Nucleon Helicity Distributions

26th International Symposium on Spin Physics (SPIN2025)

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Foreword

Thanks for the invitation!

SPIN (in QCD)

- is a fundamental quantum degree of freedom
- contributes to the basic structure of fundamental interactions
- provides a complete test of QCD
- is a unique opportunity to probe the inner structure of a composite system

Outline

1. Longitudinally polarized partons in QCD
    - Parton distributions, factorisation, evolution
2. Where we stand: global QCD analyses
    - A view on current parton distributions
3. Where we are going: the theory front
    - A selection of recent theoretical developments
4. Where we aim to get: opportunities at the EIC
    - and synergies with other facilities
5. Summary

1. Longitudinally polarised partons in QCD

Where does the proton angular momentum come from?

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \xrightarrow{\text{naive p.m.}} 2\langle S_z^{q+\bar{q}}\rangle \simeq 1$$

($a_0 \sim 0.6$ including relativistic effects [NPB 337 (1990) 509])

$\mathrm{EMC\ 1988\ a_0} = 0.098 \pm 0.076 \pm 0.113$ [PLB 206 (1998) 364; NPB 328 (1989) 1]

An anomalous gluon contribution to the singlet axial charge [PLB 212 (1988) 391]

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \overset{\overline{\text{MS}}}{=} \Delta\Sigma(\mu^2) - n_f\frac{\alpha_s(\mu^2)}{2\pi}\Delta G(\mu^2) \qquad \Delta G(\mu^2) \propto \left[\alpha_s(\mu^2)\right]^{-1}$$

The gluon does not decouple in the asymptotic limit

A realisation of the proton's total angular momentum decomposition [NPB 337 (1990) 509]

$$\mathcal{J}(\mu^2) = \sum_f \left\langle P;S|\hat{J}^z_f(\mu^2)|P;S\right\rangle = \frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2) + \mathcal{L}_q(\mu^2) + \mathcal{L}_g(\mu^2)$$

The decomposition is not unique

What should be the decompositions that lead to gauge-invariant, physically meaningful terms (and in which sense these are measurable) are discussed in [Phys.Rept. 541 (2014) 163]

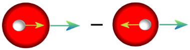
Here I focus on $\Delta\Sigma$ and Δg

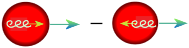
$$\Delta\Sigma(\mu^2) = \sum_{q=u,d,s}\int_0^1 [\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] \qquad \Delta G(\mu^2) = \int_0^1 dx \Delta g(x,\mu^2)$$

Polarized PDFs on the light-cone [Rev.Mod.Phys. 67 (1995) 157]

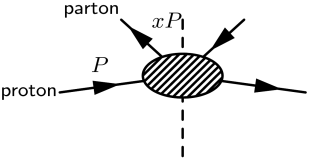
1. The momentum densities of partons with spin ($^{\uparrow}$) or ($^{\downarrow}$) *w.r.t* the nucleon

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$ 

$\Delta g(x) =$ 

2. Allow for a proper field-theoretic definition as matrix elements of bilocal operators



collinear transition
of a massles proton h
into a massless parton i
with fractional momentum x
local OPE $\Longrightarrow$ lattice formulation

$$\Delta q(x) = \frac{1}{4\pi}\int dy^- e^{-iy^- xP^+}\langle P,S|\bar{\psi}(0,y^-,\mathbf{0}_\perp)\gamma^+\gamma^5\psi(0)|P,S\rangle$$

$$\Delta g(x) = \frac{1}{4\pi x P^+}\int dy^- e^{-iy^- xP^+}\langle P,S|G^{+\alpha}(0,y^-,\mathbf{0}_\perp)\tilde{G}^{+}_{\alpha}(0)|P,S\rangle$$

with light-cone coordinates and QCD field-strength tensor G ($A^+ = 0$ gauge)

$$y = (y^+, y^-, \mathbf{y}_\perp)\,, \quad y^+ = (y^0 + y^z)/\sqrt{2}\,, \quad y^- = (y^0 - y^z)/\sqrt{2}\,, \quad \mathbf{y}_\perp = (v^x, v^y)$$

$$G^{\alpha}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$$

3. All these definitions have ultraviolet divergences which must be renormalized

Factorisation and evolution [Adv.Ser.Direct.HEP 5 (1988) 1]

1. A variety of sufficiently inclusive processes factorise

$$\text{DIS}: \qquad g_1 = \frac{\sum_q^{n_f} e_q^2}{2n_f}\left(C_{\rm NS}\otimes\Delta q_{\rm NS} + C_{\rm S}\otimes\Delta\Sigma + 2n_f C_g\otimes\Delta g\right)$$

$$\text{SIDIS}: \qquad g_1^h = \sum_{q,\bar{q}} e_q^2\left[\Delta q\otimes C_{qq}^{1,h}\otimes D_q^h + \Delta q\otimes C_{gq}^{1,h}\otimes D_g^h + \Delta g\otimes C_{qg}^{1,h}\otimes D_q^h\right]$$

$$\text{PP}: \qquad \Delta\sigma = \sigma^{(+)+} - \sigma^{(+)-} = \sum_{a,b,(c)} \Delta f_a\otimes(\Delta)f_b(\otimes D_c^h)\otimes\Delta\hat{\sigma}_{ab}^{(c)}$$

$$C_{If}(y,\alpha_s) = \sum_{k=0}\left(\frac{\alpha_s}{4\pi}\right)^k C_{If}^{(k)}(y)$$

DIS (NNLO)	[NPB 417 (1994) 61]
SIDIS (NNLO)	[PRL 133 (2024) 211904; ibid. 211905]
PP ((N)NLO)	[PRL 125 (2020) 082001,PRD 70 (2004) 034010]
	[PRD 67 (2003) 054004;054005; 81 (2010) 094020]

2. After factorisation, PDFs depend perturbatively on μ

$$\frac{\partial}{\partial\ln\mu^2}\Delta f_i(x,\mu^2) = \sum_j^{n_f}\int_x^1 \frac{dz}{z}\Delta P_{ji}\left(z,\alpha_s(\mu^2)\right)\Delta f_j\left(\frac{x}{z},\mu^2\right)$$

$$\Delta P_{ji}(z,\alpha_s) = \sum_{k=0}\left(\frac{\alpha_s}{4\pi}\right)^{k+1}\Delta P_{ji}^{(k)}(z)$$

LO	[NPB 126 (1977) 298]
NLO	[ZPC 70 (1996) 637, PRD 54 (1996) 2023]
	[NPB 475 (1996) 47]
NNLO	[NPB 889 (2014) 351]

Experimental probes

DIS: $\ell^\pm\{p,d,n\} \to \ell^\pm X$

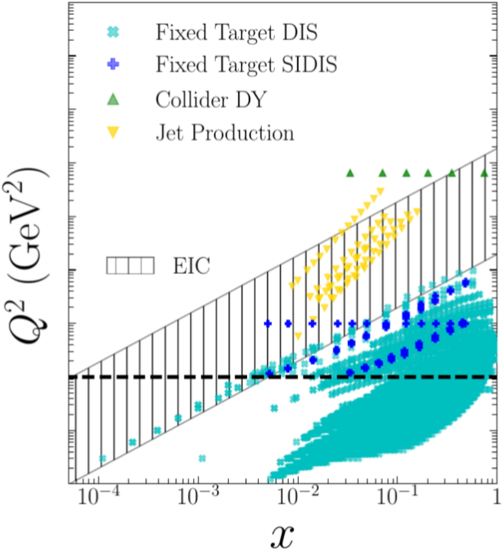
partonic process	PDF probed
$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg

SIDIS: $\ell^\pm\{p,d\} \to \ell^\pm h X$

partonic process	PDF probed
$\gamma^* q \to q$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$
$\gamma^* g \to c\bar{c}$	Δg

PP: $pp \to jet(s), h, W^\pm$

partonic process	PDF probed
$gg \to qg$ $qg \to qg$	Δg
$u_L\bar{d}_R \to W^+$ $d_L\bar{u}_R \to W^-$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$



[Figure adapted from Ann.Rev.Nucl.Part.Sci. 70 (2020) 43]

2. Where we stand: global QCD analyses

Recent determinations of polarised PDFs

	DSSV	JAM	MAP	NNPDF
DIS	☑	☑	☑	☑
SIDIS	☑	☑	☑	☒
PP	☑ (jets,π^0,W)	☑ (jets,$W^\pm$)	☒	☑ (jets,$W^\pm$)
uncertainties	Monte Carlo	Monte Carlo	Monte Carlo	Monte Carlo
parametrisation	polynomial	polynomial	neural network	neural network
hyperparametrisation	no	no	no	yes
perturbative accuracy	NNLO	NLO	NNLO	NNLO
missin ghigher orders	no	no	no	yes
mass scheme	ZM	ZM	ZM	FONLL
latest update	BDSSV24 [PRL 133 (2024) 151901]	JAM17 [PRL 119 (2017) 132001]	MAPPDFpol1.0 [PLB 865 (2025) 139497]	NNPDFpol2.0 [JHEP07 (2025) 168]

older determinations: AAC [NPB 813 (2009) 106], BB [NPB 841 (2010) 205], LSS, [PRD 82 (2010) 114018] ...

A prescription to propagate uncertainties

$$E[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)\mathcal{O}(\Delta f)$$

$$V[\mathcal{O}] = \int \mathcal{D}\Delta f \mathcal{P}(\Delta f|data)[\mathcal{O}(\Delta f) - E[\mathcal{O}]]^2$$

$$\mathcal{P}(\Delta f|data) \longrightarrow \{\Delta f_k\}$$
$$E[\mathcal{O}] \approx \tfrac{1}{N}\textstyle\sum_k \mathcal{O}(\Delta f_k)$$
$$V[\mathcal{O}] \approx \tfrac{1}{N}\textstyle\sum_k [\mathcal{O}(\Delta f_k) - E[\mathcal{O}]]^2$$

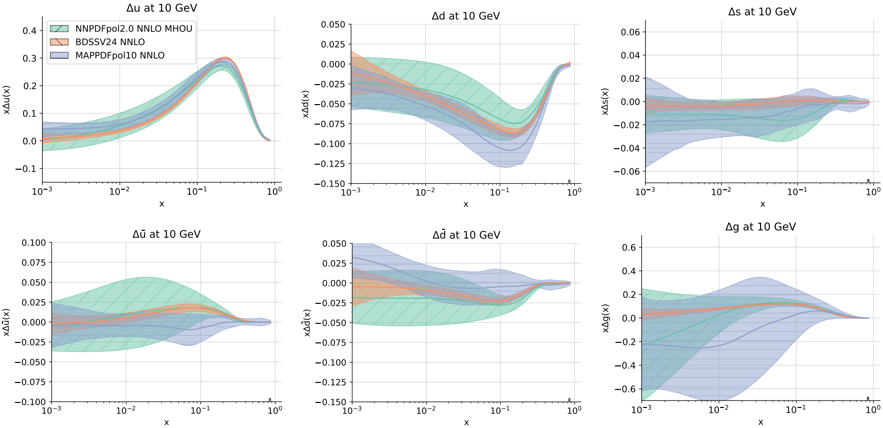
A set of theoretical constraints

$$\int dx[\Delta u^+ - \Delta d^+] = a_3$$

$$\int dx[\Delta u^+ + \Delta d^+ - 2\Delta s^+] = a_8$$

from SU(2) and SU(3) symmetries
with a_3 and a_8 determined from baryon decays
$$|\Delta f| \leq f$$
from LO positivity

Polarised PDFs at NNLO



Good agreement across different groups

Residual differences due to choice of data and methodology

Times are mature for a benchmark, as in the case of unpolarised PDFs

Fragmentation functions (FFs) accurate to NNLO are required

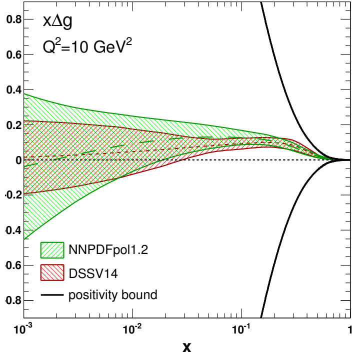
Large differences across groups occur for FFs, which do not impact polarised PDFs

[See plenary talk by A. Vossem on Monday afternoon; see talks by H.-Y. Xing and X. Shen on Tuesday morning]

Gluon helicity

High-p_T jet production

first evidence of a sizeable, positive gluon polarisation in the proton



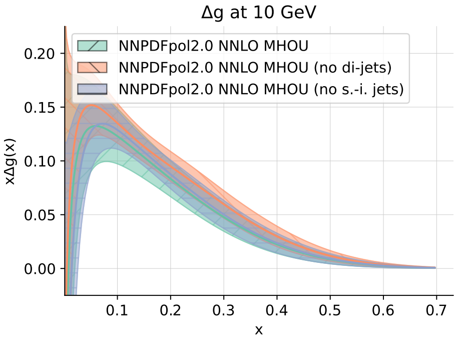
$$\langle x_{1,2}\rangle \simeq \frac{2p_T}{\sqrt{s}}e^{-\eta/2} \approx [0.05, 0.2]$$

NNPDF and DSSV results well compatible

$$\int_{0.01}^{0.2} dx\,\Delta g(x, Q^2 = 10\,\text{GeV}^2) = +0.23\pm0.15$$

High-p_T di-jets

confirm a positive gluon polarisation in the proton



$$\langle x_{1,2}\rangle \simeq \frac{p_T}{\sqrt{s}}(e^{\pm\eta_1\pm\eta_2}) \approx [0.01, 0.2]$$

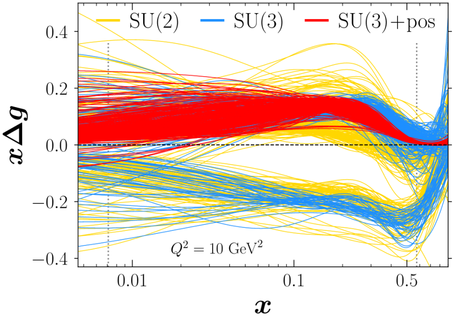
x sensitivity extended down to $x \sim 0.01$

$$\int_{0.01}^{0.2} dx\,\Delta g(x, Q^2 = 10\,\text{GeV}^2) = +0.32\pm0.13$$

Gluon helicity: a negative gluon?

Simultaneous anaysis
of unpolarised and polarised
jet production at RHIC [PRD 105 (2022) 074022]

Positive and negative Δg solutions
equally good (same fit quality)

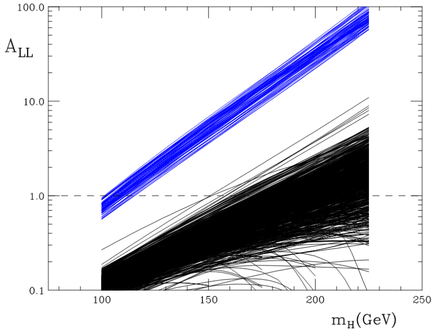


Dependence of Δg on RHIC jets is quadratic

High-p_T hadron production in SIDIS at JLab
and EIC can rule out negative solutions
[PRD 107 (2023) 034033]

Consider Higgs boson production
in polarised PP collisions at RHIC
Positivity of cross sections implies $|A_{\rm LL}| \leq 1$

A negative Δg violates positivity [PRD 109 (2024) 7]
(note the log scale on the vertical axis)

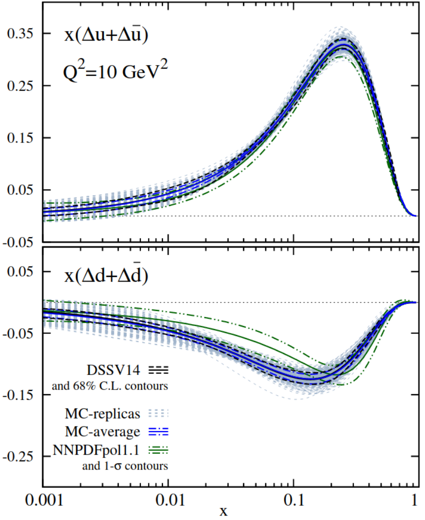


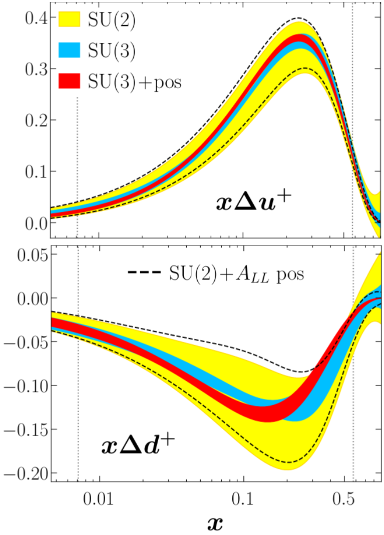
LO positivity bound marginally modified at
large x by higher order corrections
[NPB 534 (1998) 277]

Requirement $|\Delta g| \leq g$ does not bias the fit

Total up and down helicities

Mostly from DIS [PRD 100 (2019) 114027; PRD 105 (2022) 074022; NPB 887 (2014) 276]

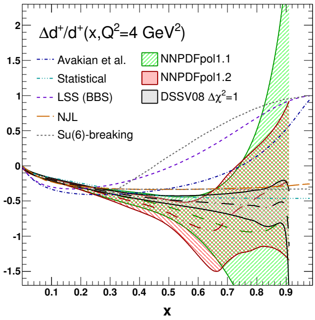




Overall good agreement across DSSV, JAM and NNPDF parton sets

Total up and down helicities at large x

Playground for models [PLB 742 (2015) 117]

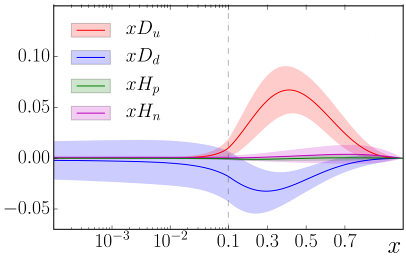


Model	$\Delta d^+/d^+$	Model	$\Delta d^+/d^+$
SU(6)	$-1/3$	NJL	-0.25
RCQM	$-1/3$	DSE (*realistic*)	-0.26
QHD ($\sigma_{1/2}$)	1	DSE (*contact*)	-0.33
QHD (ψ_ρ)	$-1/3$	pQCD	1
NNPDFpol1.1 ($x=0.9$) -0.74 ± 3.57			
NNPDFpol1.2 ($x=0.9$) -0.23 ± 1.06			

[See talk by C. Roberts on Tuesday morning]
[See talks by X. Zhao, Y. Yang and D. Hou on Wednesday]

Beyond leading-twist factorisation

Fit of higher twist terms (up to $\tau = 4$)
in JAM15 [PRD 93 (2016) 074005]

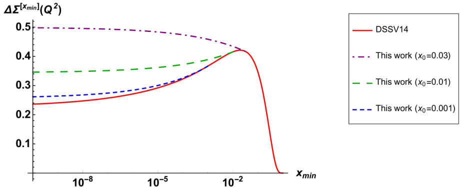


$g_1^{\tau=3} \propto D$ and $g_1^{\tau=4} = H/Q^2$

nonzero twist-3
quark distributions

twist-4 quark distributions
compatible with zero

Quark singlet helicity at small x



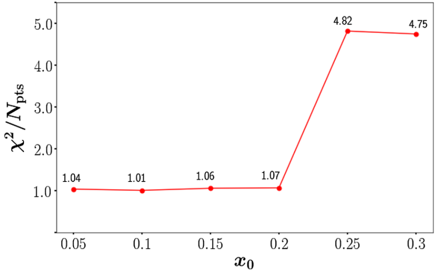
[JHEP 1601 (2016) 072; PRL 118 (2017) 052001]
[PLB 772 (2017) 136; JHEP 2207 (2022) 095]

Small-x evolution equations for g_1
based on the dipole model
resum powers of $\alpha_s \ln^2(1/x)$
become closed for N_C, n_f large
a solution for the flavor-singlet is

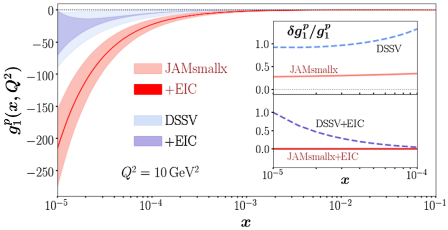
$$g_1 \sim \Delta\Sigma \sim \left(\frac{1}{x}\right)^{\alpha_h}, \qquad \alpha_h \sim 2.31\sqrt{\frac{\alpha_s N_C}{2\pi}}$$

Potential solid amount of spin at small x
attach $\Delta\hat{\Sigma}(x, Q^2) = N x^{-\alpha_h}$ at x_0 to DSSV
Should be tested at an EIC

How small is x_0?



Which precision to test small x?



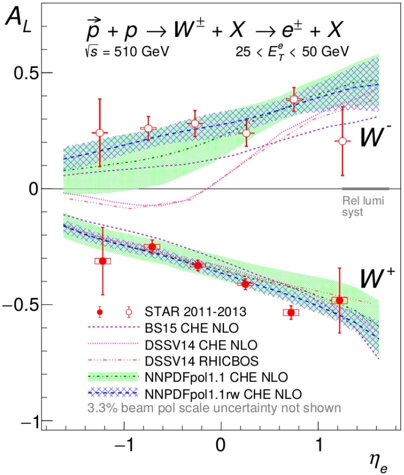
[PRD 104 (2021) L031501]

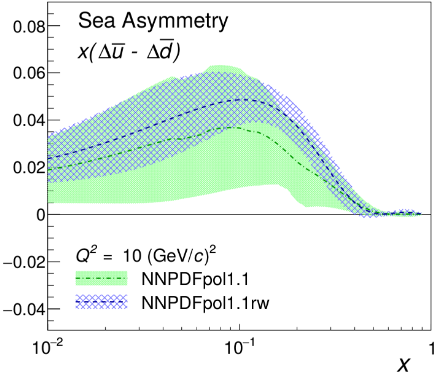
Sea quark helicity $\Delta_s = \Delta\bar{u} - \Delta\bar{d}$ [arXiv:1702.05077]

$W^{\pm}$ boson production

first evidence of broken flavor symmetry for polarised light sea quarks

New 2013 data [PRD 99 (2019) 51102]





$$\langle x_{1,2}\rangle \simeq \frac{M_W}{\sqrt{s}} e^{-\eta_l/2} \approx [0.04, 0.4]$$

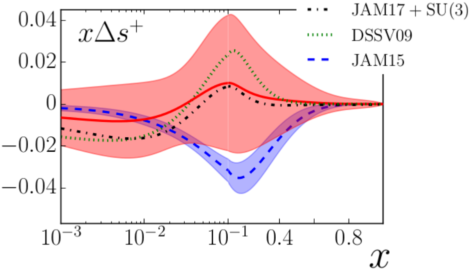
$$\Delta\bar{u} > 0 > \Delta\bar{d},\ |\Delta\bar{d}| > |\Delta\bar{u}|$$

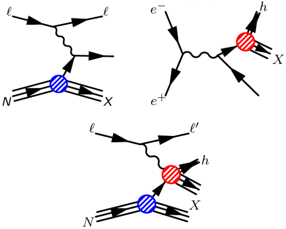
$$\int_{0.04}^{0.4} dx\, \Delta_s(x, Q^2 = 10\,\text{GeV}^2) = +0.06 \pm 0.03$$

$$\rightarrow +0.07 \pm 0.01$$

[See also PRD 106 (2022) L031502]

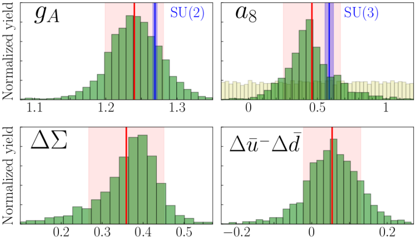
Strange helicity: DIS vs SIDIS [PRL 119 (2017) 132001]





process	target	$N_{\rm dat}$	χ^2
DIS	$p, d, {}^3$He	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES	d	18	28.1
HERMES	p	18	14.2
COMPASS	d	20	8.0
COMPASS	p	24	18.2
SIDIS $(K^\pm)$			
HERMES	d	27	18.3
COMPASS	d	20	18.7
COMPASS	p	24	12.3
Total:		**1855**	**1969.7**

[See also talk by S. Pate on Tuesday morning]



$g_A = 1.24 \pm 0.04$ $\quad a_8 = 0.46 \pm 0.21$

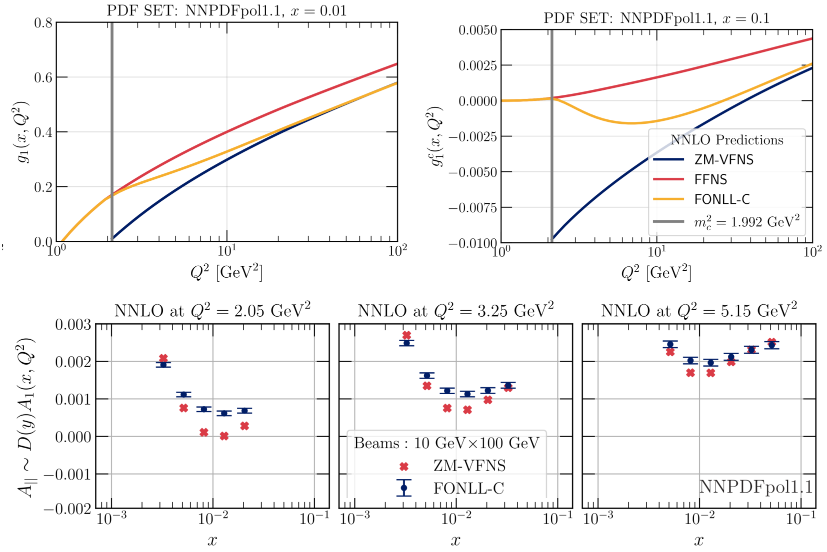
confirmation of SU(2) symmetry to $\sim 2\%$

$\sim 20\%$ SU(3) breaking $\pm 20\%$

3. Where we are going: the theory front

Heavy flavour production at NLO and a GM-VFN scheme

Calculation of the heavy flavour contribution to the differential g_1^Q at NLO and extension of FONLL to the polarised case [PRD 98 (2018) 014018; ibid. 104 (2021) 016033; EPJ C84 (2024) 189]



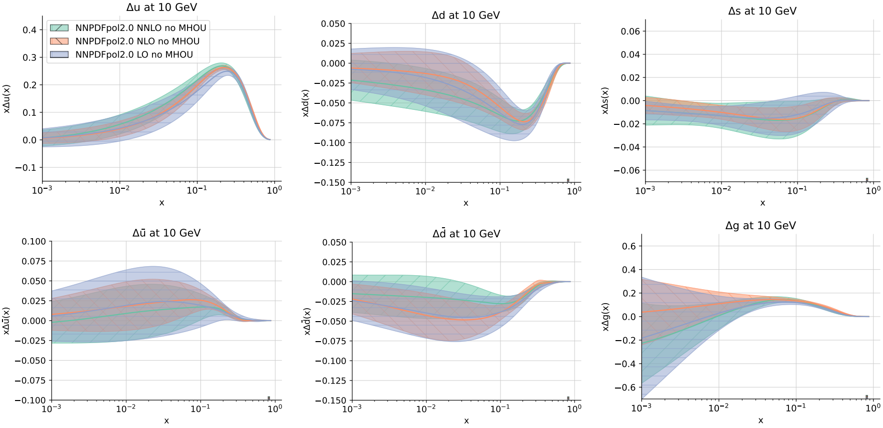
Progress on NNLO computations

$W^{\pm}$ boson production [PLB 817 (2021) 136333] from the N-jettiness method

DIS+jet [PRL 125 (2020) 082001] from the Projection-to-Born method

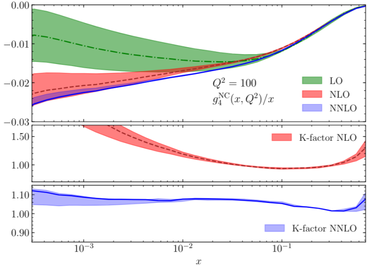
SIDIS [PRL 133 (2024) 211904; ibid. 211905] from antenna subtraction

Very good perturbative convergence of polarised PDFs; marginal role of residual MHOUs

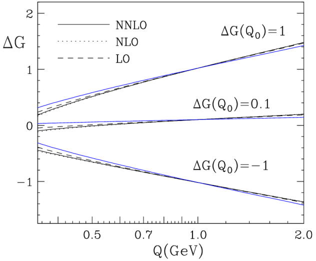


In preparation for the EIC . . .

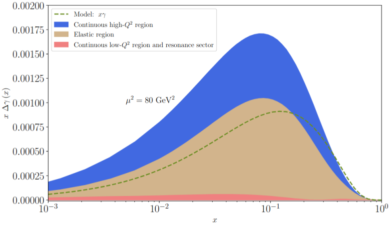
The $g_1^{\gamma,Z,W}$, $g_4^{\gamma,Z,W}$, $g_L^{\gamma,Z,W}$ DIS structure functions at NNLO [EPJ C82 (2022) 12 1167]



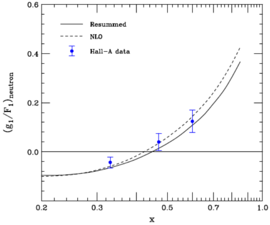
Static DGLAP solutions of ΔG at all perturbative orders [PRD 99 (2019) 054001]



The polarised photon PDF in the LuxQED formalism [EPJ C84 (2024) 905]

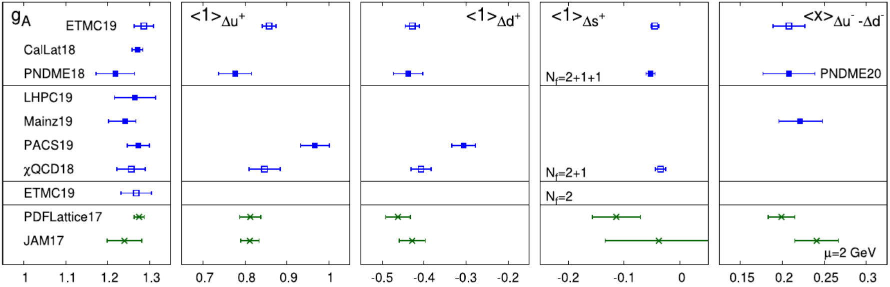


Threshold resummation in DIS and SIDIS [PRC 70 (2004) 065207; PRD 71 (2005) 012003]



Comparing lattice QCD and global PDF fits: moments

PDFLattice studies [Prog.Part.Nucl.Phys. 100 (2018) 107; ibid. 121 (2021) 103908]



$$g_A = \langle 1 \rangle_{\Delta u^+ - \Delta d^+} = \int_0^1 dx\, \left[\Delta u^+ - \Delta d^+\right](x, Q^2)$$

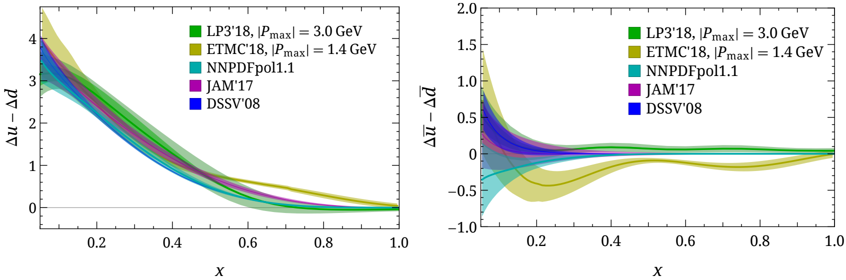
$$\langle 1 \rangle_{\Delta q^+} = \int_0^1 dx\, \Delta q^+(x, Q^2)$$

$$\langle x \rangle_{\Delta u^- - \Delta d^-} = \int_0^1 x dx\, \left[\Delta u^- - \Delta d^-\right](x, Q^2)$$

[See talks by L. Liu, D.-J. Zhao, Z. Hu on Tuesday morning]

Comparing lattice QCD and global PDF fits: PDFs

PDFLattice studies [Prog.Part.Nucl.Phys. 100 (2018) 107; ibid. 121 (2021) 103908]



Ref.	Sea quarks	Valence quarks	$N_{\Delta t}$	method	$P_{\rm max}$ (GeV)	a (fm)	M_π (MeV)	$M_\pi L$
ETMC'20	2f twisted mass	twisted mass	4	pseudo-PDF	1.38	0.09	130	3.0
JLab/W&M	2+1 clover	clover	n/a	pseudo-PDF	3.29	0.09	172–358	5.08–5.47
ETMC'18	2f twisted mass	twisted mass	4	quasi-PDF	1.38	0.09	130	3.0
LP3'18	2+1+1f HISQ	clover	4	quasi-PDF	3	0.09	135	4.0
LP3'17	2+1+1f HISQ	clover	2	quasi-PDF	1.3	0.09	135	4.0

ETMC and LP3 determinations are both at the physical pion mass
Lattice determinations are qualitatively similar (among them) and similar to global fits
Nucleon momentum is limited by lattice spacing
Different procedures lead to slightly different behaviour in x

[See talks by L. Liu, D.-J. Zhao, Z. Hu on Tuesday morning]

Which precision shall we require to lattice QCD?

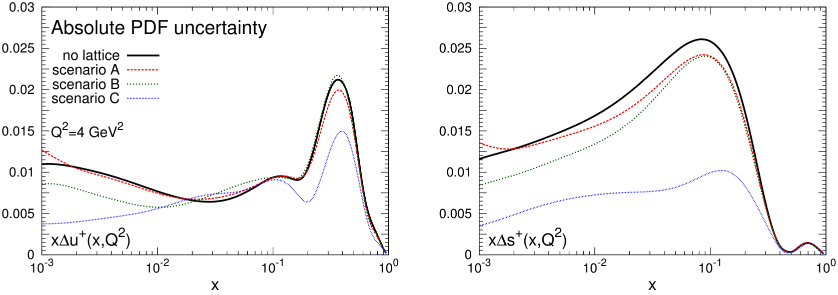
Generate lattice QCD pseudodata assuming NNPDFpol1.1 central values for

$$g_A \equiv \langle 1 \rangle_{\Delta u^+ - \Delta d^+},\ \langle 1 \rangle_{\Delta u^+},\ \langle 1 \rangle_{\Delta d^+},\ \langle 1 \rangle_{\Delta s^+},\ \langle x \rangle_{\Delta u^- - \Delta d^-}$$

Assume percentage uncertainties according to three scenarios

scenario	g_A	$\langle 1 \rangle_{\Delta u^+}$	$\langle 1 \rangle_{\Delta d^+}$	$\langle 1 \rangle_{\Delta s^+}$	$\langle x \rangle_{\Delta u^- - \Delta d^-}$
A	5%	5%	10%	100%	70%
B	3%	3%	5%	50%	30%
C	1%	1%	2%	20%	15%
current	3%	3%	5%	70%	65%

Reweight NNPDFpol1.1 with lattice pseudodata and look at the impact



What if one includes lattice data in a global fit?

Use pseudo-pdf pseudodata [PRD 103 (2021) 016003]

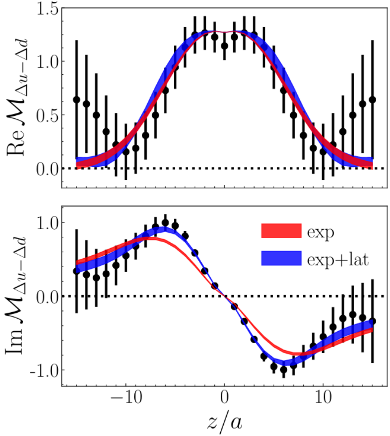
Define the matrix elements $\mathcal{M}$ on the light-cone

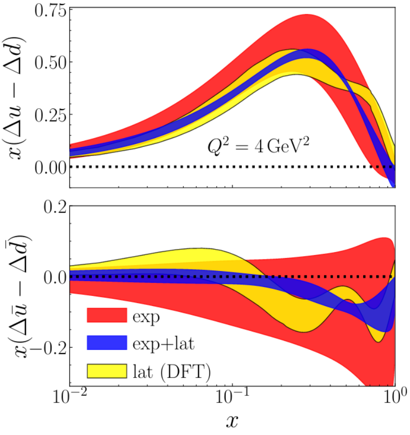
Relate them to light-front PDFs using LaMET

$$\mathrm{Re}\mathcal{M}_{\Delta u-\Delta d} = \mathcal{C}^{\mathrm{Re}}_{\Delta u-\Delta d} \otimes T_3$$

$$\mathrm{Im}\mathcal{M}_{\Delta u-\Delta d} = \mathcal{C}^{\mathrm{Im}}_{\Delta u-\Delta d} \otimes V_3$$

Observable	N_{pt}	χ^2_{pt} (exp)	χ^2_{pt} (exp+lat)
DIS ($A_1^{p,d}$)	651	1.1	—
ETMC Re$\mathcal{M}_{\Delta u-\Delta d}$	31	—	0.5
ETMC Im$\mathcal{M}_{\Delta u-\Delta d}$	30	—	0.3
Total (exp+lat)	712	—	1.0

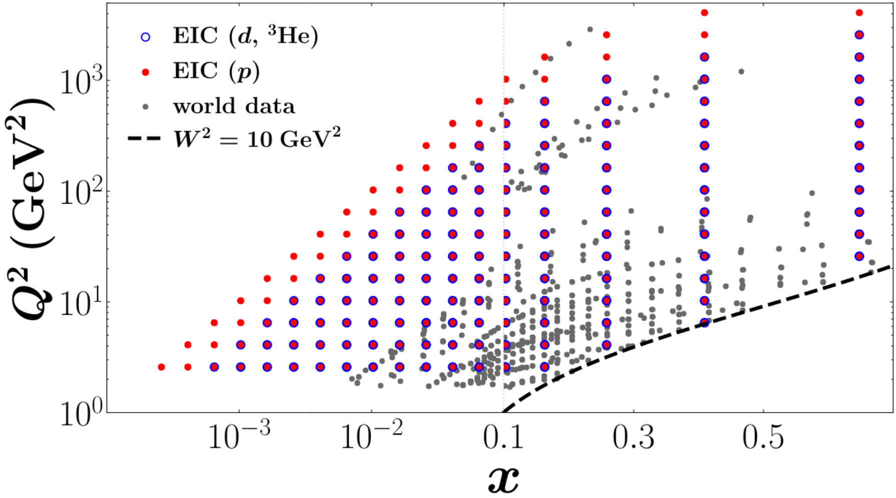




4. Where we aim to get: opportunities at the EIC

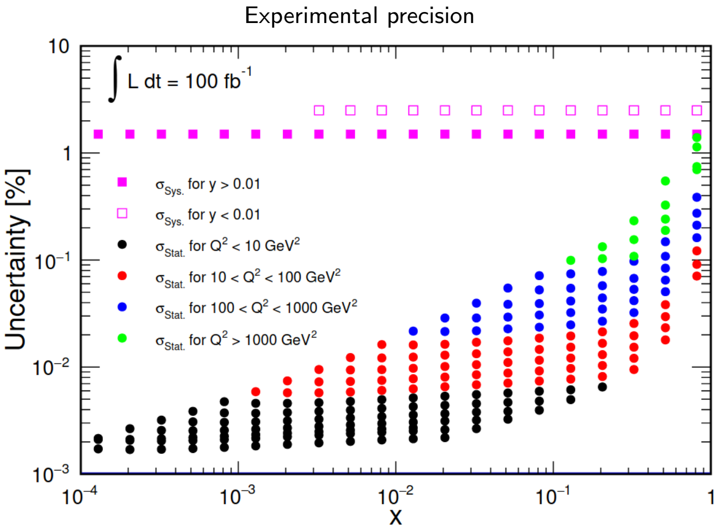
EIC measurements

Kinematic coverage



[Figure from PRD 104 (2021) 034028]

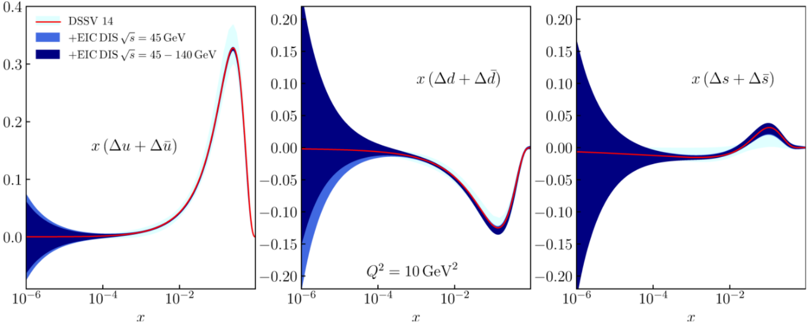
EIC measurements

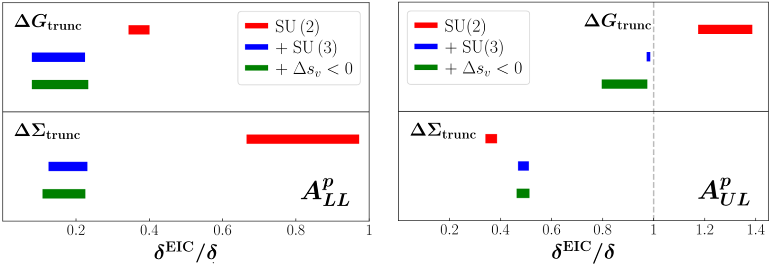


[Figure from the EIC Yellow Report arXiv:2103.05419]

Impact of neutral-current inclusive DIS

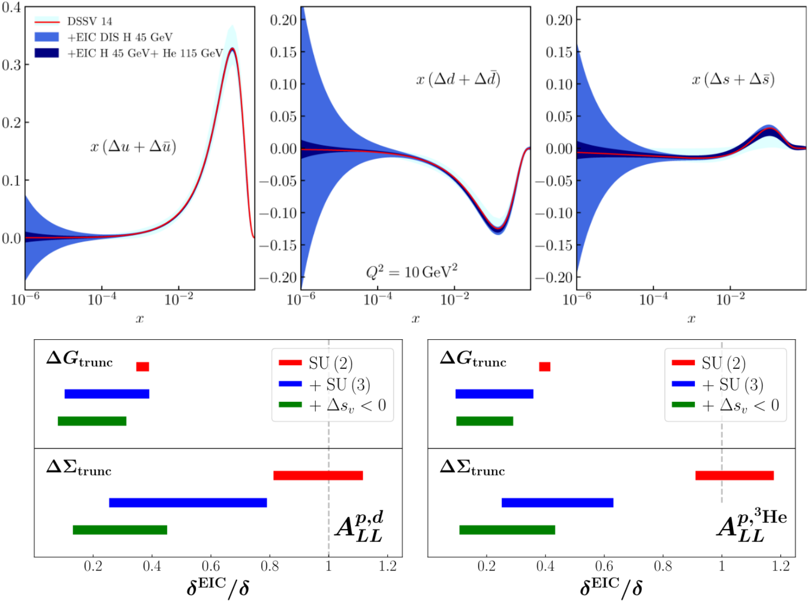
On proton [PRD 102 (2020) 094018; PRD 104 (2021) 034028]





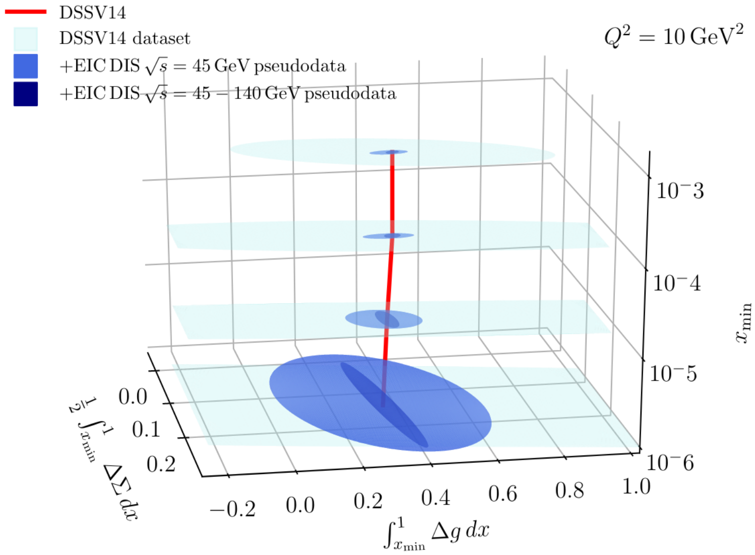
Impact of neutral-current inclusive DIS

On ^{3}He [PRD 102 (2020) 094018; PRD 104 (2021) 034028]



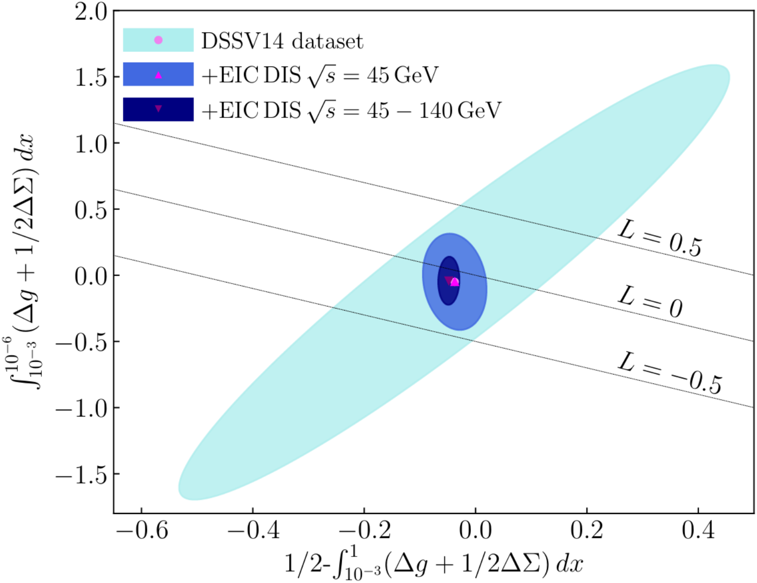
Impact of neutral-current inclusive DIS

Quark, gluon and orbital angolar momenta [PRD 88 (2013) 114025]



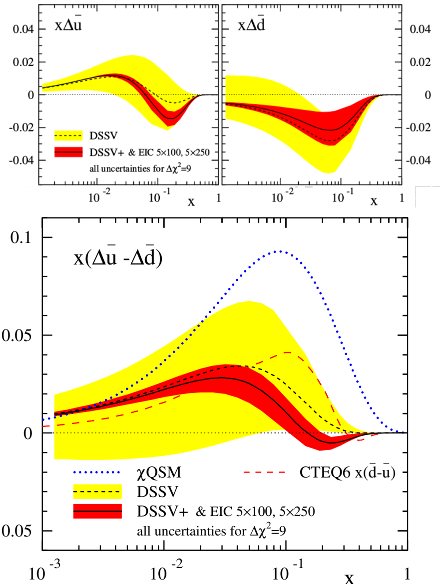
Impact of neutral-current inclusive DIS

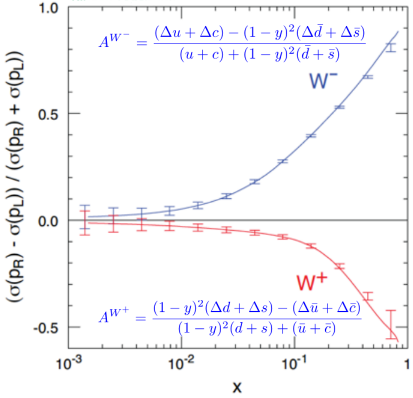
Quark, gluon and orbital angolar momenta [PRD 88 (2013) 114025]



Impact of charged-current DIS

On proton [PRD 88 (2013) 114025]





$$A_L^{W^+,p} \xrightarrow[y\to 0]{\text{LO}} \frac{\Delta u - \Delta\bar{d}}{u+\bar{d}}$$

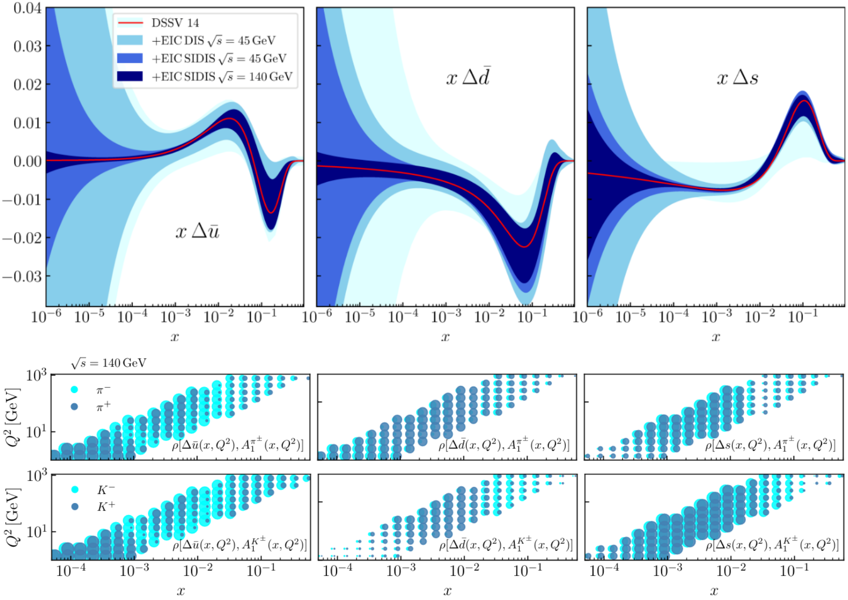
$$A_L^{W^+,p} \xrightarrow[y=1/2]{\text{LO}} \frac{4\Delta u - \Delta\bar{d}}{4u+\bar{d}}$$

$$A_L^{W^+,p} \xrightarrow[y\to 1]{\text{LO}} \frac{\Delta u}{u}$$

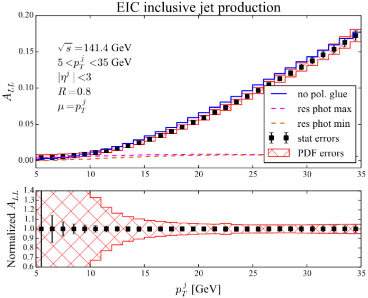
$\longleftrightarrow$ for $A_L^{W^-,n}$

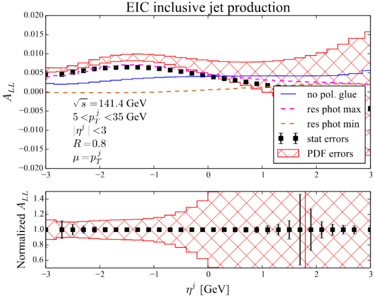
Impact of semi-inclusive DIS

With charged pions and kaons [PRD 102 (2020)]



Impact of DIS+jets



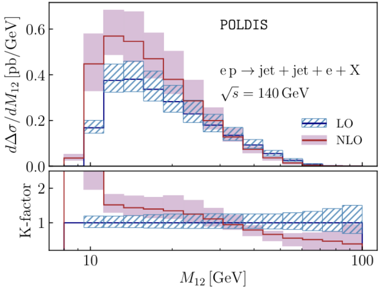


[PRD 98 (2018) 054031]

Include all relevant partonic contributions (direct and resolved photon contributions)

Intermediate-to-high tranverse momenta are sensitive to the polarised gluon PDF

Find that PDF errors are much larger than projected statistical errors

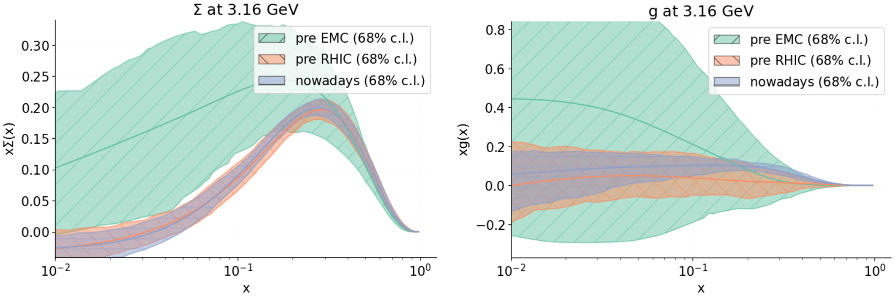


[PRD 103 (2021) 014008; ibid. 105 (2022) 074025]

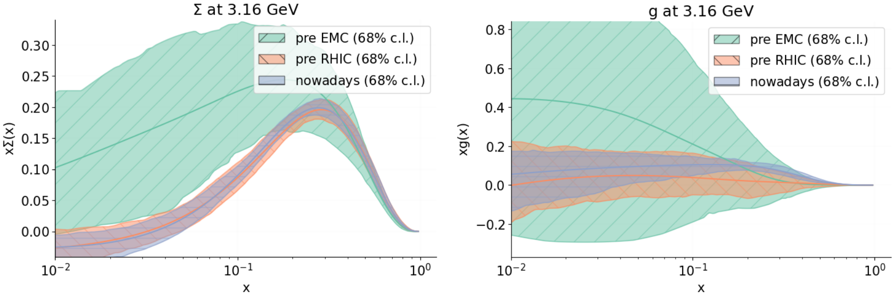
Fully differential NLO corrections to di-jet production in NC and CC DIS

5. In summary

Look back to look ahead



Look back to look ahead



[Spin] is a mysterious beast, and yet its practical effect prevails the whole of science. The existence of spin, and statistics associated with it, is the most subtle and ingenious design of Nature - without it the whole Universe would collapse.

S-I. Tomonaga, *The story of spin* 2nd ed., University of Chicago Press (1998) [from the preface]

Thank you