NANJING UNIVERSITY

Gravitational and Electromagnetic Form Factors of Pion, Kaon, and Nucleon

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100 YEARS SPIN 2025

26th International Symposium on Spin Physics

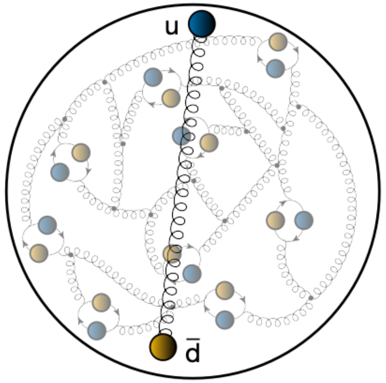
A Century of Spin

The 26th International Symposium on Spin Physics, Qingdao, Shandong, China, September 22 - 26, 2025

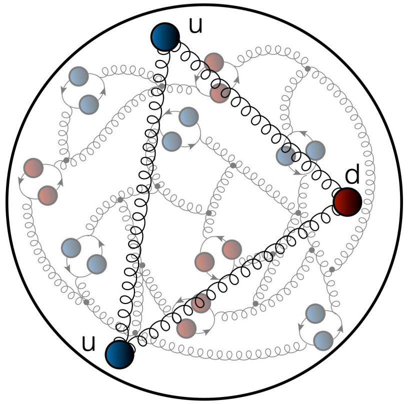
Structure of Pion, Kaon, and Nucleon

- Form factor: the closest thing we have to a snapshot, the size, shape and makeup of the hadron
  - ✓ **Electromagnetic form factor**
  - ✓ Two-photon transition form factor
  - ✓ **Gravitational form factor**
- 1D picture of how quarks move within the hadron
  - ✓ Distribution amplitude (DA)
  - ✓ Distribution function (DF)
- A multidimensional view of the hadron structure
  - ✓ Transverse momentum dependent distribution function (TMD)
  - ✓ Generalized parton distribution (GPD)

etc..



pion

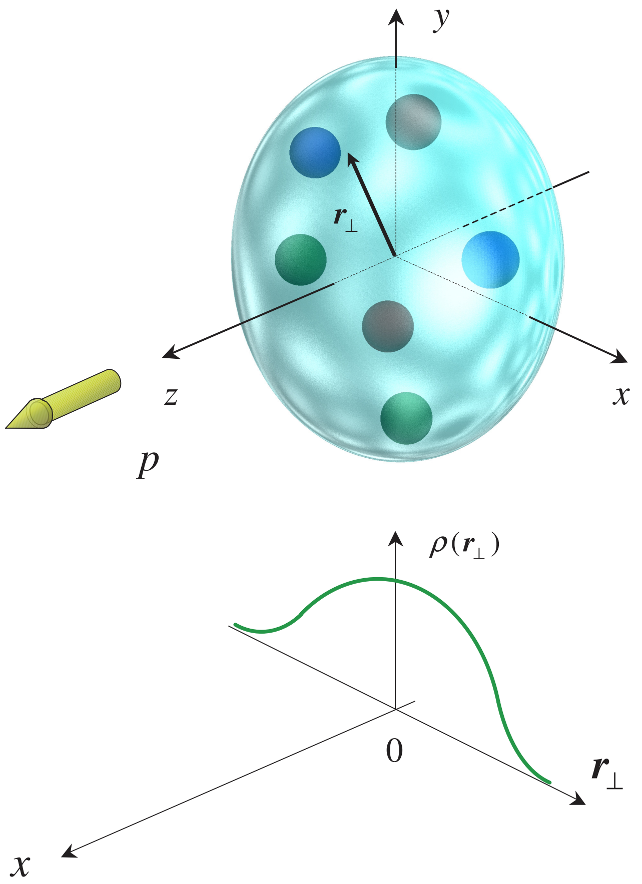


proton

Electromagnetic and gravitational form factors

- Electromagnetic form factor:
  - ✓ Electric charge radius
  - ✓ Electric charge distribution
- Gravitational form factor:
  - ✓ D-term: last global unknown property of hadron
  - ✓ Mass radius
  - ✓ Mechanical radius
  - ✓ Shear force distribution
  - ✓ Pressure distribution

etc..



Distribution Amplitude (DA) and pion electromagnetic form factor

➢ Perturbative QCD predicts charged-pion elastic form factor

G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980). A. V. Efremov and A. V. Radyushkin, Phys. Lett. B 94, 245 (1980).

$$\exists Q_0 > \Lambda_{QCD} \mid Q^2 F_\pi(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2 , \qquad w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi_\pi(x) ,$$

➢ f_π is the leptonic decay constant of pion, $\alpha_s(Q^2)$ is the is the leading-order strong running-coupling.

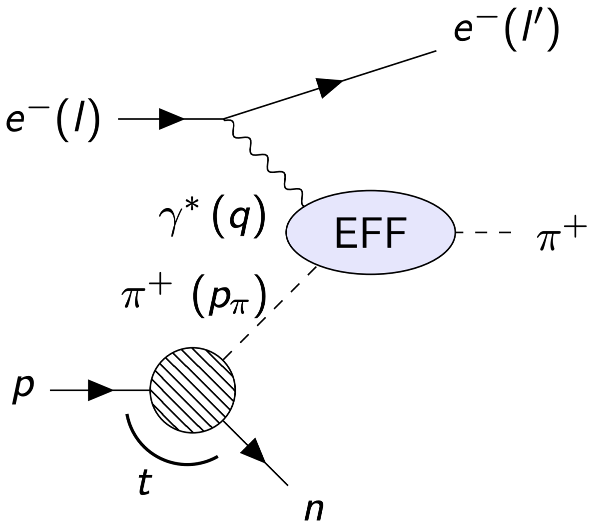
➢ $\varphi_\pi(x)$ is the meson's dressed-valence-quark distribution amplitude (DA). The value of Q_0 is not predicted by pQCD.

➢ Asymptotic DA at $\Lambda^2_{\mathrm{QCD}}/Q^2 \simeq 0$, i.e., very large values of Q^2,

$$\varphi_\pi(x) = 6x(1-x) ,$$

➢ then $w_\varphi = 1$, and

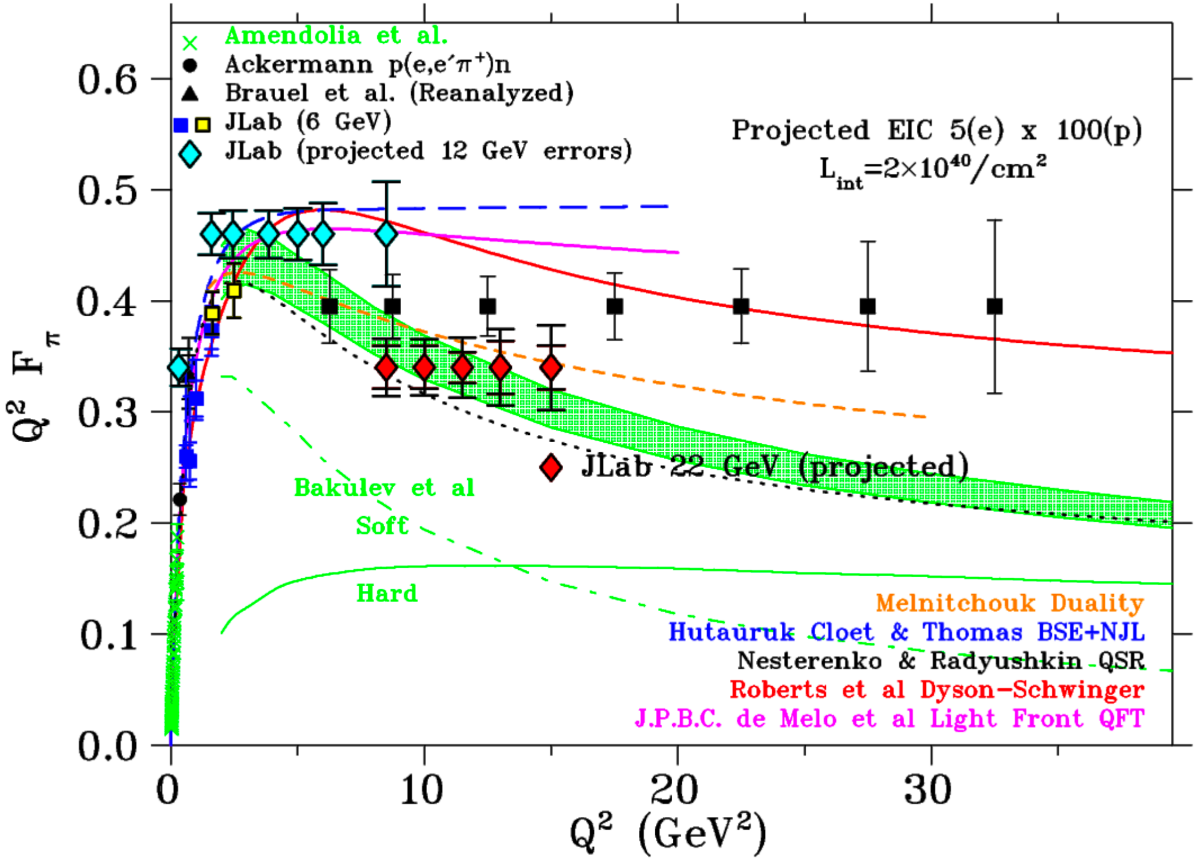
$$Q^2 F_\pi(Q^2) \overset{Q^2\to\infty}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 ,$$



No free pion target - use "virtual pion cloud" of the proton.

Pion electromagnetic form factor

- Figure is taken from JLab 22 GeV white paper
- Existing data (blue, black, yellow, green) and projected uncertainties for future data on the pion form factor from JLab (12 GeV: cyan; 22 GeV: red) and EIC (black), in comparison to a variety of hadronic structure models.



JLab 22 GeV white paper: Eur. Phys. J. A 60 (2024) 9, 173

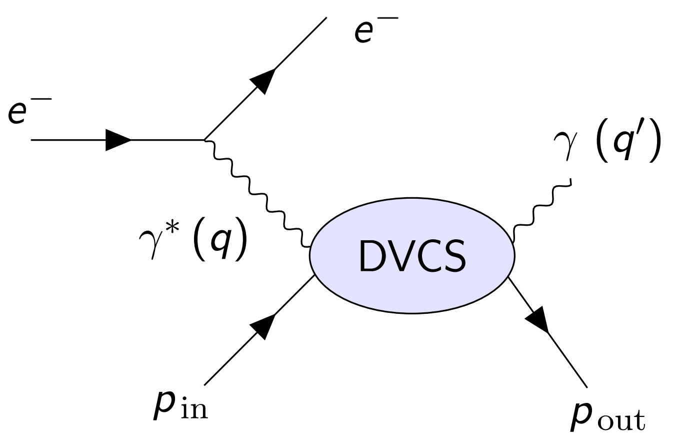
Gravitational form factor

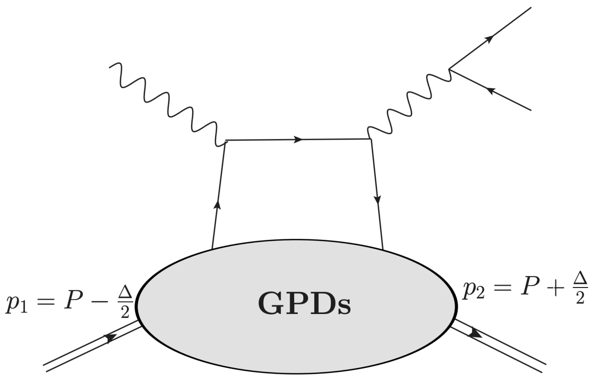
➢ Gravitational form factor (GFFs) are related to the Mellin moment of the chiral-even GPDs

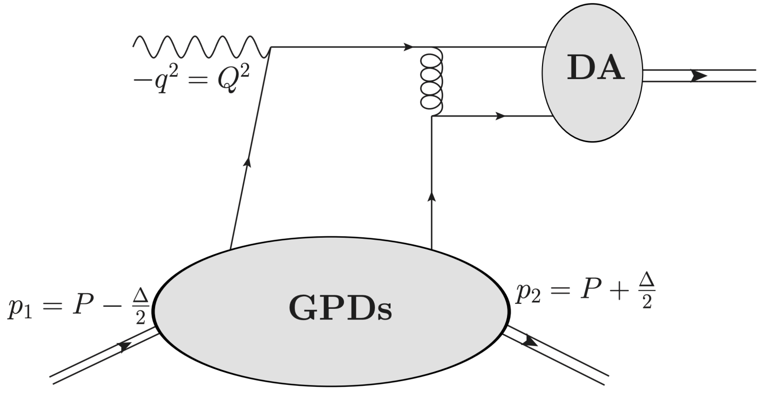
$$\int_{-1}^{1} dx\, x\, H^q(x,\xi,Q^2) = A^q(Q^2) + \xi^2 D^q(Q^2)\,,$$

➢ GPD measurements: Deeply virtual Compton scattering (DVCS), Time-like Compton scattering (TCS), Deeply virtual Meson production (DVMP) etc..

Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan, Rev. Mod. Phys. 95 (2023) 4, 041002.



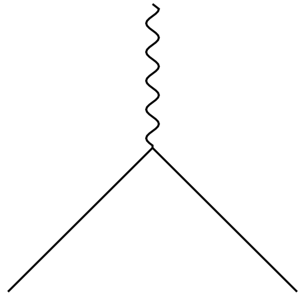


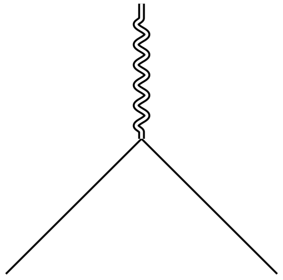


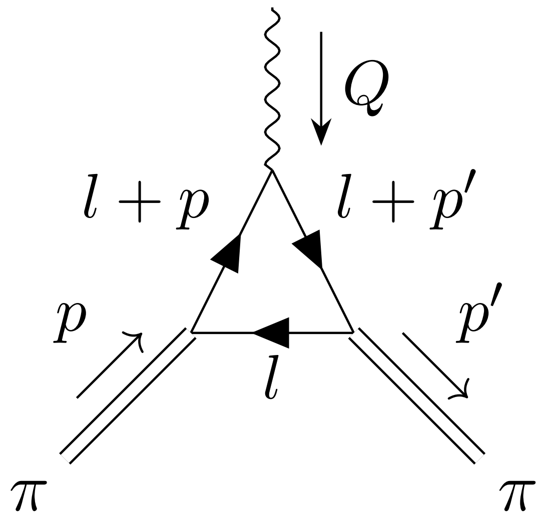
➢ GFFs may be probed indirectly in these exclusive processes.

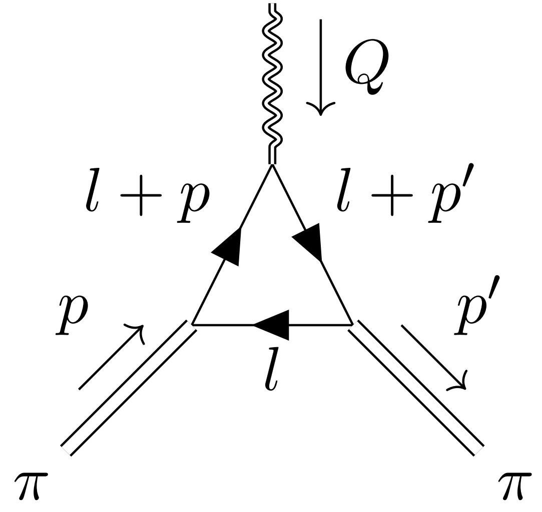
Electromagnetic and Gravitational form factors

- ➢ Interactions between hadron and different probes
  - ✓ Electromagnetic probe: photon
  - ✓ Gravitational probe: graviton
- ➢ Dressed hadron + probe vertex
  - ✓ Dressed quark photon vertex
  - ✓ Dressed quark graviton vertex

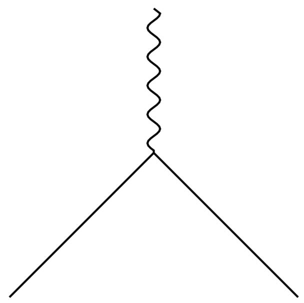








Dressed quark photon vertex



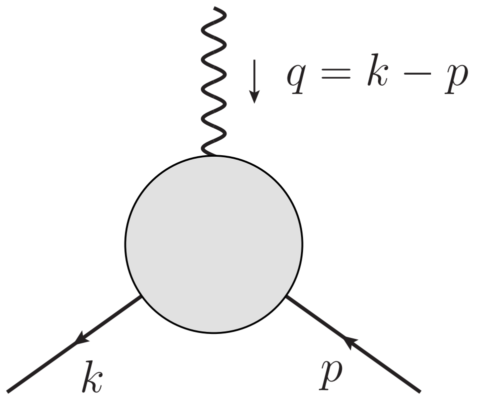
➢ In general, quark photon vertex can be divided into longitudinal and transverse parts, including twelve independent tensor structures:

$$\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)\,,$$

✓ Four tensors in longitudinal parts $\{L^j_\mu(k,p), j = 1,\ldots,4\}$ and eight tensors in transverse parts: $\{T^j_\mu(k,p), j = 1,\ldots,8\}$.

Qin, Chang, Liu, Roberts, Schmidt, Phys. Lett. B 722 (2013) 384-388

➢ Longitudinal part of the quark photon vertex is constrained by the Ward-Green-Takahashi (WGT) identity:



$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)\,,$$

✓ Ball-Chiu vertex:

$$i\Gamma^{\rm BC}_\nu(k_+,k_-) = i\gamma_\nu \Sigma_{A_\pm} + 2ik_\nu \gamma\cdot k\,\Delta_{A_\pm} + 2k_\nu \Delta_{B_\pm}\,,$$

Dressed quark photon vertex

- Γ_ν^{BC} is a matrix-valued regular function (free of kinematic singularities for real arguments).
- Transverse part can be solved with the inhomogeneous vector Bethe-Salpeter equation:

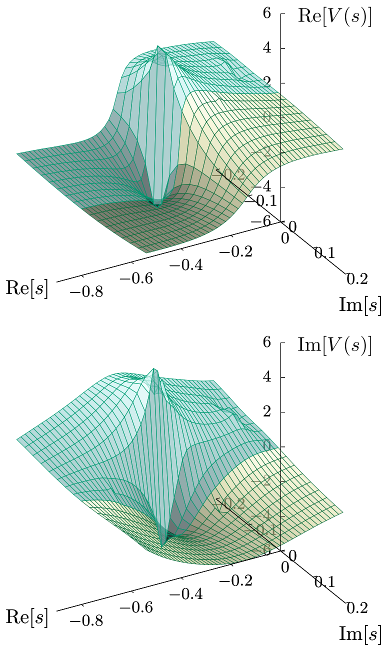
$$\Gamma_\mu(p_{out}, p_{in}; Q) \;=\; Z_2 \gamma_\mu + \int_k K(p_{out}, p_{in}; k_{out}, k_{in}) S(k_{out})\, \Gamma_\mu(k_{out}, k_{in}; Q)\, S(k_{in}) \;,$$

- Transverse part exhibits timelike-Q^2poles, one at the mass-squared of each neutral vector meson supported by the interaction. The lightest such state is the ρ^0-meson.

Bhagwat, Maris, Phys. Rev. C 77 (2008) 025203.

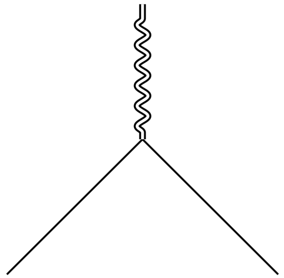
$$\Gamma_\mu(p_{out}, p_{in}) \sim \frac{\Gamma_\mu^V(p_{out}, p_{in})\, f_V\, M_V}{Q^2 + M_V^2} \;,$$

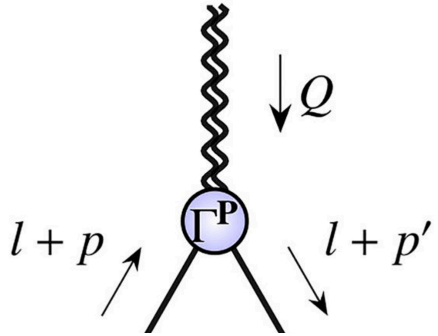
- The fact that the dressed $q\bar{q}\gamma$-vertex exhibits these vector meson poles explains the success of naive vector-meson-dominance (VMD) models.



Richard Williams, Phys. Lett. B 798 (2019) 134943

Dressed quark graviton vertex





- Longitudinal part + transverse part

$$\Gamma^{g}_{\mu\nu}(k,Q) = \Gamma^{g_M}_{\mu\nu}(k,Q) + \Gamma^{gT}_{\mu\nu}(k,Q)\,,$$

- Longitudinal part is constrained by the Ward-Green-Takahashi

R. Brout, F. Englert. Phys. Rev. 141(4), 1231–1232 (1966)

$$Q_\mu i\Gamma^{g}_{\mu\nu}(k,Q) = S^{-1}(k_+)k_{-\nu} - S^{-1}(k_-)k_{+\nu}\,.$$

- Transverse part can be solved with the inhomogeneous tensor Bethe-Salpeter equation

Yin-Zhen Xu et al. Eur. Phys. J. C 84 (2024) 2, 191

$$i\Gamma^{g}_{\mu\nu}(k_+,k_-) = Z_2[i\gamma_\mu k_\nu - \delta_{\mu\nu}(i\gamma\cdot k + Z_m^0 m^\zeta)] + Z_2^2\int_{dl}^{\Lambda} K(k-l)[S(l_+)i\Gamma^{g}_{\mu\nu}(l_+,l_-)S(l_-)]\,,$$

- and it has scalar and tensor poles

$$\left.\frac{-f_{\mathbb{S}}\Gamma^{\mathbb{S}}(k;Q)}{1+m_{\mathbb{S}}^2/Q^2}\right|_{Q^2+m_{\mathbb{S}}^2\simeq 0}\,,\quad \left.\frac{-f_{\mathbb{T}}\Gamma^{\mathbb{T}}_{\mu\nu}(k;Q)}{1+m_{\mathbb{T}}^2/Q^2}\right|_{Q^2+m_{\mathbb{T}}^2\simeq 0}\,,$$

Gravitational form factors of pion

➢ The expectation value of the energy-momentum tensor in the pion:

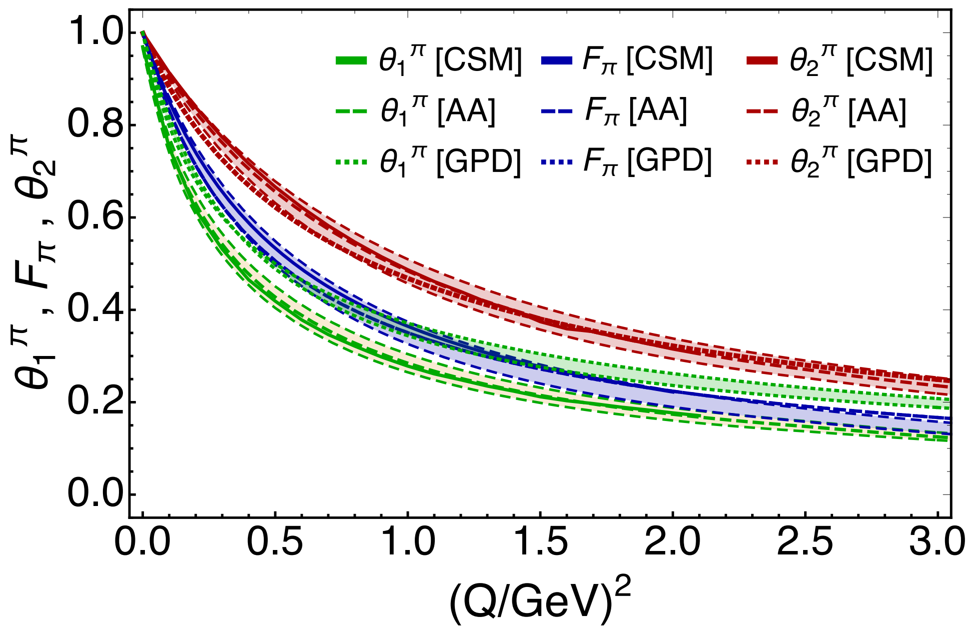
$$\Lambda^g_{\mu\nu}(P,Q) = 2P_\mu P_\nu \theta_2^\pi(Q^2) + \frac{1}{2}[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu]\theta_1^\pi(Q^2) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi(Q^2),$$

➢ $\theta^\pi_{2,1}$ are the in-pion mass and pressure distribution form factors. The relations follow from symmetries:

$$\theta_2^\pi(0) = 1, \quad \theta_1^\pi(0) \overset{m_\pi^2=0}{=} 1, \quad \bar{c}^\pi(Q^2) \equiv 0.$$

➢ The pressure distribution radius is greater than both the electromagnetic and mass radii:

$$r_\pi^{\theta_1} > r_\pi^F > r_\pi^{\theta_2}, r_\pi^{\theta_2}/r_\pi^F = 0.74, r_\pi^F/r_\pi^{\theta_1} \approx 0.79$$



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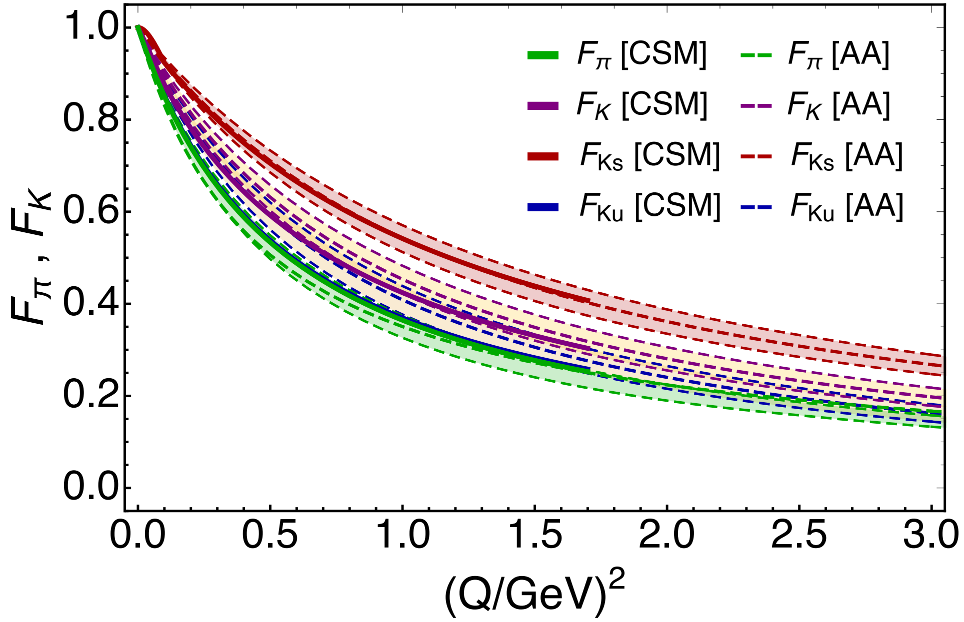
Gravitational form factors of kaon

- For pion, direct rainbow ladder (RL) results are employed within $Q^2 \lesssim 2$ GeV^2, whilst beyond this region, perturbation theory integral representation (PTIRs) is used. For kaon, RL is applied within $Q^2 \lesssim 1.7$ GeV^2.

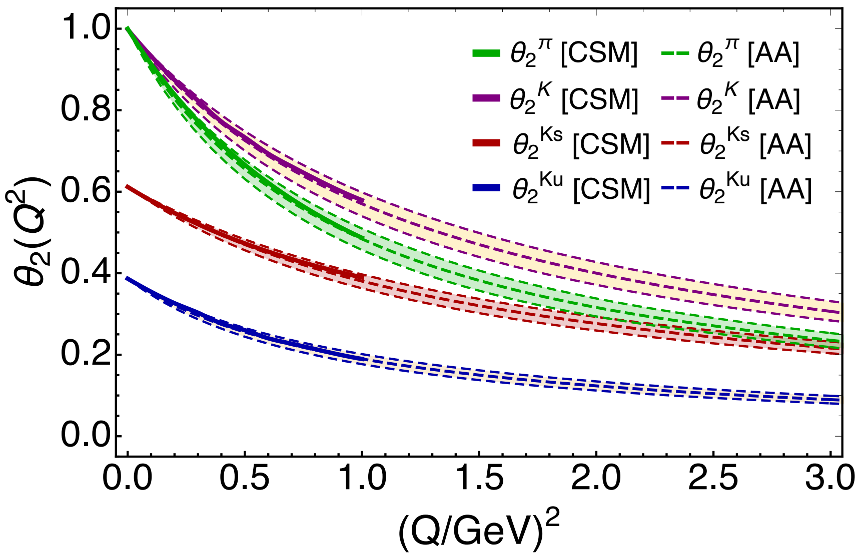
- u quark in Kaon elastic electric form factor is almost indistinguishable from the u quark in π form factor, F_π

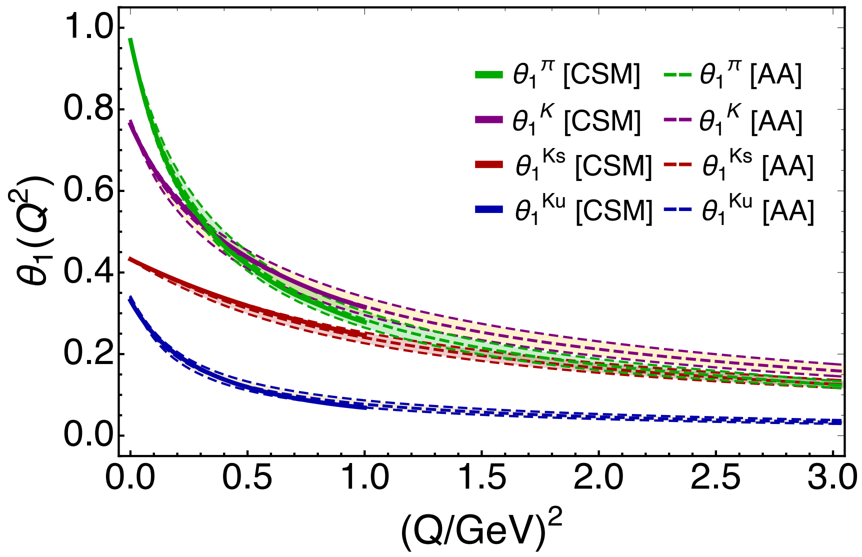
- There is a marked difference between the $\bar{s}$ in Kaon and u in Kaon electromagnetic form factors: $F_K^{\bar{s}}(Q^2), F_K^u(Q^2)$, A similar distinction is expressed in the ratio of $\bar{s}$ in Kaon and *u* in Kaon valence parton DFs.



Gravitational form factors of kaon

- $\theta_2^{K_u}(0) = 0.39, \theta_2^{K_s}(0) = 0.61$, so $\theta_2^{K}(0) = 1.$
- $\theta_1^{K}(0) = 0.77$
- The ordering of kaon radii is the same as that found for the pion: $r_K^{\theta_1} > r_K^F > r_K^{\theta_2}$
- In each case, the net kaon radius is smaller than the kindred pion radius



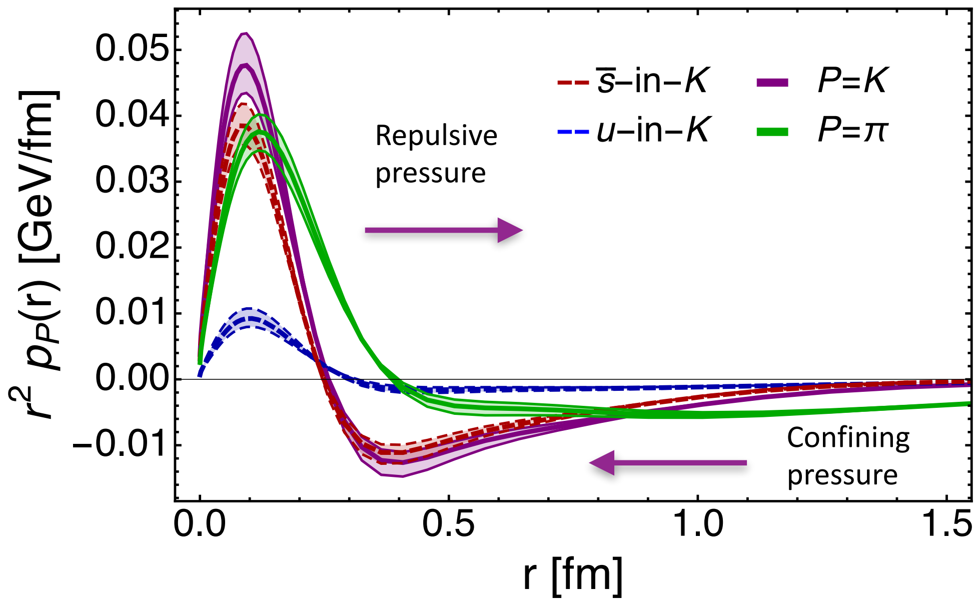


Gravitational form factors of pion and kaon

➢ Pseudoscalar meson pressure and shear force distributions may be defined.

➢ Physical interpretation: meson pressures are positive and large on the neighbourhood $r \simeq 0$, whereupon the meson's dressed-valence constituents are pushing away from each other.

➢ With increasing separation, the pressure switches sign (von Laue condition) indicating a transition to the domain wherewithin confinement forces exert their influence on the pair.



➢ The zeros - confinement radius (in fm):

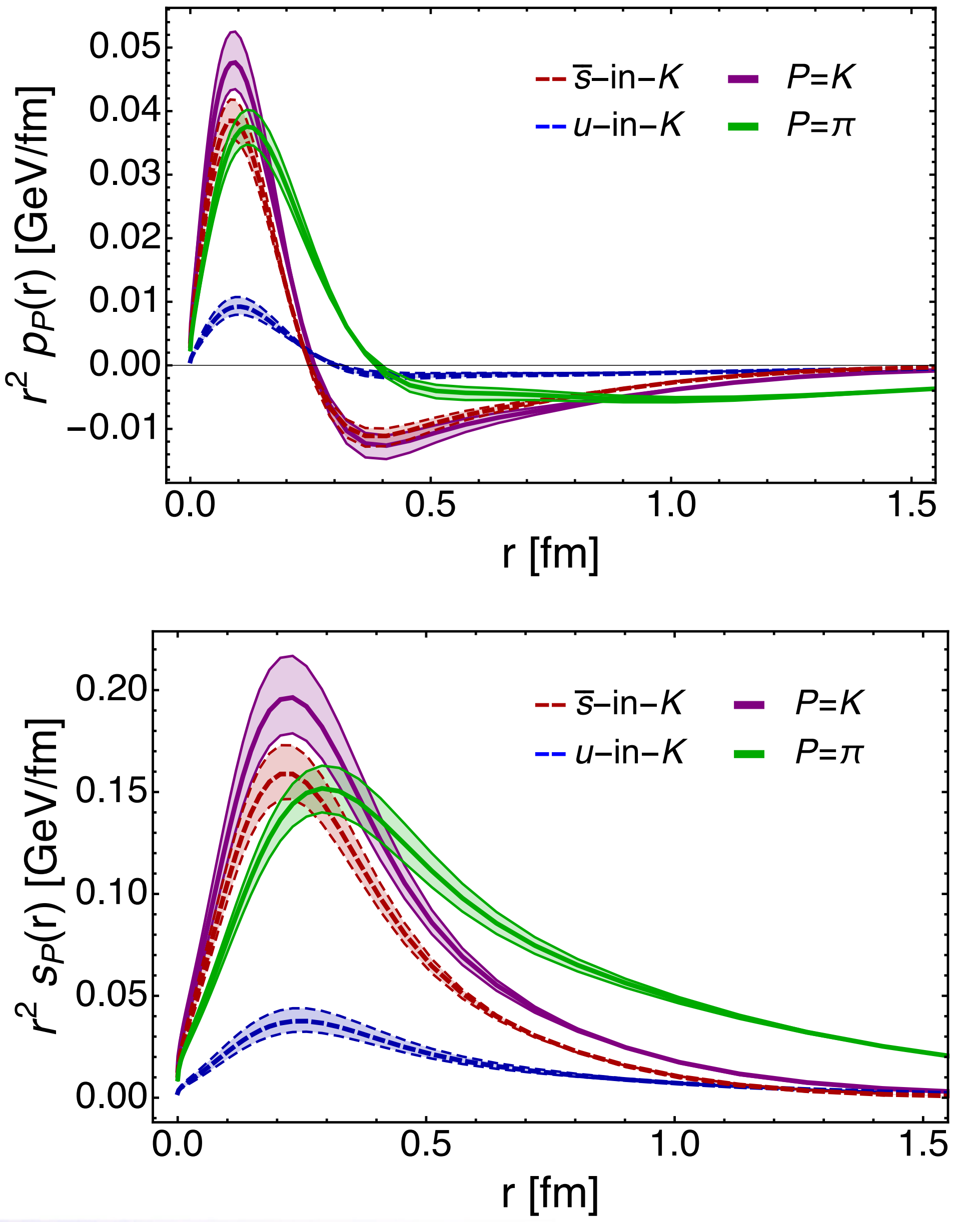
$$r_c^{\pi} = 0.39(1), r_c^{K} = 0.26(1), r_c^{K_u} = 0.30(1), r_c^{K_{\bar{s}}} = 0.25(1)$$

Gravitational form factors of pion and kaon

- ➢ The Kaon profile is more compact, so the associated peak core pressure is higher than in the pion: the ratio is ≈ 1.5.
- ➢ The shear pressures are an indicator of the strength of deformation forces within the meson.
- ➢ Shear forces are maximal in the neighbourhood upon which the pressure changes sign.
- ➢ Total pion and kaon shear forces:

$$\int_0^\infty dr\, r^2 s_K(r) = 0.77 \int_0^\infty dr\, r^2 s_\pi(r)\,.$$

the π result is greater.



Gravitational form factors of proton

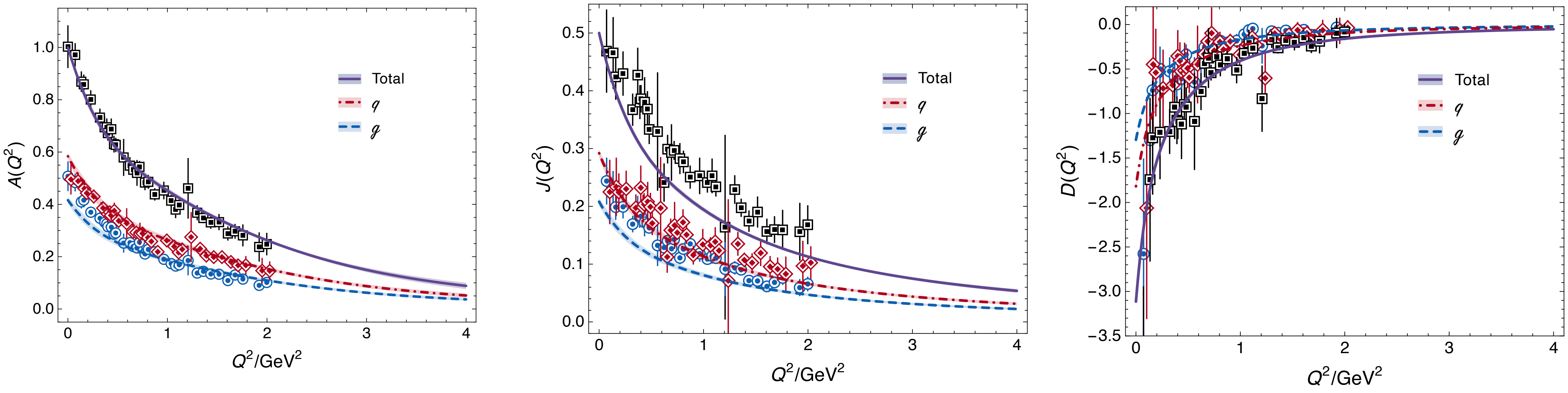
- The proton (nucleon) has three gravitational form factors:

$$m_N \Lambda^{Ng}_{\mu\nu}(Q) = -\Lambda_+(p_f)[K_\mu K_\nu A(Q^2) + iK_{\{\mu}\sigma_{\nu\}\rho}Q_\rho J(Q^2) + \frac{1}{4}(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)D(Q^2)]\Lambda_+(p_i)\,,$$

- A is the mass distribution form factor; J relates to spin distribution; D relates to pressure and shear force distributions.
- Symmetries entail $A(0) = 1, J(0) = 1/2$. $D(0)$ is the last unknown global property.
- Species decomposition:
  - ✓Total GFFs are scale independent, but species decomposition of GFFs are scale dependent.
  - ✓An all-orders (AO) evolution scheme is developed.
  - ✓For any quark or gluon sector contributions, $F^q(Q^2;\zeta)$, $F^g(Q^2;\zeta)$, the species decomposition contribution is $\langle x\rangle^p_\zeta \times F(Q^2)$, where $\langle x\rangle^p_\zeta$ is the parton species light-front momentum fraction in the hadron at ζ. We take $\zeta = 2$ GeV.

Gravitational form factors of proton

➢ Our results for the species separated form factors compared with available lattice QCD results: in all cases, they agree within mutual uncertainties.



D.C. Hackett, D.A. Pefkou, P.E. Shanahan. Phys. Rev. Lett. 132(25), 251904 (2024); Zhao-qian Yao et al. Eur. Phys. J. A 61 (2025) 5, 92.

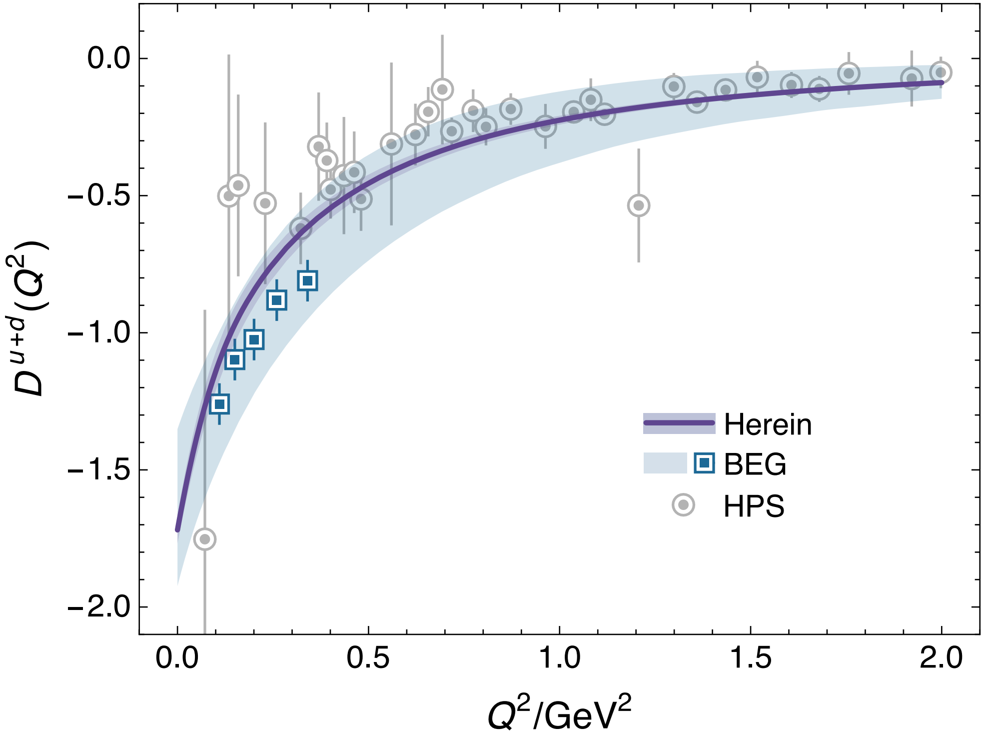
Gravitational form factors of proton

- D term: $D(0) = -3.11(1)$.

- Lattice QCD results:

  z-expansion: −3.87(97); Dipole: −3.35(58)

- Light quark alone: $D^{u+d}(0;\zeta_2) = -1.73(5)$. Inference from available DVCS data yields $D^{u+d}(0;\zeta) = -1.63(29)$[Burkert:2018bqq]

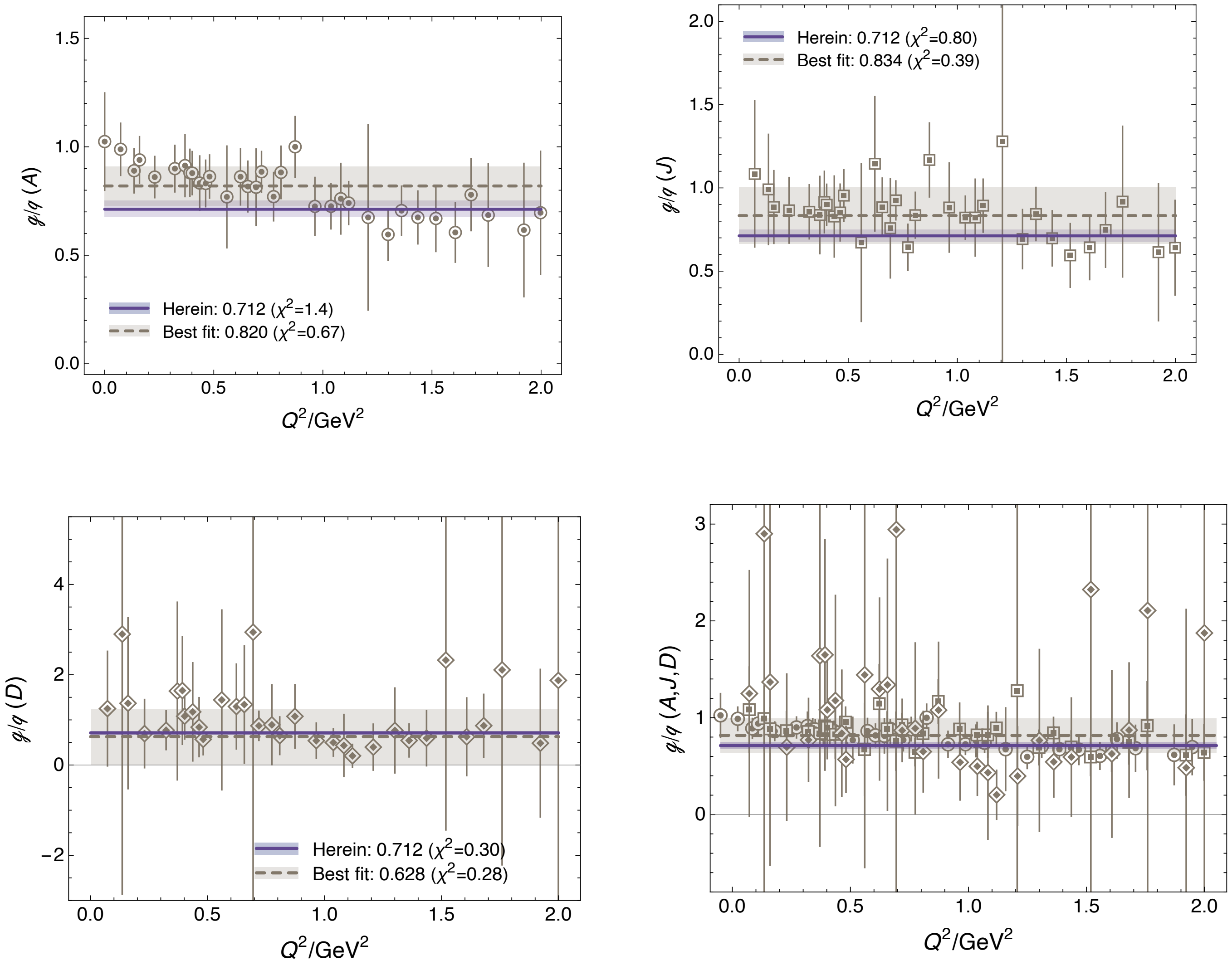


V.D. Burkert, L. Elouadrhiri, F.-X. Girod. Nature 557(7705), 396–399 (2018); D.C. Hackett, D.A. Pefkou, P.E. Shanahan. Phys. Rev. Lett. 132(25), 251904 (2024).

Gravitational form factors of proton

- All order evolution approach yields that, for each form factor, the ζ_2 contribution ratio gluon:total-quark is a fixed number (constant, independent of Q^2), viz. $g(Q^2)/q(Q^2) = 0.71(4)$.

- Within large uncertainties, available lattice QCD results are compatible with this.



Gravitational form factors of proton

➢ Breit frame density profiles (energy, pressure, shear and normal force distributions) via appropriate three-dimensional Fourier transforms.

Polyakov, Schweitzer. Int.J.Mod.Phys.A 33 (2018) 26, 1830025

➢ The pion peak values are roughly twice those in the proton, and such pressures are an order of magnitude greater than are expected at the core of neutron stars.

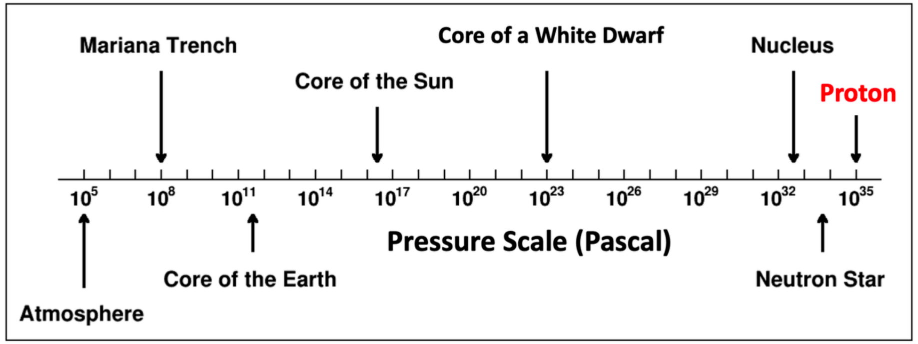
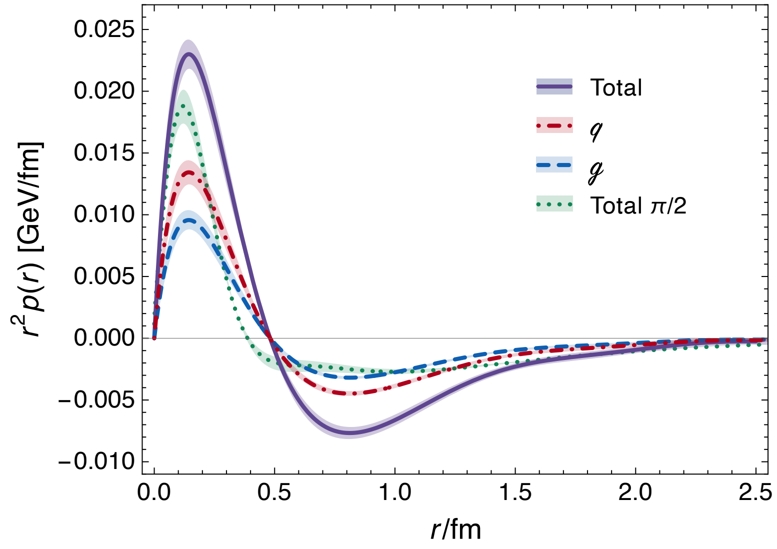
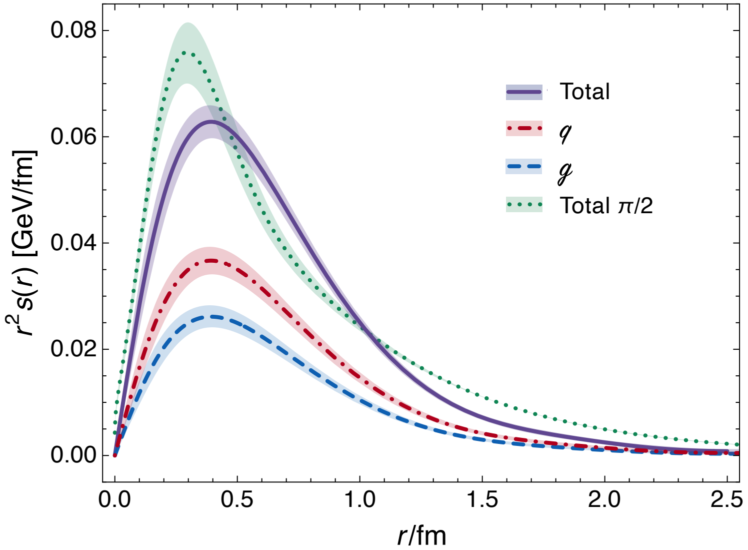


FIG. 15 in Burkert, Elouadrhiri, Girod, Lorcé, Schweitzer, Shanahan, Rev. Mod. Phys. 95 (2023) 4, 041002.





Gravitational form factors of proton

- Nucleon mass and mechanical radii can be defined in terms of $\epsilon(r)$, and Normal force distribution $F^{\|}(r)$

$$\langle r^2\rangle_{\text{mass}} = \frac{\int d^3r\, r^2 \epsilon(r)}{\int d^3r \epsilon(r)}, \quad \langle r^2\rangle_{\text{mech}} = \frac{\int d^3r\, r^2 F^{\|}(r)}{\int d^3r F^{\|}(r)}.$$

- These expressions are equivalent to (mechanical radius is not related to the slope of a form factor):

$$\langle r^2\rangle_{\text{mass}} = \left[-6\frac{d}{dt}A(t)\Big|_{t=0} - 3\frac{D(0)}{2m_N^2}\right]\frac{1}{A(0)}, \quad \langle r^2\rangle_{\text{mech}} = \frac{6}{\int_0^\infty dt\,[D(t)/D(0)]},$$

- We obtain ($r_{\text{ch}} = 0.887(3)$ fm is the proton charge radius calculated using the same framework):

$$r_{\text{mass}} = 0.81(5) r_{\text{ch}}, \quad r_{\text{mech}} = 0.72(2) r_{\text{ch}},$$

- Species decompositions at $\zeta = 2$ GeV

$$r^q_{\text{mass}} = 0.62(4) r_{\text{ch}}, r^g_{\text{mass}} = 0.52(3) r_{\text{ch}}, r^q_{\text{mech}} = 0.55(2) r_{\text{ch}}, r^g_{\text{mech}} = 0.47(2) r_{\text{ch}}.$$

Summary and Outlook

➢Summary

✓Using a Continuum Schwinger Function Methods, presented a calculation of the Gravitational Form Factors (GFFs) of Pion, Kaon, and Nucleon.

✓Pion and kaon GFFs: graviton + quark vertex, $r_\pi^{\theta_1} > r_\pi^F > r_\pi^{\theta_2}$; Kaon density and pressure profiles are more compact than pion's. Pion confinement radius ≈ 0.39 fm. Kaon confinement radius is 33% smaller.

✓Proton GFFs: A universal ratio for nucleon's GFFs: glue/quark ≈ 0.71; Near-core pressure in pion is twice that in proton, both far exceed neutron star pressures. Mechanical radius < mass radius < charge radius.

✓Future high-luminosity, high-energy facilities (JLAB at 22 GeV, EIC, and EicC) are poised to measure these GFFs.

➢Outlook

Hadron structure, such as transverse momentum dependent distribution (TMD), generalized parton distribution (GPD), fragmentation function (FF), etc..

Thank you!