# Application of covariance matrix in TMD effects

Yiyu Zhou
University of Turin

in collaboration with: P. Barry, E. Boglione, E. Nocera and A. Signori

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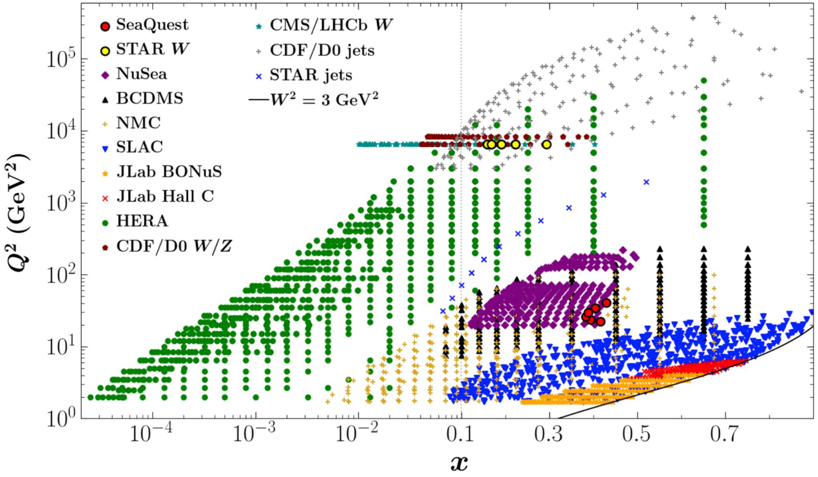
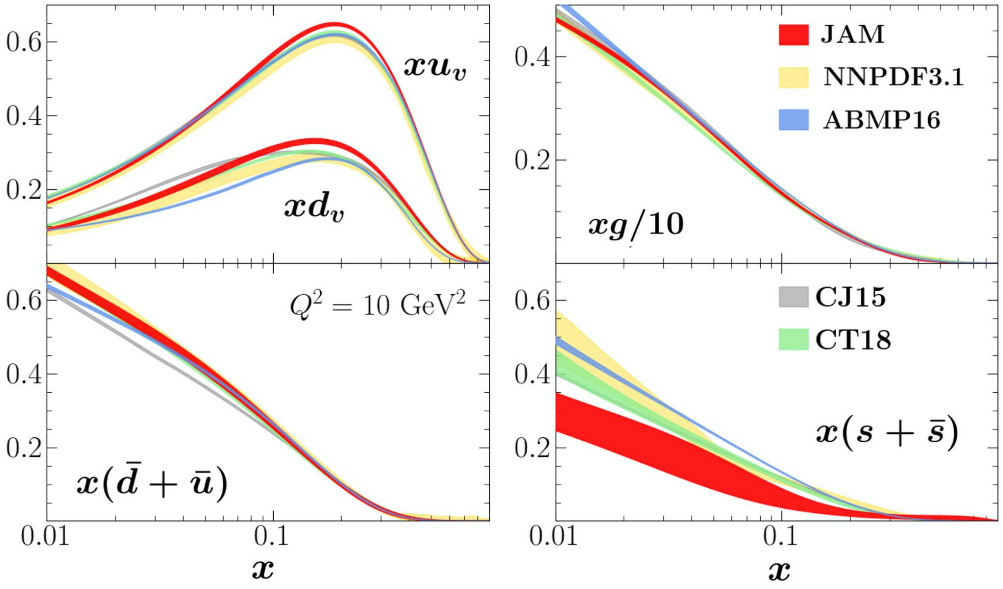
# Collinear structures

Most well-known are the collinear parton distribution functions (PDFs)

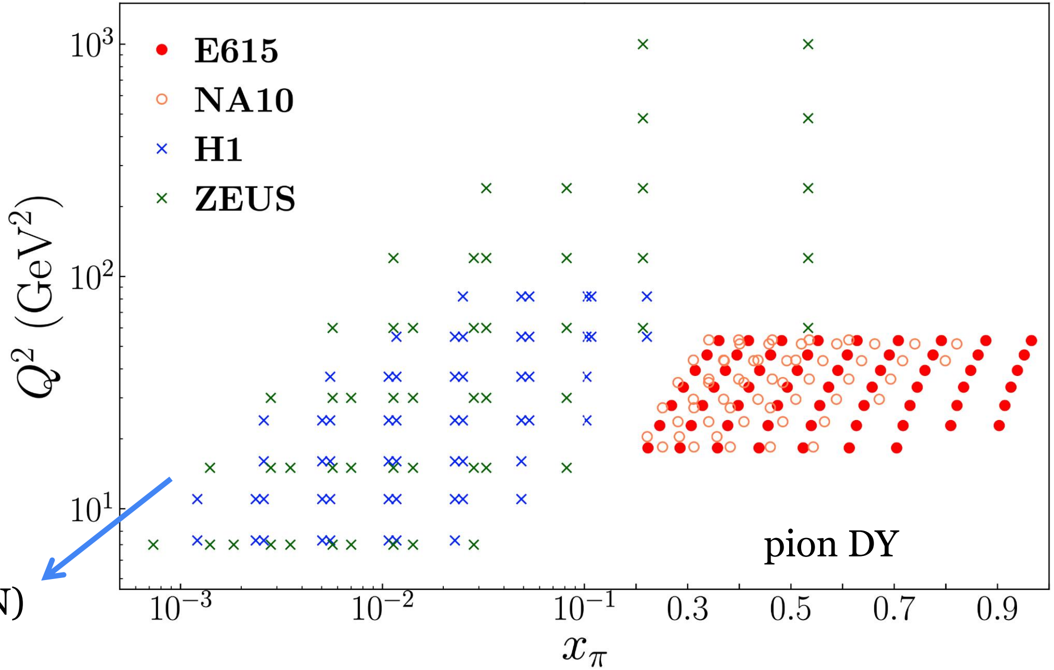



Cocuzza et al: 2109:00677

# Datasets for studying pion structures

- Less data available than in the proton case
- Still valuable to study

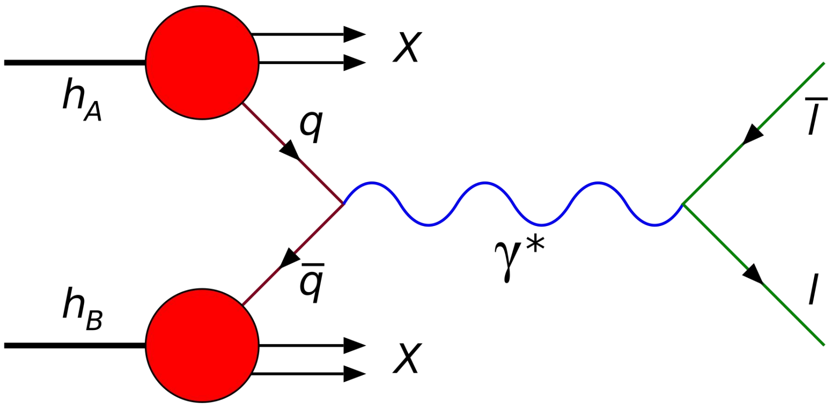



leading neutron (LN)

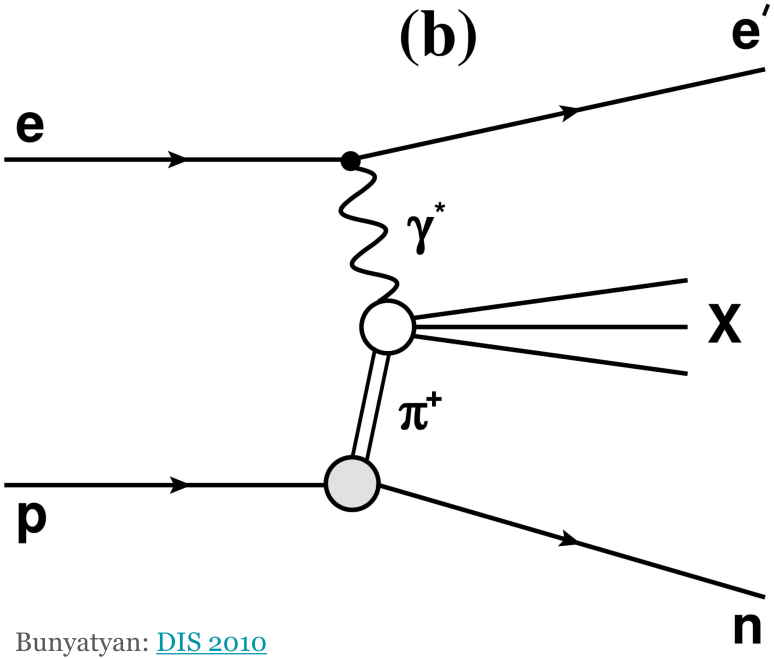
Barry: thesis

# Processes to study pion structure

- Drell-Yan (DY) and leading neutron(LN)



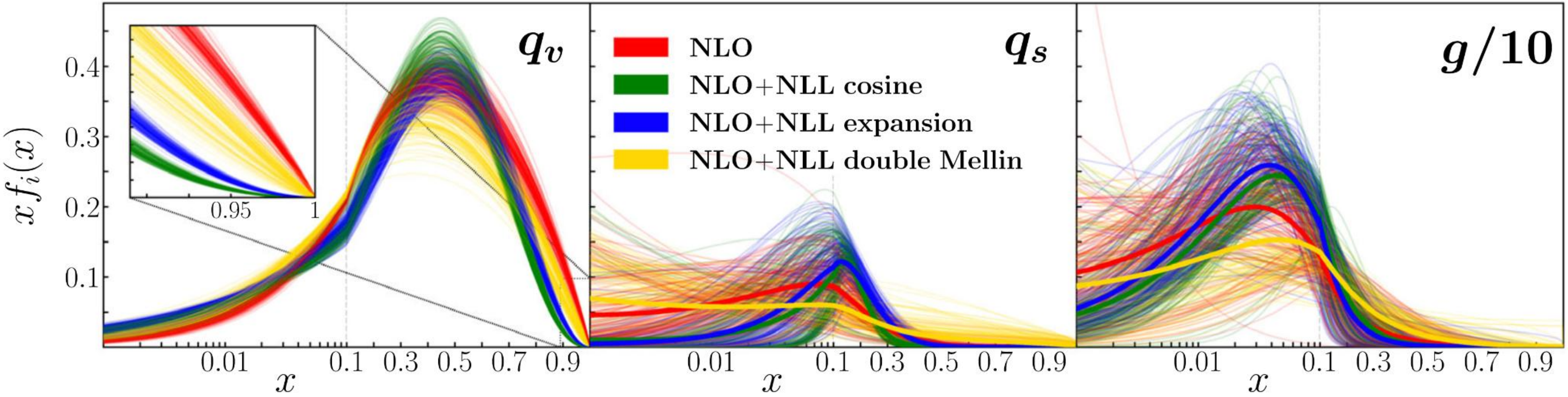

Wikipedia




Bunyatyan: DIS 2010

# Collinear pion PDFs

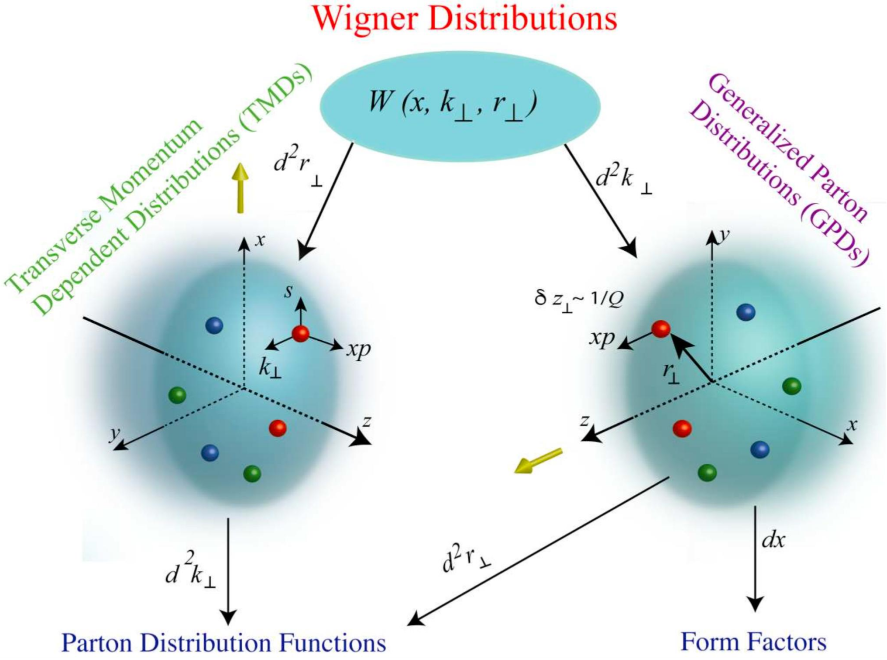
- Collinear pion PDFs are analyzed for valence, sea quarks and gluon

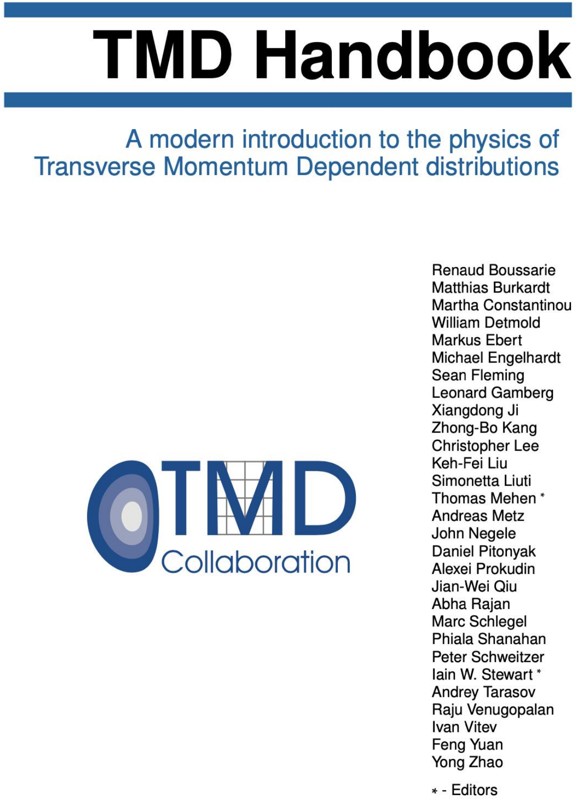



Barry, Ji, Sato & Melnitchouk: 2108.05822

# Structures in the transverse direction

- TMD (transverse momentum dependent) distributions
- GPDs (generalized parton distributions)

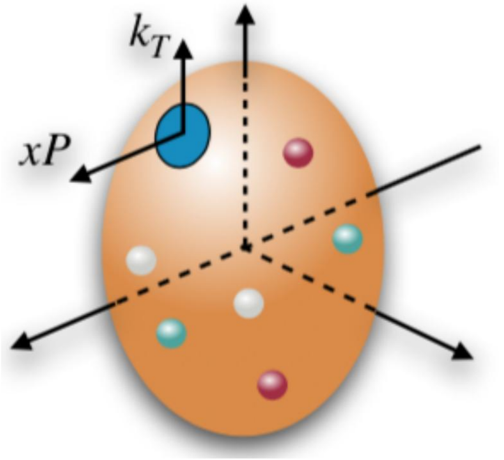


# 3D structure in momentum space

TMD (transverse momentum dependent) distributions:

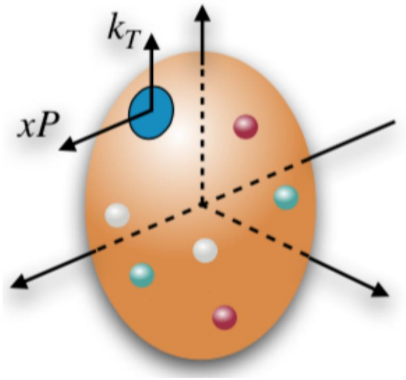
- longitudinal momentum fraction
- transverse momentum $k_T$ of partons in the hadron



# 3D structure in momentum space

TMD (transverse momentum dependent) distributions:

- longitudinal momentum fraction
- transverse momentum $k_T$ of quarks in the hadron
- $b_T$ is Fourier conjugate to the intrinsic transverse momentum $k_T$
- one can learn about coordinate space correlation of quarks

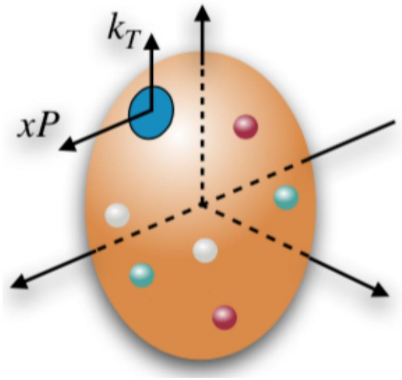



$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \,\mathrm{Tr}\left(\left\langle \mathcal{N} \middle| \overline{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \middle| \mathcal{N} \right\rangle\right)$$

# Factorization of low-$q_T$ Drell-Yan

The Drell-Yan $q_T$-dependent cross section can be factorized in the low $q_T$ region into:

- Hard part $H$ that describes partonic scattering amplitude
- TMD distributions that describe the structures of pion and proton/nucleus



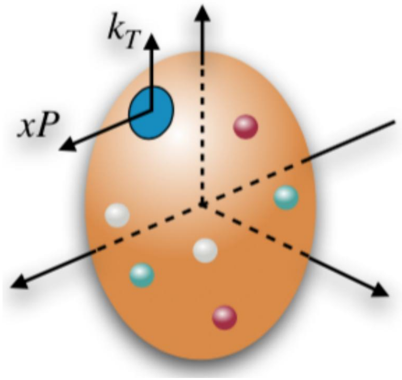

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau\,\mathrm{d}y\,\mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau s^2}\sum_q H_{q\bar{q}}^{\mathrm{DY}}\left(Q^2,\mu\right)\int \mathrm{d}^2\boldsymbol{b}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\,\widetilde{f}_{q/\pi}\left(x_\pi, b_T, \mu, Q^2\right)\widetilde{f}_{q/A}\left(x_A, b_T, \mu, Q^2\right)$$

# TMD PDFs with the $b_*$ prescription

$b_*$ prescription is applied to smoothly join low and high $b_T$ regions

$$b_* \equiv b_T / \sqrt{1 + b_T^2 / b_{\max}^2}$$

- $g_{q/N}$: intrinsic non-perturbative structure of the TMD
- $g_K$: universal non-perturbative Collins-Soper kernel
- $S_{\text{pert}}$: perturbative evolution of the TMD




$$\tilde{f}_{q/\mathcal{N}}\left(x, b_T, \mu = Q, Q^2\right) = (C \otimes f)_{q/\mathcal{N}}(x, b_*) \times \exp\left(-g_{q/\mathcal{N}}(x, b_T) - g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) - S_{\text{pert.}}(b_*, Q_0, Q, \mu = Q)\right)$$

# Nuclear TMD correction: previous approach

We model the nuclear TMD PDFs as:

$$\widetilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\widetilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\widetilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

And further modify the $g_{q/N/A}$ as [Alrashed et al: 2107.12401]:

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}}\left(1 + a_N\left(A^{1/3} - 1\right)\right)$$

We have also assumed/used:

- Bound protons and neutrons follow TMD factorization
- Isospin symmetry so that $u/p/A \leftrightarrow d/n/A$

# Nuclear TMD correction: new approach

We still model the nuclear TMD PDFs as:

$$\widetilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A}\widetilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A}\widetilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

We now introduce the nuclear covariance matrix:

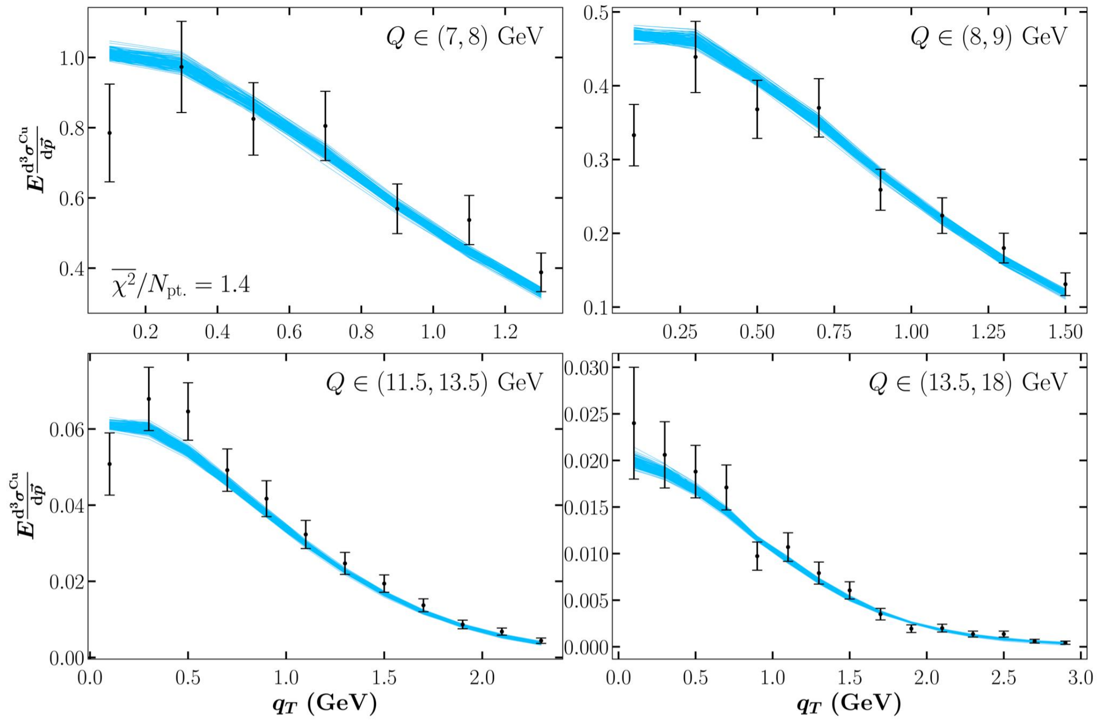
$$\frac{1}{\sqrt{N_{\text{rep.}}}}\Delta_{i,n} = \frac{1}{\sqrt{N_{\text{rep.}}}}\left(T_{i,n}^{\text{nuc.}} - T_{i,n}^{\text{vac.}}\right)$$

- $N_{\text{rep.}}$: number of replicas
- $T_{i,n}^{\text{nuc.}}$: theory value **with** nuclear TMD NP kernel
- $T_{i,n}^{\text{vac.}}$: theory value **without** nuclear TMD NP kernel

All data points are connected, across different datasets!

# Data-theory agreement

| dataset | $N_{\rm pt}$ | $\chi^2/N_{\rm pt}$ |
|---|---|---|
| E288 ($p$Pt) | 30 | 1.5 |
| E288 ($p$Pt) | 39 | 1.1 |
| E288 ($p$Pt) | 62 | 0.9 |
| E605 ($p$Cu) | 42 | 1.4 |
| E772 ($p$D) | 51 | 2.6 |
| E866 (Fe/Be) | 7 | 3.0 |
| E866 (W/Be) | 7 | 2.9 |
| E615 ($\pi$W) | 40 | 1.6 |
| E537 ($\pi$W) | 27 | 1.3 |
| **total** | **1370** | **0.90** |



# Data-theory agreement

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| E615 ($\pi$W) | 40 | 1.6 |
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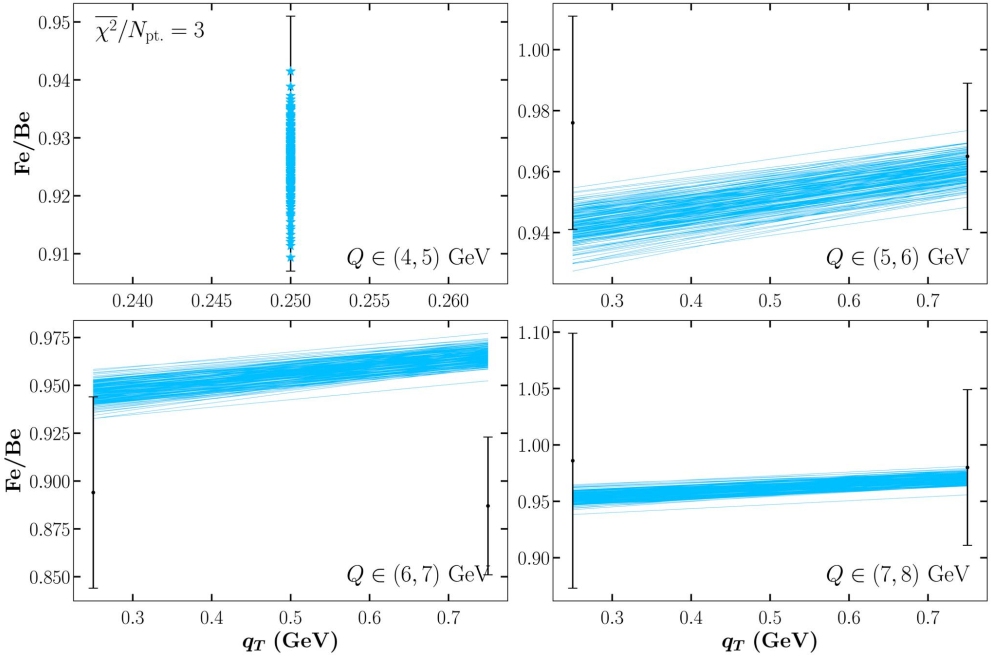
| **total** | **305** | **1.6** |




Description is actually good, large $\chi^2$ comes from penalty terms that connect data points across datasets

# Data-theory agreement: compare to baseline

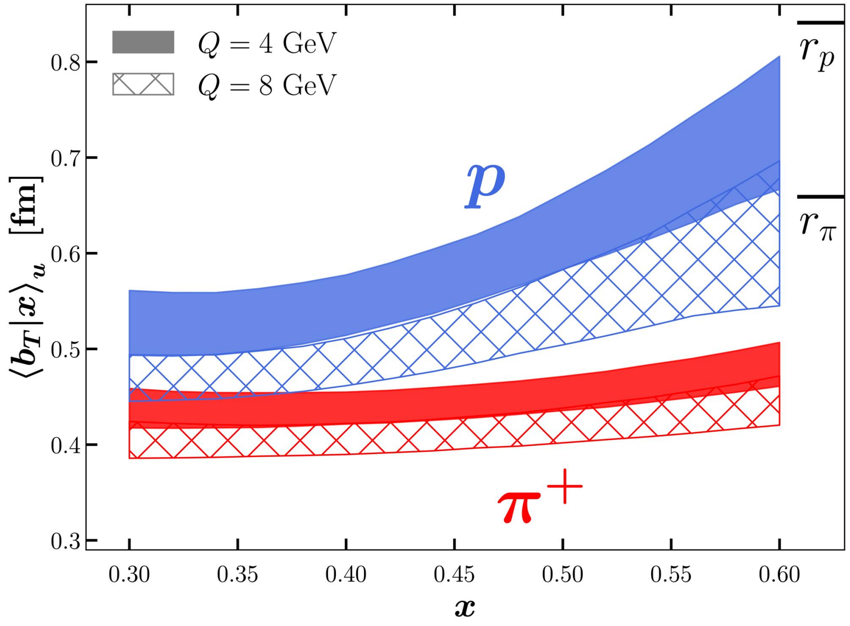
| dataset | $N_{\text{pt}}$ | $\chi^2/N_{\text{pt}}$ (baseline) | $\chi^2/N_{\text{pt}}$ |
|---|---|---|---|
| E288 ($p$Pt) | 30 | 1.1 | 1.5 |
| E288 ($p$Pt) | 39 | 1.0 | 1.1 |
| E288 ($p$Pt) | 62 | 0.8 | 0.9 |
| E605 ($p$Cu) | 42 | 1.2 | 1.4 |
| E772 ($p$D) | 51 | 2.5 | 2.6 |
| E866 (Fe/Be) | 7 | 1.1 | 3.0 |
| E866 (W/Be) | 7 | 1.0 | 2.9 |
| E615 ($\pi$W) | 40 | 1.4 | 1.6 |
| E537 ($\pi$W) | 27 | 1.0 | 1.3 |
| **total** | **305** | **1.3** | **1.6** |

# Average $b_T$ as a function of $x$

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int d^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int d^2\boldsymbol{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2),$$

- Nominal charge radius from PDG are marked
- As $x \to 1$, transverse motion phase space becomes narrower, hence $b_T$ increases
- As $Q$ increases, more gluons are radiated and $k_T$ becomes larger, hence $b_T$ decreases

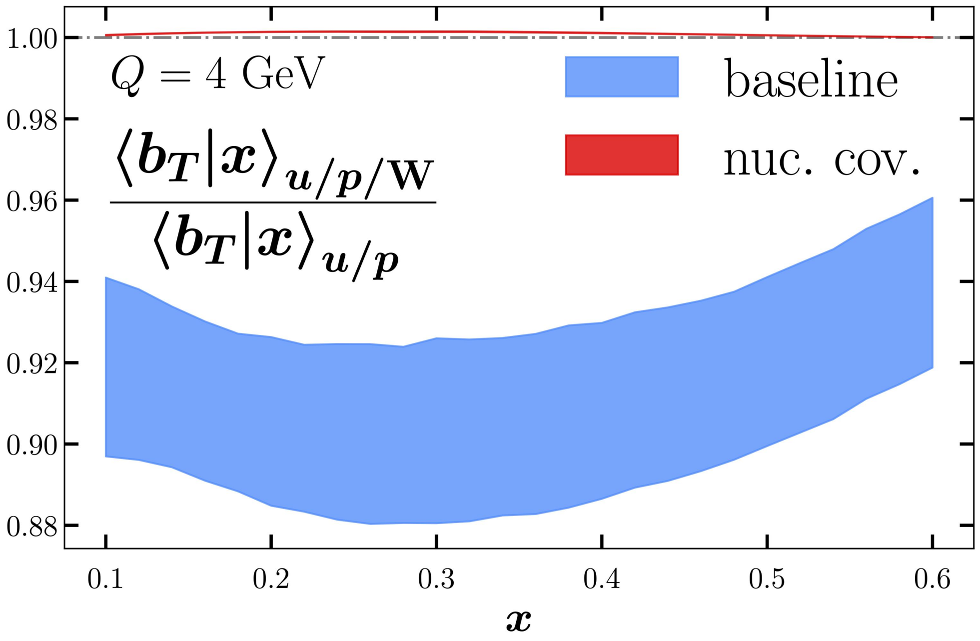


# Nuclear modified average $b_T$ as a function of $x$

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

$$\langle b_T|x\rangle_{q/\mathcal{N}} = \int \mathrm{d}^2\boldsymbol{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2),$$

- Broadening of $k_T$ (hence smaller $b_T$) is observed in baseline
- With nuclear covariance matrix, no suppression is observed and uncertainty is vanishing ($a_N$ = 0)



# Summary

- We have explored a new approach for quantifying nuclear correction in TMD PDFs
- Studying the TMD distributions in pion is as important as studying the proton
- In the future, lattice datasets can be included in the analysis
- Future tagged experiments at EIC, JLab 22 GeV and AMBER at CERN can provide flavor separation

# Thank you for your attention!