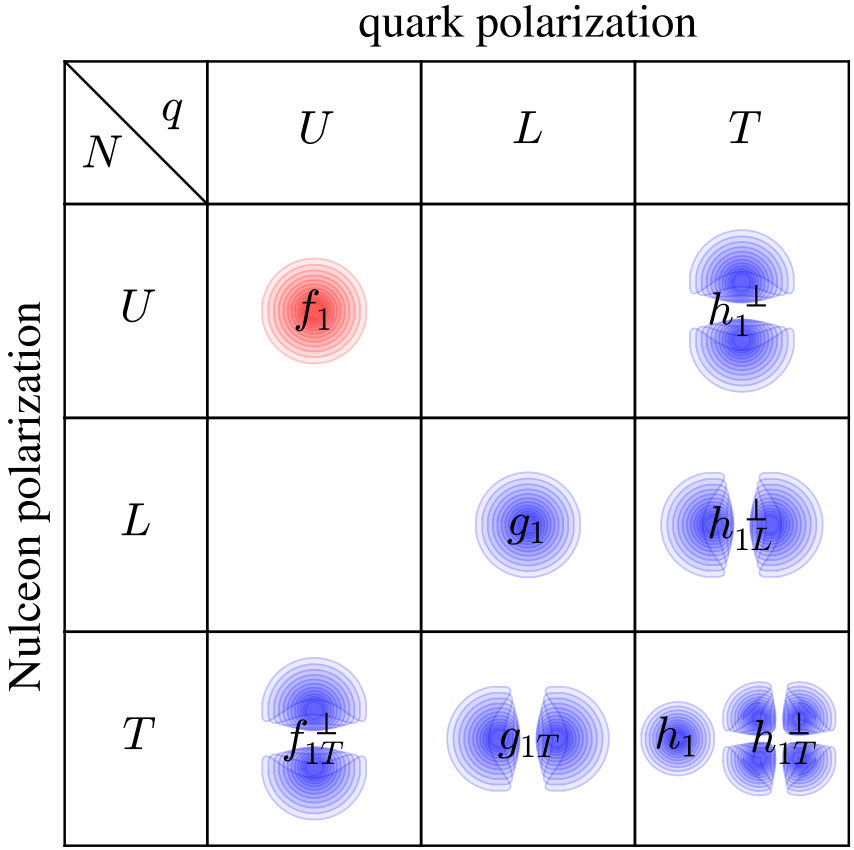
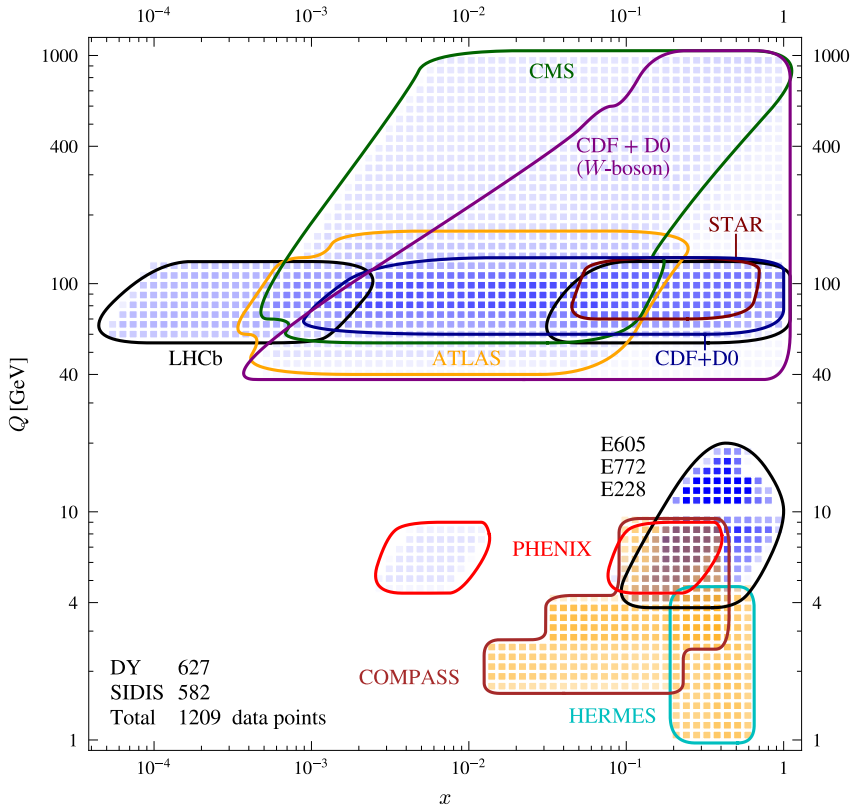
8 TMD distributions



leading twist TMDPDFs



Kinematic coverage of ART25

unpolarised SIDIS cross-section (γ^*) in TMD regime

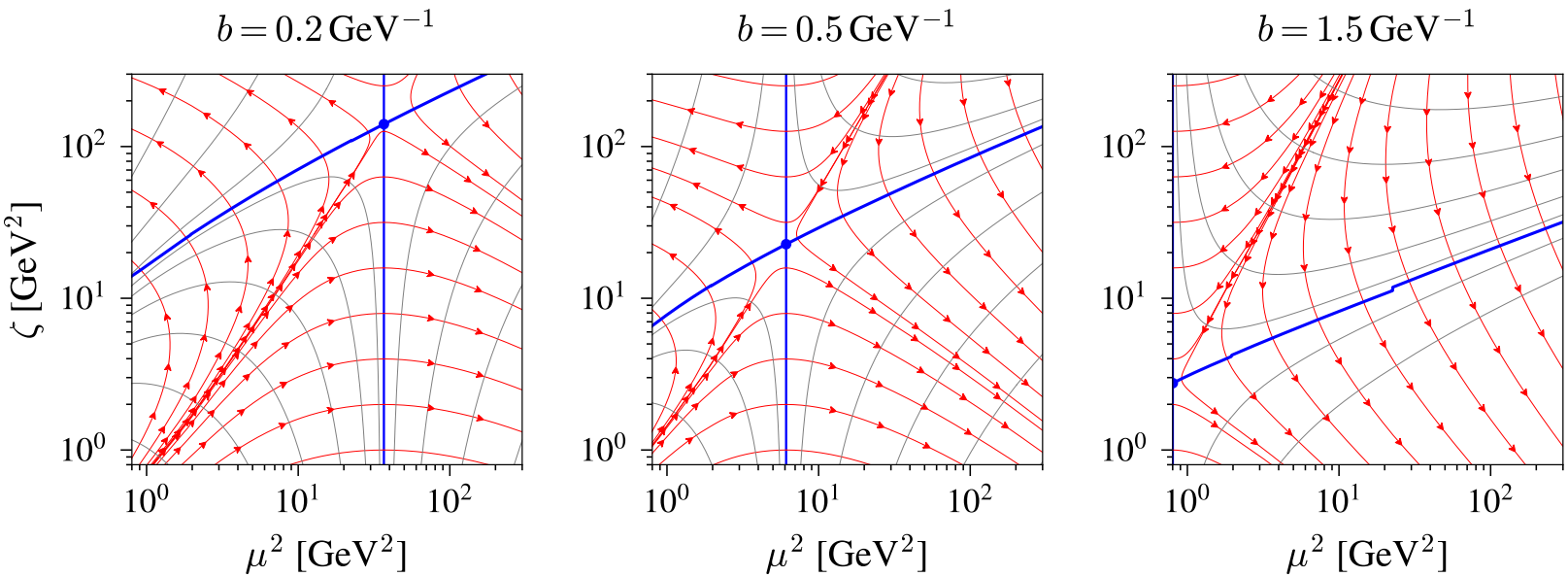
$$\frac{d\sigma^{\pi^+}_{\text{SIDIS}}}{dQ^2\,dx dz dp_T^2} = \sigma_0 \sum_f C_{\text{SIDIS}}(Q) \int_0^\infty db\, b\, J_0(b p_T/z)$$
$$\times f_{1,f}(x_s, b) D_{1,f\to\pi^+}(z, b) \left(\frac{Q^2}{\zeta(b)}\right)^{-2\mathcal{D}(b,Q)}$$

Parametrise $\rightarrow$ extract

- unpolarised TMDPDF $f_1(x, b)$
- unpolarised TMDFF $D_{1,f\to h}(z, b)$
- Collins-Soper kernel $\mathcal{D}(Q, b)$

Framework: matching TMDPDF

$$f_{1,f}(x,b) = \int_x^1 \frac{dy}{y} \sum_{f'} C_{f\to f'}(y,b,\mu_{\rm OPE})\, q_{f'}\left(\frac{x}{y},\mu_{\rm OPE}\right) f^f_{\rm NP}(x,b)$$



optimal TMD distribution: on **special null-evolution curve**

ART23 → ART25

Differences

- + SIDIS data / TMDFFs
- $\lambda_1^u = \lambda_1^{\overline{u}}, \quad \lambda_1^d = \lambda_1^{\overline{d}}$ (same small-x behaviour for q & $\overline{q}$)
- $\lambda_3^{u/d}$ ((slightly) more flexible ansatz)
- large-x resummantion in Coefficient function $C_{f\to f'}$ [2501.17274]
- $\mu_{\rm OPE} = \frac{C_0}{b} \cancel{+2\,\text{GeV}} + 5\,\text{GeV}$ (avoid quark-threshholds)
- $B_{NP} \to 1.5\,\text{GeV}^{-1}$ (constrained parameter)

Model overview for TMD distributions

$$f^f_{\mathrm{NP}}(x,b) = \cosh^{-1}\left(\left(\lambda_1^f(1-x)^{\lambda_3} + \lambda_2^f x\right) b\right)$$
$$f \in \left\{u, \overline{u}, d, \overline{d}, sea\right\}$$

$$d^{f,h}_{\mathrm{NP}}(z,b) = \cosh^{-1}\left(\eta_0^h(1-z)\frac{b}{z}\right)\left(1+\eta_1^{\{h,f\}}\frac{b^2}{z^2}\right)$$
$$h \in \left\{\pi^\pm, K^\pm\right\} \quad f \in \left\{u, \overline{d}, \overline{u}, sea\right\} \text{ for } h = \pi^+$$

$\rightarrow$ 10 parameters for PDFs, 5 for $\pi^\pm$ FFs, 5 for $K^\pm$ FFs

$$f^f_{\mathrm{NP}}(x,b),\ d^{f,h}_{\mathrm{NP}}(z,b) \sim e^{-\alpha(x/z)b} \quad \text{for} \quad b \gg 1$$

Large b behaviour similar to SV19!

Summary

data input

- $\langle Q \rangle > 2\,\text{GeV}$
- $\frac{q_T}{Q} = \lambda < 0.25$
  (TMD regime: $\lambda \ll 1$)
- technical cuts

$\rightarrow$ 627(DY) + 582(SIDIS)

collinear input:

- `MSHTPDF20` (N2LO)
  (PDFs)
- `MAPFF1.0` (N2LO)
  (FFs)

perturbative input

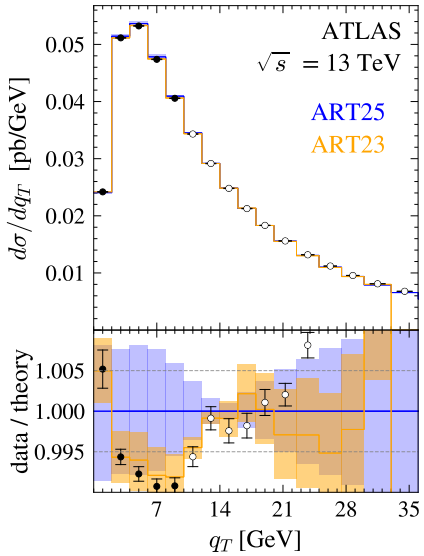
Γ_{cusp}	γ_V	$\mathcal{D}_{\text{pert}}$	$C_{f\to f'}$	C_{h}
N^{4}LO	N^{3}LO	N^{3}LO	N^{3}LO	N^{4}LO
α_s^5	α_s^4	α_s^4	α_s^3	α_s^4

output

CS-kernel, TMDPDFs, TMDFFs

$\chi^2/N_{\text{pt}} = 1.17 \oplus 0.92$	**ART25**
$1.16 \oplus 0.95$	SV19
1.00	ART23

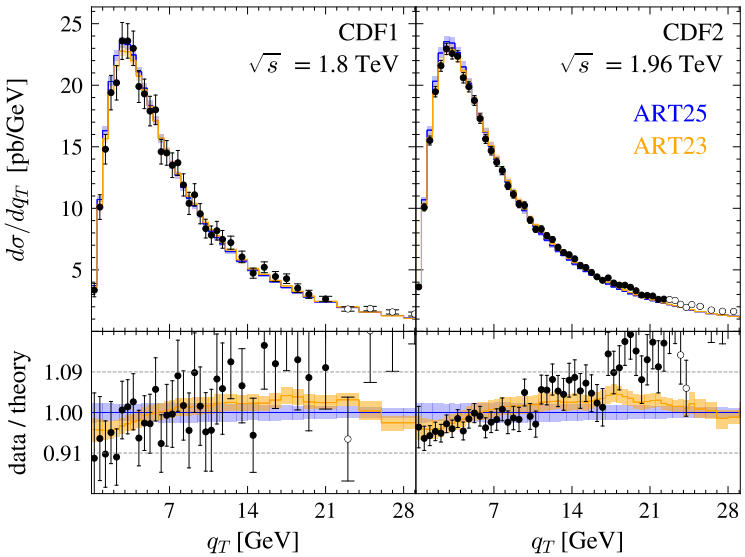
Results: Drell-Yan



← ATLAS

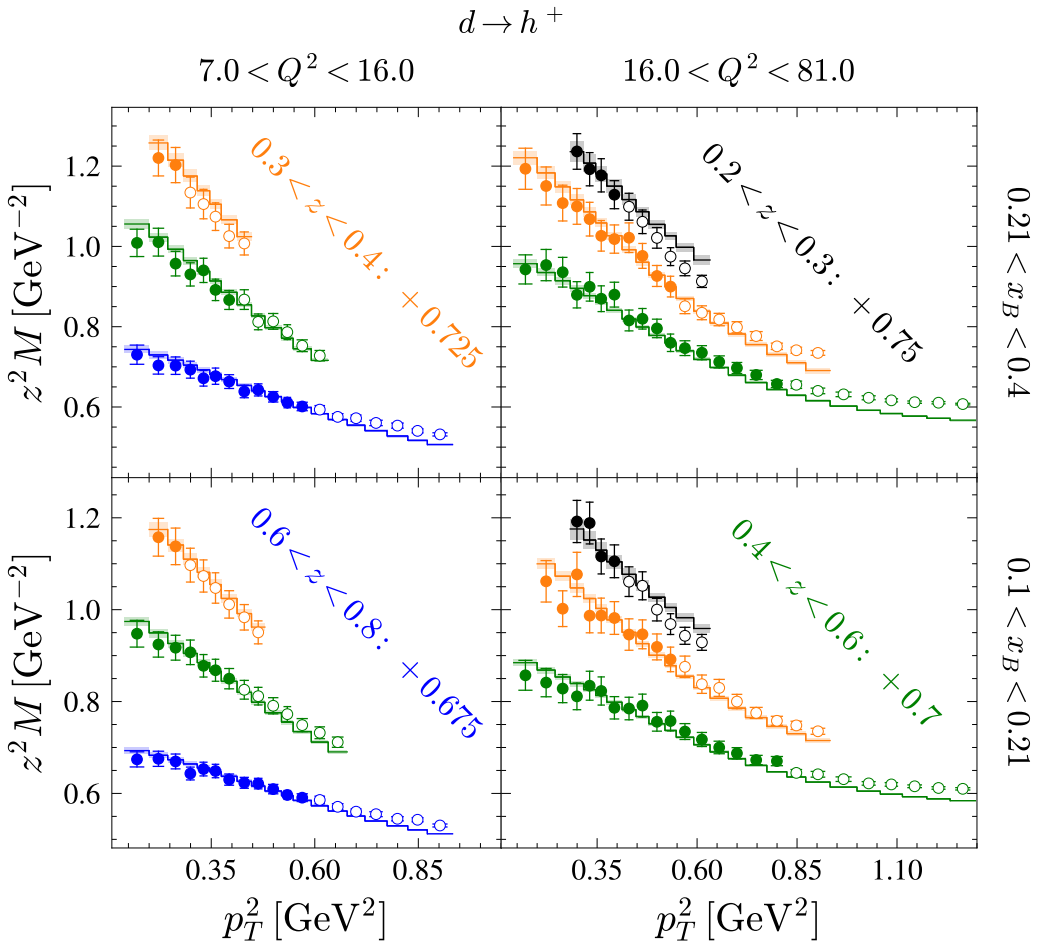
fit result in χ^2 per group of experiment

Data set	$N_{\rm pt}$	$\chi^2_D/N_{\rm pt}$	$\chi^2_\lambda/N_{\rm pt}$	$\chi^2/N_{\rm pt}$
CDF	84	2.06	0.07	2.13
D0	36	2.20	0.08	2.28
ATLAS	49	1.43	0.25	1.68
CMS	113	0.69	0.13	0.82
LHCb	68	1.06	0.26	1.32
PHENIX	3	0.41	0.08	0.48
STAR	11	1.15	0.16	1.32
collider:	364	1.34	0.15	1.49
E228	175	0.67	0.02	0.69
E772	35	1.25	0.12	1.37
E605	53	0.31	0.12	0.42
fixed-target:	263	0.67	0.05	0.73
total:	627	1.06	0.11	1.17



← CDF

Results: SIDIS

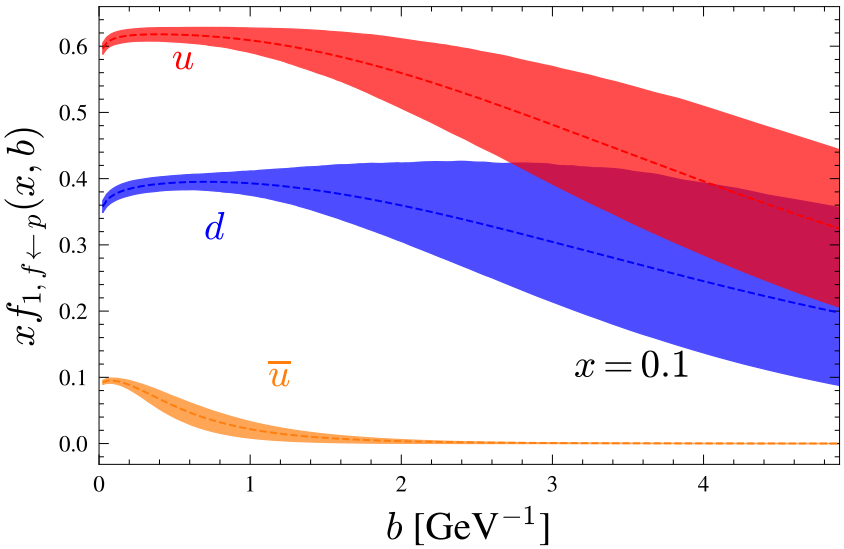


comparison with COMPASS

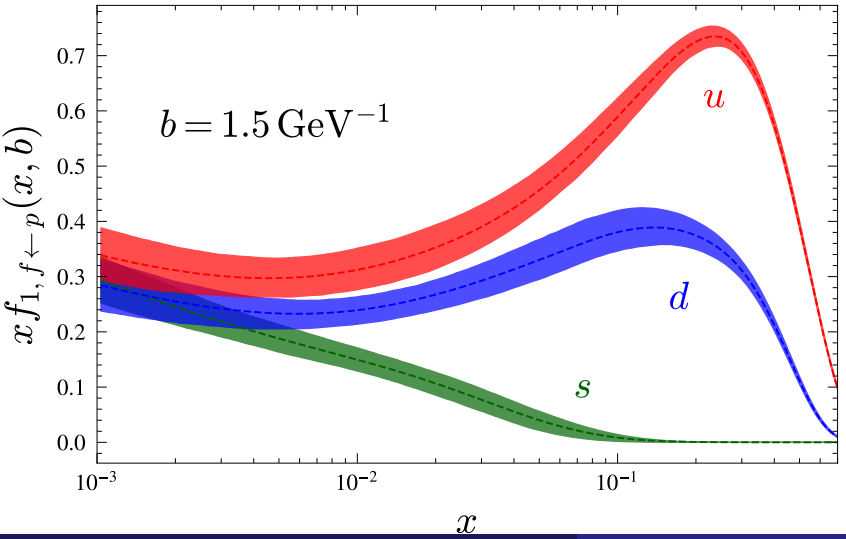
fit result in χ^2 per channel

Data set	N_{pt}	χ^2/N_{pt}
HERMES π^+	48	1.70
HERMES π^-	48	1.29
HERMES K^+	48	0.47
HERMES K^-	48	1.34
COMPASS h^+	195	0.67
COMPASS h^-	195	0.91
total:	582	0.92

RESULTS: PDFs

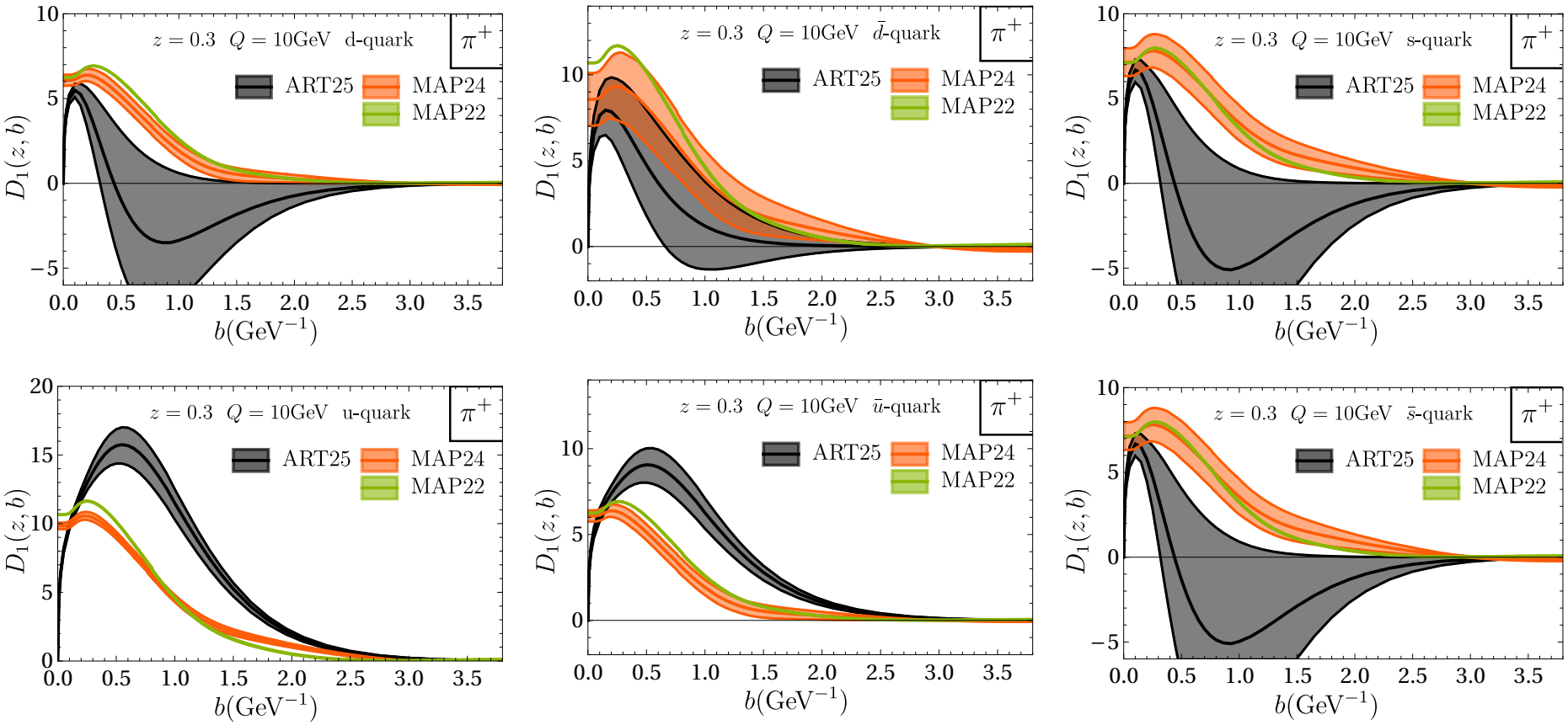


PDFs at as function of b



PDFs at as function and x

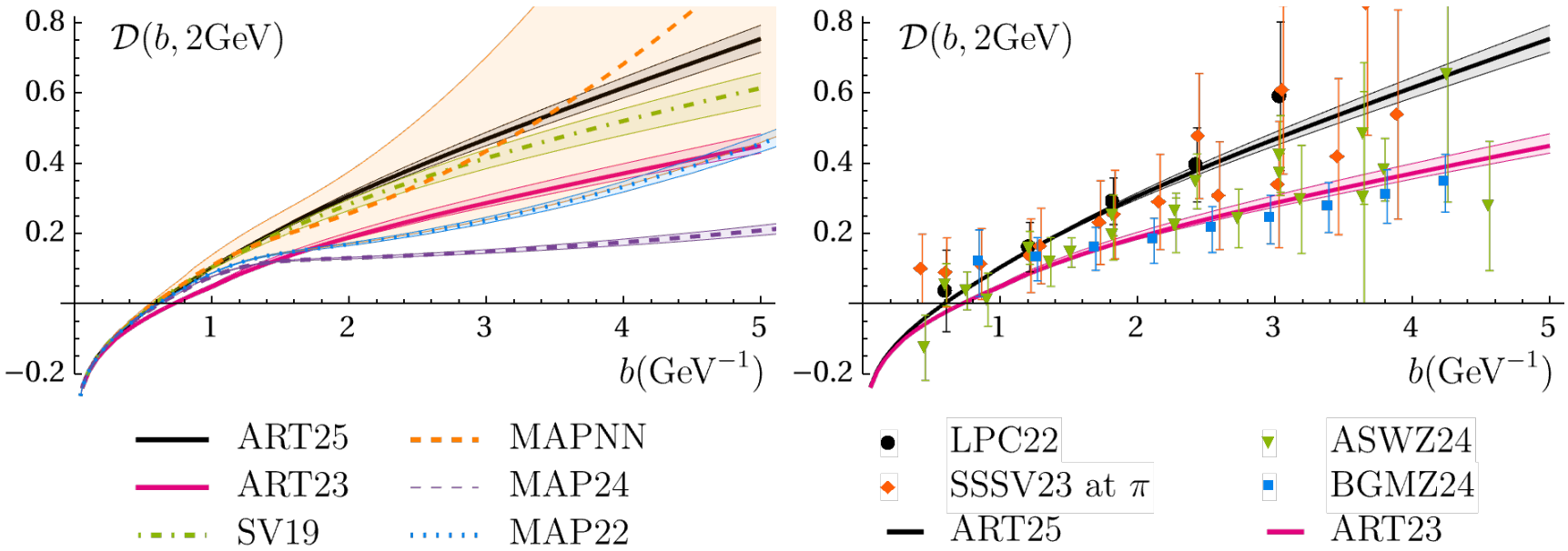
RESULTS: FFs



FFs at as function of b compared with MAP results

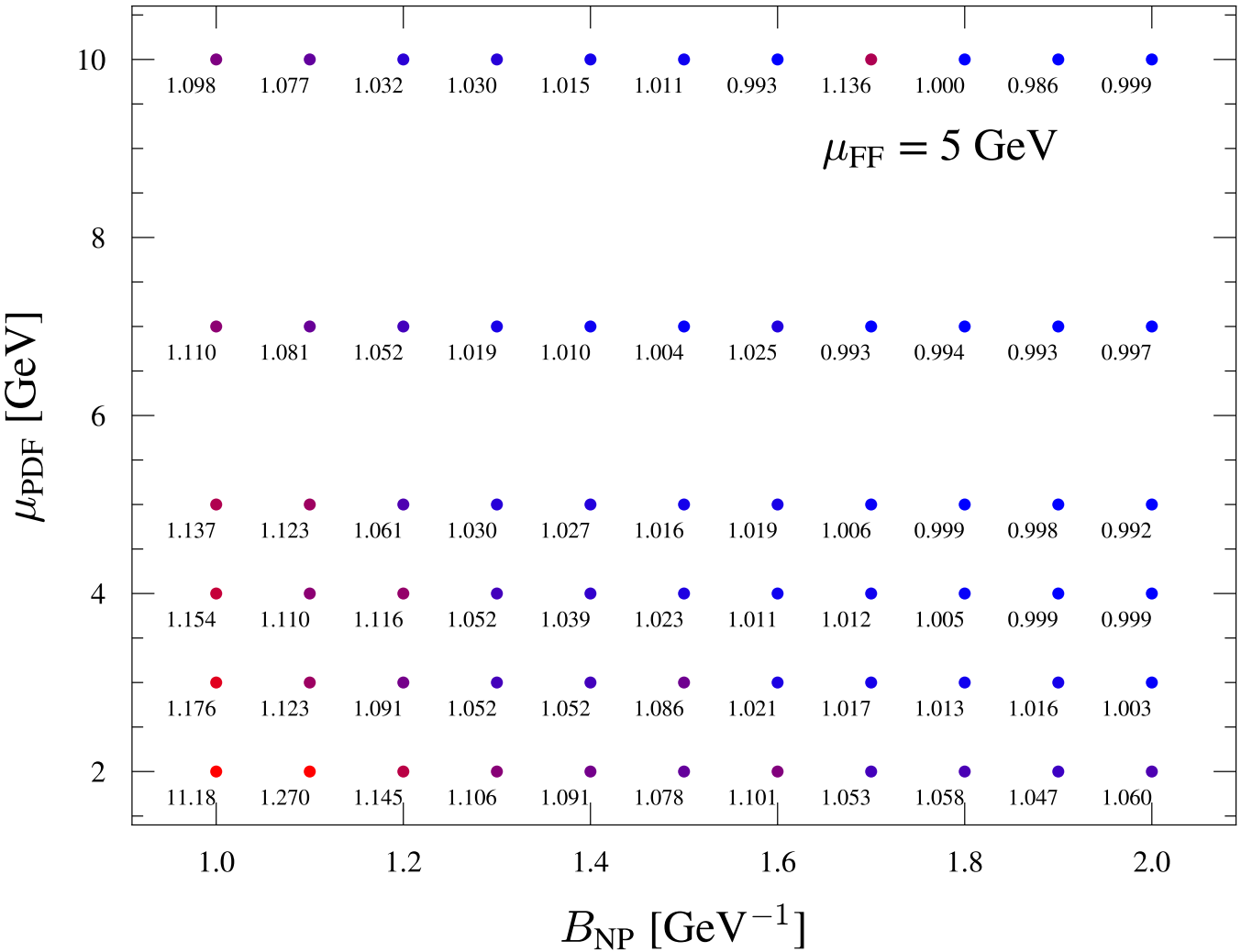
Collins-Soper kernel

$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{small-b}}(b^*,\mu) + c_0\, bb^* + c_1\, bb^* \ln\left(\frac{b^*}{B_{\text{NP}}}\right)$$



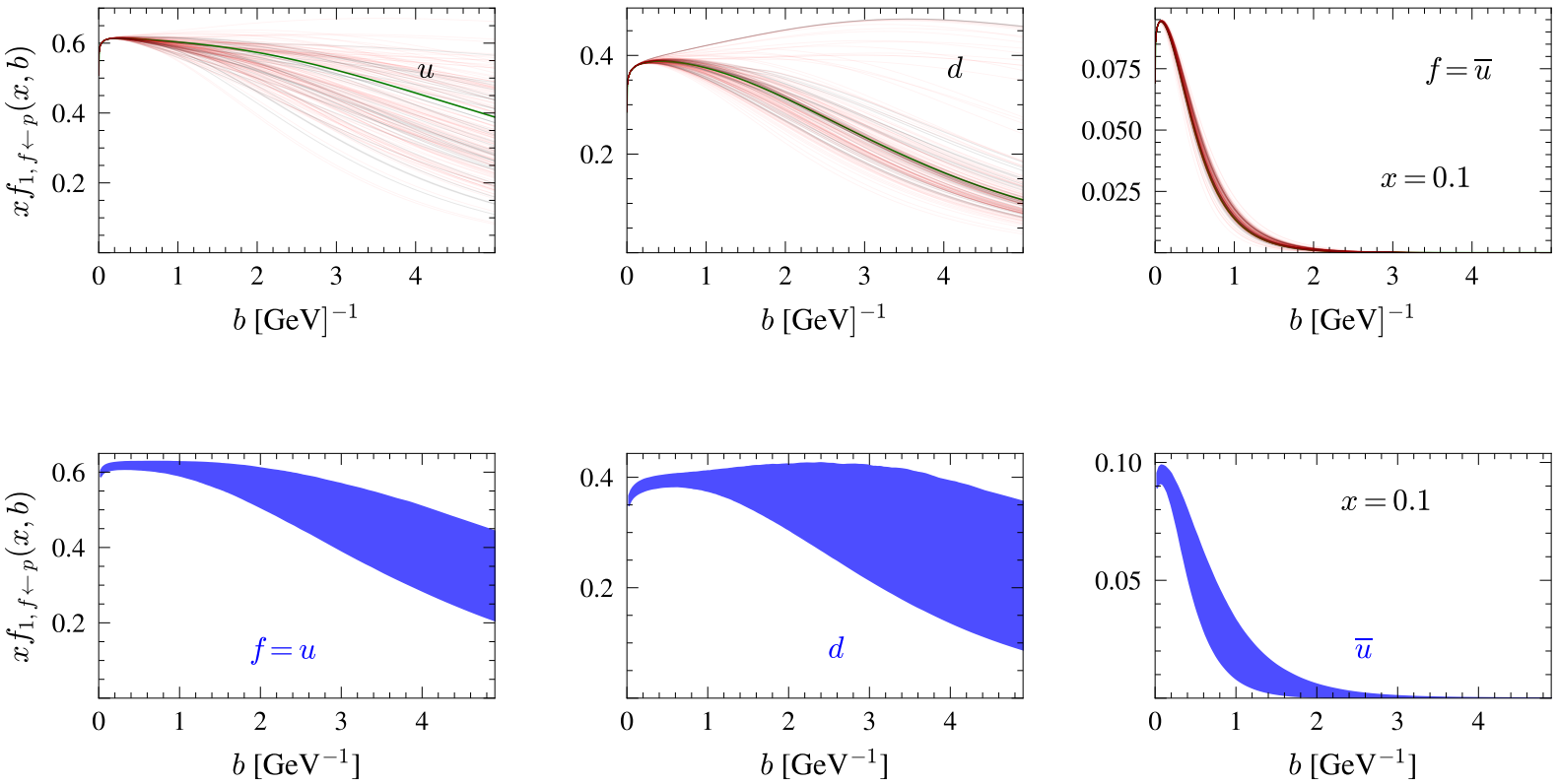
comparison with phenomenology (left) and lattice (right)

variation of fixed intrinsic parameters: μ_{OPE} and B_{NP}



χ^2/N_{dat} for different choices of *intrinsic parameters*

variation of fixed intrinsic parameters: μ_{OPE} and B_{NP}

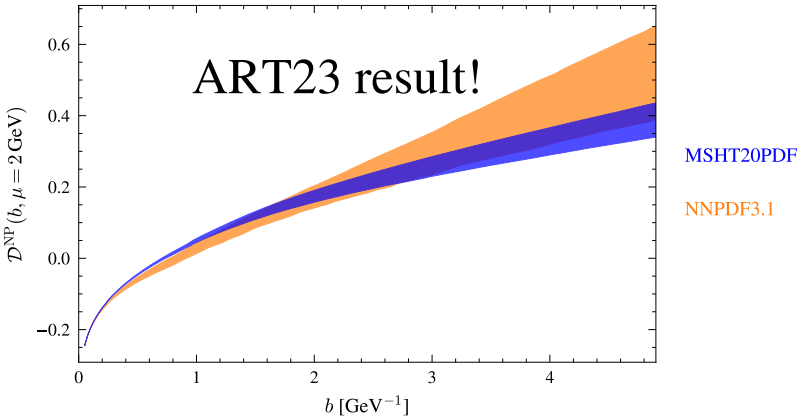


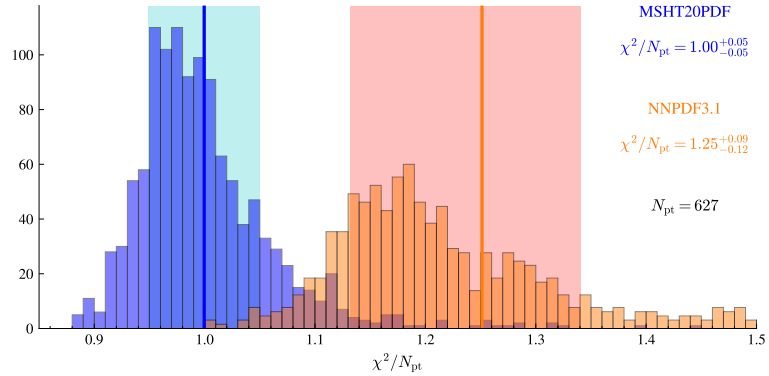
top: scale variation

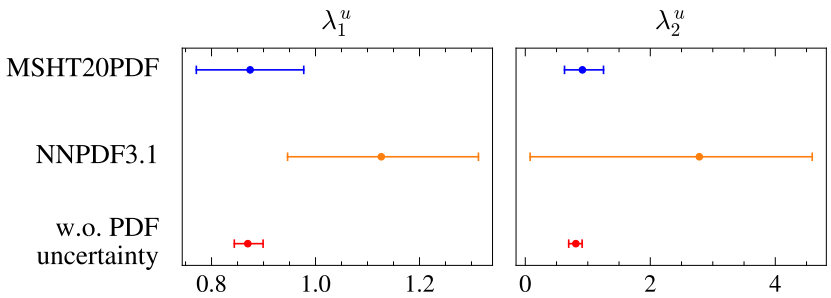
bottom: ART25 68%CI

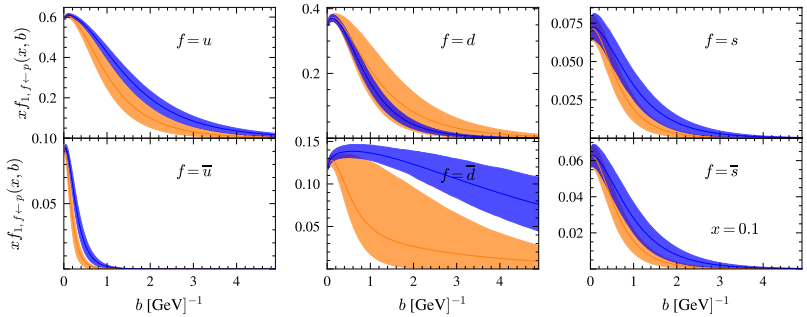
Systematics: Impact of Boundary Conditions

The PDFs are unique – yet not uniquely determined









issue addressed: combined PDF + TMDPDF fit [Talk by A. Prokudin]

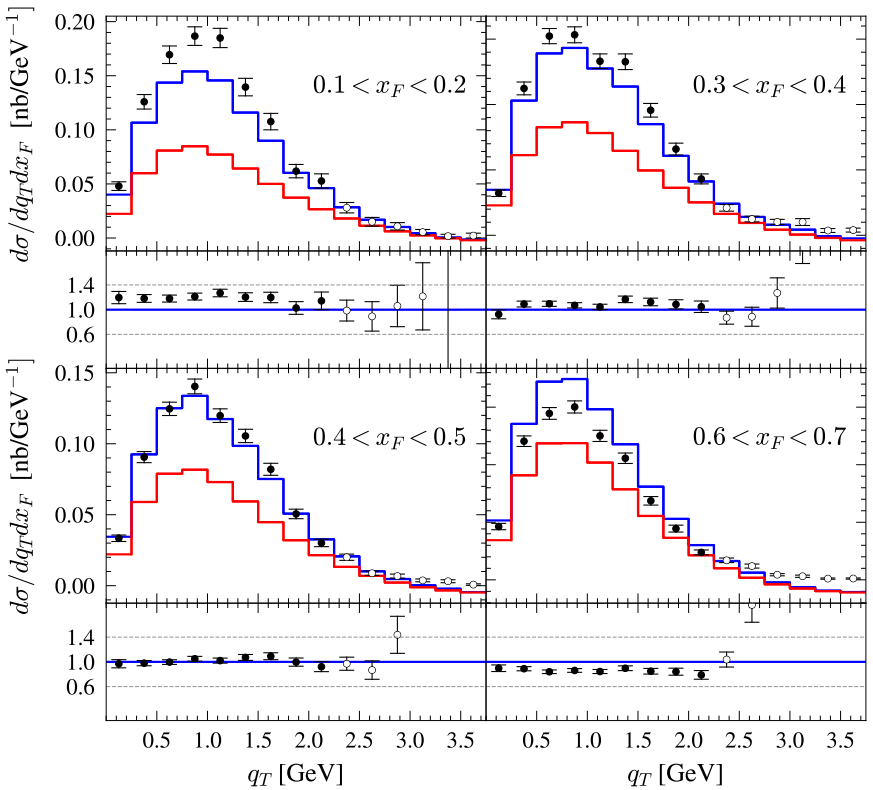
Problem with fixed target DY (&Pion induced DY) (?)

Table of χ^2 results
for fixed target DY data

Experiment	σ_{norm}	χ^2/N_{pt}	sys. shift	#dat
E228-200	25%	0.547	20%	43
E228-300	25%	0.683	26 %	53
E228-400	25%	1.241	29 %	79
E772	10%	1.233	20 %	35
E605	15%	0.357	38 %	35
PHE200	12%	0.386	-5%	3
A13-norm	0%	1.274	0 %	5

origin: higher Twist corrections?
($Q \in (4, 20)\,[\text{GeV}]$)
relation to pion induced DY?

Also for pi-DY
($Q \sim 10\,\text{GeV}$, fixed target)
predictions without shift



Summary

data input

- $\langle Q \rangle > 2\,\text{GeV}$
- $\frac{q_T}{Q} = \lambda < 0.25$
  (TMD regime: $\lambda \ll 1$)
- technical cuts

$\rightarrow$ 627(DY) + 582(SIDIS)

collinear input:

- `MSHTPDF20` (N2LO)
  (PDFs)
- `MAPFF1.0` (N2LO)
  (FFs)

perturbative input

Γ_{cusp}	γ_V	$\mathcal{D}_{\text{pert}}$	$C_{f\to f'}$	C_{h}
N^{4}LO	N^{3}LO	N^{3}LO	N^{3}LO	N^{4}LO
α_s^5	α_s^4	α_s^4	α_s^3	α_s^4

output

CS-kernel, TMDPDFs, TMDFFs

$\chi^2/N_{\text{pt}} = 1.17 \oplus 0.92$	**ART25**
$1.16 \oplus 0.95$	SV19
1.00	ART23

Recapitulation & outlook

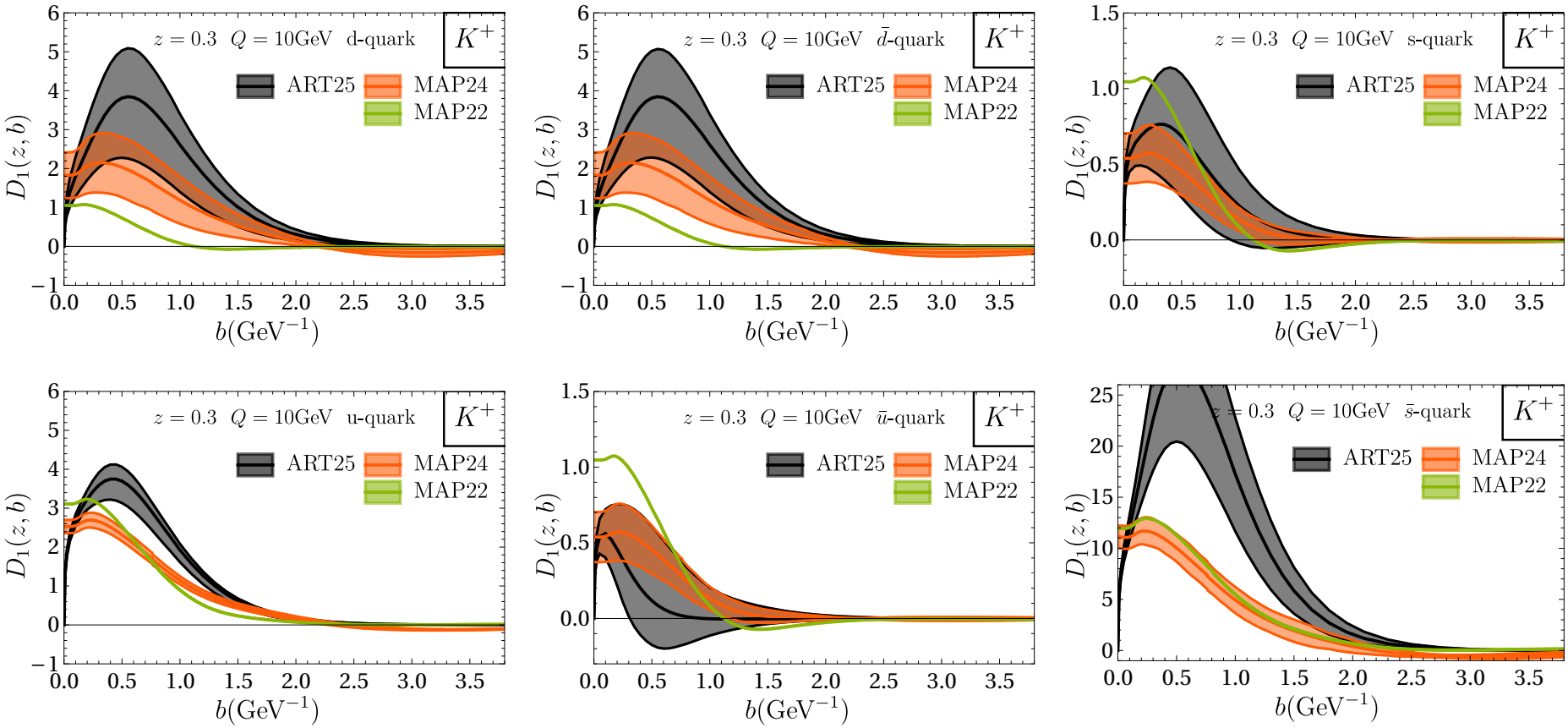
What has been done:

- good quality of fit
  $\chi^2/N_{\text{pt}} = 1.05 = 1.17 \oplus 0.92$ (for average parameters)
- more spread in replica fits affecting DY (no issue in ART23)
- SIDIS data can be described without imposing normalisation

Work in progress:

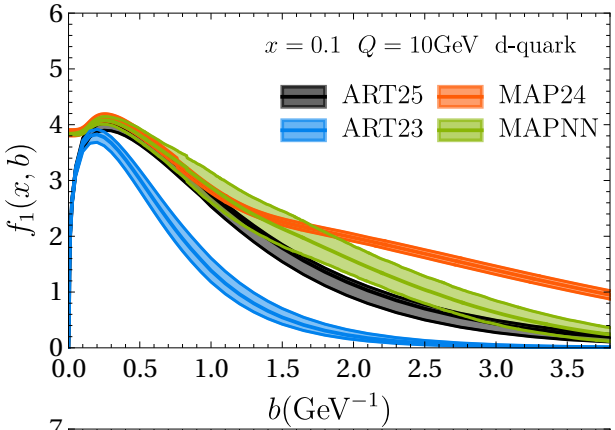
- pion TMDPDF fit

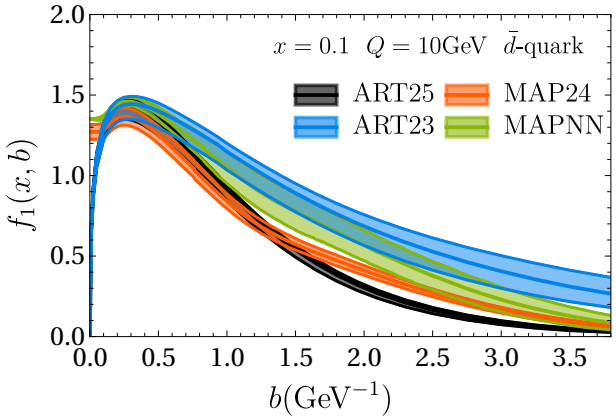
RESULTS: FFs

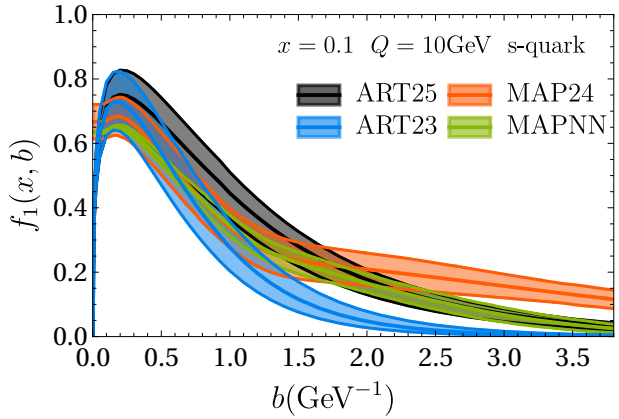


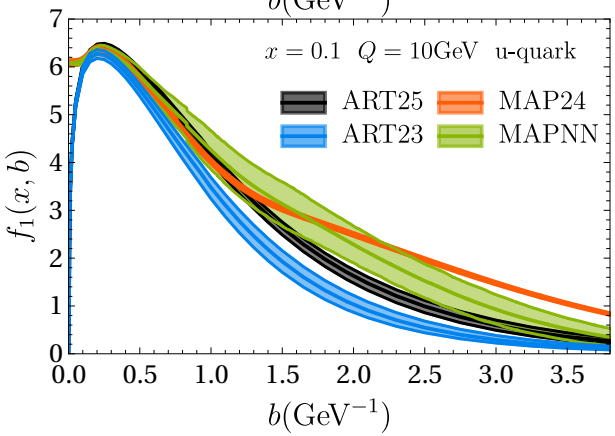
FFs at as function of b compared with MAP results

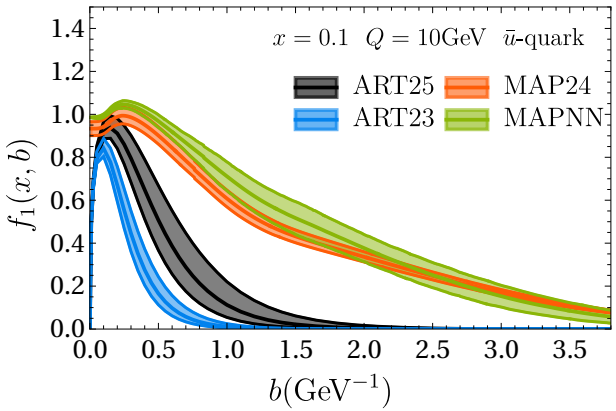
Extracted function: TMDPDF



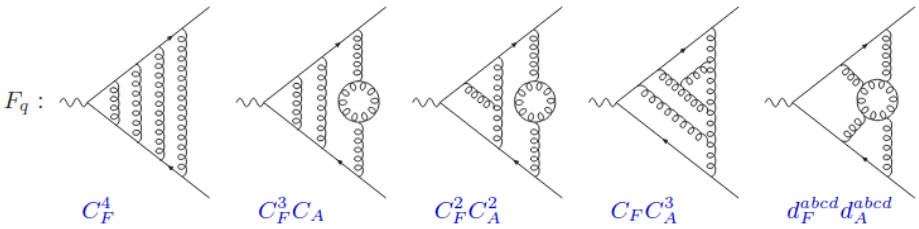




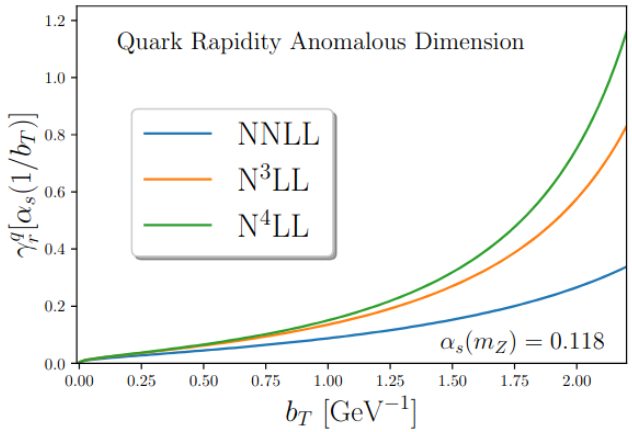




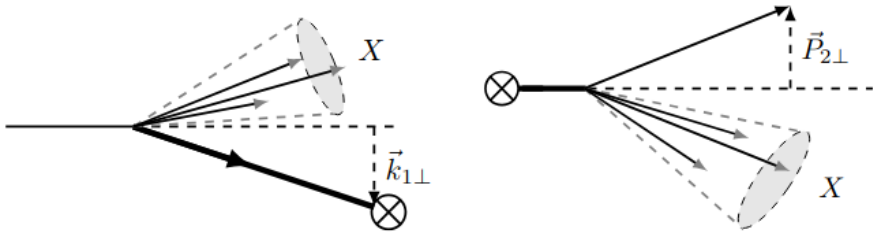
Perturbative input



"Quark and gluon form factors in four-loop QCD" [2202.04660]
$\rightarrow C(Q^2)$



Rapidity anomalous dimension at N4LL [2205.02242]
$\rightarrow \mathcal{D}_{\text{pert}}(b, \mu)$



Splitting functions at N3LL [1908.03831]
$\rightarrow C_{f \to f'}(y, \mu_{\text{OPE}})$

Parametrisation: SV19

$$f_{\rm NP}^{f}(x,b) = \exp\left(-\frac{(\lambda_1\overline{x} + \lambda_2 x + \lambda_3 x\overline{x})}{\sqrt{1+\lambda_4 x^{\lambda_5} b^2}} b^2\right) \tag{1}$$

$$d_{\rm NP}^{f}(z,b) = \exp\left(-\frac{(\eta_1 z + \eta_2 \overline{z}+)}{\sqrt{1+\eta_3 (b/z)^2}} \frac{b^2}{z^2}\right)\left(1+\eta_4 \frac{b^2}{z^2}\right) \tag{2}$$

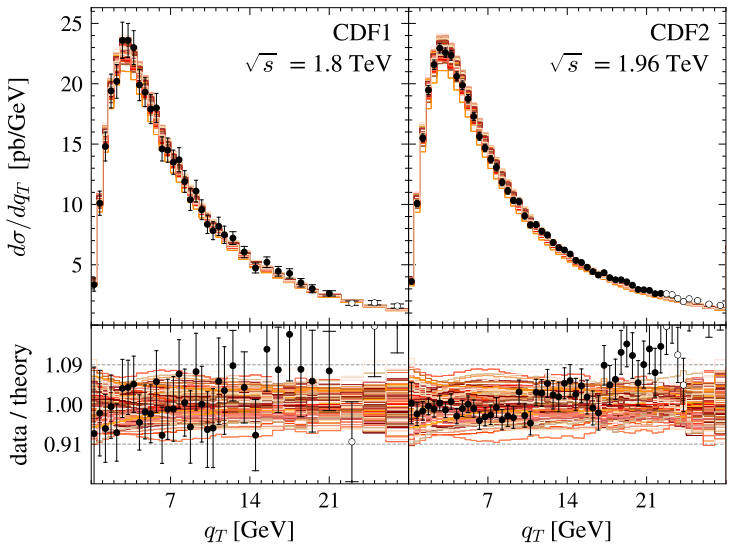
$$\mathcal{D}_{\rm NP}(b,\mu) = \mathcal{D}_{\rm resum}(\mu, b^*) + c_0 b b^* \tag{3}$$

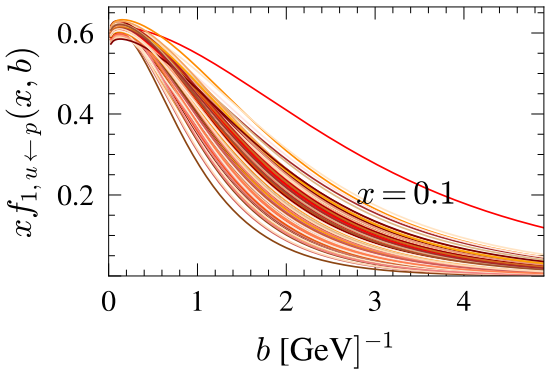
$$f_{\rm NP}^{f}(x,b) \sim e^{-\alpha b} \quad \text{for} \quad b \gg 1 \tag{4}$$

Uncertainty processing in fit (ART23 figures)

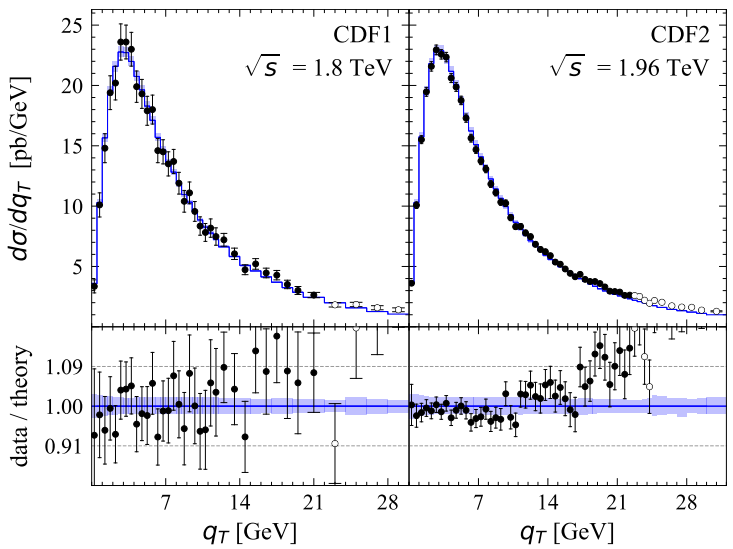
replica of data + replica of PDF $\xrightarrow{\text{fit}}$ TMDPDF replica

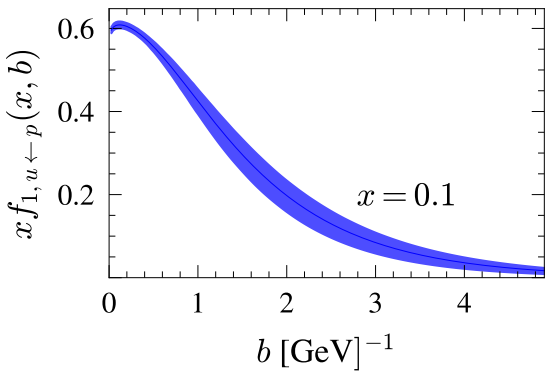
ensemble of replicas

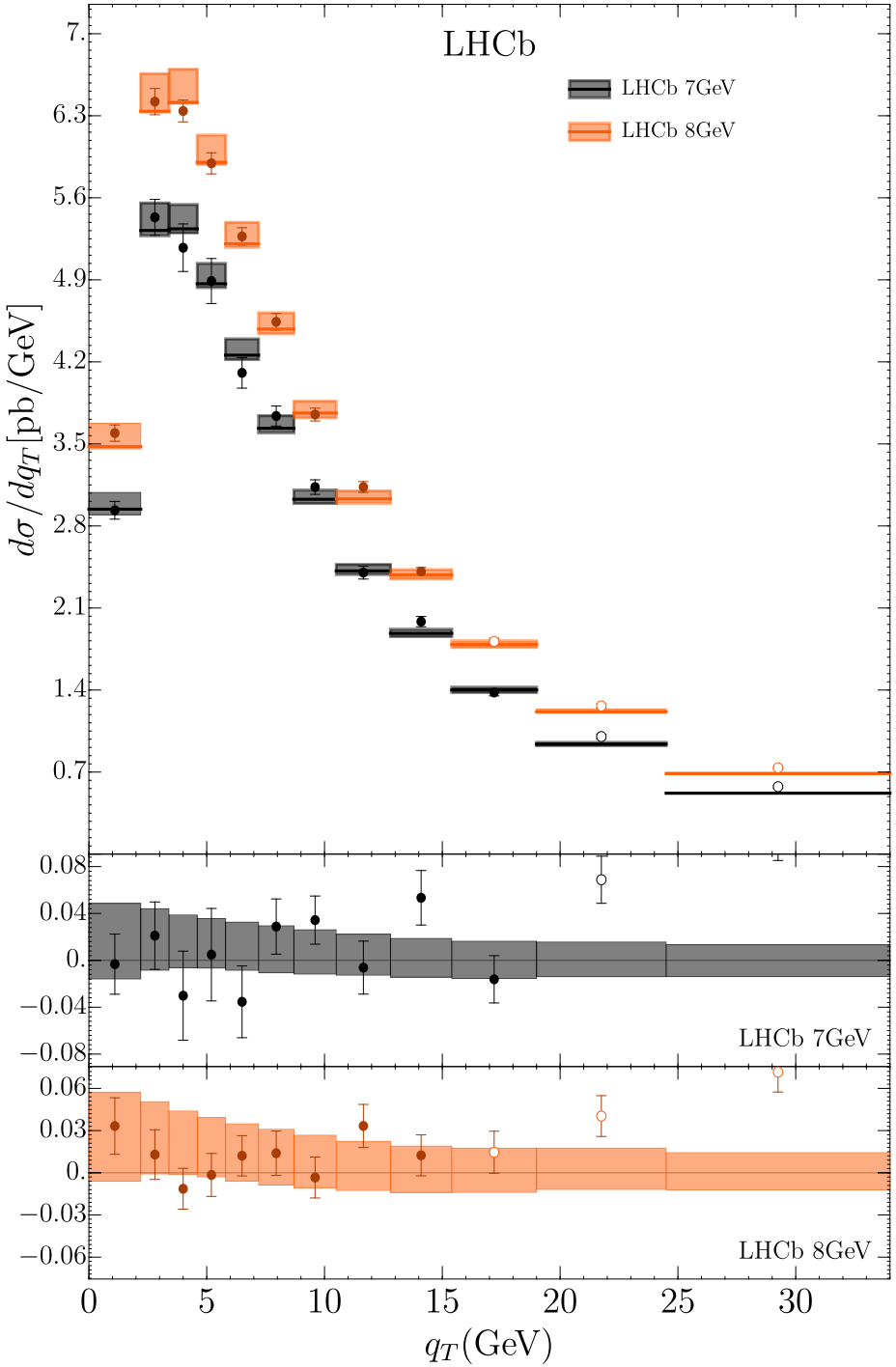


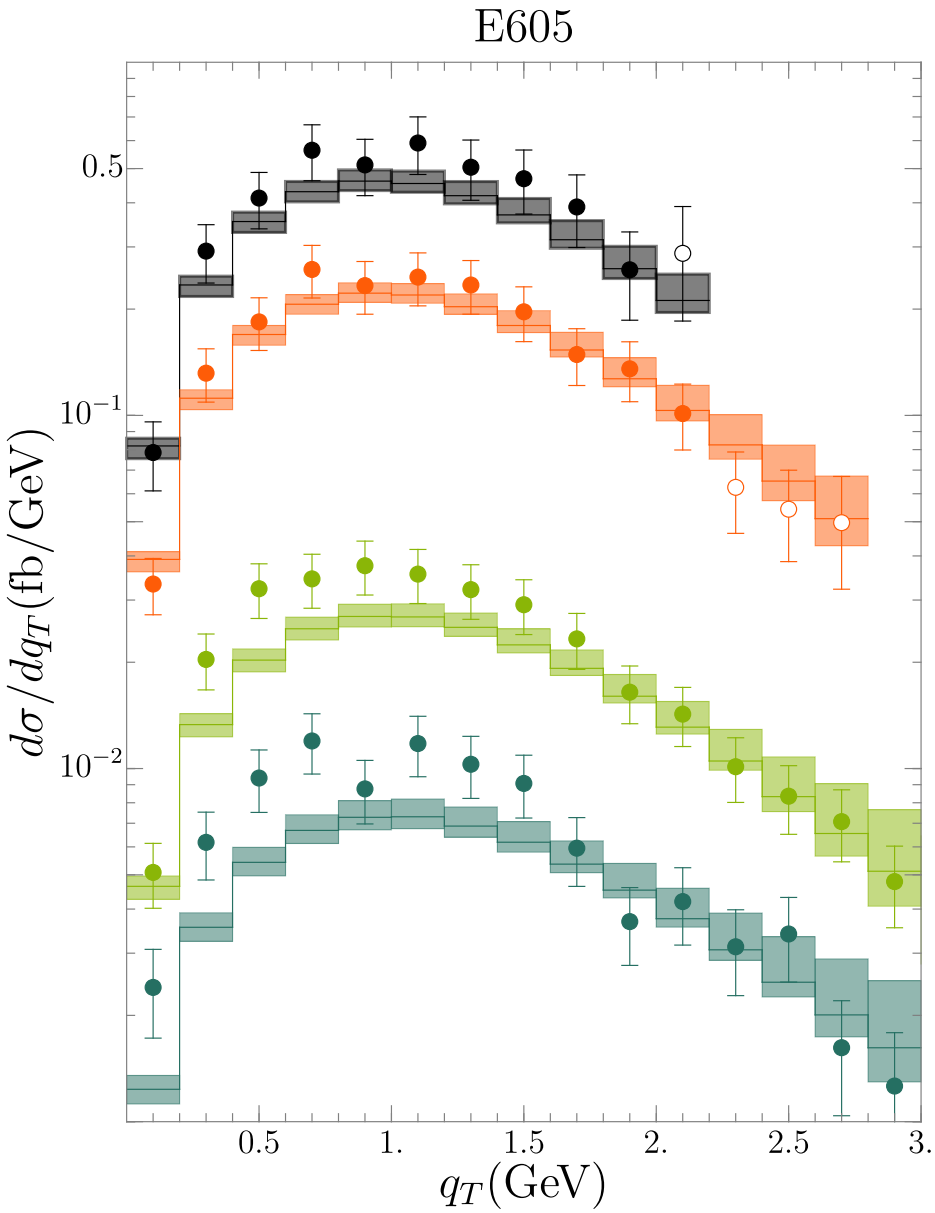


average value and 68% CI





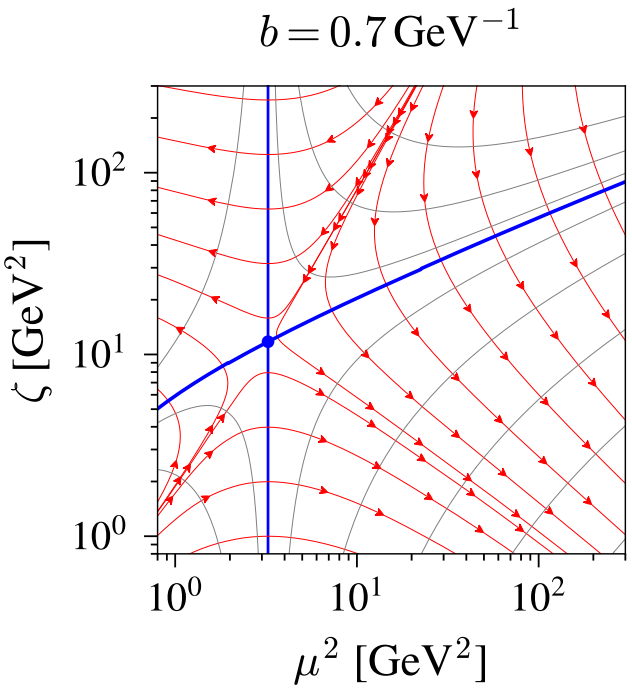

LHCb
LHCb 7GeV
LHCb 8GeV
$d\sigma/dq_T$[pb/GeV]
LHCb 7GeV
LHCb 8GeV
q_T(GeV)


E605
$d\sigma/dq_T$(fb/GeV)
q_T(GeV)
$7 < Q < 8$GeV
$8 < Q < 9$GeV
$11.5 < Q < 13.5$GeV
$13.5 < Q < 18$GeV

Scale dependence:evolution equations

$$\mu^2 \frac{d}{d\mu^2} f(x, b; \mu, \zeta) = \frac{\gamma_F(\mu, \zeta)}{2} f(x, b; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} f(x, b; \mu, \zeta) = -\mathcal{D}(b, \mu) f(x, b; \mu, \zeta)$$



To evolve $(\mu, \zeta) \to (Q, Q^2)$

- define TMDPDF at saddle point
- evolve in μ
- evolve in ζ