Baryon-antibaryon generalized distribution amplitudes

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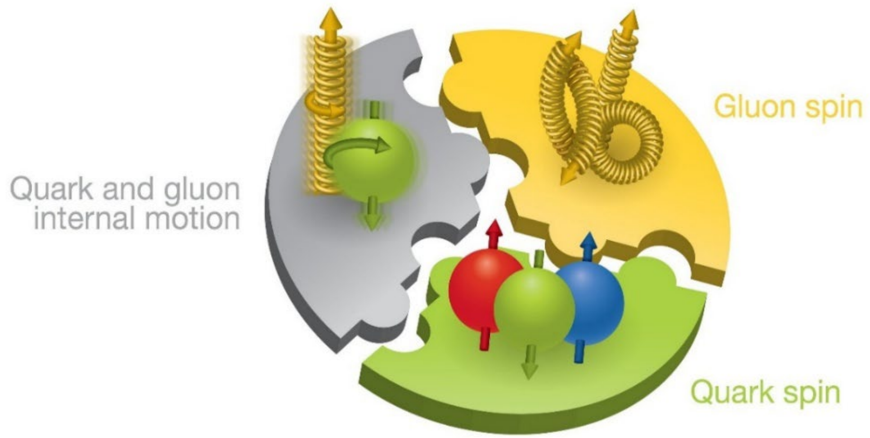
Reference : Jing Han, Bernard Pire, and Qin-Tao Song, PRD 112(2025) , 014048

GPDs and Proton spin puzzle

Proton Spin puzzle

Proton Spin:
- $\Delta u^+, \Delta d^+, \Delta s^+$: ~30%
- $\Delta L, \Delta g$: rest part



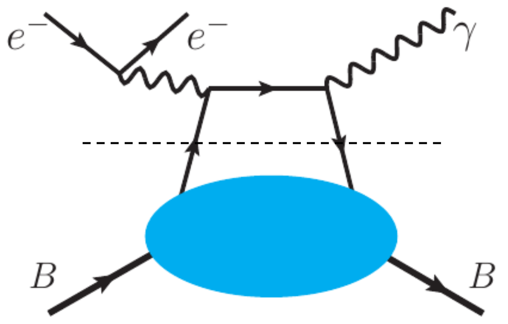
Generalized Parton Distributions (GPDs)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|\bar{q}\left(-\frac{1}{2}z\right)\gamma^+ q\left(\frac{1}{2}z\right)|p\rangle_{(z^+=0,\vec{z}=0)}$$

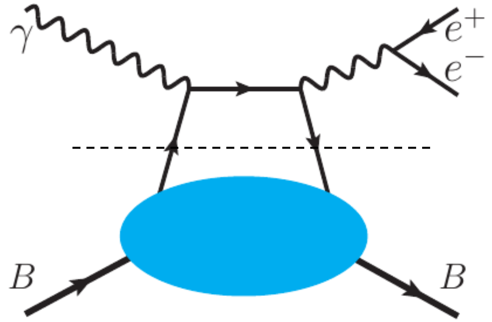
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

X.-D. Ji, PRL 78, 610 (1997)

GPDs and GFFs



$eB \to e\gamma B$ (DVCS)



$\gamma B \to e^+e^-B$ (TCS)

GPDs can be accessed in DVCS and TCS .Photon virtuality is large enough to satisfy QCD factorization, and the DVCS and TCS amplitudes can be separated into hard and soft parts. The soft part is described by GPDs.

From GPDs to Gravitational Form Factors(GFFs):

$$\int_{-1}^{1} dx\, xH^q(x,\xi,t) = A^q(t) + \xi^2 D^q(t)$$

$$\int_{-1}^{1} dx\, xE^q(x,\xi,t) = B^q(t) - \xi^2 D^q(t)$$

$$\frac{1}{2}[A^q(0) + B^q(0)] = J(0)$$

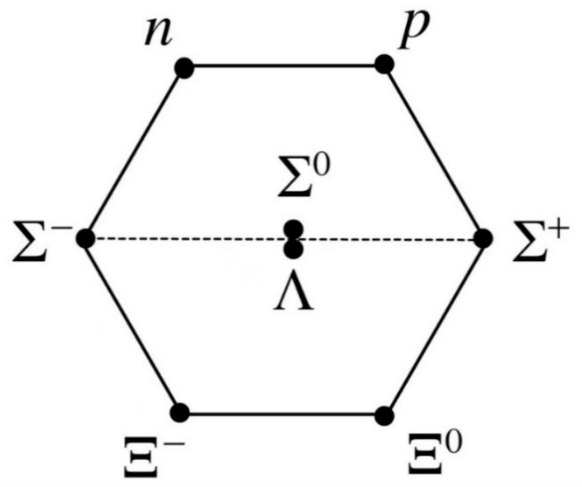
X.-D. Ji, PRL 78, 610 (1997)

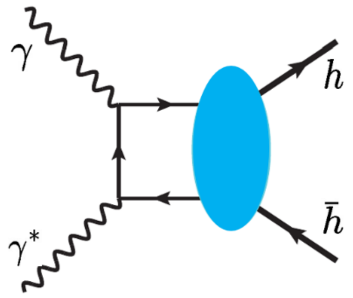
E. R. Berger, M. Diehl and B. Pire, EPJC 23 (2002) 675-689

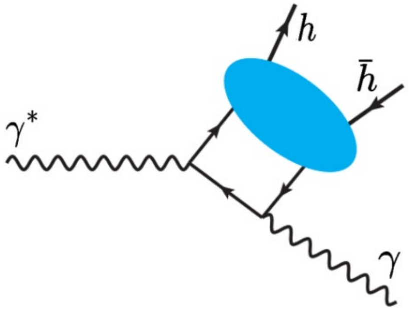
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025

Hardon GDAs

For unstable hadrons, it is difficult to measure the DVCS and TCS directly. In the baryon octet, only the proton GPDs can be probed.







By studying $\gamma\gamma^* \to h\bar{h}$ or $\gamma^* \to \gamma h\bar{h}$, one can access the Generalized Distribution Amplitudes (GDAs) of unstable hadrons.

QCD factorization: $Q^2 \gg s^2, \Lambda_{QCD}^2$.

GDAs describe the amplitude of $q\bar{q} \to h\bar{h}$.

M. Diehl, T. Gousset, B. Pire, PRD 62 (2000) 073014

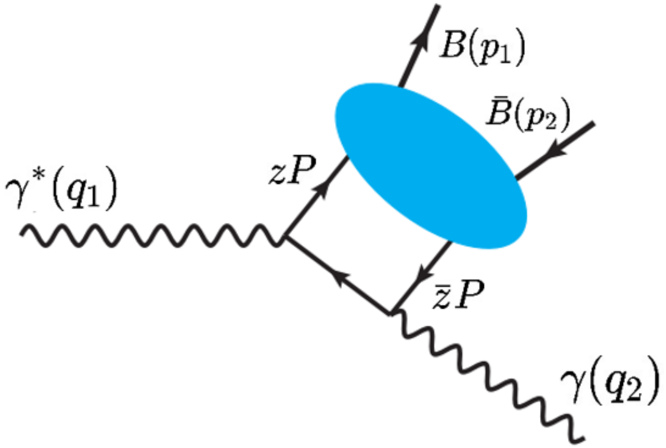
Z. Lu and I. Schmidt, PRD 73 (2006) 094021; 75 (2007) 099902(E)

Baryon GDAs

The GDAs of baryon-antibaryon pair

$$P^+ \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle B(p_1)\bar{B}(p_2) \big| \bar{q}(\bar{x})\gamma^+ q(0) |0\rangle$$
$$= \Phi_V^q(z, \zeta_0, \hat{s}) \bar{u}(p_1)\gamma^+ v(p_2) + \Phi_S^q(z, \zeta_0, \hat{s}) \frac{P^+}{2m} \bar{u}(p_1) v(p_2)$$

$$P^+ \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle B(p_1)\bar{B}(p_2) \big| \bar{q}(\bar{x})\gamma^+ \gamma_5 q(0) |0\rangle$$
$$= \Phi_A^q(z, \zeta_0, \hat{s}) \bar{u}(p_1)\gamma^+ \gamma_5 v(p_2) + \Phi_P^q(z, \zeta_0, \hat{s}) \frac{P^+}{2m} \bar{u}(p_1) \gamma_5 v(p_2)$$



The GDAs depend on following variables

$$z = \frac{k_1^+}{P^+} \qquad \zeta_0 = \frac{\Delta^+}{P^+} \qquad \hat{s} = (p_1 + p_2)^2$$

$$P = p_1 + p_2 \qquad \Delta = p_2 - p_1$$

M. Diehl, P. Kroll, and C.Vogt, EPJC 26 (2003) 567

GDAs and Timelike GFFs

From GDAs to timelike GFFs:

$$\int_0^1 dz(2z-1)\Phi_V^q(z,\zeta_0,\hat{s}) = -2\zeta_0 J^q(\hat{s})$$

$$\int_0^1 dz(2z-1)\Phi_S^q(z,\zeta_0,\hat{s}) = D^q(\hat{s}) + [A^q(\hat{s}) - 2J^q(\hat{s})](\zeta_0)^2$$

Jing Han, Bernard Pire, and Qin-Tao Song, PRD 112(2025) 014048

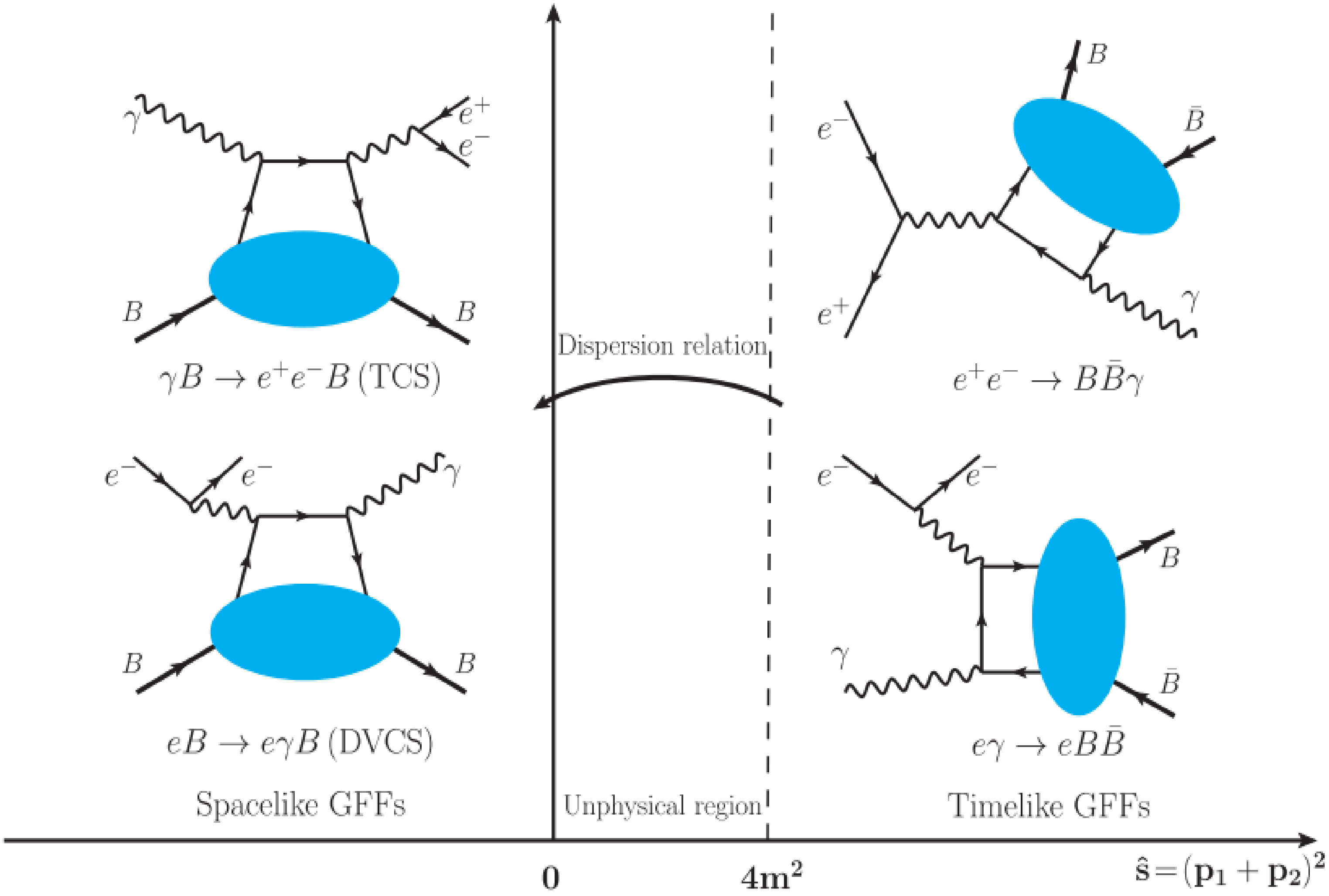
One can access spacelike GFFs by using dispersion relation

$$\mathcal{F}^B(t) = \int_{4m^2}^{\infty} \frac{d\hat{s}}{\pi} \frac{\mathrm{Im}\mathcal{F}^B(\hat{s})}{\hat{s}-t}$$

$\mathcal{F}^B$ is a GFF of the bayron

G. A. Miller, M. Strikman, and C. Weiss, PRD 83 (2011) 013006

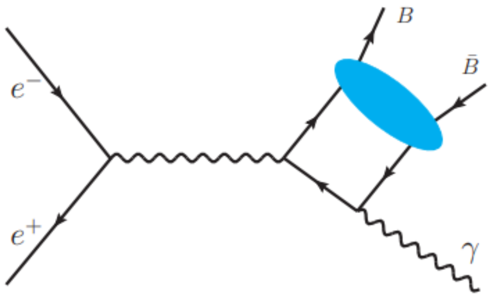
GDAs,GPDs and GFFs



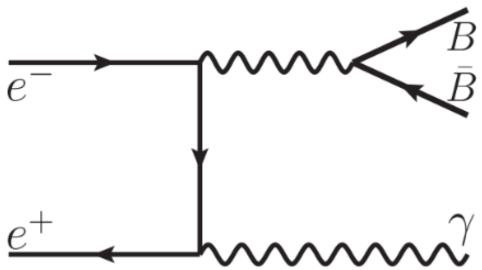
A complete image of GFFs can be accessed by GDAs

GDAs and $e^-e^+ \to B\bar{B}\gamma$

The process $e^-e^+ \to B\bar{B}\gamma$ includes two subprocesses.



$e^-e^+ \to \gamma^* \to \gamma B\bar{B}$



$e^-e^+ \to \gamma^*\gamma \to \gamma B\bar{B}$

(1) QCD subprocess: $e^-e^+ \to \gamma^* \to B\bar{B}\gamma$, the blob represents the $B\bar{B}$ GDAs

$$\Phi_i^q(z, \zeta_0, \hat{s}), \quad \text{for } i = V, S, A, P$$

(2) ISR subprocess: $e^-e^+ \to \gamma^*\gamma \to B\bar{B}$, the $\gamma^* \to B\bar{B}$ vertex is parameterized in terms of the electromagnetic(EM) FFs

$$\langle \bar{B}(p_2)B(p_1)|\, \bar{q}(0)\gamma^\mu q(0)\,|0\rangle = F_V^q(\hat{s})\bar{u}(p_1)\gamma^\mu v(p_2) + F_S^q(\hat{s})\frac{\Delta^\mu}{2m}\bar{u}(p_1)v(p_2),$$

$$G_E(\hat{s}) = F_V^q(\hat{s}) + (\tau - 1)F_S^q(\hat{s}), G_M(\hat{s}) = F_V^q(\hat{s}) \quad \tau = \hat{s}/4m^2$$

M. Diehl, P. Kroll, and C.Vogt, EPJC 26 (2003) 567

The process $e^-e^+ \to B\bar{B}\gamma$ can be measured at BESIII, Belle II, and STCF. Actually, this process has been used for the recent measurements of EM FFs.

Ablikim et al. (BESIII Collaboration), PRD 99 (2019) 092002

Ablikim et al. (BESIII Collaboration), PRL 130 (2023) 151905

Kinematics of $e^-e^+ \to B\bar{B}\gamma$

We define the following variables

$$s = (q_1)^2 = (k_1 + k_2)^2, \quad \hat{s} = W^2 = (p_1 + p_2)^2, \quad u = (k_1 - q_2)^2, \quad \Delta = p_2 - p_1.$$

The lightcone vectors are introduced

$$n = \frac{\sqrt{2s}}{s - \hat{s}} q_2, \quad \tilde{n} = \sqrt{\frac{2}{s}} \left(q_1 - \frac{s}{s - \hat{s}} q_2 \right)$$

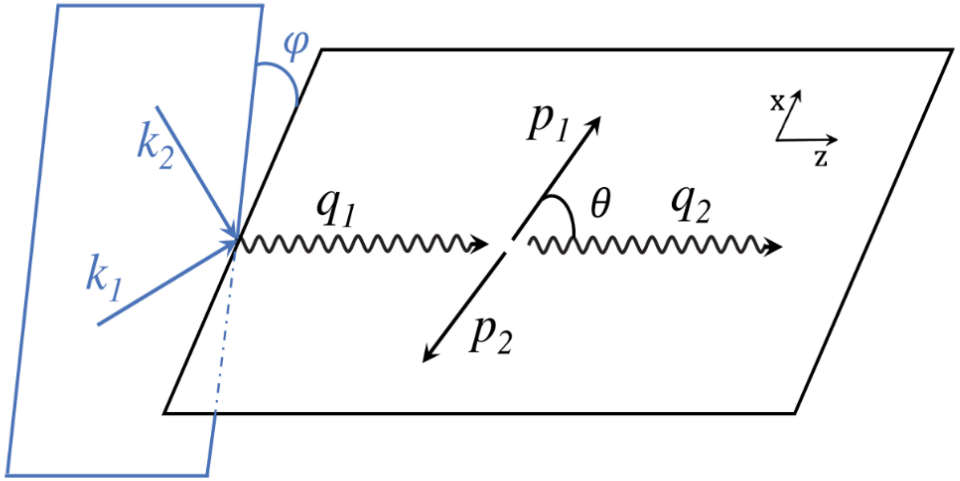
Transverse tensors are defined

$$g_T^{\alpha\beta} = g^{\alpha\beta} - n^\alpha \tilde{n}^\beta - n^\beta \tilde{n}^\alpha$$

$$\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \tilde{n}_\alpha n_\beta, \quad \epsilon^{0123} = 1$$

In the c.m. frame of the $B\bar{B}$ pair

$$\beta_0 = \sqrt{1 - \frac{4m^2}{\hat{s}}} \qquad \zeta_0 = \frac{\Delta \cdot n}{P \cdot n} = \beta_0 \cos\theta$$

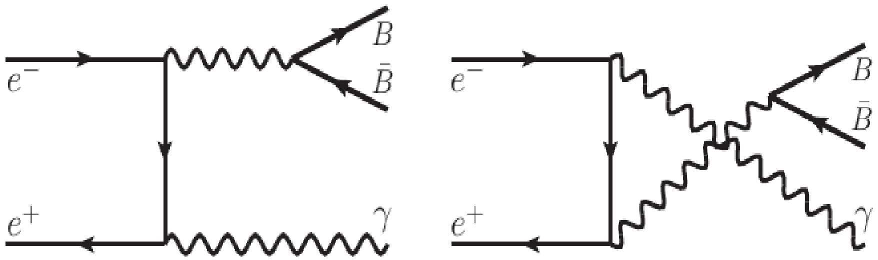


The z axis is chosen along the momentum q_1, and the momenta of the baryon pair lie in the x-z plane,

Theoretical Cross Sections

We first calculate the ISR process:

$$e^- e^+ \to \gamma^* \gamma \to B\bar{B}$$



$$\frac{d\sigma_{\rm ISR}}{d\hat{s}\,du\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{\rm em}^3 \beta_0^3}{4\pi s^2}\frac{1}{\epsilon\hat{s}}\left[b_0 + b_1\cos^2\theta + b_2\sin^2\theta + b_3\sin(2\theta)\cos\varphi + b_4\sin^2\theta\cos(2\varphi)\right]$$

$$b_0 = [1-2x(1-x)(1+\epsilon)](2\lambda-1)|G_M|^2,$$

$$b_1 = [1-2x(1-x)(1-\epsilon)]|G_M|^2 + 4\epsilon x(x-1)(\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_2 = 2\epsilon x(x-1)|G_M|^2 + [1-2x(1-x)](\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_3 = \sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}(2x-1)\mathrm{sgn}(\rho)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right],$$

$$b_4 = 2\epsilon x(1-x)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right].$$

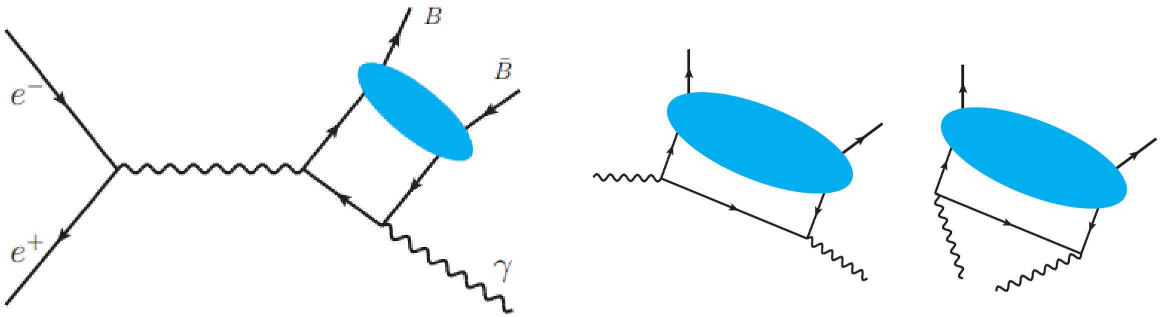
$$x = s/(s-\hat{s}),\ \lambda = 1/(\beta_0)^2,\ \rho = \hat{s} - s - 2u$$

$$\epsilon = (y-1)/\left(1 - y + \frac{y^2}{2}\right),\ \ y = q_1\cdot q_2/k_1\cdot k_2$$

Theoretical Cross Sections

Then, we calculate the subprocess of

$e^- e^+ \to \gamma^* \to B\bar{B}\gamma$



We define the Compton FFs

$$(\zeta_0)\mathcal{F}_i = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_i^q(z,\zeta,\hat{s})(i = V, S), \quad \mathcal{F}_{i'} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{1}{z(1-z)} \Phi_{i'}^q(z,\zeta,\hat{s})(i' = A, P)$$

The hardon tensor (leading twist) is given by

$$T_{\mu\nu} = \frac{-2}{\sqrt{2s}} \left\{ g_T^{\mu\nu} \left[\zeta_0 \mathcal{F}_V \bar{u}(p_1)\gamma^+ v(p_2) + \mathcal{F}_S \frac{P^+}{2m} \bar{u}(p_1) v(p_2) \right] - i\epsilon_T^{\mu\nu} \left[\mathcal{F}_A \bar{u}(p_1)\gamma^+\gamma^5 v(p_2) + \mathcal{F}_P \frac{P^+}{2m} \bar{u}(p_1)\gamma^5 v(p_2) \right] \right\}$$

$$\frac{d\sigma_{\mathrm{G}}}{d\hat{s}\,du\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{\mathrm{em}}^3 \beta_0}{8\pi s^3} \frac{1}{1+\epsilon} \left[|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2\mathrm{Re}(\mathcal{F}_A \mathcal{F}_P^*) + \frac{\hat{s}\left(|\mathcal{F}_P|^2 + |\mathcal{F}_S|^2\right)}{4m^2} \right.$$
$$\left. + (\beta_0)^2 \cos^2\theta \left[|\mathcal{F}_V|^2 + 2\mathrm{Re}(\mathcal{F}_S \mathcal{F}_V^*) - |\mathcal{F}_A|^2 \right] - (\beta_0)^4 \cos^4\theta |\mathcal{F}_V|^2 \right]$$

Theoretical Cross Sections

The interference term of two subprocesses should be also included.

$$i\mathcal{M}_{\rm ISR} = e^2\left(\frac{m_1^{\mu\alpha}}{t\hat{s}} + \frac{m_2^{\alpha\mu}}{u\hat{s}}\right)J_\alpha \varepsilon_\mu^*, \quad i\mathcal{M}_{\rm G} = \frac{e^2}{s} m_g^\alpha T_{\alpha\mu}\varepsilon_\mu^*$$

$$|\mathcal{M}_{\rm I}|^2 = \mathcal{M}_{\rm ISR}\mathcal{M}_{\rm G}^* + \mathcal{M}_{\rm ISR}^*\mathcal{M}_{\rm G}$$

One can decompose the cross section according to its dependence on angles

$$\frac{d\sigma_{\rm I}}{d\hat{s}\,du\,d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0}{8\pi s^2}\frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}\,s\epsilon(1+\epsilon)}}\left[c_0\cos\theta + c_1\cos^3\theta + c_2\sin\theta\cos\varphi + c_3\sin(2\theta)\cos\theta\cos\varphi\right]$$

$$\begin{aligned}
c_0 =& 2\,\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[\mathrm{Re}(\mathcal{F}_V G_M^*) + \mathrm{Re}(\mathcal{F}_S G_E^*)],\\
c_1 =& 2(\beta_0)^2\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)],\\
c_2 =& 2[1-(1-x)(1+\epsilon)]\mathrm{Re}(\mathcal{F}_A G_M^*) + 2[1-(1-x)(1-\epsilon)]\mathrm{Re}(\mathcal{F}_S G_E^*),\\
c_3 =& (\beta_0)^2[1-(1-x)(1-\epsilon)][(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)].
\end{aligned}$$

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Theoretical Cross Sections

After integration over azimuthal angle, the differential cross sections are expressed as

$$\frac{d\sigma_{\rm ISR}}{d\hat{s}dud(\cos\theta)} = \frac{\alpha_{\rm em}^3\beta_0^3}{2s^2}\frac{1}{\epsilon\hat{s}}[b_0 + b_1\cos^2\theta + b_2\sin^2\theta],$$

$$\frac{d\sigma_{\rm G}}{d\hat{s}dud(\cos\theta)} = \frac{\alpha_{\rm em}^3\beta_0}{4s^3}\frac{1}{1+\epsilon}\Bigg[|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2{\rm Re}(\mathcal{F}_A\mathcal{F}_P^*) + \frac{\hat{s}(|\mathcal{F}_P|^2+|\mathcal{F}_S|^2)}{4m^2} + (\beta_0)^2\cos^2\theta[|\mathcal{F}_V|^2 + 2{\rm Re}(\mathcal{F}_S\mathcal{F}_V^*) - |\mathcal{F}_A|^2] - (\beta_0)^4\cos^4\theta|\mathcal{F}_V|^2\Bigg],$$

$$\frac{d\sigma_{\rm I}}{d\hat{s}dud(\cos\theta)} = \frac{\alpha_{\rm em}^3(\beta_0)^2}{4s^2}\frac{\sqrt{2}}{\sqrt{s\hat{s}\epsilon(1+\epsilon)}}[c_0\cos\theta + c_1\cos^3\theta].$$

The timelike Compton FFs are expressed in terms of the baryon GDAs, which only appear in $d\sigma_G$ and $d\sigma_I$.

Baryon-Antibaryon Asymmetry

Generally, the contribution of $d\sigma_I$ is larger than $d\sigma_G$. To extract baryon GDAs, it is necessary to study the interference contribution.

Consider the exchange of $(\theta,\varphi) \to (\pi-\theta,\pi+\varphi)$: $d\sigma_{\mathrm{ISR}} \longrightarrow d\sigma_{\mathrm{ISR}} \quad d\sigma_{\mathrm{G}} \longrightarrow d\sigma_{\mathrm{G}} \quad d\sigma_{\mathrm{I}} \longrightarrow -d\sigma_{\mathrm{I}}$

Thus, the interference contribution can be obtained

$$d\sigma(B,\bar{B}) - d\sigma(\bar{B},B) = 2d\sigma_{\mathrm{I}}$$

We can also define a new observable, similar to the charge asymmetry in the TCS process.

The forward-backward asymmetry is expressed as

$$A_{\mathrm{FB}}(\theta) = \frac{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}$$

The numerator of A_{FB} is given by

$$\frac{d\sigma_{\mathrm{FB}}}{d\hat{s}\,du\,d(\cos\theta)} = \frac{\alpha_{\mathrm{em}}^3 \beta_0}{8\pi s^2} \frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}\,s}\,\epsilon(1+\epsilon)} \left[2\pi\left(c_0\cos\theta + c_1\cos^3\theta\right) - 4\left(c_2\sin\theta + c_3\sin(2\theta)\cos\theta\right)\right]$$

This provides another way to obtain interference contribution.

E. R. Berger, M. Diehl, and B. Pire, EPJC 23 (2002) 675

Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$

We adopted effective proton EM FF for numerical estimate

$$F_p(\hat{s}) = F_{3p}(\hat{s}) + F_{osc}(p), \qquad p = \sqrt{\hat{s}\left(\frac{\hat{s}}{4m^2} - 1\right)}.$$

$$F_{3p}(\hat{s}) = \frac{F_0}{\left(1 + \frac{\hat{s}}{m_a^2}\right)\left(1 - \frac{\hat{s}}{m_0^2}\right)^2}, \qquad |G_E(\hat{s})| = F_p(\hat{s})\sqrt{\frac{1+2\tau}{1+2\tau/R(\hat{s})^2}},$$

$$F_{osc}(p) = Ae^{-Bp}\cos(Cp + D), \qquad |G_M(\hat{s})| = F_p(\hat{s})\sqrt{\frac{1+2\tau}{R(\hat{s})^2+2\tau}}.$$

$$R(\hat{s}) = \frac{1}{1+\omega^2/r_0}[1 + r_1 e^{-r_2\omega}\sin(r_3\omega)], \qquad \omega = \sqrt{\hat{s}} - 2m.$$

We neglect the imaginary phases of EM FFs G_E and G_M

A. Bianconi and E. Tomasi-Gustafsson, PRL 114 (2015) 232301

E. Tomasi-Gustafsson, A. Bianconi, and S. Pacetti, PRC 103 (2021) 035203

Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$

For proton vector GDAs , we assume that

$$\Phi_S^q \sim \Phi_V^q \sim \Phi_{\pi\pi}^q$$

$$\Phi_{\pi\pi}^q(z, \cos\theta, \hat{s}) = \frac{3(2\alpha+3)}{5B(\alpha+1, \alpha+1)} z^\alpha (1-z)^\alpha (2z-1) \times \left[\tilde{B}_{10}^q(\hat{s}) + \tilde{B}_{12}^q(\hat{s}) P_2(\cos\theta)\right]$$

S. Kumano, Q.-T. Song, and O. V. Teryaev, PRD 97 (2018) 014020

The proton Compton FFs are of a similar magnitude to the pion Compton FFs.

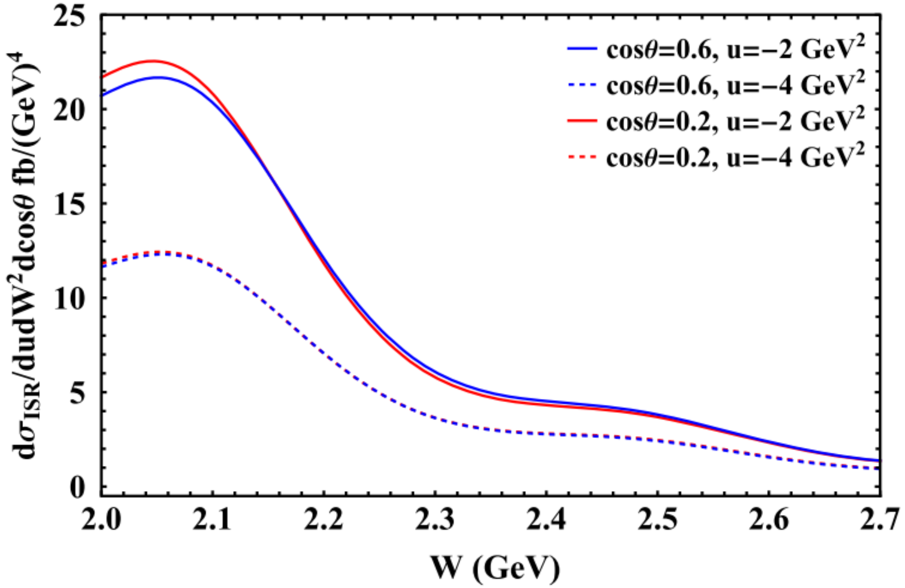
$$\mathcal{F}_\pi = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z\bar{z}} \Phi_{\pi\pi}^q(z, \zeta_0, \hat{s})$$

$$\mathcal{F}_S = \mathcal{F}_\pi, \qquad \mathcal{F}_A = \mathcal{F}_P = g_A \mathcal{F}_\pi,$$

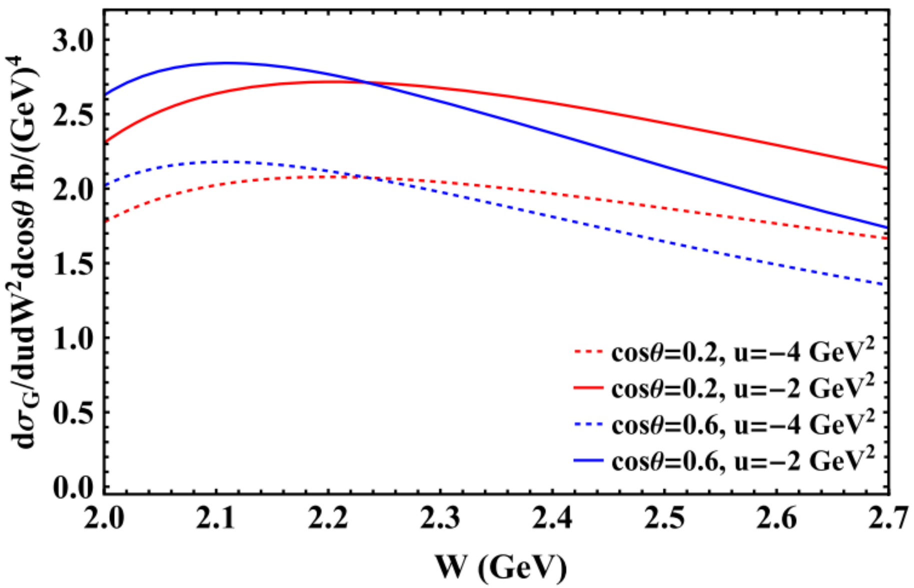
$$\zeta_0 \mathcal{F}_V = \mathcal{F}_\pi \cos\theta, \qquad g_A = e_u g_A^u + e_d g_A^d$$

Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$

We choose $\sqrt{s} = 4$ Gev. This value is typical for BESIII and the proposed STCF. To satisfy $s \gg \hat{s}$, the range of W is set between 2.0 to 2.7 Gev.

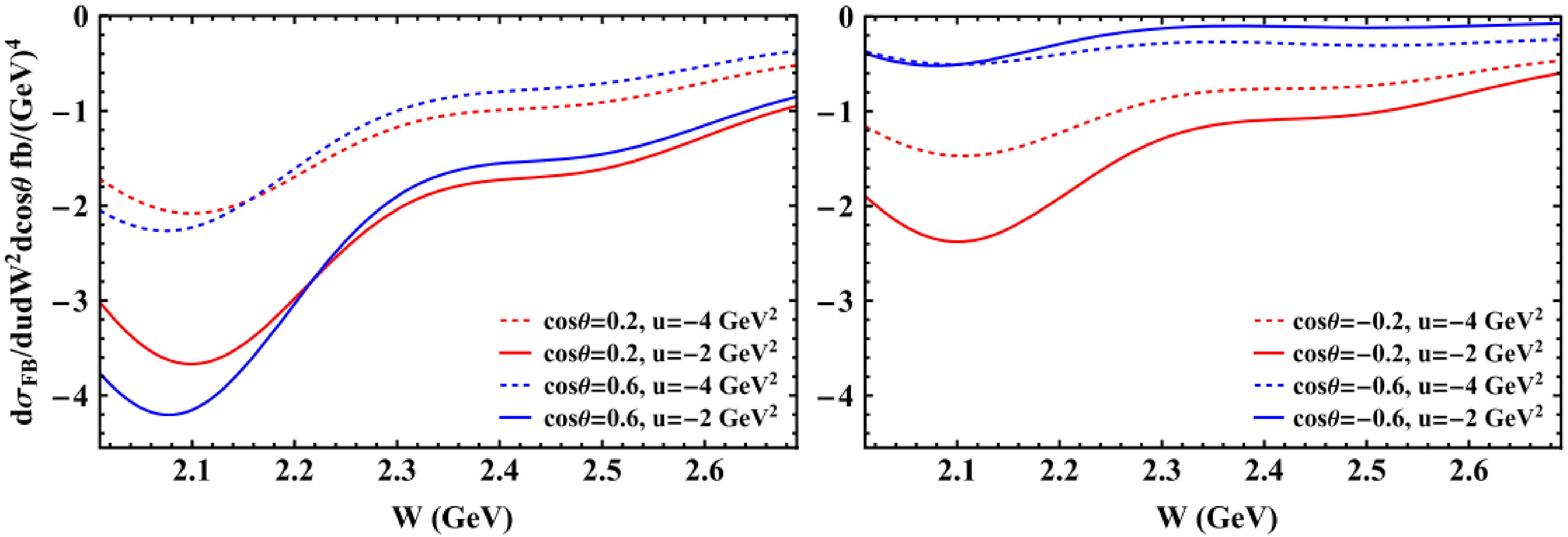


Estimate of ISR contribution



Estimate of $e^+e^- \to \gamma^* \to B\bar{B}\gamma$ contribution

Numerical estimate of $e^-e^+ \to p\bar{p}\gamma$



Estimate of interference contribution

Jing Han, Bernard Pire, and Qin-Tao Song, PRD 112(2025) 014048

Our results shows that the interference term is large than pure QCD contribution, it will play an important role in the extraction of baryon GDAs.

Summary

- ⇨ Studying baryon GDAs can help access the GFFs of unstable baryons

- ⇨ We calculated the cross section of $e^-e^+ \to B\bar{B}\gamma$, which can be expressed as Compton FFs and EM FFs

- ⇨ Using models of GDAs and EM FFs ,We provided numerical estimatie of the $e^-e^+ \to p\bar{p}\gamma$, which will be helpful for future measurements at BESIII, Belle II , and STCF.

Thank you very much