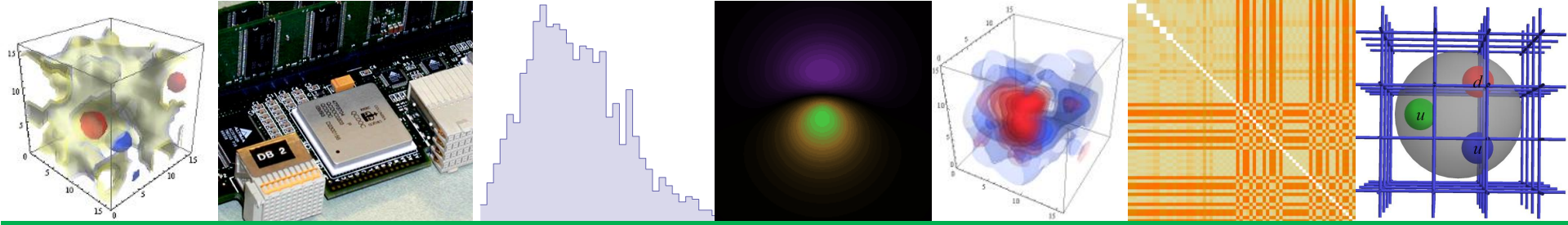


Recent Progress on x-dependent GPDs from the Lattice

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

Outline

§ Lattice QCD and Backgrounds

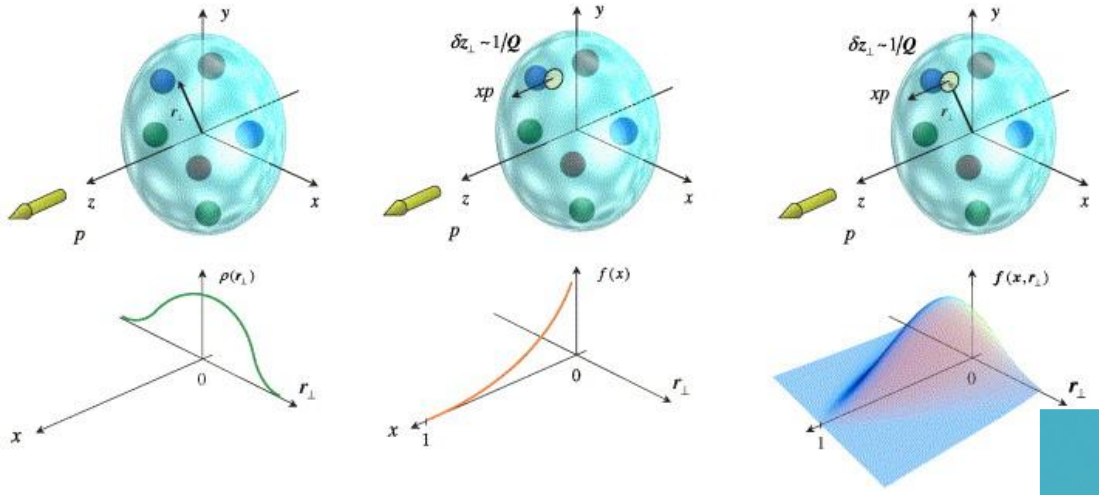
§ LaMET Method and Progress on PDFs

§ Selected GPDs Results

- Impact of Lattice-QCD PDFs on Global Fits
- Nonzero-skewness GPDs

3D Imaging

§ GPDs encode information about the spatial structure & the partonic distribution of spin and orbital angular momenta



Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

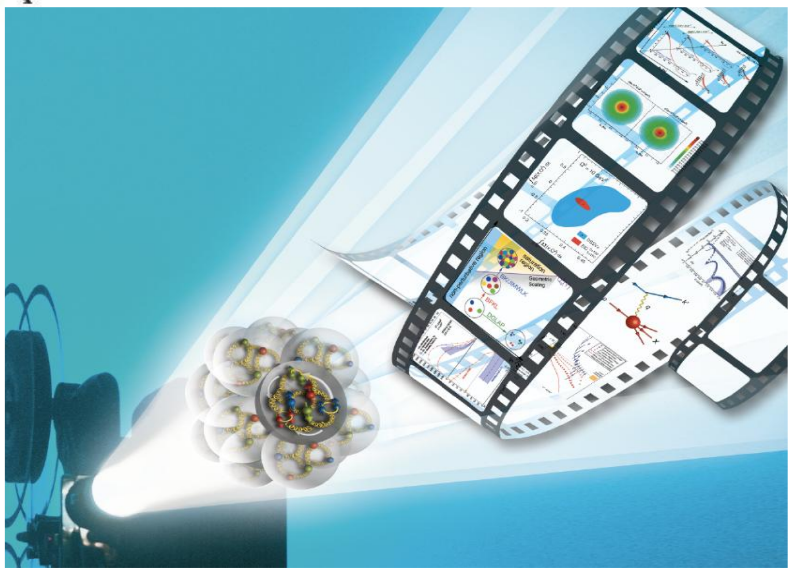
Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

EIC White Paper, 1212.1701

Yellow Report, 2103.05419

Daria Sokhan on Mon; Paweł Sznajder on Thur



What is Lattice QCD?

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

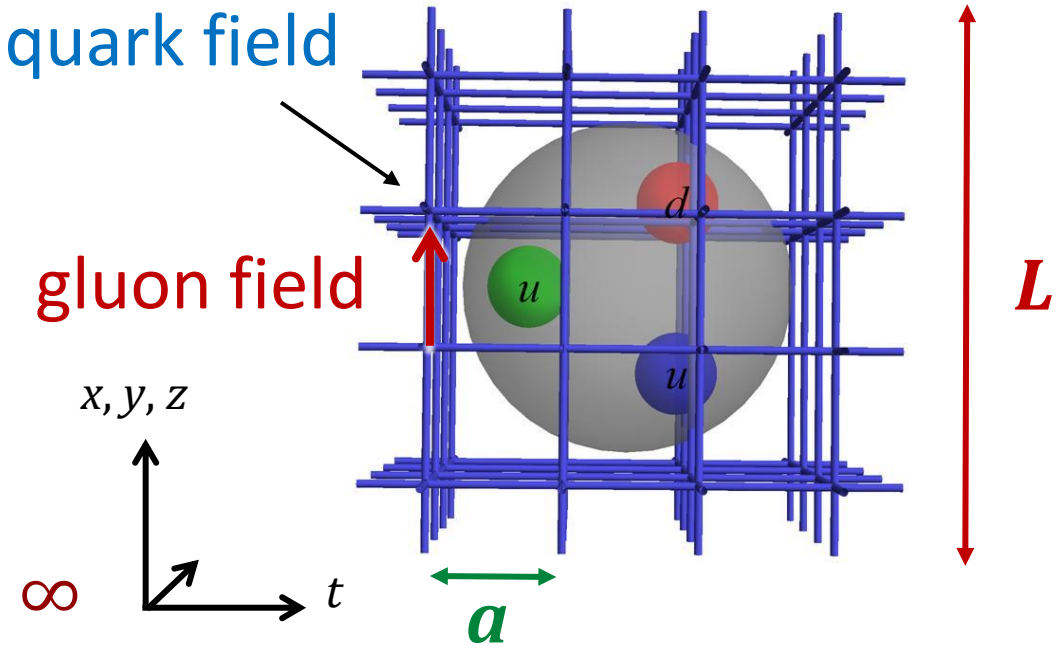
$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O\,(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  - discretize spacetime
- Impose an infrared cutoff
  - finite volume

§ Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

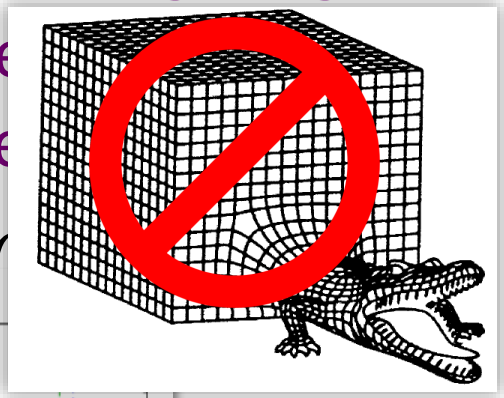


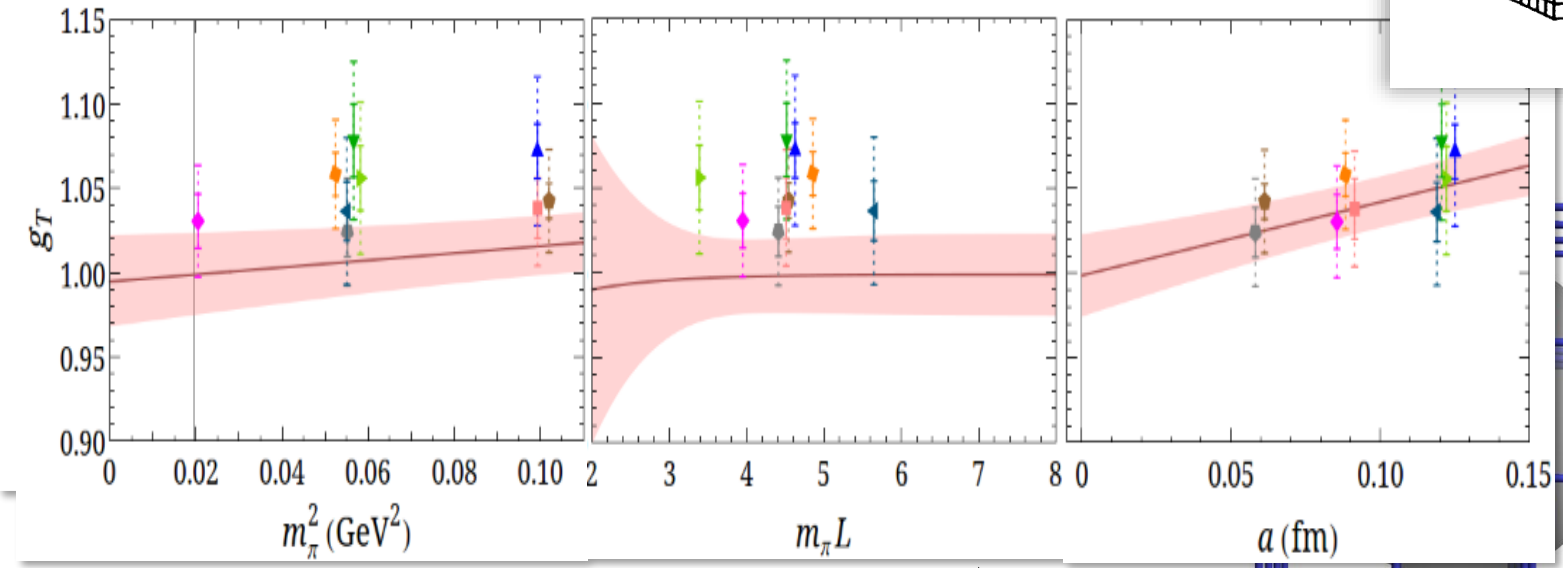
Lattice QCD 101

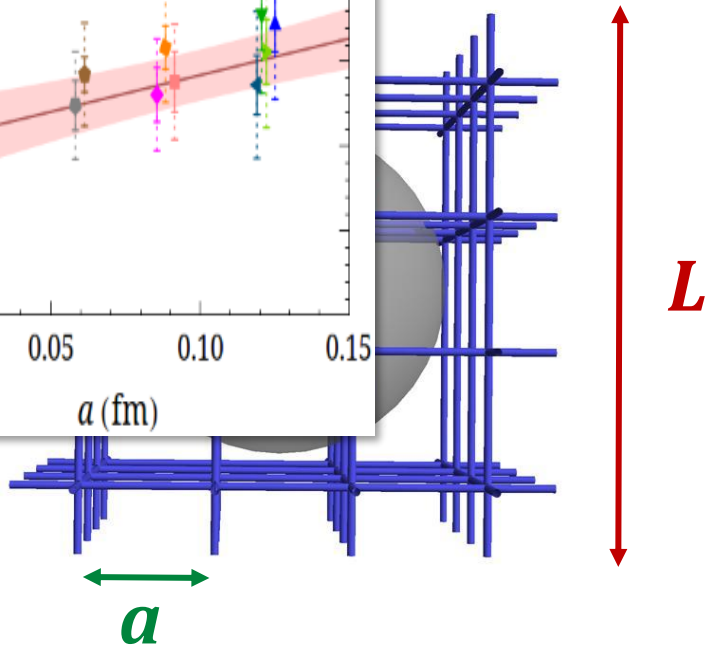
§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field the

§ Physical observables are calculated from the

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}($$

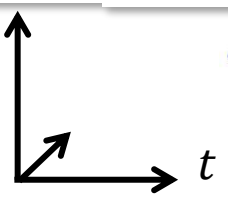




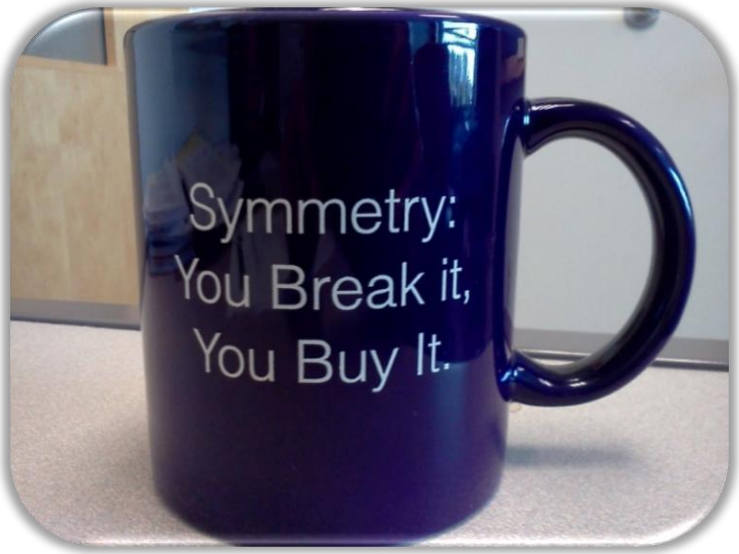


§ Recover physical limit

$m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$, $L \to \infty$



Lattice Structure Limitation



§ Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ Longstanding obstacle!

- Holy grail of structure calculations

§ Applies to many structure quantities:

- Parton distribution functions (PDF)
- Generalized parton distributions (GPD)
- Transverse-momentum distributions (TMD)
- ⋮

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

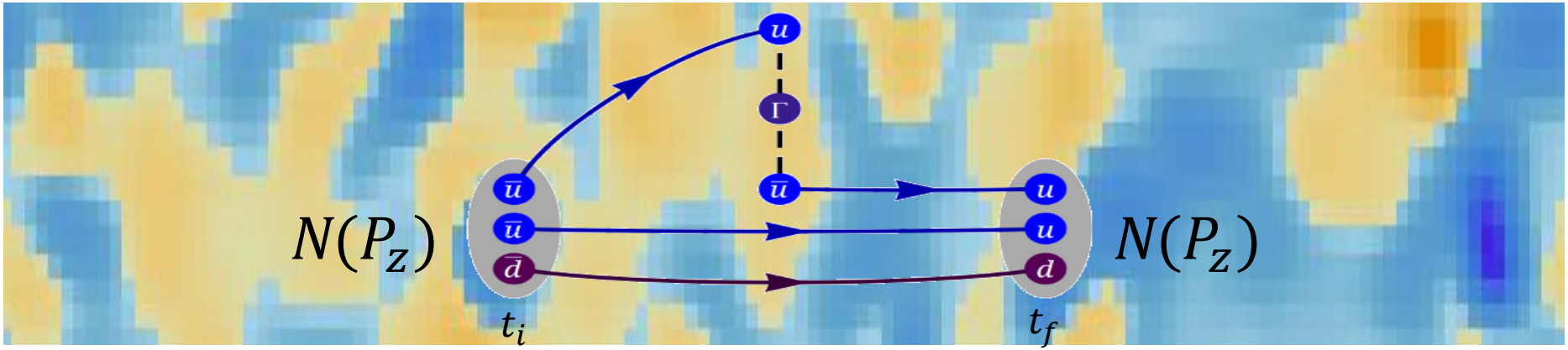
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

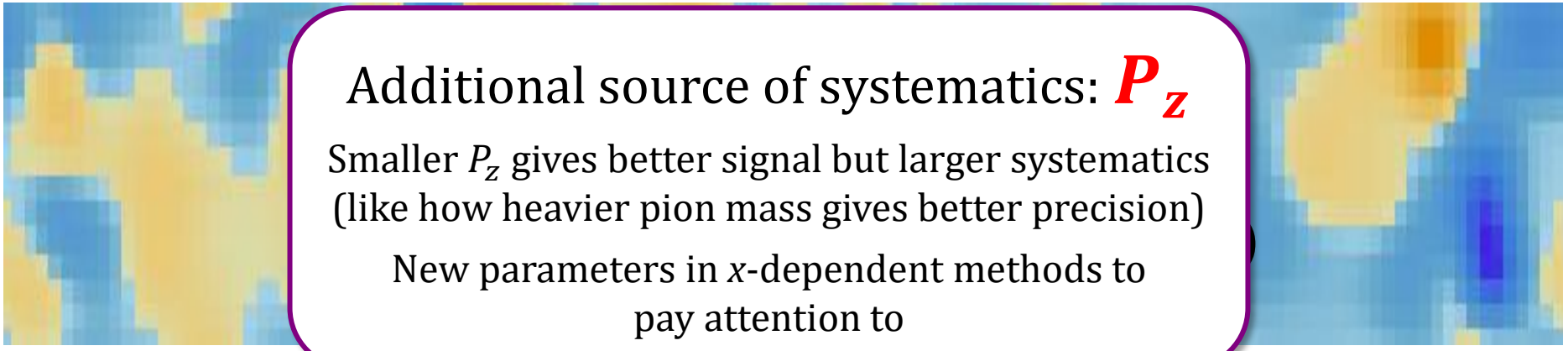
§ Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}}, \frac{\Lambda_{\rm QCD}^2}{(x\boldsymbol{P_z})^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)\boldsymbol{P_z})^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



Additional source of systematics: $\boldsymbol{P_z}$

Smaller P_z gives better signal but larger systematics (like how heavier pion mass gives better precision)

New parameters in x-dependent methods to pay attention to

§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

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X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

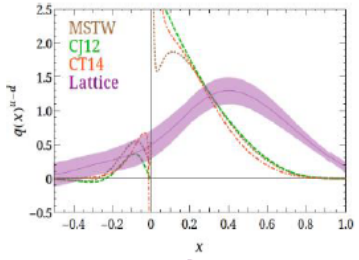
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Compton amplitude method (A.J. Chambers et al., 1703.01153)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)

Lattice Parton Calculations

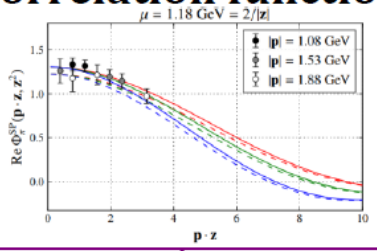
§ Rapid developments!

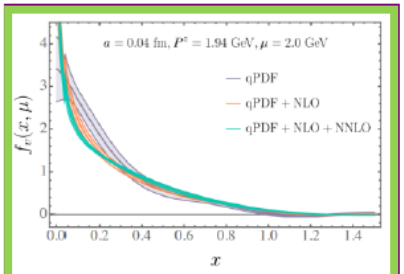
HL, Prog.Part.Nucl.Phys. 144 (2025)

LaMET/quasi-PDF lattice calculation



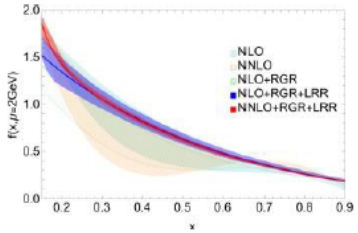
Euclidean correlation functions





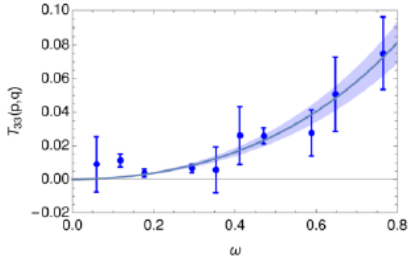
1st NNLO PDF

1stPDF w/ LRR+RGR

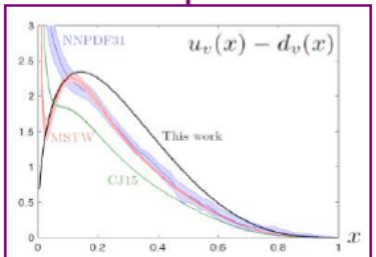


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024

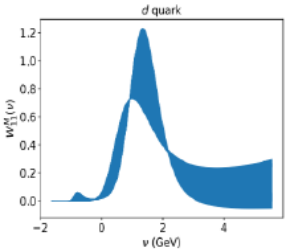
Compton amplitude



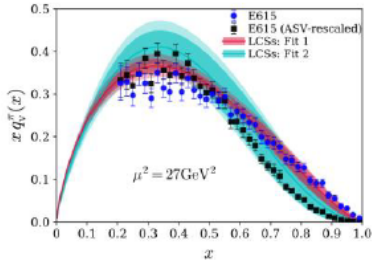
pseudo-PDFs



Hadronic tensor



LCS

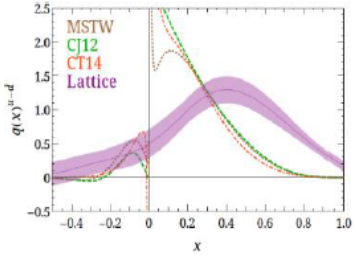


Lattice Parton Calculations

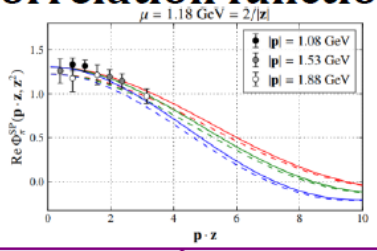
§ Rapid developments!

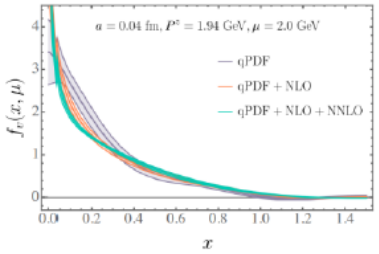
HL, Few Body Syst. 64 (2023) 3, 58

LaMET/quasi-PDF lattice calculation



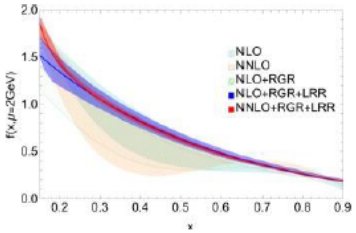
Euclidean correlation functions





1st NNLO PDF

1stPDF w/ LRR+RGR

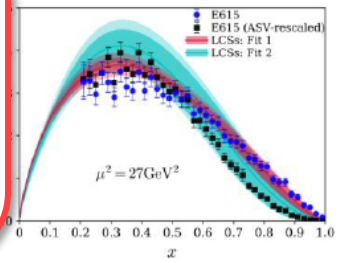


2022 2023 2024

Other LaMET talks in this conference

- Wei Wang (Lattice calculation of TMDs)@ Mon. 5:00 PM
- Heng-Tong Ding (Meson EMFF and GPD) @ Tue. 9:30 AM
- Liuming Liu (Nucleon gluon PDF) @ Tue., 10:20 AM
- Mu-hua Zhang (Baryon DAs) @ Tue., 11:00 AM
- Jin-Xin Tan (TMD wavefunction) @ Wed., 11:10 AM

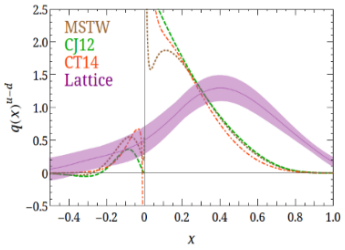
LCS



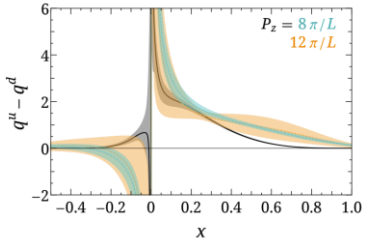
Lattice Parton Calculations

§ Physics-quantity milestones

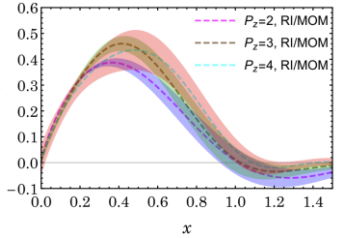
First unpol. lattice PDF



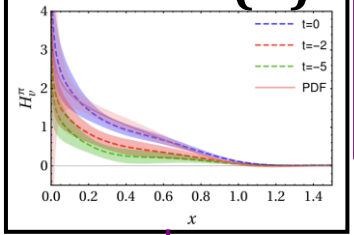
First PDFs at $M_\pi^{\rm phys}$



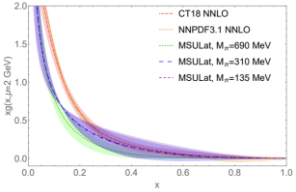
Pion v-PDF



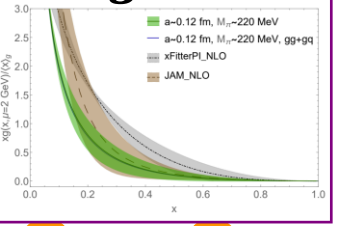
1st GPD (π)



N g-PDF

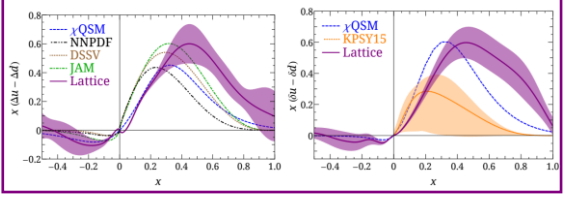


π g-PDF

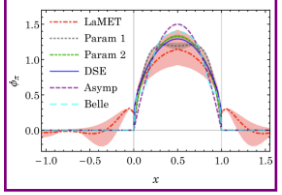


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

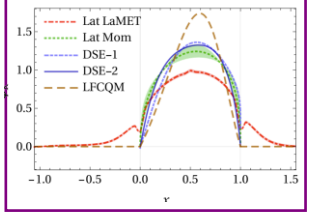
Pol. PDFs and mass corrections



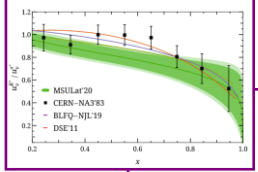
Pion DA



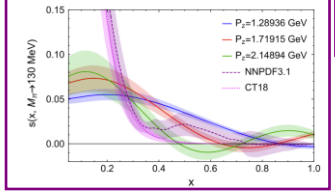
Kaon DA



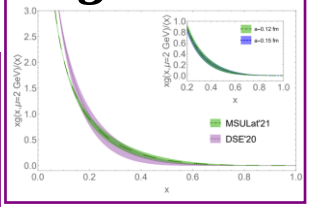
K PDF



s,c PDF



Kaon g-PDF

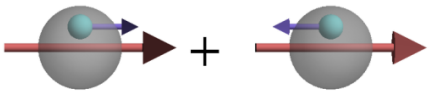


HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

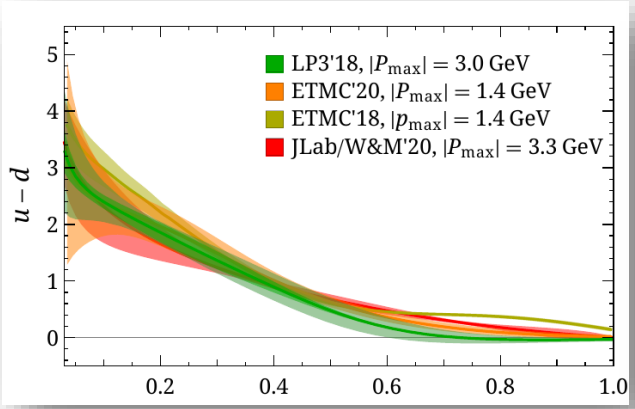
Lattice Example Results

§ Summary of PDF results at physical pion mass

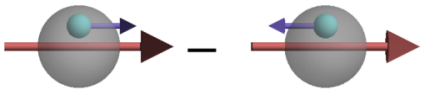
unpolarized



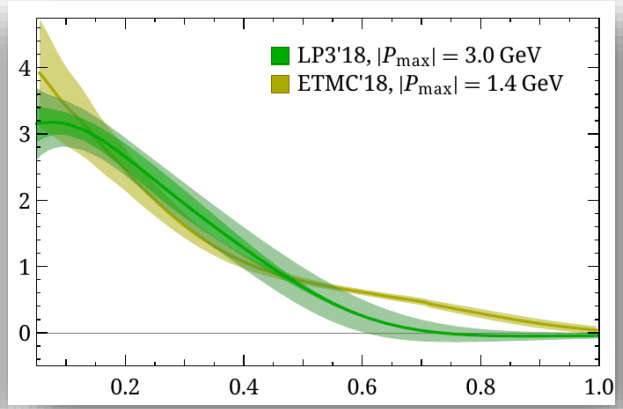
$u(x) - d(x)$



longitudinally polarized

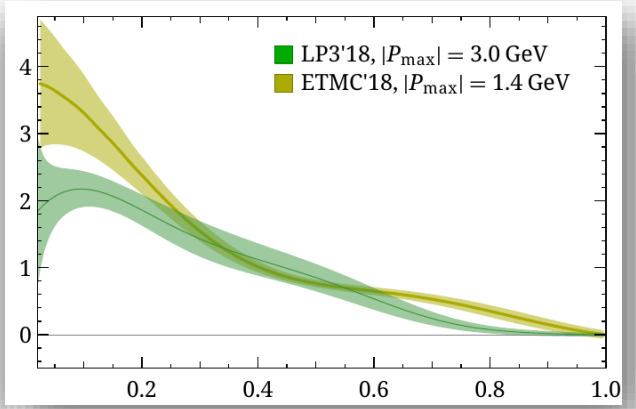


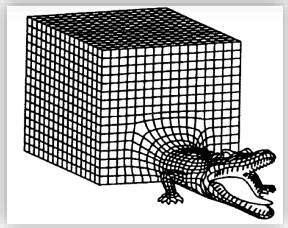
$\Delta u(x) - \Delta d(x)$



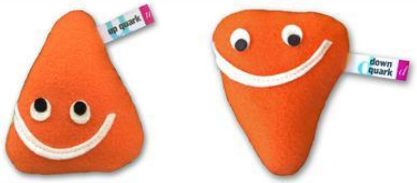
transversely polarized

$\delta u(x) - \delta d(x)$





Finite volume,
Discretization,
…

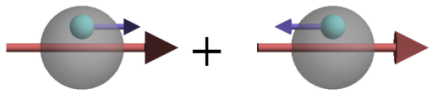


2006.08636 (PDFLattice2019)

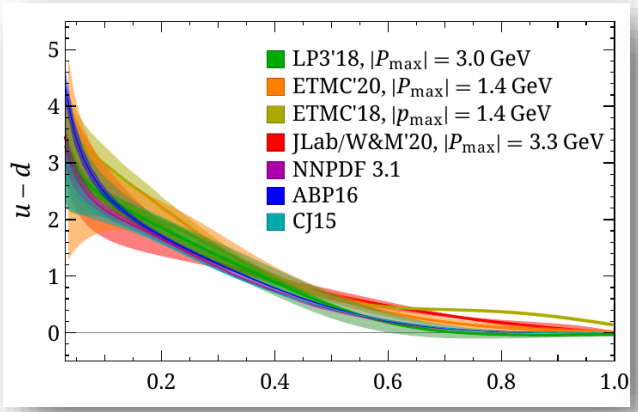
Lattice Example Results

§ Summary of PDF results at physical pion mass

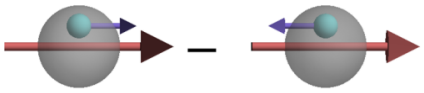
unpolarized



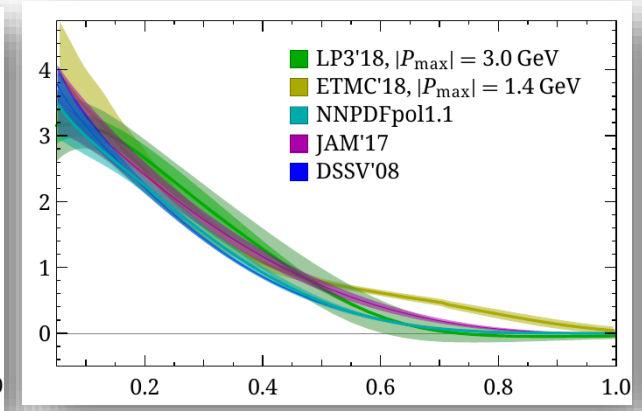
$u(x) - d(x)$



longitudinally polarized

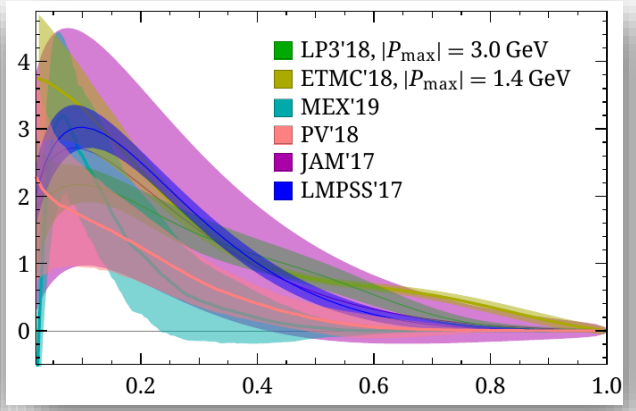


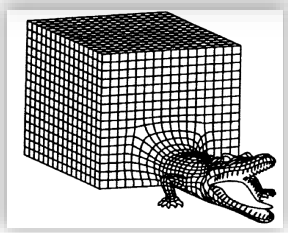
$\Delta u(x) - \Delta d(x)$



transversely polarized

$\delta u(x) - \delta d(x)$





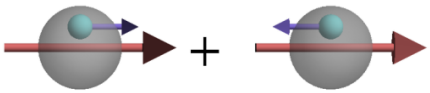
Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

Lattice Example Results

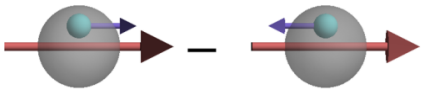
§ Summary of PDF results at physical pion mass

unpolarized



$u(x) - d(x)$

longitudinally polarized



$\Delta u(x) - \Delta d(x)$

transversely polarized

$\delta u(x) - \delta d(x)$

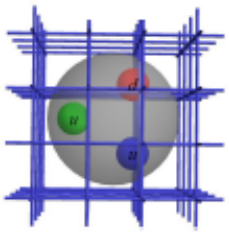
§ Complementary lattice inputs for best PDFs

Theory Input

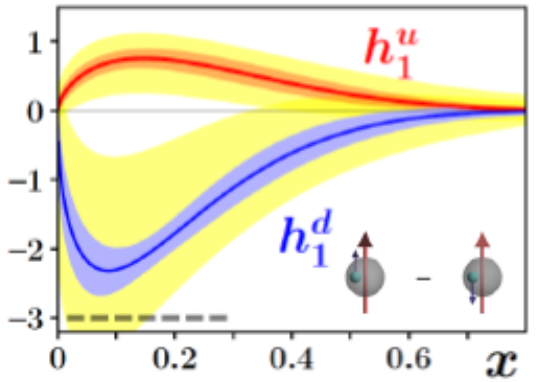
Exp't Input

Global Analysis of PDFs

+



Work has been made in this direction



Phys. Rev. Lett. 120, 152502 (2018)

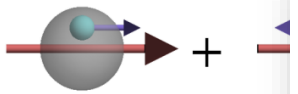
Lattice Example Results

§ Summary of PDF results at physical pion mass

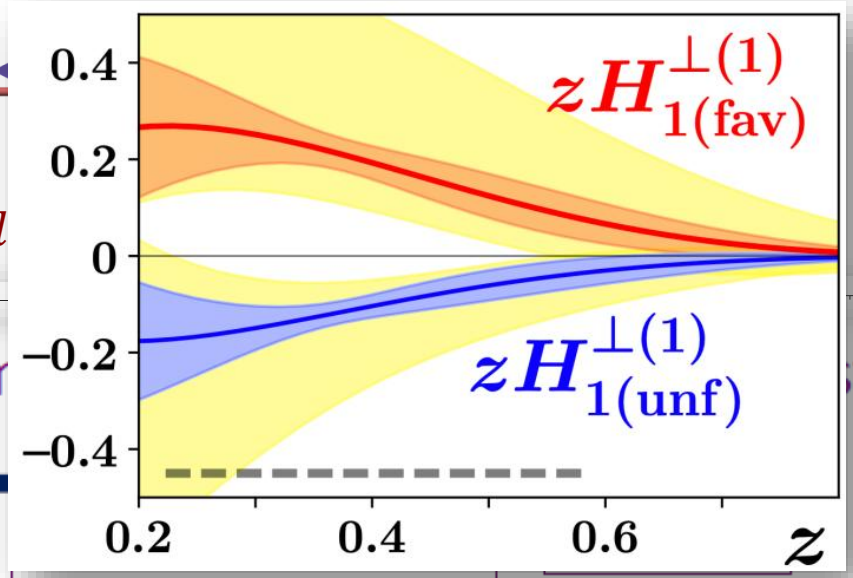
unpolarized

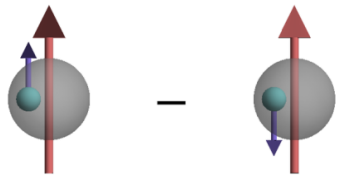
longitudinally polarized

transversely polarized



$u(x) - d$





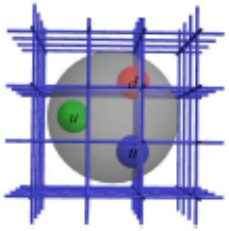
$\delta u(x) - \delta d(x)$

§ Complem ... t PDFs

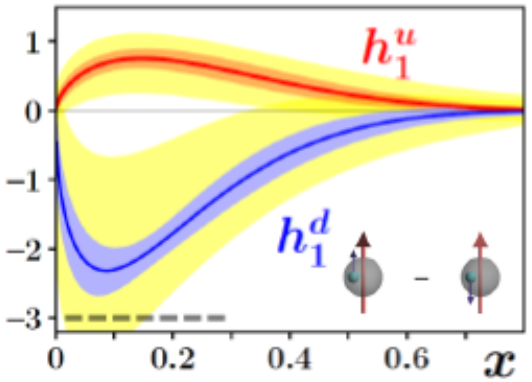
Theory Input

Global Analysis of PDFs

+



Work has been made in this direction



Phys. Rev. Lett. 120, 152502 (2018)

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

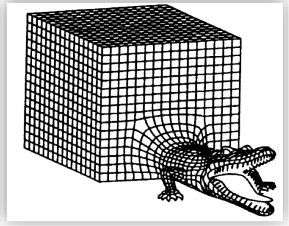
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

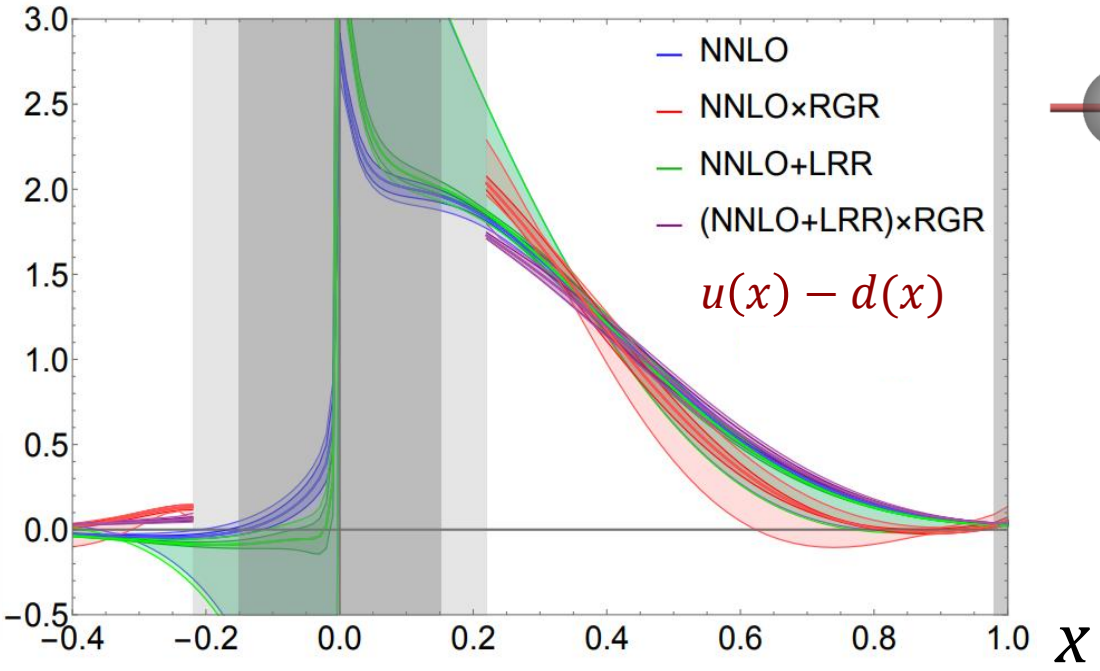
R. Zhang, et. al.
PLB 844, 138081 (2023)

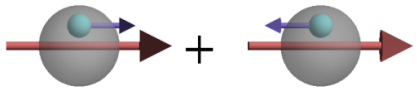
- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, $P_z \approx 2$ GeV

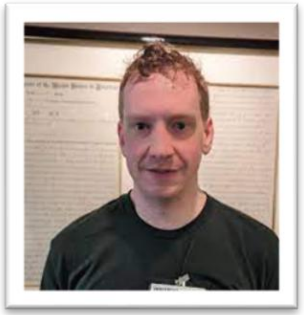
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…









P: Jack Holligan

Isovector PDFs Update

§ Nucleon isovector PDF calculated directly at **physical pion mass**

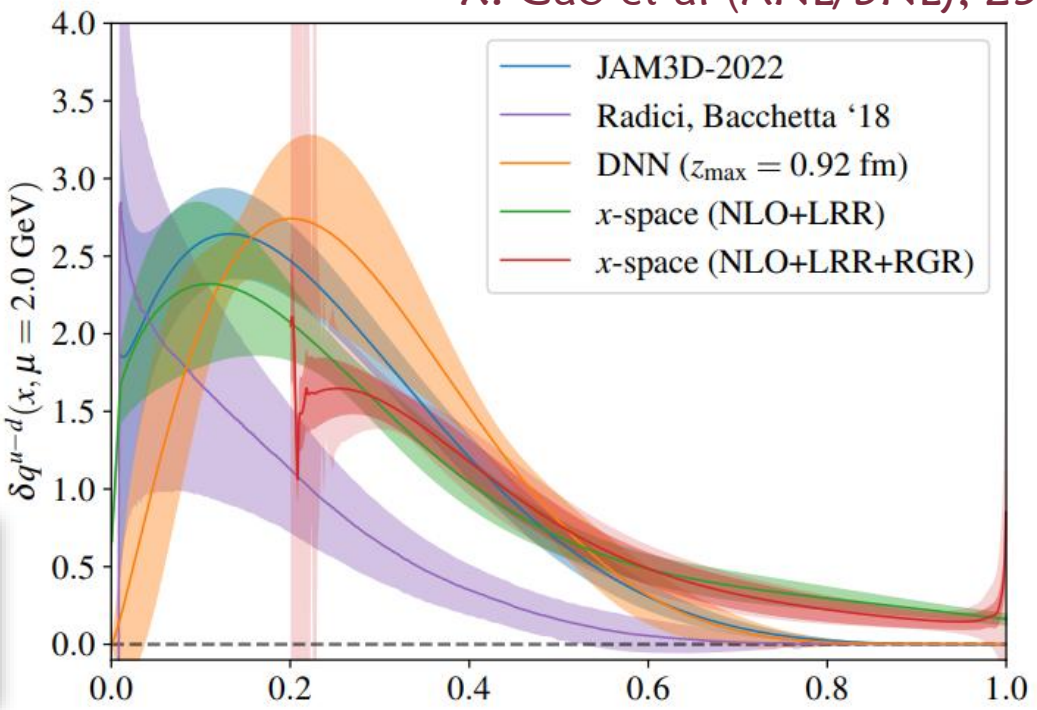
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al.
PLB 844, 138081 (2023)

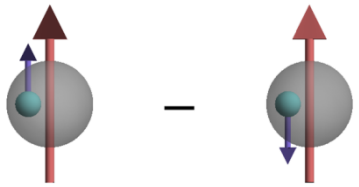
- N_f= 2+1 clover/HISQ, $a \approx 0.076$ fm, $P_z \approx 1.5$ GeV

X. Gao et al (ANL/BNL), 2310.19047 [hep-lat]

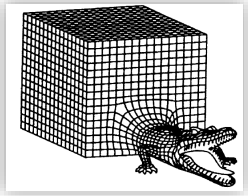
Wanted PDFs, GPDs, etc…



Transversity

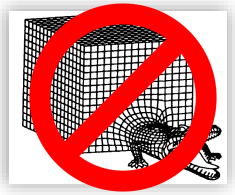


$\delta u(x) - \delta d(x)$



Continuum PDF

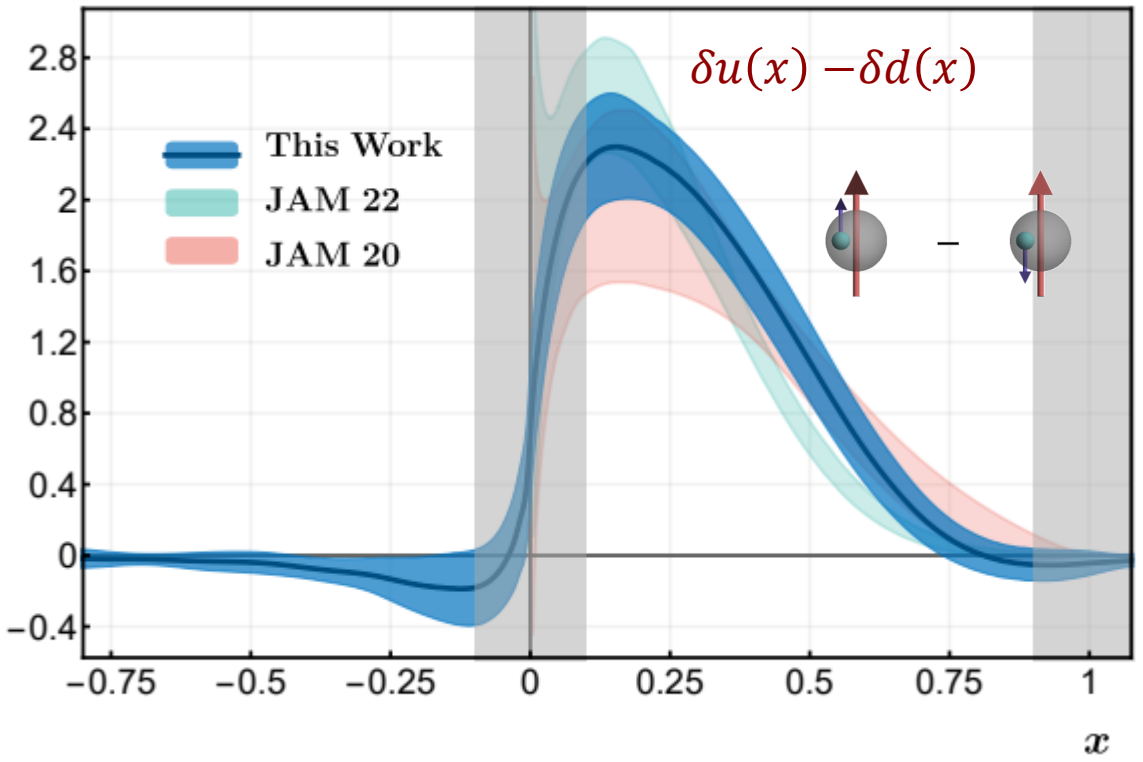
§ Nucleon PDFs using quasi-PDFs in the continuum limit



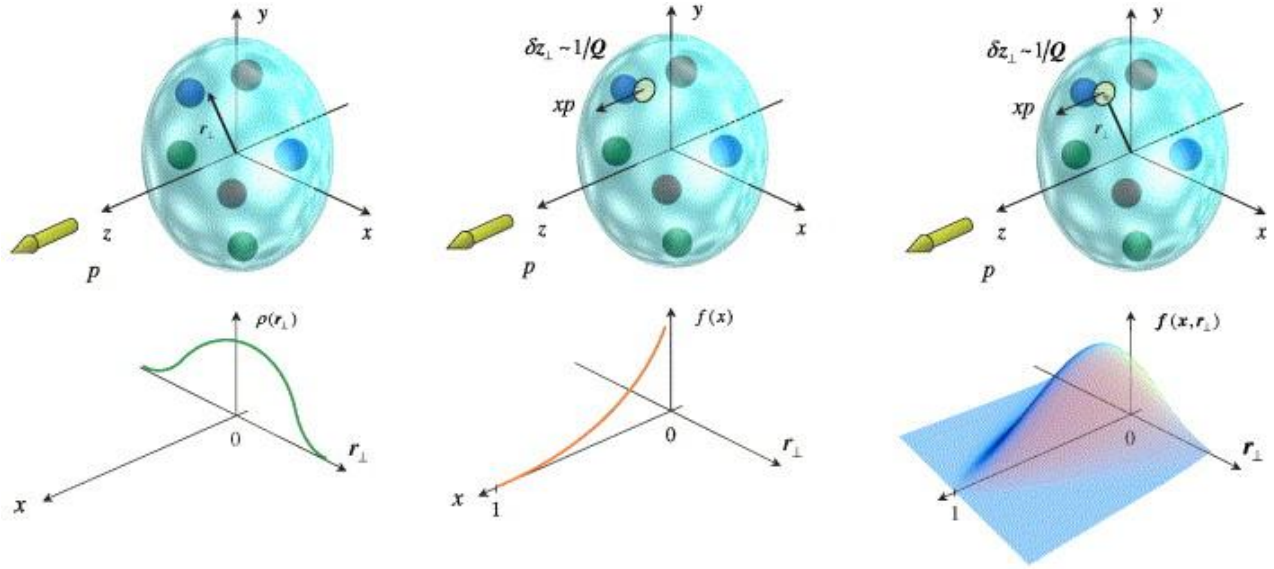
- Lattice details: clover/2+1 clover (LPC)
  - a ≈ {0.49, 0.64, 0.85, 0.98} fm,
  - $M_\pi \in [222, 354]$-MeV pion,
  - $M_\pi L \in [3.9, 8.1]$
  - $P_z \in [1.8, 2.8]$

Fei Yao et al (LPC), 2208.08008

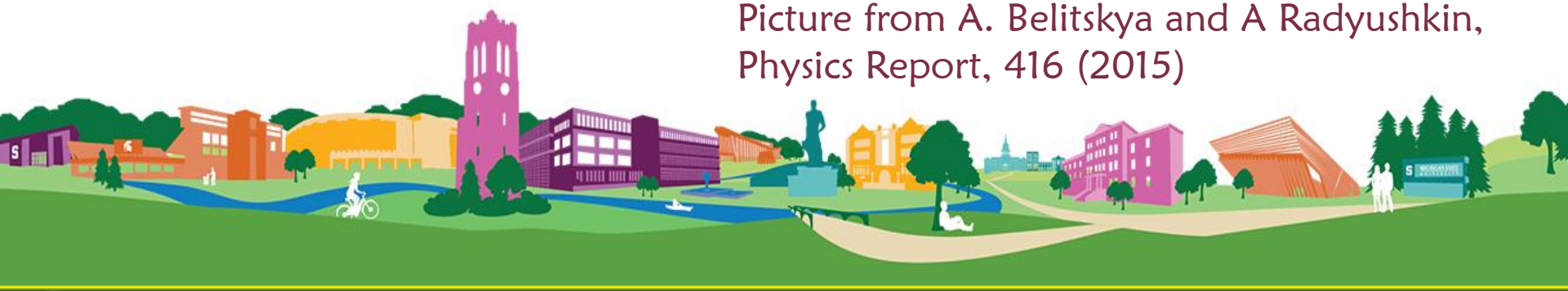
Wanted PDFs, GPDs, etc…



Bjorken-x Dependent GPDs

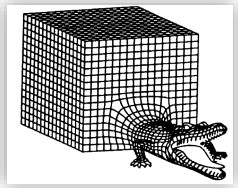


Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)



Bjorken-x Dependent GPDs

Single-ensemble result



finite-volume,
discretization,
heavier quark mass,
Smaller boosted P_z,
…

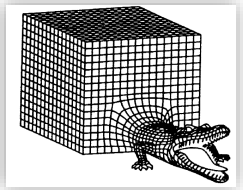
First Lattice GPDs

§ First glimpse into **pion** GPD using **Quasi-PDF**/LaMET

Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

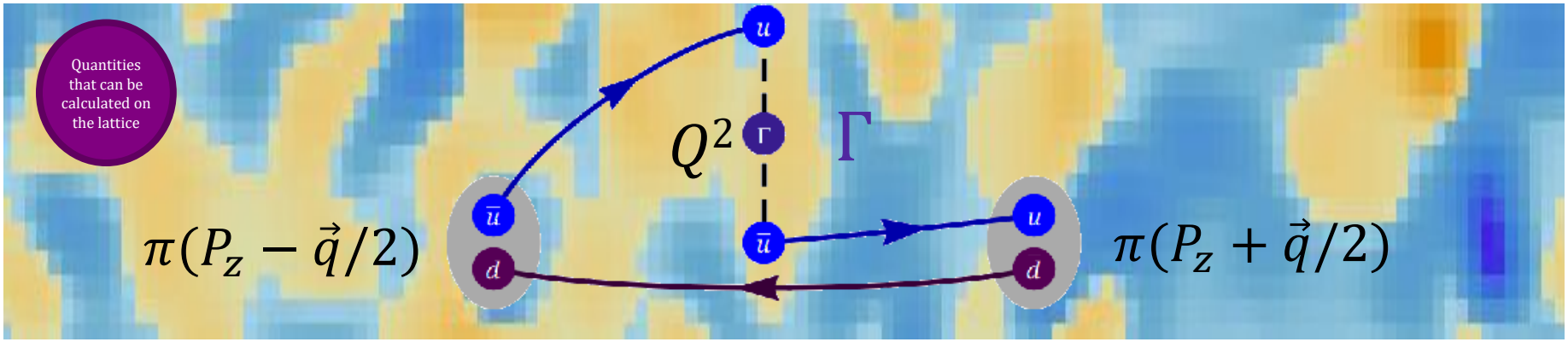
$P_z \approx \mathbf{1.6}$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056



J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



First Lattice GPDs

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MILC, Phys. Rev. D, 82 (2010), 074501;
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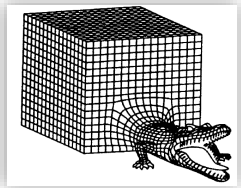
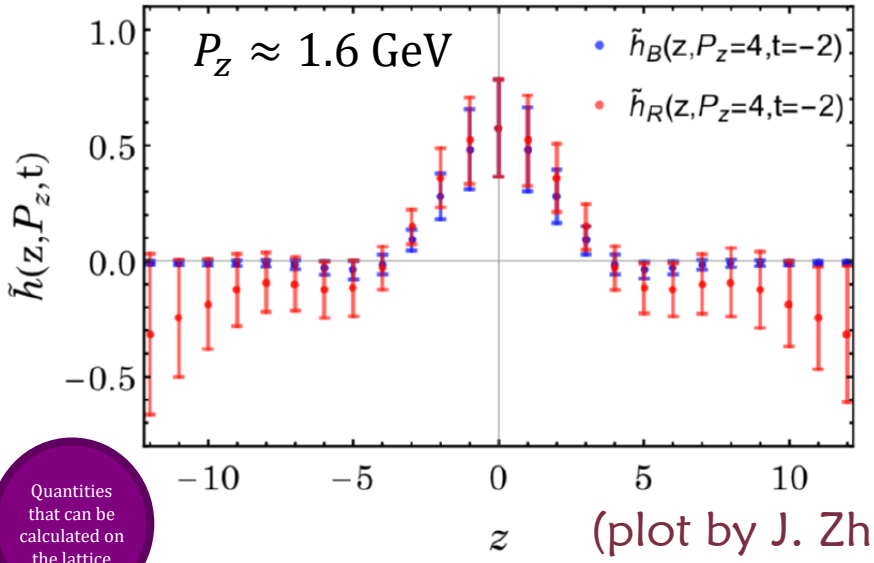
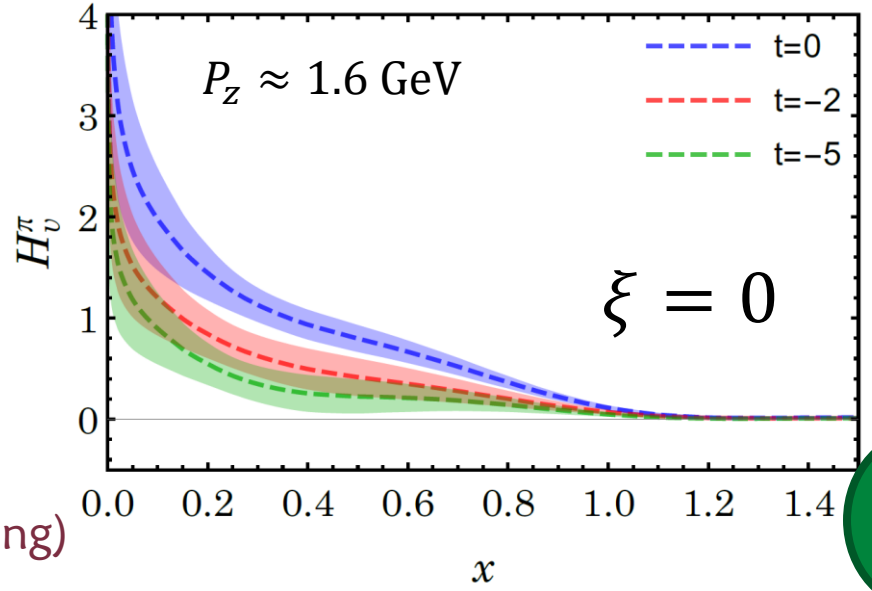


J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma \left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



(plot by J. Zhang)



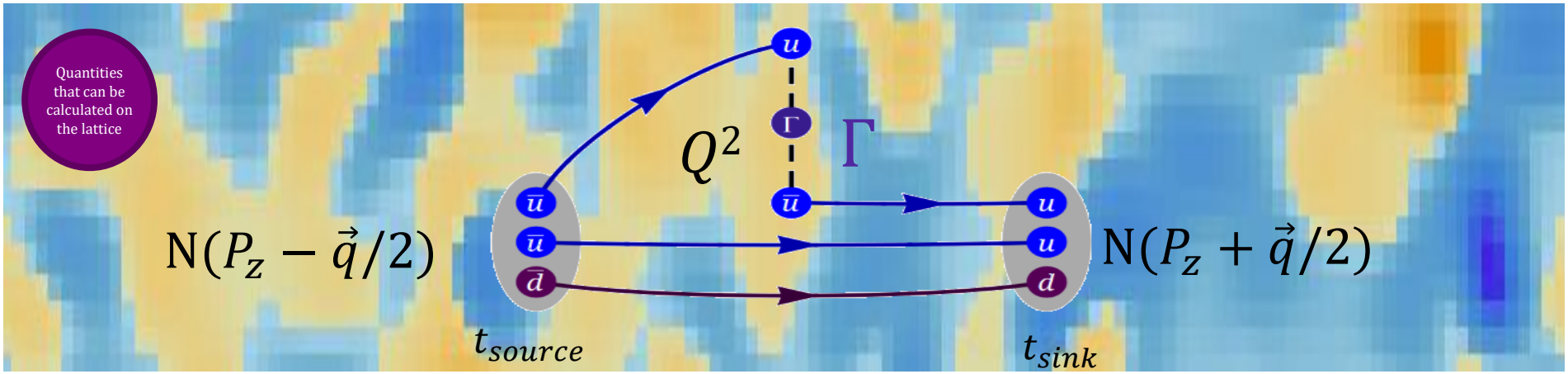
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
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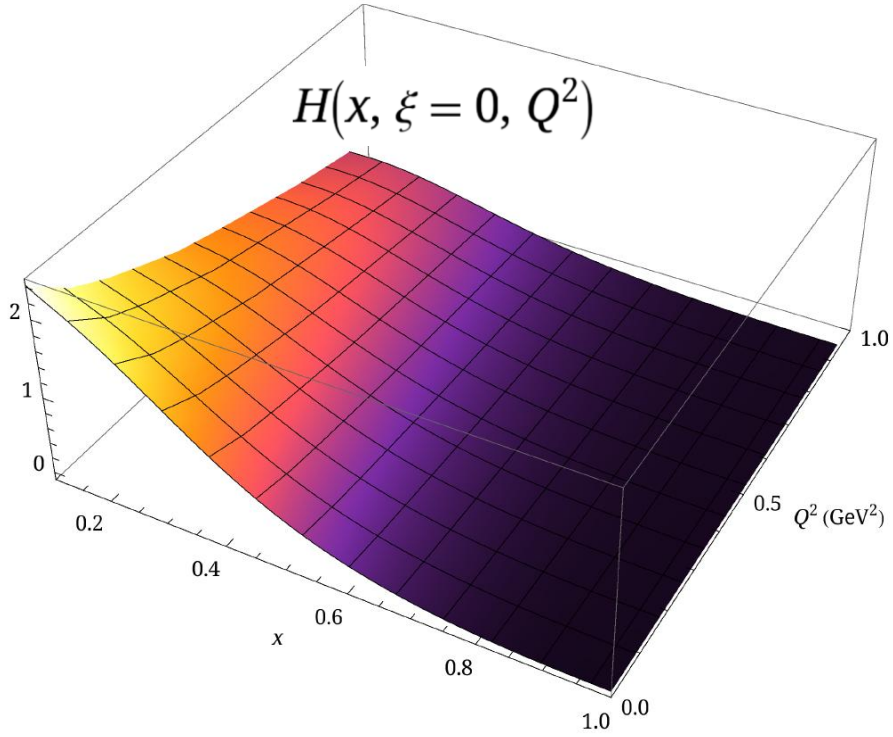
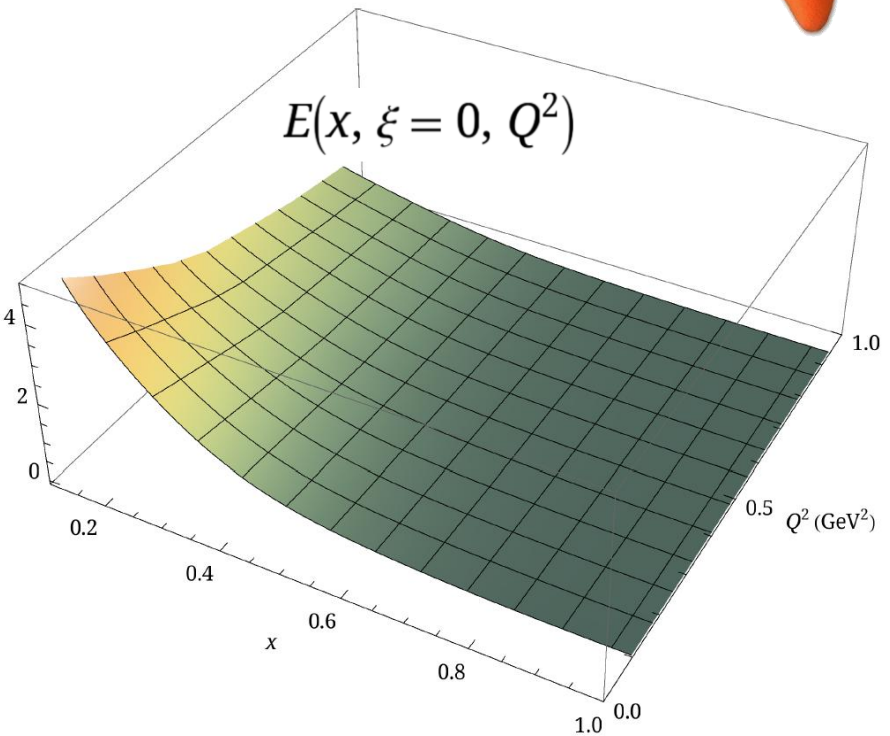




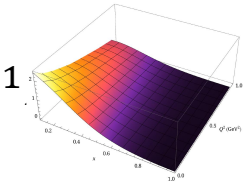
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

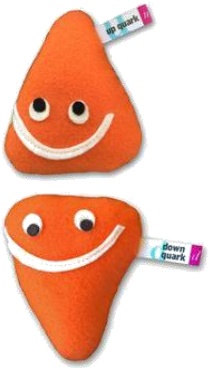
Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

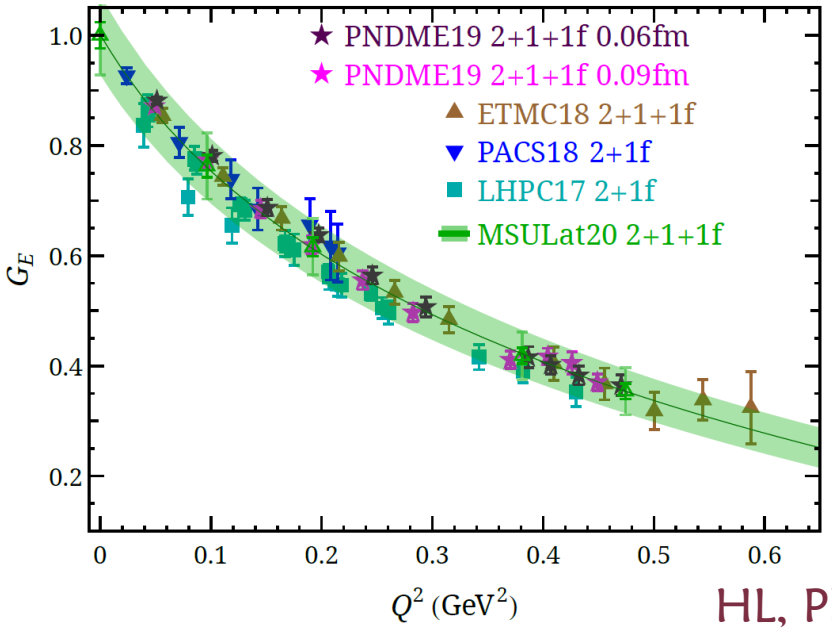
- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$





$n = 1$



Checked for the very first time on the Lattice!

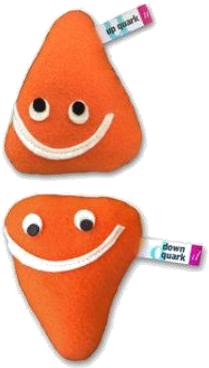
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

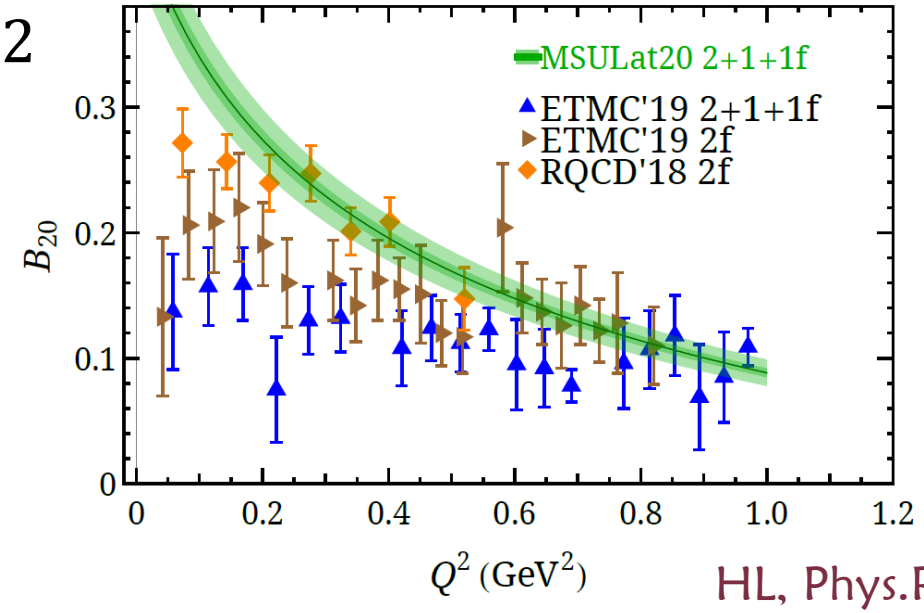
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx \mathbf{2}$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} \; = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 2$



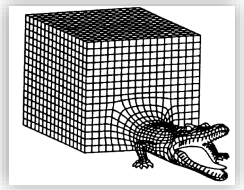
Checked for the very first time on the Lattice!

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

First LQCD Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

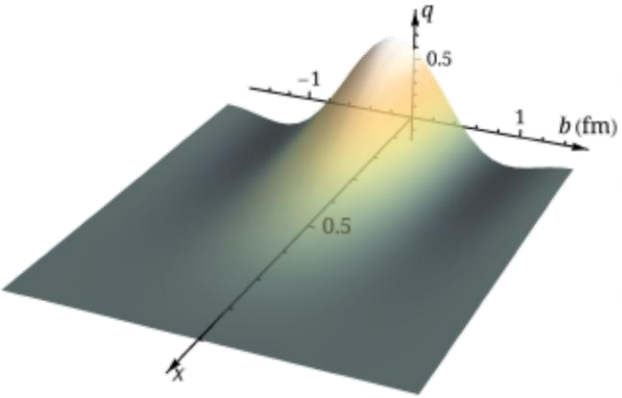
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

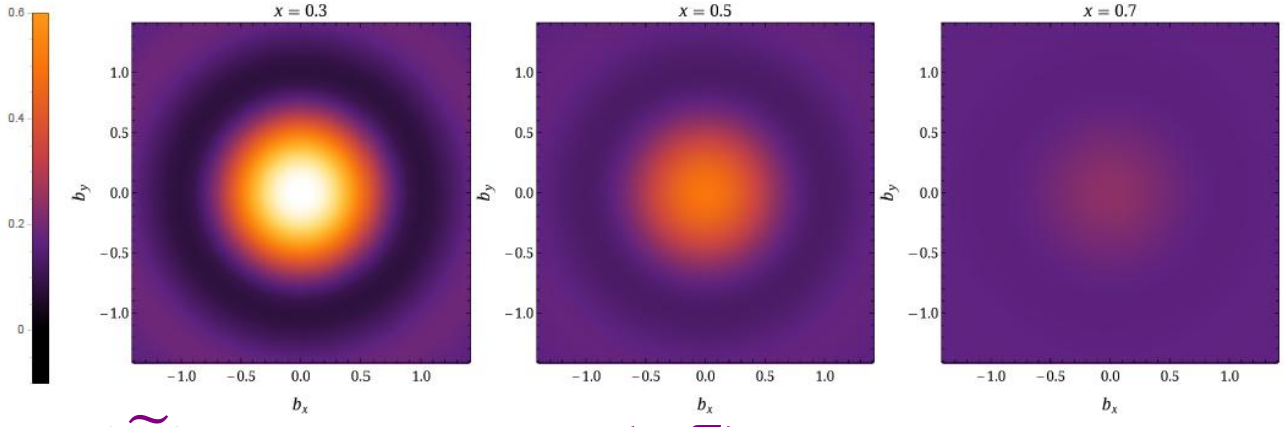


finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$

HL, Phys. Rev. Lett. 127 (2021) 18, 182001





§ Nucleon helicity GPD ($\widetilde{H}$) and pion GPD (H^{π}) using quasi-PDFs at **physical pion mass**

HL (MSULat), Phys.Lett.B 824 (2022) 136821;
Phys. Lett. B 846 (2023) 138181

Tomography @ Physical Pion Mass

§ Lots of borations

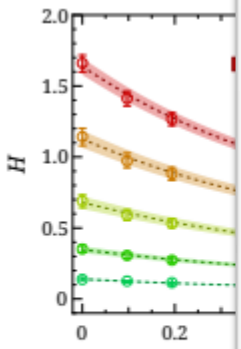
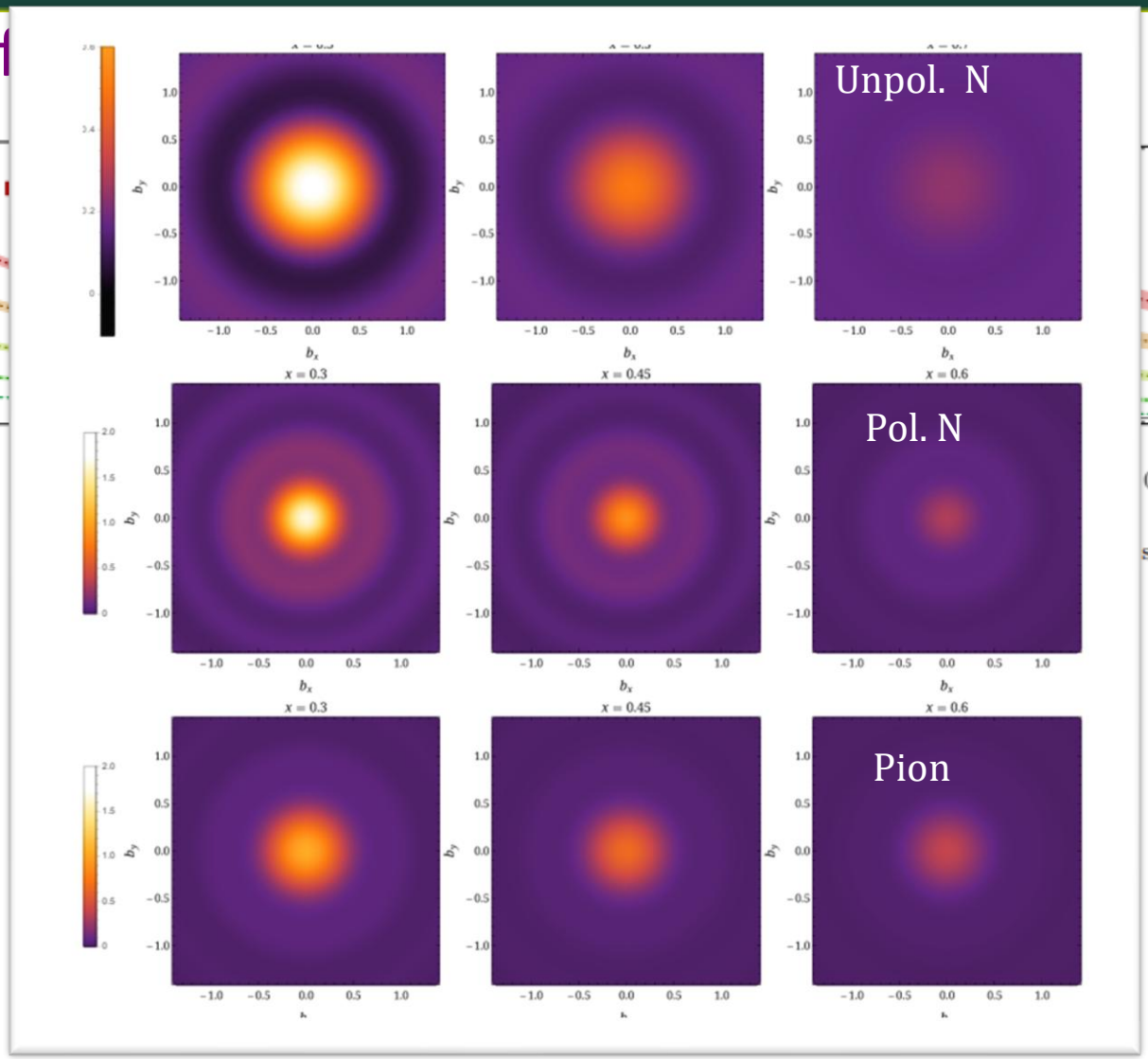
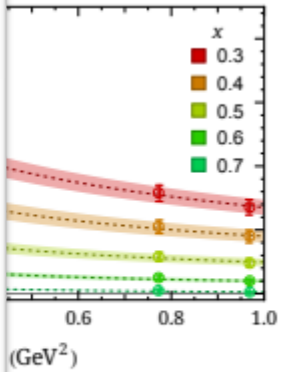


Figure 4.1:





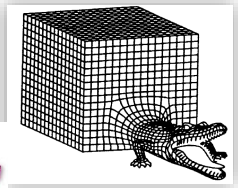
selected x values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

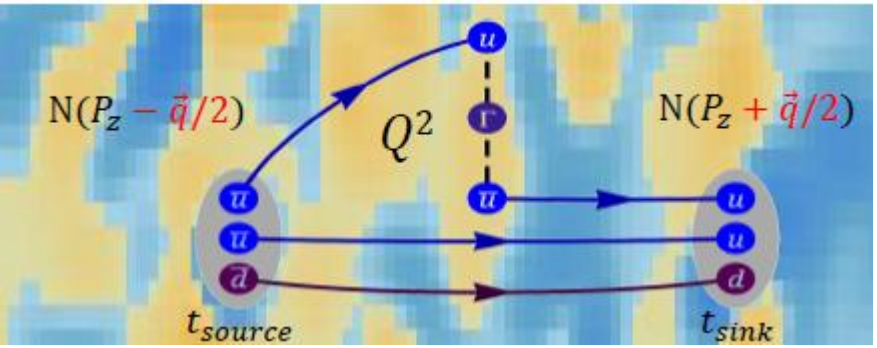
Asymmetric-Frame GPD

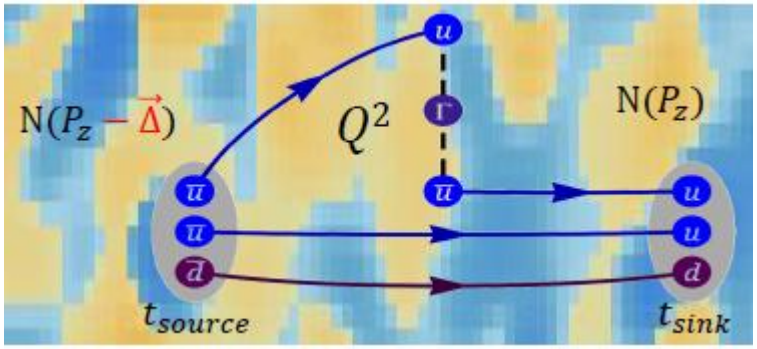
§ New calculations by ANL/BNL/ETMC using asymmetric frame

- 2+1+1 twisted-Wilson, 0.09 fm, $P_z \approx \mathbf{1.3}$ GeV
  260-MeV pion, one source-sink used
- $\xi = 0$ isovector nucleon GPD results



finite-volume, discretization, *heavy quark, excited-state, …*





$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\boldsymbol{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \boldsymbol{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1} + mz^\mu\boldsymbol{A_2} + \frac{\Delta^\mu}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

ANL/BNL/ETMC, 2209.05373 ,2310.13114

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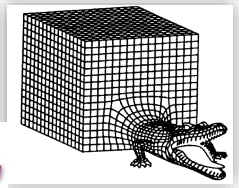

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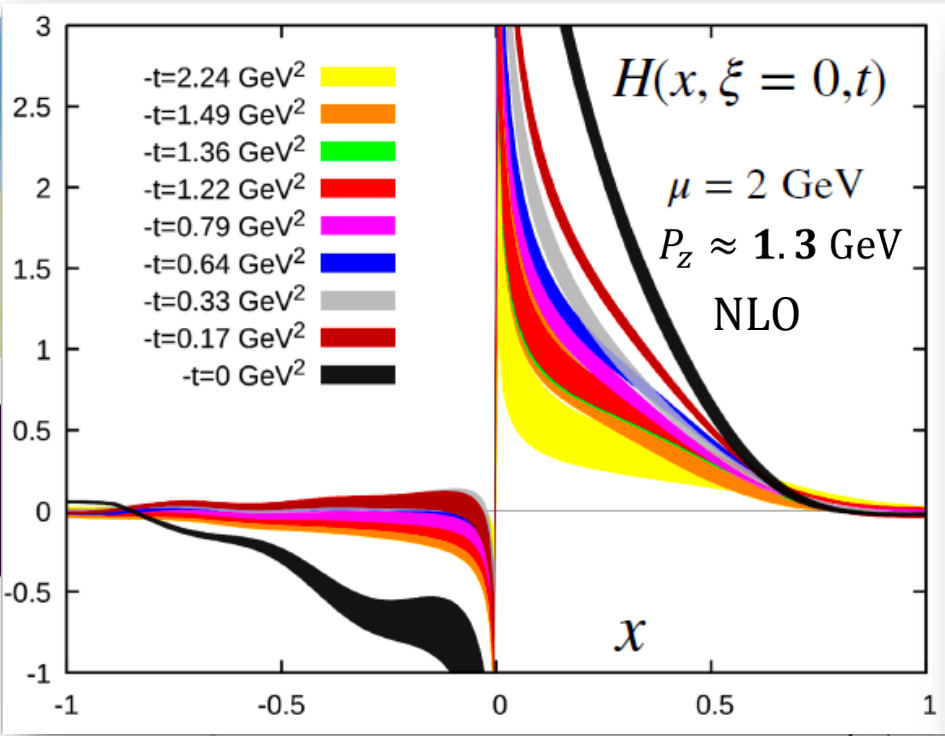

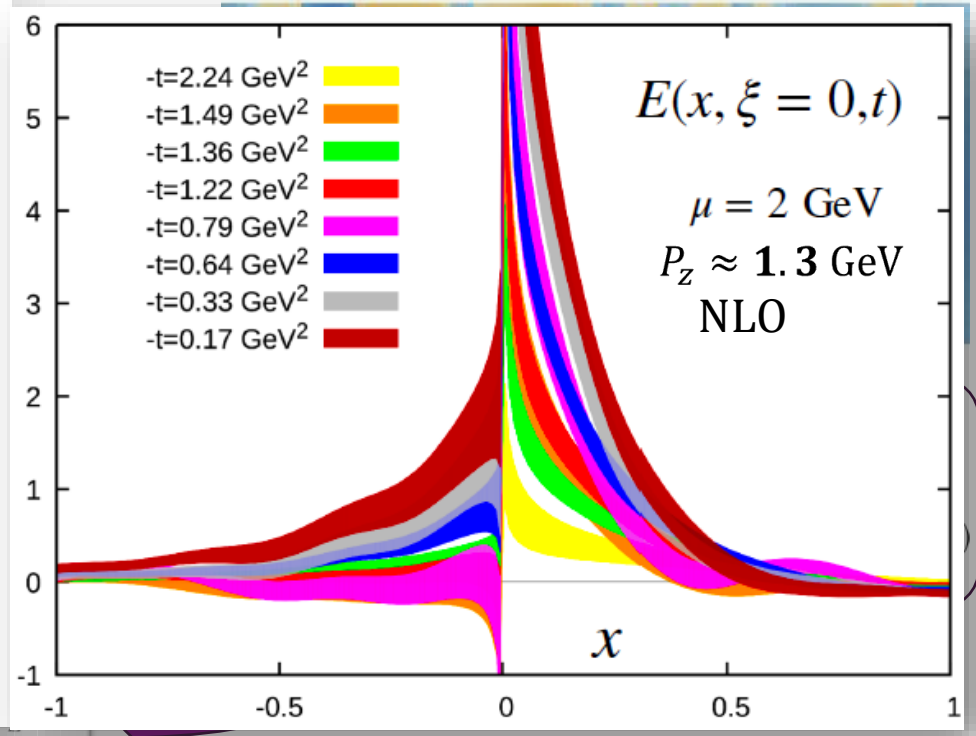

See Shohini Bhattacharya' talks@ Lattice 2024

ANL/BNL/ETMC, 2209.05373 ,2310.13114

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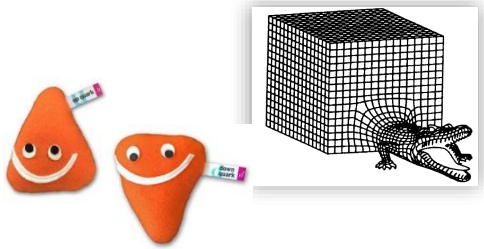

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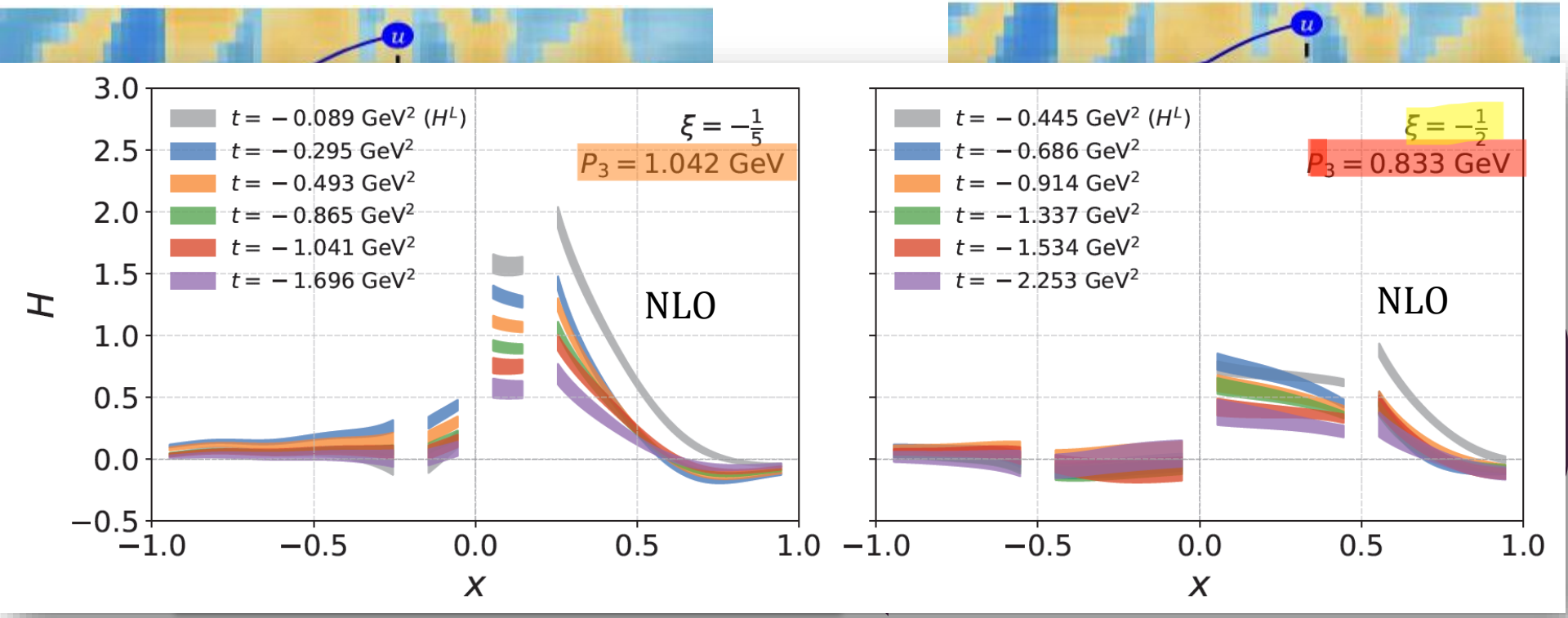


ETMC, 2508.17998

GPD Systematic Update

§ Nucleon isovector GPDs calculated directly at physical pion mass

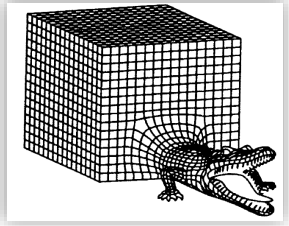
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

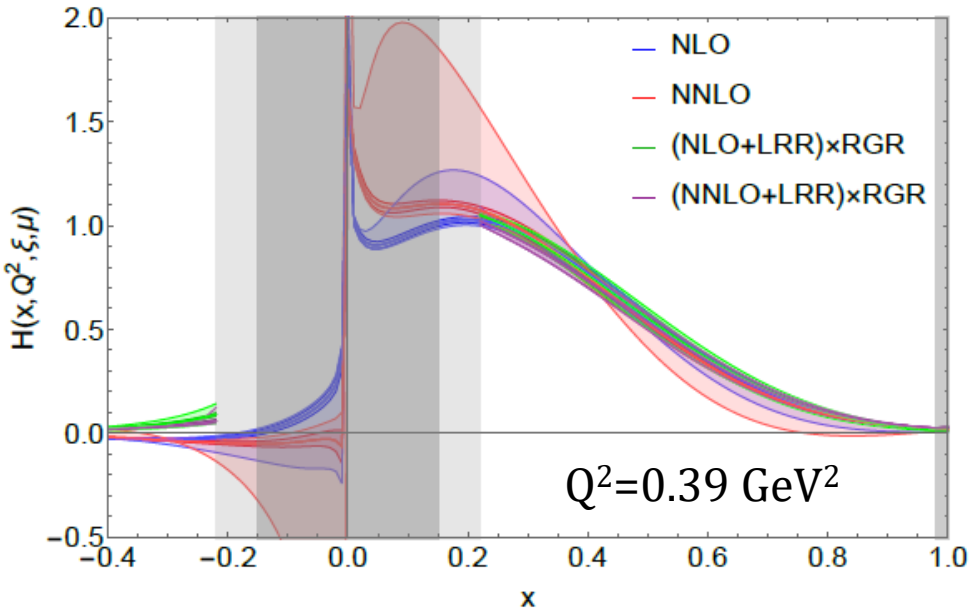
R. Zhang, et. al.
PLB 844, 138081 (2023)

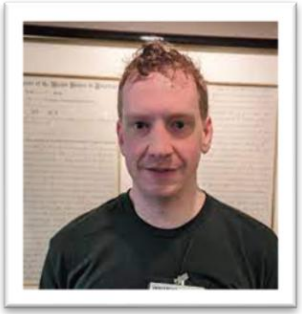
- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Wanted PDFs, GPDs, etc…







P: Jack Holligan

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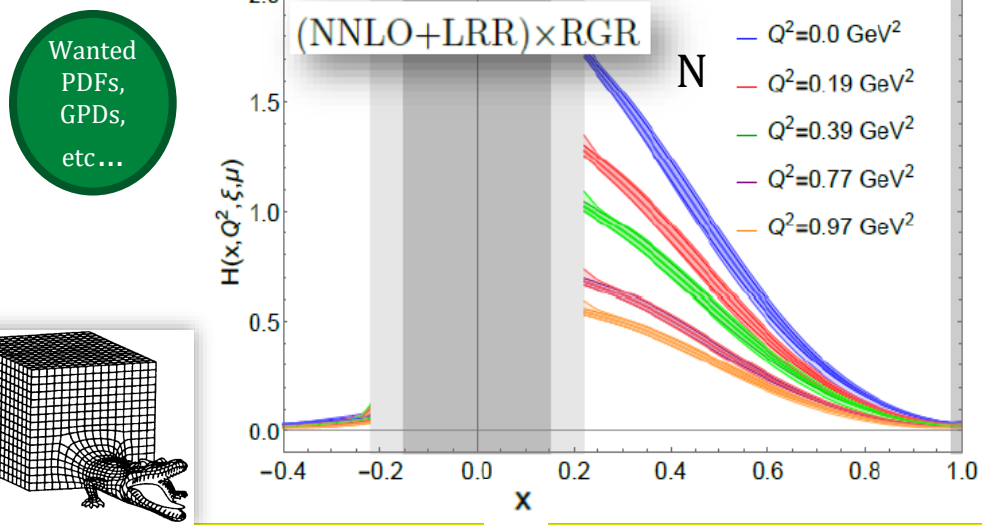

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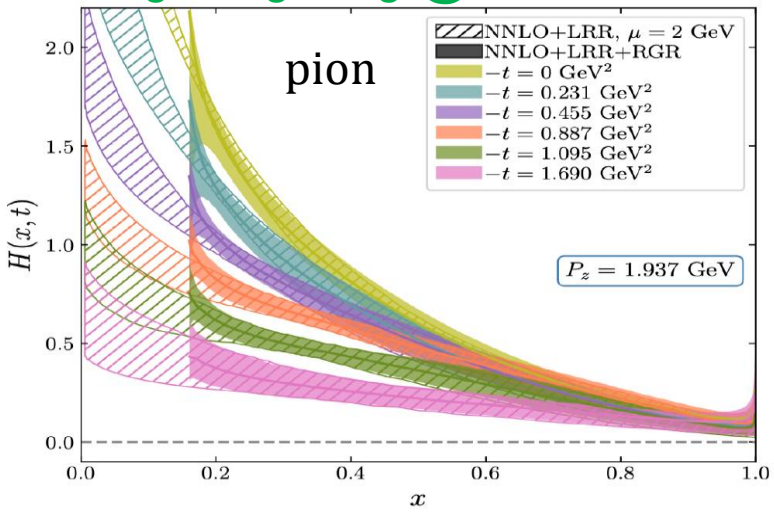
  H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]
  Heng-Tong Ding @ Tue. 9:30 AM



GPD Systematic Update

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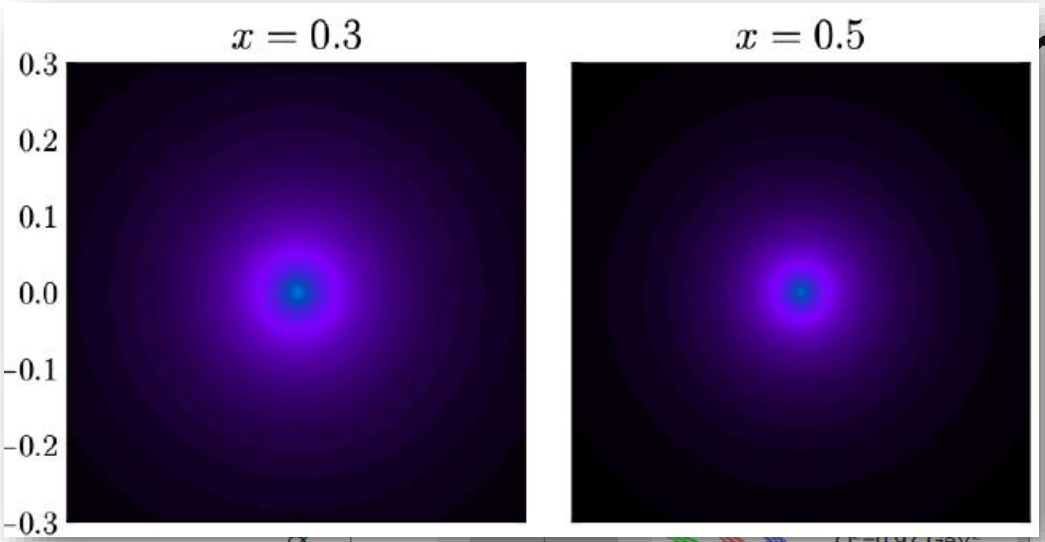

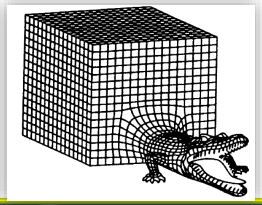

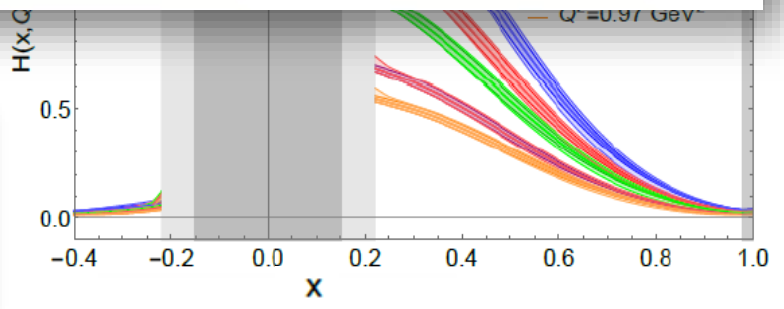

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H. Ding et al (ANL/BNL/Wuhan), 2407.03516 [hep-lat]

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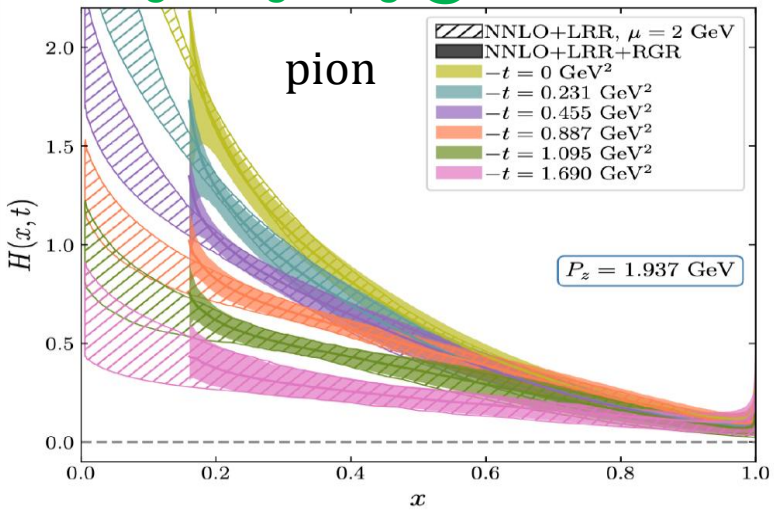


$\xi \neq 0$ GPDs

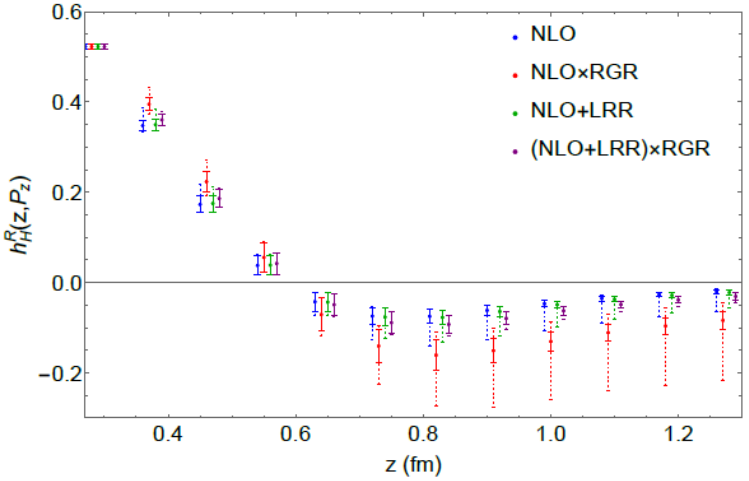
§ Only the NLO matching kernel is available

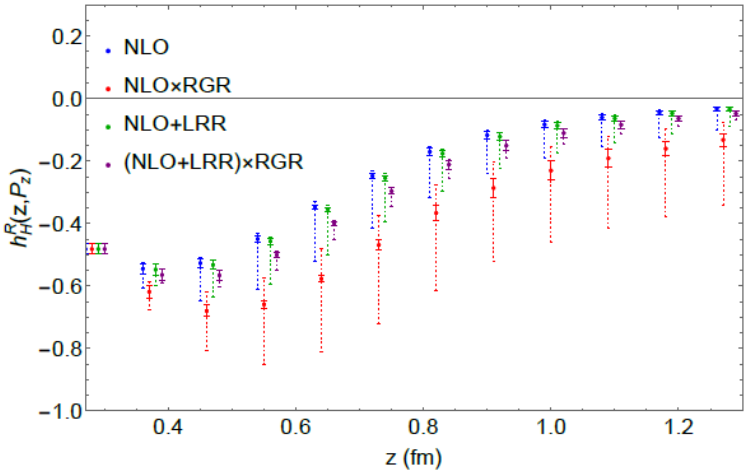
$$
\begin{aligned}
&\mathcal{K}(x, y, \mu, \xi, P_z) \\
&= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\Bigg[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4y^2(\xi+x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4y^2(\xi-x)^2P_z^2}{\mu^2}\right)-1\right) \\
&+ \left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4y^2(x-y)^2P_z^2}{\mu^2}\right)-1\right)\Bigg]
\end{aligned}
$$

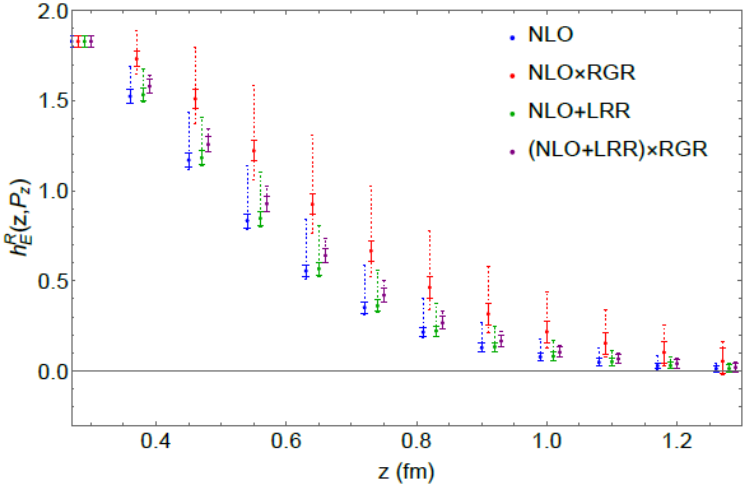
Fei Yao, Yao Ji & Jian-Hui Zhang, JHEP 11(2023) 021

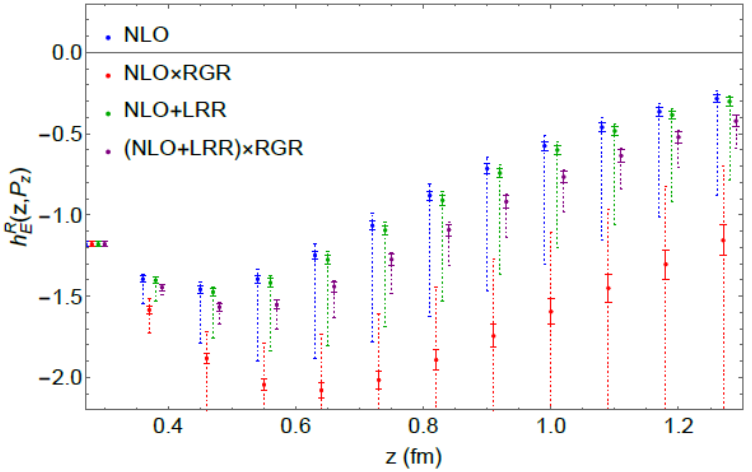
$\xi \neq 0$ GPDs

§ Physical pion mass; NLO $\xi = 0.1$, $Q^2 = 0.23$ GeV^2









Quantities that can be calculated on the lattice

J. Holligan, HL (MSULat),2312.10829 [hep-lat]

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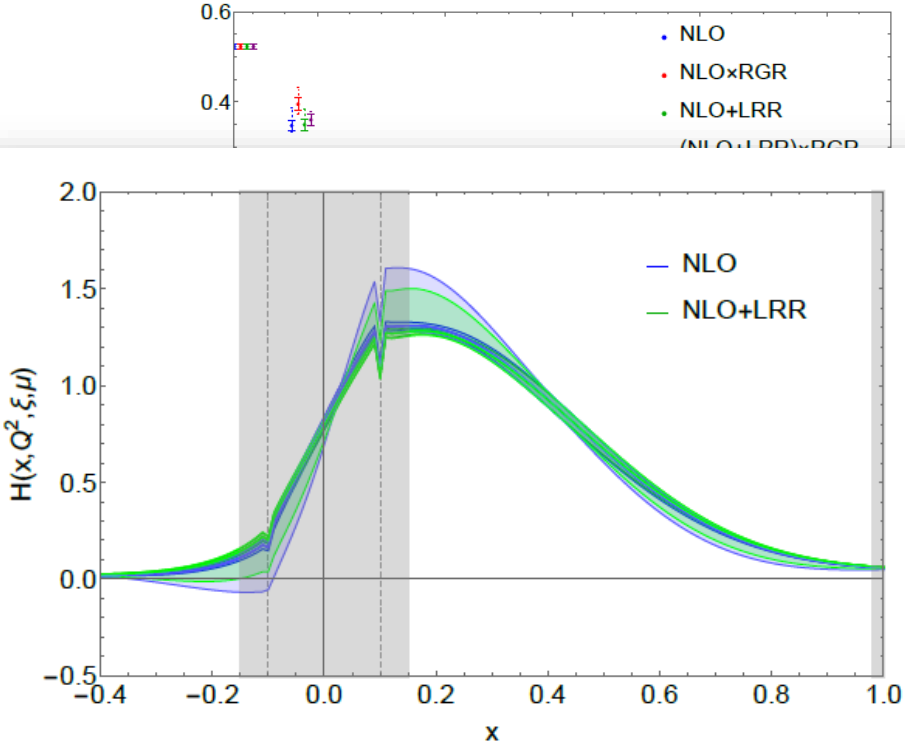
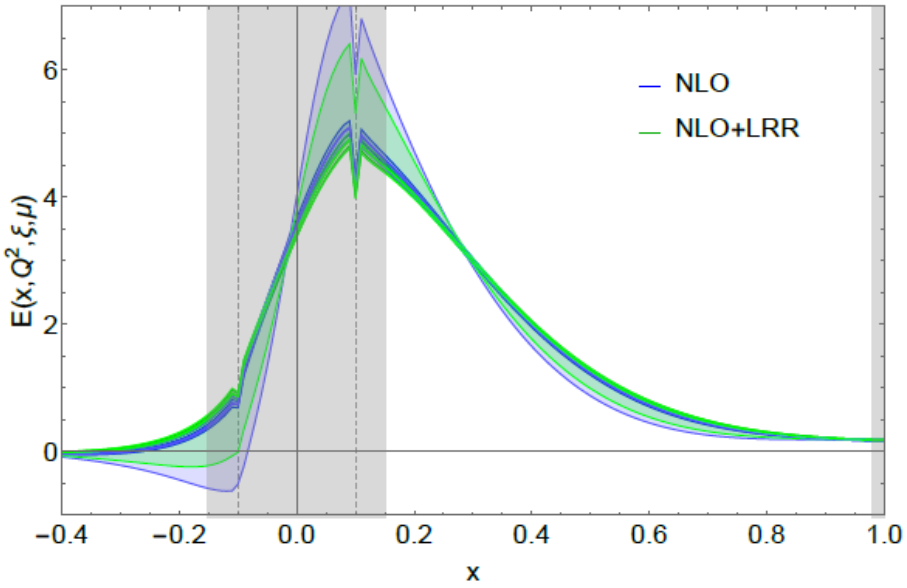
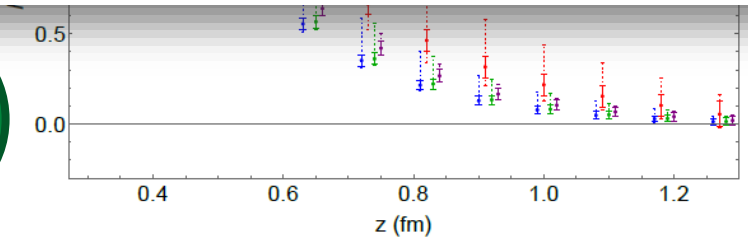
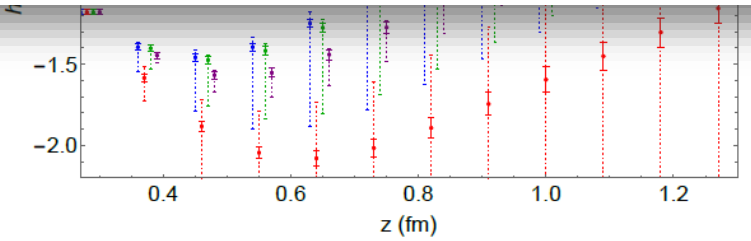




Wanted PDFs, GPDs, etc…

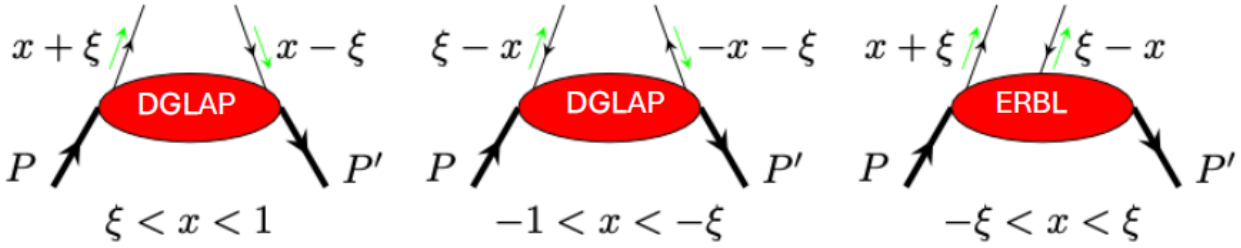




J. Holligan, HL (MSULat),2312.10829 [hep-lat]

Large logarithms in Matching

J. Holligan, HL, Rui Zhang, Y. Zhao, JHEP 207 (2025) 241

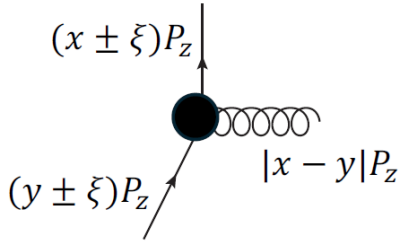


Slide by Rui Zhang (MIT)

Difficulty

Multi-scale nature

- Outgoing quark $(x+\xi)P_z$
- Incoming quark $(x-\xi)P_z$
- Gluon emission $|x-y|P_z$



Three different logarithms cannot be resummed with only one RG equation

Irrelevant logarithms

Quark log can be suppressed

- $\frac{|\xi+x|}{(\xi+y)} \ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right) \xrightarrow{\xi+x\to 0} 0$
- $\frac{|\xi-x|}{(\xi+y)} \ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right) \xrightarrow{\xi-x\to 0} 0$

ERBL region:

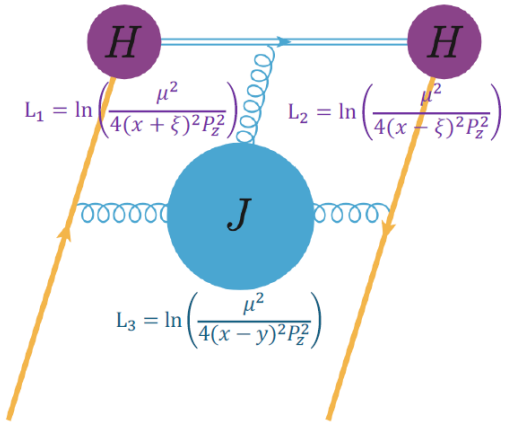
Only in the threshold limit (soft gluon emission $x \to y$), the log terms diverge

DGLAP region:

Logs diverge when $\xi \to 0$, but same as above when $\mu = 2xP_z$

Threshold Factorization

$$C(x,y,\xi,P_z) \xrightarrow{x\to y} J(|x-y|P_z) \otimes H((x\pm\xi)P_z)$$



Threshold log: soft gluon $x \to y$

Large logarithms in Matching

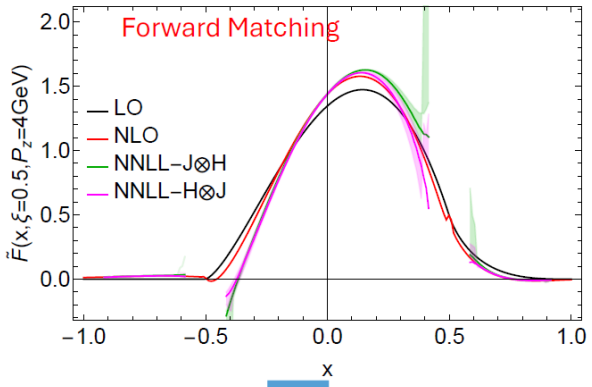
§ Application on toy GPD model

Slide by Rui Zhang (MIT)

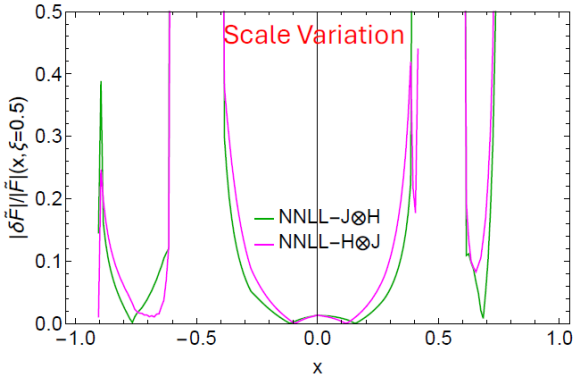
- Initial matching
  - $C_{\rm DGLAP}(\mu) = \hat{V}_C(2xP,\mu)C(2xP)$
  - $C_{\rm ERBL}(\mu) = \hat{V}_C(2\xi P,\mu)C(2\xi P)$
- Separate all scales
  - $C \xrightarrow{x\to y} H\otimes J$ (or $J\otimes H$)
- Solving RG equations
  - $H_{\rm TR}(\mu) = \hat{V}_H(\mu_h,\mu)H(\mu_h)$
  - $J_{\rm TR}(\mu) = \hat{V}_J(\mu_i,\mu)J(\mu_i)$
- Identifying physical scales
  - $\mu_{h_1} = 2|\xi + x|P$
  - $\mu_{h_2} = 2|\xi - x|P$
  - $\mu_i = 2\min\{|x|, |\xi \pm x|\}P$
- Correct Full Matching

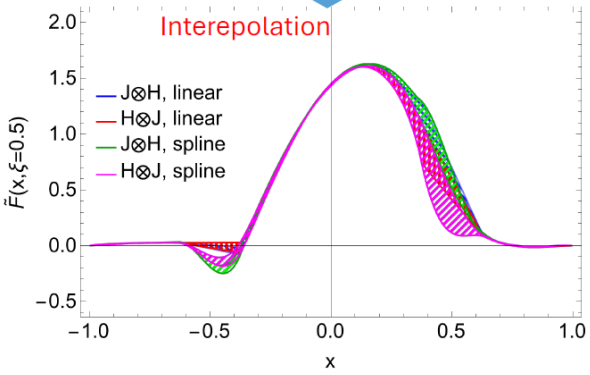
$$C_{\rm TR} = (H\otimes J)_{\rm TR}\otimes (H\otimes J)^{-1}_{\rm NLO}\otimes C_{\rm NLO}$$

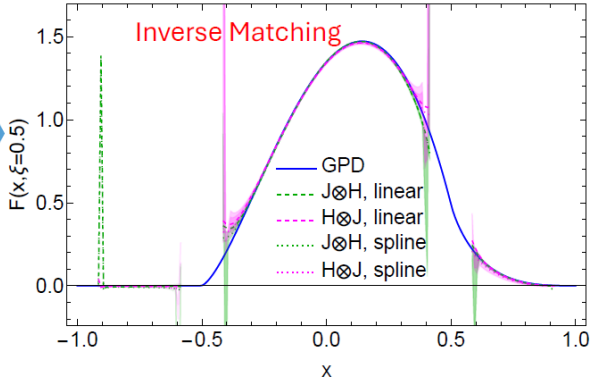
Test on GPD model



$x_0 \propto \Lambda_{\rm QCD}/P_z$





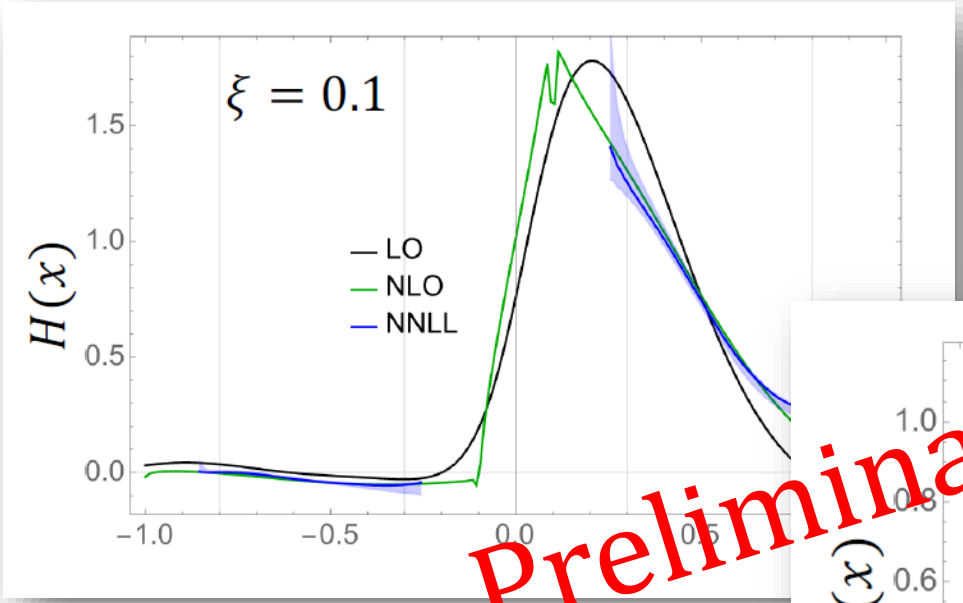


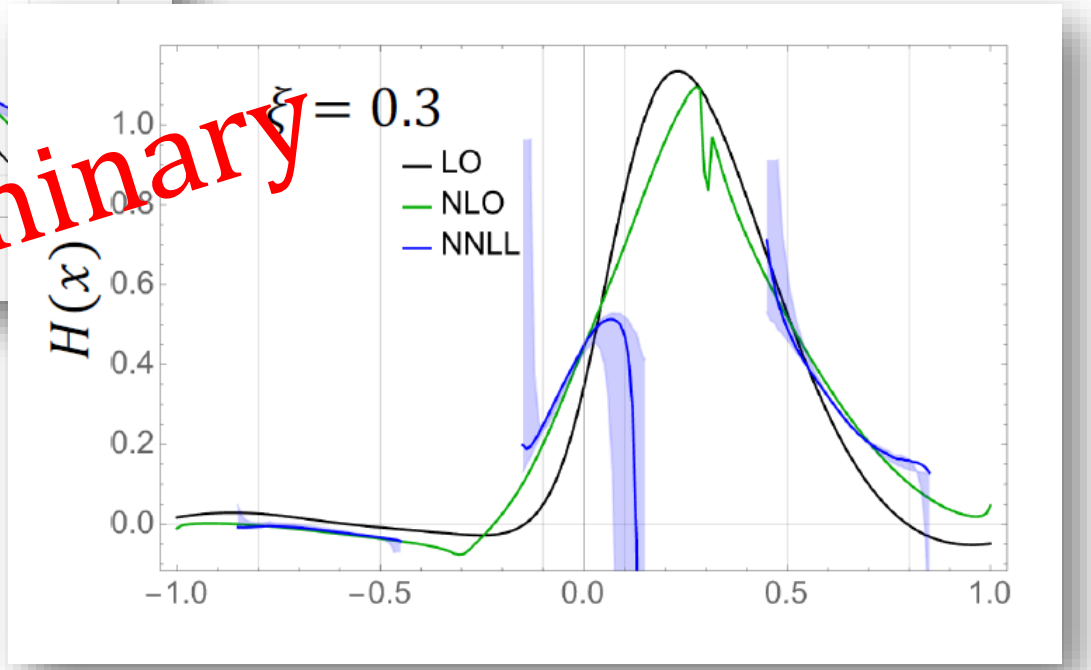
J. Holligan, HL, Rui Zhang, Y. Zhao, JHEP 207 (2025) 241

Large logarithms in Matching

§ First application to lattice data at physical pion mass

plots by Rui Zhang (MIT)





Preliminary

Lattice Progress & Challenges

§ Not enough time to cover the following

- Nucleon isovector form factors and generalized form factors (GFFs) using pseudo-PDF and traditional OPE
- (LO) Twist-3 GPDs by ETMC

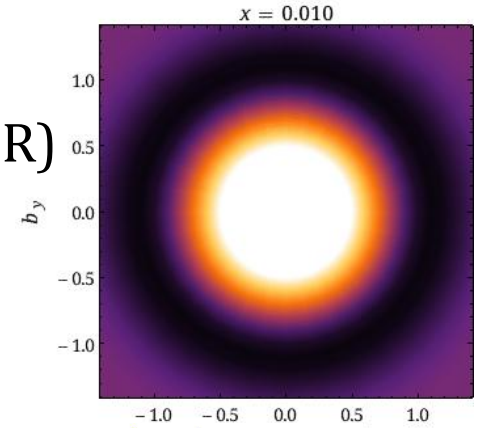
§ Challenges ahead for precision PDFs/GPDs

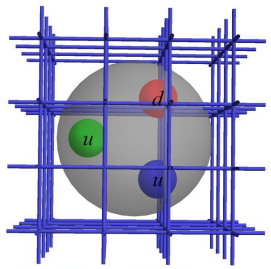
- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Access small-x physics; some methods have inverse problem in PDF extraction, more computational resources, etc.

For more details and references, refer to 2202.07193

Summary and Outlook

- § Exciting era using LQCD to study nucleon and pion GPDs
  - Overcoming longstanding obstacles: Bjorken-*x* dependent GPDs
- § Much progress in GPDs on the lattice
  - Widely studied with LaMET and its variants
  - Improvements: NNLO GPDs and systematics (LRR+RGR)
  - Recent progress made in nonzero-skewness GPDs
- § Precision and progress are limited by resources
  - Challenges = new opportunities



§ **Theory Input** → **Global Analysis of GPDs** ← **Exp't Input** + 

EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
The work of HL is sponsored by NSF grant PHY 2209424 & 1653405, DOE grant DE-SC0024053 & RCSA Cottrell Scholar Award

Students Wanted

LGT4HEP website: https://lgt4hep.github.io/

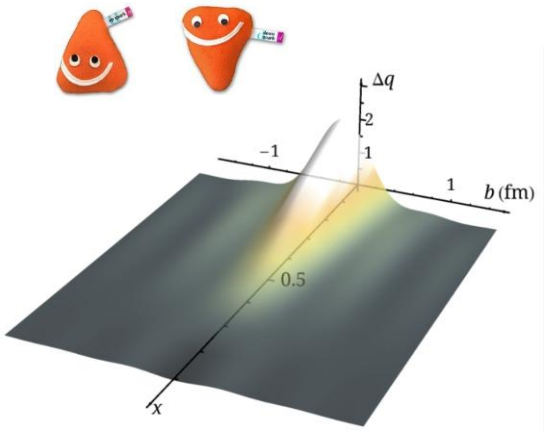
Backup Slides

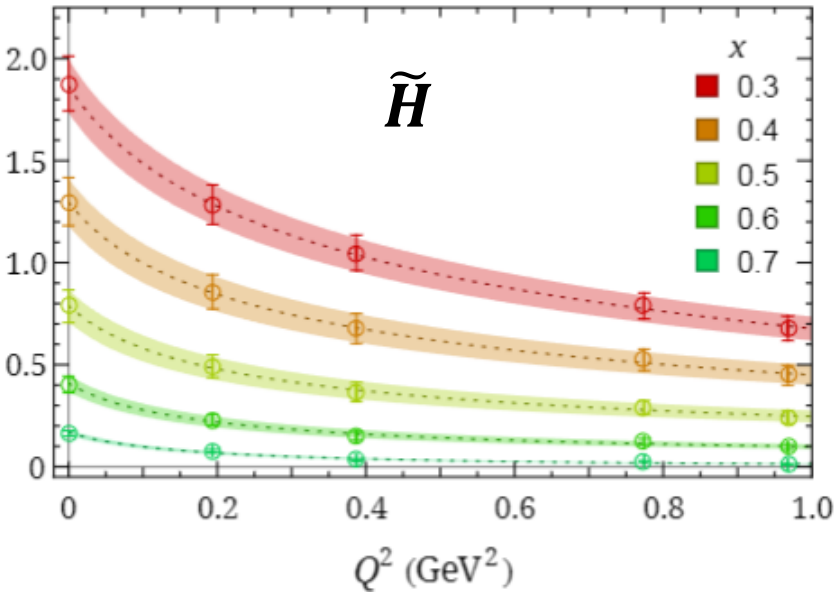
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

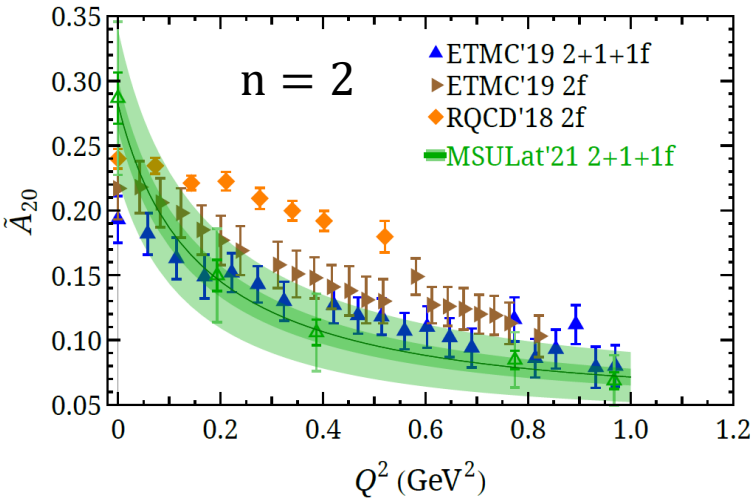
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





- Take the integral to form moments



Hadron Tomography

§ Lots of progress on tomography by many collaborations

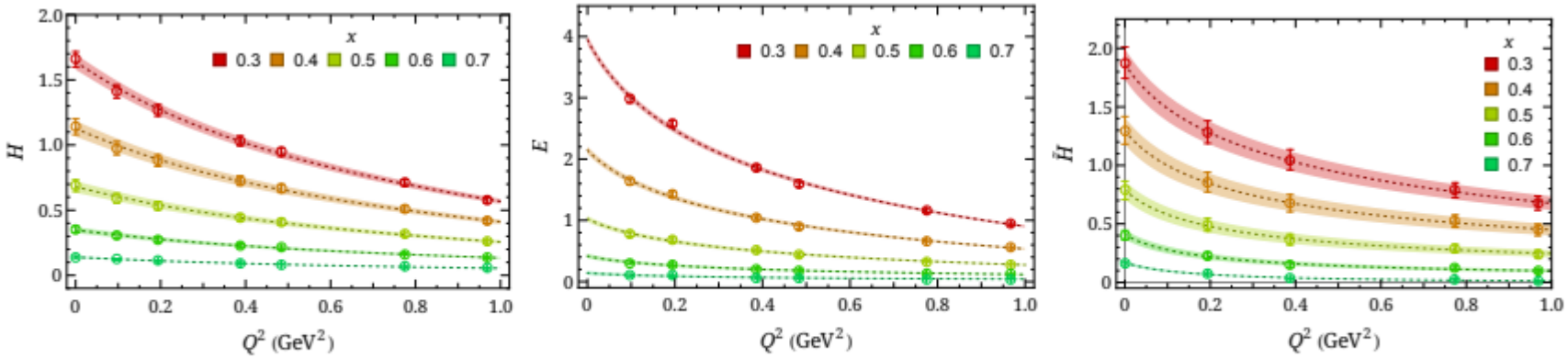
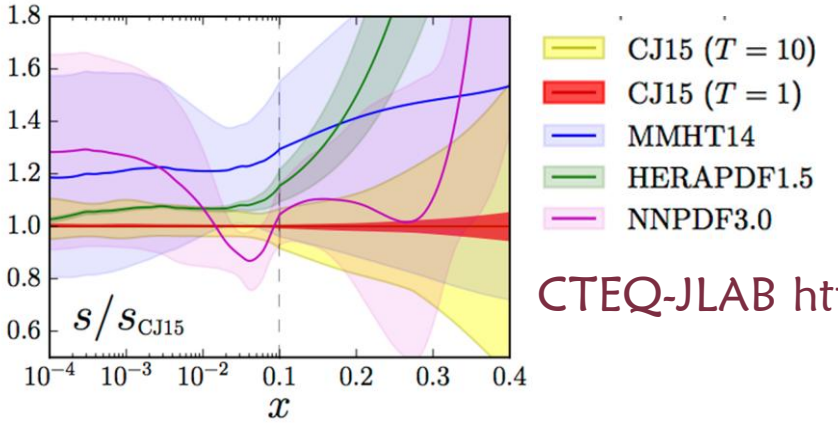


Figure 4.1: Nucleon isovector H (left), E (middle) and $\tilde{H}$ (right) GPDs at $\xi = 0$ with z-expansion to Q^2 at selected x values.

HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

First Lattice Strange PDF

§ Large uncertainties in global PDFs



- Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

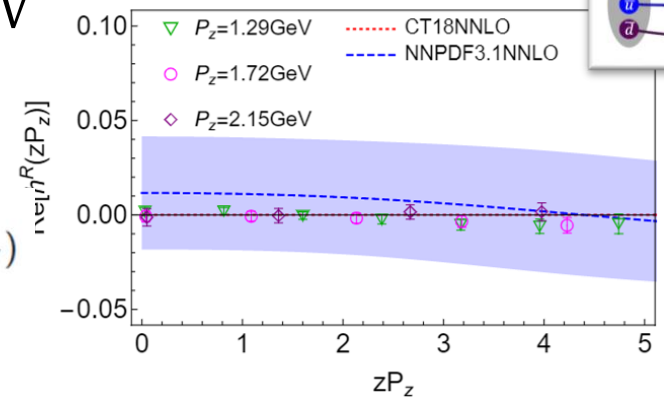
First Lattice Strange PDF

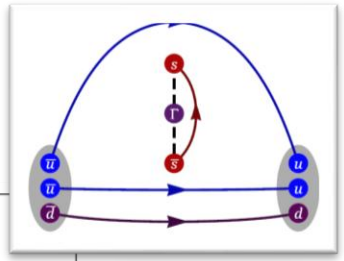
§ Results by MSULat/quasi-PDF method

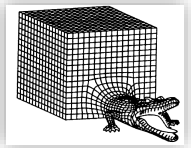
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$





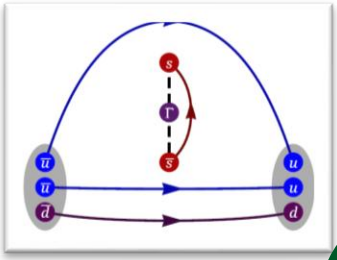


Quantities that can be calculated on the lattice

Lattice Strangeness Asymmetry Impact

§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV, $P_z \approx 1.7$ GeV



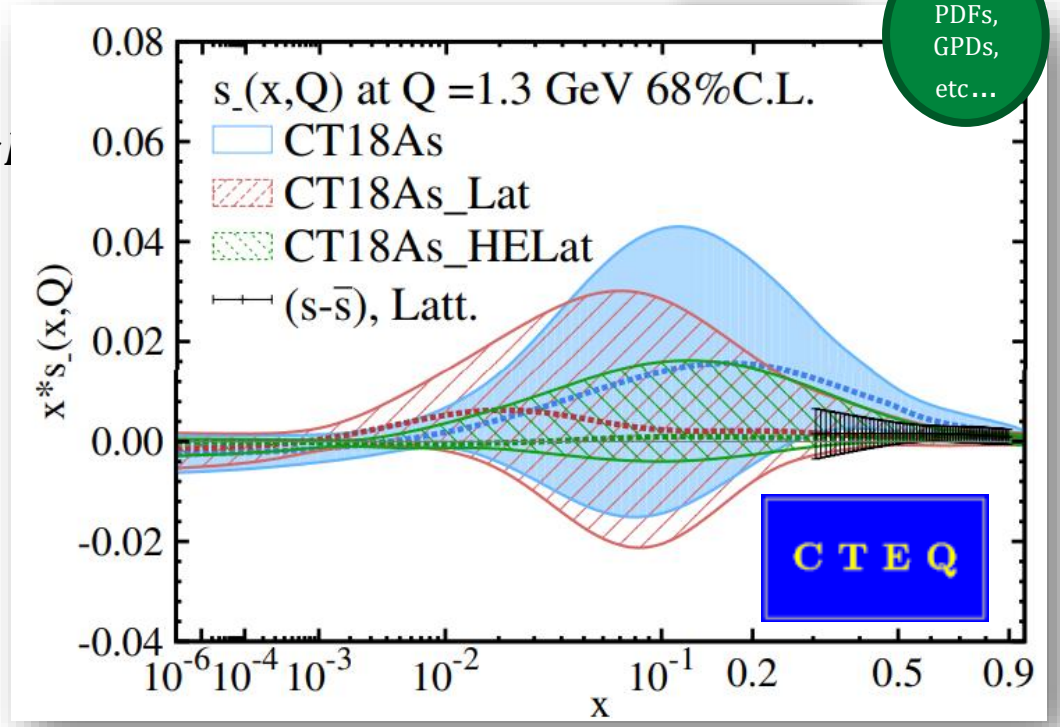
R. Zhang et al (MSULat), 2005.01124

$$\mathrm{Re}[h(z)] \propto \int \mathrm{d}x \left(s(x) - \bar{s}(x)\right) \cos(xz$$

§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{x^2 P_z^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-x)^2 P_z^2}\right)$$

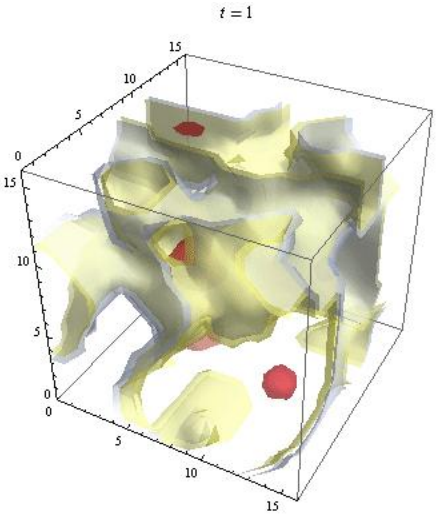
T. Hou, HL, M. Yan, C. Yuan, 2211.11064

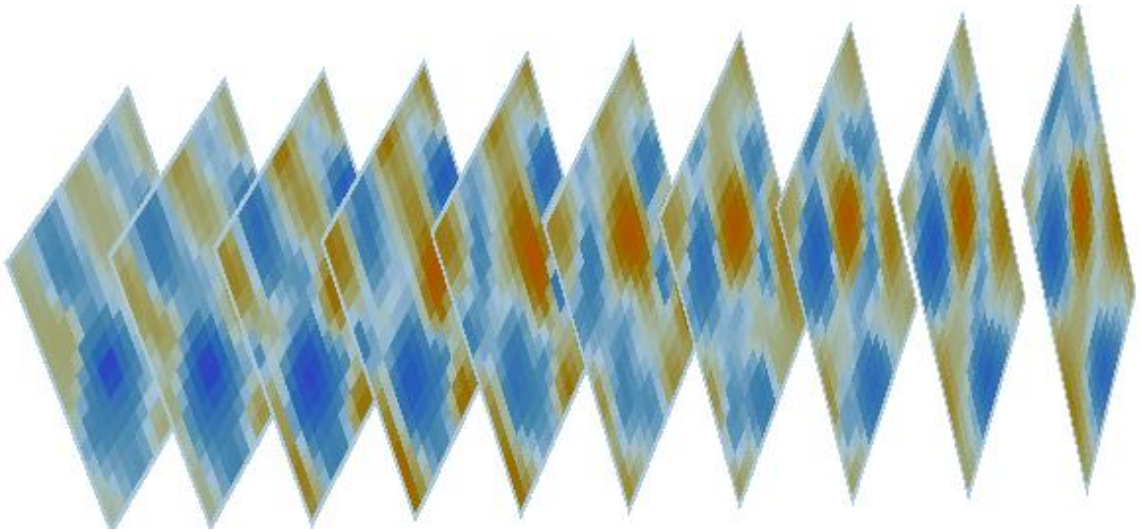


§ The strangeness asymmetry $s(x, Q) - \bar{s}(x, Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

Anatomy of a Lattice Calculation

1. Start with QCD Vacuum (gauge configurations)



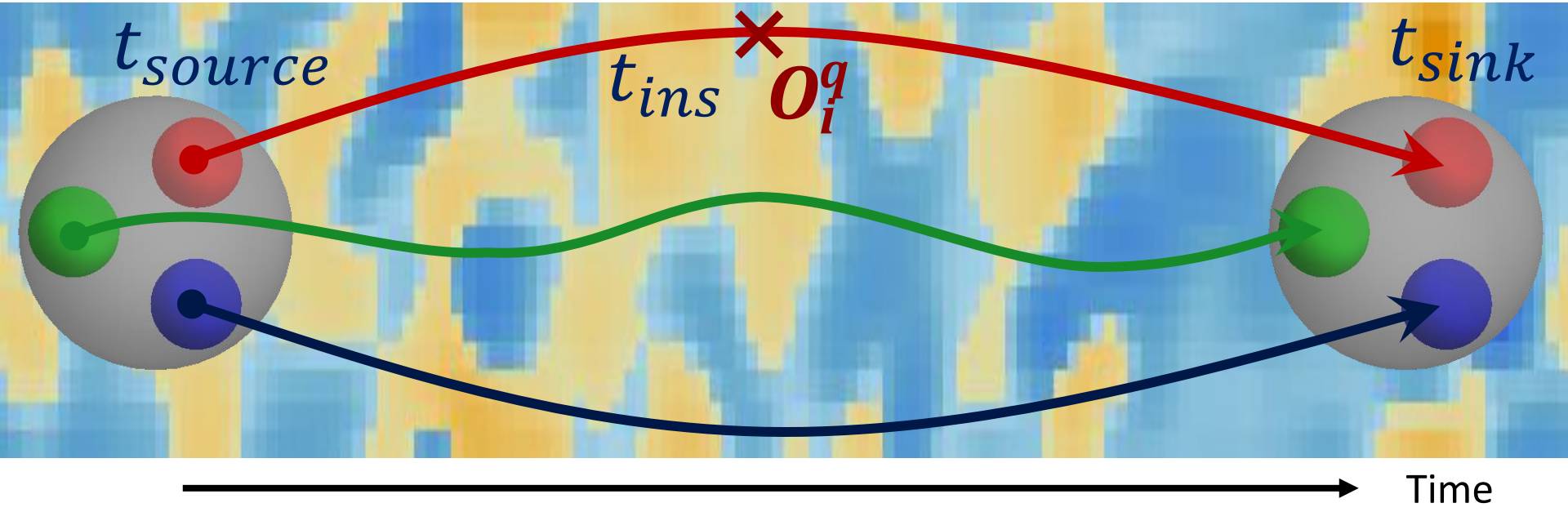


§ Pick a QCD vacuum

- Gauge/fermion actions, flavor (2, 2+1, 2+1+1), m_π, a, L, …

Anatomy of a Lattice Calculation

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

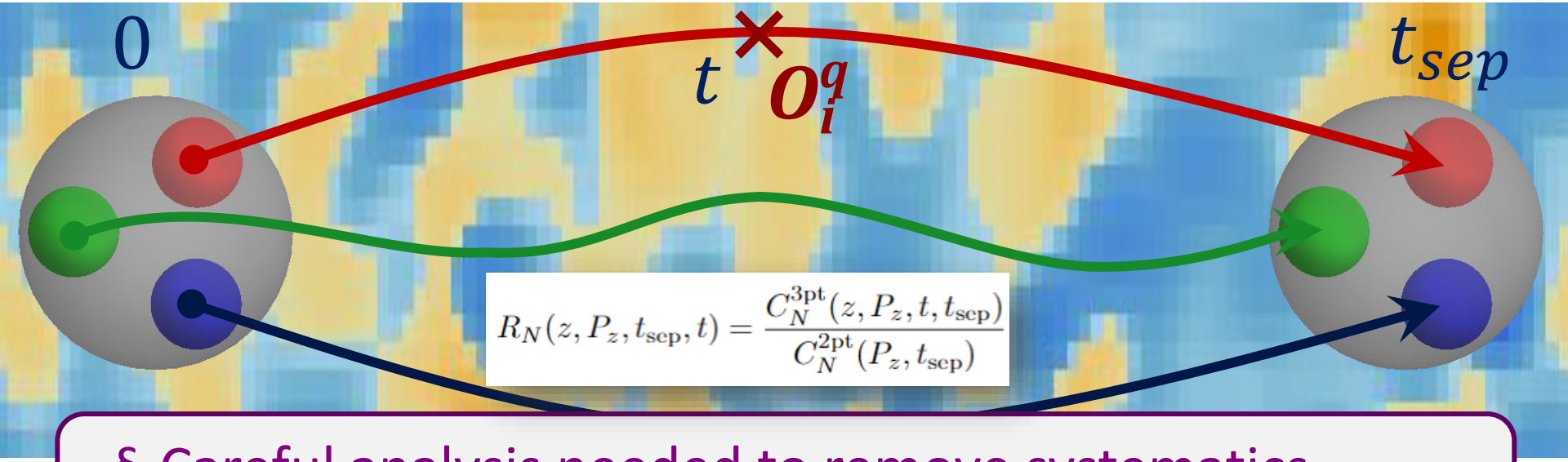


§ Construct correlators (hadronic observables)

- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Anatomy of a Lattice Calculation

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

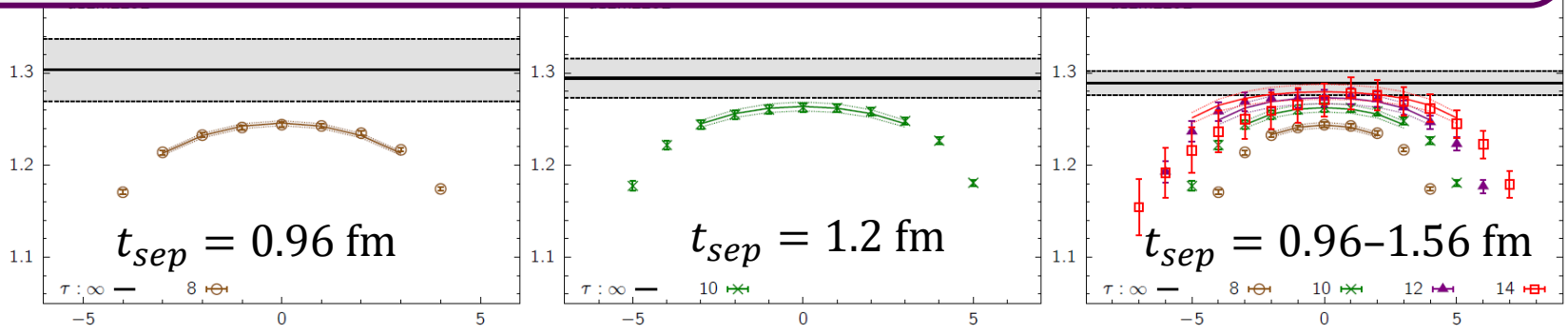


$$R_N(z, P_z, t_{\rm sep}, t) = \frac{C_N^{\rm 3pt}(z, P_z, t, t_{\rm sep})}{C_N^{\rm 2pt}(P_z, t_{\rm sep})}$$

§ Careful analysis needed to remove systematics

- Wrong results if **excited-state systematic** is not under control

Ratio Plot

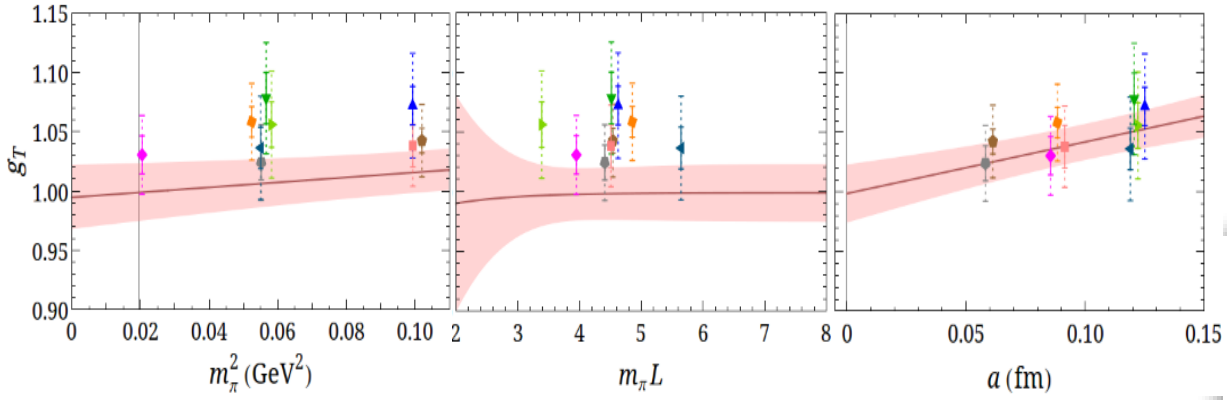


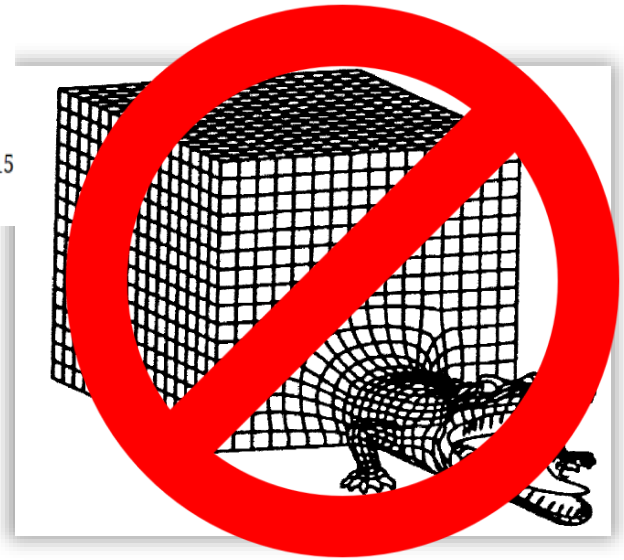
Anatomy of a Lattice Calculation

4. Systematic uncertainty (nonzero a, finite L, etc.)

- Nonperturbative renormalization, etc
- Extrapolation to the continuum limit

  $(m_\pi \to m_\pi^{\text{phys}},\ L \to \infty,\ a \to 0)$





Moments of PDFs

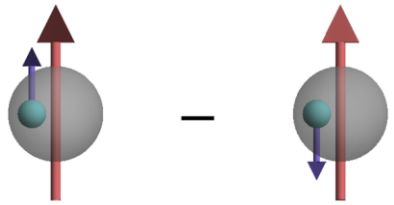
§ PDG-like rating system or average

§ LatticePDF Workshop

↝ Lattice representatives came together and devised a rating system

§ Lattice QCD/global fit status

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1 \rangle_{\delta u-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1 \rangle_{\delta d-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1 \rangle_{\delta s-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

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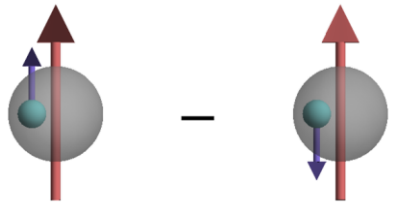

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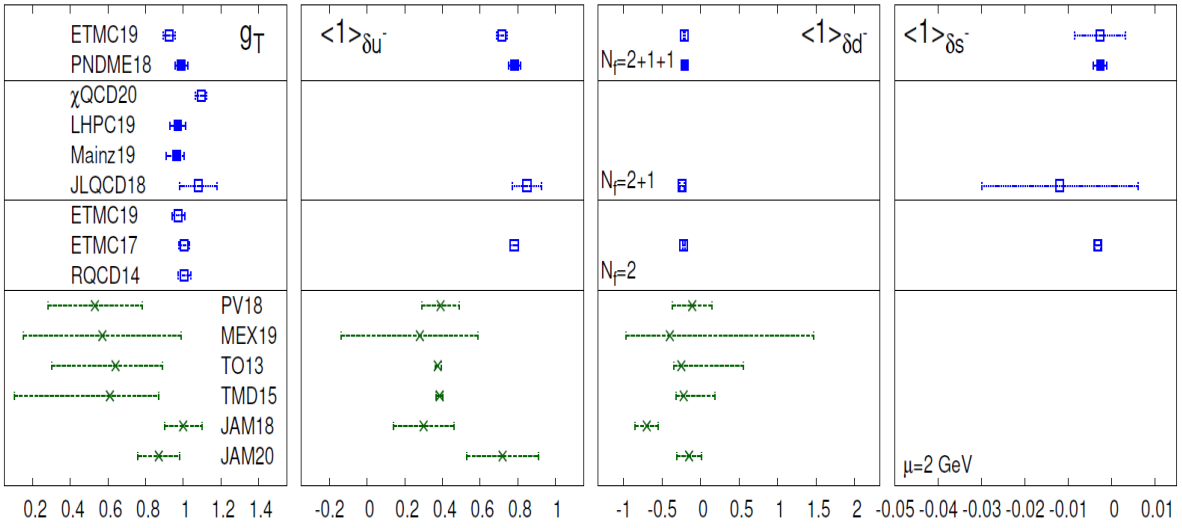
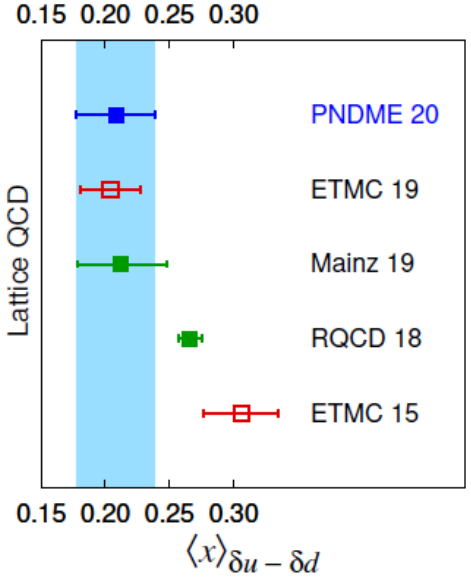


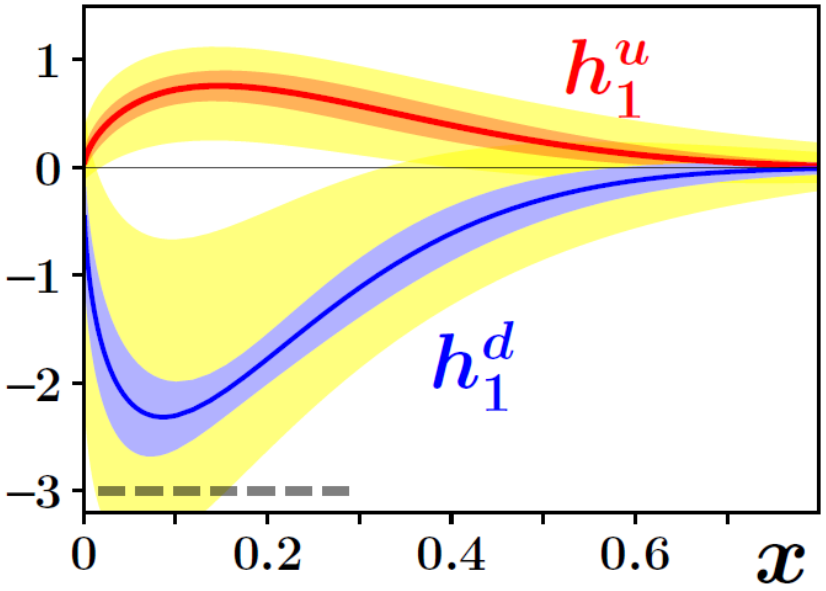


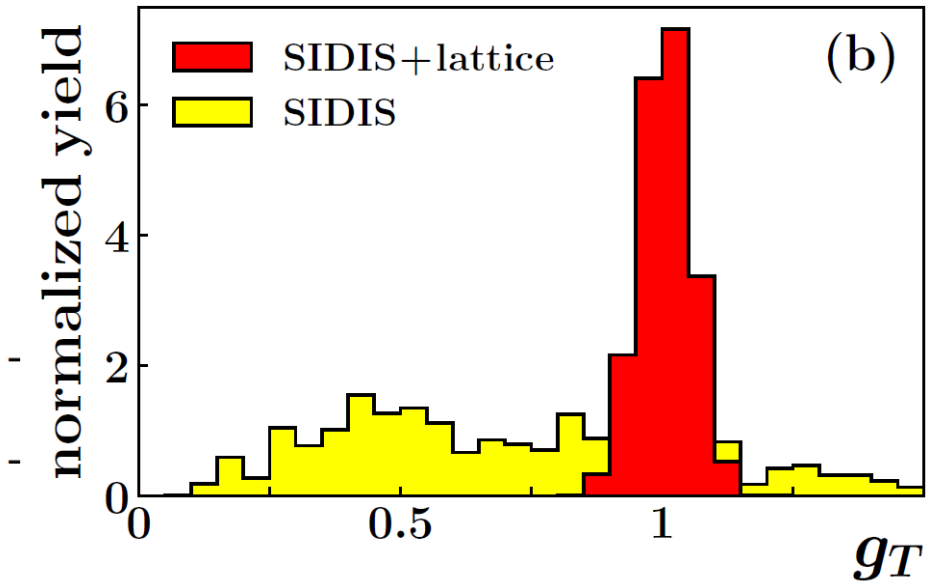
S. Mondal et al (PNDME), 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets





Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

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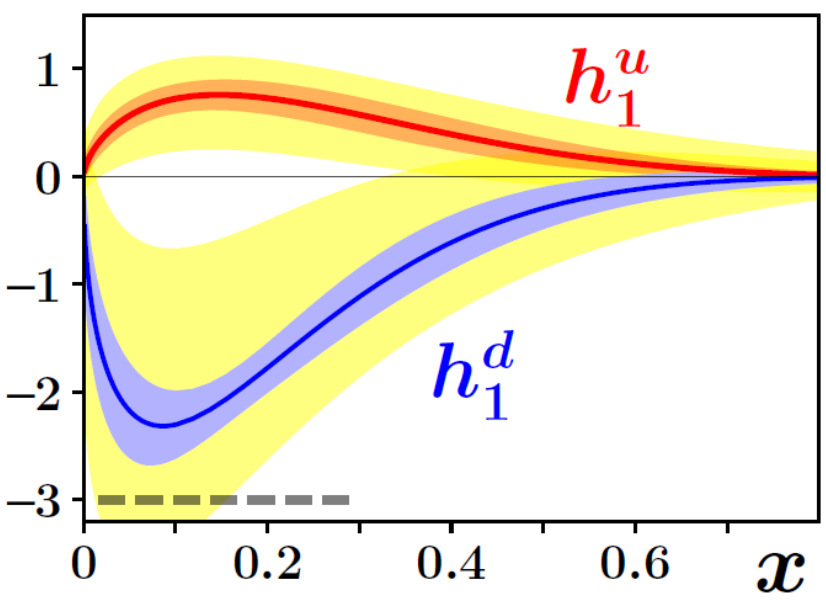
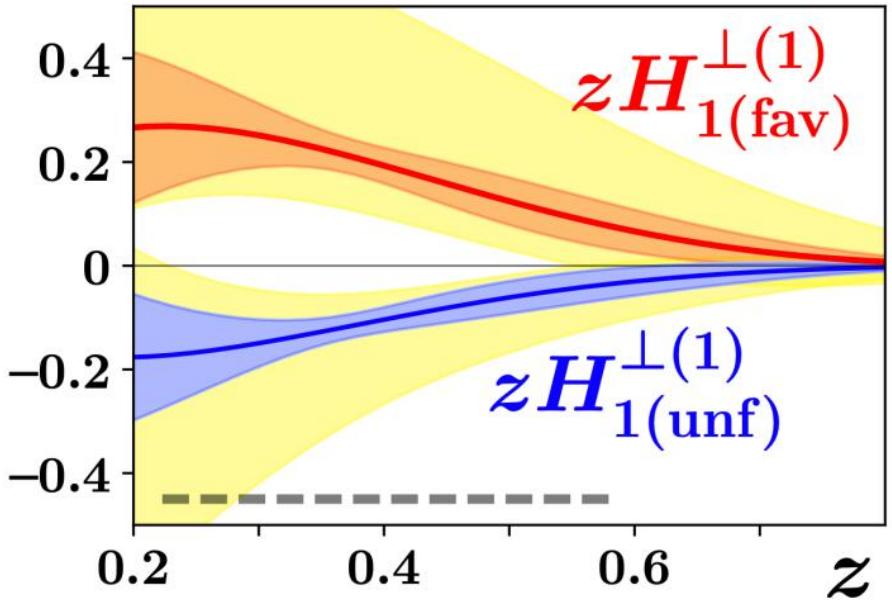


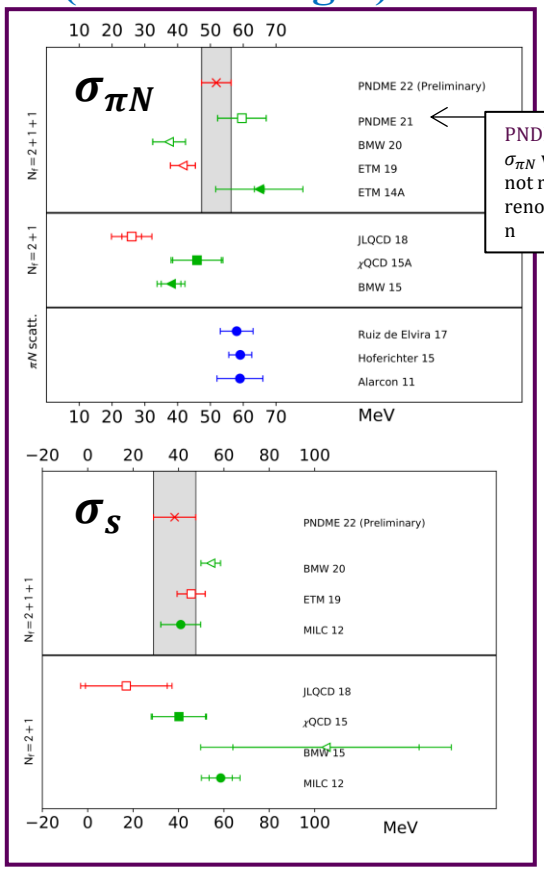


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Nucleon Flavor Diagonal Charges

Comparison with FLAG 2021 results

Nucleon sigma terms (Scalar charges)

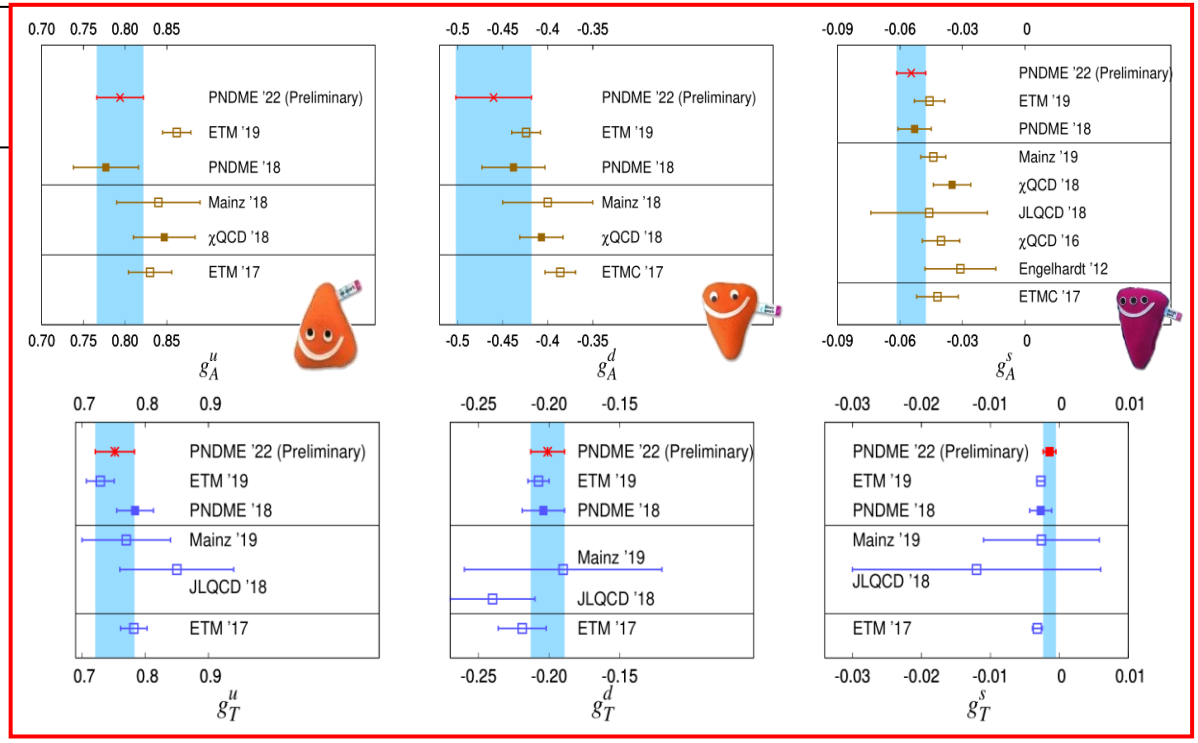


PNDME (2021) $\sigma_{\pi N}$ which does not require renormalization

[PNDME, Lattice 2022 update, **preliminary**]

- Clover fermion on $N_f = 2 + 1 + 1$ HISQ ensembles
- Flavor mixing calculated nonperturbatively
- **Chiral-Continuum extrapolation** including a data at M_π^{Phys}

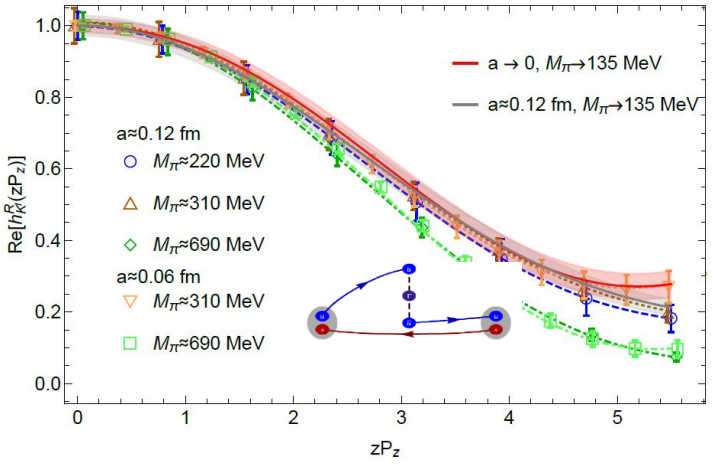
Axial and Tensor charges

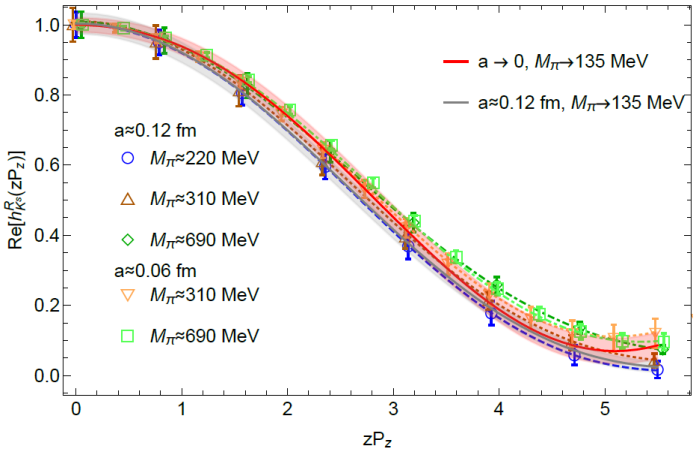


Plots by Sungwoo Park

Meson Valence-quark PDFs

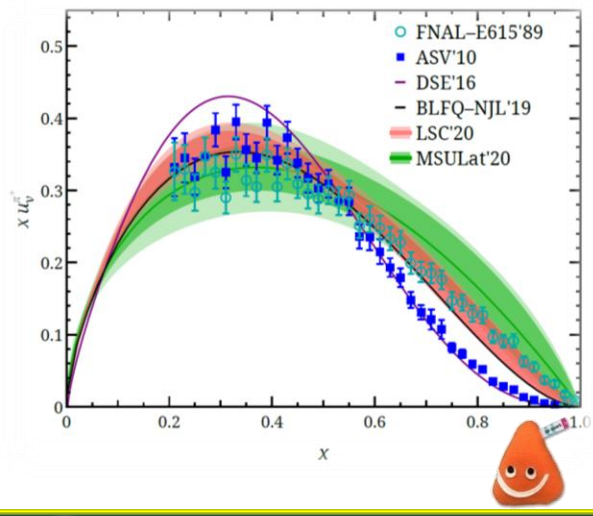
§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

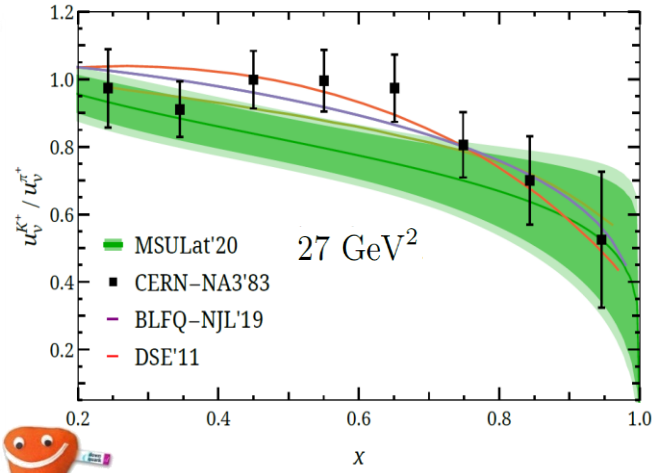


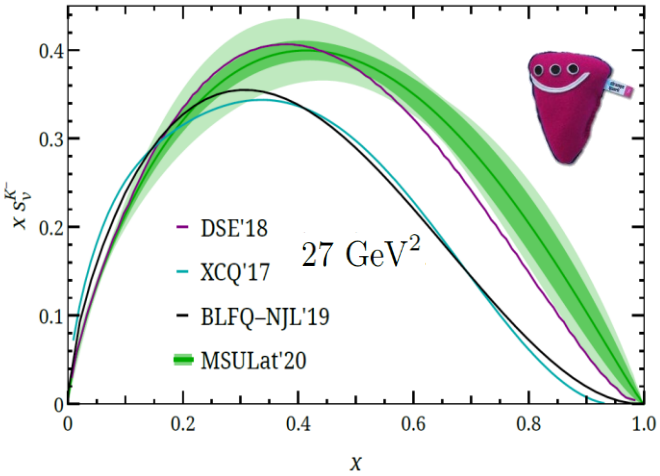


Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…



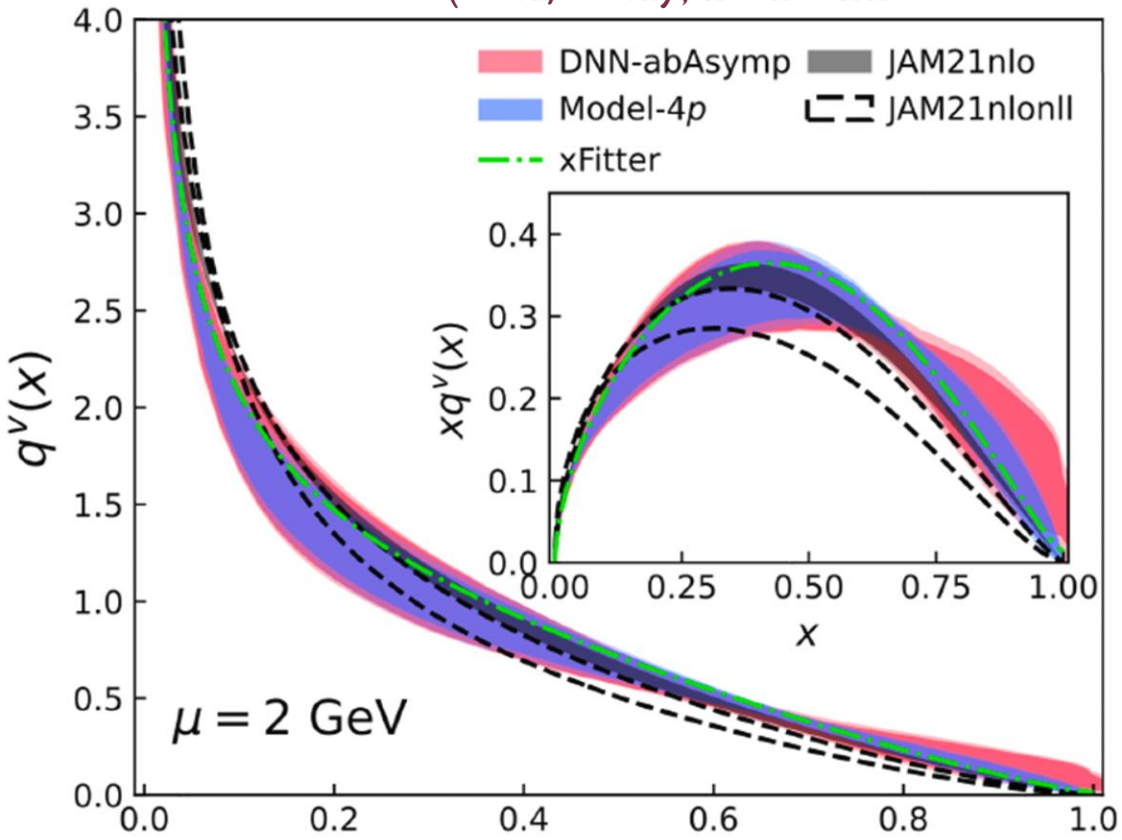




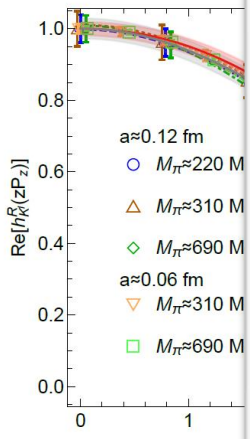
Meson Valence-quark PDFs

§ Pion/Ka … um limit

X. Gao et al (BNL/ANL), 2112.02208



Pion PDF; 1st NNLO matching



Quantities that can be calculated on the lattice

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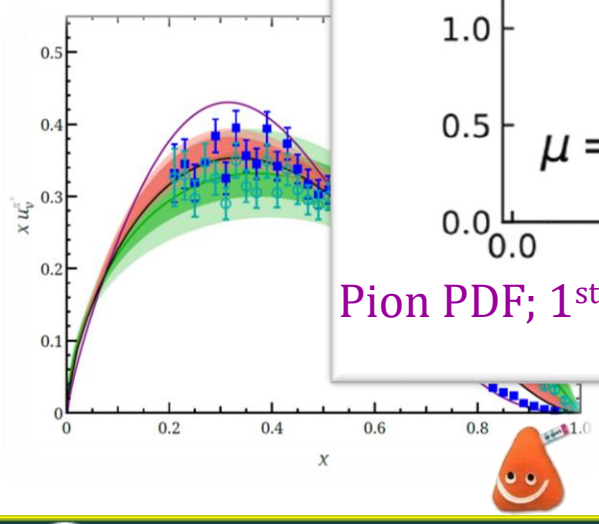
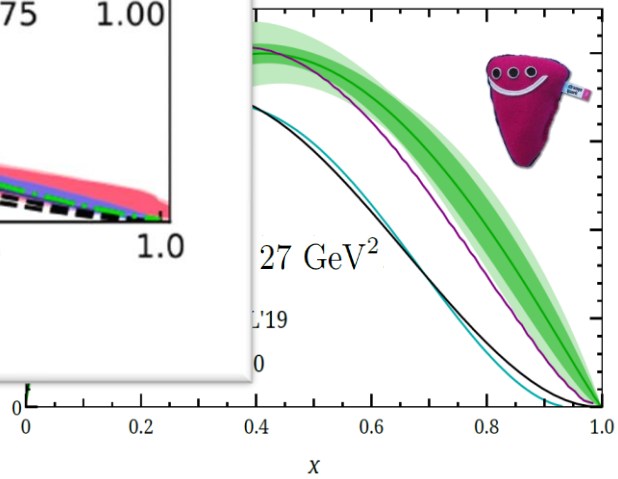




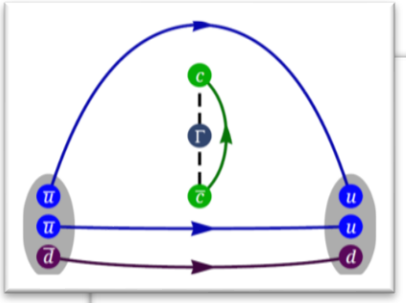
First Lattice Charm PDF

§ Large uncertainties in global PDFs

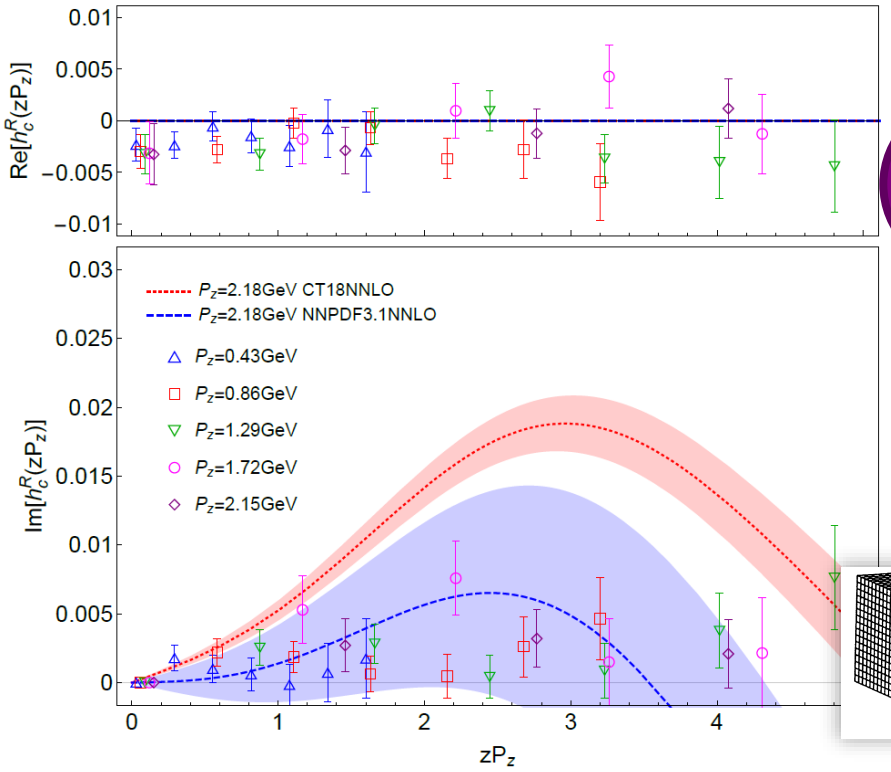
§ Results by MSULat/quasi-PDF method

Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

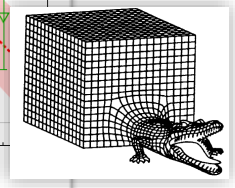
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



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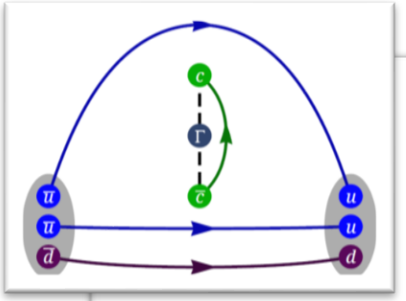
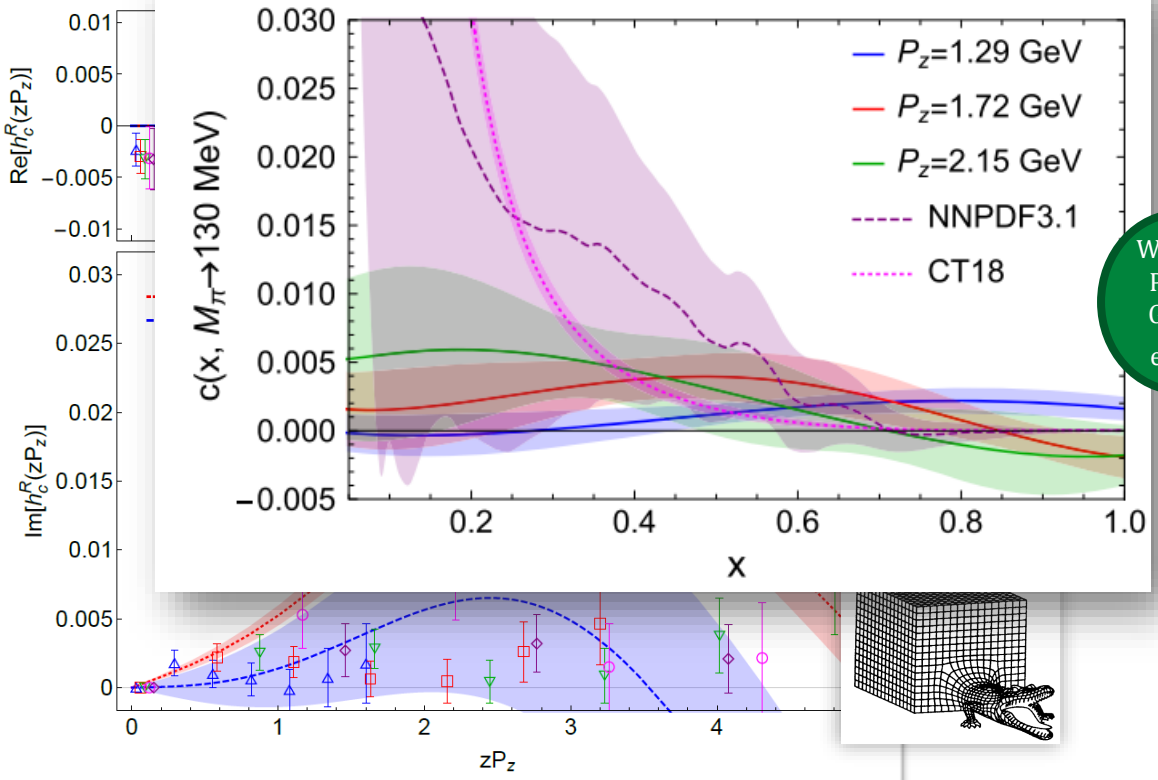


- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



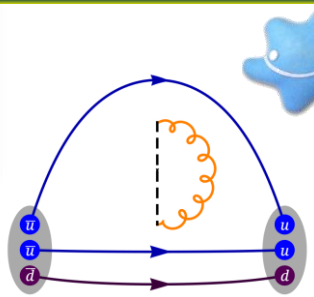
Wanted PDFs, GPDs, etc…

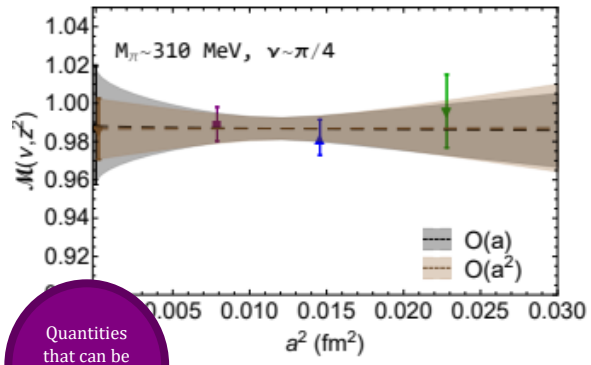
Gluon PDF in Nucleon

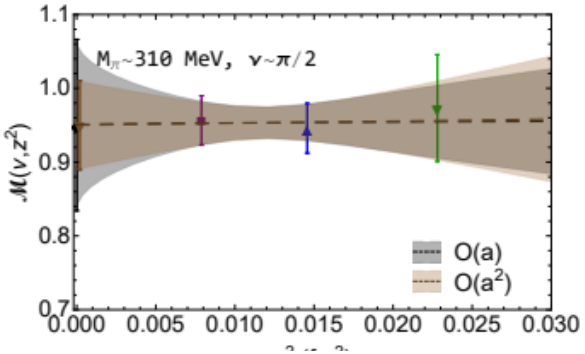
§ Continuum Gluon PDF w/ pseudo-PDF

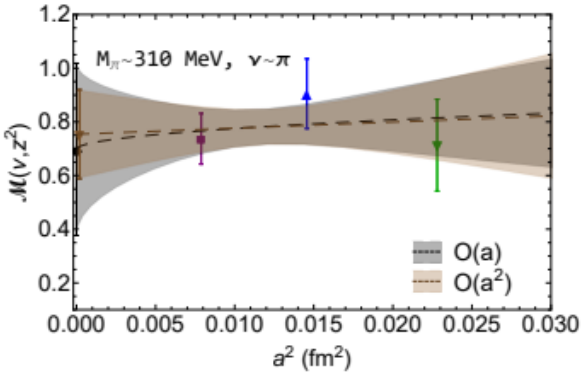
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

arXiv:2210.09985

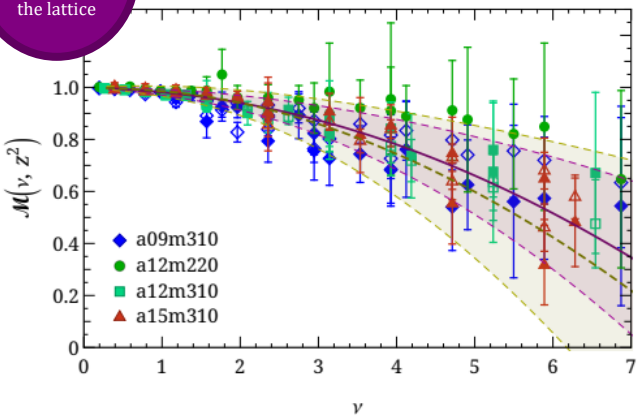


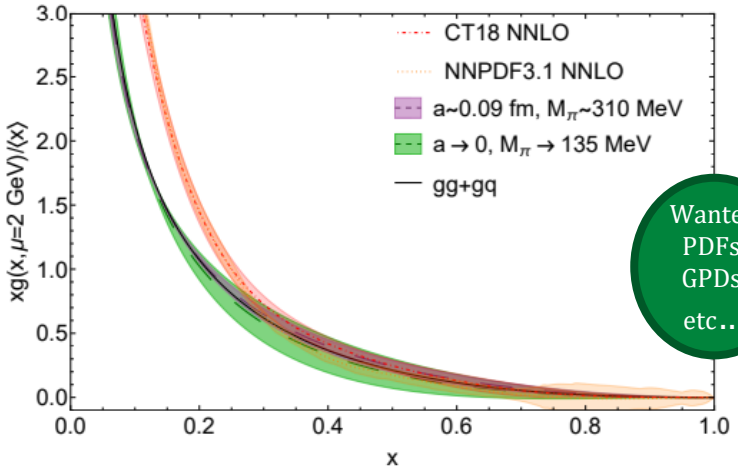






Quantities that can be calculated on the lattice





Wanted PDFs, GPDs, etc…

G: Bill Good

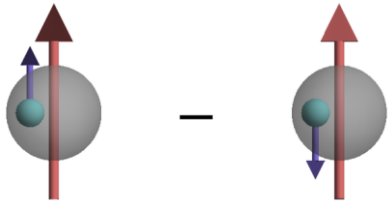
Moments of PDFs

§ Transversity first moments are most commonly done

§ State-of-the art example

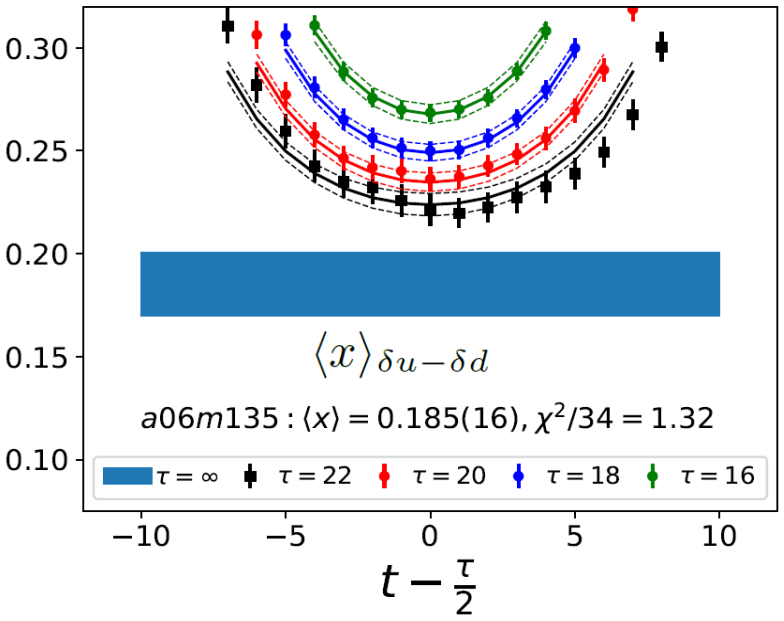
- 2 physical pion mass ensembles
- Extrapolate to the physical limit

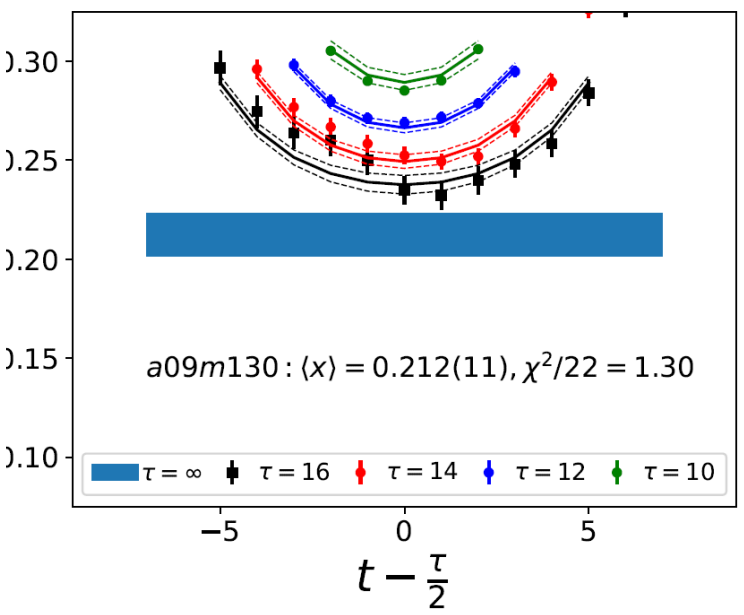
$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$



Santanu Mondal et al (PNDME collaboration), 2005.13779

Plot by Santanu Mondal



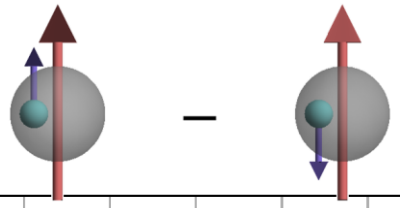


Moments of PDFs

§ Only lowest few moments

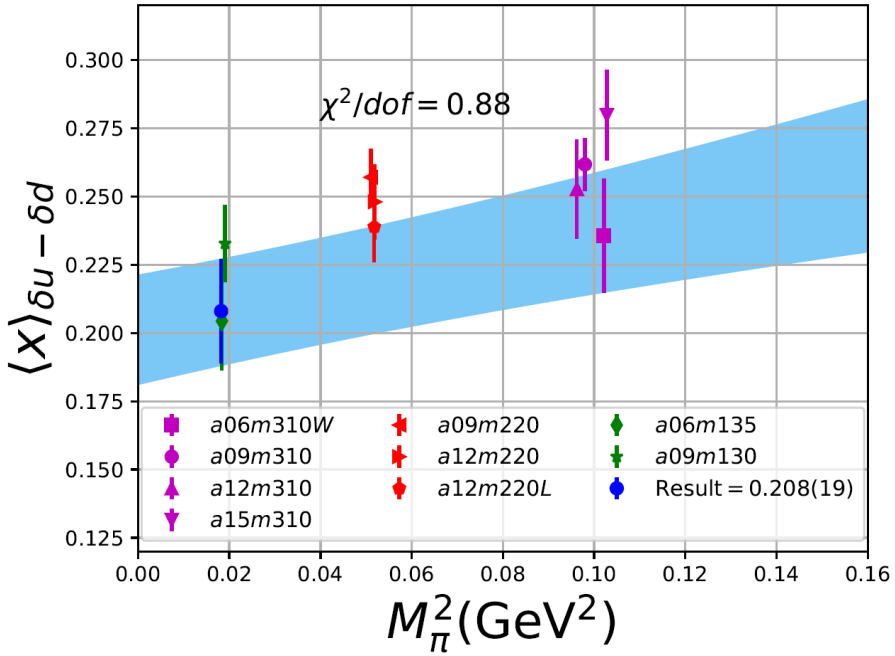
§ State-of-the art example

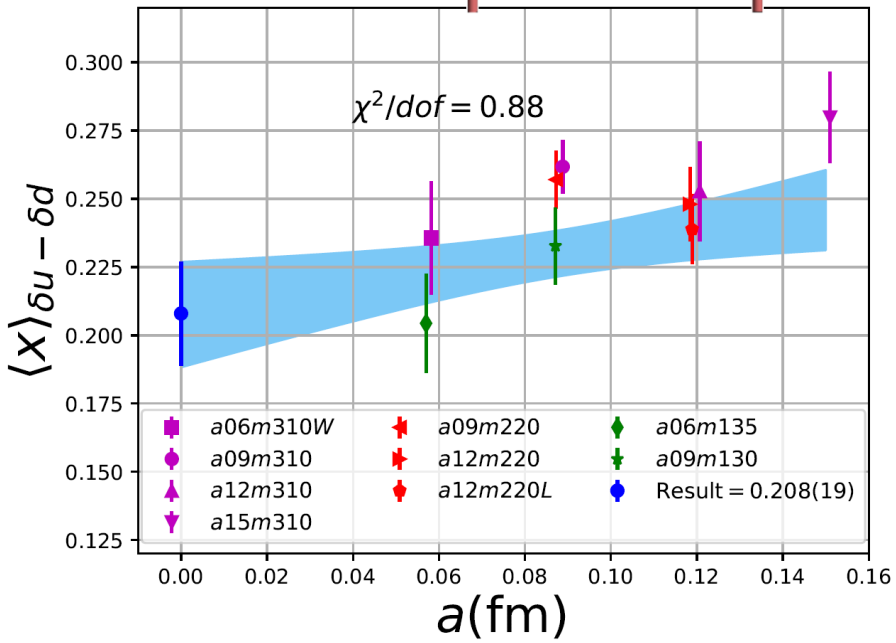
$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



- Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





§ Usually more than one LQCD calculation

- Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

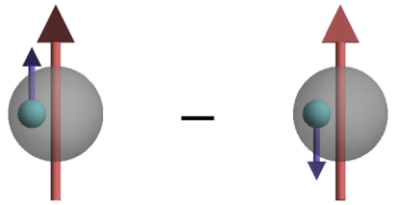
§ PDG-like rating system or average

§ LatticePDF Workshop

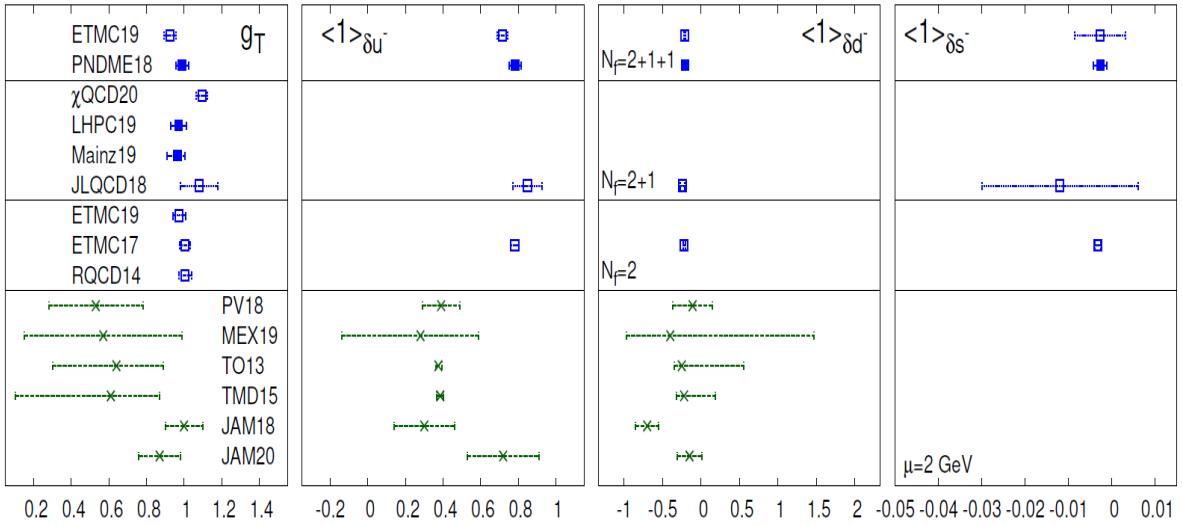
↝ Lattice representatives came together and devised a rating system

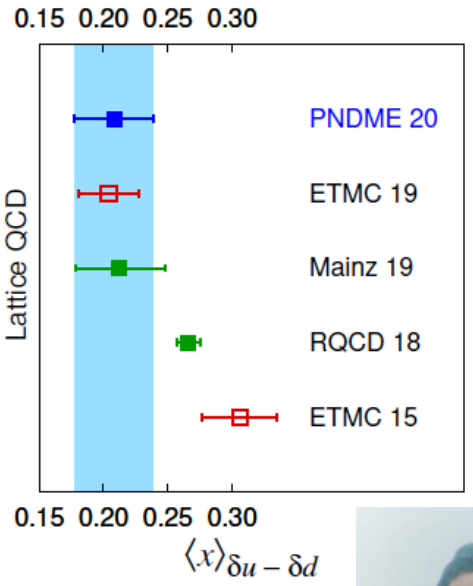
§ Recent lattice QCD/global fit status

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

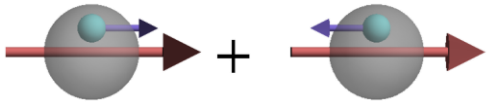




S. Mondal et al 2005.13779

PDFs on the Lattice

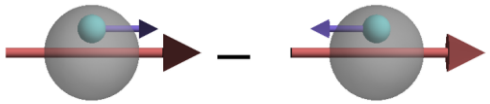
§ Traditional lattice calculations rely on operator product expansion, only provide moments



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

most well known



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

Moments of PDFs

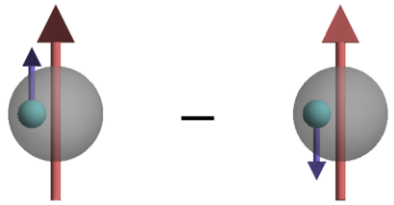
§ PDG-like rating system or average

§ LatticePDF Workshop

↝ Lattice representatives came together and devised a rating system

§ Lattice QCD/global fit status

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1 \rangle_{\delta u-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1 \rangle_{\delta d-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1 \rangle_{\delta s-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

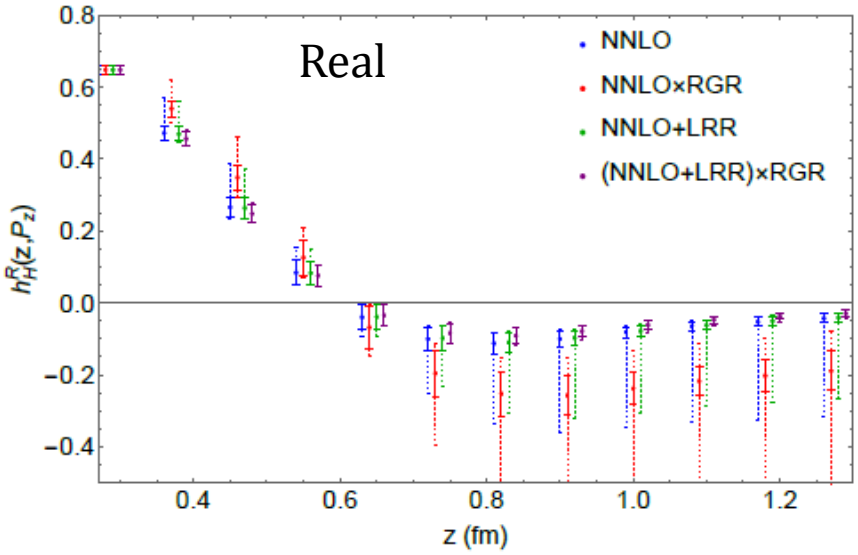
Forward-Limit Case: PDF

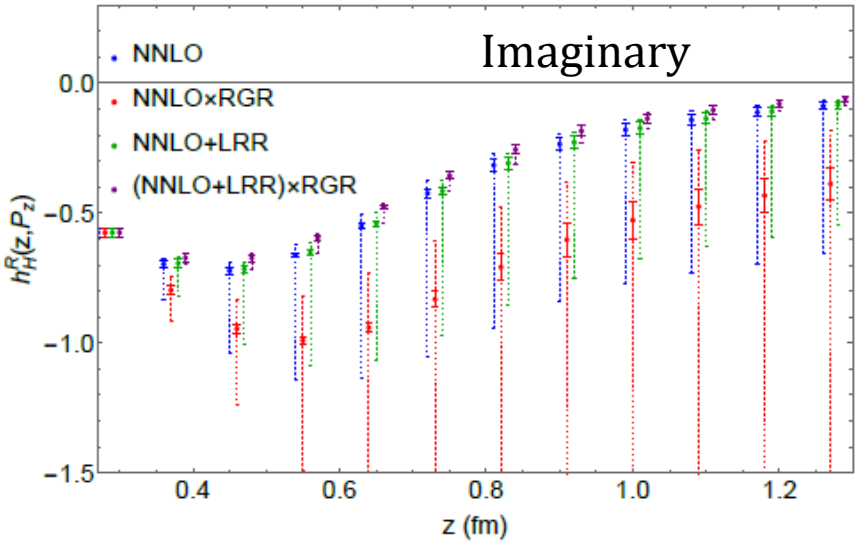
§ NNLO hybrid-ratio renormalized matrix elements

$$h^R(z,P_z) = \begin{cases} N\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z < z_s \\ Ne^{(\delta m + m_0)(z-z_s)}\dfrac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the linear divergence & renormalon ambiguity at large distances

- Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0\ \text{GeV})$
- Systematic errors shown below:

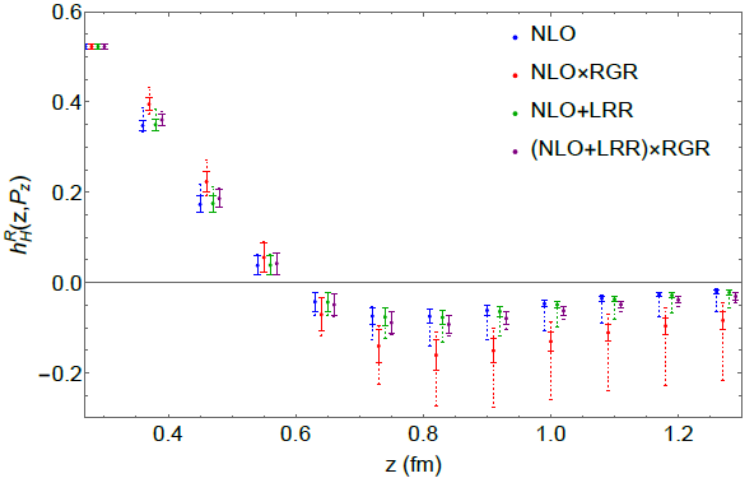


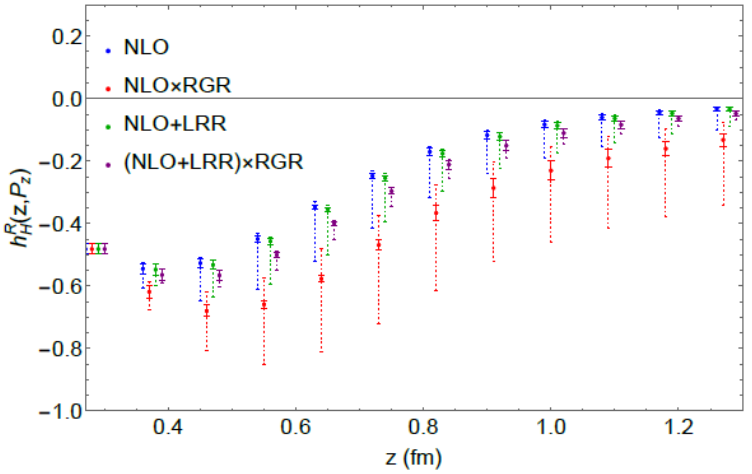


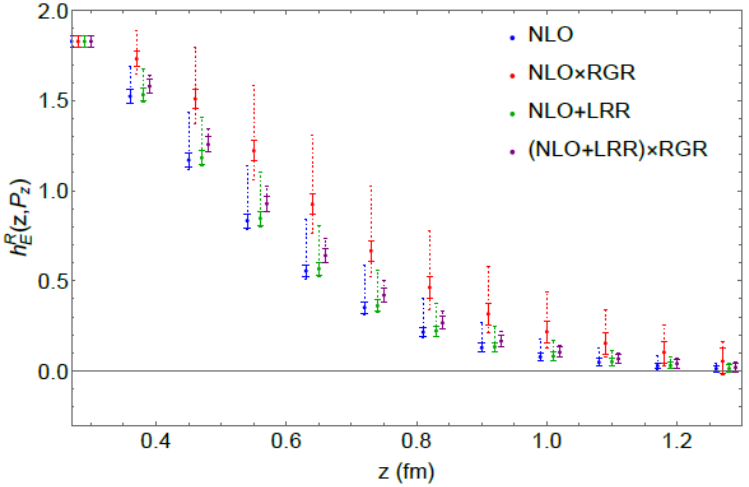
J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

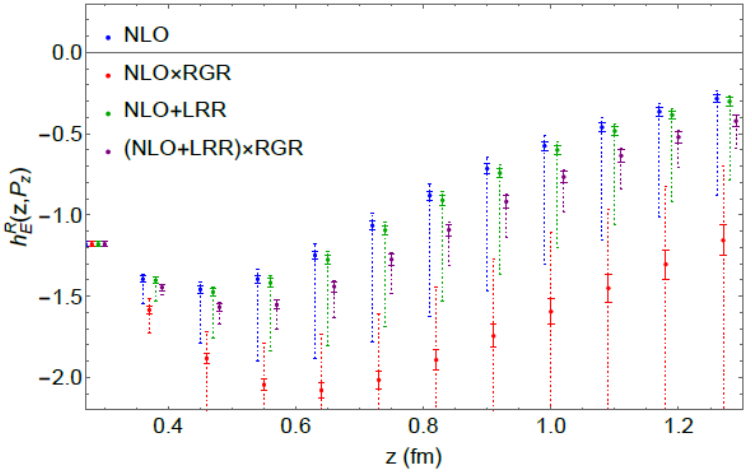
$\xi \neq 0$ GPDs

§ NLO $\xi = 0.1$, $Q^2 = 0.23\ \mathrm{GeV}^2$





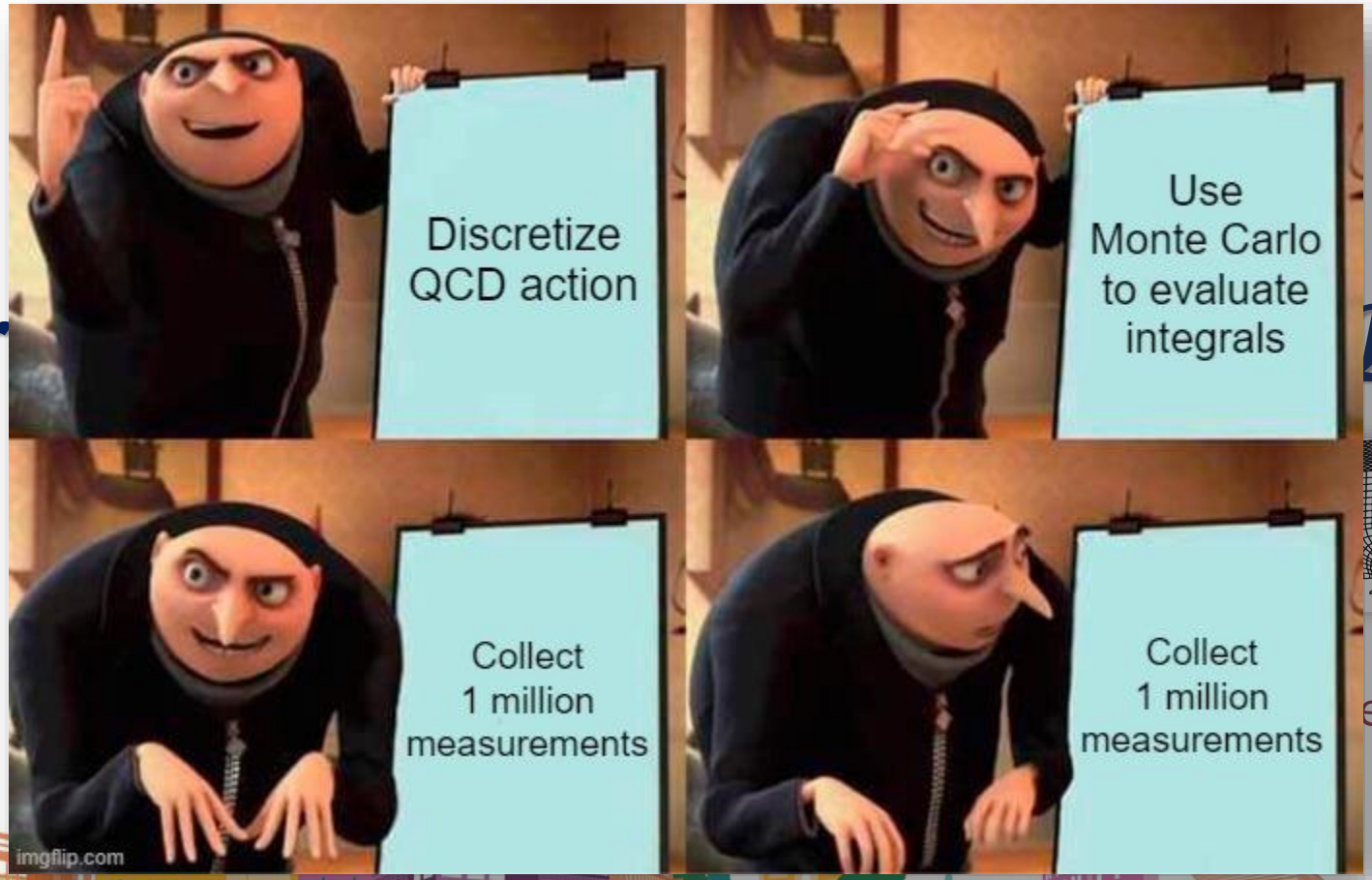


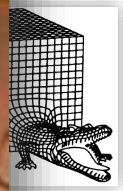


Quantities that can be calculated on the lattice

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Str DFs

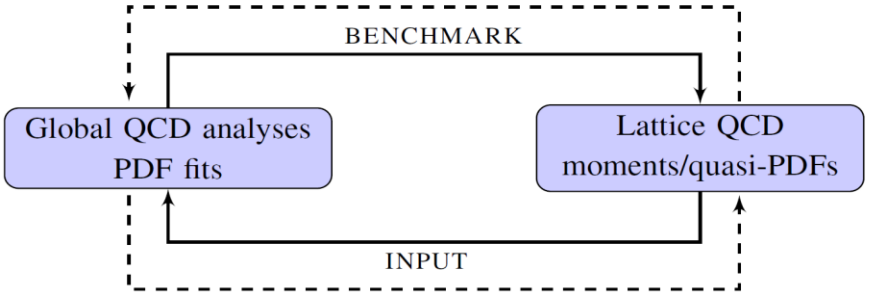




esults

How Can Lattice Help?

THE PDFLATTICE2017 WORKSHOP



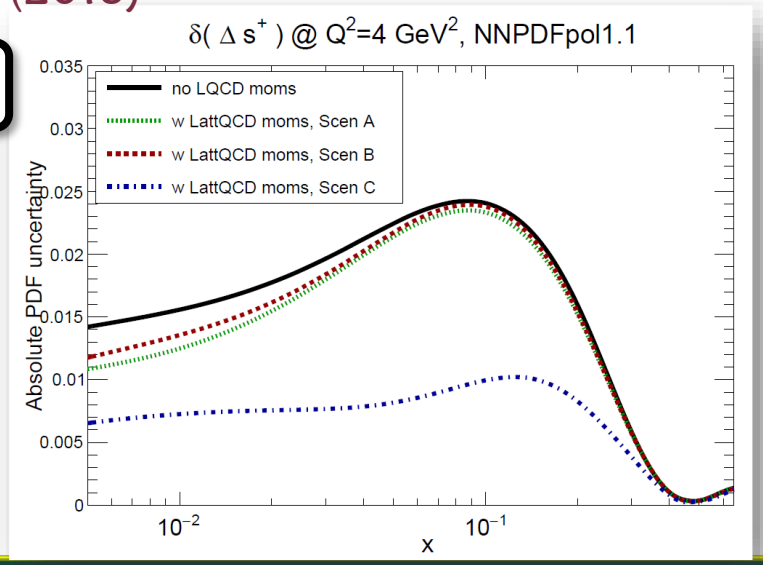
Plot by
E. Nocera

LHC (precision physics)	RHIC, JLab, . . . (hadron physics)
Higgs boson characterisation	Spin physics, nucleon structure
Precision SM measurements (*e.g.* M_W)	Large-x behaviour
BSM searches, SUSY	Nuclear modifications

Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

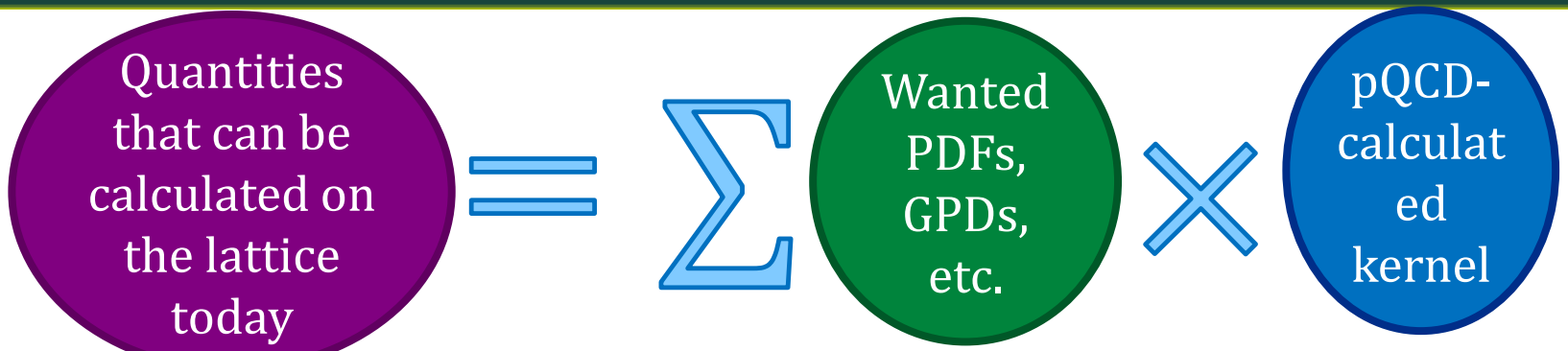
Example study

A: 70% B: 50% C: 20%



§ Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?

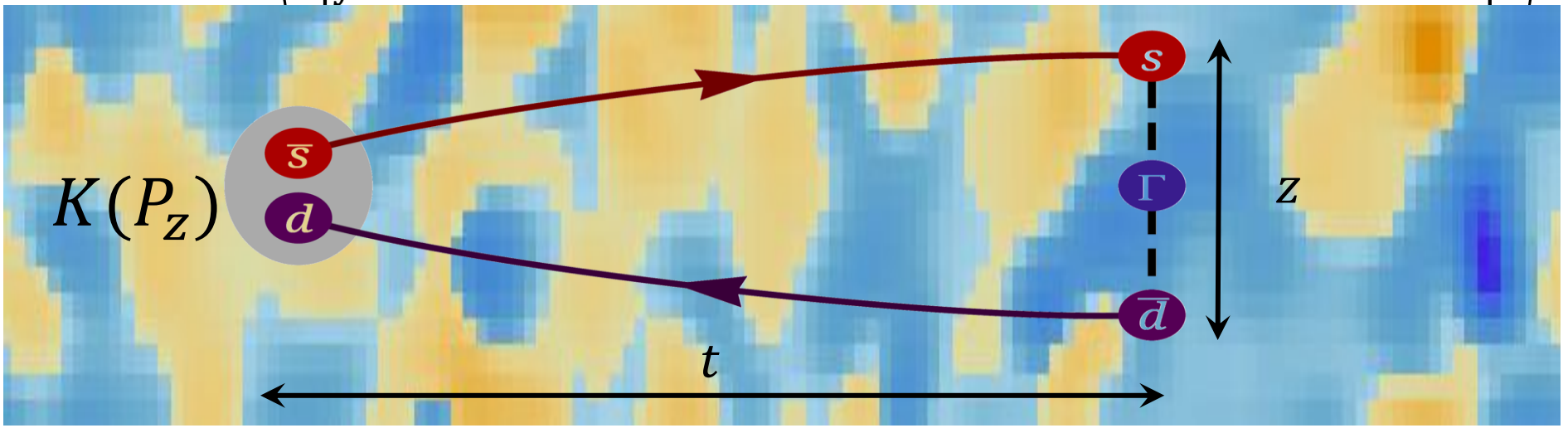
Application on Inverse Problem



R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Example: Pion/Kaon Distribution Amplitude

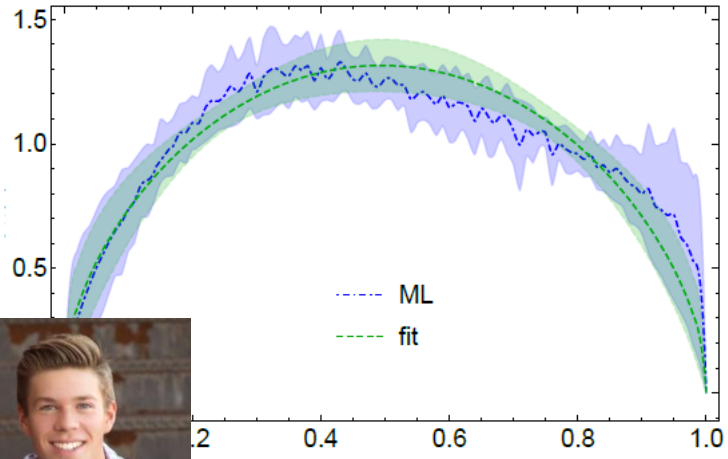
$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\,\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



Application on Inverse Problem

R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

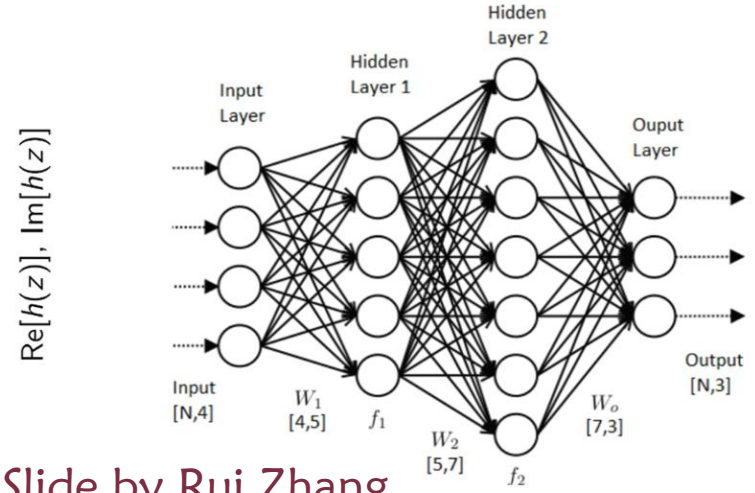
Pion Distribution Amplitude



UG: Carson Honkala

Machine Learning - A Promising Solution?

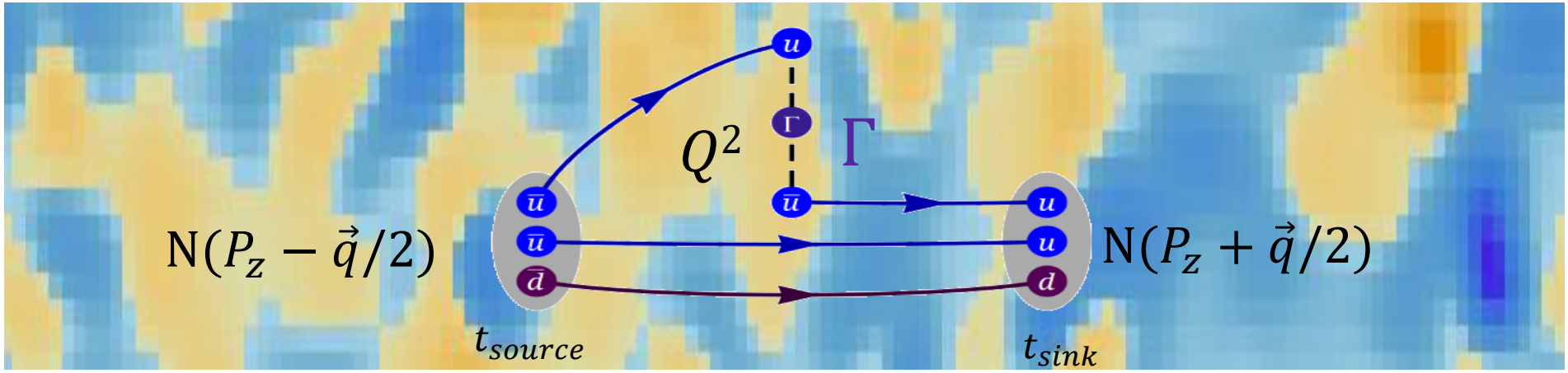
Machine learning models are effective in extracting complicated dependence of the output data on input data.



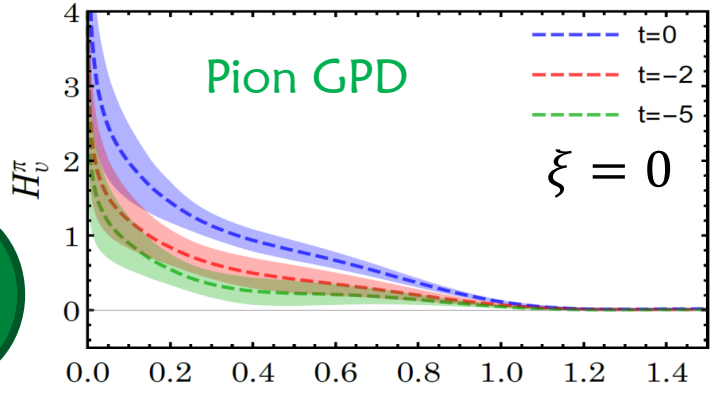
Slide by Rui Zhang

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

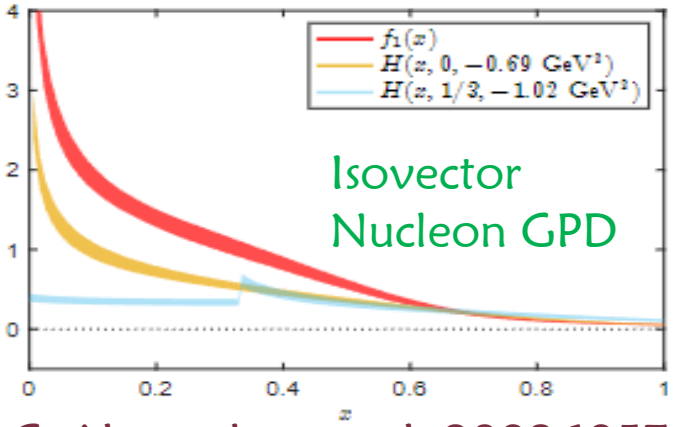


§ Heavy pion-mass results



Wanted PDFs, GPDs, etc…

J. Chen, HL, J. Zhang, 1904.12376



C. Alexandrou et al, 2008.10573

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

MSULat: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ

  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right]$$