Nucleon structure from an AdS/QCD model in the Veneziano limit

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Based on: Phys.Rev.D 112 (2025) 3, 036011

Significance of Nucleon Structure Studies

V.D.Burkert, Rev.Mod.Phys. 95 (2023) 4, 041002

1. Understanding the Origin of Mass

To answer the ultimate question: **"Where does most of our mass come from?"**
The answer lies not in the Higgs mechanism, but primarily in the **dynamical energy of QCD** and the non-perturbative structure of the nucleon.

2. Understanding the Origin of Spin

Similar to the mass puzzle: How does the proton's spin of **1/2 emerge** from the spins and orbital angular momenta of its constituent quarks and gluons?
This remains the central question of the **"spin crisis"** in particle physics.

3. Testing Fundamental Theory (QCD)

The nucleon serves as the most important **laboratory** for studying strong interactions in the non-perturbative regime.
It provides a crucial testing ground for validating and developing our understanding of QCD.

How to "See" the Internal Structure?

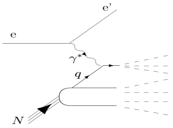
Probe Type	Process	Revealed Physical Information	Key Observables
Electroweak Probe	DIS	Parton momentum distribution	Structure Functions → PDFs
Electroweak Probe	Elastic scattering	Electromagnetic distribution	EM FFs
Gravitational Probe	(Theoretical concept)	Mechanical distribution	GFFs
Hadron Spectrum	–	Global properties: Mass of nucleon as QCD bound state	Mass spectrum

Table: Experimental probes for studying nucleon internal structure

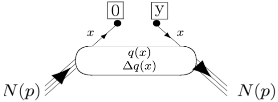
Jiali Deng, Phys.Rev.D 112 (2025) 3, 036011
S. Kumano, Phys.Rev.D 97 (2018) 1, 014020

How to access its?

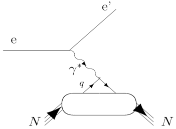
Michel Guidal, Rept.Prog.Phys. 76 (2013) 066202



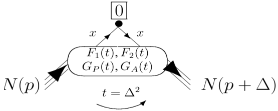
deep inelastic scattering



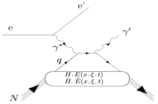
$$\langle p|\overline{\Psi}_q(0)o\Psi_q(y)|p\rangle$$



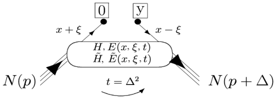
elastic scattering.



$$\langle p'|\overline{\Psi}_q(0)o\Psi_q(0)|p\rangle$$



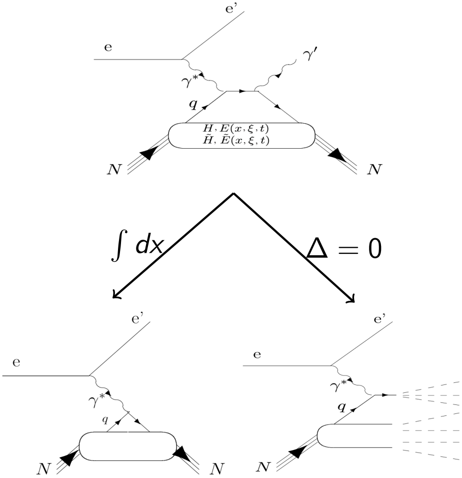
Deeply Virtual Compton Scattering



$$\langle p'|\overline{\Psi}_q(0)o\Psi_q(y)|p\rangle$$

interrelation

Rept.Prog.Phys. 76 (2013) 066202
V.D.Burkert, Rev.Mod.Phys. 95 (2023) 4, 041002



$$H(x,0,0) = \begin{cases} q(x), & x > 0 \\ -\bar{q}(-x), & x < 0 \end{cases}$$

$$\widetilde{H}(x,0,0) = \begin{cases} q(x), & x > 0 \\ \bar{q}(-x), & x < 0 \end{cases}$$

$$\int H(x,\xi,t)dx = F_1(t)$$

$$\int E(x,\xi,t)dx = F_2(t)$$

$$\int xH(x,\xi,t)dx = A(t) + \xi^2 D(t)$$

$$\int xE(x,\xi,t)dx = B_2(t) - \xi^2 D(t)$$

Outline

- Holographic VQCD Model
- Mass spectrum for proton
- Structure function of proton
- Electromagnetic form factors
- Gravitational form factors

- **Holographic VQCD Model**
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VQCD Model

Matti Jarvinen, JHEP 03 (2012) 002
U. G¨ursoy, JHEP 02 (2008) 019

Large-N_c Limits Comparison

- **Standard 't Hooft limit**: $N_c \to \infty$, $\lambda = g_{\mathrm{YM}}^2 N_c$ and N_f finite,
- **Veneziano limit**: $N_c \to \infty$, $N_f \to \infty$, $\frac{N_f}{N_c} = x$ fixed
- Preserves important quark effects while maintaining large-N_c simplifications

Holographic Construction

- **Gluon sector**: 5D Einstein-dilaton gravity

$$S_g = M^3 N_c^2 \int d^5x \sqrt{g}[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)]$$

- **Flavor sector**: Tachyonic Dirac-Born-Infeld action

$$S_{DBI} = -M^3 N_c^2 \int d^5x \sqrt{g} V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

- **Metric ansatz**:

$$ds^2 = e^{2A(z)}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

Boundary Condition and Confinement

UV boundary Condition

- **Two-loop QCD β-function**:

$$\frac{d\lambda}{dA} = \beta(g) = -\frac{g^3}{(4\pi)^2}(\frac{11}{3}N_c - \frac{2}{3}N_f) - \frac{g^5}{(4\pi)^4}(\frac{34}{3}N_c^2 - \frac{N_f}{N_c}(\frac{13}{3}N_c^2 - 1)) + \dots$$

- **Perturbative anomalous dimension**: $\frac{d\tau}{dA} = \gamma = -\frac{a_0}{4\pi}g^2 + \frac{a_1}{(4\pi)^2}g^4 + \dots$

Confinement Criterion

- **Wilson loop criterion**: Confinement requires area law behavior at large distances

$$E(L) \sim T_f e^{2A_s(r)} L, A_s(r) = A(r) + \frac{2}{3}\Phi(r)$$

confinement is equivalent to the existence of a minimum of the expression.

- Holographic VQCD Model
- **Mass spectrum for proton**
- Structure function of proton
- Electromagnetic form factors
- Gravitational form factors

Mass Spectrum

By solving the equations of motion, we obtain a Schrödinger-like equation:

$$-\varphi''_{R/L}(z) + (m_5^2 e^{2A_S(z)} \pm m_5 e^{A_S(z)} A'_S(z))\varphi_{R/L}(z) = M_n^2 \varphi_{R/L}(z)$$

	proton	M_{exp}/Gev	other/Gev	Our/Gev	%M
n=1	N(939)	0.938	0.987	0.939	0.107
n=2	N(1440)	1.360 to 1.380	1.264	1.333	2.701
n=3	N(1710)	1.680 to 1.720	1.531	1.653	2.764
n=4	N(1880)	1.820 to 1.900	1.791	1.893	1.774
n=5	N(2100)	2.050 to 2.150	2.046	2.097	0.143
n=6	N(2300)	2.300	2.296	2.273	1.174

Figure: Mass spectrum results from numerical solution of the Schrödinger equation

Jiali Deng, Phys.Rev.D 112 (2025) 3, 036011
FolcoCapossoli, Chin.Phys.C 44 (2020) 6, 064104

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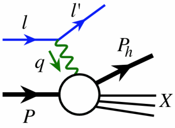
Structure Function

The hadronic tensor is defined as:

$$W^{\mu\nu} = 4\pi \int d^4x\, e^{iq\cdot x} \langle P | J^\mu(x) J^\nu(0) | P \rangle$$



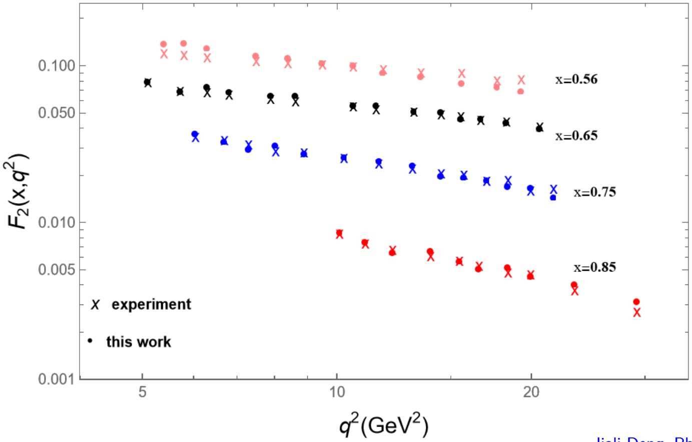
which can be decomposed into structure functions:

$$W^{\mu\nu} = F\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{2x}{q^2} F(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x})$$

where $x = \frac{Q^2}{2P\cdot q}$ is the Bjorken scaling variable.
The interaction term in the holographic description:

$$\eta_\mu \langle P + q, s_X | J^\mu(0) | P, s \rangle = S_{\text{int}} = g_V \int dz\, d^4y \sqrt{-g}\, \Phi(z, y) A_\mu(z, y) J^\mu(y)$$

Structure Function



Jiali Deng, Phys.Rev.D 112 (2025) 3, 036011
FolcoCapossoli, Phys.Rev.D 102 (2020) 8, 086004

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Electromagnetic form factors

Hadron matrix element

$$\langle p', s' | J^\mu(0) | p, s \rangle = e\bar{u}(p', s')\Gamma^\mu(p, p')u(p, s)$$

Electric charge:
$q_e = F_1(0) = G_E(0)$
Magnetic moment:
$\mu = q + F_2(0) = G_M(0)$

Vertex function:

$$\Gamma^\mu(p, p') = \gamma^\mu \underbrace{F_1(Q^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\underbrace{F_2(Q^2)}_{\text{Pauli}}, \quad \Delta = p' - p, P = (p' + p)/2$$

$$= \frac{MP^\mu}{P^2}\underbrace{G_E(Q^2)}_{\text{Electric}} + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta \gamma_\lambda \gamma^5}{2P^2}\underbrace{G_M(Q^2)}_{\text{Magnetic}}$$

Sachs EM FFs

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Electromagnetic form factors

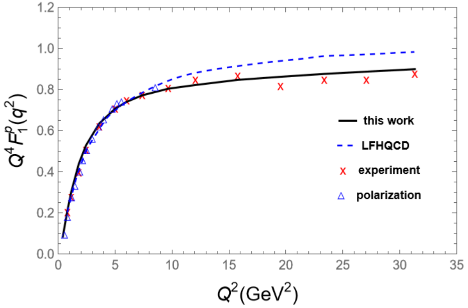
In the holographic description, spin-conserving matrix elements can be represented as:

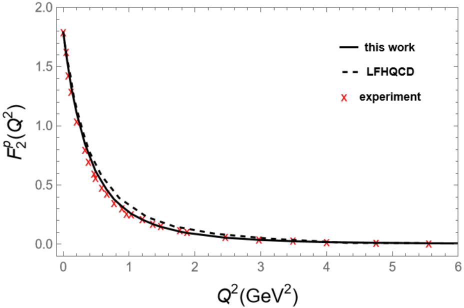
$$\int d^4x dz \sqrt{-g}\bar{\Psi}_{p'}(x,z) e^M_A \Gamma^A \phi_M(x,z)\Psi_p(x,z) \sim (2\pi)^4\delta^4(p'-p-q)\eta_\mu \phi(z)\bar{u}(p')\gamma^\mu F_1(q^2)u(p)$$

The spin-flip matrix element can be written as:

$$\int d^4x dz \sqrt{-g}\bar{\Psi}_{p'}(x,z) e^M_A e^N_B [\Gamma^A, \Gamma^B] F_{MN}(x,z)\Psi_p(x,z) \quad \sim (2\pi)^4\delta^4(p'-p-q)\eta_\mu \phi(z)\bar{u}(p')\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)u(p).$$

Raza Sabbir Sufian, Phys.Rev.D 95 (2017) 1, 014011





Jiali Deng, Phys.Rev.D 112 (2025) 3, 036011
Raza Sabbir Sufian, Phys.Rev.D 95 (2017) 1, 014011

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Gravitational Form Factors

Hadron matrix element

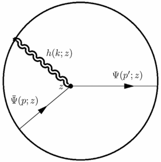
$$\langle p', s' | T^{\mu\nu}(0) | p, s \rangle = e\bar{u}(p', s')\Gamma^{\mu\nu}(p, p')u(p, s)$$

Vertex function:

$$\Gamma^{\mu\nu}(p, p') = \gamma^{(\mu} P^{\nu)} A(Q^2) + \frac{iP^{(\mu}\sigma^{\nu)\alpha} q_\alpha}{2M} B(Q^2) + \frac{q^\mu q^\nu - \eta^{\mu\nu} q^2}{2M} D(Q^2), \quad q = p' - p, P = (p' + p)/2$$

The interaction term in the holographic description:

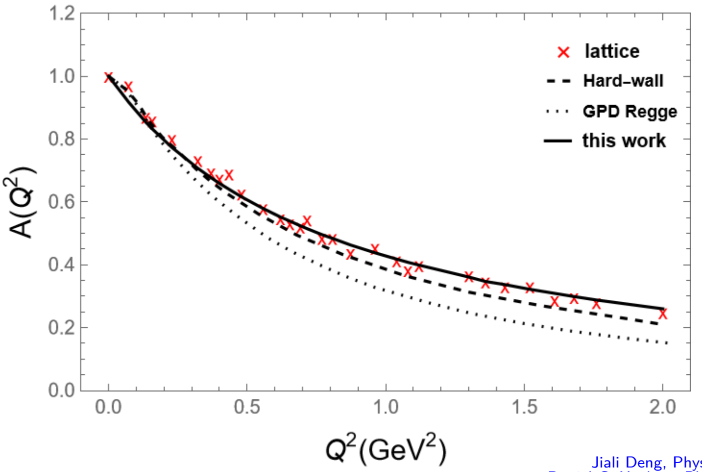
$$\int d^5 x \sqrt{g} h_{\mu\nu} T_F^{\mu\nu} \sim \langle p', s' | T^{\mu\nu}(0) | p, s \rangle$$



The equation of motion for graviton

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{mn} \partial_N h_{\mu\nu}) + m^2 h_{\mu\nu} = 0$$

Gravitational Form Factors



Jiali Deng, Phys.Rev.D 112 (2025) 3, 036011
Daniel C. Hackett, Phys.Rev.Lett. 132 (2024) 25, 25190
Zainul Abidin, Phys.Rev.D 79 (2009) 115003

Summary and outlook

1. We computed the **mass spectrum of the proton** using the VQCD model and determined the metric parameters.
2. Through **deep inelastic scattering**, we calculated the proton's structure functions. Smaller x values correspond to higher probabilities of final states being excited states.
3. We computed the **electromagnetic form factors** of the proton.
4. We computed the **gravitational form factors** of the proton.

Next, we will calculate the D-term and the mechanical properties of the proton.

Thank you!