100 YEARS SPIN 2025

26th International Symposium on Spin Physics

A Century of Spin

Semi-inclusive deep inelastic scattering off a tensor-polarized spin-1 target

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arXiv:2508.06134 [hep-ph]

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Research Center for Particle Science and Technology

Introduction

- **Spin-1/2:** $\boldsymbol{\rho}: \mathbf{2} \times \mathbf{2}$ $\qquad \boldsymbol{\rho} = \frac{1}{2}(\mathbf{1} + S^i \boldsymbol{\sigma}^i)$

S^i : spin vector $\quad \boldsymbol{S} = (S_T^x, S_T^y, S_L)$ —3 independent components

- **Spin-1:** $\boldsymbol{\rho}: \mathbf{3} \times \mathbf{3}$ $\qquad \boldsymbol{\rho} = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2} S^i \boldsymbol{\Sigma}^i + 3 T^{ij} \boldsymbol{\Sigma}^{ij}\right)$

T^{ij} : symmetric traceless spin tensor

$$T^{ij} = \frac{1}{2}\begin{pmatrix} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & -2S_{LL} \end{pmatrix}$$

$$\left.\begin{array}{lr} S_T^x, S_T^y, S_L & \mathbf{3} \\ S_{LL}, S_{LT}^x, S_{LT}^y, S_{TT}^{xx}, S_{TT}^{xy} & \mathbf{5} \end{array}\right\} \text{8 independent components}$$

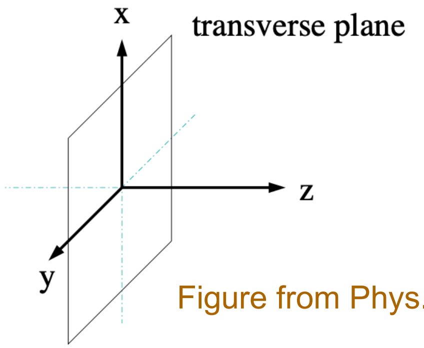
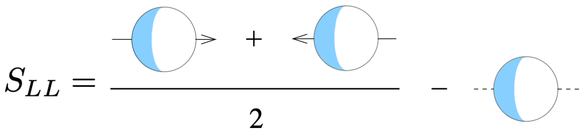
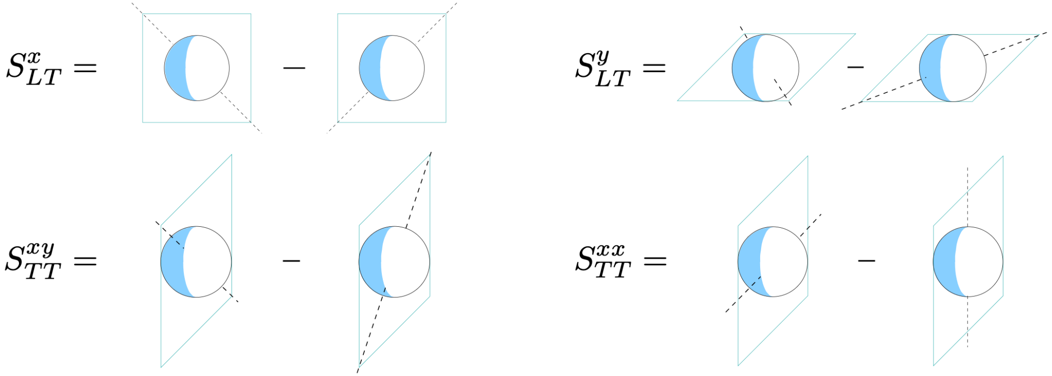


Figure from Phys. Rev. D 62, 114004 (2000).



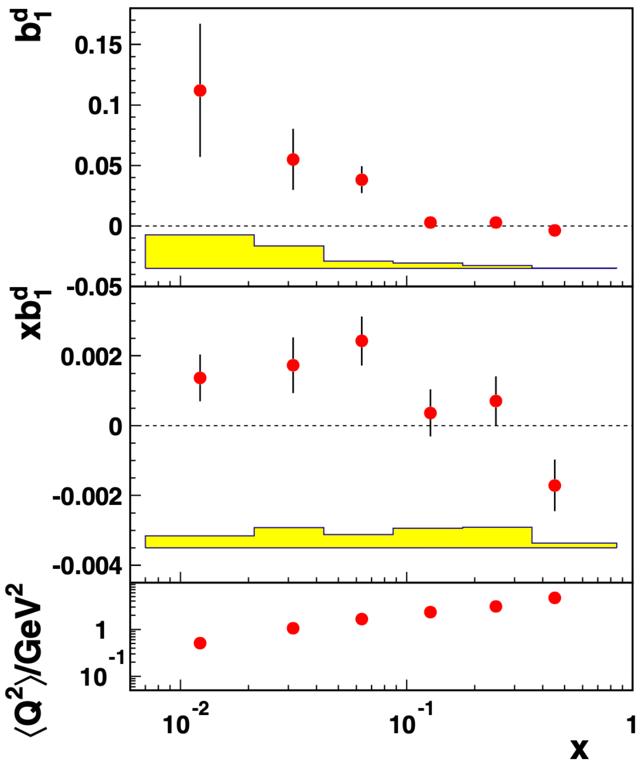


Spin-1 hadrons offer unique access to explore novel effects beyond spin-1/2 targets.

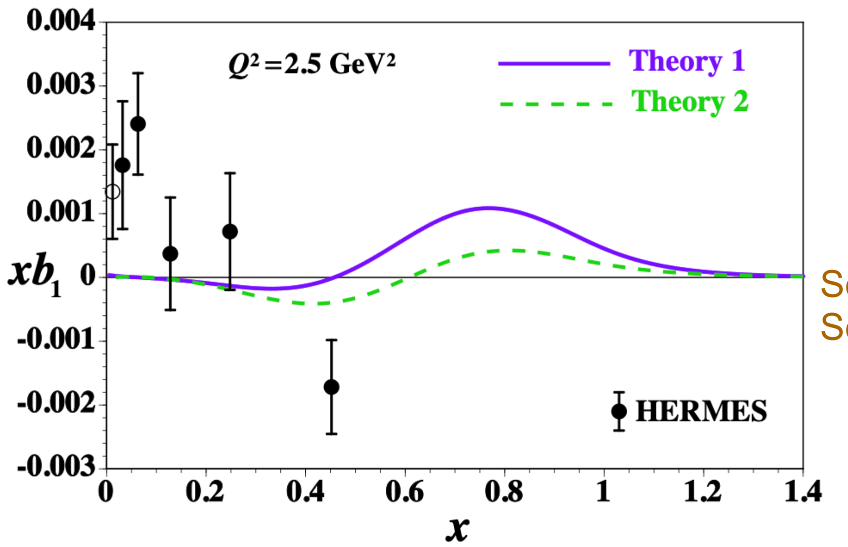
Introduction

■ Experiments with deuteron

- Deuteron is the simplest stable spin-1 nucleus (a weakly bound system of a proton and a neutron).
- HERMES experiment: First measurement of tensor polarized structure function b_1



HERMES collaboration, Phys. Rev. Lett. 95 242001 (2005).



See S. Kumano's slides, Sep. 23, 2:00 pm

W. Cosyn, Y-B. Dong, S. Kumano, and, M. Sargsian, Phys. Rev. D 95, 074036 (2017)

Understanding the tensor structure of spin-1 deuteron is required for learning partonic structure of light nuclei.

Introduction

- **Upcoming experiments: JLab, Fermilab**

The Deuteron Tensor Structure Function b_1

A Proposal to Jefferson Lab PAC-40

E12-13-011

Spin 1 Transverse Momentum Dependent Tensor Structure Functions in CLAS12

Jiwan Poudel, Alessandro Bacchetta, Jian-Ping Chen, Dustin Keller, Ishara Fernando et al. (Feb 27, 2025)

e-Print: 2502.20044 [hep-ph]

Experimental study of tensor structure function of deuteron

Jiwan Poudel (Jefferson Lab), Alessandro Bacchetta (INFN, Pavia), Jian-Ping Chen (Jefferson Lab), Nathaly Santiesteban (New Hampshire U.) (Apr 18, 2025)

Published in: *Eur.Phys.J.A* 61 (2025) 4, 81 • e-Print: 2506.04506 [nucl-ex]

Enhanced Tensor Polarization in Solid-State Targets

Dustin Keller, Don Crabb, Donal Day (Aug 21, 2020)

Published in: *Nucl.Instrum.Meth.A* 981 (2020) 164504 • e-Print: 2008.09515 [physics.ins-det]

The Transverse Structure of the Deuteron with Drell-Yan

SpinQuest Collaboration • D. Keller (Virginia U.) for the collaboration. (May 2, 2022)

e-Print: 2205.01249 [nucl-ex]

Inclusive DIS and SIDIS

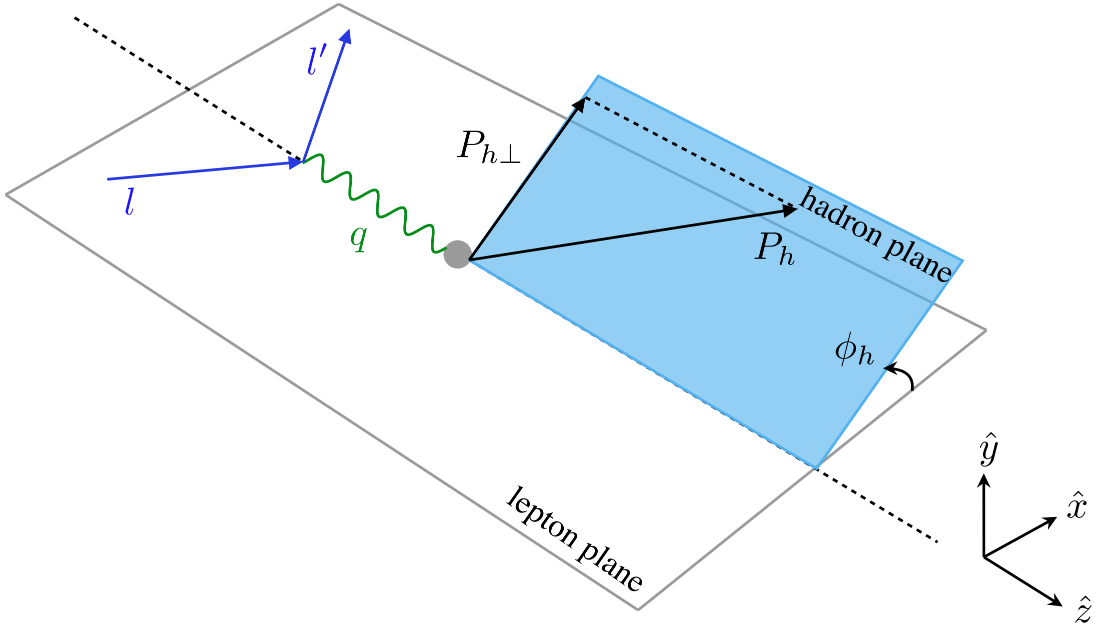
tensor polarized collinear PDFs and TMD PDFs

Proton-deuteron Drell-Yan

Corresponding theoretical study is required!

Kinematics and spin states

Trento conventions



Virtual photon-target frame

$$\ell(l) + \underline{d(P)} \to \ell(l') + h(P_h) + X(P_X)$$

Spin-1 target, e.g. deuteron

Several commonly used dimensionless variables:

$$\boxed{x_d = \frac{Q^2}{2P\cdot q},} \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_d}{Q}$$

x_d is scaling variable for the deuteron. $x_B = 2x_d$

$$\cos\phi_h = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \qquad \sin\phi_h = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

Kinematics and spin states

Basis vectors: $\hat{t}^\mu = \frac{2x_d}{\sqrt{1+\gamma^2}Q}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right), \quad \hat{x}^\mu = \frac{l_\perp^\mu}{|\boldsymbol{l}_\perp|}, \quad \hat{y}^\mu = \epsilon_\perp^{\mu\nu}\hat{x}_\nu \quad \hat{z}^\mu = -\frac{q^\mu}{Q}$

Spin vector:

$$S^\mu = S_L \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_T^\mu$$

$$S_L = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}} \qquad S_T^\mu = g_\perp^{\mu\nu}S_\nu$$

Spin tensor:

$$T^{\mu\nu} = \frac{1}{2}\left\{\frac{2}{3}S_{LL}\left(\frac{2(\gamma^2 q^\mu - 2x_d P^\mu)(\gamma^2 q^\nu - 2x_d P^\nu)}{\gamma^2(\gamma^2+1)Q^2} + g_\perp^{\mu\nu}\right) + \frac{2x_d P^{\{\mu}S_{LT}^{\nu\}} - \gamma^2 q^{\{\mu}S_{LT}^{\nu\}}}{\gamma\sqrt{\gamma^2+1}Q} + S_{TT}^{\mu\nu}\right\}$$

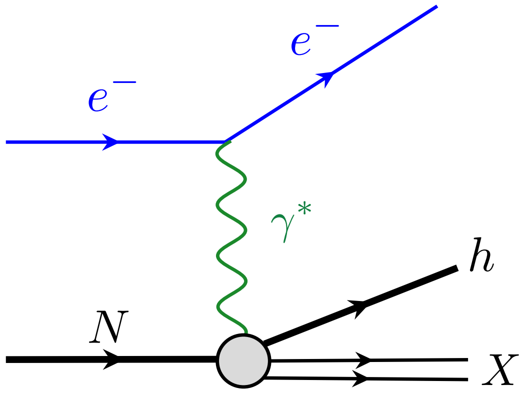
$$\begin{aligned} S_{LL} =& \frac{3}{2}T^{\mu\nu}\hat{L}_\mu\hat{L}_\nu, \\ S_{LT}^\mu =& -2T_{\alpha\beta}\hat{L}^\alpha g_\perp^{\mu\beta}, \\ S_{TT}^{\mu\nu} =& 2T_{\alpha\beta}g_\perp^{\alpha\mu}g_\perp^{\beta\nu} - T_{\alpha\beta}\hat{L}^\alpha\hat{L}^\beta g_\perp^{\mu\nu} \end{aligned}$$

$$\cos\phi_{LT} = -\frac{2T^{\mu\nu}\hat{L}_\mu\hat{x}_\nu}{|S_{LT}|}, \quad \sin\phi_{LT} = -\frac{2T^{\mu\nu}\hat{L}_\mu\hat{y}_\nu}{|S_{LT}|},$$

$$\cos 2\phi_{TT} = \frac{2T^{\mu\nu}\hat{x}_\mu\hat{x}_\nu + \frac{2}{3}T^{\mu\nu}\hat{L}_\mu\hat{L}_\nu}{|S_{TT}|}, \quad \sin 2\phi_{TT} = \frac{2T^{\mu\nu}\hat{x}_\mu\hat{y}_\nu}{|S_{TT}|}$$

Cross section in terms of structure functions

Decomposition of the hadronic tensor



$$\frac{d\sigma}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2} = \frac{\alpha^2 y}{8Q^4 z} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l \cdot l' + i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma\right)$$

Polarized lepton beam

$$W^{\mu\nu}(q;P,S,T;P_h) = \sum_X \delta^4(P+q-P_h-P_X) \langle P,S,T|J^\mu(0)|P_X,P_h\rangle \langle P_X,P_h|J^\nu(0)|P,S,T\rangle$$

Hadronic tensor must satisfy:

Hermiticity: $W^{*\mu\nu}(q;P,S,T;P_h) = W^{\nu\mu}(q;P,S,T;P_h)$

Parity invariance : $W^{\mu\nu}(q;P,S,T;P_h) = W_{\mu\nu}\left(q,P,-\bar{S},\bar{T};P_h\right)$

Gauge invariance: $q_\mu W^{\mu\nu}(q;P,S,T;P_h) = W^{\mu\nu}(q;P,S,T;P_h)\, q_\nu = 0$

Cross section in terms of structure functions

Hadronic tensor $W^{\mu\nu}$ → **Basis tensors multiplied by scaler functions.**

9 basic Lorentz tensors:

$$h_U^{S\mu\nu} = \left\{ g_q^{\mu\nu}, P_q^{\mu} P_q^{\nu}, P_q^{\{\mu} P_{hq}^{\nu\}}, P_{hq}^{\mu} P_{hq}^{\nu} \right\}$$

$$\tilde{h}_U^{S\mu\nu} = \left\{ \epsilon^{\{\mu q P P_h} P_q^{\nu\}}, \epsilon^{\{\mu q P P_h} P_{hq}^{\nu\}} \right\}$$

$$h_U^{A\mu\nu} = \left\{ P_q^{[\mu} P_{hq}^{\nu]} \right\}$$

$$\tilde{h}_U^{A\mu\nu} = \left\{ \epsilon^{\mu\nu q P}, \epsilon^{\mu\nu q P_h} \right\}$$

$$P_q^{\mu} = P^{\mu} - \frac{P \cdot q}{q^2} q^{\mu}$$

$$P_{hq}^{\mu} = P_h^{\mu} - \frac{P_h \cdot q}{q^2} q^{\mu}$$

$$g_q^{\mu\nu} = g^{\mu\nu} - \frac{q^{\mu} q^{\nu}}{q^2}$$

Polarized basis tensors = Polarization dependent (pseudo) scalars × *Basic Lorentz tensors*

Symmetric part: $h_T^{S\mu\nu} = \{T^{P_h P_h}, T^{P_h q}, T^{qq}\} h_U^{S\mu\nu}, \left\{\epsilon^{T^{P_h} P P_h q}, \epsilon^{T^q P P_h q}\right\} \tilde{h}_U^{S\mu\nu}$ **16**

Antisymmetric part: $h_T^{A\mu\nu} = \{T^{P_h P_h}, T^{P_h q}, T^{qq}\} h_U^{A\mu\nu}, \left\{\epsilon^{T^{P_h} P P_h q}, \epsilon^{T^q P P_h q}\right\} \tilde{h}_U^{A\mu\nu}$ **7**

23 basis tensors

$$W_T^{\mu\nu} = \sum_{i=1}^{16} V_{T,i}^{S} h_{T,i}^{S\mu\nu} + i \sum_{i=1}^{7} V_{T,i}^{A} h_{T,i}^{A\mu\nu}$$

subscript U, T: target polarization
superscript S, A: symmetric, antisymmetric
V_i: structure functions

K. b. Chen, W. h. Yang, S. y. Wei, and Z. t. Liang, Phys. Rev. D 94, 034003 (2016).

J. Zhao, Z. Zhang, Z-t. Liang, T. Liu, and Y-j. Zhou, Phys. Rev. D 109, 074017 (2024).

Cross section in terms of structure functions

- **General cross section**

$$d\sigma = \begin{cases} d\sigma_U & \text{unpolarized} \\ + & \\ d\sigma_V & \text{vector polarized} \\ + & \\ d\sigma_T & \textbf{tensor polarized} \end{cases}$$

41

18

23

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x_d y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_d}\right)\Big\{ S_{LL}\Big[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{U(LL)}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\, F_{U(LL)}^{\cos 2\phi_h} + \lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{L(LL)}^{\sin\phi_h}\Big]$$

S_{LL}

$$+|S_{LT}|\Big[\cos(\phi_h-\phi_{LT})\Big(F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} + \varepsilon F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})}\Big) + \sqrt{2\varepsilon(1+\varepsilon)}\Big(\cos\phi_{LT}F_{U(LT)}^{\cos\phi_{LT}} + \cos(2\phi_h-\phi_{LT})F_{U(LT)}^{\cos(2\phi_h-\phi_{LT})}\Big) + \varepsilon\Big(\cos(\phi_h+\phi_{LT})F_{U(LT)}^{\cos(\phi_h+\phi_{LT})} + \cos(3\phi_h-\phi_{LT})F_{U(LT)}^{\cos(3\phi_h-\phi_{LT})}\Big) + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\Big(\sin\phi_{LT}F_{L(LT)}^{\sin\phi_{LT}} + \sin(2\phi_h-\phi_{LT})F_{L(LT)}^{\sin(2\phi_h-\phi_{LT})}\Big) + \lambda_e\sqrt{1-\varepsilon^2}\sin(\phi_h-\phi_{LT})F_{L(LT)}^{\sin(\phi_h-\phi_{LT})}\Big]$$

S_{LT}

$$+|S_{TT}|\Big[\cos(2\phi_h-2\phi_{TT})\Big(F_{U(TT),T}^{\cos(2\phi_h-2\phi_{TT})} + \varepsilon F_{U(TT),L}^{\cos(2\phi_h-2\phi_{TT})}\Big) + \sqrt{2\varepsilon(1+\varepsilon)}\Big(\cos(\phi_h-2\phi_{TT})F_{U(TT)}^{\cos(\phi_h-2\phi_{TT})} + \cos(3\phi_h-2\phi_{TT})F_{U(TT)}^{\cos(3\phi_h-2\phi_{TT})}\Big) + \varepsilon\Big(\cos(2\phi_{TT})F_{U(TT)}^{\cos(2\phi_{TT})} + \cos(4\phi_h-2\phi_{TT})F_{U(TT)}^{\cos(4\phi_h-2\phi_{TT})}\Big) + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\Big(\sin(\phi_h-2\phi_{TT})F_{L(TT)}^{\sin(\phi_h-2\phi_{TT})} + \sin(3\phi_h-2\phi_{TT})F_{L(TT)}^{\sin(3\phi_h-2\phi_{TT})}\Big) + \lambda_e\sqrt{1-\varepsilon^2}\sin(2\phi_h-2\phi_{TT})F_{L(TT)}^{\sin(2\phi_h-2\phi_{TT})}\Big]\Big\}$$

S_{TT}

Cross section in terms of structure functions

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy dz d\phi_h d\psi dP_{h\perp}^2} \xrightarrow{\text{Integrating over } P_{h\perp}} \frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz}$$ 5

$P_{h\perp}$ integrated SIDIS:

$$\begin{aligned}\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz} = & \frac{2\alpha^2}{x_d y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_d}\right)\Big\{S_{LL}\left(F_{U(LL),T}+\varepsilon F_{U(LL),L}\right) \\ & + |S_{LT}|\left(\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}}\right) \\ & + |S_{TT}|\varepsilon\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})}\Big\}\end{aligned}$$

$$F_{U(LL),T}(x_d, z, Q^2) = \int d^2\boldsymbol{P}_{h\perp} F_{U(LL),T}(x_d, z, P_{h\perp}^2, Q^2)$$

$$\sum_h \int dz z \frac{d\sigma(\ell N \to \ell h X)}{dz dx_d dy d\psi} \longrightarrow \frac{\nu + M}{\nu}\frac{d\sigma(\ell N \to \ell X)}{dx_d dy d\psi}$$ 4

Inclusive DIS:

$$\begin{aligned}\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi} = & \frac{2\alpha^2}{x_d y Q^2}\frac{y^2}{2(1-\varepsilon)}\Big\{S_{LL}\left(F_{U(LL),T}+\varepsilon F_{U(LL),L}\right) \\ & + |S_{LT}|\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + |S_{TT}|\varepsilon\cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})}\Big\}\end{aligned}$$

$$F_{U(LL),T}(x_d, Q^2) = \sum_h \int dz\, z\, F_{U(LL),T}(x_d, z, Q^2)$$

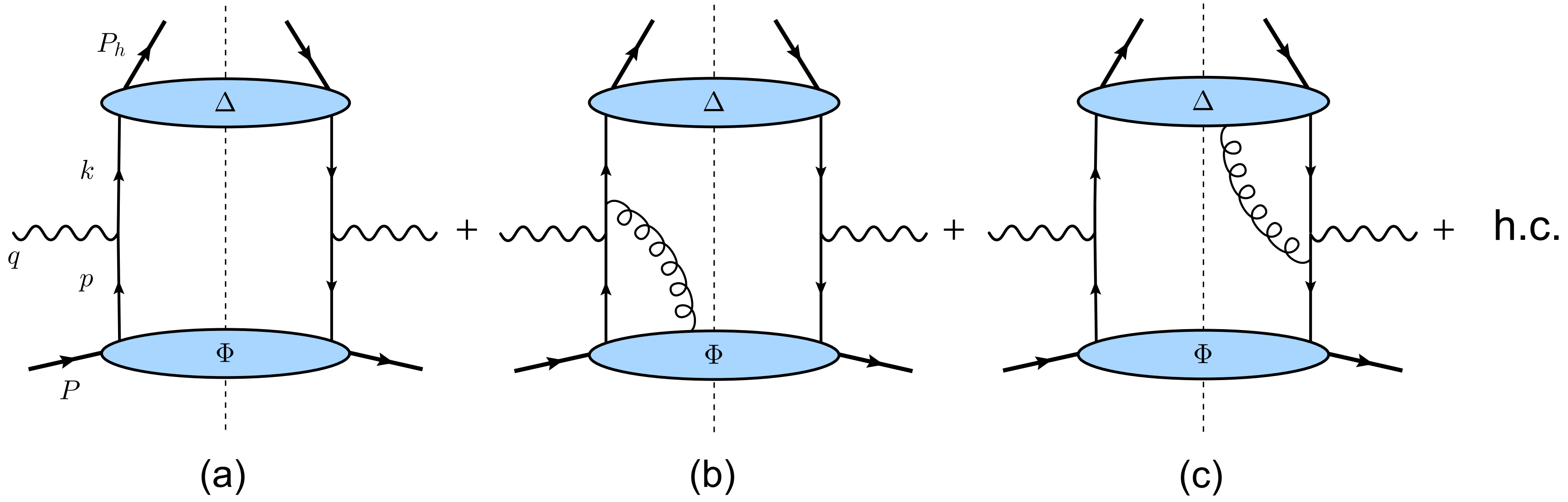
Structure functions in terms of partonic functions

Hadronic tensor up to subleading order in $1/Q$

$$W^{\mu\nu} = 2z\sum_a e_a^2 \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T)\mathrm{Tr}\Big\{\Phi^a(x,p_T)\gamma^\mu\Delta^a(z,k_T)\gamma^\nu - \frac{1}{Q\sqrt{2}}\Big[\gamma^\alpha \not{n}_+\gamma^\nu\tilde{\Phi}^a_{A\alpha}(x,p_T)\gamma^\mu\Delta^a(z,k_T) + \gamma^\alpha\not{n}_-\gamma^\mu\tilde{\Delta}^a_{A\alpha}(z,k_T)\gamma^\nu\Phi^a(x,p_T) + \mathrm{h.c.}\Big]\Big\}$$

Φ, Δ : quark-quark correlators

$\tilde{\Phi}_{A\alpha}, \tilde{\Delta}_{A\alpha}$: quark-gluon-quark correlators



Structure functions in terms of partonic functions

Quark-quark correlators

quark-quark distribution correlator: $\Phi_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\mathcal{U}^c_{(0,\xi)}\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$

$$
\begin{aligned}
\Phi(x,p_T;T) = \frac{1}{2}\Big\{ & f_{1LL}S_{LL}\not{n}_+ - f_{1LT}\frac{S_{LT}\cdot p_T}{M}\not{n}_+ + f_{1TT}\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\not{n}_+ \\
& + \left(g_{1LT}\frac{\epsilon_T^{\mu\nu}S_{LT\mu}p_{T\nu}}{M}\gamma_5\not{n}_+\right) - \left(g_{1TT}\frac{\epsilon_T^{\mu\nu}S_{TT\nu\rho}p_T^\rho p_{T\mu}}{M^2}\gamma_5\not{n}_+\right) \\
& + \left(ih_{1LL}^\perp S_{LL}\frac{[\not{p}_T,\not{n}_+]}{2M}\right) + \left(ih'_{1LT}\frac{[\not{S}_{LT},\not{n}_+]}{2}\right) - \left(ih_{1LT}^\perp\frac{S_{LT}\cdot p_T}{M}\frac{[\not{p}_T,\not{n}_+]}{2M}\right) \\
& - \left(h'_{1TT}\frac{\sigma^{\mu\nu}S_{TT\mu\rho}p_T^\rho n_{+\nu}}{M}\right) + \left(ih_{1TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{[\not{p}_T,\not{n}_+]}{2M}\right)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
+ \frac{M}{2P^+}\Big\{ & e_{LL}S_{LL} - e_{LT}^\perp\frac{S_{LT}\cdot p_T}{M} + e_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2} \\
& + f_{LL}^\perp S_{LL}\frac{\not{p}_T}{M} + f'_{LT}\not{S}_{LT} - f_{LT}^\perp\frac{\not{p}_T S_{LT}\cdot p_T}{M^2} \\
& - f'_{TT}\frac{S_{TT}^{\mu\nu}\gamma_\mu p_{T\nu}}{M} + f_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{\not{p}_T}{M} \\
& - ie_{LT}\frac{S_{LT\mu}\epsilon_T^{\mu\nu}p_{T\nu}}{M}\gamma_5 + ie_{TT}\frac{S_{TT\mu\rho}p_T^\rho\epsilon_T^{\mu\nu}p_{T\nu}}{M^2}\gamma_5 \\
& - \left(g_{LL}^\perp S_{LL}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}}{M}\right) - \left(g'_{LT}\gamma_5\epsilon_T^{\mu\nu}\gamma_\mu S_{LT\nu}\right) + \left(g_{LT}^\perp\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}S_{LT}\cdot p_T}{M^2}\right) \\
& + \left(g'_{TT}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu S_{TT\nu\alpha}p_T^\alpha}{M}\right) - \left(g_{TT}^\perp\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\gamma_5\frac{\varepsilon_T^{\mu\nu}\gamma_\mu p_{T\nu}}{M}\right) \\
& - \left(ih_{LL}S_{LL}\frac{[\not{n}_-,\not{n}_+]}{2}\right) - \left(ih_{LT}\frac{S_{LT}\cdot p_T}{M}\frac{[\not{n}_-,\not{n}_+]}{2}\right) + \left(ih_{LT}^\perp\frac{[\not{S}_{LT},\not{p}_T]}{2M}\right) \\
& + \left(ih_{TT}\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{[\not{n}_-,\not{n}_+]}{2}\right) - \left(ih_{TT}^\perp\frac{[S_{TT}^{\mu\nu}\gamma_\mu p_{T\nu},\not{p}_T]}{2M^2}\right)\Big\}
\end{aligned}
$$

10 twist-2

A. Bacchetta and P. J. Mulders, Phys. Rev. D 62, 114004 (2000).

20 twist-3

S. Kumano and Q. T. Song, Phys. Rev. D 103, 014025 (2021).

Structure functions in terms of partonic functions

Spin-1 TMD PDFs

twist-2

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$
LL	f_{1LL}		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	$h_{1LT},\ h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	$h_{1TT},\ h_{1TT}^\perp$

A. Bacchetta and P. J. Mulders, Phys. Rev. D 62, 114004 (2000).

See S. Kumano's slides, Sep. 23, 2:00 pm

twist-3

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	$f^\perp,\ e$	$g^\perp$	h
L	$f_L^\perp,\ e_L$	$g_L^\perp$	h_L
T	$f_T,\ f_T^\perp,\ e_T,\ e_T^\perp$	$g_T,\ g_T^\perp$	$h_T,\ h_T^\perp$
LL	$f_{LL}^\perp,\ e_{LL}$	$g_{LL}^\perp$	h_{LL}
LT	$f_{LT},\ f_{LT}^\perp,\ e_{LT},\ e_{LT}^\perp$	$g_{LT},\ g_{LT}^\perp$	$h_{LT},\ h_{LT}^\perp$
TT	$f_{TT},\ f_{TT}^\perp,\ e_{TT},\ e_{TT}^\perp$	$g_{TT},\ g_{TT}^\perp$	$h_{TT},\ h_{TT}^\perp$

twist-4

Hadron Polarization	Quark Polarization: Unpolarized	Longitudinally Polarized	Transversely Polarized
U	f_3		$h_3^\perp$
L		g_{3L}	$h_{3L}^\perp$
T	$f_{3T}^\perp$	g_{3T}	$h_{3T},\ h_{3T}^\perp$
LL	f_{3LL}		$h_{3LL}^\perp$
LT	f_{3LT}	g_{3LT}	$h_{3LT},\ h_{3LT}^\perp$
TT	f_{3TT}	g_{3TT}	$h_{3TT},\ h_{3TT}^\perp$

S. Kumano and Q. T. Song, Phys. Rev. D 103, 014025 (2021).

Structure functions in terms of partonic functions

quark-quark fragmentation correlator:

$$\Delta_{ij}(z,k_T) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \langle 0|\mathcal{U}^{n_+}_{(+\infty,\xi)}\psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^+=0}$$

$$\text{TMD PDFs} \xrightarrow[f\to D,\quad e\to E,\quad h\to H,\quad g\to G]{n_+\to n_-,\quad \epsilon_T\to-\epsilon_T,\quad P^+\to P_h^-,\quad M\to M_h,\quad x\to 1/z} \text{TMD FFs}$$

The final-state hadron is unpolarized

$$\Delta(z,k_T) = \frac{1}{2}\left\{\boxed{D_1 \not{n}_- + iH_1^\perp \frac{[\not{k}_T,\not{n}_-]}{2M_h}}\right\} \qquad \text{twist-2} \qquad 2$$

$$+\boxed{\frac{M_h}{2P_h^-}\left\{E + D^\perp \frac{\not{k}_T}{2M_h} + iH\frac{[\not{n}_-,\not{n}_+]}{2} + G^\perp\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\right\}} \qquad \text{twist-3} \qquad 4$$

Structure functions in terms of partonic functions

Quark-gluon-quark correlators

quark-gluon-quark correlator:

$$\left(\Phi_D^\mu\right)_{ij}(x,p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\mathcal{U}^{n_-}_{(0,\xi)} iD^\mu(\xi)\psi_i(\xi)|P\rangle\Big|_{\xi^+=0} \qquad iD^\mu(\xi) = i\partial^\mu + gA^\mu$$

plus-component: $\Phi_D^+(x,p_T) = xP^+\Phi(x,p_T)$

transverse component: $\tilde{\Phi}_A^\alpha(x,p_T) = \Phi_D^\alpha(x,p_T) - p_T^\alpha\Phi(x,p_T)$

Under the constraint of parity conservation

$$\begin{aligned}
\tilde{\Phi}_A^\alpha(x,p_T;T) = \frac{xM}{2}\Bigg\{&\Bigg[(\tilde{f}_{LL}^\perp - i\tilde{g}_{LL}^\perp)S_{LL}\frac{p_{T\rho}}{M} + (\tilde{f}'_{LT} - i\tilde{g}'_{LT})S_{LT\rho} - (\tilde{f}'_{TT} - i\tilde{g}'_{TT})S_{TT\rho\sigma}\frac{p_T^\sigma}{M} \\
&- (\tilde{f}_{LT}^\perp - i\,\tilde{g}_{LT}^\perp)\frac{S_{LT}\cdot p_T}{M}\frac{p_{T\rho}}{M} + (\tilde{f}_{TT}^\perp - i\,\tilde{g}_{TT}^\perp)\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\frac{p_{T\rho}}{M}\Bigg](g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5) \\
&- \Bigg[(\tilde{h}_{LT}^\perp + i\,\tilde{e}_{LT}^\perp)\frac{\epsilon_T^{\rho\sigma}S_{LT\rho}p_{T\sigma}}{M} + (\tilde{h}_{TT}^\perp + i\,\tilde{e}_{TT}^\perp)\frac{\epsilon_T^{\beta\rho}S_{TT\rho\sigma}p_{T\beta}p_T^\sigma}{M^2}\Bigg]\gamma_T^\alpha\gamma_5 \\
&+ \Bigg[(\tilde{h}_{LL} + i\tilde{e}_{LL})S_{LL} - (\tilde{h}_{LT} - i\tilde{e}_{LT})\frac{S_{LT}\cdot p_T}{M} - (\tilde{h}_{TT} - i\tilde{e}_{TT})\frac{p_T\cdot S_{TT}\cdot p_T}{M^2}\Bigg]i\gamma_T^\alpha \\
&+ \ldots(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5)\Bigg\}\frac{\not{n}_+}{2}
\end{aligned}$$

twist-3

Structure functions in terms of partonic functions

QCD equation of motion: $[i\not{D}(\xi)-m]\psi(\xi)=\left[\gamma^+ iD^-(\xi)+\gamma^- iD^+(\xi)+\gamma_T^\alpha iD_\alpha(\xi)-m\right]\psi(\xi)=0$

$$\mathrm{Tr}\Gamma^+\left[x\gamma^-\Phi_3+\gamma_{T\rho}\tilde{\Phi}^\rho_{A,3}+\frac{\not{p}_T}{M}\Phi_2-\frac{m}{M}\Phi_2\right]=0 \qquad \Gamma^+=\{\gamma^+,\gamma^+\gamma_5,i\sigma^{\alpha+}\gamma_5\}$$

The relations between the twist-2 and twist-3 functions:

T-even

$$\begin{aligned}
xe_{LL}&=x\tilde{e}_{LL}+\frac{m}{M}f_{1LL},\\
xf^\perp_{LL}&=x\tilde{f}^\perp_{LL}+f_{1LL},\\
xe^\perp_{LT}&=x\tilde{e}_{LT}+\frac{m}{M}f_{1LT},\\
xe^\perp_{TT}&=x\tilde{e}_{TT}+\frac{m}{M}f_{1TT},\\
xe_{LT}&=x\tilde{e}^\perp_{LT},\\
xe_{TT}&=x\tilde{e}^\perp_{TT},\\
xf'_{LT}&=x\tilde{f}'_{LT},\\
xf_{LT}&=x\tilde{f}_{LT}-\frac{p_T^2}{2M^2}f_{1LT},\\
xf^\perp_{LT}&=x\tilde{f}^\perp_{LT}+f_{1LT},\\
xf'_{TT}&=x\tilde{f}'_{TT},\\
xf_{TT}&=x\tilde{f}_{TT}-\frac{p_T^2}{2M^2}f_{1TT},\\
xf^\perp_{TT}&=x\tilde{f}^\perp_{TT}+f_{1TT},
\end{aligned}$$

T-odd

$$\begin{aligned}
xg^\perp_{LL}&=x\tilde{g}^\perp_{LL}+\frac{m}{M}h^\perp_{1LL},\\
xg'_{LT}&=x\tilde{g}'_{LT}+\frac{p_T^2}{M^2}g_{1LT}+\frac{m}{M}h_{1LT}+\frac{m}{M}\frac{p_T^2}{2M^2}h^\perp_{1LT},\\
xg^\perp_{LT}&=x\tilde{g}^\perp_{LT}+g_{1LT}+\frac{m}{M}h^\perp_{1LT},\\
xg_{LT}&=x\tilde{g}_{LT}-\frac{p_T^2}{2M^2}g_{1LT}+\frac{m}{M}h_{1LT},\\
xg'_{TT}&=x\tilde{g}'_{TT}-\frac{p_T^2}{M^2}g_{1TT}+\frac{m}{M}h_{1TT}+\frac{m}{M}\frac{p_T^2}{2M^2}h^\perp_{1TT},\\
xg^\perp_{TT}&=x\tilde{g}^\perp_{TT}-g_{1TT}+\frac{m}{M}h^\perp_{1TT},\\
xg_{TT}&=x\tilde{g}_{TT}-\frac{p_T^2}{2M^2}g_{1TT}+\frac{m}{M}h_{1TT},\\
xh_{LL}&=x\tilde{h}_{LL}+\frac{p_T^2}{M^2}h^\perp_{1LL}.\\
xh_{LT}&=x\tilde{h}_{LT}+h_{1LT}-\frac{p_T^2}{2M^2}h^\perp_{1LT},\\
xh^\perp_{LT}&=x\tilde{h}^\perp_{LT}+h_{1LT}+\frac{p_T^2}{2M^2}h^\perp_{1LT}+\frac{m}{M}g_{1LT},\\
xh_{TT}&=x\tilde{h}_{TT}+h_{1TT}-\frac{p_T^2}{2M^2}h^\perp_{1TT},\\
xh^\perp_{TT}&=x\tilde{h}^\perp_{TT}+h_{1TT}+\frac{p_T^2}{2M^2}h^\perp_{1TT}-\frac{m}{M}g_{1TT},
\end{aligned}$$

Each of the TMD PDFs defined by quark-quark correlator have the corresponding relations.

Structure functions in terms of partonic functions

Results for structure functions

$$\mathcal{C}[wfD] = x\sum_a e_a^2 \int d^2\boldsymbol{k}_T d^2\boldsymbol{p}_T \delta^{(2)}\left(\boldsymbol{k}_T - \boldsymbol{p}_T - \boldsymbol{P}_{h\perp}/z\right) w\left(\boldsymbol{k}_T, \boldsymbol{p}_T\right) f^a\left(x, k_T^2\right) D^a\left(z, p_T^2\right)$$

Up to twist-3, we have 21 nonzero structure functions:

$$F_{U(LL),L} = 0,$$

$$F_{U(LL)}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{H}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{D}^{\perp}}{z}\right)\right],$$

$$F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1LL}^{\perp}H_1^{\perp}\right],$$

$$F_{L(LL)}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{E}}{z}\right)\right],$$

$$F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1LT}D_1\right],$$

$$F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})} = 0,$$

$$F_{U(LT)}^{\cos\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\left\{-\left[\left(xf_{LT}D_1 + \frac{M_h}{M}h_{1LT}\frac{\tilde{H}}{z}\right)\right] + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(xh_{LT}H_1^{\perp} + \frac{M_h}{M}g_{1LT}\frac{\tilde{G}^{\perp}}{z}\right) + \left(xh_{LT}^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1LT}\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$\vdots$

$\hat{\boldsymbol{h}} = \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$: the direction of the transverse momentum

- In the parton model, S_{LL}-dependent structure functions "=" unpolarized structure functions
- **twist-2:** even number of ϕ_h, ϕ_{LT}, ϕ_{TT} **twist-3:** odd number of ϕ_h, ϕ_{LT}, ϕ_{TT}
- nonvanishing structure functions ⟹ study the tensor-polarized TMD PDFs

Structure functions in terms of partonic functions

$P_{h\perp}$ integrated SIDIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi dz} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_d}\right) \Big\{ S_{LL} \left(F_{U(LL),T} + \varepsilon F_{U(LL),L} \right) + |S_{LT}| \left(\sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_{LT} F_{L(LT)}^{\sin\phi_{LT}} \right) + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$\boxed{F_{U(LL),T}(x_d, z) = x \sum_a e_a^2 f_{1LL}^a(x) D_1^a(z),}$$

$$F_{U(LL),L}(x_d, z) = 0,$$

$$\boxed{F_{U(LT)}^{\cos\phi_{LT}}(x_d, z) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x) D_1^a(z),}$$

$$F_{U(TT)}^{\cos(2\phi_{TT})}(x_d, z) = 0.$$

Inclusive DIS:

$$\frac{d\sigma_{\text{Tens}}}{dx_d dy d\psi} = \frac{2\alpha^2}{x_d y Q^2} \frac{y^2}{2(1-\varepsilon)} \Big\{ S_{LL} \left(F_{U(LL),T} + \varepsilon F_{U(LL),L} \right) + |S_{LT}| \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_{LT} F_{U(LT)}^{\cos\phi_{LT}} + |S_{TT}| \varepsilon \cos(2\phi_{TT}) F_{U(TT)}^{\cos(2\phi_{TT})} \Big\}$$

$$\boxed{F_{U(LL),T}(x_d) = x \sum_a e_a^2 f_{1LL}^a(x),}$$ twist-2

$$F_{U(LL),L}(x_d) = 0,$$

$$\boxed{F_{U(LT)}^{\cos\phi_{LT}}(x_d) = -x \sum_a e_a^2 \frac{2M}{Q} f_{LT}^a(x),}$$ twist-3

$$F_{U(TT)}^{\cos(2\phi_{TT})}(x_d) = 0.$$

Extract tensor-polarized collinear PDFs

Summary

- We express the tensor-polarized part of the SIDIS cross section in terms of 23 structure functions.
- In the parton model, we compute the structure functions up to the subleading power in $1/Q$ (twist-3).

  The 21 structure functions have nontrivial expressions in SIDIS.

  Only two structure functions contribute to the cross section in inclusive DIS.
- Upcoming experiments, such as those at JLab using tensor-polarized deuteron targets, will facilitate measurements, enhancing our understanding of nucleon spin dynamics and potential novel nuclear phenomena.

Thank you!

Back up

Cross section in terms of structure functions

The relations between these two sets of structure functions:

$$
\begin{aligned}
F_{U(LL),T} &= -\left[2(1+\gamma^2)b_1 - \frac{\gamma^2}{x_d}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right)\right],\\
F_{U(LL),L} &= \frac{1}{x_d}\left[2(1+\gamma^2)x_d b_1 - (1+\gamma^2)^2\left(\frac{1}{3}b_2 + b_3 + b_4\right)\right.\\
&\qquad \left. - (1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) - \left(\frac{1}{3}b_2 - b_3\right)\right],\\
F_{U(LT)}^{\cos\phi_{LT}} &= -\frac{\gamma}{2x_d}\left[(1+\gamma^2)\left(\frac{1}{3}b_2 - b_4\right) + \left(\frac{2}{3}b_2 - 2b_3\right)\right],\\
F_{U(TT)}^{\cos(2\phi_{TT})} &= -\frac{\gamma^2}{2x_d}\left(\frac{1}{6}b_2 - \frac{1}{2}b_3\right).
\end{aligned}
$$

$$
\begin{aligned}
b_1 &= -\frac{1}{2(\gamma^2+1)}\left(F_{U(LL),T} + F_{U(TT)}^{\cos(2\phi_{TT})}\right),\\
b_2 &= -\frac{x_d}{(\gamma^2+1)^2}\left[F_{U(LL),L} + F_{U(LL),T} + 2\gamma F_{U(LT)}^{\cos\phi_{LT}} + (2\gamma^2+1)F_{U(TT)}^{\cos(2\phi_{TT})}\right],\\
\gamma^2 b_3 &= \frac{x_d}{3(\gamma^2+1)^2}\left[-\gamma^2 F_{U(LL),L} - \gamma^2 F_{U(LL),T} - 2\gamma^3 F_{U(LT)}^{\cos\phi_{LT}}\right.\\
&\qquad \left. + (4\gamma^4 + 11\gamma^2 + 6)F_{U(TT)}^{\cos(2\phi_{TT})}\right],\\
\gamma^2 b_4 &= -\frac{x_d}{3(\gamma^2+1)^2}\left[\gamma^2 F_{U(LL),L} + \gamma^2 F_{U(LL),T} - 2\gamma(2\gamma^2+3)F_{U(LT)}^{\cos\phi_{LT}}\right.\\
&\qquad \left. + (2\gamma^4 + 13\gamma^2 + 12)F_{U(TT)}^{\cos(2\phi_{TT})}\right].
\end{aligned}
$$

P. Hoodbhoy, R. L. Jaffe, and A. Manohar, Nucl. Phys. B 312, 571 (1989).

W. Cosyn, B. R. Tomei, A. Sosa, and A. Zec, Eur. Phys. J. A 61, 83 (2025).

Structure functions in terms of partonic functions

QCD equation of motion

$$[i\not{D}(\xi) - m]\psi(\xi) = \left[\gamma^+ iD^-(\xi) + \gamma^- iD^+(\xi) + \gamma_T^\alpha iD_\alpha(\xi) - m\right]\psi(\xi) = 0$$

$$\Phi = \Phi_2 + \frac{M}{P^+}\Phi_3 + \left(\frac{M}{P^+}\right)^2\Phi_4 \qquad \frac{1}{M}\tilde{\Phi}_A^\alpha = \tilde{\Phi}_{A,3}^\alpha + \frac{M}{P^+}\tilde{\Phi}_{A,4}^\alpha + \left(\frac{M}{P^+}\right)^2\tilde{\Phi}_{A,5}^\alpha$$

$$\mathcal{P}_+\left[xM\gamma^-\Phi_3 + M\gamma_{T\rho}\,\tilde{\Phi}_{A,3}^\rho + \not{p}_T\Phi_2 - m\Phi_2\right] + \mathcal{P}_+\left[xM\gamma^-\Phi_4 + M\gamma_{T\rho}\tilde{\Phi}_{A,4}^\rho + \not{p}_T\Phi_3 - m\Phi_3\right]\frac{M}{P^+} = 0,$$

where we use the relations $\mathcal{P}_+\Phi_4 = \mathcal{P}_-\Phi_2 = 0$ $\quad \mathcal{P}_+\tilde{\Phi}_{A,5}^\alpha = \mathcal{P}_-\tilde{\Phi}_{A,3}^\alpha = 0$

$$\mathrm{Tr}\Gamma^+\left[x\gamma^-\Phi_3 + \gamma_{T\rho}\tilde{\Phi}_{A,3}^\rho + \frac{\not{p}_T}{M}\Phi_2 - \frac{m}{M}\Phi_2\right] = 0 \qquad \Gamma^+ = \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$$

Structure functions in terms of partonic functions

$$(\Delta_D^\mu)_{ij}(z, k_T) =$$
$$\frac{1}{2z}\sum_X \int \frac{d\xi^+}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\xi)}\, iD^\mu(\xi)\ \psi_i(\xi)|h,X\rangle\, \langle h,X|\bar{\psi}_j\,(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=0}$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = \Delta_D^\alpha(z,k_T) - k_T^\alpha \Delta(z,k_T)$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = \frac{M_h}{2z}\left\{\left(\tilde{D}^\perp - i\tilde{G}^\perp\right)\frac{k_{T\rho}}{M_h}\left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\right) + \left(\tilde{H} + i\tilde{E}\right) i\gamma_T^\alpha + \ldots \left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\right)\right\}\frac{\not{n}_-}{2}.$$

$$\frac{E}{z} = \frac{\tilde{E}}{z} + \frac{m}{M_h} D_1,$$
$$\frac{D^\perp}{z} = \frac{\tilde{D}^\perp}{z} + D_1,$$
$$\frac{G^\perp}{z} = \frac{\tilde{G}^\perp}{z} + \frac{m}{M_h} H_1^\perp,$$
$$\frac{H}{z} = \frac{\tilde{H}}{z} + \frac{k_T^2}{M_h^2} H_1^\perp.$$

Structure functions in terms of partonic functions

21 nonzero structure functions:

$$
\begin{aligned}
&F_{U(LL),L} = 0,\\
&F_{U(LL)}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{H}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{D}^{\perp}}{z}\Big)\Big],\\
&F_{U(LL)}^{\cos 2\phi_h} = \mathcal{C}\Big[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1LL}^{\perp}H_1^{\perp}\Big],\\
&F_{L(LL)}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe_{LL}H_1^{\perp} + \frac{M_h}{M}f_{1LL}\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg_{LL}^{\perp}D_1 + \frac{M_h}{M}h_{1LL}^{\perp}\frac{\tilde{E}}{z}\Big)\Big],\\
&F_{U(LT),T}^{\cos(\phi_h-\phi_{LT})} = \mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1LT}D_1\Big],\\
&F_{U(LT),L}^{\cos(\phi_h-\phi_{LT})} = 0,\\
&F_{U(LT)}^{\cos\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\Big\{-\Big[\Big(xf_{LT}D_1 + \frac{M_h}{M}h_{1LT}\frac{\tilde{H}}{z}\Big)\Big] + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xh_{LT}H_1^{\perp} + \frac{M_h}{M}g_{1LT}\frac{\tilde{G}^{\perp}}{z}\Big) + \Big(xh_{LT}^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1LT}\frac{\tilde{D}^{\perp}}{z}\Big)\Big]\Big\},
\end{aligned}
$$

Structure functions in terms of partonic functions

$$F_{U(LT)}^{\cos(\phi_h+\phi_{LT})} = \mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_{1LT}H_1^\perp\Big],$$

$$F_{U(LT)}^{\cos(3\phi_h-\phi_{LT})} = \mathcal{C}\Big[-\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}h_{1LT}^\perp H_1^\perp\Big],$$

$$F_{L(LT)}^{\sin\phi_{LT}} = \frac{2M}{Q}\mathcal{C}\Big\{\Big(xg_{LT}D_1+\frac{M_h}{M}h_{1LT}\frac{\tilde{E}}{z}\Big)+\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_{LT}H_1^\perp-\frac{M_h}{M}g_{1LT}\frac{\tilde{D}^\perp}{z}\Big)-\Big(xe_{LT}^\perp H_1^\perp+\frac{M_h}{M}f_{1LT}\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},$$

$$F_{L(LT)}^{\sin(2\phi_h-\phi_{LT})} = \frac{2M}{Q}\mathcal{C}\Big\{-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\Big[\Big(xe_{LT}H_1^\perp+\frac{M_h}{M}f_{1LT}\frac{\tilde{G}^\perp}{z}\Big)+\Big(xe_{LT}^\perp H_1^\perp-\frac{M_h}{M}g_{1LT}\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},$$

$$F_{L(LT)}^{\sin(\phi_h-\phi_{LT})} = \mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}g_{1LT}D_1\Big],$$

$$F_{U(TT),T}^{\cos(2\phi_h-2\phi_{LT})} = \mathcal{C}\Big[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}f_{1TT}D_1\Big],$$

$$F_{U(TT),L}^{\cos(2\phi_h-2\phi_{LT})} = 0,$$

$$F_{U(TT)}^{\cos(\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\Big\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\big(xf_{TT}D_1-\frac{M_h}{M}h_{1TT}\frac{\tilde{H}}{z}\big)+\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}\Big[\Big(xh_{TT}H_1^\perp-\frac{M_h}{M}g_{1TT}\frac{\tilde{G}^\perp}{z}\Big)+\Big(xh_{TT}^\perp H_1^\perp-\frac{M_h}{M}f_{1TT}\frac{\tilde{D}^\perp}{z}\Big)\Big]\Big\},$$

$$\begin{aligned}F_{U(TT)}^{\cos(3\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\Big\{&\frac{3(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\big(2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2\big)}{2M^3}\Big(xf_{TT}^\perp D_1+\frac{M_h}{M}h_{1TT}^\perp\frac{\tilde{H}^\perp}{z}\Big)\\&+\frac{4(\boldsymbol{h}\cdot\boldsymbol{k}_T)(\boldsymbol{h}\cdot\boldsymbol{p}_T)^2-2(\boldsymbol{h}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)-(\boldsymbol{h}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2}{2M^2M_h}\times\Big[\Big(xh_{TT}^\perp H_1^\perp+\frac{M_h}{M}f_{1TT}\frac{\tilde{D}^\perp}{z}\Big)-\Big(xh_{TT}H_1^\perp+\frac{M_h}{M}g_{1TT}\frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},\end{aligned}$$

Structure functions in terms of partonic functions

$$F_{U(TT)}^{\cos(2\phi_{TT})} = \mathcal{C}\left[\frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h}h_{1TT}H_1^\perp\right],$$

$$F_{U(TT)}^{\cos(4\phi_h-2\phi_{TT})} = \mathcal{C}\left[-\left(\frac{4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2-8(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^3}{2M^3M_h}+\frac{4(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)\boldsymbol{p}_T^2}{2M^3M_h}\right)h_{1TT}^\perp H_1^\perp\right],$$

$$F_{L(TT)}^{\sin(\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\left\{\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_{TT}D_1+\frac{M_h}{M}h_{1TT}\frac{\tilde{E}}{z}\right)-\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2-2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)}{2M^2M_h}\left[\left(xe_{TT}H_1^\perp-\frac{M_h}{M}f_{1TT}\frac{\tilde{G}^\perp}{z}\right)-\left(xe_{TT}^\perp H_1^\perp-\frac{M_h}{M}g_{1TT}\frac{\tilde{D}^\perp}{z}\right)\right]\right\},$$

$$\begin{aligned}F_{L(TT)}^{\sin(3\phi_h-2\phi_{TT})} = \frac{2M}{Q}\mathcal{C}\Bigg\{&-\frac{3(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\big(2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2\big)}{2M^3}\left(xg_{TT}^\perp D_1+\frac{M_h}{M}h_{1TT}^\perp\frac{\tilde{E}}{z}\right)\\ &+\frac{4(\boldsymbol{h}\cdot\boldsymbol{k}_T)(\boldsymbol{h}\cdot\boldsymbol{p}_T)^2-2(\boldsymbol{h}\cdot\boldsymbol{p}_T)(\boldsymbol{k}_T\cdot\boldsymbol{p}_T)-(\boldsymbol{h}\cdot\boldsymbol{k}_T)\boldsymbol{p}_T^2}{2M^2M_h}\times\left[\left(xe_{TT}H_1^\perp+\frac{M_h}{M}f_{1TT}\frac{\tilde{G}^\perp}{z}\right)+\left(xe_{TT}^\perp H_1^\perp+\frac{M_h}{M}g_{1TT}\frac{\tilde{D}^\perp}{z}\right)\right]\Bigg\},\end{aligned}$$

$$F_{L(TT)}^{\sin(2\phi_h-2\phi_{TT})} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{M^2}g_{1TT}D_1\right]$$