# Unifying the study of leading and subleading twist PDFs within Dyson-Schwinger equations.

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**2025.09.24@青岛《The 26th international symposium on spin physics (SPIN2025)》**

# Outline

- A brief introduction to $f(x)$ and $e(x)$.
- A new approach to $f(x)$ and $e(x)$ through Dyson-Schwinger equations.
- Some new results.
- Summary & Outlook.

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# f(x) & e(x)

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{\mathrm{i}x P^+ \xi^-} \langle \mathrm{P}|\bar{\psi}(0)\gamma^+\psi(\xi)|\mathrm{P}\rangle\Big|_{\xi^+=0,\boldsymbol{\xi}_\perp=0} \qquad e(x) = \frac{P^+}{M}\int \frac{d\xi^-}{4\pi} e^{\mathrm{i}x P^+ \xi^-} \langle \mathrm{P}|\bar{\psi}(0)\mathrm{I}_4\psi(\xi)|\mathrm{P}\rangle\Big|_{\xi^+=0,\boldsymbol{\xi}_\perp=0}$$

$$\Phi_{ij}(k,P,S) = \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\langle PS|\bar{\psi}_j(0)\psi_i(\xi)|PS\rangle.$$

$$\Phi_{ij}(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\Phi_{ij}(k,P,S)\delta(x-k^+/P^+)$$

$$\Phi(x) = \underbrace{\tfrac{1}{2}\{f(x)\not{P} + \lambda_N \Delta f(x)\gamma_5\not{P} + \Delta_T f(x)\not{P}\gamma_5\not{S}_\perp\}}_{\text{Twist-2}} + \underbrace{\frac{M}{2}\left\{e(x) + g_T(x)\gamma_5\not{S}_\perp + \frac{\lambda_N}{2}h_L(x)\gamma_5[\not{p},\not{n}]\right\}}_{\text{Twist-3}} + \ldots$$

| Hadron Polarization | 0 | L | T |
|---|---|---|---|
| Twist 2 | $f(x)$ | $\Delta f(x)$ | $\Delta_T f(x)$ |
| Twist 3 | $e(x)$ | $h_L(x)$ | $g_T(x)$ |

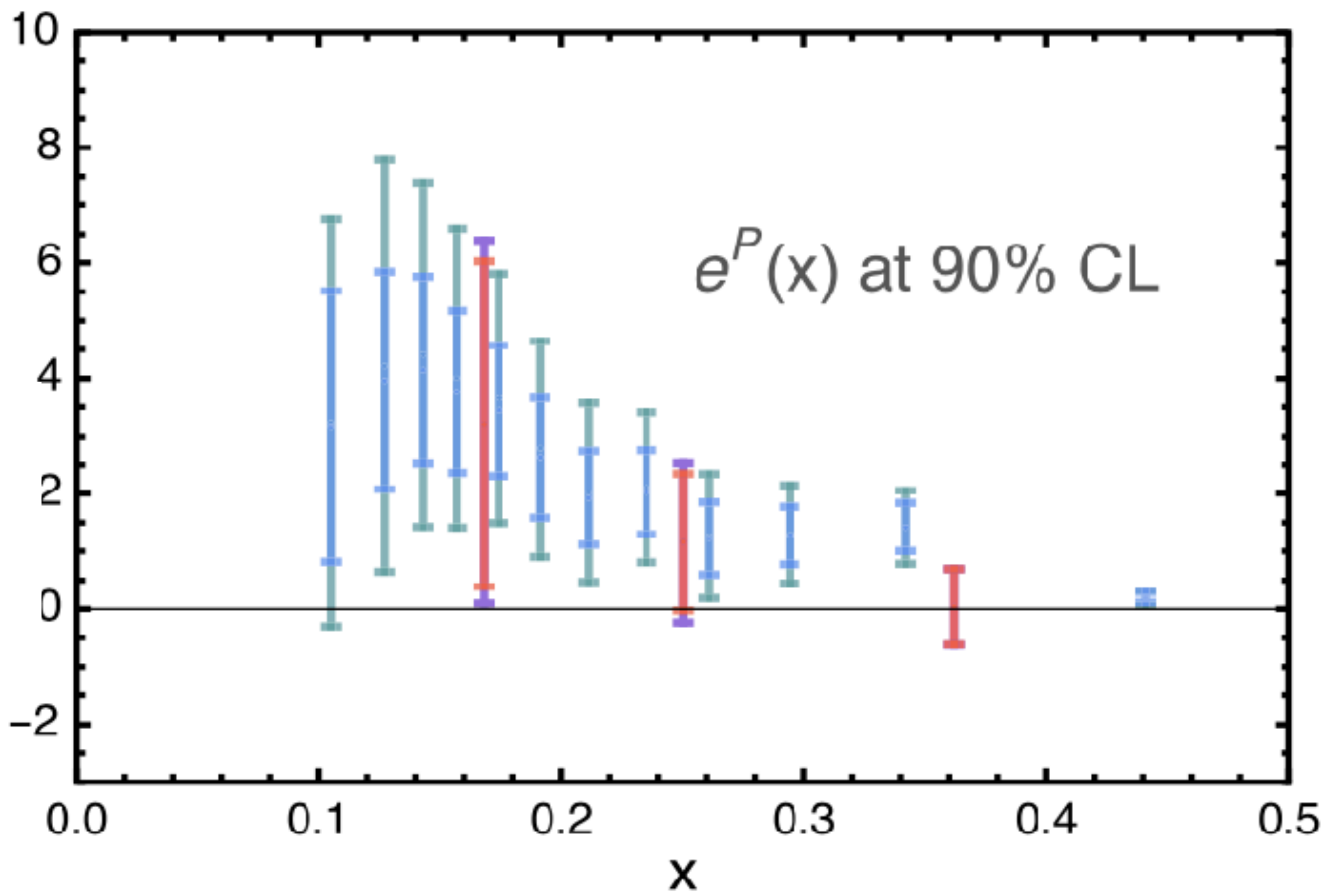
# e(x) in experiment.

- Dihadron SIDIS — beam single-spin asymmetry

$$e(\ell) + p(P) \to e'(\ell') + \pi^+(P_1) + \pi^-(P_2) + X,$$

$$A_{LU}^{\sin\phi_R} = \frac{\sum_q e_q^2 [x e^q(x,Q^2) H_{1,sp}^{\sphericalangle,q}(z,M_h,Q^2) + \frac{M_h}{zM} f_1^q(x,Q^2) \tilde{G}_{sp}^{\sphericalangle,q}(z,M_h,Q^2)]}{\sum_q e_q^2 f_1^q(x,Q^2) D_{1,ss+pp}^q(z,M_h,Q^2)}$$

Aurore Courtoy et al, PRD 106, 014027 (2022)
CLAS Collaboration, PRL 126, 152501 (2021)

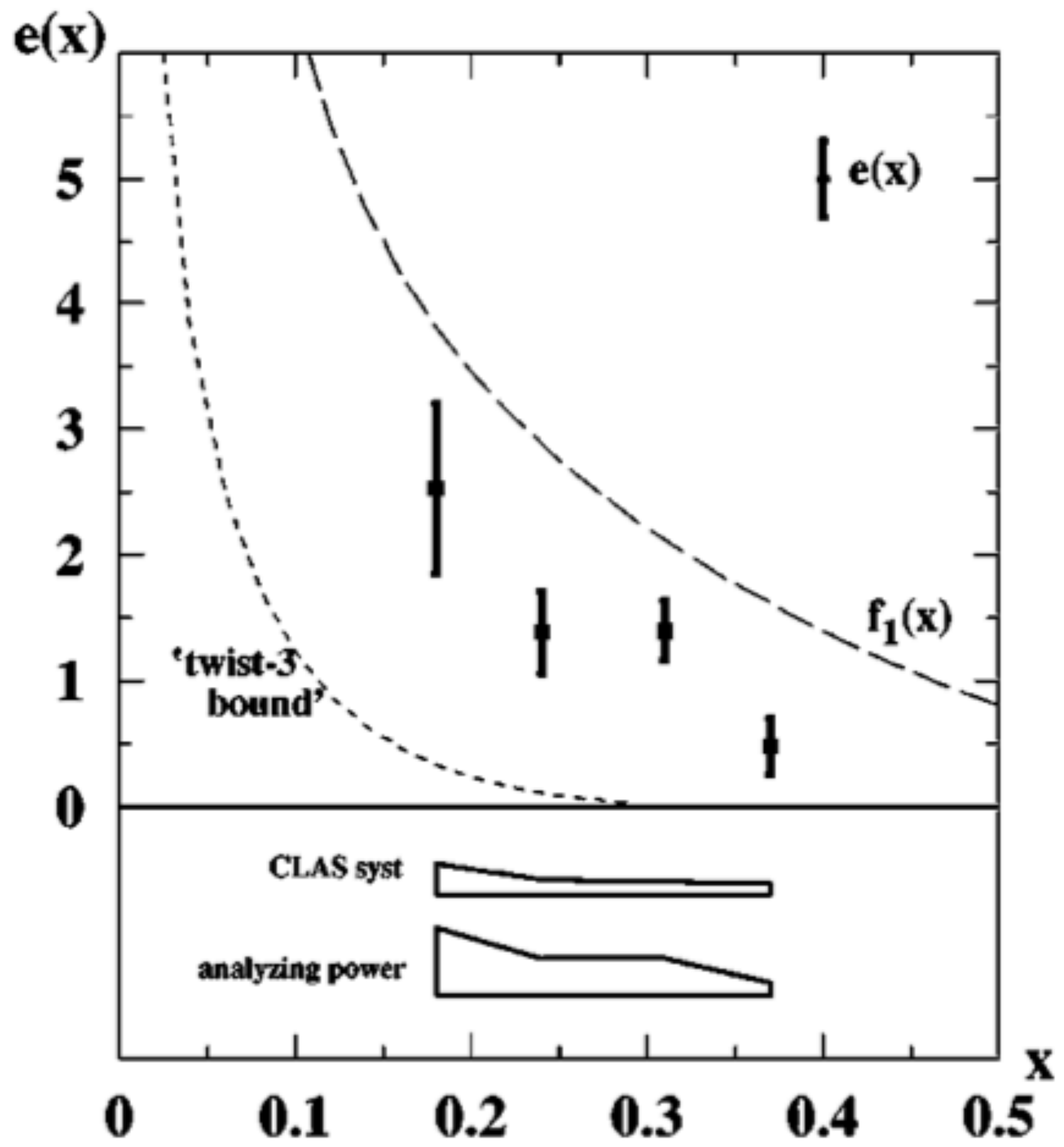



- Single-hadron SIDIS — beam single-spin asymmetry

$$ep \to e'hX$$

$$A_{LU}^{\sin\phi}(x) = \frac{1}{\langle z \rangle \sqrt{1 + \langle \mathbf{P}_{N\perp}^2 \rangle / \langle \mathbf{k}_\perp^2 \rangle}} \frac{\int dy 4y\sqrt{1-y} M_N/Q^5 \sum_a e_a^2 x^2 e^a(x) \langle H_1^{\perp a} \rangle}{\int dy (1+(1-y)^2)/Q^4 \sum_b e_b^2 x f_1^b(x) \langle D_1^b \rangle}$$

A. Efremov, et al, PRD 67, 114014 (2003)



# e(x) decomposition through QCD's EOM

$$e_q(x) = \frac{P\cdot n}{M_h}\int \frac{d\lambda}{4\pi} e^{ixP\cdot n\lambda}\langle P|\bar{\psi}_q(0)[0,\lambda n]\psi_q(\lambda n)|P\rangle$$

$$\bar{\psi}(0)[0,z]\psi(z) = \bar{\psi}(0)\psi(0) + $$

$$+\frac{1}{2}\int_0^1 du \int_0^u dv \bar{\psi}(0)\sigma^{\alpha\beta}z_\beta[0,vz]gG_{\alpha\nu}(vz)z^\nu[vz,uz]\,\psi(uz) -$$

$$-im_q\int_0^1 du\bar{\psi}(0)\ \not{z}[0,uz]\,\psi(uz) -$$

$$-\frac{i}{2}\int_0^1 du\Big(\bar{\psi}(0)(i\not{D}-m_q)\ \not{z}[0,uz]\psi(uz) + \bar{\psi}(0)\ \not{z}[0,uz](i\not{D}-m_q)\psi(uz)\Big).$$

$$e_q(x) = \frac{\langle h|\bar{\psi}_q(0)\psi_q(0)|h\rangle}{2M_h}\delta(x) + \frac{m_q}{xm_h}f_q(x) + e_q^{tw3}(x)$$

Novel features of twist-3 PDF!

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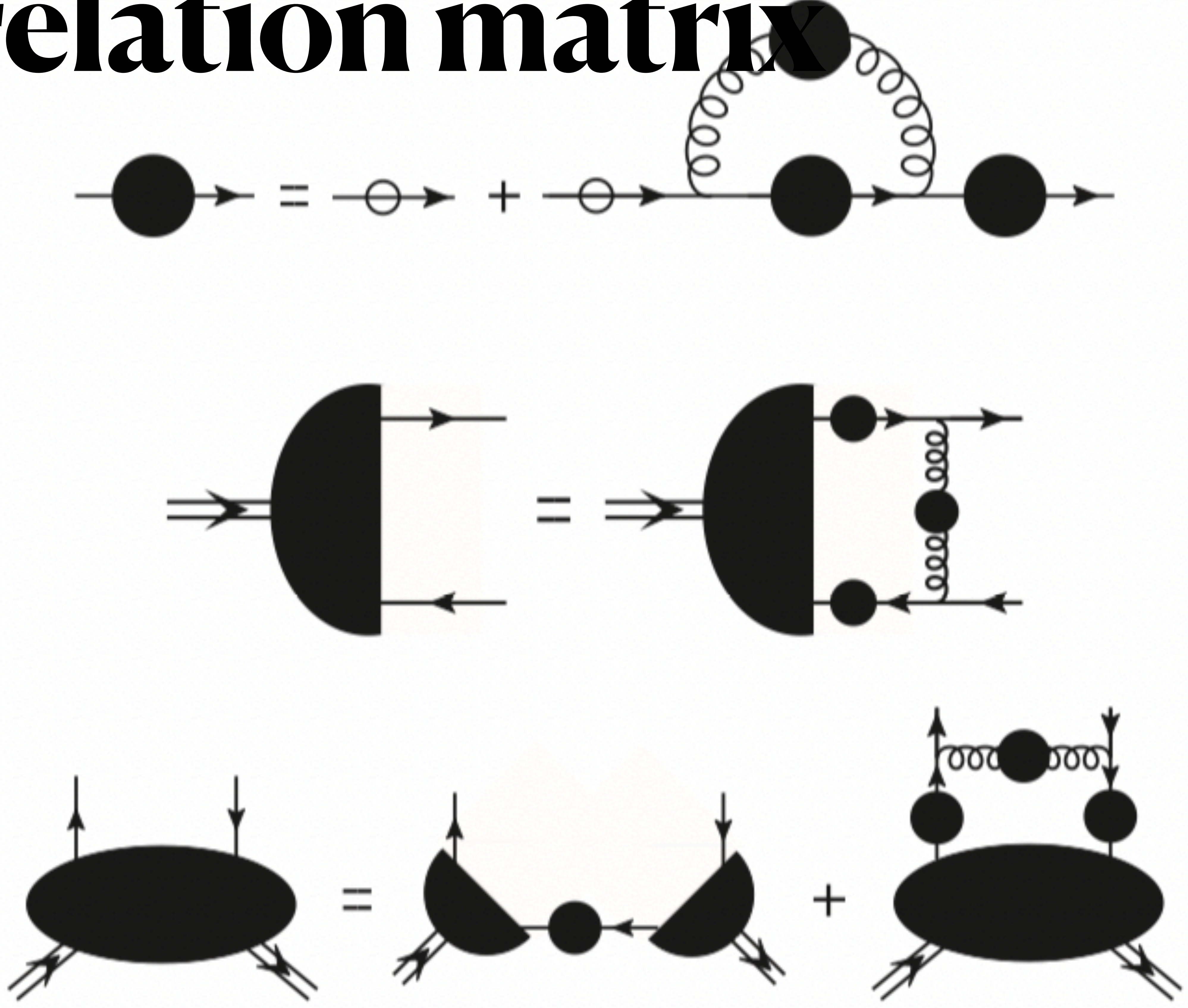
# RL-DSEs for quark-quark correlation matrix

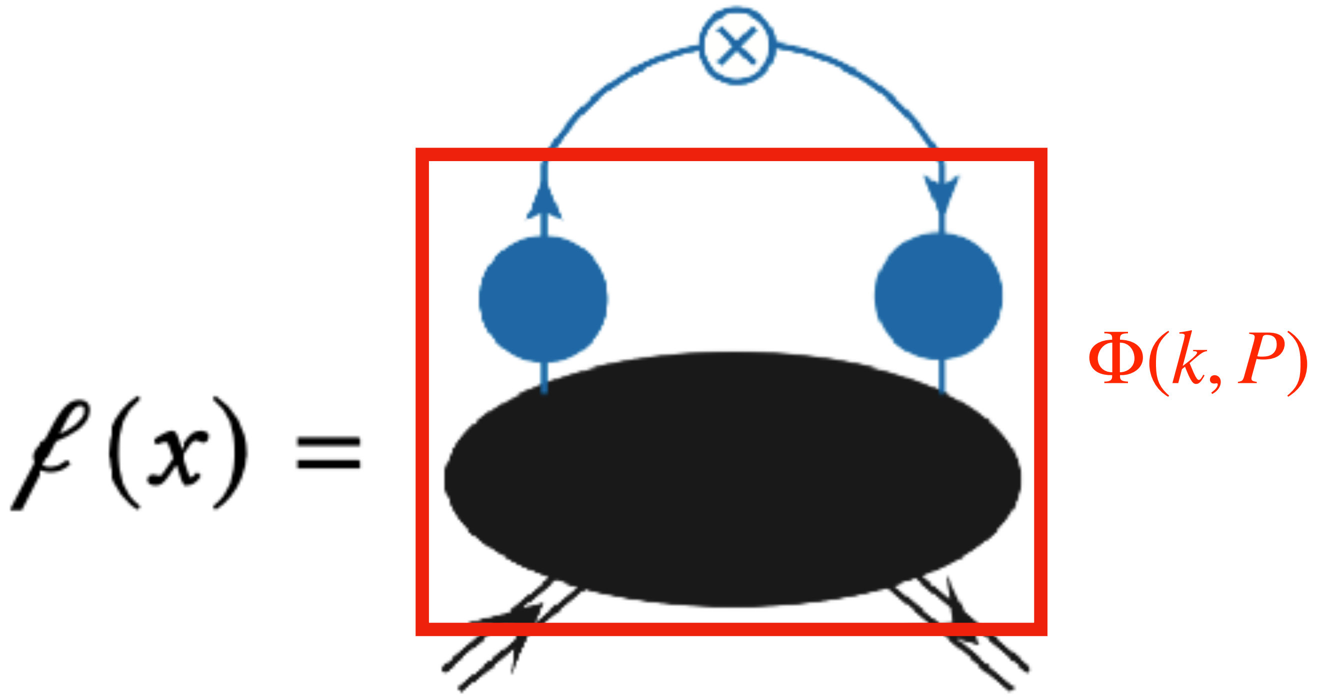


- DSEs are EOMs of Green functions.
- Gap equation: quark propagator's DSE.
- Bethe-Salpeter equation: DSE of meson->$q\bar{q}$ vertex.
- We introduce a new DSE aimed for quark-quark correlation matrix, which is the mother function to various leading and subleading-twist PDFs.

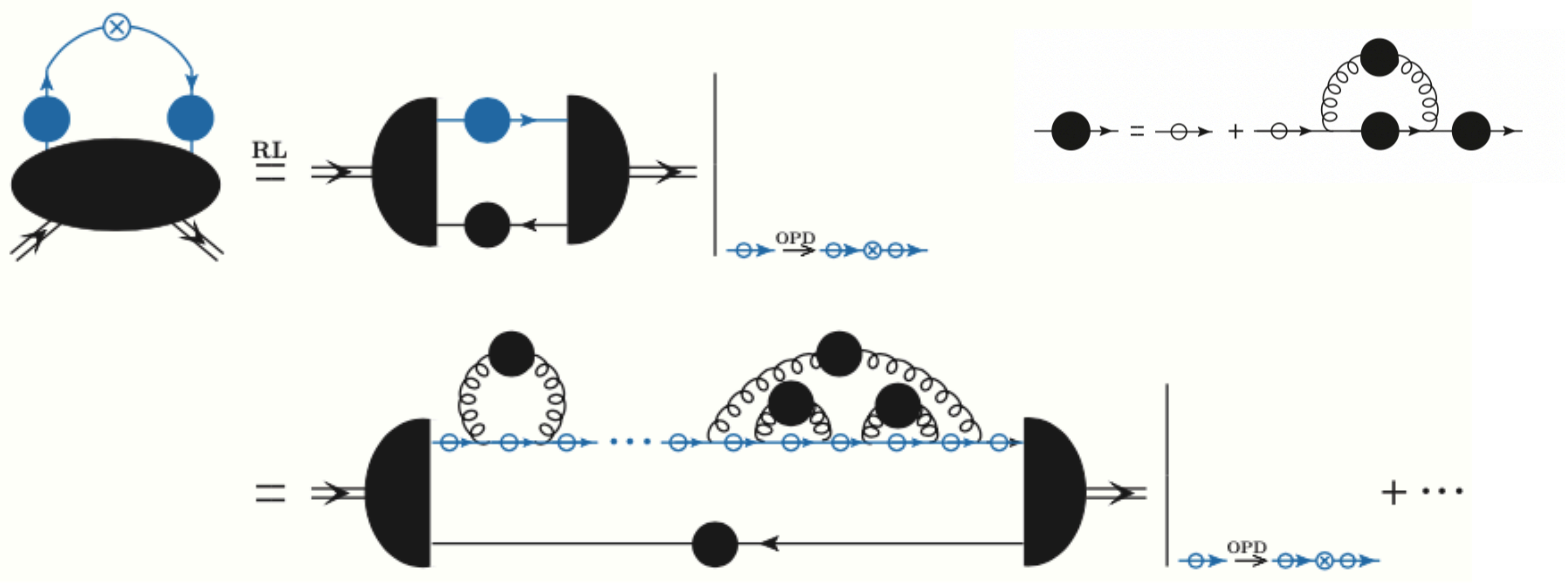
$$\Phi_{ij}(k,P) = \int d^4\xi \, e^{ik\cdot\xi} \langle P|\bar{\psi}_j(0)\psi_i(\xi)|P\rangle.$$

- The twist-2 and -3 PDFs can be simultaneously extracted from $\Phi$

$$f(x) = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\delta(k_\eta^+ - xP^+)\mathrm{Tr}\left[\Phi(k,P)\Gamma_f\right]$$



# Link Pert. and Nonpert. objects

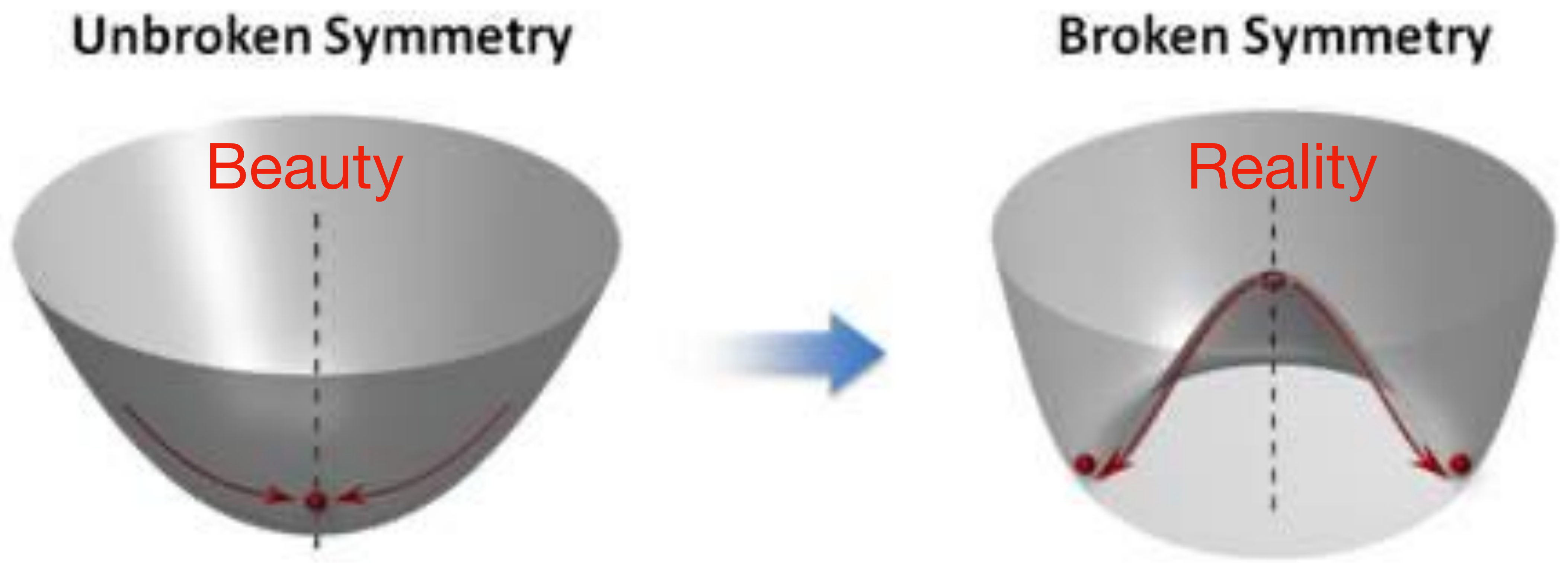



- Every bare quark propagator in the dressed quark propagator is probed by the light-cone vertex concerned.

# Symmetries preserved

- Poincare symmetry (Continuum QFT+Regularization)
  - translation, boost, rotation
- Chiral symmetry. (Axial-vector Ward Identity)
  - $m_q$~0 MeV
  - Spontaneous(dynamical) breaking.
  - Evidence: Goldstone boson, etc.
- Global $U(1)$ symmetry. (Vector Ward Identity)
  - Baryon number conservation(quark number sum rule)

$$SU_V(3) \otimes \cancel{SU_A(3)}$$



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# Mellin moments

$$\ell(x) = \frac{1}{2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\delta(k_\eta^+ - xP^+)\mathrm{Tr}\left[\Phi(k,P)\Gamma_\ell\right]$$

$$\langle x^n \rangle = \int dx x^n \ell(x)$$

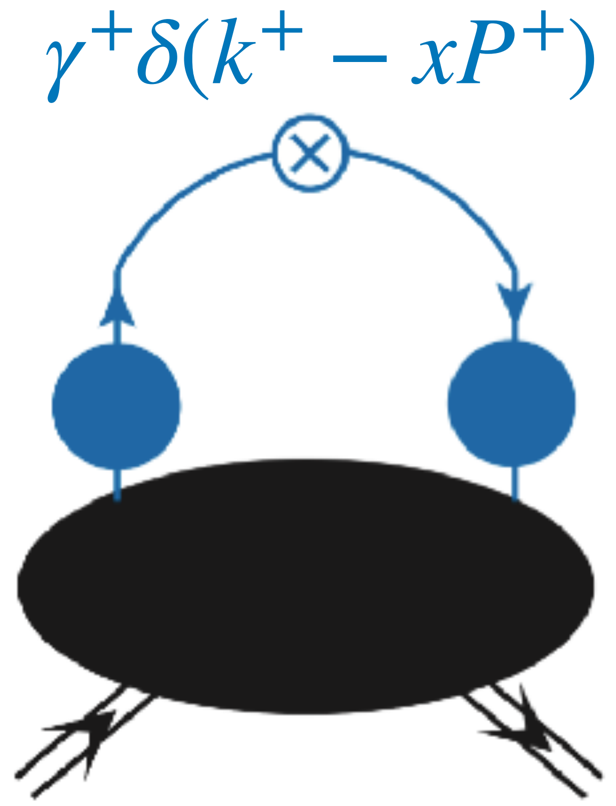
- Compute the Mellin-moments of f(x) and e(x).
- Reconstruct the PDFs by fitting the moments with parameterized ansatz.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $\langle x^n \rangle_{f(x)}$ | 1.00 | 0.372 | 0.202 | 0.129 | 0.0910 | 0.0680 | 0.0531 | 0.0428 | 0.0358 |
| $\langle x^n \rangle_{e(x)}$ | 6.45 | 0.113 | 0.303 | 0.278 | 0.231 | 0.191 | 0.159 | 0.134 | 0.115 |
| $\langle x^n \rangle_{e_{\mathrm{mass}}^{\mathrm{Max}}(x)}$ | | 0.113 | 0.0419 | 0.0228 | 0.0146 | 0.0103 | 0.00773 | 0.00604 | 0.00488 |
| $\langle x^n \rangle_{e_{\mathrm{tw3}}(x)}$ | | 0 | 0.261 | 0.255 | 0.216 | 0.181 | 0.151 | 0.128 | 0.110 |

TABLE I. Computed Mellin moments of pion PDFs.

# Pion f(x) at hadronic scale

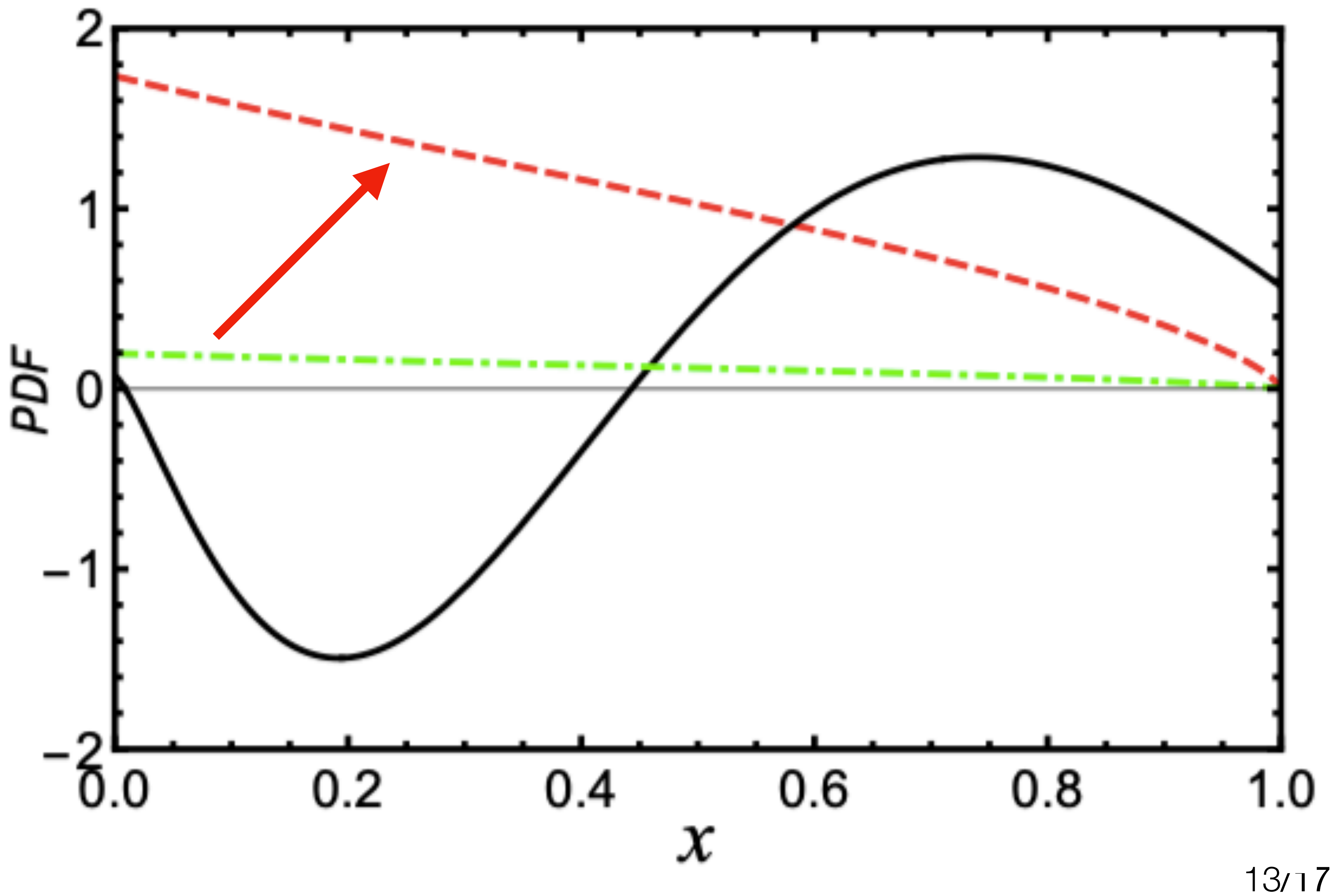
$$f(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0)\gamma^+\psi(\xi)|P\rangle\Big|_{\xi^+=0,\xi_\perp=0}$$

$f(x)=$ 


| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $\langle x^n\rangle_{f(x)}$ | 1.00 | 0.372 | 0.202 | 0.129 | 0.0910 | 0.0680 | 0.0531 | 0.0428 | 0.0358 |

$$\langle x^n\rangle = \int dx x^n f(x)$$

- $\langle x^0\rangle_f \equiv \int dx x^0 f(x) = 1$: quark number sum rule.
- $\langle x^1\rangle_f < 0.5$: gluon carries away 26% momentum fraction.
- The RL-DSE is associated with a low scale, where sea quark distributions are absent.



# Pion e(x) at initial scale

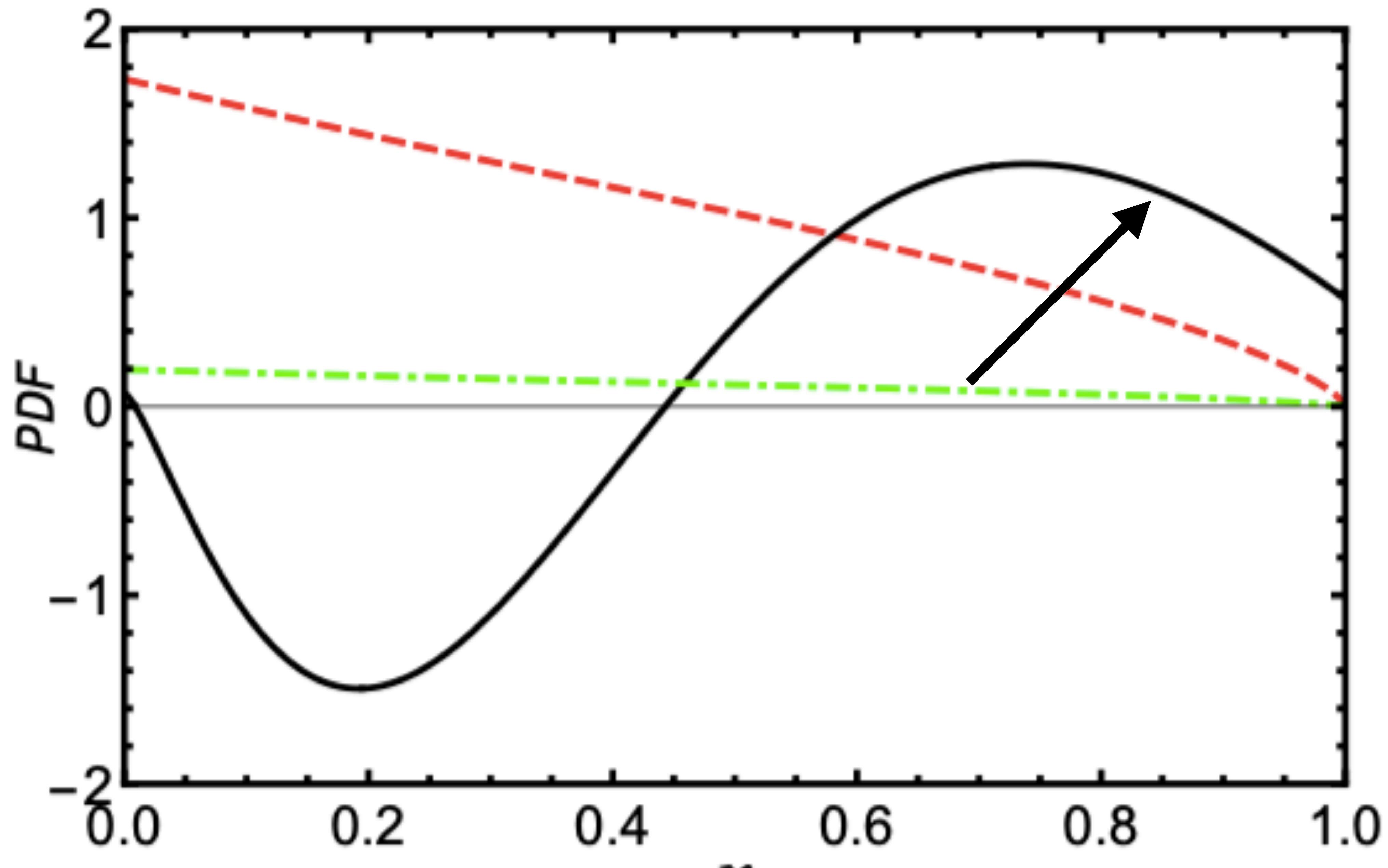
$$e_q(x) = \frac{\langle h|\bar{\psi}_q(0)\psi_q(0)|h\rangle}{2M_h}\delta(x) + \frac{m_q}{x m_h} f_q(x) + e_q^{tw3}(x)$$

$$e(x) = \frac{P^+}{M}\int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0)I_4\psi(\xi)|P\rangle\Big|_{\xi^+=0,\xi_\perp=0}$$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $\langle x^n\rangle_{f(x)}$ | 1.00 | 0.372 | 0.202 | 0.129 | 0.0910 | 0.0680 | 0.0531 | 0.0428 | 0.0358 |
| $\langle x^n\rangle_{e(x)}$ | 6.45 | 0.113 | 0.303 | 0.278 | 0.231 | 0.191 | 0.159 | 0.134 | 0.115 |
| $\langle x^n\rangle_{e_{\rm mass}^{\rm Max}(x)}$ | | 0.113 | 0.0419 | 0.0228 | 0.0146 | 0.0103 | 0.00773 | 0.00604 | 0.00488 |
| $\langle x^n\rangle_{e_{\rm tw3}(x)}$ | | 0 | 0.261 | 0.255 | 0.216 | 0.181 | 0.151 | 0.128 | 0.110 |

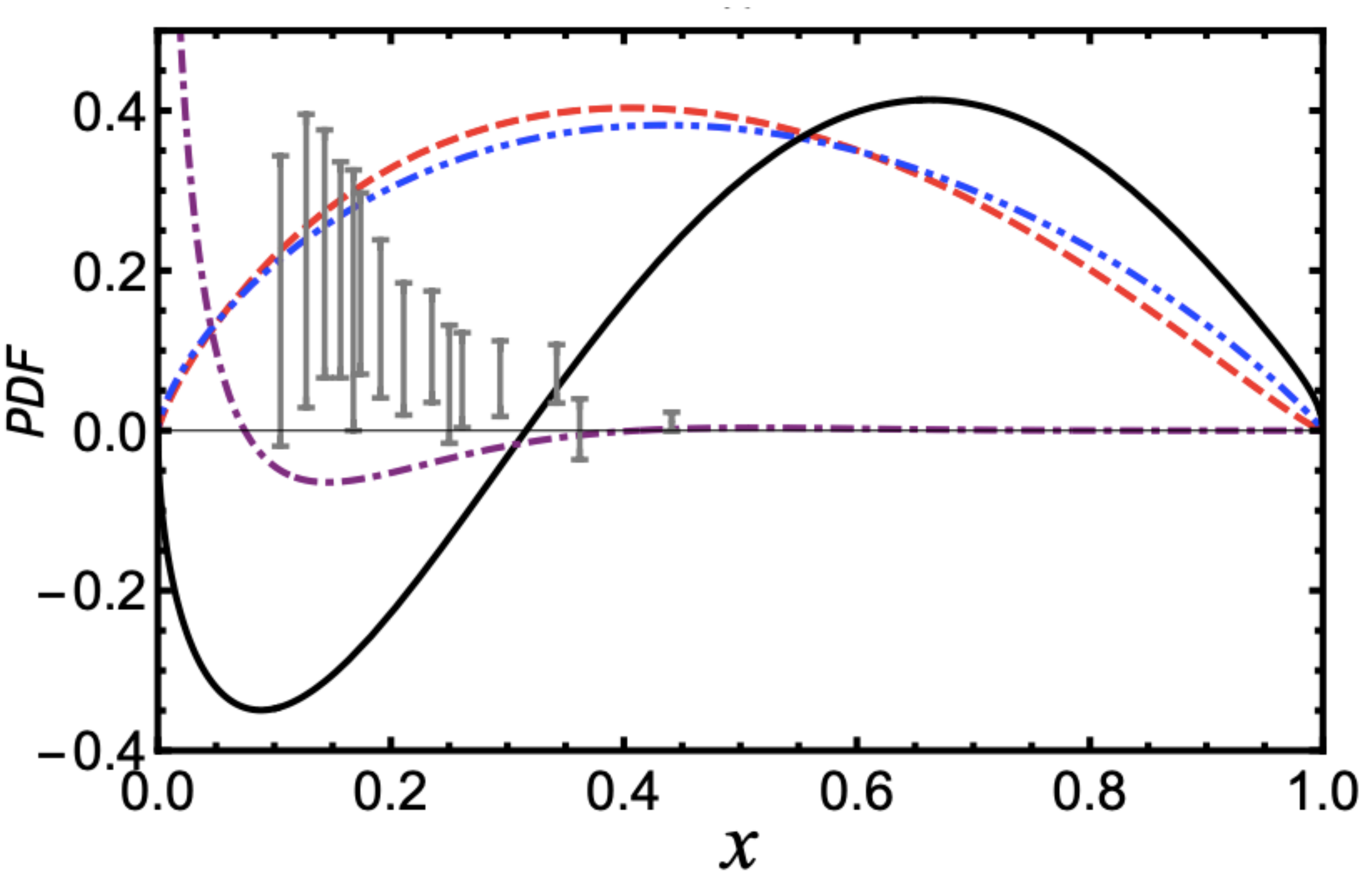
- Strong indication of $\delta(x)$ is found! (Zero mode)
- Strong suppression of first moment is found! (Chiral symmetry) $\langle x\rangle_e = m_q/m_h$
- Significant pure twist-3 contribution is found! Arising from $qA\bar{q}$ correlation.

by zero modes in the light-cone formalism [11,16–19]. The scalar PDF has been studied in nonperturbative models for hadron structure [20–28]. In most quark models, the dominant contribution is found to be the mass contribution, which originates from the equation of motions for free



# Evolved PDFs and Implications for Nucleon

- Evolution of twist-3 PDFs is rather complicated: mixing between two and three partons
- In large $N_C$ limit the e(x) evolution becomes simpler.
- The f(x) and e(x) are evolved to experimental scale ~ 1 GeV.
- f(x) compares well with JAM global fit.
- The node of e(x) persists under evolution.
- We notice hints of similar pattern in the case of nucleon.

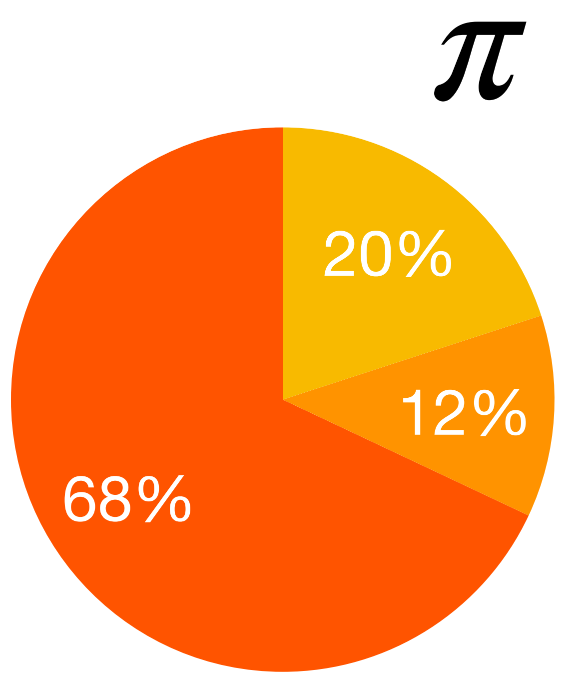


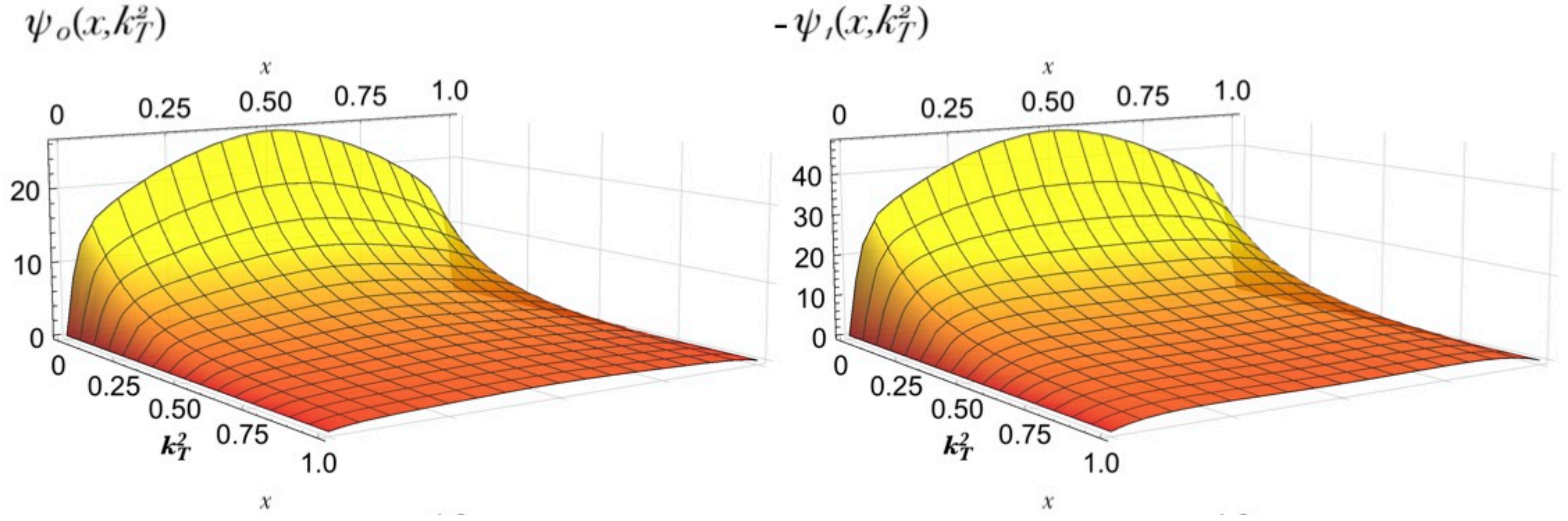

$$\langle x \rangle_e = m_q / m_h$$

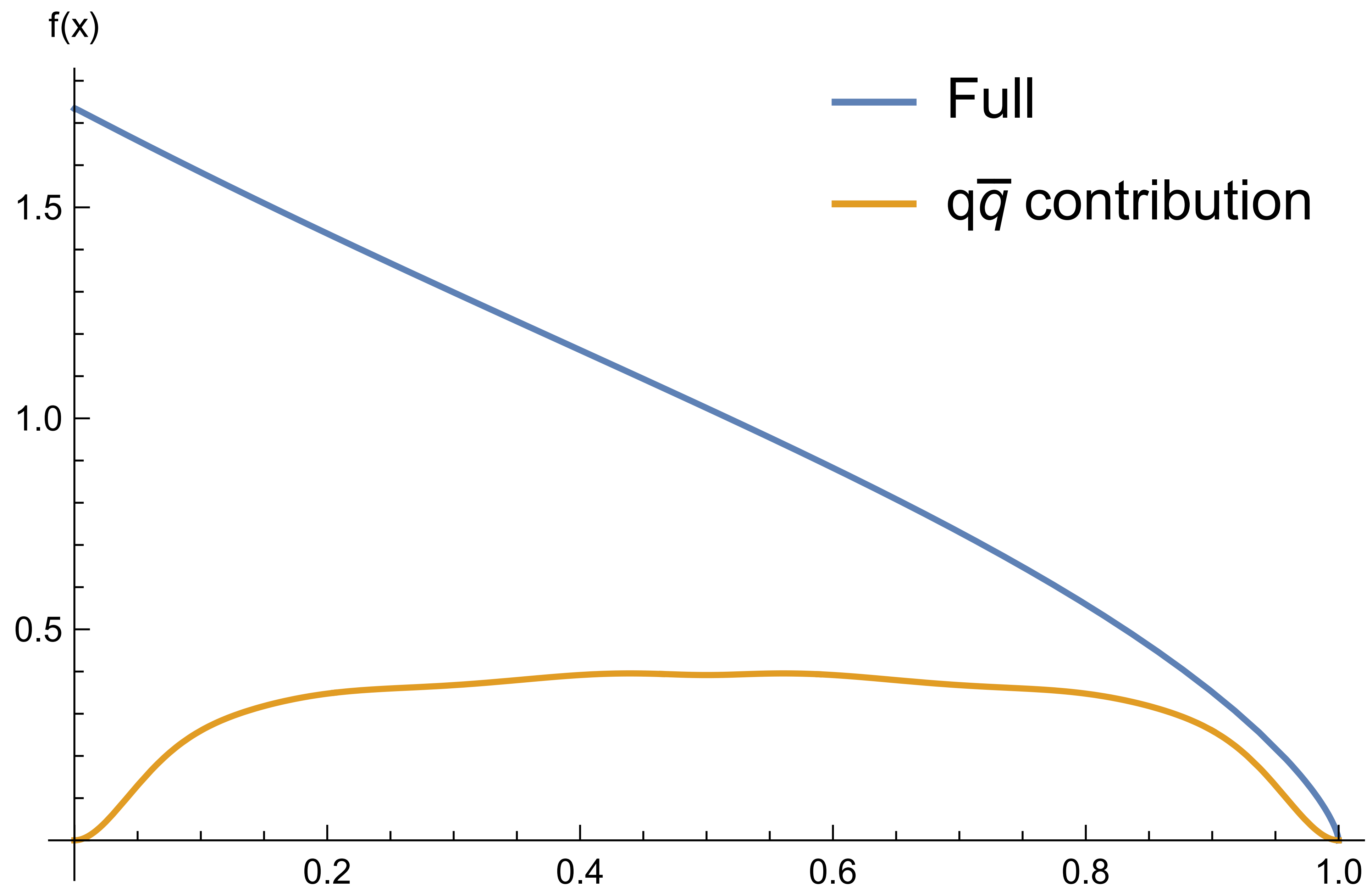
# Origin: Higher Fock-states

$$|M\rangle = \phi_2|q\bar{q}\rangle + \phi_3|q\bar{q}g\rangle + \phi_4|q\bar{q}gg\rangle + \ldots$$

$$\phi_i(x,\vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+)\mathrm{Tr}[\Gamma_i \chi(k,P)]$$




*Phys.Rev.Lett.* 122 (2019) 8, 082301
*Phys.Rev.D* 101 (2020) 7, 074014
*Phys.Rev.D* 104 (2021) 9, 094016

- The $q\bar{q}$-LFWFs only constitute about 30% to the total normalization.
- Many gluon components reside in pion.

# Conclusion

- A new Φ-DSE is introduced to render a unified study of f(x) and e(x).
- e(x) contains a Dirac-$\delta$ function, through nonperturbative dynamics.
- e(x) is dominated by pure twist-3 part.
- e(x) contains a node.

# Outlook

- More PDFs
- TMD PDFs
- Nucleon

**Thank you for your attention!**