Fragmentation functions at NNLO & constraints on proton PDFs

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Based on *PRL 135, 041902, 2025*. In collaboration with
Jun Gao, Hongxi Xing, Yuxiang Zhao, Bin Zhou

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A Century of Spin

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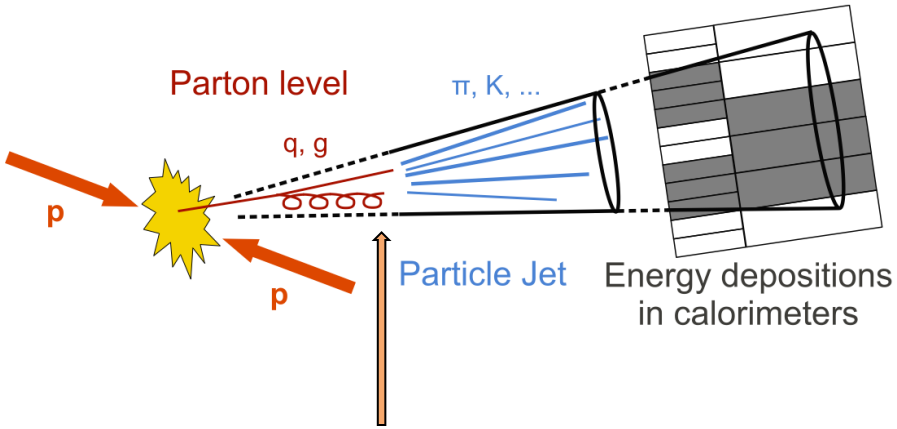
Fragmentation Functions (FFs) as extension of the parton model

Bjorken, et al., '71
FFs proposed

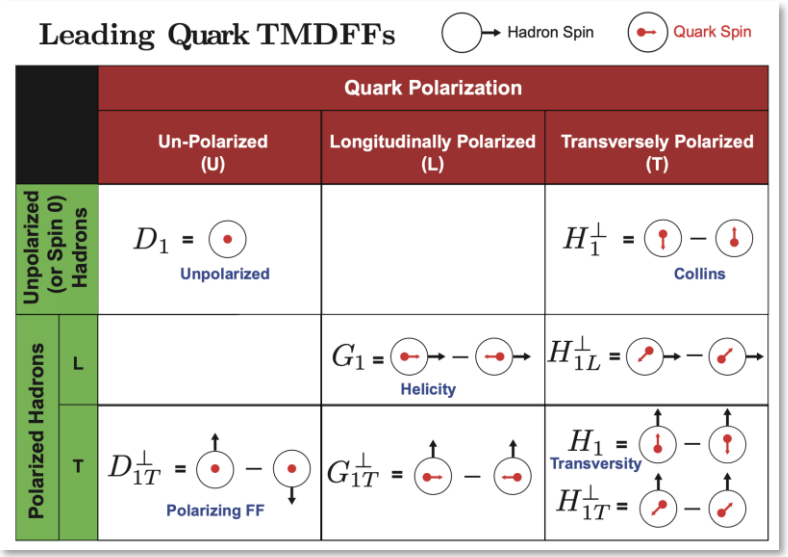
1969 **1971** **1977**

Feynman, '69
Parton model

Field, Feynman, '77
First determination



Collinear FFs: $D_{h\leftarrow i}(z\equiv p_h^+/p_i^+)$

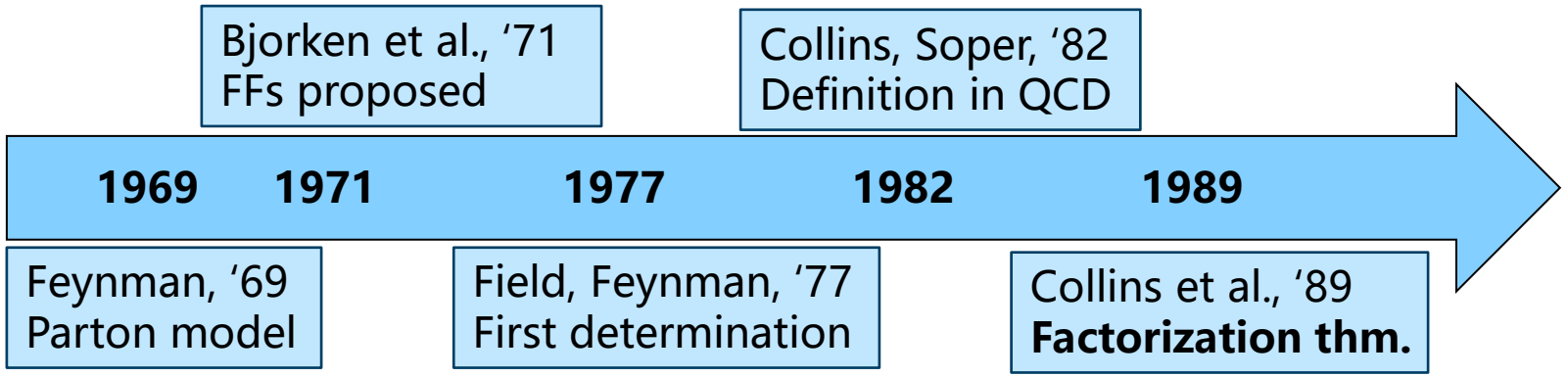


[2304.03302]

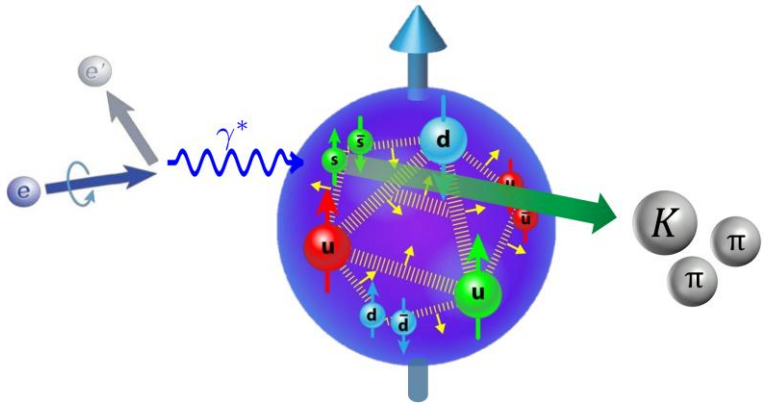
[See also talks of A. Vossen & Y.-K. Song]

$D_{h\leftarrow i}(z)$: number density of hadron h carrying a fraction of momentum z of parton i

Why FFs: key ingredients of QCD factorization framework



- Semi-Inclusive DIS (SIDIS) : e + N -> e + **h** + X:

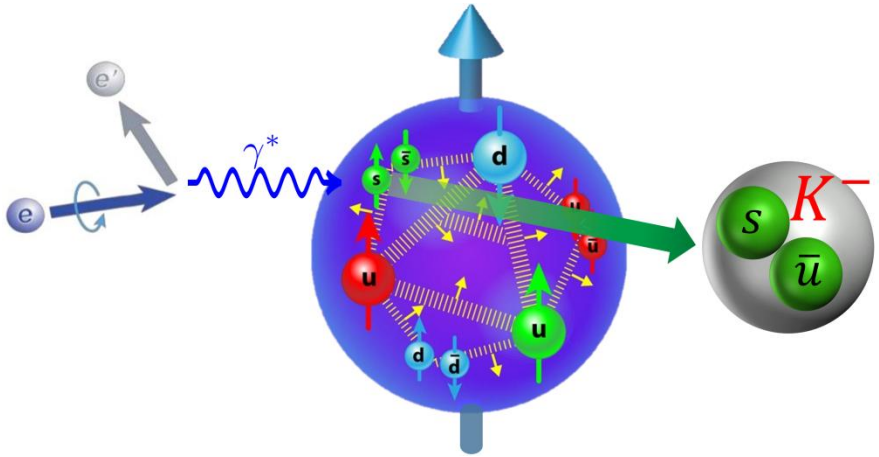


$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

PDF FF

Why FFs: inputs of nucleon structure studies

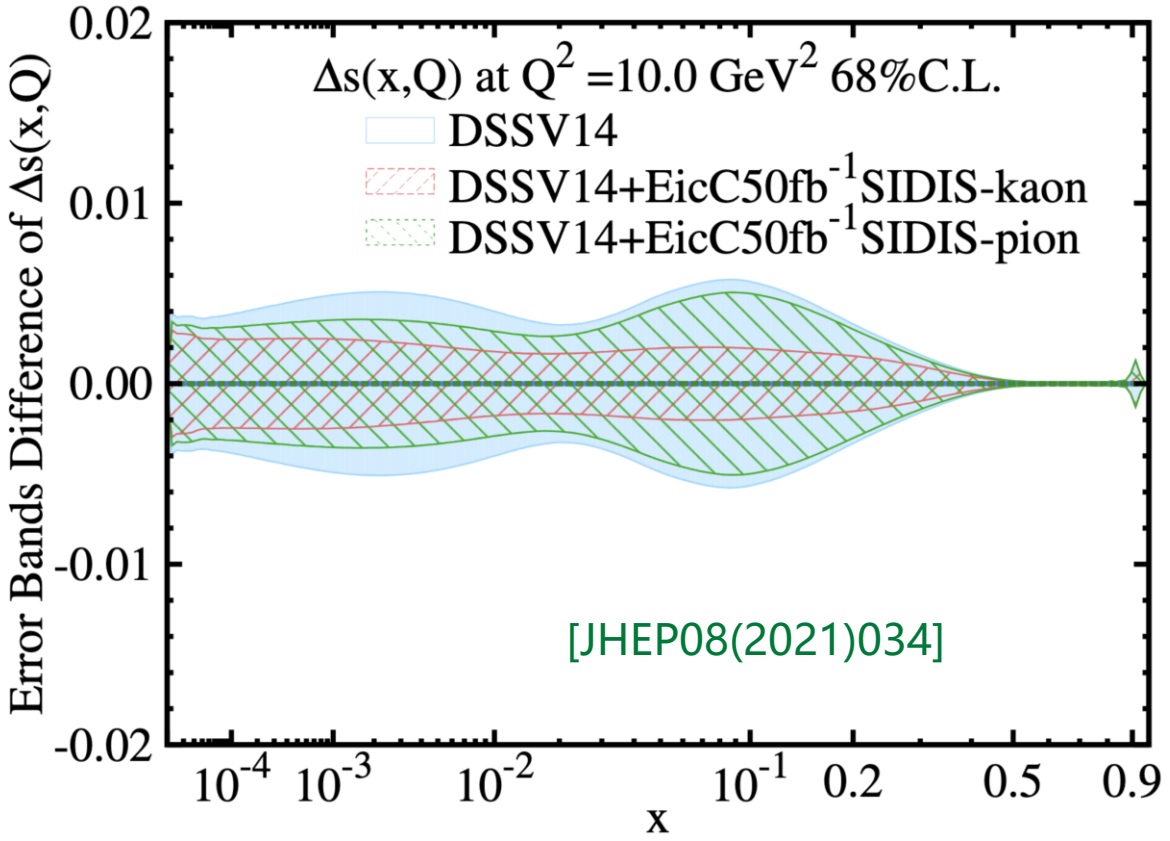
Identified hadron in SIDIS helps discriminate the initial parton



FFs are key inputs of pPDFs determination

$$g_1^h(x,Q^2,z)=\frac{1}{2}\sum_q e_q^2\left[\Delta q(x,Q^2)\,D_q^h(Q^2,z)+\Delta\bar{q}(x,Q^2)\,D_{\bar{q}}^h(Q^2,z)\right]+\mathcal{O}(\alpha_s)$$

pPDF FF

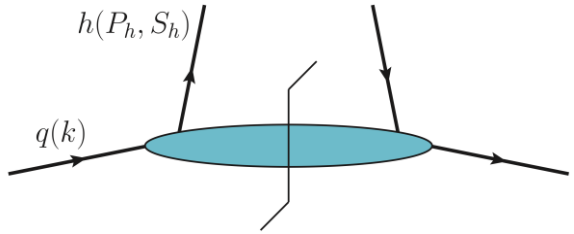


[See also E. Nocera's talk on Monday]

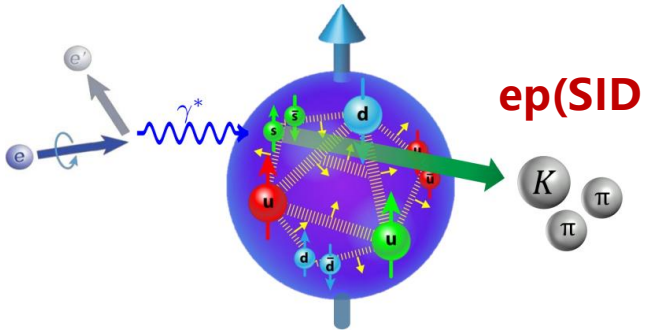
Determination of FFs from global data fit

❖ Field theory definiton of the collinear (integrated) quark FFs [Collins, Soper '82]

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{\mathrm{d}\xi^+}{2\pi} e^{iP_h^-\xi^+/z}\,\mathrm{Tr}\Big[\langle 0\,|\,\mathcal{W}(\infty^+,\xi^+)\;\psi_q(\xi^+,0^-,\vec{0}_T)\,|\,P_h,S_h;X\rangle \times \langle P_h,S_h;X\,|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\;\mathcal{W}(0^+,\infty^+)\,|\,0\rangle\;\gamma^-\Big]$$



❖ Global analyses based on factorization formula



ep(SIDIS) **+ ee(SIA), & pp collisions**

$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x)\otimes\hat{\sigma}_{j\leftarrow i}(x,y,z)\otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

measurement — input/output — pQCD — to be determined. Universal !

[See also H.-Y. Xing's talk]

Outline

- Introduction
- Global analyses of unpolarized collinear FFs
- NPC analyses of FFs at NNLO

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
pp hadron in jet	✗	✗	✗	✗	✗	✓
FFs	$\pi^{\pm},K^{\pm},p$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

Only some of the recent global analyses are shown here.

Efforts on global data fitting of parton FFs

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✘	✓	✓	✓	✓	✓
pp incl. hadron	✘	✘	✓	✘	✘	✓
pp hadron in jet	✘	✘	✘	✘	✘	✓
FFs	$\pi^{\pm},K^{\pm},p$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

Only some of the recent global analyses are shown here.

- FFs determination at NLO from Nonperturbative Physics Collaboration (NPC)
  - NPC23 FFs to light **charged** hadrons:

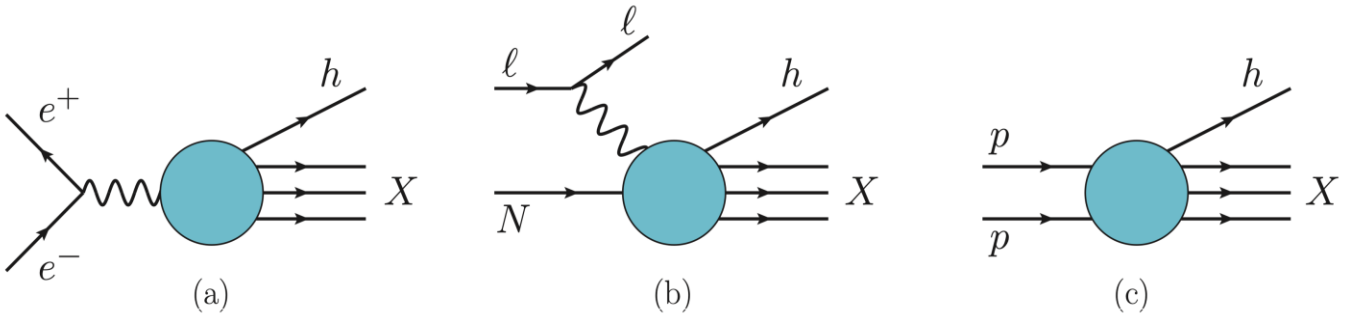
    Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
    Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019,* '24 (Editors' suggestion)

  - NPC23 FFs to light **neutral** hadrons:

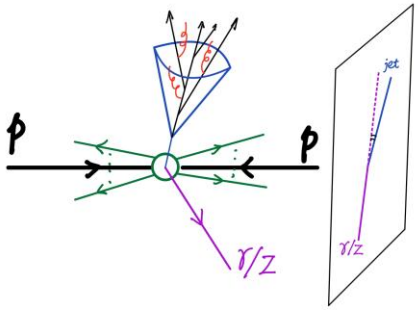
    Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, 2503.21311 (*PRD*)

NPC23 NLO analyses incorporates various types of data

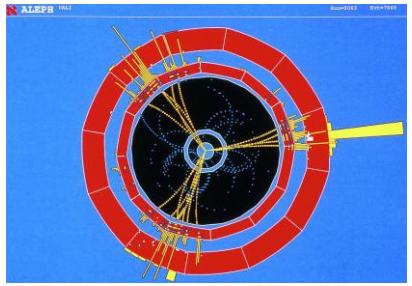
- various types of hadron production data in NPC FFs determination



ee ep pp



Hadron in jet (pp)



Hadron in jet (ee)

Hadron-in-jet data provides direct probe of z dependence: $\dfrac{p_{T,h}}{p_{T,j}} \xrightarrow{\text{LO}} z$

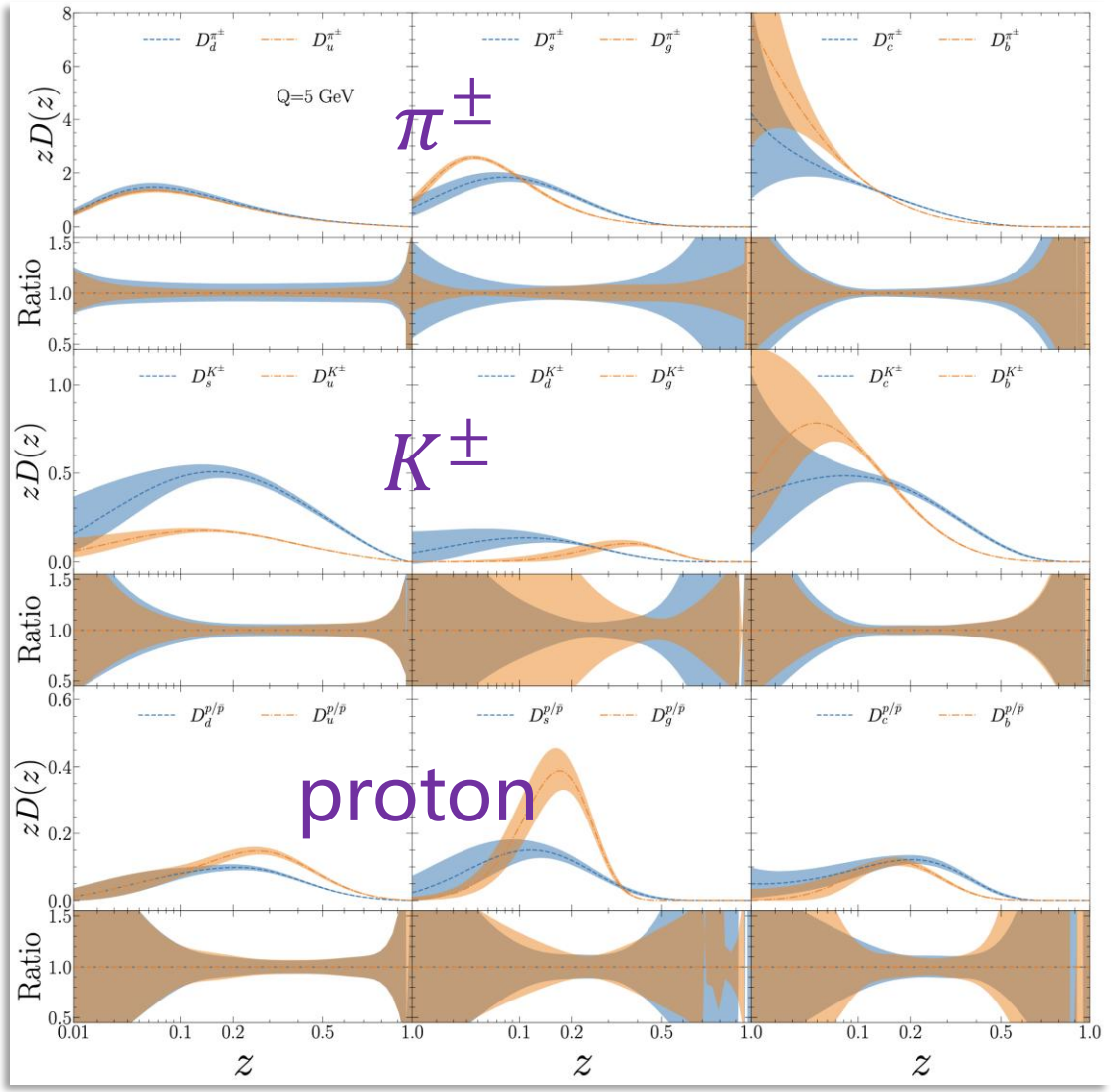
- All theoretical predictions calculated with **FMNLO.**

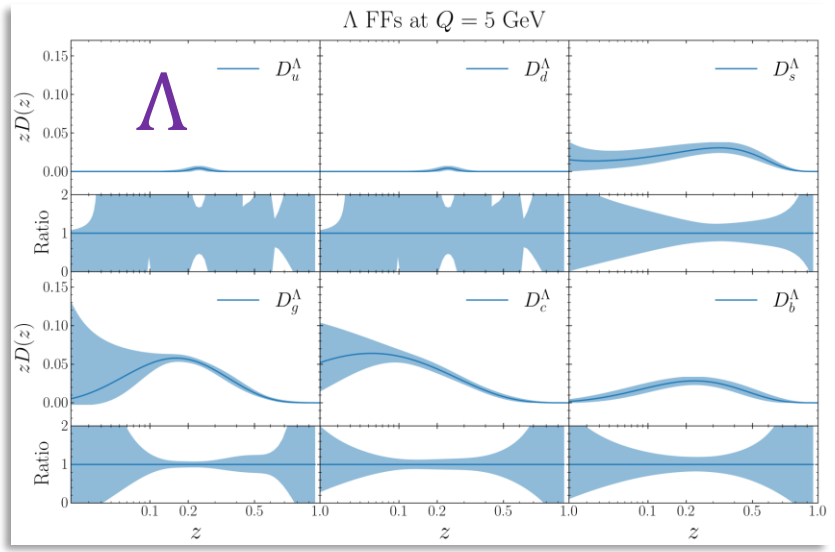
[Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]

figure credit: A. Metz, A. Vossen,1607.02521; ALEPH Collaboration

The NPC23 NLO FF sets

Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
Gao, Liu, **XS**, Xing, Zhao**,** *PRD 110, 114019,* '24 (Editors' suggestion)
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, 2503.21311 (*PRD)*





NPC23 FFs are publicly available:

LHAPDF 6.5.5

Main page	PDF sets	Class hierarchy	Examples	More...		
2070000	NPC23_PIp_nlo	(tarball)	(info file)	127
2070200	NPC23_KAp_nlo	(tarball)	(info file)	127
2070400	NPC23_PRp_nlo	(tarball)	(info file)	127
2070600	NPC23_PIm_nlo	(tarball)	(info file)	127
2070800	NPC23_KAm_nlo	(tarball)	(info file)	127
2071000	NPC23_PRm_nlo	(tarball)	(info file)	127
2071200	NPC23_PIsum_nlo	(tarball)	(info file)	127
2071400	NPC23_KAsum_nlo	(tarball)	(info file)	127
2071600	NPC23_PRsum_nlo	(tarball)	(info file)	127
2071800	NPC23_CHHAp_nlo	(tarball)	(info file)	127
2072000	NPC23_CHHAm_nlo	(tarball)	(info file)	127
2072200	NPC23_CHHAsum_nlo	(tarball)	(info file)	127

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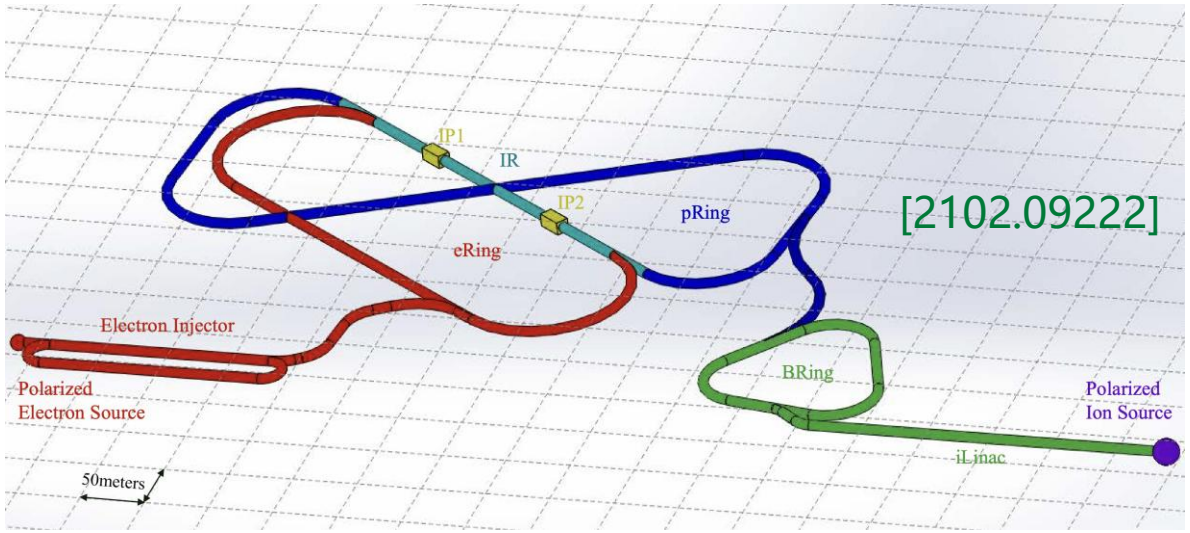
- NPC analyses of FFs at NNLO

The need for high-precision FFs extractions



- The Electron-Ion Collider (EIC)
  - start operation in the early 2030s
  - unprecedented access to nucleon structure
  - **FFs** as keys ingredients of SIDIS at the EIC

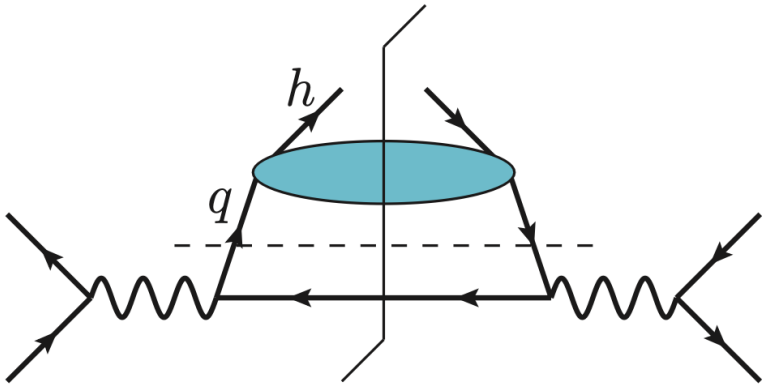
- Efforts from China
  - ep collisions: EicC
  - ee collisions: BESIII measurements

    [BESIII, 2502.16084 (**PRL**)]

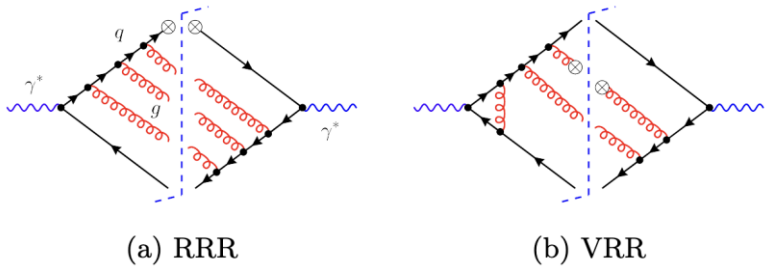
    [See Yateng Zhang's talk]

High-precision FFs as key output & input

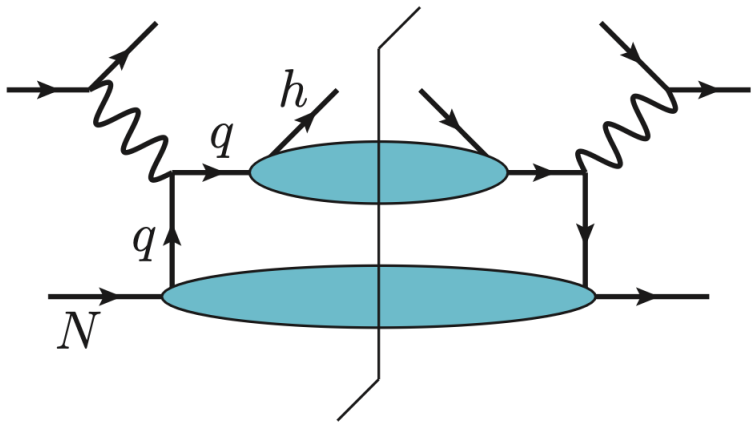
Recent progresses from pQCD



❖SIA(e^+e^-) at N3LO



[He, Xing, Yang, Zhu, PRL.135.101901(2025)]

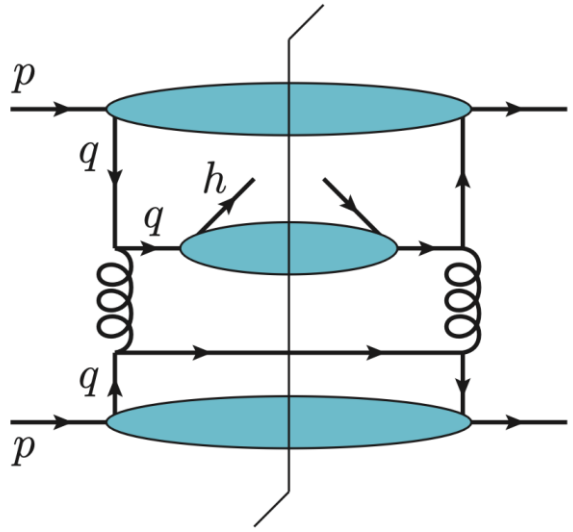


❖ SIDIS(ep) at NNLO

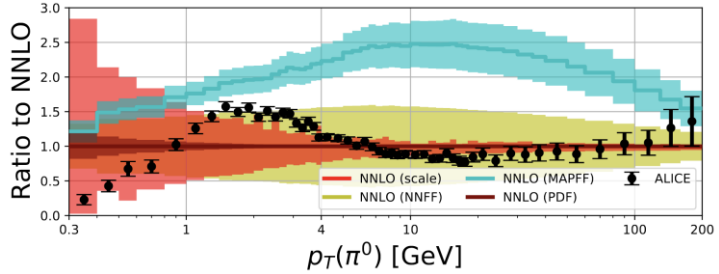
[Bonino, Gehrmann, et al.

& Goyal, Moch, et al.

PRL.132.251901, '24, PRL.132.251902, '24, PRL.133.211904, '24, PRL.133.211905, '24, 2504.05376]



❖ pp at NNLO



[Czakon, Generet, Mitov, Poncelet, 2503.11489]

Figure credit: A. Metz, A. Vossen,1607.02521

NPC analyses of FFs at NNLO + constraints on proton PDFs

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025]

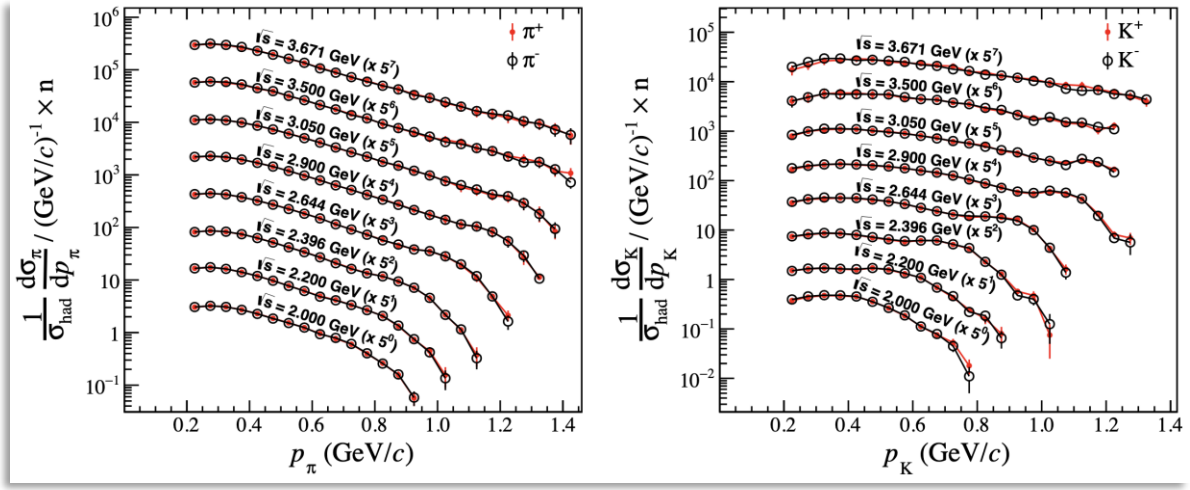
collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
pp hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	$\pi^{\pm},K^{\pm},p$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ η	$\pi^{\pm}$	$\pi^{\pm},K^{\pm}$	$\pi^{\pm},K^{\pm},p,h^{\pm}$ K^0,η,Λ	$\pi^{\pm},K^{\pm}$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

Only some of the recent global analyses are shown here.

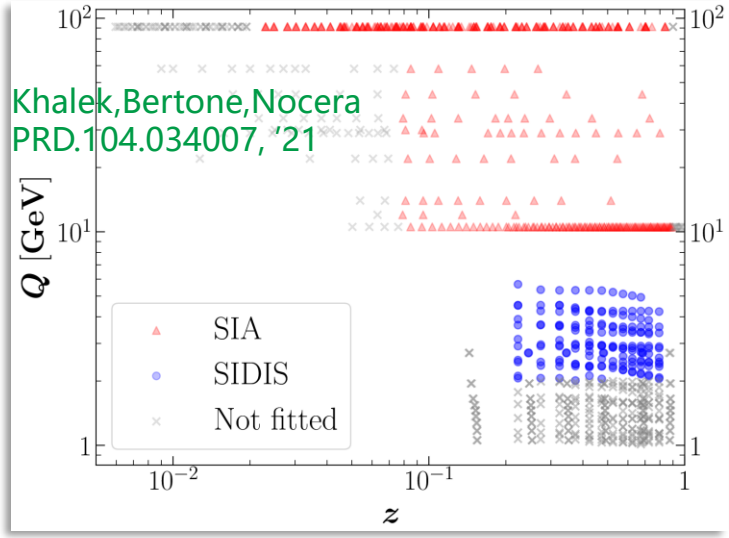
Global analysis of FFs at full NNLO: the datasets

SIA(e^+e^-) data used in the fit:

exp.	$\sqrt{s}$/GeV	lum.(n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb^{-1}	2002	inc. had.	$\pi^{\pm}$,$K^{\pm}$
OPAL	m_Z	780 000	1994	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
ALEPH	m_Z	520 000	1995	$Z \to q\bar{q}$	$\pi^{\pm}$, $K^{\pm}$
DELPHI	m_Z	1 400 000	1998	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$,$K^{\pm}$
SLD	m_Z	400 000	2004	$Z \to q\bar{q}$	$\pi^{\pm}$,$K^{\pm}$
				$Z \to b\bar{b}$	$\pi^{\pm}$, $K^{\pm}$
				$Z \to c\bar{c}$	$\pi^{\pm}$,$K^{\pm}$
TASSO	44	34 pb^{-1}	1989	inc. had.	$\pi^{\pm}$, π^0
TASSO	34	77 pb^{-1}	1989	inc. had.	$\pi^{\pm}$,$K^{\pm}$
TPC/2γ	29	70 pb^{-1}	1988	inc. had.	$\pi^{\pm}$,$K^{\pm}$
Belle	10.52	68 fb^{-1}	2013	inc. had.	$\pi^{\pm}$, $K^{\pm}$
BaBar	10.54	0.91 fb^{-1}	2013	inc. had.	$\pi^{\pm}$,$K^{\pm}$
BESIII	2.0-3.671	253 pb^{-1}	2025	inc. had.	$\pi^{\pm}$, $K^{\pm}$



[BESIII, 2502.16084 (**PRL**)]



SIA

SIDIS / BESIII

- Kinematic cuts in our analyses:
  - $Q > 3$ GeV (SIA)
  - $Q > 2$ GeV(SIDIS)
  - $z > 0.01, E_h > E_{h,min}$(0.8 GeV by default)

Global analysis of FFs at full NNLO: theoretical prediction

FFs at starting scale $Q_0 = 1.4$ GeV

- parameterized as

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

- charge/isospin symmetries suppress number of free parameters (54 in total)

$$D_{u\to\pi^+}(z,Q) = D_{\bar{u}\to\pi^-}(z,Q)$$

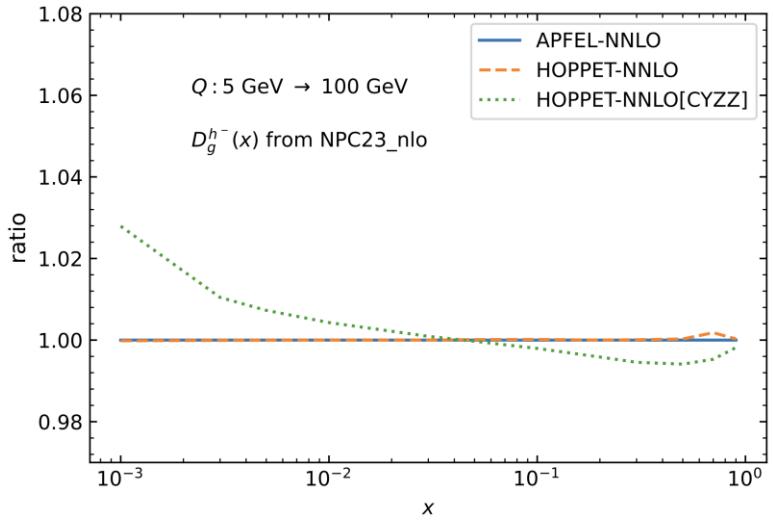
$$D_{u\to\pi^+}(z,Q_0) = D_{\bar{d}\to\pi^+}(z,Q_0)$$

FFs at arbitrary energy scale ($Q_0 \to Q$)

- 3-loop timelike DGLAP evolution

  [Mitov, Moch, Vogt, Almasy]

- $+P_{qg}^{T(2)}$ correction [Chen, Yang, Zhu, Zhu, '20]



- Heavy quark FFs are frozen below mass threshold.

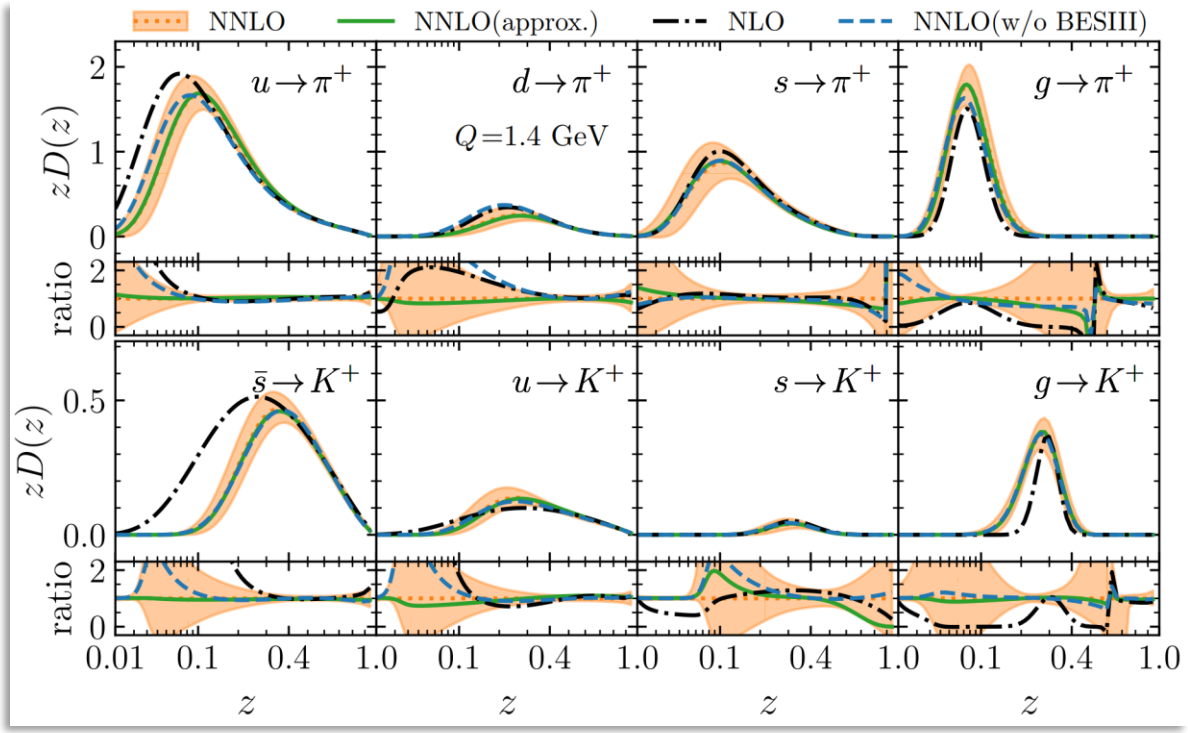
Global analysis of FFs at full NNLO: results

- Fit quality of the NNLO analyses

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

⇑

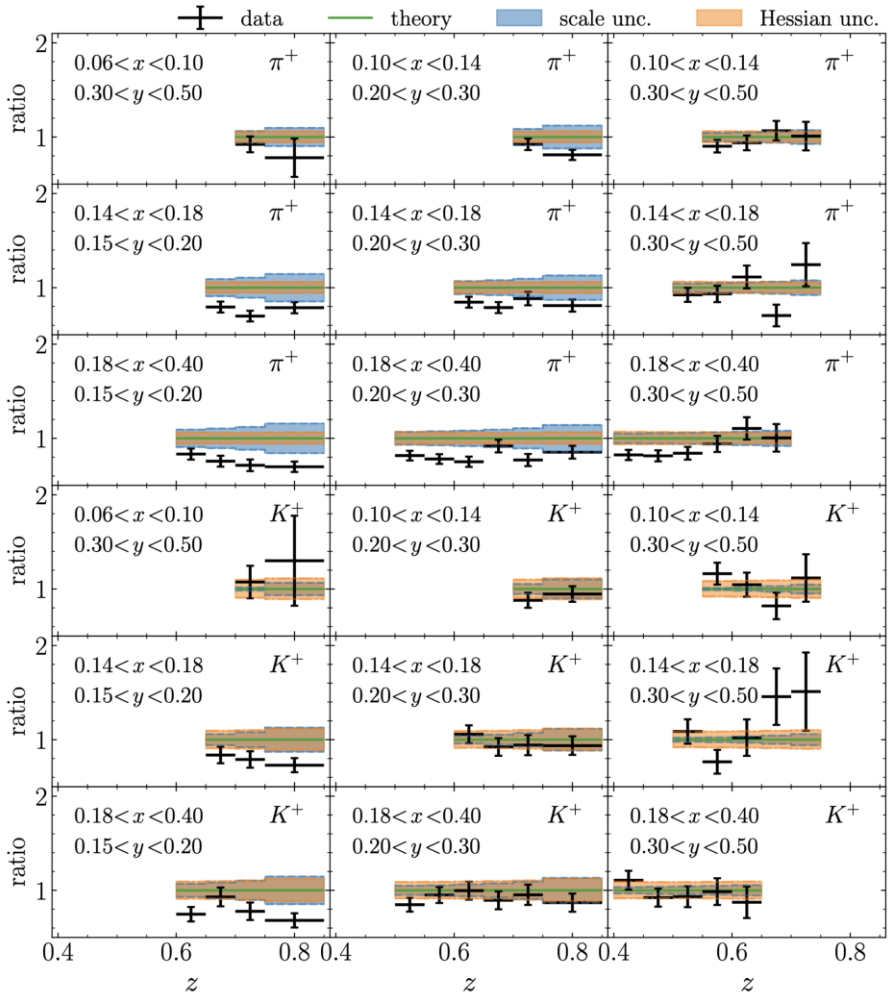
energy cut of the identified hadron



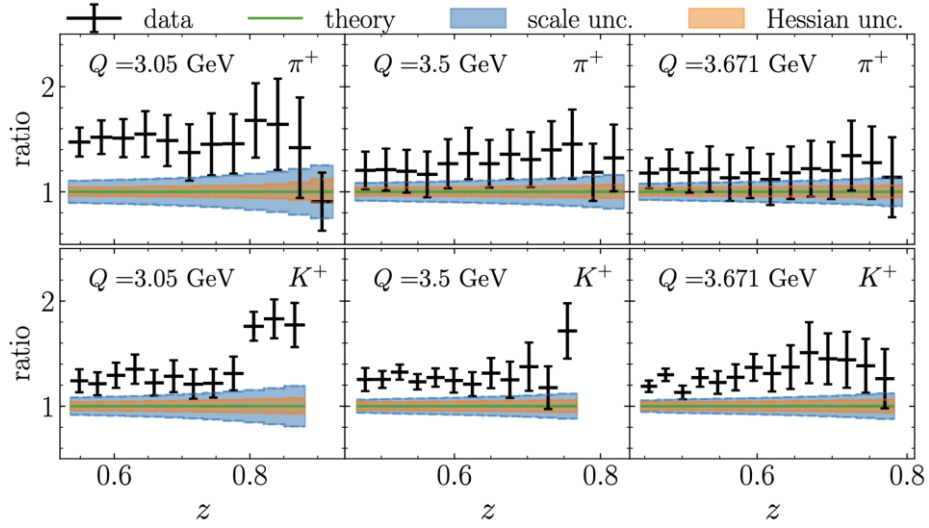
LHAgrids of our FFs have been submitted to the LHAPDF repository.

Test on leading-twist collinear factorization at low Q

Theory v.s. data for COMPASS06 (SIDIS: 2~5GeV)



Theory v.s. data for BESIII (SIA: ~3GeV)



- Kinematic cuts in our analyses:
  - $Q > 3$ GeV (SIA)
  - $Q > 2$ GeV(SIDIS)
  - $z > 0.01, E_h > 0.8$ GeV

the **first** test of **universality of FFs** at Q~3 GeV using ee and SIDIS data

Application: constraining proton PDFs at NNLO

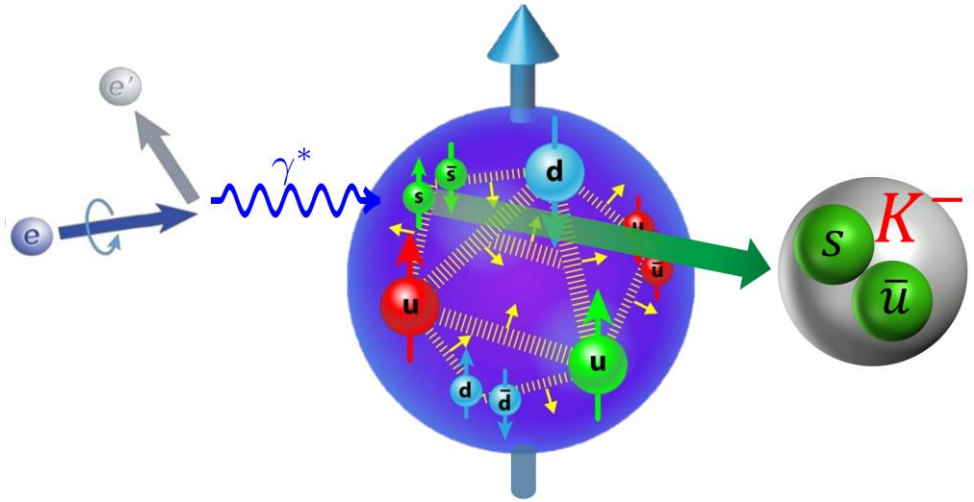
SIDIS may also constrain PDFs:

$$\frac{\mathrm{d}^3\sigma_h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

unpol. PDF FF

$$g_1^h(x,Q^2,z) = \frac{1}{2}\sum_q e_q^2 \left[\Delta q(x,Q^2)\, D_q^h(Q^2,z) + \Delta\bar{q}(x,Q^2)\, D_{\bar{q}}^h(Q^2,z)\right] + \mathcal{O}(\alpha_s)$$

pPDF FF



- LO x-section of iso-scalar target SIDIS(COMPASS)

$$\frac{\mathrm{d}\sigma^{K^+}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} - \frac{\mathrm{d}\sigma^{K^-}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z}$$

$$\sim 2\Big(u_v(x)+d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$

$$+\Big(s(x)-\bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

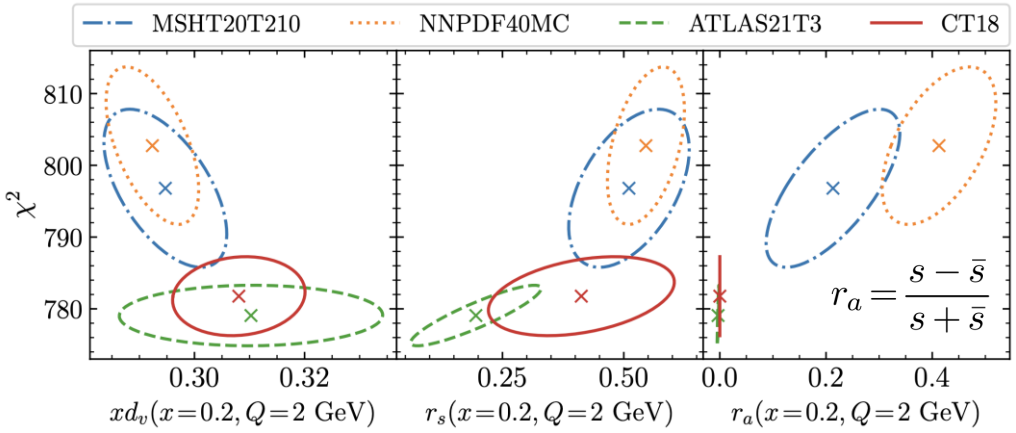
PDF FF

is sensitive to **strangeness asymmetry**

$$r_a = \frac{s-\bar{s}}{s+\bar{s}}$$

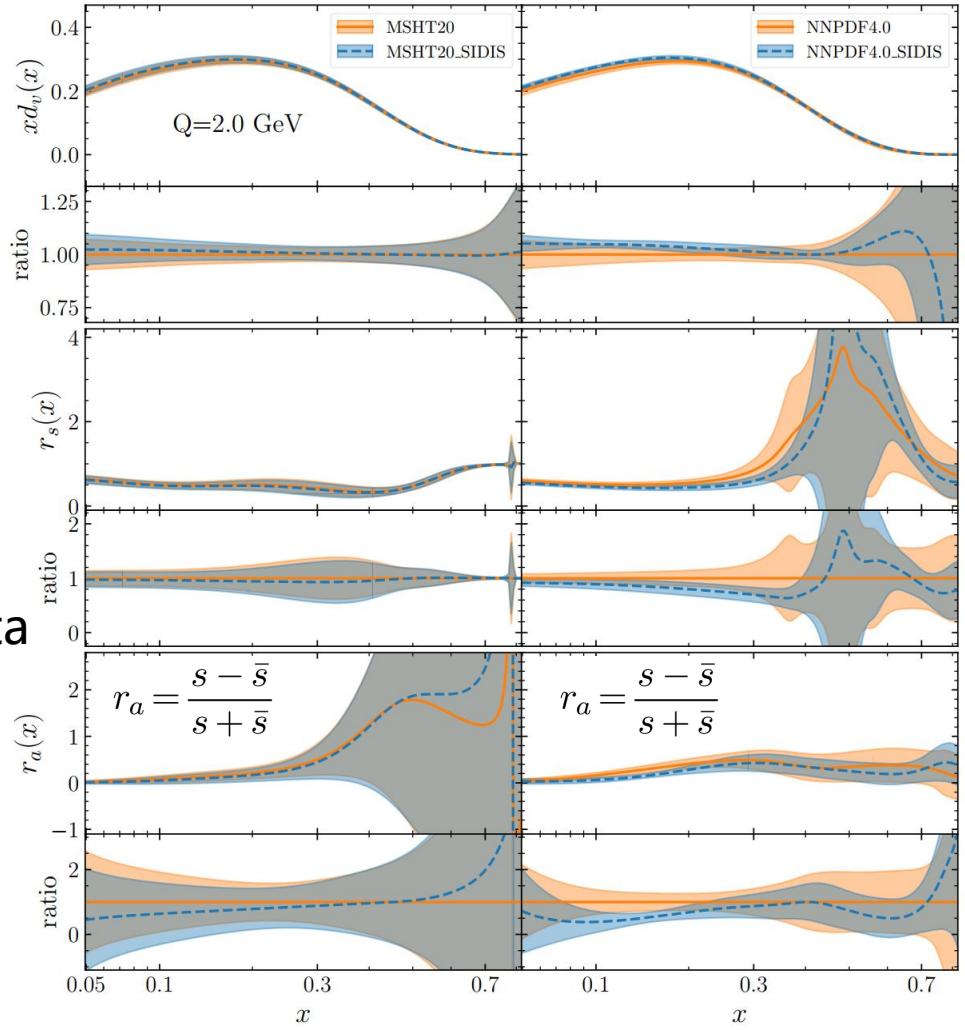
Application: constraining unpolarized PDFs at NNLO

- Correlation between χ^2 and PDFs



- **Modified PDFs** which reflect the impact of SIDIS data
  - Reweighting of the NNPDF4.0 PDF set
  - Profiling of the MSHT20 PDF set

PDF sets before and after reweighting/profiling



[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]

Summary

- NPC collaboration has delivered precise and comprehensive FF sets at NLO.
- This work presents the first global (SIA+SIDIS) FFs determination at full NNLO.
- **FF** studies may also contribute to **nucleon structures analyses**.

	NPC23 NLO	this talk
SIA (ee)	✓	✓
SIDIS (ep)	✓	✓
pp incl. hadron	✓	✗
pp hadron in jet	✓	✗
FFs	$\pi^{\pm}$,$K^{\pm}$,p,$h^{\pm}$ K^0,η,Λ	$\pi^{\pm}$,$K^{\pm}$
pQCD order	NLO	NNLO

Global FF analyses from NPC (submitted to LHAPDF repository):

NLO charged hadron:

Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903*, 2024

Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019*, (Editors' suggestion), 2024

NLO neutral hadron:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, 2503.21311 (*PRD)*

NNLO: Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025 (this talk)

Thank you for your attention!

Backup slides

The parameterization

- Joint determination of FFs to charged pion, kaon at NNLO in QCD
- Parameterization at $Q_0 = 1.4$ GeV

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

- Charge conjugation symmetry

$$D_{u\to\pi^+}(z,Q) = D_{\bar{u}\to\pi^-}(z,Q)$$

- Isospin symmetry

$$D_{u\to\pi^+}(z,Q_0) = D_{\bar{d}\to\pi^+}(z,Q_0)$$

[Gao, **XS**, Xing, Zhao, Zhou, *PRL 135, 041902*,]

flavor	favored	a_0	α	β	a_1	a_2
$u=\bar{d}$	✓	✓	✓	✓	✓	✓
$d=\bar{u}$	✗	✓	✓	✓	✓	✓
$s=\bar{s}$	✗	✓	$=\alpha_d$	✓	✓	✓
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

flavor	favored	a_0	α	β	a_1	a_2
u	✓	✓	✓	✓	✓	✓
$\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_u$	✓	✓
$s=\bar{u}=d=\bar{d}$	✗	✓	✓	✓	✓	✗
$c=\bar{c}$	✗	✓	✓	✓	✓	✓
$b=\bar{b}$	✗	✓	✓	✓	✓	✓
g	✗	✓	✓	✓	✓	✗

pQCD order

- Fit quality of the NNLO analyses

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

- Alternative fits at NLO

$E_{h,\min}$[GeV]	BESIII		COMPASS		B-factories		HE-SIA		global		
	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.38	358	1.50	233	1.01	426	1.23	1259	1631.2	1.30
0.6	212	1.26	290	1.44	228	0.87	423	1.06	1153	1333.2	1.16
0.7	182	1.12	214	1.43	223	0.67	413	0.97	1032	1057.9	1.03
0.8	152	1.03	142	1.26	218	0.54	407	0.85	919	801.6	0.87
0.9	122	1.08	94	1.22	213	0.52	407	0.84	836	697.5	0.83
1.0	98	1.18	54	0.93	209	0.49	403	0.83	764	603.7	0.79