q_T-slicing method and factorization structures for multi-jet processes

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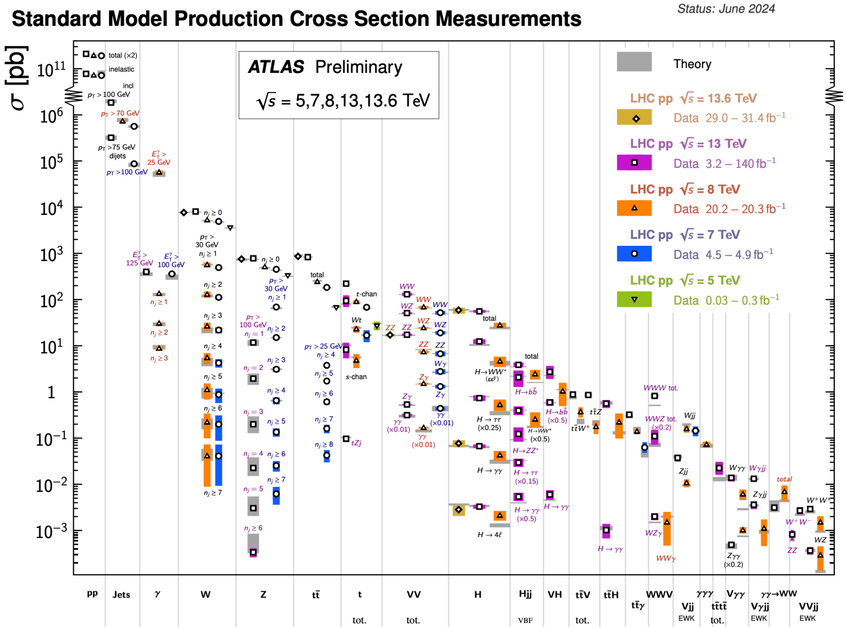
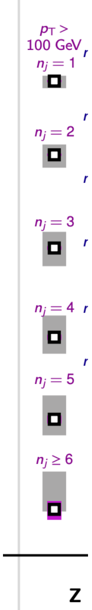
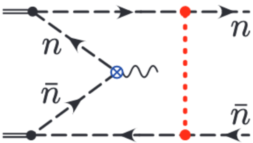
Standard Model Summary Plots June 2024 [ATLAS Collaboration '24]

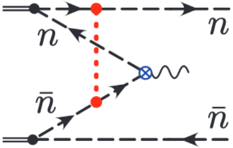




Factorization-violation

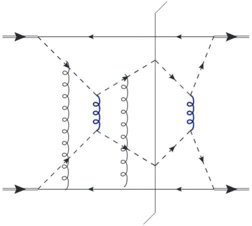
All proton collisions include forward component (proton remnants).





dashed red line: Glauber [Rothstein, Stewart, '16]

Absence of factorization-violation due to Glauber gluons is important element of factorization proof for Drell-Yan process. [Bodwin '85; Collins, Soper, Sterman '85]
e.g. Transverse momentum dependent (TMD) factorization is violated in dijet production [Collins, Qiu '07; Collins '07, Vogelsang, Yuan '07; Rogers, Mulders 10, ...]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

Slicing approach

$$\frac{\mathrm{d}\sigma^{(m)}_{\mathrm{N^kLO}}}{\mathrm{d}X} = \int_0^{\delta} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{(m)}_{\mathrm{N^kLO}}}{\mathrm{d}X\,\mathrm{d}q_T} + \int_{\delta}^{\infty} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{(m+1)}_{\mathrm{N^{k-1}LO}}}{\mathrm{d}X\,\mathrm{d}q_T}$$

- δ (or q_T^{cut}) plays a role as slicing variable here;
- The factorization formula, obtained e.g. through Soft-Collinear Effective Theory (SCET), can be used to handle the cancellation of infrared (IR) divergences in

  $$\frac{\mathrm{d}\sigma^{(m)}_{\mathrm{N^kLO}}}{\mathrm{d}q_T} \equiv \frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}q_T}[1 + \mathcal{O}(\delta^p)],$$

  whereas

  $$\frac{\mathrm{d}\sigma^{(m+1)}_{\mathrm{N^{k-1}LO}}}{\mathrm{d}q_T} \equiv \frac{\mathrm{d}\sigma_{\mathrm{QCD}}}{\mathrm{d}q_T}$$

  is numerically easier to compute.
- Large cancellations between singular and regular terms.

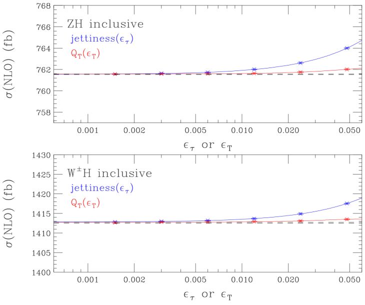
Modern slicing variables

1. Transverse momentum of a colorless or colored (but massive) system. [Catani, Grazzini ’ 07]

   For processes with jets, such as $pp \to V$+jets or $pp \to 2$jets, q_T is unsuitable because $q_T = 0$ for radiation emitted inside jets.

2. Jettiness[Boughezal, Focke, Liu, Petriello ’ 15; Gaunt, Stahlhofen, Tackmann, Walsh ’ 15]

$$\tau_N = \sum_i \min_{j=a,b,\cdots N} \left\{ \frac{2q_j k_i}{Q^2} \right\}$$



3. For most color-singlet processes, q_T performs better than 0-jettiness as slicing variable [Campbell, Ellis, Seth ’ 22], which motivates the exploration of extending q_T to processes involving jets.

q_T-slicing for jets

- We propose two generalizations of q_T that can be used for jet processes. [RJF, Rahn, Shao, Waalewijn, Wu ' 24]

The key ingredient is the use of a recoil-free jet axis!

Utilizing the Winner-Take-All (WTA) scheme to define slicing variables.

- Azimuthal decorrelation $\delta\phi = q_x/p_{T,1}$:

$$q_x = p_{x,1}^{\mathrm{WTA}} + p_{x,2}^{\mathrm{WTA}}$$

- Magnitude of total transverse momentum $|\vec{q}_T|$:

$$\vec{q}_T = \sum_{i=\mathrm{jets},V,\cdots} \vec{p}_{T,i}^{\,\mathrm{WTA}}$$

Why the use of WTA scheme enables slicing?

A jet definition includes a jet algorithm and a **recombination scheme**. The former defines how some particles are grouped into jets, while the latter specifies how a momentum is assigned to a jet.

- **Standard E-scheme** simply sums the four-momenta of all particles within the jet.
- p_T^n**-weighted recombination scheme**: [Banfi, Dasgupta, Delenda ' 08]

$$p_{T,r} = p_{T,i} + p_{T,j}, \quad \phi_r = \frac{\omega_i \phi_i + \omega_j \phi_j}{\omega_i + \omega_j}, \quad y_r = \frac{\omega_i y_i + \omega_j y_j}{\omega_i + \omega_j}, \quad \omega = p_T^n$$

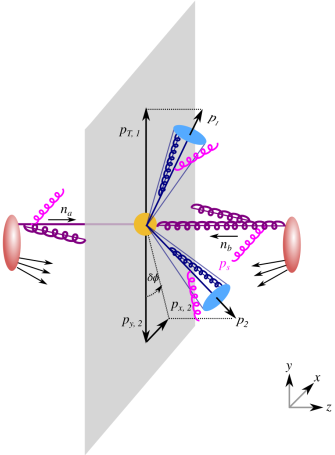
If $n \to \infty$: **Winner-take-all scheme**: [Bertolini, Chan, Thaler ' 13]

$$p_{T,r} = p_{T,i} + p_{T,j}, \quad \hat{n}_r = \begin{cases} \hat{n}_i, & p_{T,i} \geq p_{T,j} \\ \hat{n}_j, & p_{T,i} < p_{T,j} \end{cases}$$

- In the WTA scheme, soft radiation inside the jet can also influence the jet axis through momentum conservation, similar to radiation outside the jet, leading to a non-zero q_T.

Azimuthal decorrelation ($\delta\phi = q_x/p_{T,1}$)

- A simple factorization formula. [Chien, Rahn, Shao, Waalewijn & Wu ’ 22 + Schrinder ’ 21]
- Suitable for processes that are planar at Born level, such as $pp \to V$+jet, $pp \to 2$jets, $e^+e^- \to 3$jets, etc.



The $pp \to 2$ jet process. By using the WTA scheme, the transverse momentum perpendicular to the scattering plane q_x (equal to $p_{x,2}$ in the picture), or equivalently the azimuthal decorrelation $\delta\phi$, is a suitable slicing variable.

The factorization ingredients are: the hard scattering (yellow), collinear initial- (purple) and final-state (blue) radiation, and soft radiation (pink).

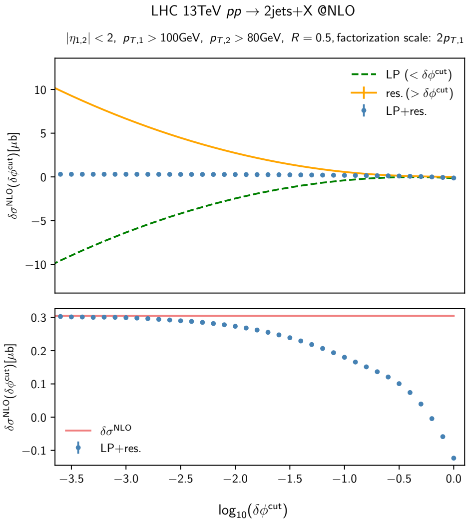
Azimuthal decorrelation

- Factorization formula ($pp \to 2$jets):

$$\frac{d\sigma_{\rm SCET}}{dp_{T,1}\, d\eta_1\, d\eta_2\, dq_x} = \int \frac{db_x}{2\pi}\, e^{iq_x b_x} \sum_{i,j,k,\ell} B_i(x_a, b_x)\, B_j(x_b, b_x) \mathcal{J}_k(b_x) \mathcal{J}_\ell(b_x)$$
$$\times\, {\rm tr}\Big[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1}, \eta_1 - \eta_2)\, \hat{S}_{ijk\ell}(b_x, \eta_1, \eta_2)\Big]\,.$$

- Factorization ingredients:
  - Standard TMD PDFs $B_{i,j}$: known at N^{3}LO; [Luo, Yang, Zhu, Zhu ’ 19; Ebert, Mistlberger, Vita ’ 20]
  - Soft function $\hat{S}_{ijk\ell}$: can directly be obtained from the standard TMD soft function at NNLO; [Gao, Li, Moult, Zhu ’ 19]
  - TMD jet functions $\mathcal{J}_{k,\ell}$: partially known at NNLO. [Reyes, Scimemi, Waalewijn, Zoppi ’ 19; Bell, Brune, Das, Wald ’ 23; Fang, Gao, Li, Shao ’ 24]

q_x-slicing for $pp \to 2$jets



- Slicing capitalizes on this,

$$\frac{d\sigma}{dX} = \int_0^\delta dq_x \frac{d\sigma_{\rm SCET}}{dX\,dq_x}[1 + \mathcal{O}(\delta^p)] + \int_\delta^\infty dq_x \frac{d\sigma_{\rm QCD}}{dX\,dq_x}.$$

- $\sigma_{\rm QCD}$ is obtained from NLOJET++. [Nagy, Trocsanyi '01]
- Jets are defined in a standard way with partons clustered by anti-k_T algorithm and momentum recombined by standard E-scheme.
- Large cancellation and nice convergence at NLO.

Total transverse momentum ($|\vec{q}_T|$)

- We can still use the magnitude of the total transverse momentum $q_T = |\vec{q}_T|$ of the color-singlets and jets as a slicing variable, when utilizing the WTA scheme.
- q_T-slicing can be applied to processes with non-planar kinematics, such as $pp \to Z + 2\text{jets}$, $pp \to 3\text{jets}$, etc.
- Factorization formula:

$$\frac{\mathrm{d}\sigma_{\mathrm{SCET}}}{\mathrm{d}p_{T,1}\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}q_T} = q_T \int \frac{\mathrm{d}^2\vec{b}_T}{2\pi} J_0(q_T|\vec{b}_T|) \sum_{i,j,k,\ell} B_i(x_a, \vec{b}_T)\, B_j(x_b, \vec{b}_T) \mathcal{J}_k(b_x) \\ \times \mathcal{J}_\ell(b_x)\, \mathrm{tr}\left[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1}, \eta_1 - \eta_2)\, \hat{S}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, R)\right].$$

- Only soft function is new: Outside the jet (b_T), inside the jet (b_x).

Refactorization of soft function

The q_T-soft function refactorizes into global and collinear-soft contributions in small-R limit, while the remaining power corrections appear as a series in R^{2n},

$$\hat{S}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, R) = \hat{S}^{\text{global}}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, R) S^{cs}_k(\vec{b}_T, \eta_1, R) S^{cs}_\ell(\vec{b}_T, \eta_2, R) + \mathcal{O}(R^{2n})$$

We find that, after boosting along the jet axis, the collinear-soft function reduces to the hemisphere soft function, with an in-cone dependence on $b_\perp$ and an out-of-cone dependence on b_-.

$$S^{cs}_j(b_\perp,\, b_-) = 1 + \frac{Z_\alpha \alpha_s(\mu)}{4\pi} S^{\text{in}}_j(b_\perp) + \frac{Z_\alpha \alpha_s(\mu)}{4\pi} S^{\text{out}}_j(b_-)$$
$$+ \left(\frac{Z_\alpha \alpha_s(\mu)}{4\pi}\right)^2 S^{cs,(2)}_j(b_\perp,\, b_-) + \mathcal{O}(\alpha_s^3)$$

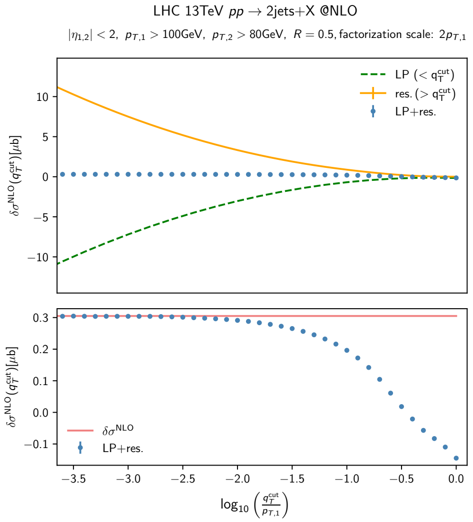
Soft function

$$\hat{\hat{S}}^{(1)}_{\text{finite}}(q_T^{\text{cut}},\eta_1,\eta_2,R,\mu,\nu) = \frac{\alpha_s(\mu)}{4\pi}\Bigg\{-4L_\mu^2\sum_i \mathbf{T}_i^2 + L_\mu\Bigg[4\ln\frac{\mu^2}{\nu^2}\sum_i\mathbf{T}_i^2 + 8\sum_{i<j}\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{n_i\cdot n_j}{2}$$
$$-8\ln 2\,(\mathbf{T}_a+\mathbf{T}_b)\cdot(\mathbf{T}_1+\mathbf{T}_2) - 16\ln 2\;\mathbf{T}_1\cdot\mathbf{T}_2\Bigg] - \frac{\pi^2}{6}\sum_i\mathbf{T}_i^2$$
$$+\left[(\mathbf{T}_a+\mathbf{T}_b)\cdot(\mathbf{T}_1+\mathbf{T}_2)+2\;\mathbf{T}_1\cdot\mathbf{T}_2\right]\left(4\ln 2\ln\frac{\mu^2}{\nu^2}+\frac{\pi^2}{3}+4\ln^2\frac{R}{2}\right)$$
$$+\sum_{j\in\text{jets}}(\mathbf{T}_a+\mathbf{T}_b)\cdot\mathbf{T}_j\;8\ln 2\ln(2\cosh\eta_j)$$
$$+\mathbf{T}_1\cdot\mathbf{T}_2\left[8\ln 2\ln(4\cosh\eta_1\cosh\eta_2)-2\ln^2(2+2\cosh(\eta_1-\eta_2))+2(\eta_1-\eta_2)^2\right]$$
$$-\sum_{i<j}\mathbf{T}_i\cdot\mathbf{T}_j\,S_{ij}^{\text{corr}}(\eta_1,\eta_2,R)\Bigg\}$$

R-correction for jet-jet dipole:

$$S_{12}^{\text{corr}}(\eta_1,\eta_2,R) = -2R^2\ln\frac{R}{2}\tanh^2\left(\frac{\eta_1-\eta_2}{2}\right)+R^2\left[\frac{7}{3}-\frac{6}{1+\cosh(\eta_1-\eta_2)}\right]$$
$$+R^4\left[\frac{49}{720}-\frac{e^{\eta_1+\eta_2}\left(3e^{2\eta_1}+3e^{2\eta_2}-8e^{\eta_1+\eta_2}\right)}{2(e^{\eta_1}+e^{\eta_2})^4}-\ln\left(\frac{R}{2}\right)\frac{\left(e^{2\eta_1}+e^{2\eta_2}-10e^{\eta_1+\eta_2}\right)^2}{36(e^{\eta_1}+e^{\eta_2})^4}\right]+\mathcal{O}(R^6)$$

q_T-slicing for $pp \to 2$jets

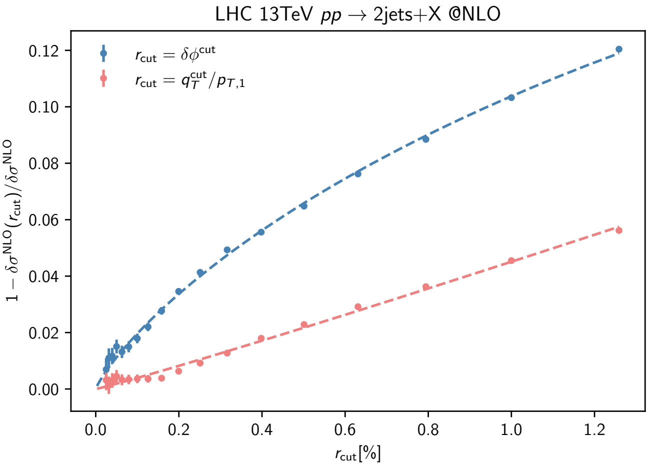


- Slicing capitalizes on this,

$$\frac{d\sigma}{dX} = \int_0^\delta dq_T \frac{d\sigma_{\rm SCET}}{dX\, dq_T}[1+\mathcal{O}(\delta^p)] + \int_\delta^\infty dq_T \frac{d\sigma_{\rm QCD}}{dX\, dq_T}.$$

- We include finite terms up to $\mathcal{O}(R^4)$, achieving a 1% precision for the cross section at $R = 0.5$.

Comparison between two slicing variables



The curves are obtained from fitting to $a\, r_{\rm cut} \ln r_{\rm cut} + b\, r_{\rm cut}$.

q_T converges faster than q_x slicing at the expense of a more complicated soft function, and can also be extended to non-planar Born processes.

2-loop ingredient: NNLO collinear-soft function

$$
\begin{aligned}
\bar{S}_j^{cs,(2)}(b_T) \; &= \left(\frac{\mu b_T}{b_0}\right)^{4\epsilon}\Bigg[\left(\frac{\nu b_T R_j}{b_0}\right)^{\eta}\omega^2 C_F C_A \bar{v}_A^{\text{in}} + R^{-4\epsilon} C_F C_A \bar{v}_A^{\text{out}} \\
&+ \left(\frac{\nu b_T R_j}{b_0}\right)^{\eta}\omega^2\left[\omega^2\left(\frac{\nu b_T R_j}{b_0}\right)^{\eta} C_F^2 \bar{h}_{2F} + C_F C_A \bar{h}_A + C_F n_f T_F \bar{h}_f\right] \\
&+ \left(\frac{\nu b_T R_j}{b_0}\right)^{\eta} R^{-2\epsilon}\omega^2 C_F^2 \bar{p}_{2F} + R^{-4\epsilon}(C_F C_A \bar{p}_A + C_F n_f T_F \bar{p}_f) + R^{-2\epsilon} C_F C_A \bar{p}_{\text{NGL}} \\
&+ R^{-4\epsilon}\left(C_F^2 \bar{g}_{2F} + C_F C_A \bar{g}_A + C_F n_f T_F \bar{g}_f\right)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\bar{v}_A^{\text{in}} &= -\frac{1}{\epsilon^4} + \frac{4}{\eta\epsilon^3} - \frac{16\ln 2}{\eta\epsilon^2} + \frac{\frac{7\pi^2}{6} + 8\ln^2 2}{\epsilon^2} + \frac{2\left(\pi^2 + 16\ln^2 2\right)}{\eta\epsilon} - \frac{2\left(10\pi^2\ln 2 + 32\ln^3 2 + 34\zeta_3\right)}{3\epsilon} \\
&\quad - \frac{4\left(6\pi^2\ln 2 + 32\ln^3 2 + 46\zeta_3\right)}{3\eta} + \frac{1009\pi^4}{360} + \frac{52}{3}\pi^2\ln^2 2 + 32\ln^4 2 + 152\zeta_3\ln 2 \\
\bar{v}_A^{\text{out}} &= \frac{1}{\epsilon^4} - \frac{\pi^2}{2\epsilon^2} + \frac{8\zeta_3}{3\epsilon} + \frac{\pi^4}{120}
\end{aligned}
$$

2-loop ingredient: NNLO collinear-soft function

$$\bar{h}_A = \frac{1}{\epsilon^4} + \frac{11}{6\epsilon^3} + \left(\frac{67}{18} - \frac{5\pi^2}{3} - 8\ln^2 2\right)\frac{1}{\epsilon^2} + \left(\frac{211}{27} - \frac{121\pi^2}{36} + 8\pi^2\ln 2 - \frac{44\ln^2 2}{3} + \frac{64\ln^3 2}{3} + \frac{35\zeta_3}{3}\right)\frac{1}{\epsilon}$$
$$+ \left[-\frac{4}{\epsilon^3} + \left(-\frac{22}{3} + 16\ln 2\right)\frac{1}{\epsilon^2} + \left(-\frac{134}{9} - \frac{4\pi^2}{3} + \frac{88\ln 2}{3} - 32\ln^2 2\right)\frac{1}{\epsilon}\right.$$
$$\left. + \left(-\frac{808}{27} - \frac{55\pi^2}{9} + \frac{536\ln 2}{9} + \frac{16\pi^2\ln 2}{3} - \frac{176\ln^2 2}{3} + \frac{128\ln^3 2}{3} + \frac{268\zeta_3}{3}\right)\right]\frac{1}{\eta} - 365.293(48),$$

$$\bar{h}_f = -\frac{2}{3\epsilon^3} - \frac{10}{9\epsilon^2} + \left(-\frac{74}{27} + \frac{11\pi^2}{9} + \frac{16\ln^2 2}{3}\right)\frac{1}{\epsilon} + \left[\frac{8}{3\epsilon^2} + \left(\frac{40}{9} - \frac{32\ln 2}{3}\right)\frac{1}{\epsilon}\right.$$
$$\left. \left(\frac{224}{27} + \frac{20\pi^2}{9} - \frac{160\ln 2}{9} + \frac{64\ln^2 2}{3}\right)\right]\frac{1}{\eta} - 40.8677(14)$$

$$\bar{g}_A = -\frac{1}{\epsilon^4} - \frac{11}{6\epsilon^3} + \frac{-67+6\pi^2}{18\epsilon^2} + \frac{-47840 - 2010\pi^2 + 1233\pi^4 - 75240\zeta_3}{3240} + \frac{-772 - 33\pi^2 + 468\zeta_3}{108\epsilon}$$
$$\bar{g}_f = \frac{2}{3\epsilon^3} + \frac{10}{9\epsilon^2} + \frac{38+3\pi^2}{27\epsilon} + \frac{238 + 15\pi^2 + 684\zeta_3}{81}.$$

$$\bar{p}_f = \frac{4(3-2\pi^2)}{9\epsilon} - \frac{68}{9} + \frac{64\pi^2}{27} - \frac{16\zeta_3}{3}, \quad \bar{p}_A = \left(-\frac{2}{3} + \frac{22\pi^2}{9} - 4\zeta_3\right)\frac{1}{\epsilon} + \frac{40}{9} - \frac{134\pi^2}{27} + \frac{44\zeta_3}{3} + \frac{8\pi^4}{45},$$
$$\bar{p}_{\mathrm{NGL}} = \frac{2\pi^2}{3\epsilon^2} + \left(8\zeta_3 - \frac{4}{3}\pi^2\ln 2\right)\frac{1}{\epsilon} + \left(\frac{13\pi^4}{45} + \frac{4}{3}\pi^2\ln^2 2 - 16\zeta_3\ln 2\right)$$

In-Out radiations: Full theory vs EFT [Becher, Pecjak, Shao ’ 16]

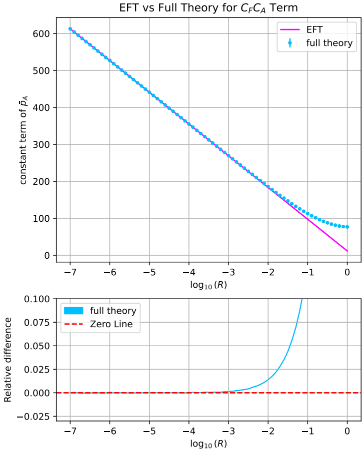


Figure: Comparison between the constant term of $\bar{p}_{A,\text{full}}$ in the full theory and $\bar{p}_{\text{NGL}} + R^{-2\epsilon}\bar{p}_A$ in the EFT.

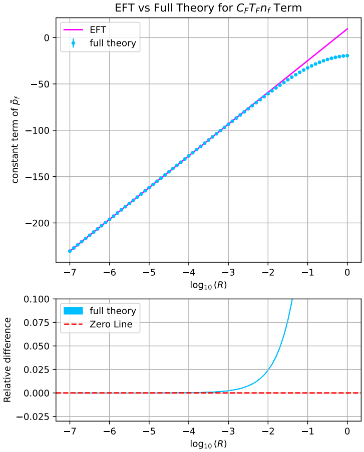
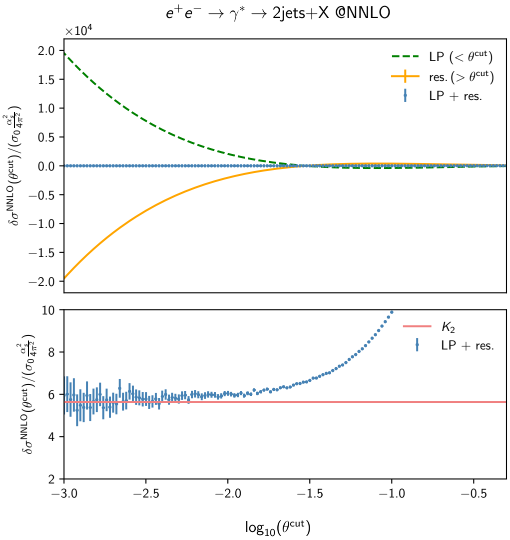


Figure: Comparison between the constant term of $\bar{p}_{f,\text{full}}$ in the full theory and $R^{-2\epsilon}\bar{p}_f$ in the EFT.

Summary and outlook

1. IR divergences present a significant challenge in precision QCD calculations, especially for processes involving multi-jet final states.
2. Two novel extensions of transverse momentum slicing variables (q_x and q_T) are proposed, specifically tailored for jet processes.
   - q_x slicing only applies to planar Born processes, such as $pp \to 2$ jets, but offers a dramatic simplification of the soft function.
   - q_T slicing necessitates a more complex soft function but offers faster convergence and can be applied to non-planar cases.
   - Two slicing approaches are successfully demonstrated at NLO.
3. The NNLO collinear-soft function has been calculated and successfully passes both the RG and RRG consistency checks.
4. These developments offer a promising framework for tackling the challenges of multi-jet final states at NNLO, paving the way for further advancements in precision QCD calculations.

Back up: NNLO slicing in $e^+e^- \to 2$ jet process



we define our slicing variable θ as

$$\theta = \arctan\left(\frac{2q_T}{Q}\right) \approx \frac{2q_T}{Q},$$

which is equal to π minus the angle between the leading and subleading jets.