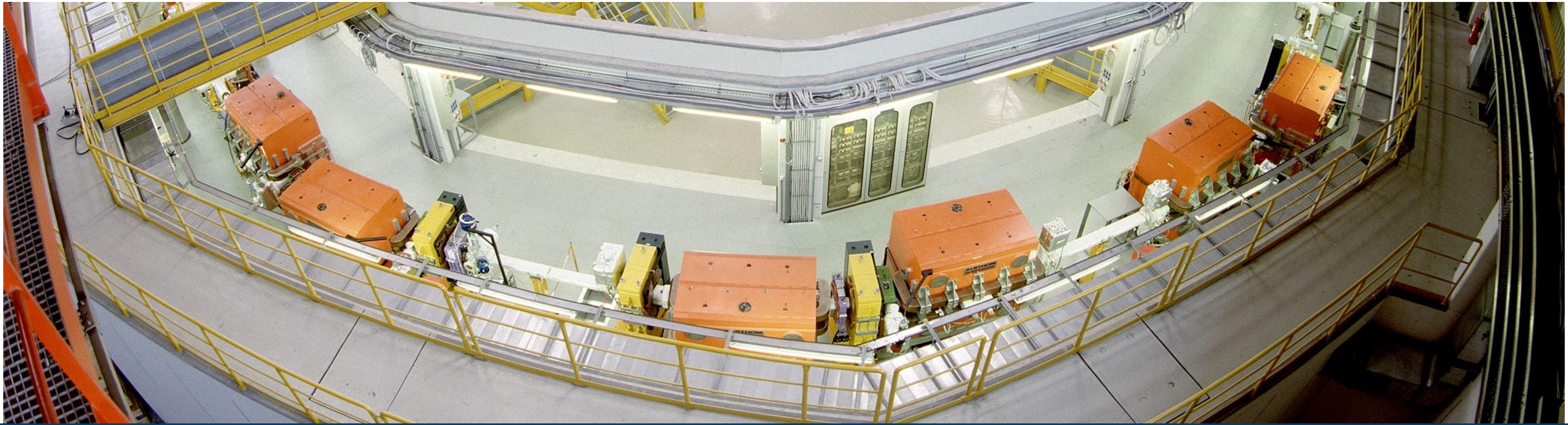


THE FIRST DIRECT MEASUREMENT OF THE DEUTERON ELECTRIC DIPOLE MOMENT AT THE COOLER SYNCHROTRON COSY

26th International Symposium on Spin Physics (SPIN2025)

23.09.2025 I ACHIM ANDRES

JEDI

Physics Institute III B | RWTH AACHEN UNIVERSITY

JÜLICH Forschungszentrum

MATTER ANTIMATTER ASYMMETRY

- Big Bang: Equal amount of matter and antimatter

$$N_B = N_{\bar{B}}$$

- Early Universe:

Theory: $B + \bar{B} \rightarrow \gamma + \gamma + \cdots$

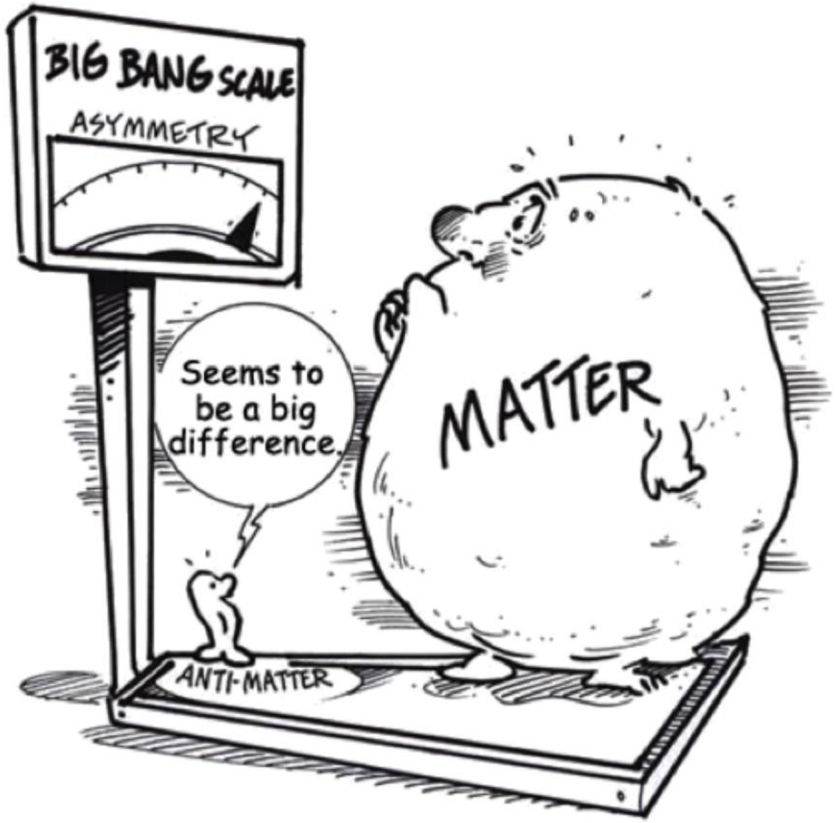
Measurement: $B + \bar{B} \rightarrow \gamma + \gamma + \cdots + B + \cdots ?$

- Today: Asymmetry between matter and antimatter

$$\eta = \frac{N_B - N_{\bar{B}}}{N_\gamma}$$

- Mismatch between expectation and measurement

$$\eta^{\text{SCM}} \approx 10^{-18} \quad \text{versus} \quad \eta^{\text{meas}} \approx 10^{-10}$$



JÜLICH
Forschungszentrum

MATTER ANTIMATTER ASYMMETRY

- Big Bang: Equal amount of matter and antimatter

$$N_B = N_{\bar{B}}$$

- Early Universe:

Theory: $B + \bar{B} \rightarrow \gamma + \gamma + \cdots$

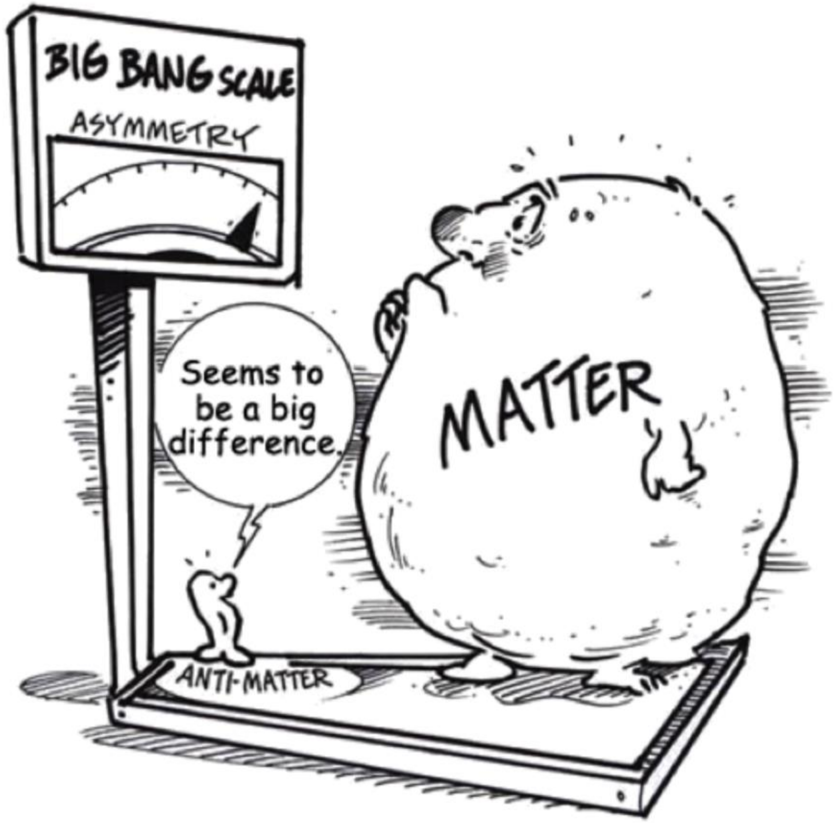
Measurement: $B + \bar{B} \rightarrow \gamma + \gamma + \cdots + B + \cdots ?$

- Today: Asymmetry between matter and antimatter

$$\eta = \frac{N_B - N_{\bar{B}}}{N_\gamma}$$

- Mismatch between expectation and measurement

$\eta^{\mathrm{SCM}} \approx 10^{-18}$ versus $\eta^{\mathrm{meas}} \approx 10^{-10}$



- Andrei Sakharov (1976)
  - Baryon Number Violation
  - *C* and *CP* Violation
  - Deviation from thermal equilibrium

JÜLICH Forschungszentrum

ELECTRIC DIPOLE MOMENTS

- EDM is a **vectorial** property aligned with **the particles' spin**
- Magnetic Dipole Moment (MDM): $\vec{\mu} = \mu \cdot \vec{s}$ with $\mu = g \frac{q}{2m}$
- Electric Dipole Moment (EDM): $\vec{d} = d \cdot \vec{s}$ with $d = \eta^{\mathrm{EDM}} \frac{q}{2mc}$

JÜLICH Forschungszentrum

ELECTRIC DIPOLE MOMENTS AND CP VIOLATION

- EDM is a **vectorial** property aligned with **the particles' spin**
- Magnetic Dipole Moment (MDM): $\vec{\mu} = \mu \cdot \vec{s}$ with $\mu = g\,\frac{q}{2m}$
- Electric Dipole Moment (EDM): $\vec{d} = d \cdot \vec{s}$ with $d = \eta^{\mathrm{EDM}}\,\frac{q}{2mc}$

$$H = -\vec{d} \cdot \vec{E} - \vec{\mu} \cdot \vec{B}$$

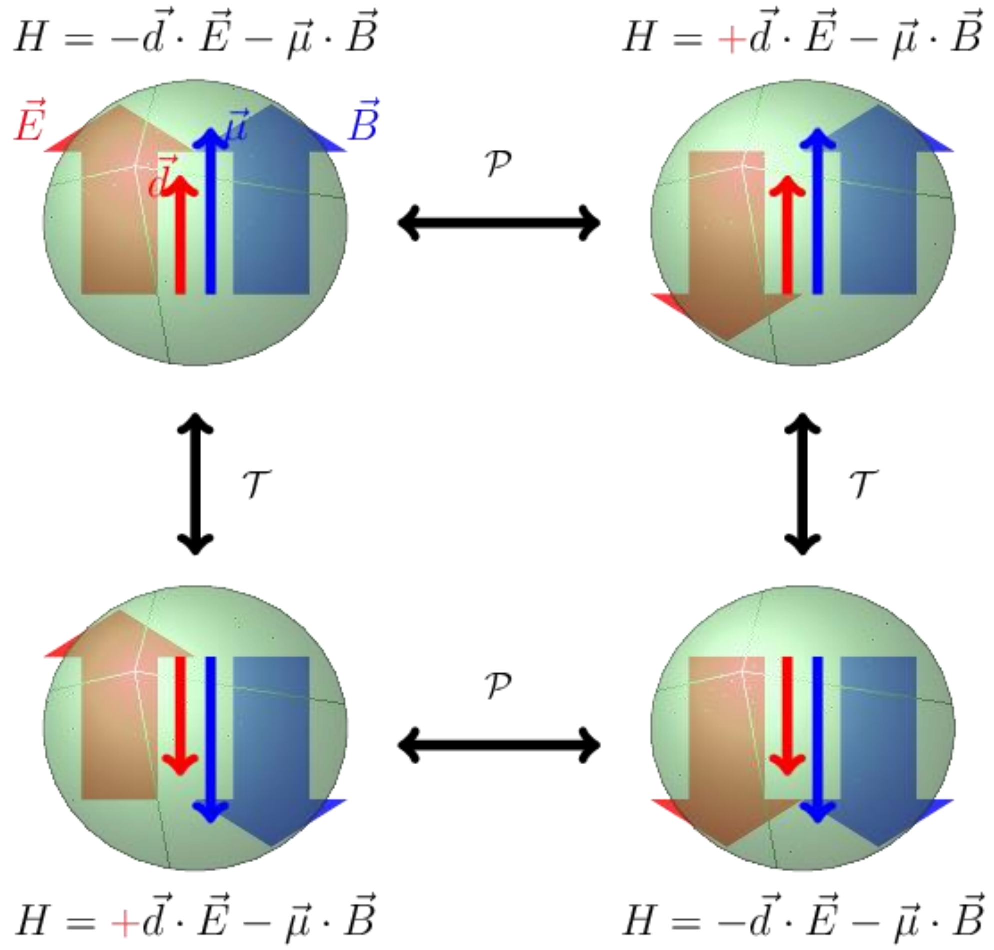
Parity P : $H = +\vec{d} \cdot \vec{E} - \vec{\mu} \cdot \vec{B}$

Time T : $H = +\vec{d} \cdot \vec{E} - \vec{\mu} \cdot \vec{B}$

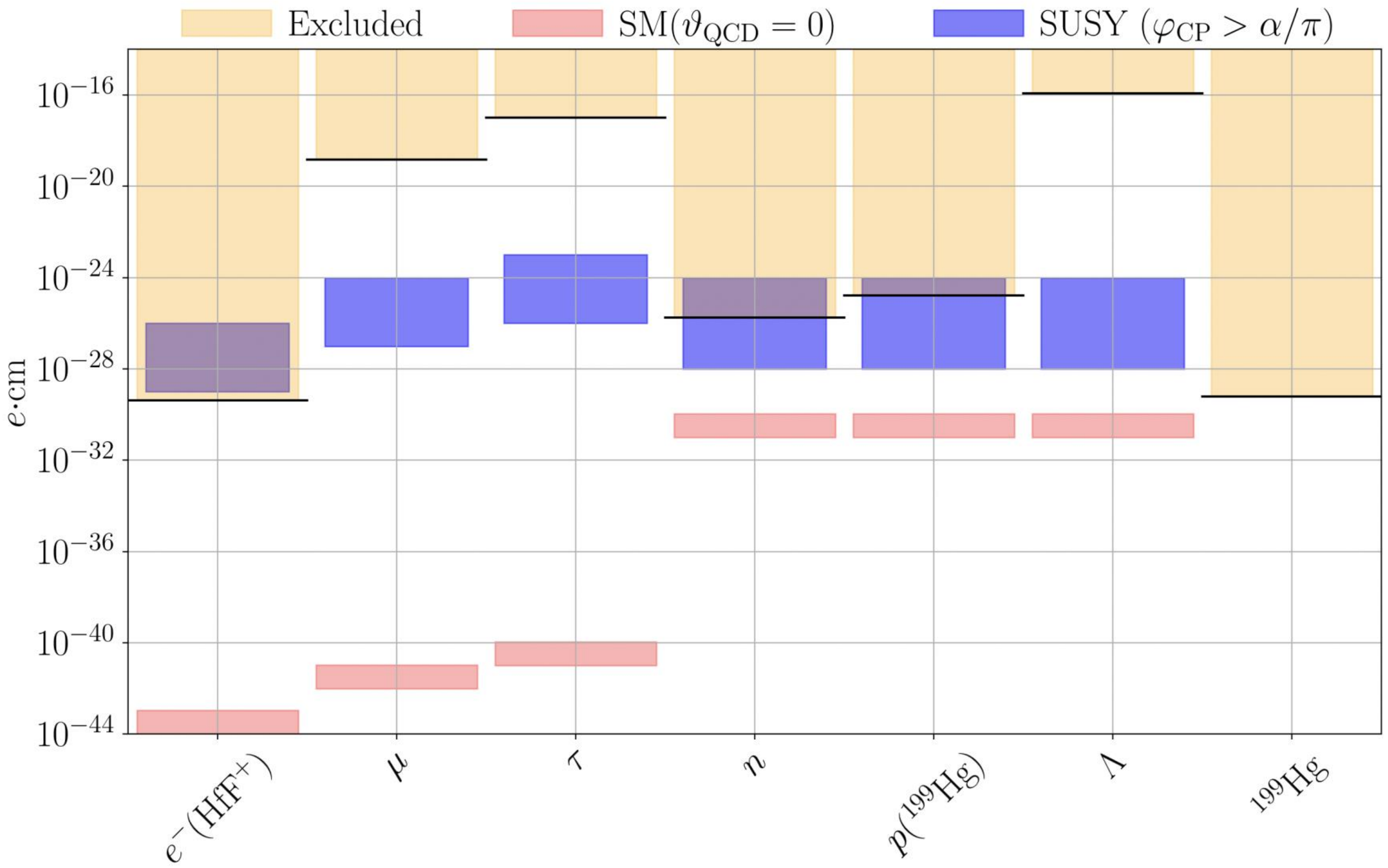
- According to CPT - Theorem:

T Violation = CP Violation

⇒ EDM violates both P and CP symmetry

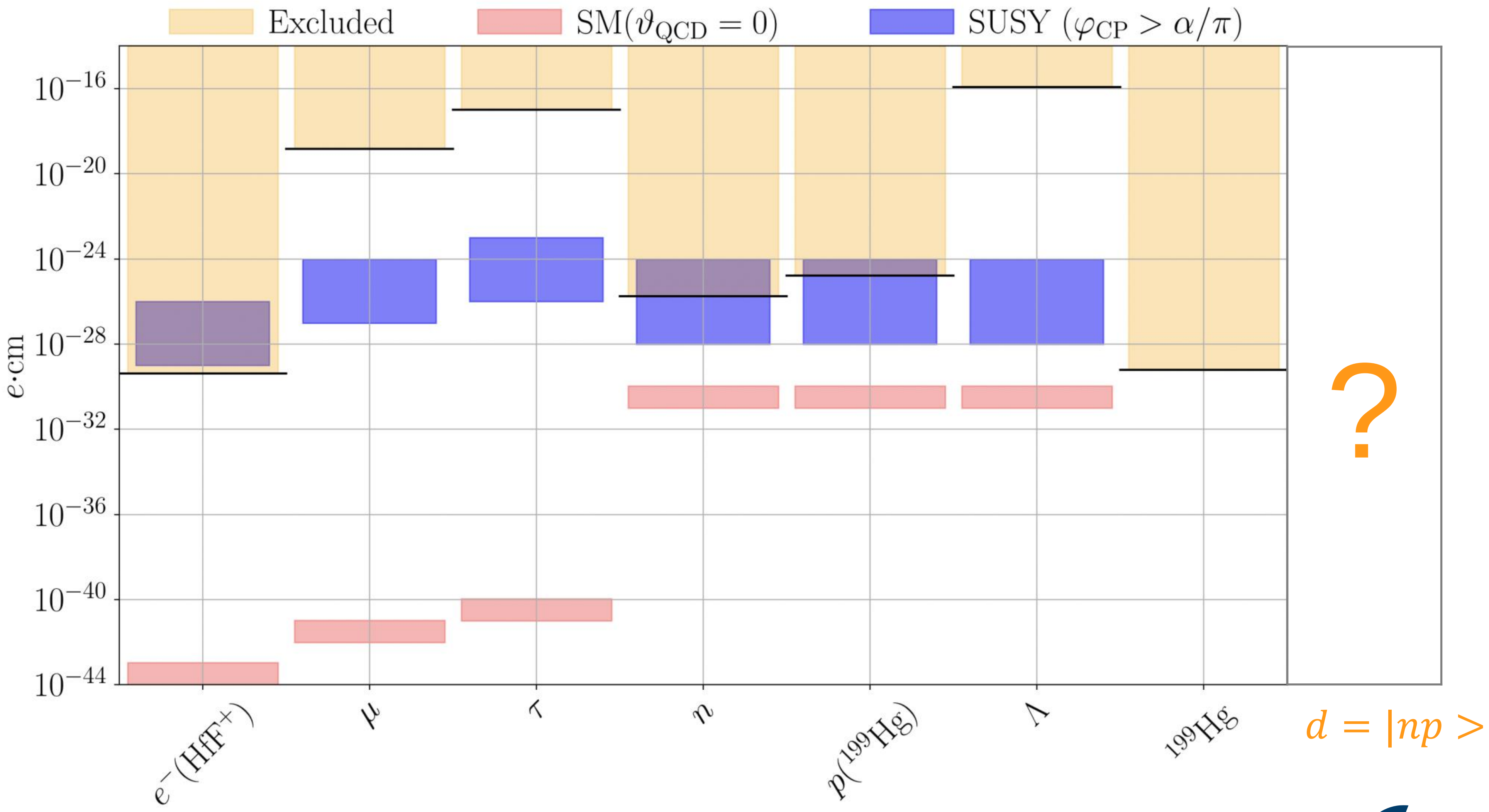


PERMANENT EDM SEARCH



JÜLICH Forschungszentrum

PERMANENT EDM SEARCH



JÜLICH Forschungszentrum

SPIN DYNAMICS

Thomas – BMT Equation

$\boldsymbol{G_d}$	-0.143
G_μ	0.001

$$\vec{d} = d\frac{\vec{s}}{|\vec{s}|} \text{ with } d = \eta^{\text{EDM}}\frac{q\hbar}{2mc}$$

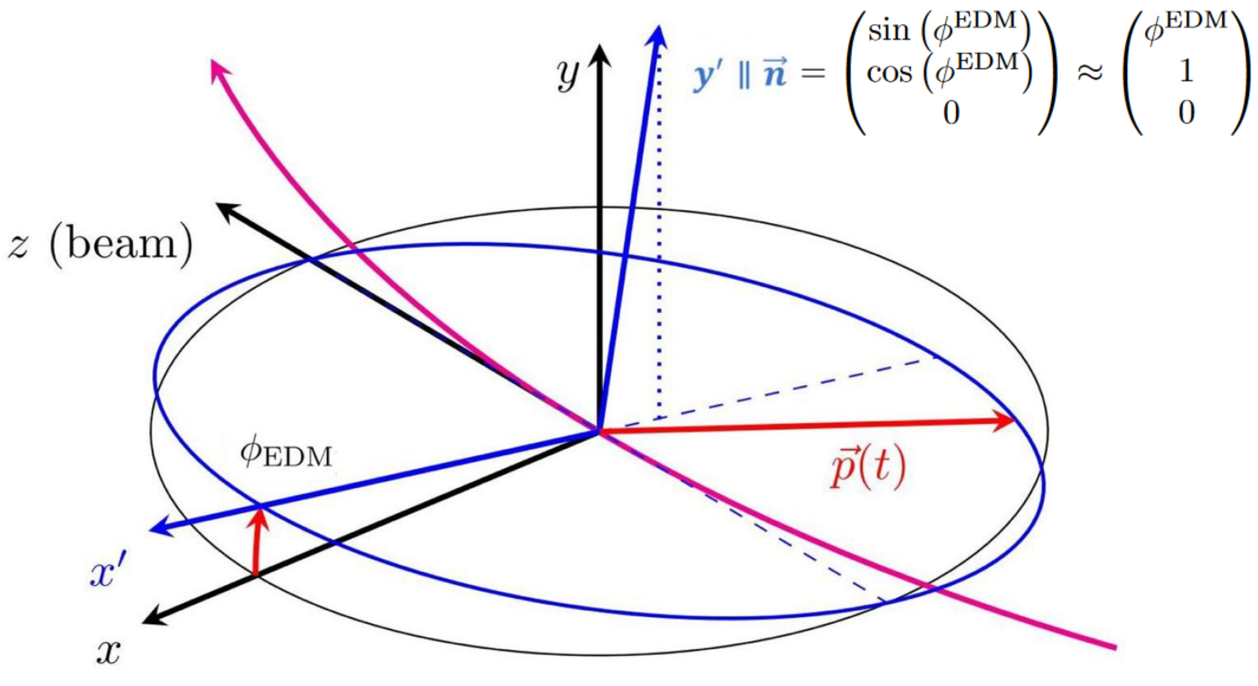
- Measure the influence of the EDM on the spin motion

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}n} = \left(\vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}}\right) \times \vec{S}$$

$$\vec{\Omega}_{\text{MDM}} = -2\pi\gamma G\vec{e}_y \rightarrow f_{\text{MDM}} = 120\text{ kHz}$$

$$\vec{\Omega}_{\text{EDM}} = \eta^{\text{EDM}}\gamma\beta\vec{e}_x$$

$$\tan\left(\frac{|\vec{\Omega}_{\text{EDM}}|}{|\vec{\Omega}_{\text{MDM}}|}\right) \approx \phi_{\text{EDM}} \approx -\eta^{\text{EDM}}\frac{2\beta}{G}$$



JÜLICH Forschungszentrum

SPIN DYNAMICS

Thomas – BMT Equation

$$\vec{d} = d\frac{\vec{s}}{|\vec{s}|} \text{ with } d = \eta^{\text{EDM}}\frac{q\hbar}{2mc}$$

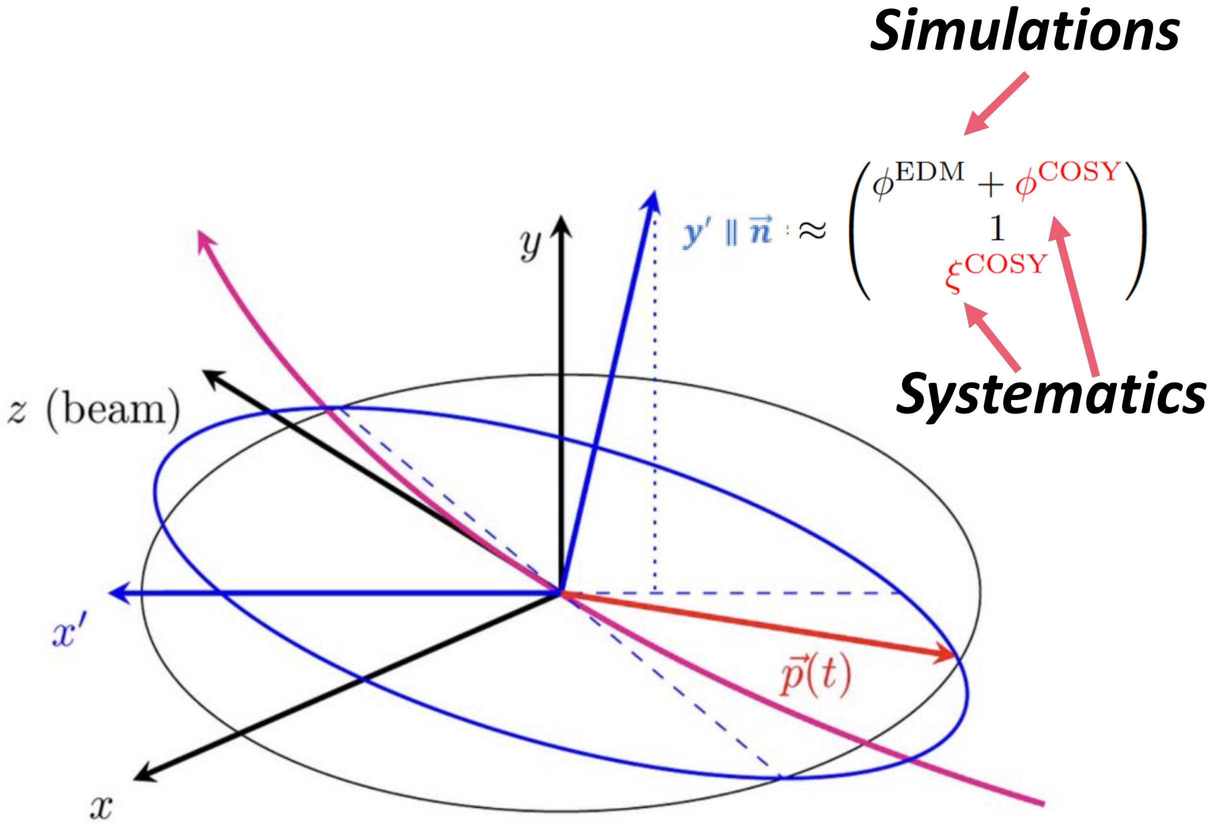
- Measure the influence of the EDM on the spin motion

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}n} = (\vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}}) \times \vec{S}$$

$$\vec{\Omega}_{\text{MDM}} = -2\pi\gamma G\vec{e}_y$$

$$\vec{\Omega}_{\text{EDM}} = \eta^{\text{EDM}}\gamma\beta\vec{e}_x$$

$$\tan\left(\frac{|\vec{\Omega}_{\text{EDM}}|}{|\vec{\Omega}_{\text{MDM}}|}\right) \approx \phi_{\text{EDM}} \approx -\eta^{\text{EDM}}\frac{\beta}{2G}$$



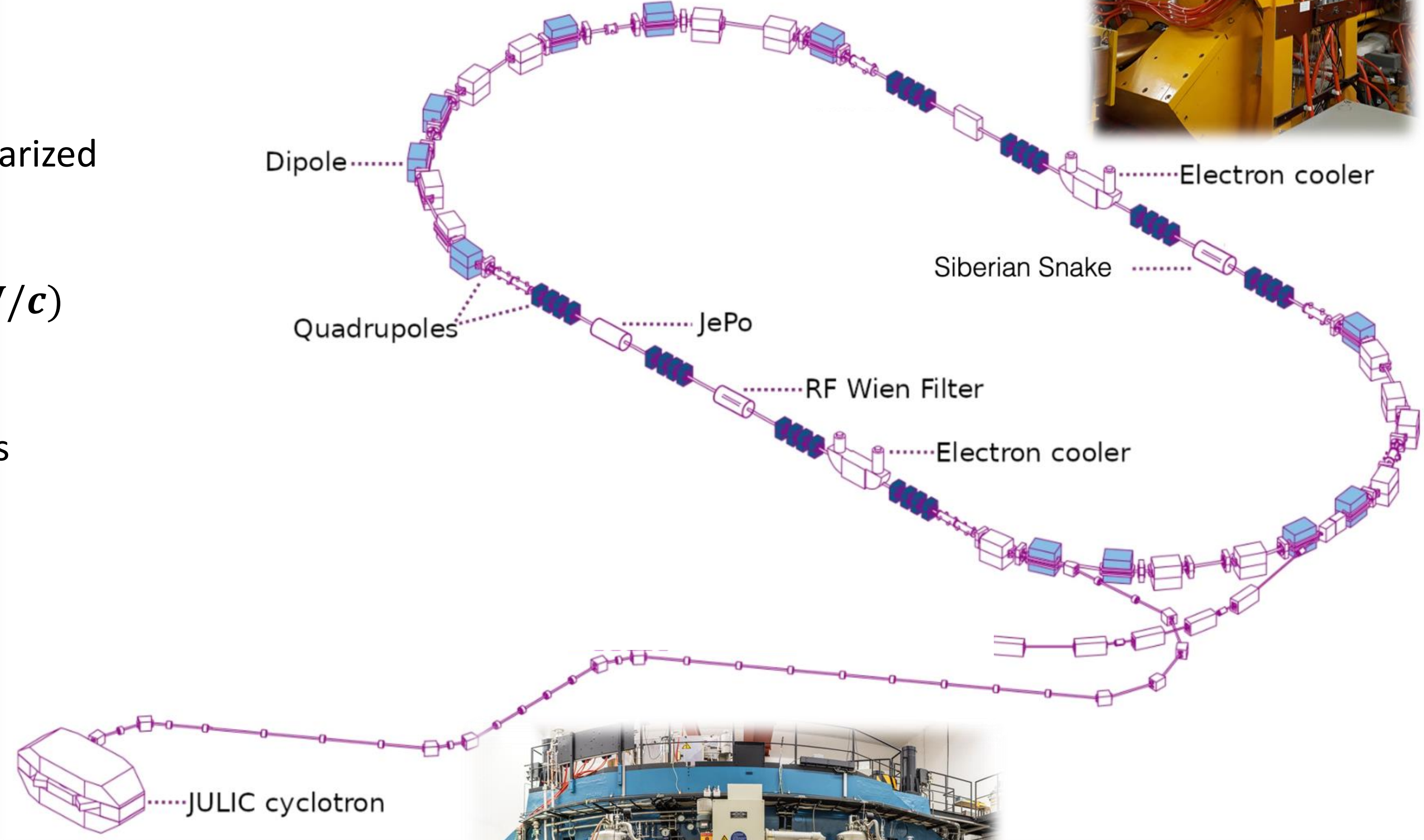
- **Problem**: Ring **imperfections** (magnet misalignments,..) lead to rotations of $\hat{n}$ in **radial** (x) and **longitudinal** (z) direction

JÜLICH Forschungszentrum

COSY - COOLER SYNCHROTRON (1993 – 2023)

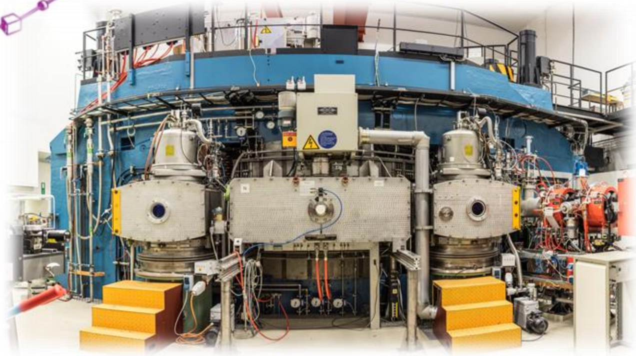
Overview

- Circumference 184 m
- Accelerate and Store **Polarized** / Unpolarized **Deuterons** and Protons
- $p = 0.3 - 3.7$ GeV/c $(\boldsymbol{p_d = 970\ \mathrm{MeV}/c})$
- Excellent Beam Quality
- Hadron Physics / **Precision** Experiments



$\vec{s}$ $\qquad \vec{P} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

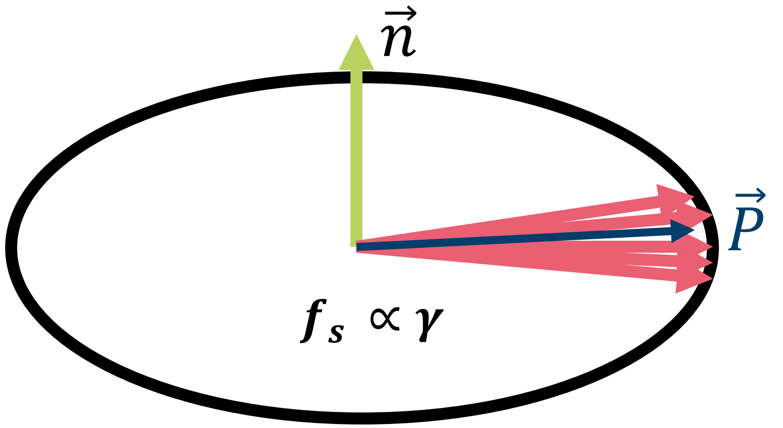
$$\vec{P} = \frac{1}{n}\left\langle \sum \vec{s} \right\rangle$$

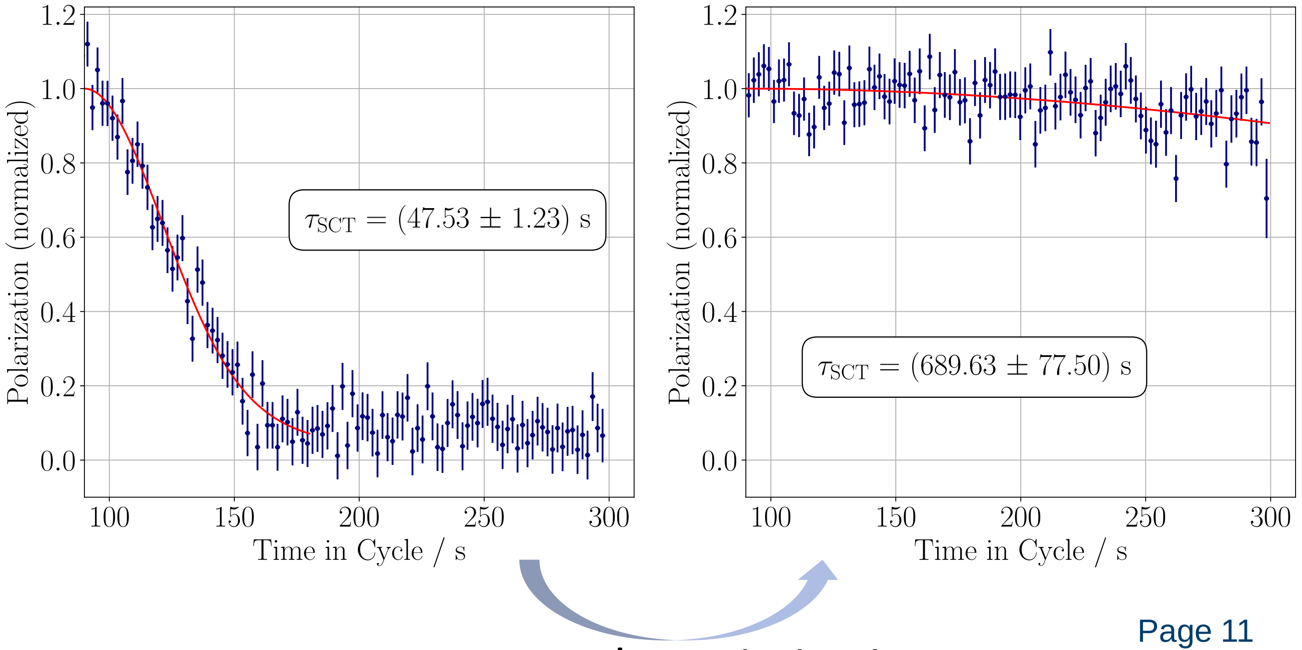


JÜLICH Forschungszentrum

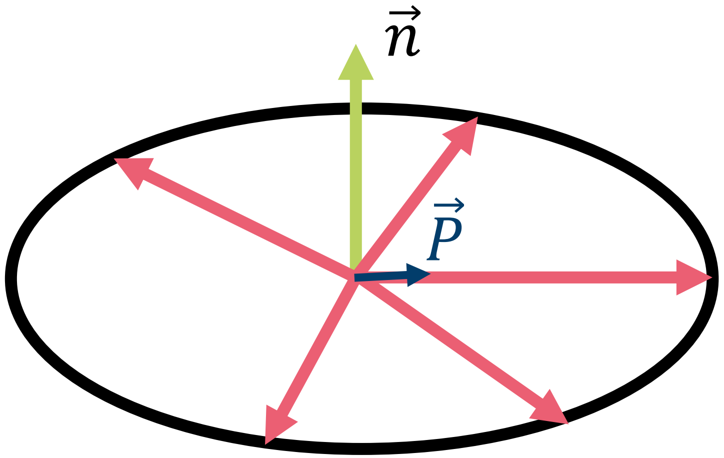
ACHIEVEMENTS AT COSY

- A **spread** in particles energy distribution leads to **decoherence** over time
- Measure 120 kHz spin tune precession to 10^{-10} in 100 s
- Development of polarization feed back system
- RF Wien filter (Single bunch spin manipulation)



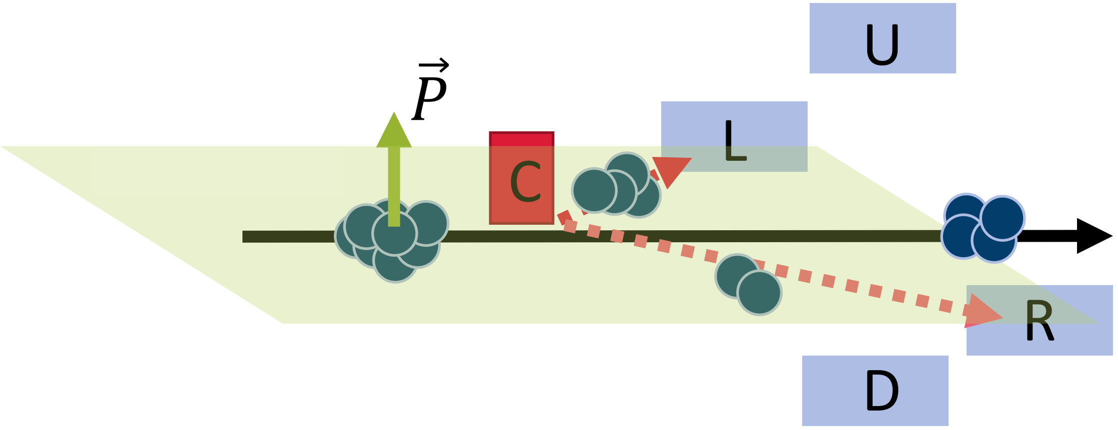


sextupole optimization



POLARIMETER

$$p_y \propto \frac{N_L - N_R}{N_L + N_R}$$



POLARIMETER

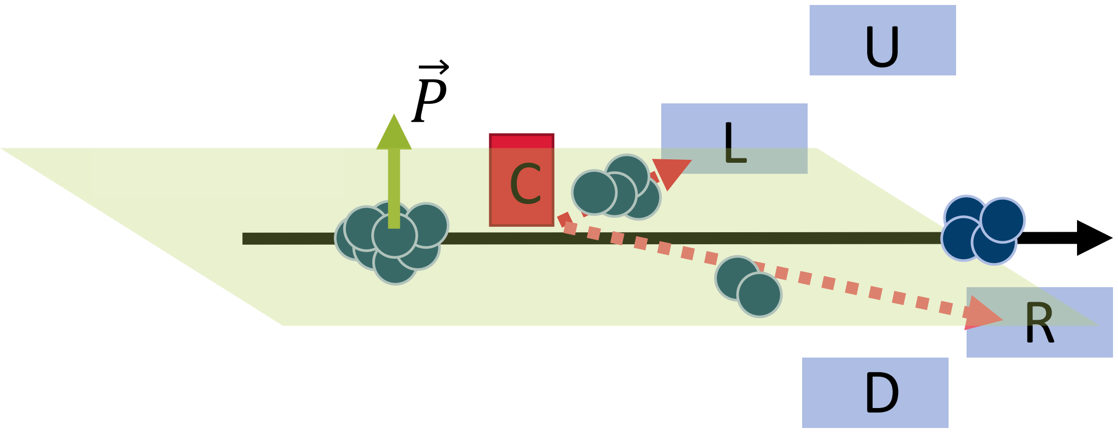
$$N_U \propto p_{xz} \cdot \sin(2\pi f_s t)$$

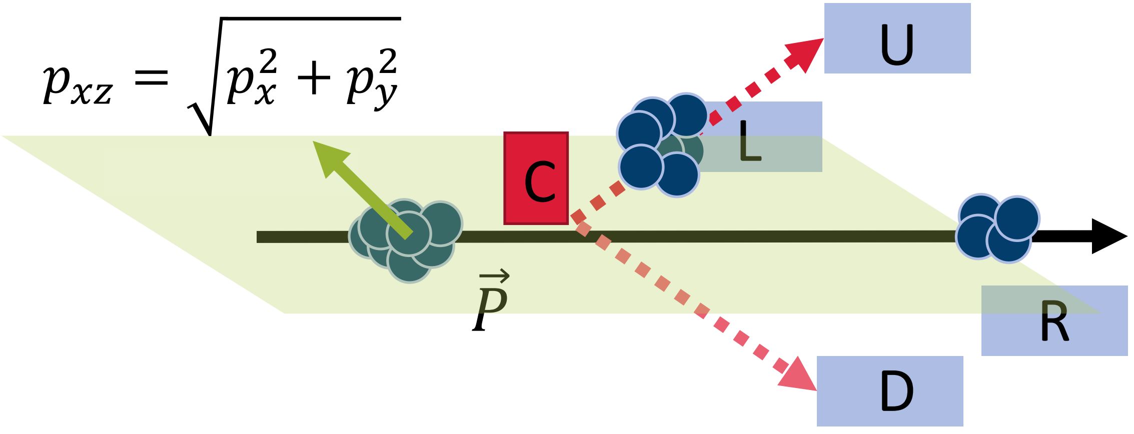
$$N_D \propto p_{xz} \cdot \sin(2\pi f_s t + \pi)$$

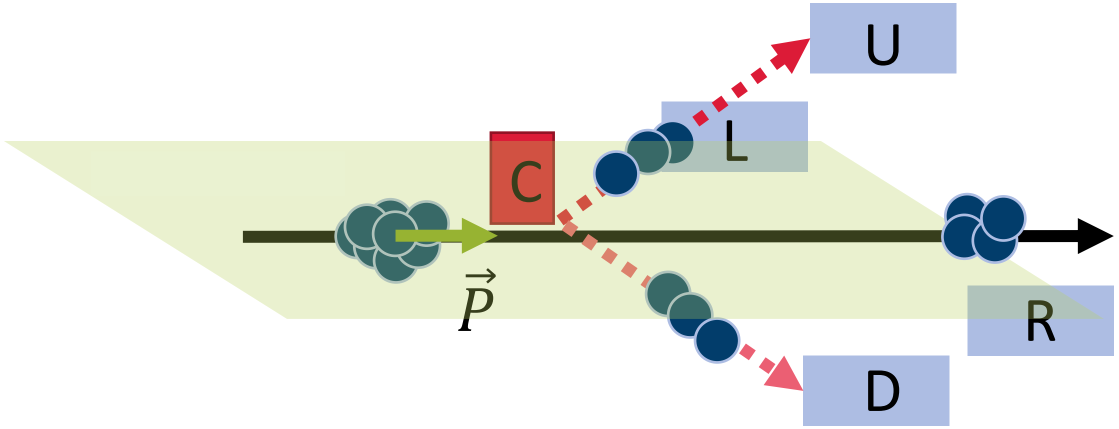
$$p_{xz} \propto \frac{N_U - N_D}{N_U + N_D}$$

$$p_{xz} = \sqrt{p_x^2 + p_y^2}$$

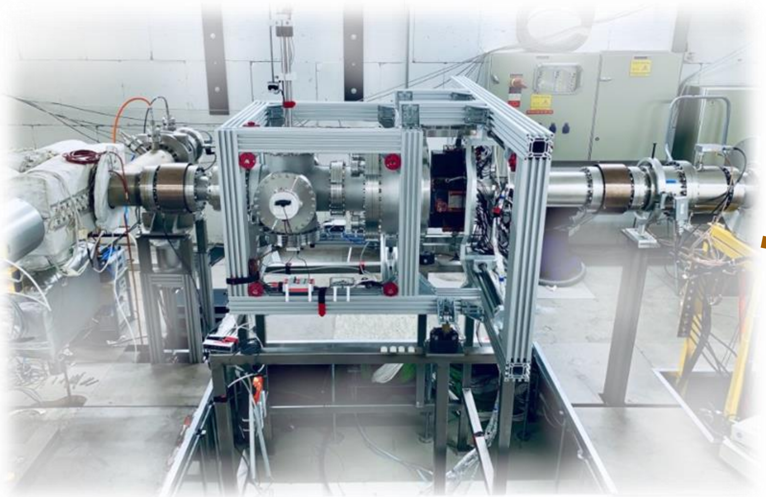
$$p_y \propto \frac{N_L - N_R}{N_L + N_R}$$

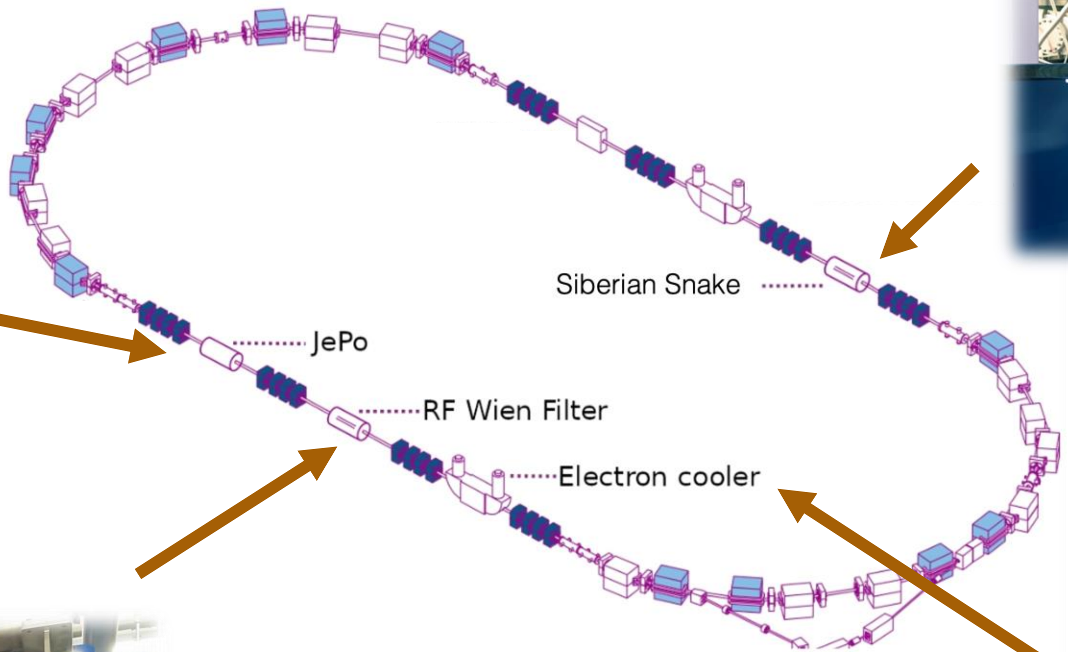


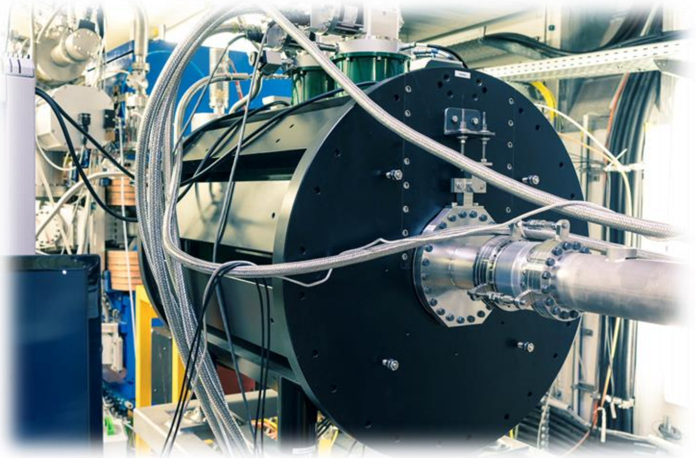




COSY

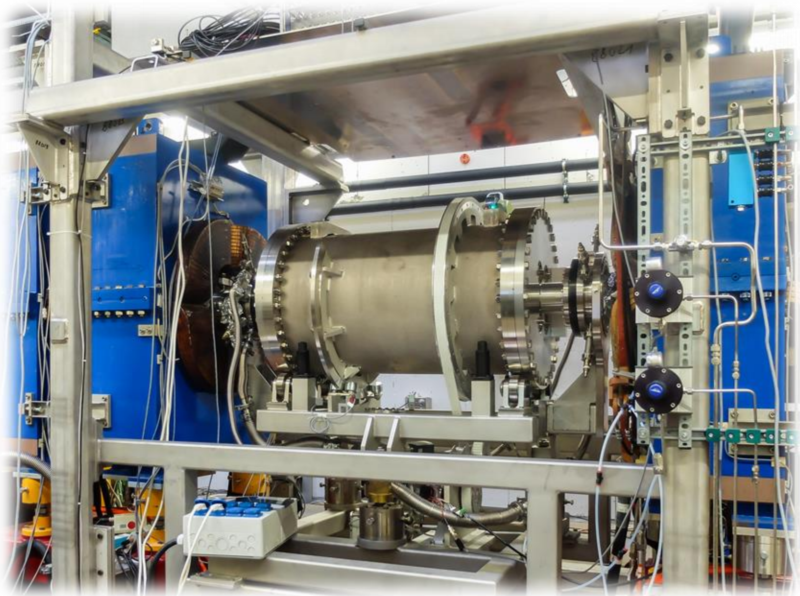






$\hat{n}_{z,\,\text{Snake}}$

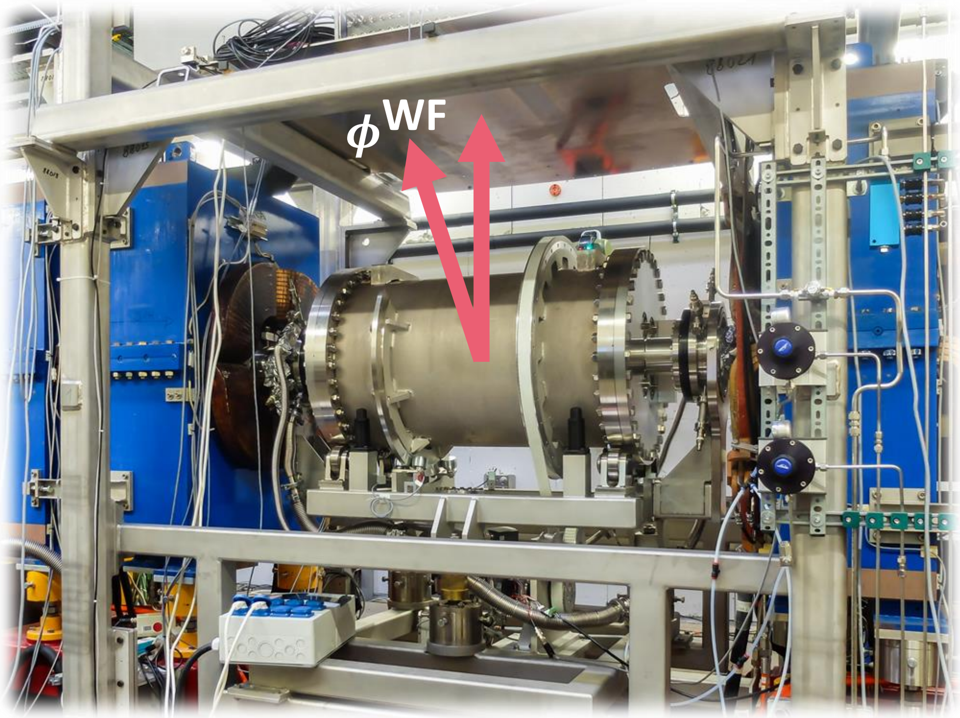
$\hat{n}_{z,\text{Sol}}$



$\hat{n}_{x,\text{WF}}$ & $\hat{n}_{z,\text{WF}}$

EDM signal!

RF WIEN FILTER



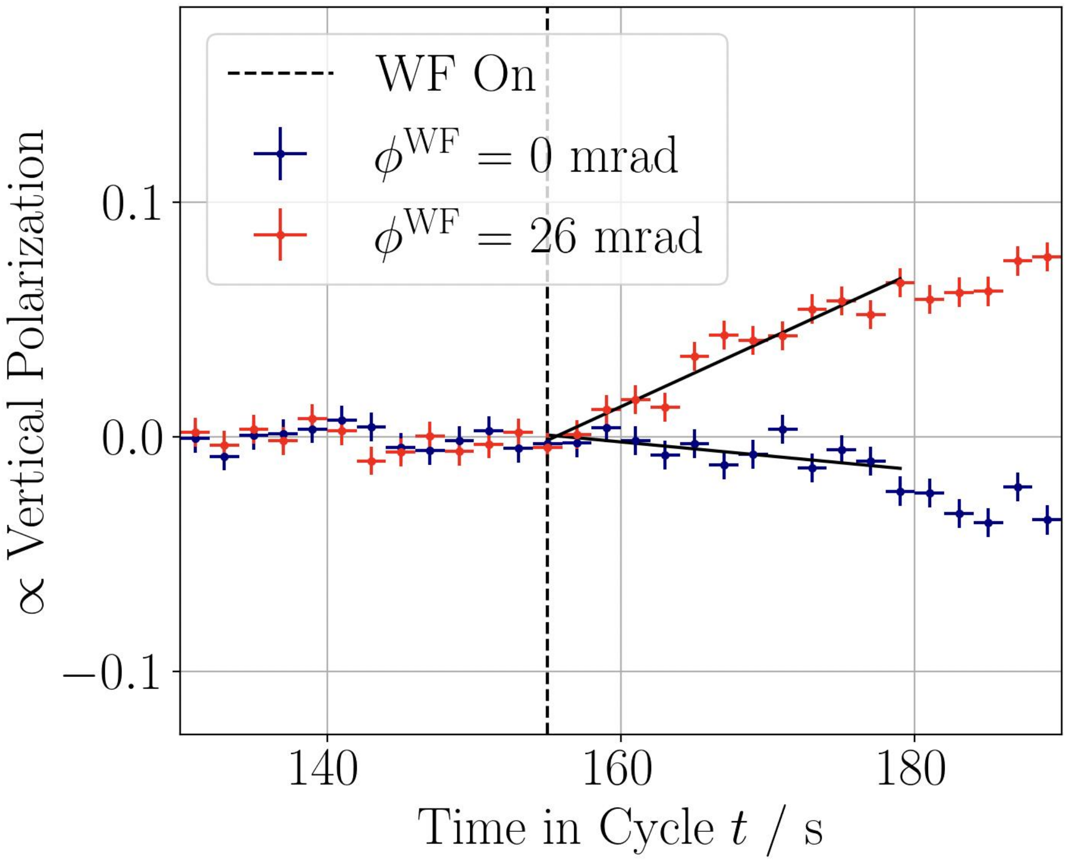
- Goal: **Measure $\boldsymbol{n_x}$**
- $\vec{E} \perp \vec{B} \perp$ Beam $\rightarrow \vec{F}_L = q \cdot (\vec{E} + \vec{v} \times \vec{B}) = 0$
- $\vec{B}$- Field can be rotated around the beam pipe by $\boldsymbol{\phi}^{\mathbf{WF}}$
- Needs to run on a harmonic of the spin precession frequency

$$f_{\mathrm{WF}} = f_s$$

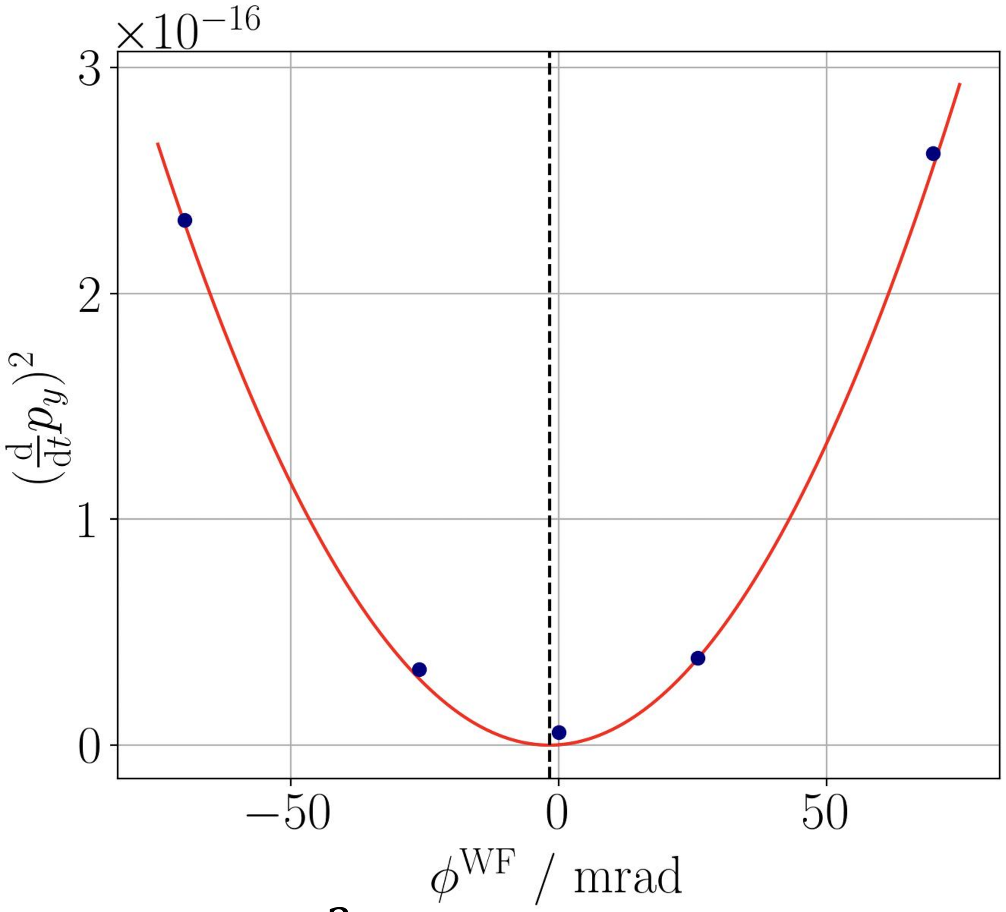
- $\vec{E} = \vec{E}_0 \cos(2\pi f_s + \phi)$ and $\vec{B} = \vec{B}_0 \cos(2\pi f_s + \phi)$
- Both frequencies need to have an adjustable phase relation

JÜLICH Forschungszentrum

MEASUREMENT PRINCIPLE



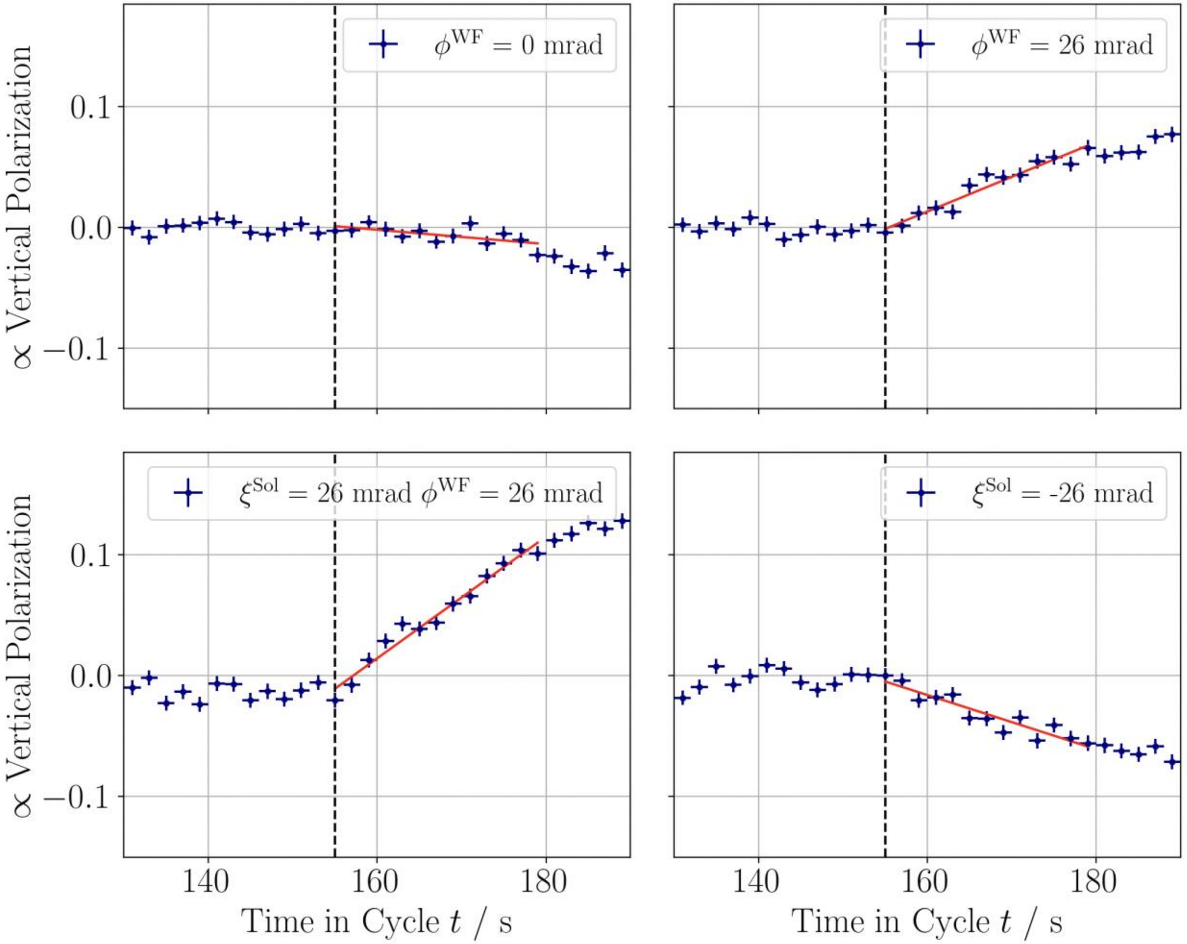
Build up rate $\frac{\mathrm{d}}{\mathrm{d}t} p_y(t)$

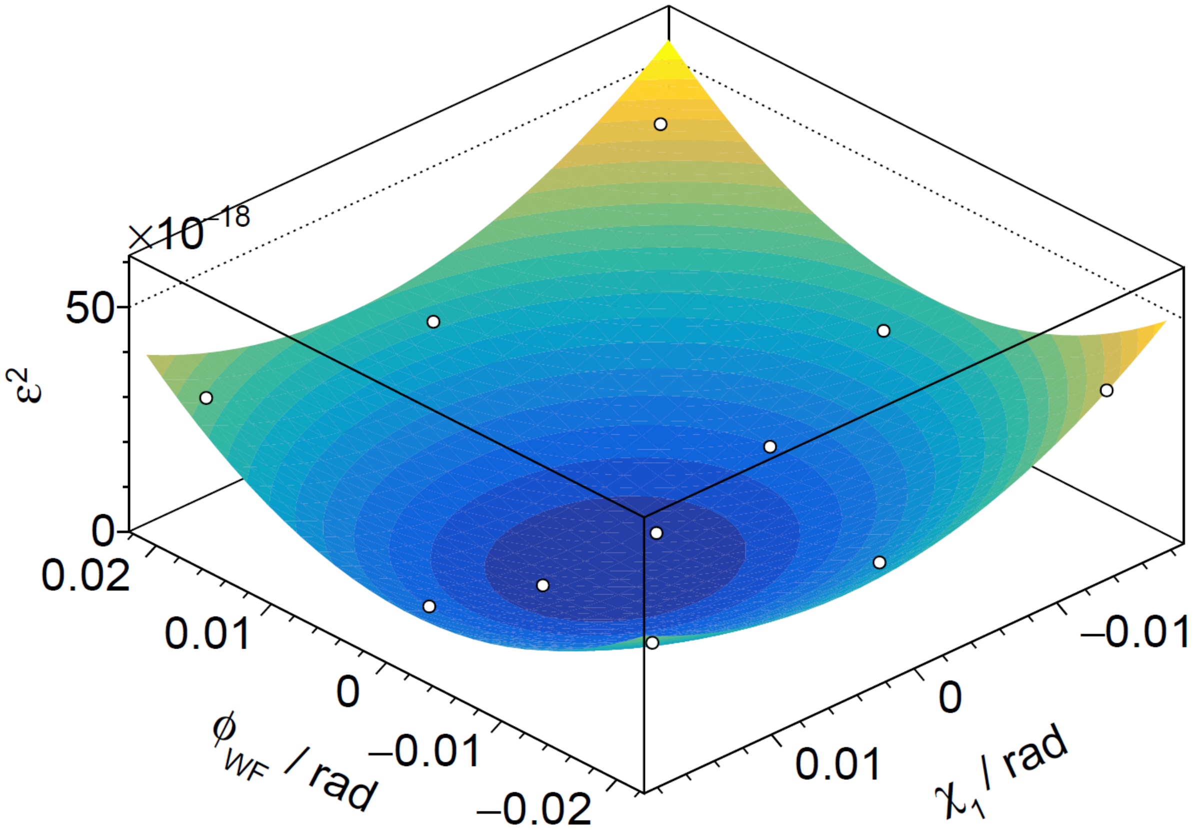


$$\left(\frac{\mathrm{d}}{\mathrm{d}t} p_y\right)^2 \propto |\vec{n} \times \vec{m}| \propto \left(n_x - \phi^{\mathrm{WF}}\right)^2 + \ldots$$

JÜLICH Forschungszentrum

MEASUREMENT PRINCIPLE



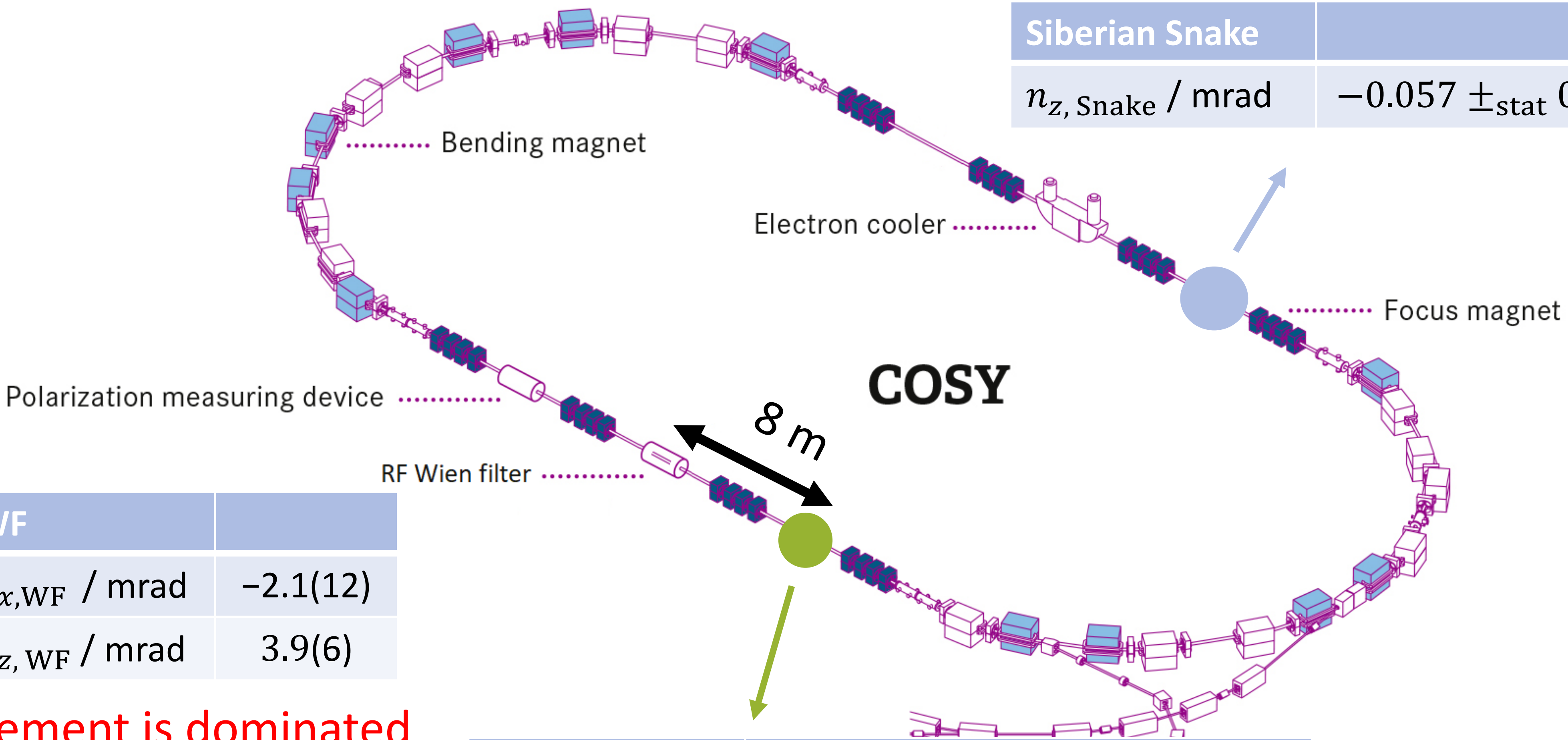


$$\left(\frac{\mathrm{d}}{\mathrm{d}t}p_y\right)^2 \propto |\vec{n}\times\vec{m}| \propto \left[\left(n_x-\phi^{\mathrm{WF}}\right)^2+\left(n_z-\xi^{\mathrm{Sol}}\right)^2\right]$$

RESULTS

n_z: Systematics

n_x: EDM + Systematics



Siberian Snake	
$n_{z,\,\mathrm{Snake}}$ / mrad	$-0.057 \pm_{\mathrm{stat}} 0.001 \pm_{\mathrm{sys}} 0.032$

WF	
$n_{x,\mathrm{WF}}$ / mrad	−2.1(12)
$n_{z,\,\mathrm{WF}}$ / mrad	3.9(6)

2MV Solenoid	
$n_{z,\,\mathrm{Sol}}$ / mrad	$-0.070 \pm_{\mathrm{stat}} 0.001 \pm_{\mathrm{sys}} 0.032$

Measurement is dominated by systematics!

JÜLICH Forschungszentrum

SPIN DYNAMICS

Thomas – BMT Equation

$$\vec{d} = d\frac{\vec{s}}{|\vec{s}|} \text{ with } d = \eta^{\text{EDM}}\frac{q\hbar}{2mc}$$

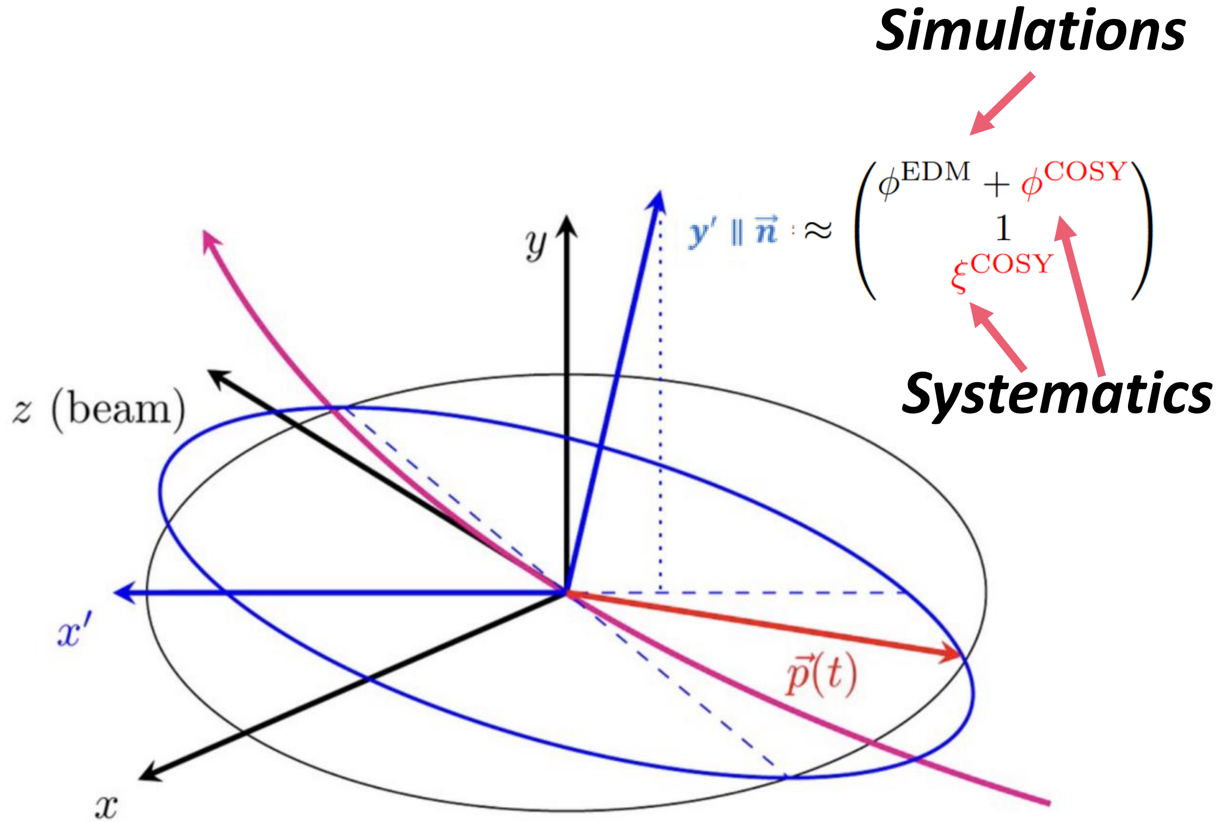
- Measure the influence of the EDM on the spin motion

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}n} = \left(\vec{\Omega}_{\text{MDM}} + \vec{\Omega}_{\text{EDM}}\right) \times \vec{S}$$

$$\vec{\Omega}_{\text{MDM}} = -2\pi\gamma G\vec{e}_y$$

$$\vec{\Omega}_{\text{EDM}} = \eta^{\text{EDM}}\gamma\beta\vec{e}_x$$

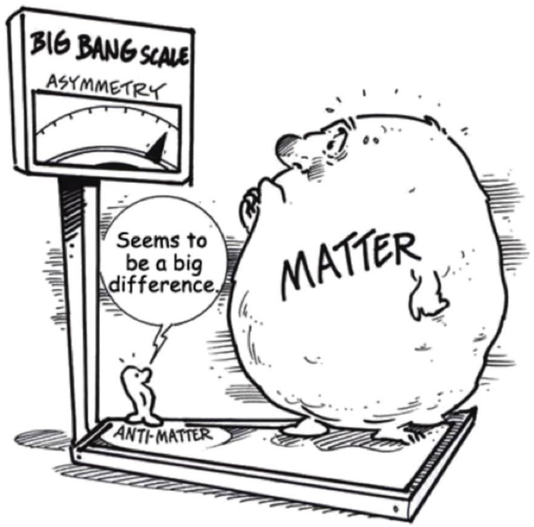
$$\tan\left(\frac{|\vec{\Omega}_{\text{EDM}}|}{|\vec{\Omega}_{\text{MDM}}|}\right) \approx \phi_{\text{EDM}} \approx -\eta^{\text{EDM}}\frac{\beta}{2G}$$



- $|d| < 3.0 \times 10^{-17}\ e \cdot \text{cm}$ (95% C.L.) (Preliminary)

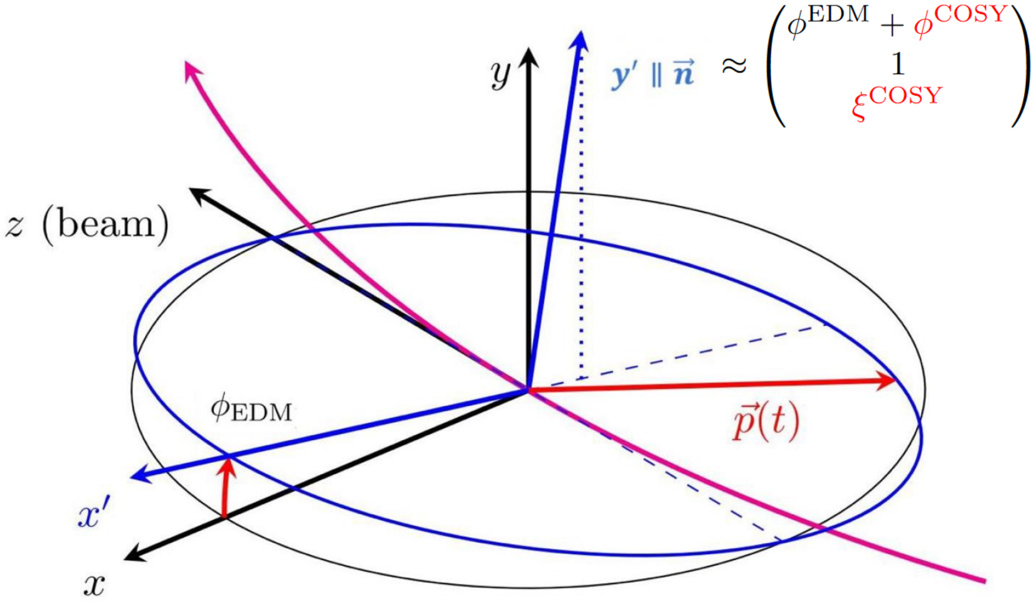
JÜLICH Forschungszentrum

SUMMARY

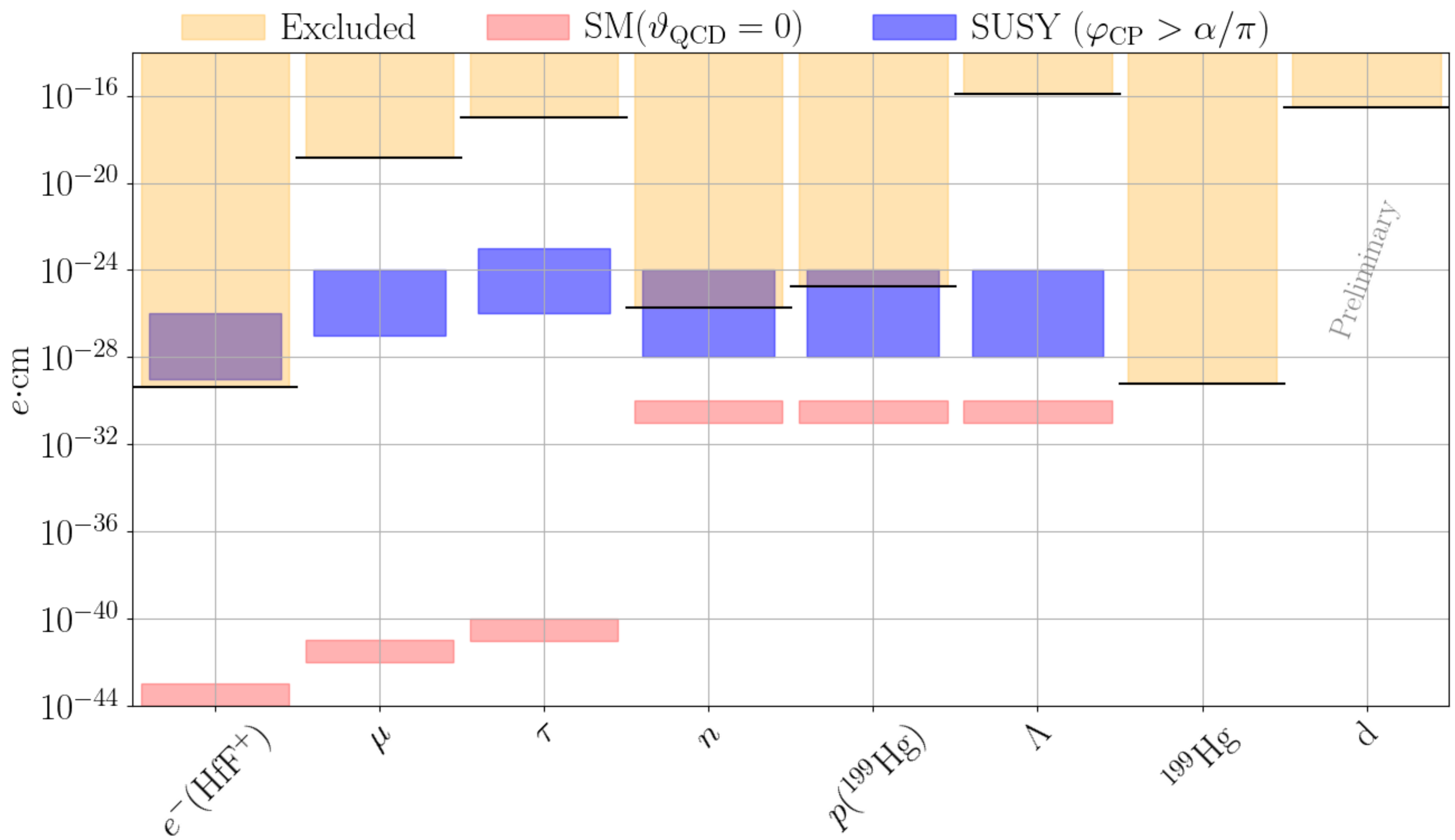


- **Motivation:** EDMs as a source of CP violation and a problem solver
- **Goal**: Measure the **influence** of the deuteron EDM on the **beam polarization**
- While the method works, the data **cannot be interpret correctly**
- We determine a **first limit** of the permanent deuteron EDM

$|d| < 3.0 \times 10^{-17}\ e \cdot \mathrm{cm}$ (95% C.L.) (Preliminary)



PERMANENT DEUTERON EDM SEARCH

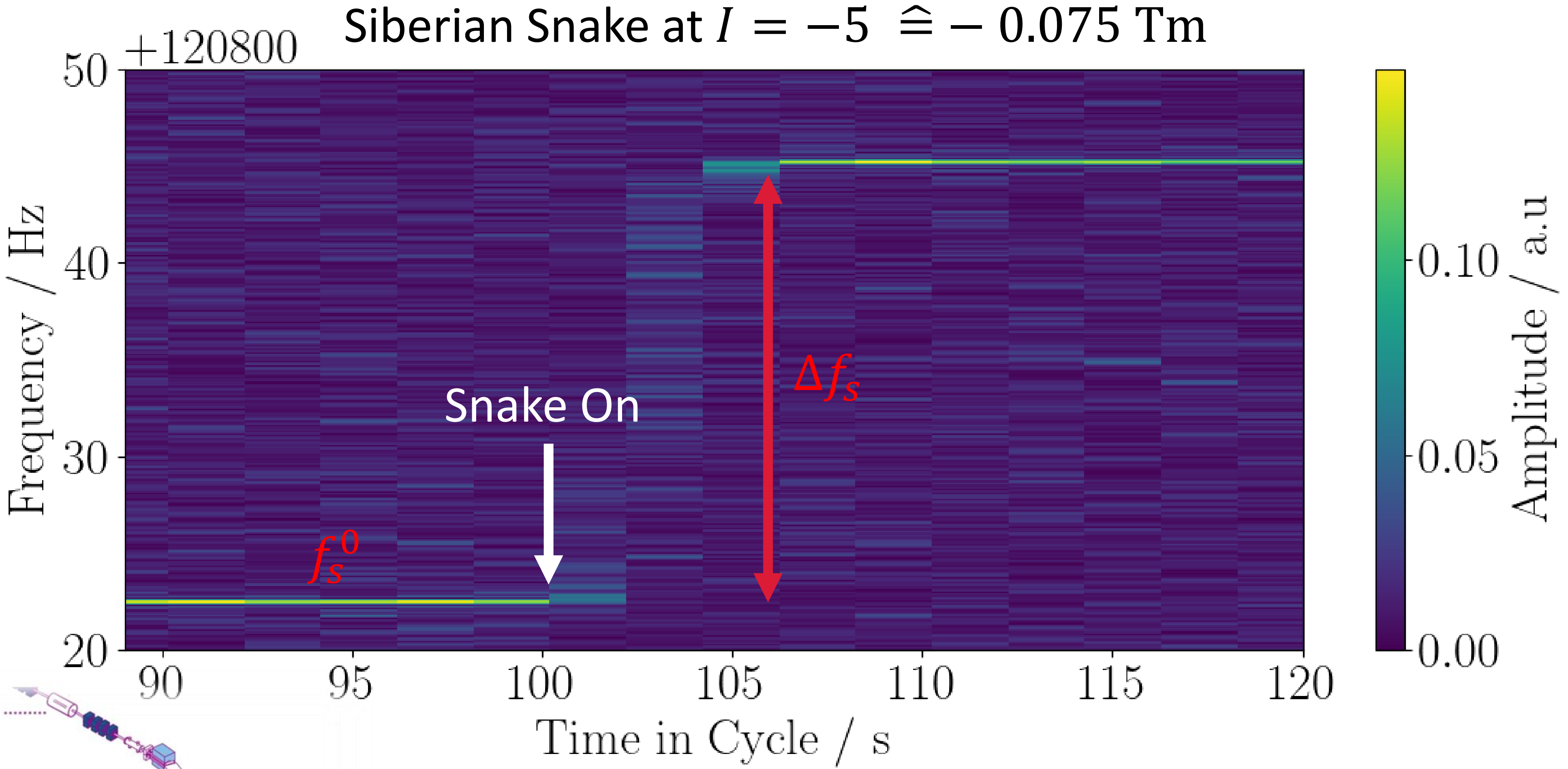


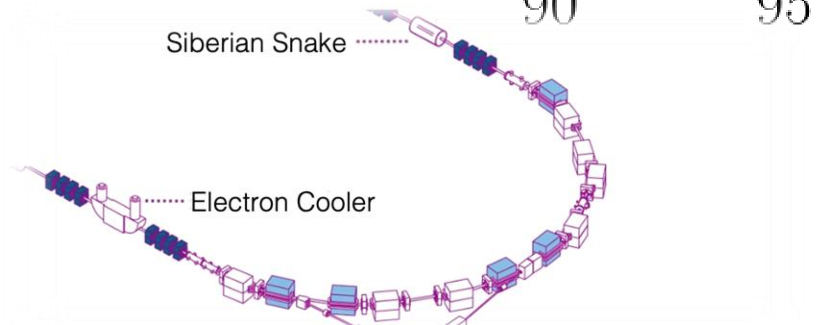
Publication in progess..

BACK UP

INVARIANT SPIN AXIS AT THE SOLENOIDS

Methodology



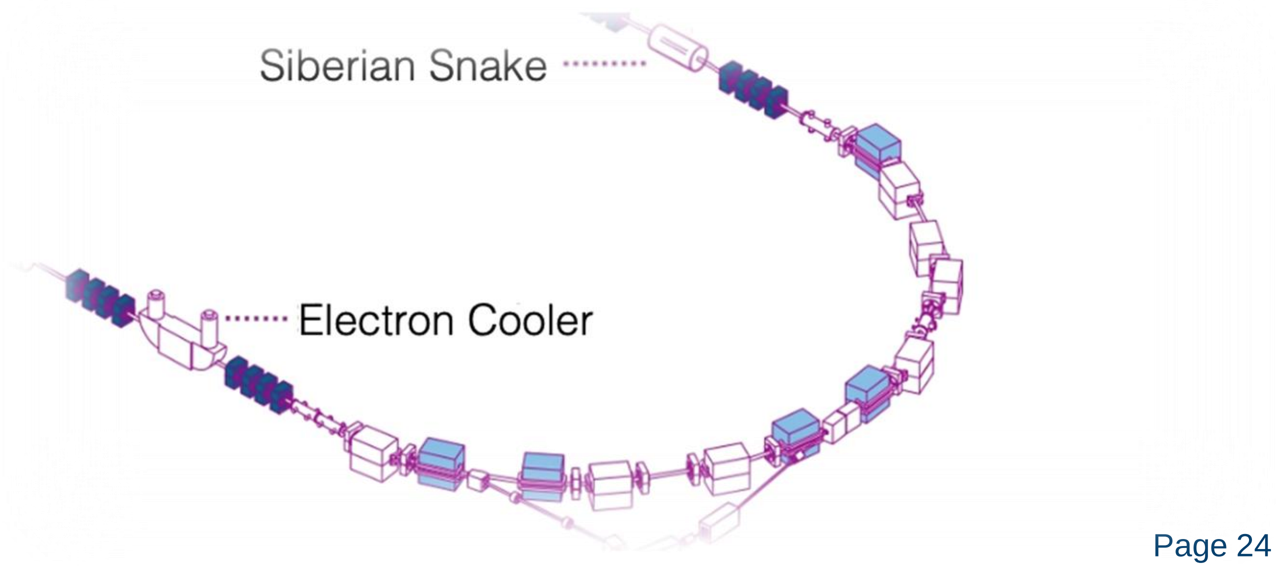


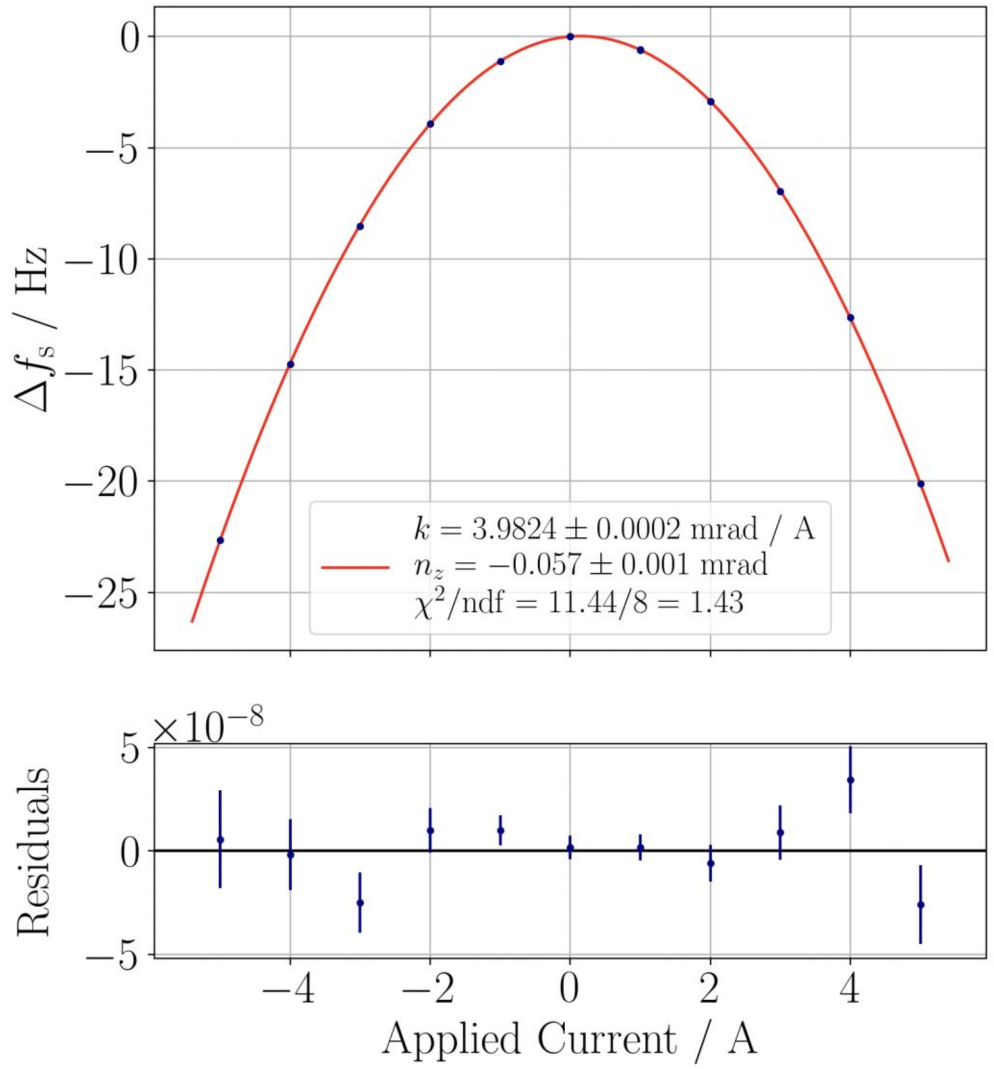
JÜLICH
Forschungszentrum

INVARIANT SPIN AXIS AT THE SOLENOIDS

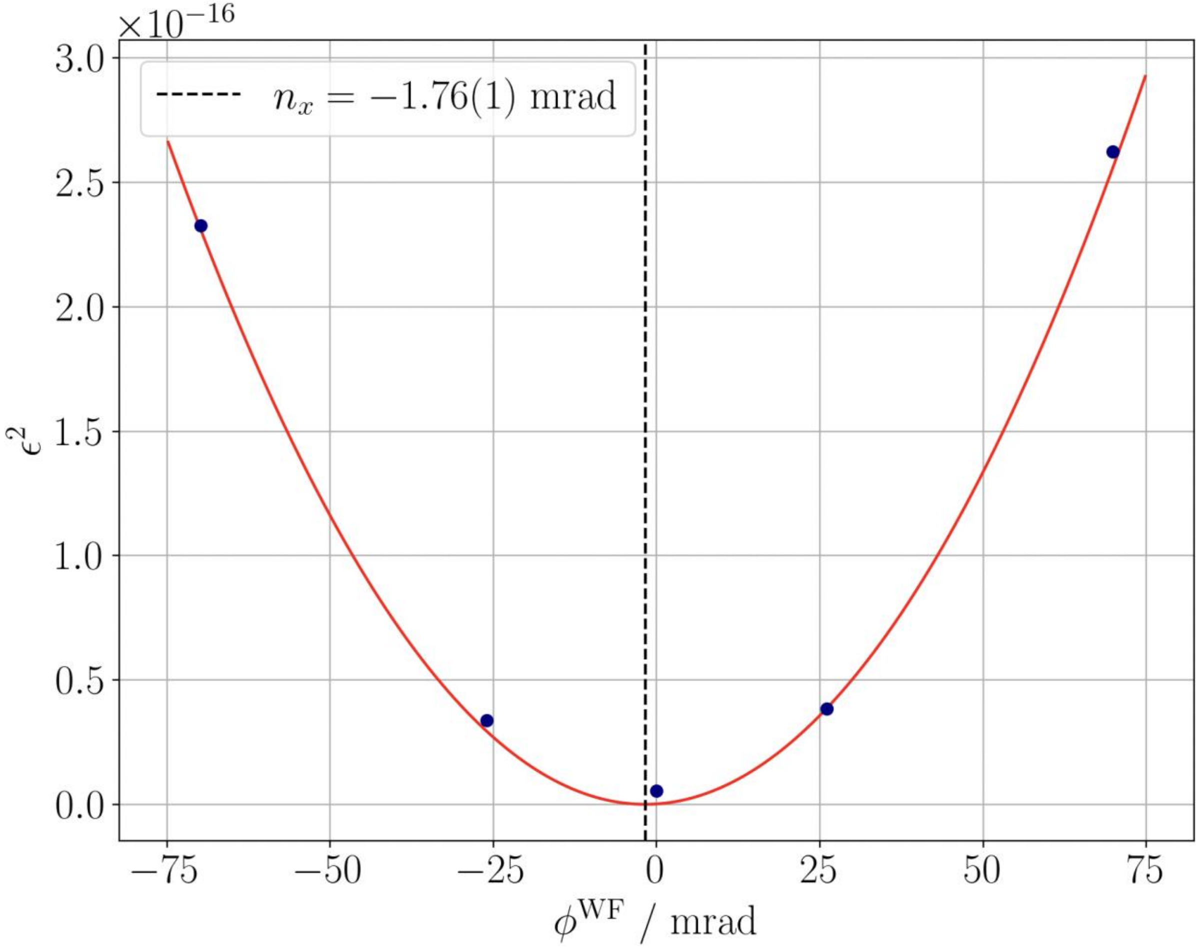
$$\Delta f_s = \frac{f_{\mathrm{rev}}}{4\pi}\left[\frac{\cos\left(\pi\frac{f_s^0}{f_{\mathrm{rev}}}\right)}{2} k^2 I^2 + n_z k I\right]$$

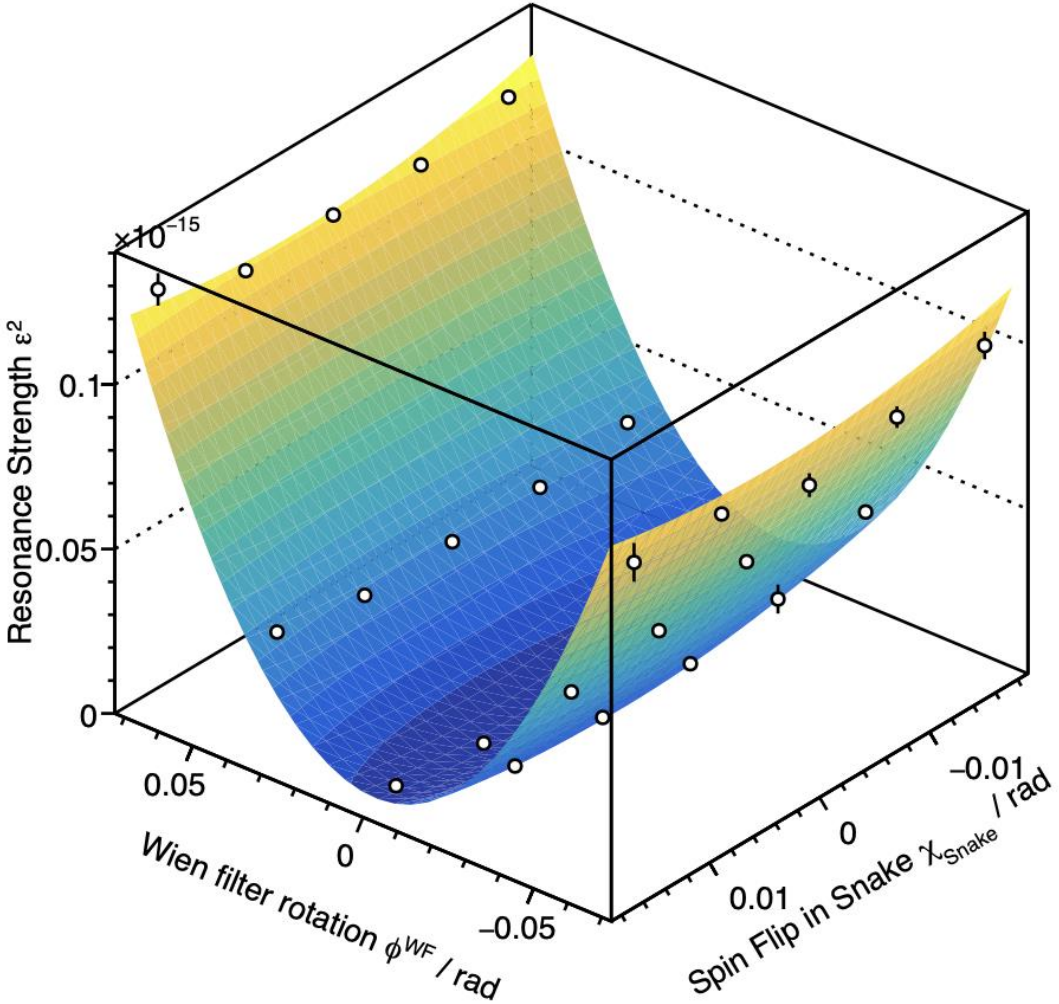
- k translates the coil current into a spin tilit angle
- n_z denotes the z component of the invariant spin axis



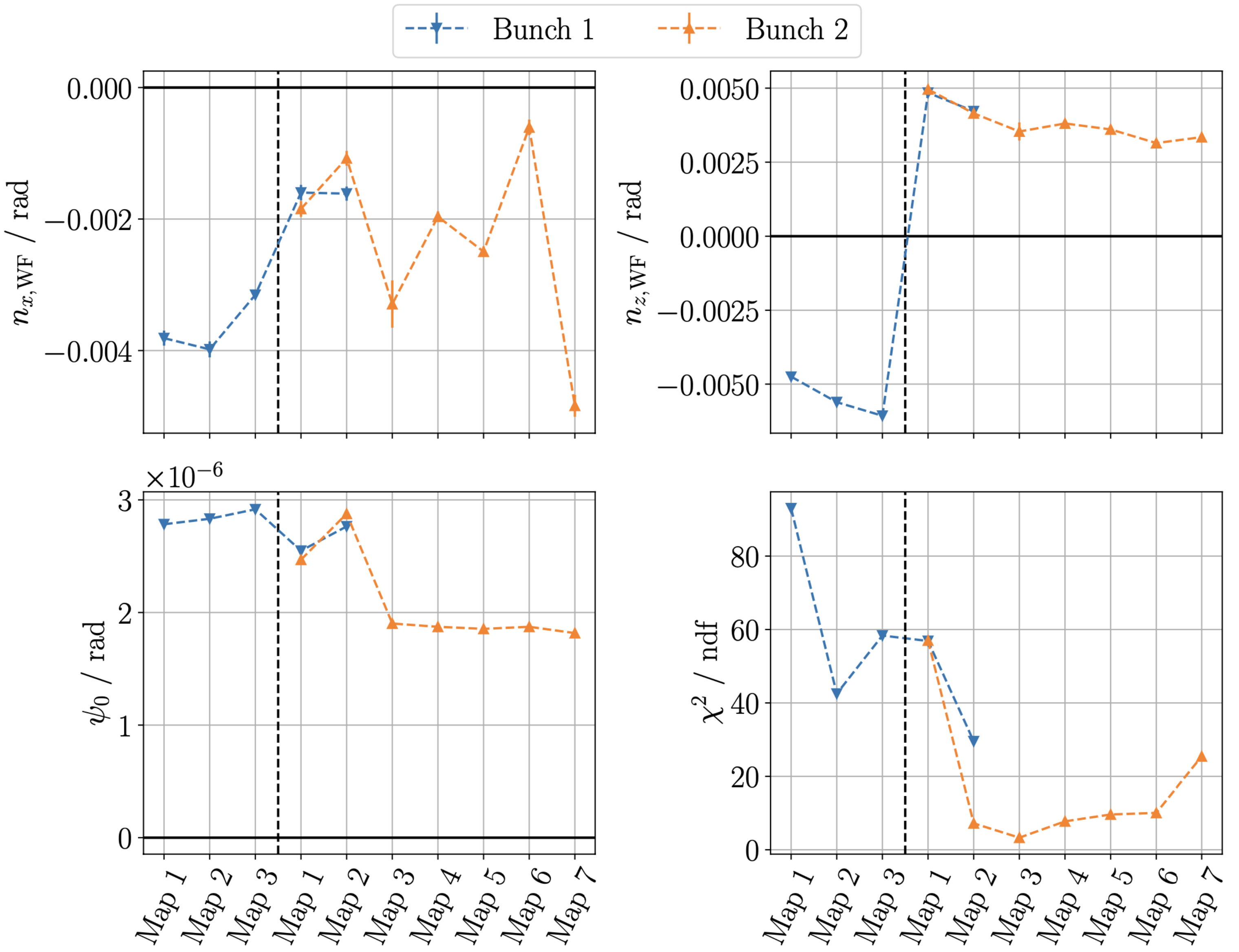


DETAILED LOOK INTO DATA





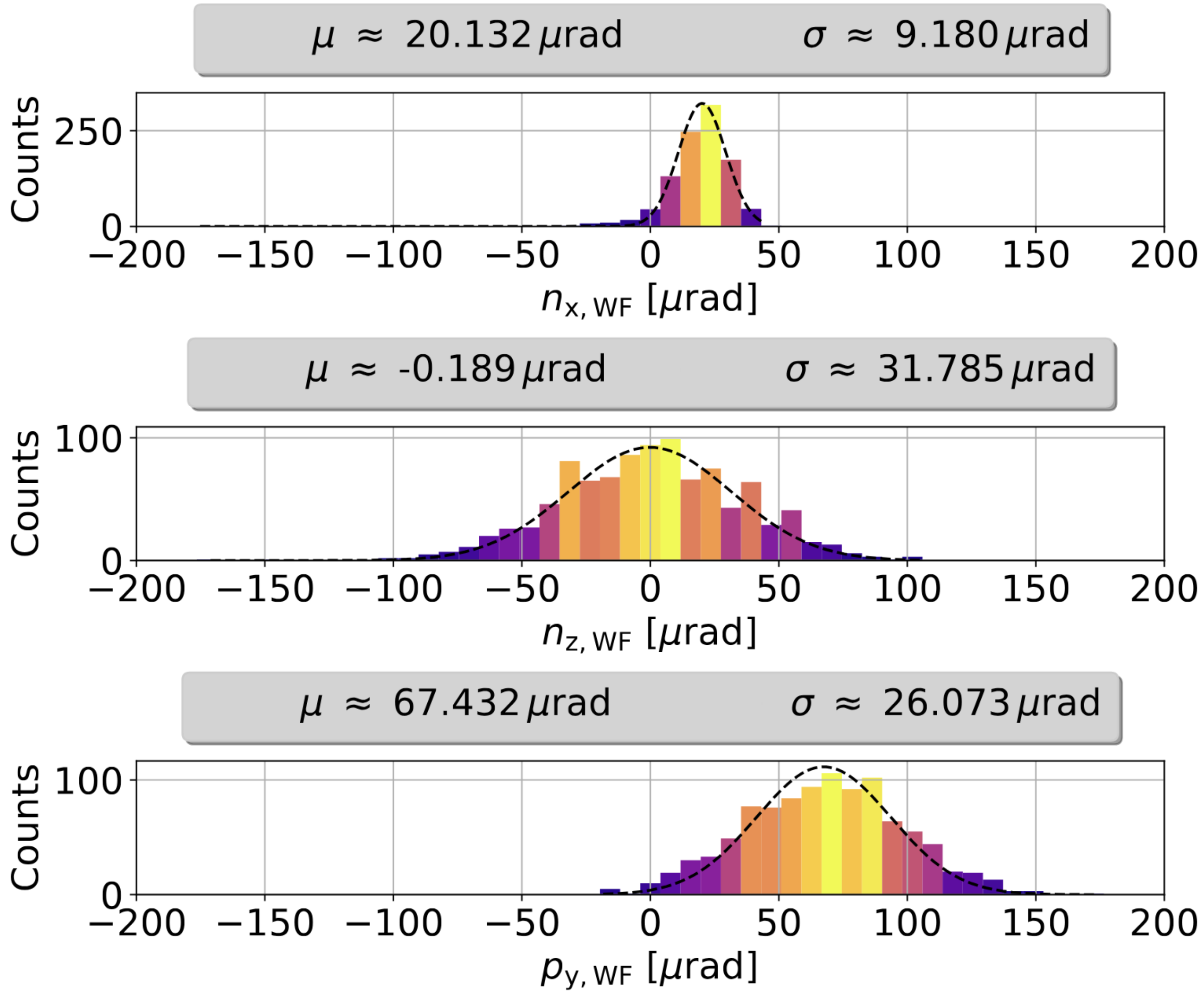
RESULTS



Bunch 1
Bunch 2
$n_{x,\mathrm{WF}}$ / rad
0.000
−0.002
−0.004
$n_{z,\mathrm{WF}}$ / rad
0.0050
0.0025
0.0000
−0.0025
−0.0050
$\times 10^{-6}$
3
2
1
0
ψ_0 / rad
χ^2 / ndf
80
60
40
20
0
Map 1
Map 2
Map 3
Map 1
Map 2
Map 3
Map 4
Map 5
Map 6
Map 7

SYSTEMATIC ESTIMATES FROM SIMULATIONS

Max Vitz (PhD)



ELECTRIC DIPOLE MOMENTS

Axion Search

- Violation of symmetries was observed in the weak sector
- However: not sufficient
- **CP** violation in the strong sector

$$L_{\bar{\theta}_{\text{QCD}}} = -\bar{\theta}_{\text{QCD}} \frac{g_s^2}{64\pi^2} \varepsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta}$$

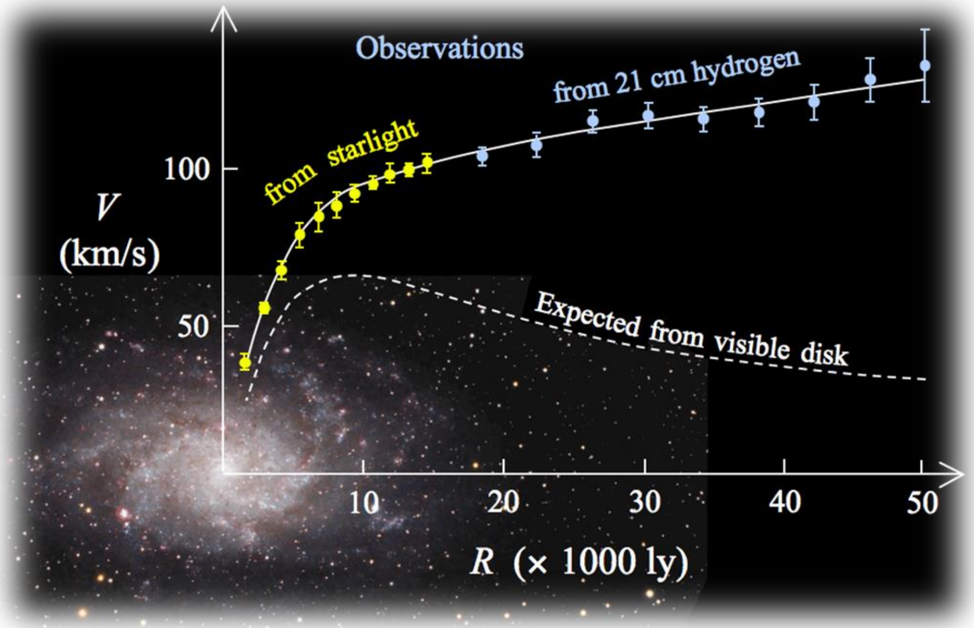
- Limits from neutron EDM measurements limit constrain

$$\bar{\theta}_{\text{QCD}} \leq 10^{-10} \Rightarrow \text{Strong } CP \text{ Problem}$$

- Problem solver: **Axion** or **Axion Like Particles** (ALPs)

$$\vec{d} = d \frac{\vec{s}}{|\vec{s}|} \quad \text{with } d = d_{\text{DC}} + d_{\text{AC}} \cos(\omega_a + \phi_a) \text{ and } \omega_a = \frac{m_a c^2}{\hbar}$$

- Existence of an axion leads to an additional oscillating EDM component
- Axion could explain the strong CP - problem
- Axion are potential candidate for Dark Matter



JÜLICH Forschungszentrum

AXION SEARCH @ COSY

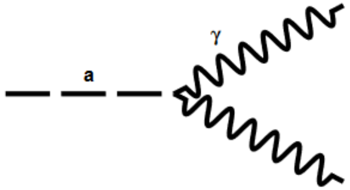
$$d = d_{\mathrm{DC}} + d^{d}_{\mathrm{AC}} \cos(\omega_a + \phi_a)$$

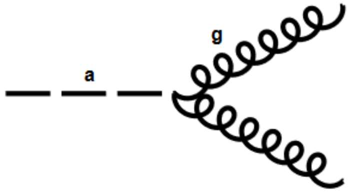
$$\omega_a = \frac{m_a c^2}{\hbar}$$

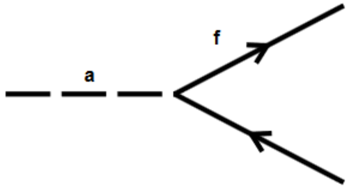
- Constraints for the axion gluon coupling:

$$|g_{ad\gamma}| < 1.7 \times 10^{-7}\,\mathrm{GeV}^{-2}$$

$$\mathcal{L}: -\frac{\alpha}{8\pi}\frac{C_\gamma}{f_a}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu} \quad -\frac{\alpha_s}{8\pi}\frac{C_G}{f_a}\, a\, G^b_{\mu\nu}\tilde{G}^{b,\mu\nu} \quad -\frac{1}{2}\frac{C_N}{f_a}\,\partial_\mu a\, \bar{\Psi}_f \gamma^\mu \gamma^5 \Psi_f$$







studied by many experiments

Electric Dipole Moment (EDM)

axion wind term

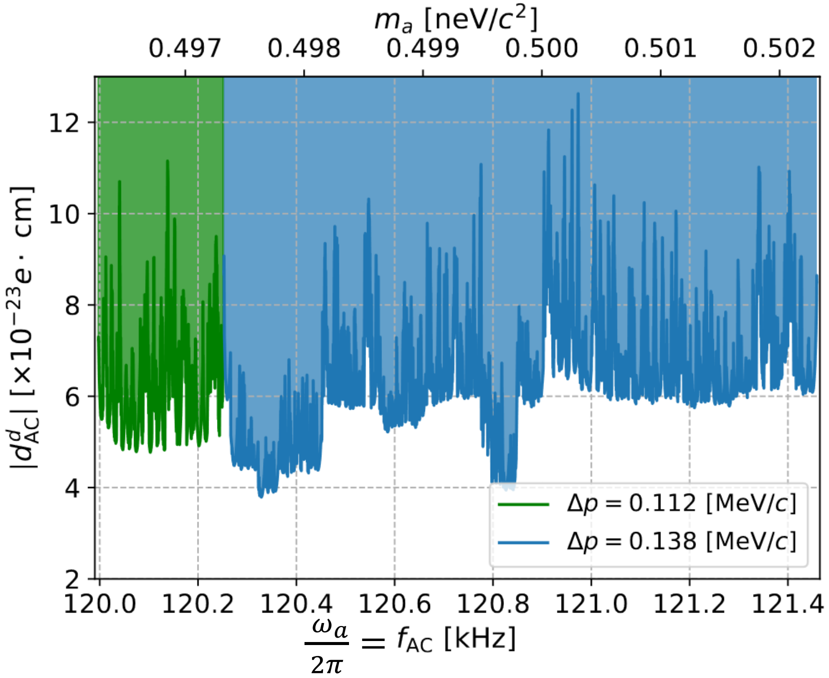
accessible in storage ring experiments with spin polarized beams

PHYSICAL REVIEW X **13**, 031004 (2023)

First Search for Axionlike Particles in a Storage Ring Using a Polarized Deuteron Beam

S. Karanth,[1,*] E. J. Stephenson,[2,†] S. P. Chang,[3,4] V. Hejny,[5] S. Park,[4] J. Pretz,[5,6,7] Y. K. Semertzidis,[3,4] A. Wirzba,[5,8] A. Wrońska,[1] F. Abusaif,[6,5,¶] A. Aggarwal,[1] A. Aksentev,[9] B. Alberdi,[6,5,**,††] A. Andres,[6,5] L. Barion,[10] I. Bekman,[5,‡] M. Beyß,[6,5] C. Böhme,[5] B. Breitkreutz,[5,§] C. von Byern,[6,5] N. Canale,[10] G. Ciullo,[10] S. Dymov,[10] N.-O. Fröhlich,[5,‖] R. Gebel,[5,11] K. Grigoryev,[5,§] D. Grzonka,[5] J. Hetzel,[5] O. Javakhishvili,[12] H. Jeong,[13] A. Kacharava,[5] V. Kamerdzhiev,[5,§] I. Keshelashvili,[5,§] A. Kononov,[10] K. Laihem,[6,§] A. Lehrach,[5,7] P. Lenisa,[10] N. Lomidze,[14] B. Lorentz,[11] A. Magiera,[1] D. Mchedlishvili,[14,19] F. Müller,[6,5] A. Nass,[5] N. N. Nikolaev,[15,16] A. Pesce,[5] V. Poncza,[6,5] D. Prasuhn,[5,§] F. Rathmann,[5] A. Saleev,[10] D. Shergelashvili,[14] V. Shmakova,[10,§] N. Shurkhno,[5,§] S. Siddique,[6,5,§] J. Slim,[6,11,‖] H. Soltner,[17] R. Stassen,[5] H. Ströher,[5,7] M. Tabidze,[14] G. Tagliente,[18] Y. Valdau,[5,§] M. Vitz,[5,6] T. Wagner,[5,6,§] and P. Wüstner[17]

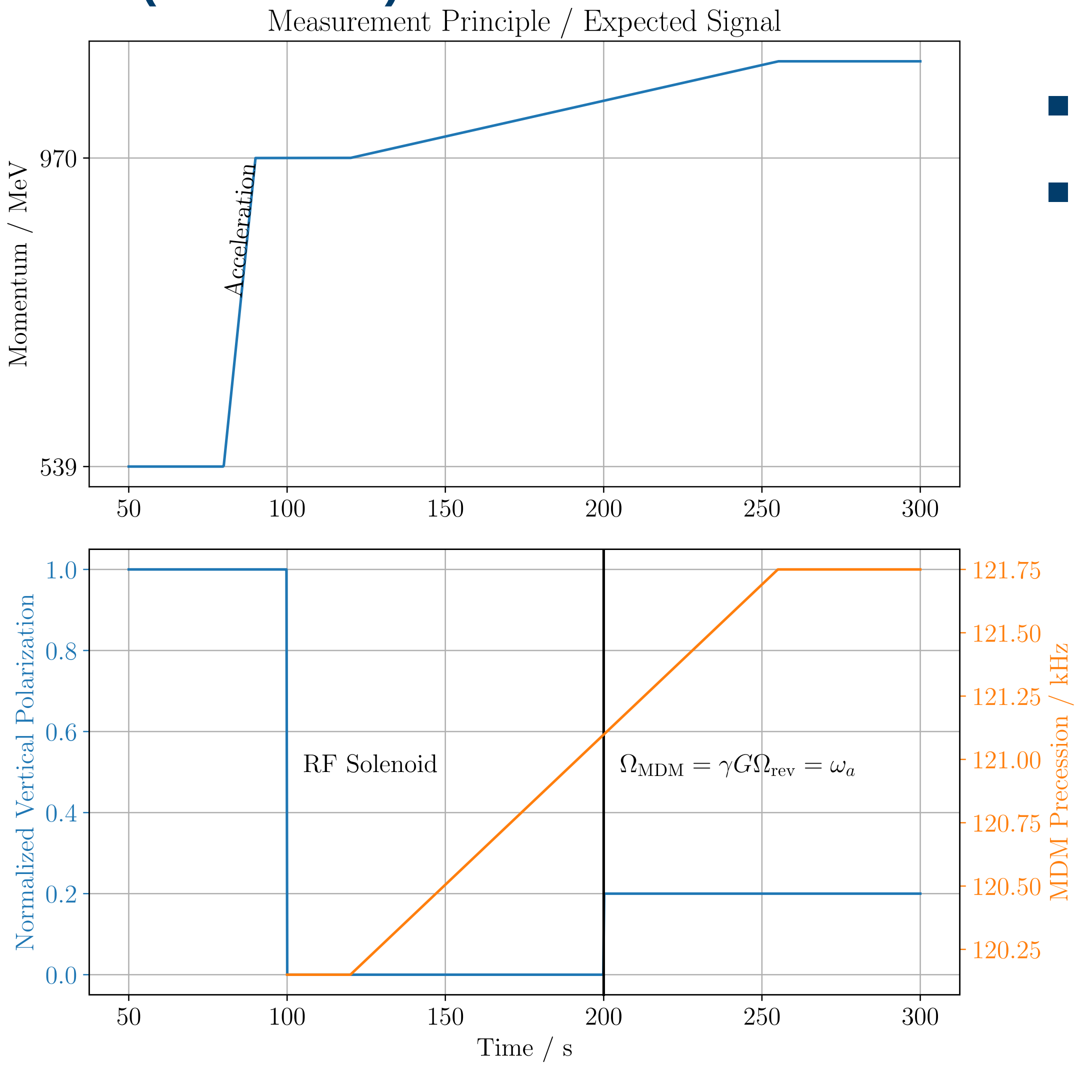
(JEDI Collaboration)



- 90% CL upper limit on the ALPs induced oscillating EDM
- Average of $|d^d_{\mathrm{AC}}| < 6.4 \times 10^{-23} e \cdot \mathrm{cm}$

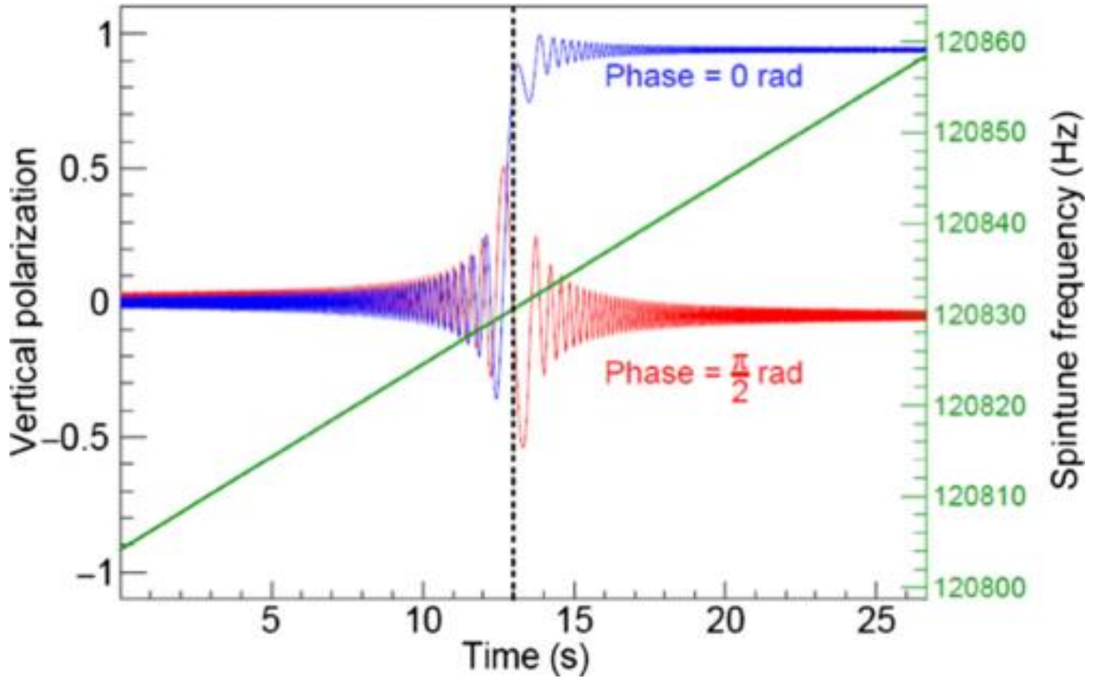
AXION SEARCH

S. Karanth (PhD Work)



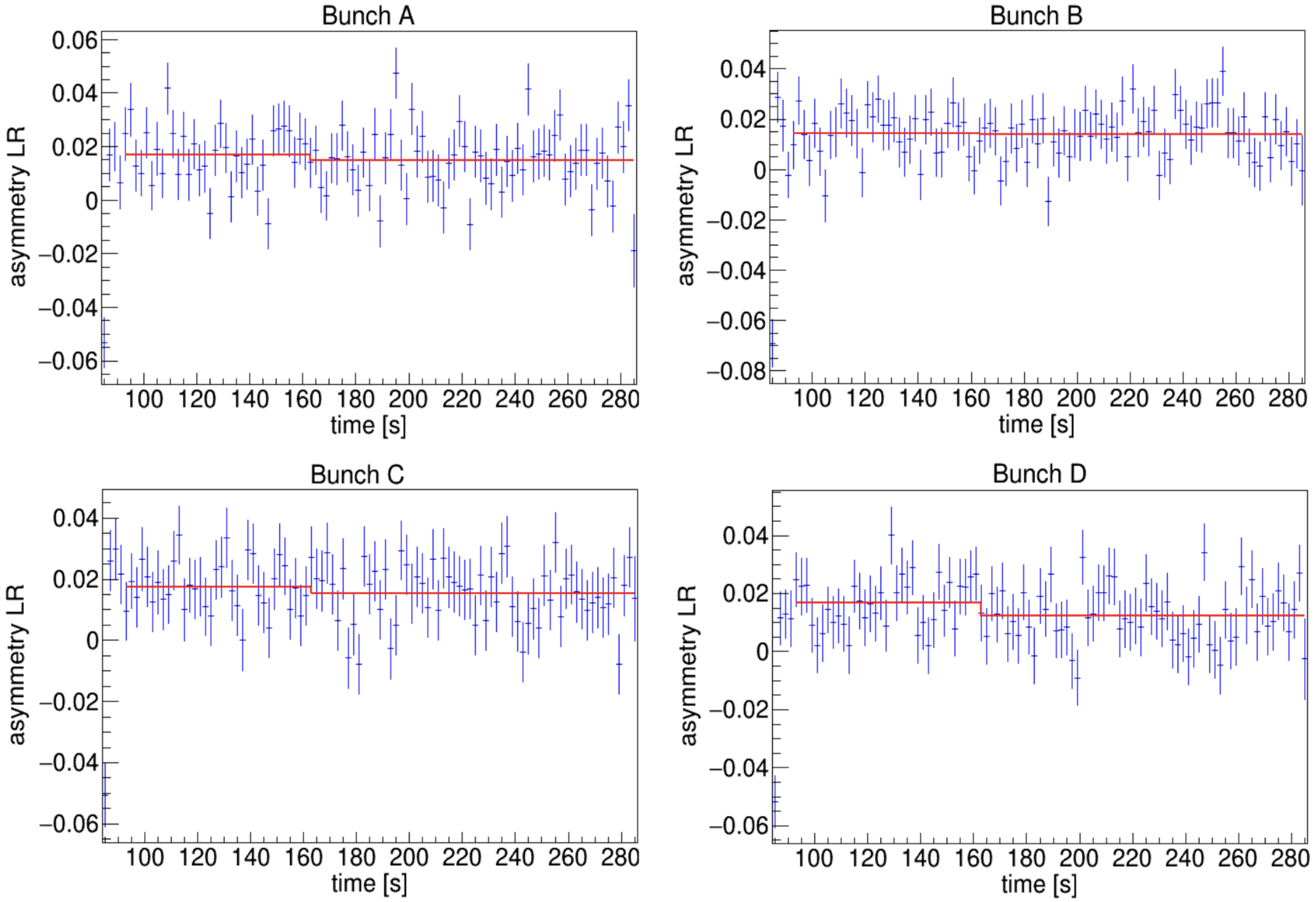
$$d = d_{\mathrm{DC}} + d^{d}_{\mathrm{AC}} \cos(\omega_a + \phi_a)$$

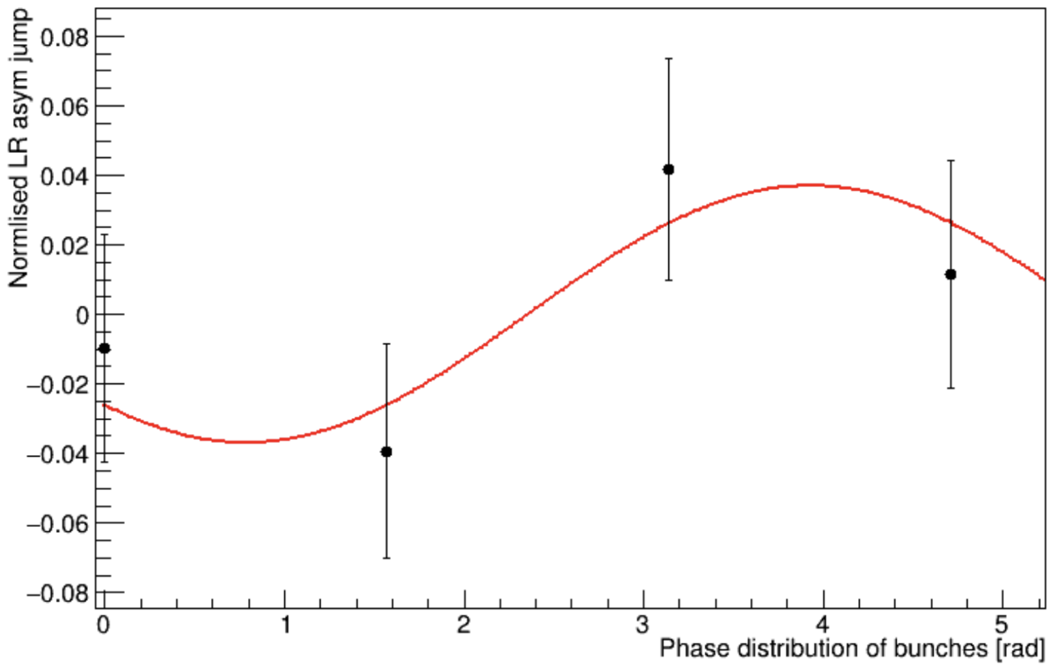
- Problem: Phase is unknown!
- Solution: Inject 4 bunches with different spin directionality!



AXION SEARCH

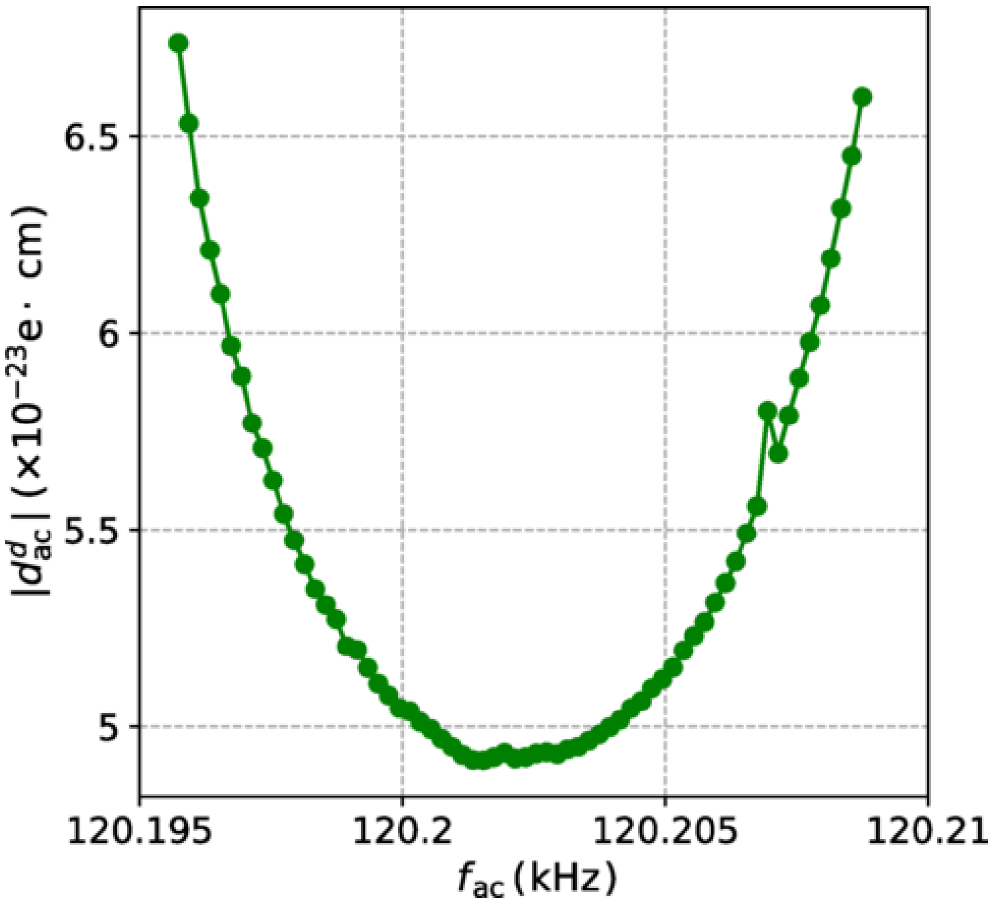
S. Karanth (PhD Work)

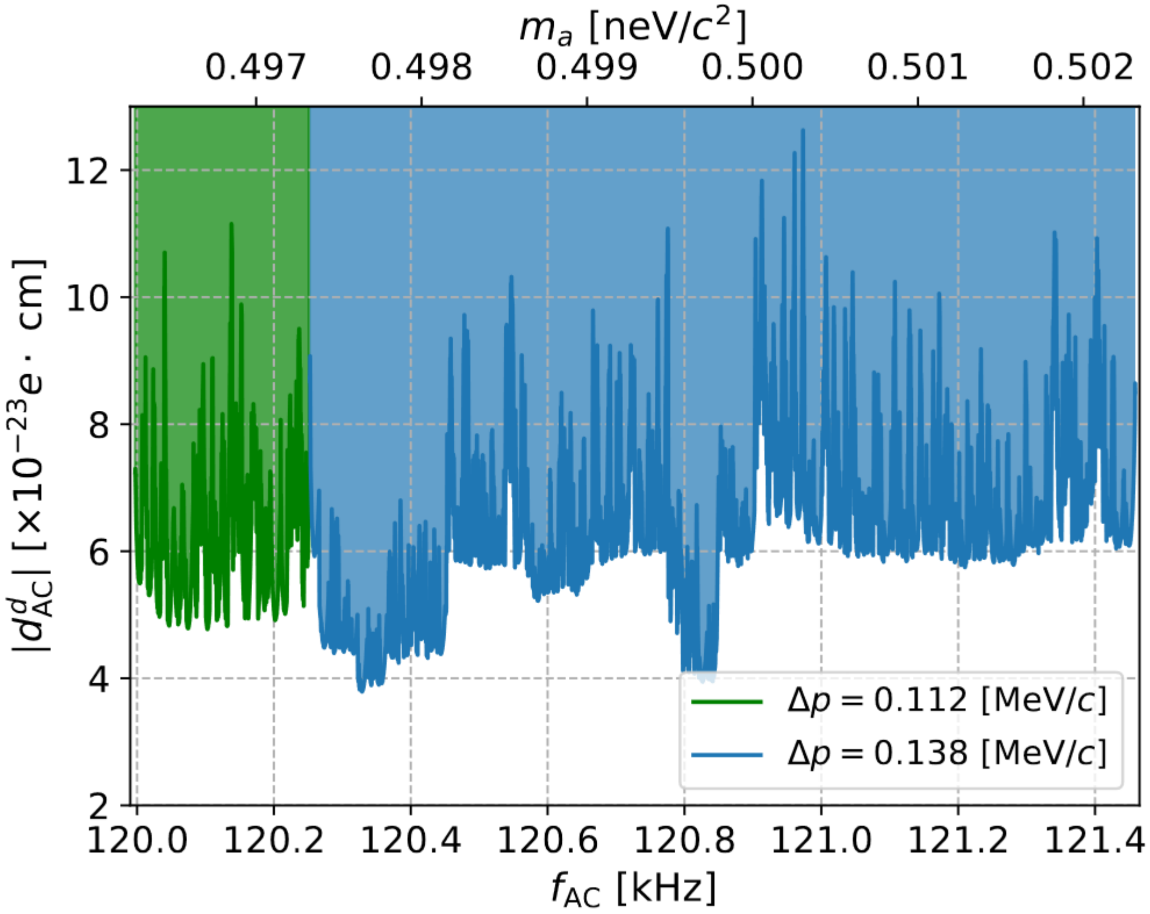




AXION SEARCH

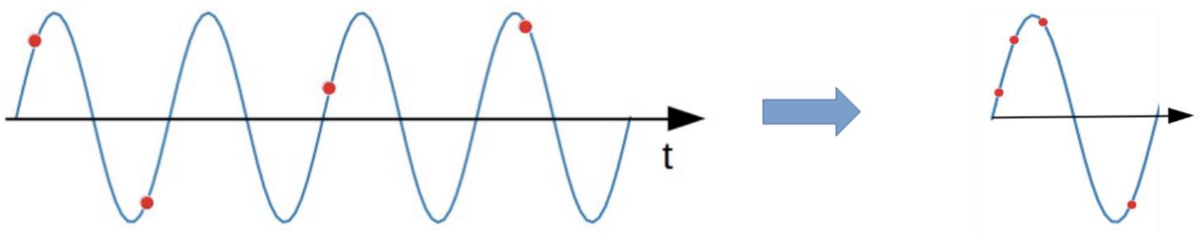
S. Karanth (PhD Work)





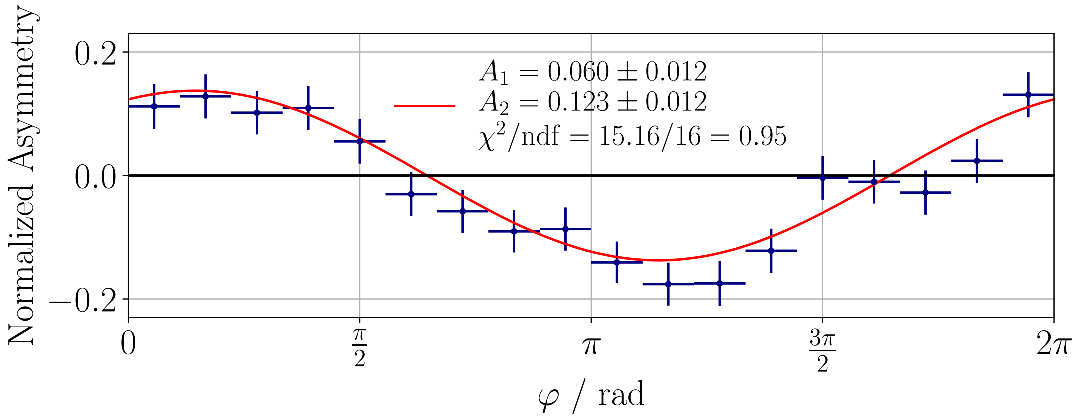
SPIN TUNE MEASUREMENT

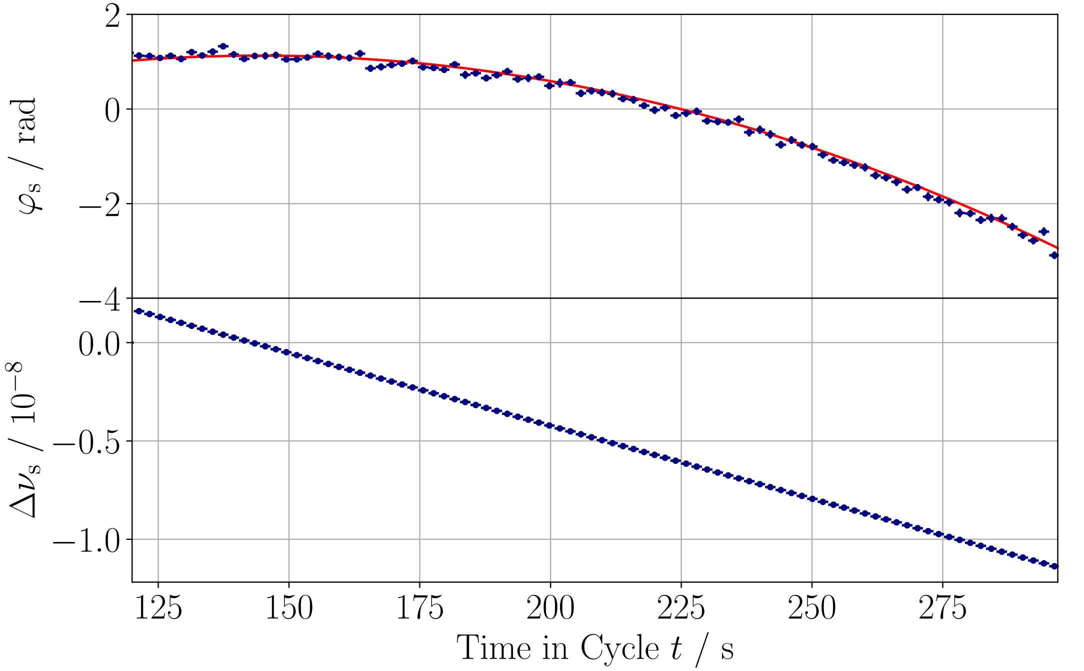
- Spin precesses with $f_s = \gamma G f_{\text{rev}} \approx 121\ \text{kHz}$
- Detector handles event rates $15000\,\frac{1}{s}$
  - 1 hit per 10 precessions
  - No direct fit possible
- Assume a fixed frequency ν_s^{fixed}



- The change of spin tune is given by

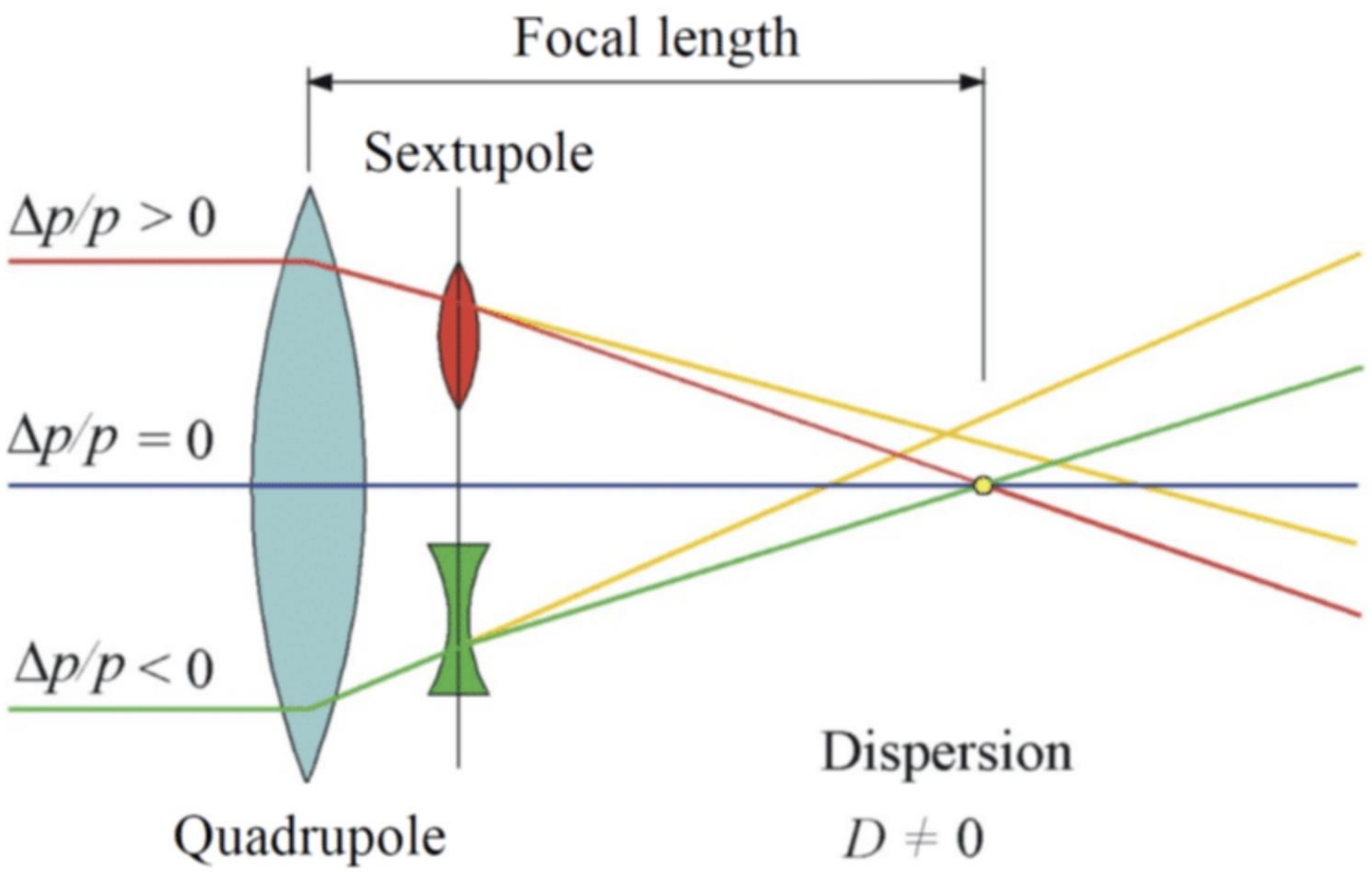
$$\nu_s(t) = \nu_s^{\text{fixed}} + \frac{1}{2\pi f_{\text{rev}}}\frac{d\varphi_s}{dt} = \nu_s^{\text{fixed}} + \Delta\nu_s(t)$$



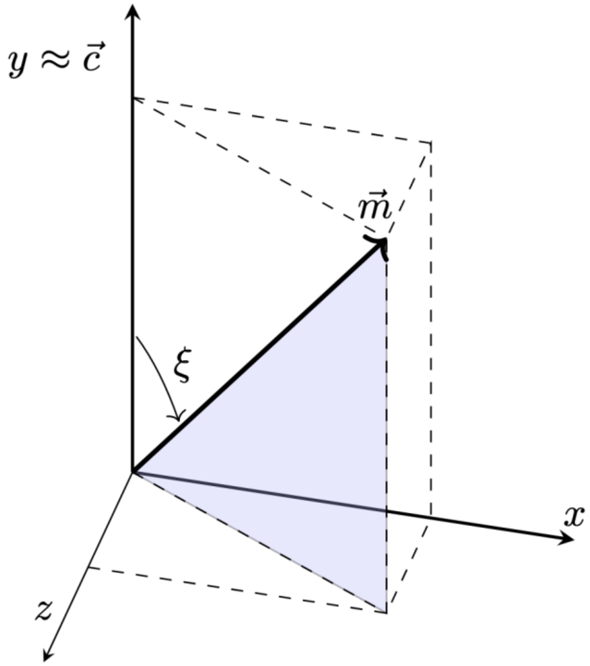


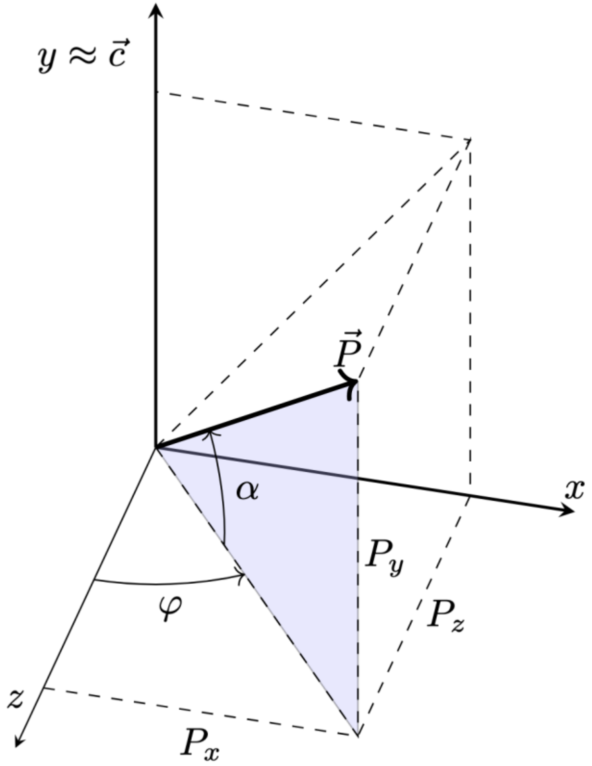
JÜLICH Forschungszentrum

SEXTUPOLE CORRECTION FOR SCT ENHANCEMENT



PHASE FEEDBACK





- Tasks of the phase feedback:
  - Measures the spin precession frequency and adjusts the Wien filter frequency to it
  - Provides a fixed phase relation between both frequencies

$$\frac{\mathrm{d}\alpha}{\mathrm{d}n} = \frac{k}{2}\cos\varphi_{\mathrm{rel}}$$

$$\frac{\mathrm{d}\varphi_{\mathrm{rel}}}{\mathrm{d}n} = \frac{k}{2}\left(\sin\varphi_{\mathrm{rel}}\tan\alpha + q\right)$$

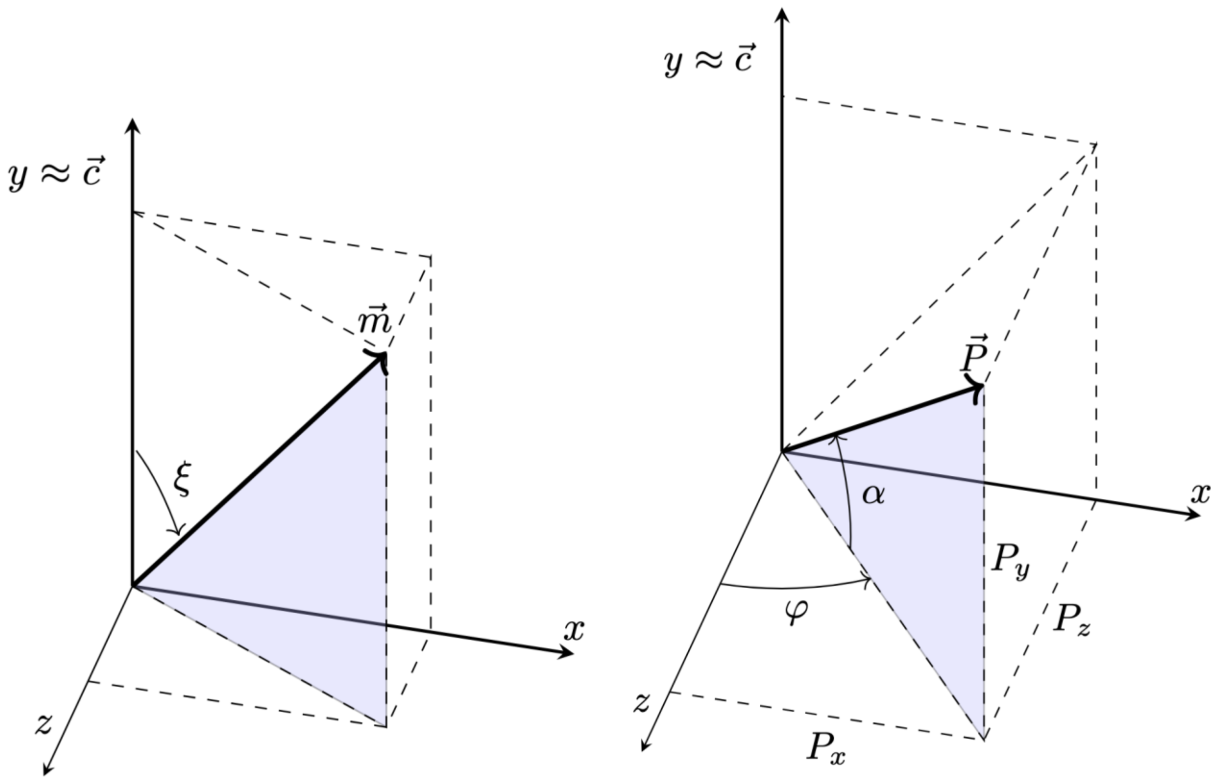
$$k = -\chi_0 \sin\xi = -4\pi\epsilon$$

$$q = \frac{4\pi\Delta f}{k f_{\mathrm{rev}}}$$

$$\epsilon = \frac{f_{\mathrm{v}}}{f_{\mathrm{rev}}} = \frac{1}{4\pi}\chi_0 \left|\vec{c}\times\vec{m}\right| = \frac{1}{4\pi}\chi_0 \sin\xi$$

PHASE FEEDBACK

Slope Method



- Phase Feedback prevents the relative phase from changing

$$\frac{d\alpha}{dn} = \frac{k}{2}\cos\varphi_{\mathrm{rel}}$$

$$\frac{d\varphi_{\mathrm{rel}}}{dn} = \frac{k}{2}(\sin\varphi_{\mathrm{rel}}\tan\alpha + q) \rightarrow \frac{d\varphi_{\mathrm{rel}}}{dn} = 0$$

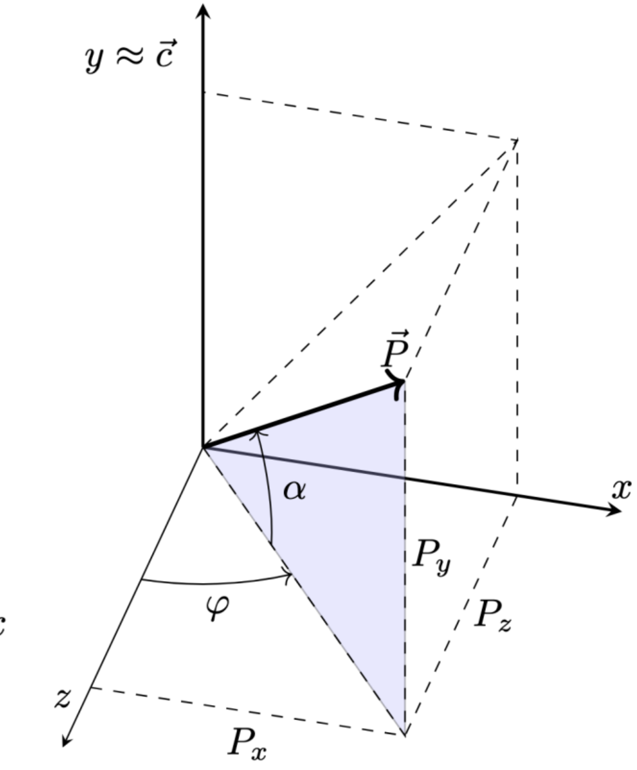
$$\frac{d\alpha}{dt} = 2\pi\epsilon f_{\mathrm{rev}}\cos\varphi_{\mathrm{rel}}$$

$$\epsilon = \frac{f_{\mathrm{v}}}{f_{\mathrm{rev}}} = \frac{1}{4\pi}\chi_0\,|\vec{c}\times\vec{m}| = \frac{1}{4\pi}\chi_0\,\sin\xi$$

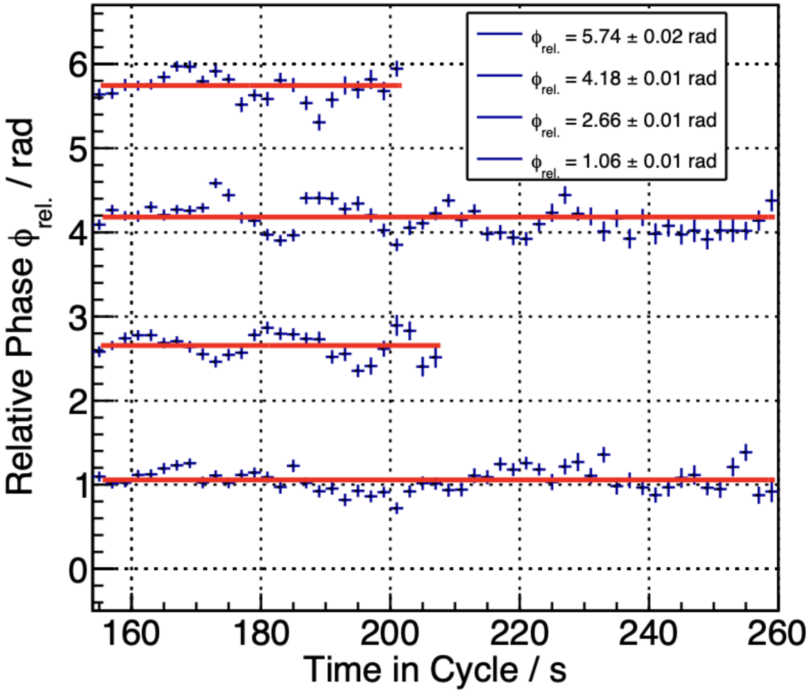
PHASE FEEDBACK

Slope Method

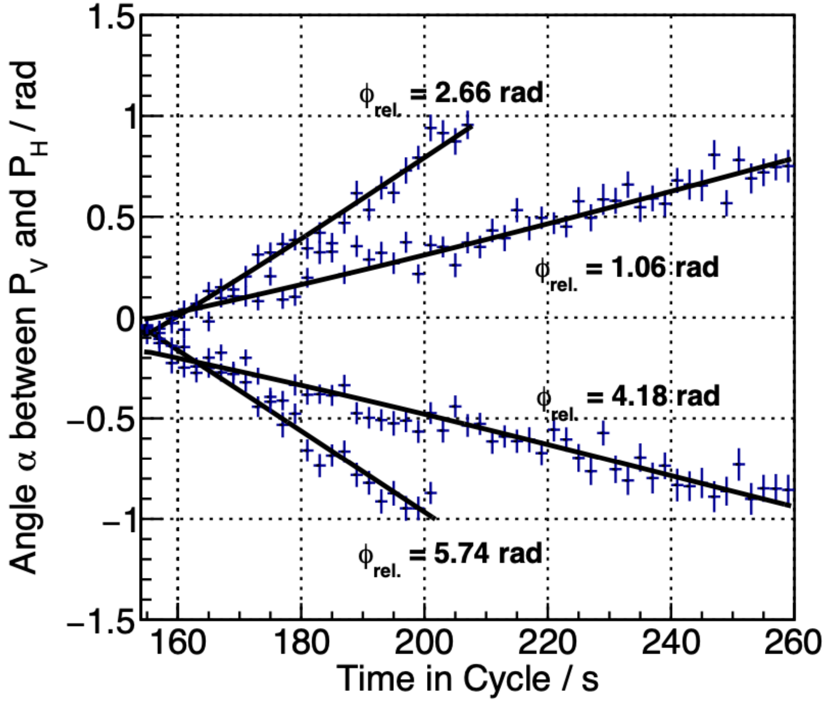
$$\epsilon = \frac{f_{\mathrm{v}}}{f_{\mathrm{rev}}} = \frac{1}{4\pi}\chi_0\,|\vec{c}\times\vec{m}| = \frac{1}{4\pi}\chi_0\,\sin\xi$$



$$\frac{\mathrm{d}\varphi_{\mathrm{rel}}}{\mathrm{d}n} = 0$$



$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 2\pi\epsilon f_{\mathrm{rev}}\cos\varphi_{\mathrm{rel}}$$

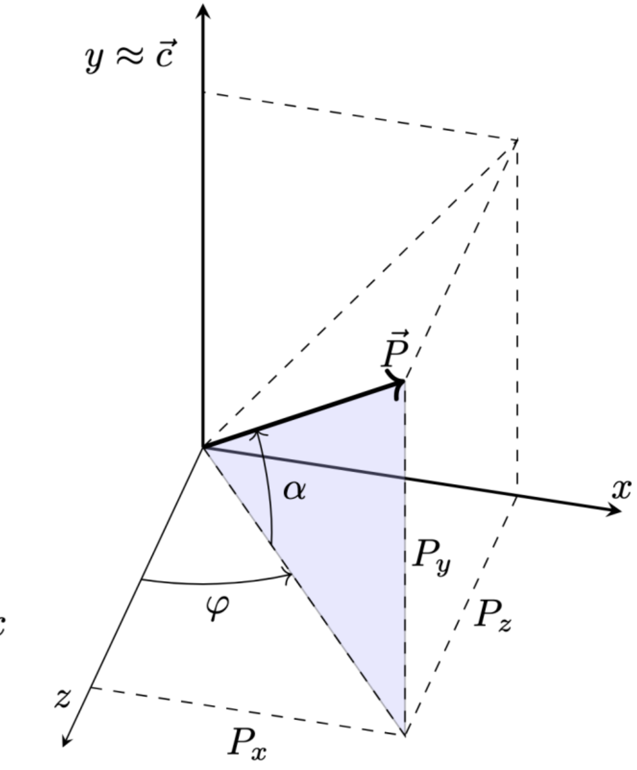


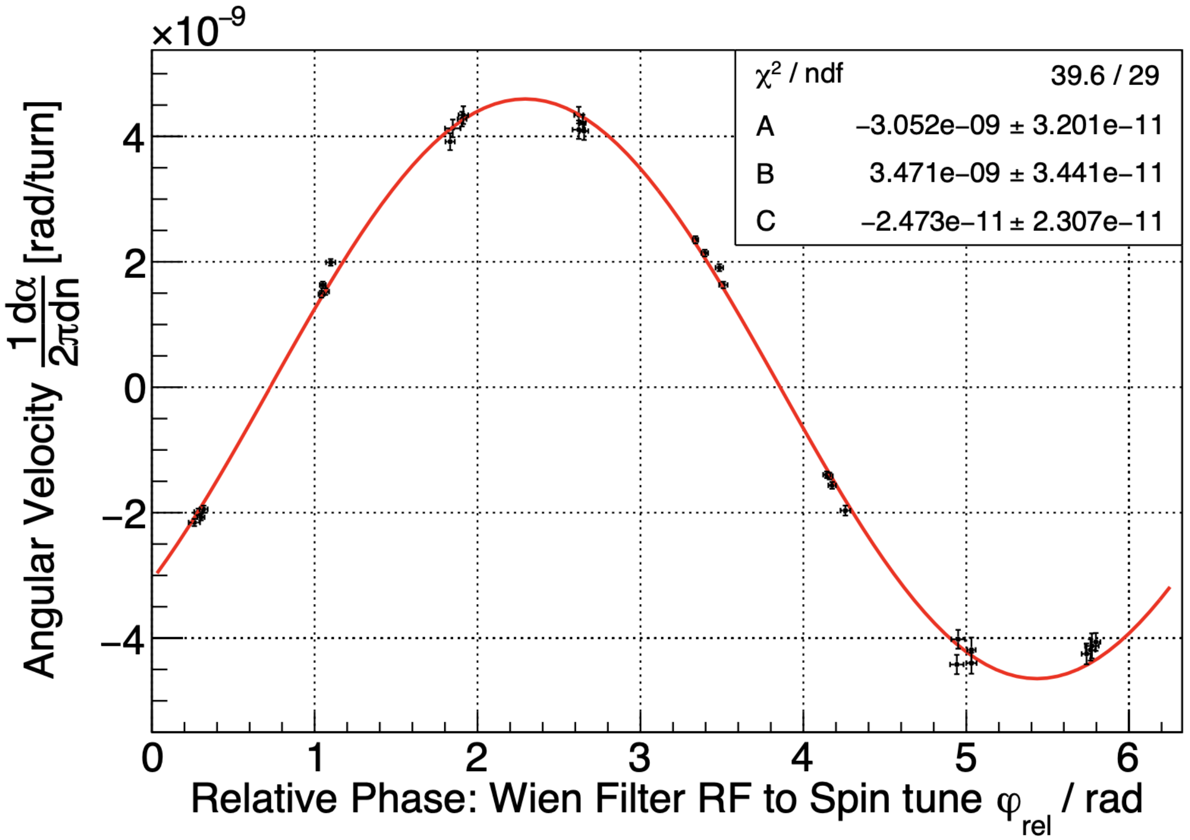
PHASE FEEDBACK

Slope Method

$$\epsilon = \frac{f_\mathrm{v}}{f_\mathrm{rev}} = \frac{1}{4\pi}\chi_0 \left|\vec{c} \times \vec{m}\right| = \frac{1}{4\pi}\chi_0 \sin\xi$$

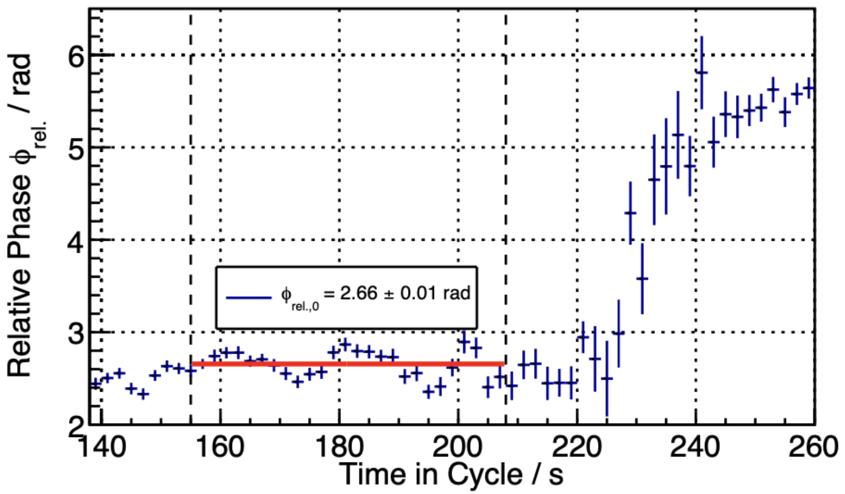
$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 2\pi\epsilon f_\mathrm{rev} \cos\varphi_\mathrm{rel}$$

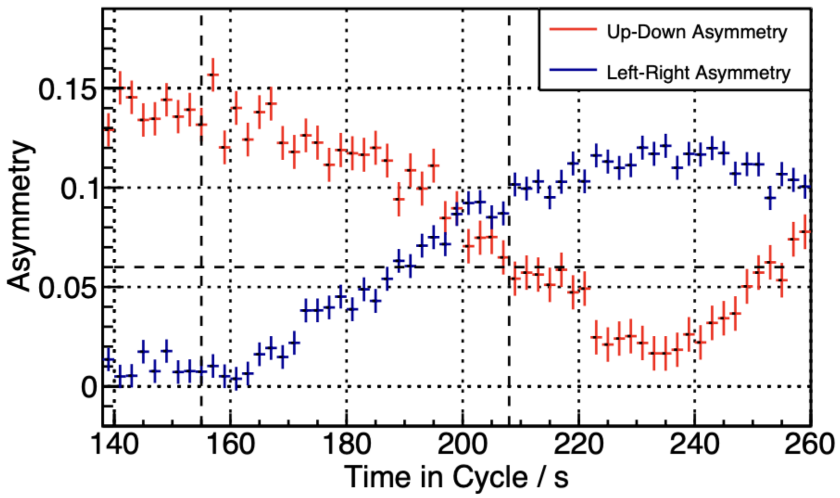


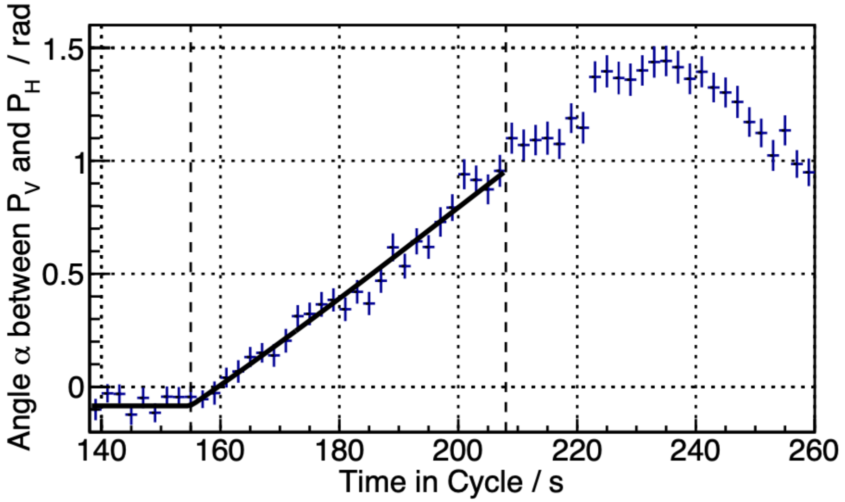


PHASE FEEDBACK

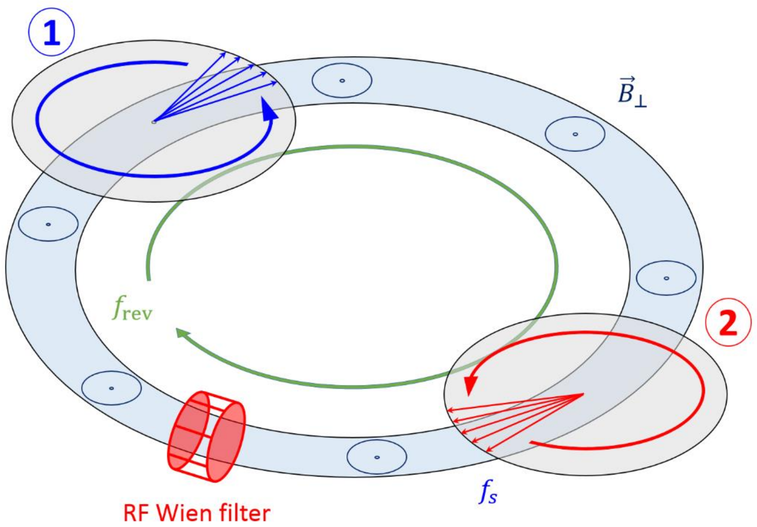
Slope Method - Limitations







PHASE FEEDBACK



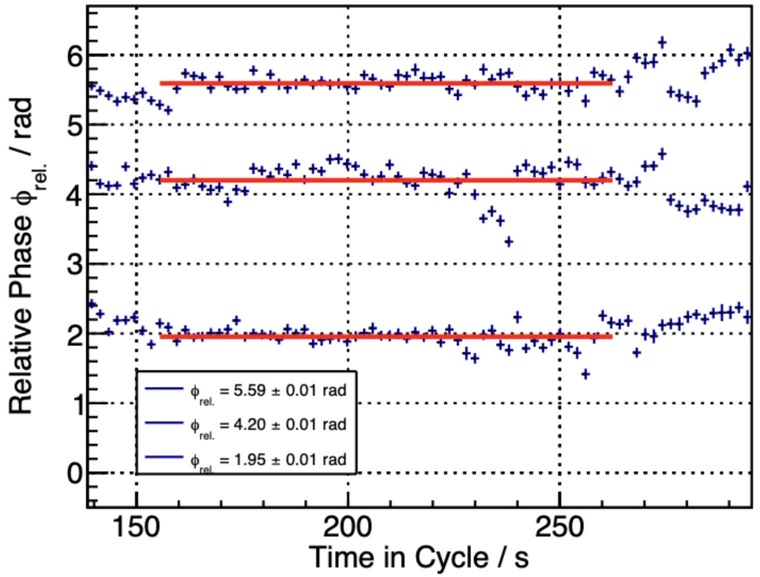
Blue Bunch

$$\frac{d\alpha}{dn} = \frac{k}{2}\cos\varphi_{rel}$$
$$\frac{d\varphi_{rel}}{dn} = \frac{k}{2}(\sin\varphi_{rel}\tan\alpha + q) \quad \rightarrow \quad \frac{d\varphi_{rel}}{dn} = 0$$

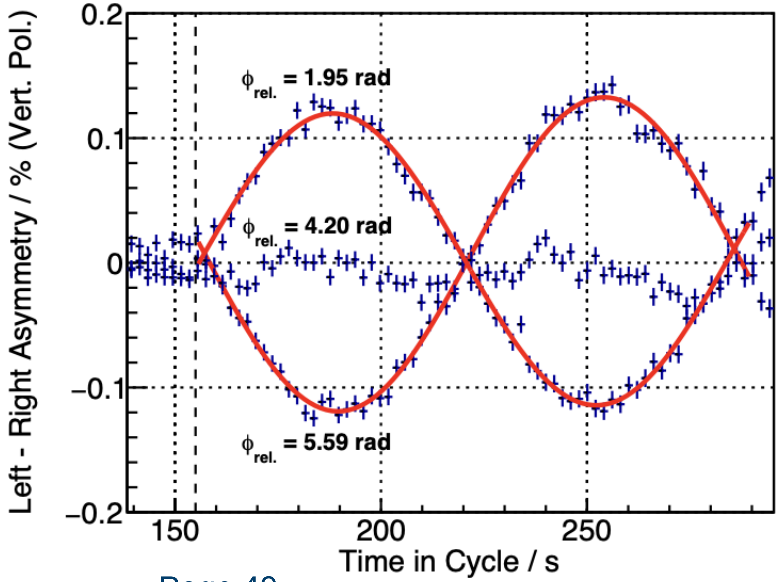
Red Bunch

$$\frac{d\alpha}{dn} = \frac{k}{2}\cos\varphi_{rel}$$
$$\frac{d\varphi_{rel}}{dn} = \frac{k}{2}(\sin\varphi_{rel}\tan\alpha + q)$$

$$p_y(t) = \sin\alpha(t) = \cos\varphi_{rel}(0)\sin(2\pi\epsilon f_{rev}t)\,.$$



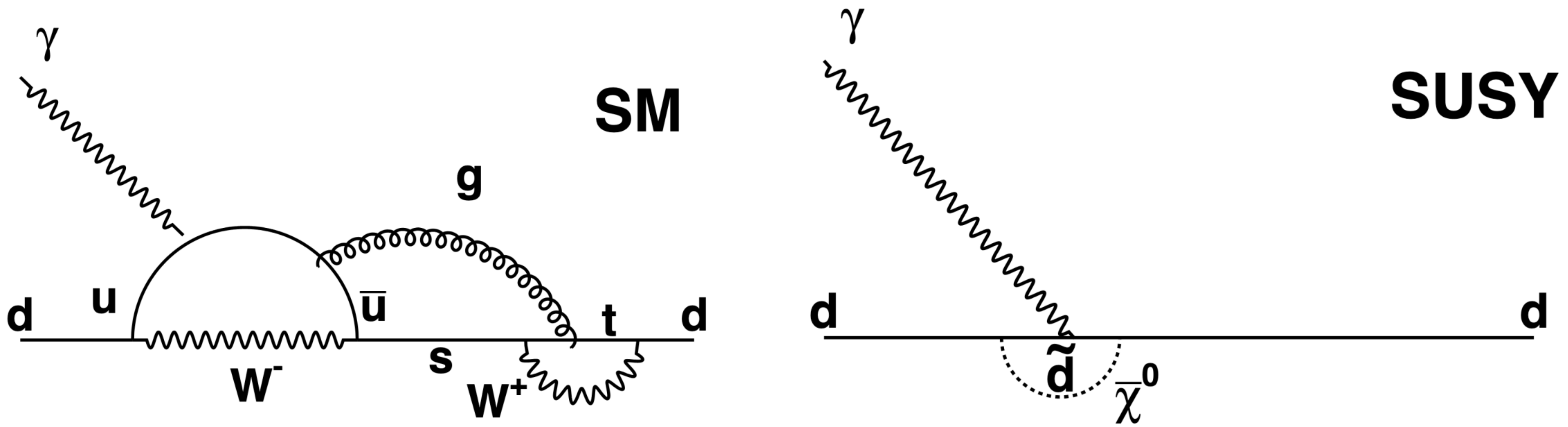
Blue Bunch



Red Bunch

JÜLICH Forschungszentrum

EDM IN THE STANDARD MODEL AND SUSY



WHATS NEXT?

$$\frac{\mathrm{d}\vec{S}}{\mathrm{d}t} = \left(\vec{\Omega}_{\mathrm{MDM}} + \vec{\Omega}_{\mathrm{EDM}}\right) \times \vec{S},$$

$$\vec{\Omega}_{\mathrm{MDM}} = -\frac{q}{m}\left[\left(G + \frac{1}{\gamma}\right)\vec{B} - \left(G + \frac{1}{\gamma+1}\right)\vec{\beta} \times \frac{\vec{E}}{c}\right]$$

$$\vec{\Omega}_{\mathrm{EDM}} = -\frac{q}{mc}\frac{\eta_{\mathrm{EDM}}}{2}\left[\vec{E} + c\vec{\beta} \times \vec{B}\right].$$

Frozen Spin Condition

$$\vec{\Omega}_{\mathrm{MDM}} - \vec{\Omega}_{\mathrm{rev}} = -\frac{q}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right] \stackrel{!}{=} 0.$$

For a pure magnetic ring (E=0)

$$G\vec{B}\gamma \stackrel{!}{=} 0 \;\lightning.$$

In an all – electric ring

$$\vec{\Omega}_{\mathrm{MDM}} - \vec{\Omega}_{\mathrm{rev}} = \frac{q}{m}\left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c} \stackrel{!}{=} 0.$$

$$G - \frac{1}{\gamma^2 - 1} \stackrel{!}{=} 0 \;\Rightarrow\; p_{\mathrm{magic}} = \frac{mc}{\sqrt{G}}.$$

Combined E – B Ring

$$B = E \cdot \frac{\beta^2\gamma^2 G - 1}{c\beta\gamma^2 G}$$

SPIN TUNE MAPPING

$$\frac{d\vec{S}}{d\theta} = \frac{d\vec{S}}{dt}\frac{dt}{d\theta} = \frac{1}{\omega_c}\vec{S}\times\vec{\Omega}_{\text{rs}} = -\frac{1}{\omega_c}\vec{\Omega}_{\text{rs}}\times\vec{S} = \vec{\omega}\times\vec{S}.$$

Spinor formalism

$$\psi = (u\ d)^T$$

$$\vec{S} \equiv \langle\psi|\vec{\sigma}|\psi\rangle = \psi^\dagger\vec{\sigma}\psi,$$

The TBMT equation of spin motion becomes

$$\frac{d\psi}{d\theta} = \frac{i}{2}(\vec{\sigma}\cdot\vec{\omega})\psi = \frac{i}{2}\begin{pmatrix}\omega_z, & \omega_x - i\omega_s \\ \omega_x + i\omega_s, & -\omega_z\end{pmatrix}\psi.$$

$$\psi(\theta) = t_{\text{Ring}}\psi(\theta_0)$$

$$\mathbf{t}_{\text{Ring}} = e^{-i\pi\nu_s^0\vec{\sigma}\cdot\vec{n}} = \cos(\pi\nu_s^0)I - i(\vec{\sigma}\cdot\vec{n})\sin(\pi\nu_s^0)$$

$$\mathbf{t}_X = e^{-i\frac{\chi_X}{2}\vec{\sigma}\cdot\vec{k}} = \cos\left(\frac{\chi_X}{2}\right)I - i\left(\vec{\sigma}\cdot\vec{k}\right)\sin\left(\frac{\chi_X}{2}\right)$$

$$\begin{aligned}\mathbf{T} &= \mathbf{t}_{\text{Ring}}\mathbf{t}_X \\ &= \cos(\pi\nu_s^0)\cos\left(\frac{\chi_X}{2}\right)I - i\left(\vec{\sigma}\cdot\vec{k}\right)\sin\left(\frac{\chi_X}{2}\right)\cos(\pi\nu_s^0) \\ &\quad - i(\vec{\sigma}\cdot\vec{n})\sin(\pi\nu_s^0)\cos\left(\frac{\chi_X}{2}\right) - (\vec{\sigma}\cdot\vec{n})\left(\vec{\sigma}\cdot\vec{k}\right)\sin(\pi\nu_s^0)\sin\left(\frac{\chi_X}{2}\right) \\ &\overset{\text{Eq. (C.3)}}{=} \cos(\pi\nu_s^0)\cos\left(\frac{\chi_X}{2}\right)I - \left(\vec{n}\cdot\vec{k}\right)I\sin(\pi\nu_s^0)\sin\left(\frac{\chi_X}{2}\right) \\ &\quad - i\vec{\sigma}\cdot\left[\left(\vec{n}\times\vec{k}\right)\sin(\pi\nu_s^0)\sin\left(\frac{\chi_X}{2}\right) + \vec{k}\sin\left(\frac{\chi_X}{2}\right)\cos(\pi\nu_s^0) + \vec{n}\sin(\nu_s^0)\cos\left(\frac{\chi_X}{2}\right)\right] \\ &\overset{!}{=} \cos(\pi\nu_s(\chi_X))I - i(\vec{\sigma}\cdot\vec{n}(\chi_X))\sin(\pi\nu_s(\chi_X)).\end{aligned}$$

$$\begin{aligned}\vec{n}(\chi_X) = \frac{1}{\sin(\pi\nu_s(\chi_X))}\cdot\Bigg[&\left(\vec{n}\times\vec{k}\right)\sin(\pi\nu_s^0)\sin\left(\frac{\chi_X}{2}\right) \\ &+\vec{k}\sin\left(\frac{\chi_X}{2}\right)\cos(\pi\nu_s^0) + \vec{n}\sin(\nu_s^0)\cos\left(\frac{\chi_X}{2}\right)\Bigg],\end{aligned}$$

$$\cos(\pi\nu_s(\chi_X)) = \cos(\pi\nu_s^0)\cos\left(\frac{\chi_X}{2}\right) - \left(\vec{n}\cdot\vec{k}\right)\sin(\pi\nu_s^0)\sin\left(\frac{\chi_X}{2}\right).$$