26th International Symposium on Spin Physics

A Century of Spin

Relativistic dynamics of charmonia in strong magnetic fields

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Based on: Wen, Li, Zhou, YL, Vary, Phys. Rev. D 112 (2025)

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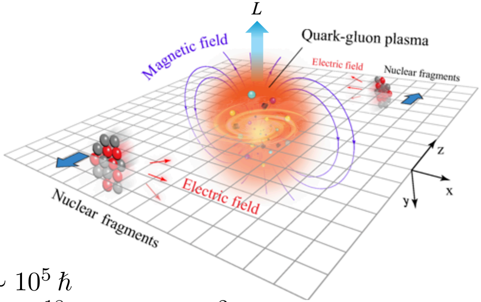
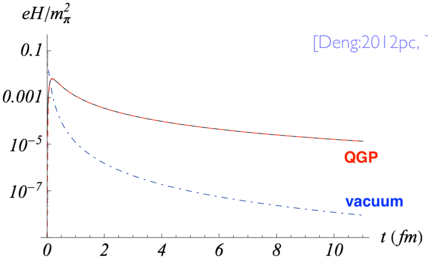
Magnetic fields in relativistic heavy ion collision

- Macroscopic spin:

- Subatomic spin:



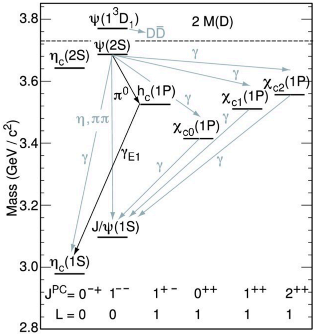


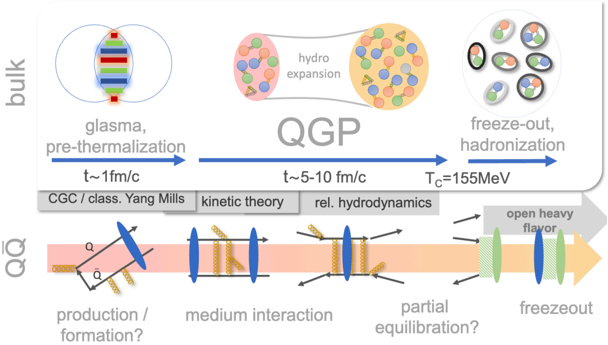
[Deng:2012pc, Tuchin:2013apa]

$L \sim 10^5\,\hbar$

$B \sim 10^{18}$ Gauss $\sim 10 m_\pi^2$

Quarkonia in heavy-ion collisions

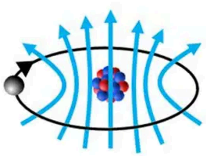




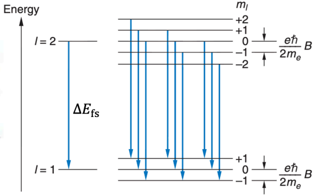
- Quarkonium: the hydrogen atom of QCD
- Ideal probe for quark-gluon plasma

Quarkonia in magnetic fields

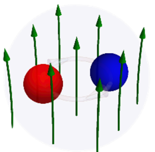
[Review: Iwasaki:2021nrz]



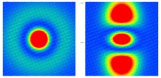
$B_{ext} \sim 10^4$ G
$\mu B_{ext} \sim 10^{-4}$ eV



B=0 B≠0

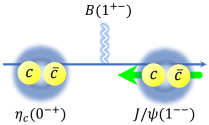


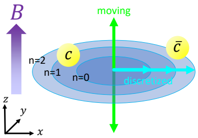
$B_{ext} \sim 10^{18}$ G
$\mu B_{ext} \sim 10$ MeV



B=0 B≠0

- Atomic physics: nonrelativistic and perturbation
- Hadronic physics: sizeable relativistic and non-perturbative corrections
- Physics:
  - Zeeman effect, Paschen-Back effect, motional Stark effect
  - State mixing, Landau levels, Lorentz deformation
  - ......
- Theoretical methods: non-relativistic quark model, effective Lagrangians, QCD sum rule, holography, ...





Light-Front Hamiltonian formalism

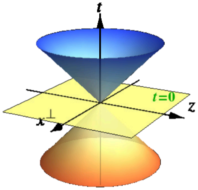
[Reviews: Brodsky:1997de]

- Full quantum information is contained in the hadronic wave function $|\Psi\rangle$

$$i\frac{\partial}{\partial x^\mu}|\Psi\rangle = P_\mu|\Psi\rangle, \quad (\mu = 0,1,2,3)$$

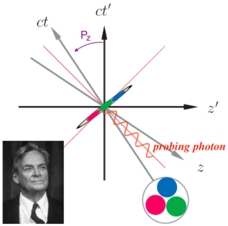
- Relativity allows a free choice of the time variable, e.g. x^0, $x^+ = x^0 + x^3$
- Infinite momentum frame = light-front quantization: retaining the partonic picture

equal-time quantization
$t = 0$

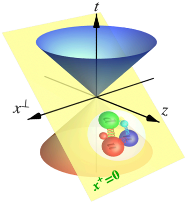


$$i\frac{\partial}{\partial t}|\Psi\rangle = P^0|\Psi\rangle$$

infinite momentum frame
$P_z \to \infty$



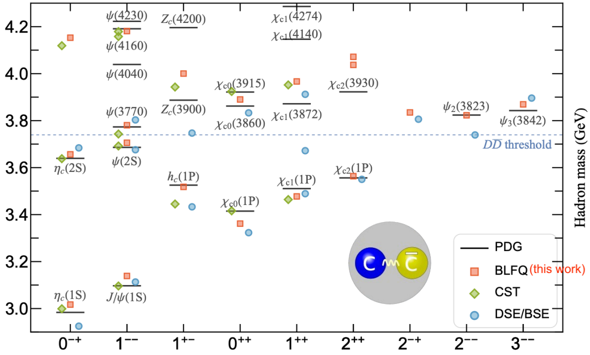
light-front quantization
$x^+ = t + z/c = 0$

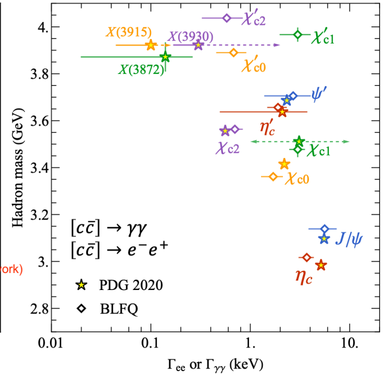


$$i\frac{\partial}{\partial x^+}|\Psi\rangle = P_+|\Psi\rangle$$

Charmonium on the light front

[Li:2015zda, Li:2017mlw, Li:2021ejv]





- Basis light-front quantization: effective Hamiltonian from one-gluon exchange + holographic confinement
- Two free parameters (m_c, κ), rms deviation: 30 MeV [Gross:2022hyw]
- Good agreement with the PDG data for both the masses and the widths

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

Coupling to external fields

[Wen:2025dwy]

- Minimal coupling: $\partial^\mu \to \partial^\mu - ie\mathcal{A}^\mu \;\Rightarrow\; \mathcal{L}_{\text{int}} = -J^\mu \mathcal{A}_\mu$
- Gauge fixing: $\mathcal{A}^+ = 0, \vec{\mathcal{A}} = -\frac{1}{2}\vec{r}\times\vec{B}$, where $\vec{B} = B\hat{z}$
- The light-front Hamiltonian:

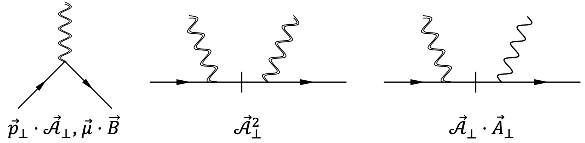
$$P^- = P^-_{\text{qcd}} + e\int \mathrm{d}^3x\, J^\mu \mathcal{A}_\mu + \frac{e^2}{2}\int \mathrm{d}^3x\, \overline{\psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \psi,$$
$$+ \frac{e^2}{2}\int \mathrm{d}^3x\, \overline{\psi}\gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu \psi + \frac{e^2}{2}\int \mathrm{d}^3x\, \overline{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+}\gamma^\nu \mathcal{A}_\nu \psi.$$

Extra terms in P^-: the Seagull term, and the terms that modifies the quark potential.

- Exact quantum many-body form:

$$P^- = \sum_i \frac{(\vec{p}_{i\perp} - q_i\vec{\mathcal{A}}_\perp)^2 + m_i^2}{p_i^+} - \vec{\mu}_i\cdot\vec{B} + V_{\text{QCD}}(B)$$

where, $\vec{\mu}_i = g_S(q_i/p_i^+)\vec{S}_i$ is the light-front magnetic moment.



One-particle state in magnetic fields

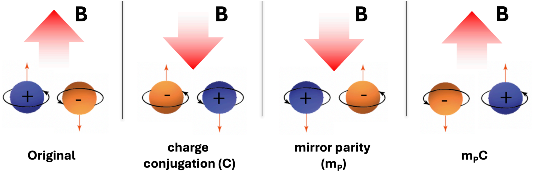
- A pointlike particle in magnetic fields: exact relativistic Landau levels

$$P^- = \frac{\vec{p}_\perp^2 + M_q^2 + \frac{1}{4} e_q^2 B^2 r_\perp^2}{p^+} - (\vec{m} + \vec{\mu}) \cdot \vec{B}$$

where, $\vec{m} = g_L (q/p^+) \vec{L}$ is the orbital magnetic moment. The eigen-energies are,

$$p^- = \frac{M_q^2 + |e_q B|(2n + |m| + m + 1 + 2m_s)}{p^+} \quad \Rightarrow \quad E^2 = p_z^2 + M_q^2 + |e_q B|(2N + 1 + 2m_s)$$

- A composite particle in magnetic fields: Wigner representation & max. compatible operator set
  - In vacuum: $\{P^\mu, \vec{S}^2, S_z, \mathsf{P}, \mathsf{C}\}$, BLFQ: $\{\mathcal{M}^2, P^+, H_{\text{cm}}, L_{\text{cm}}, \vec{S}^2, S_z, \mathsf{m}_P, \mathsf{C}\}$
  - In magnetic fields in BLFQ: $\{\mathcal{M}^2, P^+, \cancel{H_{\text{cm}}}, \cancel{L_{\text{cm}}}, \cancel{\vec{S}^2}, S_z, \mathsf{m}_P \mathsf{C}\}$



Charmonium in magnetic fields

- Two-body Hamiltonian:

$$H = H_{\text{qcd}} - q_f eB\Big(\frac{m_1 + 2s_1}{x} - \frac{m_2 + 2s_2}{1-x}\Big) + \frac{(q_f eB)^2}{4}\Big(\frac{r_{1\perp}^2}{x} + \frac{r_{2\perp}^2}{1-x}\Big) \equiv H_{\text{qcd}} + H_{\text{int}}$$

- Separating center-of-mass motion and relative motion:

$$H_{\text{int}} = -q_f eB\Big(\frac{(1-x)m + 2s_1}{x} - \frac{xm + 2s_2}{1-x}\Big) + \frac{(q_f eB)^2}{4}\frac{(1-3x+3x^2)r_\perp^2}{x(1-x)}$$
$$+ \frac{(q_f eB)^2}{4x(1-x)}\Big(\vec{R}_\perp^2 + 2(1-2x)\vec{R}_\perp \cdot \vec{r}_\perp\Big) - q_f e\vec{r}_\perp \cdot (\vec{P}_\perp \times \vec{B}) + \frac{q_f e}{x(1-x)}\vec{k}_\perp \cdot (\vec{R}_\perp \times \vec{B})$$

- In non-relativistic limit $x \approx 1/2$, the relative OAM ($m = \vec{r}_\perp \times \vec{k}_\perp$) does not contribute to the interaction
- Motional Stark effect: electric field $\vec{E}_\perp = \vec{V}_\perp \times \vec{B}$ induced by c.m. motion where $\vec{V}_\perp = \vec{P}_\perp / P^+$
- A-B effect: gauge field $\vec{\mathsf{A}}_\perp = -(1/2)\vec{R}_\perp \times \vec{B}$ induced by c.m. motion

Basis light-front quantization

[Vary:2009gt]

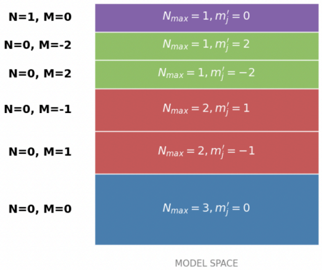
- Adopt a basis $\{|N, M\rangle_{\text{cm}}|n, m, l, s_1, s_2\rangle_{\text{rel}}\}$ with basis functions

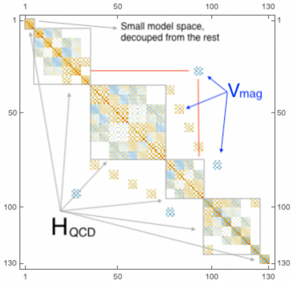
$$\langle p_1, p_2|N, M\rangle = \phi_{NM}(\vec{P}_\perp)\phi_{nm}(\vec{k}_\perp/\sqrt{x(1-x)})\chi_l(x),$$

where $P = p_1 + p_2$, $x = p_1^+/P^+$, $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{P}_\perp$. ϕ is quantum HO function, and χ is Jacobi polynomials.

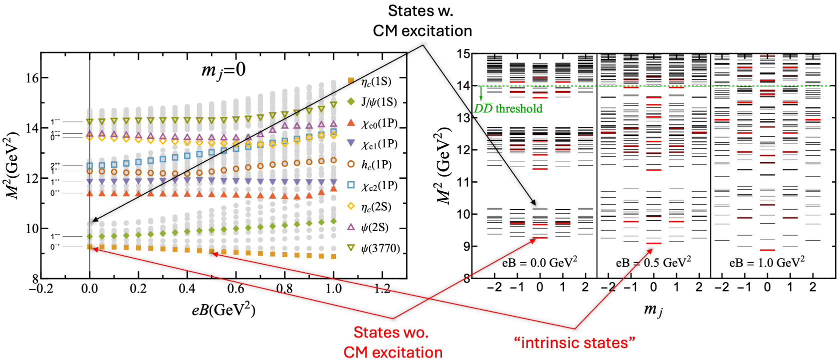
- Basis truncations:

$$2N + |M| + 1 + 2n + |m| + 1 \leq \bar{N}_{\text{max}},$$
$$l \leq L_{\text{max}},$$
$$M + m + s_1 + s_2 = m_j$$



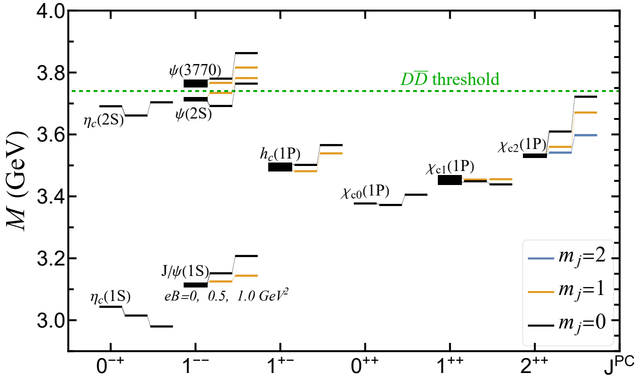


Charmonium spectrum in weak magnetic fields



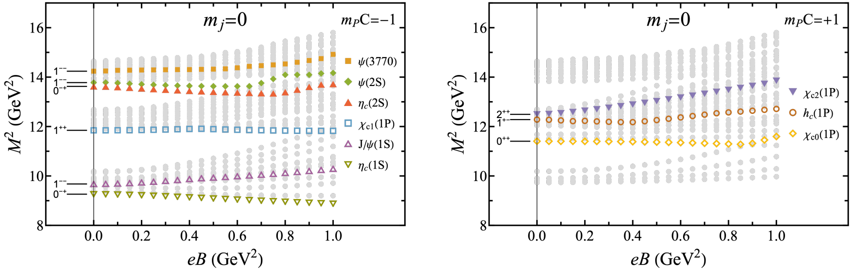
- In vacuum, states with c.m. excitation can be identified by examining $\langle H_{\rm cm} \rangle$
- In magnetic fields, c.m. motion couples with relative motion
- In weak fields, we define ``intrinsic states" as those smoothly connected to the genuine intrinsic states at $eB = 0$

Charmonium spectrum in weak magnetic fields



- Non-linear Zeeman effect: splitting of magnetic projections m_j in magnetic fields
- Level repulsion due to state mixing, e.g. J/ψ-η_c

Charmonium spectrum in magnetic fields



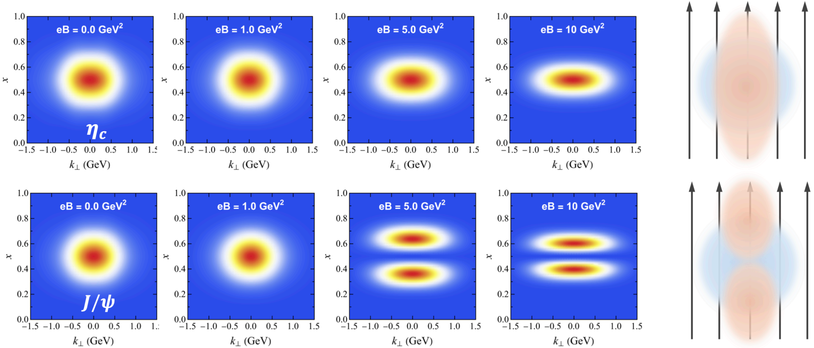
- At strong magnetic fields, states strongly mix and only exact quantum numbers such as $m_P C$ can be used to identify states
- Note that the g.s. of $m_P C = +1$ sector is a J/ψ-like state with c.m. excitation

Charmonium density in magnetic fields

$$\rho(x, \vec{k}_\perp) = \sum_{s,\bar{s}} \int \frac{d^2 P_\perp}{(2\pi)^2} \left| \psi_{s\bar{s}}(\vec{P}_\perp, x, \vec{k}_\perp) \right|^2$$

$\boldsymbol{m_P C = -1}$
ground state

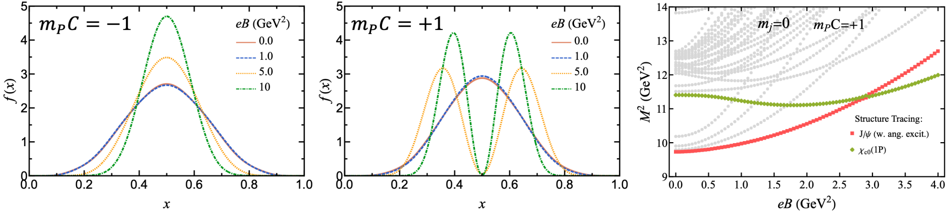
$\boldsymbol{m_P C = +1}$
ground state



- Deformation of η_c $(m_P C = -1)$ as the strength of the magnetic field B increases
- Transition of the ground state of $m_P C = +1$ from J/ψ w. OAM to χ_{c0} like state with longitudinal excitation at a magnetic field $B \gtrsim 2.8\,\mathrm{GeV}^2$

Charmonium density in magnetic fields

$$f(x) = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \rho(x, \vec{k}_\perp)$$



- Deformation in the longitudinal direction as the strength of the magnetic field B increases
- Transition of a singly peaked distribution to double-hump distribution of the ground state of $m_P C = +1$: consistent with a transition from J/ψ w. OAM to χ_{c0} like state with longitudinal excitation at a magnetic field $B \gtrsim 2.8\,\mathrm{GeV}^2$

Summary and outlooks

- We investigate the properties of charmonium systems in strong external magnetic fields using a relativistic light-front Hamiltonian approach
- Reveal key effects of the magnetic fields: Zeeman effect, spin mixing, deformation, and structural shifts
- Significance of the relativistic corrections and the consistent treatment of the center-of-mass coupling
- Extension to the vortical motion of the medium
- Possible applications to the spin polarization of baryon and vector meson J/ψ

Thank you!


I think we’re gonna need
Backup