26th International Symposium on Spin Physics
A Century of Spin

Hadronic tensor in (1+1)-dimensional lattice gauge theory by quantum computing

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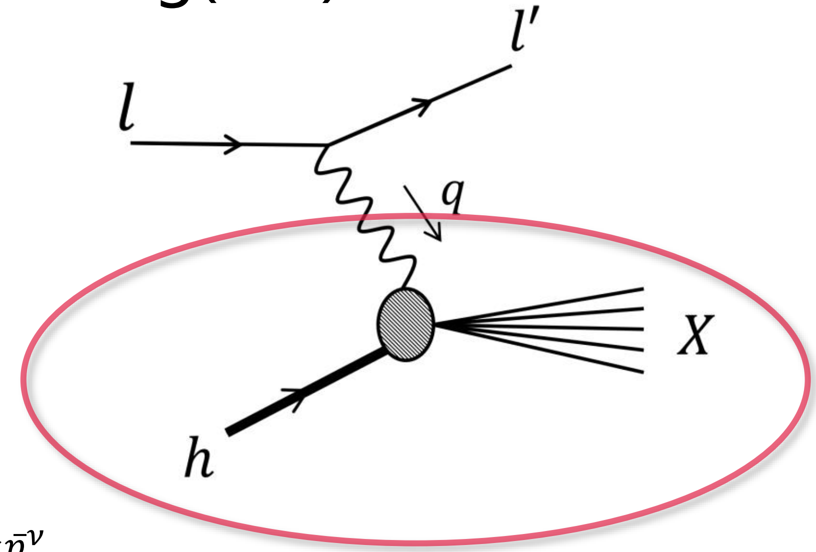
Outline

- Introduction
- Hadronic tensor in (1+1)-dimensional lattice gauge theory(LGT)
  - Map the lattice quantum fields to qubits
  - Preparation of the hadron state
  - Evaluation of the current-current correlation functions
- Results and summary

Introduction

■ The differential cross section of deep inelastic scattering(DIS) :

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha_{em}^2 y}{Q^4} L_{\mu\nu} W^{\mu\nu}$$



- leptonic tensor : $L_{\mu\nu}(l, l')$
- Hadronic tensor : $W^{\mu\nu}(q)$

  decomposed into structure function : $W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{\bar{p}^\mu \bar{p}^\nu}{p\cdot q} F_2$

- QCD factorization : $\sigma_{DIS} \sim f_{a/h} \otimes \sigma_a$
- The PDFs can be extracted from structure function: $F_i = \sum_a c_i^a \otimes f_{a/h}$

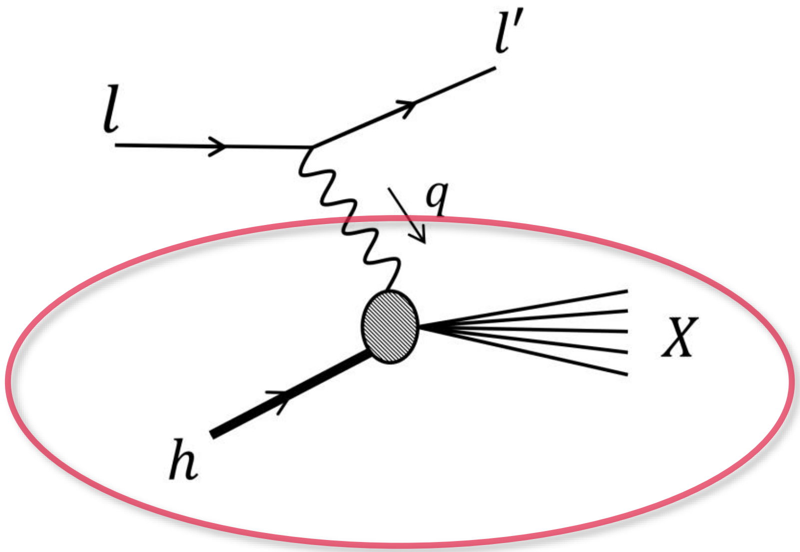
Hadronic tensor

—— A time-dependance non-perturbative quantity

- Hadronic tensor in d dimensional space-time:

$$W^{\mu\nu}(q) = \int d^{\mathrm{d}}z\, \mathrm{e}^{\mathrm{i}qz} \langle h| J^{\mu}(z) J^{\nu}(0)|h\rangle$$

$J^{\mu} = \bar{\psi}\gamma^{\mu}\psi$, is electric current when the lepton interact with hadron by exchanging a photon



➡ The time-dependance quantities can not be directly calculated by Lattice QCD

➡ QC can simulate real-time dynamics naturally

Procedures of simulating hadronic tensor on quantum computers

- $$W^{\mu\nu}(q) = \int d^2z \ \mathrm{e}^{\mathrm{i}qz} \langle h| \, \mathrm{e}^{\mathrm{i}Ht} \, J^{\mu}(0,\vec{z}\,) \, \mathrm{e}^{\mathrm{i}Ht} \, J^{\nu}(0,0) \big| h \rangle$$

1. Map the quantum fields to qubits

Models : (1+1) D Abelian and non-Abelian LGT

2. Preparation of the hadron state

Quantum-number-resolving variational quantum eigensolver(VQE)

3. Evaluation of the current-current correlation functions

1. Map the lattice quantum fields to qubits

■ Discretization： Kogut-Suskind formalism

$$H_{K.S.} = -\frac{i}{2a}\sum_n \left(\psi_{n+1}^\dagger U_n \psi_n - h.c.\right) + \sum_n (-1)^n \psi_n^\dagger \psi_n + \frac{ag^2}{2}\sum_n \boldsymbol{L}_n^2$$

- Matter field： $\psi_n = \begin{pmatrix} \psi_n^r \\ \psi_n^g \end{pmatrix}$

  Gauge field： $U_n = \exp\left(iagA_n^{\mu,a}T^a\right),\quad a = 1,2,3$

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

- Non-abelian charges：

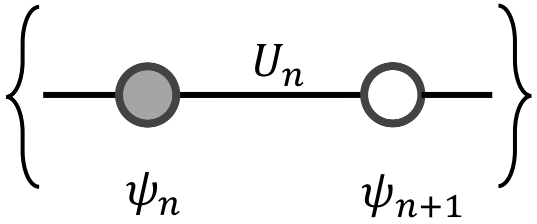
$$Q_n^a = \sum_{i,j} \psi_n^{i\dagger}(T^a)_{ij}\psi_n^j,\quad i,j = r,g$$

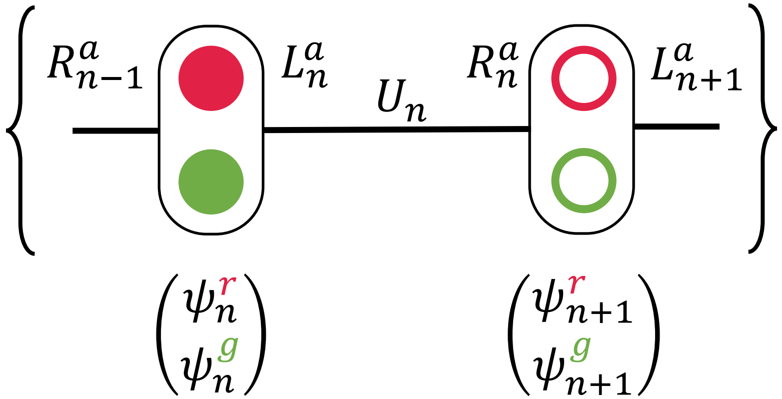
- Baryon number：

$$B = \sum_n \frac{1}{2}\left(\psi_n^{r\dagger}\psi_n^r + \psi_n^{g\dagger}\psi_n^g\right)$$

- Gauss law： $L_n^a - R_{n-1}^a = Q_n^a$

- One physical site：





1. Map the lattice quantum fields to qubits

Map the LGT to qubits

- Jordan-Wigner transformation : $\phi_n \to \left(\prod_{m<n} \sigma_m^3\right)\sigma_n^-$, in SU(2) : $\psi_n = \begin{pmatrix}\psi_n^r \\ \psi_n^g\end{pmatrix} = \begin{pmatrix}\phi_{2n} \\ \phi_{2n+1}\end{pmatrix}$

Map the hadronic tensor to qubits

$$W^{\mu\nu}(q^0, q^1) = \int \mathrm{d}t \sum_{\vec{z}} \mathrm{e}^{\mathrm{i}q^0 t}\, \mathrm{e}^{-\mathrm{i}q^1 z} \, \langle h| \, \mathrm{e}^{\mathrm{i}Ht} J_n^\mu \, \mathrm{e}^{-\mathrm{i}Ht} J_m^\nu \, |h\rangle, \; \vec{z} = n - m$$

U(1) LGT: $\begin{cases} J_n^0 = \frac{1}{2}[(-1)^n + \sigma_n^3] \\ J_n^1 = \sigma_n^+ \sigma_{n+1}^- + \sigma_{n+1}^+ \sigma_n^- \end{cases}$

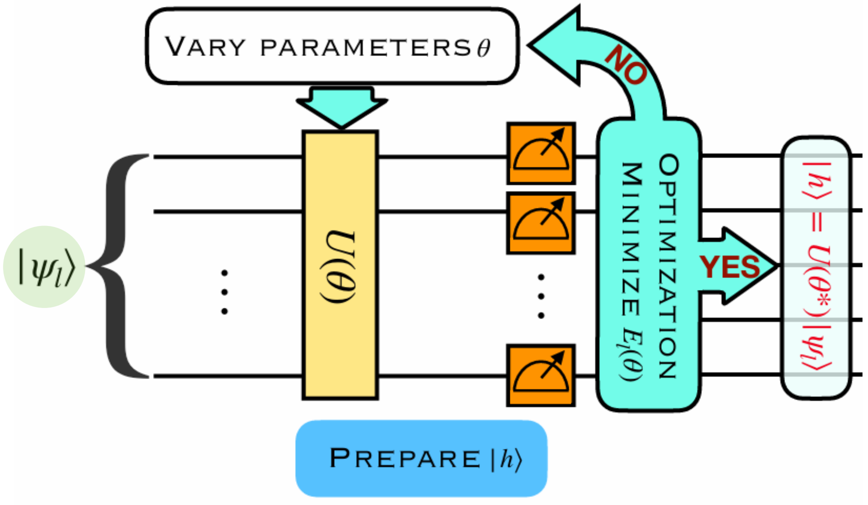
SU(2) LGT : $\begin{cases} J_n^0 = \frac{1}{2}(\sigma_{2n}^3 + \sigma_{2n+1}^3) \\ J_n^1 = \sigma_{2n}^+ \sigma_{2n+1}^- + \sigma_{2n+1}^+ \sigma_{2n}^- \end{cases}$

2. Preparation of the hadron state

—— Quantum-number-resolving VQE :

- Target hadron state :

  the k-th excited state with quantum number l



Li et al. (QuNu), Phys. Rev. D 105, L111502

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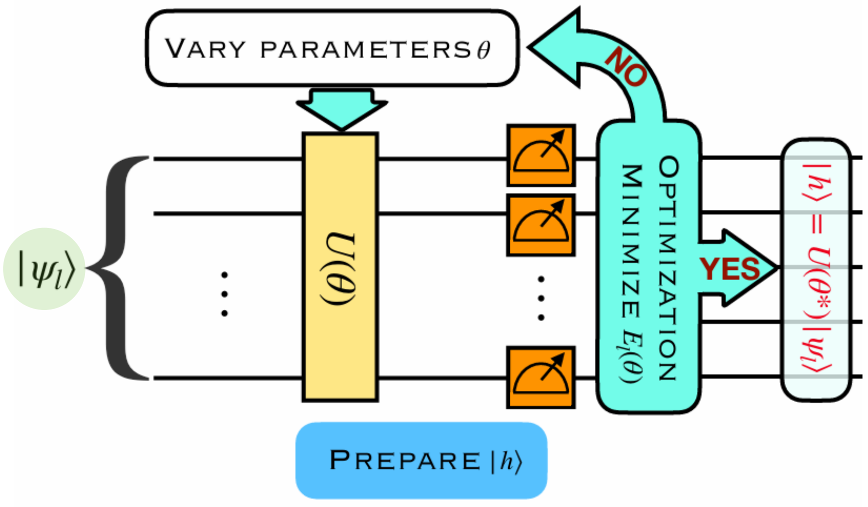
(1) Prepare k reference states $|\psi_{li}\rangle,\ i = 0,1 \cdots k-1$

Have the same quantum number l as the hadron state



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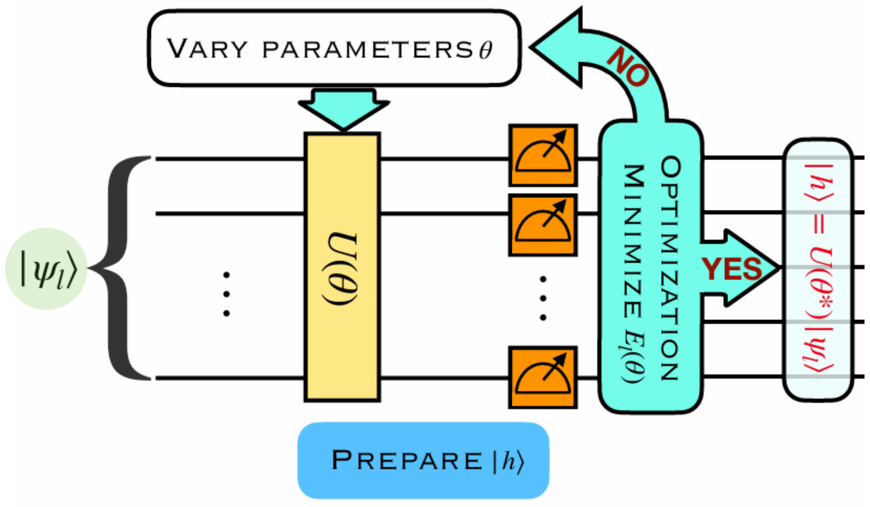

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(2) Construct $U(\theta)$: Quantum alternating operator ansatz(QAOA)

Divide the Hamiltonian

$H = H_1 + H_2 + \cdots + H_k$

- Each H_i has the symmetry of H
- $[H_i, H_{i+1}] \neq 0$

Parameterized symmetry-preserving operator

$$U(\theta) = \prod_{i=1}^{p} \prod_{j=1}^{k} \exp\left(\mathrm{i}\theta_{ij} H_j\right)$$

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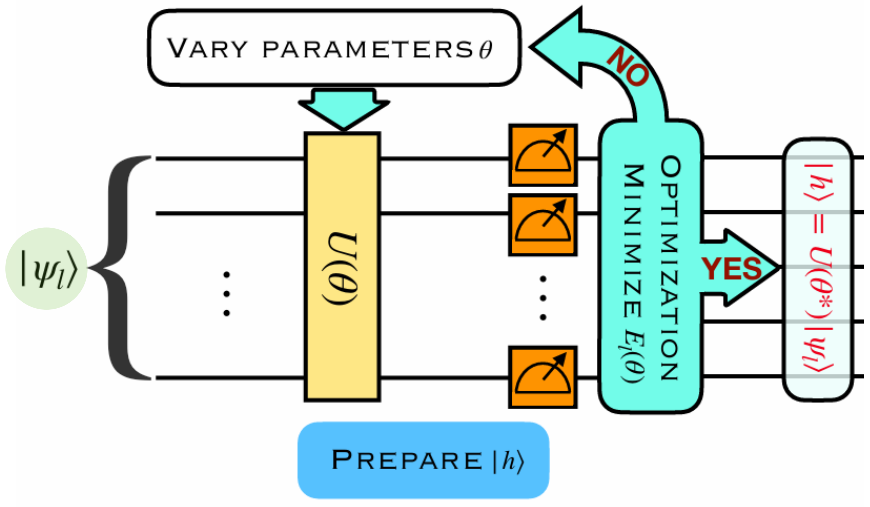

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Parameterized symmetry-preserving operator

$$U(\theta) = \prod_{i=1}^{p} \prod_{j=1}^{k} \exp(i\theta_{ij} H_j)$$

(3) Minimize the cost function $E_l(\theta)$

$$E_l(\theta) = \sum_{i=0}^{k-1} \omega_{li} \langle \psi_{li}(\theta)|H|\psi_{li}(\theta)\rangle$$

$$\omega_{l0} > \omega_{l1} > \cdots > \omega_{li} > 0$$

(4) The hadron state is prepared as $|h\rangle = U(\theta^*)|\psi_{l,k-1}\rangle$

—— Quantum-number-resolving VQE :

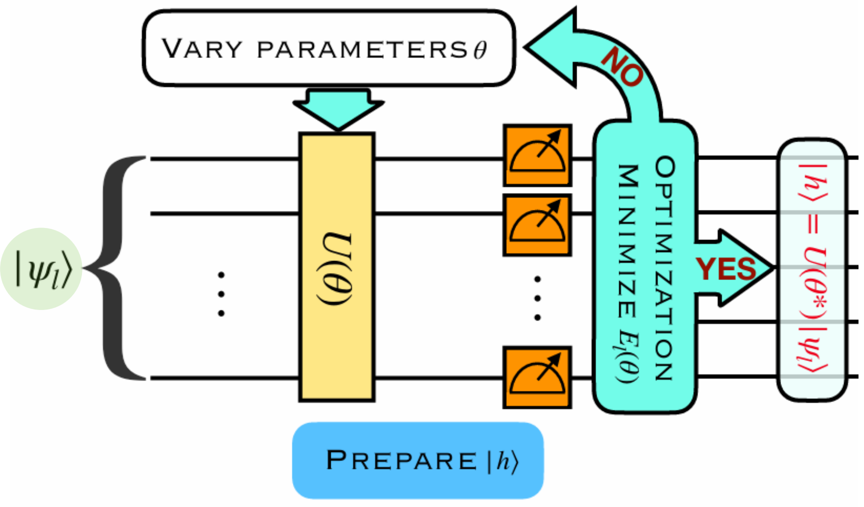
- **Target state :** $|q\bar{q}\rangle, |qq\rangle, |\Omega\rangle$

  Meson : 1st excited state in $B = 0$ sector

  Baryon : ground state in $B = 1$ sector

- **Conserve quantity :** $(\mathcal{Q}_{tot})^2$, $\mathcal{Q}^3_{tot}$, B, $\vec{p}$

$$(\mathcal{Q}_{tot})^2 = \mathcal{Q}^3_{tot} = 0, \quad \vec{p} = 0$$



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The reference states : $|\psi_{Bi}\rangle$

$$|\psi_{00}\rangle = |\underline{0011}\ \underline{0011} \cdots \underline{0011}\rangle$$

$$|\psi_{01}\rangle = \sqrt{\frac{2}{N}}\left(|\underline{1100}\ \underline{0011} \cdots \underline{0011}\rangle + |\underline{0011}\ \underline{1100} \cdots \underline{0011}\rangle + \cdots + |\underline{0011}\ \underline{0011} \cdots \underline{1100}\rangle\right)$$

$$|\psi_{10}\rangle = \sqrt{\frac{2}{N}}\left(|\underline{1111}\ \underline{0011} \cdots \underline{0011}\rangle + |\underline{0011}\ \underline{1111} \cdots \underline{0011}\rangle + \cdots + |\underline{0011}\ \underline{0011} \cdots \underline{1111}\rangle\right)$$

$$|\Omega\rangle = U(\theta_0^*)\,|\psi_{00}\rangle$$

$$|q\bar{q}\rangle = U(\theta_0^*)\,|\psi_{01}\rangle$$

$$|qq\rangle = U(\theta_1^*)\,|\psi_{10}\rangle$$

3. Evaluation of the current-current correlation functions

- The current-current correlation functions

$$\mathcal{O}^{\mu\nu}(\vec{z},t) = \langle h|\, \mathrm{e}^{\mathrm{i}Ht}\, J_n^{\mu}\, \mathrm{e}^{-\mathrm{i}Ht}\, J_m^{\nu}\, |h\rangle$$

U(1) :

$$J_n^0 = \frac{1}{2}[(-1)^n + \sigma_n^3]$$

$$J_n^1 = \sigma_n^+\, \sigma_{n+1}^- + \sigma_{n+1}^+ \sigma_n^-$$

SU(2) :

$$J_n^0 = \frac{1}{2}(\sigma_{2n}^3 + \sigma_{2n+1}^3)$$

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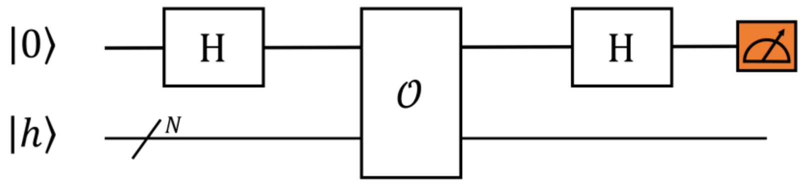
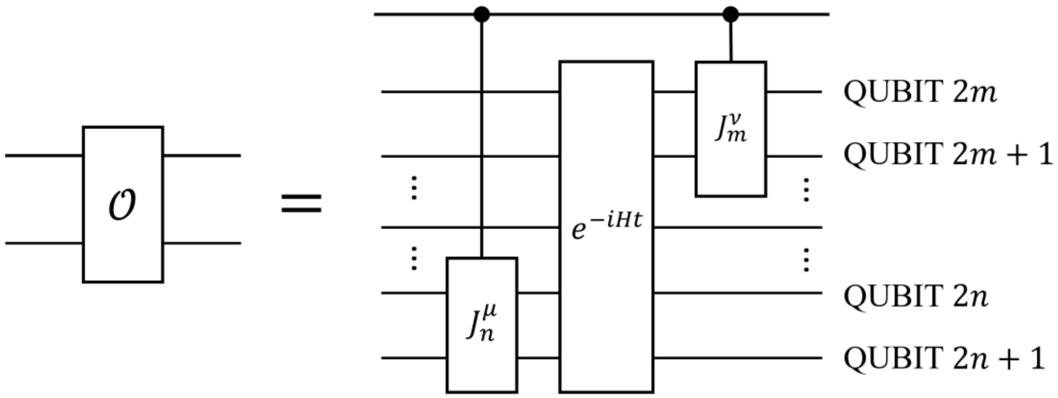
Pedernales et al, Phys. Rev. Lett. 113, 020505 (2014)





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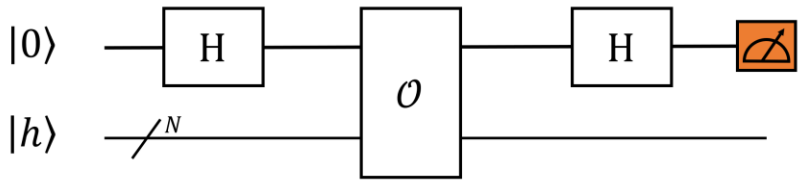
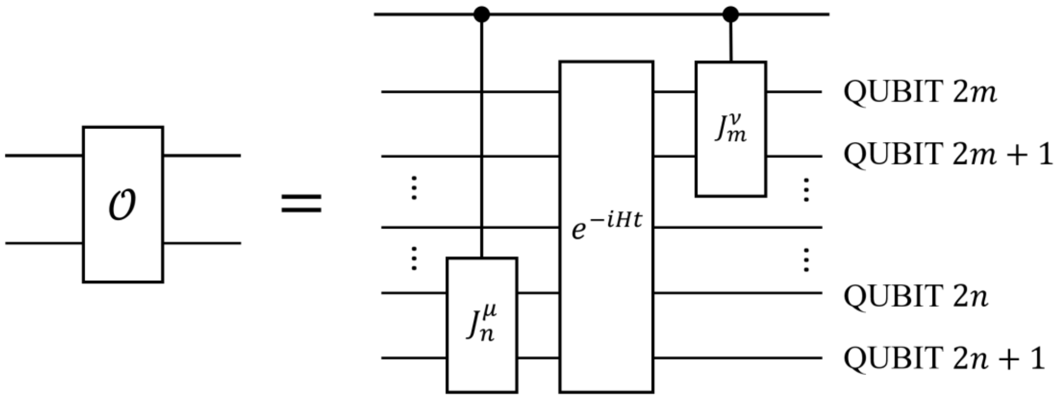


- The Fourier transformation

$$W^{\mu\nu}(q^0,q^1) = \int \mathrm{d}t \sum_{\vec{z}} e^{iq^0 t}\, e^{-iq^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t),$$

$$\vec{z} = n - m$$

$$W^{\mu\nu}(q^0,q^1) = \int_{-n_t}^{n_t} \mathrm{d}t \sum_{\vec{z}} e^{iq^0 t}\, e^{-iq^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t)$$



Results of the hadronic tensor

- The current-current correlation functions

$$\mathcal{O}^{\mu\nu}(\vec{z},t) = \langle h|\, e^{iHt}\, J_n^{\mu}\, e^{-iHt}\, J_m^{\nu}\, |h\rangle$$

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- The Fourier transformation

$$W^{\mu\nu}(q^0,q^1) = \int dt \sum_{\vec{z}} e^{iq^0 t}\, e^{-iq^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t),$$

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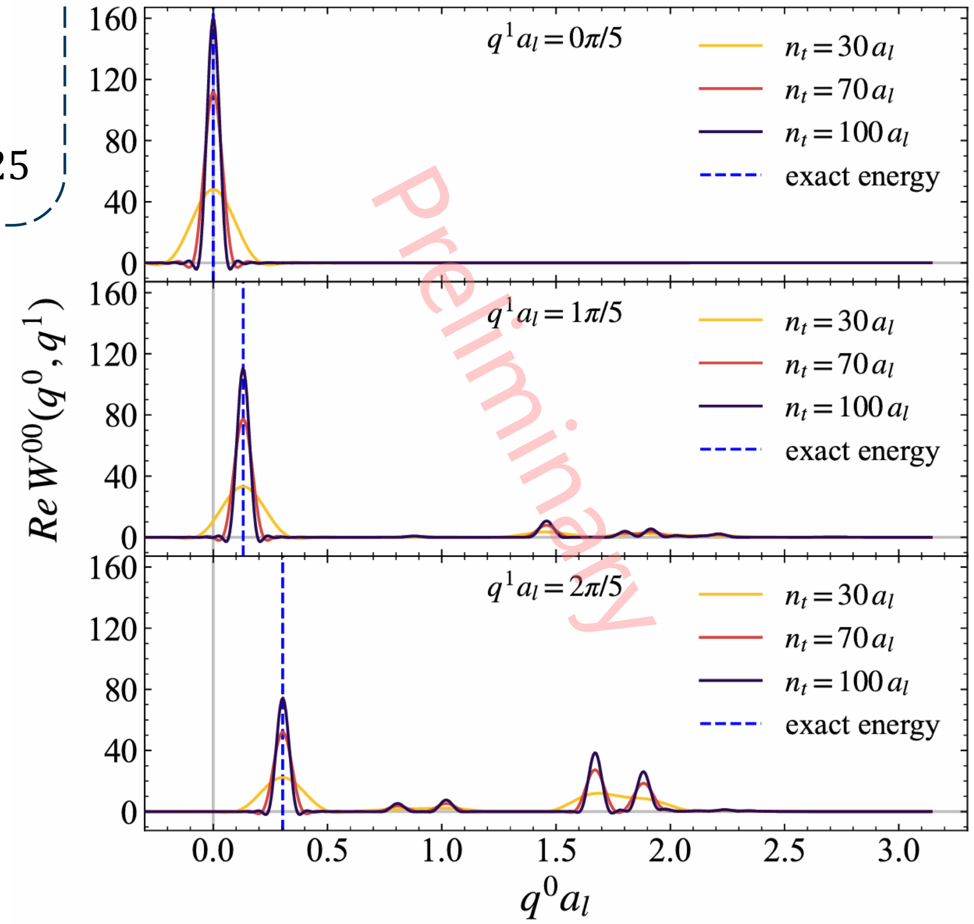
$$W^{\mu\nu}(q^0,q^1) = \int_{-n_t}^{n_t} dt \sum_{\vec{z}} e^{iq^0 t}\, e^{-iq^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t)$$

SU(2) baryon

$L = 20,$
$m = 0.5,$
$g = 1.0,$

$m_{qq}a = 1.25$



Results of the hadronic tensor

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$$W^{\mu\nu}(q^0,q^1) = \int \mathrm{d}t \sum_{\vec{z}} \mathrm{e}^{\mathrm{i}q^0 t}\, \mathrm{e}^{-\mathrm{i}q^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t),$$

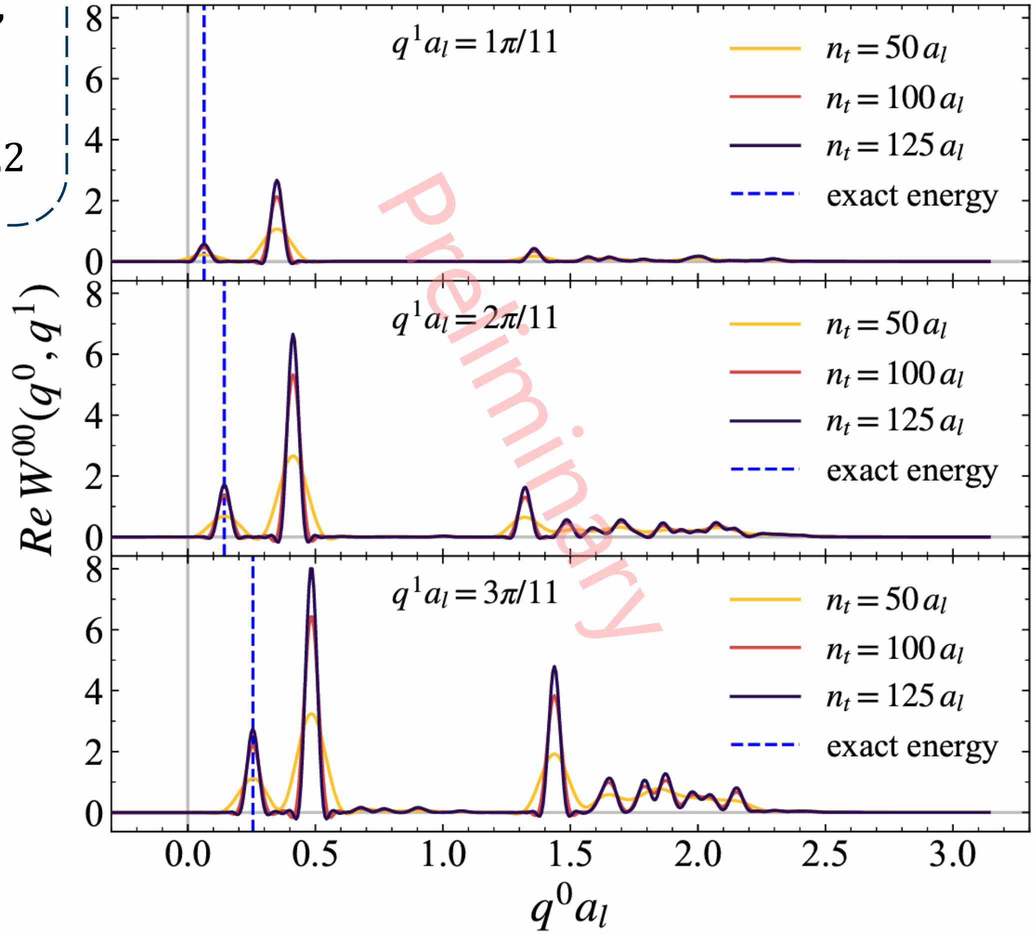
$$\vec{z} = n - m$$

$$W^{\mu\nu}(q^0,q^1) = \int_{-n_t}^{n_t} \mathrm{d}t \sum_{\vec{z}} \mathrm{e}^{\mathrm{i}q^0 t}\, \mathrm{e}^{-\mathrm{i}q^1 z}\, \mathcal{O}^{\mu\nu}(\vec{z},t)$$

U(1) meson

$L = 22,$
$m = 0.45,$
$g = 0.6,$

$m_{q\bar{q}}a = 1.2$



Results of the hadronic tensor

$$W^{\mu\nu}(q) = \int d^2z\ e^{iqz} \langle h|\, J^\mu(z)\, J^\nu(0) |h\rangle$$

Inserting a complete set of intermediate states : $\mathbb{I} = \sum_X \int d\Pi_X |X\rangle\langle X|$

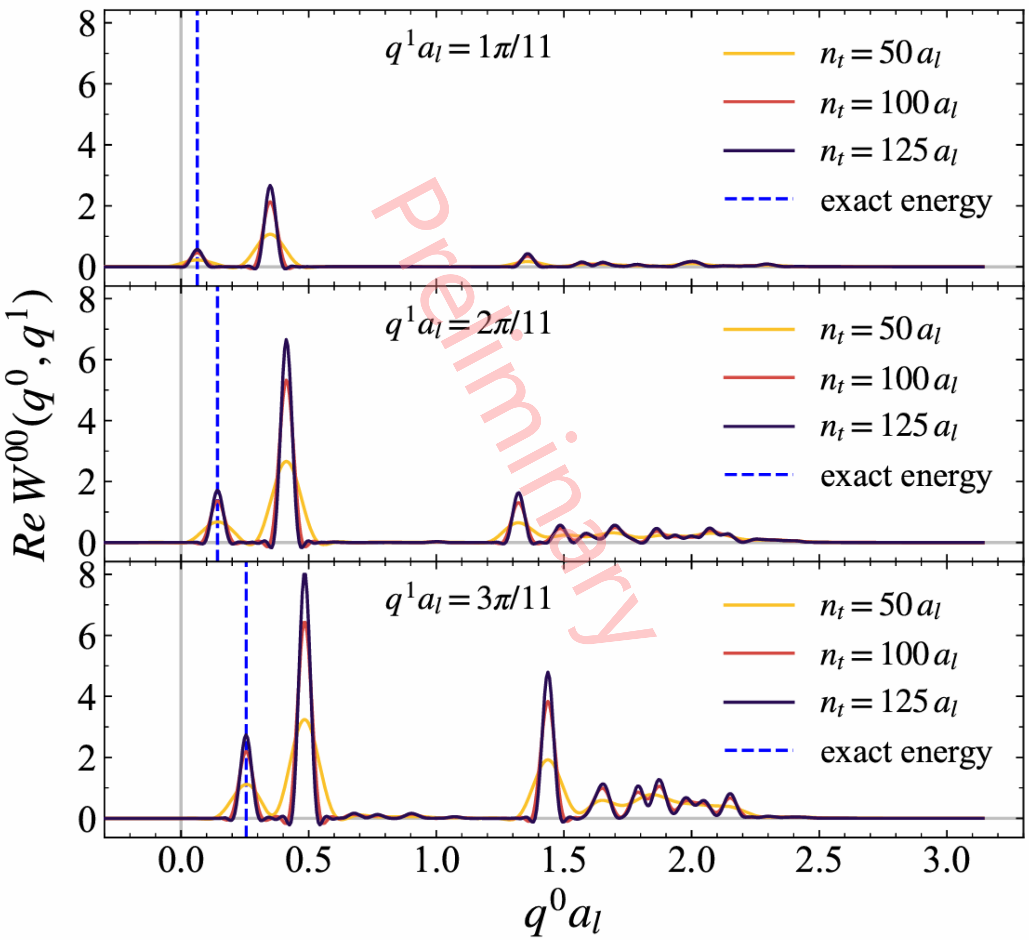
translation invariant : $\langle h|\, J^\mu(z)|X\rangle = e^{i(p-p_X)z} \langle h|\, J^\mu(0)|X\rangle$

$$W^{00}(q^0,q^1) = \sum_X \int d\Pi_X\, \delta^{(2)}(q+p-p_X)\, \langle h|\, J^0(0)\, |X\rangle\langle X|\, J^0(0)\, |h\rangle$$

One-particle final state : $|h'(p')\rangle \in |X\rangle$

$$= \sum_{h'} \frac{2\pi}{2E_{h'}} \delta^{(1)}\left(q^0+p^0-p'^0\right)\Big|_{p'^1=q^1+p^1} \langle h|\, J^0(0)\, |h'\rangle\langle h'|\, J^0(0)\, |h\rangle$$

Two- or multi-particle final state : continuous spectrum



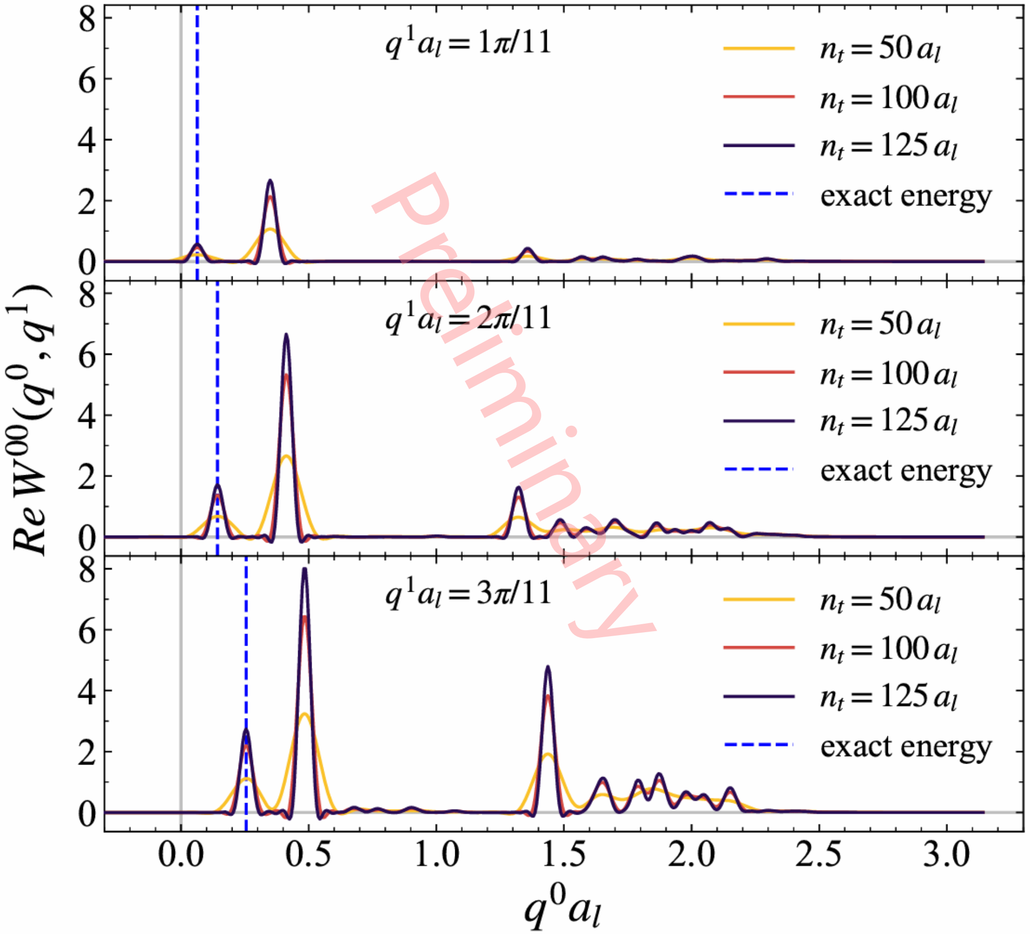
Extract form factor from hadronic tensor

- $$W^{00}(q^0,q^1) = \sum_{h'} \frac{2\pi}{2E_{h'}} \delta^{(1)}\left(q^0 + p^0 - p'^0\right)\Big|_{p'^1=q^1+p^1} \times \langle h| J^0(0) |h'\rangle\langle h'| J^0(0) |h\rangle + \text{others}$$

Integrating over a region R in q^0 to extract the matrix element square :

$$\int_R dq^0\, W^{00} = \frac{2\pi}{2E_{h'}} \langle h| J^0(0) |h'\rangle\langle h'| J^0(0) |h\rangle$$

The first peak ~ $\langle h(p)| J^0(0) |h(p')\rangle\langle h(p')| J^0(0) |h(p)\rangle$



Results of form factor

- $$W^{00}(q^0,q^1) = \sum_{h'} \frac{2\pi}{2E_{h'}} \delta^{(1)}\left(q^0 + p^0 - p'^0\right)\Big|_{p'^1=q^1+p^1} \times \langle h| J^0(0) |h'\rangle\langle h'| J^0(0) |h\rangle + \text{others}$$

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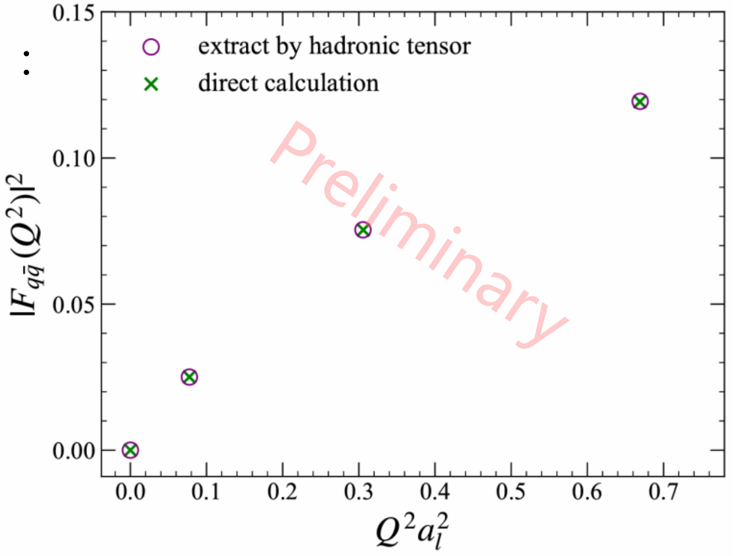
- In the case of scalar particles :

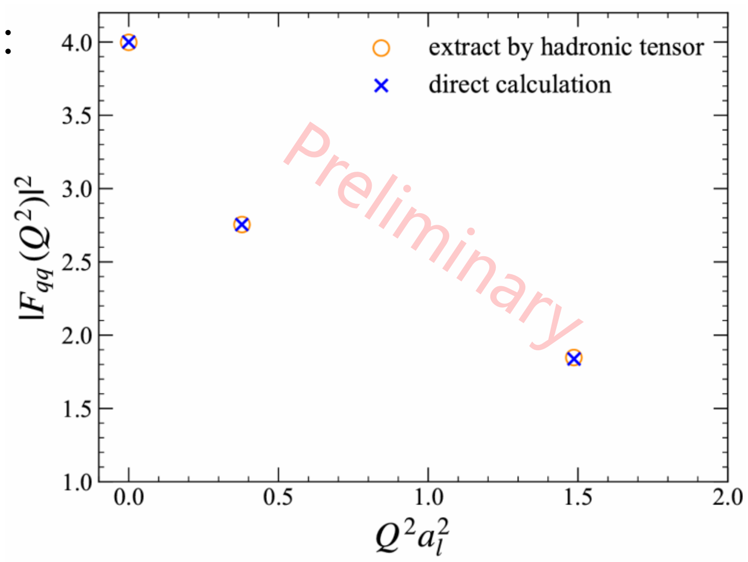
  $$\langle h(p')| J^\mu(0) |h(p)\rangle = (p+p')^\mu F(Q^2)$$

  $$F(0) = charge$$

U(1) meson :



SU(2) baryon :



Summary

- We simulate the hadronic tensor of $(1+1)$-dimensional LGT by quantum algorithms.
  - meson state in $U(1)$ and $SU(2)$ LGT and baryon state in $SU(2)$ LGT
  - extract the form factor from hadronic tensor
- The results shows the ability of QC to study nucleon structure and scattering.
- The study of larger energy transfers requires an increased number of qubits.

Thank you!

Back up

Extract form factor from hadronic tensor

- $$W^{00}(q^0, q^1) = \sum_{h'} \frac{2\pi}{2E_{h'}} \delta^{(1)}\left(q^0 + p^0 - p'^0\right)\Big|_{p'^1=q^1+p^1} \times \langle h| J^0(0) |h'\rangle\langle h'| J^0(0) |h\rangle + \text{others}$$

  Integrating over a region R in q^0 to extract the matrix element square :

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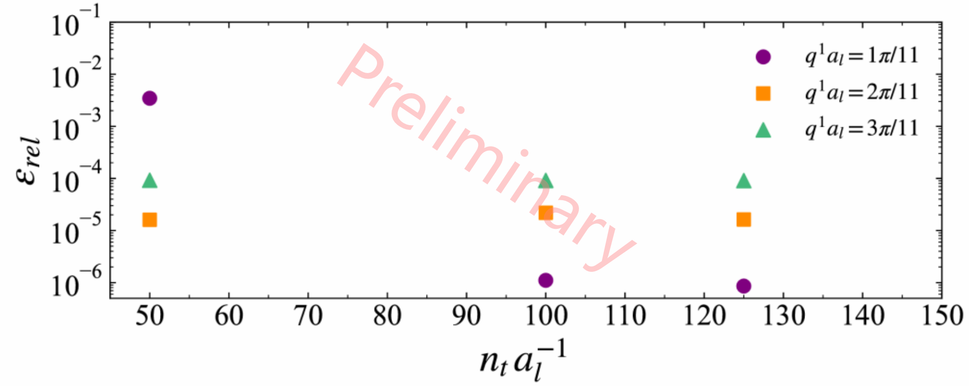
  The first peak ~ $\langle h(p)| J^0(0) |h(p')\rangle\langle h(p')| J^0(0) |h(p)\rangle$

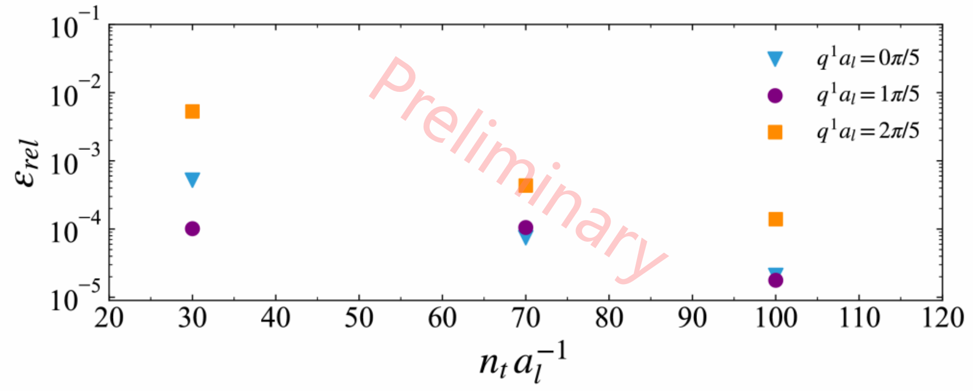
- Relative error for time truncation :

  $$\varepsilon_{rel} = \frac{\left|\,|\mathcal{J}^0_{DC}|^2 - |\mathcal{J}^0_{HT}|^2\right|}{|\mathcal{J}^0_{DC}|^2}$$

U(1) LGT :



SU(2) LGT :



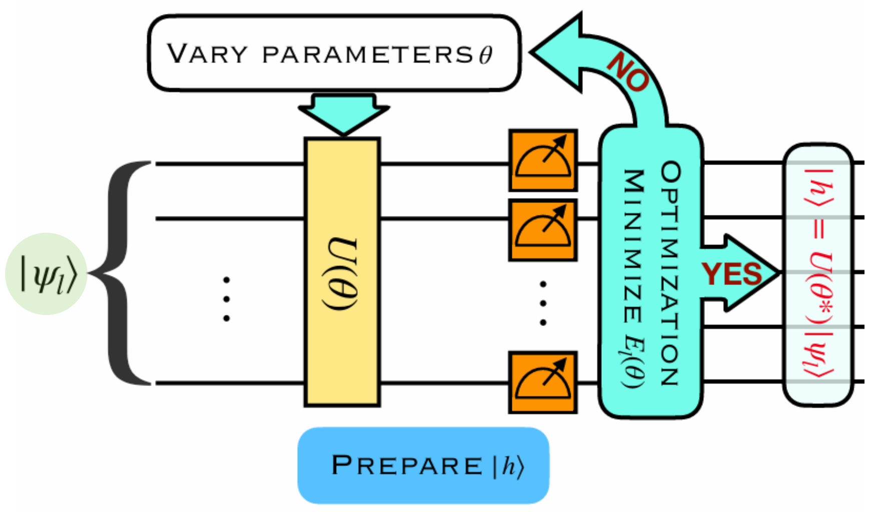
Quantum alternating operator ansatz(QAOA)

- SU(2) Hamiltonian :

$$\bar{H}_{\rm kin} = -\frac{i}{2a_l}\sum_{n=0}^{N-1}(\sigma^+_{2n}\sigma^3_{2n+1}\sigma^-_{2n+2} + \sigma^+_{2n+1}\sigma^3_{2n+1}\sigma^-_{2n+2} - h.c.)\,,$$

$$\bar{H}_{\rm m} = m\sum_{n=0}^{N-1}(-1)^n\left[\frac{1}{2}(\sigma^3_{2n}+\sigma^3_{2n+1})+1\right]\,,$$

$$\bar{H}_{\rm e} = \frac{g^2 a_l}{2N^2}\sum_a\sum_{n=0}^{N-1}\left[\sum_{m=1}^{N}(m-N\theta_{m>n})Q^a_m\right]^2$$



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Divide the Hamiltonian :

$$H_1 = -\frac{i}{2a_l}\sum_{n={\rm even}}^{N-1}(\sigma^+_{2n}\sigma^3_{2n+1}\sigma^-_{2n+2} + \sigma^+_{2n+1}\sigma^3_{2n+1}\sigma^-_{2n+2} - h.c.)\,,$$

$$H_2 = \bar{H}_{\rm m}\,,$$

$$H_3 = H_1(n={\rm even}\to n={\rm odd})\,,$$

$$H_4 = \bar{H}_{\rm e}\,.$$

Satisfying : $[H_i, H_{i+1}]\neq 0\,,\quad [\mathcal{C}, H_i]=0\,,\quad [T, H_i]=0\,,$

Solve the Gauss law in PBC

- Gauss Law in lattice : $L_n^a - R_{n-1}^a = Q_n^a$

average electric field : $\mathcal{E} = \frac{1}{N}\sum_n L_n^a$, then: $L_n^a = \mathcal{E} + \delta L_n^a$

average electric field: $\sum_n \delta L_n^a = 0 \quad (1)$

gauss law: $\delta L_n^a - U_{n-1}^{adj.}\,\delta L_{n-1}^a = Q_m^a \quad (2)$

from (2) , recursively : $$\delta L_n^a = \left(U_{n-1}^{adj.} \dots U_0^{adj.}\right)\delta L_0^a + \sum_{m=1}^{n} Q_m^a \quad (3)$$

then (1) : $$\sum_n \delta L_n^a = \sum_n \left(U_{n-1}^{adj.} \dots U_0^{adj.}\right)\delta L_0^a + \sum_n \sum_{m=1}^{n} Q_m^a$$

$$0 = N\left(U_{n-1}^{adj.} \dots U_0^{adj.}\right)\delta L_0^a + \sum_{m=1}^{N-1}(N-m)Q_m^a$$

We can get : $$\left(U_{n-1}^{adj.} \dots U_0^{adj.}\right)\delta L_0^a = -\frac{1}{N}\sum_{m=1}^{N-1}(N-m)Q_m^a \quad (4)$$

substituting (4) into (3) :

$$\delta L_n^a = -\frac{1}{N}\sum_{m=1}^{N-1}(N-m)Q_m^a + \sum_{m=1}^{n} Q_m^a$$

$$= \frac{1}{N}\sum_{m=1}^{N}(m - N\theta_{m>n})Q_m^a$$

$$L_n^a = \mathcal{E} + \frac{1}{N}\sum_{m=1}^{N}(m - N\theta_{m>n})Q_m^a$$

The lattice gauge theory in (1+1)-dimension

Eliminate the d.o.f. of gauge fields

- Solve the Gauss law in lattice : $L_n^a - R_{n-1}^a = Q_n^a$

$$L_n^a = \mathcal{E}^a + \frac{1}{N}\sum_{m=1}^{N}(m - N\theta_{m>n})Q_m^a$$

average electric field : $\mathcal{E}^a = \frac{1}{N}\sum_n L_n^a$

- Gauge transformation :

$$\psi_n \to \left(\prod_{\mathrm{m}<n} U_\mathrm{m}\right)\psi_n$$

$$\psi_{n+1}^\dagger U_n \psi_n \to \psi_{n+1}^\dagger \psi_n$$

Map the LGT to Pauli matrices

- Jordan-Wigner transformation: $\phi_n \to \left(\prod_{\mathrm{m}<n} \sigma_m^3\right)\sigma_n^-$ where $\psi_n = \begin{pmatrix}\psi_n^r \\ \psi_n^g\end{pmatrix} = \begin{pmatrix}\phi_{2n} \\ \phi_{2n+1}\end{pmatrix}$

The hadronic tensor mapped to qubits :

$$W^{\mu\nu}(q^0, q^1) = \frac{1}{4\pi}\sum_{t,\vec{z}} \mathrm{e}^{\mathrm{i}q^0 t}\,\mathrm{e}^{-\mathrm{i}q^1 z}\,\langle h|\mathrm{e}^{\mathrm{i}Ht}J_n^\mu \mathrm{e}^{-\mathrm{i}Ht}J_m^\nu|h\rangle$$

$$\begin{cases} J_n^0 = \frac{1}{2}(\sigma_{2n}^3 + \sigma_{2n+1}^3) \\ J_n^1 = \sigma_{2n}^+\,\sigma_{2n+1}^- + \sigma_{2n+1}^+\sigma_{2n}^- \end{cases}$$

Extract form factor from hadronic tensor

$$W^{\mu\nu}(q) = \int d^2z\, \mathrm{e}^{\mathrm{i}qz} \langle h|J^\mu(z)J^\nu(0)|h\rangle$$

Inserting a complete set of intermediate states :

$$\mathbb{I} = \sum_X \int d\Pi_X |X\rangle\langle X|$$

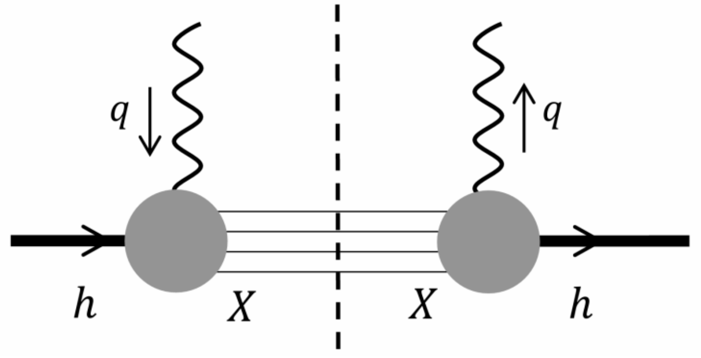
$$W^{\mu\nu}(q) = \sum_X \int d^2z\, d\Pi_X\ \mathrm{e}^{\mathrm{i}qz} \langle h|J^\mu(z)|X\rangle\langle X|J^\nu(0)|h\rangle$$

$$= \sum_X \int d^2z\, d\Pi_X\ \mathrm{e}^{\mathrm{i}qz}\mathrm{e}^{\mathrm{i}(p-p_X)z} \langle h|J^\mu(0)|X\rangle\langle X|J^\nu(0)|h\rangle$$

$$= \sum_X \int d\Pi_X\ (2\pi)^2\delta^{(2)}(q+p-p_X) \langle h|J^\mu(0)|X\rangle\langle X|J^\nu(0)|h\rangle$$

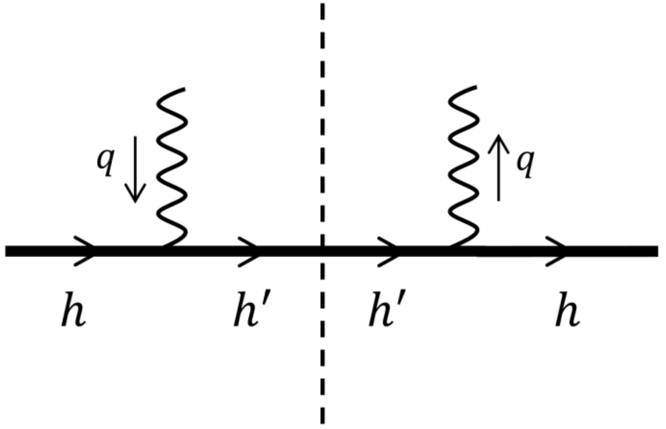
One-particle final state : $|h'(p')\rangle \in |X\rangle$

$$= \sum_{h'} \delta^{(1)}\left(q^0+p^0-p'^0\right)\Big|_{q^1=p'^1-p^1} \langle h|J^\mu(0)|h'(p')\rangle\langle h'(p')|J^\nu(0)|h\rangle + \text{ others}$$



$|X\rangle \to |h'(p')\rangle$

$and\ \ \delta^{(1)}\left(q^0+p^0-p'^0\right)\Big|_{q^1=p'^1-p^1}$



Results

- Hadronic vacuum polarization:

HVP: $\Pi^{\mu\nu} = \int d^2 z \ \mathrm{e}^{\mathrm{i}qz} \langle\Omega|\, T\{\, J^{\mu}(z)\, J^{\nu}(0)\, \}\, |\Omega\rangle$