Nucleon Gluon PDF from Lattice QCD

Liuming Liu(刘柳明)
Institute of Modern Physics, CAS

In collaboration with
Chen Chen， Hong-Xin Dong, Peng Sun, Yi-Bo Yang,
Fei Yao, Jian-Hui Zhang, Shiyi Zhong

CLQCD

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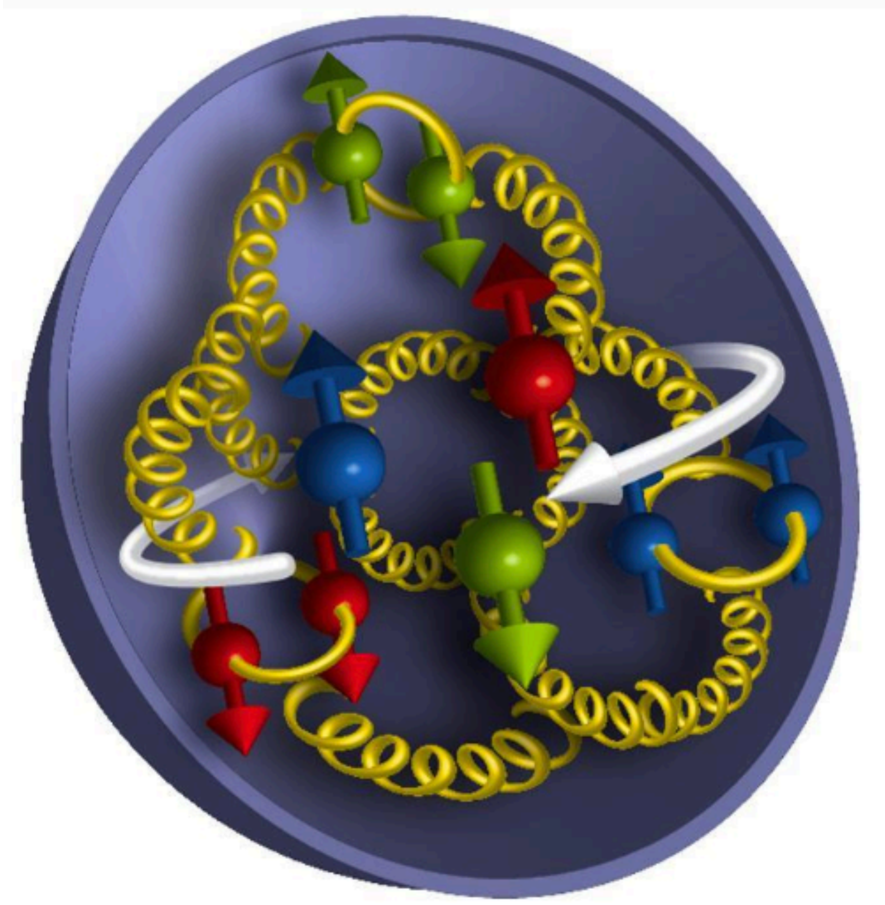
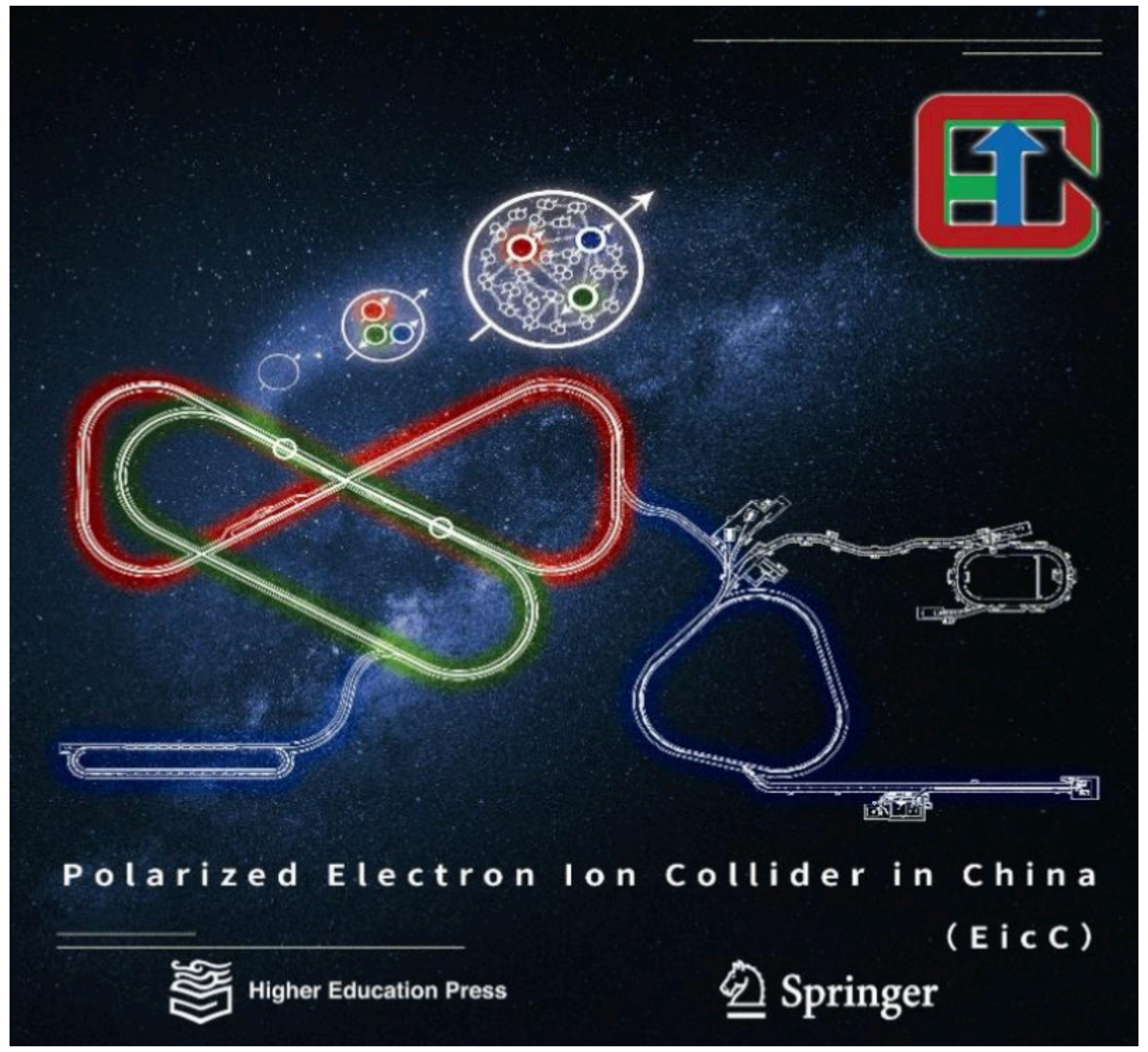
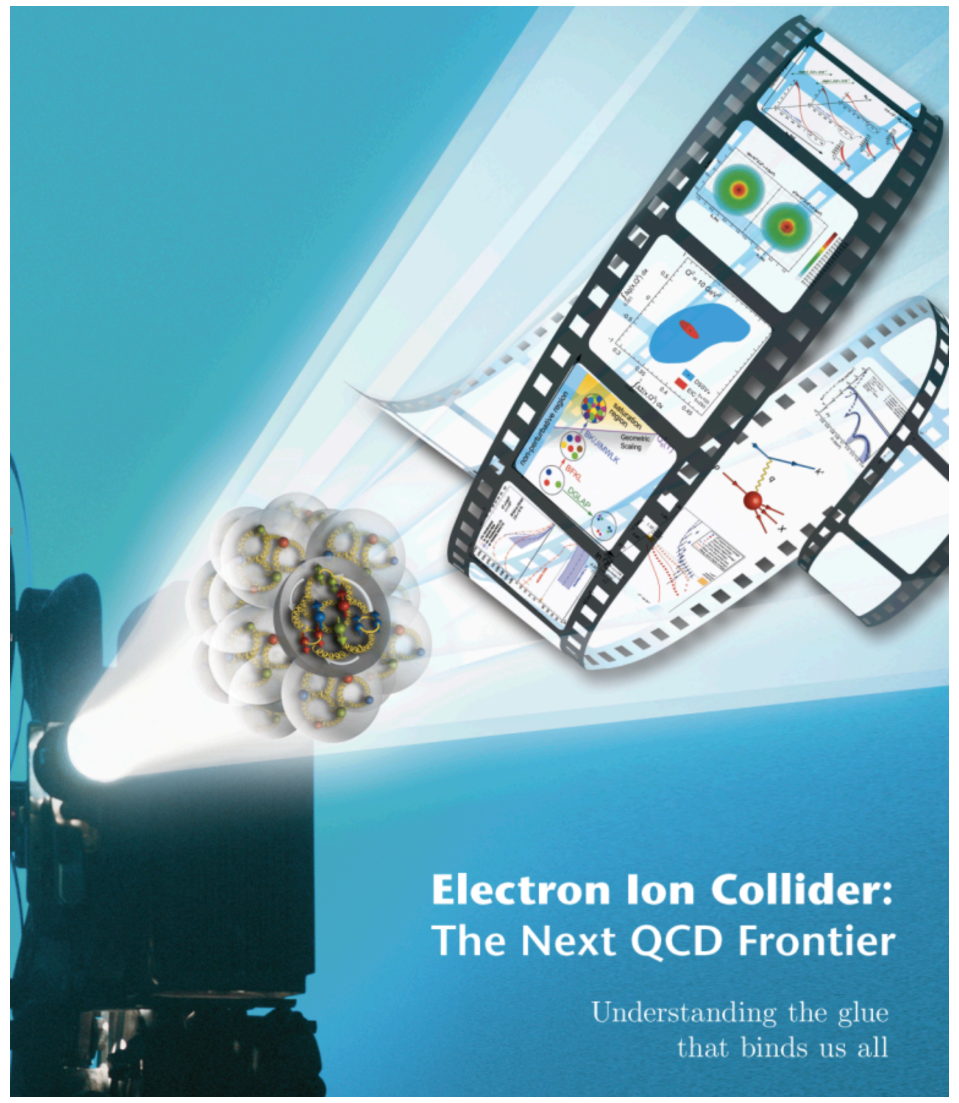
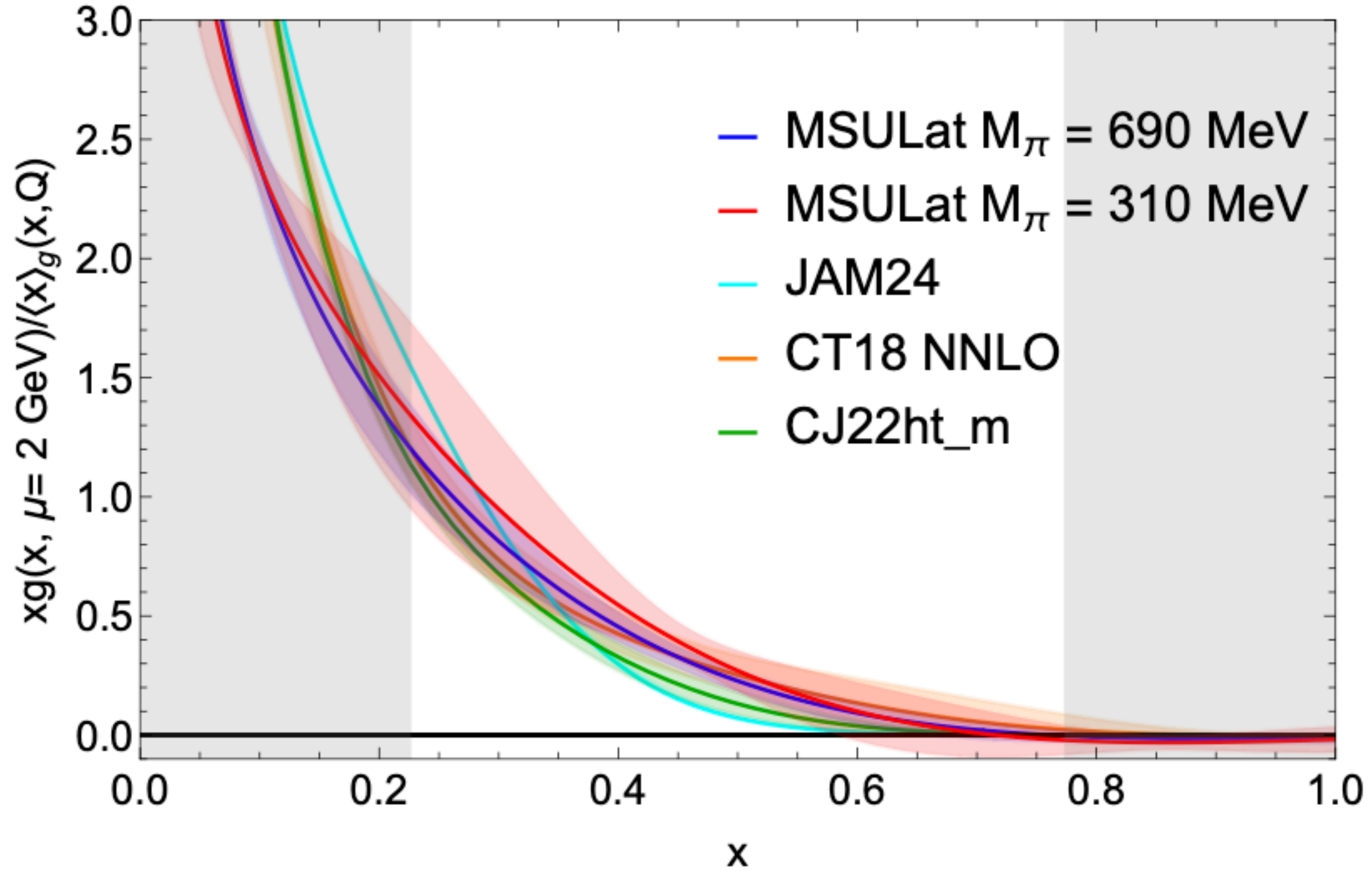
Introduction

- Gluons: the mediators of the strong force, play a significnat role in the internal structure of hadrons.
- Global fit: model dependent, weak constraints at large x region.
- Lattice QCD: first principle inputs





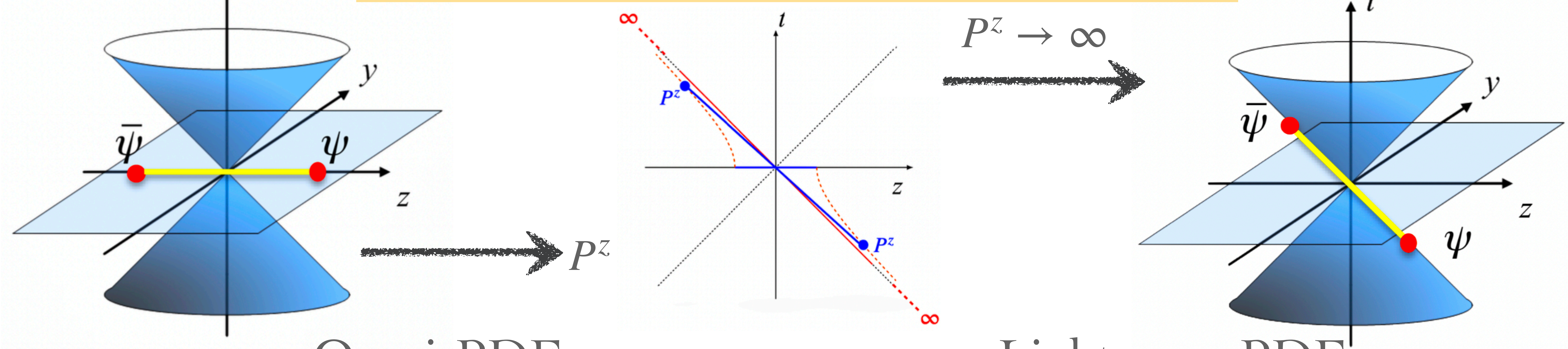




W. Good el al., arXiv:2505.13321

PDFs on lattice

Large momentum effective theory(LaMET)



Quasi-PDF Light-cone PDF

- Light-cone: take limit $P_z \to \infty$ before the renormalization.
- Lattice: renormalize before taking limit $P_z \to \infty$
- The two limits do no commute.
- The difference is UV physics, perturbative matching can connect the quasi-PDFs (lattice) and the light-cone PDFs.

CLQCD Configurations

CLQCD

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
0.105 fm	$24^3 \times 72$	292	1000
	$32^3 \times 64$		1000
	$48^3 \times 96$		800
	$64^3 \times 128$		70
	$32^3 \times 64$	225	450
	$48^3 \times 96$		700
	$48^3 \times 96$	135	700
	$64^3 \times 128$		200
0.090 fm	$32^3 \times 64$	287	900
0.077 fm	$24^3 \times 72$	300	250
	$32^3 \times 96$		480
	$48^3 \times 96$		200
	$32^3 \times 64$	210	460
	$48^3 \times 96$		200
	$64^3 \times 128$	135	180
0.069 fm	$36^3 \times 108$	300	700
0.052 fm	$48^3 \times 144$	317	1000
	$64^3 \times 128$		100

- $2 + 1$ dynamical quark flavor
- Symanzik-improved gauge action.
- Wilson-clover quark action.
- Quark masses m_u, m_d, m_s are computed based on these ensembles and the results agree with FLAG average value.
- Quark propagators using distillation smearing.

Z.-C.Hu el al., (CLQCD), PRD109(2024)5,054507

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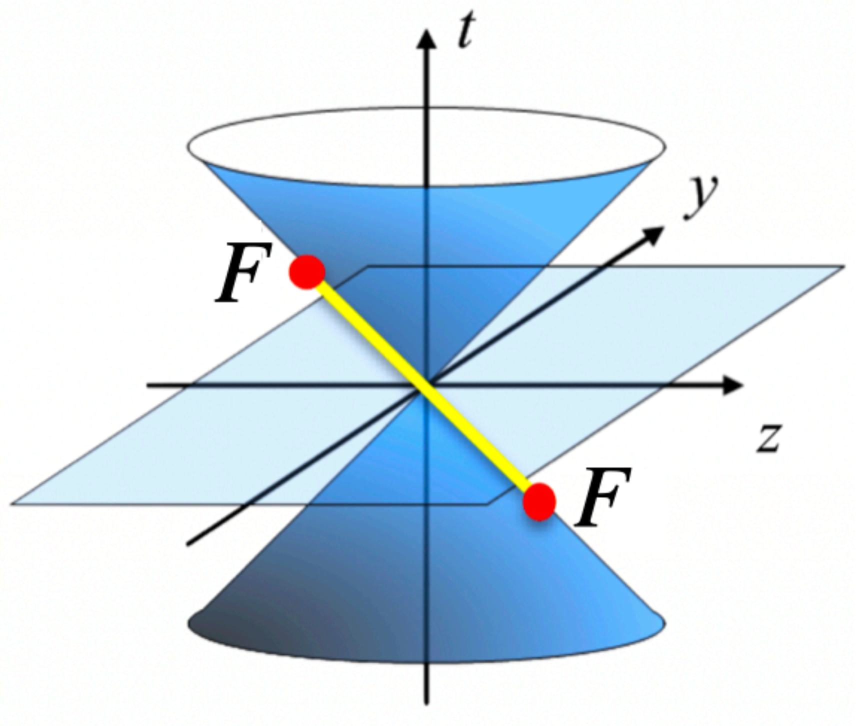
Gluon quasi-PDF matrix element

- The light-cone gluon PDF is defined as:

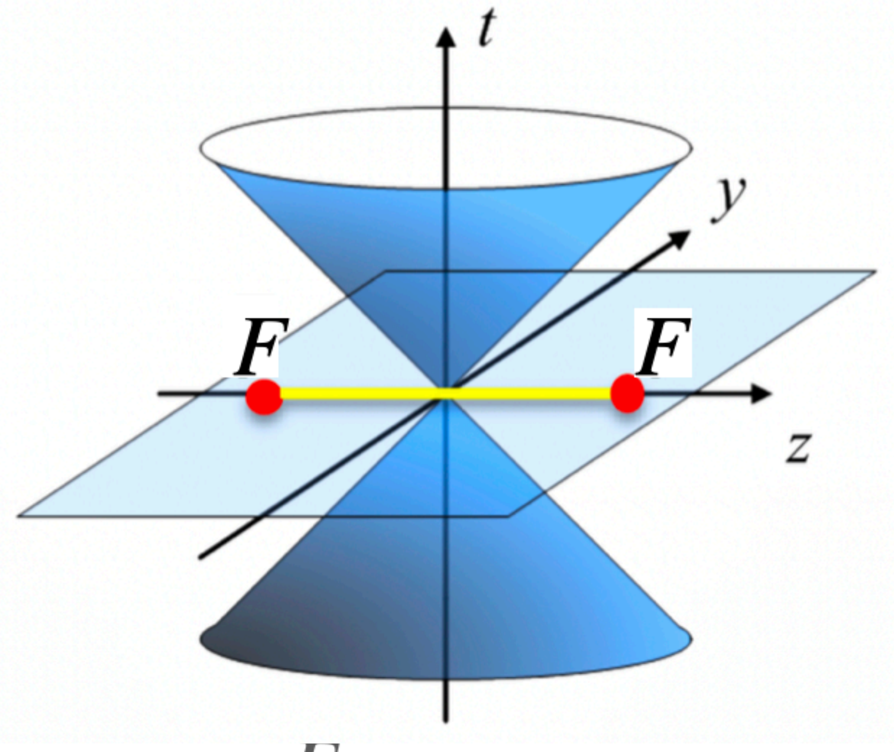
$$g(x,\mu) = \frac{1}{2xP^+}\int_{-\infty}^{+\infty}\frac{d\xi^-}{2\pi}e^{-ixP^+\xi^-}\langle P|F_a^{+\mu}(\xi^-)\mathscr{W}_{ab}(\xi^-,0)F^+_{b,\mu}(0)|P\rangle$$



- According to LaMET, the gluon pdf can be extracted from a spatially correlated matrix element:

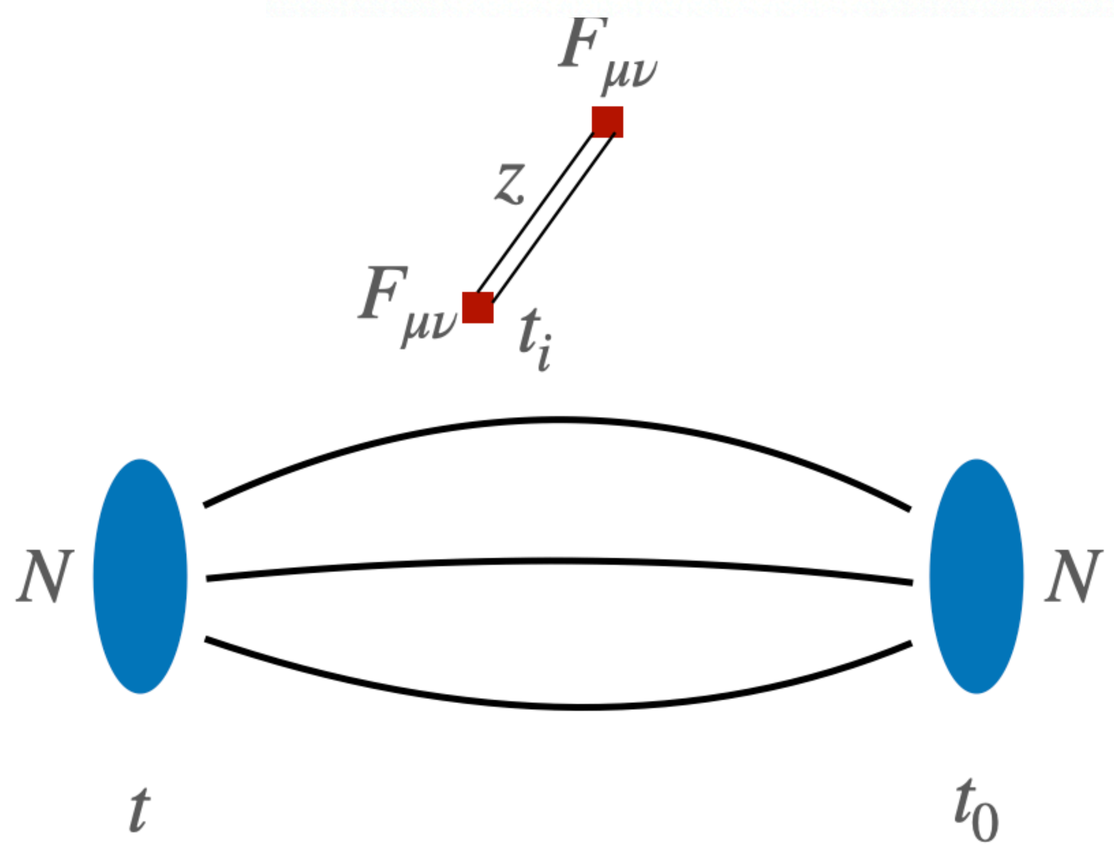
$$C_{3pt}(z,P_z,t,t_i) = \langle 0|N(t,P_z)\mathcal{O}_{\mu\nu}(z,t_i)\bar{N}(t_0,P_z)|0\rangle$$

$$\mathcal{O}_{\mu\nu}(z,t_i) = \sum_x Tr[F_{\mu\nu}(x,t_i)L(x,x+z)F_{\nu\mu}(x+z,t_i)L(x+z,x)]$$



The gluon operator:

$$\mathcal{O}_G = \mathcal{O}_{tx} + \mathcal{O}_{ty} - 2\mathcal{O}_{xy}$$



Noise reduction techniques

CLQCD

❖ Distillation quark smearing method

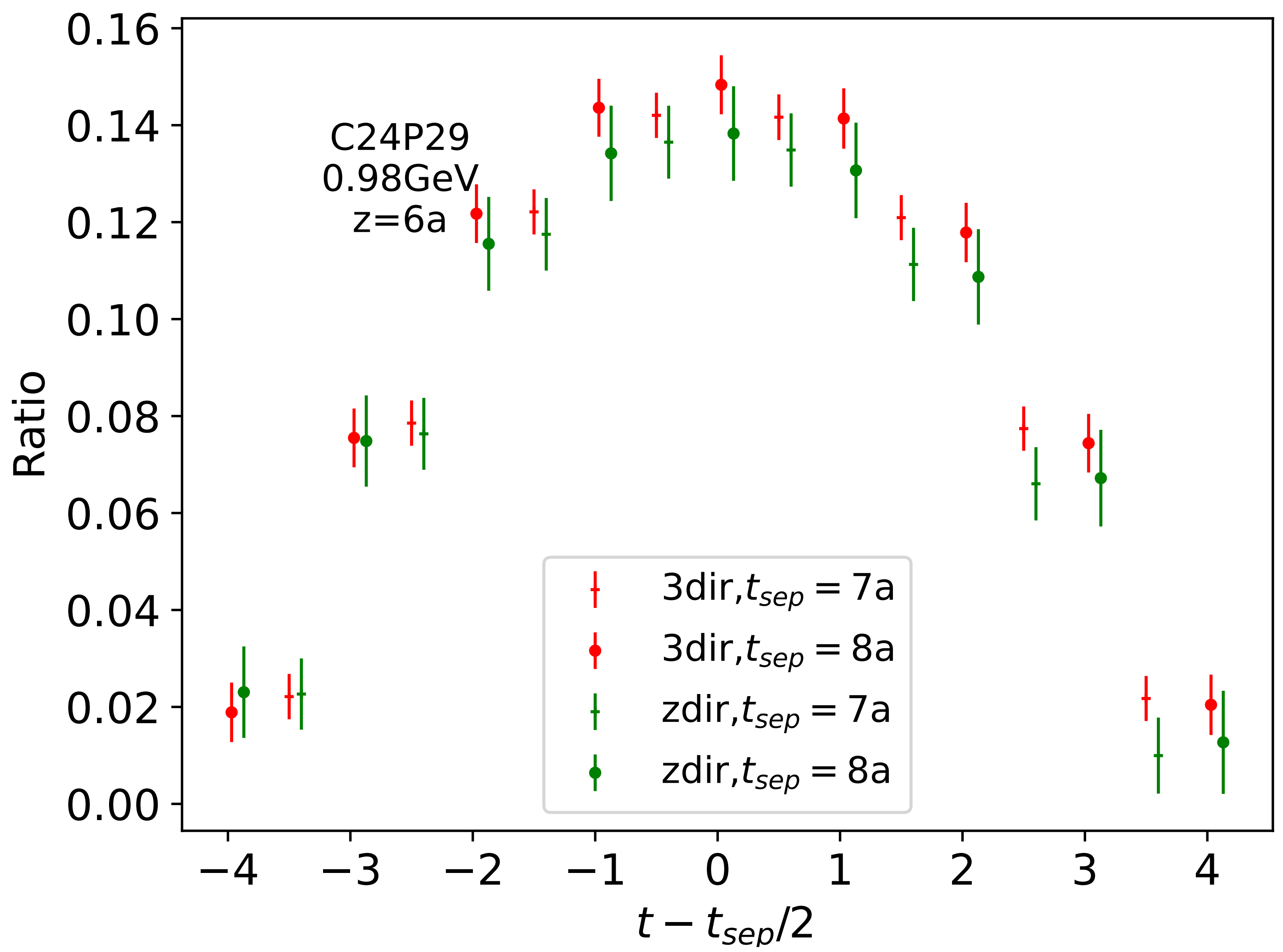
- Smearing: an operator that effectively projects onto the hadronic states of interest.
- Distillation smearing operator: $\square = \sum_{k=1}^{N} v_k \otimes v_k^*$

v_k is the k'th eigenvector of the laplacian operator defined on the gauge configuration.

- Improve precision, suppress excited-states contaminations, all-to-all propagators, efficient computation with many interpolating operators…

❖ Use the rotational symmetry:

$$C_{3pt} = \frac{1}{3}[C_{3pt}(z, p_z, t, t_i) + C_{3pt}(x, p_x, t, t_i) + C_{3pt}(y, p_y, t, t_i)]$$



Bare matrix elements

Keeping the ground state and the first excited-state, the three- and two-point functions can be decomposed as:

$$C^{3pt}(z;P_z;t,t_{sep}) = |A_0|^2 \langle 0|\mathcal{O}_G(z;t)|0\rangle e^{-E_0 t_{sep}} + |A_1|^2 \langle 1|\mathcal{O}_G(z;t)|1\rangle e^{-E_1 t_{sep}}$$

$$+ A_1 A_0^* \langle 1|\mathcal{O}_G(z;t)|0\rangle e^{-E_1(t_{sep}-t)} E^{-E_0 t} + A_0 A_1^* \langle 0|\mathcal{O}_G(z;t)|1\rangle e^{-E_0(t_{sep}-t)} E^{-E_1 t} + \cdots$$

$$C^{2pt}(P_z;t_{sep}) = |A_0|^2 e^{-E_0 t_{sep}} + |A_1|^2 e^{-E_1 t_{sep}} + \cdots$$

The ground state matrix elements can be extracted from the ratio of three- to two-point function :

$$R = \frac{C^{3pt}}{C^{2pt}} = c_0 + c_1 e^{-\Delta E(t_{sep}-t)} + c_1 e^{-\Delta E\, t} + c_2 e^{-\Delta E\, t_{sep}} + \cdots$$

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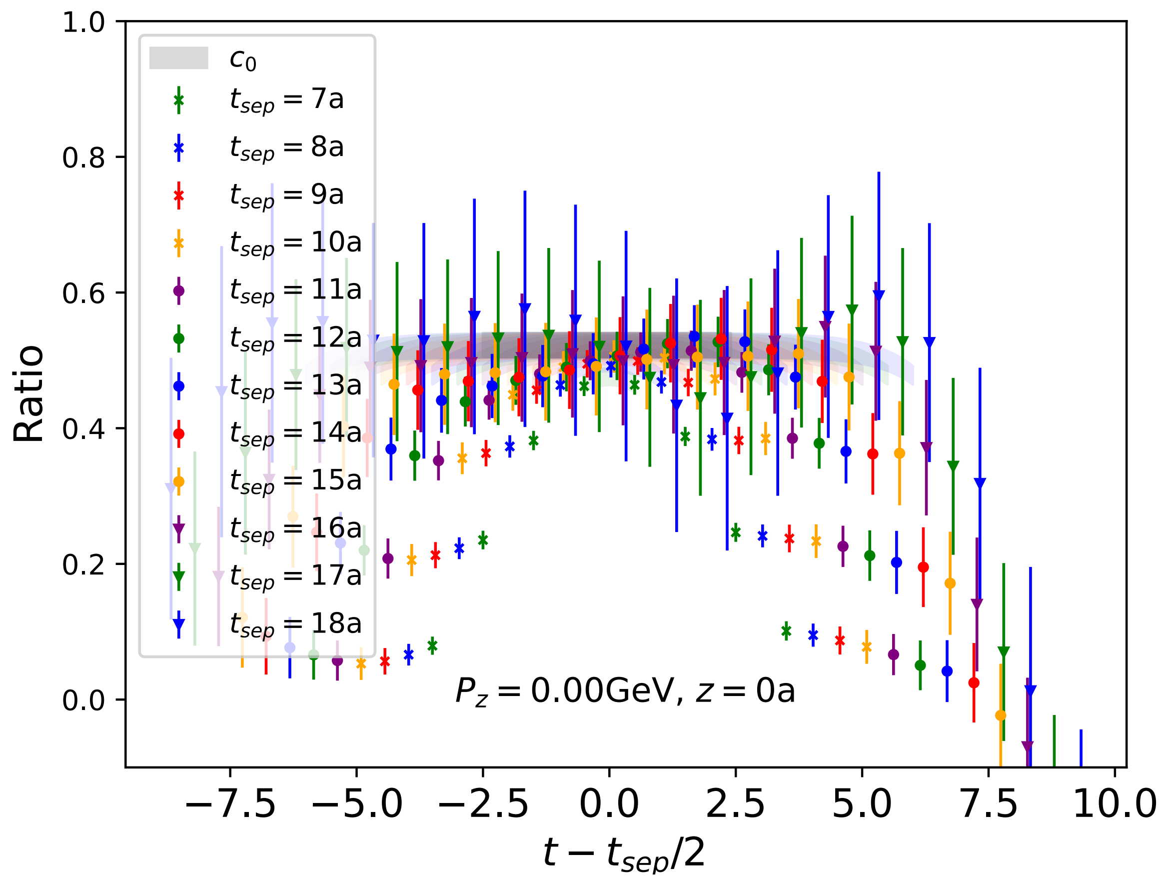
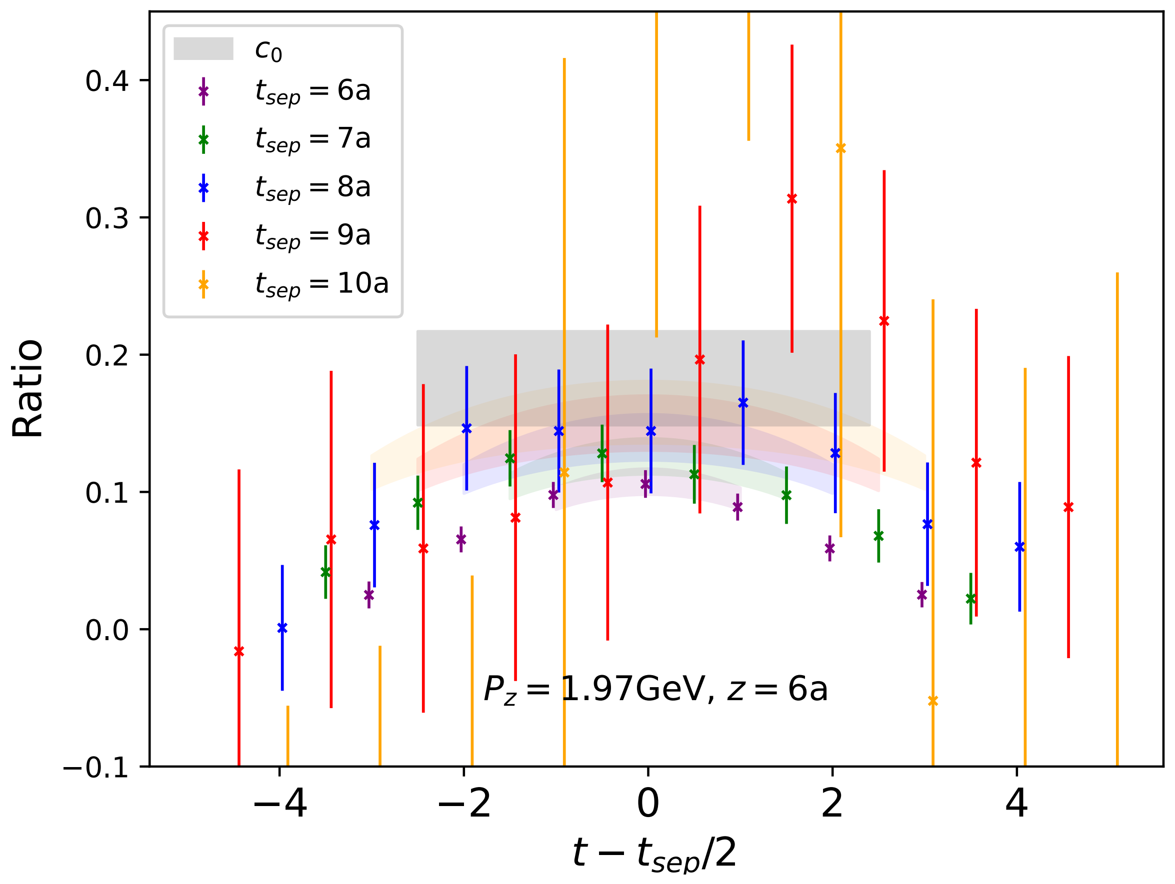
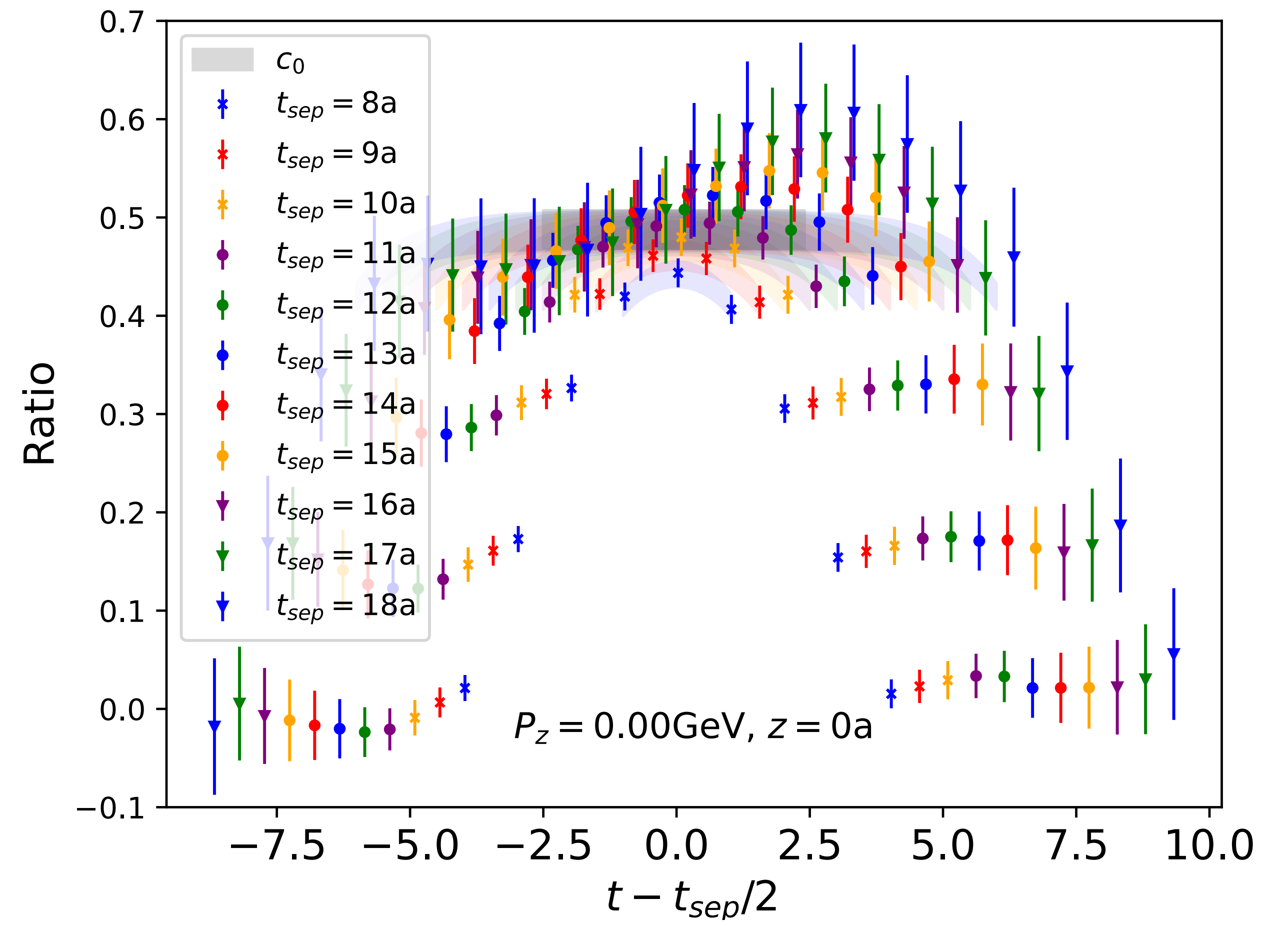
Bare matrix elements

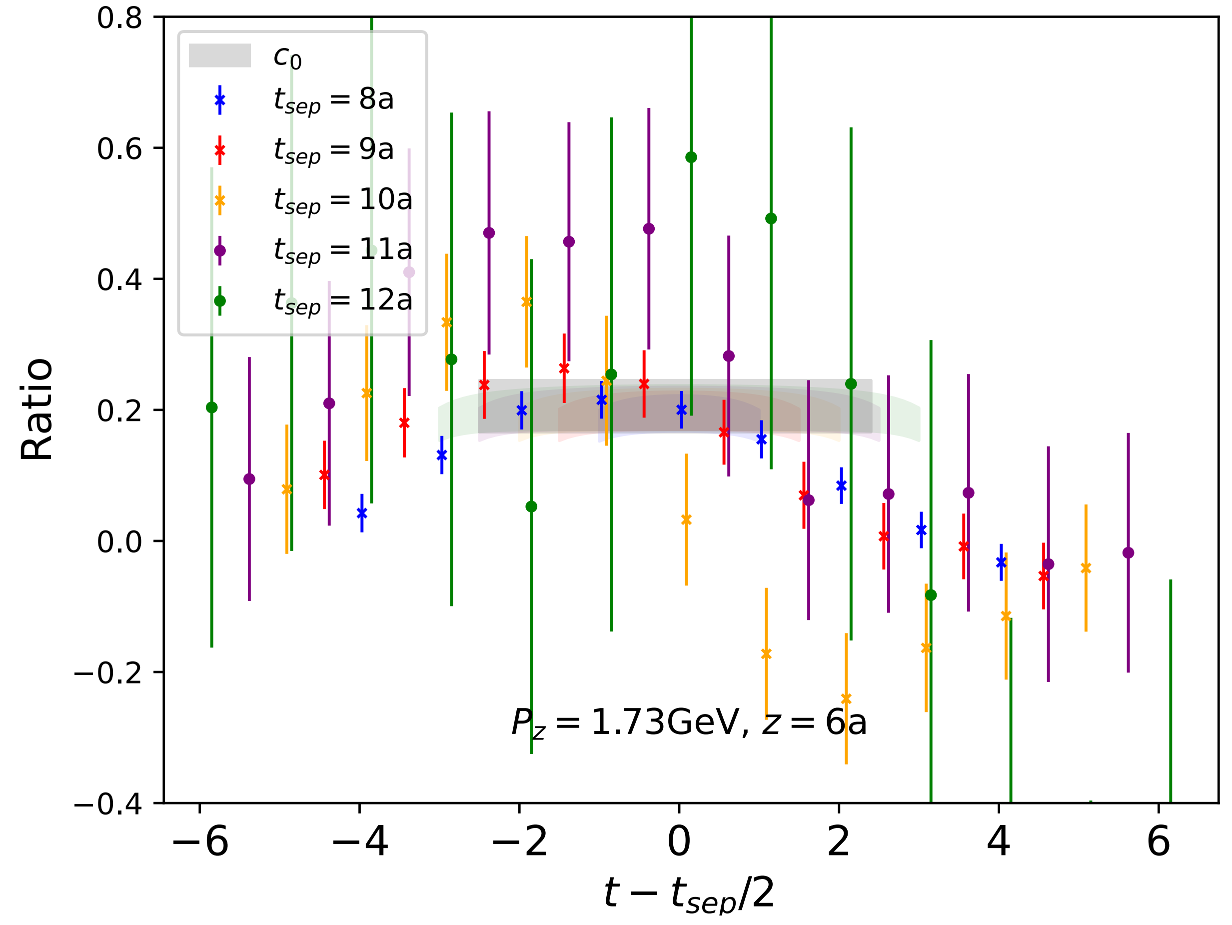
$a = 0.105$fm



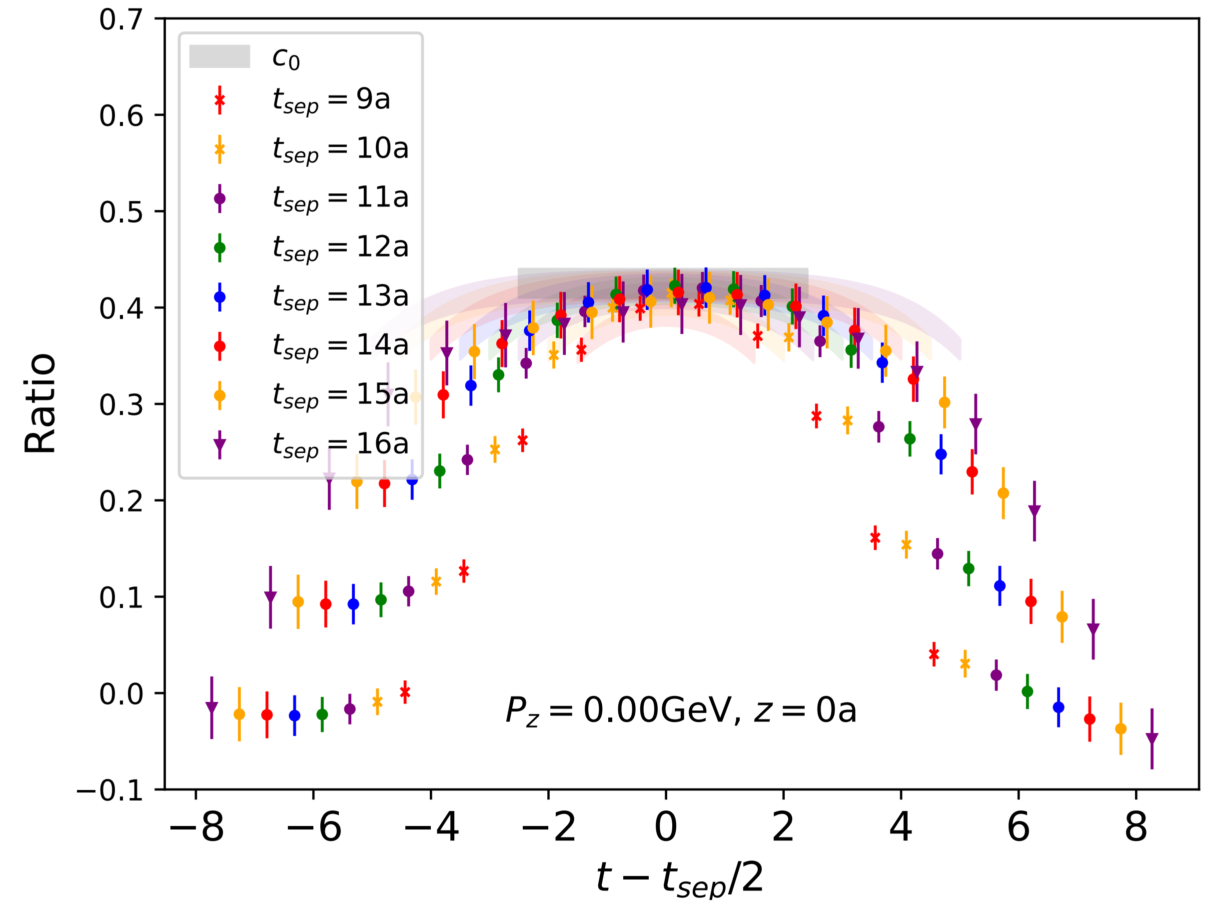


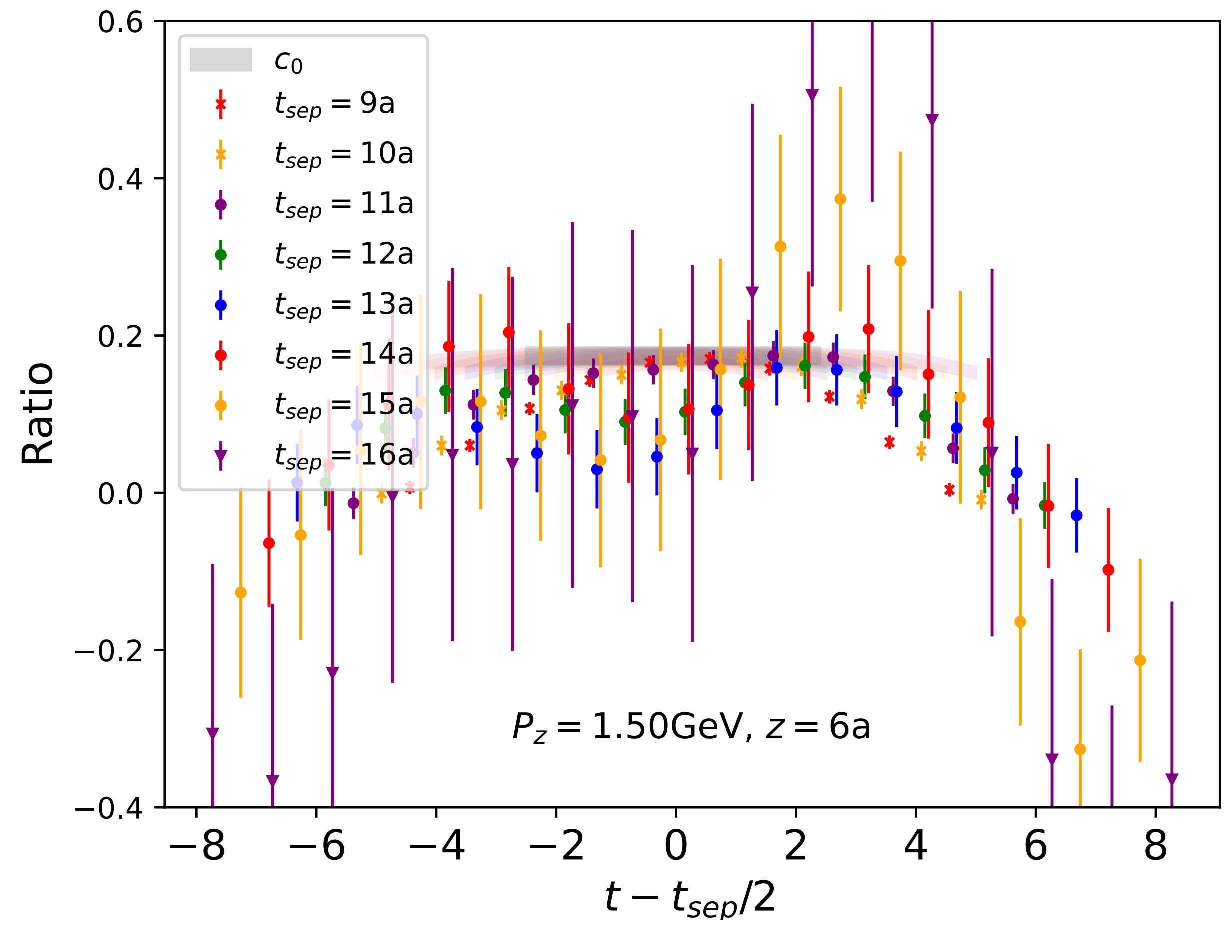
$a = 0.090$fm





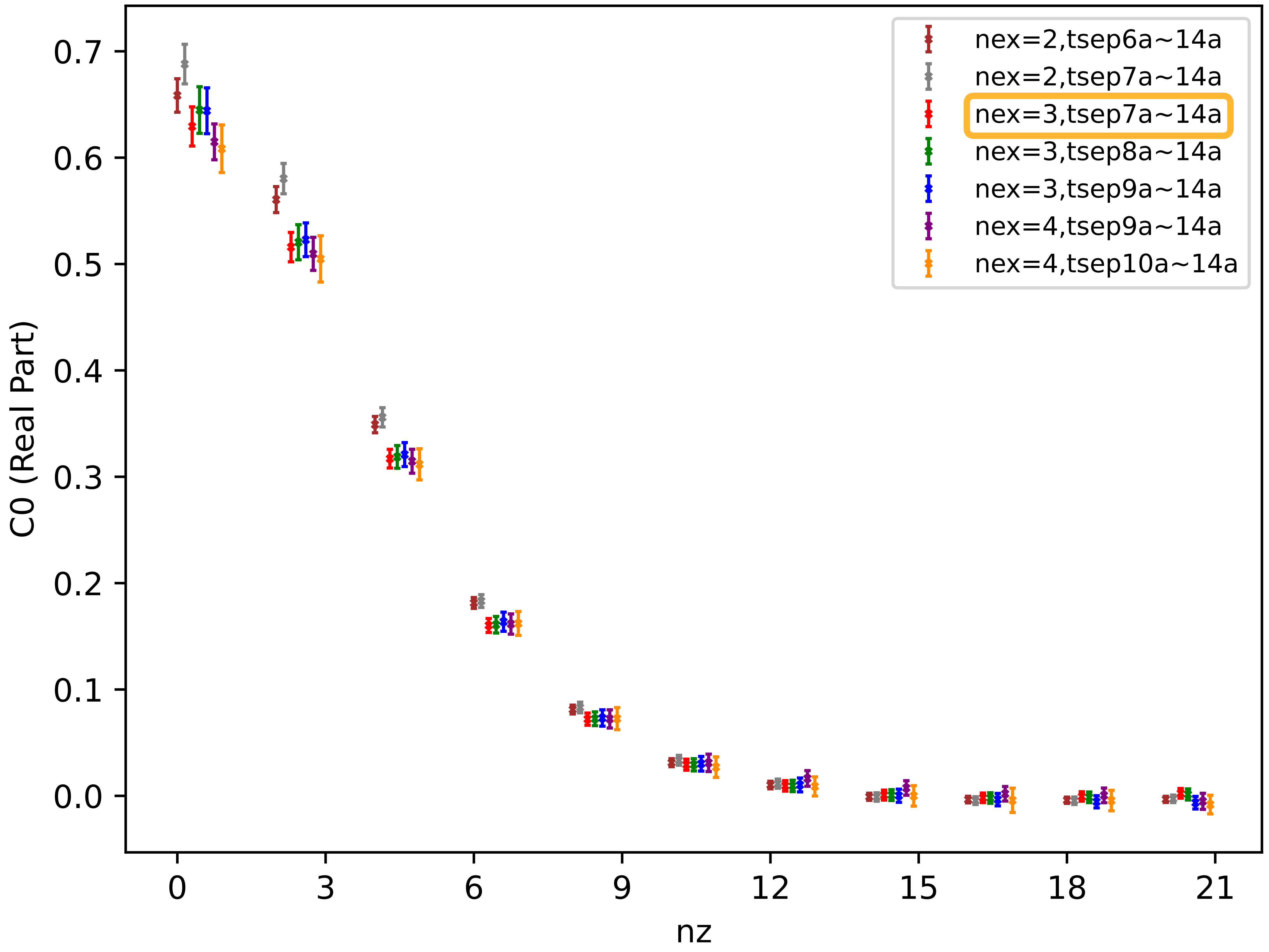
$a = 0.077$fm





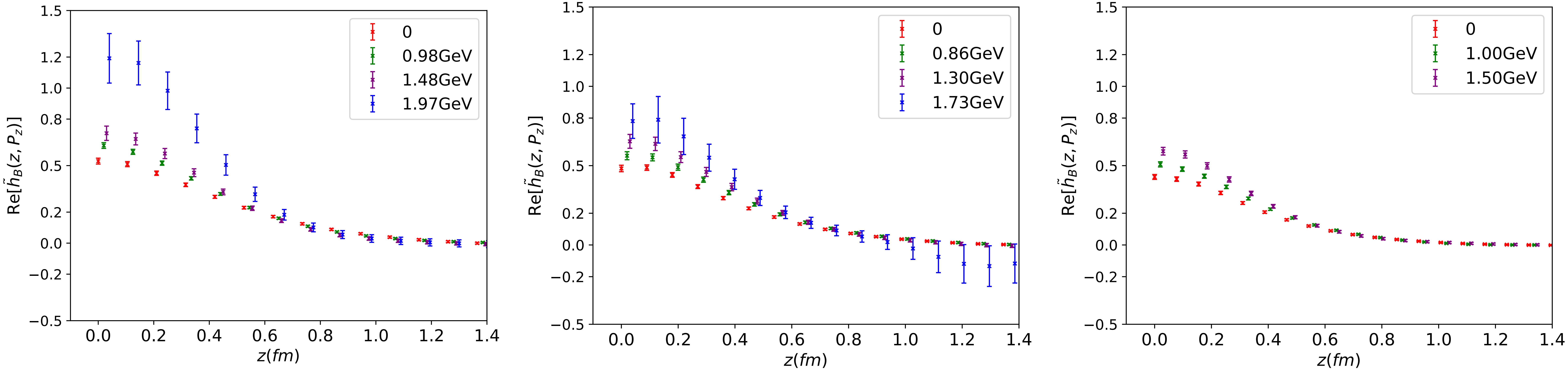
Bare matrix elements

Choose the fit ranges:



Bare matrix elements

$a = 0.105\text{fm}$ $a = 0.090\text{fm}$ $a = 0.077\text{fm}$



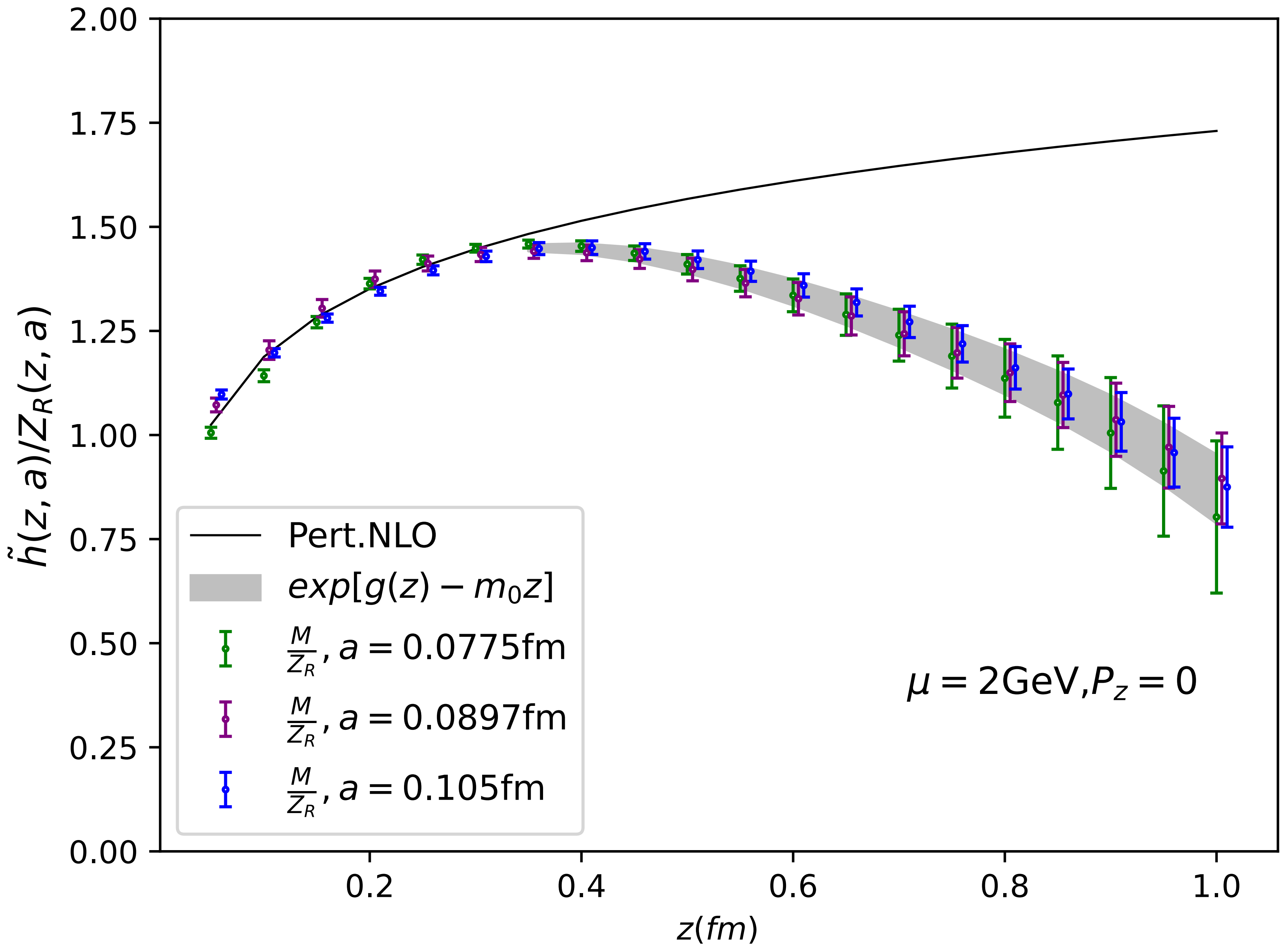
Renormalization

Hybrid renormalization：

$$\tilde{h}_R(z,P_z) = \frac{\tilde{h}^n_B(z,P_z,1/a)}{\tilde{h}^n_B(z,P_z=0,1/a)}\theta(z_s-|z|) \quad \text{(Ratio scheme)}$$

$$+\eta_s\frac{\tilde{h}^n_B(z,P_z,1/a)}{Z_R(z,1/a)}\theta(|z|-z_s) \quad \text{(Self-renormalization)}$$

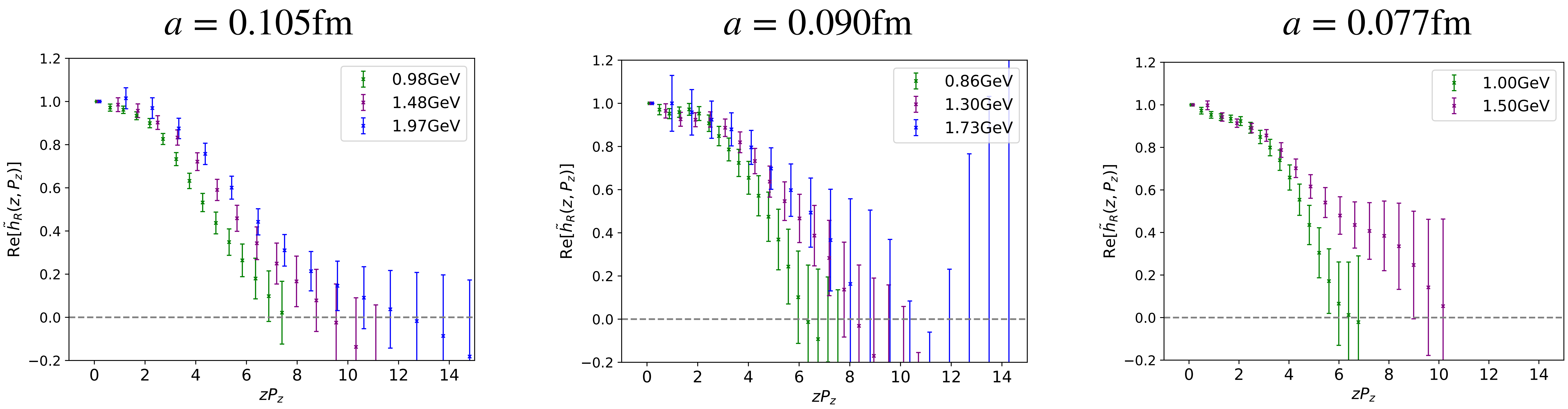
$$Z_R(z,1/a,\mu) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]}+m_0 z + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right]+\frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2\right\}$$



parameter	fit result
k	1.93(0.61)
$\Lambda_{\rm QCD}$(GeV)	0.61(0.09)
m_0(GeV)	2.94(1.36)
d	−0.051(0.02)
χ^2/d.o.f.	0.09

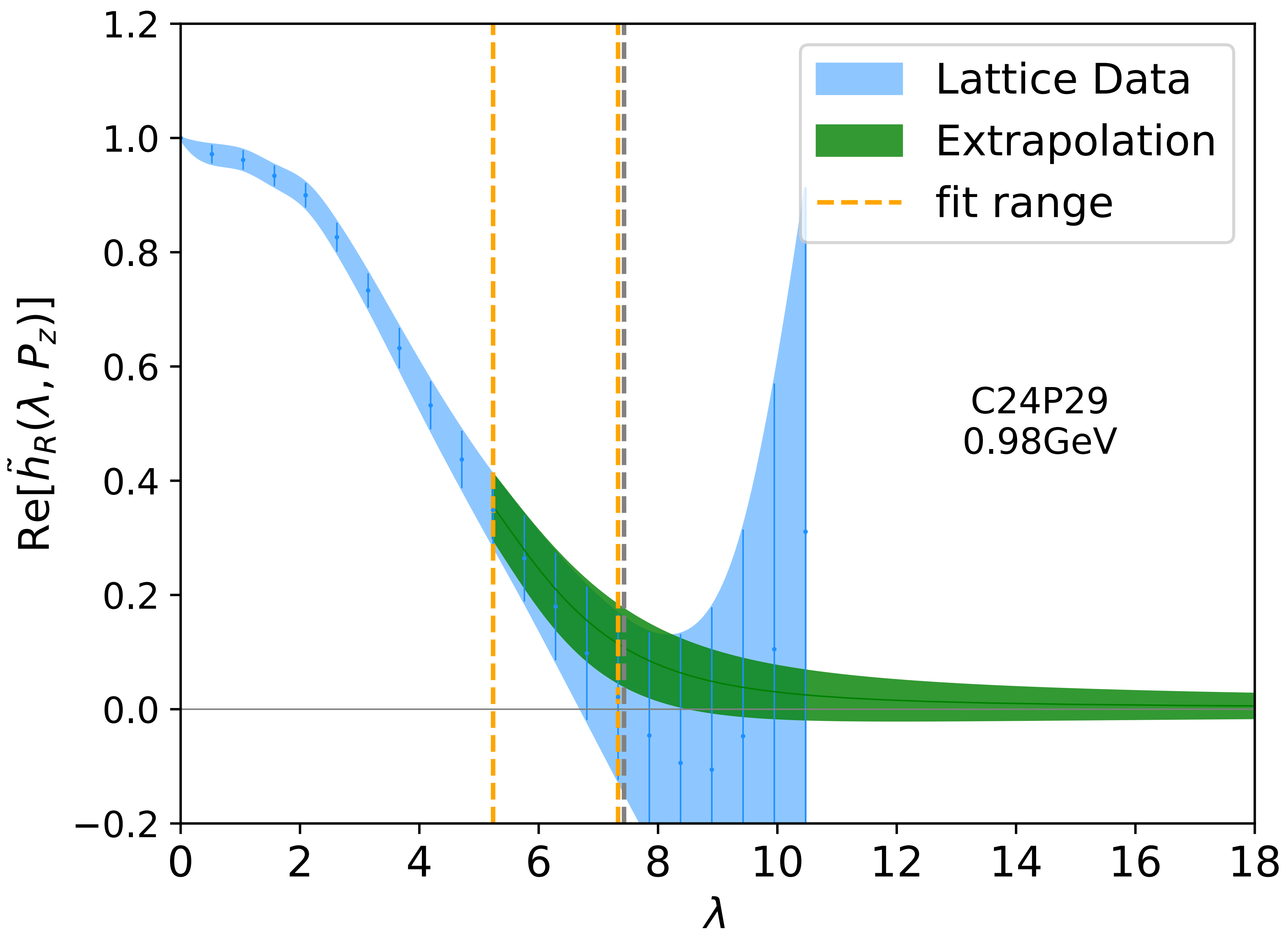
Renormalization

Renormalized matrix elements:



Renormalization

Large $\lambda = zP_z$ extrapolation: $\tilde{h}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0}$



Matching to light-cone PDF

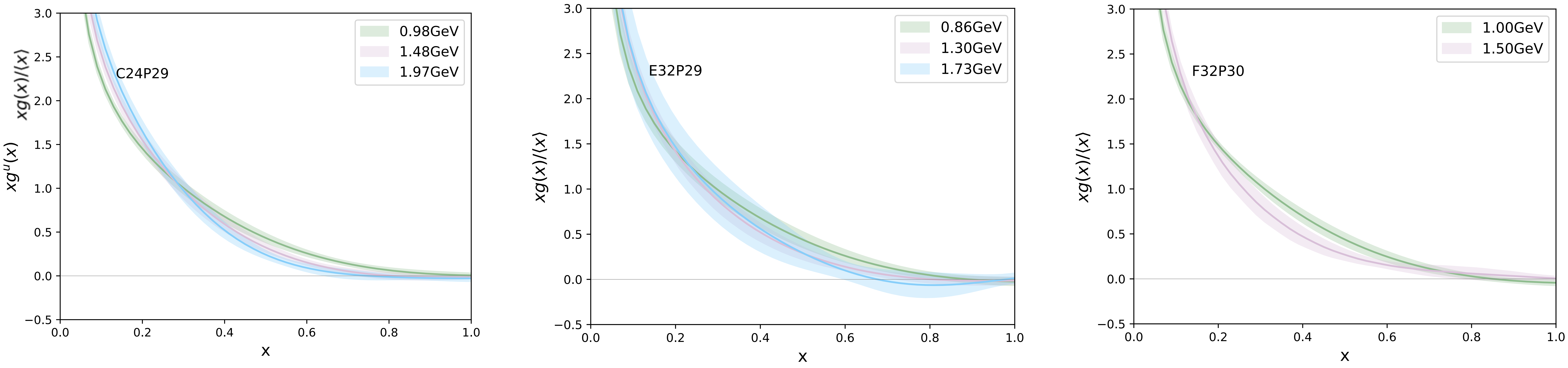
$$x\tilde{g}(x,P^z) = \int_{-1}^{1} \frac{dy}{|y|} C_{\text{hybrid}}\left(\frac{x}{y}, \lambda_s, \frac{\mu}{yP^z}\right) yg(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{(xP^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)P^z)^2}\right)$$

Quasi PDF (Lattice)

Matching kernel (Perturbative)

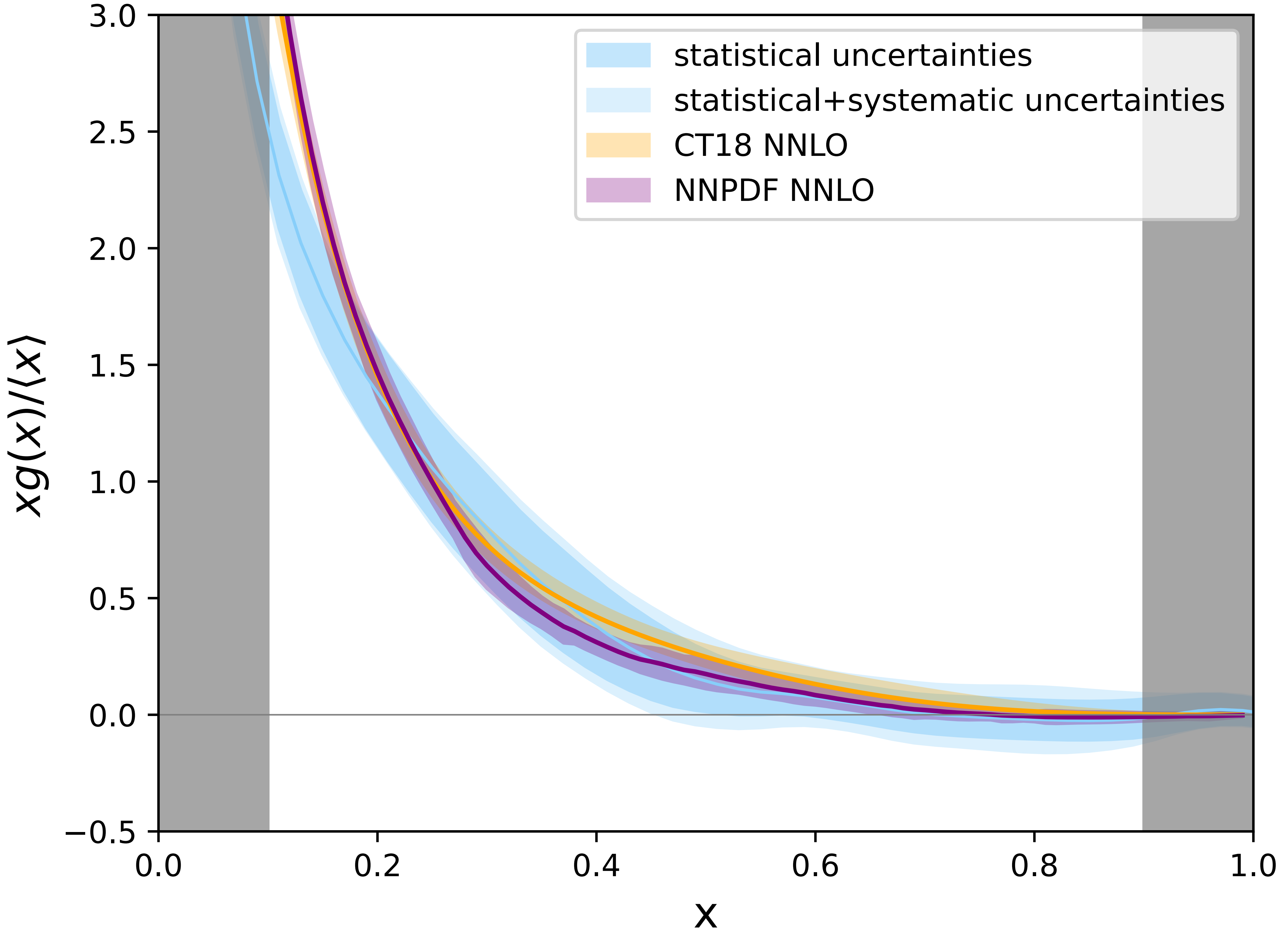
Light-cone PDF

$a = 0.105\text{fm}$ $a = 0.090\text{fm}$ $a = 0.077\text{fm}$



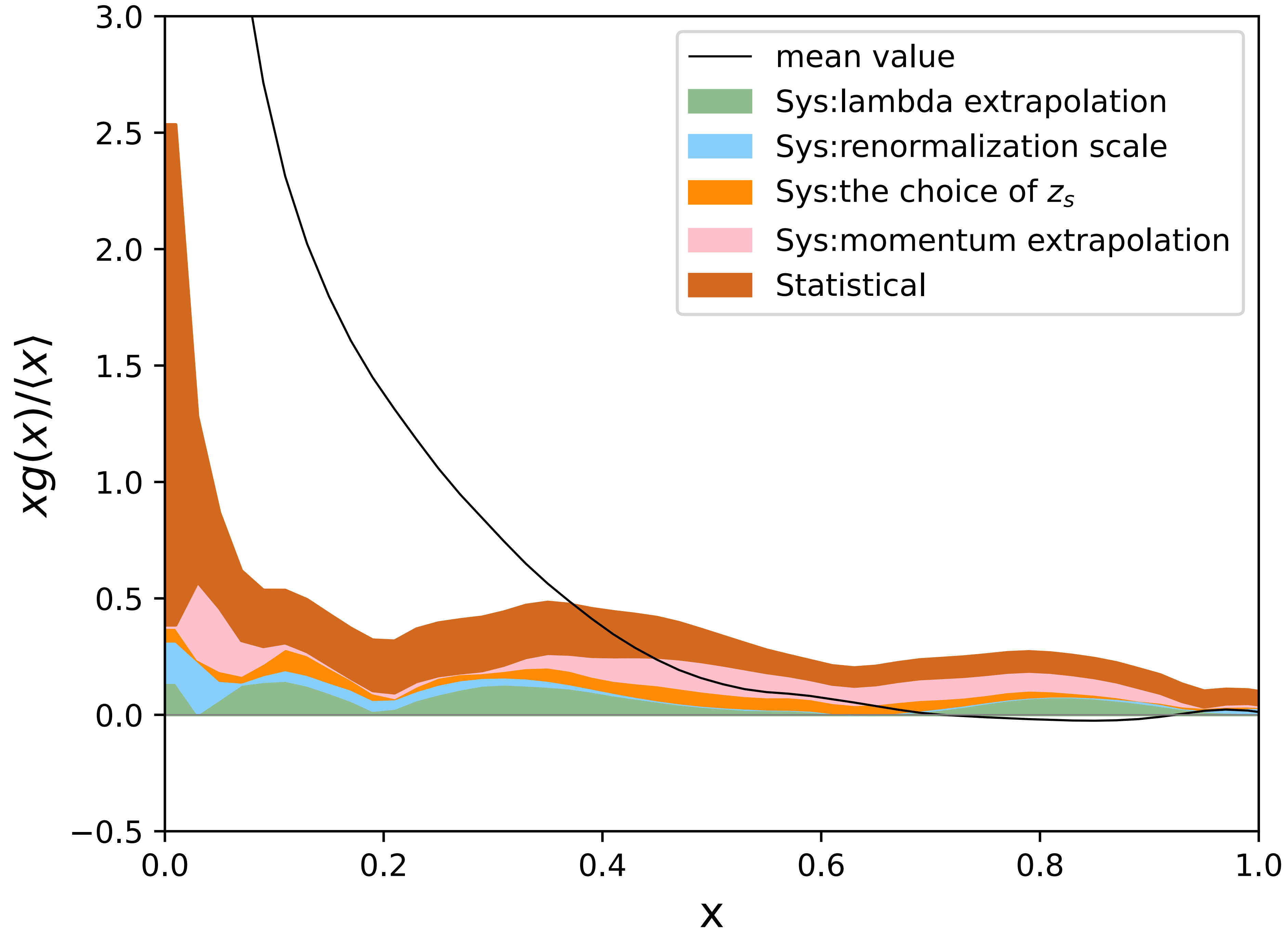
Large momentum and continuum extrapolations

$$xg(x,P_z,a) = xg_0(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2}$$



Systematics:

- λ extrapolation
- Renormalization scale dependence
- Infinite momentum and continuum extrap.
- Hybrid scheme z_s



Summary and outlook

- Unpolorized gluon pdf of the nucleon is computed at $M_\pi \sim 300$MeV in the framework of LaMET. The result agrees with the global fit results.
- Mixing with quarks is ignored.
- Furthur improve the signal: momentum smearing for large momentum, gradient flow…
- More lattice spacings, physical pion mass.
- ……

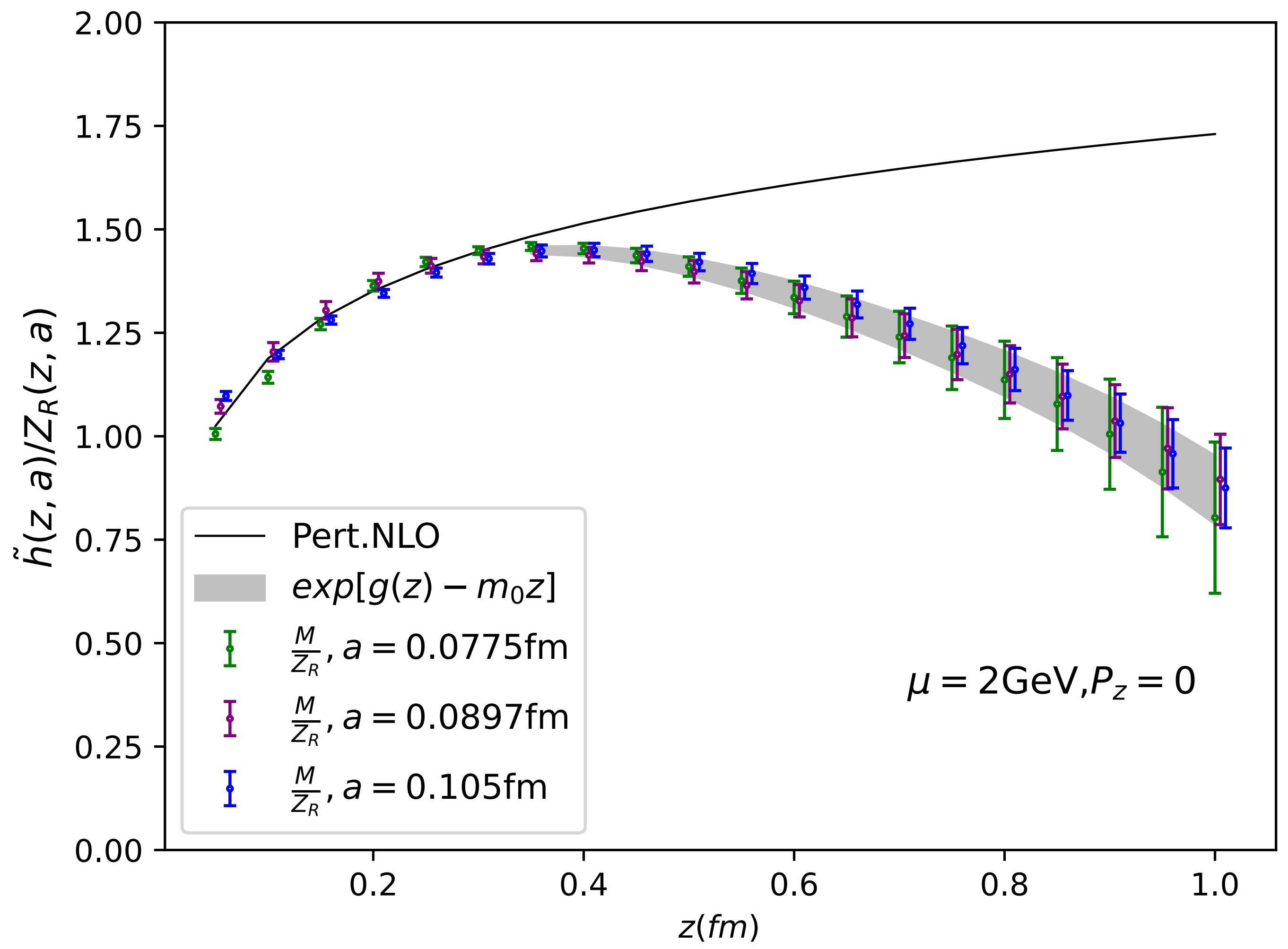
Thanks!

Backups

CLQCD

Self renormalization factor

$$Z_R(z,1/a,\mu) = \exp\left\{ \frac{kz}{a\ln[a\Lambda_{QCD}]} + m_0 z + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{QCD})}{\ln(\mu/\Lambda_{QCD})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{QCD}]}\right]^2 \right\}$$



Parameters in Z_R is determined by fitting the zero-momentum bare matrix elements:

$$\ln\tilde{h}^n_B(z,P_z=0,1/a) = \frac{kz}{a\ln[a\Lambda_{QCD}]} + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{QCD})}{\ln(\mu/\Lambda_{QCD})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{QCD}]}\right]^2 + \begin{cases} \ln[Z_{\overline{MS}}(z,\mu)] + m_0 z & \text{if } z_0 \le z \le z_1 \\ g(z) & \text{if } z_1 < z \end{cases}$$

$$Z_{\overline{MS}}(z,\mu) = 1 + \frac{\alpha_s C_A}{4\pi}\left(\frac{5}{3}\ln\frac{z^2\mu^2}{4e^{-2\gamma_E}} + 3\right)$$

Backups

Matching kernel:

$$C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi>1\\ \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\left(-\ln\frac{\mu^2}{4y^2P_z^2}+\ln(\xi(1-\xi))\right)-\frac{15-56\xi+102\xi^2-96\xi^3+48\xi^4}{6(1-\xi)}\right]_+ & 0<\xi<1\\ \left[\frac{-2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}-\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi<0,\end{cases}$$

$$C_{\text{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z})=C_{\text{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)+\frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left(-\frac{1}{|1-\xi|}+\frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+$$