Nuclear Science Computing Center at CCNU

Probing Meson Structure via Lattice QCD: EMFF at high Q^2 and GPD

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EMFF at large Q^2 and GPD

- “QCD Predictions for Meson Electromagnetic Form Factors at High Momenta: Testing Factorization in Exclusive Processes,”

  HTD, **Xiang Gao(高翔)**, A. Hanlon, S. Mukherjee, P. Petreczky, **Qi Shi(施岐)**, S. Syritsyn, R. Zhang, Y. Zhao, Phys.Rev.Lett. 133 (2024) 18, arXiv: 2404.04412

- “Three-dimensional Imaging of Pion using Lattice QCD: Generalized Parton Distributions,”

  HTD, Xiang Gao(高翔), S. Mukherjee, P. Petreczky, **Qi Shi(施岐)**, S. Syritsyn and Y. Zhao, JHEP 02 (2025) 056, arXiv: 2407.03516

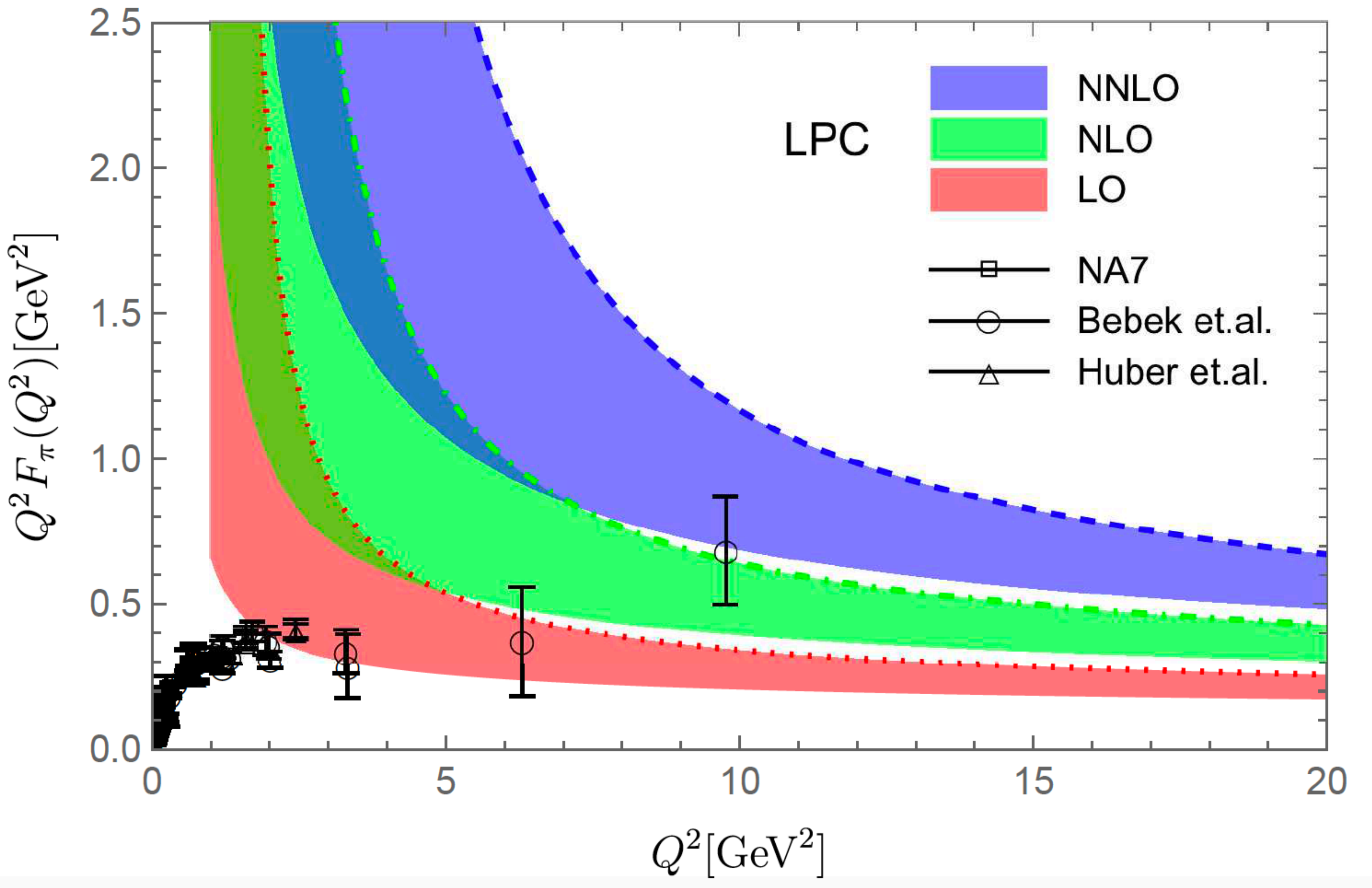
QCD factorization for hard exclusive processes

At leading twist, the collinear factorization of EM form factors

Farrar & Jackson, PRL 79'
Lepage & Brodsky, PRD 80'
Efremov & Radyushkin, PLB 80'

$$F_M(Q^2) = \int_0^1 \int_0^1 dxdy \ \phi_M^*(y, \mu_F^2) \ T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \ \phi_M(x, \mu_F^2)$$

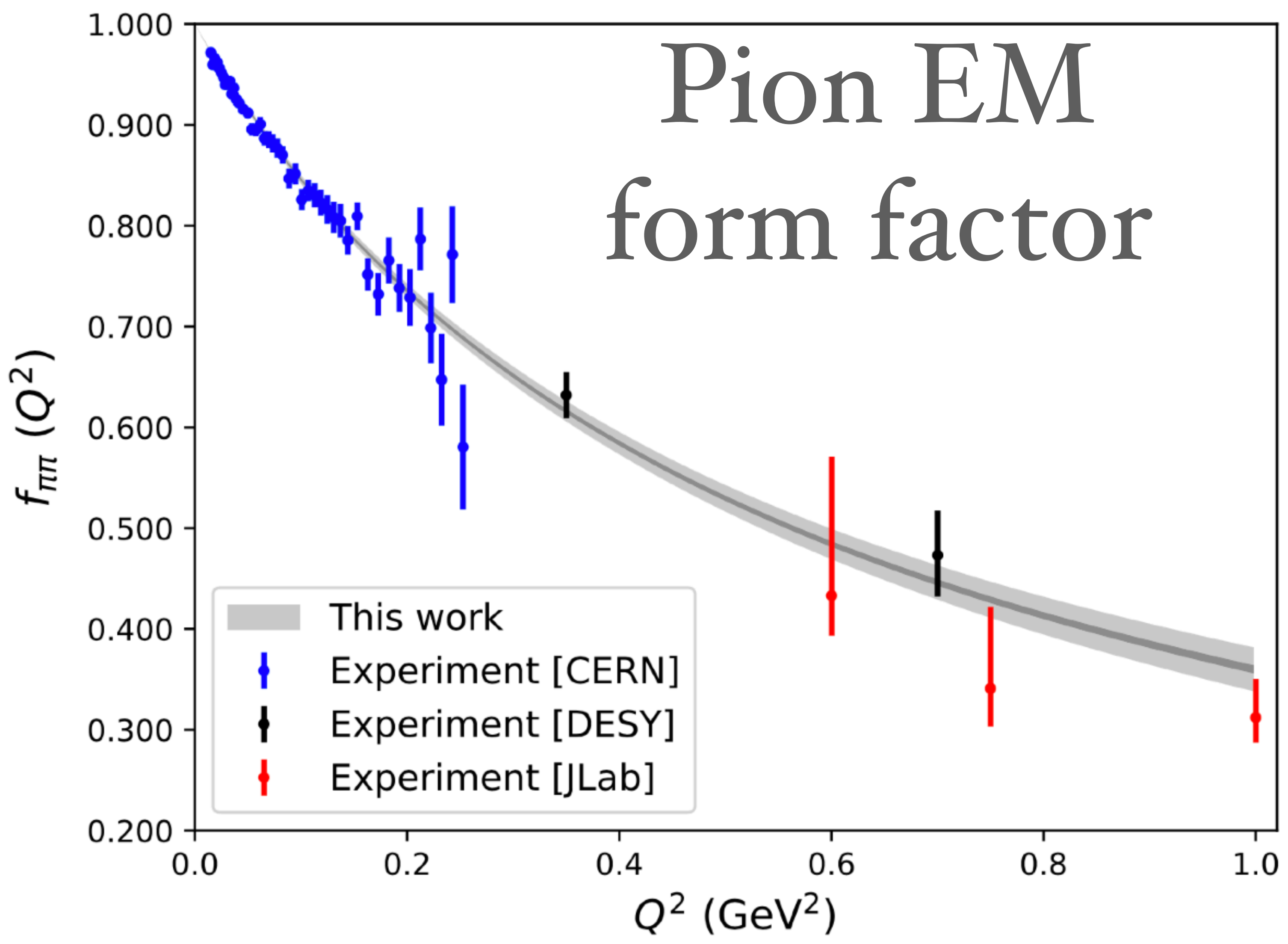
DA: Non-perturbative physics — Hard-process kernel obtained in pQCD — DA: Non-perturbative physics



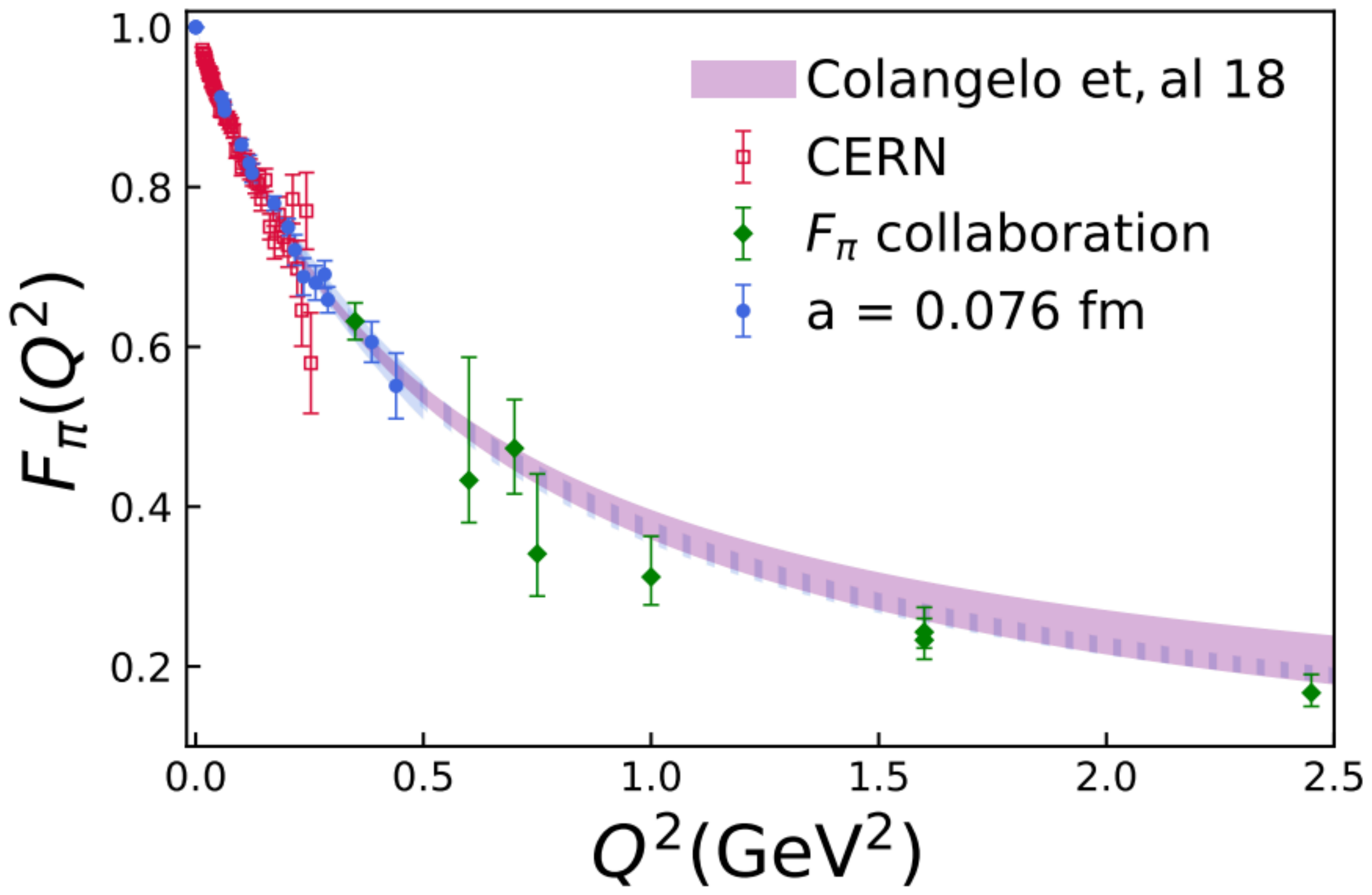
Chen, Chen, Feng & Jia, PRL 132 (2024) 20
Ji, Shi, Wang, Wang, Wang & Yu, PRL 134 (2025) 22, 221901

- NA7: $Q^2 \lesssim 0.25$ GeV^2, elastic scattering of pion from atomic electron — NPB 277 (1986)168
- Huber et al. (Jlab F_π collaboration): $Q^2 \lesssim 2.5$ GeV^2 — PRC 78 (2008) 045203
- Bebek et al. (Cornell): $Q^2 \lesssim 10$ GeV^2, large statistical and systematic uncertainties — PRD 17 (1978) 1693
- Jlab, EicC & EIC, Q^2 up to ~ 30 GeV^2 — white papers arXiv:2102.09222, 2103.05419

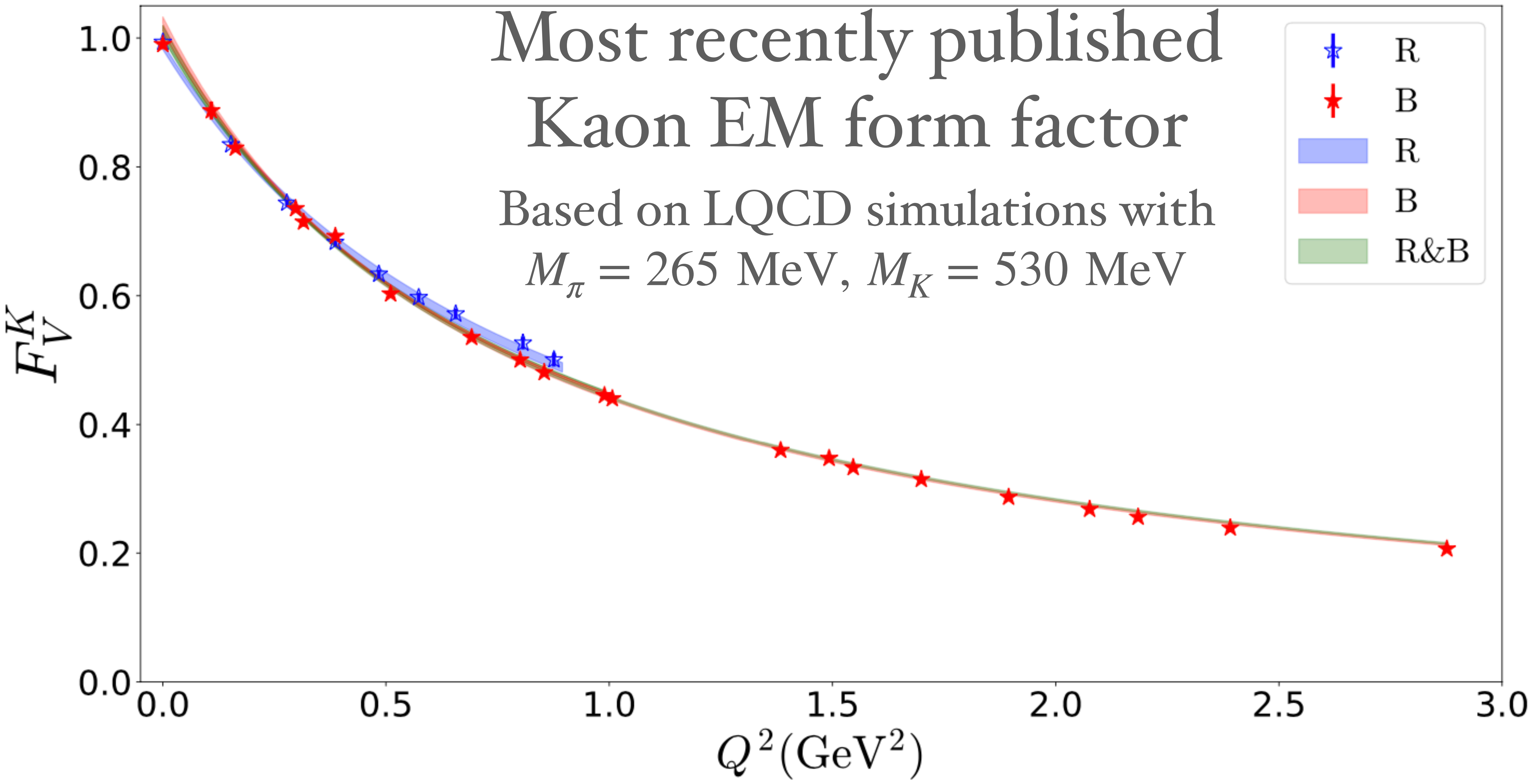
Current status: pion/kaon EM form factors from Lattice QCD



高翔 et al.Tsinghua-BNL-ANL, PRD 104 (2021) 114515



G. Wang et al,, [χQCD], PRD 104 (2021) 074502



Alexandrou et al., [ETMC], Phys.Rev.D 105 (2022) 5, 054502

Many computations on the pion form factor, but much less on kaon

Mostly restricted to $Q^2 \lesssim 3$ GeV^2

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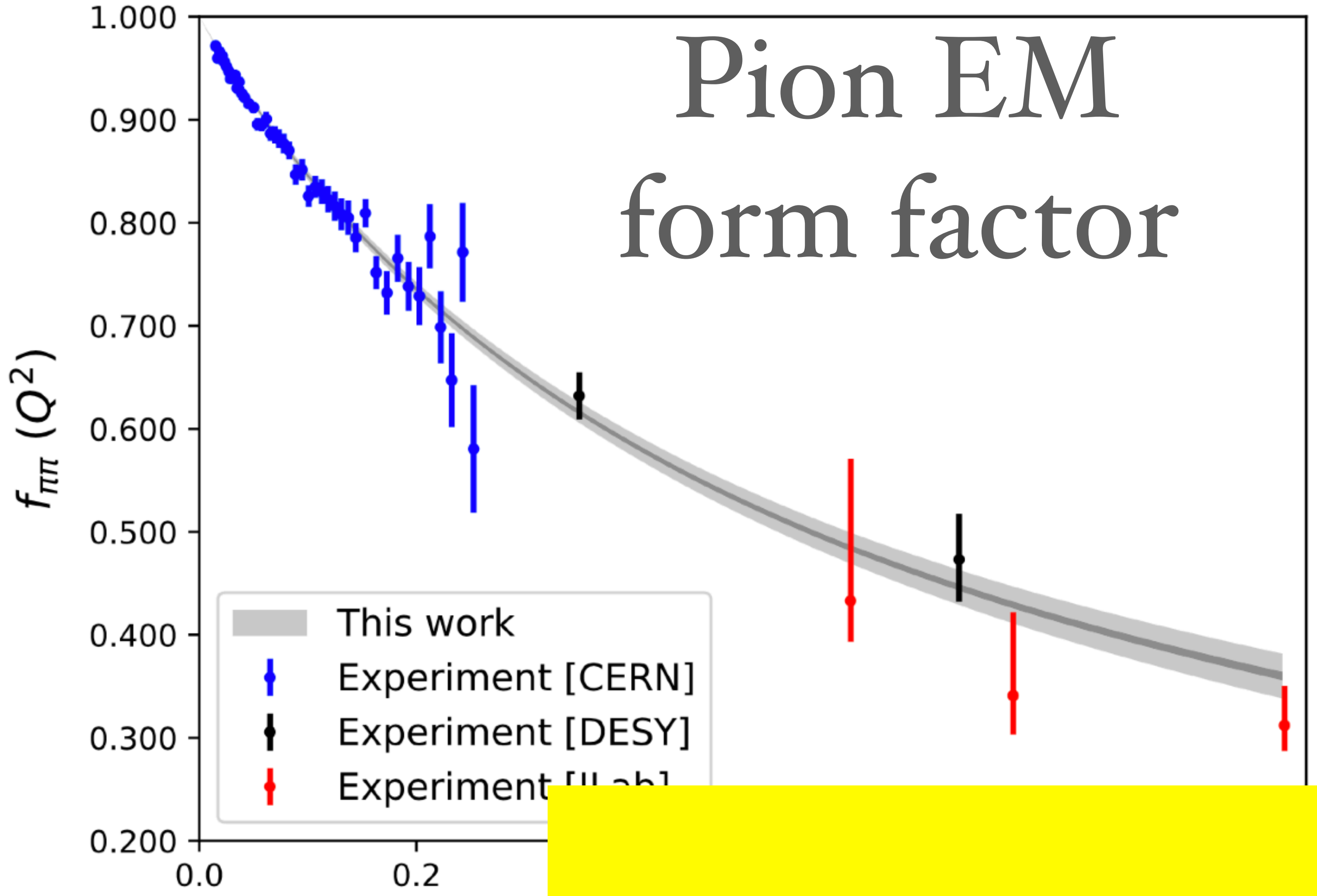
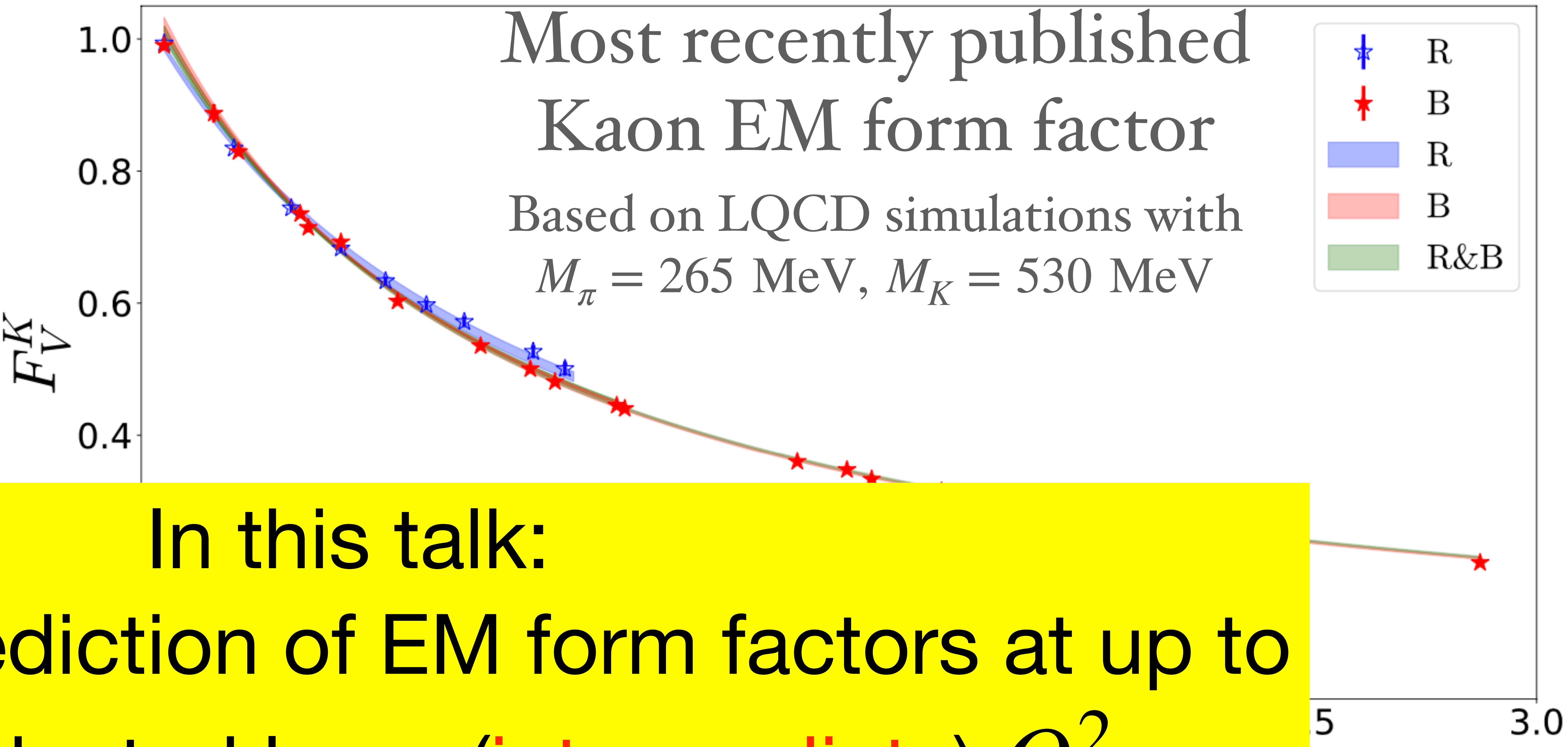


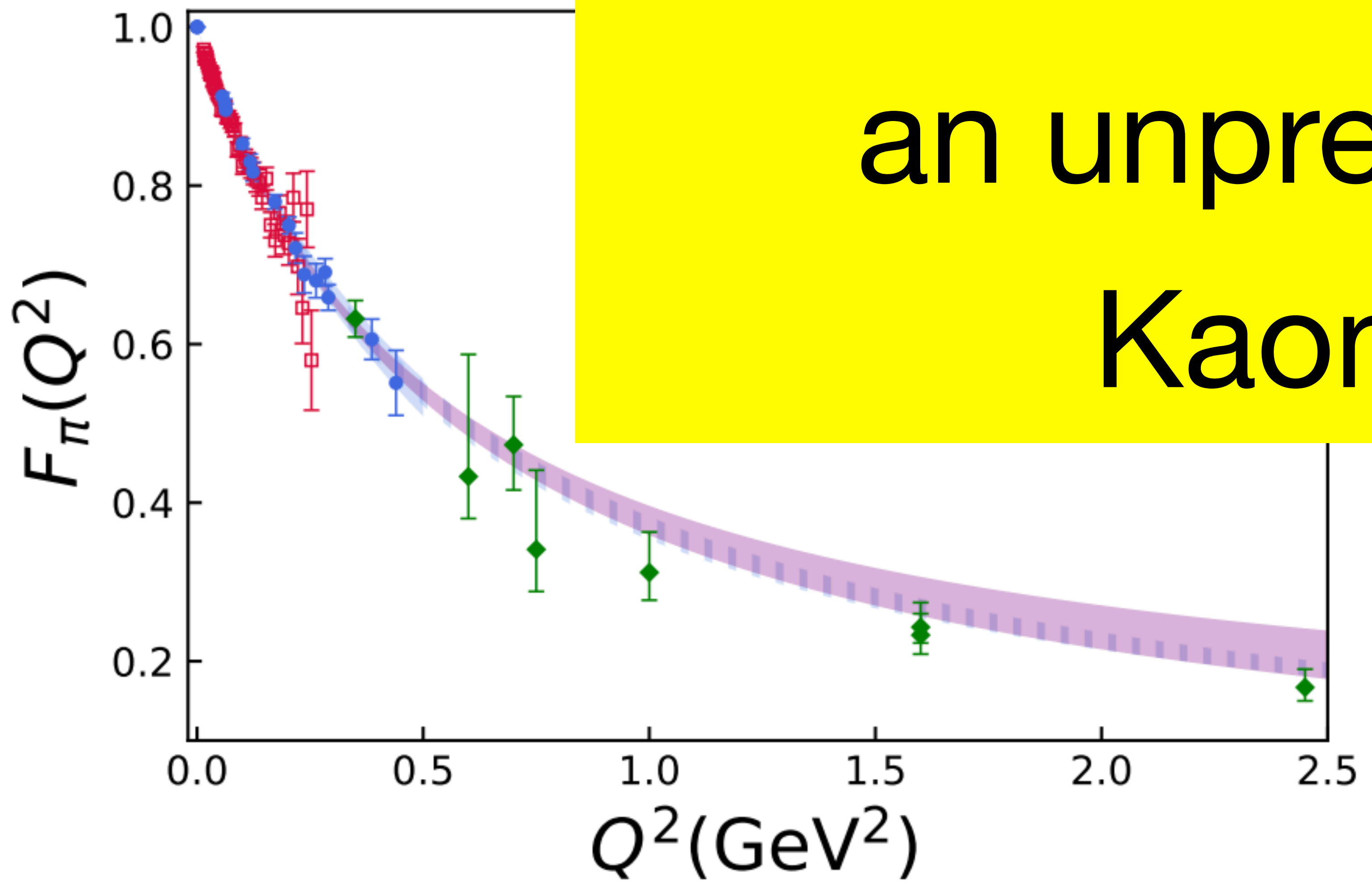
高翔 et al.Tsinghua-BN



In this talk:

Lattice QCD prediction of EM form factors at up to an unprecedented large (intermediate) Q^2:

Kaon ~28 GeV^2 , Pion ~ 10 GeV^2



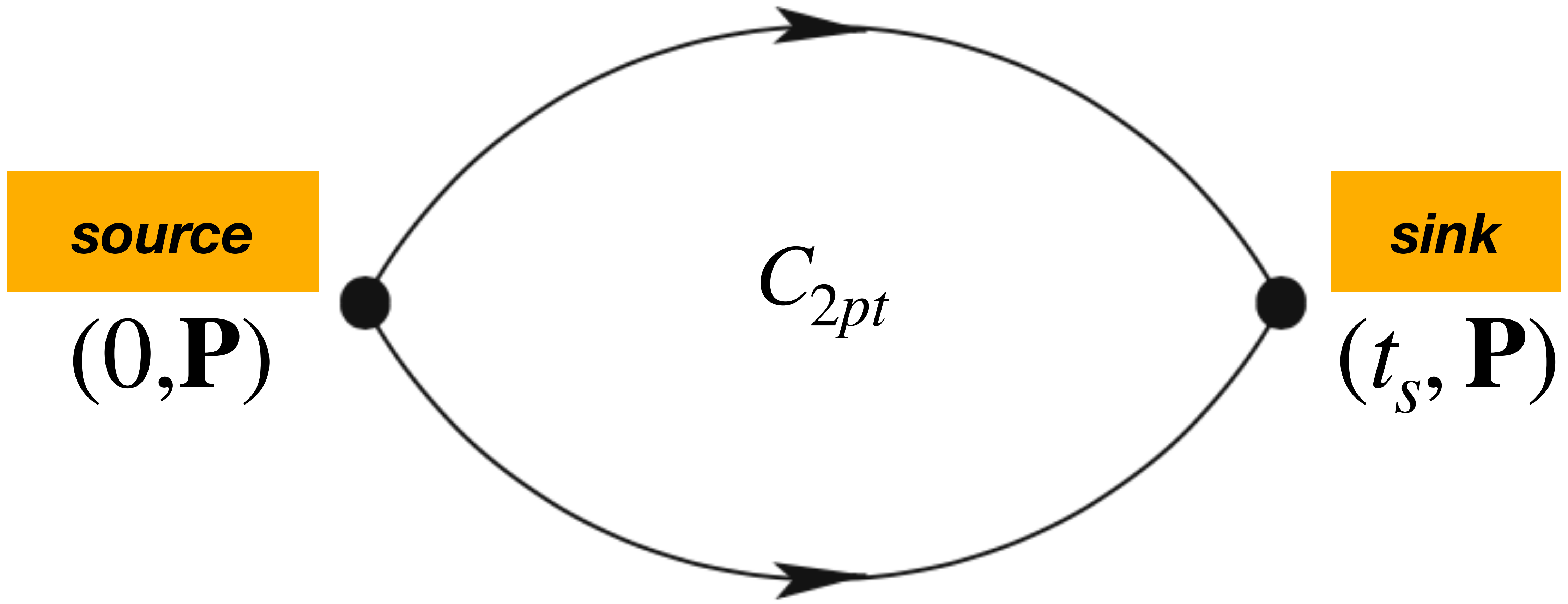
G. Wang et al,, [χQCD], PRD 104 (2021) 074502

Many computations on the pion form factor, but much less on kaon

Mostly restricted to $Q^2 \lesssim 3$ GeV^2

Kaon at nonzero momentum

- Two point kaon correlation function



$$C_{\rm 2pt}(\mathbf{P}, t_s) = \left\langle [K(\mathbf{P}, t_s)][K(\mathbf{P},0)]^\dagger \right\rangle$$

$$K(\mathbf{P}, t) = \sum_{\mathbf{x}} \bar{s}(\mathbf{x}, t)\gamma_5 u(\mathbf{x}, t) e^{-i\mathbf{P}\cdot\mathbf{x}}$$

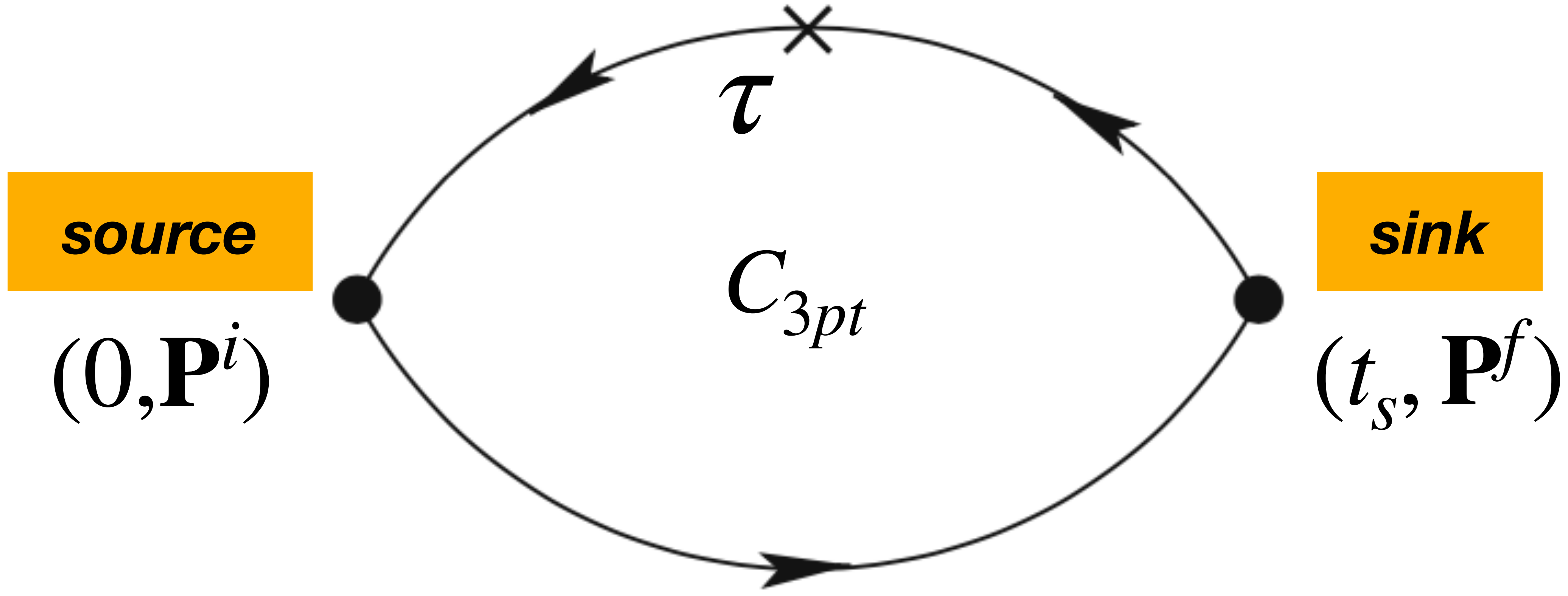
$$\mathbf{P} = \frac{2\pi}{N_\sigma}\mathbf{n}\, a^{-1}$$

- Determine energy of states from the energy decomposition:

$$C_{\rm 2pt}(\mathbf{P}, t_s) = \sum_{n=0}^{N_{\rm state}-1} |\langle \Omega | K_S | n; \mathbf{P}\rangle|^2 (e^{-E_n t_s} + e^{-E_n(aL_t - t_s)})$$

Three point correlation function

$$O_\Gamma = (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{s}\gamma_\mu s)$$



$$C_{3\text{pt}}(\mathbf{P}^f,\mathbf{P}^i,\tau,t_s) = \left\langle [K_{S^f}(\mathbf{P}^f,t_s)]O_\Gamma(\mathbf{q},\tau)[K_{S^i}(\mathbf{P}^i,0)]^\dagger \right\rangle$$

$$\mathbf{P}^i = \mathbf{P}^f - \mathbf{q}$$

$$Q^2 = -(\mathbf{P}^i - \mathbf{P}^f)^2$$

$$C_{3\text{pt}}(\mathbf{P}^f,\mathbf{P}^i;\tau,t_s) = \sum_{m,n} \langle \Omega | K_{S^f} | m;\mathbf{P}^f\rangle \langle m;\mathbf{P}^f | O_\Gamma | n;\mathbf{P}^i\rangle \langle n;\mathbf{P}^i | K^\dagger_{S^i} | \Omega\rangle \times e^{-(t_s-\tau)E^f_m} e^{-\tau E^i_n}$$

EM form factor:

Bare matrix element of kaon ground state $F^B(Q^2) = \langle 0;\mathbf{P}^f | O_\Gamma | 0;\mathbf{P}^i\rangle$

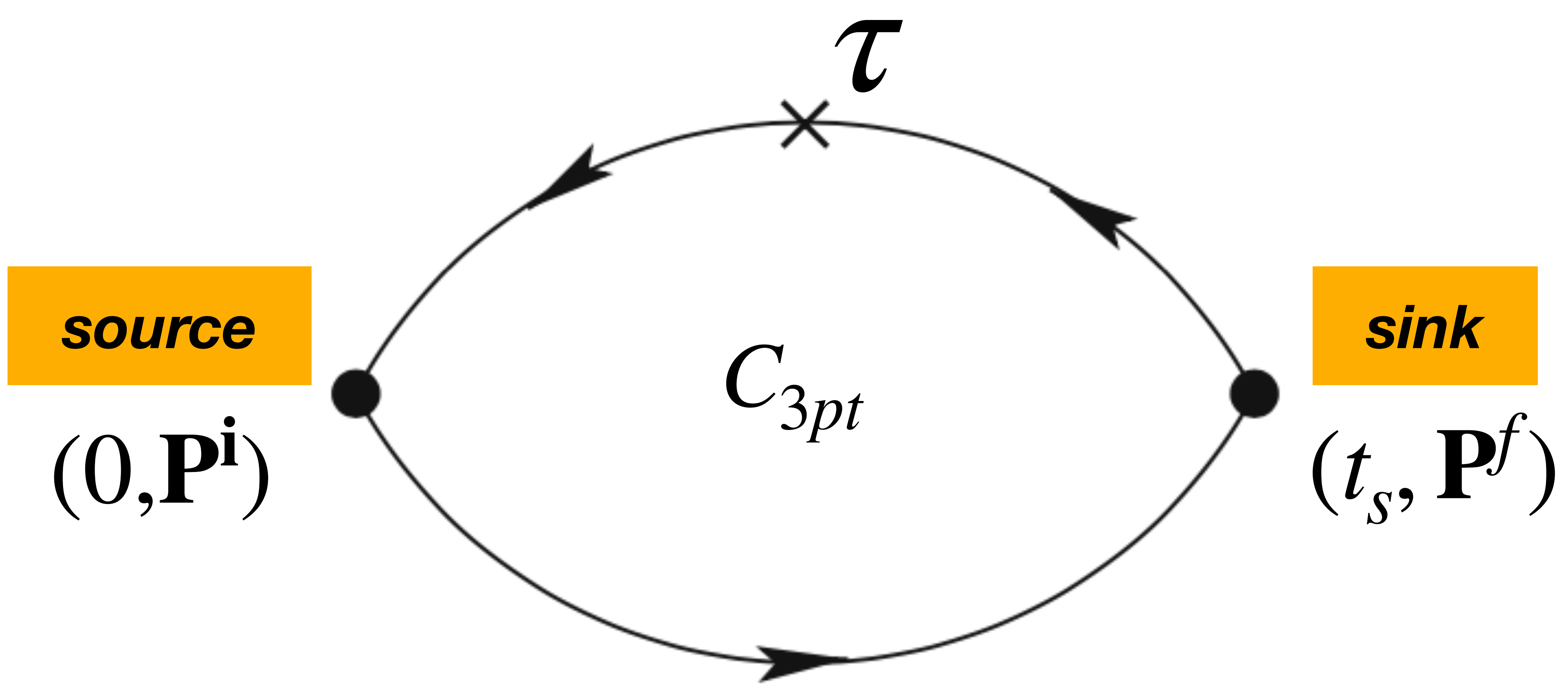
Extraction of bare form factor

- Construct the ratio between 3 and 2-pt corr.:

$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)}{C_{2pt}(t_s, \mathbf{P}^f)} \times \left[\frac{C_{2pt}(t_s - \tau, \mathbf{P}^i) C_{2pt}(\tau, \mathbf{P}^f) C_{2pt}(t_s, \mathbf{P}^f)}{C_{2pt}(t_s - \tau, \mathbf{P}^f) C_{2pt}(\tau, \mathbf{P}^i) C_{2pt}(t_s, \mathbf{P}^i)} \right]^{1/2}$$

- Bare form factor:

$$F^B(Q^2) = \lim_{\tau \to \infty, t_s \to \infty} R^{fi}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s)$$



- Form factor: $F(Q^2) = F^B \times Z_V$

Lattice setup

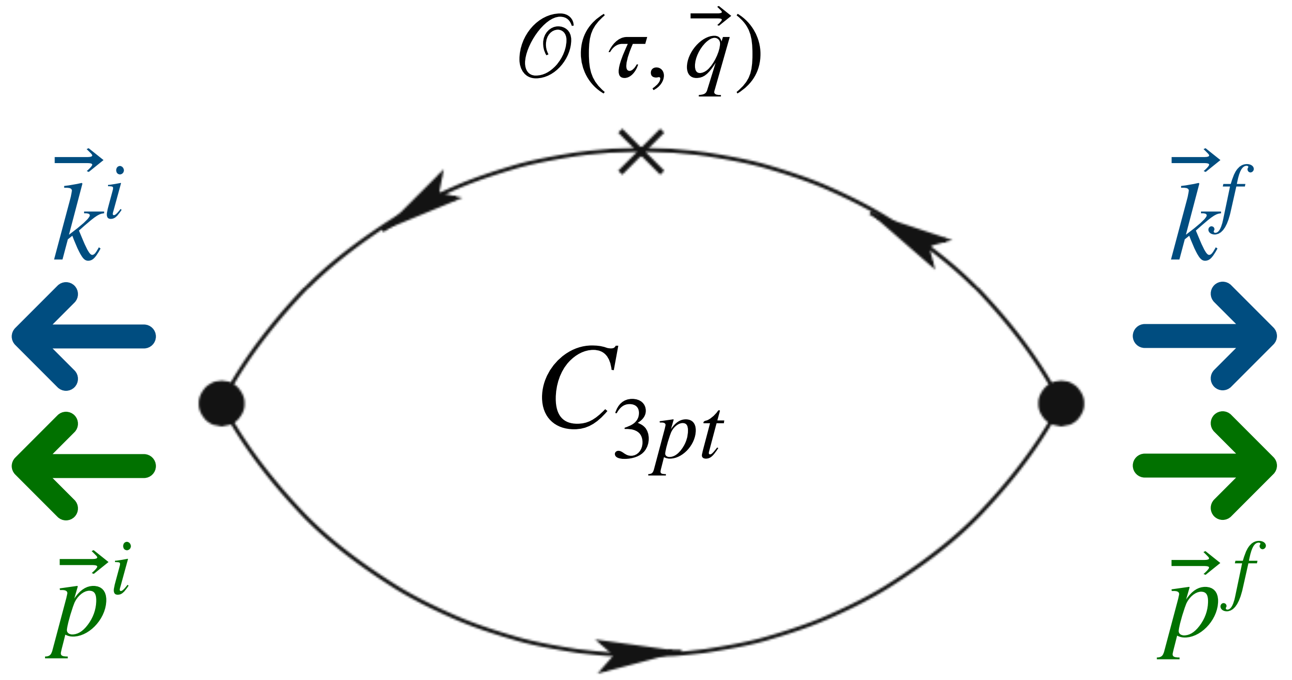
- N_f=2+1 QCD on 64^3x64 lattices with a=0.076 & 0.04 fm ([HotQCD] configurations)

  Sea quark: Highly Improved Staggered Quark (HISQ) action
  Valence quark: Wilson-Clover action

- At the physical point: $M_{\pi^+} = 140$ MeV, $M_{K^+} = 497$ MeV

- Boost smearing with the corresponding signs of the quark momenta at source & sink

  - Pion: up to 10 GeV^2 with $a = 0.076$ fm
  - Kaon: up to 28 GeV^2 with $a = 0.076$ & 0.04 fm

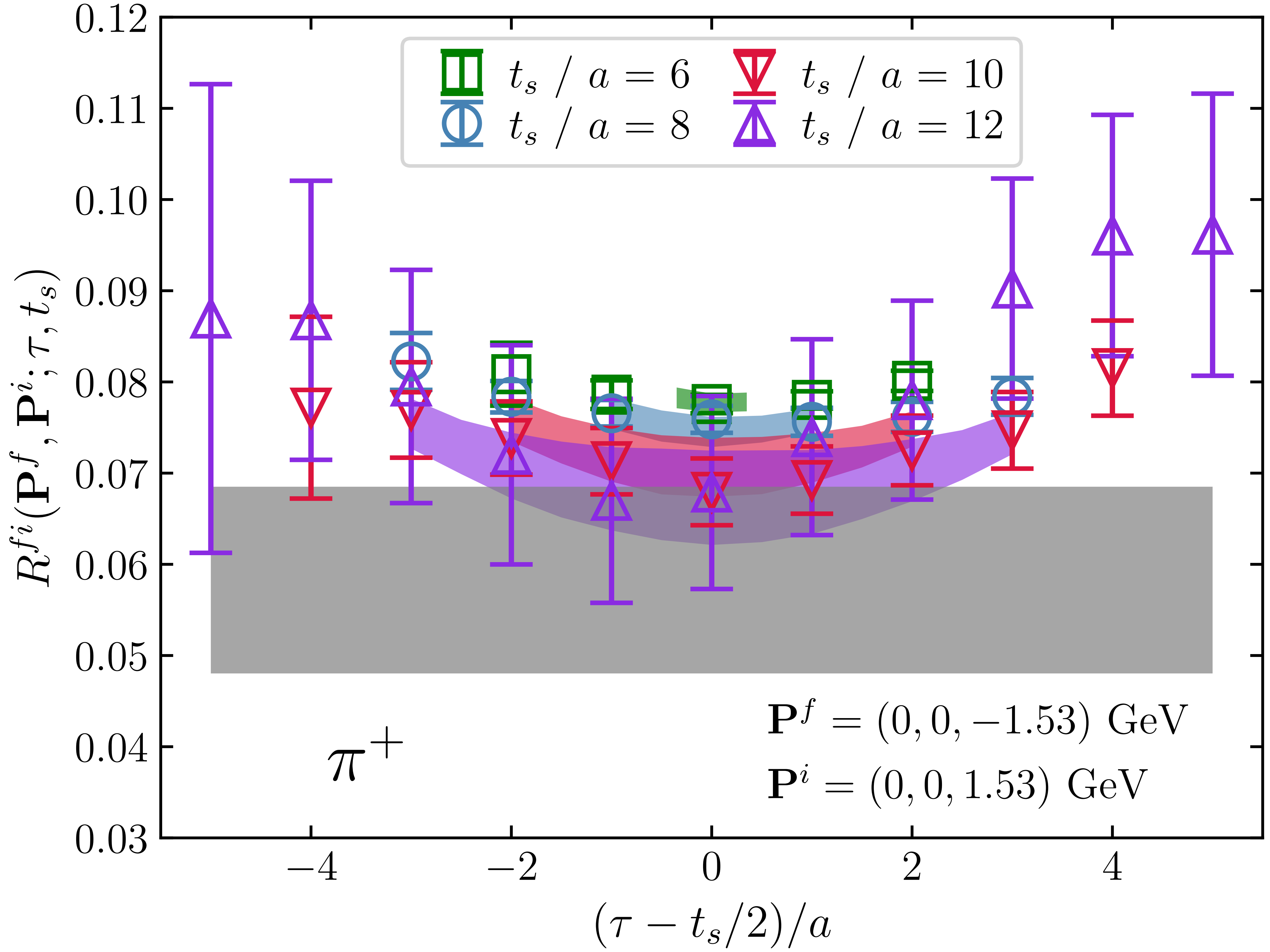


Extraction of the form factor

$$N_{state} = 2: R^{fi}(\tau, t_s) = \left(\mathcal{O}_{00} + \frac{A_1}{A_0}\mathcal{O}_{11} e^{-t_s \Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01} e^{-\tau \Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10} e^{-(t_s-\tau)\Delta E} \right) \Big/ \left(1 + \frac{A_1}{A_0} e^{-t_s \Delta E} \right), \quad \Delta E = E_1 - E_0$$

$\mathcal{O}_{00}$: F^B

π^+, $Q^2 = 9.4\ \mathrm{GeV}^2$

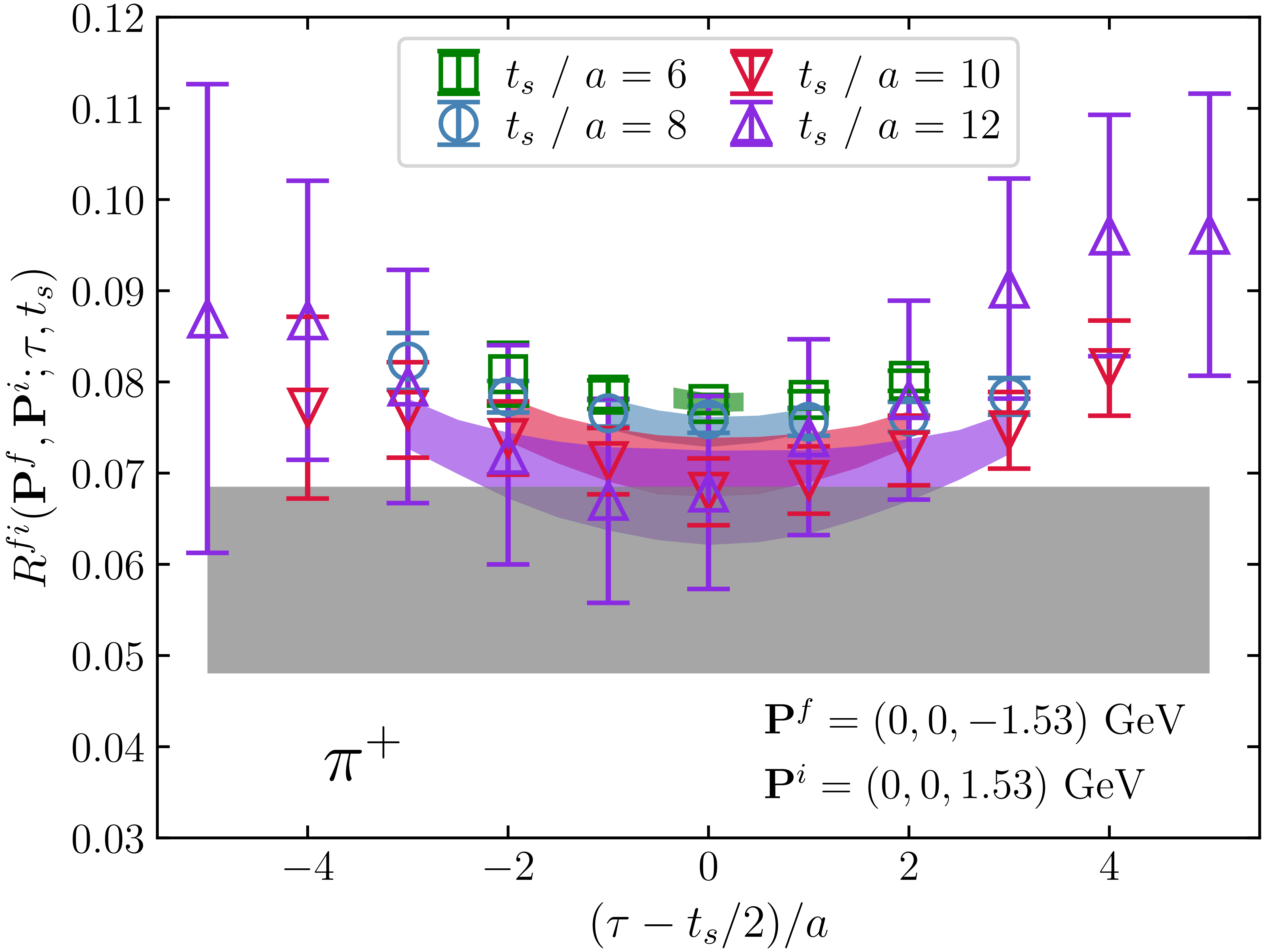


- Use the values of energy E_n and amplitude A_n extracted from C_{2pt}
- Perform a 4-parameter fit to the ratio R^{fi} to extract F^B

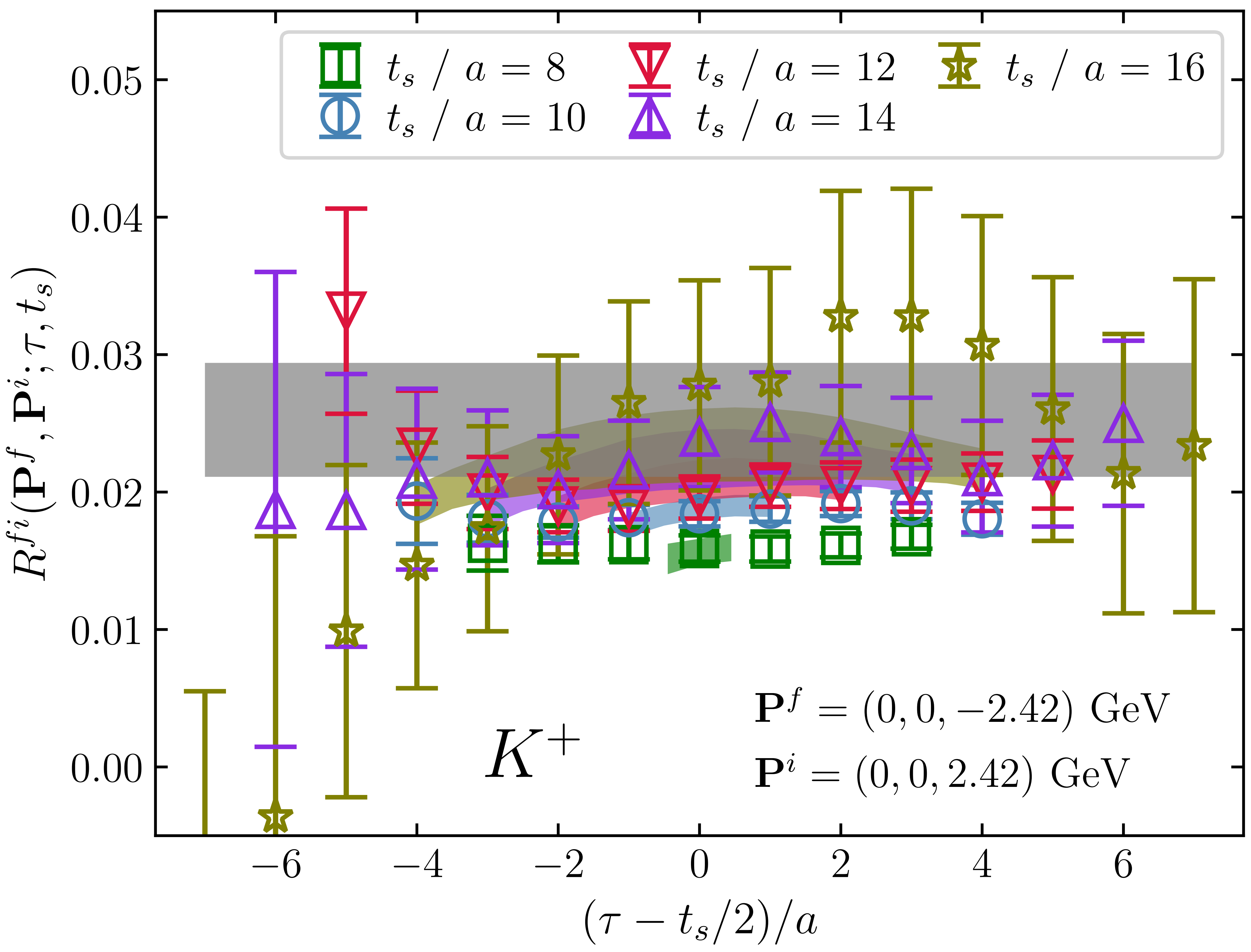
Extraction of the form factor

$$N_{state} = 2:\ R^{fi}(\tau, t_s) = \left(\underbrace{\mathcal{O}_{00}}_{F^B} + \frac{A_1}{A_0}\mathcal{O}_{11}e^{-t_s\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{01}e^{-\tau\Delta E} + \sqrt{\frac{A_1}{A_0}}\mathcal{O}_{10}e^{-(t_s-\tau)\Delta E}\right) \Big/ \left(1 + \frac{A_1}{A_0}e^{-t_s\Delta E}\right),\ \Delta E = E_1 - E_0$$

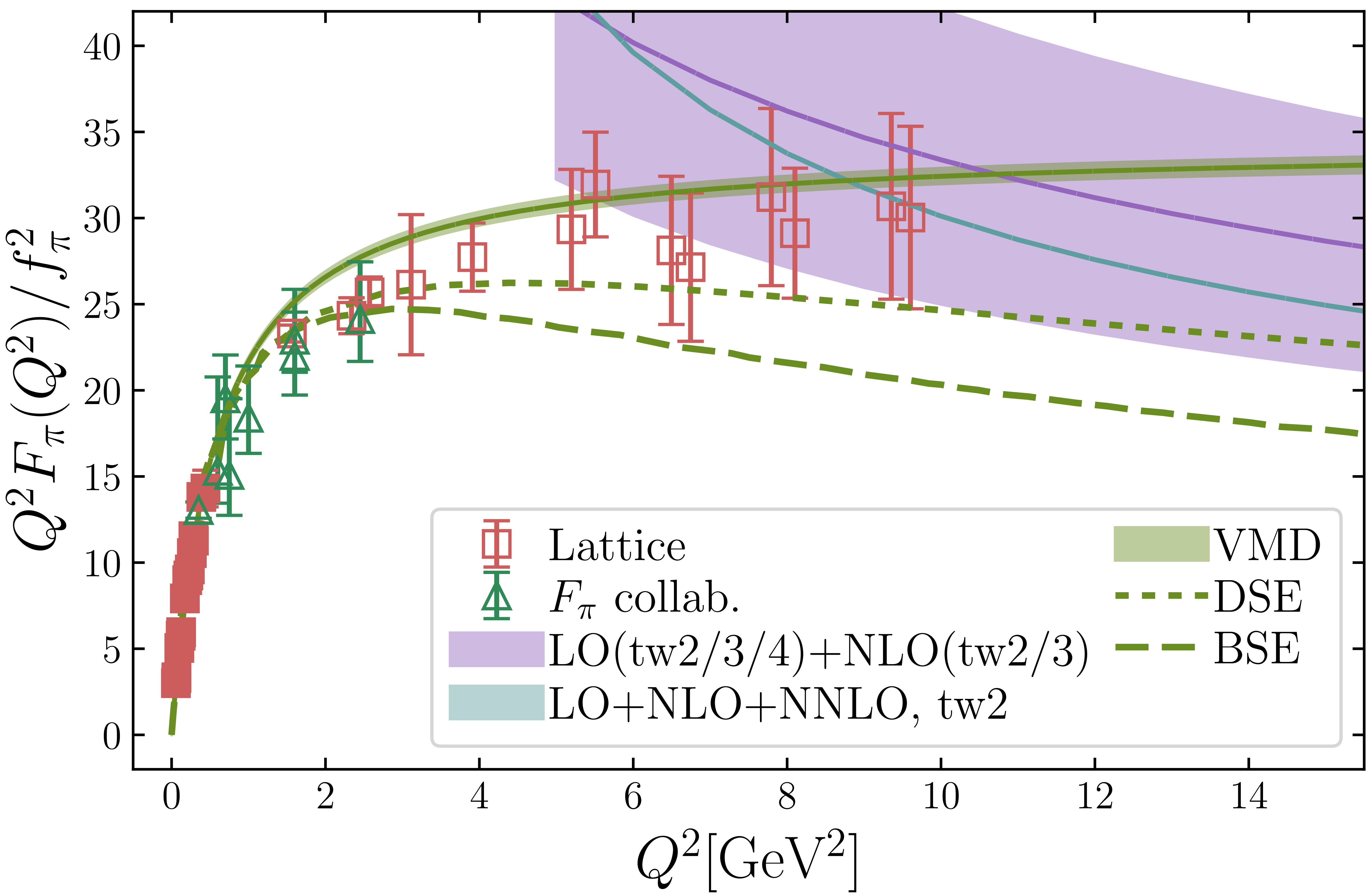
$\pi^+,\ Q^2 = 9.4\ \mathrm{GeV}^2$



$K^+,\ Q^2 = 23.4\ \mathrm{GeV}^2$



Pion form factor up to $Q^2 \sim 10\ \mathrm{GeV}^2$



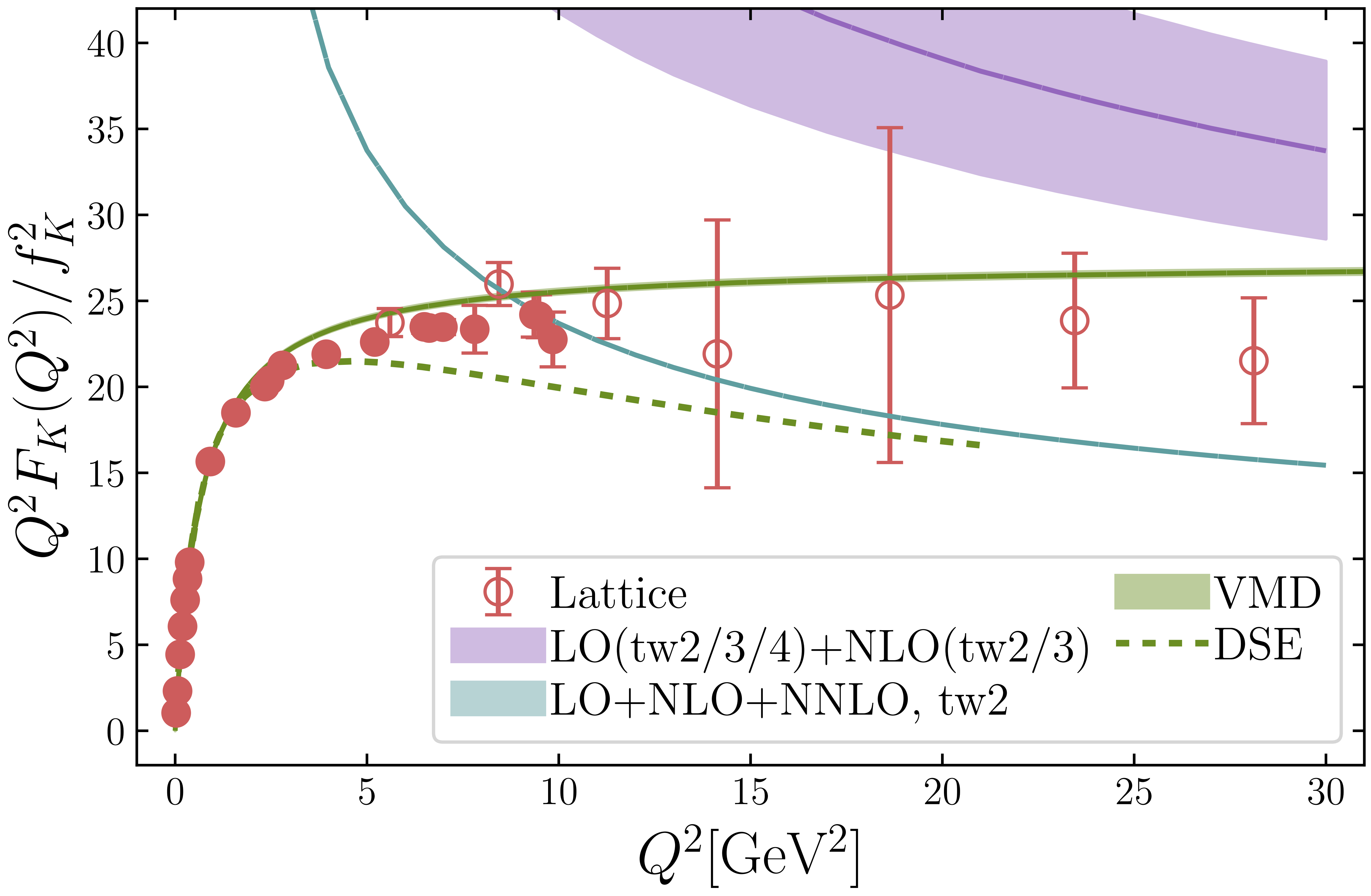
- Blue band: collinear factorization, Chen et al., 2312.17228
- Purple band: k_T factorization, Cheng et al., PRD 19', EPJC23'
- DSE (Dyson-Schwinger Eq.): Gao et al., PRD 96(2017)034024
- BSE (Bethe–Salpeter Eq.): Ydrefors et al., PLB 820 (2021)136494
- VMD (Vector Meson Dominance): $F_\pi(Q^2) = 1/(1 + Q^2/M^2)$

Gao et al., 2102.06047

LO asymptotic result: $Q^2 F_{\pi^+}(Q^2)/f_\pi^2 \simeq 8.6$

HTD, **Xiang Gao(高翔)**, A. Hanlon, S. Mukherjee, P. Petreczky, **Qi Shi(施岐)**, S. Syritsyn, R. Zhang, Y. Zhao, Phys.Rev.Lett. 133 (2024) 18

Kaon form factor up to $Q^2 \sim 28$ GeV^2



- Blue band: collinear factorization, Chen et al., 2312.17228
- Purple band: k_T factorization, Cheng, priv. com.
- DSE: Gao et al., PRD 96(2017)034024
- VMD: $F_{K^+}(Q^2) = \sum_{v=\rho,\phi,\omega} c_v/(1+Q^2/m_v^2)$

fit in low $Q^2 \lesssim 0.4$ GeV^2, resulting $\langle r_K^2 \rangle = 0.360(2)\mathrm{fm}^2$

Consistent with $\langle r_K^2 \rangle = 0.359(3)\mathrm{fm}^2$ Stamen et al., EPJC 82(2022)432

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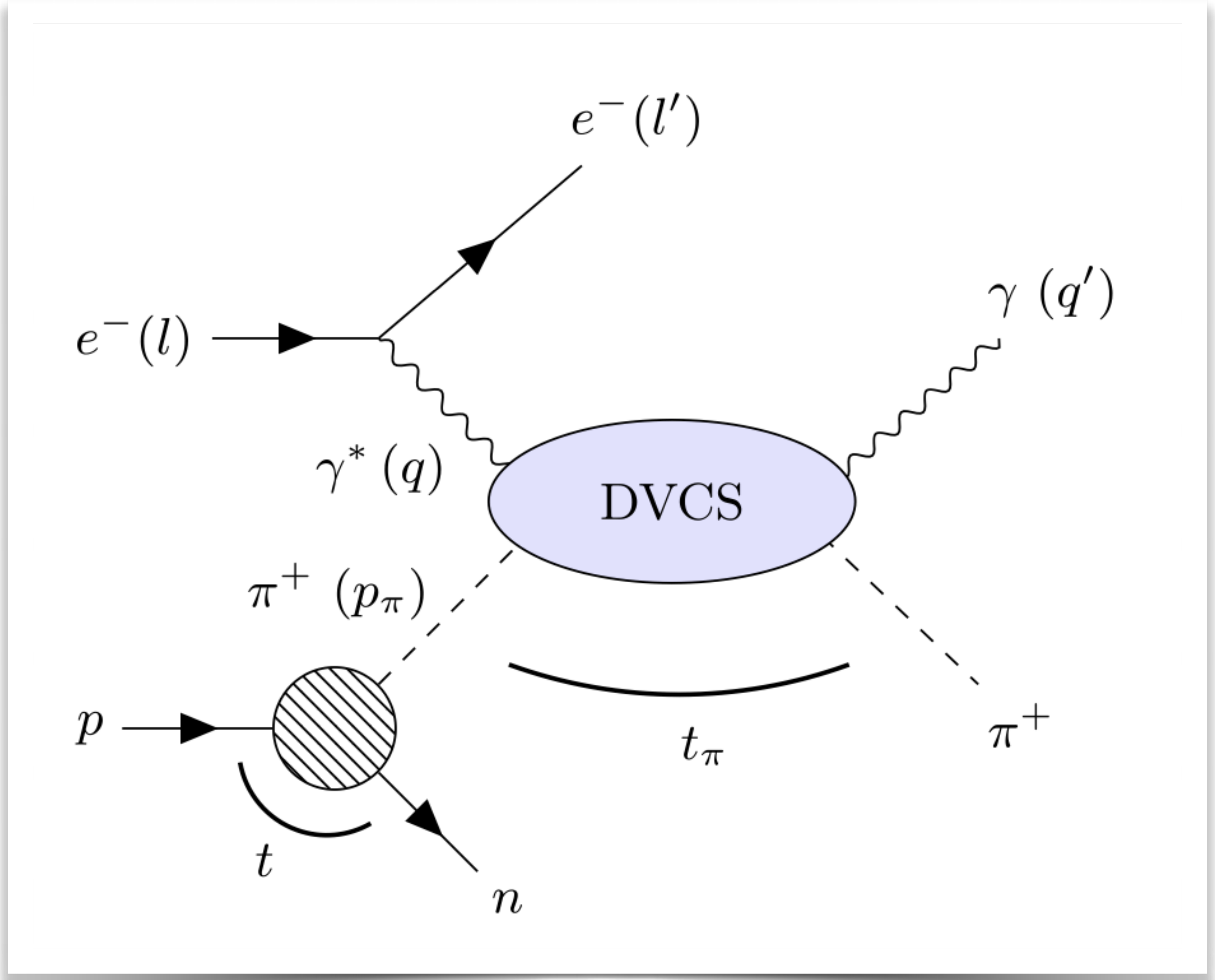
Generalized Portion Distributions (GPDs)

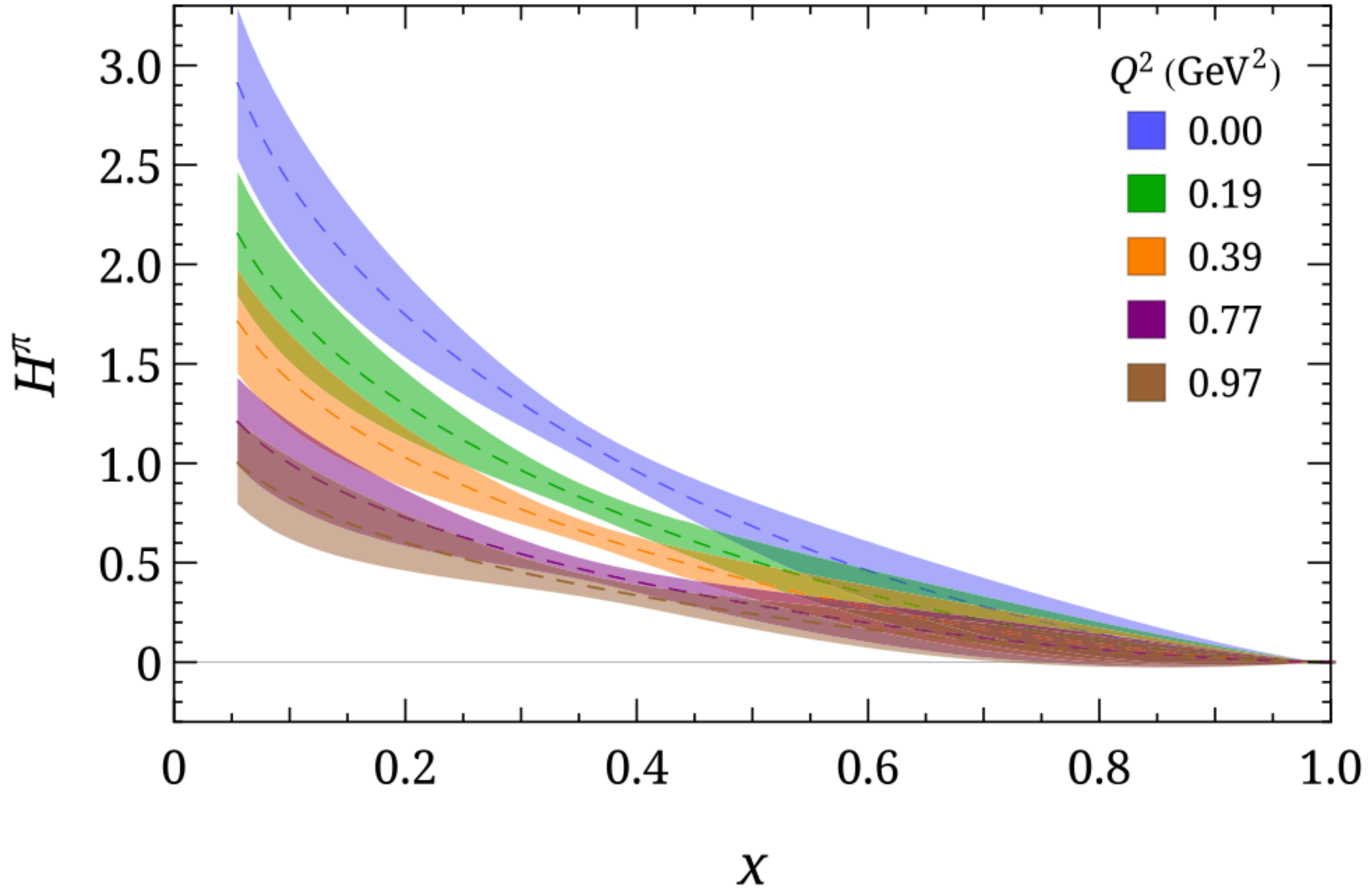
Accessing the Pion 3D Structure at
US and China Electron-Ion Colliders
Model needed



Chávez et al., PRL 128 (2022) 20, 202501

Pion valence GPDs obtained using LaMET

a≈0.09fm and Pz= 1.73 GeV in the Breit frame



Limited LQCD results on Pion GPD
H.-W. Lin, PLB 846 (2023) 138181
J.-W. Chen et al., NPB 952 (2020)114940

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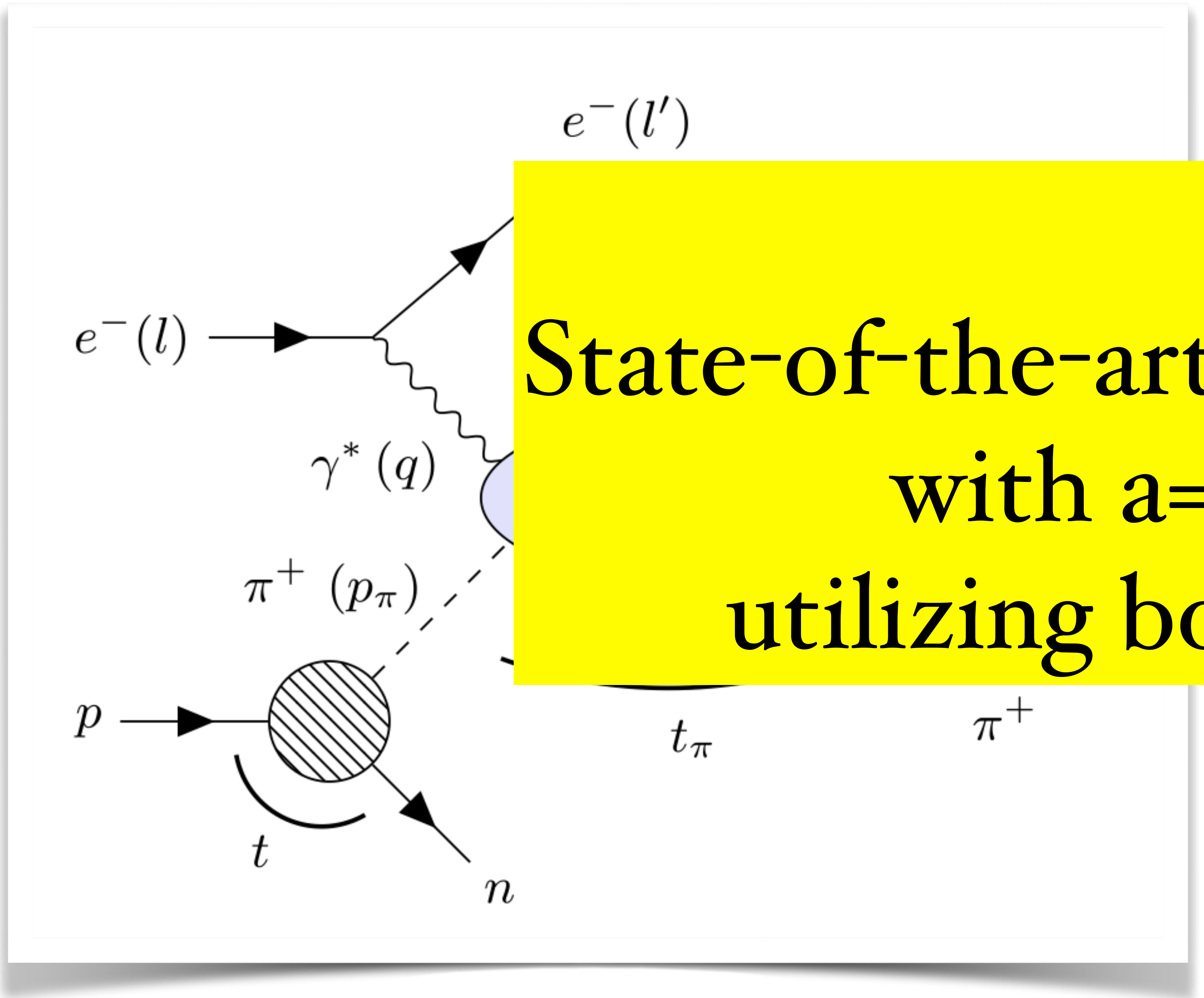

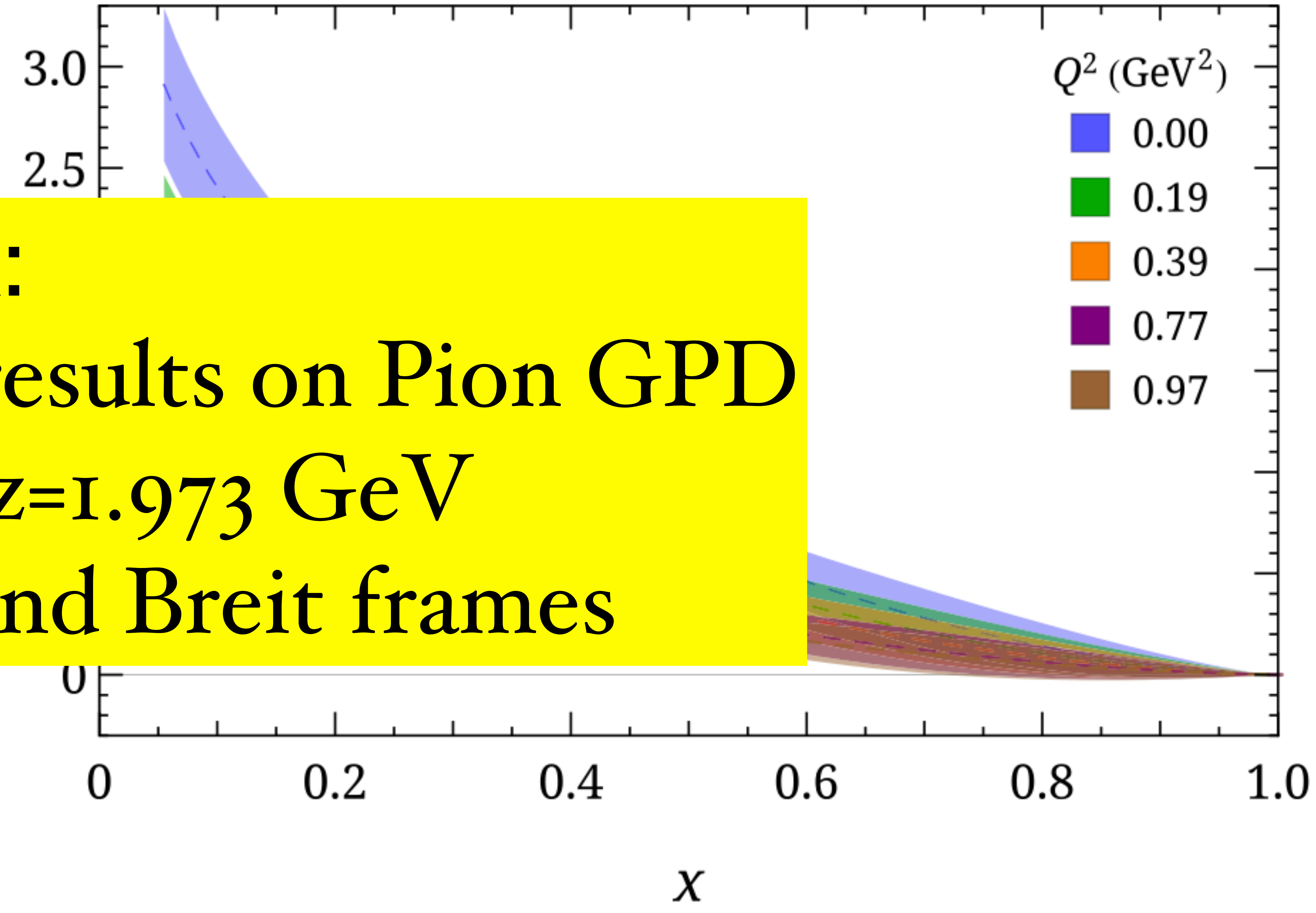

In this talk:
State-of-the-art lattice QCD results on Pion GPD
with a=0.04 fm and Pz=1.973 GeV
utilizing both non-Breit and Breit frames

Chávez et al., PRL 128 (2022) 20, 202501

Limited LQCD results on Pion GPD
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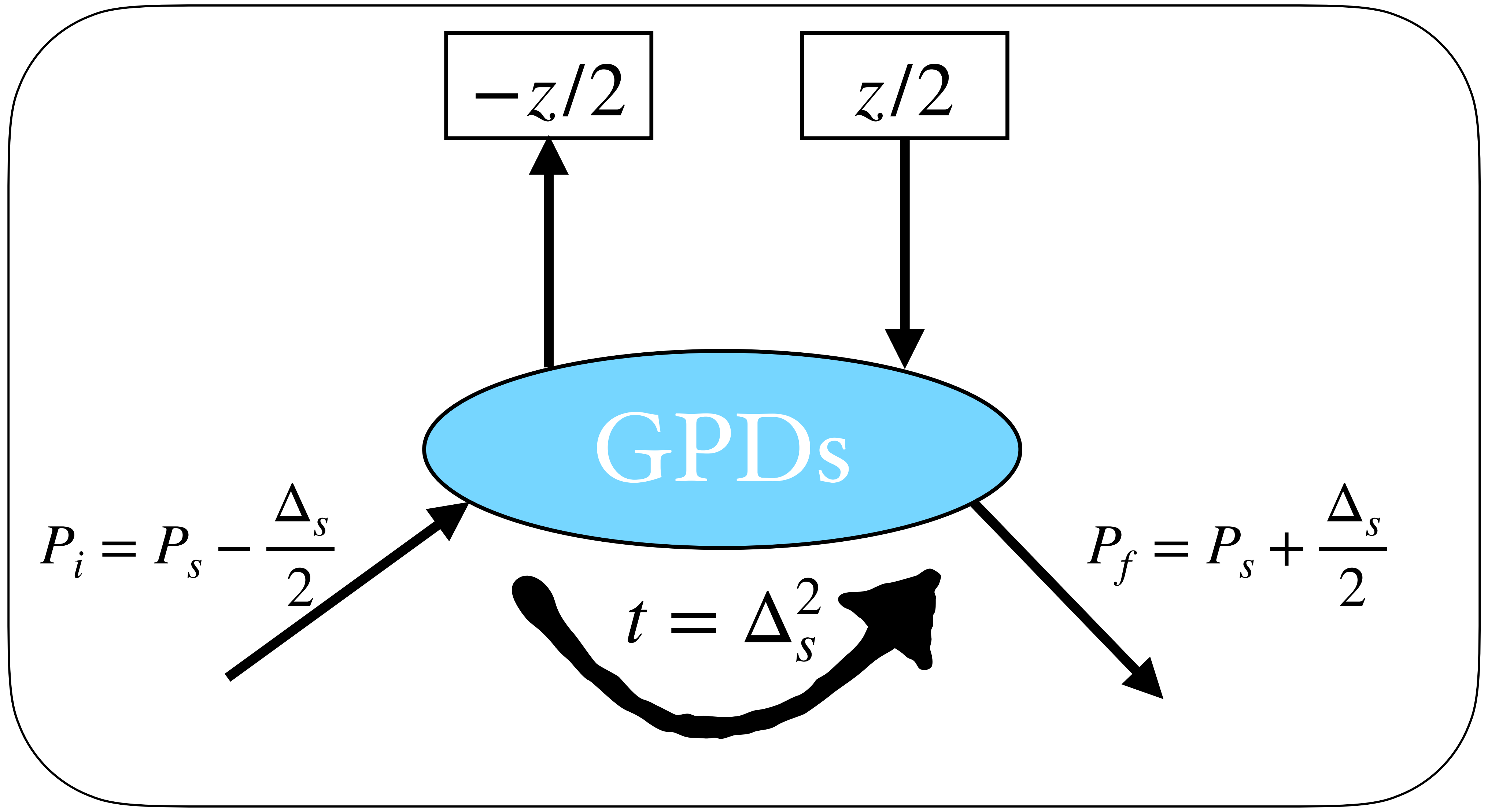
Pion Generalized Parton Distributions

$$M^{\mu}(z, \bar{P}, \Delta) = \langle \pi(P_f) \,|\, O^{\gamma_\mu}(z) \,|\, \pi(P_i) \rangle$$

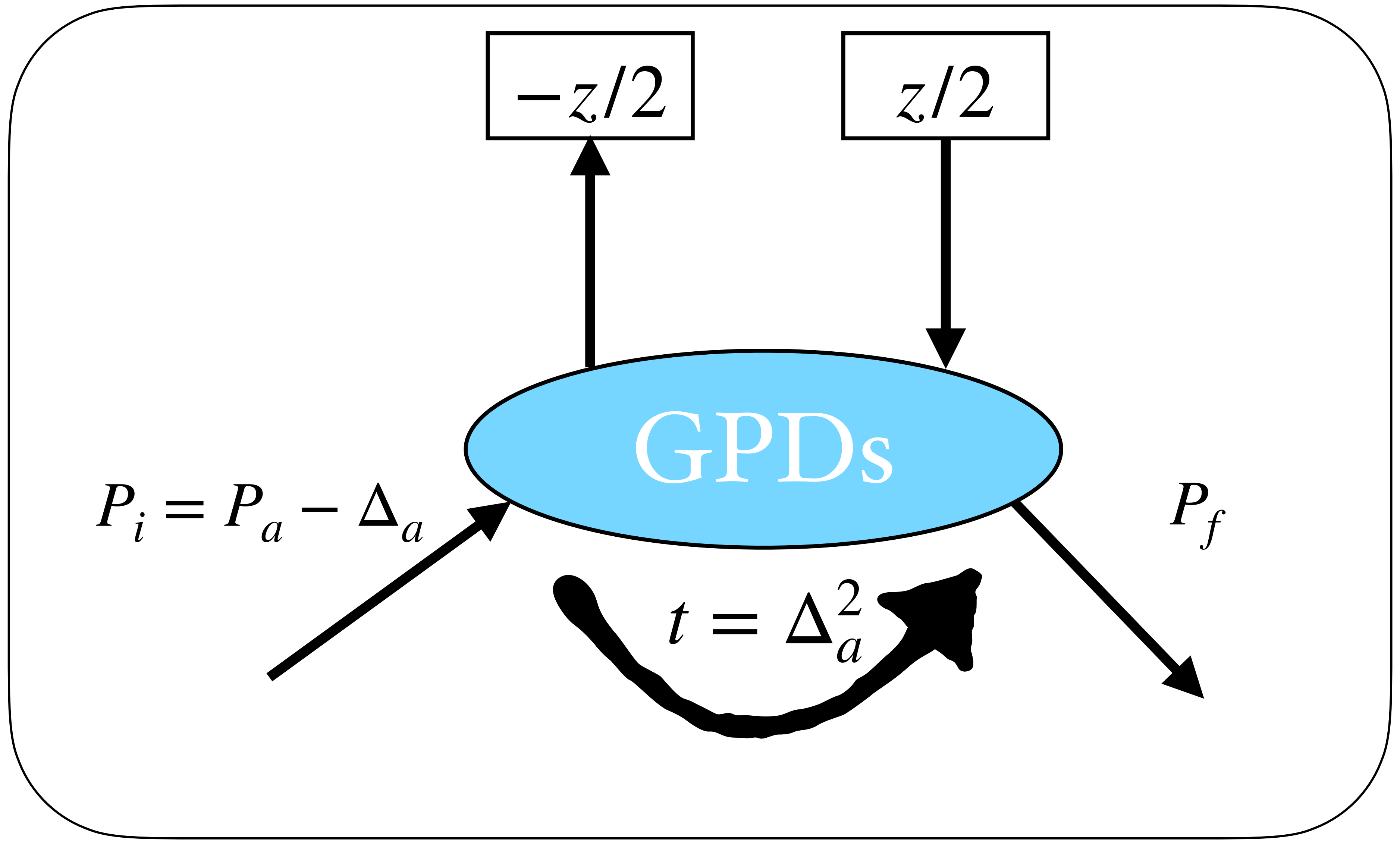
$$O^{\gamma_\mu}(z) = \frac{1}{2}\left[\bar{u}(-\frac{z}{2})\gamma^{\mu}\mathcal{W}_{-\frac{z}{2},\frac{z}{2}}u(\frac{z}{2}) - \bar{d}(-\frac{z}{2})\gamma^{\mu}\mathcal{W}_{-\frac{z}{2},\frac{z}{2}}d(\frac{z}{2})\right]$$

In Lattice QCD, each P_f requires a separate computation



Symmetric frame

Traditional way: vary P_f to obtain different momentum transfer



Asymmetric frame

Novel way: fix P_f, vary t

Bhattacharya et al., PRD 106 (2022) 114512

Frame independent approach

Bhattacharya et al., PRD 106 (2022) 114512

$$M^\mu(z, \bar{P}, \Delta) = \langle \pi(P_f) | O^{\gamma_\mu}(z) | \pi(P_i) \rangle \qquad O^{\gamma_\mu}(z) = \frac{1}{2}\left[\bar{u}(-\frac{z}{2})\gamma^\mu \mathcal{W}_{-\frac{z}{2},\frac{z}{2}} u(\frac{z}{2}) - \bar{d}(-\frac{z}{2})\gamma^\mu \mathcal{W}_{-\frac{z}{2},\frac{z}{2}} d(\frac{z}{2})\right]$$

- Express the matrix element in terms of Lorentz-invariant A_i

$$M^\mu(z, \bar{P}, \Delta) = \bar{P}^\mu A_1 + m_\pi^2 z^\mu A_2 + \Delta^\mu A_3 \qquad \bar{P}^\mu = (p_f^\mu + p_i^\mu)/2,\ \Delta^\mu = p_f^\mu - p_i^\mu$$

M can be extracted from the ratio of 2-pt and 3-pt correlation functions of pions

- Frame independent GPD H can be expressed as

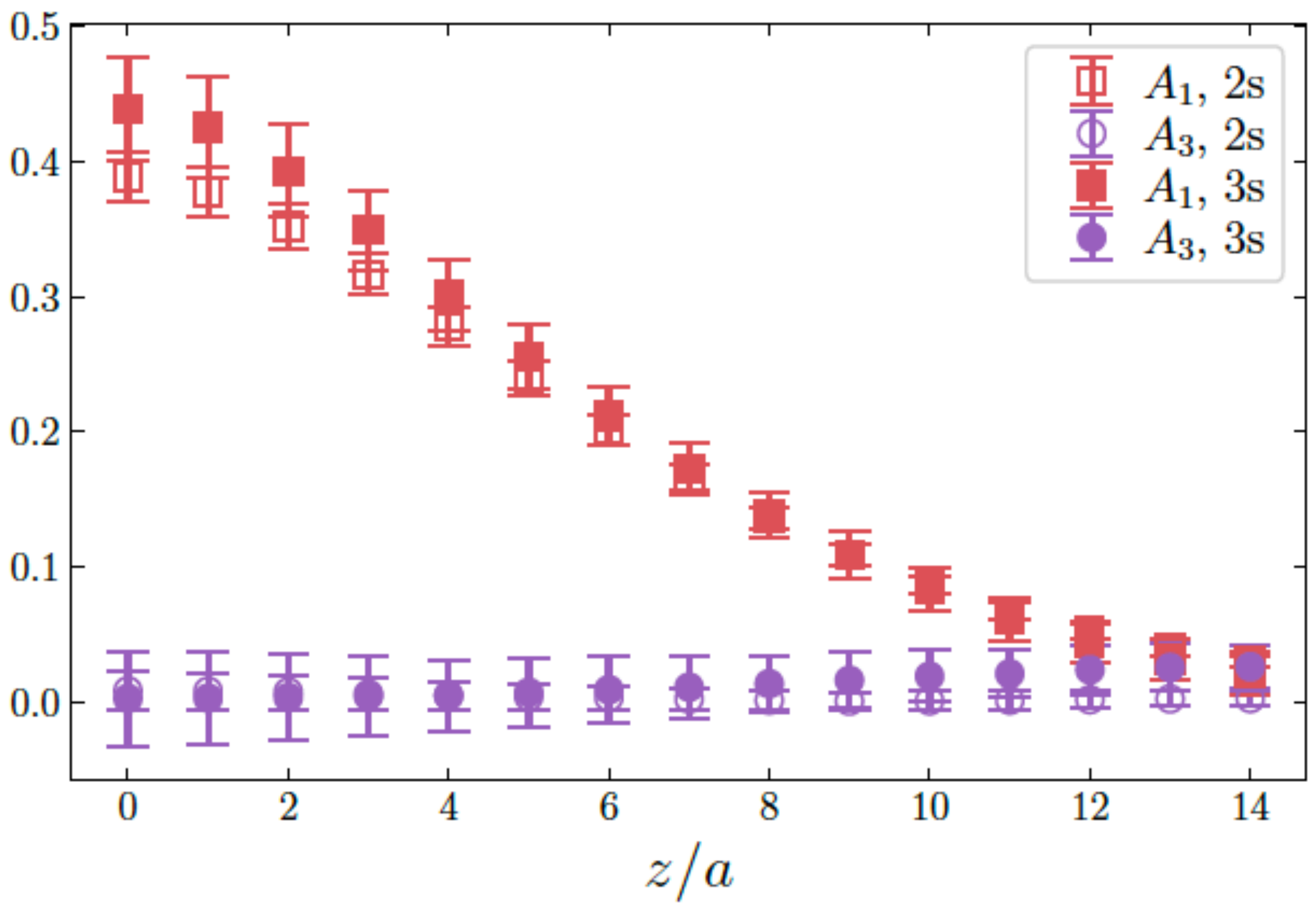
$$H(P, z, \Delta) = A_1 + \frac{z \cdot Q}{z \cdot P} A_3 \quad \text{with} \quad A_3(-z \cdot Q) = -A_3(z \cdot Q) \longrightarrow A_3(z \cdot Q = 0) = 0$$

$$A_1(\text{Sym}) \sim A_1(\text{Asym})$$

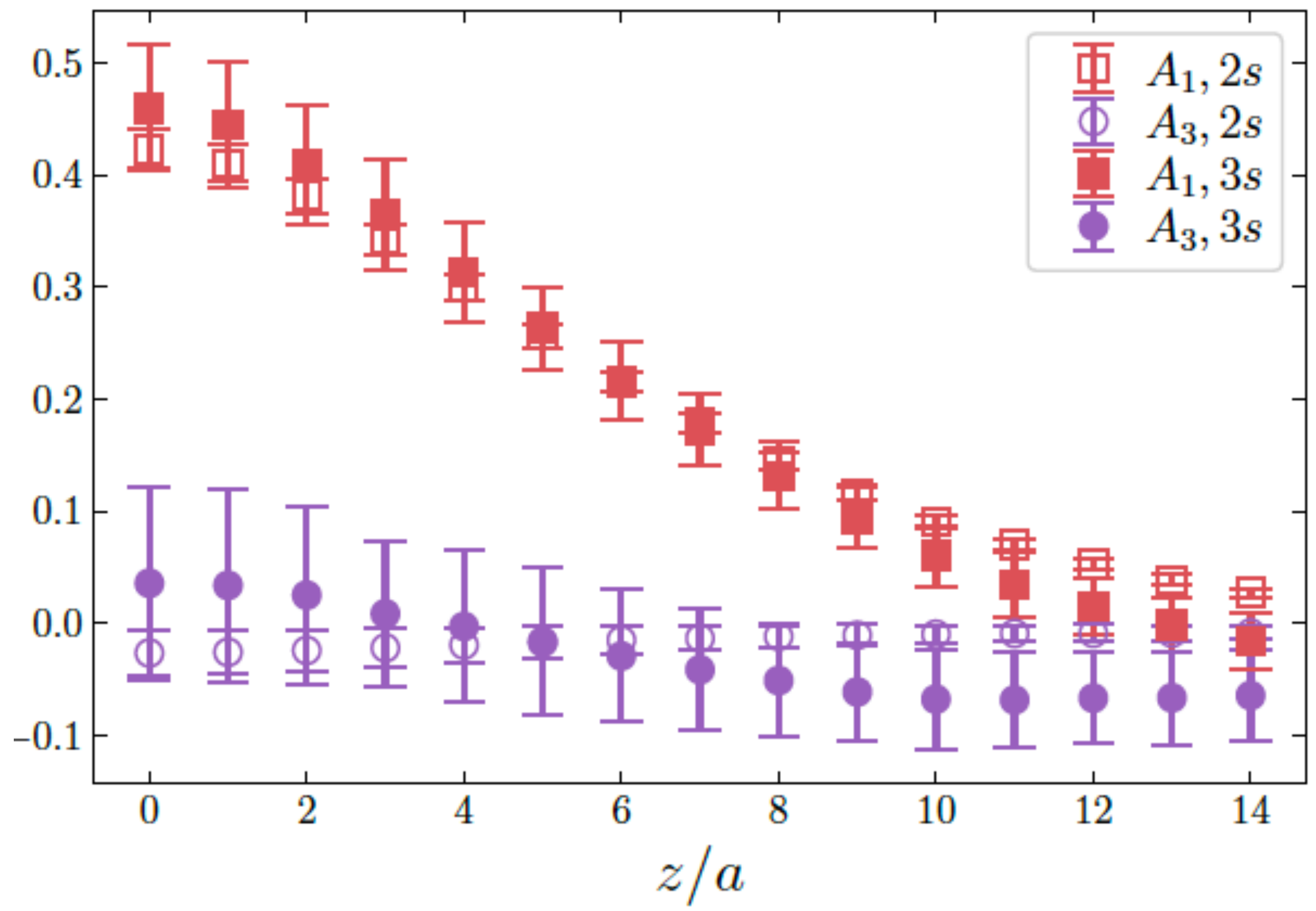
A_i in Breit and non-Breit frames

Pz=0.968 GeV

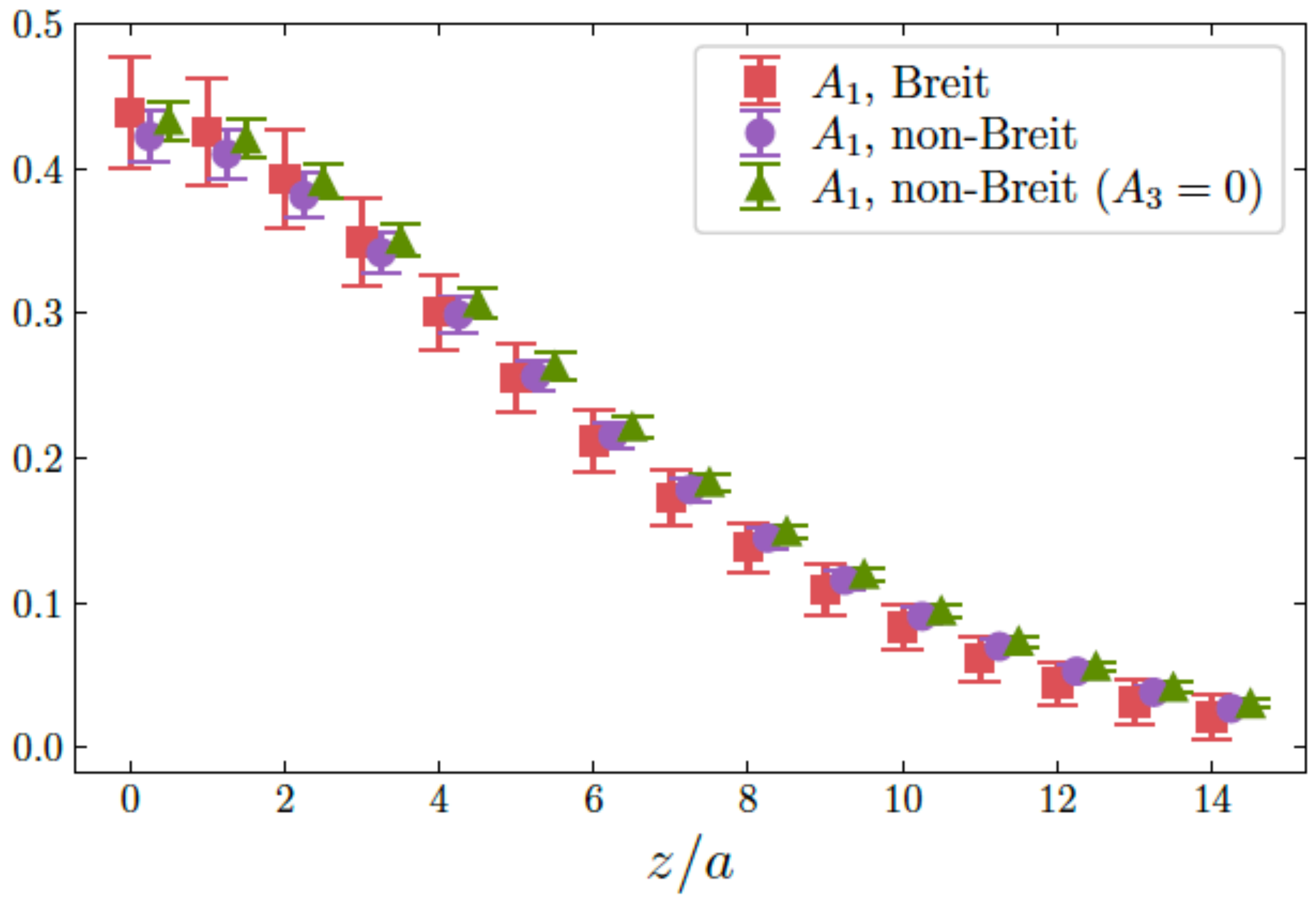
A1 & A3 in the Breit frame



A1 & A3 in the non-Breit frame



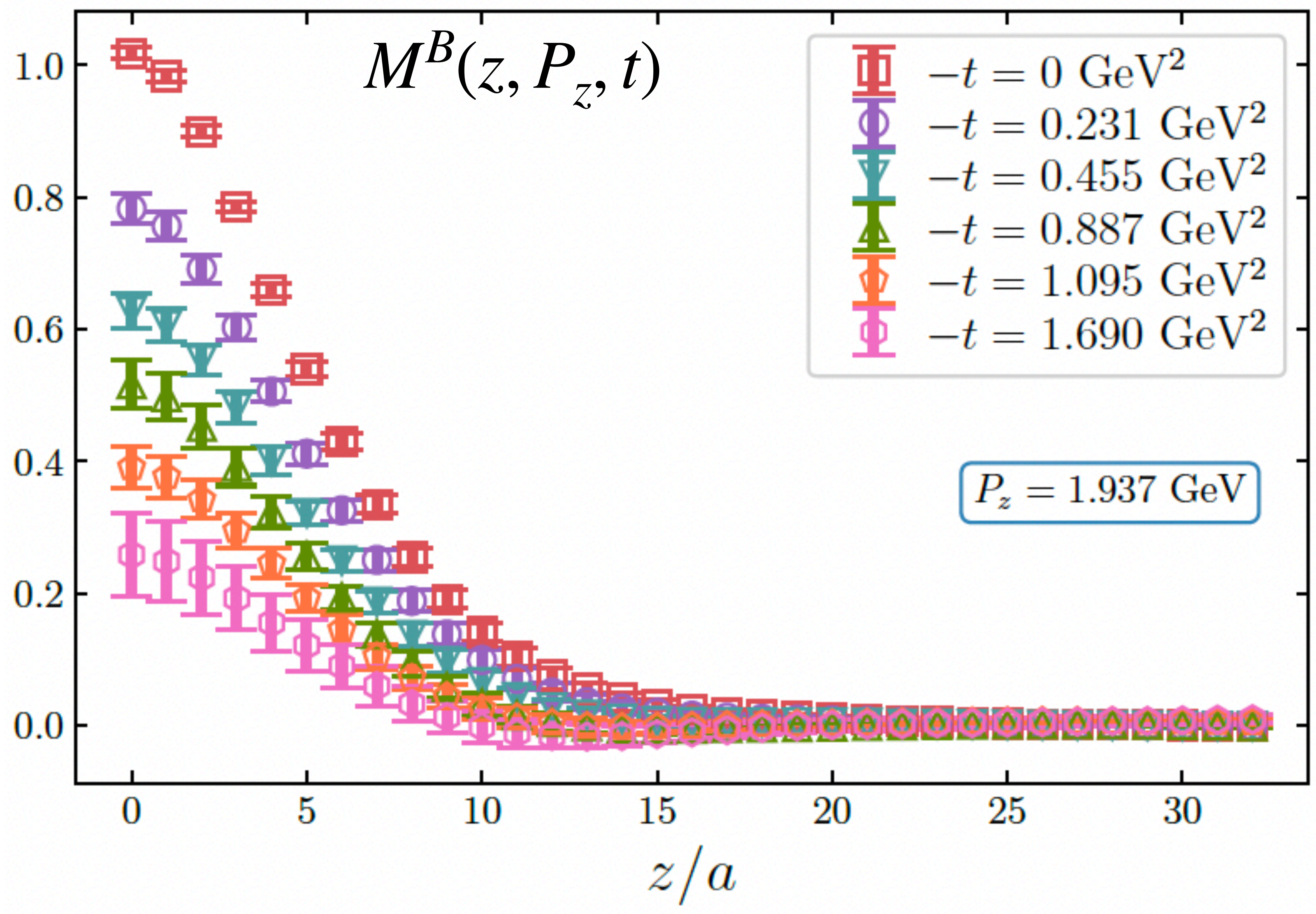
A1 in both frames



In both frames: $A_3 \approx 0$; A_1(Breit) ≈ A_1(non-Breit)

Feasible to extract A_1 from non-Breit frame

Recipe to obtain light-cone GPD



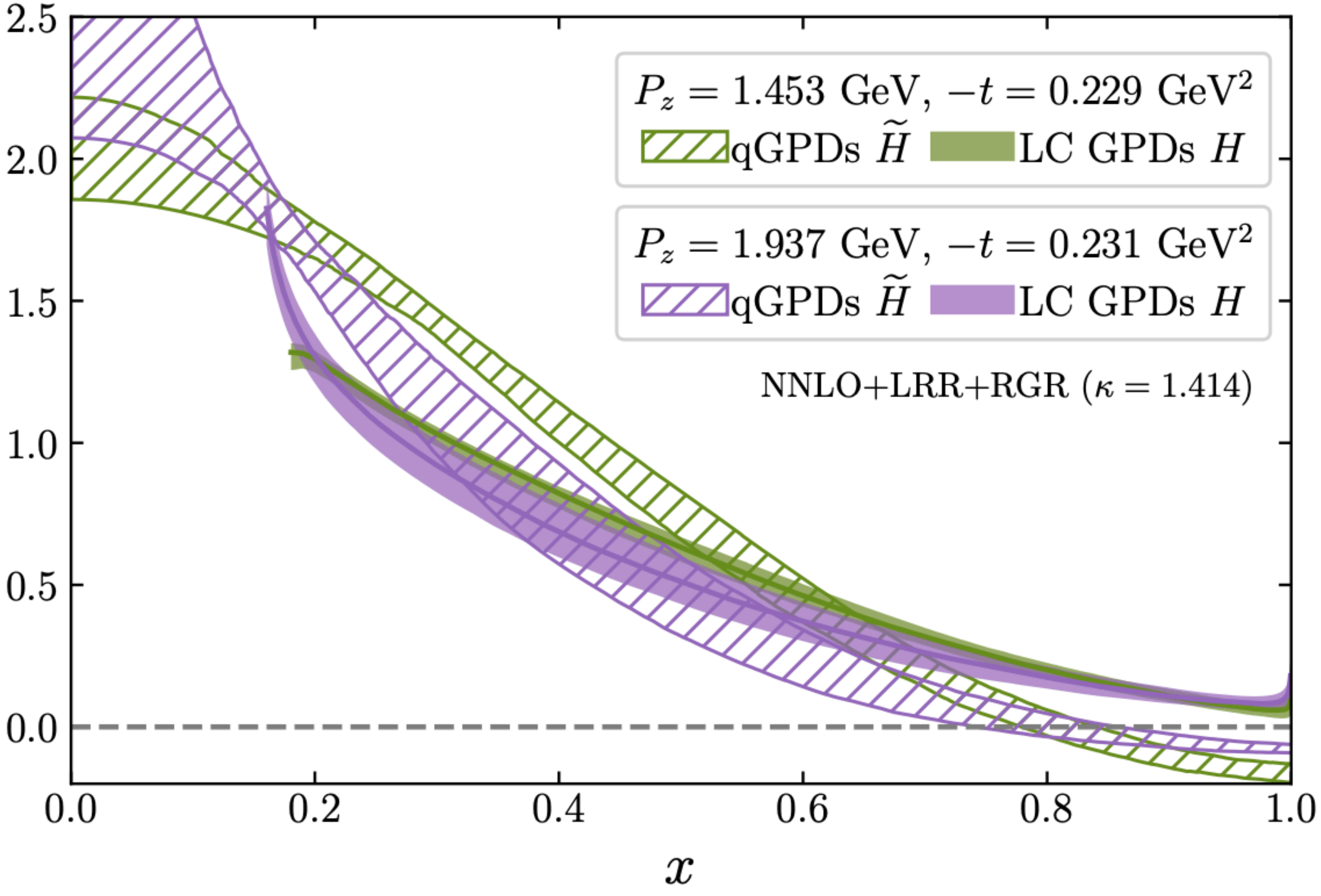
I: Renormalization of $M^B \longrightarrow M^R$

II: $M^R \xrightarrow{\text{Fourier transform}}$ quasi-GPD

III: quasi-GPD $\xrightarrow{\text{LaMET}}$ Light cone GPD

III: Light-cone GPD H from quasi-GPD $\tilde{H}$

$$H(x,\mu,t)=\int_{-\infty}^{\infty}\frac{\mathrm{d}k}{|k|}\int_{-\infty}^{\infty}\frac{\mathrm{d}y}{|y|}\mathcal{C}_{\mathrm{evo}}^{-1}\left(\frac{x}{k},\frac{\mu}{\mu_0}\right)\times\mathcal{C}^{-1}\left(\frac{k}{y},\frac{\mu_0}{yP_z},|y|\lambda_s\right)\widetilde{H}(y,P_z,t,z_s,\mu_0)$$

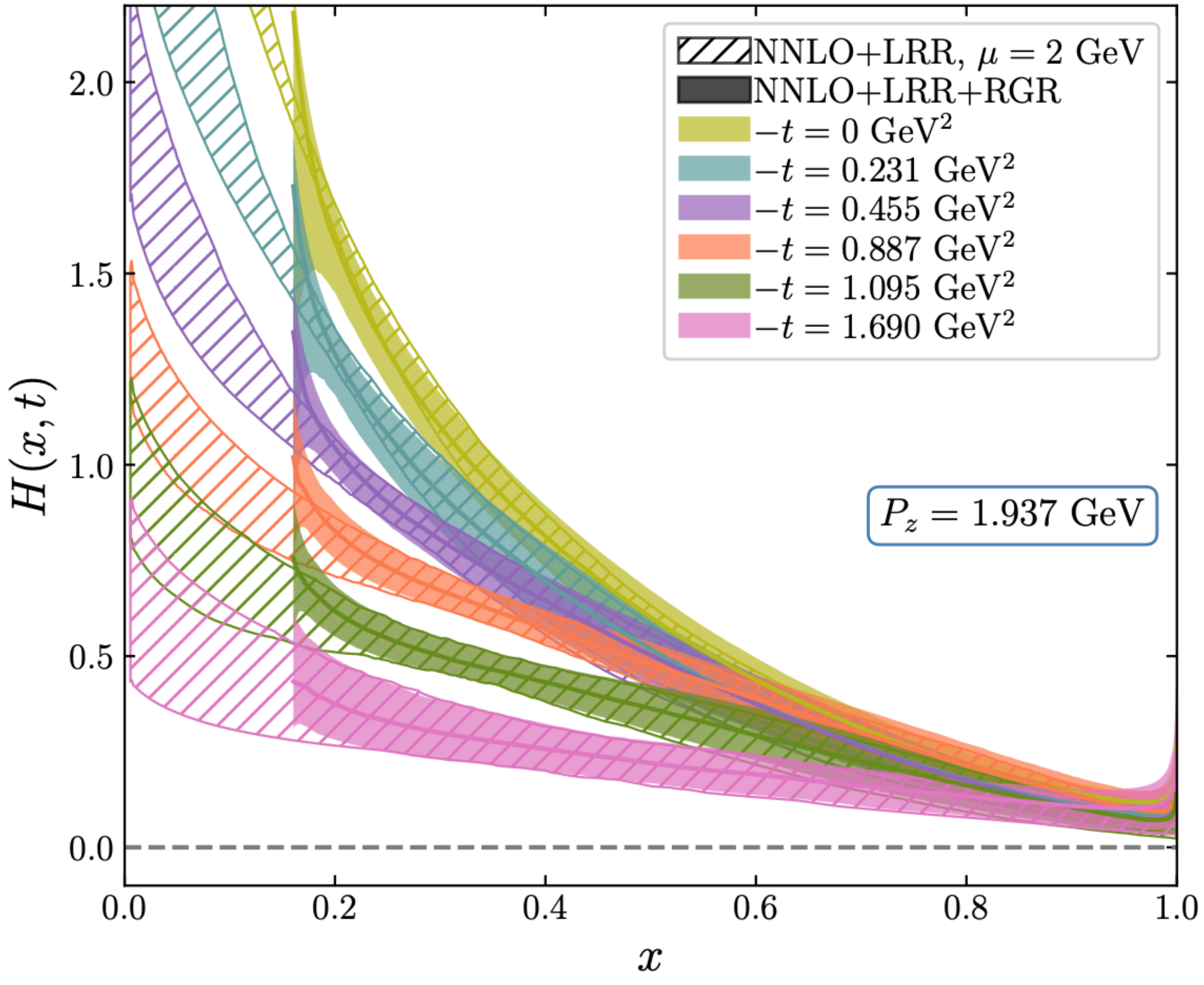


Perturbative matching

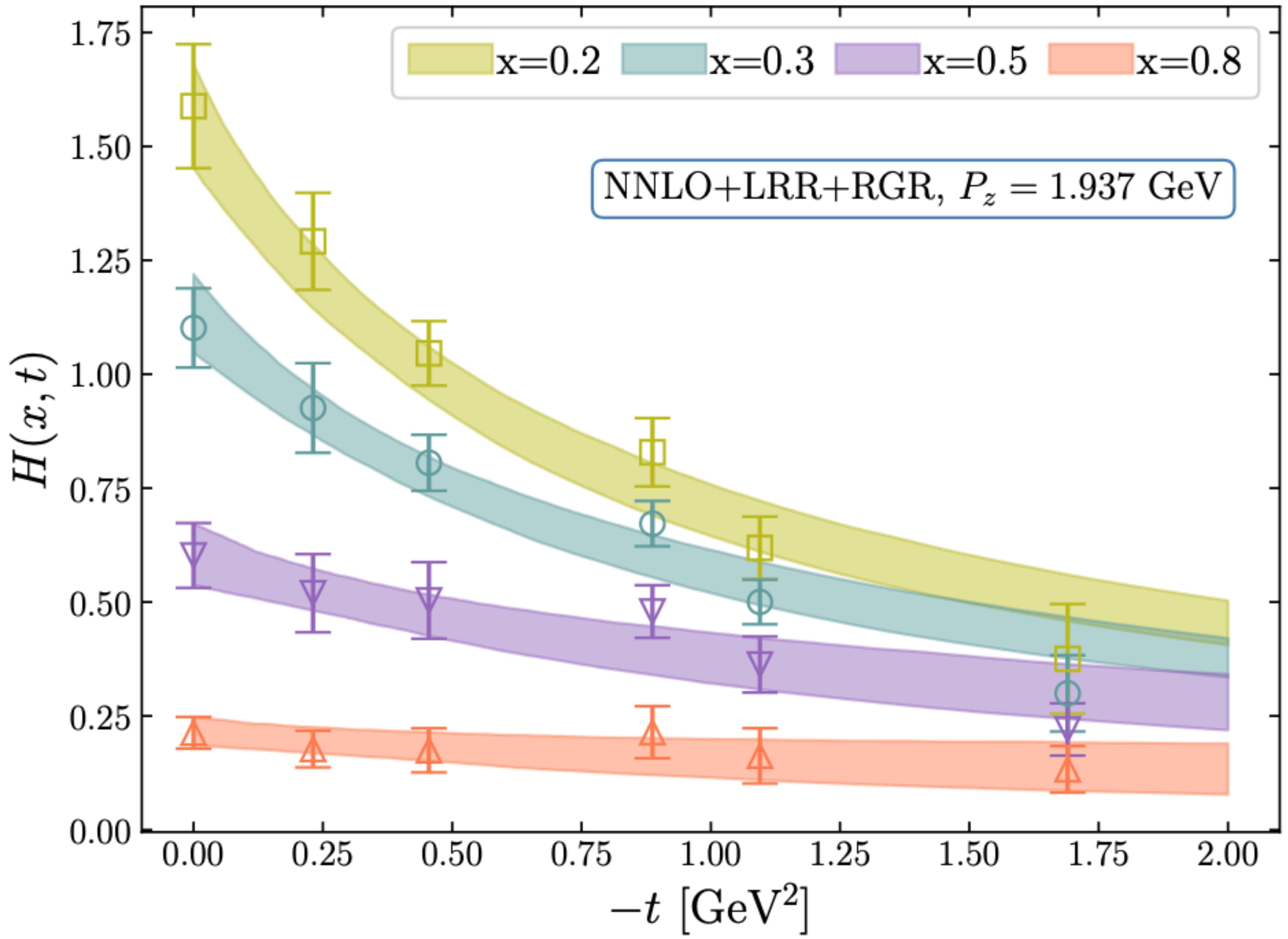
Significant at small and large x

Reduction of Pz dependence

Pion light-cone GPDs



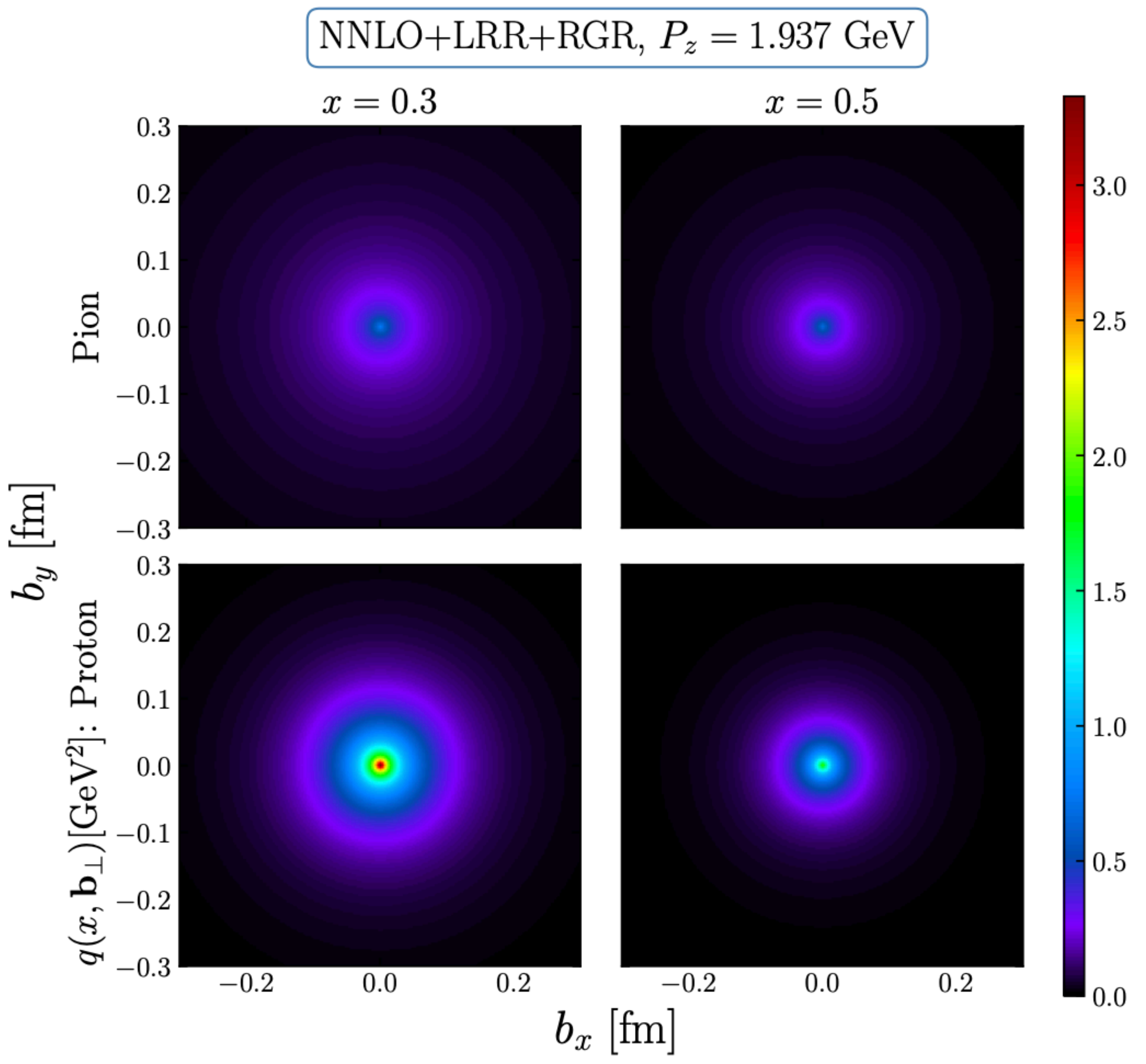
Monopole form: $H(x,t) = \dfrac{H(x,0)}{1 - t/M^2(x)}$



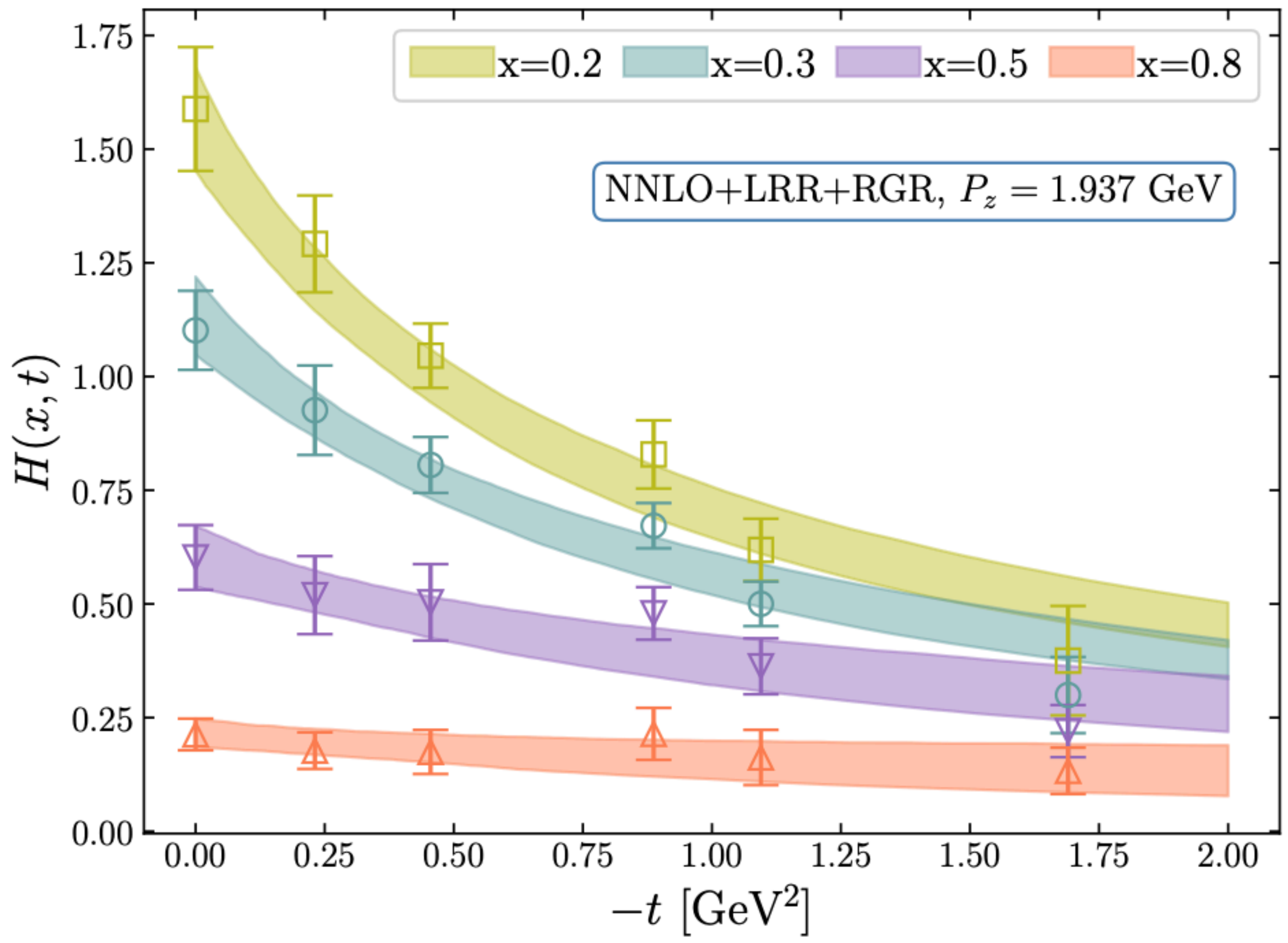
HTD, Xiang Gao(高翔), S. Mukherjee, P. Petreczky, **Qi Shi(施岐)**, S. Syritsyn and Y. Zhao, JHEP 02 (2025) 056, arXiv: 2407.03516

Impact-parameter-space Parton Distributions (IPD)

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, \mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$



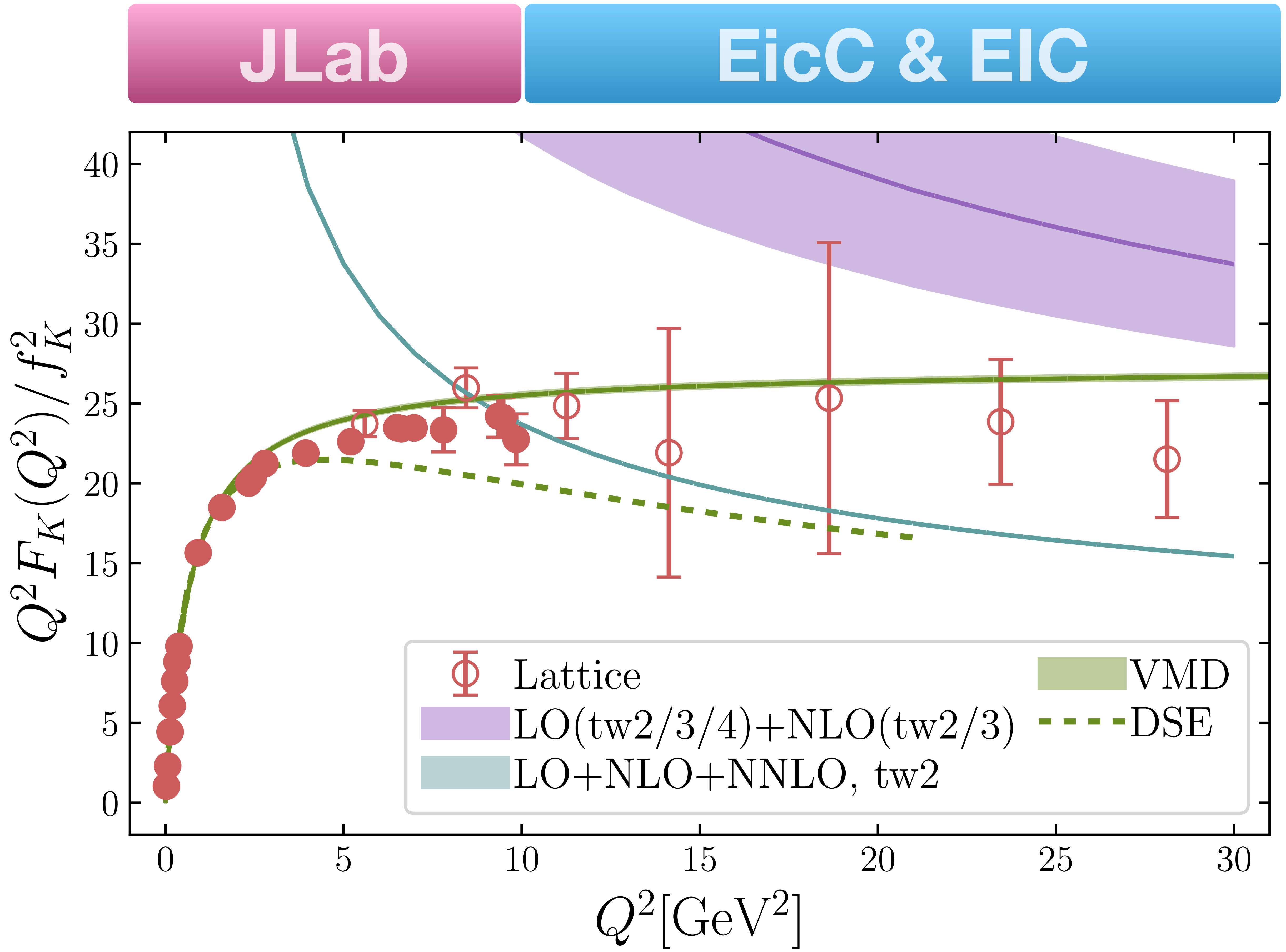
Monopole form: $H(x, t) = \dfrac{H(x,0)}{1 - t/M^2(x)}$

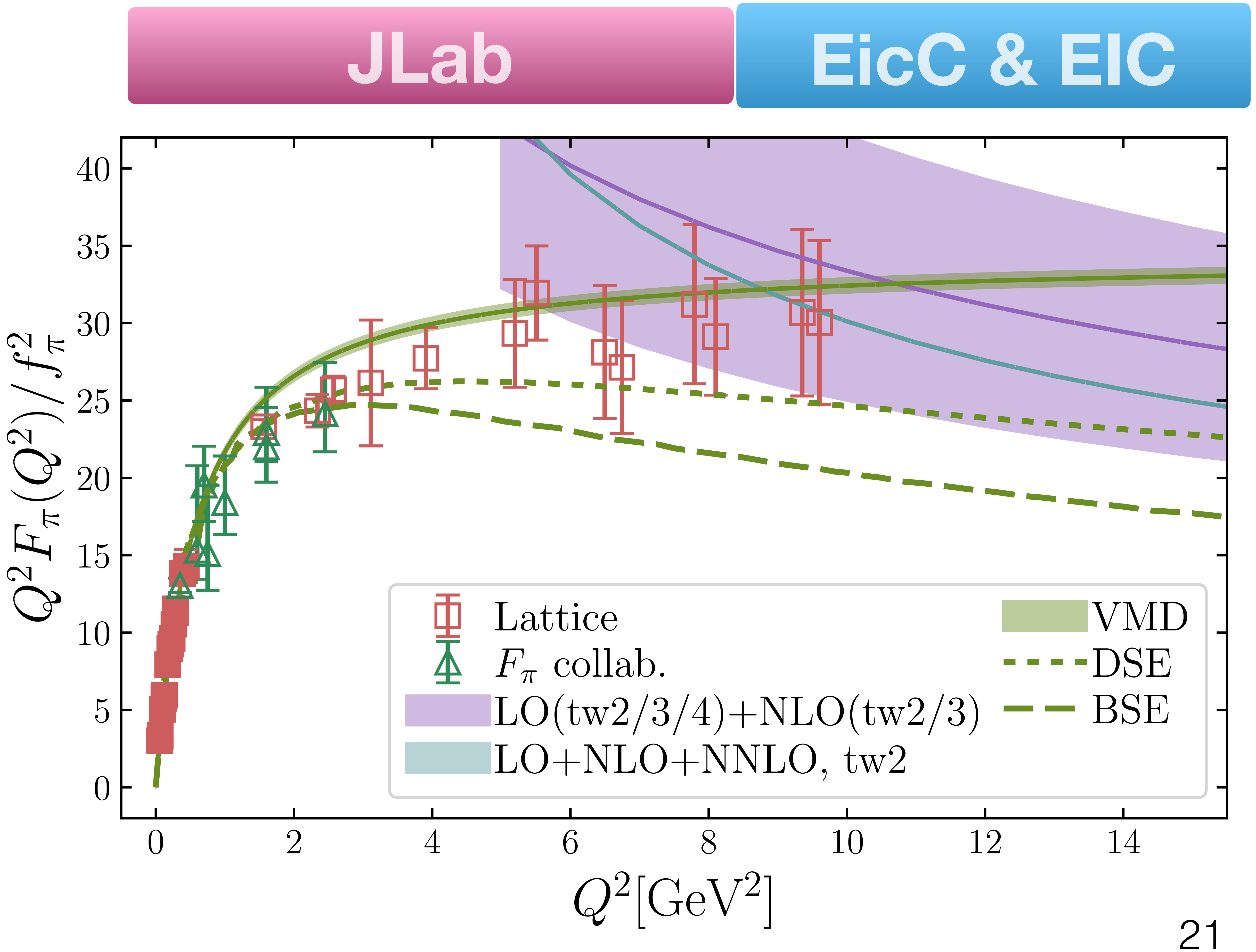


Data of proton from Cichy et al., arXiv:2304.14970

Summary

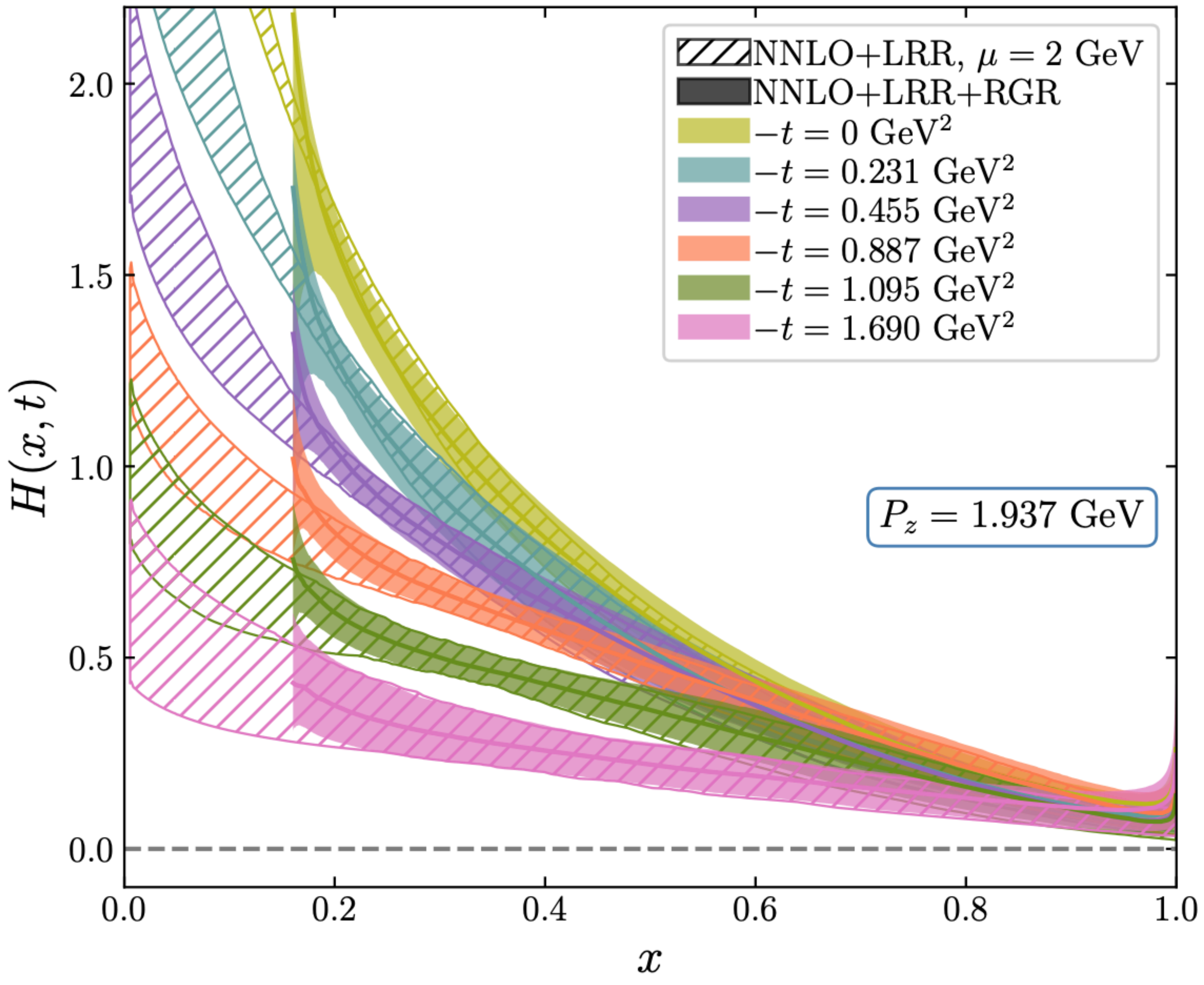
☑ A first LQCD prediction of Kaon and Pion electromagnetic form factors with Q^2 up to ~28 and 10 GeV^2, respectively

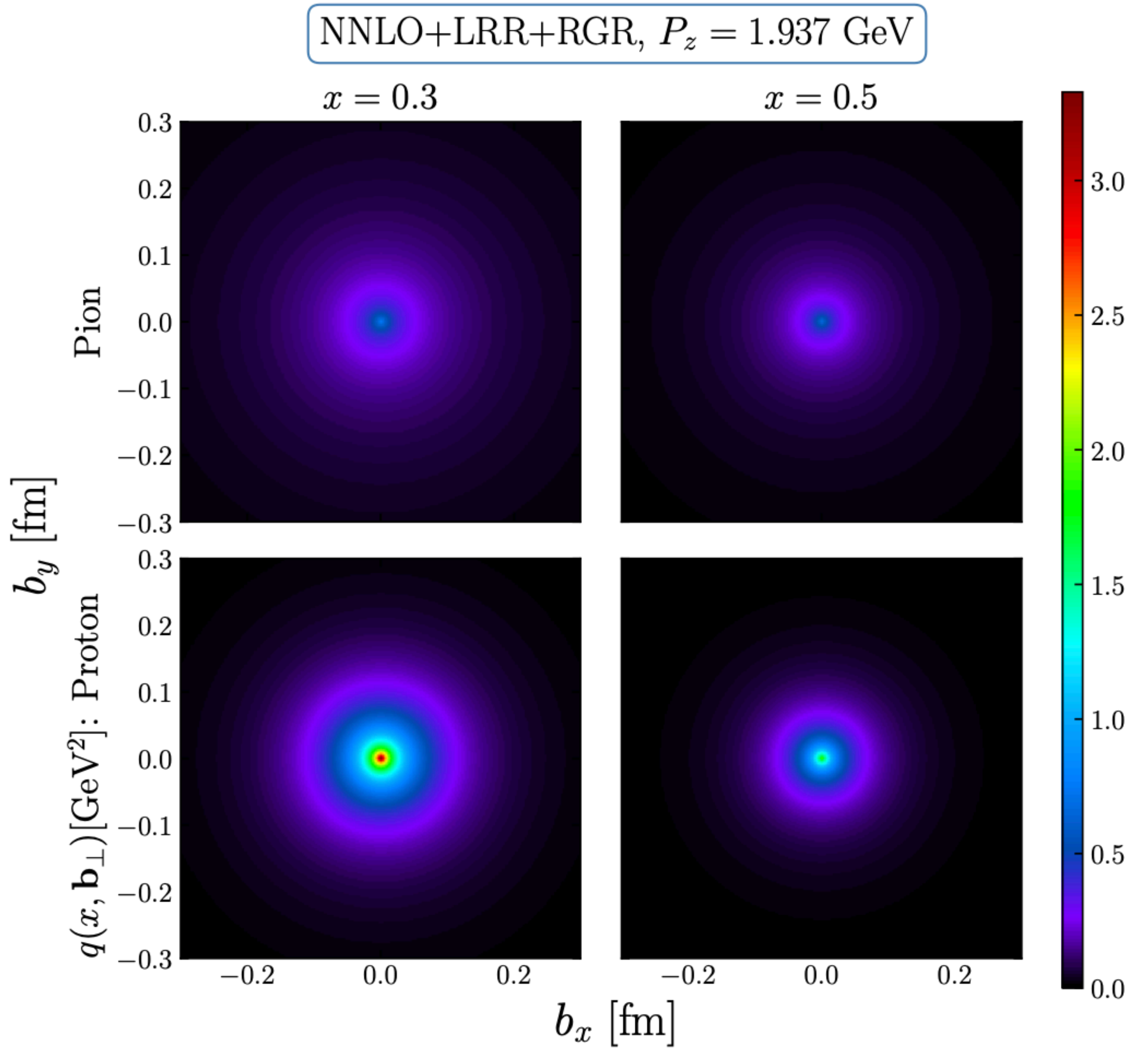




Summary

☑ State-of-the-art lattice QCD computations of the Pion GPD in the non-Breit frame





Backup

I: Renormalization: Hybrid scheme

linear divergence

$$M^{\mathrm{B}}(z,a) = Z(a) e^{-\delta m(a)|z|} e^{-\bar{m}_0|z|} M^{\mathrm{R}}(z,a)$$

z-independent logarithmic divergence

address the scheme dependence of δm & to match the lattice scheme to the $\overline{\mathrm{MS}}$ scheme

- Hybrid scheme: short and long distances Ji et al., NPB 964 (2021) 115311

$$M^{\mathrm{R}}(z, z_s; P_z, t) = \begin{cases} \dfrac{M^{\mathrm{B}}(z, P_z, t)}{M^{\mathrm{B}}(z, 0, 0)}, & |z| \le |z_s|; \\ \dfrac{M^{\mathrm{B}}(z, P_z, t)}{M^{\mathrm{B}}(z_s, 0, 0)} e^{(\delta m + \bar{m}_0)|z - z_s|}, & |z| > |z_s|. \end{cases}$$

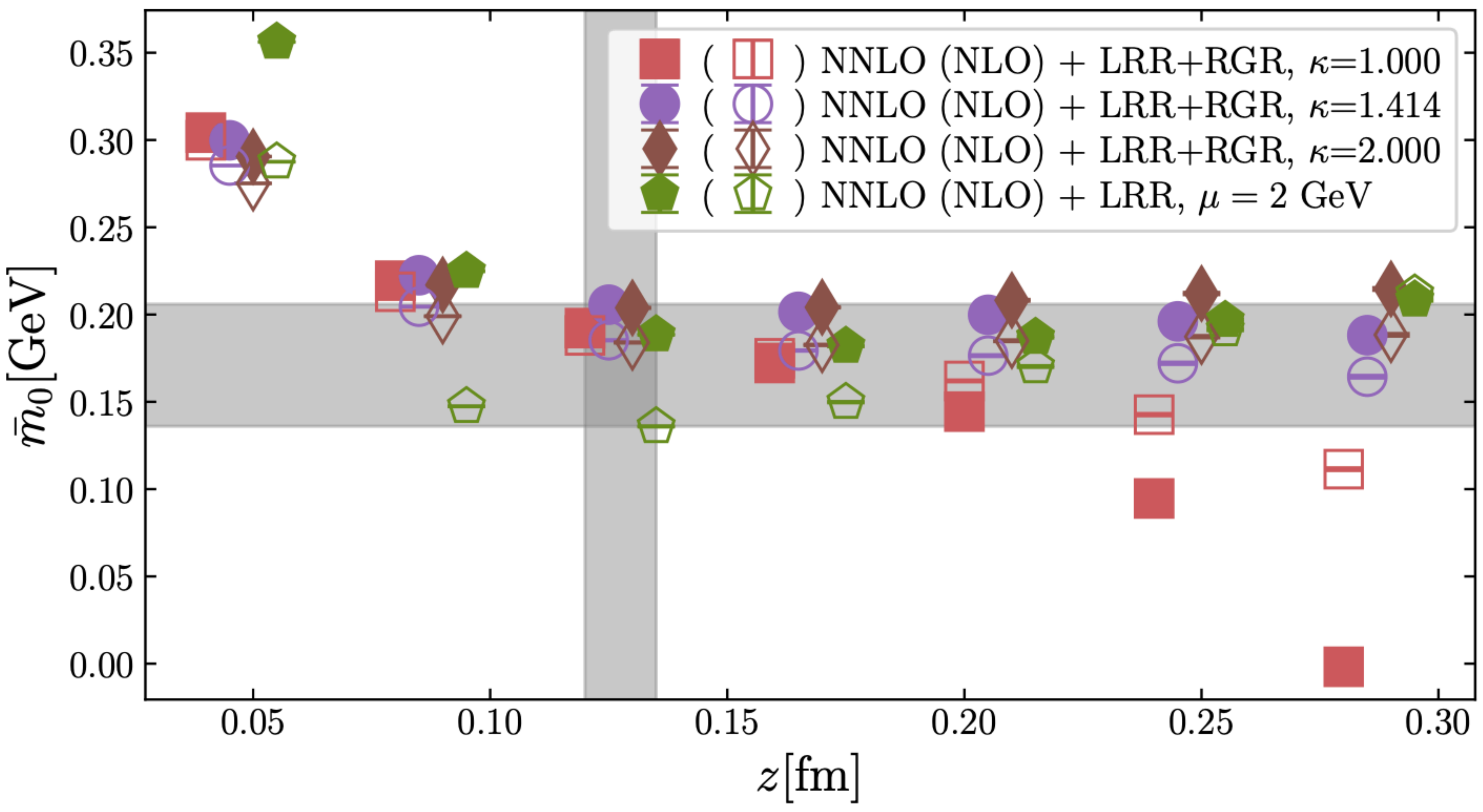
- $a\delta m$ = 0.1508(12) for a =0.04fm Gao et al., PRL 128 (2022) 142003

Determination of $\bar{m}_0$

Lattice results at Pz=t=0

$\overline{\text{MS}}$ OPE expression

$$e^{(\delta m+\bar{m}_0)\delta z}\frac{M^{\rm B}(z+\delta z)}{M^{\rm B}(z)}=\frac{C_0(\alpha_s(\mu_0(z+\delta z)),\mu_0^2(z+\delta z)^2)}{C_0(\alpha_s(\mu_0(z)),\mu_0^2z^2)}\times\exp\left[\int_{\alpha_s(\mu_0(z+\delta z))}^{\alpha_s(\mu_0(z))}\frac{d\alpha_s(\mu')}{\beta[\alpha_s(\mu')]}\gamma_O[\alpha_s(\mu')]\right]$$



- Wilson coefficient: C_0

  NNLO or NLO

  + Leading Renormalon Resummation (LRR)

Li et al., PRL 126(2021)072001
Chen et al., PRL 126(2001)072002
Holligan et al., NPB993(2023)116282
Zhang et al., PLB 844(2023)138081

- Renormalization Group Resummation (RGR)

  NNLL

Gao et al., PRD 103(2021)094504

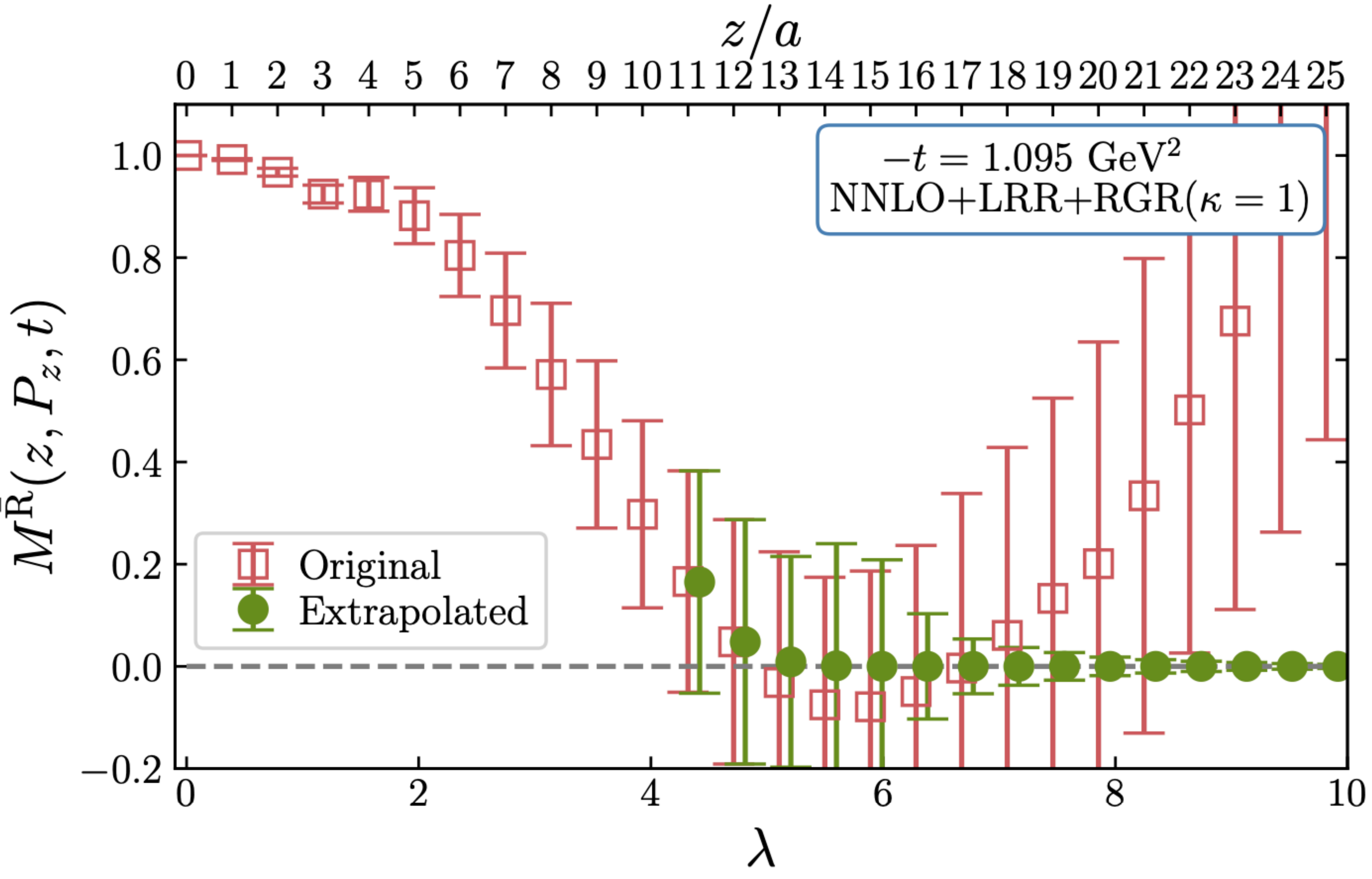
- $\mu_0 = 2\ \kappa\ e^{-\gamma_E}/\,|z|$

- γ_0 : the anomalous dimension of the quark bilinear operator, up to 3 loops

Braun et al., JHEP 07(2004)161

II: quasi-GPD $\tilde{H}$ from the $M^{\bar{R}}$

$$\widetilde{H}(x, P_z, t) = 2\int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{ix\lambda} \widetilde{H}_{\mathrm{LI}}(zP_z, t, z^2) = F(P_z, t) \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{\pi} e^{ix\lambda} M^{\bar{\mathrm{R}}}(z, P_z, t)$$



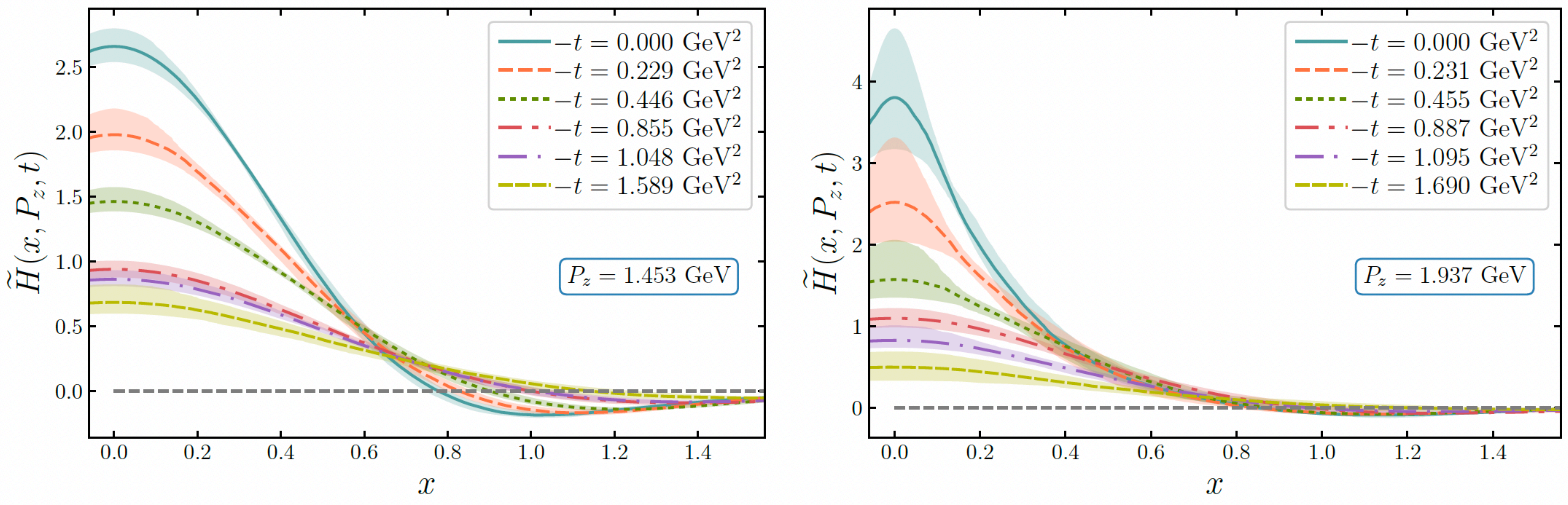
Unphysical oscillation in large λ

Extrapolation

$$M^{\bar{\mathrm{R}}} = A \frac{e^{-mz}}{(zP_z)^d}$$

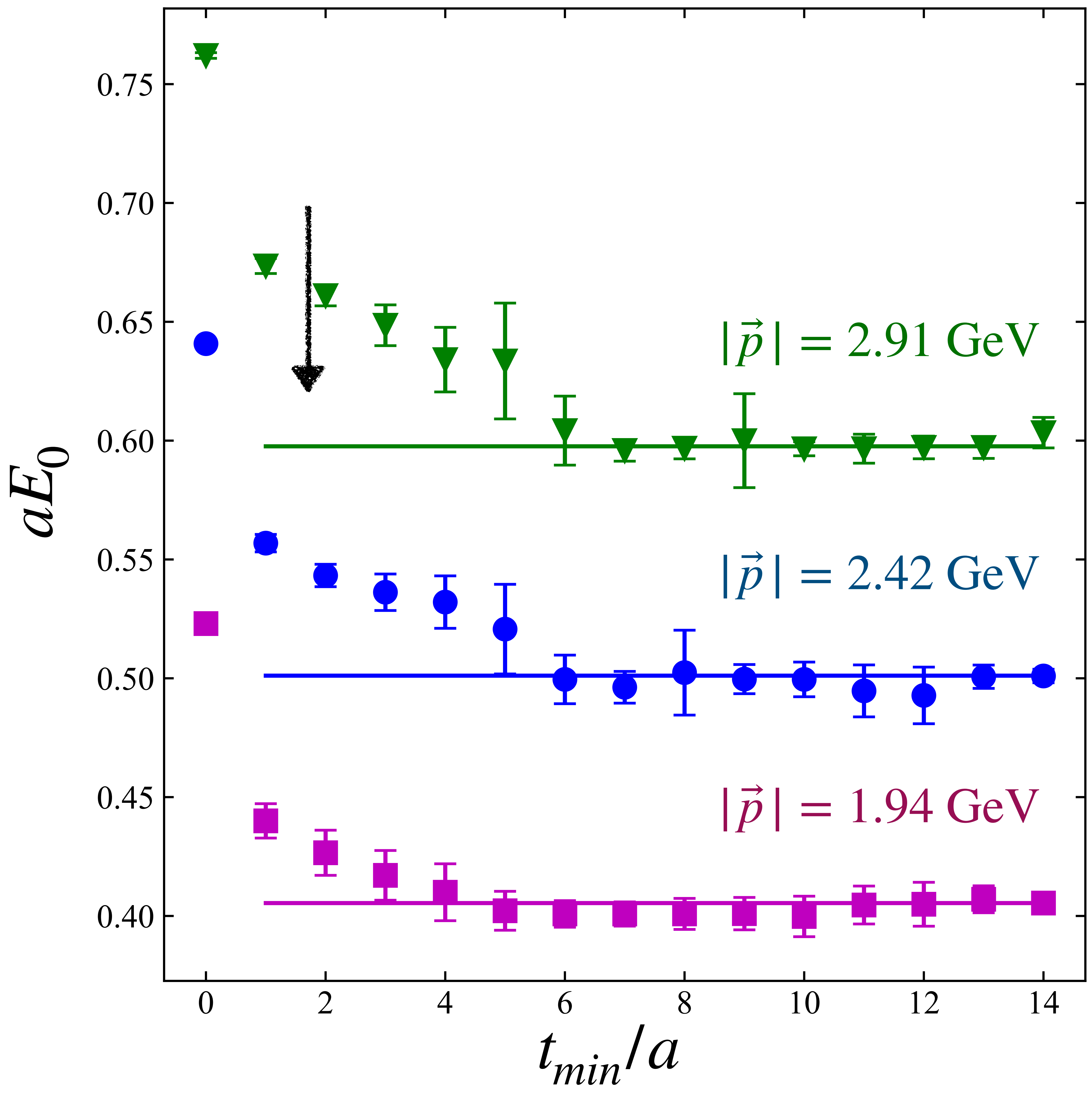
II: quasi-GPD $\tilde{H}$ from the $M^{\bar{R}}$

$$\widetilde{H}(x, P_z, t) = 2\int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{ix\lambda} \widetilde{H}_{\mathrm{LI}}(zP_z, t, z^2) = F(P_z, t) \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{\pi} e^{ix\lambda} M^{\bar{\mathrm{R}}}(z, P_z, t)$$

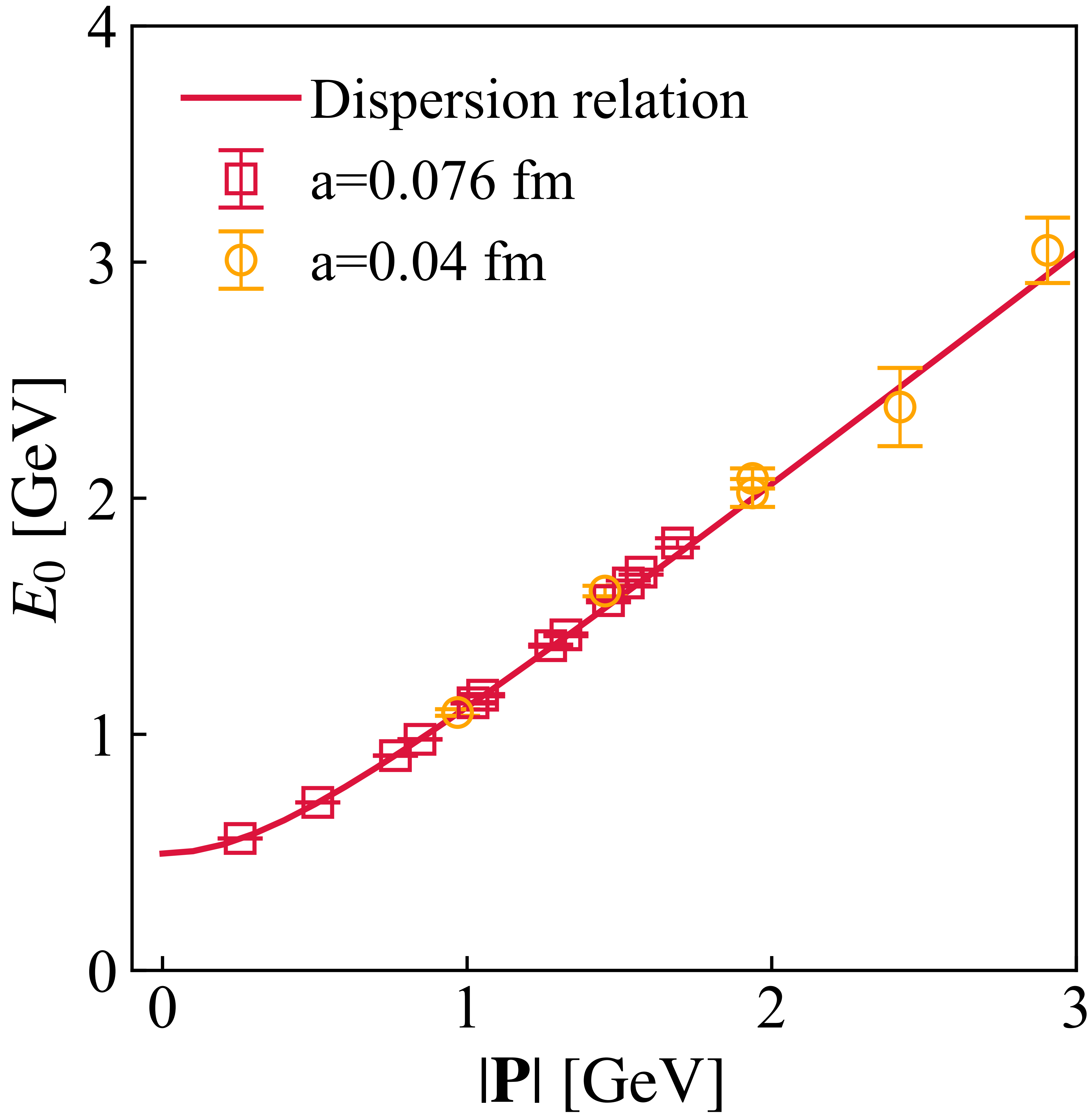


Kaon at large momentum

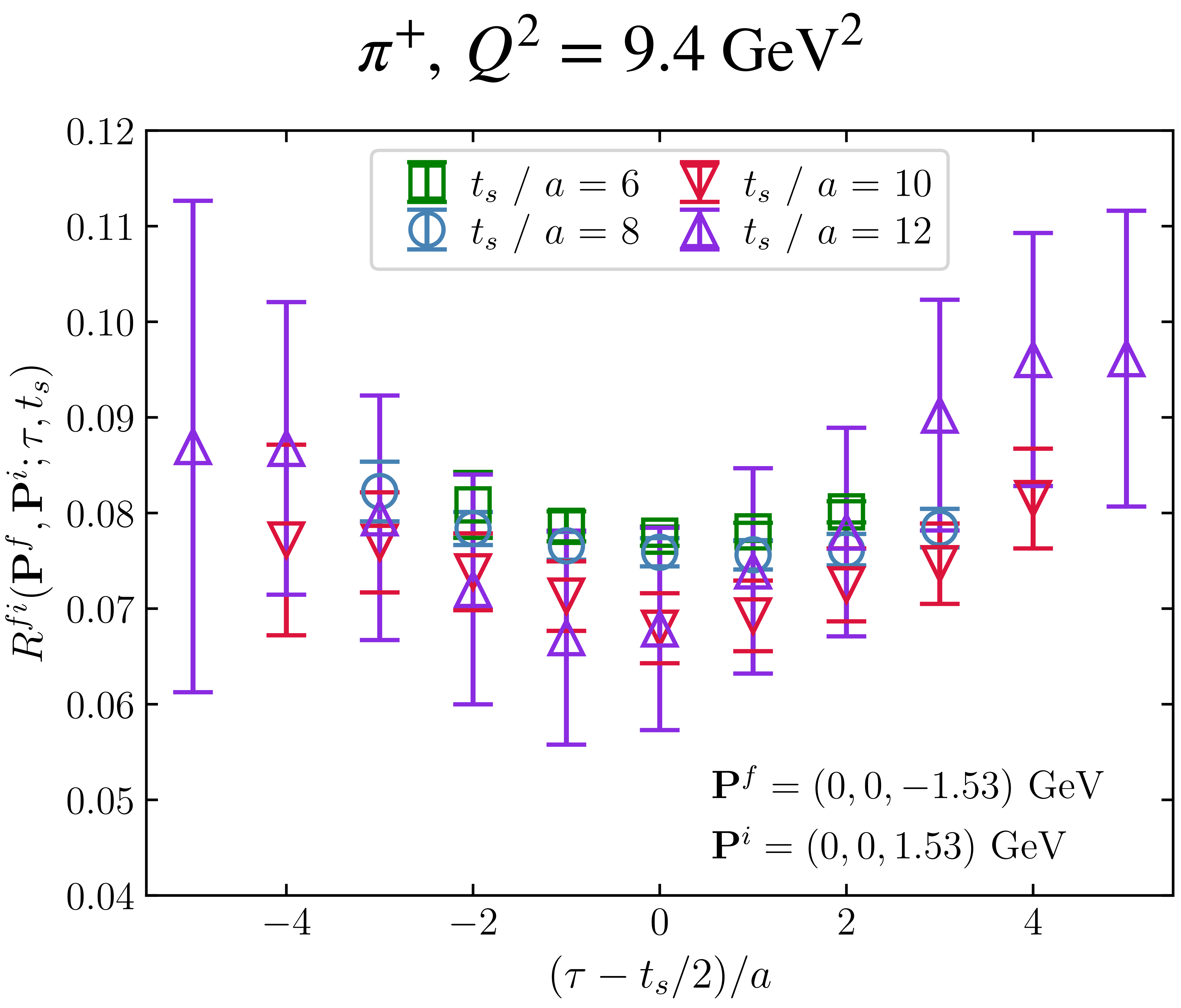
Dispersion relation $E_0(\vec{p}) = \sqrt{m_{K^+}^2 + \vec{p}^2}$

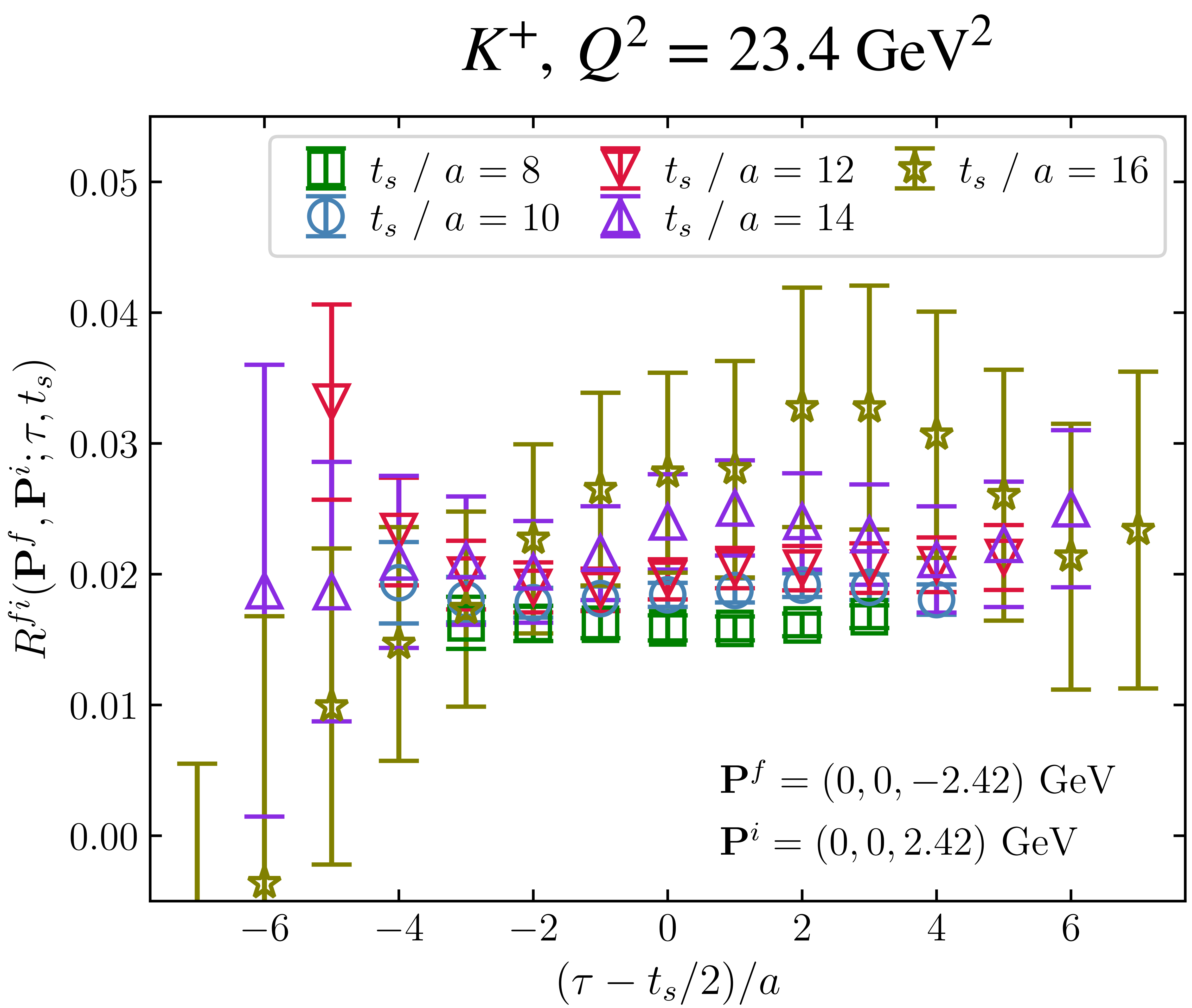


P up to ~3GeV

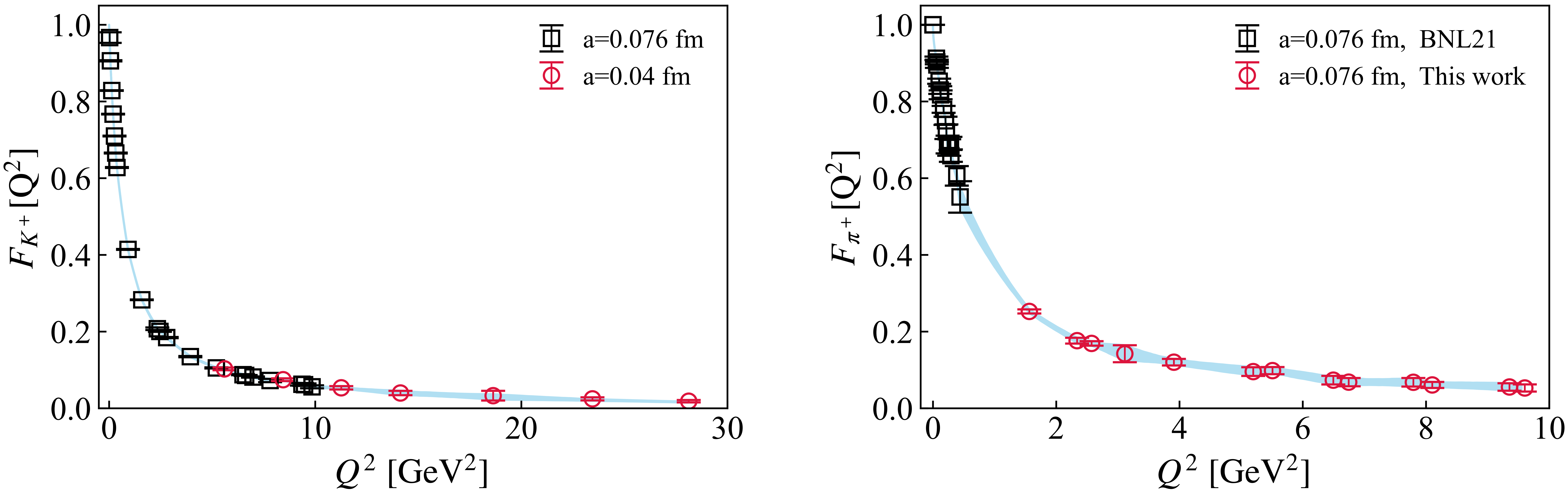


Lattice data of $R^{fi} \sim C_{3pt}/C_{2pt}$





Renormalized Pion and Kaon form factors



$$F_M(Q^2 \to \infty) = 8\pi\alpha_s(Q^2)f_M^2/Q^2$$

$$Q^2F_M(Q^2)/f_M^2 = 8\pi\alpha_s(Q^2)$$

Lattice QCD setup for GPD

- N_f=2+1 QCD on 64^3x64 lattices with a= 0.04 fm ([HotQCD] configurations)

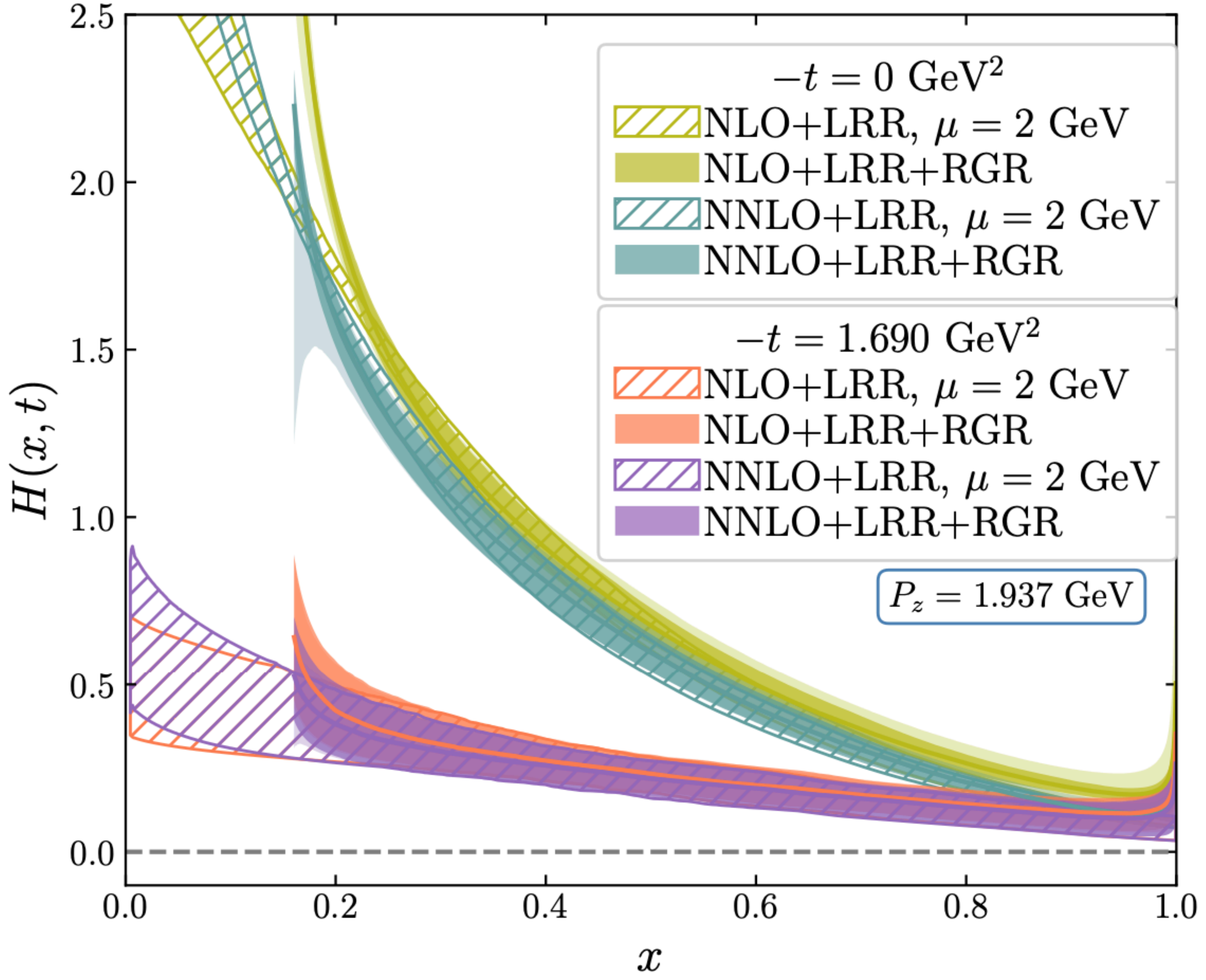
  Sea quark: Highly Improved Staggered Quark (HISQ) action

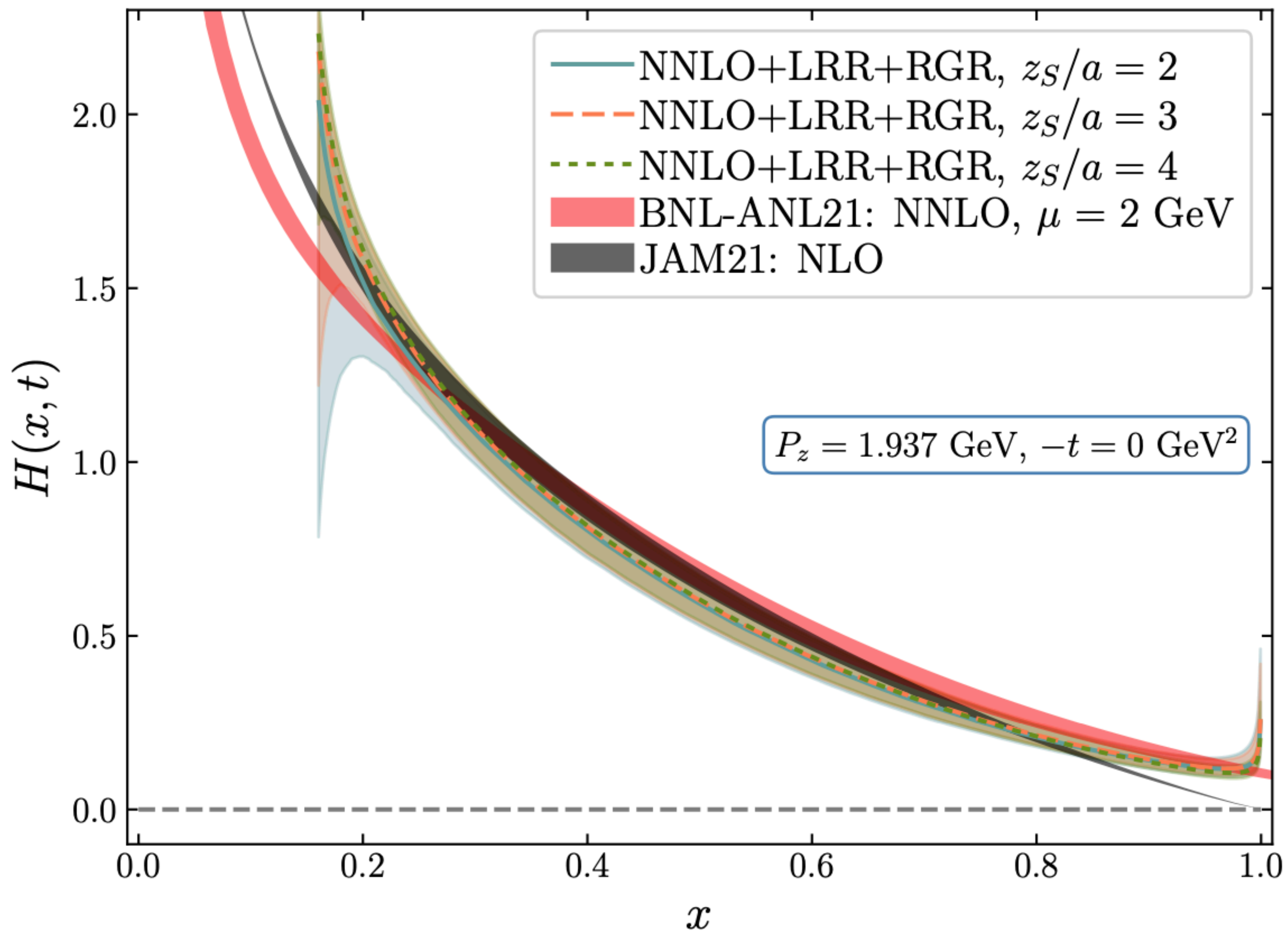
  Valence quark: Wilson-Clover action

- Sea quark pion mass: 160 MeV, valence pion mass 300 MeV

Frame	t_s/a	$\mathbf{n}^f = (n_x^f, n_y^f, n_z^f)$	m_z	P_z[GeV]	$\mathbf{n}^\Delta = (\lvert n_x^\Delta\rvert, \lvert n_y^\Delta\rvert, \lvert n_z^\Delta\rvert)$	$-t$[GeV^2]	#cfgs	(#ex, #sl)
Breit	9,12,15,18	(1, 0, 2)	2	0.968	(2, 0, 0)	0.938	115	(1, 32)
non-Breit	9,12,15,18	(0,0,0)	0	0	(0,0,0)	0	314	(3, 96)
	9,12,15,18	(0,0,2)	2	0.968	(1,2,0)	0.952	314	(4, 128)
	9,12,15	(0,0,3)	2	1.453	[(0,0,0), (1,0,0) (1,1,0), (2,0,0) (2,1,0), (2,2,0)]	[0, 0.229, 0.446, 0.855, 1.048, 1.589]	314	(4, 128)
	9,12,15	(0,0,4)	3	1.937		[0, 0.231, 0.455, 0.887, 1.095, 1.690]	564	(4, 128)

Dependences on the perturbative matching and renormalization scales

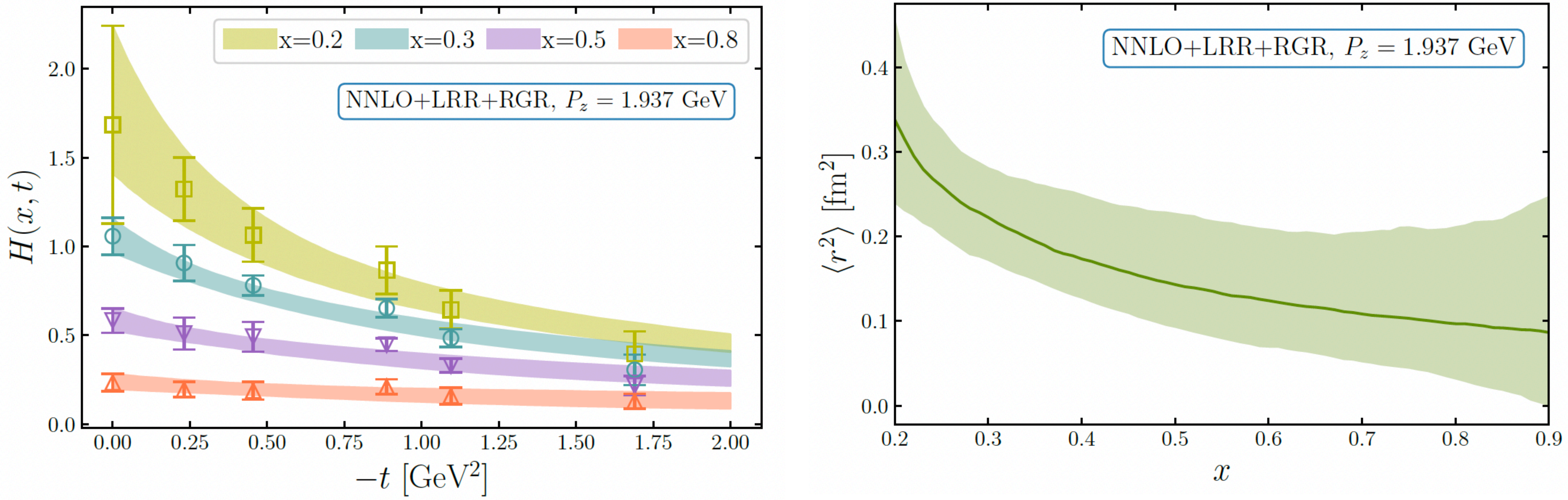




Parameterization of pion GPDs

Monopole form: $H(x,t) = \dfrac{H(x,0)}{1 - t/M^2(x)}$

$\langle r^2(x) \rangle = 6/M^2(x)$



x=0.2, comparable to pion charge radius obtained from the Pion EMFF, $\langle r_\pi^2 \rangle = 0.313(27)\text{fm}^2$

Gao et al., PRD 104(2021)114515

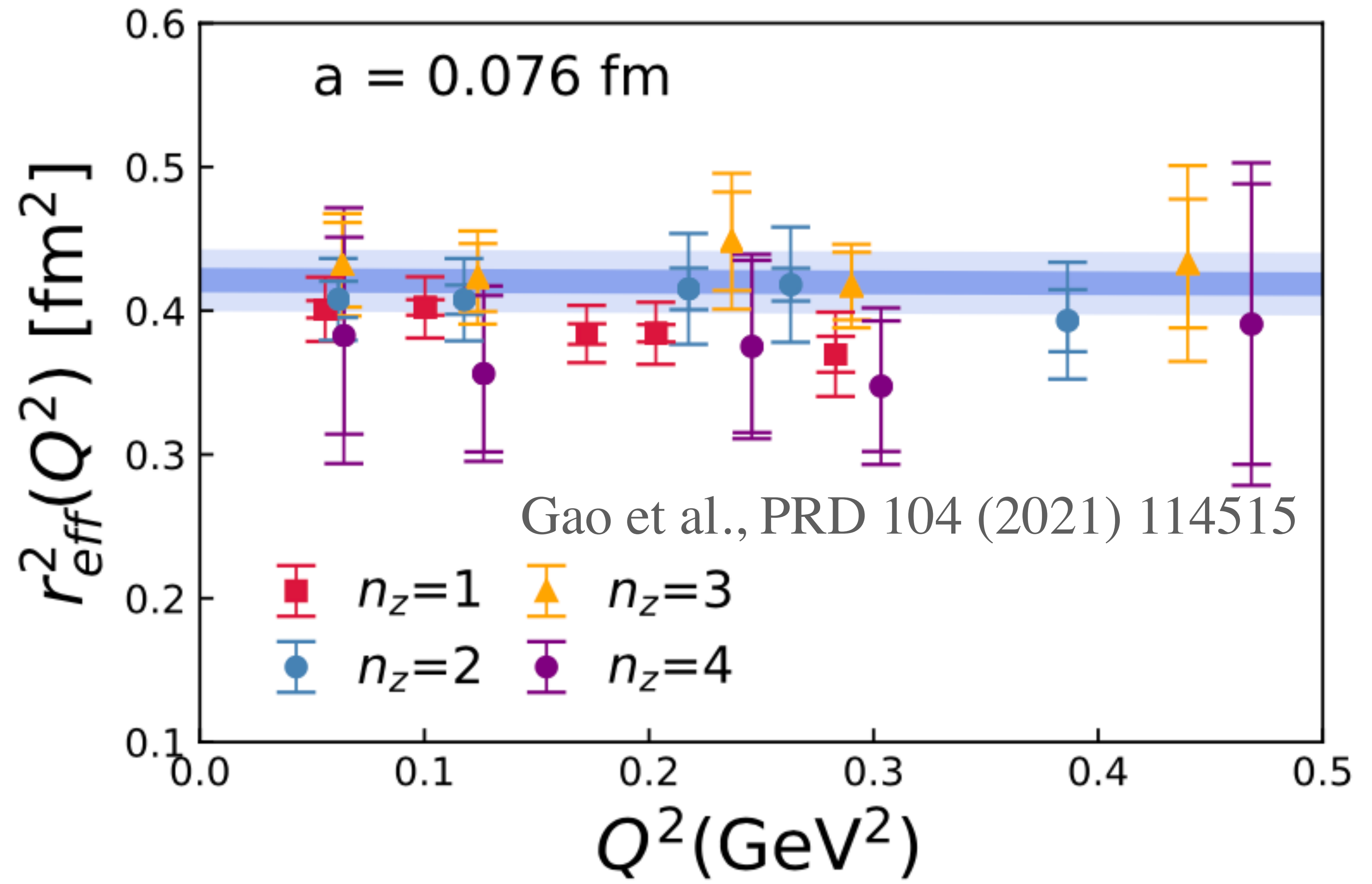
Pion/kaon EM form factors

Small Q^2 limit: hadronic picture

- Vector Meson Dominance -> Charge radius

$$r^2_{eff}(Q^2) = \frac{6(1/F_\pi(Q^2)-1)}{Q^2}.$$

$$\langle r_\pi^2 \rangle = 0.42(2)\ \mathrm{fm}^2, \langle r_\pi^2 \rangle_{PDG} = 0.434(5)\ \mathrm{fm}^2$$



Large Q^2 limit: partonic picture

$$Q^2F_M(Q^2) \approx 16\pi\,\alpha_s(Q^2)\, f_M^2\ \omega_M^2(Q^2), \quad \omega_M^2(Q^2) = e_{\bar{q}}\omega_{\bar{q}}^2(Q^2) + e_u\omega_u^2(Q^2)$$

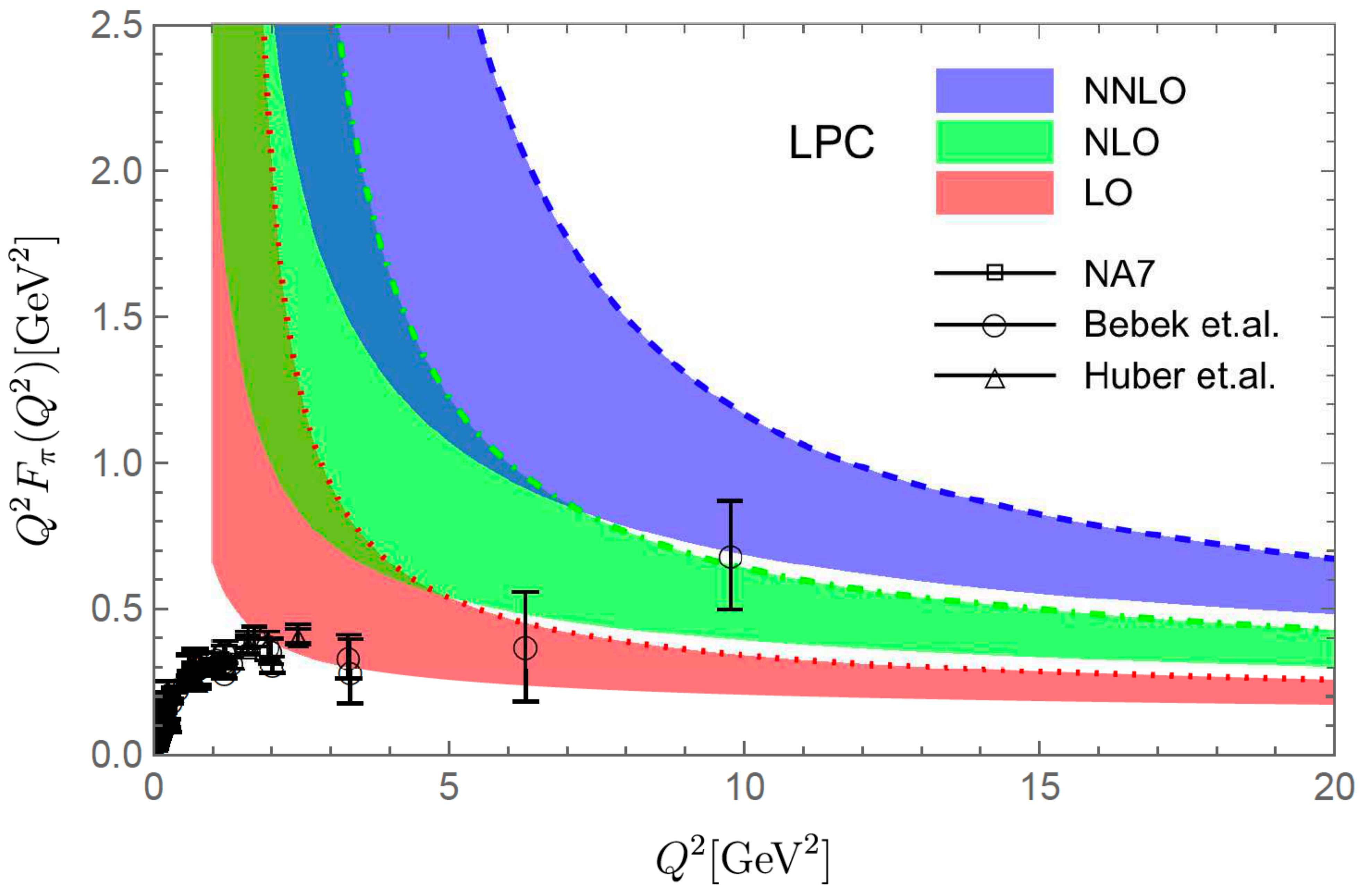
Lepage & Brodsky, 79', 80'
Efremov & Radyushkin 80'

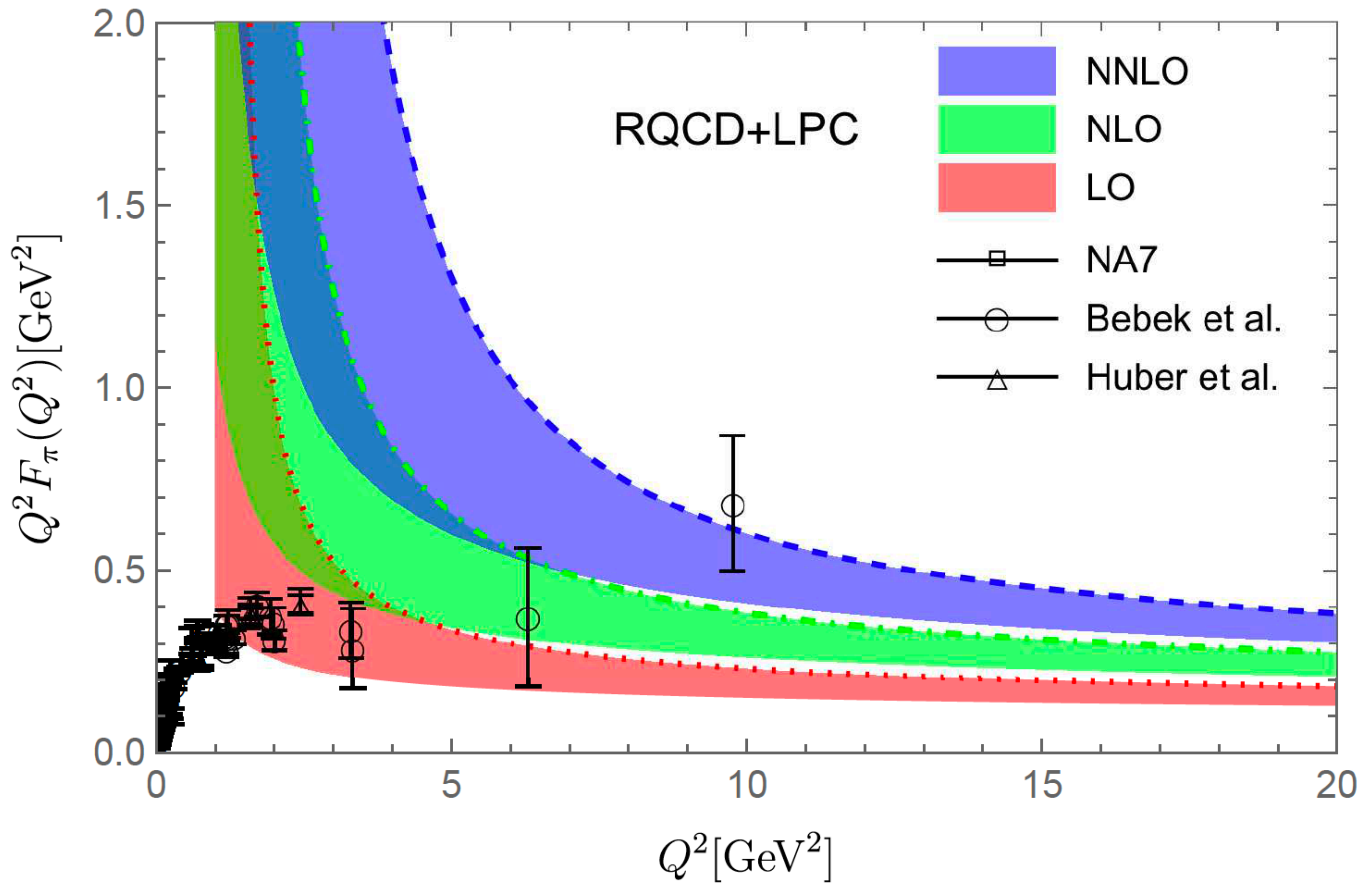
$$\omega_f = \frac{1}{3}\int_0^1 dx q_f(x)\phi_M(x,Q^2)$$

leading-twist parton distribution amplitude (DA)

- Asymptotic DA: $\phi_M(x, Q^2 \to \infty) = 6x(1-x)$
- DA from LQCD: pion & kaon etc. J. Hua et al. [LPC], Phys.Rev.Lett. 129 (2022) 13
Gao et al., PRD 106 (2022) 074505,G. Bali et al., JHEP 08 (2019) 065, …

Impact of Gegenbauer moments from DAs





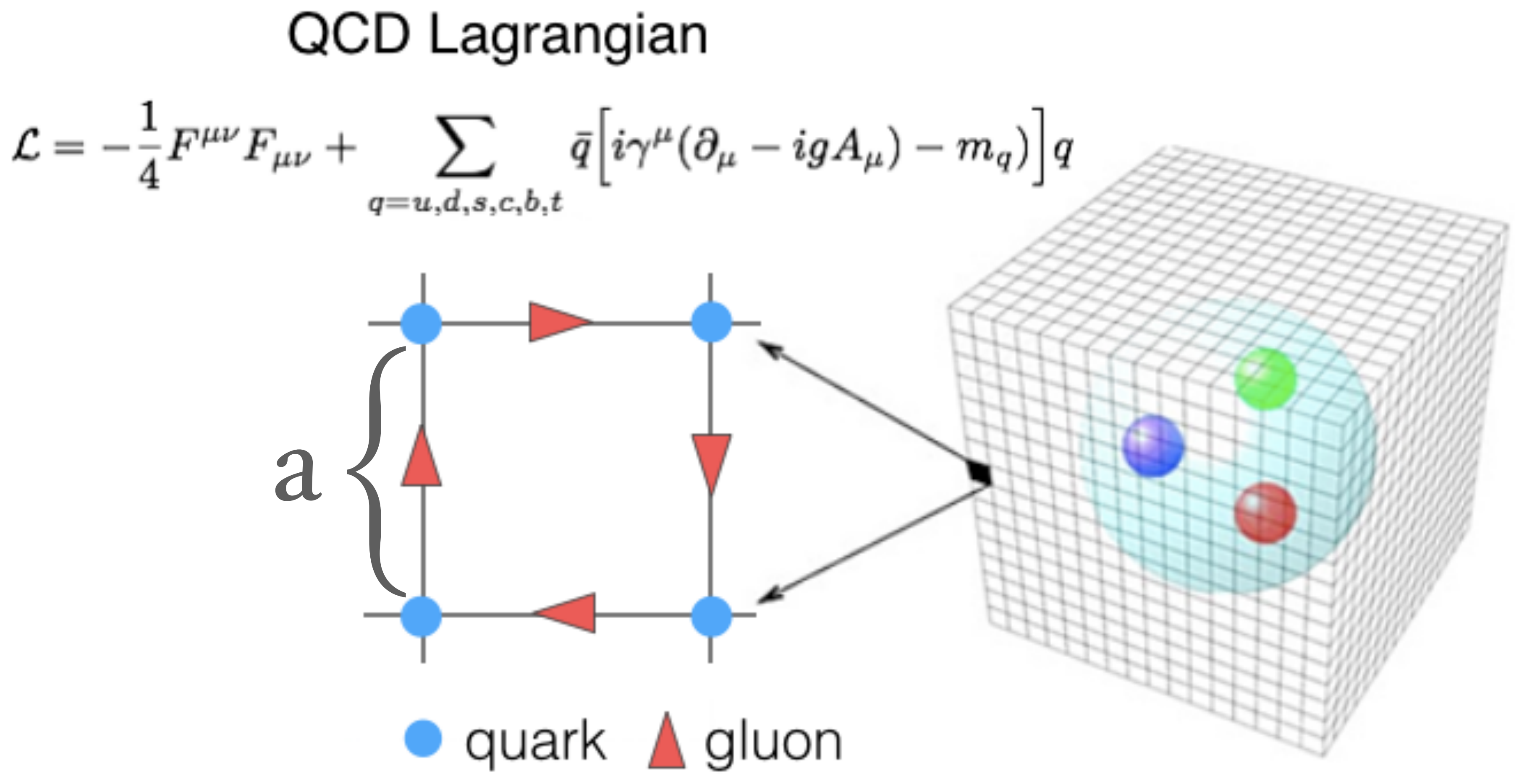
Chen, Chen, Feng & Jia, arXiv:2312.17228

RQCD: $a_2(2\ \mathrm{GeV}) = 0.116^{+0.019}_{-0.020}$

LPC: $a_2(2\ \mathrm{GeV}) = 0.258 \pm 0.087, \quad a_4(2\ \mathrm{GeV}) = 0.122 \pm 0.056, \quad a_6(2\ \mathrm{GeV}) = 0.068 \pm 0.038.$

In this work, pion: $a_2 = 0.196(32)$, $a_4 = 0.085(26)$, $a_6 = 0.056(15)$

Lattice QCD



- Lattice simulations of QCD give first principle results
- But need to have control of — "Goal"
  - ✤ Thermodynamic limit — $V = 2 \sim 4$ fm — $V \to \infty$
  - ✤ Continuum limit — $a = 0.1 \sim 0.04$ fm — $a \to 0$
  - ✤ Chiral extrapolation — $M_\pi \sim 500 \to 200$MeV — $M_\pi = 140$ MeV (Physical Point)
  - ✤ Statistical errors — $N_{conf} \sim \mathcal{O}(1000)$ — $N_{conf} \to \infty$
- Fast computers and algorithms are essential

EM form factor of Kaon: $< K(P_1)|J_\mu|K(P_2) > = (P_1 + P_2)_\mu F_K(Q^2)$