

Bjorken x weighted Energy-Energy Correlators from the Target Fragmentation Region to the Current Fragmentation Region

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Phy.Rev.D 109(2024) 9,096004



September 22-26

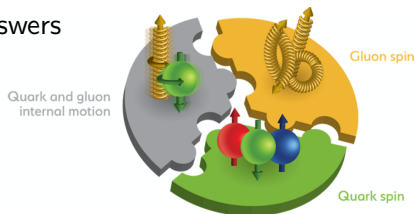


Outline

- Conventional approach to nucleon structure
- Concept and feature of the Bjorken x weighted EEC
- Numerical result
- Summary and Outlook

Why are we interested in Nucleon Structure?

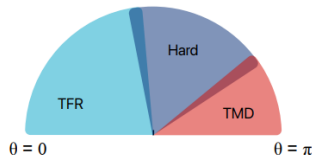
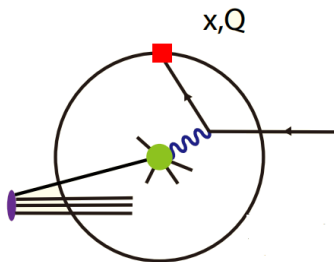
- Still many question yet to answers
 - Partons hadronize
 - Spin components
 - Mass decomposition
 - gluon saturation
 - ...
- Major focus of EIC, EicC



Conventional approach to nucleon structure

Parton distribution function(PDF)

- Inclusive
- Clean in both experiment and theory
- Lose information

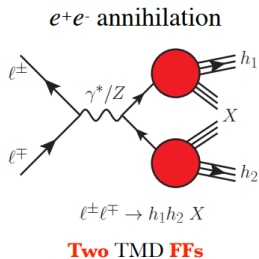
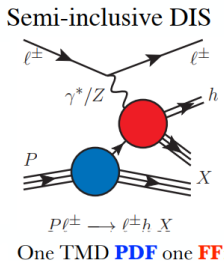
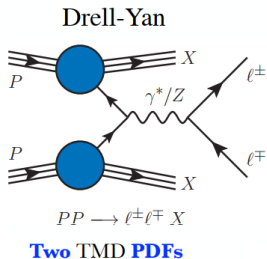


Conventional approach to nucleon structure

Transverse Momentum Dependent (TMD) Distribution

TMD Handbook, [arxiv:2303.01530](https://arxiv.org/abs/2303.01530)

- Two TMD ;
- Sudakov suppression.



The Nucleon Energy Correlators

Nucleon EEC [X.Liu and H.X.Zhu, PRL 130,091901 \(2023\) 9,9](#)

–Bjorken x weighted EEC of the target fragmentation region.

- The active parton k_t unconstrained, not TMD;
- Energy weighted collinear PDF;
- Transverse dynamics through energy flow operator $\mathcal{E}$;
- Insensitive to soft radiations, no Sudakov suppression

$$f_{q,EEC}(z, \theta) = \int \frac{dy^-}{4\pi} e^{-izP^+ \frac{b^-}{2}} \langle p | \bar{\chi}_n(y^-) \frac{\gamma^+}{2} \hat{\mathcal{E}}(\theta) \chi_n(0) | P \rangle$$

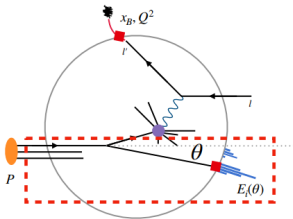
$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r \rightarrow \infty} r^2 n^i T_{0i}(t, r\hat{n})$$

$$\hat{\mathcal{E}}(\theta) | X \rangle = \sum_{i \in X} \frac{E_i}{E_P} \Theta(\theta - \theta_i) | X \rangle$$

The Nucleon Energy Correlators

- NEEC can be observed through the following process

$$\Sigma(x_B, Q^2, \theta) = \sum_i \int \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i) \Theta(\theta - \theta_i)$$



- When $\theta Q \ll Q$, factorization can be shown using SCET

H.T.Cao, X.Liu, H.X.Zhu, Phys.Rev.D 107 (2023) 11,114008

$$\Sigma(x_B, Q^2, \theta) = \int_{x_B}^1 \frac{dz}{z} \hat{\sigma}_i\left(\frac{x_B}{z}, Q^2, \mu\right) f_{i,EEC}(z, \theta, \mu)$$

The Nucleon Energy Correlators

- $\theta \ll 1$, hard($p_H \sim Q(1, 1, 1)$), soft($p_S \sim Q(\theta^a, \theta^a, \theta^a)$),
collinear($p_C \sim Q(1, \theta^2, \theta)$)

$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P} \sum_{i \in X} \left(E_{H,i} \Theta(\theta - \theta_{H,i}) + E_{C,i} \Theta(\theta - \theta_{C,i}) \right. \\ \left. + E_{S,i} \Theta(\theta - \theta_{S,i}) \right) |X_H, X_C, X_S\rangle$$

- $\Lambda_{QCD} \ll \theta Q$, hard collinear mode(C_1), $p_t \sim Q(1, \theta^2, \theta)$
SCET_{II} mode (C_2), $p_t \sim Q(1, \lambda^2, \lambda)$.

$$\hat{\mathcal{E}}(\theta)|X\rangle = \frac{1}{E_P} \sum_{i \in C_1, j \in C_2} (E_{C_1,i} \Theta(\theta - \theta_{C_1,i}) + E_{C_2,j} \Theta(\theta - \theta_{C_2,j})) |X_{C_1} X_{C_2}\rangle \\ = \frac{1}{E_P} \sum_{i \in X} (E_X - E_{C_1,i} \Theta(\theta_{C_1,i} - \theta)) |X_{C_1} X_{C_2}\rangle$$

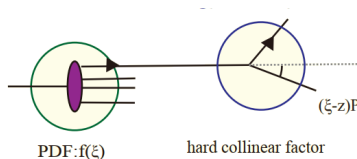
The Nucleon Energy Correlators

- NEEC can be further matched to pdf.

$$f_{i,EEC}(z, \ln \frac{Q\theta}{u\mu}) = f_i(z) - \int_z^1 \frac{d\xi}{\xi} l_{i,j}(\frac{z}{\xi}, \ln \frac{Q\theta}{u\mu}) \xi f_j(\xi)$$

- We can understand why this object is called NEEC, which is obvious from the fix order calculation.

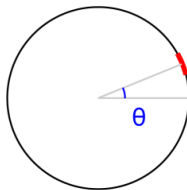
$$\frac{df_{EEC}^{(1)}(z,\theta)}{d\theta} \propto \xi f(\xi) [(1 - \frac{z}{\xi}) \frac{1}{\theta} P(\frac{z}{\theta})]$$



The Nucleon Energy Correlators

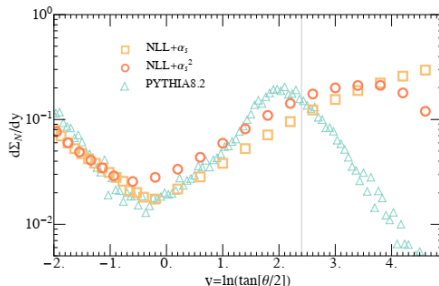
- When the angle θ is very small, we expect to see the region of the free hadron gas phase. In this phase, the energy is evenly distributed.

$$\frac{d\Sigma(\theta)}{d\theta} \propto \theta$$



TFR to the CFR

- Target Fragmentation Region(TFR)



- Current Fragmentation Region(CFR)

The observed particles result from the fragmentation of the parton struck by the virtual photon.

- TMD region
- Hard region

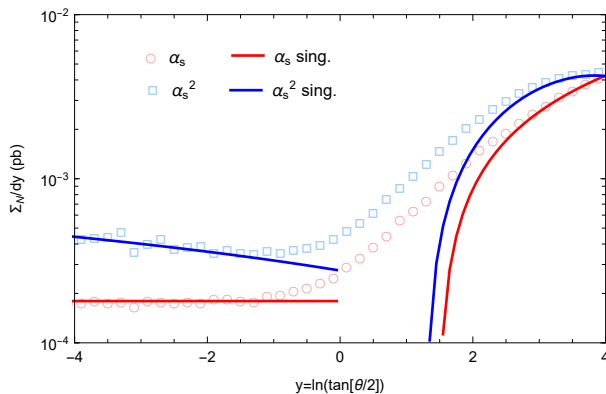
- When $\pi - \theta \ll 1$, the factorization theorem for this observation in this region has already been derived as follows:

H.T.Li, Makris, Vitev, Phys.Rev.D 103 (2021) 9,094005

$$\begin{aligned} \frac{d\Sigma_N(Q^2, \theta)}{d\theta} &= \int dx_B x_B^N H(Q^2) \int d^2\mathbf{q}_T \frac{d^2\mathbf{b}}{(2\pi)^2} \exp[-i\mathbf{q}_T \cdot \mathbf{b}] \\ &\times B_{f/p}(b, x_B) S(b) J_{f,\text{EEC}}(b) \delta\left(\frac{2|\mathbf{q}_T|}{Q} - \theta\right) \end{aligned}$$

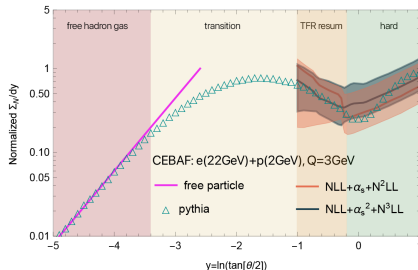
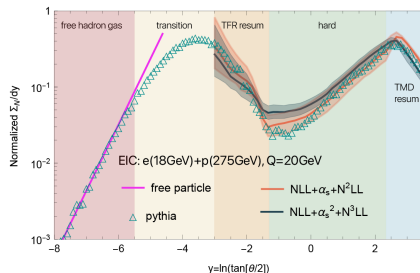
Numerical result

- Comparison between the $\ln\theta$ leading singular contributions with the fixed-order calculations.



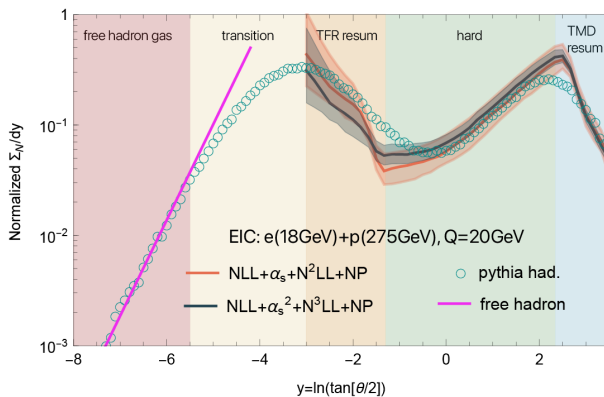
Numerical result

- Comparison between the result of fixed-order calculation + resummation + free hadron gas model and the Pythia simulation result at the partonic level.



Numerical result

- The comparison of EEC between the SCET predictions with hadronic PYTHIA simulations for EIC kinematics.



Summary and Outlook

- Provides a new window into nucleon structure, bridging CFR and TFR.
- It is factorizable, measurable, and computable within SCET.
- Application: Unraveling the gluon saturation by small- x physics
H.-Y.Liu, X.Liu, F.Yuan, H.X.Zhu, Phys.Rev.L 130 (2023) 18,18
- Application: Exhibits linearly polarized gluons inside the nucleon
X.L.Li, X.Liu, F.Yuan, H.X.Zhu, Phys.Rev.D 108 (2023) 9,L0915028
- Future work will includes higher-order calculations.

Thanks for your attention!

Matching Scheme

- In EIC kinematics

$$\frac{d\Sigma_N}{dQ^2 dy} = (1 - f^2) \left. \frac{d\Sigma_N}{dQ^2 dy} \right|_{\text{QCD}} + f^2 \left(\left. \frac{d\Sigma_N}{dQ^2 dy} \right|_{\text{non-sing}} + \left. \frac{d\Sigma_N}{dQ^2 dy} \right|_{\text{res}} \right)$$

where

$$f = \frac{1}{2} \left[\cos \left(\frac{\cos\theta - a}{a - b} \pi \right) + 1 \right] \quad (2.3 < y < 3)$$

$$f = \frac{1}{2} \left[\cos \left(\frac{\cos\theta - c}{c - d} \pi \right) + 1 \right] \quad (-2 < y < -1.3)$$

a, b, c, d equal to $\cos\theta$ with θ associated with $y = 3, 3.2, -2, -1.3$

- In CEBAF kinematics

$$f = \frac{1}{2} \left[\cos \left(\frac{\cos\theta - a}{a - b} \pi \right) + 1 \right]$$

a and b equal to $\cos\theta$ with θ associated with $y = 0.2, 0.1$