Proton Spin Structure on the Light Front

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With

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Outline

- Basis Light-front Quantization
  - What is it
  - Why we need it
  - How we do it

- Proton spin structure
  - Form factors
  - Parton distribution functions
  - Spin decomposition

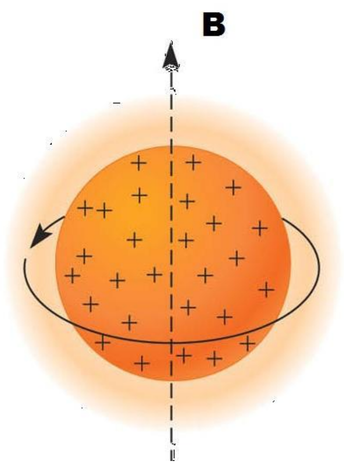
Discovery of Proton

- Proton was discovered by Rutherford in 1919

$$^{14}N + \alpha \rightarrow {}^{17}O + p$$

- Anomalous magnetic moment → not point-like

[Otto Stern, 1933]



If proton were point-like spin-½ particle

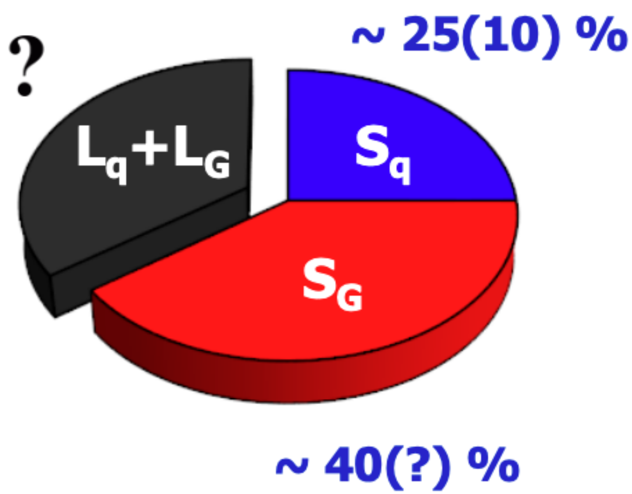
$$\mu_p = g\frac{e}{2m_N}S = \frac{e}{2m_N} \equiv \mu_N$$

→

Experimental value

$$\mu_p = 2.5\mu_N$$

Spin Puzzle



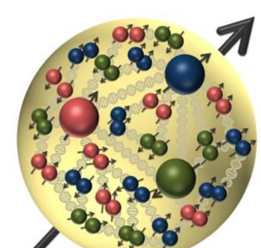
Orbital angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

Need to know 3D tomography of nucleon from QCD

$$\mathcal{L}_{QCD} = \left(\bar{\psi}_q (i\not{D} - m_q)\psi_q \right) - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha}$$

?



Hamiltonian Formalism

- Hamiltonian formalism describes bound-state structure

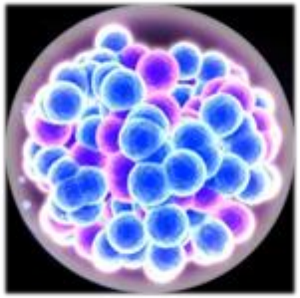
$$H|\psi\rangle = E|\psi\rangle$$

- Eigenstates $|\psi\rangle$ encode full information of the system

Nonrelativistic

atom

Nonrelativistic



nucleus

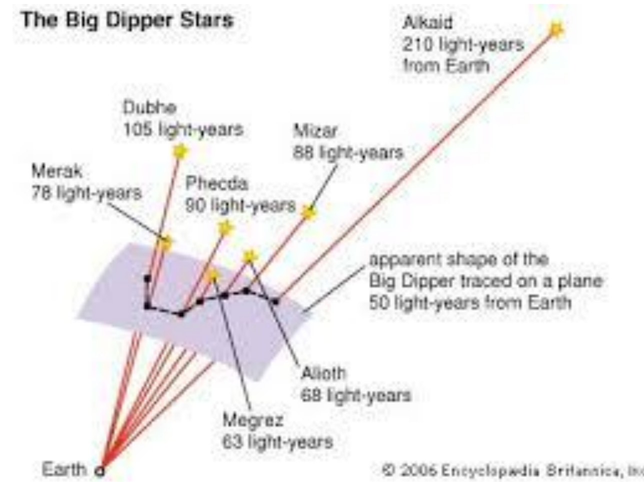
Relativistic

nucleon

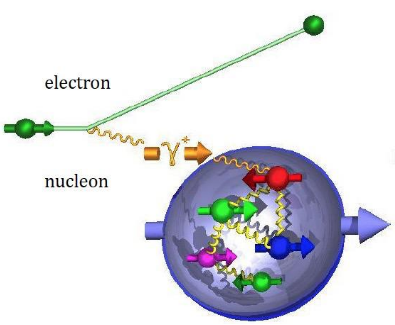
- Relativity ➡ retardation effect

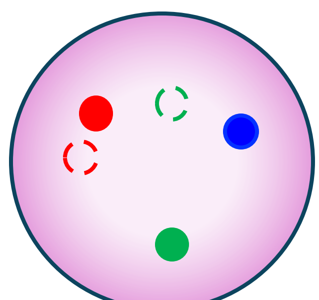
Structure of Relativistic Bound States

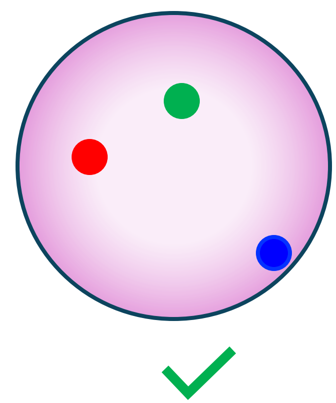
- Challenge: retardation effect



- Solution: light-front time

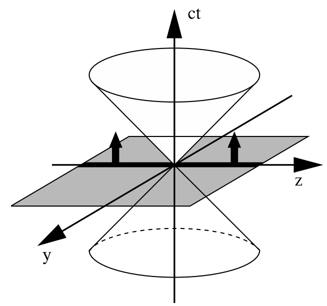
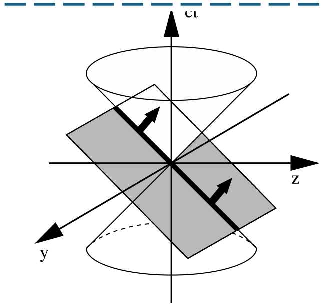






Light-front Quantization

[Dirac, 1949]

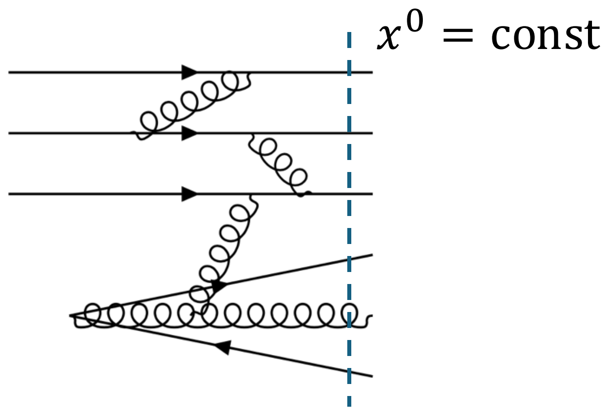
	Equal time quantization	Light-front quantization
Time variable	$t \square x^0$	$t \square x^+ = x^0 + x^3$
Quantization surface		
Coordinate space	x^1, x^2, x^3	$x^- = x^0 - x^3$, $x^\perp = x^{1,2}$
Momentum Space	$P^0, \vec{P}$	$P^- = P^0 - P^3$, $P^+ = P^0 + P^3$, $P^\perp = P^{1,2}$
	$i\frac{\square}{\square t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\square}{\square x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
Dispersion relation	$P^0 = \sqrt{m^2 + P^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

Main Advantage I

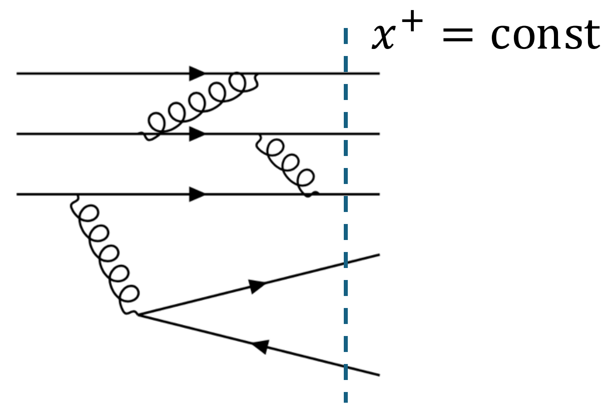
1. Simple vacuum

Proton wave function in Fock space:

$$|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\,q\bar{q}\rangle + \cdots\cdots$$



Equal-time quantization



Light-front quantization

- Bound-state wave functions are not contaminated by vacuum
- Higher Fock sectors reflect retardation effect

Main Advantage II

2. Frame-independent wave function

Lorentz boost transformation is kinematic on light front:

Equal-time		Light-front
$\tilde{x}^0 = \gamma(x^0 + \beta x^3)$	➡	$\tilde{x}^+ = \gamma(1+\beta)x^+$
$\tilde{x}^3 = \gamma(x^3 + \beta x^0)$		$\tilde{x}^- = \gamma(1-\beta)x^-$

Light-front quantization

➡ Hamiltonian framework for quantum field theory

Basis Light-Front Quantization

$$P^{-}|\psi\rangle = P_{\psi}^{-}|\psi\rangle$$

[Vary, et.al, 2010]

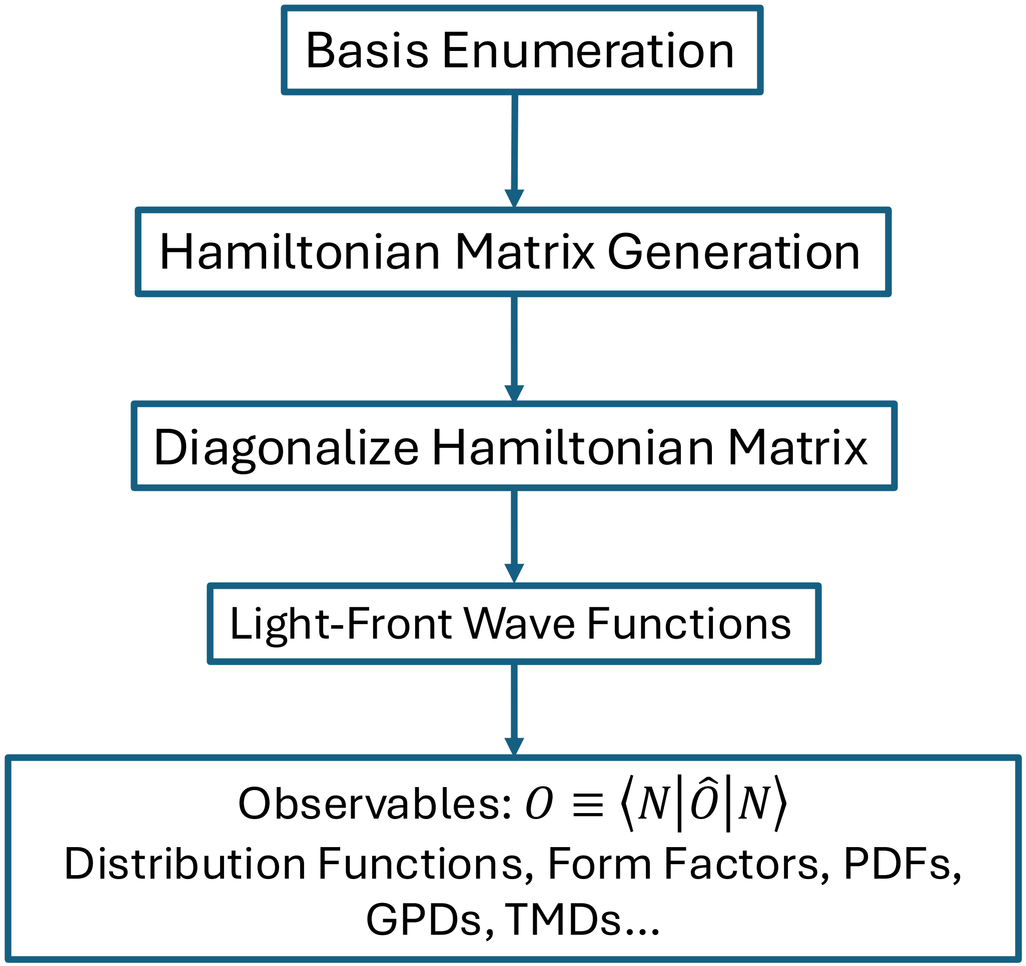
- Guiding principle: preserve symmetries in Hamiltonian
  - rotational symmetry in transverse directions
  - symmetry among identical particles

- Basis setup and truncation:
  - Fock sector expansion: $|N\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, q\bar{q}\rangle + \cdots\cdots$
  - single particle basis:

$$|qqq\rangle = |n_1, m_1, n_2, m_2, n_3, m_3\rangle \otimes |k_1^+, k_2^+, k_3^+\rangle \otimes |\lambda_1, \lambda_2, \lambda_3, C\rangle$$

2-d harmonic oscillator (2DHO)	Discretized longitudinal momentum	Helicity and color
$\sum_i (2n_i + \lvert m_i\rvert + 1) \leq N_{\max}$	$\sum_i k_i^+ = K_{\max}$	$\mathrm{m}_J = \sum_i (\lambda_i + m_i)$

BLFQ Algorithm Flowchart



Progress toward First Principles

$$|\mathrm{N}\rangle = |qqq\rangle + |qqqg\rangle + |qqq\, u\bar{u}\rangle + |qqq\, d\bar{d}\rangle + |qqq\, s\bar{s}\rangle + \cdots$$

See Chandan Mondal's talk on Tuesday morning

- **Wave Functions:**

  [PRD,102,016008] (2019) **[PRD,108 9, 094002] (2023)** **[PLB, 867,139599] (2025)**

- GPDs:

  [PRD,104,094036] (2021) **[PLB,847,138305] (2023)**

  [PRD,105,094018] (2022) **[PRD,110.056027] (2024)**

  [PRD,109,014015] (2024) **[PLB,860,139153] (2025)**

  [PLB,855,138809] (2024)

- **TMDs:**

  [PLB,833,137360] (2022) **[PLB,855 138831] (2024)**

  [PRD,108,036009] (2023)

- Higher-twist Distribution (GPD,TMD,DPD):

  [PRD,109,034031] (2024) **[PLB,855 138829] (2024)**

  [arXiv:2410.11574] (2024)

- Gravitational Form Factors:

  [PRD,110,056027] (2024)

QCD Light-front Hamiltonian

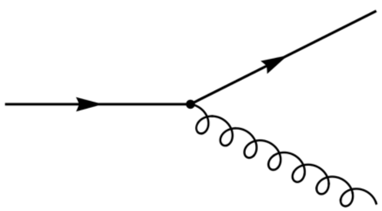
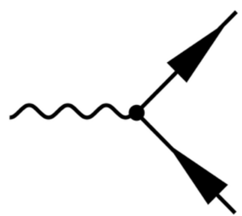
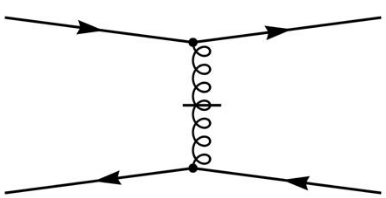
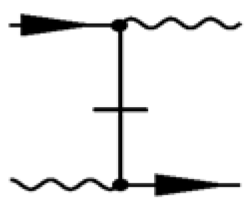
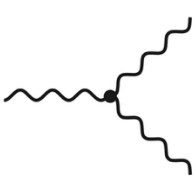
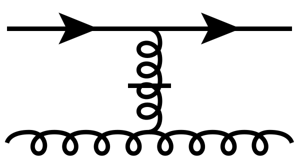
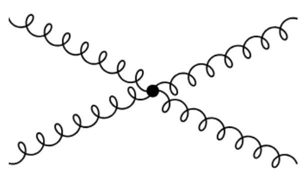
[S. Brodsky, H-C Pauli, S. Pinsky, '97]

- QCD light-front Hamiltonian from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \quad \Longrightarrow \quad P^{-}_{QCD} = H_K + H_I \qquad A^{+} = 0$$

$$H_K = \tfrac{1}{2}\int \mathrm{d}^3x\, \bar{\psi}\gamma^{+}\frac{(\mathrm{i}\partial^{\perp})^2 + m^2}{\mathrm{i}\partial^{+}}\psi - \tfrac{1}{2}\int \mathrm{d}^3x\, A^{i}_{a}(\mathrm{i}\partial^{\perp})^2 A^{i}{}_{a}$$

$$\begin{aligned}
H_I = &+ g\int \mathrm{d}^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\psi \\
&+ \tfrac{1}{2}g^2\int \mathrm{d}^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\frac{\gamma^{+}}{\mathrm{i}\partial^{+}}\gamma_{\nu}A^{\nu}\psi \\
&- \mathrm{i}g^2\int \mathrm{d}^3x\, f^{abc}\bar{\psi}\gamma^{+}T^{c}\psi\frac{1}{(\mathrm{i}\partial^{+})^2}\left(\mathrm{i}\partial^{+}A^{\mu}_{a}A_{\mu b}\right) \\
&+ \tfrac{1}{2}g^2\int \mathrm{d}^3x\, \bar{\psi}\gamma^{+}T^{a}\psi\frac{1}{(\mathrm{i}\partial^{+})^2}\bar{\psi}\gamma^{+}T^{a}\psi \\
&+ \mathrm{i}g\int \mathrm{d}^3x\, f^{abc}\mathrm{i}\partial^{\mu}A^{\nu a}A^{b}_{\mu}A^{c}_{\nu} \\
&- \tfrac{1}{2}g^2\int \mathrm{d}^3x\, f^{abc}f^{ade}\mathrm{i}\partial^{+}A^{\mu}_{b}A_{\mu c}\frac{1}{(\mathrm{i}\partial^{+})^2}\left(\mathrm{i}\partial^{+}A^{\nu}_{d}A_{\nu e}\right) \\
&+ \tfrac{1}{4}g^2\int \mathrm{d}^3x\, f^{abc}f^{ade}A^{\mu}_{b}A^{\nu}_{c}A_{\mu d}A_{\nu e}.
\end{aligned}$$

ψ: quark field operator
A^{a}_{μ}: gluon field operator

7 terms in H_I

Input Parameters

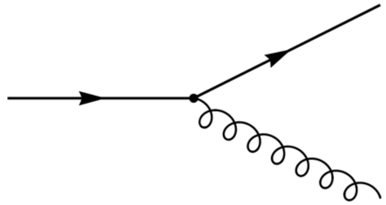
- Truncation Parameters: up to 6-parton Fock sectors with $N_{\max} = 7$ & $K = 10$

$$|\mathrm{N}\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

- Input Parameters:

m_u	m_d	m_s	m_f	g	b_{inst}	b
0.401 GeV	0.4 GeV	0.6 GeV	1.8 GeV	2.5	3.0 GeV	0.6 GeV

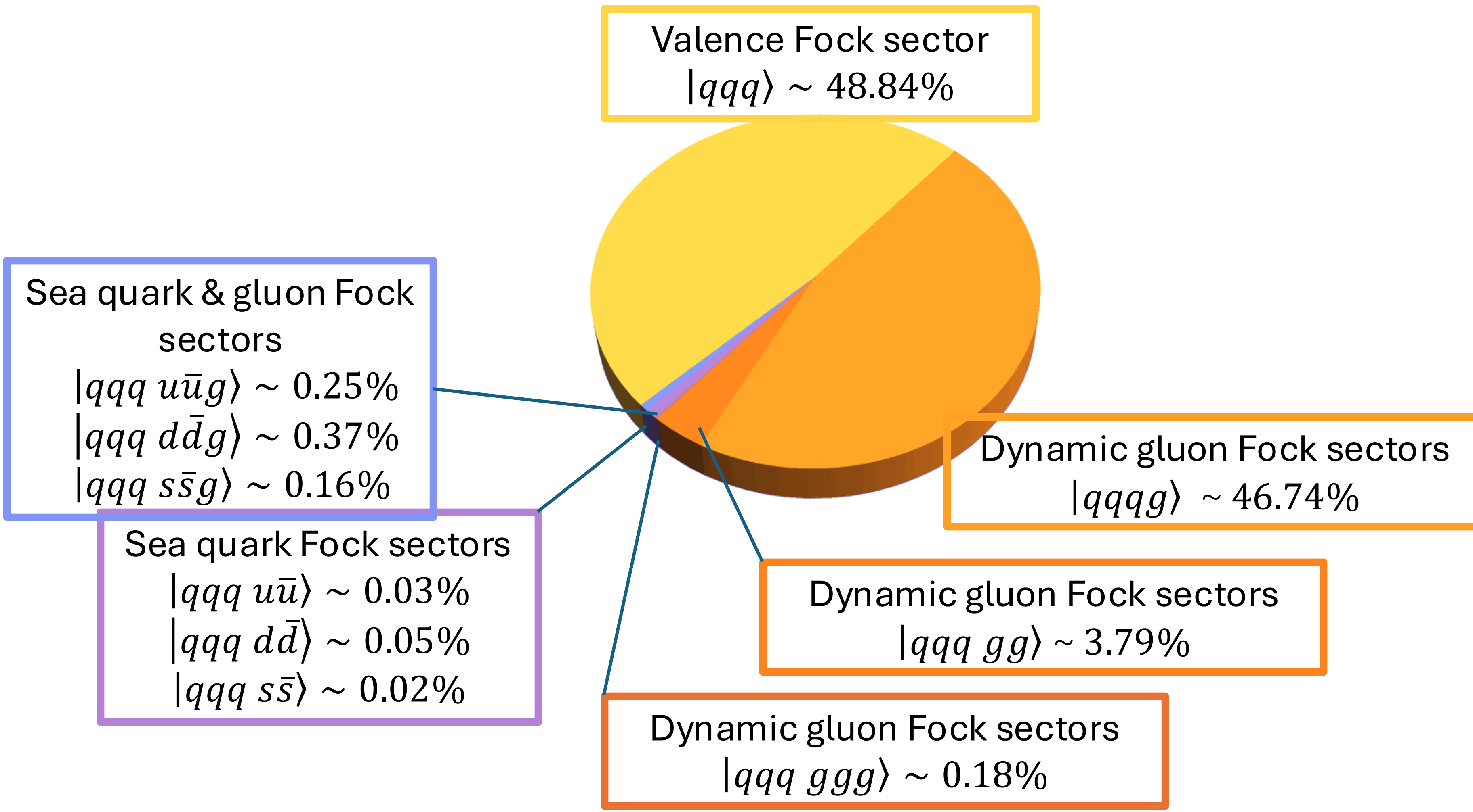
- determined by fitting electromagnetic Form Factors and proton mass $M_p = 0.93\text{GeV}$
- separate quark mass in the quark-gluon vertex m_f is needed



$$\propto g m_f + \cdots$$

Fock Sector Contribution

$$|N\rangle \rightarrow |qqq\rangle + |qqqu\bar{u}\rangle + |qqqd\bar{d}\rangle + |qqqs\bar{s}\rangle + |qqqu\bar{u}g\rangle + |qqqd\bar{d}g\rangle + |qqqs\bar{s}g\rangle + |qqqg\rangle + |qqqgg\rangle + |qqqggg\rangle$$

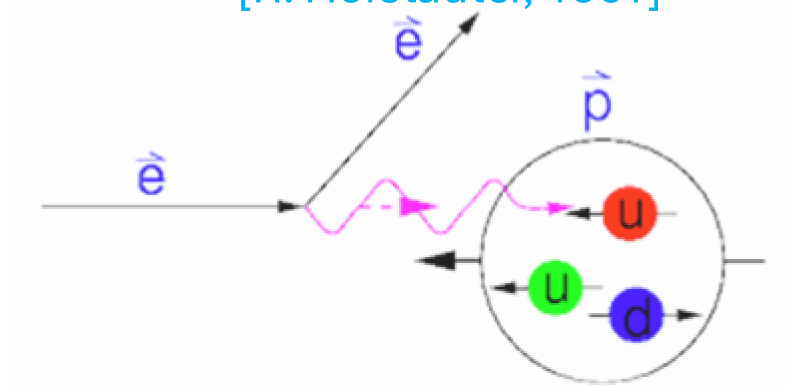


Electromagnetic Form Factor

- Elastic scattering of proton

$$e(p) + h(P) \rightarrow e(p') + h(P')$$

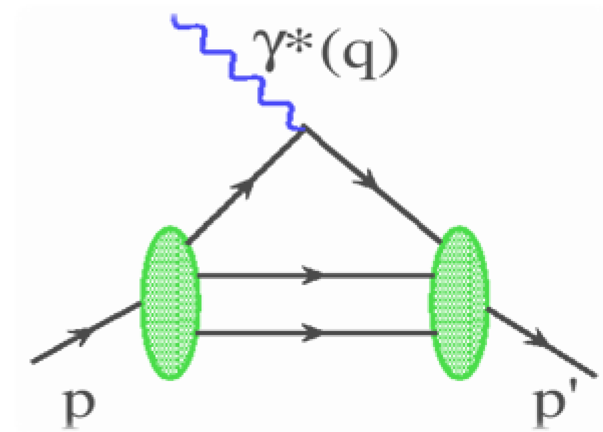
[R. Hofstadter, 1961]



- Elastic electron scattering established the extended nature of the proton (proton radius)

$$\langle N(p')|J^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu}\underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N}q_{\nu}\underbrace{F_2(q^2)}\right]u(p)$$

Dirac Form Factor Pauli Form Factor



- The Fourier transformation of these form factors provide spatial distributions (charge and magnetization distributions)

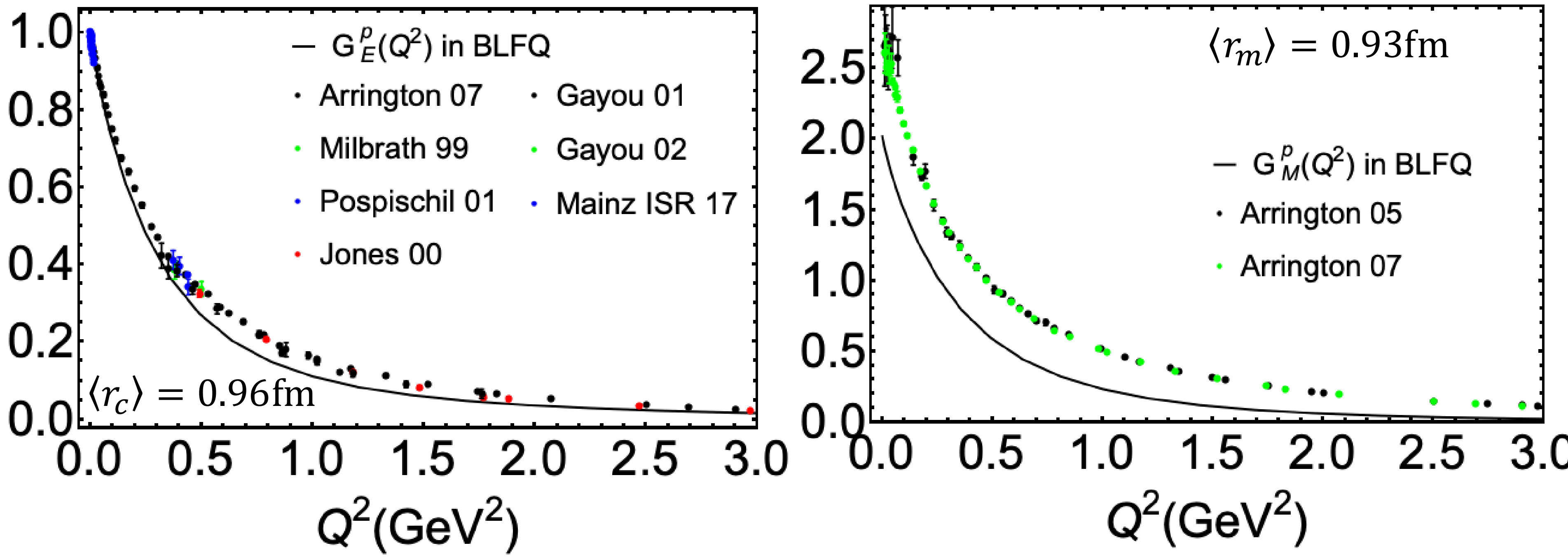
Electromagnetic Form Factors

$$\langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2(Q^2)\right]u(p)$$

$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M_N^2}\sum_q e_q F_2^q(Q^2)\,,$$

$$G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2).$$

Preliminary



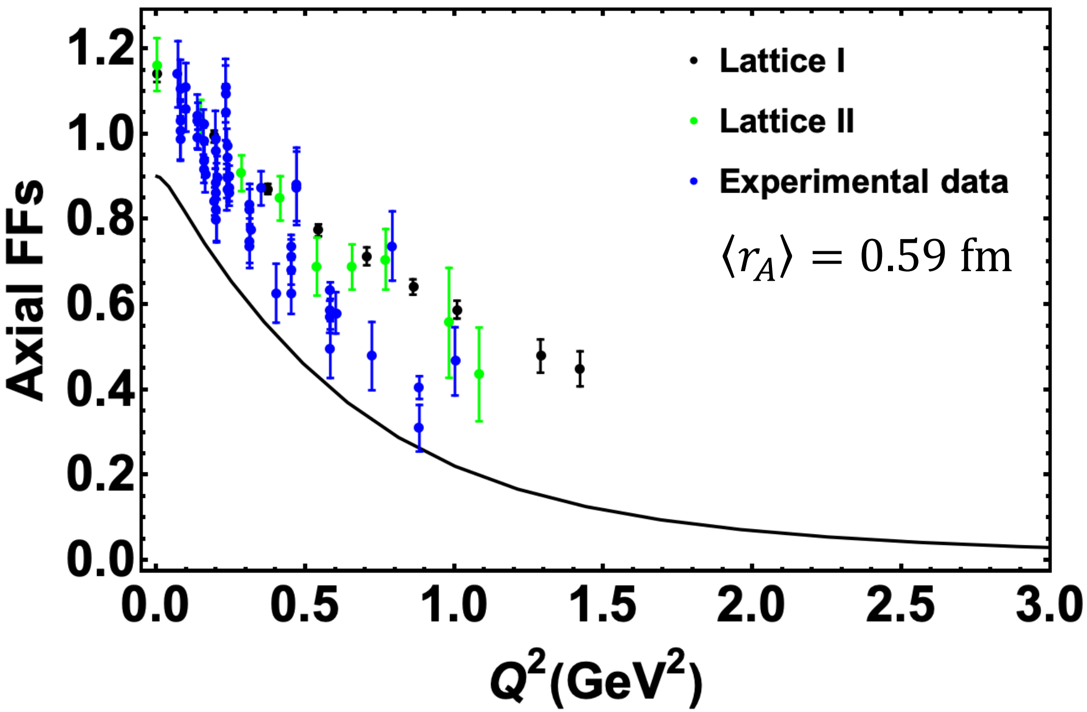
- Qualitatively agree with the experimental data for charge and magnetic FFs

Axial Form Factor

- Provide information on axial charge distributions

$$\langle N(p')|A^a_\mu|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu G_A(t) + \frac{(p'-p)_\mu}{2m} G_P(t)\right]\gamma_5 \frac{\tau^a}{2} u(p)$$

$$A^a_\mu = \bar{q}\gamma_\mu\gamma_5 T^a q \qquad G_A(Q^2) = G_u(Q^2) - G_d(Q^2)$$



- Black line: valence quark
- Qualitatively agree with the experimental data

Exp. value

$\Delta\Sigma_u = 0.73 \quad (0.82)$

$\Delta\Sigma_d = -0.17\ (-0.45)$

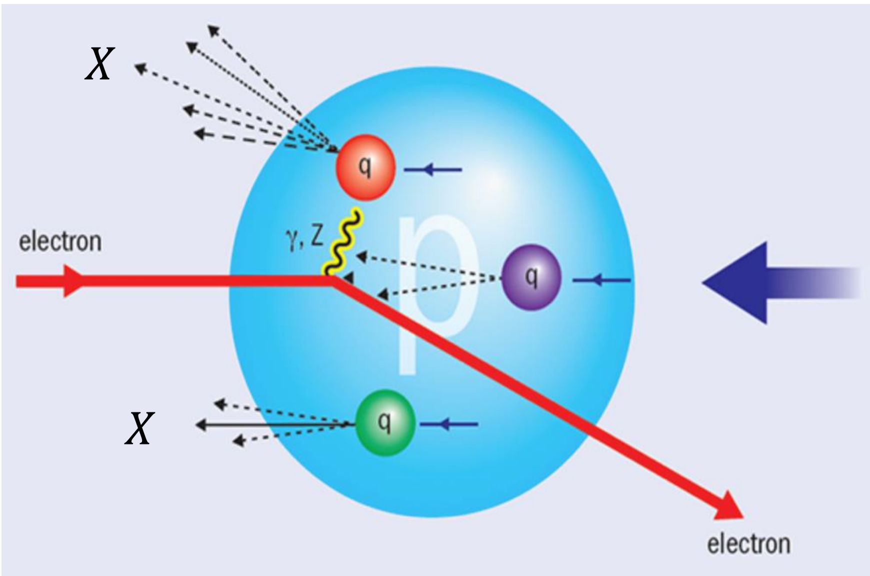
$\Delta\Sigma_{u-d} = 0.90\ \ (1.27)$

$\Delta\Sigma_{u+d} = 0.56\ \ (0.37)$

$\Delta G = 0.13$

Parton Distribution Functions (PDF)

- **Deep Inelastic Scattering (SLAC 1968)**



$$e(p) + h(P) = e'(p') + X(P')$$

✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1 \text{ fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1 \text{ fm}$$

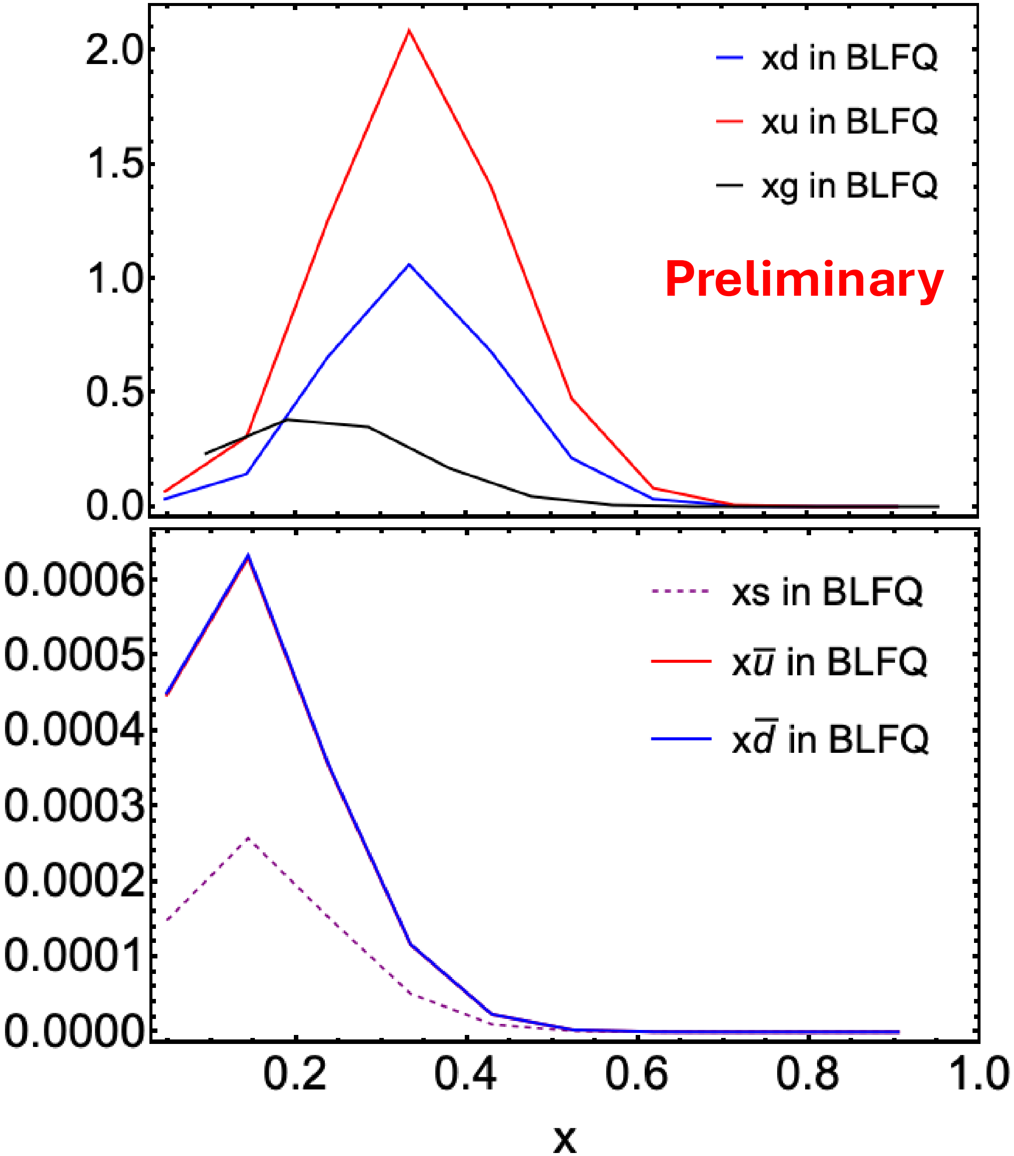
Discovery of spin ½ quarks and partonic structure

- **Parton distribution functions** (PDFs) are extracted from DIS processes.

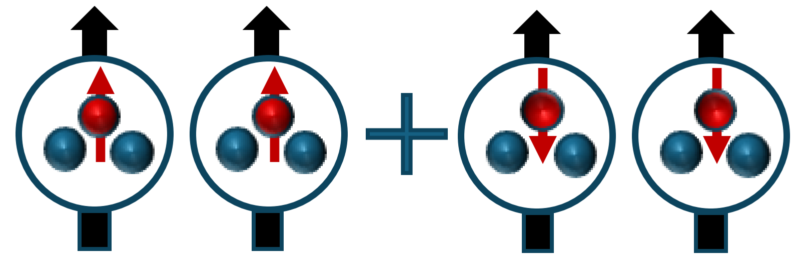
$$\Phi^{[\gamma^+]}(x, Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P, \Lambda | \bar{\psi}(z)\gamma^+\psi(0) | P, \Lambda \rangle$$

PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents

Unpolarized PDF



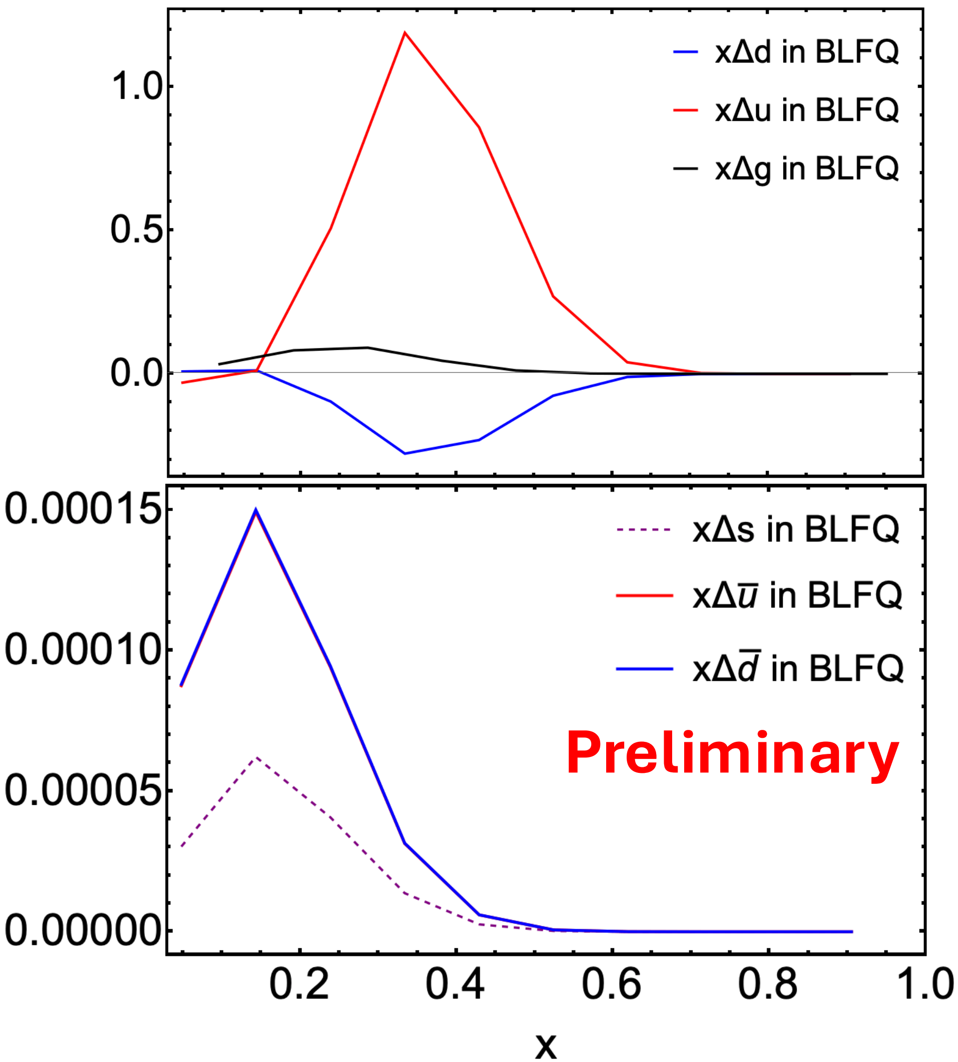
Unpolarized PDFs:



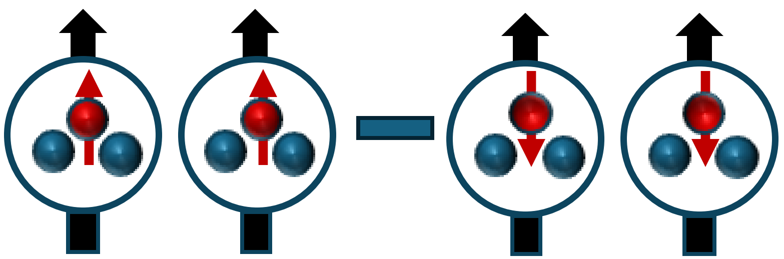
- Parton number density
- Valence quarks contribute mainly at $x \sim 0.3$
- Gluon distributions are larger than those of sea quark
- Gluon and sea quark dominate in small x region

All results are at the initial scale

Helicity Parton Distribution Functions



Helicity PDFs:

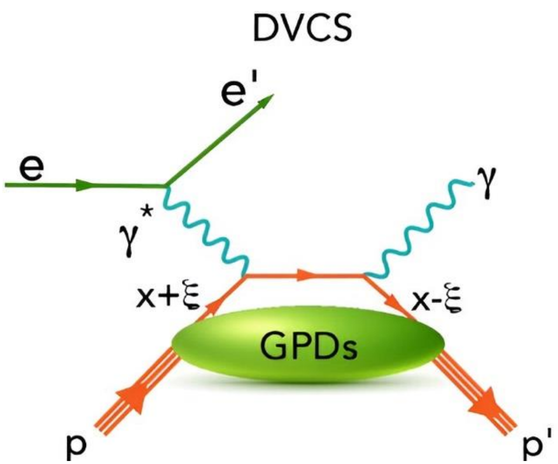


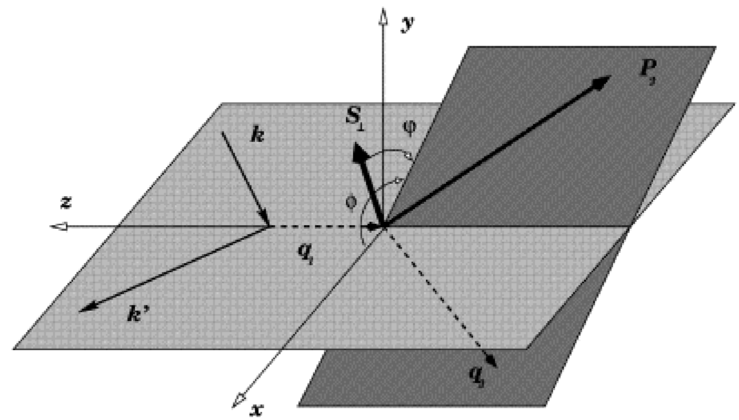
- Opposite signs for u and d quark
- Gluon contribution is positive
- Qualitatively agree with expectation

3D Structure in Coordinate Space

- Deeply Virtual Compton Scattering (DVCS) [X. Ji, Phys. Rev. D 55, 7114 (1997)]

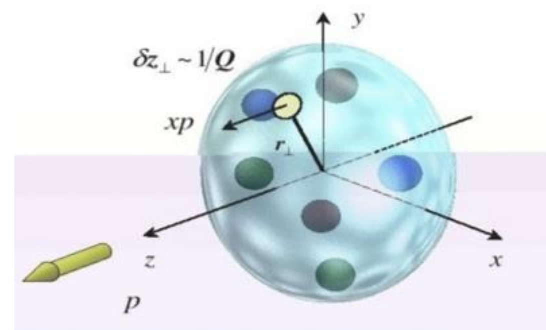
$$e(p) + P(P) \to e'(p') + P'(P') + \gamma$$

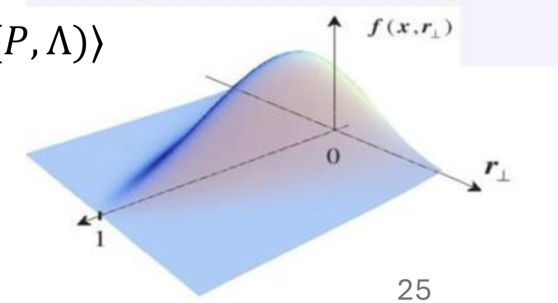




- Generalized Parton Distribution Functions (GPDs)

$$\Phi^{[\gamma^+]}(x, \Delta; Q^2) = \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle \psi_h(P+\Delta, \Lambda) | \bar{\psi}(z) \gamma^+ \psi(0) | \psi_h(P, \Lambda) \rangle$$

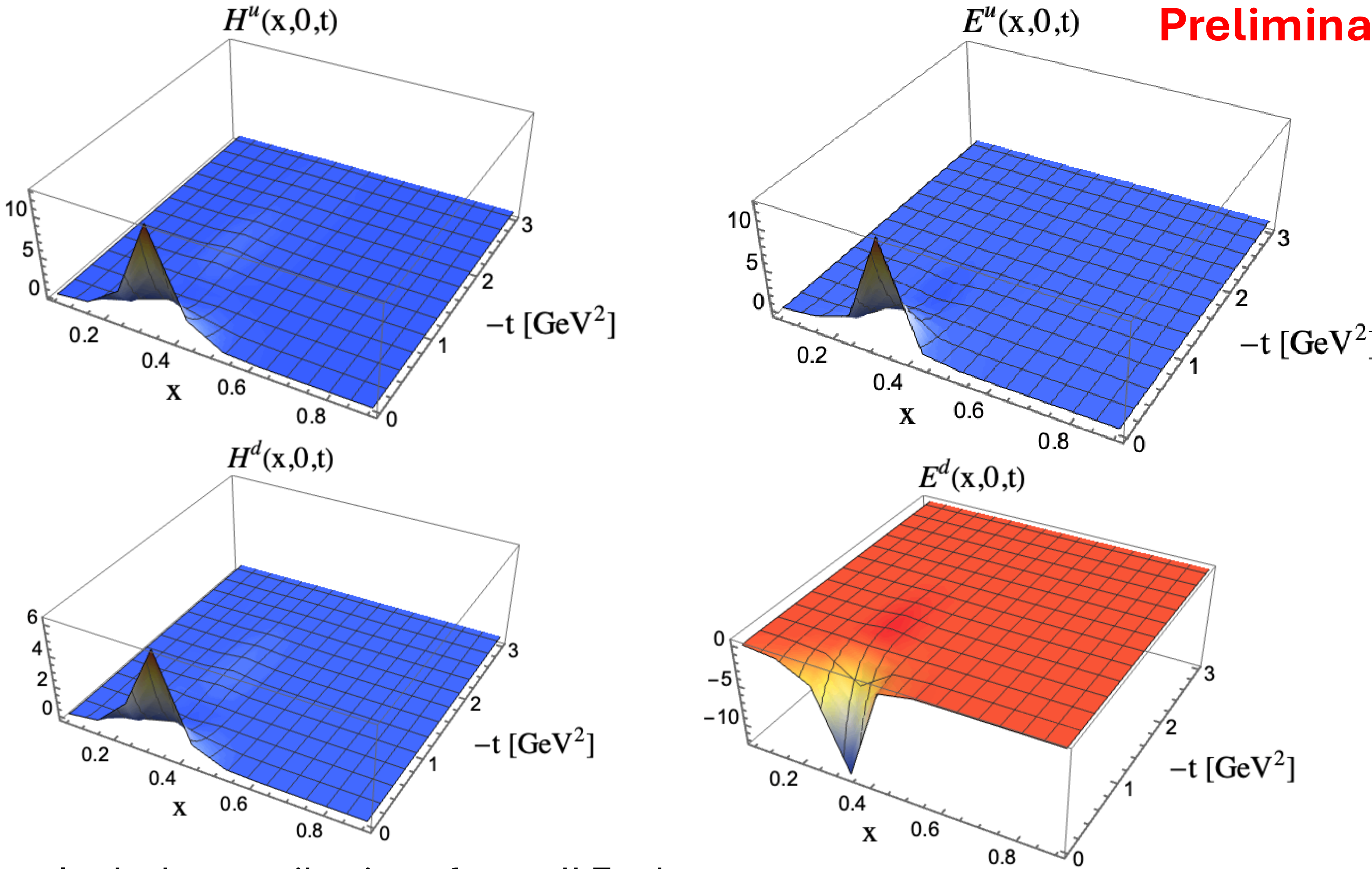




- Encode the information about three-dimensional spatial structure of a hadron

GPDs for u and d Quarks

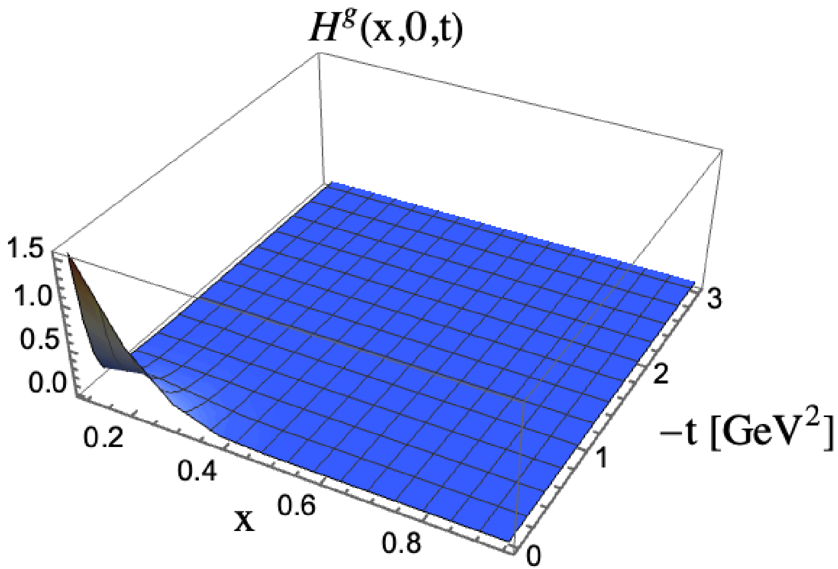
Preliminary

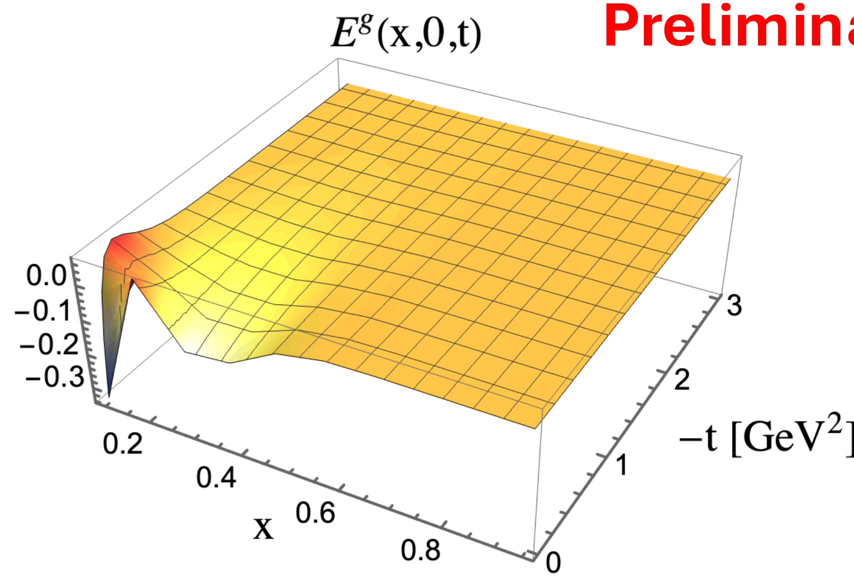


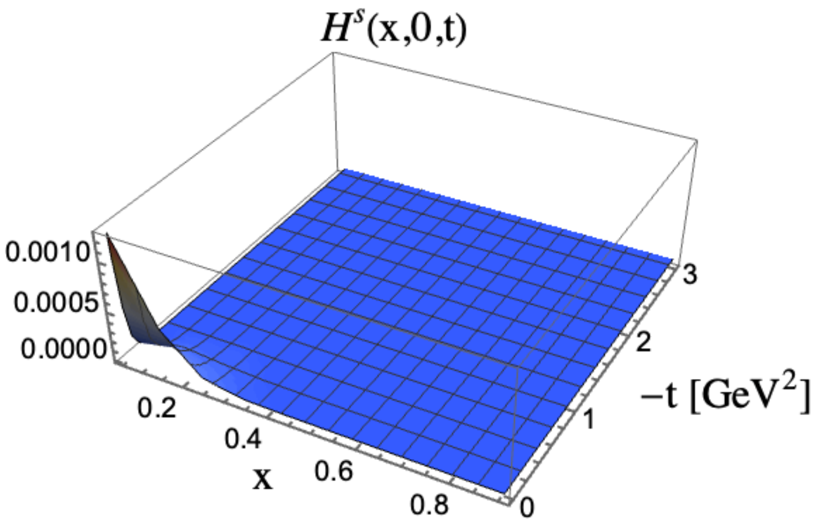
- Include contributions from all Fock sectors
- E_u is positive while E_d is negative

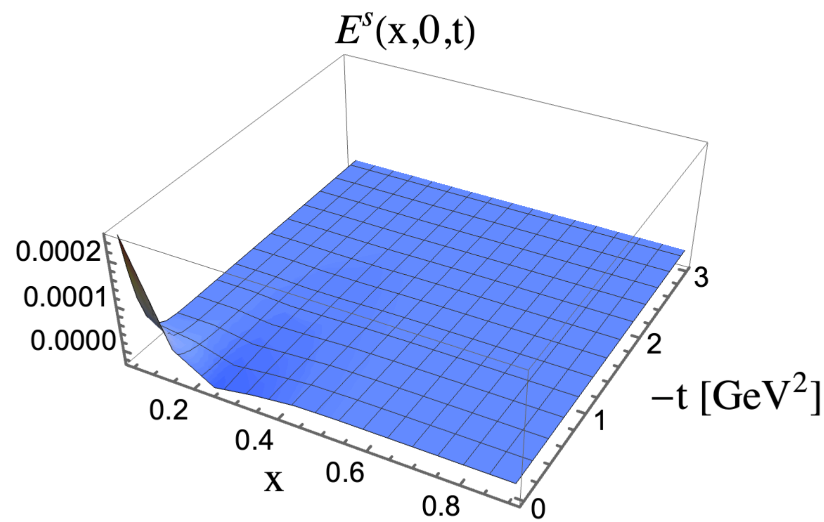
GPDs for Gluon and Strange Quark

Preliminary



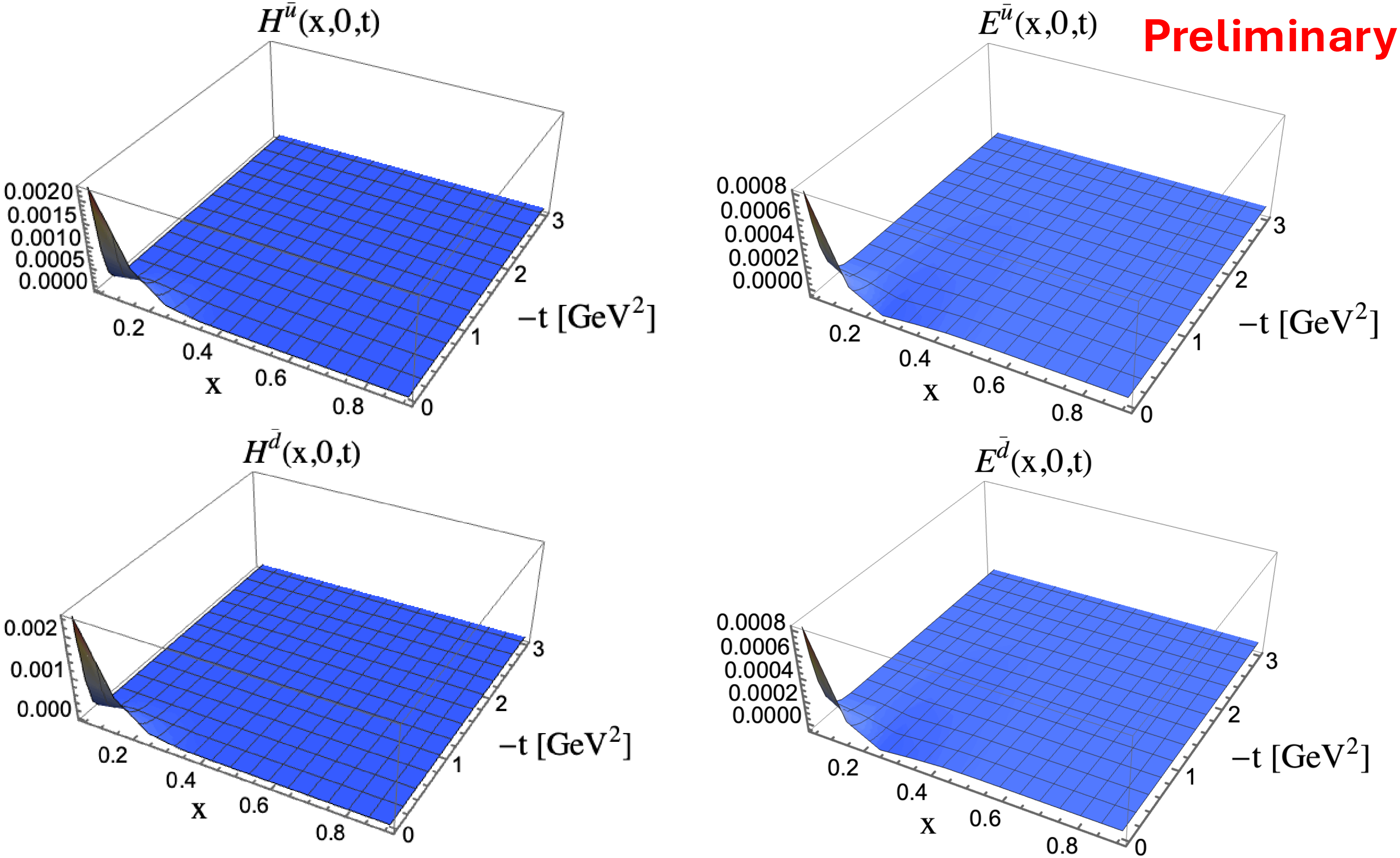






- E_s and E_g mainly contribute at small x region

GPDs for Sea Quarks



- GPD H and E for $\bar{u}$ and $\bar{d}$ peak at small x
- GPD E for $\bar{u}$ and $\bar{d}$ are positive

Orbital Angular Momentum

- Ji Sum Rule

Total Angular Momentum: $J_{q,g} = \frac{1}{2}\int dx\, x[H(x,0,0) + E(x,0,0)]$

Light-Cone Gauge $A^+ = 0$

- Spin Contribution: $S_{q,g} = \int dx\, \tilde{H}(x,0,0)$
- OAM Contribution: $L_q = \mathrm{J_q} - \mathrm{S}_q/2$, $L_g = \mathrm{J_g} - \mathrm{S}_g$

At initial scale μ_0

	u	d	s	g	$\bar{u}$	$\bar{d}$
J	0.4318	−0.0013	0.00003	0.053	0.00008	0.00008
S	0.7323	−0.1692	0.00013	0.125	0.00034	0.00034
L	0.0657	0.0833	−0.00010	−0.072	−0.00026	−0.00026

Conclusions

- Basis Light-front Quantization
  - A nonperturbative approach to quantum field theory
  - Calculate proton structure from QCD first principles
  - Spin structure qualitatively agree with expectation
  - Separate quark mass for vertex interaction is needed

Thank you!

Outlook

Current status

Full QCD interaction

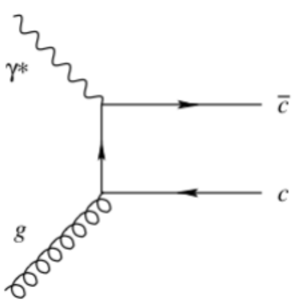
Deutron calculation
$|qqq\ qqq\rangle + |qqq\ qqq\ g\rangle$

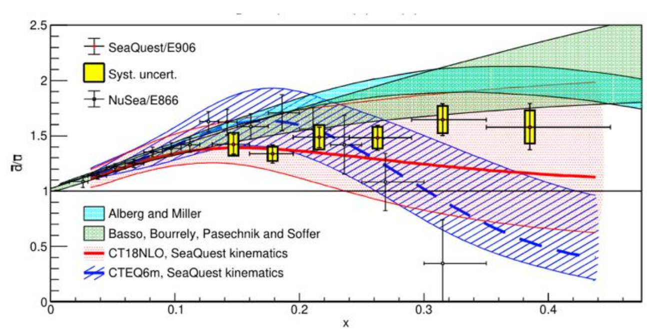
EMC effect

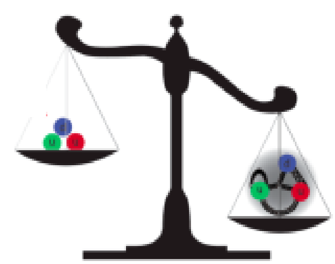
Intrinsic charm

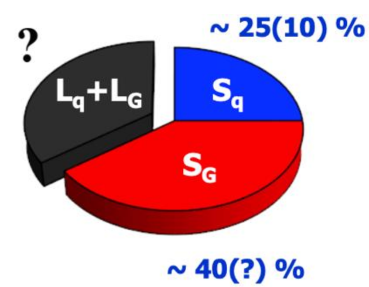
Sea asymmetry

Origin of spin and mass









Thank you!