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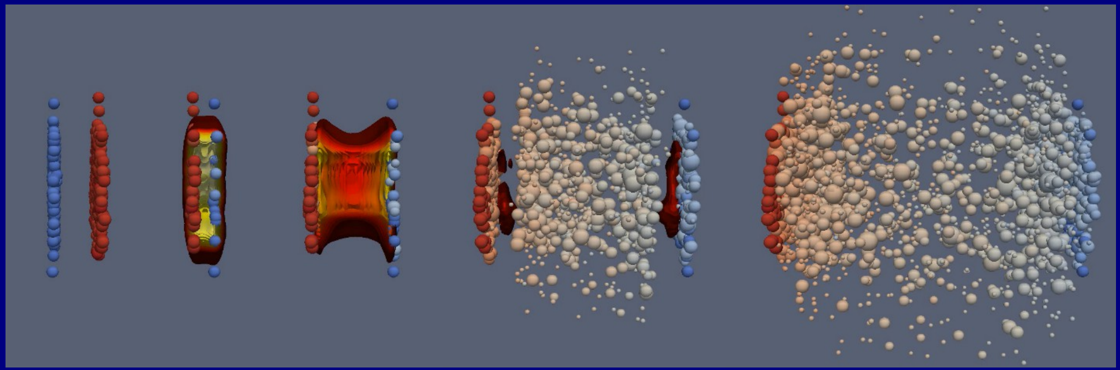
Spin physics in relativistic heavy ion collisions from a theoretical viewpoint

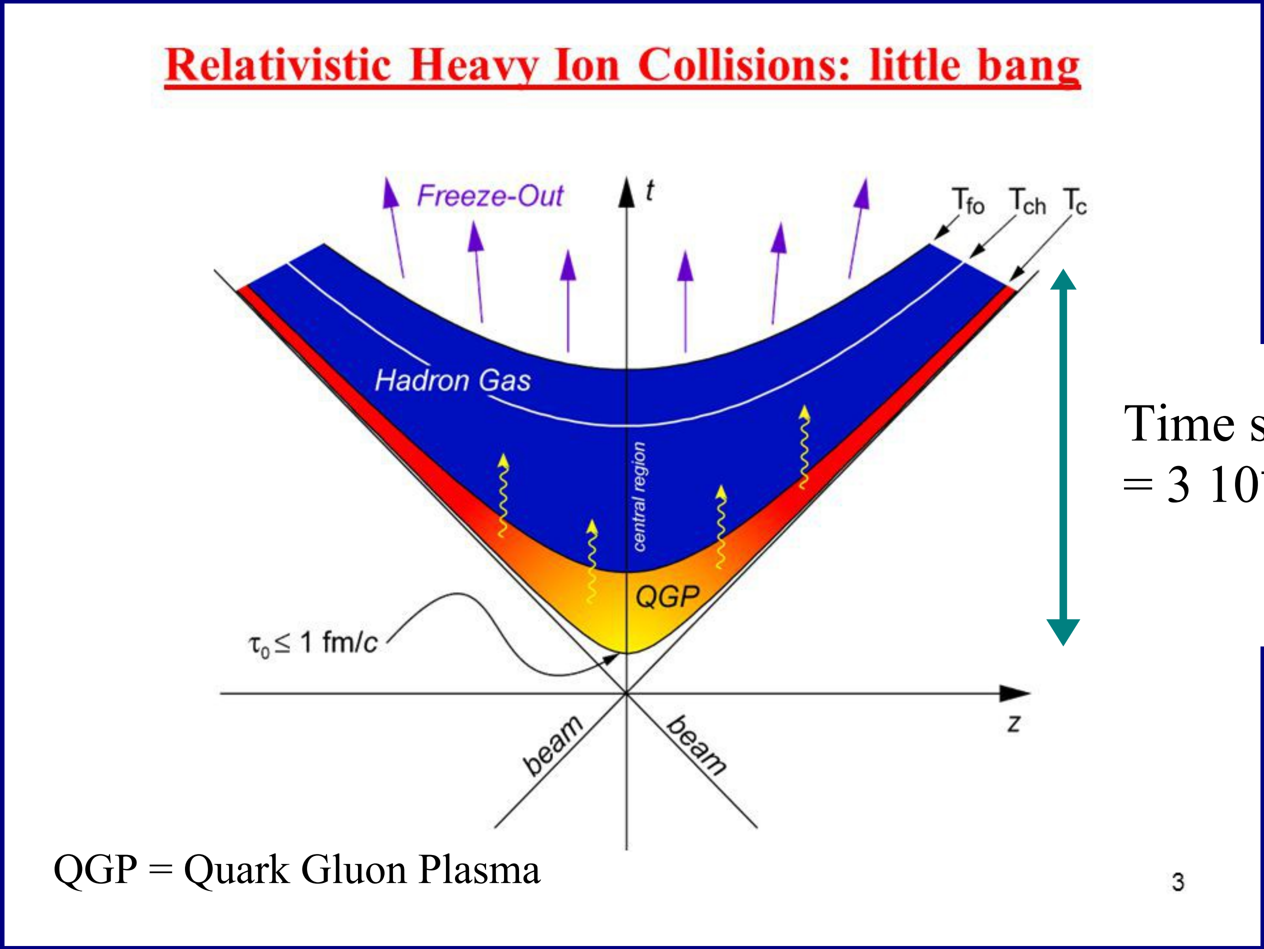
OUTLINE

- Introduction: relativistic heavy ion collisions and QGP
- Polarization and local thermodynamic equilibrium
- Spin polarization vector and spin alignment
- Spin as a probe of QGP in relativistic nuclear collisions
- Conclusions

SPIN 2025, Qingdao 22-26 September 2025

Relativistic nuclear collision





Time scale ~ 10 fm/c
= 3 10^{-23} sec

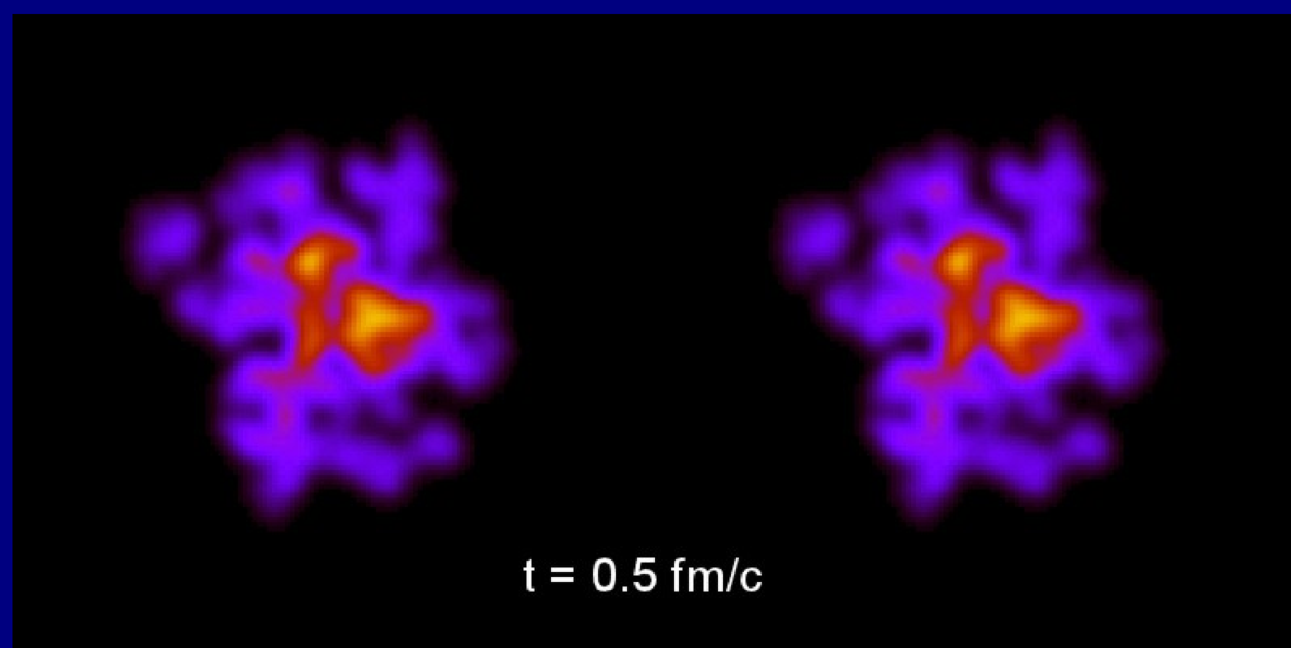
Quark Gluon Plasma and relativistic hydrodynamics

To study the dynamics of the QGP, we rely on relativistic hydrodynamics

Effective theory working under the key assumption of *local thermodynamic equilibrium*
Separation of scales: microscopic interaction length << lenght of variation of quantities describing thermodynamic equilibrium (T, u, μ_B,...)

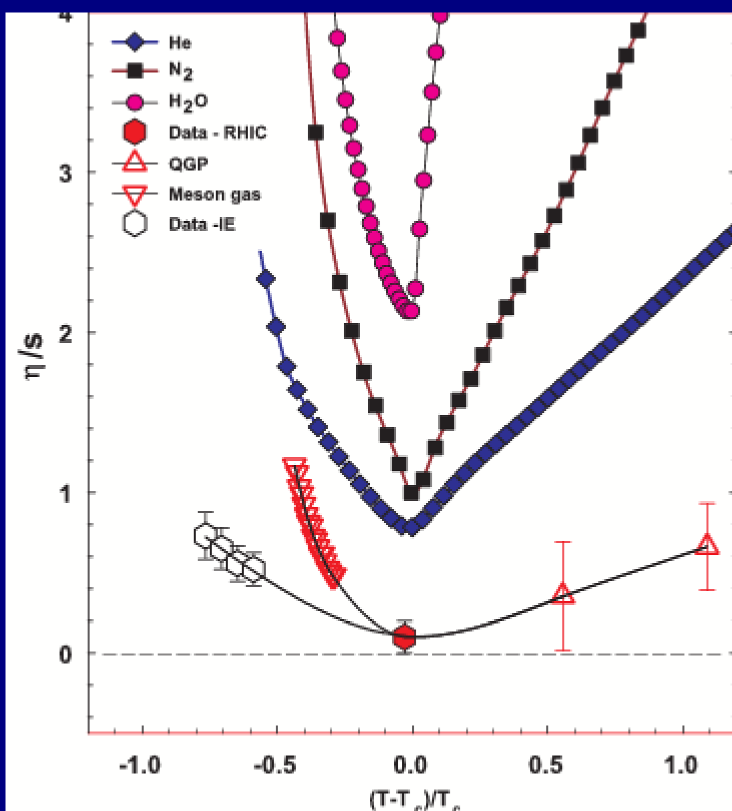
Formidable reduction of dynamical degrees of freedom:
interacting quantum fields $\longrightarrow$ few classical fields $T(x)$, $u(x)$, etc.

$$\partial_\mu \langle \widehat{T}^{\mu\nu} \rangle = 0$$
$$\partial_\mu \langle \widehat{j}^{\mu} \rangle = 0$$



QGP is an extraordinary fluid

- It is the hottest ever made: $T \sim 5\ 10^{12}$ K
- It is the tiniest ever made: ~10 fm across
- It has the largest initial pressure, energy density and largest initial acceleration ($a \sim 10^{30}\ g$) Surface gravity of a black hole ~ 3 10^{12} *g/(M/Ms)*
- It has the lowest viscosity/entropy density ratio ever observed



$$\eta/s \sim \lambda/\lambda_T$$

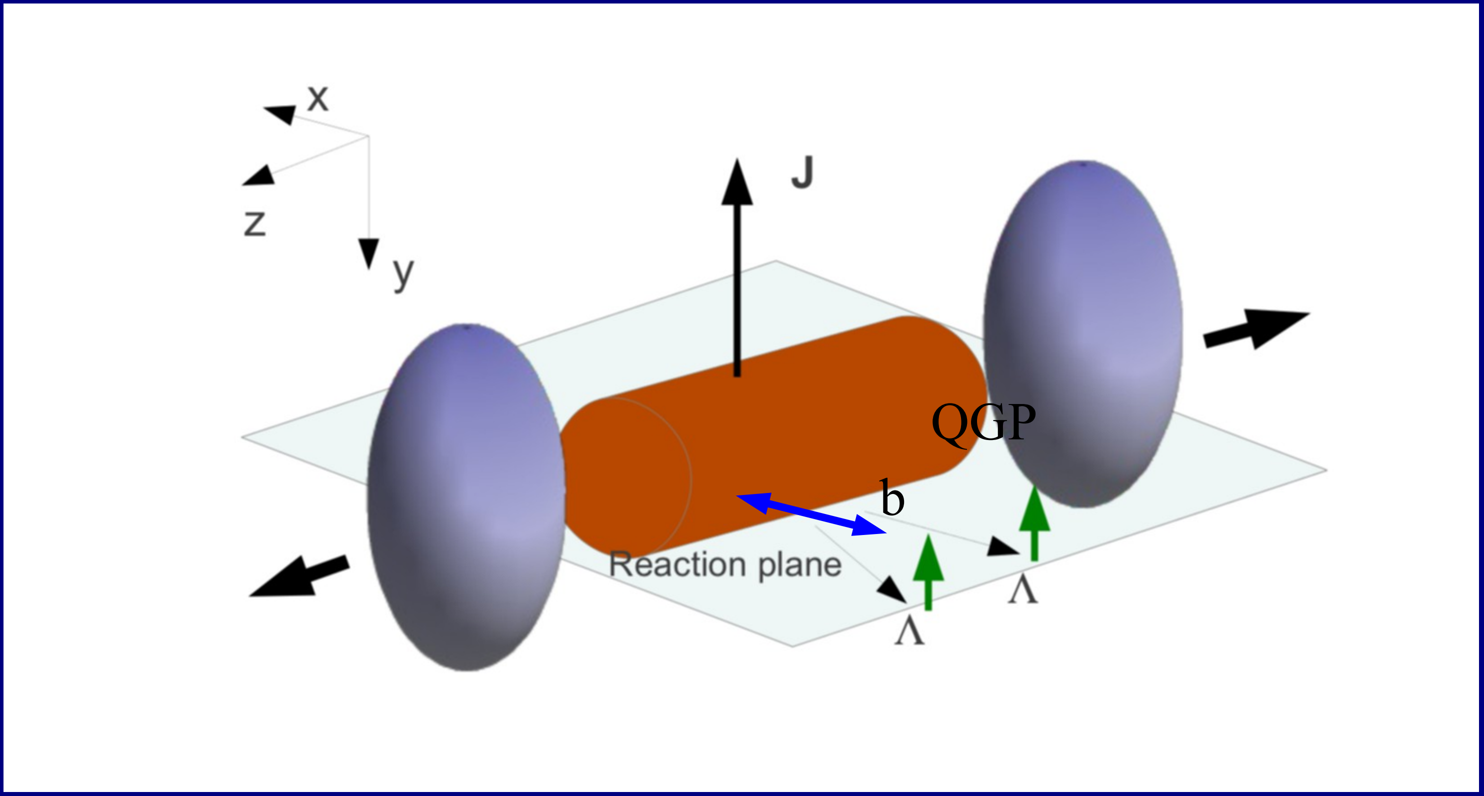
If the mean free path ~ mean wavelength
the fluid cannot be described as a kinetic system of colliding quasi-free particles, unlike most systems known in physics.

QGP cannot be described in terms of colliding particles or quasiparticles and yet local thermodynamic equilibrium can be defined, hence hydrodynamics

Global polarization in relativistic nuclear collisions

Peripheral collisions ➡ Angular momentum ➡ Global polarization w.r.t reaction plane

By parton spin-orbit coupling: Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301
By local equilibration (equipartition): F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906



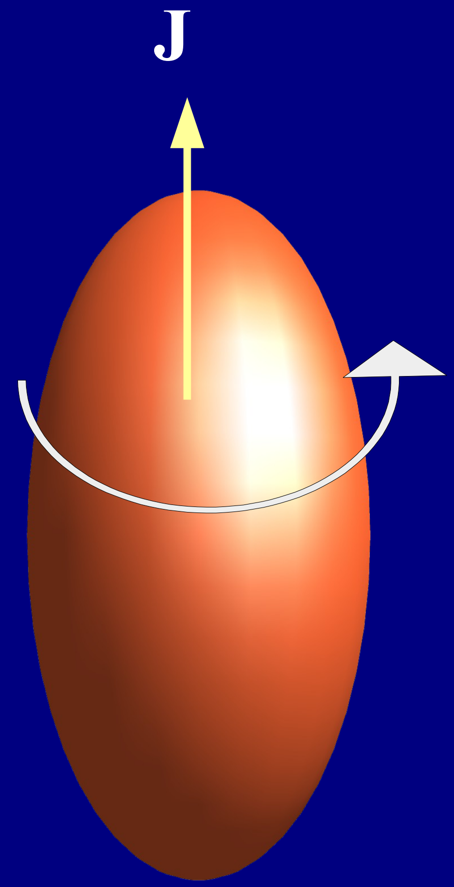
Polarization by rotation

A rigidly rotating non-relativistic Boltzmann gas in thermodynamic equilibrium

For a single non-relativistic particle:

$$\widehat{\rho} = \frac{1}{Z_\omega} \mathrm{e}^{-\widehat{H}/T + \boldsymbol{\omega}\cdot\widehat{\mathbf{J}}/T}$$

$$\widehat{\mathbf{J}} = \widehat{\mathbf{L}} + \widehat{\mathbf{S}}$$



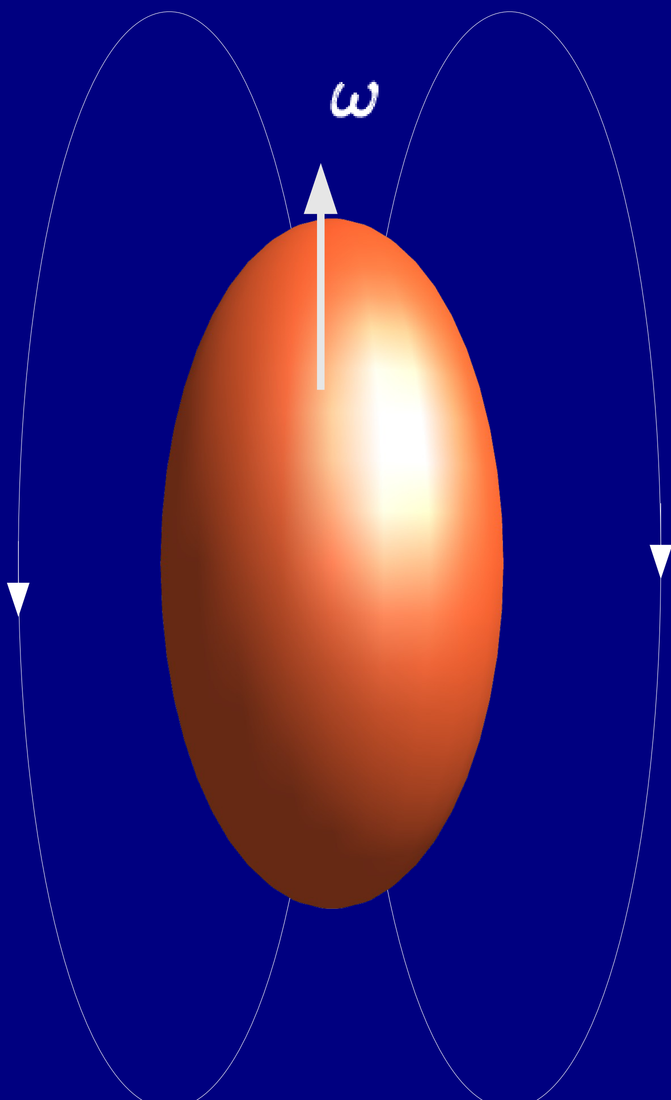
$$\Rightarrow \quad P \simeq \frac{S+1}{3} \frac{\hbar\omega}{KT}$$

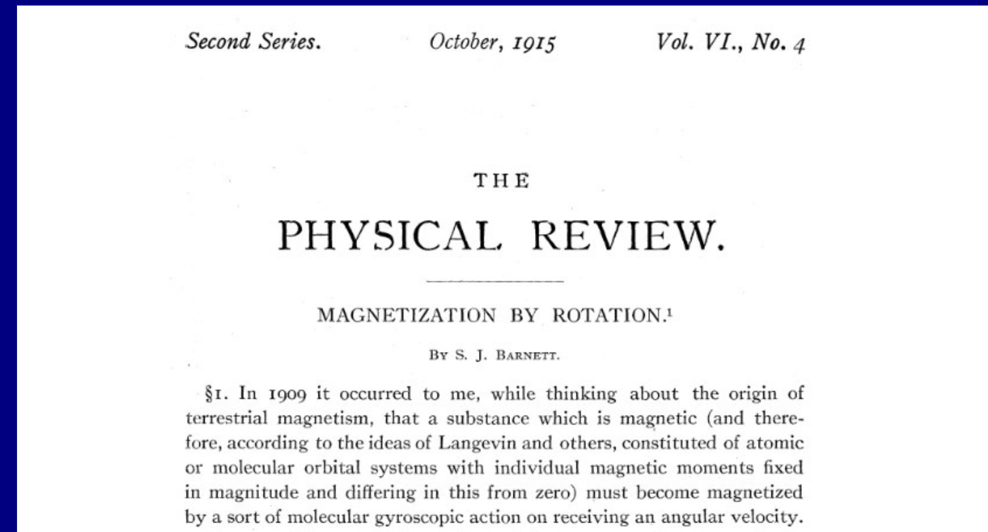
At room temperature, even for very large angular velocity, it is a tiny polarization

Barnett effect

S. J. Barnett, *Magnetization by Rotation,*

Phys. Rev. 6, 239–270 (1915).




Second Series. October, 1915 Vol. VI., No. 4

THE

PHYSICAL REVIEW.

MAGNETIZATION BY ROTATION.[1]

BY S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

Spontaneous magnetization of an uncharged body when spun around its axis

$$P \simeq \frac{S+1}{3}\frac{\hbar\omega}{KT} \Rightarrow M = \frac{\chi}{g}\omega$$

It can be seen as a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.

Polarization in a relativistic fluid: theoretical ingredients

1) *Local (not global!) thermodynamic equilibrium*

2) *Relativity*

3) *Spin* ⇒ *quantum mechanics*

4) *Dense, strongly interacting system (not a kinetic system)*

We have to find the appropriate tools extending

$$\widehat{\rho} = \frac{1}{Z_\omega} \mathrm{e}^{-\widehat{H}/T + \boldsymbol{\omega} \cdot \widehat{\mathbf{J}}/T}$$

to make theoretical predictions about spin polarization

Local equilibrium in QFT

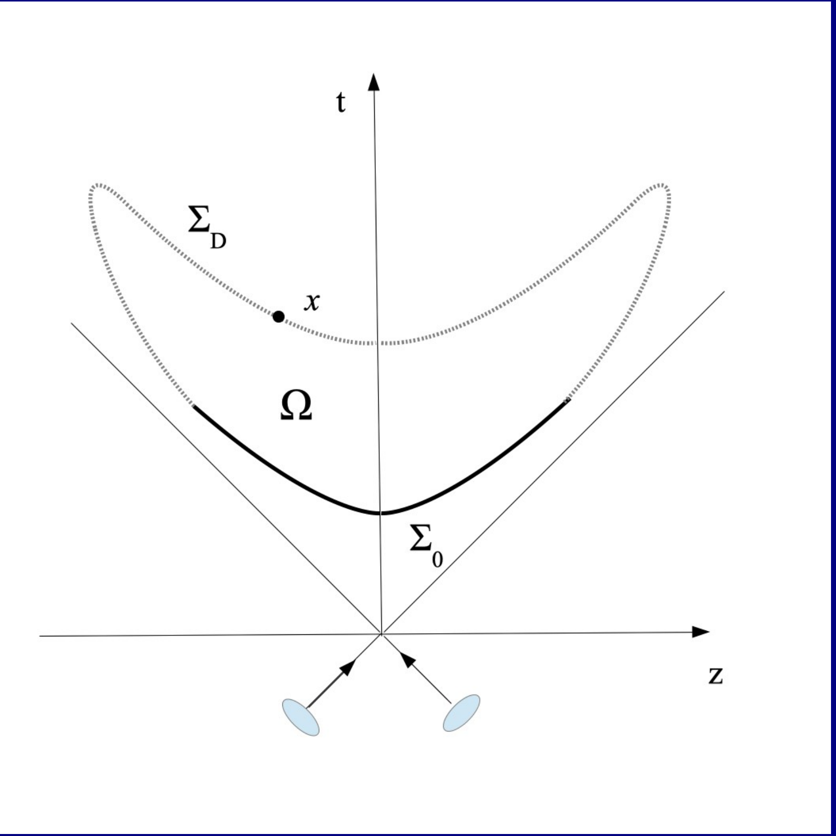
QGP around T_c cannot be described in terms of colliding particles or quasiparticles and yet local thermodynamic equilibrium can be defined

$$\widehat{\rho} = \widehat{\rho}_{\rm LE}(\tau_0) = \frac{1}{Z} \exp\left[-\int_{\Sigma_0} {\rm d}\Sigma_\mu(y) \left(\widehat{T}^{\mu\nu}(y)\beta_\nu(y) - \widehat{j}^\mu(y)\zeta(y) \right) \right] ,$$

$$\beta^\mu = \frac{1}{T} u^\mu \qquad \zeta = \frac{\mu}{T}$$

Extends global equilibrium to local equilibrium in a relativistic framework

Obtained by maximizing $S = -{\rm Tr}(\widehat{\rho} \log \widehat{\rho})$
constraining energy, momentum and charge *density*



By using Gauss theorem

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\int_{\Sigma_D} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Omega {\rm d}^4 y \left(\widehat{T}^{\mu\nu}(y)\partial_\mu \beta_\nu(y) - \partial_\mu \zeta(y) \widehat{j}^\mu(y) \right) \right]$$

local equilibrium *dissipation*

Spin and Wigner function

$$\Theta(p)_{\sigma\tau} = \frac{\mathrm{Tr}(\widehat{\rho}\,\widehat{a}_\tau^\dagger(p)\widehat{a}_\sigma(p))}{\sum_\sigma \mathrm{Tr}(\widehat{\rho}\,\widehat{a}_\sigma^\dagger(p)\widehat{a}_\sigma(p))},$$

Spin density matrix

Because of local thermodynamic equilibrium, it should be written in terms of local fields

Wigner operator and Wigner function (example, Dirac field)

$$\begin{aligned}\widehat{W}(x,k)_{AB} &= -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y} : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \\ &= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\, \mathrm{e}^{-ik\cdot y} : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\end{aligned}$$

$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

Particle, antiparticle and space-like decomposition:

$$W(x,k) = W(x,k)\theta(k^2)\theta(k^0) + W(x,k)\theta(k^2)\theta(-k^0) + W(x,k)\theta(-k^2) \equiv W(x,k)_+ + W(x,k)_- + W(x,k)_S$$

Dirac field

$$\Theta(p)_{\sigma\tau} = \frac{\int_\Sigma \mathrm{d}\Sigma_\mu p^\mu \bar{u}_\sigma(p) W_+(x,p) u_\tau(p)}{\sum_\sigma \int_\Sigma \mathrm{d}\Sigma_\mu p^\mu \bar{u}_\sigma(p) W_+(x,p) u_\tau(p)}$$

Spin polarization vector

$$S^\mu(p) = \frac{\int_{\Sigma_\mathrm{D}} \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W^+(x,p)\right]}{2\int_{\Sigma_\mathrm{D}} \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[W^+(x,p)\right]},$$

Spin polarization of fermions

Local equilibrium at the freeze-out implies a connection between spin polarization and (thermal) vorticity

$$\widehat{\rho} = \frac{1}{Z} \exp \left[- \int_{\Sigma_D} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) + \int_\Omega \mathrm{d}^4 y \left(\widehat{T}^{\mu\nu}(y) \partial_\mu \beta_\nu(y) - \partial_\mu \zeta(y) \widehat{j}^\mu(y) \right) \right]$$

local equilibrium

Dissipation: hopefully a small correction

free field approximation

$$S^\mu(p) = \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F (1 - n_F) \partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \; n_F}$$

$$n_F = (\mathrm{e}^{\beta \cdot p - \xi} + 1)^{-1}$$

$$\beta = \frac{1}{T} u$$

F.B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)

Leading term relating the spin polarization to thermo-hydrodynamic fields (up to 2021)

Contributions of vorticity, acceleration and Grad T

$$\partial_\mu \beta_\nu = \partial_\mu \left(\frac{1}{T} \right) + \frac{1}{T} \partial_\mu u_\nu$$

$$\begin{aligned} A^\mu &= u \cdot \partial u^\mu \\ \omega^\mu &= \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \partial_\nu u_\rho u_\sigma \end{aligned}$$

$$\begin{aligned} S^\mu(p) \int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \, n_F &= \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) \nabla_\nu (1/T) u_\rho \\ &+ \frac{1}{8m} \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) 2 \frac{\omega^\mu u \cdot p - u^\mu \omega \cdot p}{T} \\ &- \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \int_\Sigma \mathrm{d}\Sigma \cdot p \, n_F (1 - n_F) \frac{1}{T} A_\nu u_\rho \end{aligned}$$

Grad T

Vorticity

Acceleration

In the rest frame of the particle:

$$\mathbf{S}^* \propto \frac{\hbar}{KT^2} \mathbf{u} \times \nabla T + \frac{\hbar}{KT} (\boldsymbol{\omega} - \boldsymbol{\omega} \cdot \mathbf{v} \mathbf{u}/c^2) + \frac{\hbar}{KT} \mathbf{A} \times \mathbf{u}/c^2$$

Thermal term (new)

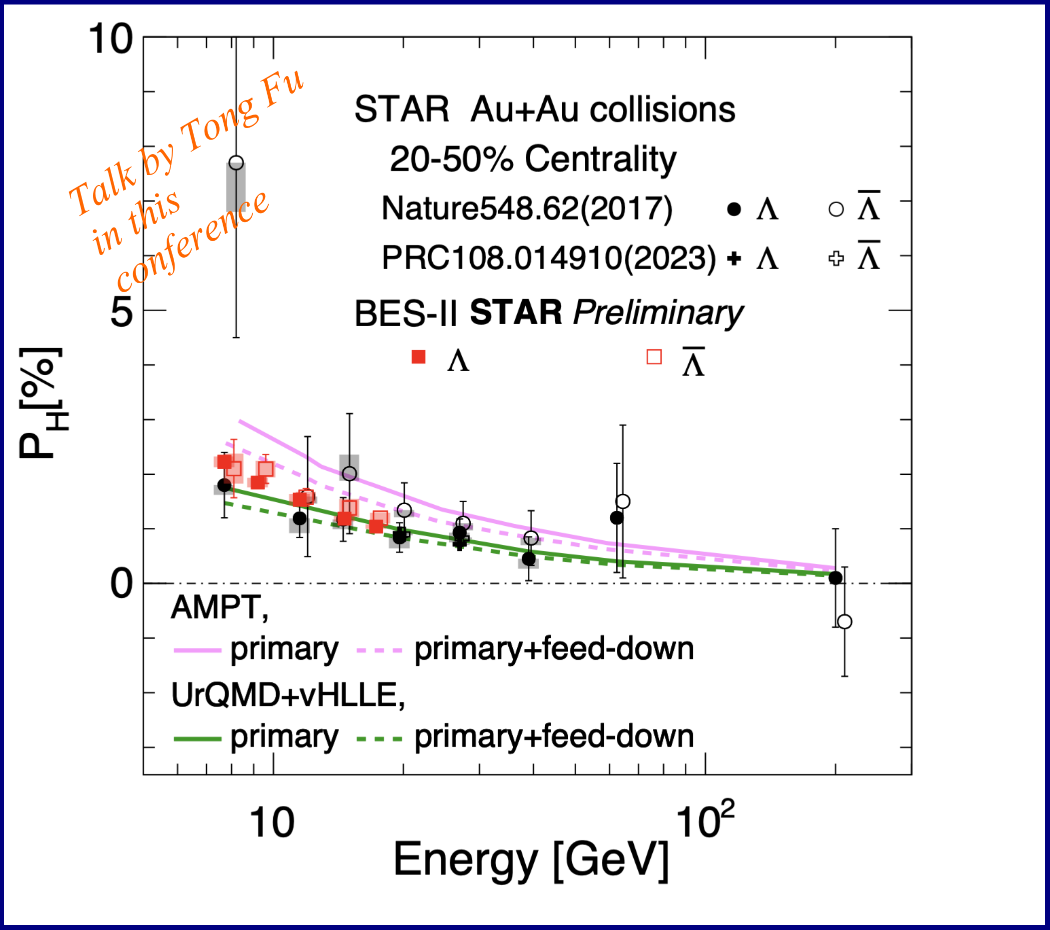
Vorticous term (known)

Acceleration term (purely relativistic)

$$\frac{\hbar \omega}{KT} \approx \frac{c}{12\mathrm{fm} 200\mathrm{MeV}} \approx 0.08 \qquad a \approx 10^{30} g \implies \frac{\hbar a}{cKT} \approx 0.06$$

This formula reproduces global polarization

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions,* Nature 548 62-65, 2017



The fluid with highest vorticity: $\sim 10^{21}$ Hz

Two important facts:

- Particle and antiparticle have the same polarization sign
- Different from an incoherent superposition of NN collisions

A new term: spin-thermal shear coupling

Spin polarization at local thermodynamic equilibrium, leading order:

< 2021

$$S^{\mu}_{\varpi}(p) = -\frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_{\sigma}\frac{\int_{\Sigma}\mathrm{d}\Sigma_{\tau}p^{\tau}n_F(1-n_F)\varpi_{\nu\rho}}{\int_{\Sigma}\mathrm{d}\Sigma_{\tau}p^{\tau}\ n_F}$$

$$\varpi_{\rho\lambda} = -\frac{1}{2}\left(\partial_{\rho}\beta_{\lambda} - \partial_{\lambda}\beta_{\rho}\right)$$

$$\xi_{\rho\lambda} = \frac{1}{2}\left(\partial_{\rho}\beta_{\lambda} + \partial_{\lambda}\beta_{\rho}\right)$$

Spin-thermal shear coupling or Shear-induced polarization, leading order:

> 2021

$$S^{\mu} = S^{\mu}_{\varpi} + S^{\mu}_{\xi}$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519

$$S^{\mu}_{\xi}(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_{\tau}p^{\rho}}{\varepsilon}\frac{\int_{\Sigma}\mathrm{d}\Sigma\cdot p\ n_F(1-n_F)\hat{t}_{\nu}\xi_{\sigma\rho}}{\int_{\Sigma}\mathrm{d}\Sigma\cdot p\ n_F},$$

t

$$S^{\mu}_{\xi} = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_{\tau}p^{\rho}}{p\cdot u}\frac{\int_{\Sigma}\mathrm{d}\Sigma\cdot p\ n_F(1-n_F)u_{\nu}\xi_{\sigma\rho}}{\int_{\Sigma}\mathrm{d}\Sigma\cdot p\ n_F}$$

u

S. Liu, Y. Yin, JHEP 07 (2021) 188

Why this difference?
Geometric assumptions

There is a new formula!
ArXiv:2509.14301
X.L. Sheng's talk in this conference

Confirmed in:
C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, arXiv 2109.15301, Sci. China Phys.Mech.Astron. 65 (2022) 7, 272011

Numerical calculation of polarization with hydrodynamic event simulation codes

Recent hydro calculations of Λ polarization in relativistic heavy ion collisions

S. Alzhrani, S. Ryu, and C. Shen, Phys. Rev. C **106**, 014905 (2022), arXiv:2203.15718 [nucl-th].
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. **127**, 272302 (2021), arXiv:2103.14621 [nucl-th].
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Phys. Rev. Lett. **127**, 142301 (2021), arXiv:2103.10403 [hep-ph].
X.-Y. Wu, C. Yi, G.-Y. Qin, and S. Pu, Phys. Rev. C **105**, 064909 (2022), arXiv:2204.02218 [hep-ph].
Z.-F. Jiang, X.-Y. Wu, H.-Q. Yu, S.-S. Cao, and B.-W. Zhang, Acta Phys. Sin. **72**, 072504 (2023).
Z.-F. Jiang, X.-Y. Wu, S. Cao, and B.-W. Zhang, Phys. Rev. C **108**, 064904 (2023), arXiv:2307.04257 [nucl-th].
V. H. Ribeiro, D. Dobrigkeit Chinellato, M. A. Lisa, W. Matioli Serenone, C. Shen, J. Takahashi, and G. Torrieri, Phys. Rev. C **109**, 014905 (2024), arXiv:2305.02428 [hep-ph].

B. Sahoo, C. R. Singh and R. Sahoo, Phys. Scripta 100 (2025), 065310 arXiv:2404.15138 [hep-ph]
Z. F. Jiang, S. Cao and B. W. Zhang,Phys. Rev. C 111 (2025) 034906 arXiv:2408.05774 [nucl-th]
C. Yi, X. Y. Wu, J. Zhu, S. Pu and G. Y. Qin, Phys. Rev. C 111 (2025) 044901 arXiv:2408.04296 [hep-ph]

Numerical implementation of 3+1 D causal viscous hydrodynamics with statistical hadronization and particle rescattering. Polarization transferred to Λ in secondary decays of Σ^0 and Σ^* taken into account

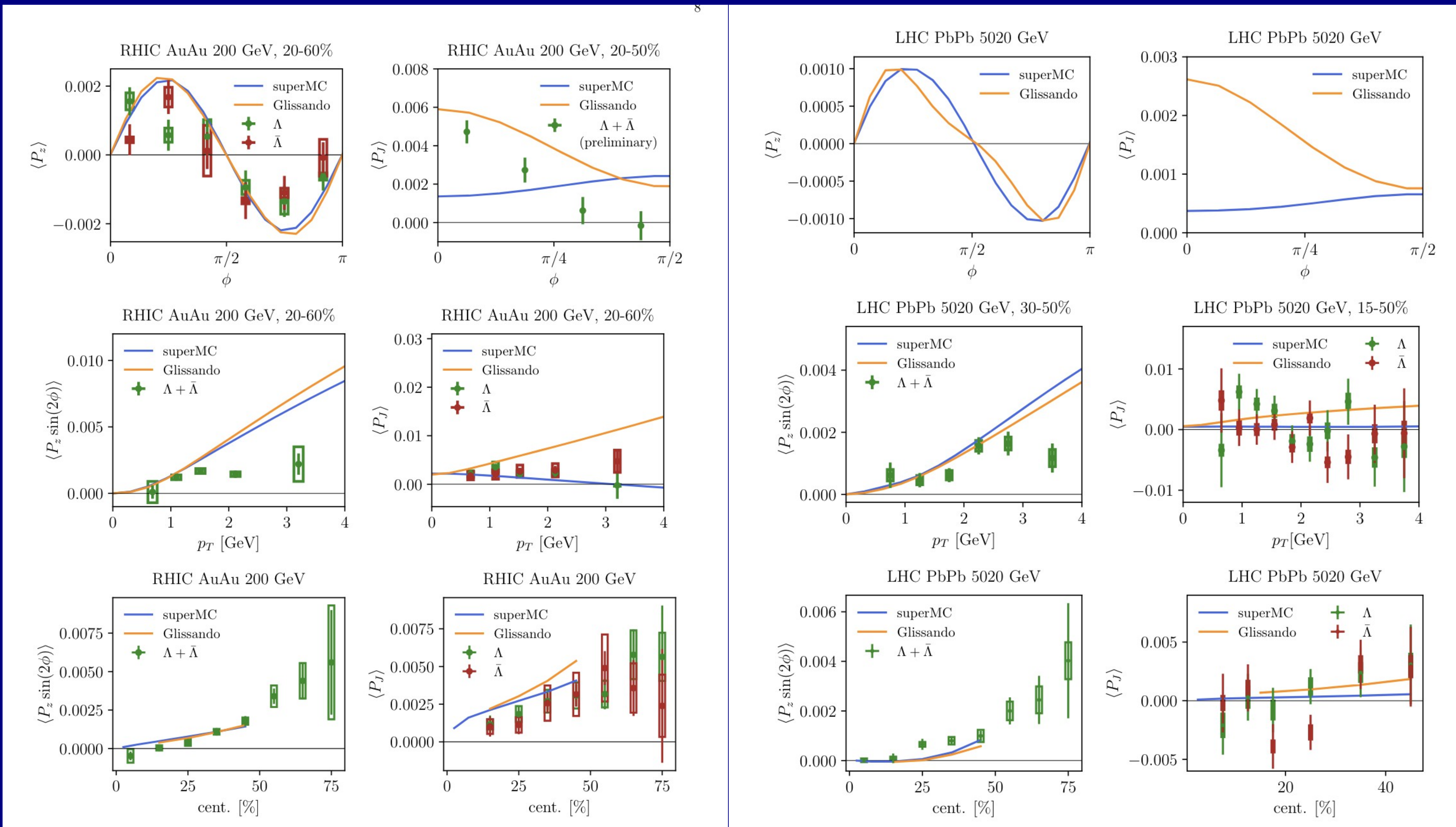
Talks by C. Yi and B. Fu this conference

Analytically solvable model:

A. Arslan, W. B. Dong, G. L. Ma, S. Pu and Q. Wang, Phys. Rev. C 111 (2025)

Talk by Q. Wang in this conference

The combination of these two contributions (at least the BBP version) is in good agreement with the data



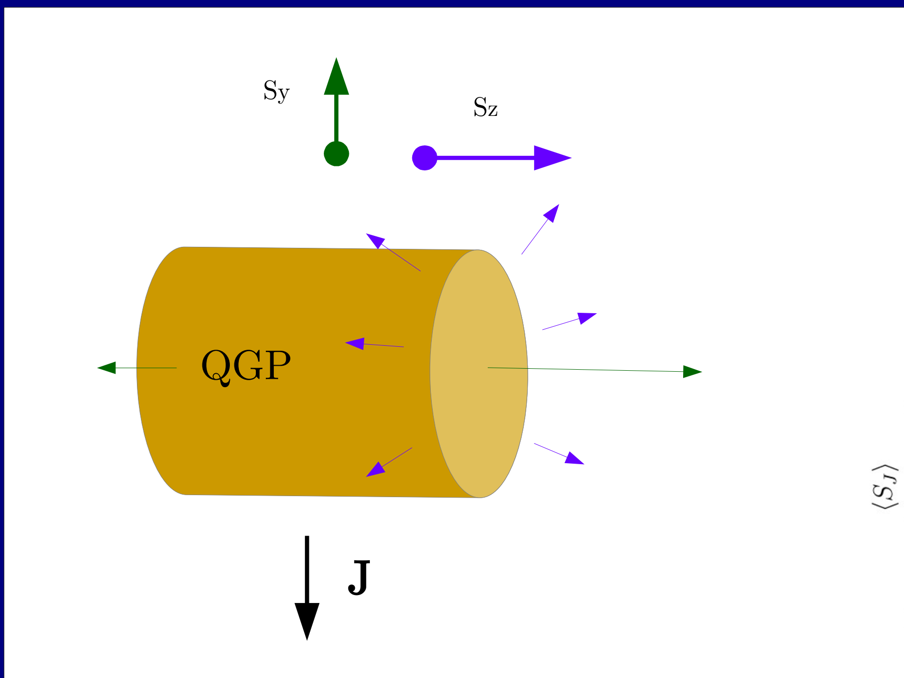
A. Palermo, F.B., E. Grossi, I. Karpenko, Eur. Phys. J. 84 (2024) 9, 920

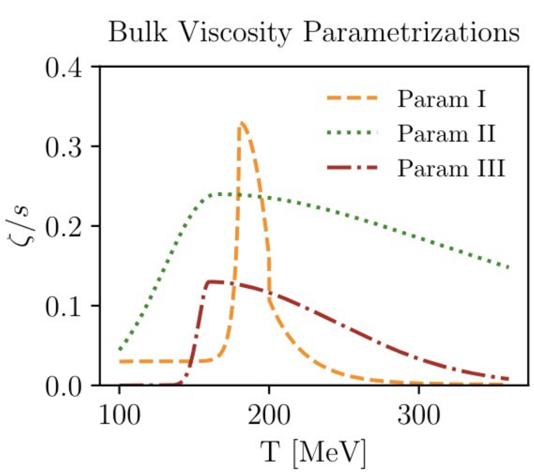
What can polarization tell us about QGP?

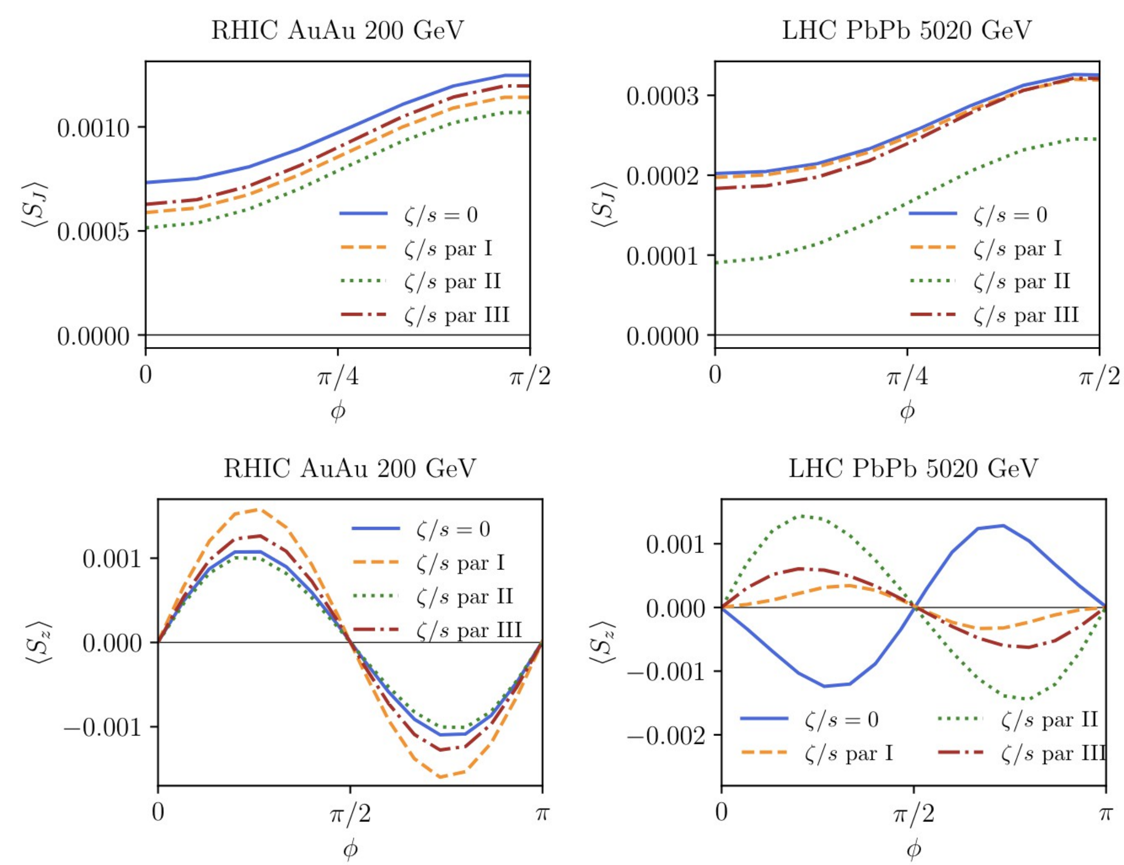
Spin polarization, unlike any other observable, at the leading order depends on hydrodynamic GRADIENTS, therefore it is a very sensitive probe of hydrodynamic motion

Longitudinal polarization seems to be very sensitive to the bulk viscosity of the QGP at the highest energy

Talk by P. Das in this conference with new ALICE data

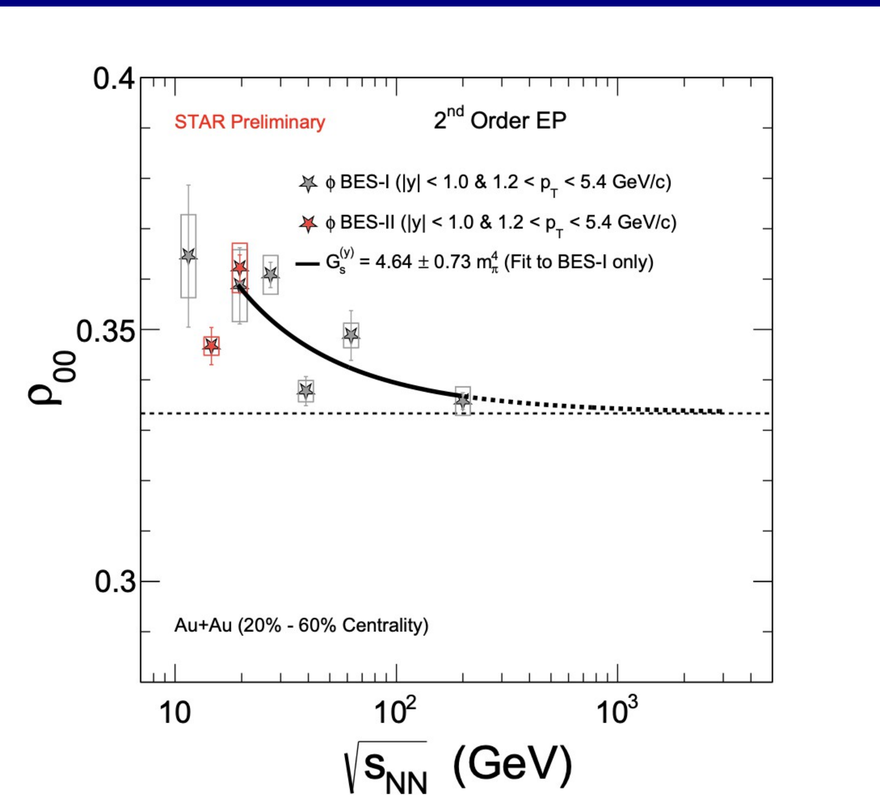






Vector meson spin alignment

$$\phi \longrightarrow K^+ \, K^-$$



G. Wilks (STAR coll.) SQM 2024

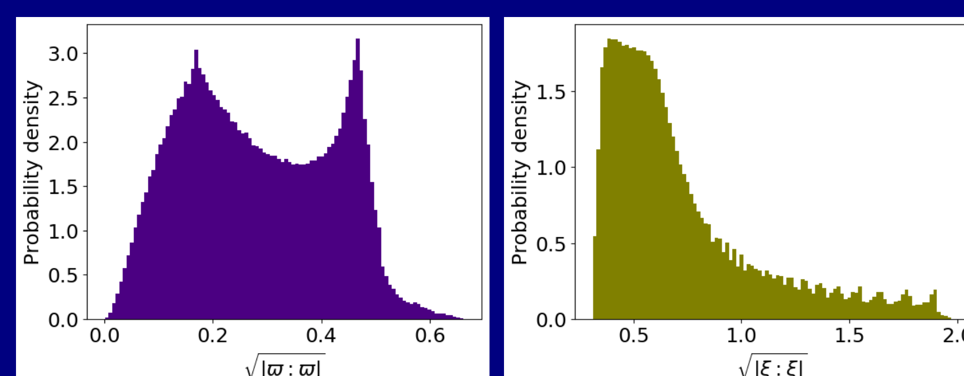
Spin density matrix:

$$\Theta(\boldsymbol{k}) = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{i=1}^{3} P^i(\boldsymbol{k}) S^i + \frac{1}{\sqrt{6}} \sum_{i,j=1}^{3} \mathfrak{T}^{ij}(\boldsymbol{k})(S^i S^j + S^j S^i),$$

Tensor component

Spin alignment much larger than expected from LTE calculations which reproduce well L polarization

Leading order in gradients at LTE = 0; quadratic contribution at LTE too small



Distribution of the magnitude of the gradients

What drives the large alignment ?

- *Dissipative effect*

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_D} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^{\mu}\right) + \int_{\Omega} \mathrm{d}^4 y \left(\widehat{T}^{\mu\nu}(y)\partial_\mu\beta_\nu(y) - \partial_\mu\zeta(y)\widehat{j}^{\mu}(y)\right)\right]$$

local equilibrium — *dissipation*

S. Y. F. Liu *et al.*, arXiv: 2206.11890
2501.17861, 2503.13408 ?

D. Wagner, N. Weickgennant, E. Speranza,
Phys.Rev.Res. 5 (2023) 1, 013187 [calculated within relativistic kinetic theory]

W. B. Dong, Y. L. Yin, X. L. Sheng, S. Z. Yang and Q. Wang,
Phys. Rev. D 109 (2024) 056025

Y. L. Yin, W. B. Dong, C. Yi and Q. Wang,
arXiv:2503.09937 [for the ρ meson]

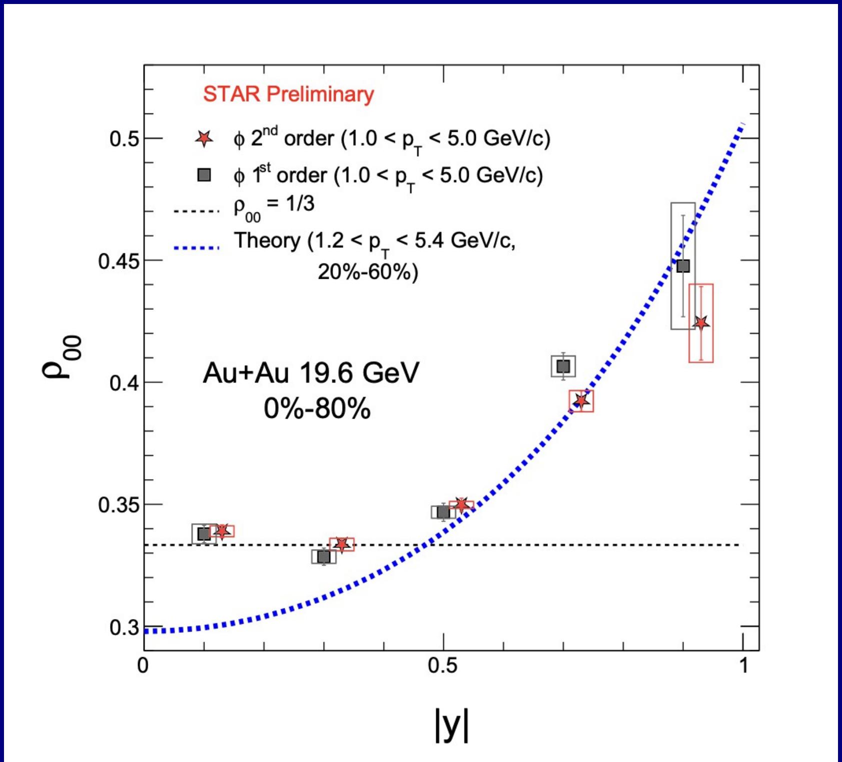
- *Interaction (so far neglected) effects*

X.L.Sheng, L. Oliva and Q. Wang,
Phys. Rev. D 101 (2020) 096005

X. L. Sheng, L. Oliva, Z. T. Liang, Q. Wang and X.N. Wang,
Phys. Rev. Lett. 131 (2023) no.4, 042304

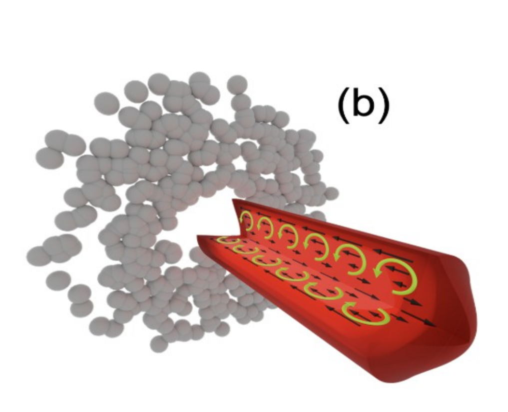
D. L. Yang, Phys. Rev. D 111 (2025), 056005 and refs. therein
[interaction and coalescence]

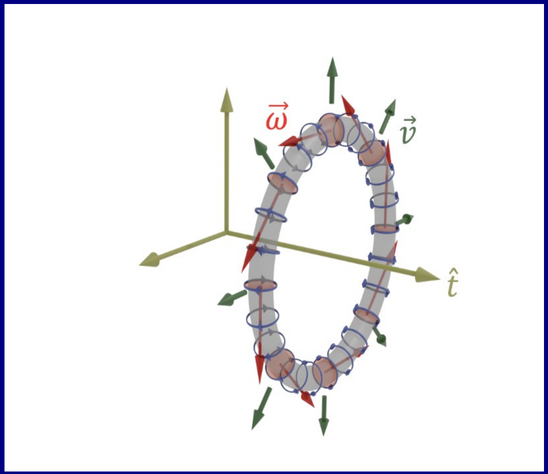
Talks by D.L. Yang and D. Hou in this conference



G. Wilks (STAR coll.) SQM 2024
Theory line from:
X. L. Sheng, S. Pu and Q. Wang, Phys. Rev. C 108 (2023) 054902

What else polarization can tell about QGP?





$$\mathcal{R}_{\Lambda}^{\hat{t}} \equiv \frac{\epsilon^{\mu\nu\rho\sigma} S_\mu n_\nu \hat{t}_\rho p_\sigma}{|S||\epsilon^{\mu\nu\rho\sigma} n_\nu \hat{t}_\rho p_\sigma|} \quad .$$

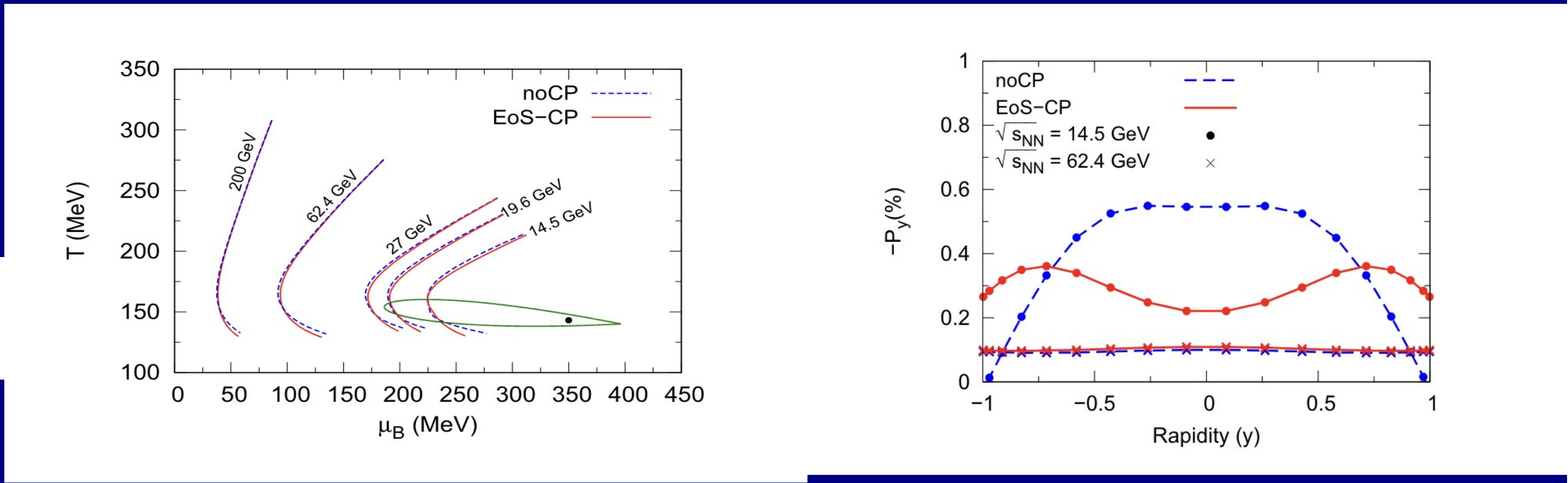
Shooting a proton or a jet through a heavy nucleus is expected to produce vortex rings, which can possibly be detected through spin polarization

V. H. Ribeiro et al., Phys.Rev.C 109 (2024) 1, 014905; M. Lisa et al., Phys.Rev.C 104 (2021) 1, 011901

Polarization as a probe of the QCD critical point

Critical behaviour of viscous coefficients

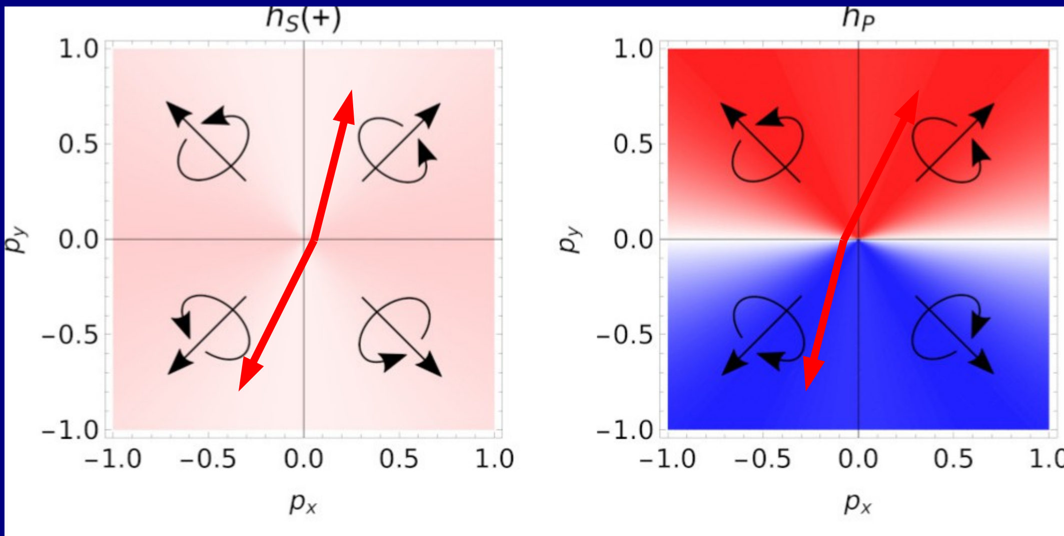
$$\zeta = \zeta_0 \left(\frac{\xi}{\xi_0}\right)^3 \;,\; \eta = \eta_0 \left(\frac{\xi}{\xi_0}\right)^{0.05}$$

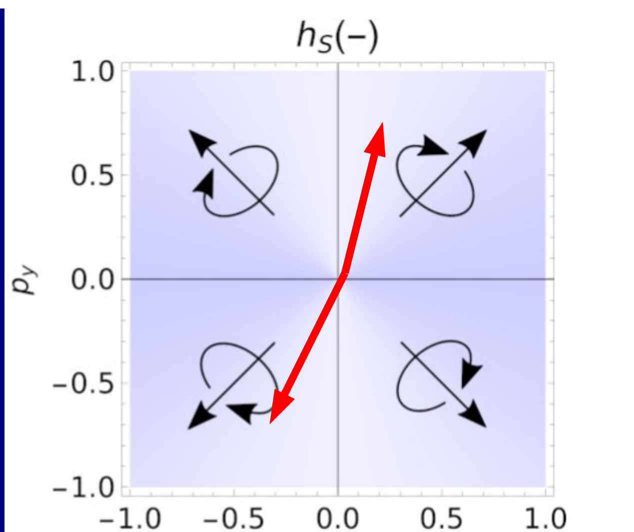


S. K. Singh, Jan e-Alam, Eur. Phys. J. C 83 (2023) 585

Helicity correlations: a probe of local parity violation

F. B., M. Buzzegoli, A. Palermo and G. Prokhorov, Phys. Lett. B 822 (2021) 136706

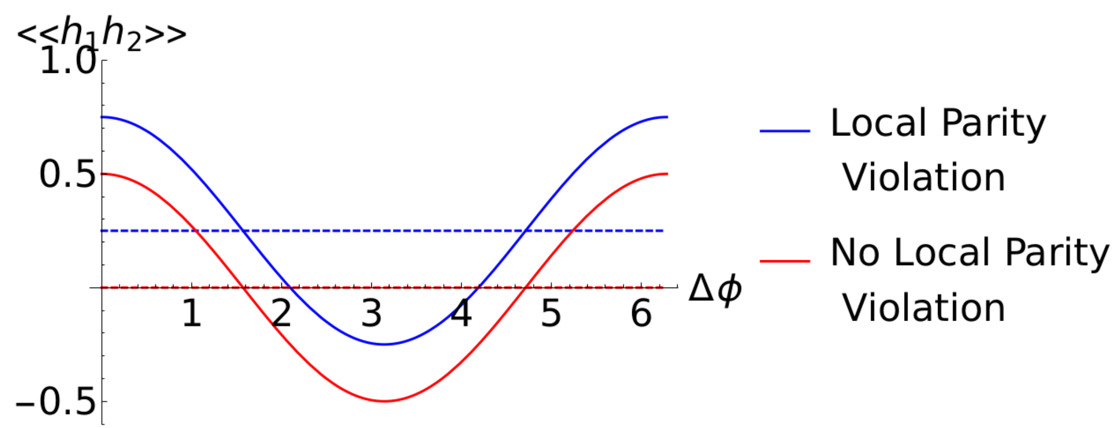




Studying the helicity of hyperons with large angle difference in the same event can reveal the local parity violation

Helicity-helicity azimuthal 2P correlation

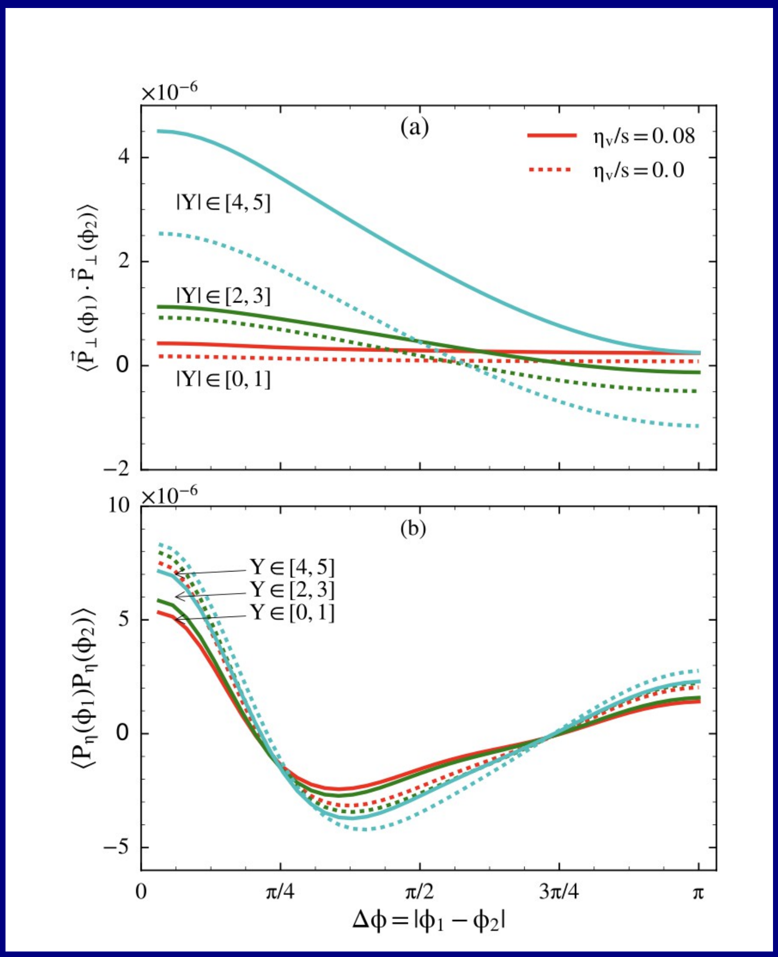
$$\langle h_1 h_2(\Delta\phi)\rangle \simeq \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \left(\bar{S}_0^2 + \bar{P}_0^2 \sin^2\phi \cos\Delta\phi\right) = \bar{S}_0^2 + \frac{1}{2}\bar{P}_0^2 \cos\Delta\phi$$



Spin-spin correlations

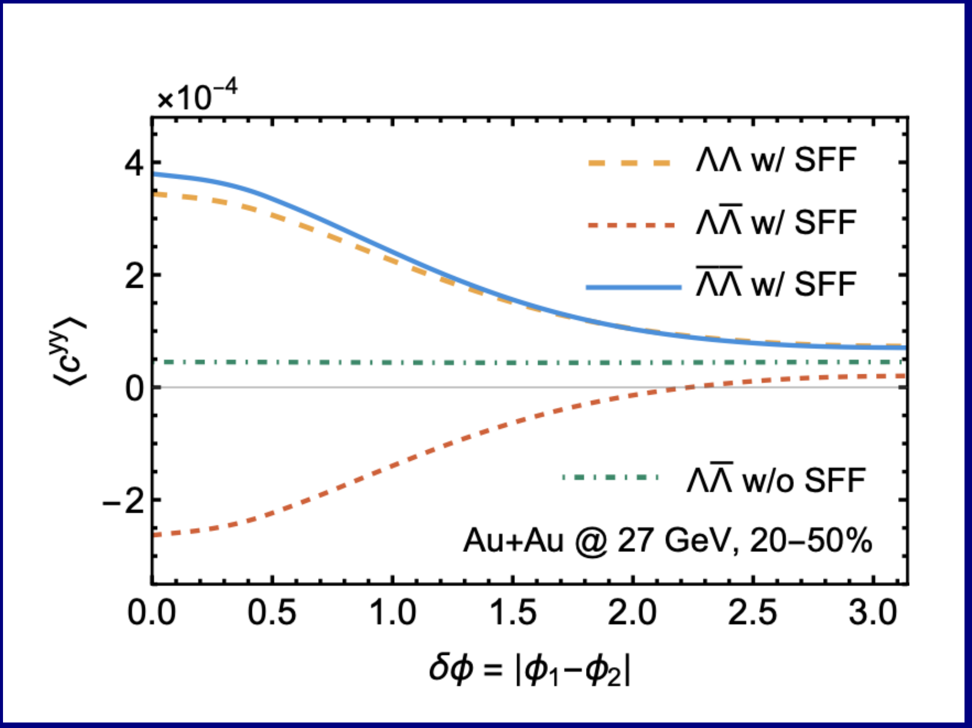
$$\int d^3p\, S^\mu(p) S^\nu(p+\Delta p) - \int d^3p\, S^\mu(p) \int d^3p\, S^\nu(p)$$

Spin polarization of Λ pairs is correlated if they are emitted from regions with similar vorticity-shear field



L. G. Pang, H. Petersen, Q. Wang and X. N. Wang, Phys. Rev. Lett. 117 (2016) 192301

Correlation is predicted to be enhanced by the same interaction effects driving the large ϕ alignment



X. L. Sheng, X. Y. Wu, D. H. Rischke and X. N. Wang,[arXiv:2508.03496 [hep-ph]].

Talk by X. Gou in this conference

Spin-spin correlations at constituent quark level

Talks by J.P. Lv, Shi Pu, K. Xu and Z. Yu in this conference

- Vector polarization

$$S_L = \boxed{\frac{5}{2}\bar{P}_q} + \frac{3}{\bar{C}_3}\left[\bar{P}_q(\bar{c}_{zz}^{(qq)} - 2\bar{c}_{ii}^{(qq)} - 2\bar{P}_q^2) + \frac{1}{2}\bar{c}_{zii}^{(qqq)}\right]$$

- Rank-2 tensor polarization

$$S_{LL} = \frac{3}{\bar{C}_3}\left(\boxed{3\bar{c}_{zz}^{(qq)} - \bar{c}_{ii}^{(qq)}} + 2\bar{P}_q^2\right)$$

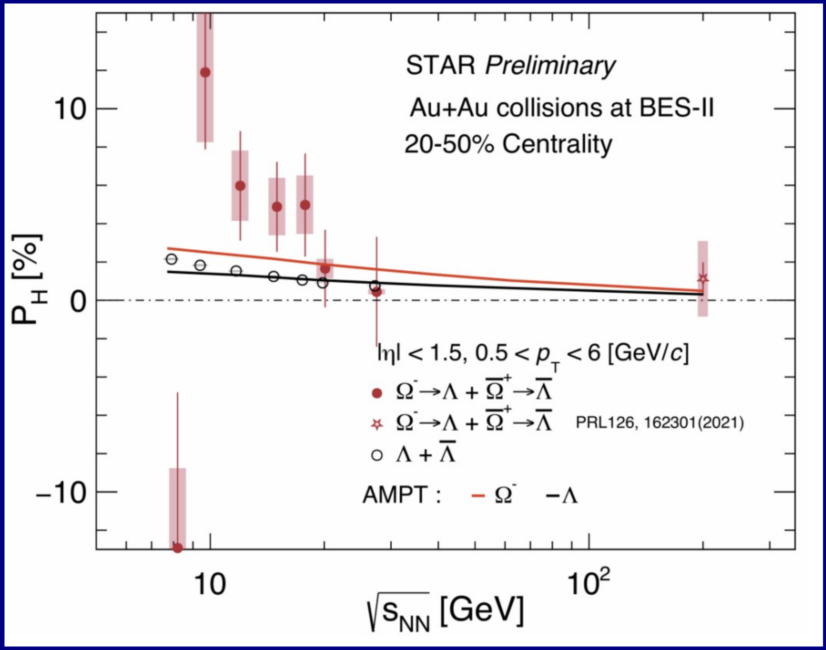
- Rank-3 tensor polarization

$$S_{LLL} = \frac{9}{10\bar{C}_3}\left[\boxed{5\bar{c}_{zzz}^{(qqq)} - 3\bar{c}_{zii}^{(qqq)}} + 3\bar{P}_q(3\bar{c}_{zz}^{(qq)} - \bar{c}_{ii}^{(qq)}) + 2\bar{P}_q^3\right]$$

Hadron	Measurables	sensitive quantities
spin-1/2 (hyperon H)	Hyperon polarization P_H	**average** quark polarization $\langle P_q \rangle$
	Hyperon spin correlation $c_{H_1H_2}, c_{H_1\bar{H}_2}$	**long range** spin correlations $c_{qq}, c_{q\bar{q}}$
spin-1 (Vector mesons)	Spin alignment ρ_{00}	**local** spin correlations $c_{q\bar{q}}$
	off-diagnal elements $\rho_{m'm}$	**local** spin correlations $c_{q\bar{q}}$
spin-3/2 $J^P = \left(\frac{3}{2}\right)^+$ baryons	Hyperon polarization S_L	**average** quark polarization $\langle P_q \rangle$
	Rank 2 tensor polarization S_{LL}	**local** spin correlations c_{qq}
	Rank 3 tensor polarization S_{LLL}	**local** spin correlations c_{qqq}

Z.Yu, SPIN 2025 conference and
Z. Zhang, J.P. Lv, Z. Yu and Z.T. Liang,
Phys. Rev. D 110 (2024) 074019

Can we distinguish between local equilibrium of hadronic fields and quark coalescence by using polarization measurements?



T. Fu, SPIN 2025 conference

Relativistic spin hydrodynamics

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

Pseudo-gauge transformations of stress-energy and spin tensor

F. Halbwachs, *Theorie relativiste des fluids a spin*, Gauthier-Villars (1960)
F. W. Hehl, Rept. Math. Phys. 9 (1976) 55 (see also F. B., L. Tinti Phys. Rev. D 84 (2011) 025013)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2} \partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

Φ = superpotential

Talks by X. G. Huang and D. L. Wang in this conference

Summary and outlook

- Spin polarization is a new powerful probe of Quark Gluon Plas: in the hydrodynamic paradigm, it is a probe of the *gradients* in the fluid

- Local equilibrium+hydrodynamic model reproduces the measured Λ polarization

- Vector mesons spin alignment larger than expected: a dissipative correction to local equilibrium, an effect of interactions or a hint of other mechanisms?

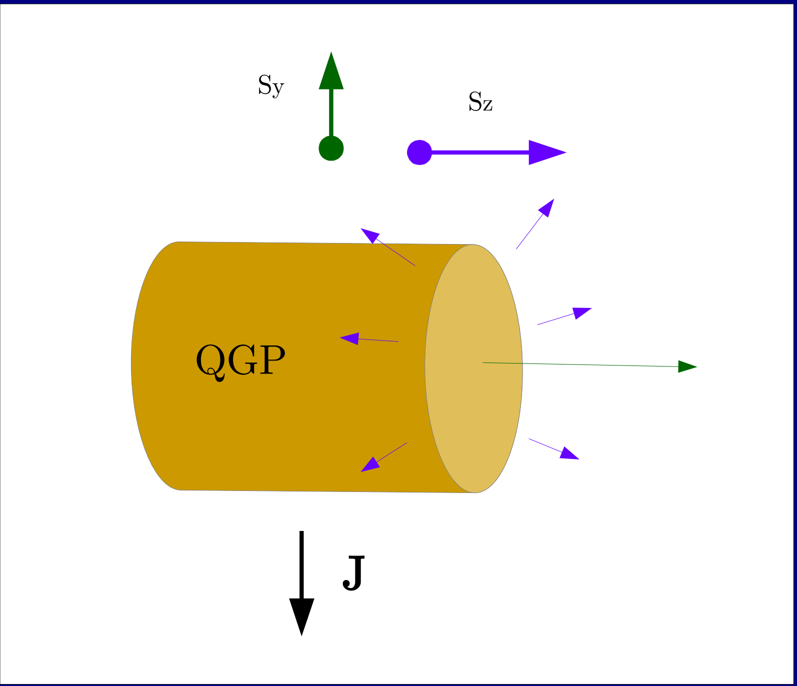
- Spin correlations: a new important observable

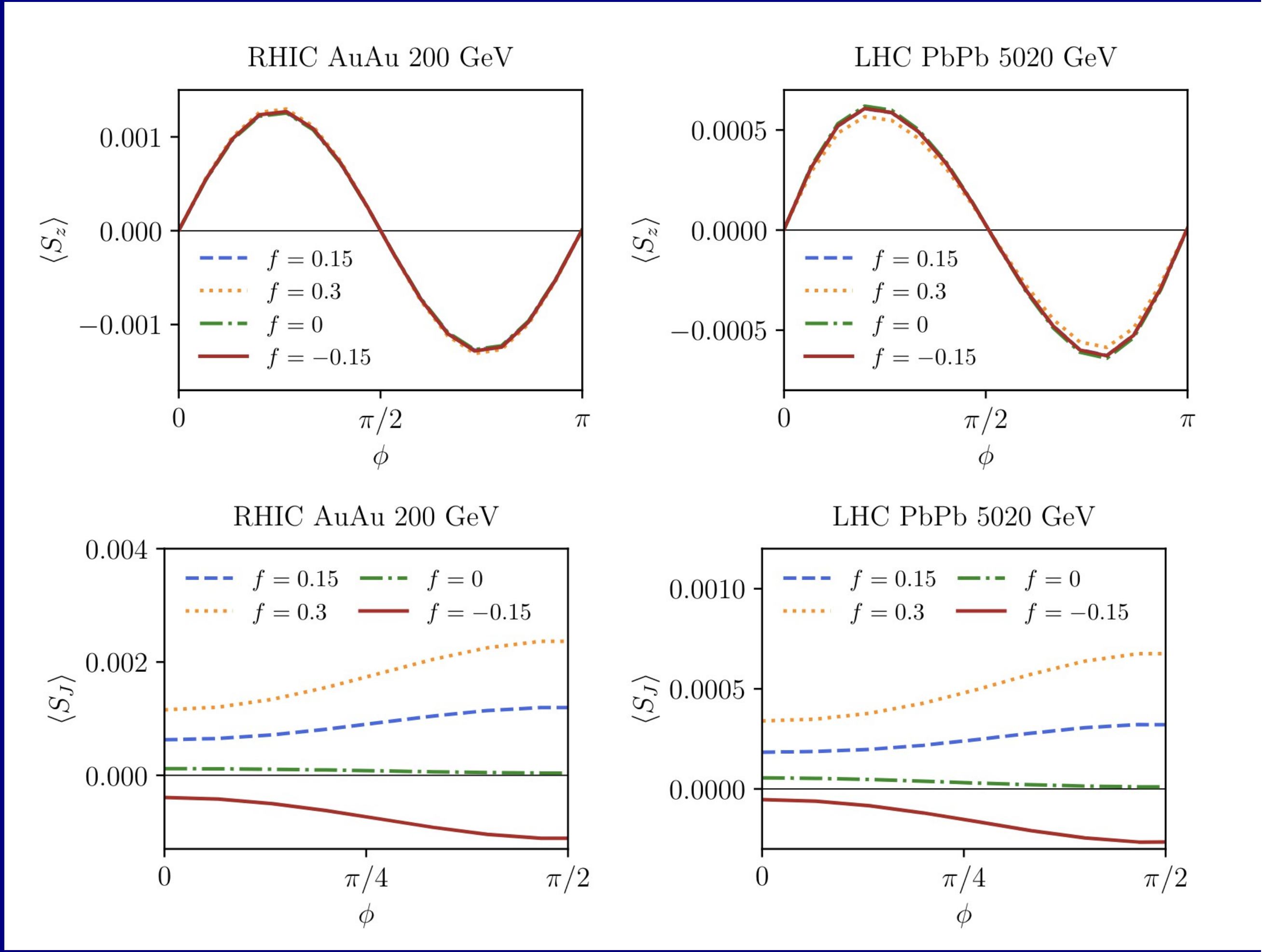
- Full potential of this observable is yet to be explored

Sensitivity to initial longitudinal flow

Variation of SUPERMC flow parameter

$$T^{\tau\tau} = \rho \cosh(f\, y_{CM})$$
$$T^{\tau\eta} = \frac{\rho}{\tau} \sinh(f\, y_{CM})$$



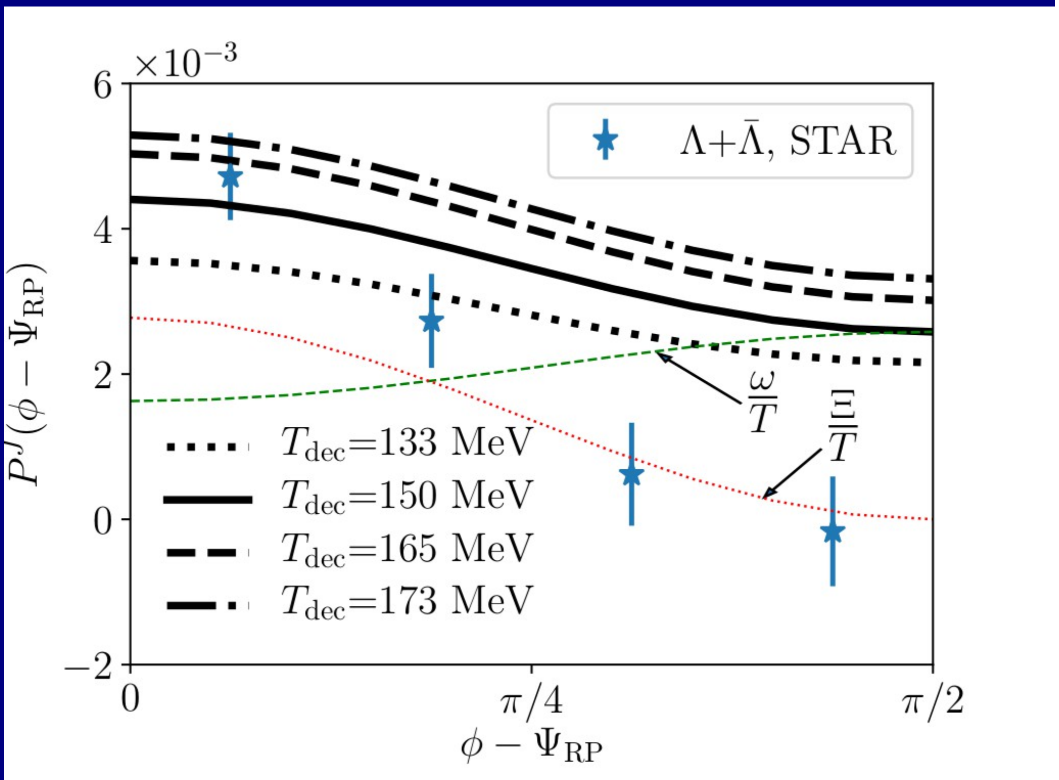


New term found: spin-thermal shear coupling

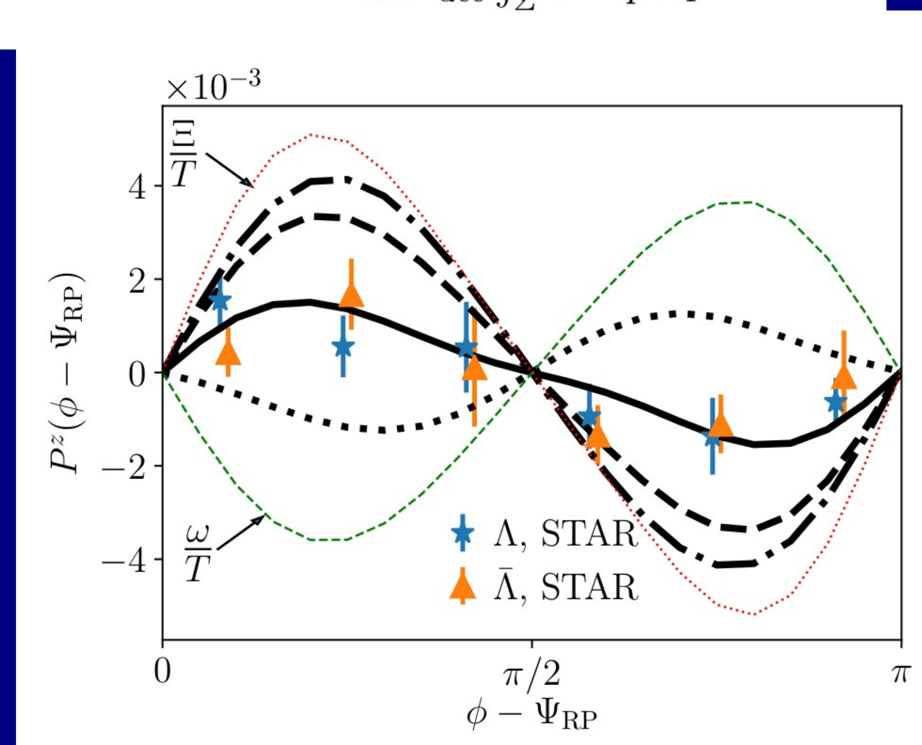
$$S^{\mu}_{\xi}(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F},$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right).$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519
S. Liu, Y. Yin, JHEP 07 (2021) 188
Confirmed by C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011



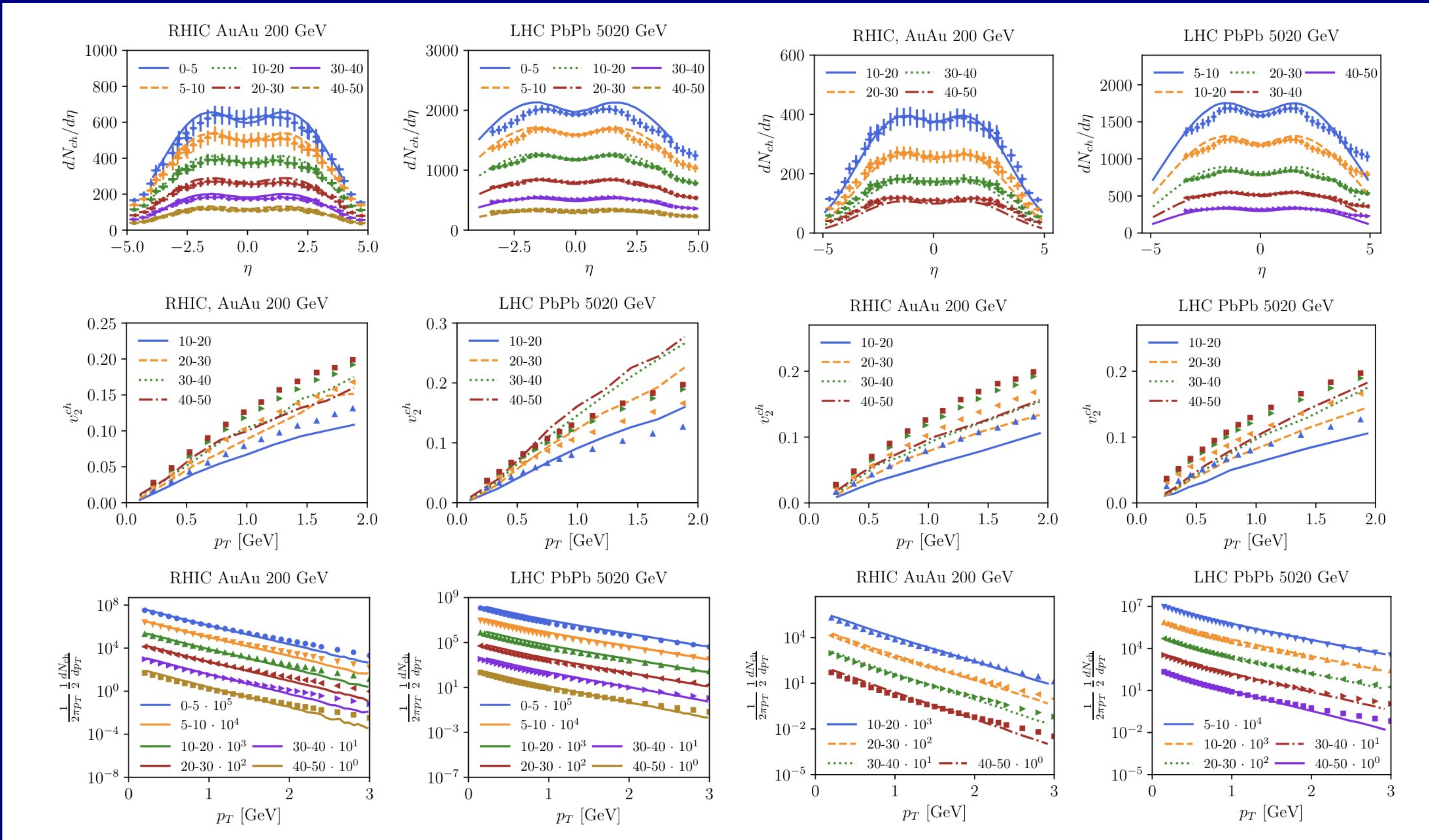
$$S^{\mu}_{\mathrm{ILE}}(p) = \qquad ($$
$$- \epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\left[\omega_{\rho\sigma} + 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon}\Xi_{\lambda\sigma}\right]}{8mT_{\mathrm{dec}}\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$



F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko,
Phys. Rev. Lett. 127 (2021) 272302

Qualification of the code

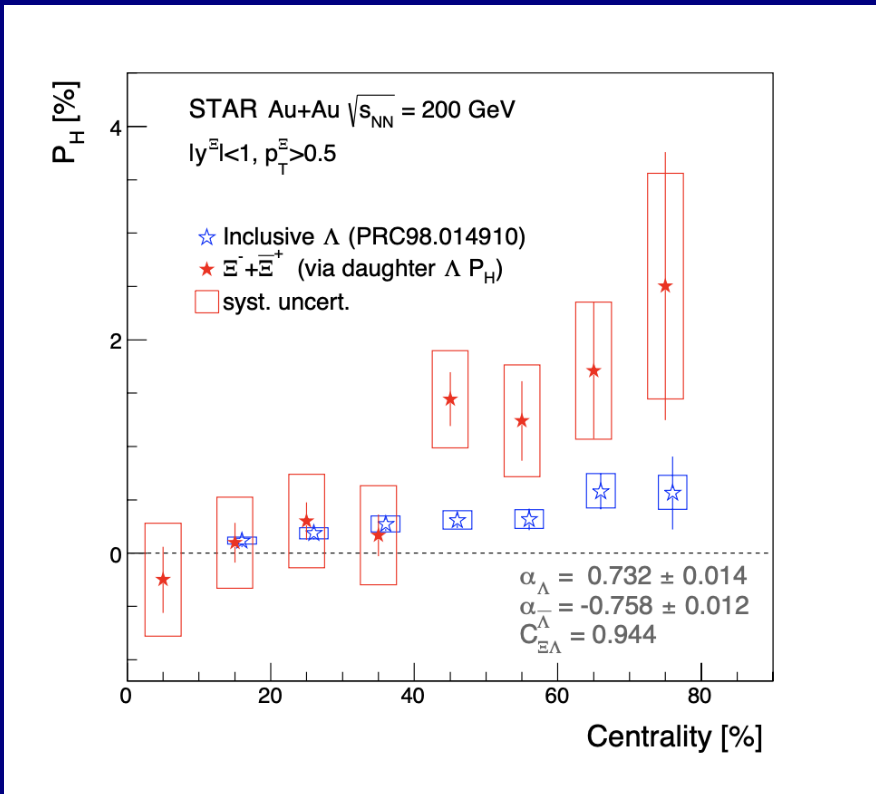
Benchmark distributions

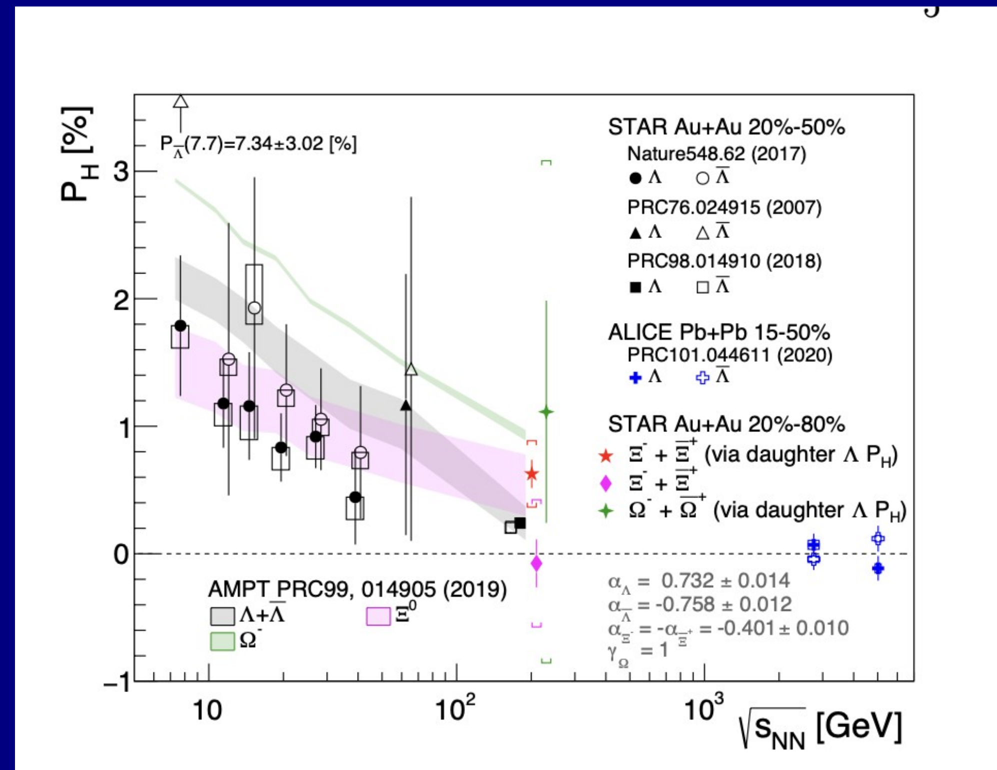


SUPERMC GLISSANDO

Heavier hyperon polarization

STAR Collaboration, Phys. Rev. Lett. 126 (2021) 16, 162301



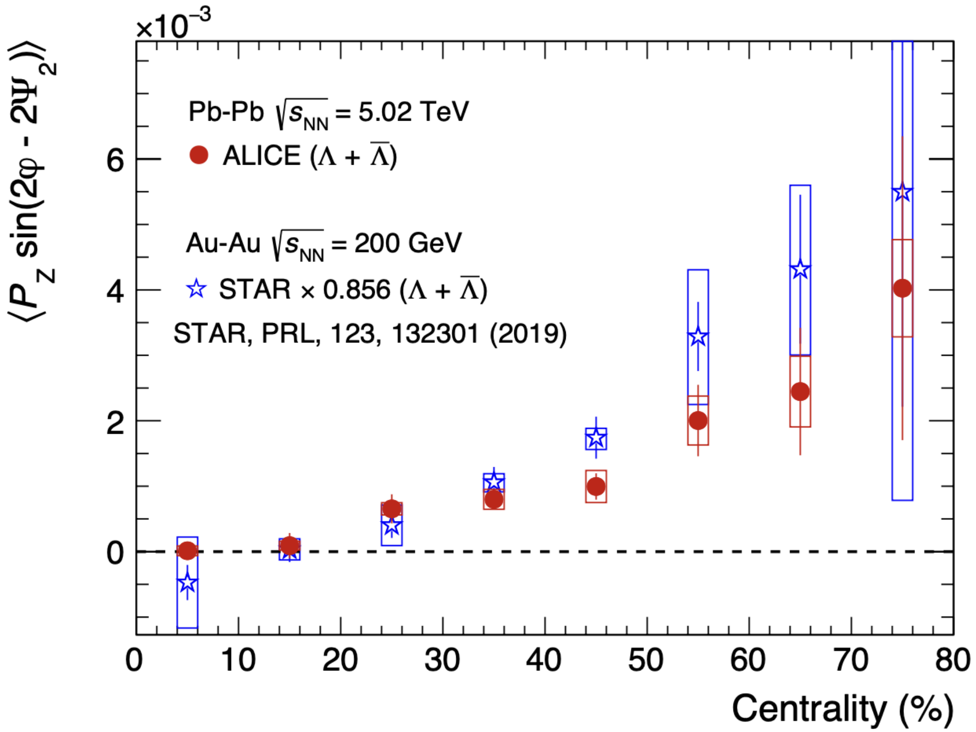


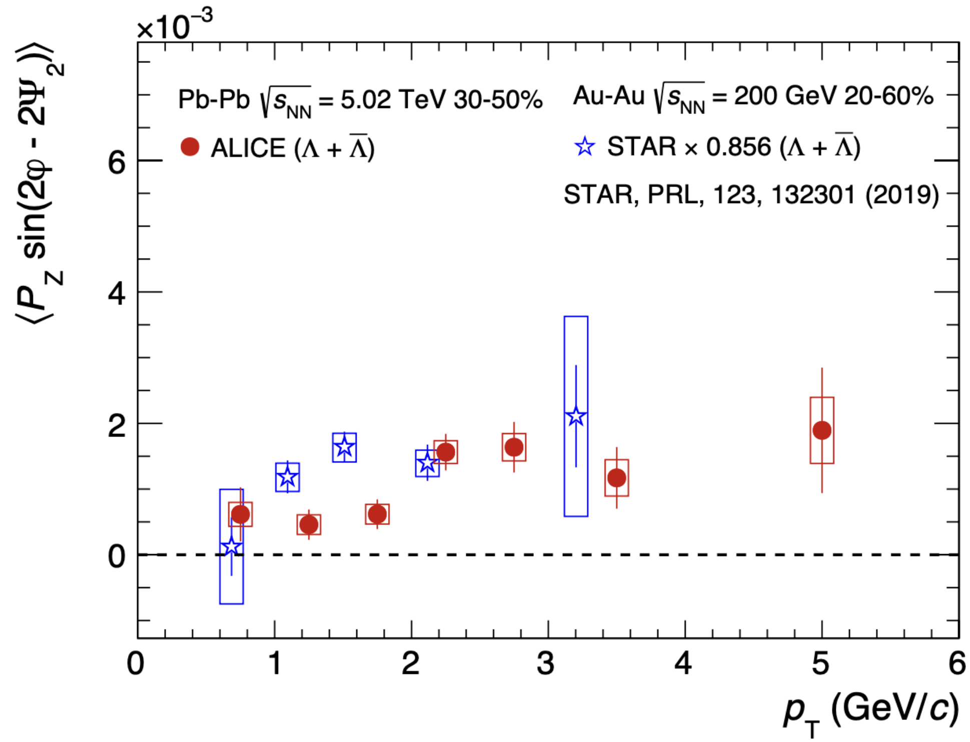
Polarization consistent with S+1 scaling, though with a large statistical error

Will become an important probe with high statistics

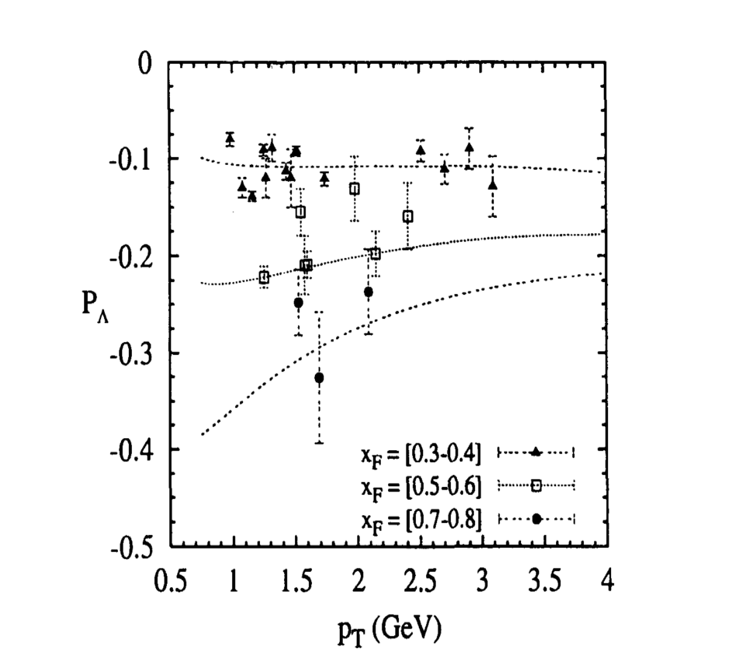
Measurement at the LHC energy

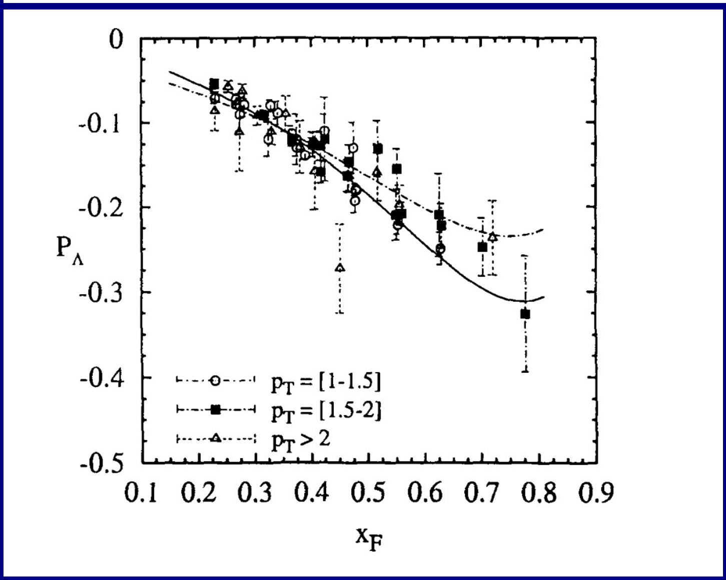
ALICE, Phys. Rev. Lett. 128, 172005. (2022)





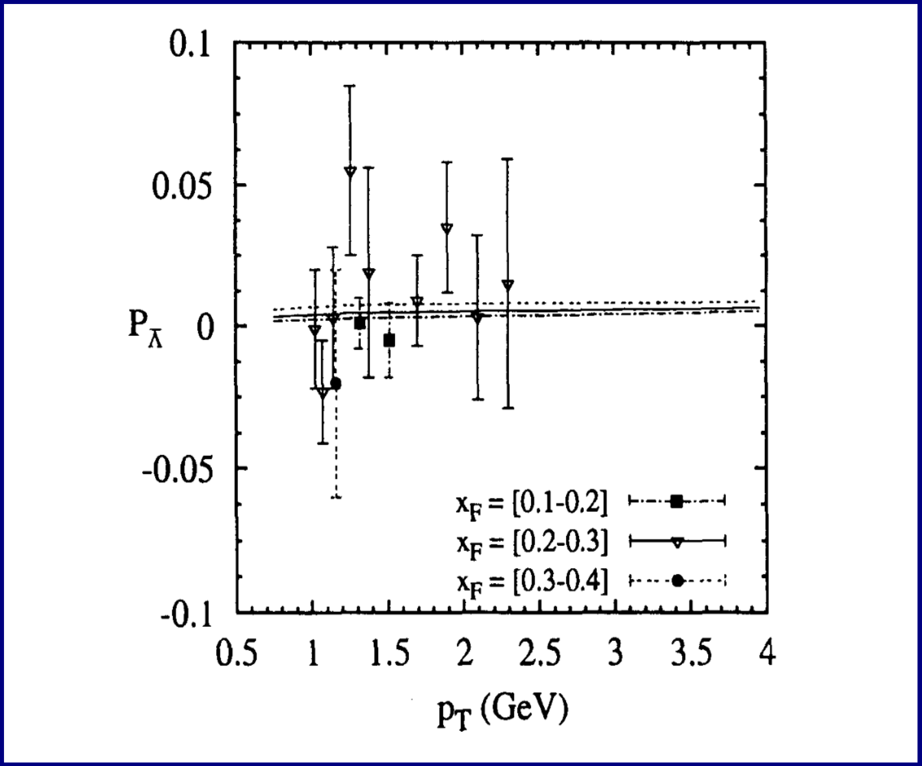
Comparison with NN collisions





Λ is polarized perpendicular to the production plane (no global polarization)

$$x_F = \frac{p_z}{|p_{zMAX}|}$$



Polarization of anti-Λ almost vanishing compared to Λ

M. Anselmino et al., Czech. J. Phys. 51 (2001) Suppl. A, compilation of data from pBe collisions

Why?

Analysis of the different gradient components of the polarization

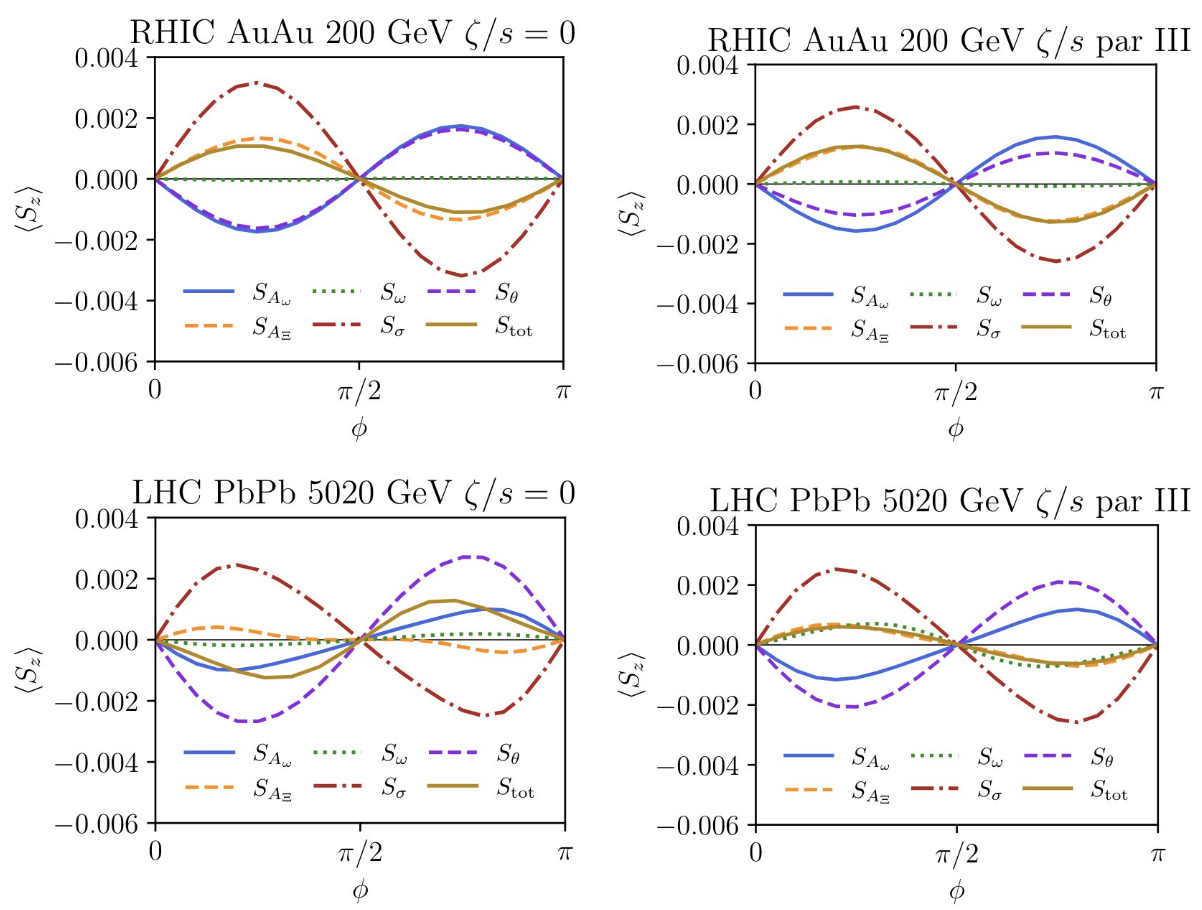
$$S^\mu_{A_\omega} = -\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\; A_\nu u_\rho}{8mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

$$S^\mu_{\omega} = \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\; [\omega^\mu u\cdot p - u^\nu \omega \cdot p]}{4mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

$$S^\mu_{A_\Xi} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho \frac{p_\tau}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\, [u_\sigma A\cdot p + A_\sigma u\cdot p]}{8mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

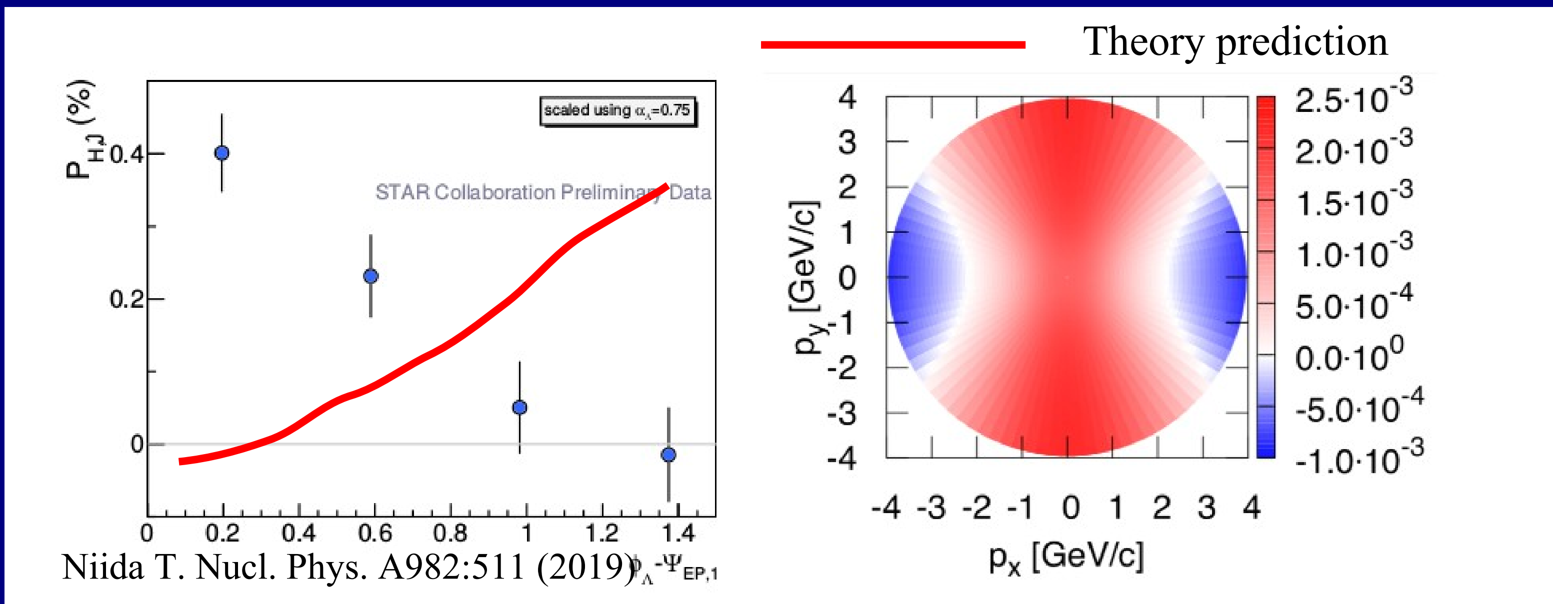
$$S^\mu_{\sigma} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \sigma_{\lambda\sigma}}{4mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F},$$

$$S^\mu_{\theta} = -\epsilon^{\mu\rho\sigma\tau} \hat{t}_\rho p_\tau \frac{p^\lambda}{\varepsilon} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) \theta \Delta_{\lambda\sigma}}{12mT_H \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}.$$

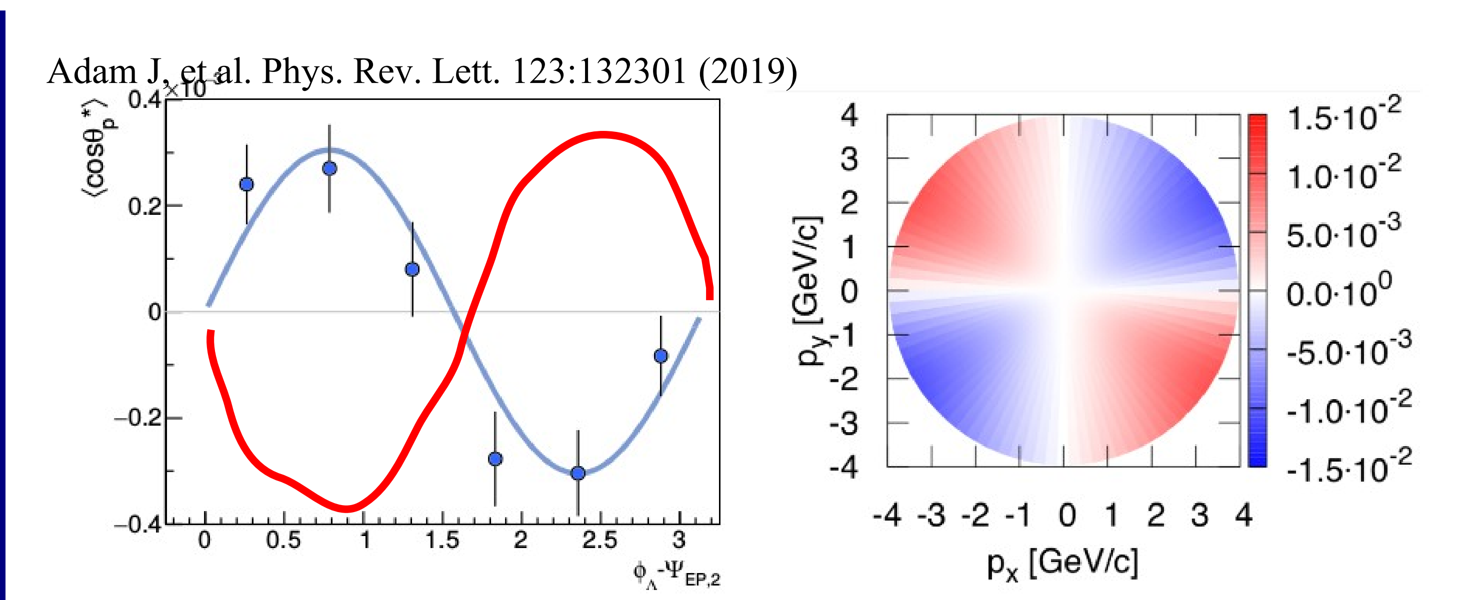


(old) Puzzle: momentum dependence of polarization

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu \beta_\rho}{\int_\Sigma \mathrm{d}\Sigma_\tau p^\tau \ n_F}$$



Spin component along J at p_z=0



Spin component along beam line at p_z=0

Why do we have a dependence on Σ ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{I\ K}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

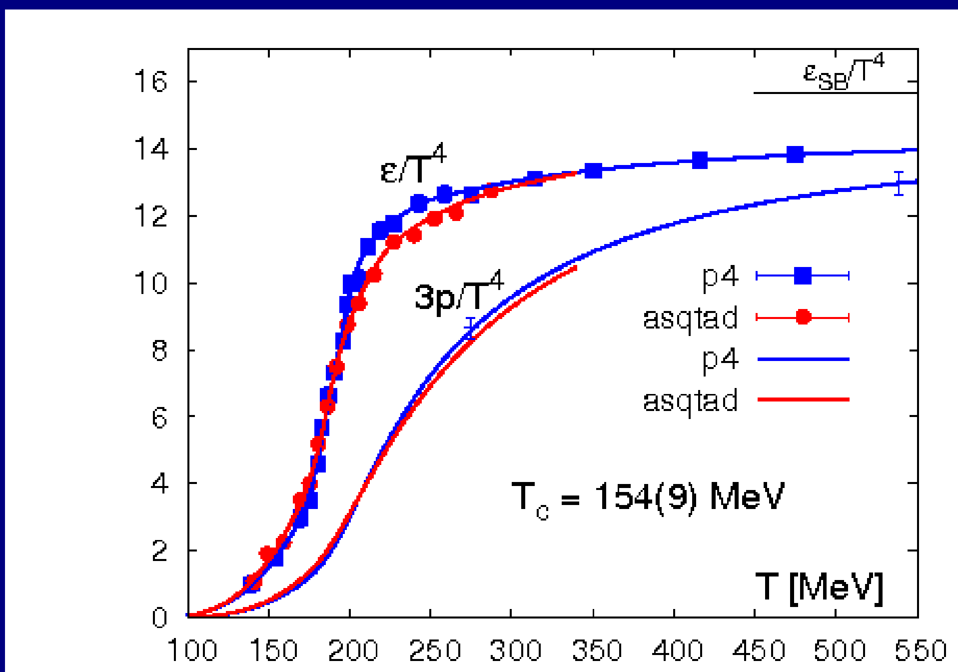
$$\widehat{\Lambda}\widehat{J}_x^{\mu\nu}\widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu}\Lambda_\beta^{-1\nu}\widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{I\ K}$ does not vanish, therefore it does depend on the integration hypersurface and

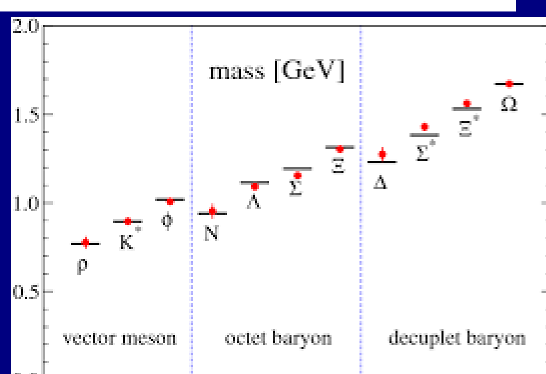
$$\widehat{\Lambda}\widehat{Q}^{\mu\nu}\widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu}\Lambda_\beta^{-1\nu}\widehat{Q}^{\alpha\beta}$$

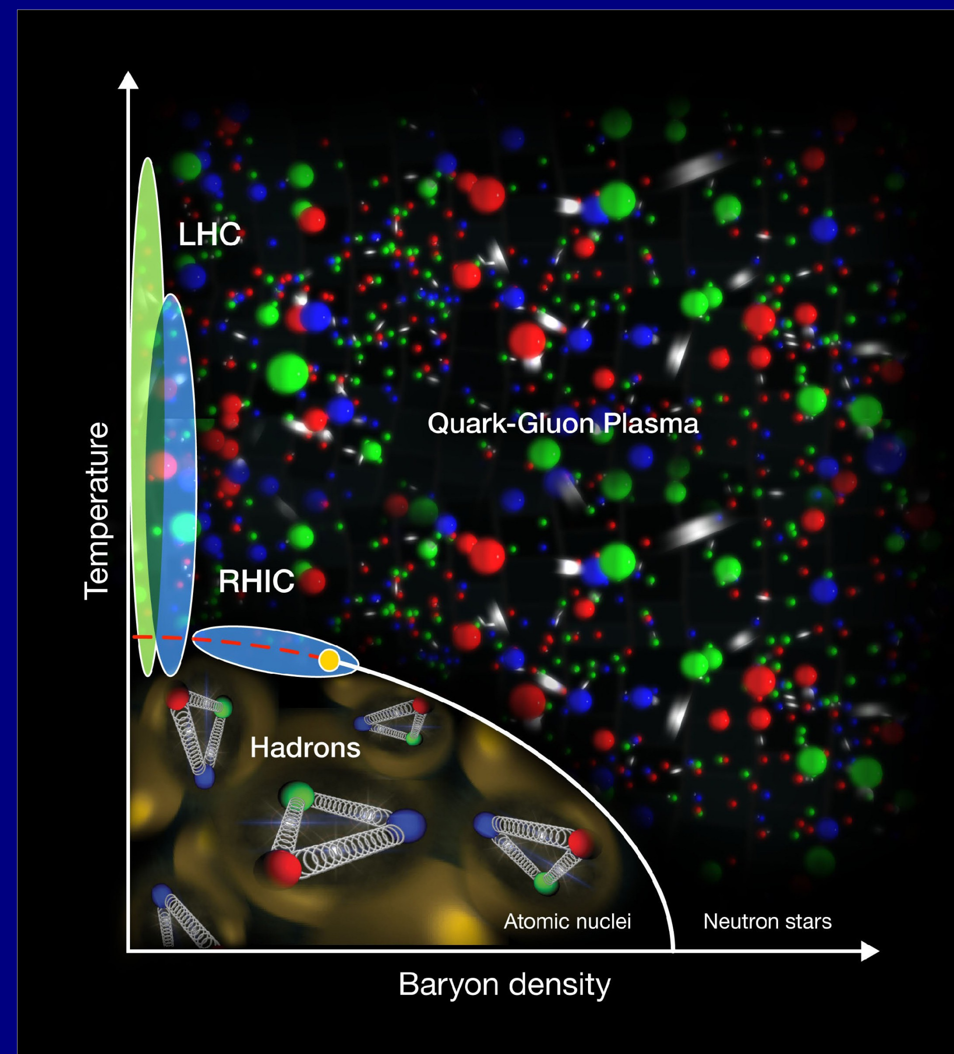
QCD phase diagram: lattice-QCD

Equation of state



QCD reproduces baryon masses





Incidentally: global thermodynamic equilibrium

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu \, \left(\widehat{T}^{\mu\nu} \beta_\nu - \zeta \widehat{j}^\mu \right) \right]$$

Independent of the 3D hypersurface Δ if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \qquad \partial_\mu \zeta = 0$$

Killing equation

$$\Rightarrow \quad \beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

The density operator becomes

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \widehat{J}^{\mu\nu} + \zeta \widehat{Q} \right]$$

What is this new term?

Does it have a non-relativistic limit?
Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

All terms are relativistic (they vanish in the infinite *c* limit) EXCEPT grad T terms, which give rise to:

$$\mathbf{S}_\xi = \frac{1}{8}\mathbf{v} \times \frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}$$

There is an equal contribution in the NR limit from thermal vorticity

Application to relativistic heavy ion collisions

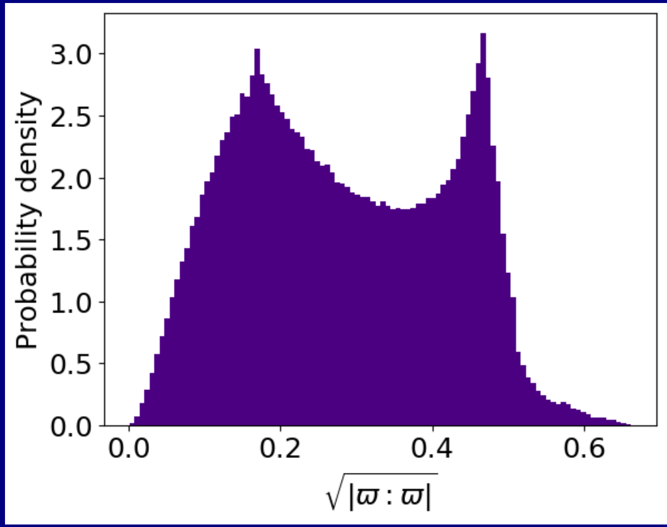
F. B., M. Buzzegoli, A. Palermo, G. Inghirami and I. Karpenko, arXiv:2103.14621

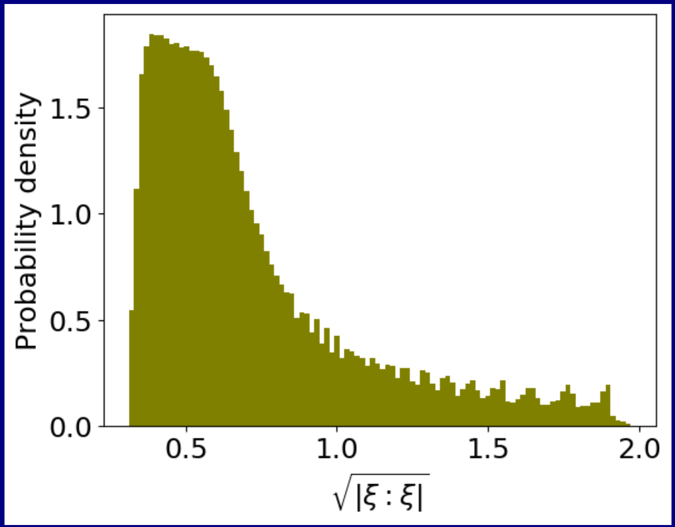
$$S^\mu = S^\mu_\varpi + S^\mu_\xi$$

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau}\frac{p_\tau p^\lambda}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\rho\xi_{\sigma\lambda}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

Is linear response theory adequate?





Isothermal hadronization

At high energy, Δ_{FO} expected to be T= constant!

$$\beta^\mu = (1/T) u^\mu$$

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} \beta_\nu \right] = \frac{1}{Z} \exp\left[- \frac{1}{T} \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \; \widehat{T}^{\mu\nu} u_\nu \right]$$

Only NOW u can be expanded!

$$u_\nu(y) = u_\nu(x) + \partial_\lambda u_\nu(x) (y-x)^\lambda + \ldots$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{2T} (\partial_\mu u_\nu(x) - \partial_\nu u_\mu(x)) \widehat{J}^{\mu\nu}_x - \frac{1}{2T} (\partial_\mu u_\nu(x) + \partial_\nu u_\mu(x)) \widehat{Q}^{\mu\nu}_x + \ldots]$$

A short theory summary

F. B., Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

Spin polarization vector for spin ½ particles:

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4(\gamma^\mu\gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}_4 W_+(x,p)}$$

Wigner function:

$$\begin{aligned}\widehat{W}(x,k)_{AB} &= -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y\ \mathrm{e}^{-ik\cdot y} : \Psi_A(x-y/2)\overline{\Psi}_B(x+y/2) : \\ &= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y\ \mathrm{e}^{-ik\cdot y} : \overline{\Psi}_B(x+y/2)\Psi_A(x-y/2) :\end{aligned}$$

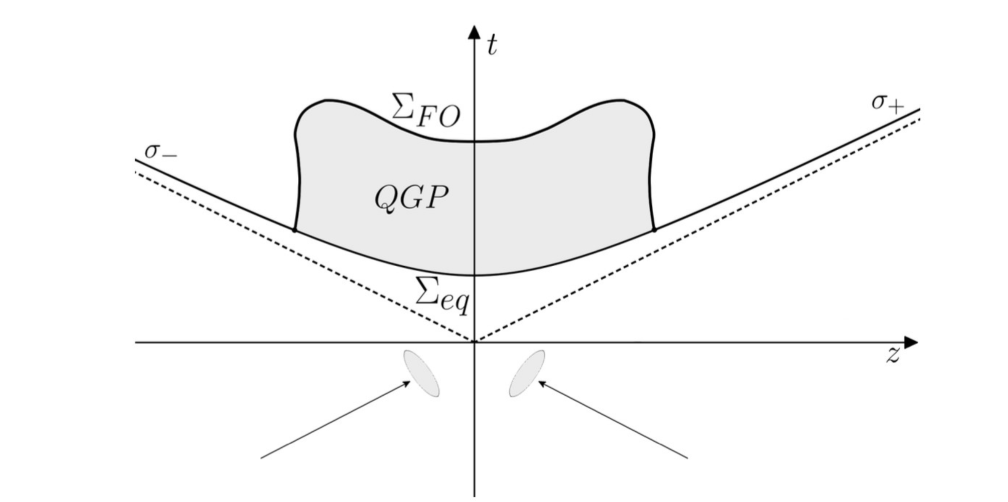
$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

Local equilibrium density operator:

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu\ \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right]$$

$$\beta = \frac{1}{T}u$$

$$\zeta = \frac{\mu}{T}$$



Hydrodynamic limit: Taylor expansion

$$W(x,k)_{\rm LE} = \frac{1}{Z}{\rm Tr}\left(\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y)\widehat{T}^{\mu\nu}_B(y)\beta_\nu(y) - \zeta(y)\widehat{j}^\mu(y)\right]\widehat{W}(x,k)\right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y)\beta_\nu(x) = \beta_\nu(x)\int_\Sigma {\rm d}\Sigma_\mu T^{\mu\nu}_B(y) = \beta_\nu(x)\widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp\left[-\beta_\mu(x)\widehat{P}^\mu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}^{\mu\nu}_x - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}^{\mu\nu}_x + \ldots\right]$$

$$\widehat{J}^{\mu\nu}_x = \int {\rm d}\Sigma_\lambda (y-x)^\mu\widehat{T}^{\lambda\nu}_B(y) - (y-x)^\nu\widehat{T}^{\lambda\mu}_B(y)$$

$$\widehat{Q}^{\mu\nu}_x = \int {\rm d}\Sigma_\lambda (y-x)^\mu\widehat{T}^{\lambda\nu}_B(y) + (y-x)^\nu\widehat{T}^{\lambda\mu}_B(y)$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x) \widehat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu}(x) \widehat{J}^{\mu\nu}_x - \frac{1}{2} \xi_{\mu\nu}(x) \widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Linear response theory

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\; \mathrm{e}^{z(\widehat{A}+\widehat{B})}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}} \simeq \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\; \mathrm{e}^{z\widehat{A}}\widehat{B}\mathrm{e}^{-z\widehat{A}}\mathrm{e}^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(\mathrm{e}^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

$$\langle \widehat{Q}_x^{\mu\nu}\widehat{W}(x,p)\rangle \qquad \langle \widehat{J}_x^{\mu\nu}\widehat{W}(x,p)\rangle$$

Spin mean vector at leading order in thermal vorticity

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z}\exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

Neglected by "prejudice" until 2021

$$S^\mu(p) = -\frac{1}{2m}\epsilon^{\mu\beta\gamma\delta}p_\delta \frac{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4(\Sigma_{\beta\gamma}W_+(x,p))}{\int {\rm d}\Sigma_\lambda p^\lambda {\rm tr}_4 W_+(x,p)}$$

+ Linear response theory

$$n_F = ({\rm e}^{\beta\cdot p - \xi}+1)^{-1}$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \; n_F}$$

See also

R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904

W. Florkowski, A. Kumar and R. Ryblewski, Phys. Rev. C 98 (2018) 044906

Y. C. Liu, L. L. Gao, K. Mameda and X. G. Huang, Phys. Rev. D 99 (2019) 085014

N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, Phys. Rev. D 100 (2019) 056018