Bottom Meson Masses and Decay Constants From the Anisotropic Quark Action on CLQCD Ensembles

The 26th international symposium on spin physics

speaker: **Mengchu Cai**

collaborators: Yi-Bo Yang, Hai-Yang Du

Institute of Theoretical Physics, Chinese Academy of Sciences

Sept 24, 2025

Outline

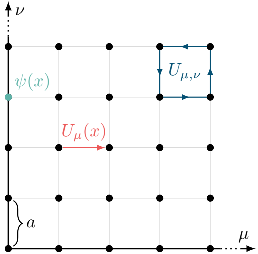
Background

- Leptonic decays of bottom mesons probe the quark-flavor-changing transitions. CKM matrix element can be determined with an experimental measurement of the decay rate.

$$\Gamma(B \to \ell\nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

- Heavy quark mass $am_b > 1$ causes large discretization errors. Heavy-quark effective theory (HQET) or Nonrelativistic QCD (NRQCD) can be implemented on the lattice.
- Utilize finer lattice spacing $a \sim 0.03$ fm. *Bazavov et al.*, 2018, *Phys. Rev. D* MILC *Hatton et al.*, 2021, *Phys. Rev. D* HPQCD ...
- Relativistic anisotropic heavy quark action improves discretization errors.
  Chen, 2001, *Phys. Rev. D* *Lin et al.*, 2007, *LATTICE2007* *Liu et al.*, 2010, *Phys. Rev. D*
  Christ et al., 2007, *Phys. Rev. D* *Aoki et al.*, 2012, *Phys. Rev. D* *Christ et al.*, 2015, *Phys. Rev. D*
  Leskovec et al., 2025, *Phys. Rev. Lett.*

Anisotropic Quark Action on the Lattice



isotropic clover quark action

$$S = a^4 \sum_x \bar{\psi}(x) \left[m_b + \gamma_\mu D_\mu - \frac{a}{2} D^2 - \frac{a}{2} c_{sw} \left(\sum_{\mu<\nu} \sigma_{\mu\nu} F_{\mu\nu} \right) \right] \psi(x)$$

covariant derivative

$$\left[U_\mu(x)\psi(x+\mu) - U_{-\mu}(x)\psi(x-\mu) \right] / \, 2a \quad \Rightarrow \quad D_\mu \psi(x)$$

field strength tensor

$$U_{\mu\nu}(x) \quad \Rightarrow \quad e^{ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)} \quad U_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x)$$

anisotropic quark action

$$S = a^4 \sum_x \bar{\psi}(x) \left[m_q + \left(\gamma_0 D_0 - \frac{a}{2} D_0^2 \right) + \nu \sum_i \left(\gamma_i D_i - \frac{a}{2} D_i^2 \right) - \frac{a}{2} c_B \left(\sum_{i<j} \sigma_{ij} F_{ij} \right) - \frac{a}{2} c_E \left(\sum_i \sigma_{0i} F_{0i} \right) \right] \psi(x)$$

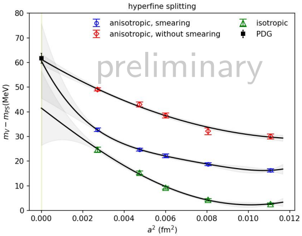
Chen, 2001, *Phys. Rev. D*

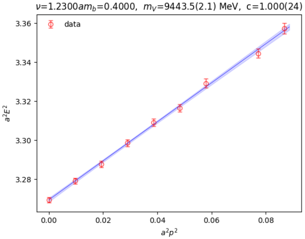
Liu et al., 2010, *Phys. Rev. D*

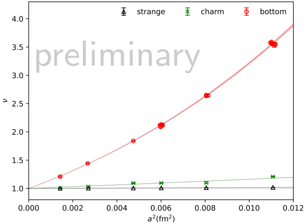
$$c_B = \frac{\nu}{u_0^3}, \quad c_E = \frac{1}{2}(1+\nu)\frac{1}{u_0^3}, \quad u_0 = \left\langle \frac{\mathrm{Re\,Tr} \sum_{x,\mu<\nu} U_{\mu\nu}(x)}{6 N_c V} \right\rangle^{1/4}$$

Anisotropic Parameter Tuning

solve bottom quark propagators with the original gauge field (no stout smearing)







at each (m_b^k, ν^l), fit the dispersion relation $E^{kl}(\boldsymbol{p})^2 = M_V^2(m_b^k, \nu^l) + (c^{kl})^2\boldsymbol{p}^2$.

$$\begin{pmatrix}\nu^{k_1}, \nu^{k_2}, \nu^{k_3}\end{pmatrix} \quad \begin{pmatrix}m_b^{l_1}, m_b^{l_2}, m_b^{l_3}\end{pmatrix} \Rightarrow \begin{aligned} M_V^{kl} &= M^i + d_1(\nu^k - \nu^i) + d_2(m_b^l - {m_b}^i), \\ (c^{kl})^2 &= 1 + d_3(\nu^k - \nu^i) + d_4(m_b^l - {m_b}^i). \end{aligned} \Rightarrow [(\nu^1, m_b^1), (\nu^2, m_b^2), (\nu^3, m_b^3)]$$

utilize the above formula to constrain $c \Rightarrow 1 \qquad M_V(m_b, \nu) \Rightarrow M_\Upsilon^{\text{phys}}$

$$\nu = \frac{\text{Sinh}(c_0 m_V a)}{c_0 m_V a} \times [1 + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2(m_{\eta_s}^2 - m_{\eta_s,phy}^2) + c_3 e^{-m_\pi L}]$$

Correlation Functions

decay constant

$$\langle\Omega|\hat{A}_\mu(0)|P(p)\rangle = i f_P p_\mu \,,$$
$$\langle\Omega|\hat{V}_\mu(0)|V(p,\epsilon)\rangle = f_V m_V \epsilon_\mu ,$$
$$\langle\Omega|\partial_\mu \hat{A}^\mu(x)|P(p)\rangle = (m_1^{\mathrm{PC}} + m_2^{\mathrm{PC}})\langle\Omega|\hat{P}(x)|P(p)\rangle$$

$$C_2(t_f) = \sum_{\mathbf{x}} \langle\Omega|\hat{O}(t_f,\boldsymbol{x})\hat{O}^\dagger(0,0)|\Omega\rangle = \sum_{n=0}^{\infty} |\langle\Omega|\hat{O}(t_f,\boldsymbol{x})|n\rangle|^2 e^{-E_n t_f}$$

nonsinglet local operator required for B, B_s, B_c

$$\hat{A}^\mu_{f_1 f_2}(x) = \bar{\psi}_{f_1}(x)\gamma^\mu\gamma^5\psi_{f_2}(x) \quad \hat{V}^\mu_{f_1 f_2}(x) = \bar{\psi}_{f_1}(x)\gamma^\mu\psi_{f_2}(x) \quad \hat{P}_{f_1 f_2}(x) = \bar{\psi}_{f_1}(x)\gamma^5\psi_{f_2}(x)$$

one field is of the light flavor and the other is of the heavy flavor

$$\hat{O}^R_{f_1 f_2} = Z_O^{f_1 f_2}\hat{O}^B_{f_1 f_2} \qquad Z_P^{f_1 f_2} = Z_V^{f_1 f_2}\left.\frac{Z_P}{Z_V}\right|_{m_q\to 0} \qquad Z_V^{lh} = \sqrt{Z_V^{ll} Z_V^{hh}}$$

wall-to-point(wp) and wall-to-wall(ww) two point correlation function

$$C^{\Gamma_1\Gamma_2}_{2,wp}(t) = \frac{1}{L^3}\sum_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}}\left\langle \bar{\psi}_1(\boldsymbol{x},t_0+t)\Gamma_1\psi_2(\boldsymbol{x},t_0+t)\left[\bar{\psi}_1(\boldsymbol{y},t_0)\Gamma_2\psi_2(\boldsymbol{z},t_0)\right]^\dagger\right\rangle$$
$$C^{\Gamma_1\Gamma_2}_{2,ww}(t) = \frac{1}{L^3}\sum_{\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{y},\boldsymbol{z}}\left\langle \bar{\psi}_1(\boldsymbol{x_1},t_0+t)\Gamma_1\psi_2(\boldsymbol{x_2},t_0+t)\left[\bar{\psi}_1(\boldsymbol{y},t_0)\Gamma_2\psi_2(\boldsymbol{z},t_0)\right]^\dagger\right\rangle$$

Pseudoscalar Meson Correlation Function

CLQCD ensembles used in the calculation

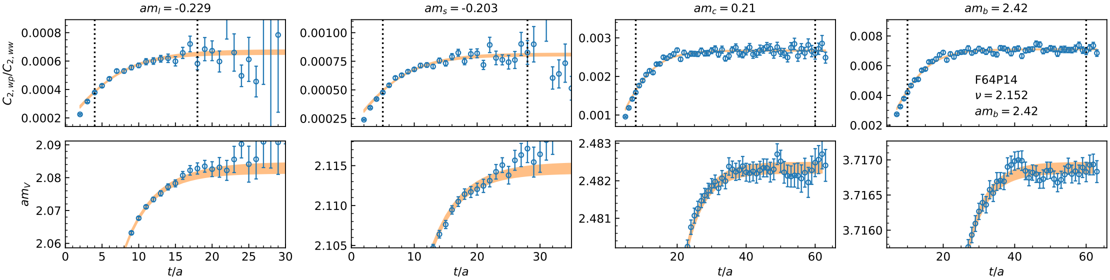
Symbol	$\hat{\beta}$	a (fm)	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	$m_\pi L$
C24P34	6.200	0.10524(05)(62)	$24^3 \times 64$	340.2(1.7)	748.61(75)	4.360(22)
C24P29			$24^3 \times 72$	292.3(1.0)	657.83(64)	3.746(13)
C32P29			$32^3 \times 64$	293.1(0.8)	658.80(43)	5.008(14)
C32P23			$32^3 \times 64$	227.9(1.2)	643.93(45)	3.894(20)
C48P23			$48^3 \times 96$	224.1(1.2)	644.08(62)	5.743(30)
C48P14			$48^3 \times 96$	136.4(1.7)	706.55(39)	3.495(44)
E28P35	6.308	0.08973(20)(53)	$28^3 \times 64$	351.4(1.4)	717.94(93)	4.462(17)
E32P29			$32^3 \times 64$	285.9(2.5)	699.6(4.4)	4.171(26)
E32P22			$32^3 \times 96$	215.5(2.6)	687.2(4.3)	3.145(33)
F32P30	6.410	0.07753(03)(45)	$32^3 \times 96$	300.4(1.2)	675.98(97)	3.780(15)
F48P30			$48^3 \times 96$	302.7(0.9)	674.76(58)	5.713(16)
F32P21			$32^3 \times 64$	210.3(2.3)	658.79(94)	2.646(28)
F48P21			$48^3 \times 96$	207.5(1.1)	661.94(64)	3.917(22)
F64P14			$64^3 \times 128$	135.6(1.5)	681.2(4.2)	3.413(31)
G36P29	6.498	0.06887(12)(41)	$36^3 \times 108$	297.2(0.9)	693.05(46)	3.731(11)
H48P32	6.720	0.05199(08)(31)	$48^3 \times 144$	316.6(1.0)	691.88(65)	4.000(12)
I64P30*	7.02	0.03780(06)(23)	$64^3 \times 128$	310.6(2.5)	668.1(4.3)	3.808(20)

*calculation on I64P30 is in process

Vector Meson Correlation Function

$$C_{2,wp}^{V_iV_i}(t) = \frac{Z_{wp}}{2M_V}\left(e^{-M_V t} + e^{-M_V(T-t)}\right)\left(1 + R_{wp}^{V_iV_i}(t)\right) \qquad C_{2,wp}^{V_iV_i}(t)/C_{2,ww}^{V_iV_i}(t) = \frac{f_P^2 M_V^2}{Z_{wp}} \frac{1 + R_{wp}^{V_iV_i}(t)}{1 + R_{ww}^{V_iV_i}(t)}$$

$$R_{wp(ww)}^{\Gamma_1\Gamma_2}(t) = \sum_{n=1}^{N} r_{wp(ww),\,n}^{\Gamma_1\Gamma_2}\left[e^{-(M+\delta E_n)t} + e^{-(M+\delta E_n)(T-t)}\right] \Big/ \left[e^{-Mt} + e^{-M(T-t)}\right]$$



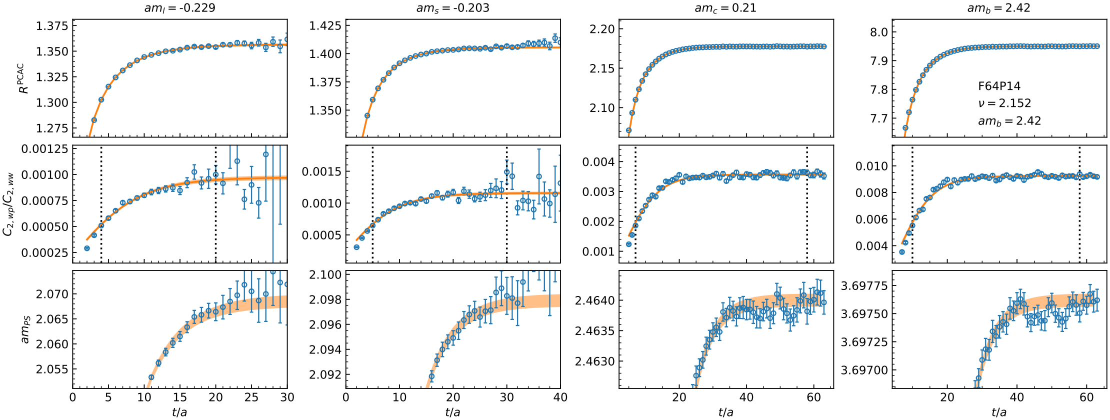
The two-state fitting is needed to describe the correlator data. Signals are still good at $t \sim T/2$ so backward propagation is considered.

$$m_{eff} = \cosh^{-1}\left[C_2(t-1) + C_2(t+1)\right] / \left[2C_2(t)\right]$$

Pseudoscalar Meson Correlation Function

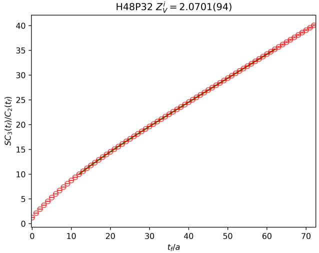
$$C^{\mathrm{PP}}_{2,wp}(t) = \frac{Z_{wp}}{2M_{\mathrm{PS}}}\left(e^{-M_{\mathrm{PS}}t} + e^{-M_{\mathrm{PS}}(T-t)}\right)\left(1 + R^{\mathrm{PP}}_{wp}(t)\right) \qquad C^{\mathrm{PP}}_{2,wp}(t)/C^{\mathrm{PP}}_{2,ww}(t) = \frac{\left(f_{\mathrm{P}} M^2_{\mathrm{PS}}\right)^2}{(m^{\mathrm{PC}}_1 + m^{\mathrm{PC}}_2)^2}\frac{1}{Z_{wp}}\frac{1 + R^{\mathrm{PP}}_{wp}(t)}{1 + R^{\mathrm{PP}}_{ww}(t)}$$

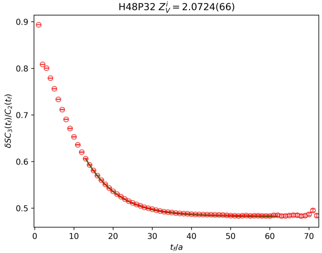
$$\left[C^{\mathrm{A_4P}}_{2,wp}(t-1) - C^{\mathrm{A_4P}}_{2,wp}(t+1)\right]\Big/ 2C^{\mathrm{PP}}_{2,wp}(t) = \frac{\mathrm{Sinh}\left(M_{\mathrm{PS}}\right)}{M_{\mathrm{PS}}}(m^{\mathrm{PC}}_1 + m^{\mathrm{PC}}_2)\frac{1 + R^{\mathrm{A_4P}}_{wp}(t)}{1 + R^{\mathrm{PP}}_{wp}(t)}$$

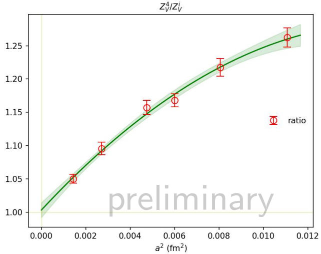


Renormalization

$$Z_V^i \langle P|V^i|P\rangle = 2P^i \qquad Z_V^t \langle P|V^t|P\rangle = 2P^0 \qquad Z_V^t \text{ maybe different from } Z_V^i$$







$$C_3^{PVP}(t_0,t,t_f) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle P(\boldsymbol{x},t_f)V(\boldsymbol{y},t)P(\vec{0},t_0)\rangle \quad R(t_f) = \frac{\sum_t C_3(t_f,t)}{C_2(t_f)} = \frac{c_0 t_f + (c_1 + c_2 t_f)e^{-\Delta E t_f} + c_3}{1 + d_1 e^{-\Delta E t_f}}$$

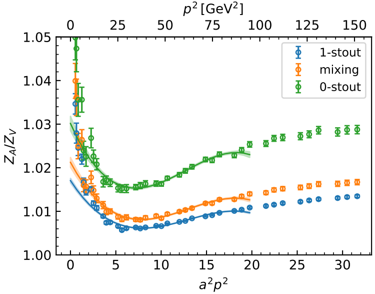
$$\delta R(t_f) = R(t_f+1) - R(t_f) = c_0 + (c_1 + c_2 t_f)e^{-\Delta E t_f} + O(e^{-2\Delta E t_f})$$

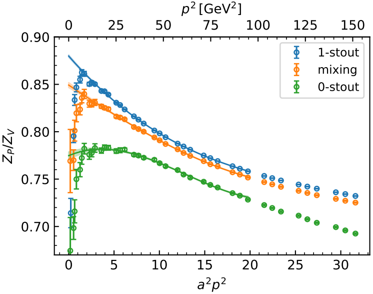
Numerical calculation shows that Z_V^0/Z_V^i goes to unity in the continuum limit

Renormalization

$$G_{\mathcal{O}_{f_1 f_2}}(p_1,p_2) = \sum_{x,y} e^{-i(p_1\cdot x - p_2\cdot y)} \langle \psi_{f_1}(x)\,[\bar\psi_{f_1}(x)\Gamma\psi_{f_2}(x)]\,\bar\psi_{f_2}(y)\rangle = S_{f_1}(p_1)\Lambda_{\mathcal{O}_{f_1 f_2}}(p_1,p_2)S_{f_2}(p_2)$$

$$S_{f_1}(p) = \sum_x \mathrm{e}^{-\mathrm{i}p\cdot x}\,\langle\psi_{f_1}(x)\bar\psi_{f_1}(0)\rangle \qquad p_1^2 = p_2^2 = (p_1-p_2)^2 = \mu^2$$





gauge field
with smearing $V_\mu(x)$
without smearing $U_\mu(x)$

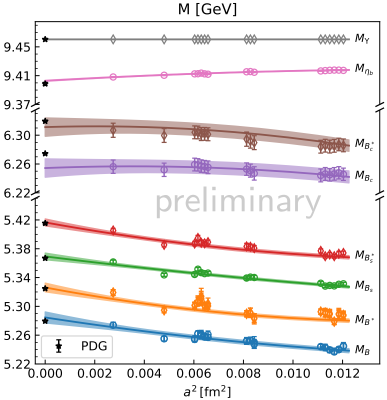
1-stout: $S_{f_1}(V)$, $S_{f_2}(V)$
mixing: $S_{f_1}(U)$, $S_{f_2}(V)$
0-stout: $S_{f_1}(U)$, $S_{f_2}(U)$

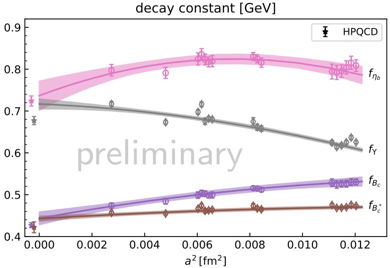
$$Z_P^{f_1 f_2} = Z_V^{f_1 f_2}\,\frac{Z_P}{Z_V}\bigg|_{m_q\to 0}$$

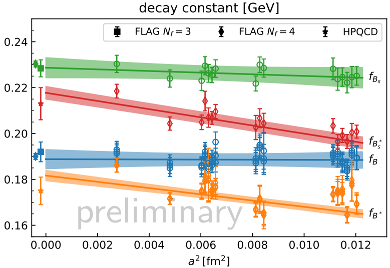
$$\frac{1}{12q^2}\mathrm{Tr}[q_\mu\Lambda^\mu_{V,B}(p_1,p_2)\not{q}] = \frac{Z_q^{\mathrm{SMOM}}}{Z_V^{\mathrm{SMOM}}} \qquad \frac{1}{12i}\mathrm{Tr}\left[\Lambda_{P,B}(p_1,p_2)\gamma_5\right] = \frac{Z_q^{\mathrm{SMOM}}}{Z_P^{\mathrm{SMOM}}}$$

$$Z^{\overline{\mathrm{MS}}}(\mu_0) = R(\mu_0,\mu)C^{\overline{\mathrm{MS}}}_{\mathrm{SMOM}}(\mu)Z^{\mathrm{SMOM}}(\mu)$$

Extrapolation to the Continuum







$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0) + \sum_{n=1}^{N} c_n^X a^{2n}$$

$$+ d_1^X (m_\pi^2 - (m_\pi^{\text{phys}})^2) + d_2^X (m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2)$$

Conclusion and Outlook

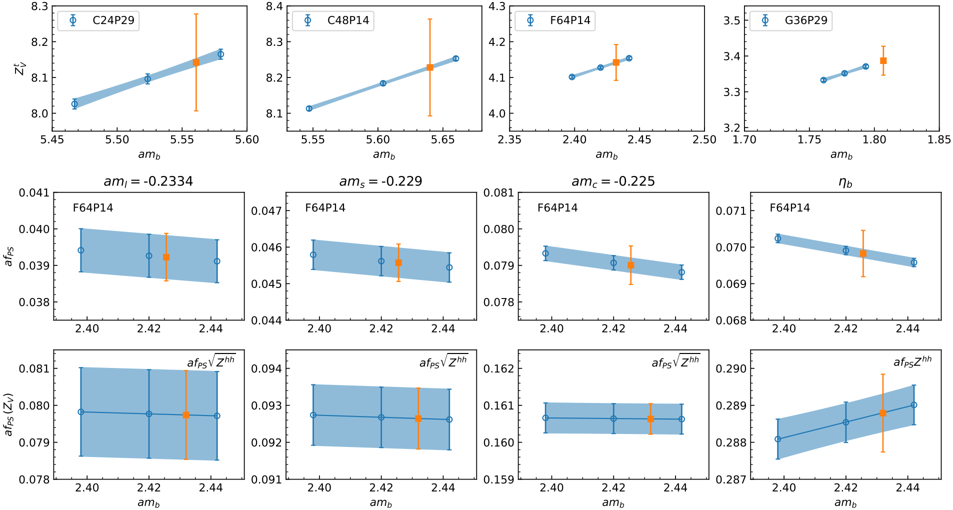
- We present a systematic study of masses and decay constants of the mesons with bottom quark using the anisotropic fermion action on CLQCD ensembles at $a \in [0.05, 0.11]$ fm and $m_\pi \in [130, 360]$ MeV
- We propose a systematic framework to renormalize the quark bi-linear operator with the bottom quark field and verify it through the renormalized decay constants
- Calculation on the $a = 0.04$ fm ensemble is in process. We will obtain more reliable continuum extrapolation results and the error estimation.

THANKS

Bottom Quark Mass Interpolation

$$M_V^i = d^0_{M_V} m_b^i + d^1_{M_V} \qquad X^i = d^0_X m_b^i + d^1_X$$

Bare bottom quark values contain error from lattice spacing. $af_{PS}^{B_l}\sqrt{Z^{hh}}$ show less m_b dependence



Reference I

Aoki, Y., et al., Nonperturbative tuning of an improved relativistic heavy-quark action with application to bottom spectroscopy, *Phys. Rev. D*, *86*, 116,003, doi:10.1103/PhysRevD.86.116003, 2012.

Bazavov, A., et al., B- and D-meson leptonic decay constants from four-flavor lattice QCD, *Phys. Rev. D*, *98*(7), 074,512, doi:10.1103/PhysRevD.98.074512, 2018.

Chen, P., Heavy quarks on anisotropic lattices: The Charmonium spectrum, *Phys. Rev. D*, *64*, 034,509, doi:10.1103/PhysRevD.64.034509, 2001.

Christ, N. H., M. Li, and H.-W. Lin, Relativistic Heavy Quark Effective Action, *Phys. Rev. D*, *76*, 074,505, doi:10.1103/PhysRevD.76.074505, 2007.

Christ, N. H., J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water, and O. Witzel, B-Meson Decay Constants from 2+1-Flavor Lattice QCD with Domain-Wall Light Quarks and Relativistic Heavy Quarks, *Phys. Rev. D*, *91*(5), 054,502, doi:10.1103/PhysRevD.91.054502, 2015.

Hatton, D., C. T. H. Davies, J. Koponen, G. P. Lepage, and A. T. Lytle, Bottomonium precision tests from full lattice QCD: Hyperfine splitting, Υ leptonic width, and b quark contribution to $e^+e^- \to$ hadrons, *Phys. Rev. D*, *103*(5), 054,512, doi:10.1103/PhysRevD.103.054512, 2021.

Leskovec, L., S. Meinel, M. Petschlies, J. Negele, S. Paul, and A. Pochinsky, B$\to\rho\ell\nu^-$ Resonance Form Factors from B$\to\pi\pi\ell\nu^-$ in Lattice QCD, *Phys. Rev. Lett.*, *134*(16), 161,901, doi:10.1103/PhysRevLett.134.161901, 2025.

Lin, H.-W., R. G. Edwards, and B. Joo, Parameter tuning of three-flavor dynamical anisotropic clover action, *PoS*, *LATTICE2007*, 119, doi:10.22323/1.042.0119, 2007.

Liu, L., H.-W. Lin, K. Orginos, and A. Walker-Loud, Singly and Doubly Charmed J=1/2 Baryon Spectrum from Lattice QCD, *Phys. Rev. D*, *81*, 094,505, doi:10.1103/PhysRevD.81.094505, 2010.