Linearly Polarized Photon Fusion as a Precision Probe of the Tau Lepton Dipole Moments at Lepton Colliders

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OUTLINE

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Background and Motivation

Lepton Dipole Moments and New Physics:

- The anomalous magnetic dipole moment (MDM) and electric dipole moment (EDM) of leptons are among the most important observables for testing the Standard Model (SM).
- Any deviation between experimental measurements and SM predictions could indicate new physics (NP).
- A nonzero EDM violates CP symmetry and provides direct evidence for new CP-violating sources beyond the SM.

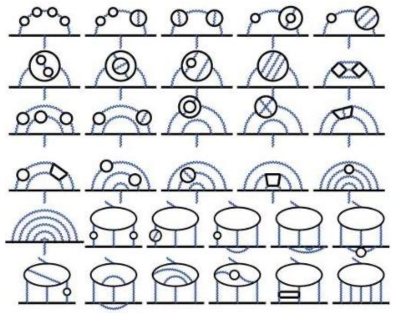
τ Lepton:

- τ is the heaviest of the three generations of charged leptons.
- The sensitivity to potential new-physics effects typically scales with the square of the mass.

Background and Motivation

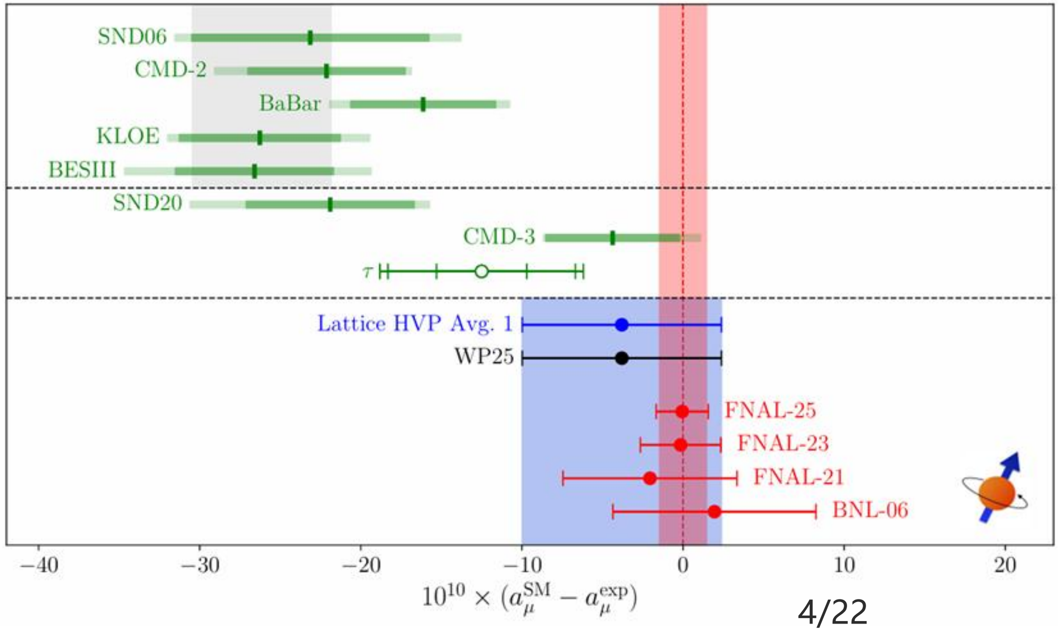
Anomalous magnetic dipole moment (MDM):

- In the SM, the leading contribution to the lepton MDM arises from the one-loop Schwinger term $\frac{\alpha_e}{2\pi} \simeq 0.00116$, and QED corrections have been computed up to the five-loop level.
- The discrepancy of muon g-2 between the world-average experimental measurement (FNAL-25 + BNL-06) and the updated SM prediction (lattice-QCD HVP + refined HLbL) is currently about $0.6\,\sigma$.



Aoyama, Hayakawa, Kinoshita, Nio (2012)

R. Aliberti et al. (2025)

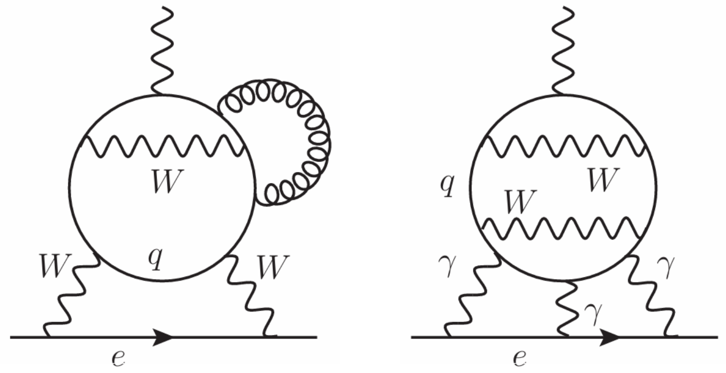


$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 38(63) \times 10^{-11}$$

Background and Motivation

Electric dipole moment (EDM) :

- In the SM, lepton EDM arises only at the four-loop level: three electro-weak loops + one additional gluon loop (to switch from leptons to quarks and to access the CKM phase). Khriplovich, Pospelov (1991)
- The SM prediction for lepton EDMs is many orders of magnitude below current experimental sensitivities.



$d_e^{\rm SM} \simeq 10^{-44}\ e{\rm cm}$

Pospelov, Ritz (2013)

$d_e^{\rm equiv} \sim 10^{-35}\ e{\rm cm}$

Ema, Gao, Pospelov (2022)

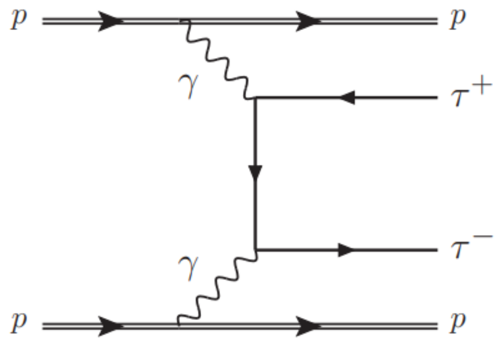
Observable	Direct limit	Indirect limit	Projected sensitivity
$\|d_e\|$	$< 4.8 \times 10^{-30}\ e\,{\rm cm}$ [9]		$\sim 10^{-30}\ e\,{\rm cm}$ [64]
$\|d_\mu\|$	$< 1.9 \times 10^{-19}\ e\,{\rm cm}$ [66]	$< 1.7 \times 10^{-20}\ e\,{\rm cm}$ [64, 67, 68]	$\sim 10^{-21}\ e\,{\rm cm}$ [69, 70] $\sim 6 \times 10^{-23}\ e\,{\rm cm}$ [71, 72]
$\|{\rm Re}(d_\tau)\|$	$< 1.7 \times 10^{-17}\ e\,{\rm cm}$ [73]	$< 1.1 \times 10^{-18}\ e\,{\rm cm}$ [64, 67, 68]	$\sim 10^{-19}\ e\,{\rm cm}$ [74] $\sim 10^{-20}\ e\,{\rm cm}$ [75]

Altmannshofer, Dev, Soni, FX arXiv:2507.23722

Background and Motivation

Limitations in τ Lepton Measurements:

- The τ lepton has an extremely short lifetime, so unlike the electron and the muon, its dipole moments cannot be measured directly through spin precession, but must instead be reconstructed indirectly from the kinematic distributions of its decay products.



Current precision:

- MDM: the most precise result comes from the CMS experiment.

$$a_\tau = 0.0009^{+0.0032}_{-0.0031}$$

CMS Collaboration, Rep. Prog. Phys.(2024)

- EDM: the strongest constraint is provided by the Belle experiment.

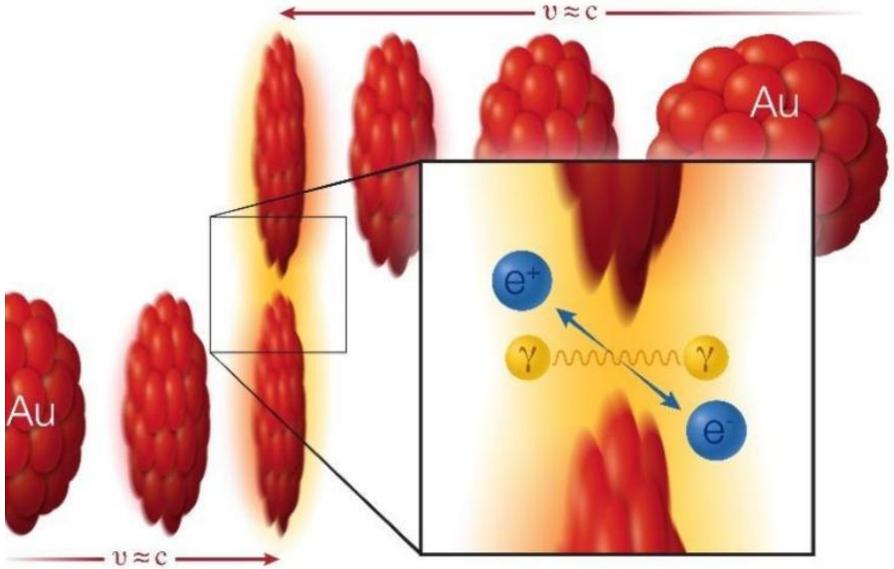
$$\mathrm{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17} e \cdot \mathrm{cm}$$

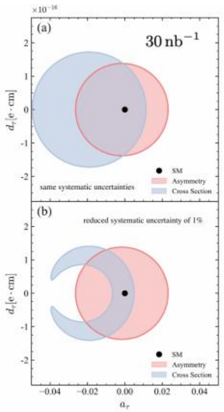
The BELLE collaboration, JHEP(2022)

Background and Motivation

Ultra-peripheral collisions (UPC):

- Two relativistic heavy ions passing close to each other generate extremely strong electromagnetic fields. These fields can be treated as a flux of quasi-real photons, known as coherent photons.
- Compared to direct collisions, UPC processes avoid large hadronic backgrounds.
- The linear polarization of coherent photons induces azimuthal asymmetries in the production of final-state particle pairs => can be used to improve the sensitivity to both MDM and EDM measurements.
- Limitation: the calculation involves non-perturbative nuclear effects, and is model dependent.



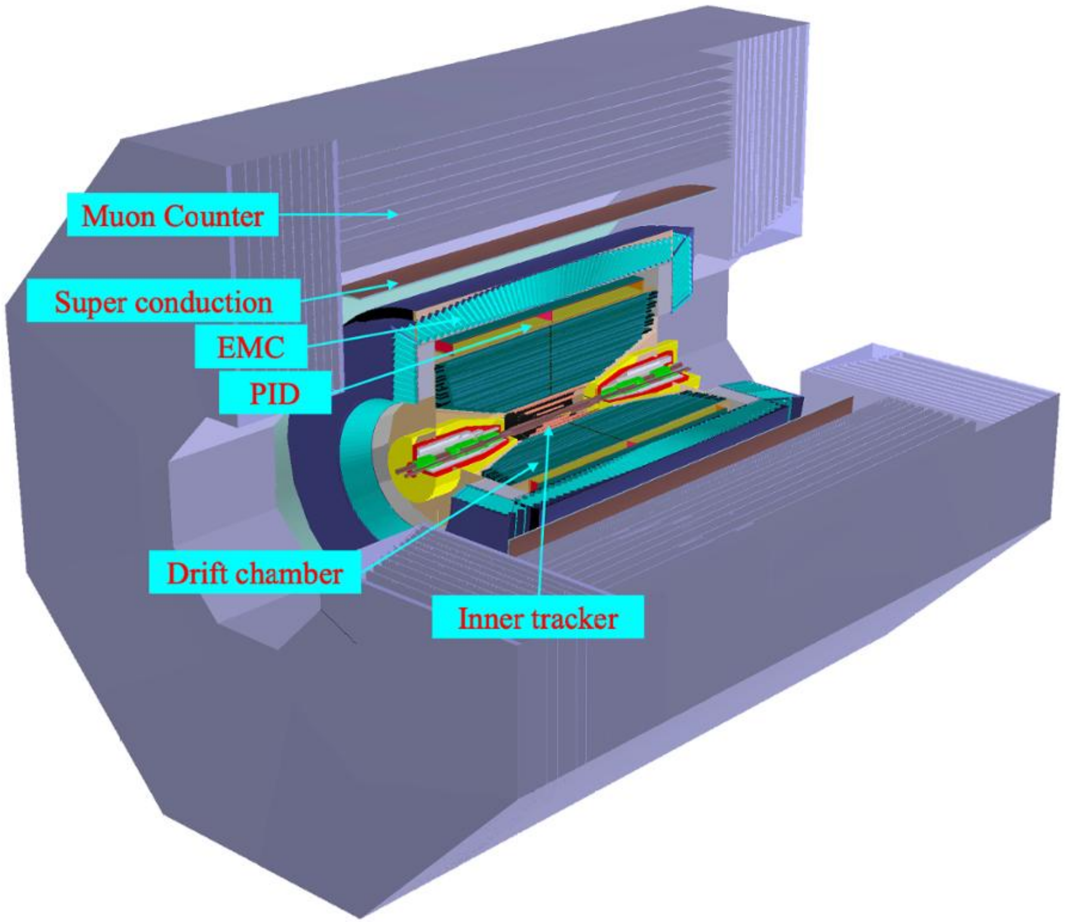


Shao, Yan, Yuan, Zhang (2023)

Background and Motivation

Super τ-Charm Facility (STCF):

- Advantage of e^+e^- colliders: relatively low background levels.
- With its high luminosity, STCF will produce 3.5×10^9 $\tau^+\tau^-$ pairs per year, representing an improvement of three orders of magnitude compared to BESIII.
- Energy range: optimized for τ physics (2–7 GeV), avoiding the Υ(4S) resonance region of B mesons, which leads to a cleaner background environment.

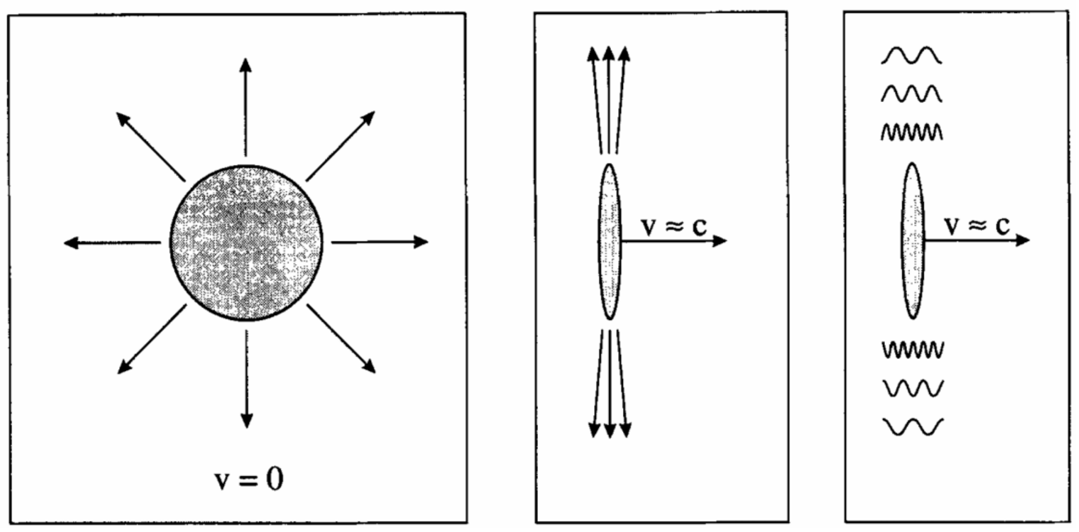


Achasov et al., Frontiers of Physics(2023)

Theoretical Framework

Equivalent Photon Approximation (EPA):

- Advantage: Treats the ultra-strong electromagnetic field as a flux of quasi-real photons, providing a transparent physical picture and simplifying the calculation.
- Limitation: To simplify the calculation, EPA integrates over the transverse momentum of photons, thereby losing information about their transverse momentum.



Fermi (1924), Williams (1933) , Weizsacker (1934)

Theoretical Framework

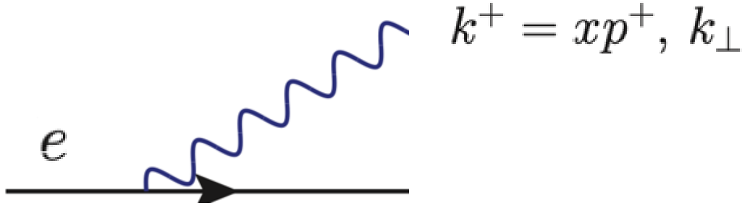
Transverse-Momentum-Dependent (TMD) Factorization:

- Retains the full transverse-momentum information of particles in the scattering process, enabling a precise description of the angular distributions of the final-state particles.
- Factorization formula: $$\mathrm{d}\sigma \propto \sum_{i,j} \int \mathrm{d}^2\boldsymbol{k}_{1\perp} \mathrm{d}^2\boldsymbol{k}_{2\perp}\, \mathcal{H} \varPhi_i(x_1, \boldsymbol{k}_{1\perp}^2) \varPhi_j(x_2, \boldsymbol{k}_{2\perp}^2) + \mathcal{O}\left(\frac{q_\perp^2}{Q^2}\right)$$
- Photon TMD distribution function: $$\begin{aligned} \varPhi^{\mu\nu}(x, \boldsymbol{k}_\perp) &= \int \frac{\mathrm{d}b^- \mathrm{d}^2\boldsymbol{b}_\perp}{P^+(2\pi)^3} e^{-ib^-(xP^+) + i\boldsymbol{b}_\perp \cdot \boldsymbol{k}_\perp} \\ &\times \langle e(P)| F^{+\mu}(b) F^{+\nu}(0) |e(P)\rangle \big|_{b^+=0} \\ &= -\frac{g_\perp^{\mu\nu}}{2} x f(x, \boldsymbol{k}_\perp^2) + \left(\frac{g_\perp^{\mu\nu}}{2} + \frac{k_\perp^\mu k_\perp^\nu}{\boldsymbol{k}_\perp^2}\right) x h_1^\perp(x, \boldsymbol{k}_\perp^2) \end{aligned}$$

Theoretical Framework

Photon TMD Distribution Functions:

- At leading order in QED, the photon TMD distribution functions can be explicitly obtained from the electron splitting process.



Unpolarized photon TMD distribution $$f(x, \boldsymbol{k}_\perp^2) = \frac{\alpha_e}{2\pi^2} \frac{1+(1-x)^2}{x} \frac{\boldsymbol{k}_\perp^2}{\left(\boldsymbol{k}_\perp^2 + x^2 m_e^2\right)^2}$$

Linearly polarized photon TMD distribution $$h_1^\perp(x, \boldsymbol{k}_\perp^2) = \frac{\alpha_e}{\pi^2} \frac{1-x}{x} \frac{\boldsymbol{k}_\perp^2}{(\boldsymbol{k}_\perp^2 + x^2 m_e^2)^2}$$

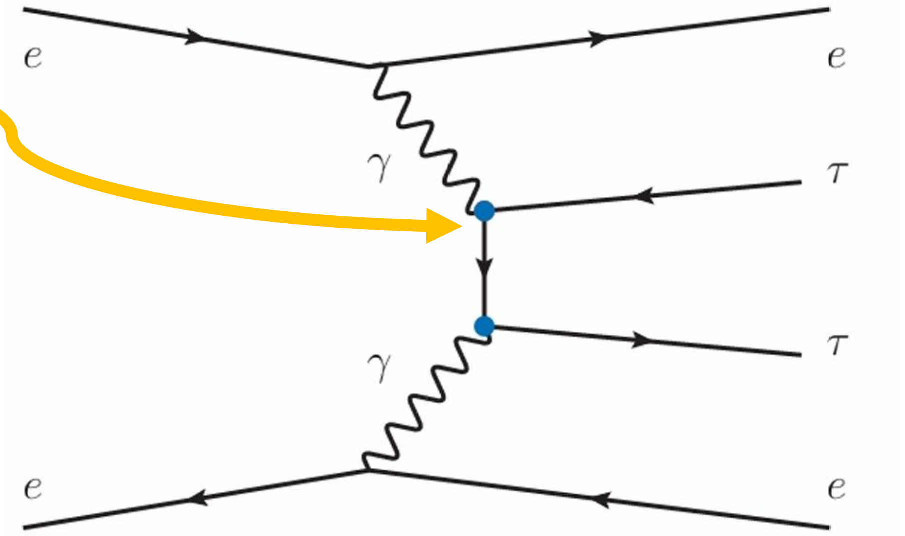
Theoretical Framework

Effective $\tau^+\tau^-\gamma$ vertex

$$\Gamma^{\mu}_{\text{eff}}(q^2) = -ie[F_1(q^2)\gamma^{\mu} + (iF_2(q^2) + F_3(q^2)\gamma^5)\frac{\sigma^{\mu\nu}q_{\nu}}{2m_{\tau}}]$$

$q^2 \to 0$

a_{τ} $\quad$ $2m_{\tau}d_{\tau}/e$



$$\gamma(k_1,\lambda_1)\gamma(k_2,\lambda_2) \to \tau^-(p_1)\tau^+(p_2)$$

Theoretical Framework

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}y_1 \mathrm{d}y_2} = \frac{1}{64\pi^2 s^2}\int \mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\delta^2(\boldsymbol{q}_\perp - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})x_1 x_2\{$$
$$(|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2 + |\mathcal{M}_{+-}|^2 + |\mathcal{M}_{-+}|^2) f(x_1, \boldsymbol{k}_{1\perp}^2) f(x_2, \boldsymbol{k}_{2\perp}^2)$$
$$-2\mathrm{Re}[\mathrm{e}^{2i(\phi_1-\phi_P)}(\mathcal{M}_{--}\mathcal{M}_{+-}^* + \mathcal{M}_{-+}\mathcal{M}_{++}^*)] h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) f(x_2, \boldsymbol{k}_{2\perp}^2)$$
$$-2\mathrm{Re}[\mathrm{e}^{2i(\phi_2-\phi_P)}(\mathcal{M}_{++}\mathcal{M}_{+-}^* + \mathcal{M}_{-+}\mathcal{M}_{--}^*)] f(x_1, \boldsymbol{k}_{1\perp}^2) h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2)$$
$$+2\mathrm{Re}[\mathrm{e}^{2i(\phi_1-\phi_2)}(\mathcal{M}_{--}\mathcal{M}_{++}^*)] h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2)$$
$$+2\mathrm{Re}[\mathrm{e}^{2i(\phi_1+\phi_2-2\phi_P)}(\mathcal{M}_{-+}\mathcal{M}_{+-}^*)] h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2)\}$$

Semi-inclusive di-tau differential cross section of $e^+e^- \to e^+e^- + \tau^+\tau^-$

$\mathcal{M}_{\lambda_1\lambda_2}$: helicity amplitude of the process $\gamma(k_1,\lambda_1)\gamma(k_2,\lambda_2) \to \tau^-(p_1)\tau^+(p_2)$

azimuthal angle of $\boldsymbol{k}_{1\perp}$, $\boldsymbol{k}_{2\perp}$ and $\boldsymbol{P}_\perp$

$$\boldsymbol{P}_\perp \equiv (\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp})/2$$
$$\boldsymbol{q}_\perp \equiv \boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

longitudinal momentum fractions of the photon

$$x_1 = \sqrt{\frac{|\boldsymbol{P}_\perp|^2 + m_\tau^2}{S_{ee}}}(e^{y_1} + e^{y_2})$$
$$x_2 = \sqrt{\frac{|\boldsymbol{P}_\perp|^2 + m_\tau^2}{S_{ee}}}(e^{-y_1} + e^{-y_2})$$

Theoretical Framework

Angular Modulation:

- Only terms up to quadratic order in the dipole form factors $F_{2,3}$ are retained.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{P}_\perp \mathrm{d}^2\boldsymbol{q}_\perp \mathrm{d}y_1 \mathrm{d}y_2} = \frac{\alpha_e^2}{2\pi^2 M^4}\Big[$$

$$\left(C_0 + C_0^{\mathrm{Re}(F_2)}\mathrm{Re}(F_2) + C_0^{\mathrm{Re}(F_2)^2}\mathrm{Re}(F_2)^2 + C_0^{\mathrm{Re}(F_3)^2}\mathrm{Re}(F_3)^2 + C_0^{\mathrm{Im}(F_2)^2}\mathrm{Im}(F_2)^2 + C_0^{\mathrm{Im}(F_3)^2}\mathrm{Im}(F_3)^2\right)$$

$$+\left(C_{c2\phi} + C_{c2\phi}^{\mathrm{Re}(F_2)^2}\mathrm{Re}(F_2)^2 + C_{c2\phi}^{\mathrm{Re}(F_3)^2}\mathrm{Re}(F_3)^2 + C_{c2\phi}^{\mathrm{Im}(F_2)^2}\mathrm{Im}(F_2)^2 + C_{c2\phi}^{\mathrm{Im}(F_3)^2}\mathrm{Im}(F_3)^2\right)\cos(2\phi)$$

$$+C_{s2\phi}^{\mathrm{Im}(F_2)\mathrm{Im}(F_3)}\mathrm{Im}(F_2)\mathrm{Im}(F_3)\sin(2\phi)$$

$$+\left(C_{c4\phi} + C_{c4\phi}^{\mathrm{Im}(F_2)^2}\mathrm{Im}(F_2)^2 + C_{c4\phi}^{\mathrm{Im}(F_3)^2}\mathrm{Im}(F_3)^2\right)\cos(4\phi)\Big]$$

$$\phi \equiv \phi_q - \phi_P$$

No contribution from the real parts

Theoretical Framework

Standard Model

$$C_0 = \frac{4m_\tau^4}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp] \Big(-2 + \frac{M^2}{m_\tau^2} + \frac{\boldsymbol{P}_\perp^2 M^2}{m_\tau^4} - 2\frac{\boldsymbol{P}_\perp^4}{m_\tau^4} \Big) x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) - 2 x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_{c2\phi} = \frac{16\boldsymbol{P}_\perp^2 m_\tau^2}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp] \, x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_q) + x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_2 - 2\phi_q)$$

$$C_{c4\phi} = \frac{-8\boldsymbol{P}_\perp^4}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp] \, x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 + 2\phi_2 - 4\phi_q)$$

Real parts of F_2 and F_3

$$C_0^{\mathrm{Re}(F_2)} = \frac{8M^2}{\boldsymbol{P}_\perp^2 + m_\tau^2} \int [\mathrm{d}\mathcal{K}_\perp] x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) - x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_0^{\mathrm{Re}(F_2)^2} = \frac{2M^2}{\boldsymbol{P}_\perp^2 + m_\tau^2} \int [\mathrm{d}\mathcal{K}_\perp] \Big(5 + 4\frac{\boldsymbol{P}_\perp^2}{m_\tau^2}\Big) x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) - 5 x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_0^{\mathrm{Re}(F_3)^2} = \frac{2M^2}{\boldsymbol{P}_\perp^2 + m_\tau^2} \int [\mathrm{d}\mathcal{K}_\perp] \Big(3 + 4\frac{\boldsymbol{P}_\perp^2}{m_\tau^2}\Big) x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) + 5 x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_{c2\phi}^{\mathrm{Re}(F_2)^2} = C_{c2\phi}^{\mathrm{Re}(F_3)^2} = \frac{-4\boldsymbol{P}_\perp^2 M^2}{m_\tau^2(\boldsymbol{P}_\perp^2 + m_\tau^2)} \int [\mathrm{d}\mathcal{K}_\perp] \, x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_q) + x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_2 - 2\phi_q)$$

Theoretical Framework

Imaginary parts of F_2 and F_3

$$C_0^{\mathrm{Im}(F_2)^2} = \frac{2M^2 m_\tau^2}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$(3 + 5\frac{\boldsymbol{P}_\perp^2}{m_\tau^2} + 2\frac{\boldsymbol{P}_\perp^4}{m_\tau^4}) x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) - (3$$
$$+\frac{\boldsymbol{P}_\perp^2}{m_\tau^2}) x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_0^{\mathrm{Im}(F_3)^2} = \frac{2M^2 m_\tau^2}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$(5 + 3\frac{\boldsymbol{P}_\perp^2}{m_\tau^2} + 2\frac{\boldsymbol{P}_\perp^4}{m_\tau^4}) x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) + (3$$
$$+\frac{\boldsymbol{P}_\perp^2}{m_\tau^2}) x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_2)$$

$$C_{s2\phi}^{\mathrm{Im}(F_2)\mathrm{Im}(F_3)} = \frac{-8\boldsymbol{P}_\perp^2 M^2}{(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_q)$$
$$-x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_2 - 2\phi_q)$$

$$C_{c2\phi}^{\mathrm{Im}(F_2)^2} = \frac{-4\boldsymbol{P}_\perp^2 M^2}{m_\tau^2(\boldsymbol{P}_\perp^2 + m_\tau^2)} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_q)$$
$$+ x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_2 - 2\phi_q)$$

$$C_{c2\phi}^{\mathrm{Im}(F_3)^2} = \frac{-4\boldsymbol{P}_\perp^2 M^2(\boldsymbol{P}_\perp^2 + 3m_\tau^2)}{m_\tau^2(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 f(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 - 2\phi_q)$$
$$+ x_1 f(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_2 - 2\phi_q)$$

$$C_{c4\phi}^{\mathrm{Im}(F_2)^2} = C_{c4\phi}^{\mathrm{Im}(F_3)^2} = \frac{4\boldsymbol{P}_\perp^4 M^2}{m_\tau^2(\boldsymbol{P}_\perp^2 + m_\tau^2)^2} \int [\mathrm{d}\mathcal{K}_\perp]$$
$$x_1 h_1^\perp(x_1, \boldsymbol{k}_{1\perp}^2) x_2 h_1^\perp(x_2, \boldsymbol{k}_{2\perp}^2) \cos(2\phi_1 + 2\phi_2 - 4\phi_q)$$

$$\int [\mathrm{d}\mathcal{K}_\perp] \equiv \pi^2 \int \mathrm{d}^2\boldsymbol{k}_{1\perp} \mathrm{d}^2\boldsymbol{k}_{2\perp} \delta^2(\boldsymbol{q}_\perp - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})$$

Numerical Results

Benchmark Machine: Super τ-Charm facility (STCF)

CME 7.0 GeV
Luminosity 3 ab^{-1}
Overall signal efficiency ε=20%

Observables related to azimuthal asymmetries:

$$A_{c2\phi} = \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)},$$

$$A_{s2\phi} = \frac{\sigma(y \times \sin 2\phi > 0) - \sigma(y \times \sin 2\phi < 0)}{\sigma(\sin 2\phi > 0) + \sigma(\sin 2\phi < 0)},$$

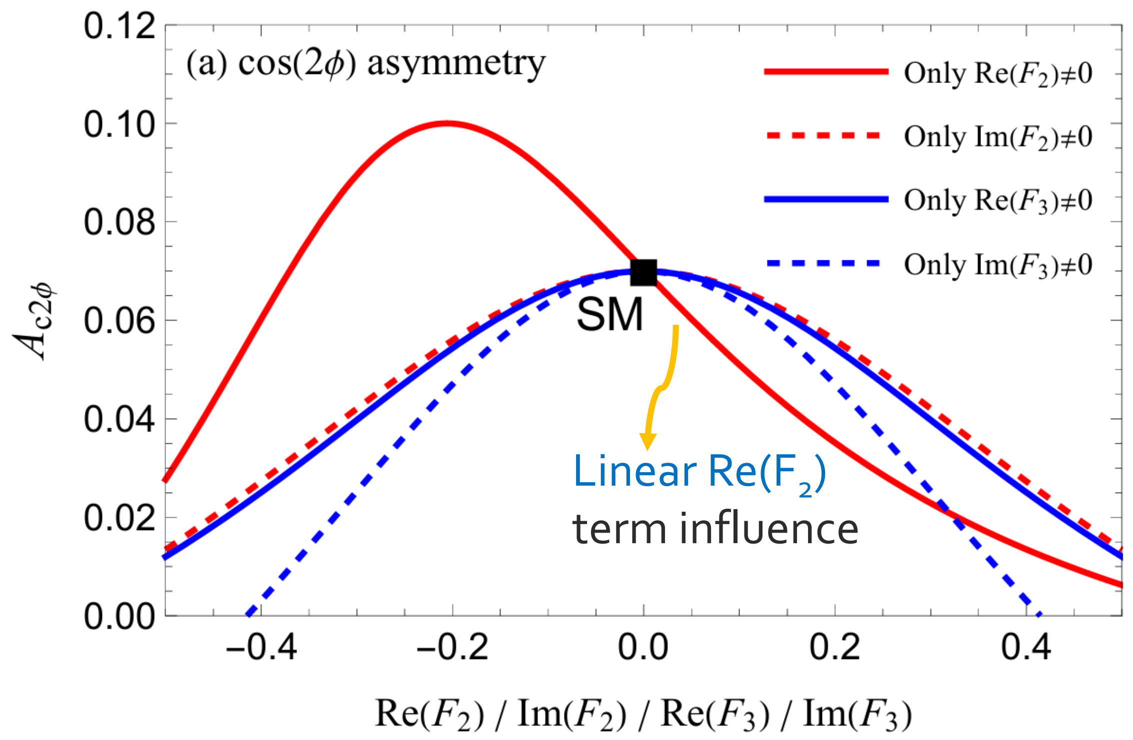
$$y = (y_1 + y_2)/2$$

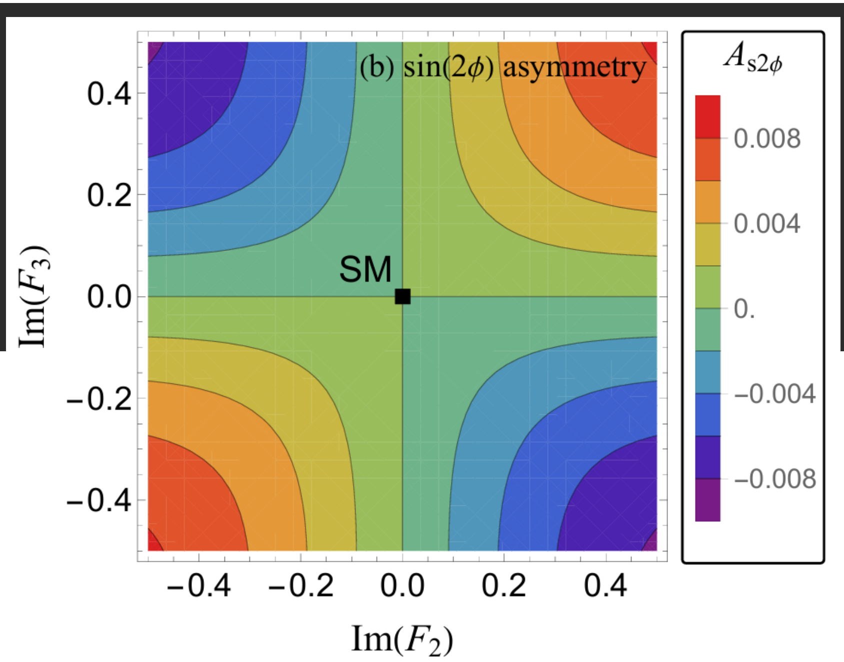
$$A_{c4\phi} = \frac{\sigma(\cos 4\phi > 0) - \sigma(\cos 4\phi < 0)}{\sigma(\cos 4\phi > 0) + \sigma(\cos 4\phi < 0)},$$

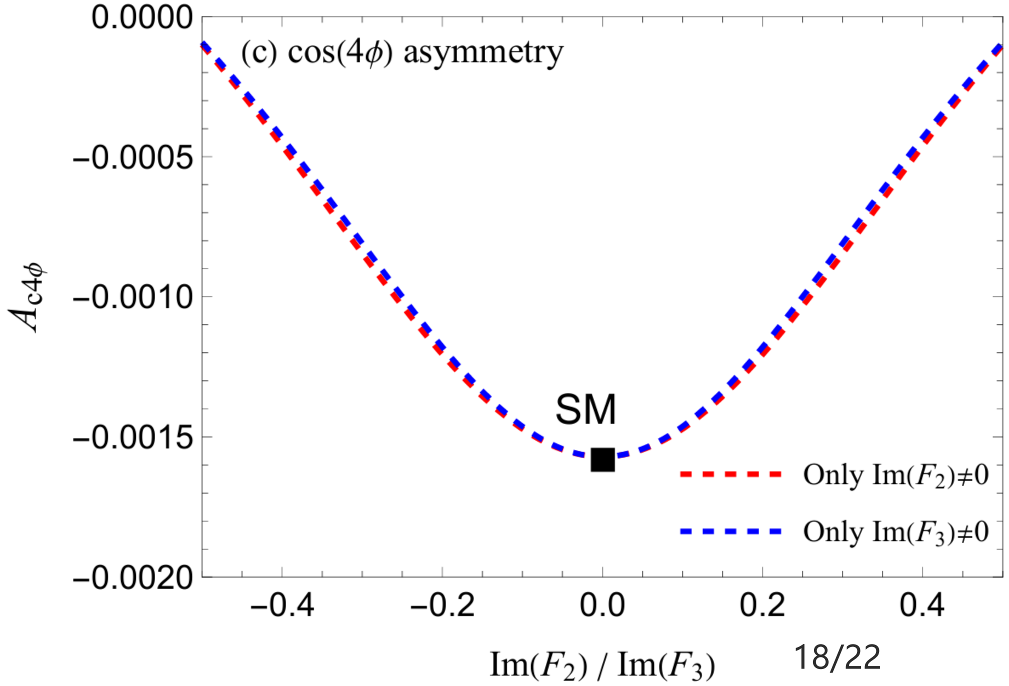
Uncertainties

$$\delta A_{\mathrm{i}} = \sqrt{\frac{1-(A_{\mathrm{i}})^2}{\sigma \cdot \mathcal{L}}} \simeq \frac{1}{\sqrt{\sigma \cdot \mathcal{L}}}$$

Numerical Results

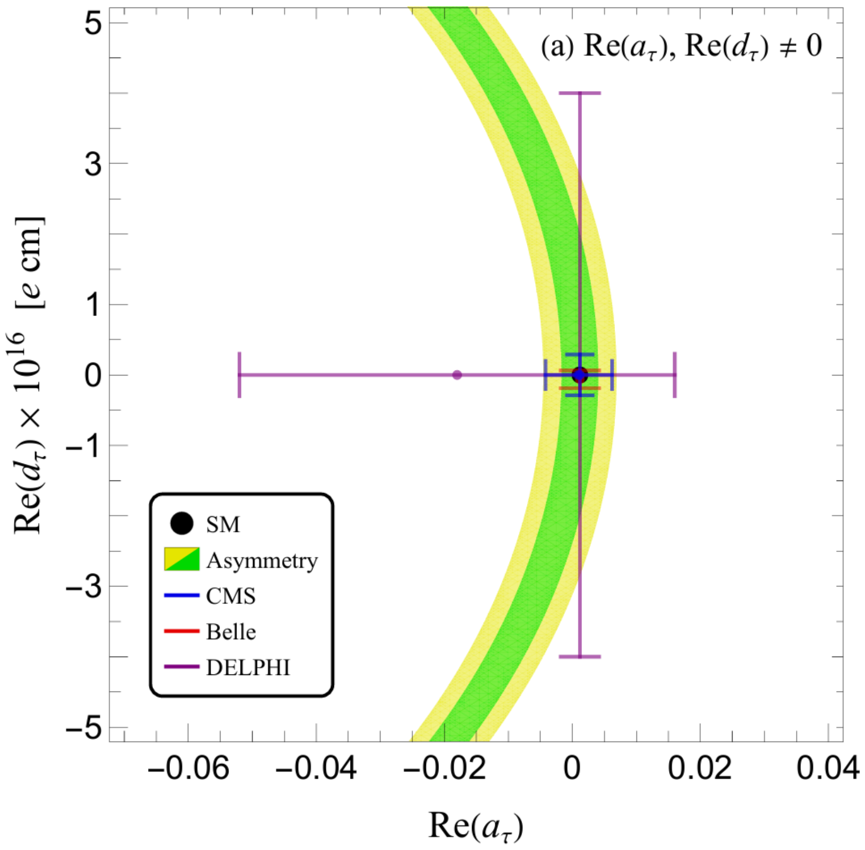


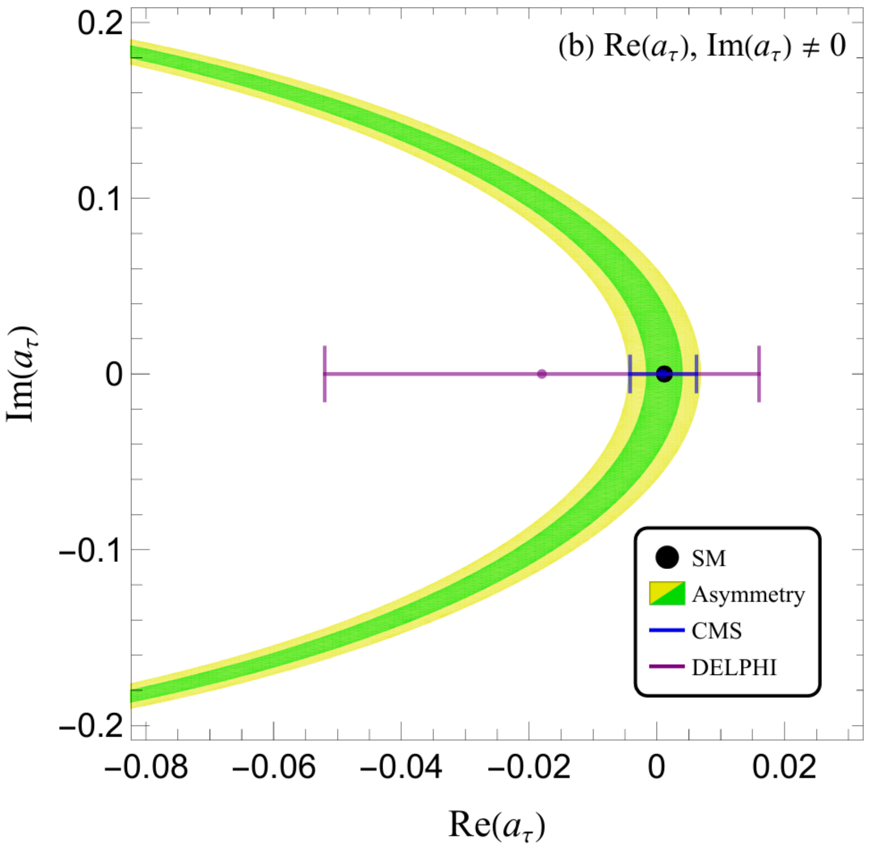




Theoretical predictions for the $\cos(2\phi)$, $\sin(2\phi)$ and $\cos(4\phi)$ azimuthal asymmetry in $\tau^+\tau^-$ production from two-photon collisions at an e^+e^- collider with E_{cm} = 7 GeV, L= 3 ab^{-1}

Numerical Results





1σ and 2σ sensitivities from the cos(2ϕ) azimuthal asymmetry

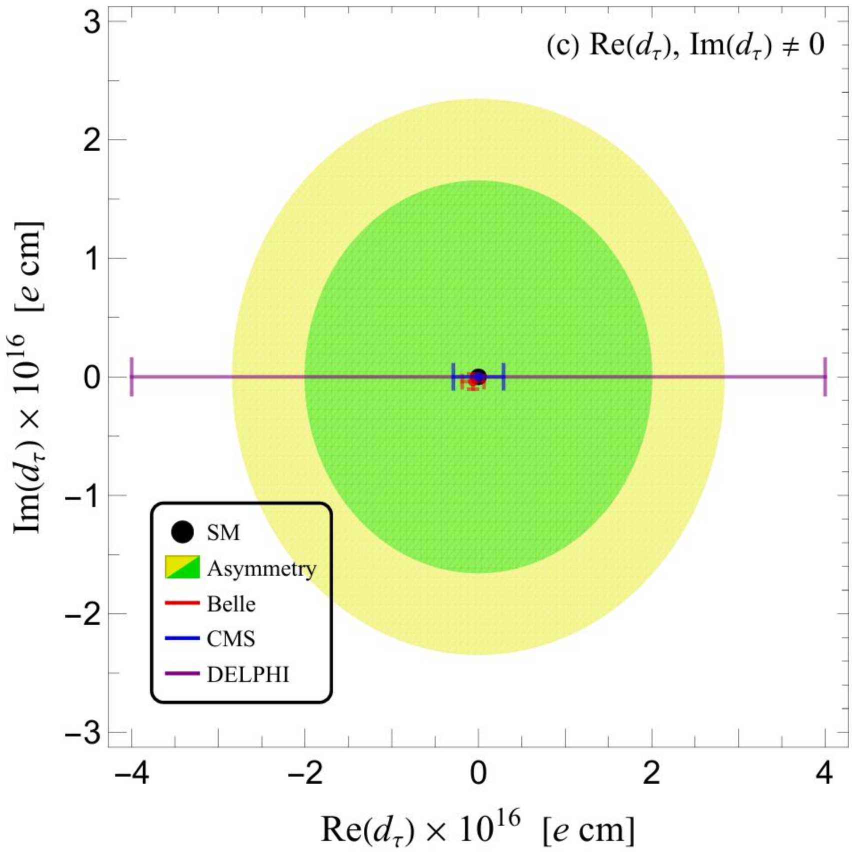
$$-4.5 \times 10^{-3} < \mathrm{Re}(a_\tau) < 6.9 \times 10^{-3}$$ @2σ CL

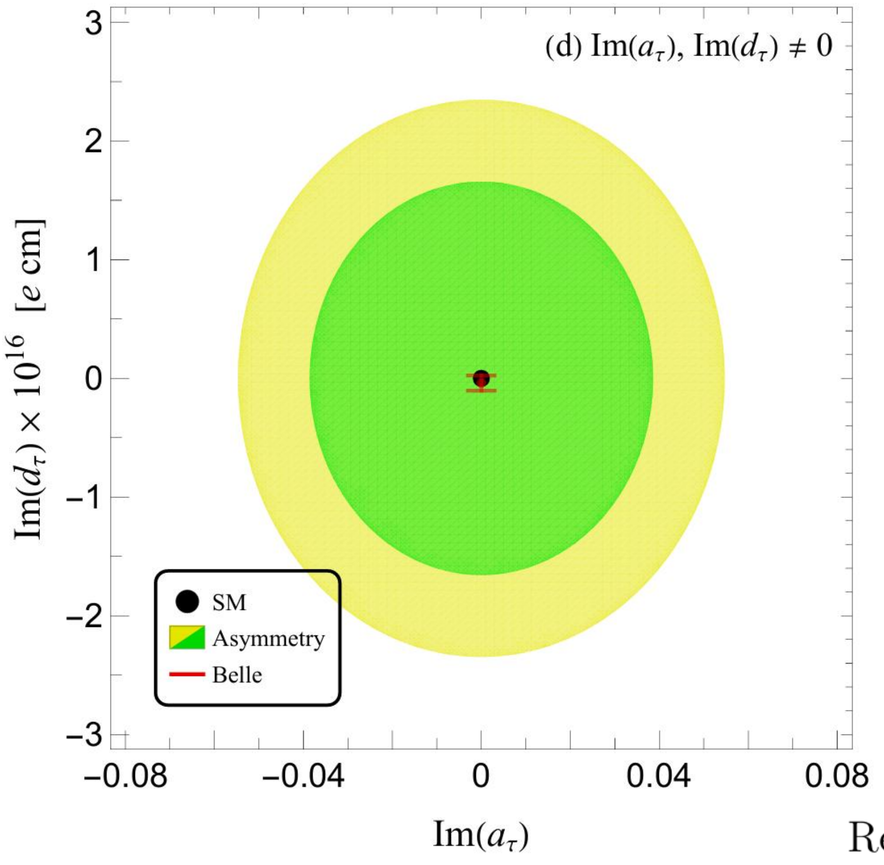
CMS

$$a_\tau = 0.0009^{+0.0032}_{-0.0031}$$

CMS Collaboration, Rep. Prog. Phys.(2024)

Numerical Results





1σ and 2σ sensitivities from the cos(2ϕ) azimuthal asymmetry

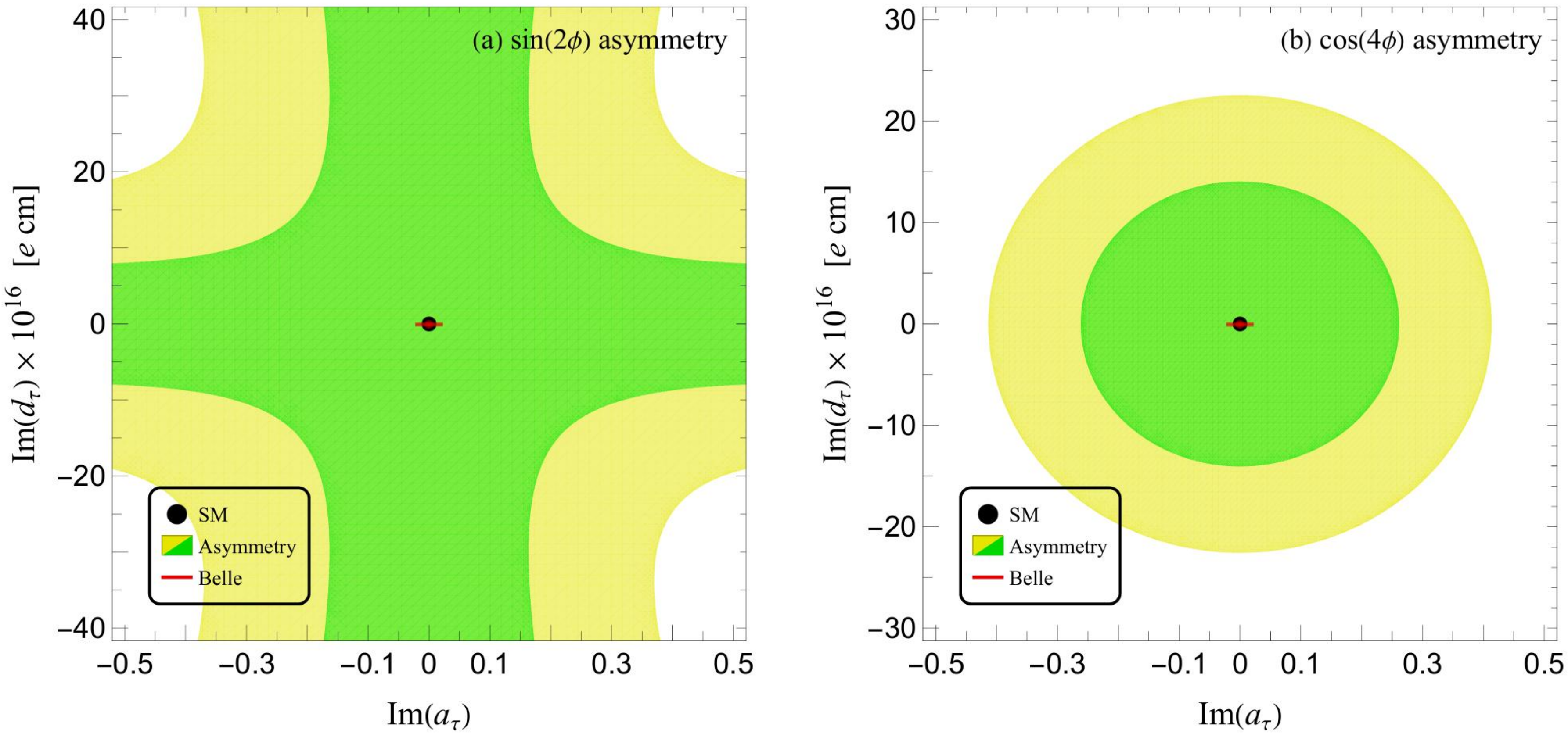
$$|\mathrm{Re}(d_\tau)| < 2.8\times 10^{-16}\, e\cdot\mathrm{cm}$$ @2σ CL

Belle

$$\mathrm{Re}(d_\tau) = (-0.62 \pm 0.63)\times 10^{-17} e\cdot\mathrm{cm}$$

The BELLE collaboration, JHEP(2022)

Numerical Results



1σ and 2σ sensitivities from the sin(2ϕ) and cos(4ϕ) azimuthal asymmetries

$$+C_{s2\phi}^{\mathrm{Im}(F_2)\mathrm{Im}(F_3)}\mathrm{Im}(F_2)\mathrm{Im}(F_3)\sin(2\phi)$$

$$+(C_{c4\phi} + C_{c4\phi}^{\mathrm{Im}(F_2)^2}\mathrm{Im}(F_2)^2 + C_{c4\phi}^{\mathrm{Im}(F_3)^2}\mathrm{Im}(F_3)^2)\cos(4\phi)]$$

Completely independent of real components and thus serve as distinctive probes of potential CP-violating effects

Summary

By introducing TMD factorization, we systematically investigated the azimuthal asymmetries in τ-pair production via two-photon processes at e^+e^- colliders.

Based on the design parameters of STCF, we made projections for the measurement precision of the MDM and EDM. In particular, the projected sensitivity to a_τ is comparable to the strongest existing limit from CMS, while relying on minimal theoretical assumptions and subject to small systematic uncertainties.

To enhance EDM sensitivity in the $\gamma\gamma\to\tau\tau$ process, we plan to implement CP-violating observables, which could introduce linear dependence on F_3 and thereby improve the achievable sensitivity.