香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

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A Century of Spin

Total Gluon Helicity from Lattice

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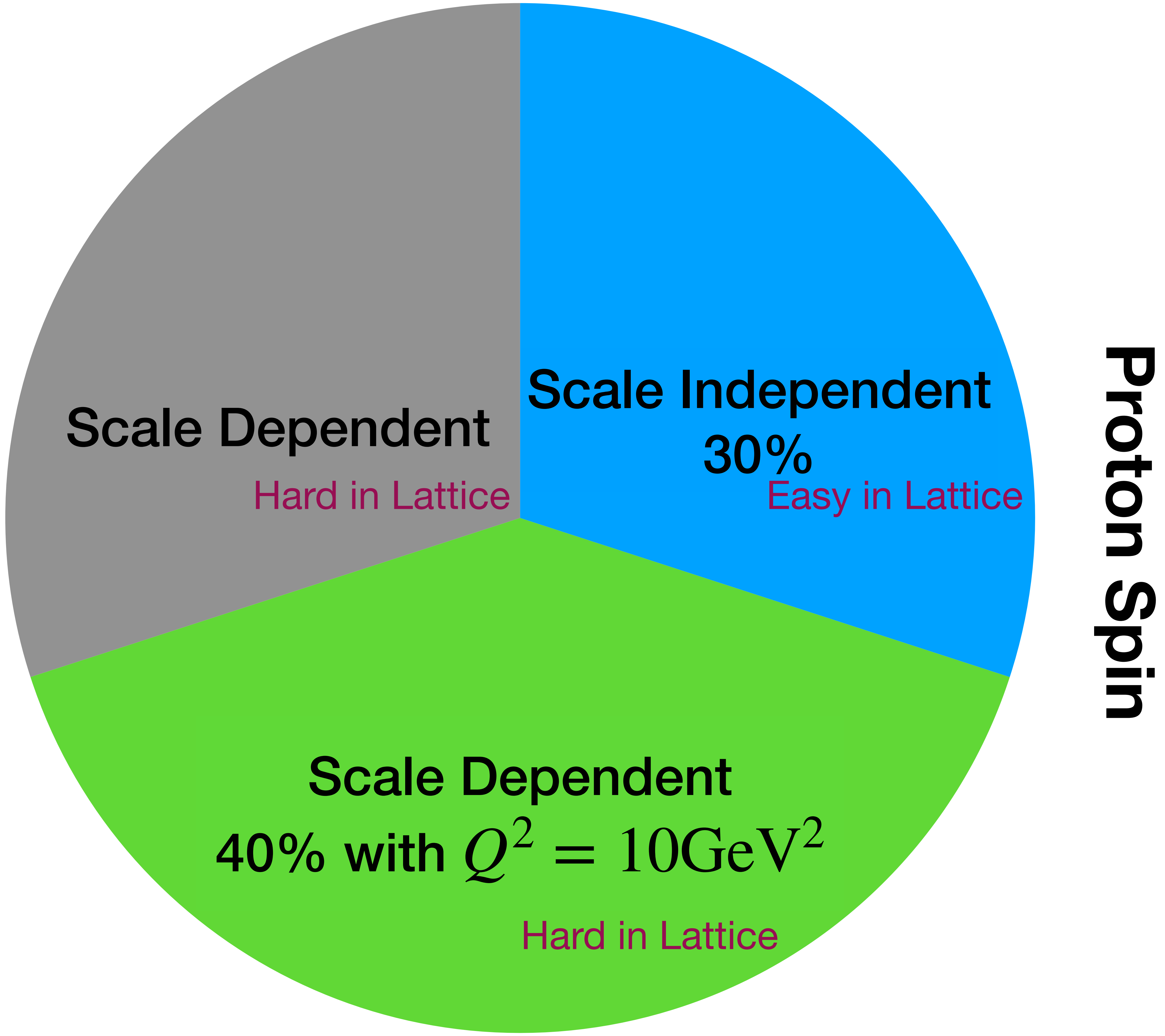
Speaker: Dian-Jun Zhao

In Collaboration with Hong-Xin Dong, Liuming Liu, Zhuo-Yi Pang, Peng Sun, Yi-Bo Yang, Jianhui Zhang, Shi-Yi Zhong et. al.

Introduction on proton spin



Quark helicity in lattice QCD

$$\Delta q = \langle \text{PS} \,|\, \bar{q}\gamma_5 \vec{\gamma} \cdot \vec{S} q \,|\, \text{PS} \rangle$$

$$\int d^3x (\bar{q}\gamma_\mu \gamma_5 q)(x) = 2m_f \int d^3x \vec{x} \vec{P}(x) - 2i \int d^3x \vec{x} \vec{q}(x)$$

$$\Delta u = 0.835(15), \qquad \Delta d = -0.435(15),$$
$$\Delta s = -0.095(15), \quad \Delta c \simeq 0.00$$

$$\boxed{\Delta \Sigma = \frac{1}{2} \sum \Delta q = 0.155(25)}$$

C. Adolph et al. Phys. Lett., B753:18–28, 2016.
Daniel de Florian et al. Phys. Rev., D80:034030, 2009.
Emanuele R. Nocera, et al. Nucl. Phys., B887:276–308, 2014.
Daniel de Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

Jian Liang, el al. Phys. Rev., D98(7):074505, 2018.
Huey-Wen Lin, et al. Phys. Rev. D, 98:094512, 2018.
C. Alexandrou, et al. Phys. Rev. Lett., 119(14):142002, 2017.
Ming Gong, et al. Phys. Rev., D95(11):114509, 2017.

Lattice interpretation of gluon helicity

Gluon helicity in lattice QCD

$$\Delta G = \int dx \Delta g(x) = \int dx \frac{i}{2xP^+} \int \frac{d\varepsilon^-}{2\pi} e^{-ix\varepsilon^- P^+} \langle \mathrm{PS} \,|\, F_a^{+\mu}(\varepsilon^-) \mathscr{L}_{ab}(\varepsilon^-, 0) \tilde{F}^+_{b,\mu}(0) \,|\, \mathrm{PS} \rangle$$

Integrating PDF

$$\Delta G^{\overline{\mathrm{MS}}\, 10\mathrm{GeV}^2} = \int_{0.05}^{1} dx g(x) = 0.2$$

Daniel de Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

ΔG is difficult to calculate in LQCD because light-cone gauge (L.G.) $\not\leftrightarrow$ Euclidean space!

LaMET suggests that Coulomb gauge fixing (C.G.) condition become to L.G. when nucleon to IMF, then $\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\mathrm{C.G.}} = S_G|_{\mathrm{IMF}}$ with matching for their difference in UV behavior.

Xiangdong Ji et al. 10.1103/PhysRevLett.111.112002

Lattice interpretation of gluon helicity

Selected as APS Highlights

Gluon helicity in lattice QCD

Yi-Bo Yang et al. 10.1103/PhysRevLett.118.102001

$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)(16)$$

quark mass and momentum dependence of the gluon spin

Lattice interpretation of gluon helicity

Potential problems with ΔG extraction using $\langle \overrightarrow{E} \times \overrightarrow{A} \rangle_{\text{C.G.}}$

1. Renormalization matching using lattice perturbation theory;

2. Poor convergence of matching from gluon spin to gluon helicity (UV cutoff $\alpha_s(\pi/a)$).

Gluon helicity ΔG from topological current

We propose a scheme that relates ΔG to **local topological current** $K^{\mu}_{\text{C.G.}}$. This scheme solves all of these problems and doesn't require perturbative matching.

$$\boxed{\text{Topological Current } K^{\mu}(x) = \epsilon^{\mu\nu\rho\sigma}\text{Tr}[A_\nu F_{\rho\sigma} - 2ig_s A_\nu A_\rho A_\sigma/3](x)}$$

$$\text{Target Three-PT } \langle \text{PS}_{\text{Proj.Z}} | K^{t/z} | \text{PS}_{\text{Proj.Z}} \rangle_{\text{C.G.}} \propto S^{t/z} \Delta G + \text{h.t.}$$

Zhuo-Yi Pang et al. 10.1007/JHEP07(2024)222

Lattice Setup

Zhi-Cheng Hu et al. 10.1103/PhysRevD.109.054507

CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing(fm)	$m_\pi^{(s)}$(MeV)	Valence Type	Ncfg	Nsource
C24P29 (B.M.E.)	Clover (2+1) + TITLS	$24^3 \times 72$	0.1052	292.3(1.0)	Clover	880	72 (Distillation +M.S.)
C48P23 (Renorm.)		$48^3 \times 96$		224.1(1.2)		380	Volume (CDER)

B.M.E.: Bare Matrix Element

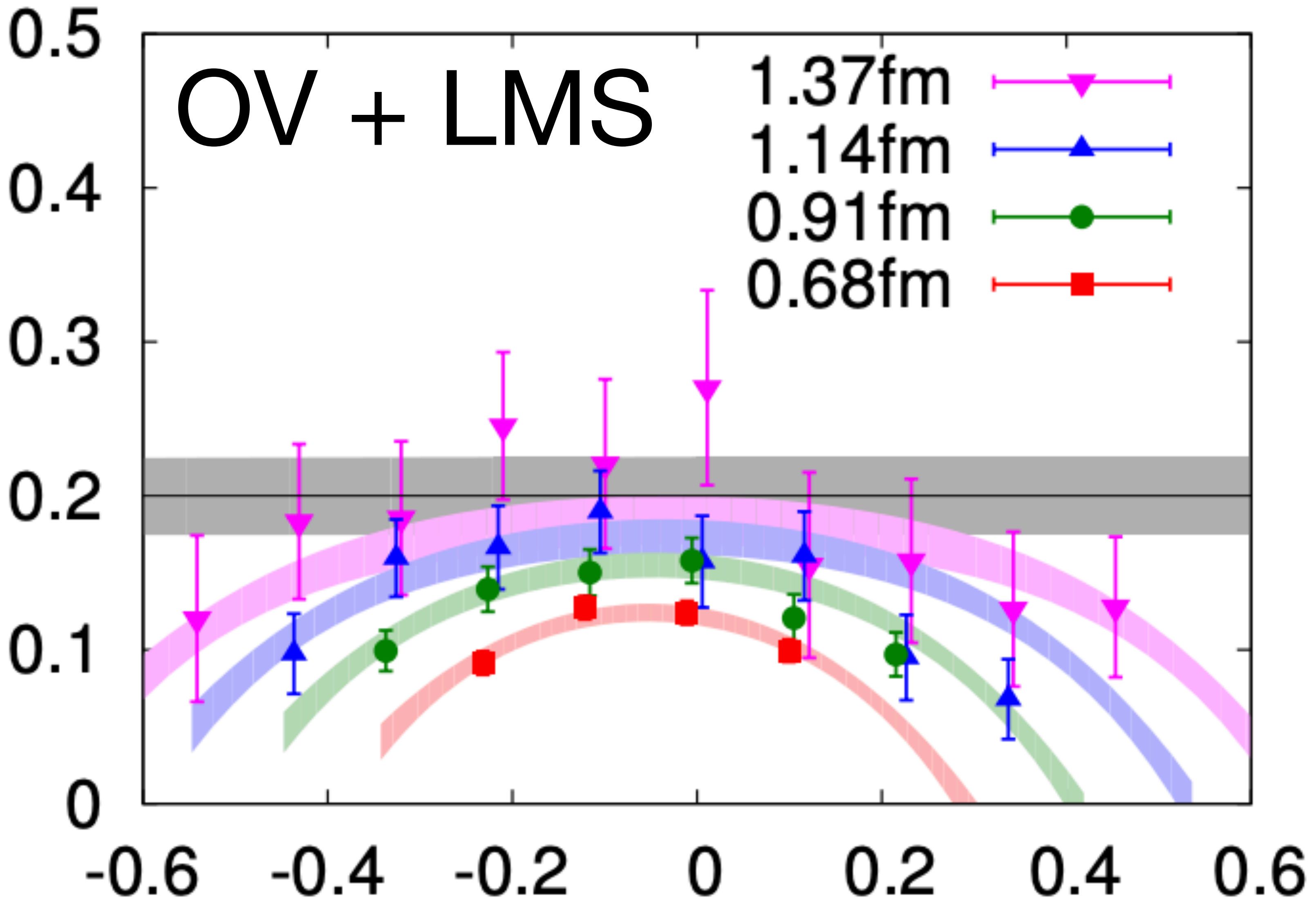
M.S.: Momentum smear

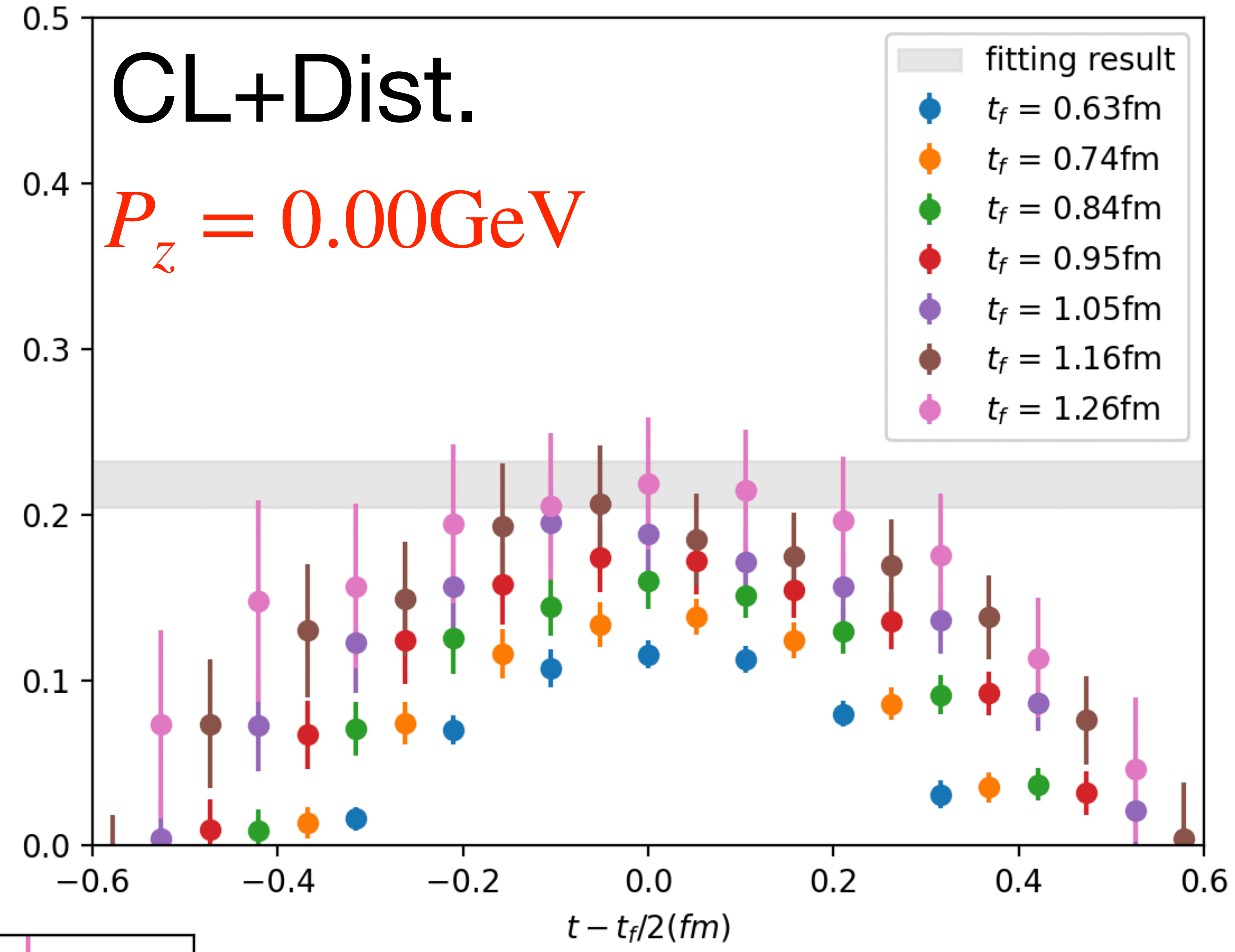
CDER.: Cluster Decomposition Error Reduction

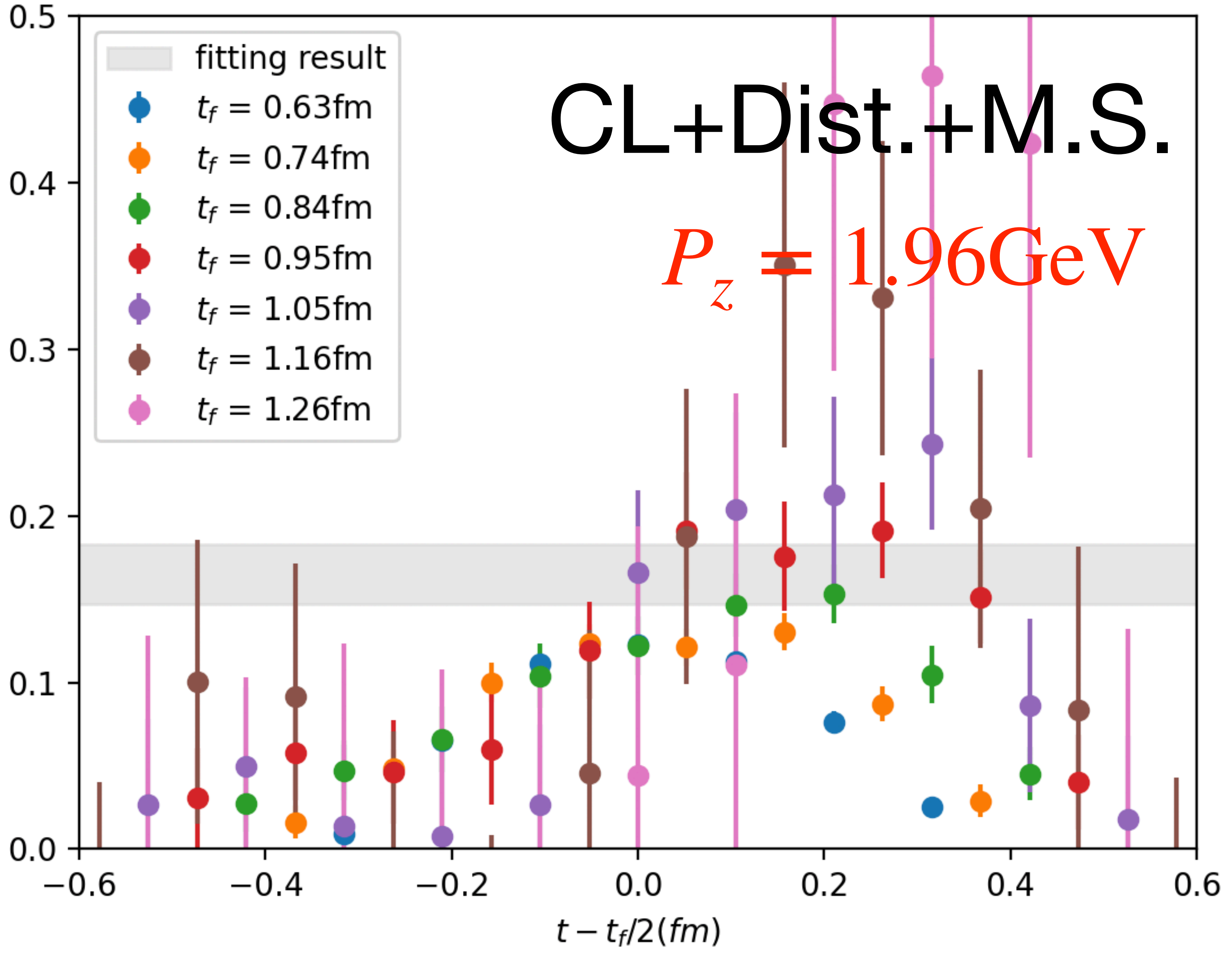
Ensemble used for renormalization has same UV properties as ensemble used for bare matrix element calculations.

$\langle \vec{E} \times \vec{A} \rangle^{\text{Bare.}}$ with optimization smear methods

Yi-Bo Yang et al. 10.1103/PhysRevLett.118.102001







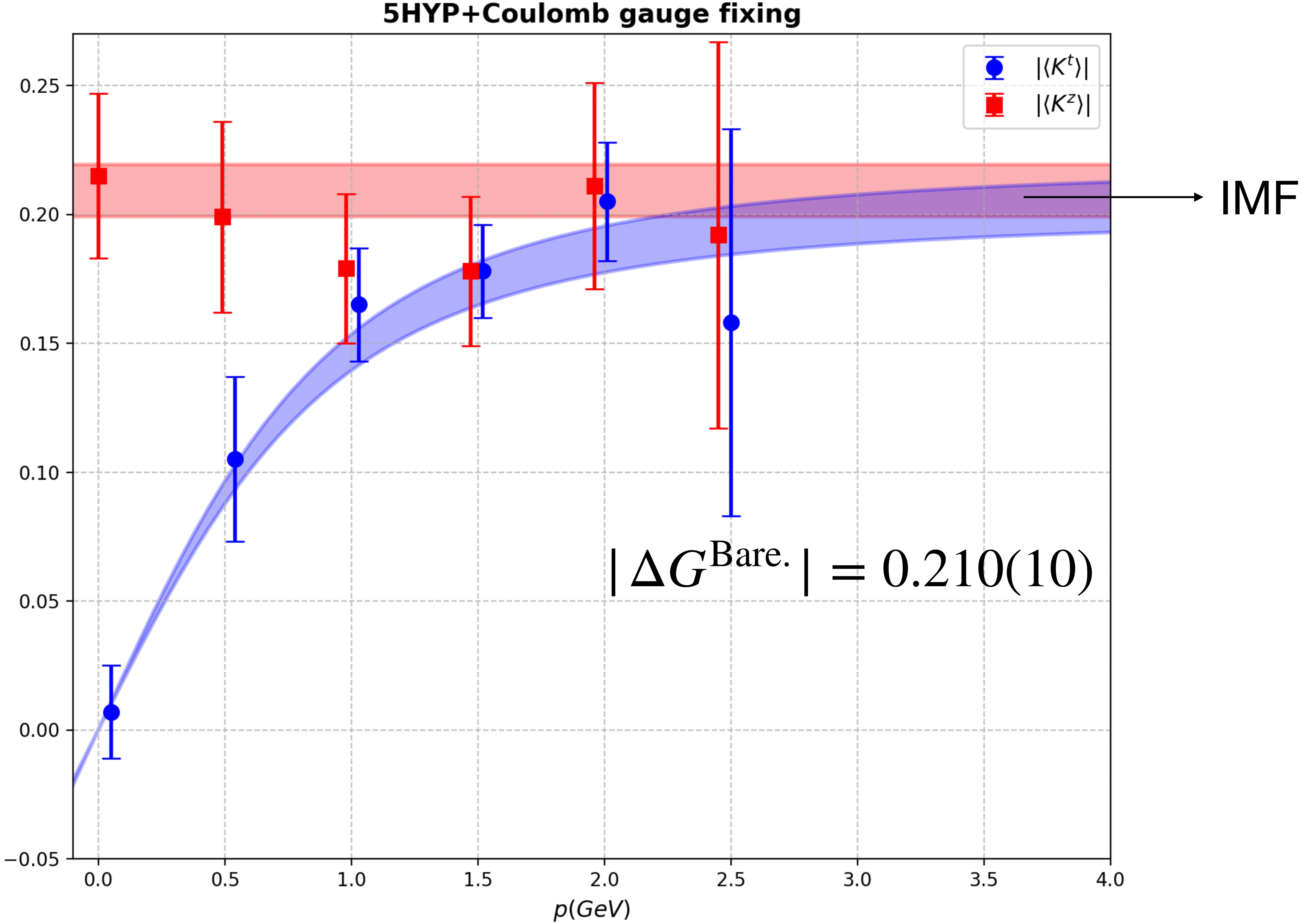
Appropriate smearing scheme can improve SNR of bare matrix elements.

Fitting Form:

$$\frac{C_3(t_f, t)}{C_2(t_f)} = \langle \vec{E} \times \vec{A} \rangle^{\text{Bare.}} + A_1 e^{-\Delta E(t_f - t)} + A_2 e^{-\Delta E t}$$

Dist. for small p_z, Dist. + M.S. for large p_z.

Relationship between B.M.E. $\langle K^{t/z} \rangle^{\text{Bare.}}$



Renormalization of topological current K^μ

RI/MOM is non-perturbative renormalization scheme on LQCD.

RI/MOM on lattice

Zhuo-Yi Pang et al. [10.1007/JHEP07(2024)222](https://doi.org/10.1007/JHEP07(2024)222)

$$\langle PS|\{\widehat{K^\rho, J^\rho}\}|PS\rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1}\langle PS|K^\rho|PS\rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2}\langle PS|J^\rho|PS\rangle^{\text{lat.}}$$

Coupled

$|\text{PS}\rangle$: Parton state (Gluon or Quark) with momentum and polarization; $J^\rho = \bar{q}\gamma_5\gamma^\rho q$: Axial-vector current operator

$\mathcal{O}^{\text{X.},g} \equiv \langle g|\mathcal{O}|g\rangle^{\text{X.}}$ $\quad\mathcal{O}^{\text{X.},q} \equiv \langle q|\mathcal{O}|q\rangle^{\text{X.}}$ $\quad J^{\text{lat.},g} \to 0$, also $K^{\text{tree.},q} = J^{\text{tree.},g} = 0$

then $\quad Z^{\text{RI}}_{11} = \dfrac{K^{\text{tree.,g}}}{K^{\text{lat.,g}}},\qquad Z^{\text{RI}}_{12} = -\dfrac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}}Z^{\text{RI}}_{11} = \mathcal{O}(\alpha_s)\times Z^{\text{RI}}_{11}$

$Z^{\text{RI}}_{21} = 0,\qquad Z^{\text{RI}}_{22} = \dfrac{J^{\text{tree.,q}}}{J^{\text{lat.,q}}} = Z^{\text{RI}}_A \longleftarrow$ Zhi-Cheng Hu et al. [10.1103/PhysRevD.109.054507](https://doi.org/10.1103/PhysRevD.109.054507)

$Z^{\text{RI}}_A = 0.857$ under C48P23

$$\boxed{\Delta G^{\text{RI}} = Z^{\text{RI}}_{11}\left(\Delta G^B - \frac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}}\Delta\Sigma^{\text{B}}\right),\qquad \Delta\Sigma^{\text{RI}} = Z^{\text{RI}}_A\Delta\Sigma^{\text{B}}}$$

Renormalization of topological current K^μ

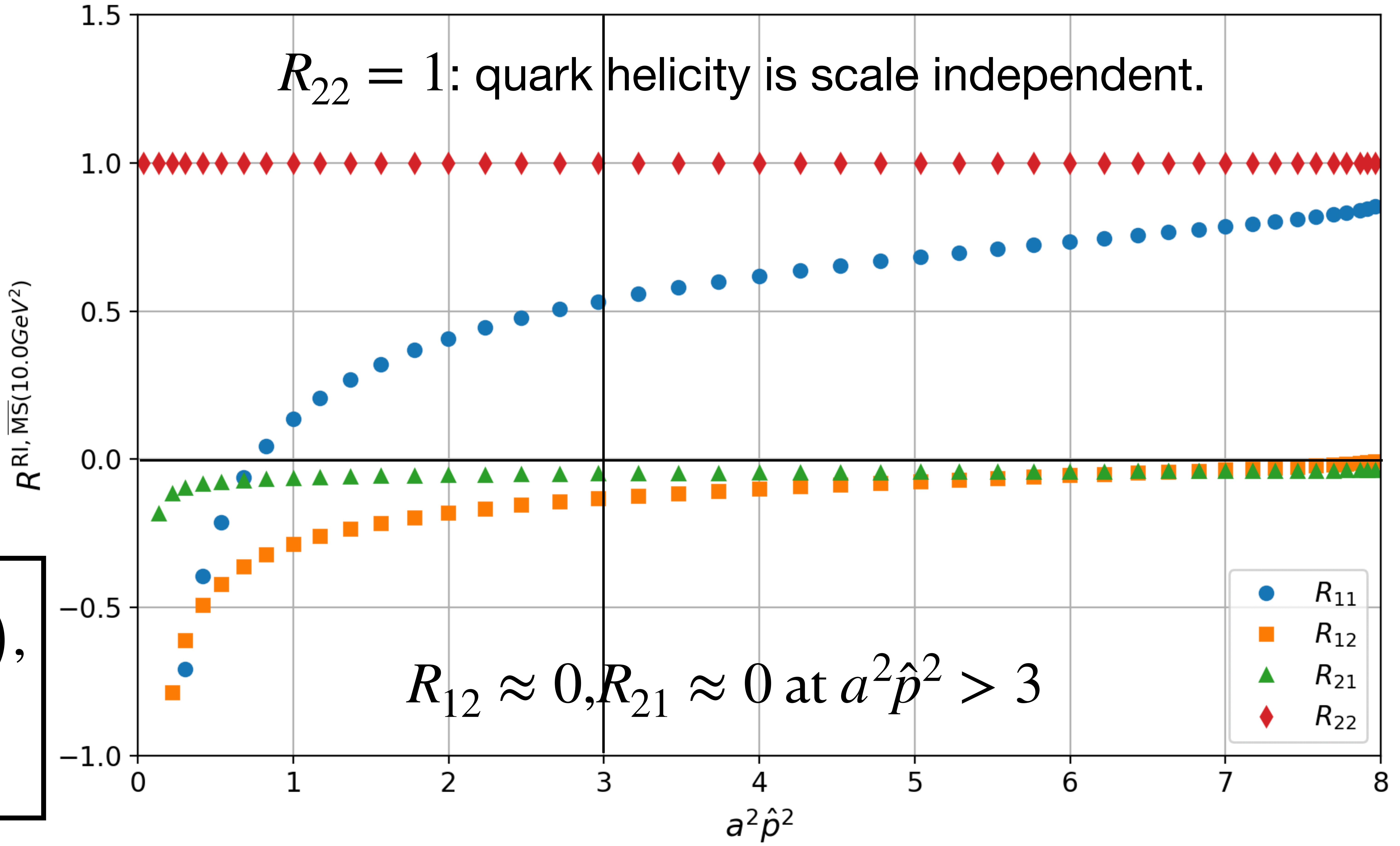
RI/MOM to $\overline{\mathrm{MS}}$

Zhuo-Yi Pang et al. 10.1007/JHEP07(2024)222

$$\Delta G^{\overline{\mathrm{MS}}} = R_{11}\Delta G^{\mathrm{RI}} + R_{12}\Delta\Sigma^{\mathrm{RI}},$$
$$\Delta\Sigma^{\overline{\mathrm{MS}}} = R_{21}\Delta G^{\mathrm{RI}} + R_{22}\Delta\Sigma^{\mathrm{RI}}$$

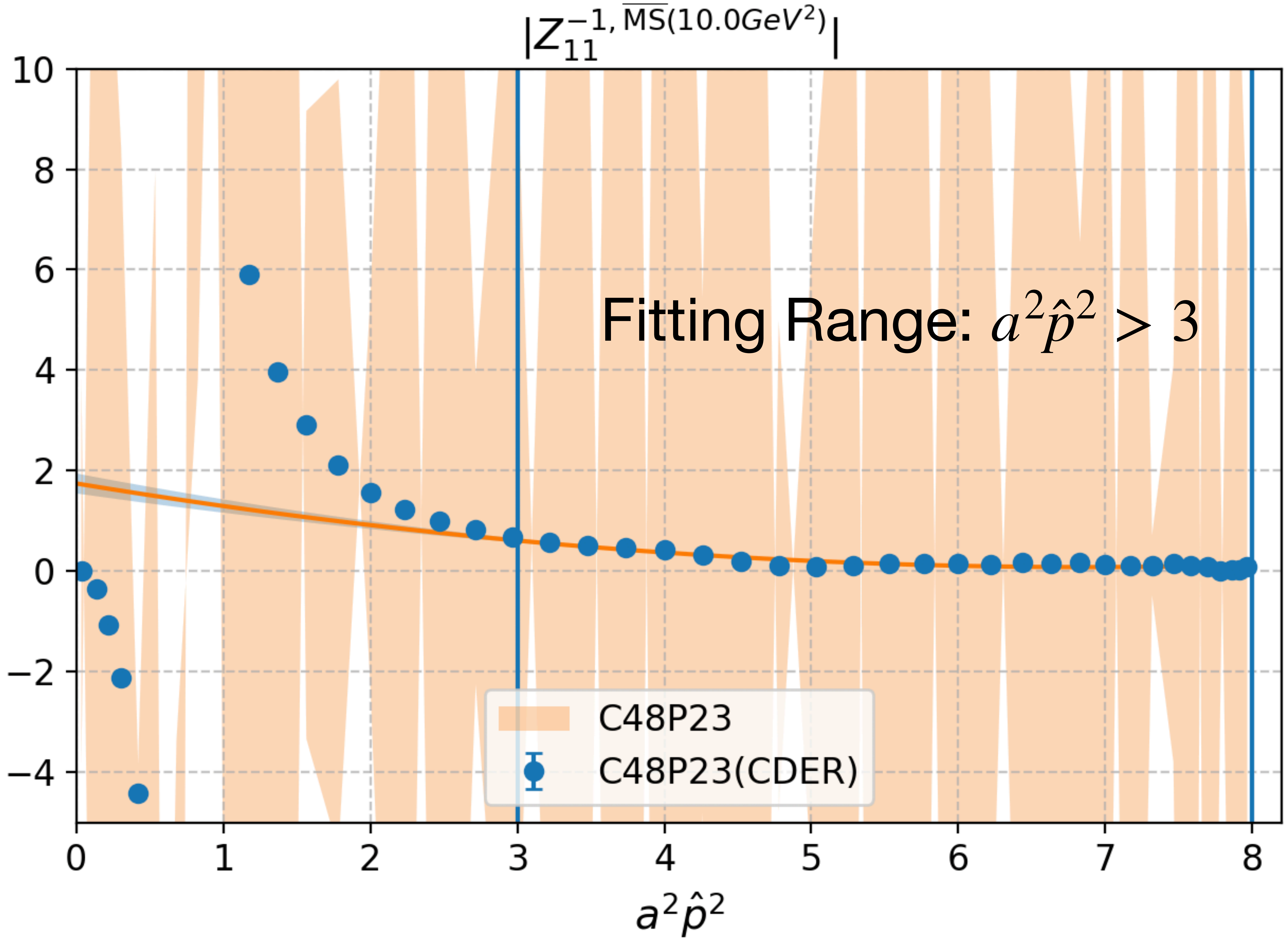
By selecting RI/MOM high scale, coupling matching coefficient R_{12}, R_{21} is made much smaller than diagonal matching coefficient.

$$\Delta G^{\overline{\mathrm{MS}}}_{(a^2\hat{p}^2)_{\mathrm{RI}}>3} = R_{11}Z^{\mathrm{RI}}_{11}\left(\Delta G^{B} - \frac{K^{\mathrm{lat.,q}}}{J^{\mathrm{lat.,q}}}\Delta\Sigma^{\mathrm{B}}\right),$$
$$\Delta\Sigma^{\overline{\mathrm{MS}}}_{(a^2\hat{p}^2)_{\mathrm{RI}}>3} = R_{22}Z^{\mathrm{RI}}_{A}\Delta\Sigma^{\mathrm{B}} = Z^{\mathrm{RI}}_{A}\Delta\Sigma^{B}$$



Preliminary results of $\Delta G^{\overline{\mathrm{MS}},10\mathrm{GeV}^2}$

$Z_{11}^{\overline{\mathrm{MS}}} \equiv R_{11} Z_{11}^{\mathrm{RI}}$ contributed by pure gluons is difficult to see signal, so we use **CDER** scheme to enhance signal-to-noise ratio. Yi-Bo Yang et al. 10.1103/PhysRevD.98.074506



Improvement in signal-to-noise ratio using **CDER** method is very significant.

$$|Z_{11}^{\overline{\mathrm{MS}},10\mathrm{GeV}^2}| \equiv |R_{11}^{10\mathrm{GeV}^2} Z_{11}^{\mathrm{RI}}| = 0.575(66)$$

$$\Delta G^{\overline{\mathrm{MS}}} = Z_{11}^{\overline{\mathrm{MS}}}\left(\Delta G^B - \frac{K^{\mathrm{lat.,q}}}{J^{\mathrm{lat.,q}}}\Delta\Sigma^{\mathrm{B}}\right)$$

$$\Delta G_{a=0.105\mathrm{fm}}^{\overline{\mathrm{MS}},10\mathrm{GeV}^2} = 0.120(15) + \mathcal{O}(0.1\alpha_s)$$

$$\sim 0.15$$

negative

Summary

1. **Distillation + Momentum smear** (for B.M.E.) and **CDER** (for Renorm.) scheme.

2. After non-perturbative matching, $\Delta G^{\overline{\mathrm{MS}},10\mathrm{GeV}^2}_{a=0.105\mathrm{fm}} \sim 0.15$, which accounts for $30\,\%$ of proton spin, which is latest result from scheme designed entirely for LQCD.

Outlook

1. Need to supplement calculation of $K^{\mathrm{lat.,q}}/J^{\mathrm{lat.,q}}$.
2. Continuous extrapolation of gluon helicity need to obtain confident results.
3. More rigorous systematic error analysis is needed.

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THANKS FOR LISTENING!

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Backup

Derivation on RI/MOM helicity renormalization

$$S_q(p) = \sum_x e^{-ip\cdot x}\langle \psi(x)\bar{\psi}(0)\rangle$$

$$\langle PS|\{K^\rho, J^\rho\}|PS\rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1}\langle PS|K^\rho|PS\rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2}\langle PS|J^\rho|PS\rangle^{\text{lat.}}$$

$$\langle q\mathcal{O}q\rangle = \sum_{x,y} e^{-i(p_1\cdot x - p_2\cdot y)}\langle \psi(x)\mathcal{O}(0)\bar{\psi}(y)\rangle$$

$J^{\text{lat.},g} \to 0$, also $K^{\text{tree.},q} = 0$ then $Z^{\text{RI}}_{11} = \dfrac{K^{\text{tree.,g}}}{K^{\text{lat.,g}}}$, $Z^{\text{RI}}_{12} = \dfrac{K^{\text{tree.,q}}}{J^{\text{lat.,q}}} - \dfrac{K^{\text{lat.,q}}K^{\text{tree.,g}}}{J^{\text{lat.,q}}K^{\text{lat.,g}}} = -\dfrac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}}Z^{\text{RI}}_{11} \propto \alpha_s Z^{\text{RI}}_{11}$

$J^{\text{lat.},g} \to 0$, also $J^{\text{tree.},g} = 0$ then $Z^{\text{RI}}_{21} = 0$, $Z^{\text{RI}}_{22} = \dfrac{J^{\text{tree.,q}}}{J^{\text{lat.,q}}}$

$K^{-1,\text{lat.},g} = \dfrac{Z_g}{\text{Tr}[S_g^{-1}\langle gKg\rangle^{\text{lat.}}S_g^{-1}]}$ where $Z_g = (S_g p^2)^{-1}$ then $K^{\text{lat.},g} = \dfrac{p^2\langle gKg\rangle^{\text{lat.}}}{\langle S_g\rangle} = \dfrac{p^2\text{Im}\{\langle A_\mu K^\rho A_\nu\rangle^{\text{lat.}}\}}{\langle S_g\rangle}$

$\mathcal{O}^{-1,\text{lat.},q} = \dfrac{Z_q}{\frac{1}{12}\text{Tr}[S_q^{-1}\langle q\mathcal{O}q\rangle^{\text{lat.}}S_q^{-1}]}$ where $Z_q = S_q^{-1}\not{p}$ then $\dfrac{K^{\text{lat.},q}}{J^{\text{lat.},q}} = \dfrac{\text{Tr}[S_q^{-1}\langle qKq\rangle^{\text{lat.}}S_q^{-1}]}{\text{Tr}[S_q^{-1}\langle qJq\rangle^{\text{lat.}}S_q^{-1}]}$

$K^{\text{tree.},g} = ip^\sigma$

$J^{\text{tree.},q} = \gamma^\mu\gamma_5\delta$

$$\langle A_yK_xA_t\rangle|_{p=1010}, \langle A_zK_xA_t\rangle|_{p=1100}, \langle A_xK_yA_t\rangle|_{p=0110}, \langle A_zK_yA_t\rangle|_{p=1100}, \langle A_xK_zA_t\rangle|_{p=0110}, \langle A_yK_zA_t\rangle|_{p=1010}$$

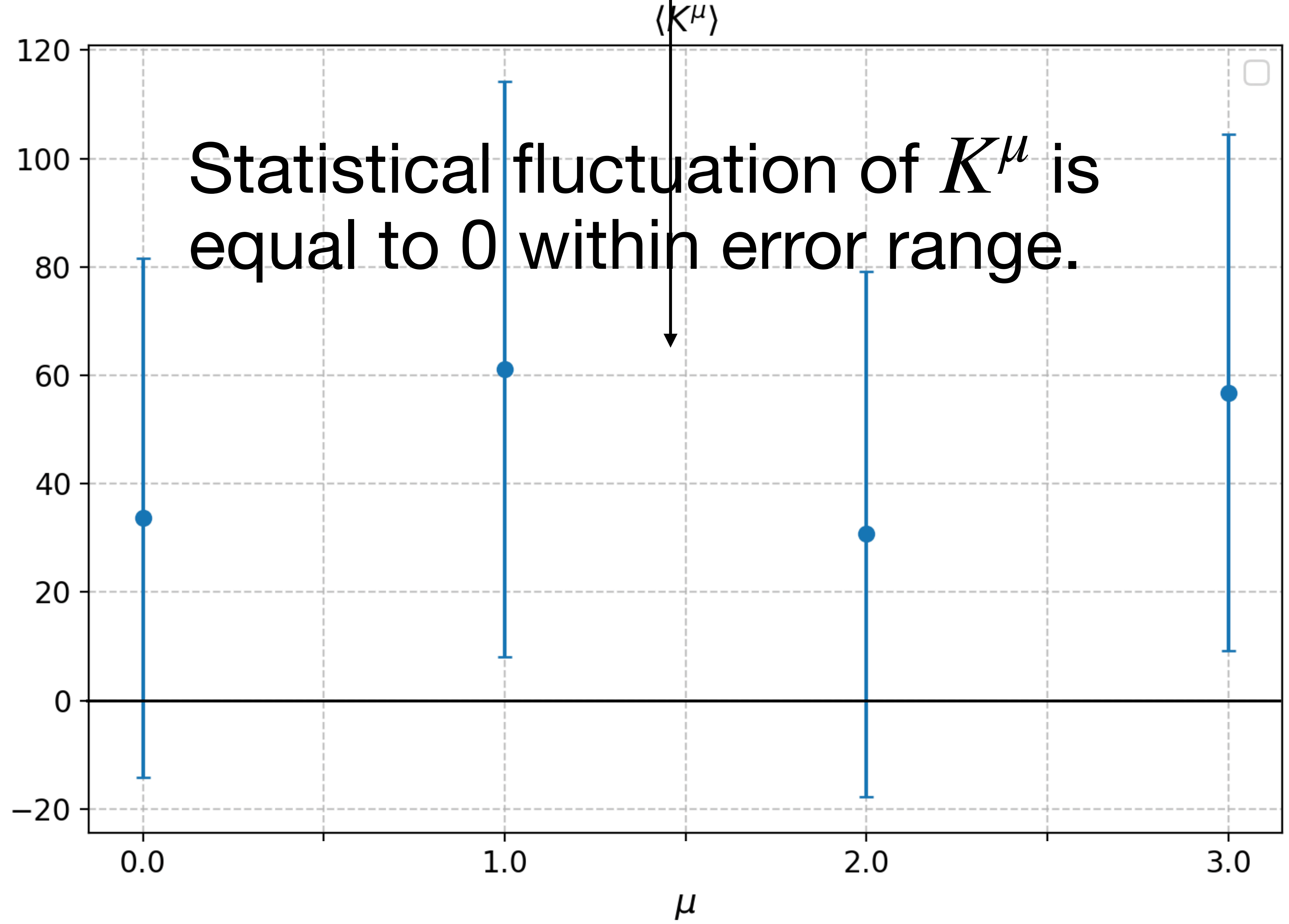
$$\langle A_xK_tA_y\rangle|_{p=0011}, \langle A_xK_tA_z\rangle|_{p=0101}, \langle A_yK_tA_z\rangle|_{p=1001}, \langle A_yK_xA_z\rangle|_{p=1001}, \langle A_xK_yA_z\rangle|_{p=0101}, \langle A_xK_zA_y\rangle|_{p=0011}$$

All of these 12 Three-PT calculate Z^{RI}_{11}, reduce error to $1/\sqrt{12}$ of each configuration.

Notes on Z_{11} Statistical Fluctuations

$$Z_{11}^{-1,\mathrm{RI}}(\mu_R^2) = \frac{p^2\langle \mathrm{Im}\{K^\rho \mathrm{Tr}[A_\mu(p)A_\nu(-p)]\}\rangle}{\epsilon^{\rho\mu\sigma\nu}p_\sigma\langle S_g\rangle}\Big|_{p^2=\mu_R^2,\mu\neq\nu\neq\rho\neq\sigma,p_\mu=p_\nu=0,p_\rho\neq 0}$$

$$= \frac{p^2\langle \mathrm{Im}\{(K^\rho - \langle K^\rho\rangle)(\mathrm{Tr}[A_\mu(p)A_\nu(-p)] - \langle \mathrm{Tr}[A_\mu(p)A_\nu(-p)]\rangle)\}\rangle}{\epsilon^{\rho\mu\sigma\nu}p_\sigma\langle S_g\rangle}\Big|_{p^2=\mu_R^2,\mu\neq\nu\neq\rho\neq\sigma,p_\mu=p_\nu=0,p_\rho\neq 0}$$



Statistical fluctuation of K^μ is equal to 0 within error range.

$$S^b_{\mu\nu}(p^2) = \frac{2}{(N_c^2-1)V}\mathrm{Tr}[A_\mu(p)A_\nu(-p)]$$

Transverse off-diagonal mode $\mathrm{Tr}[A_\mu(p)A_\nu(-p)]$, so the vacuum expectation value is originally 0.

Cluster Decomposition Error Reduction

Consider 3PT $\{\mathrm{Tr}[AA]\mathcal{O}\}(p)$

Cut operator $\mathcal{O}_{\mathrm{cut}}^{R_{sO}}(x) = \int_{|r|<R_{sO}} d^4 r \mathcal{O}(x+r)$

$$\mathcal{O}(p) = \mathcal{F}\left(\mathcal{O}(x)\right), \quad f(p, R_{sO}) = \mathcal{F}\left(f(x, R_{sO})\right) = \mathcal{F}\left(\theta(R_{sO} - |x|)\right)$$

$$\mathcal{F}\left(\mathcal{O} * f\right) = \mathcal{O}(p) \cdot f(p, R_{sO}), \quad \mathcal{O}_{\mathrm{cut}}(x) = \mathcal{F}^{-1}\mathcal{F}\left(\mathcal{O} * f\right) = \int_{||r||<R_{sO}} \mathcal{O}(x)$$

Cut gauge potential $A(x)$ for 3PT

$$B(x) = A(x)\mathcal{O}_{\mathrm{cut}}(x) \qquad B(-p) = \mathcal{F}^{-1}\left(B(x)\right) \times V, \quad A(p) = \mathcal{F}\left(A(x)\right)$$

$$g(x, R_{sA}) = \theta(R_{sA} - |x|) \qquad \{\mathrm{Tr}[AA]\mathcal{O}\}_{\mathrm{cut}}(p) = \mathcal{F}\left\{\mathcal{F}^{-1}\left(A(p) \cdot B(-p)\right) \cdot g(x, R_{sA})\right\}$$