Gravitational form factors in the perturbative limit

Qin-Tao Song (Zhengzhou University)

The 26th International Symposium on Spin Physics

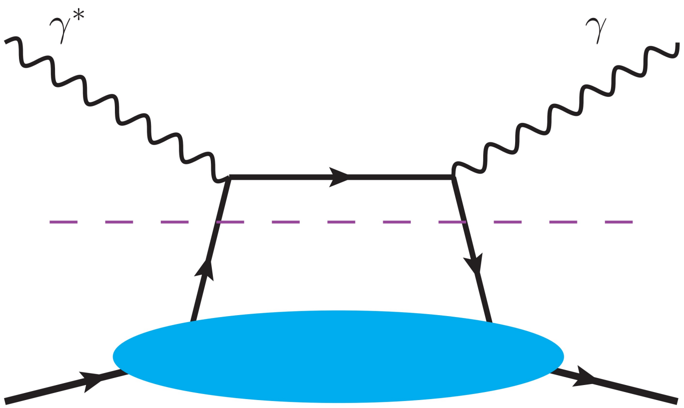
September 23, 2025, Qingdao, China

Reference: Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

Outline

- GPDs, GDAs and gravitational FFs
- GDAs and gravitational FFs in the perturbative limit: existence of a new gravitational FF, the Θ_3 term
- Exotic hybrid mesons in $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^* \gamma \to M_1 M_2$

GPDs and EMT



$\gamma^* + h \rightarrow \gamma + \mathrm{h}$

Deeply Virtual Compton Scattering (DVCS)

X.D. Ji, PRL 78(1997), 610.

GPDs can help us access the hadronic matrix elements of energy momentum tensor (EMT) indirectly.

Second moments of GPDs → Hadronic matrix elements of EMT →

- Proton spin puzzle
- EMT FFs of hadrons

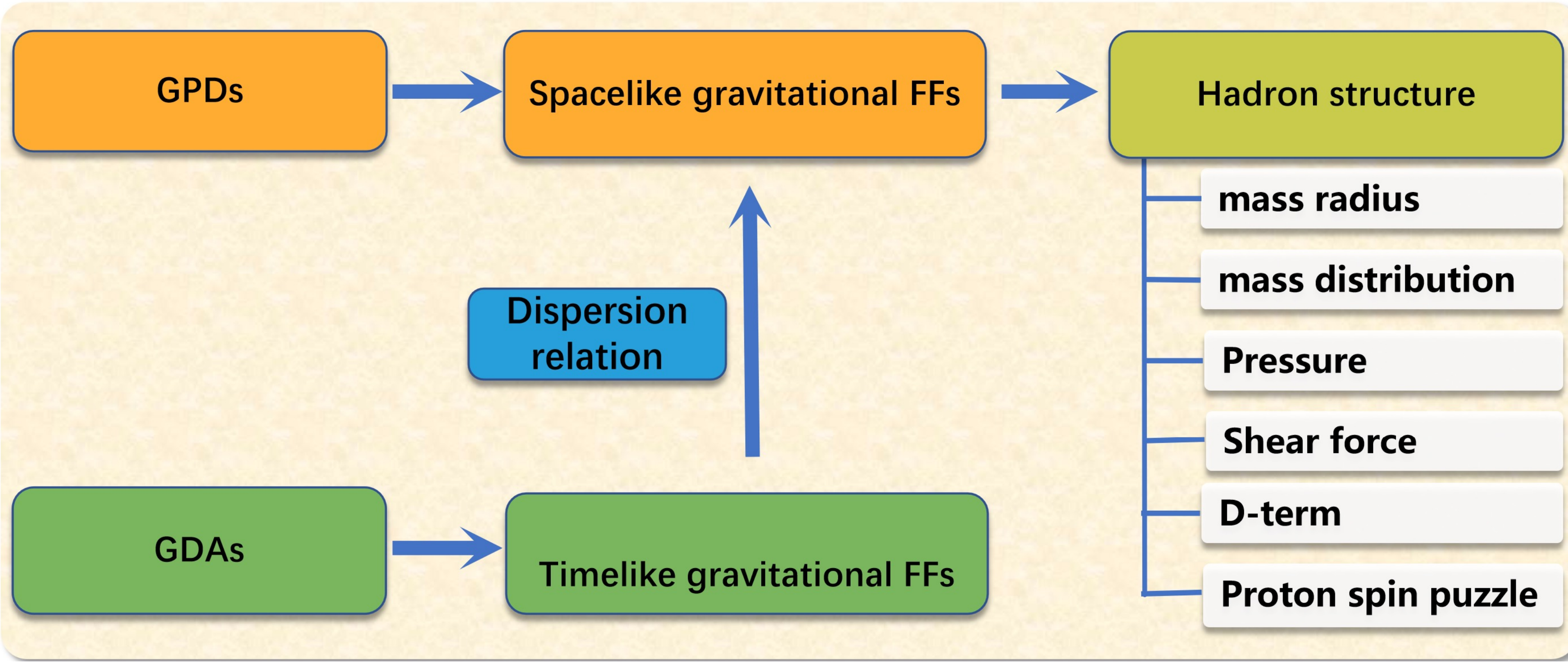
Most hadrons are not stable, so we can not use DVCS to study their GPDs. How to obtain EMT FFs for these unstable hadrons?

EMT FFs of unstable hadrons?

Generalized distribution amplitudes (GDAs) are the s-t crossed quantities of GPDs, the second moments of GDAs lead to timelike EMT FFs.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. V. Polyakov, NPB **555** (1999) 231.
S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

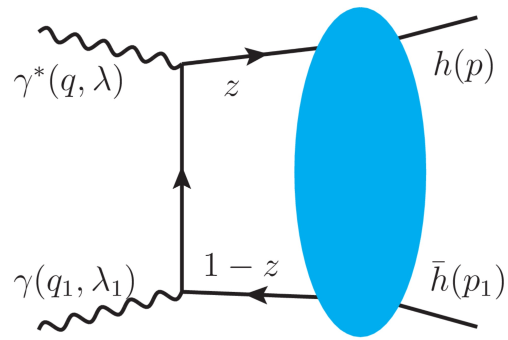
From GPDs and GDAs to hadron gravitational FFs:



M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025.
V. D. Burkert, L. Elouadrhiri, F. Girod, C. Lorce, P. Schweitzer and P. Shanahan, Rev. Mod. Phys. 95 (2023), 041002.

GDAs and timelike EMT FFs

GDAs in $\gamma^*\gamma \to h\bar{h}$:



QCD collinear factorization

Hard part: $\gamma^*\gamma \to q\bar{q}$

Soft part: $q\bar{q} \to h\bar{h}$, quark GDAs.

Quark GDA of a scalar hadron pair is defined as:

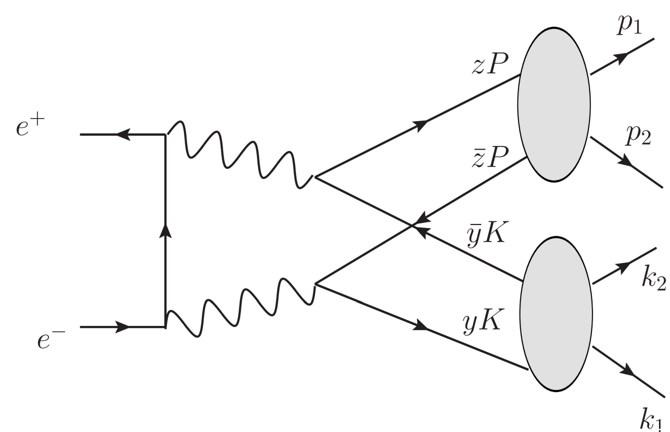
$$\Phi(z,\xi,s) = \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle h(p)\bar{h}(p_1) | \bar{q}(x^-)\gamma^+ q(0) | 0 \rangle$$

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030.

For chiral-odd two-meson GDAs, they can be accessed in $e^+e^- \to (\pi\pi)(\pi\pi)$.

Chiral-odd two-meson GDAs:



S. Bhattacharya, R. Boussarie, B. Pire and L.Szymanowski, arXiv:2507.23529.

5

GDAs and gravitational FFs

For a scalar meson pair $M_1 M_2$

$$\langle M_2(p_2)M_1(p_1)|T_q^{\mu\nu}|0\rangle = \frac{1}{2}[\Theta_1(s)(sg^{\mu\nu} - P^\mu P^\nu) + \Theta_2(s)\Delta^\mu\Delta^\nu + \Theta_3(s)P^{\{\mu}\Delta^{\nu\}}]$$

$$p_1 + p_2 = P, p_2 - p_1 = \Delta$$

The "shear viscosity" term Θ_3 does not exist for $\pi\pi$, however, it could exist for $\pi\eta, \pi\eta'$ and $\eta\eta'$. The Θ_3 term breaks the conservation law of EMT for each quark flavor in its hadronic matrix element, and its existence has not been verified.

H. Pagels, Phys. Rev. 144, 1250 (1966).
O. Teryaev, JPS Conf. Proc. 37(2022), 020406.

The Θ_3 term vanishes when we sum over the quark flavors and the gluon:

$$\sum_{i=q,g} \Theta_3^i(s) = 0.$$

The Θ_3 term is related to the P-wave GDAs:

$$\int_0^1 dz(2z-1)\Phi^q_{M_1M_2}(z,\xi,s) = \frac{2}{(P^+)^2}\langle M_1(p_2)M_1(p_1)\,|T_q^{++}(0)|\,0\rangle\,,$$

If the P-wave $M_1 M_2$ is observed in $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^*\gamma \to M_1 M_2$, it will verify the existence of the Θ_3 term by experiment.

How to investigate the Θ_3 term theoretically?

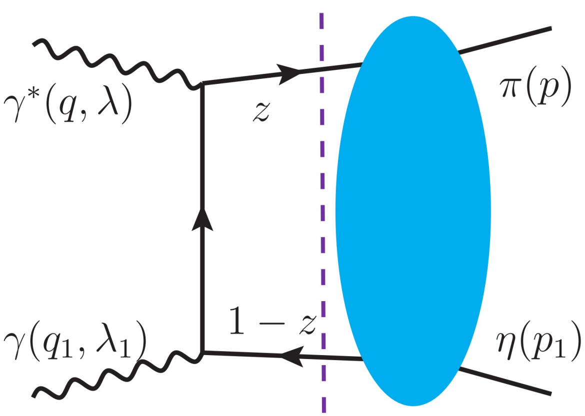
$$\gamma^*(q) + \gamma(q_1) \to \pi^+(p) + \pi^-(p_1) \qquad q^2 = -Q^2, \quad (q_1)^2 = 0,$$

In the perturbative limit : $Q^2 \gg s \gg \Lambda^2_{\rm QCD}$

The two-pion GDA can be expressed in terms of pion DAs, and meson DAs are relatively well-known quantities.

M. Diehl, T. Feldmann, P. Kroll and C. Vogt, PRD 61 (2000), 074029

We use $\pi\eta$ as an example to investigate the Θ_3 term, and try to express the pion-eta GDA in terms of meson DAs in the perturbative limit.



The amplitude of $\gamma^*\gamma \to \pi\eta$ is given by the pion-eta GDA.

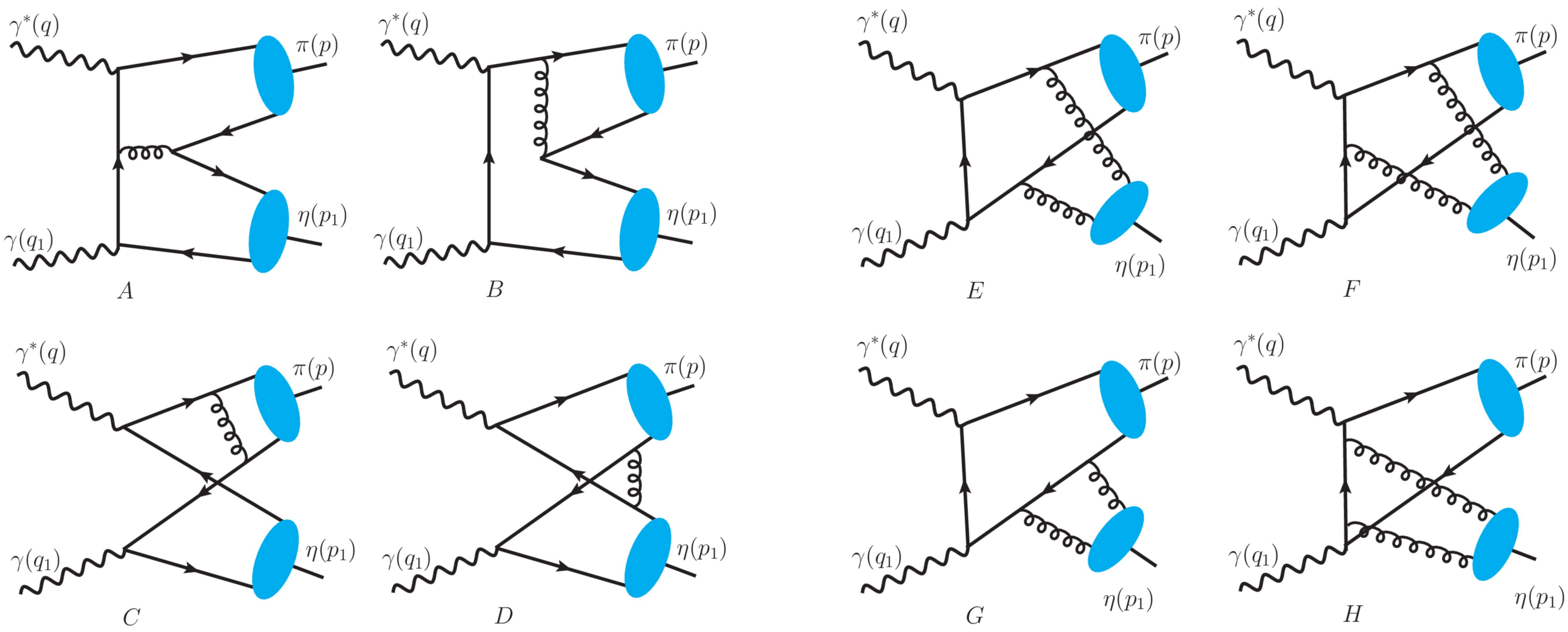
The QCD collinear factorization:

$$Q^2 \gg s, \Lambda^2_{\rm QCD}$$

The pion-eta GDA describes the amplitude from quark-antiquark pair to $\pi\eta$.

Helicity amplitudes in the perturbative limit

Feynman diagrams for $\gamma^*\gamma \to \pi\eta$ in perturbative limit:



Part 1: the amplitude is expressed in terms of quark DAs of mesons, similar as $\gamma^*\gamma \to \pi^+\pi^-$.

Part 2: the amplitude is expressed in terms of pion quark DA and eta gluon DA, which does not exist for $\gamma^*\gamma \to \pi^+\pi^-$.

GDAs in the perturbative limit

Part 1 contribution,
quark DAs of mesons:

$$\hat{\Phi}^q_{\pi\eta}(z,\xi,s)\Big|_{\text{quark DAs}} = -\frac{16\pi\alpha_s}{9}\left\{\theta(z-\xi)\frac{\bar{\xi}}{z-\xi}\int_0^1 \frac{dx}{s}\frac{z+\bar{x}\xi}{z-x\xi}\frac{\phi^q_\pi(x)}{\bar{x}}\phi^q_\eta(\tfrac{\bar{z}}{\bar{\xi}}) + \theta(\xi-z)\frac{\xi}{z-\xi}\int_0^1 \frac{dx}{s}\frac{\bar{z}+\bar{x}\bar{\xi}}{\bar{z}-x\bar{\xi}}\frac{\phi^q_\eta(x)}{\bar{x}}\phi^q_\pi(\tfrac{z}{\xi})\right\}$$

Part 2 contribution,
pion quark DA and eta gluon DA:

$$\hat{\Phi}^q_{\pi\eta}(z,\xi,s)\Big|_{\text{quark}-\text{gluon DAs}} = \frac{4\pi\alpha_s}{9\sqrt{3}}\frac{\xi}{s}\left\{\int_{S_1}\frac{dy}{\bar{y}y}\frac{y^2-(2y-1)z+\xi\bar{y}y}{(z-y-\xi\bar{y})(z-\bar{\xi}y)}\phi^g_\eta(y)\phi^q_\pi(x) - \int_0^1 dy\frac{\bar{y}}{y}\left[\frac{\theta(\xi-z)}{z-\xi}\phi^q_\pi(\tfrac{z}{\xi}) - \frac{\theta(\xi-\bar{z})}{\bar{z}-\xi}\phi^q_\pi(\tfrac{\bar{z}}{\xi})\right]\phi^g_\eta(y)\right\}$$

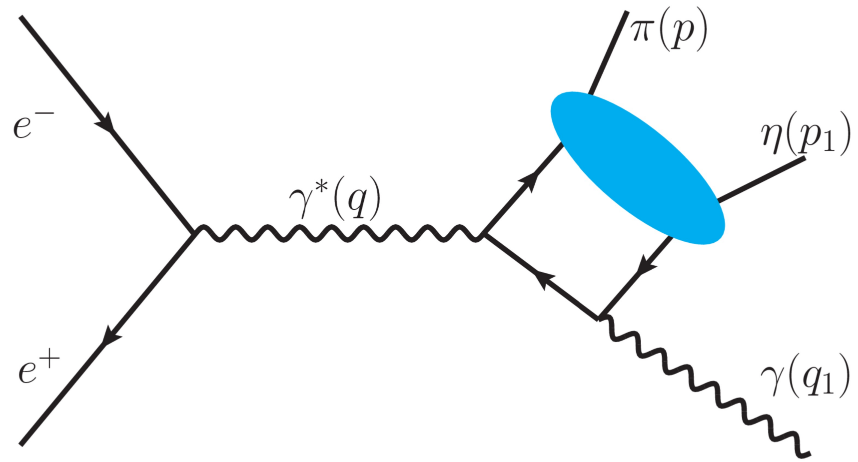
GDAs in the perturbative limit:

$$\hat{\Phi}^q_{\pi\eta}(z,\xi,s) = \hat{\Phi}^q_{\pi\eta}(z,\xi,s)\Big|_{\text{quark DAs}} + \hat{\Phi}^q_{\pi\eta}(z,\xi,s)\Big|_{\text{quark}-\text{gluon DAs}}$$

Universality of GDAs

At current stage, there are no experimental facilities to measure $\gamma^*\gamma \to \pi\eta$ in the perturbative limit, and the obtained GDAs cannot be tested by experiment.

The process $\gamma^* \to \pi + \eta + \gamma$ can be measured at Belle II in the perturbative limit.



$$Q^2 \gg \hat{s} \gg \Lambda^2_{\text{QCD}}$$

Z. Lu and I. Schmidt, PRD 73 (2006), 094021
B. Pire and Qin-Tao Song, PRD 107 (2023), 114014
B. Pire and Qin-Tao Song, PRD 109 (2024), 074016.

$\gamma^* \to \pi + \eta + \gamma$: timelike photon
$\gamma^* + \gamma \to \pi + \eta$: spacelike photon

→ Test the universality of GDAs

D. Mueller, B. Pire, L. Szymanowski and J. Wagner, RD 86 (2012), 031502.

After a similar calculation, we find the GDAs are identical for these two processes, which verifies the universality of GDAs

Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

From the obtained GDAs to gravitational FFs

$$\int_0^1 dz(2z-1)\Phi^q_{\pi\eta}(z,\xi,s) = \frac{2}{(P^+)^2}\left\langle \eta(p_1)\pi(p)\left|T_q^{++}(0)\right|0\right\rangle$$

The first moment of the GDA

$$\left\langle \eta(p_1)\pi(p)\left|T_q^{\mu\nu}(0)\right|0\right\rangle = \frac{1}{2}\left[\Theta^q_1(s)(sg^{\mu\nu} - P^\mu P^\nu) + \Theta^q_2(s)\Delta^\mu\Delta^\nu + \Theta^q_3(s)P^{\{\mu}\Delta^{\nu\}}\right].$$

The Θ_3 term does not exist for $\pi\pi$, which breaks the conservation law of EMT for each quark flavor in its hadronic matrix element.

The gravitation FFs are expressed in terms of meson DAs

$$\Theta^q_1 = -\frac{c}{s}\int dxdy\left[\frac{1+\bar{x}+y}{\bar{x}y}\phi^q_\eta(y) + \frac{\tilde{c}y}{\bar{x}x}\phi^g_\eta(y)\right]\phi^q_\pi(x),$$

$$\Theta^q_2 = -\frac{c}{s}\int dxdy\left[\frac{1+x+\bar{y}}{\bar{x}y}\phi^q_\eta(y) - \frac{\tilde{c}y}{\bar{x}x}\phi^g_\eta(y)\right]\phi^q_\pi(x),$$

$$\Theta^q_3 = \frac{2c}{s}\int dxdy\left[\frac{x-\bar{y}}{\bar{x}y}\phi^q_\eta(y) + \frac{\tilde{c}y}{\bar{x}x}\phi^g_\eta(y)\right]\phi^q_\pi(x),$$

Gravitational FFs in the perturbative limit

Isospin symmetry: $\Theta_3^u(s) = -\Theta_3^d(s)$

The Θ_3 term vanishes when summing over quark flavors, and the conserved hadronic matrix elements of EMT is recovered.

We need to use the meson DAs for a single flavor q.

$$\phi_M^{u,d}(z) = 6 f_M^{u,d} z\bar{z} \sum_{i=0} a_{2i}^M C_{2i}^{3/2}(2z-1), \qquad M = \pi, \eta, \eta'$$

$$\phi_{\eta^{(\prime)}}^s(z) = 6 f_{\eta^{(\prime)}}^s z\bar{z} \sum_{i=0} \tilde{a}_{2i}^{\eta^{(\prime)}} C_{2i}^{3/2}(2z-1), \qquad \text{for } \eta \text{ and } \eta'$$

Gegenbauer coefficients

Flavor u and d in π: a_{2i}^π does not mix with gluon DA due to isospin.

Flavor u, d and s in η: a_{2i}^η and $\tilde{a}_{2i}^\eta$ contain both SU(3) flavor-singlet and flavor-octet components.

DAs of mesons

SU(3) flavor-singlet and flavor-octet DAs in η

$$\phi^1_{\eta^{(\prime)}}(z) = 6 f^1_{\eta^{(\prime)}} z\bar{z} \sum_{i=0} \bar{a}_{2i} C^{3/2}_{2i}(2z-1),$$

$$\phi^8_{\eta^{(\prime)}}(z) = 6 f^8_{\eta^{(\prime)}} z\bar{z} \sum_{i=0} \hat{a}_{2i} C^{3/2}_{2i}(2z-1).$$

The relations among the Gegenbauer coefficients are given by

Flavor u, and d : $$a^{\eta^{(\prime)}}_{2i} = \frac{\sqrt{2} f^1_{\eta^{(\prime)}} \bar{a}_{2i} + f^8_{\eta^{(\prime)}} \hat{a}_{2i}}{\sqrt{2} f^1_{\eta^{(\prime)}} + f^8_{\eta^{(\prime)}}},$$

Flavor s : $$\tilde{a}^{\eta^{(\prime)}}_{2i} = \frac{f^1_{\eta^{(\prime)}} \bar{a}_{2i} - \sqrt{2} f^8_{\eta^{(\prime)}} \hat{a}_{2i}}{f^1_{\eta^{(\prime)}} - \sqrt{2} f^8_{\eta^{(\prime)}}}.$$

The gluon DA of is also expressed in terms of Gegenbauer polynomials.

$$\Psi^g_{\eta^{(\prime)}}(z) = f^1_{\eta^{(\prime)}} z^2(1-z)^2 \sum_{i=1} b_{2i} C^{5/2}_{2i-1}(2z-1),$$

The flavor-singlet DA mixes with gluon DA under the evolution.

Existence of the Θ_3 term

We substitute the meson DAs into these EMT FFs.

$$\Theta_1^q = -\frac{c f_\pi^q}{2s}\left\{6\left[5+4(a_2^\pi + a_2^\eta)+3a_2^\pi a_2^\eta\right] f_\eta^q + \tilde{c}(1+a_2^\pi) b_2 f_\eta^1\right\},$$

$$\Theta_2^q = -\frac{c f_\pi^q}{2s}\left\{6\left[7+8(a_2^\pi + a_2^\eta)+9a_2^\pi a_2^\eta\right] f_\eta^q - \tilde{c}(1+a_2^\pi) b_2 f_\eta^1\right\},$$

$$\Theta_3^q = \frac{c f_\pi^q}{s}\sum_{i=1}\left[6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1+\sum_{j=1} a_{2j}^\pi) b_{2i} f_\eta^1\right],$$

The first term arises only when the quark DA of the pion meson differs from that of the eta meson.

The second term will be nonzero provided that the gluon DA does not vanish.

$$b_{2i}^\eta \neq 0$$

The evolution equations are different for a_{2i}^π and a_{2i}^η, so their DAs can not be same. Thus, the existence of the Θ_3 term seems quite plausible for the $\pi\eta$ and $\pi\eta'$ pairs

Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

Numerical estimate of Gegenbauer coefficients

$$\Theta_3^q = \frac{c f_\pi^q}{s} \sum_{i=1} \left[6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1 + \sum_{j=1} a_{2j}^\pi) b_{2i}^\eta f_\eta^1 \right]$$

$$a_2^\pi \sim 0.16$$

I. Cloet, L. Chang, C.D.Roberts,et. al.,PRL111(2013), 092001.
J. Hua et. al., [Lattice Parton], PRL 129(2022) no.13, 132001.
T. Zhong, Z.H.Zhu, H.B.Fu et. al., PRD 104(2021), 016021.
C. Shi, M. Li, X. Chen and W.Jia,PRD 104(2021), 094016.
X. Gao, et. al., PRD 106(2022), 074505.
…

$$\mu_F = \sqrt{30}\ \text{GeV}$$

$$a_2^\eta \sim -0.03$$

P. Kroll and K. Passek-Kumericki, J. Phys. G 40 (2013), 075005.

The evolution effects are included.

The gluon DA: $b_2^\eta \neq 0$

Yeo-Yie Charng, T. Kurimoto, and Hsiang-nan Li,PRD 74(2006), 074024.
S. Agaev, V. Braun, N. Offen, A. Porkert and A. Schäfer, PRD 90(2014), 074019.
P. Kroll and K. Passek-Kumericki,PRD 67(2003), 054017.

For the $\eta\eta'$ pair

$$\Theta_1^q|_{\eta'\eta} = -\frac{c}{s}\int dxdy \frac{1+\bar{x}+y}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) - \frac{c\tilde{c}}{s}\int dxdy\left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) + \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

$$\Theta_2^q|_{\eta'\eta} = -\frac{c}{s}\int dxdy \frac{1+x+\bar{y}}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) + \frac{c\tilde{c}}{s}\int dxdy\left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) + \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

$$\Theta_3^q|_{\eta'\eta} = \frac{2c}{s}\int dxdy \frac{x-\bar{y}}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) + \frac{2c\tilde{c}}{s}\int dxdy\left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) - \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

Exotic hybrid mesons and P-wave GDA

The Θ_3 term is related to the P-wave GDA. Thus, we can search for the exotic mesons from the P-wave $M_1 M_2$ in $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^* \gamma \to M_1 M_2$.

$M_1 M_2$: $\pi\eta, \pi\eta'$

Isovector hybrid mesons

$I^G(J^{PC}) = 1^-(1^{-+})$

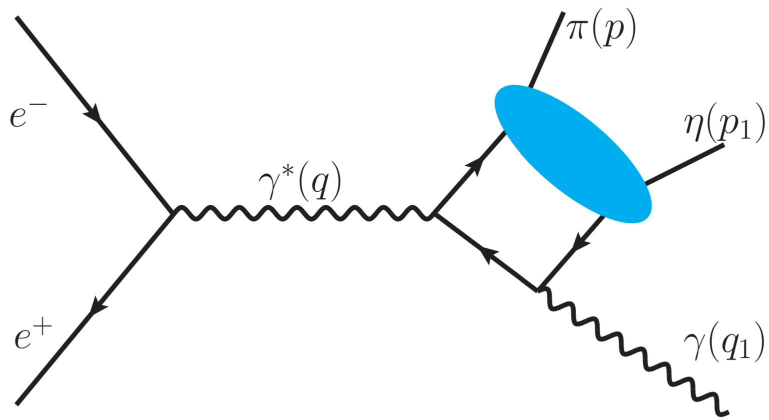
$\pi_1(1400), \pi_1(1600)$

$M_1 M_2$: $\eta\eta'$

Isoscalar hybrid mesons

$I^G(J^{PC}) = 0^+(1^{-+})$

$\eta_1(1855)$



$\gamma^* \to \pi + \eta + \gamma$ at BESIII

The exotic quantum number ($J^{PC} = 1^{-+}$) does not exit in quark model.

$\eta_1(1855)$ was observed by BESIII in $J/\psi \to \eta + \eta' + \gamma$ recently.

M. Ablikim et al. [BESIII], PRL 129 (2022), 192002.
M. Ablikim et al. [BESIII], PRD 106 (2022), 072012.

$J/\psi \to \gamma^*$: $\gamma^* \to \eta + \eta' + \gamma$ can be also measured by BESIII.

B. Pire and Q. T. Song, PRD 107 (2023), 114014.

Summary

- GDAs can be used to investigate the EMT FFs of unstable hadrons.

- GDAs and gravitational FFs in the perturbative limit: existence of the Θ_3 term term and the universality of GDAs are verified.

- The Θ_3 term indicates P-wave GDAs, thus, it is promising to search for exotic hybrid mesons in $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^* \gamma \to M_1 M_2$.

Thank you very much