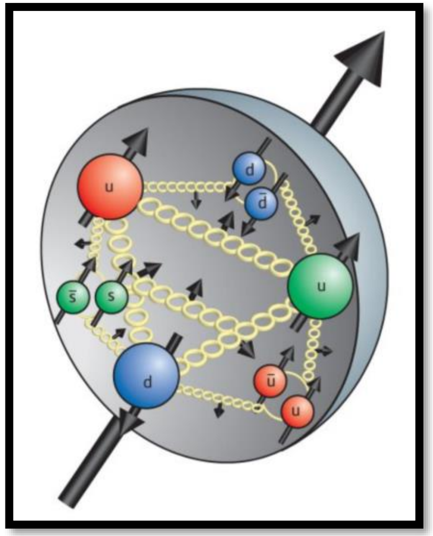
Context

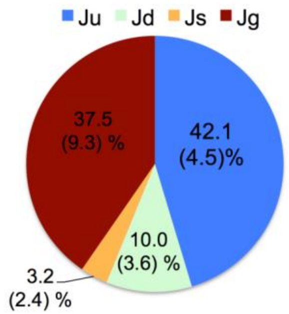
- **Chapter 1: Introduction**
- **Chapter 2: Theoretical Framework**
- **Chapter 3: Numerical result of Quark Spin and Discussion**
- **Chapter 4: Conclusion and Outlook**

Context

1. Introduction: Intrinsic Quark Spin



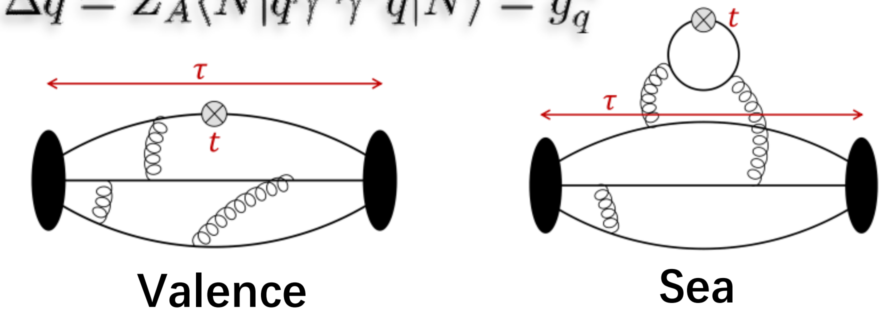
Ji's sum rule: $$\frac{1}{2} = \sum_{q=u,d,s,\cdots} (\frac{1}{2}\Delta q + L_q) + J_g$$ *X.D. Ji, PRL **78**, 610(1997)*



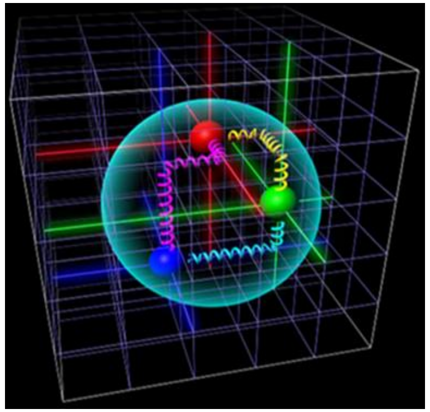
*Nature Rev.Phys. **3**, 27-38(2021)*

- Total intrinsic quark spin contribute ~30-40% to nucleon;

$$\Delta q = Z_A \langle N|\bar{q}\gamma^i\gamma^5 q|N\rangle = g_q^A$$

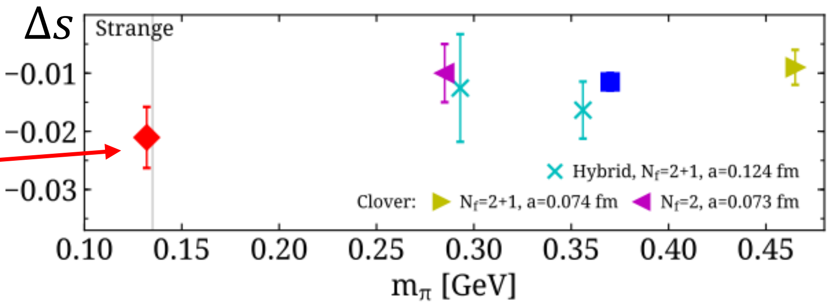


✓ Lattice QCD provides a first-principle, non-perturbative description of the strong interaction. *K. G. Wilson, P.R.D. **10**, 8 (1974);*

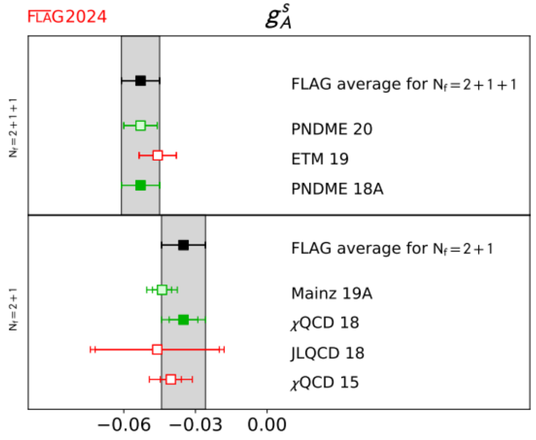


- LQCD faces large challenge on calculate sea quark contribution(Quark disconnected insertion);

$$n_{cfg} = 2153$$
$$n_{src} = 100$$
$$n_{stoc} = 1087$$



*C. Alexandrou et al., PRL **119**, 142002 (2017)*

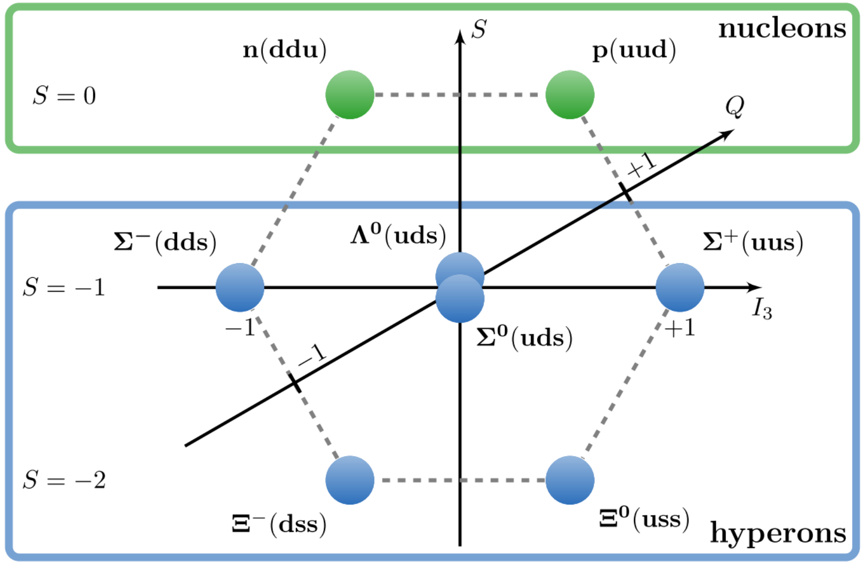


*PNDME, PRD **98**, 094512 (2018)*

*χQCD, PRD **98**, 074505 (2018)*

Mainz, arXiv 1911.01177 (2019)

1. Introduction: Flavour-singlet Quark Spin for Baryon Octet and SU(3) symmetry

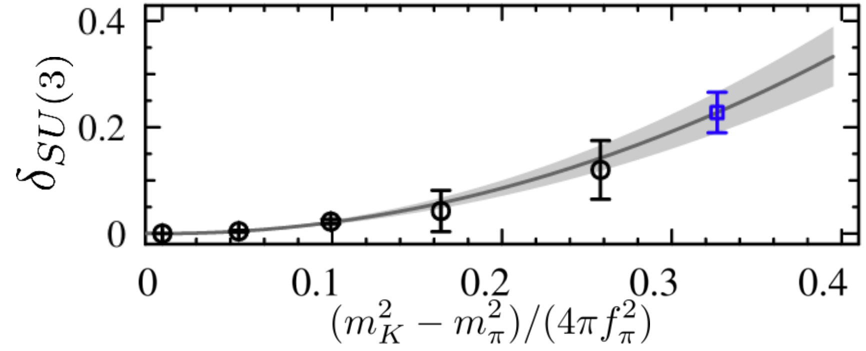


$$m_u^{\overline{\mathrm{MS}}(2\mathrm{GeV})} \simeq 2.1\mathrm{MeV}$$
$$m_d^{\overline{\mathrm{MS}}(2\mathrm{GeV})} \simeq 4.7\mathrm{MeV}$$
$$m_s^{\overline{\mathrm{MS}}(2\mathrm{GeV})} \simeq 93\mathrm{MeV}$$

FLAG(2024), arXiv:2411.04268

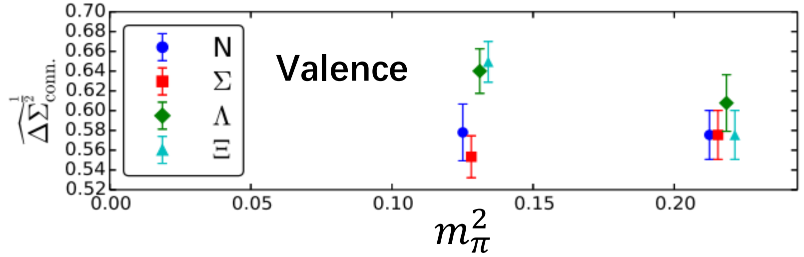
- There is moderate SU(3) symmetry breaking effects for the iso-vector axial charges, $\delta_{SU(3)}/g_A^n \sim 9\%$;

$$\delta_{SU(3)} = g^n_{A(u-d)} - g^{\Sigma}_{A(u-d)} + g^{\Xi}_{A(u-d)}$$



*H.W. Lin et al, PRD **79**, 034507 (2009)*
*A. Savanur, PRD **102**, 014501 (2020)*
*RQCD, PRD **108**, 034512 (2023)*

- How does SU(3) flavour symmetry and breaking affect Hyperons Spin Structure? *A. J. Chambers, P.R.D **90**, 014510 (2014)*



✓ In this study we use a novel ***Blending method*** to study **"Quark Spin in Octet Baryons"** (including sea quark contribution) at 3 ensembles (include 2 pion mass and 2 lattice spacing).

Context

2: Theoretical Framework: Blending Method

- *Distillation* can significantly improve the signal-to-noise ratio and suppress excited-state contamination;

$$-\nabla^2|\phi\rangle = \lambda|\phi\rangle,$$

$$L = \{|\lambda\rangle; -\nabla^2|\phi\rangle = \lambda|\phi\rangle\}.$$

PRD. ***80****, 054506 (2009)*

- Distillation space $\mathcal{L}_1$ is not complete, resulting in bias of local current ME;

- *Blending* method(BLD) can provide an unbiased estimation of identity matrix, based on samples in $\mathcal{L}_2$;

Z.C. Hu et al. , arXiv: 2505.01719(2025)

✓ All-to-All fermion Propagator could be stored and reused.

$$S_{ij}(t_1,t_2) = \int \mathrm{d}^3x\mathrm{d}^3y\langle\phi_i(\vec{x},t_1)|S(\vec{x},t_1;\vec{y},t_2)|\phi_j(\vec{y},t_2)\rangle,$$

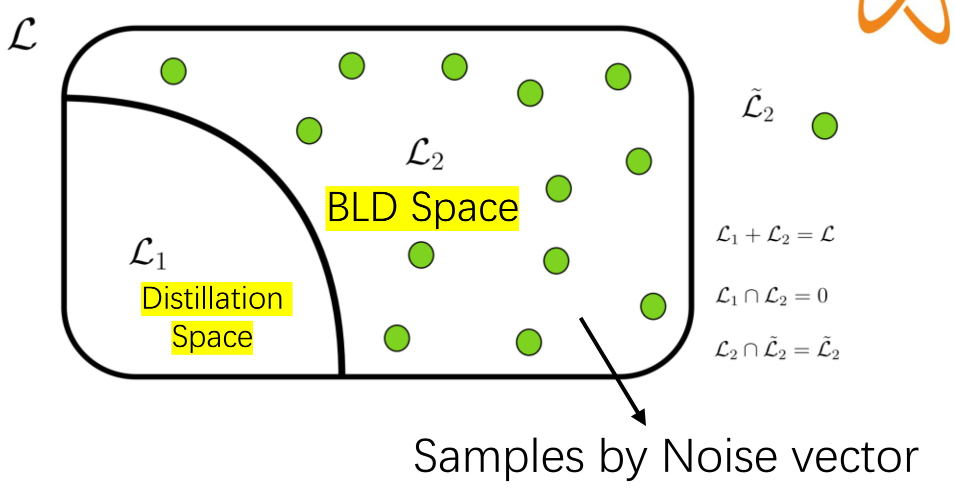
All-to-all propagator size: $(4N_cN_TN_V)^2$ ✗ Unacceptable!

1. Vectors → $(N_cN_TN_V)(N_e+N_{st})$

2.Perambulator → $[4N_T(N_e+N_{st})]^2$

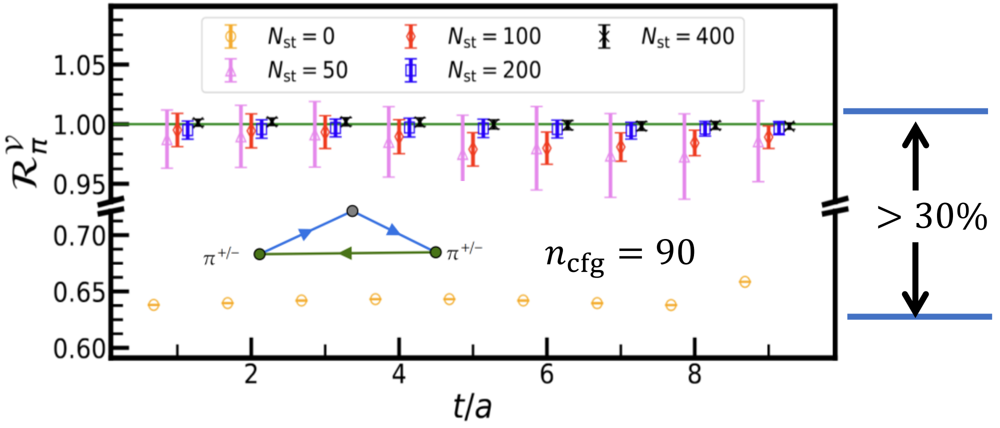
$$\frac{N_e+N_{st}}{[\mathcal{L}]} \approx 1\%$$

✓ Acceptable!



Conserved Current Test:

$$\langle H|V^c|H\rangle/\langle H|H\rangle = 1$$

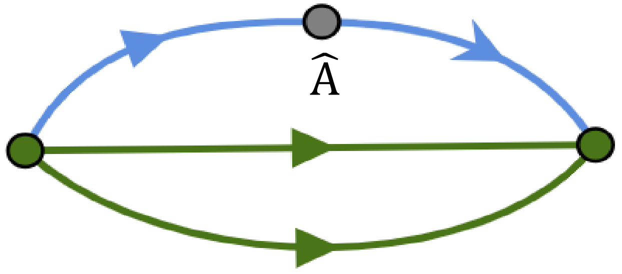


✓ $\langle\pi^+|V^c|\pi^+\rangle = 1.0002(29)$ with $N_{st} = 400$, $m_\pi \approx 290\text{MeV}$;

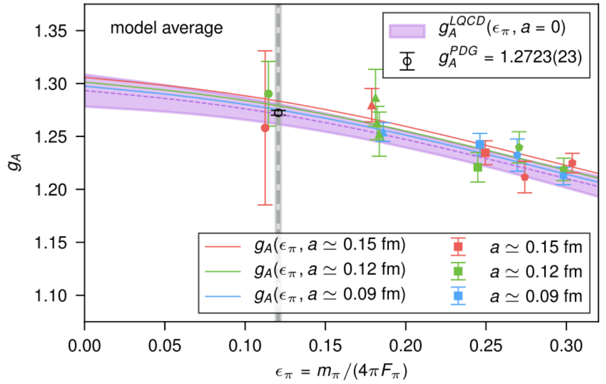
2: Theoretical Framework: Blending Method

◆ **Low-lying space of Laplace (Distillation) can get high precision nucleon external state without momentum;**

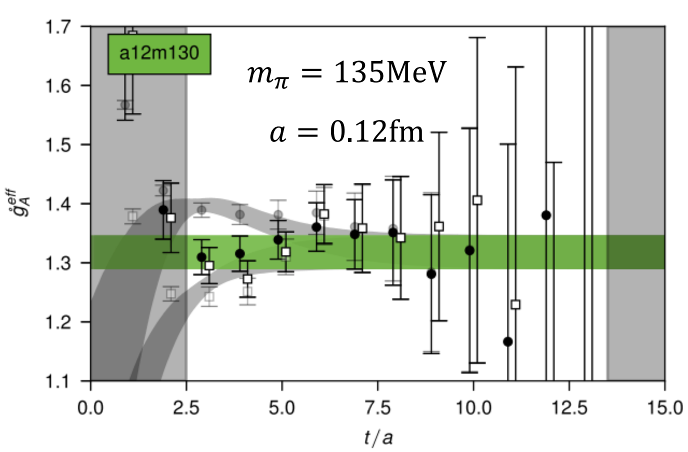
✓ **We observe substantially improved results at the physical point** ($m_\pi = 135\text{MeV}$);



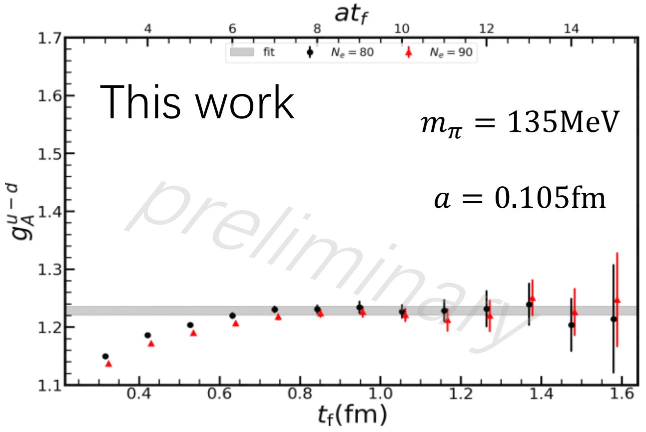
$$\langle P | \bar{u}\gamma^5\gamma^i u - \bar{d}\gamma^5\gamma^i d | P\rangle$$



Severe model dependent !!!



Nature 558,91 (2018)

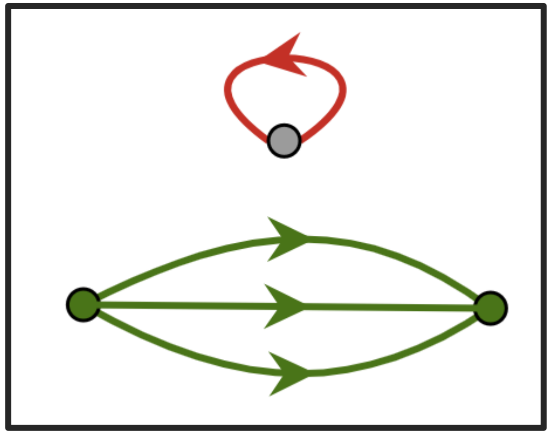


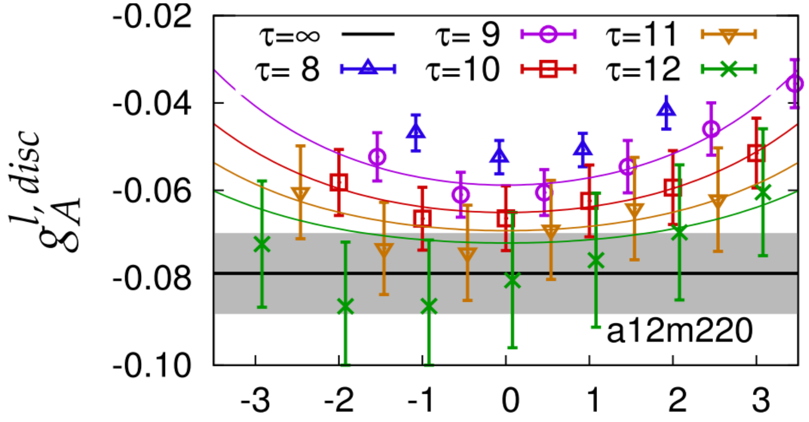
	Ensemble	n_{cfg}	g_A^{u-d}	$\delta x/\lvert x\rvert$
CLQCD(This work)	C48P14	46	1.221(07)	0.57%
CalLat	a15m130	1000	1.270(72)	5.66%
[Nature 558, 91(2018)]	a12m130	1000	1.292(30)	2.32%
RQCD *[PRD 108, 034512 (2023)]*	D452	1000	1.17(24)	20.5%

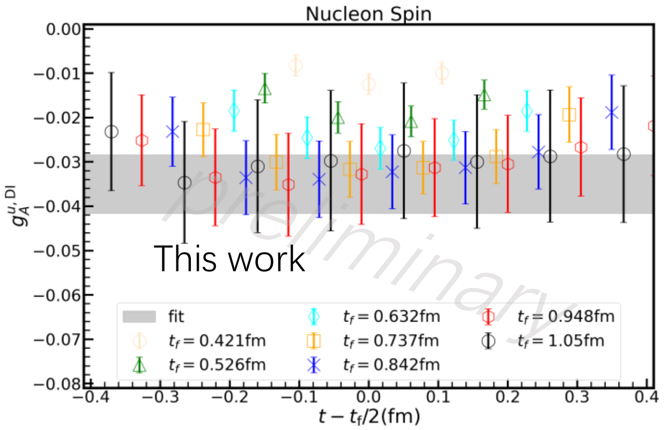
preliminary

✓ It is expected to achieve experimental-level precision of 0.1% with an internationally comparable level of statistics (e.g., 1000 configurations);

2: Theoretical Framework: Blending Method



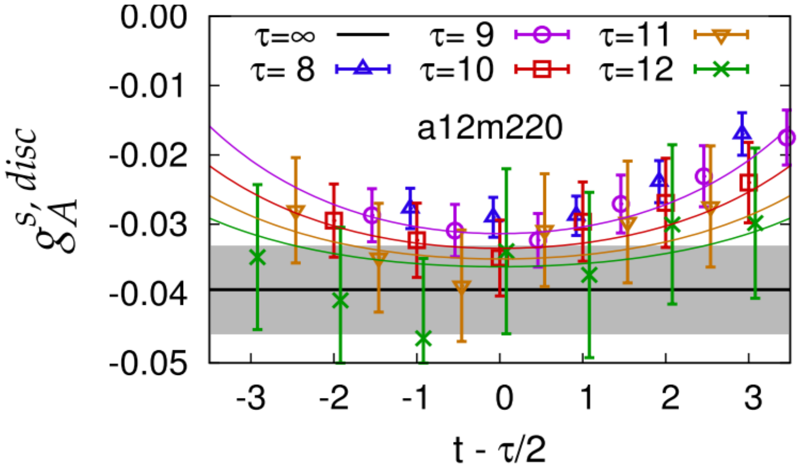


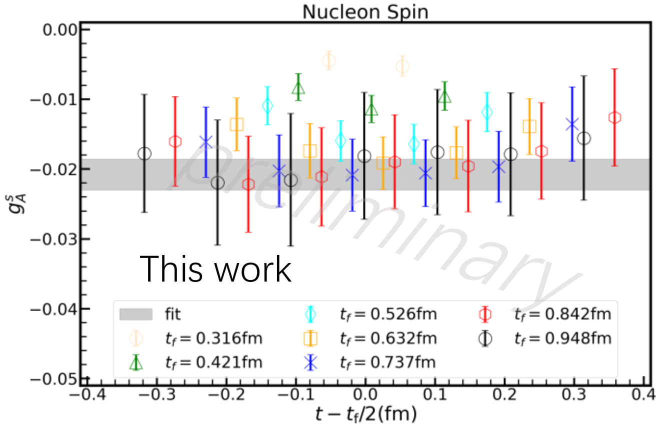


a) A better ground-state convergence!

b) The cost could be control very well.

c) Data of fermion Propagator could be stored and reused.





PNDME, P.R.D 98, 094512 (2018)

	Ensemble	m_π	$L^3 \times T$	n_{cfg}	n_{inv}	$g_A^{u,DI}$	$g_A^{s,DI}$
PNDME Collaboration	a12m220	228MeV	32x64	958	11000	-0.075(9)	-0.037(6)
CLQCD(This work)	C32P23	227MeV	32x64	80	3200*	-0.035(7)	-0.021(2)

➢ 2+1 flavor tadpole-improved **Clover Fermion** with Symanzik gauge action;

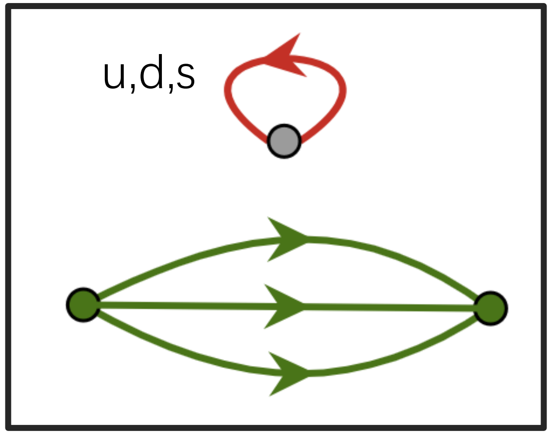
	a(fm)	m_π(MeV)	$L_x \times L_t$	$m_\pi L$	$n_{\rm cfg}$
C32P29	0.105	292	32x64	5.0	200
C32P23	0.105	227	32x64	3.9	80
F48P30	0.077	303	48x96	5.7	40

*CLQCD, PRD **109**, 054507 (2024)*
*CLQCD, PRD **111**, 054504 (2025)*

Context

3.1 Renormalization and Flavour mixing

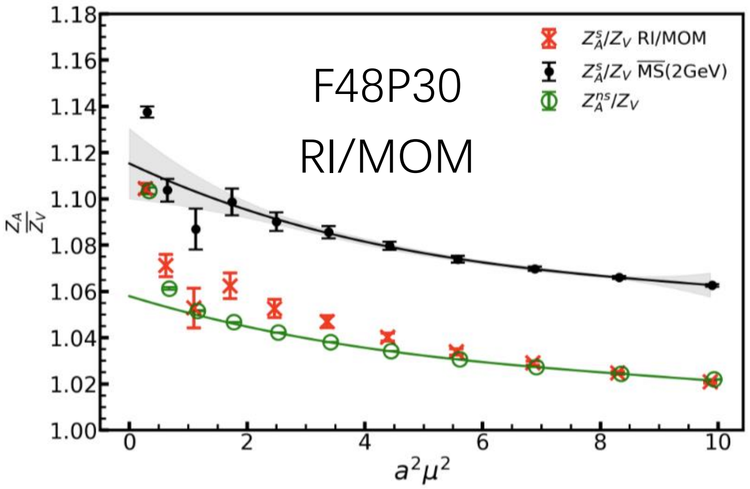
- Flavour Non-singlet and Singlet renormalized constant;

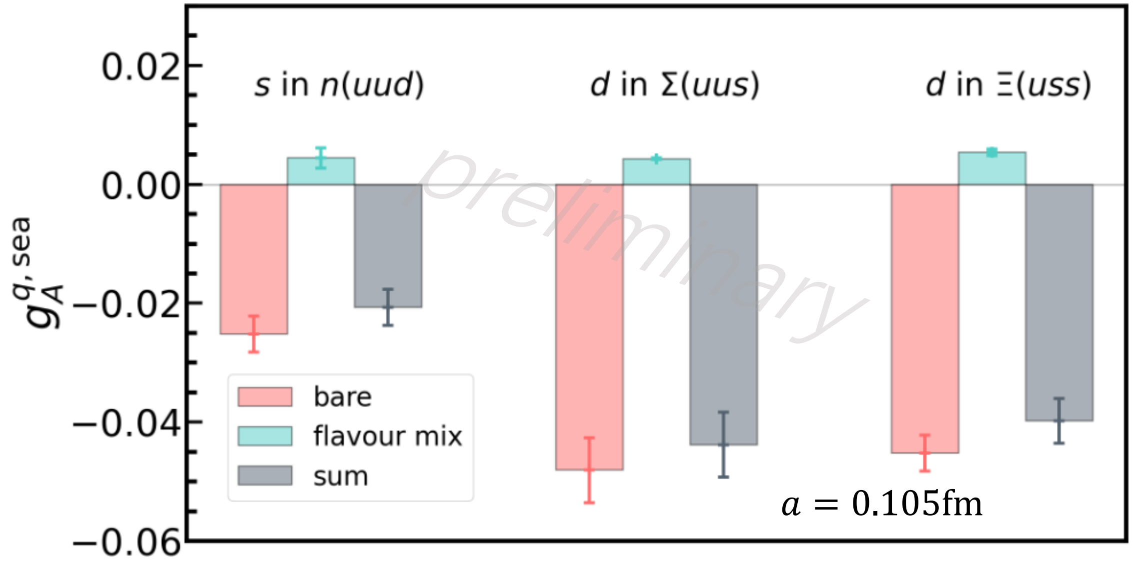


$$g_A^{q,\mathrm{bare}} \equiv \langle N|\bar{q}\gamma_i\gamma_5 q|N\rangle;$$

Flavor Mixing: $g_A^{q,\mathrm{R}} = Z_A^{\mathrm{ns}} g_A^{q,\mathrm{bare}} + \frac{Z_A^{\mathrm{s}} - Z_A^{\mathrm{ns}}}{N_f} \sum_{q=u,d,s} g_A^{q,\mathrm{bare}},$

A. J. Chambers et al. , PRD 92, 114517 (2015)



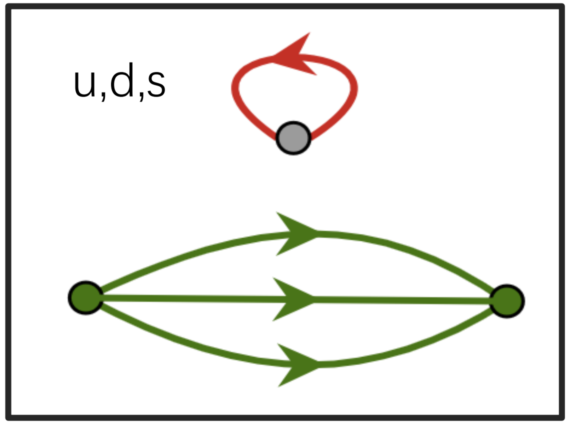


$m_\pi = 290\mathrm{MeV}$

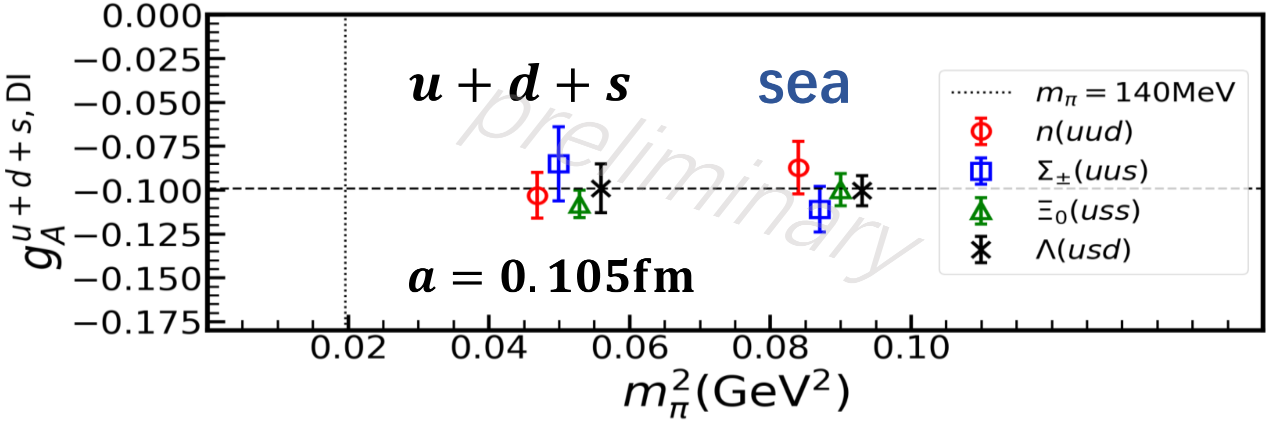
- Flavour mixing will lead to 1σ shift at current precision globally;

- $\delta g_A^{\mathrm{mix}} = \frac{Z_A^S - Z_A^{ns}}{Nf} \sum_q g_A^{q,B}$ couldn't be ignored for strange quark in nucleon, which has 20%~30% contribution;

3.2 Chiral behavior of Sea Quark Contribution to Spin

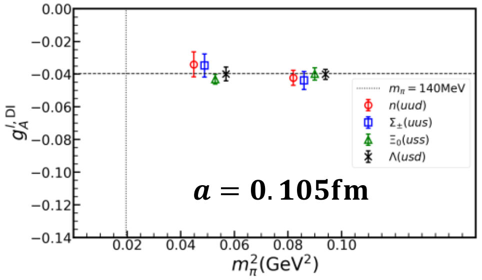


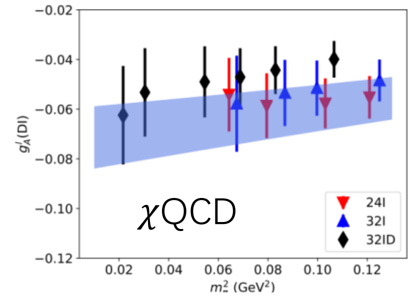
✓ The contribution of sea quarks is basically consistent in all octet baryons;

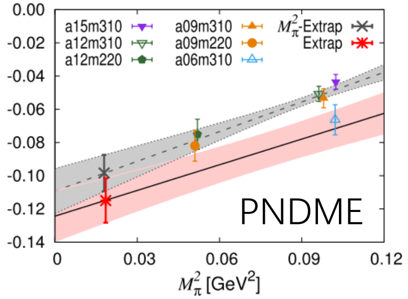


➢ We didn't see apparent m_π dependence at $m_\pi \approx 220\mathrm{MeV}$ and $m_\pi \approx 300\mathrm{MeV}$ for sea quark contribution;

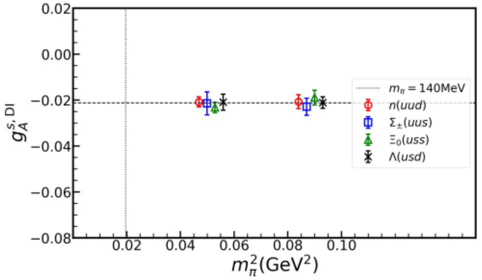
up/down:

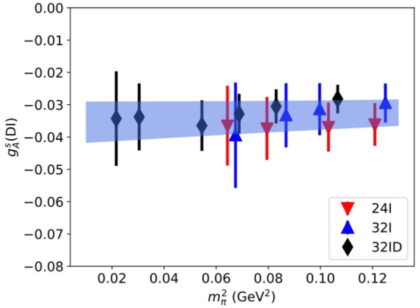


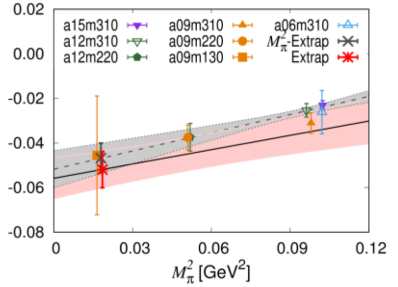




strange:







■ CLQCD result is more closed χQCD collaboration which is support for weaker m_π dependence.

PNDME, PRD ***98****, 094512 (2018)*

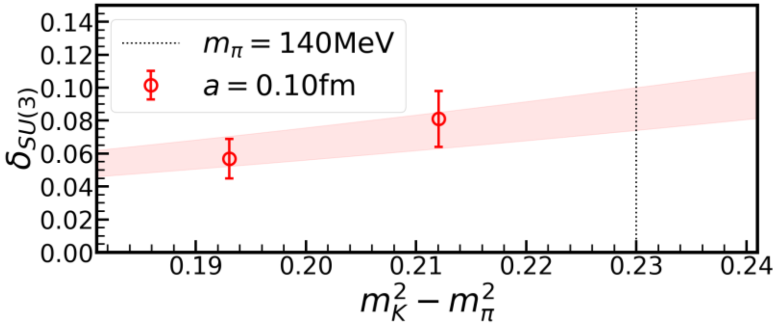
χQCD, PRD ***98****, 074505 (2018)*

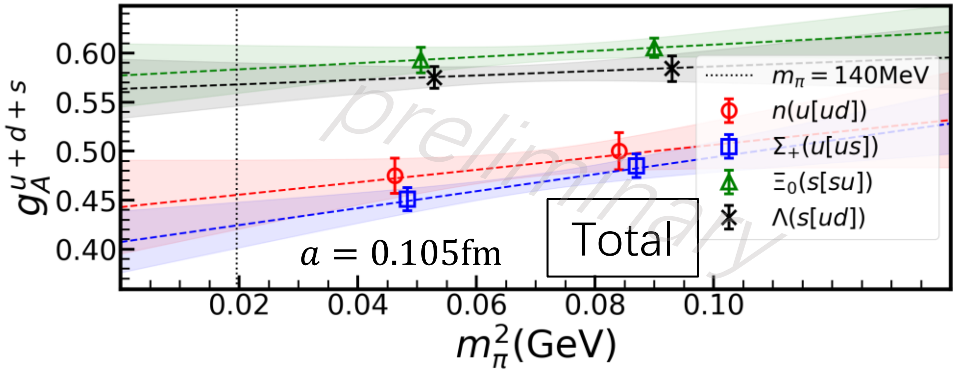
3.3 The total singlet quark spin and SU(3)-flavour symmetry

up/down dominant
$$n = u(u^T C\gamma^5 d)$$
$$\Sigma^+ = u(u^T C\gamma^5 s)$$

strange dominant
$$\Xi_0 = s(s^T C\gamma^5 u)$$
$$\Lambda_0 = 2s[u^T d] + d[u^T s] - u[d^T s]$$

$$\delta_{SU(3)} = g^n_{A(u-d)} - g^{\Sigma}_{A(u-d)} + g^{\Xi}_{A(u-d)}$$

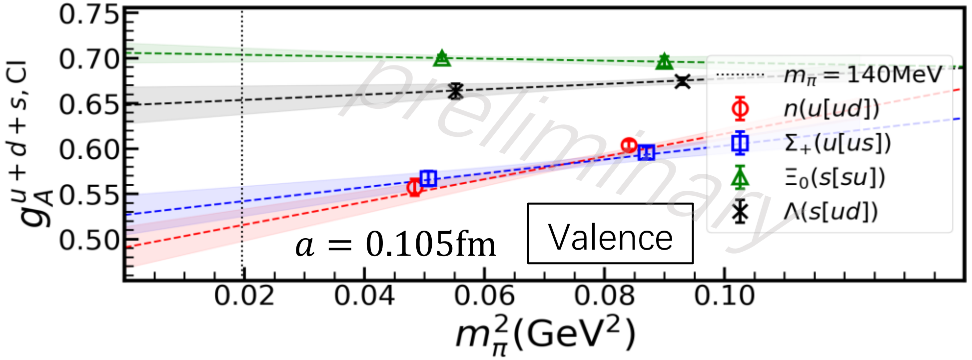




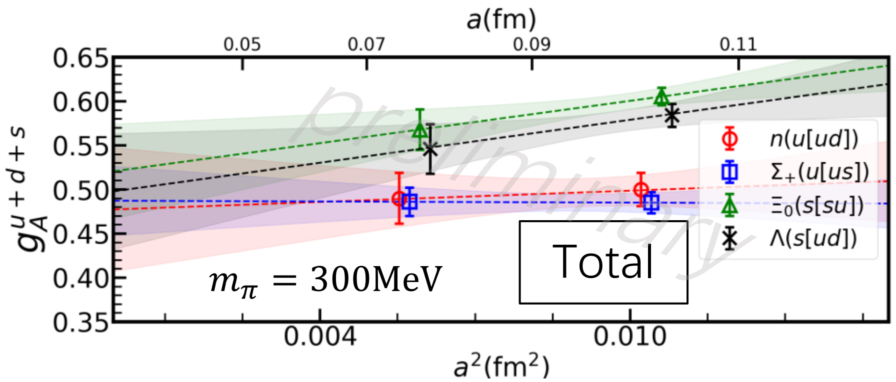
◆ SU(3) symmetry keep well inside di-quark operator;

$$n(u[ud]) \underset{\mathrm{d}}{\overset{\mathrm{s}}{\rightleftarrows}} \Sigma(u[us])\,;\ \Xi(s[su]) \underset{\mathrm{s}}{\overset{\mathrm{d}}{\rightleftarrows}} \Lambda(s[ud])$$

◆ Connected Diagram has larger influence for SU(3) symmetry breaking on 20% level on quark spin.



◆ After continuum extrapolation, there is a tendency for the SU(3) symmetry to partially recover.



Context

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Conclusion

1) The sea quark contribution is consistent in Octet baryons ranging $m_\pi \in [220,300]\mathrm{MeV}$;

2) SU(3)-flavour symmetry breaking has ~20% influence in valence quark helicity;

3) The symmetry is better preserved within specific diquark structures, as seen in the pairs like, (n, Σ) (Λ, Ξ);

4) There is a tendency for the SU(3) symmetry to partially recover after continuum extrapolation.

Outlook

1) The chiral behavior of sea quark helicity should be checked carefully;

2) More systematical computation in different lattice spacing such as $a = 0.077\mathrm{fm}$ and $a = 0.052\mathrm{fm}$ will be done in the near future;

3) Calculation directly at physical point is also in progress.

Thanks you!