GPD theory

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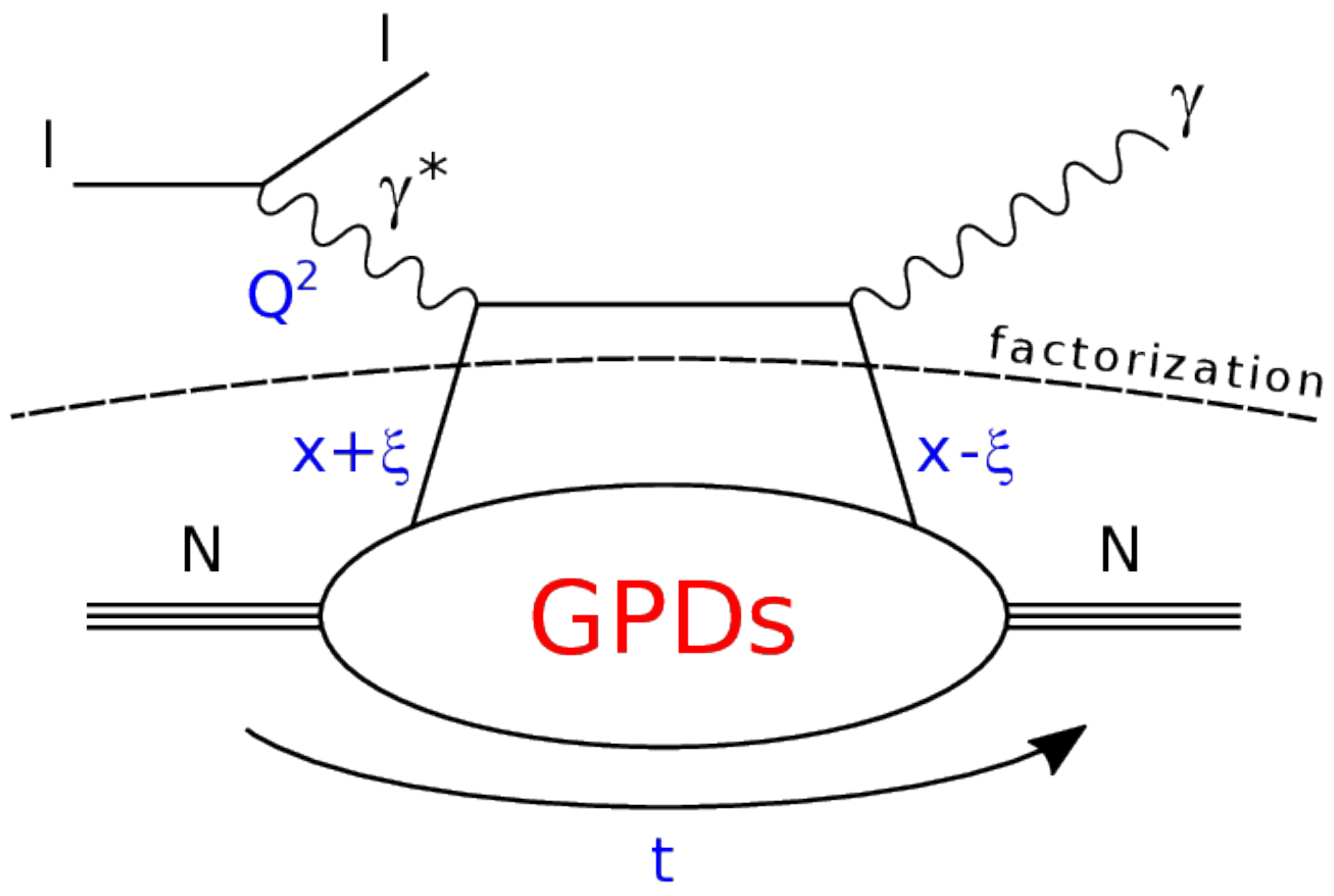
Deeply Virtual Compton Scattering (DVCS)



factorisation for |t|/Q² ≪ 1

Chiral-even GPDs:
(helicity of parton conserved)

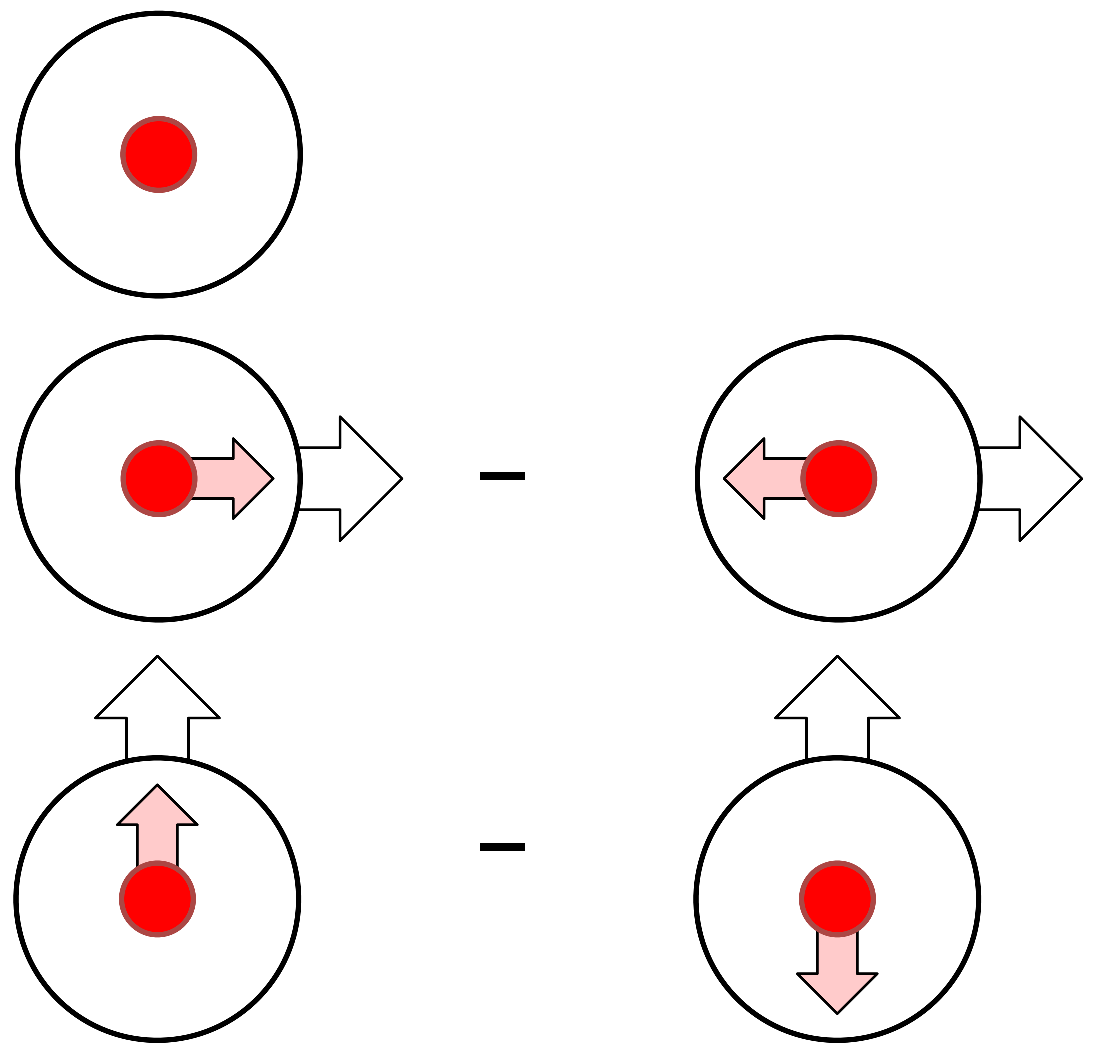
$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

Reduction to PDFs:

$$H^q(x,0,0) \equiv q(x)$$

$$\widetilde{H}^q(x,0,0) \equiv \Delta q(x)$$

$$H_T^q(x,0,0) \equiv h_1(x)$$



no corresponding relations exist for other GPDs

Reduction to Elastic Form Factors (EFFs):

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) \equiv F_1^q(t) \qquad \int_{-1}^{1} dx\, E^q(x,\xi,t) \equiv F_2^q(t)$$

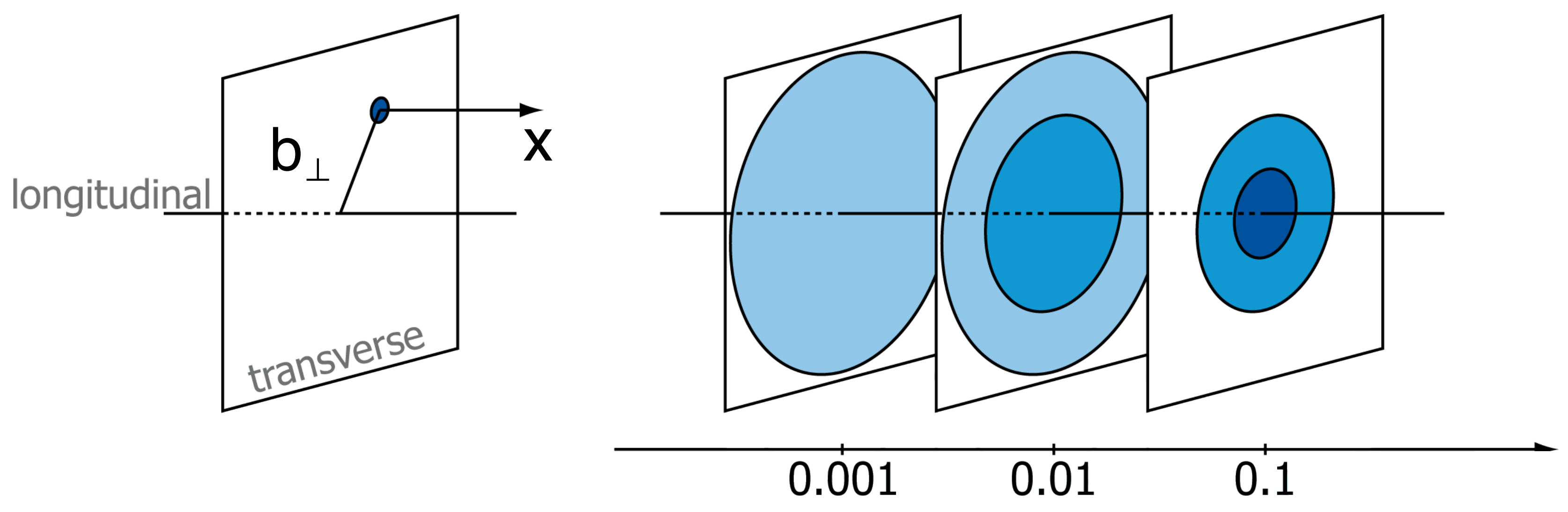
$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) \equiv g_A^q(t) \qquad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) \equiv g_P^q(t)$$

See Tuesday talks:

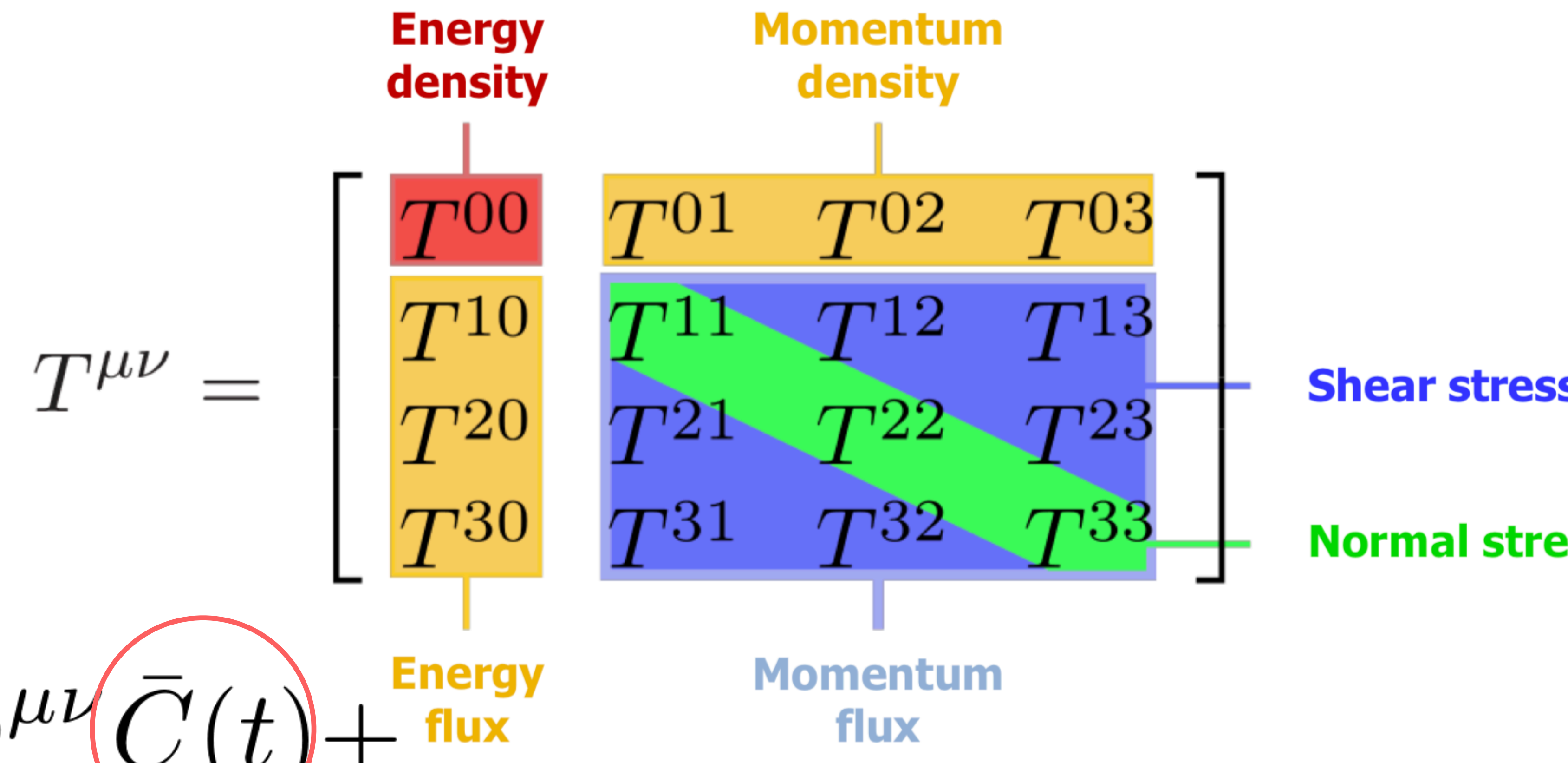
Mi Wang
Weizhi Xiong
Yi Chen
Heng-Tong Ding

Nucleon tomography:

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H_q(x, 0, -\mathbf{\Delta}_\perp^2)$$



Energy momentum tensor in terms of form factors (OAM and "mechanical" properties):



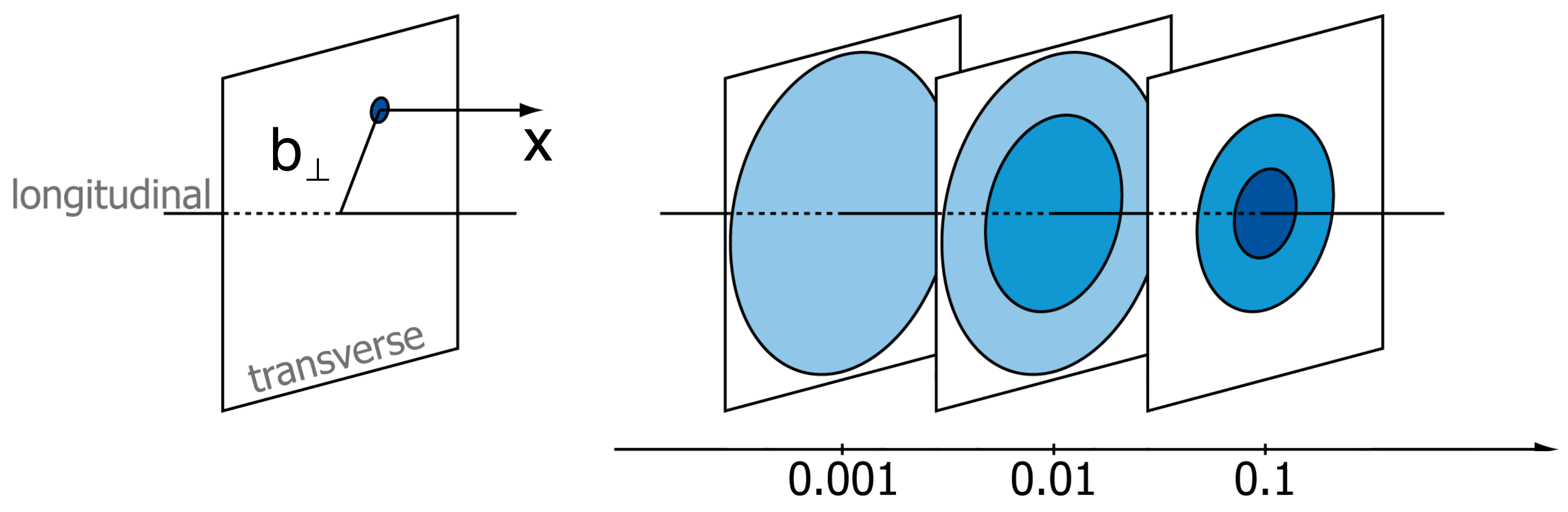
$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)$$

$$A^q(0) + B^q(0) = \int_{-1}^{1} x \left[H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] = 2J^q$$

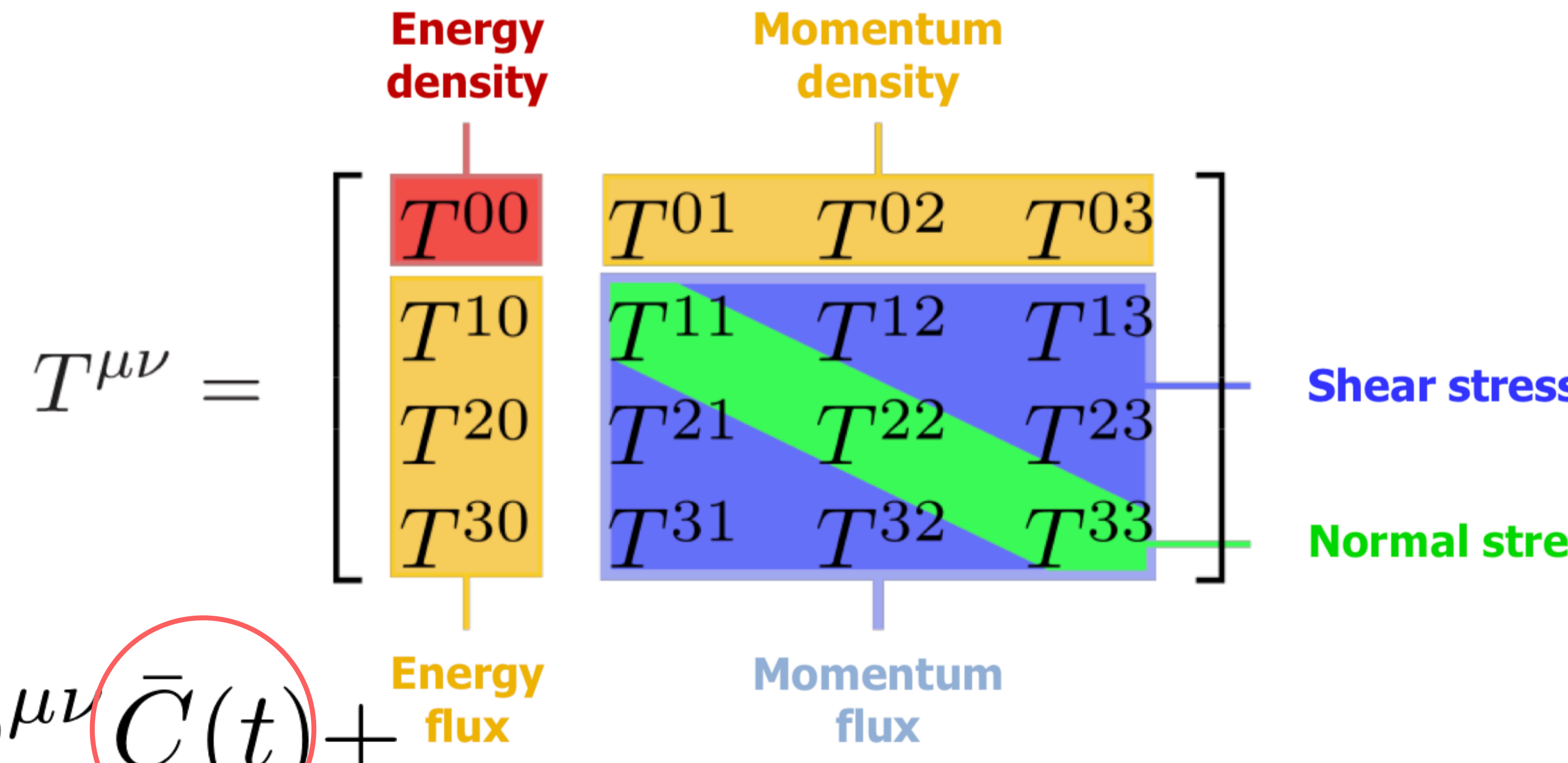
Ji's sum rule

Nucleon tomography:

$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H_q(x, 0, -\mathbf{\Delta}_\perp^2)$$



Energy momentum tensor in terms of form factors (OAM and “mechanical” properties):



$$\langle p', s'|\widehat{T}^{\mu\nu}|p, s\rangle = \bar{u}(p', s')\left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu}\bar{C}(t) + \right.$$

$$\frac{P^\mu \ldots}{\ldots} \ldots + \frac{P^\nu i \ldots}{\ldots} \ldots s)$$

See Tuesday talks:

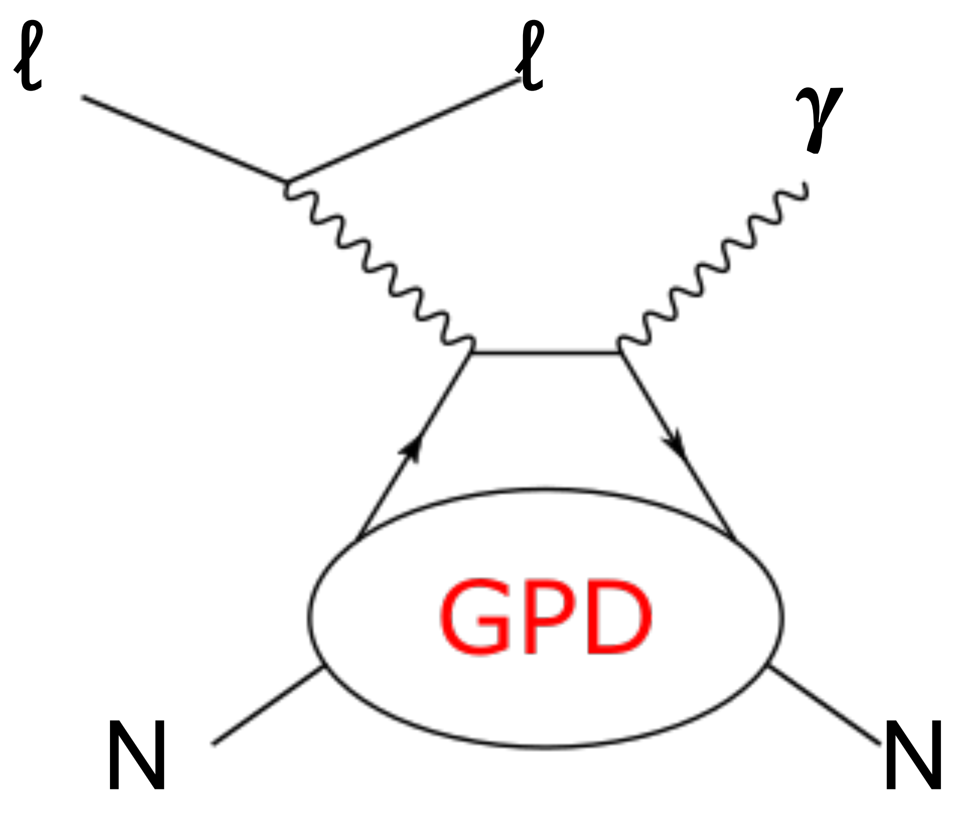
Xianghui Cao
Qin-Tao Song
Minghui Ding
Jing Han

See X. Ji, C. Yang
arXiv: hep-ph/2508.16727 for a recent discussion of “mechanical” properties

$$A^q(0) + B^q(0) = \int_{-1} x\,[H^q(x, \xi, 0) + E^q(x, \xi, 0)] = 2J^q$$

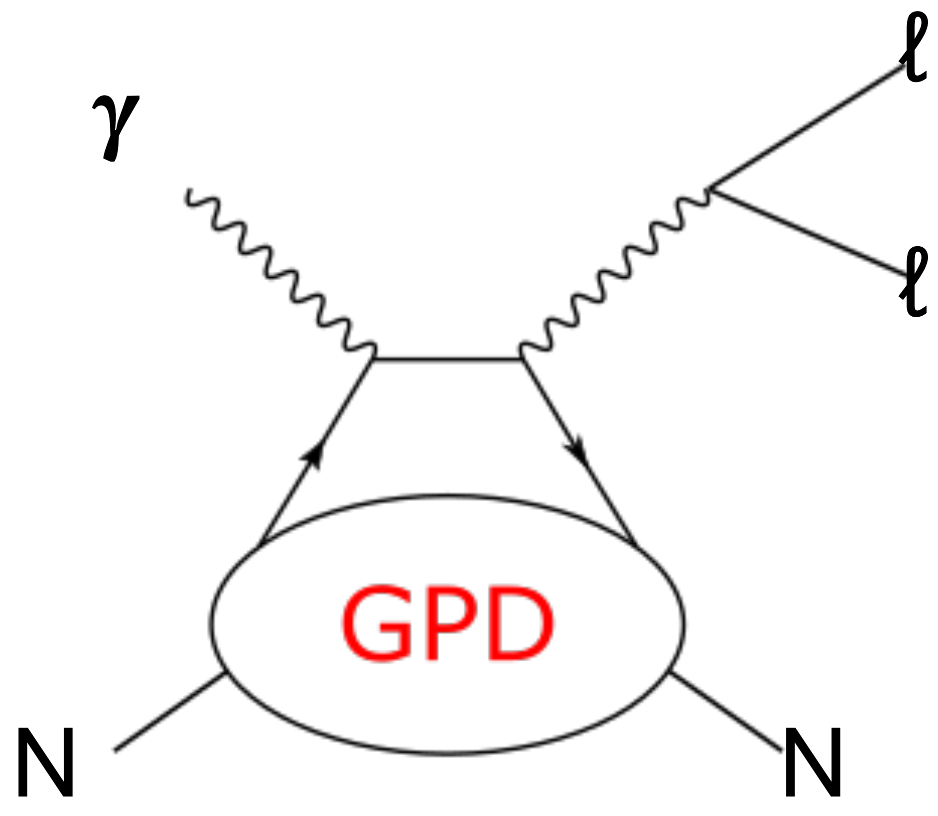
Ji’s sum rule

VCS processes provide the most straightforward way to access GPDs (at least from theory side...)



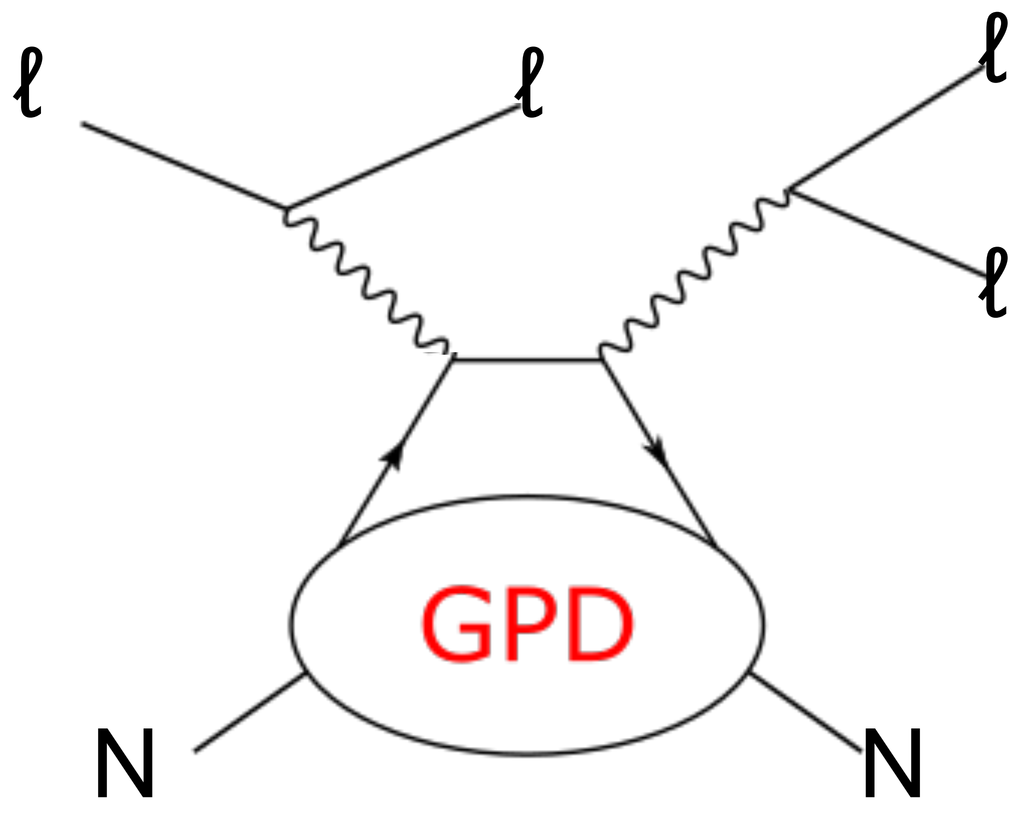
DVCS
Deeply Virtual Compton Scattering

- many measurements, see e.g.:
  EPJA 52 (2016) 6, 157
- description up to NNLO and twist-4 available
  PRL 129 (2022) 17, 172001
  JHEP 01 (2023) 078



TCS
Timelike Compton Scattering

- first measurement by CLAS
  PRL 127 (2021) 26, 262501
- description up to NLO and twist-4 available
  PRD 111 (2025) 7, 074034 (NEW)



DDVCS
Double Deeply Virtual Compton Scattering

- never measured
- description up to NLO and twist-4 available
  PRD 111 (2025) 7, 074034 (NEW)

more production channels sensitive to GPDs exist!

V. Martínez-Fernández, et al.
PRD 111 (2025) 7, 074034

DDVCS used as a unified framework for DVCS and TCS

Starting point: OPE + CFT (Braun-Ji-Manashov result)
(see: JHEP 03 (2021) 051 and JHEP 01 (2023) 078)

See Tuesday talks: Yao Ji

$$T^{\mu\nu} = i \int d^4z \, e^{iq'z} \langle p' | \mathcal{T}\{ j^\mu(z) j^\nu(0) \} | p \rangle$$

$$= \frac{1}{i\pi^2} i \int d^4z \, e^{iq'z} \left\{ \frac{1}{(-z^2+i0)^2} \left[g^{\mu\nu} \mathscr{O}(1,0) - z^\mu \partial^\nu \int_0^1 du \, \mathscr{O}(\bar{u},0) - z^\nu (\partial^\mu - i\Delta^\mu) \int_0^1 dv \, \mathscr{O}(1,v) \right] \right. + \ldots$$

where $\mathscr{O}, \mathscr{O}_1, \mathscr{O}_2$ are matrix elements $\langle p' | \mathscr{O} | p \rangle, \langle p' | \mathscr{O}_1 | p \rangle, \langle p' | \mathscr{O}_2 | p \rangle$ containing information about GPDs

For spin-0 target:

$$T^{\mu\nu} = \mathcal{A}^{00} \frac{-i}{QQ'R^2} \left[(qq')(Q'^2 q^\mu q^\nu - Q^2 q'^\mu q'^\nu) + Q^2 Q'^2 q^\mu q'^\nu - (qq')^2 q'^\mu q^\nu \right]$$
$$+ \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu \right] \bar{p}_\perp^\nu + \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu \right]$$
$$+ \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \tilde{\bar{p}}_\perp^\mu \tilde{\bar{p}}_\perp^\nu \right] + \mathcal{A}^{++} g_\perp^{\mu\nu},$$

$$R = \sqrt{(qq')^2 + Q^2 Q'^2}$$

V. Martínez-Fernández, et al.
PRD 111 (2025) 7, 074034

Result for A++

leading twist

$$\mathcal{A}^{++}_{\rm DVCS} = \lim_{\rho\to\xi} \mathcal{A}^{++}$$
$$= \int_{-1}^{1} dx \Bigg\{ -\left(1 - \frac{t}{2\mathbb{Q}^2}\right)\frac{H^{(+)}}{x-\xi+i0} - \frac{2t}{\mathbb{Q}^2}\left[\frac{1}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right) + \frac{L_{\rm DVCS}}{4}\right]H^{(+)} + \frac{t}{\mathbb{Q}^2}\partial_\xi\left[\left(\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right)+\frac{\xi L_{\rm DVCS}}{2}\right)H^{(+)}\right] - \frac{\bar{p}_\perp^2}{\mathbb{Q}^2}2\xi^3\partial_\xi^2\left[\left(\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right)+\frac{\xi L_{\rm DVCS}}{2}\right)H^{(+)}\right]\Bigg\} + O(\text{tw-6})\,,$$

$$\mathcal{A}^{++}_{\rm TCS} = \lim_{Q^2\to 0} \mathcal{A}^{++}$$
$$= \int_{-1}^{1} dx \Bigg\{ -\frac{1}{x+\xi(1-2t/\mathbb{Q}^2)+i0}H^{(+)} + \frac{t}{2\mathbb{Q}^2}\frac{1}{x+\xi+i0}H^{(+)} - \frac{2t}{\mathbb{Q}^2}\xi\partial_\xi\left(\frac{1}{x+\xi+i0}H^{(+)}\right) - \frac{2t}{\mathbb{Q}^2}\left[-\frac{1}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right) - \frac{1}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right)-\mathrm{Li}_2(1)\right)+\frac{L_{\rm TCS}}{4}\right]H^{(+)} + \frac{t}{\mathbb{Q}^2}\partial_\xi\left[\left(-\frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right)-\frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right)-\mathrm{Li}_2(1)\right)+\frac{\xi L_{\rm TCS}}{2} + \frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi+i0}\right)\right)H^{(+)}\right] - \frac{\bar{p}_\perp^2}{\mathbb{Q}^2}2\xi^3\partial_\xi^2\left[\left(-\frac{2\xi}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right)-\frac{2\xi}{x+\xi}\left(\mathrm{Li}_2\left(-\frac{x-\xi}{2\xi+i0}\right)-\mathrm{Li}_2(1)\right)+\frac{\xi L_{\rm TCS}}{2} + \frac{\xi}{x-\xi}\ln\left(\frac{x+\xi+i0}{2\xi+i0}\right)\right)H^{(+)}\right]\Bigg\} + O(\text{tw-6})\,,$$

$$L_{\rm DVCS} = \frac{4}{x-\xi}\left[\mathrm{Li}_2\left(\frac{x+\xi}{2\xi-i0}\right)-\mathrm{Li}_2(1)\right]$$

$L_{\rm TCS}$ computed numerically

TCS at twist-4

V. Martínez-Fernández, et al.
PRD 111 (2025) 7, 074034

Legend:

	twist-2	twist-4
DVCS	solid	dotted
TCS	solid	dotted

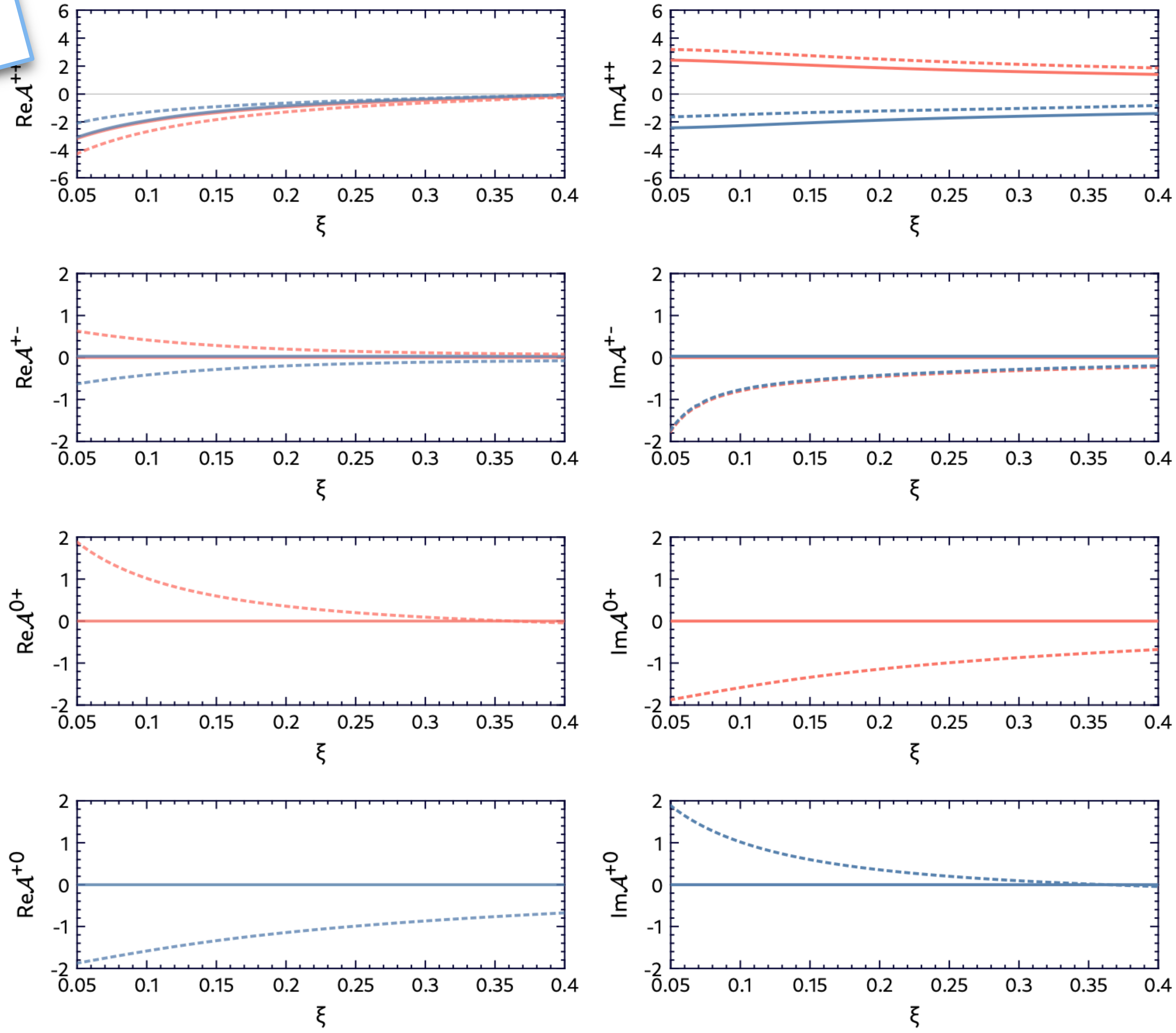
Kinematics:

$-t = 0.6$ GeV^2, $Q^2 = 1.9$ GeV^2

GPD model for pion from:

PRD 105, 094012 (2022)

formulae for A^{00} (only relevant for DDVCS) available too



Shadow GPDs

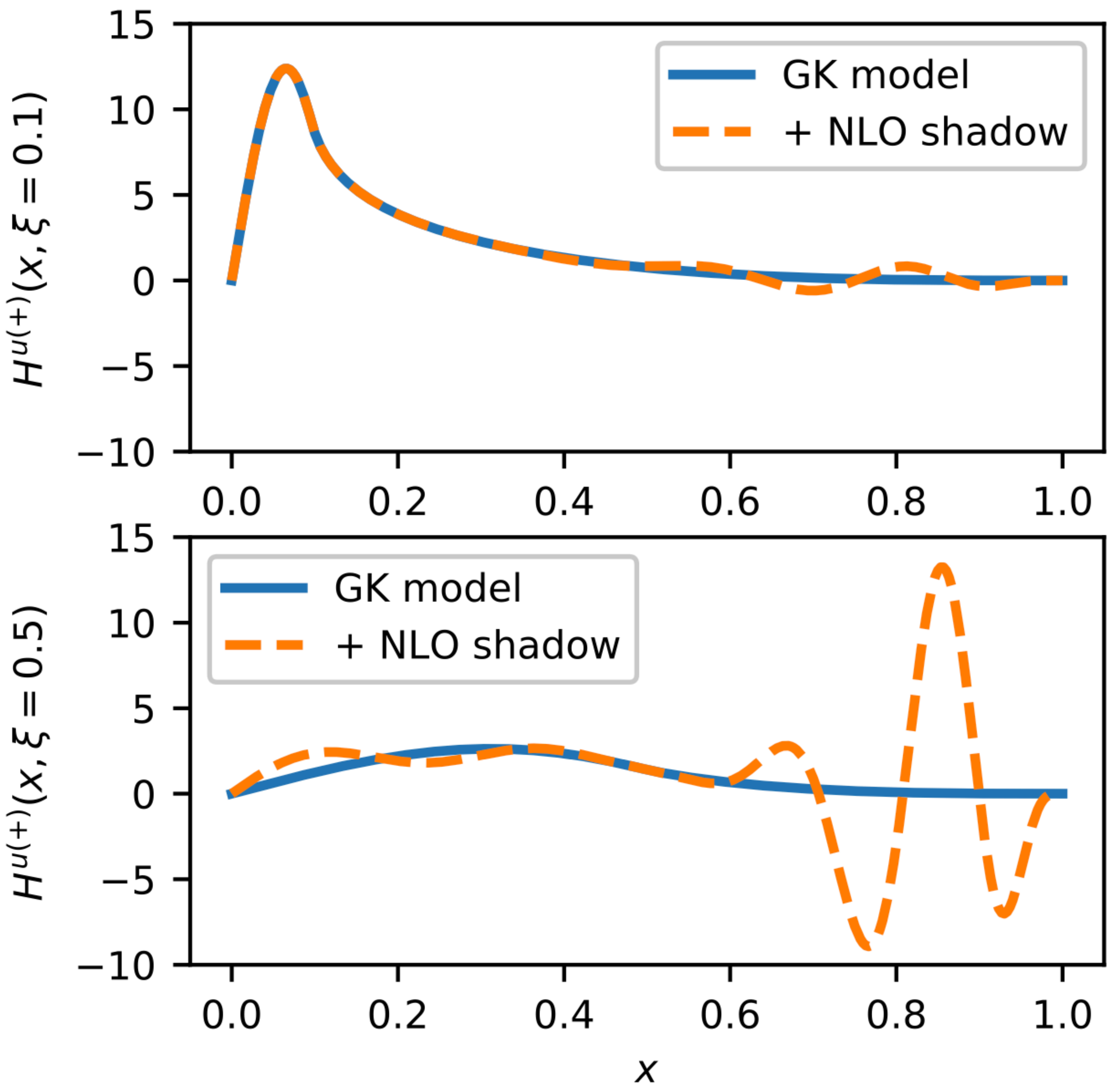
V. Bertone et al.,
Phys. Rev. D 103 (2021) 11, 114019

Shadow GPDs have considerable size, but:

- at arbitrary initial scale do not contribute to PDFs and CFFs
- at other scales contribute negligibly

making the deconvolution of CFFs ill-posed problem

Such GPDs were found for DVCS and both LO and NLO
(for discussion see also PRD 108 (2023) 3, 036027)



New type of shadow GPD

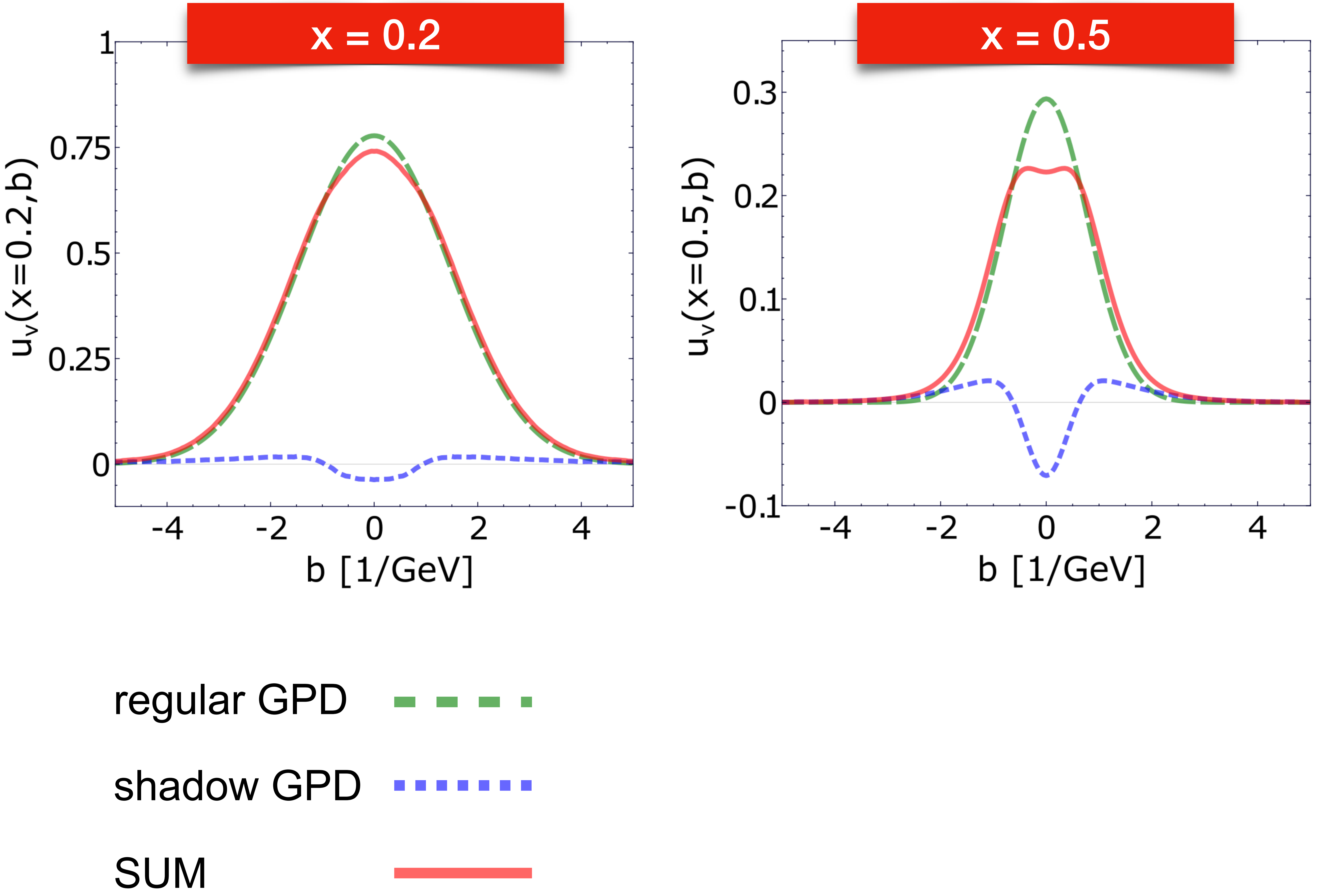
K. Cichy et al.,
PRD 110 (2024) 11, 114025

Previously, shadow GPDs were considered in (x, ξ) space, but they can also be defined in (x, t) space.

In such a case, shadow GPDs:

- at an arbitrary initial scale, do not contribute to PDFs and EFFs
- at other scales, contribute negligibly

This type of shadow GPDs can be used to study local deformations of parton densities.



Question

If deconvolution of GPDs from processes like DVCS is ill-defined, what can we do?

1. Measure, e.g., DDVCS, which probes GPDs in unexplored kinematic regions
2. We can still learn a lot from processes like DVCS alone
3. Employ lattice QCD

Double DVCS

K. Deja, et al.
Phys. Rev. D 107 (2023) 9, 094035

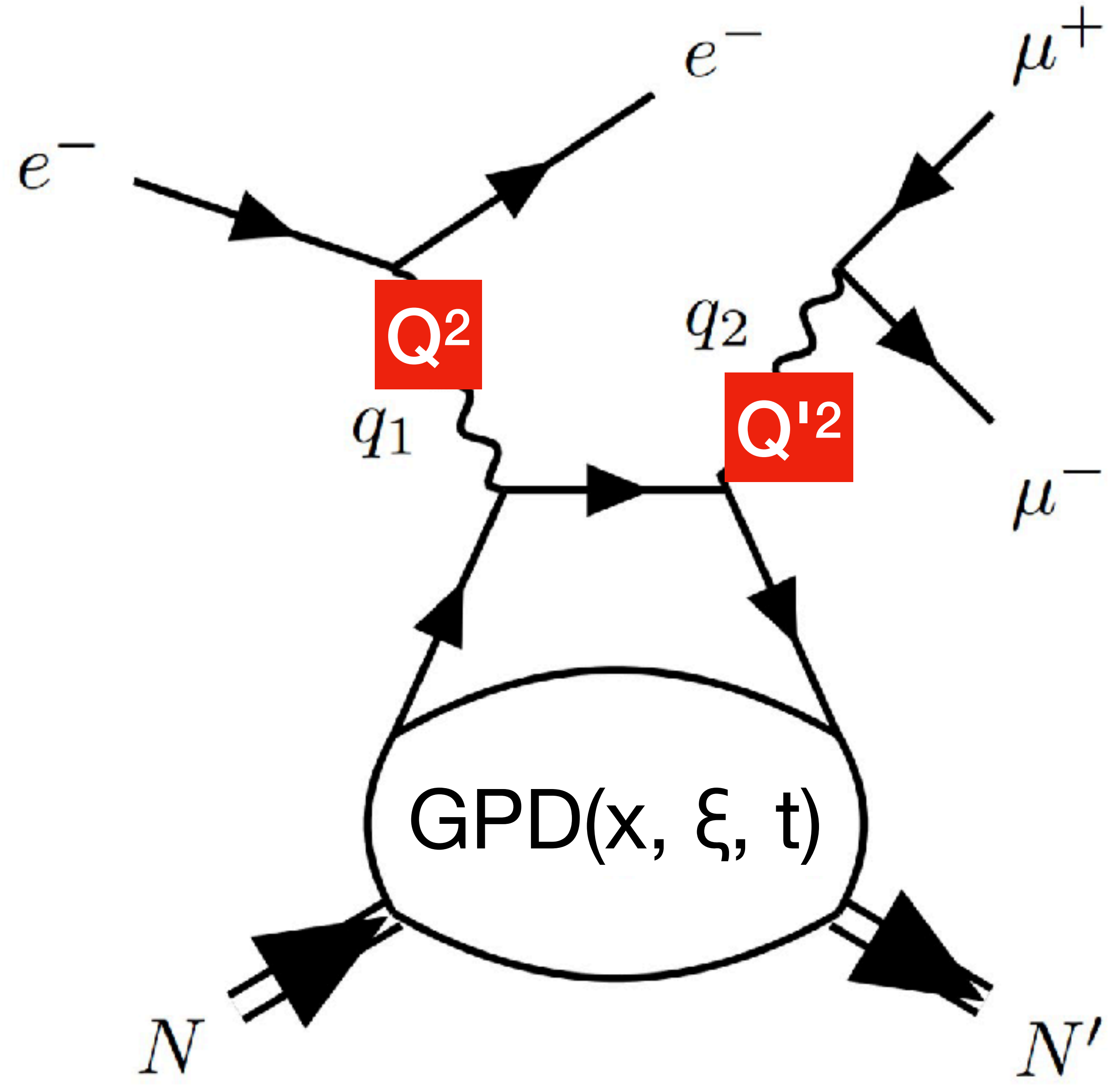
- The process allows to directly probe GPDs outside x = ξ line, but is much more challenging experimentally

$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2}$$

$$\rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$

$$(\mathcal{H}, \mathcal{E})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^{1} dx\, C_f^{(-)}(x, \rho)(H_f, E_f)(x, \xi, t)$$

$$C_f^{(\pm)}(x, \rho) \overset{LO}{=} \left(\frac{e_f}{e}\right)^2 \left(\frac{1}{\rho - x - i0} \pm \frac{1}{\rho + x - i0}\right)$$



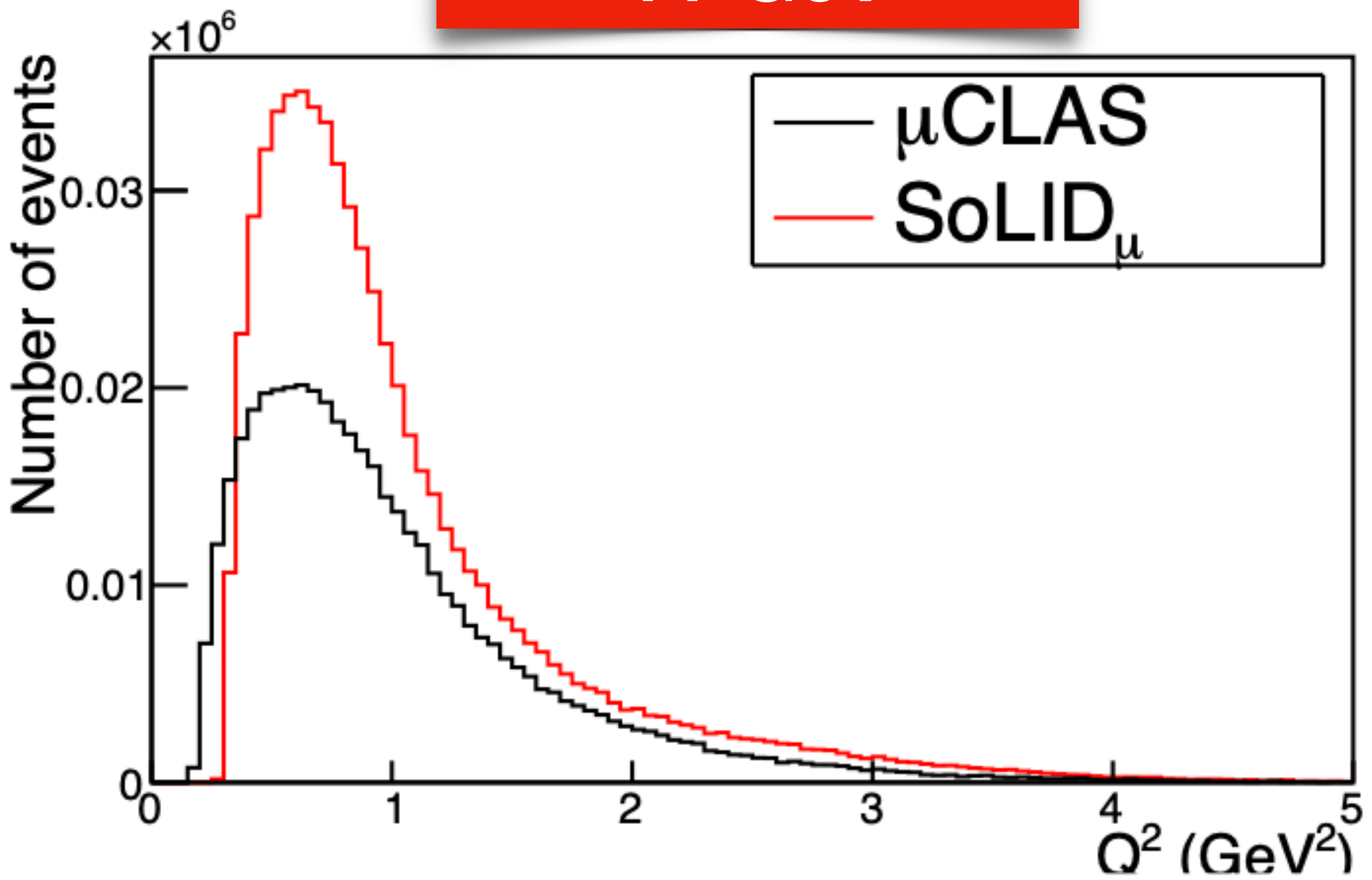
- DDVCS description revisited in view of new experiments, including reevaluation of DDVCS and BH cross-sections with Kleiss-Stirling spinor techniques

- Obtained results are available in PARTONS and EpIC MC generator

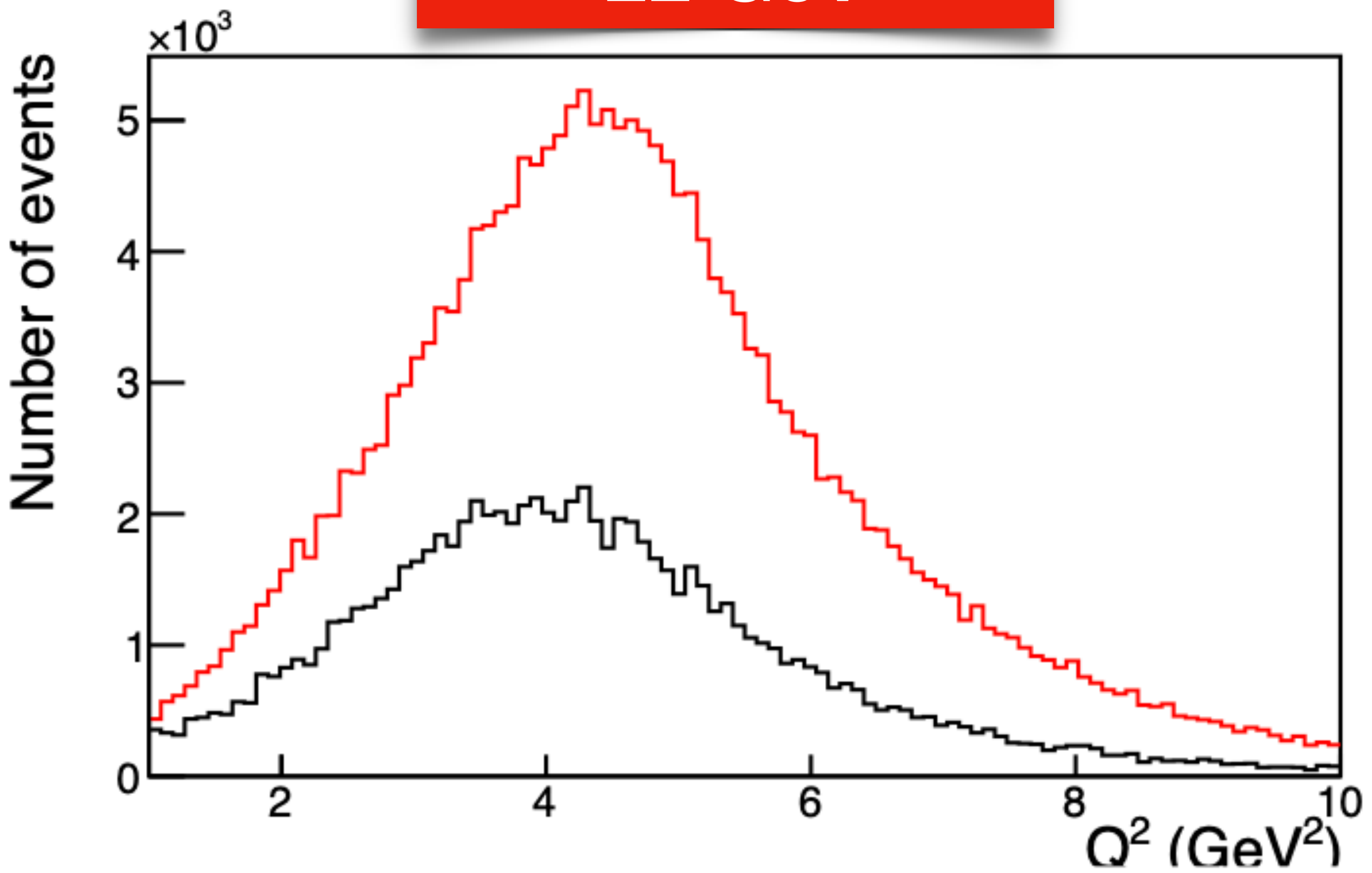
Recent impact studies

J. S. Alvarado, et al.
PRC 111 (2025) 6, 065205

11 GeV



22 GeV



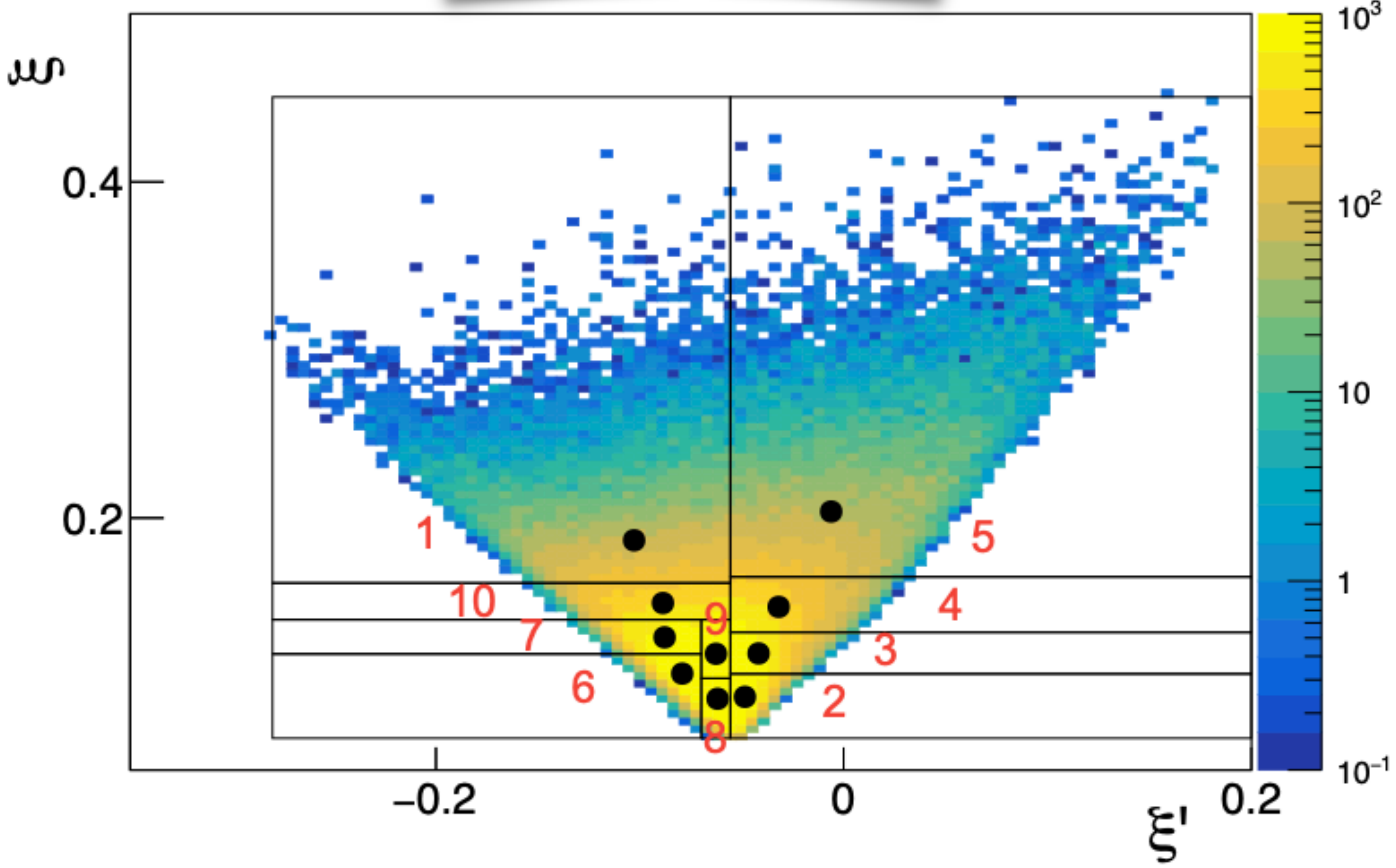
- 11 GeV at JLab

  100 days and $\mathcal{L} = 10^{37}$ cm^{-2} ·s^{-1}

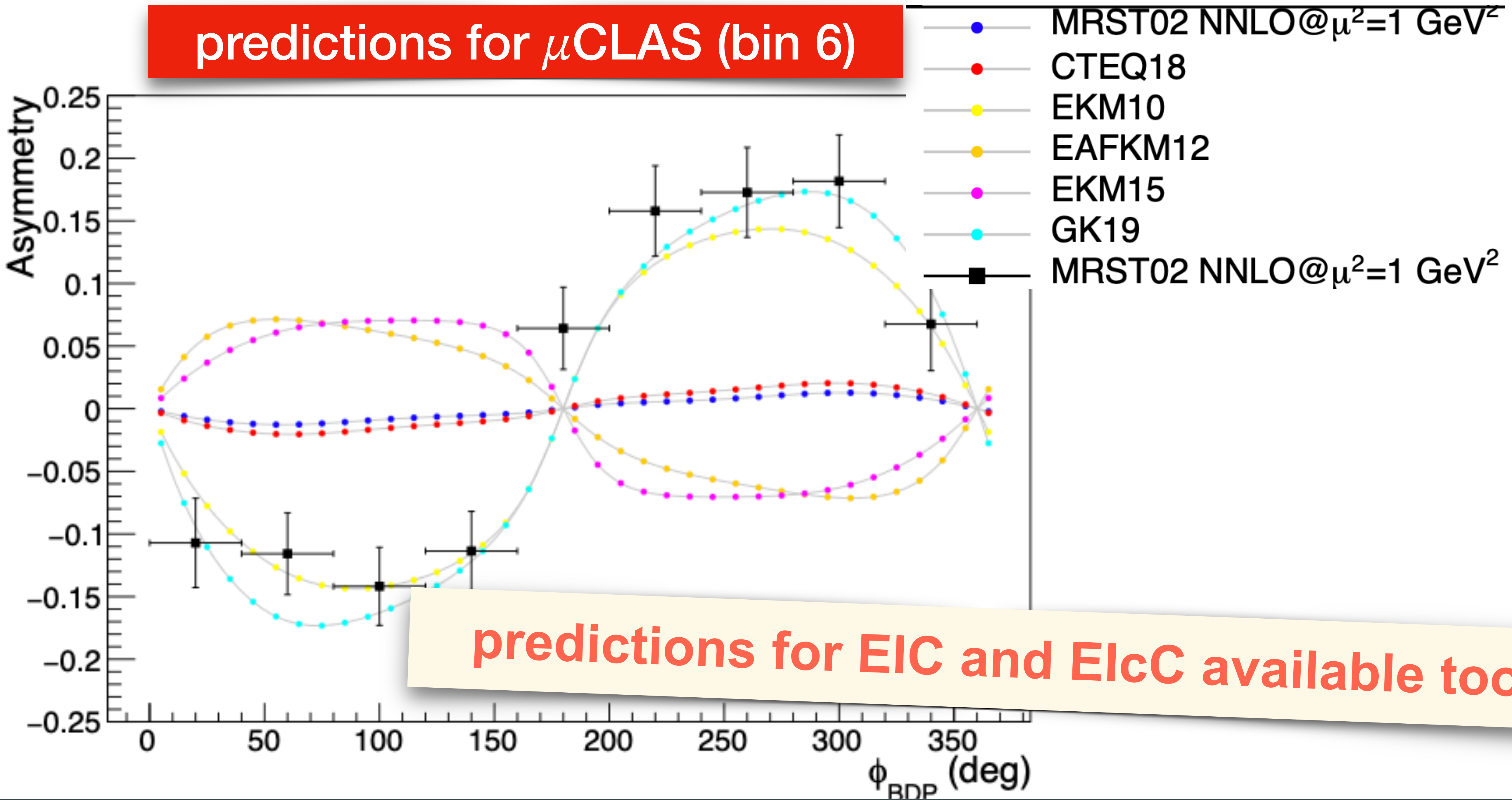
- 22 GeV at JLab

  200 days and $\mathcal{L} = 10^{37}$ cm^{-2} ·s^{-1}

11 GeV



predictions for μCLAS (bin 6)

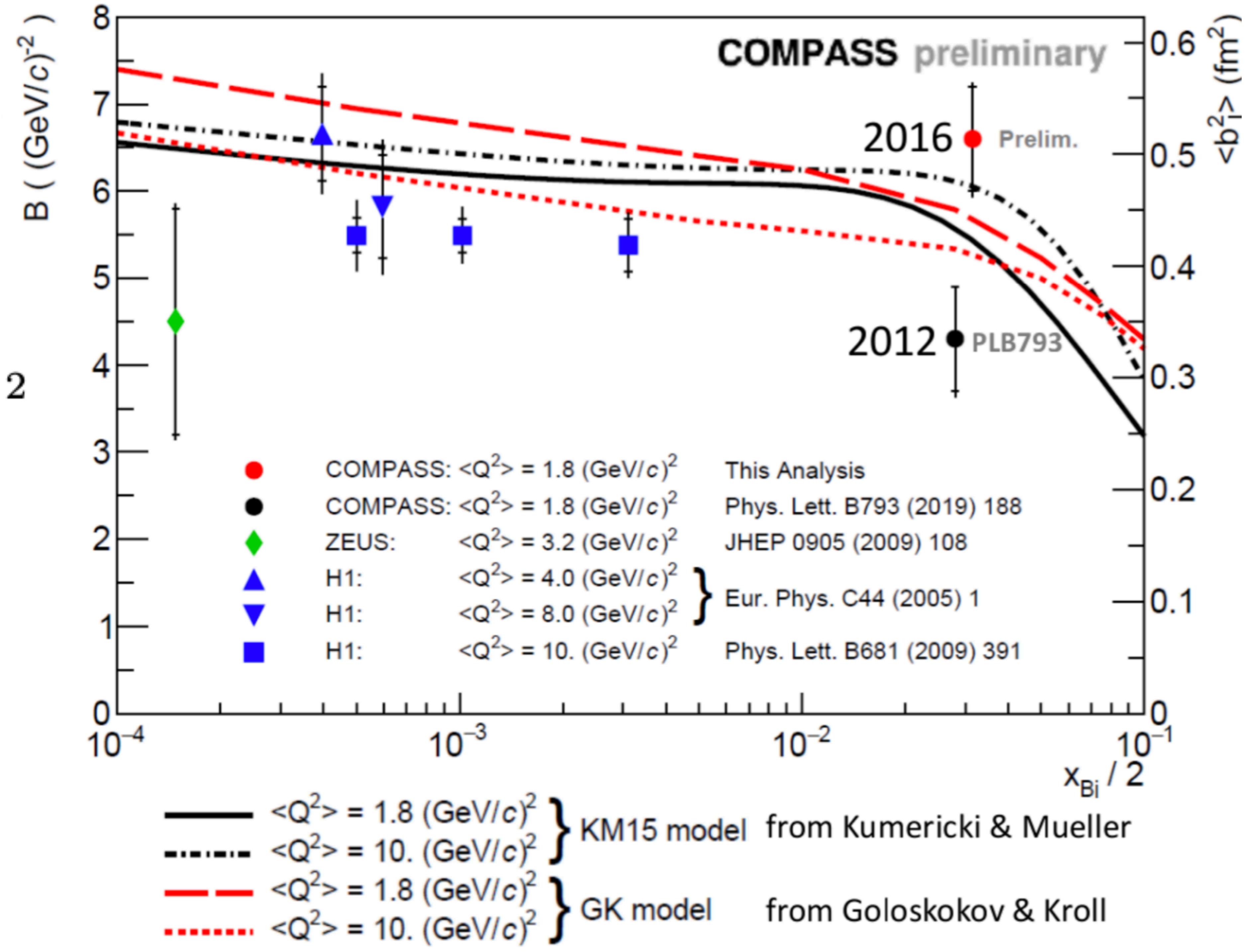


predictions for EIC and EIcC available too

Analyses not requiring de-convolution

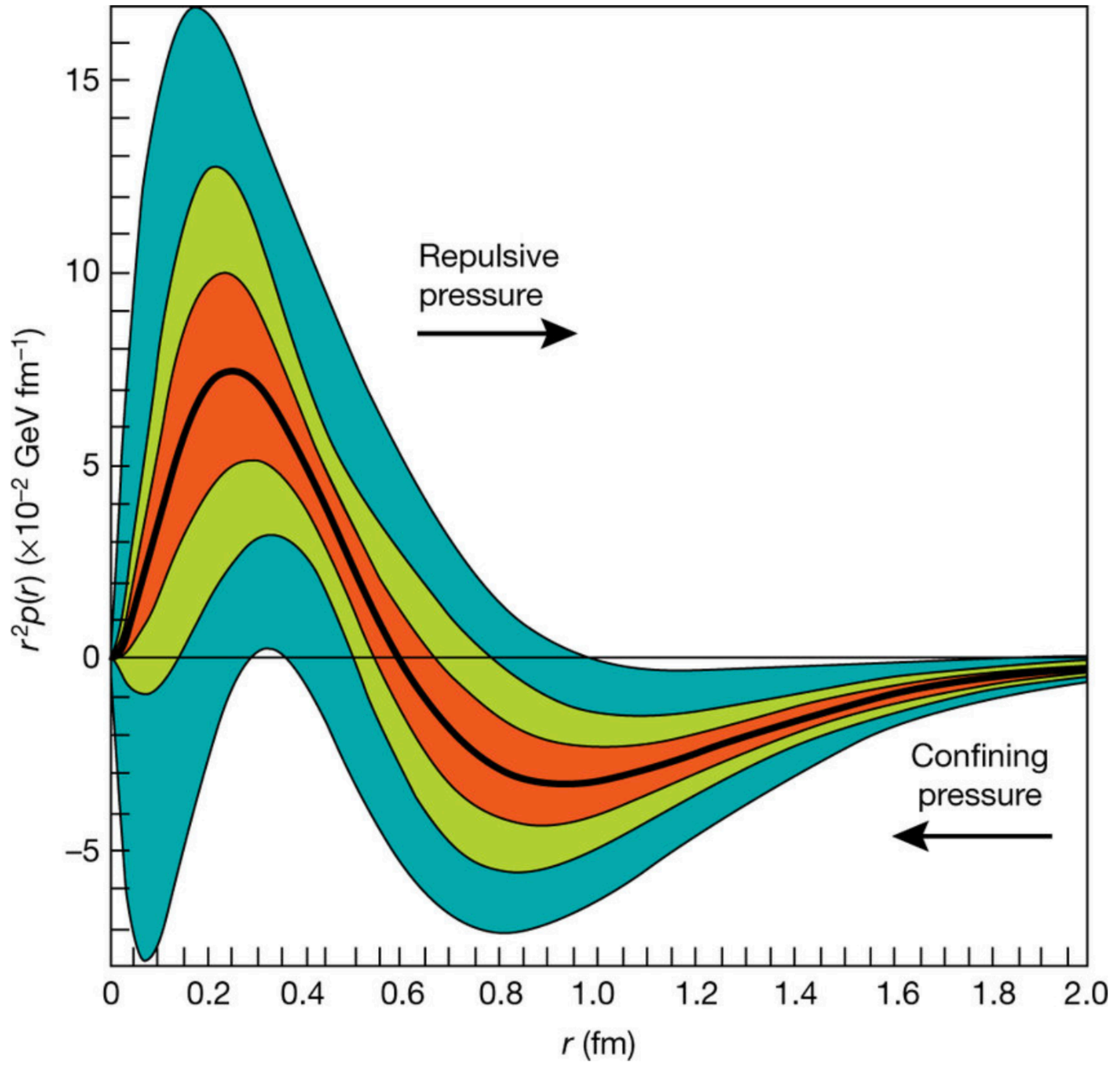
- Probing nucleon tomography at low-xB (see: *JHEP 09 (2013) 093*)

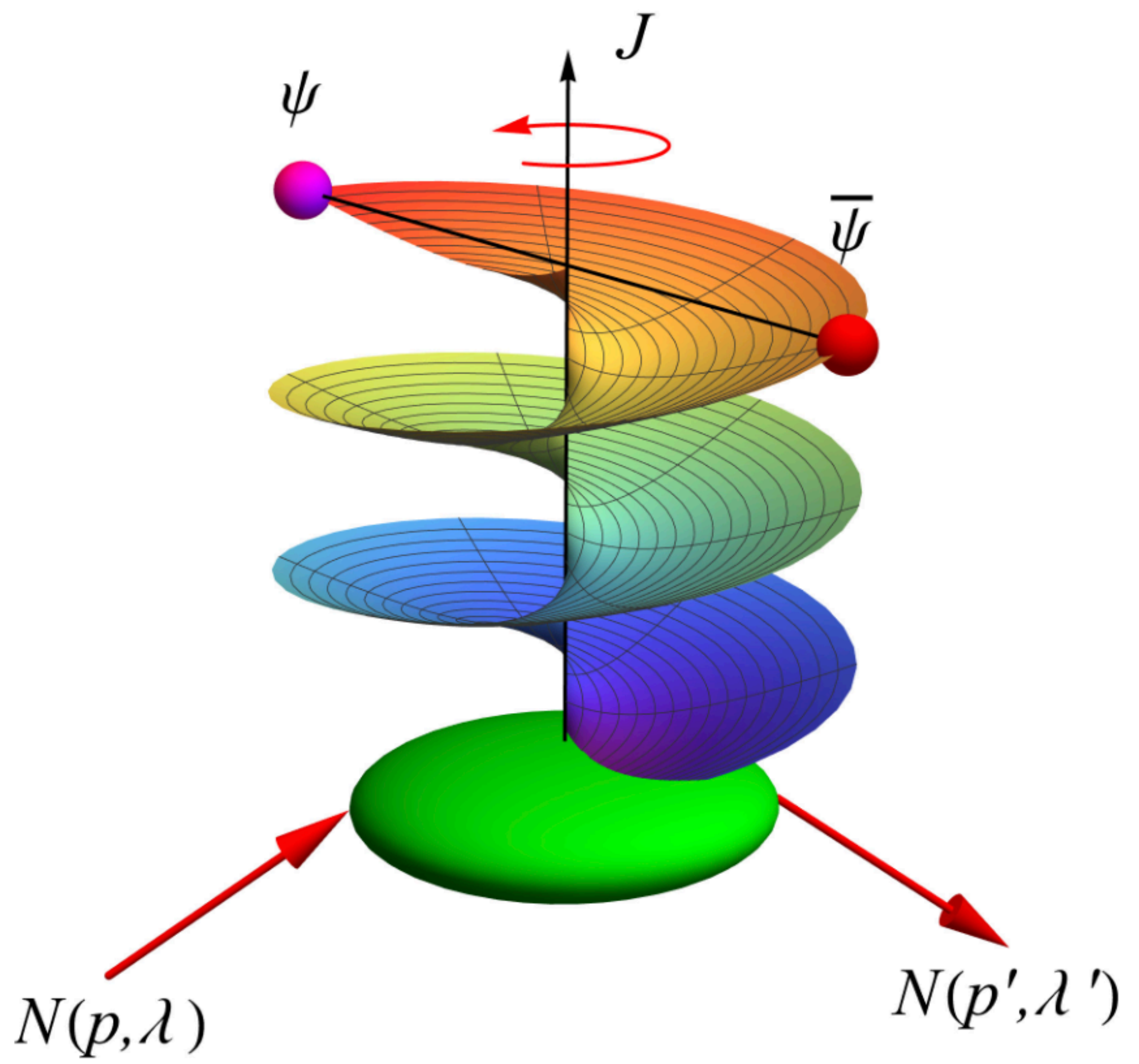
$$d^3\sigma/(dx_{Bj}\,dQ^2\,dt) \propto (\mathrm{Im}\mathcal{H}(\xi,t))^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,\xi,t)\right)^2 \propto \left(\sum_q e_q^2 H^{q(+)}(\xi,0,t)\right)^2$$



- Extraction of D-term

  (see: *Nature 570 (2019) 7759, E1*, *EPJC 81 (2021) 4, 300*)





- Froissart-Gribov projections (see: *PRD 109 (2024) 5, 054010*)

Recent impact study for EIC

E. C. Aschenauer, et al.
PRD 112 (2025) 3, 036010

- New projections for EIC
- Based on state-of-the-art detector simulation and Monte Carlo generator
- Include estimation of pi0 background and radiative corrections
- Extraction of CFFs and tomography information inspired by machine learning techniques

Projections for nucleon tomography:
L = 10 fb^{-1}, 10 GeV x 100 GeV

Dispersion relation for DVCS revisited

- N-times subtracted dispersion relation (holds at any order of perturbation theory)

$$\sum_{j\ \mathrm{even}}^{n} h_j \xi^{-j} = \Re\mathcal{H}(\xi) - \frac{2}{\pi} \fint_0^1 \left(\frac{x}{\xi}\right)^n \frac{x \Im\mathcal{H}(x)}{(\xi - x)(\xi + x)} \mathrm{d}x$$

H. Dutrieux, et al.
EPJC 85 (2025) 1, 105

h_0 proportional to convolution of D-term with the hard scattering kernel

$h_{j>0}$ proportional to β-moments of the Double Distribution convoluted with derivatives of the hard scattering kernel

- Analysis of subtraction constant with NLO coefficient functions

$$h_0(t) \equiv S(t) = \sum_q e_q^2 S^q(t) + S^g(t) \qquad S(t) \overset{\mathrm{LO}}{=} 4 \sum_q e_q^2 (d_1^q(t) + d_3^q(t) + \ldots)$$

$$S^q(t) \overset{\mathrm{NLO}}{=} d_1^q(t)\left(4 - \frac{4}{9}\frac{\alpha_s C_F}{4\pi}\right) + d_3^q(t)\left(4 + \frac{14759}{450}\frac{\alpha_s C_F}{4\pi}\right) + \ldots$$

$$S^g(t) \overset{\mathrm{NLO}}{=} -\frac{\sum_q e_q^2 \alpha_s T_F}{4\pi}\left(\frac{172}{9} d_1^g(t) + \frac{3317}{150} d_3^g(t)\right) + \ldots$$

- Higher twist corrections to subtraction constant

$$S(t) \overset{\mathrm{LO}}{=} 4 \sum_q e_q^2 \sum_{\mathrm{odd}\ n} d_n(t)\left(1 - \frac{t}{2\mathbb{Q}^2}\left(1 - \frac{1}{(2n+3)(n+1)}\right)\right)$$

V. Martínez-Fernández, et al.
arXiv: hep-ph/2509.06669

Froissart-Gribov projections - spin 0

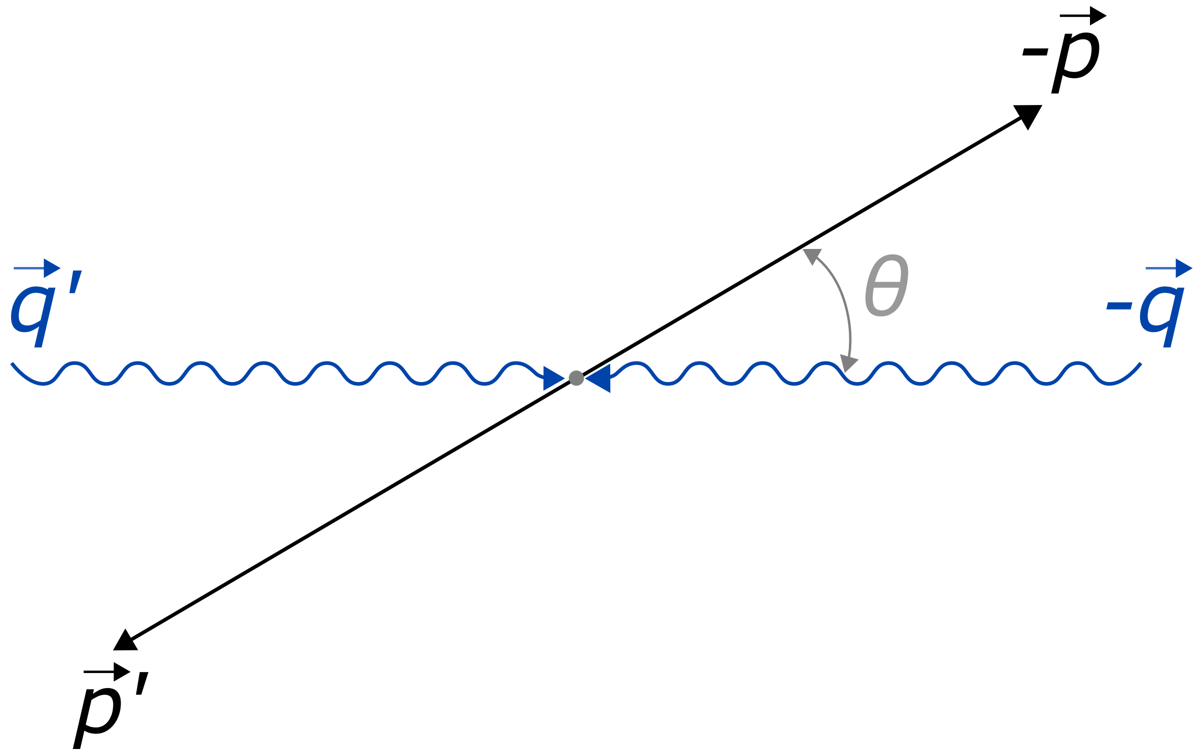
K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

FG projections are obtained by reconstructing cross-channel partial wave expansion amplitudes from the dispersive representation of the amplitude in the direct channel.

In cross-channel: $\gamma^*(q) + \gamma(-q') \to h(p') + \bar{h}(-p)$

Expansion in the cross channel SO(3) partial waves: $$\mathcal{H}_+(\cos\theta_t, t) = \sum_{\substack{J=0 \\ \text{even}}}^{\infty} F_J(t) P_J(\cos\theta_t)$$

which gives: $$F_J(t) = \frac{2J+1}{2} \int_{-1}^{1} d(\cos\theta_t) \, P_J(\cos\theta_t) \, \mathcal{H}_+(\cos\theta_t, t)$$



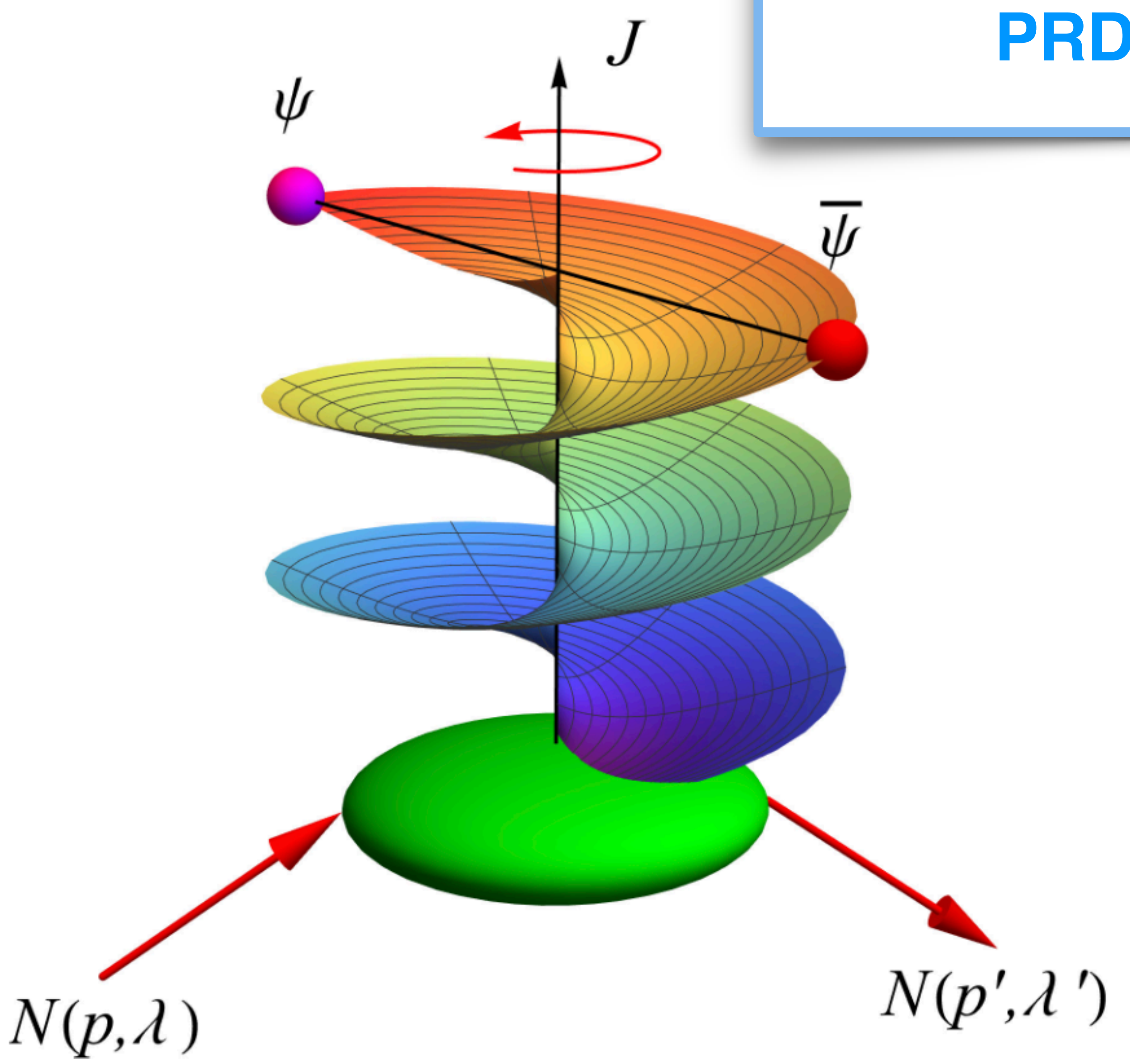
In direct-channel: $\gamma^*(q) + h(p) \to \gamma(q') + h(p')$

Dispersion relation: $$\mathrm{Re}\,\mathcal{H}_+(\xi, t) = \mathcal{P} \int_0^1 dx \frac{2x H_+(x, x, t)}{\xi^2 - x^2} + 4D(t)$$

where: $$\cos\theta_t \to -\frac{1}{\xi\beta} + \mathcal{O}(1/Q^2) \qquad \beta = \sqrt{1 - \frac{4m^2}{t}}$$

β = 1 in the current analysis (see the publication for discussion of consequences)

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

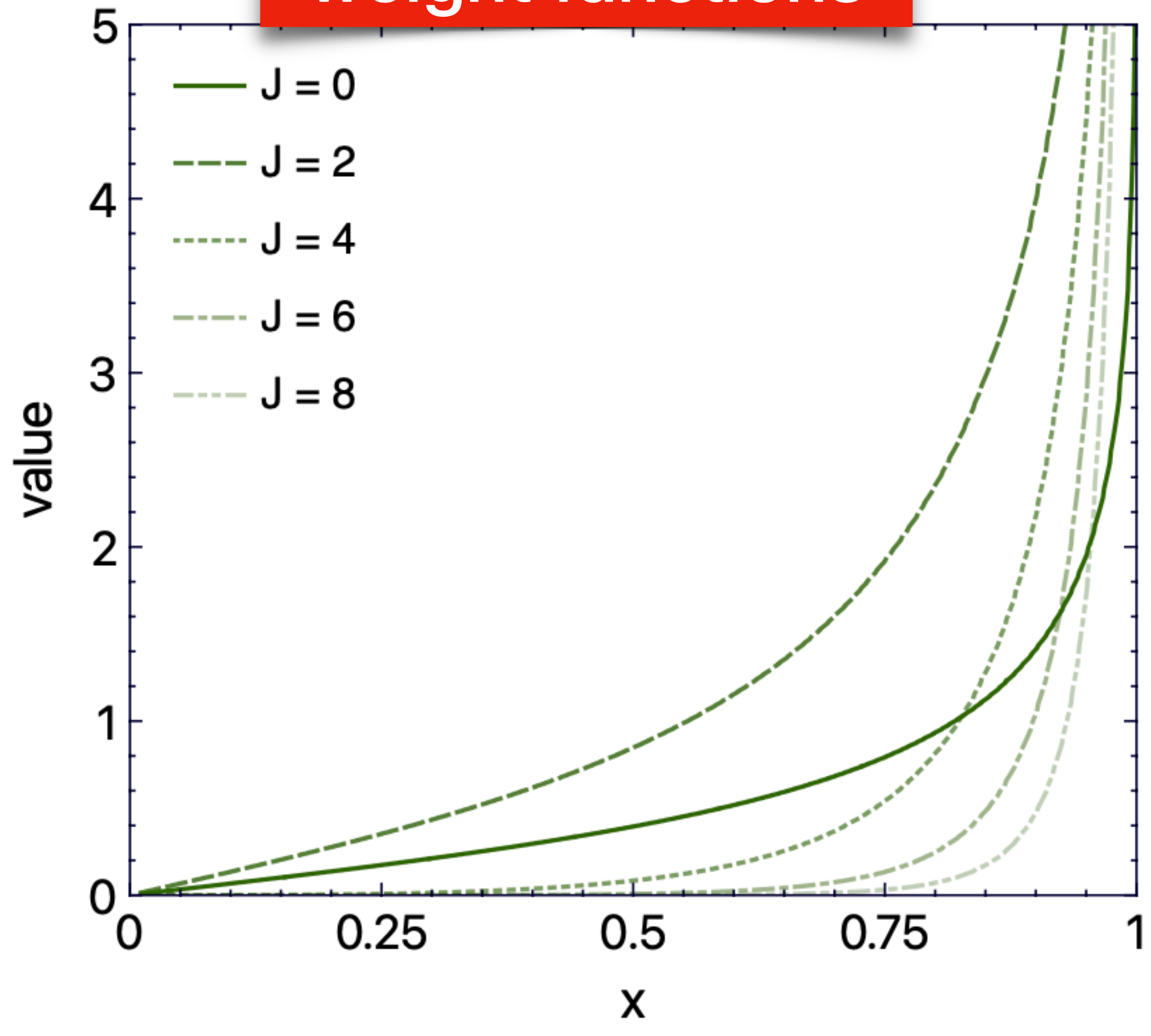


Final result:

$$F_{J=0}(t) = 2\int_0^1 dx \left(\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right) H_+(x,x,t) + 4D(t)$$

$$F_{J>0}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_J(1/x)}{x^2} H_+(x,x,t)$$

weight functions



Results for spin-0 target already obtained in
K. Kumericki, D. Mueller, K. Passek-Kumericki, EPJC 58, 193 (2008)

K. Semenov-Tian-Shansky and P.S.
PRD 109 (2024) 5, 054010

Electric combination:

$$H_{\pm}^{(E)}(x,\cos\theta_t,t) = H_{\pm}(x,\cos\theta_t,t) + \tau E_{\pm}(x,\cos\theta_t,t)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 0 $\tau \equiv t/(4m^2)$

has to be expanded in $P_J(\cos\theta_t)$ rotation function

$$F_{J=0}^{(E)}(t) = 2\int_0^1 dx \left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H_{+}^{(E)}(x,x,t) + 4(1-\tau)D(t)$$

$$F_{J>0}^{(E)}(t) = 2(2J+1)\int_0^1 dx \frac{\mathcal{Q}_0(1/x)}{x^2} H_{+}^{(E)}(x,x,t)$$

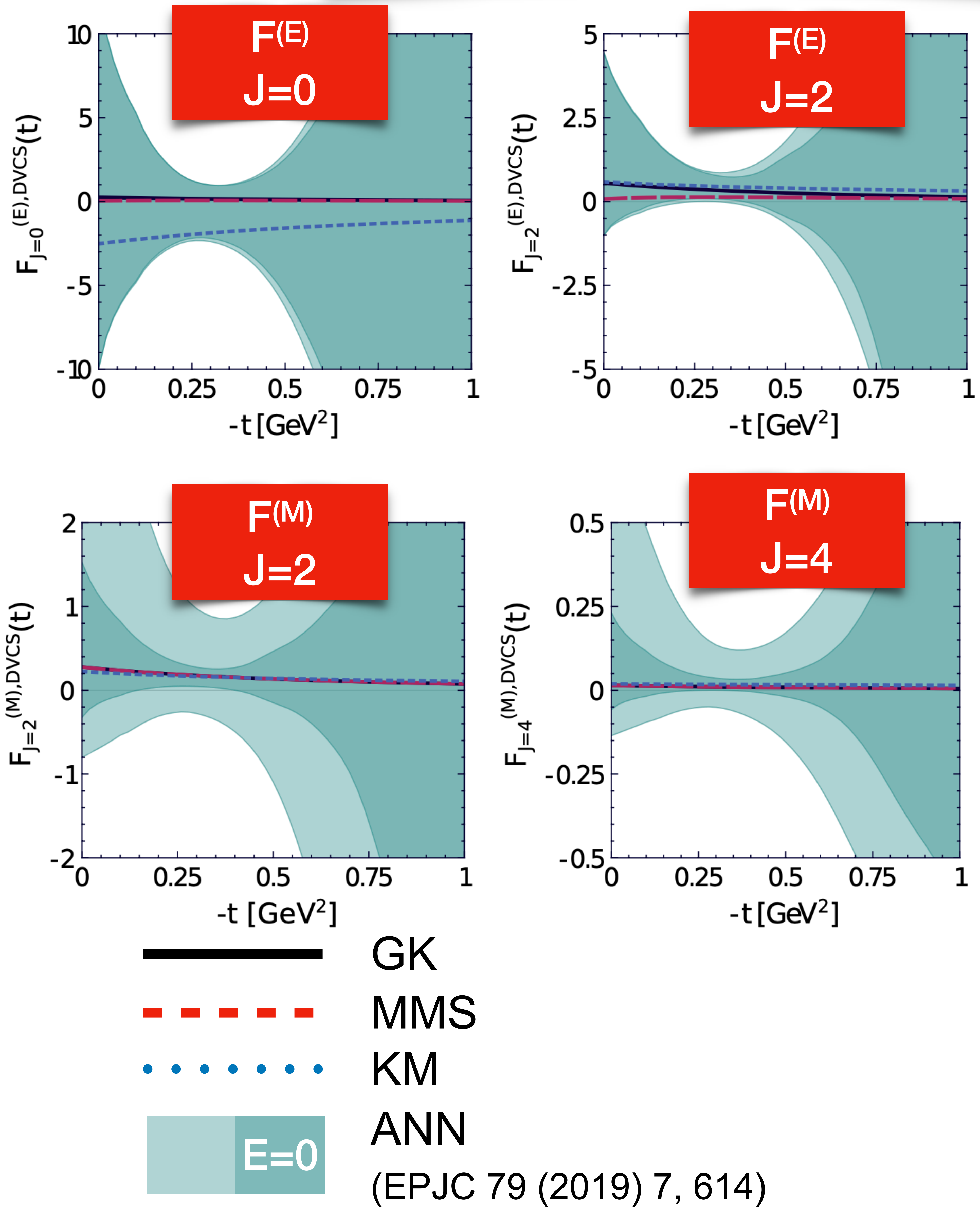
Magnetic combination:

$$H_{\pm}^{(M)}(x,\cos\theta_t,t) = H_{\pm}(x,\cos\theta_t,t) + E_{\pm}(x,\cos\theta_t,t)$$

helicities of $p\bar{p}$ couple to |λ−λ'| = 1

has to be expanded in $\sin\theta_t P'_J(\cos\theta_t)/\sqrt{J(J+1)}$ rotation function

$$F_{J}^{(M)}(t) = 2\int_0^1 dx H_{+}^{(M)}(x,x,t)\frac{2J+1}{J(J+1)}\frac{(-1)}{x}\sqrt{\frac{1}{x^2}-1}\,\mathcal{Q}_J^1(1/x)$$



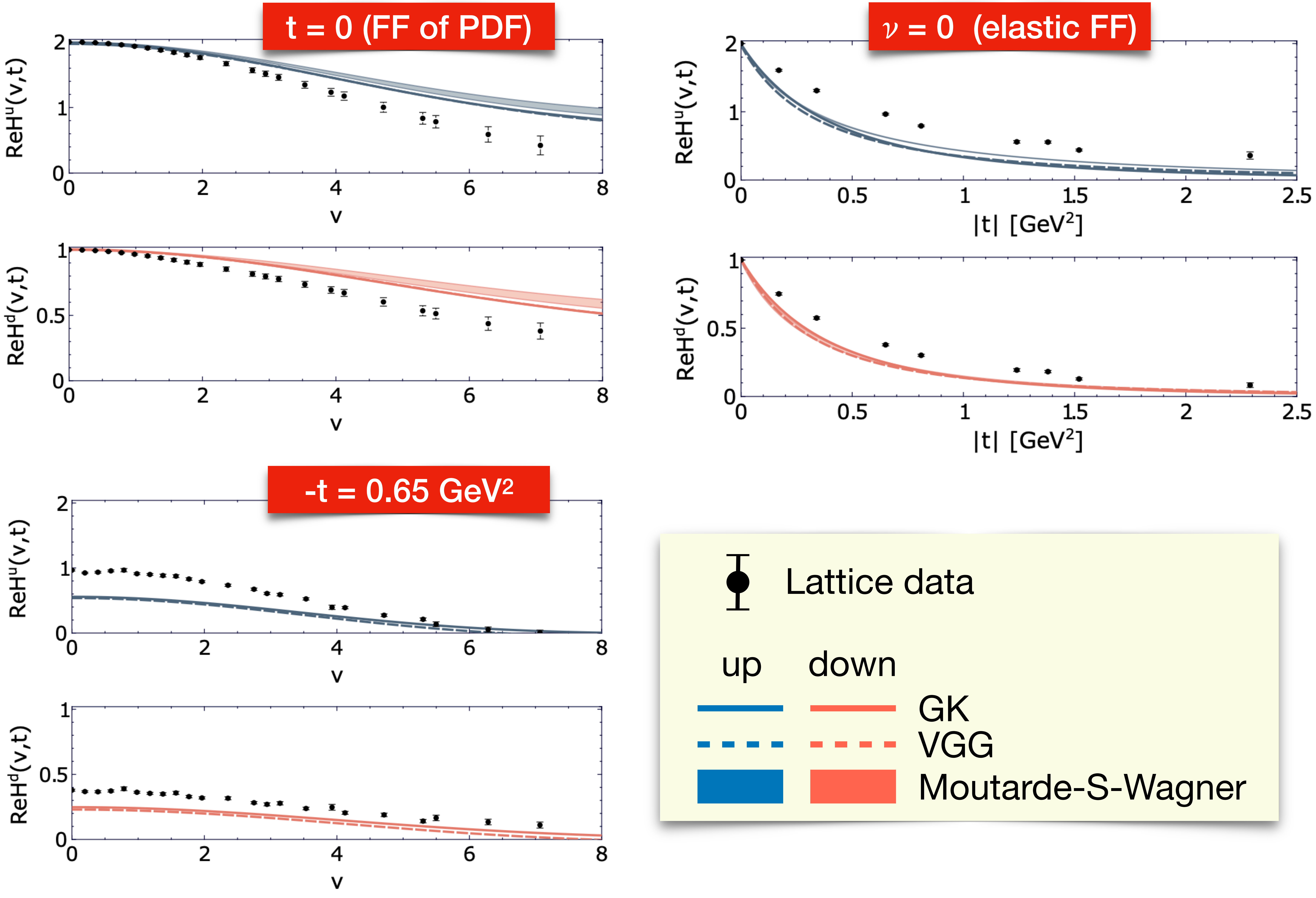
K. Cichy et al.,
PRD 110 (2024) 11, 114025

Lattice data for H(ν, xi = 0, t)
(a ≈ 0.093 fm, m_π = 260 MeV,
SDE approach)

$$H^q(\nu,\xi,t) = \int_{-1}^{1} \mathrm{d}x\, e^{ix\nu} H^q(x,\xi,t)$$

Poor agreement between lattice data and phono. parameterizations of PDFs and elastic FFs
→ will be improved over time

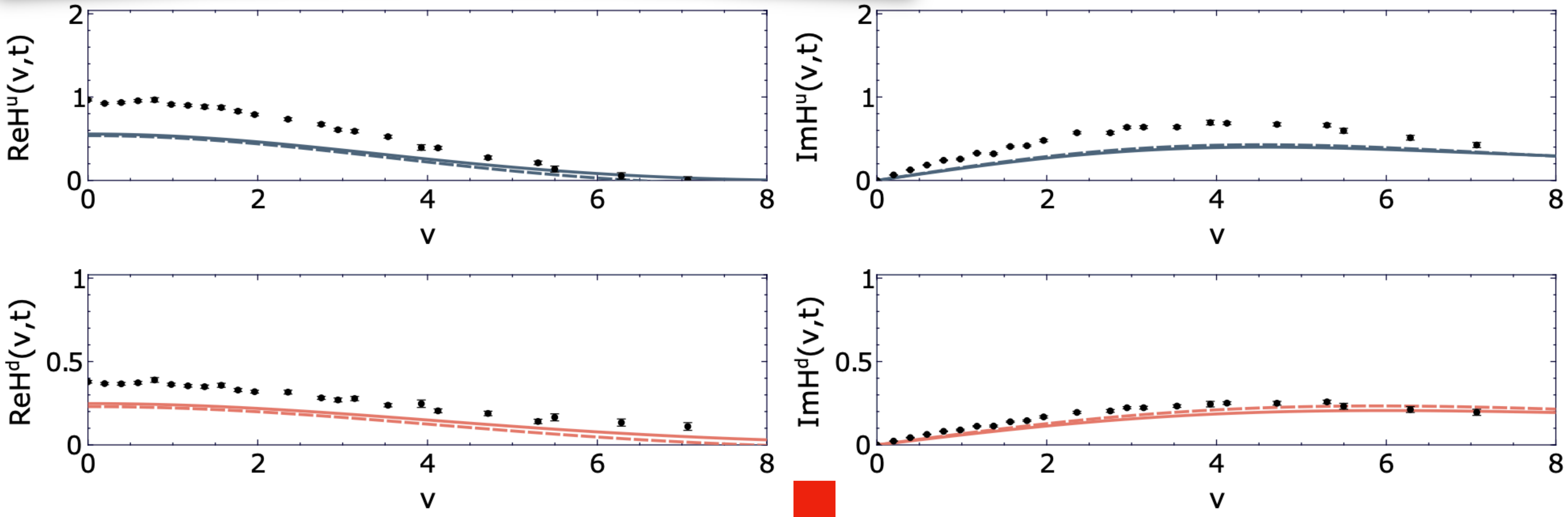
Can lattice data be meaningfully incorporated in global analyses of GPDs?



lattice-QCD

K. Cichy et al.,
PRD 110 (2024) 11, 114025

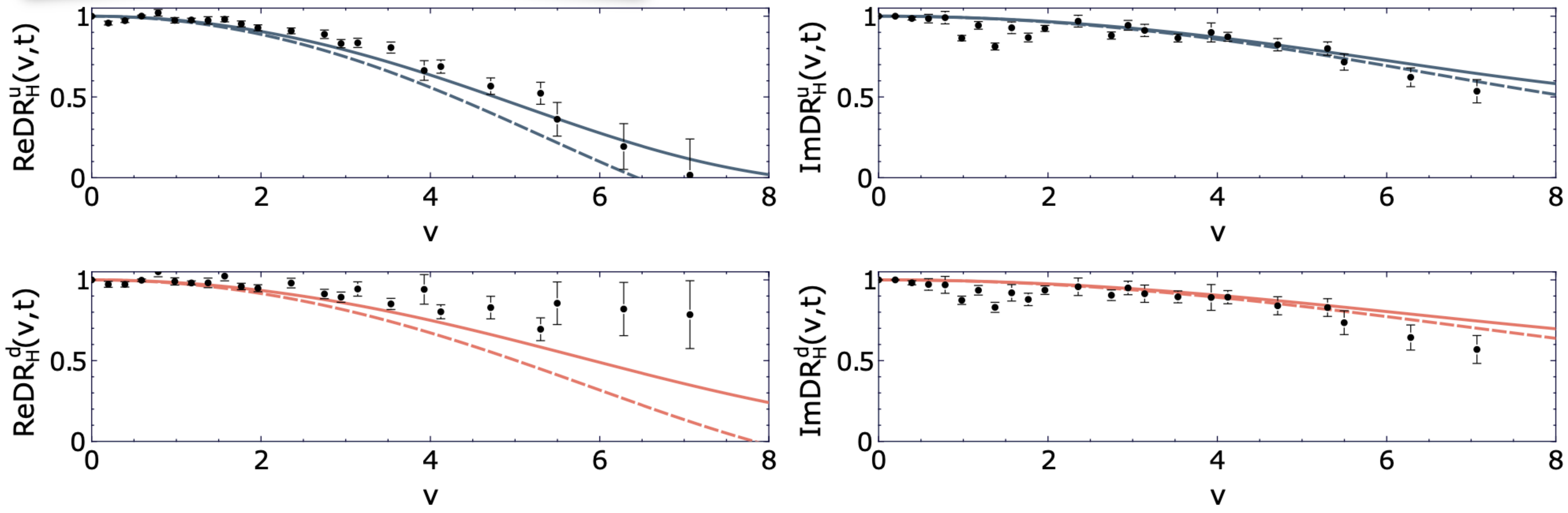
-t = 0.65 GeV² ("original" lattice-QCD results)

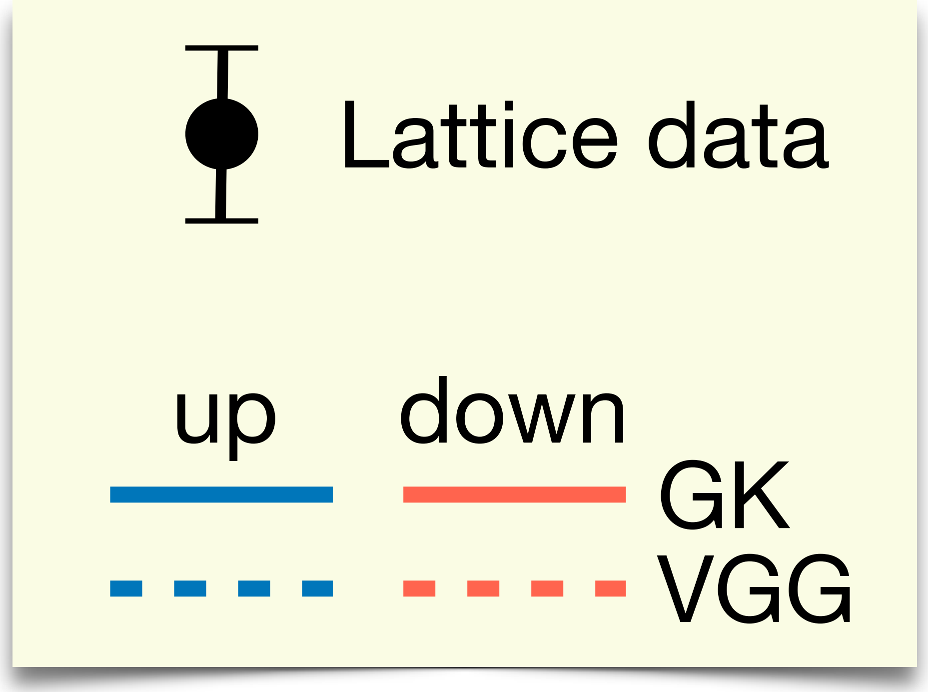


$$\mathrm{DR}_{\mathrm{Re}}(\nu, t) = \frac{\mathrm{Re}H(\nu, t)}{\mathrm{Re}H(\nu,0)} \frac{\mathrm{Re}H(0,0)}{\mathrm{Re}H(0,t)}$$

$$\mathrm{DR}_{\mathrm{Im}}(\nu, t) = \lim_{\nu' \to 0} \frac{\mathrm{Im}H(\nu, t)}{\mathrm{Im}H(\nu,0)} \frac{\mathrm{Im}H(\nu',0)}{\mathrm{Im}H(\nu', t)}$$

-t = 0.65 GeV² (double ratios)





Fit to elastic and lattice data

K. Cichy et al.,
PRD 110 (2024) 11, 114025

- Simultaneous fit to lattice data (double ratios) and elastic FF data
  → **only valance quarks!**

- General Ansatz

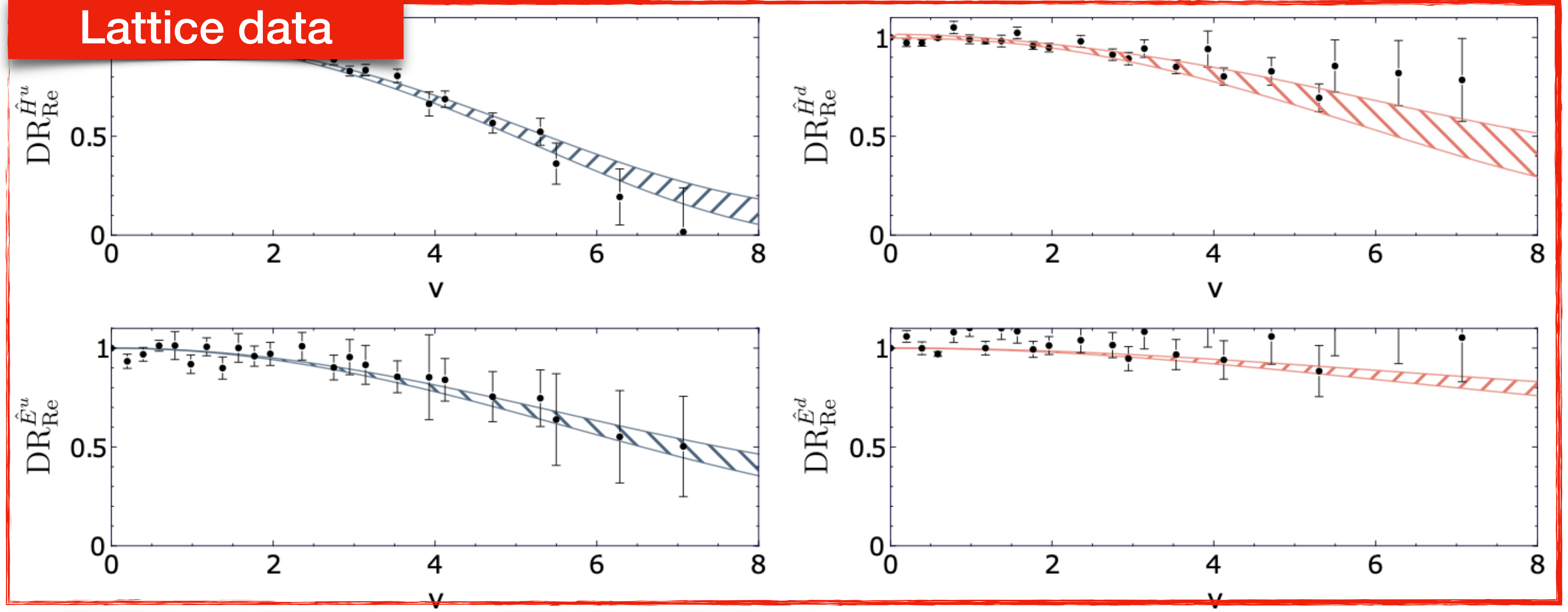
$$H^q(x,0,t) = q(x) \times f_H^q(x,t)$$

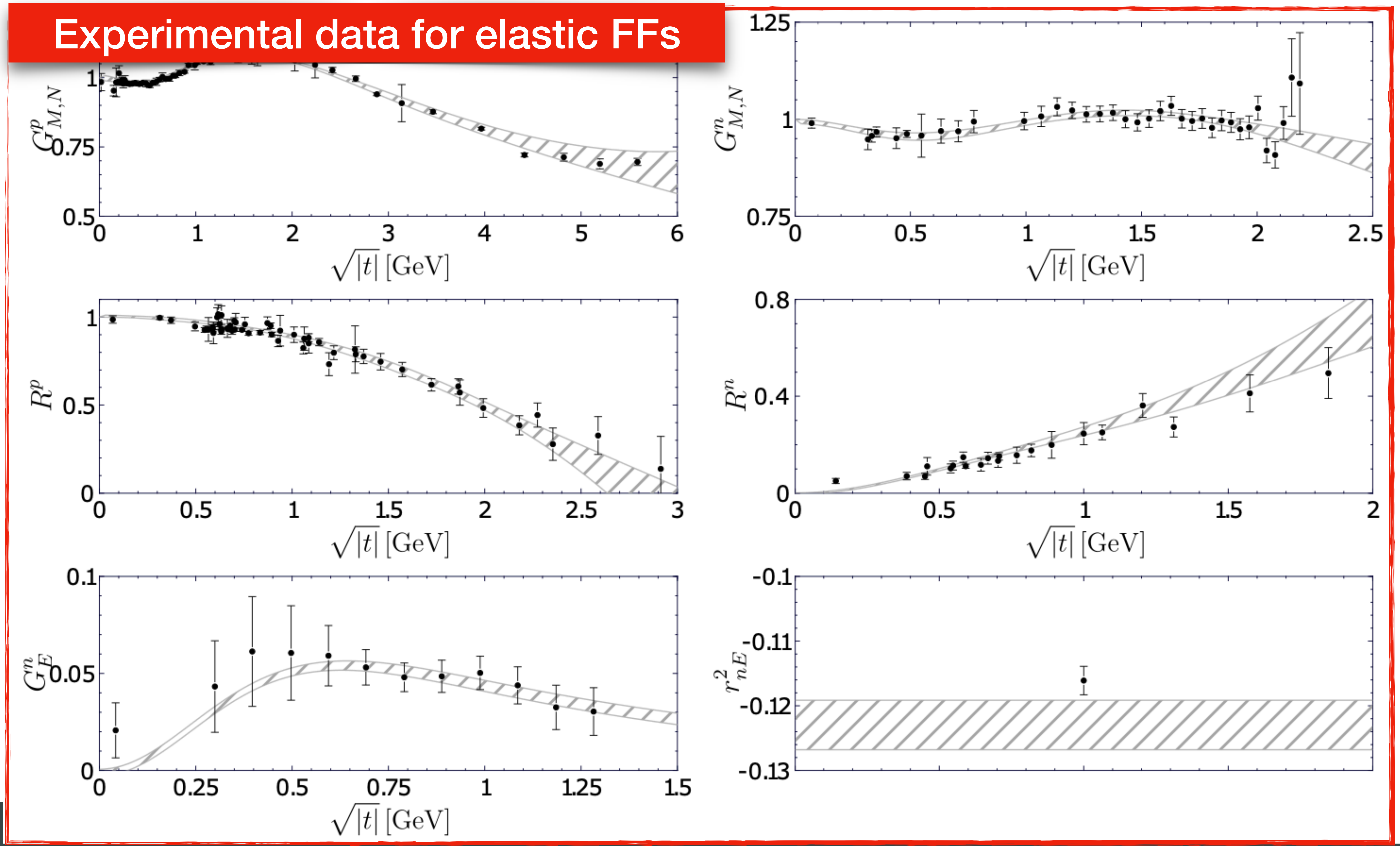
$$E^q(x,0,t) = e_q(x) \times f_E^q(x,t)$$

where q(x) taken from NNPDF set

- In the case of GPD H we also attempt to constrain shadow GPDs

- Quality of fit: $\chi^2/\mathrm{nPoints} \approx 1.34$

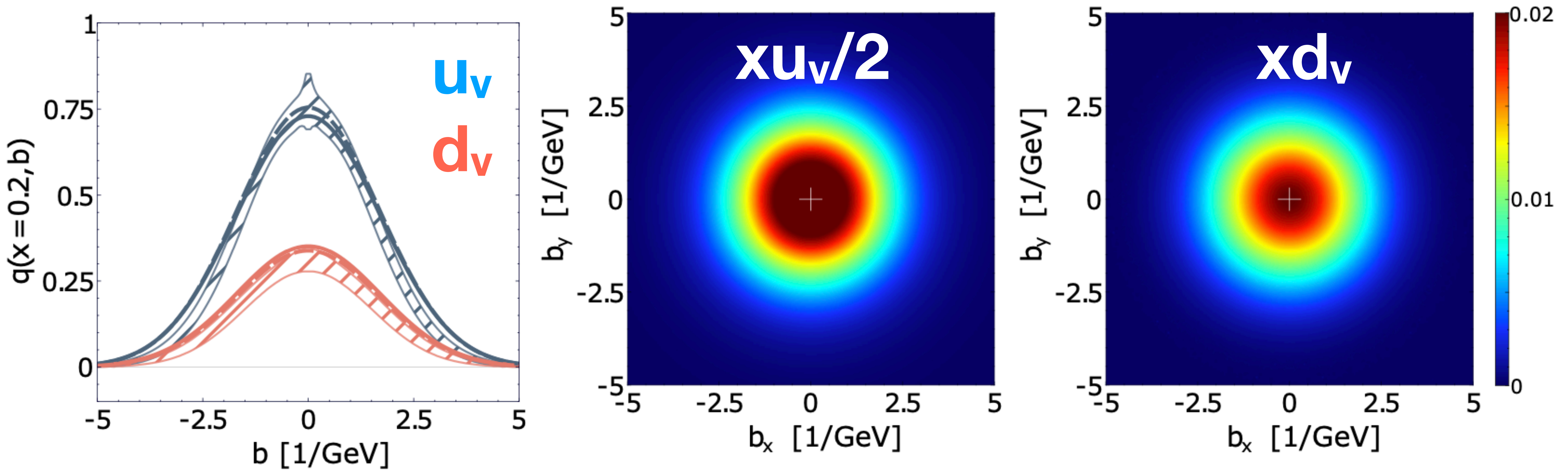




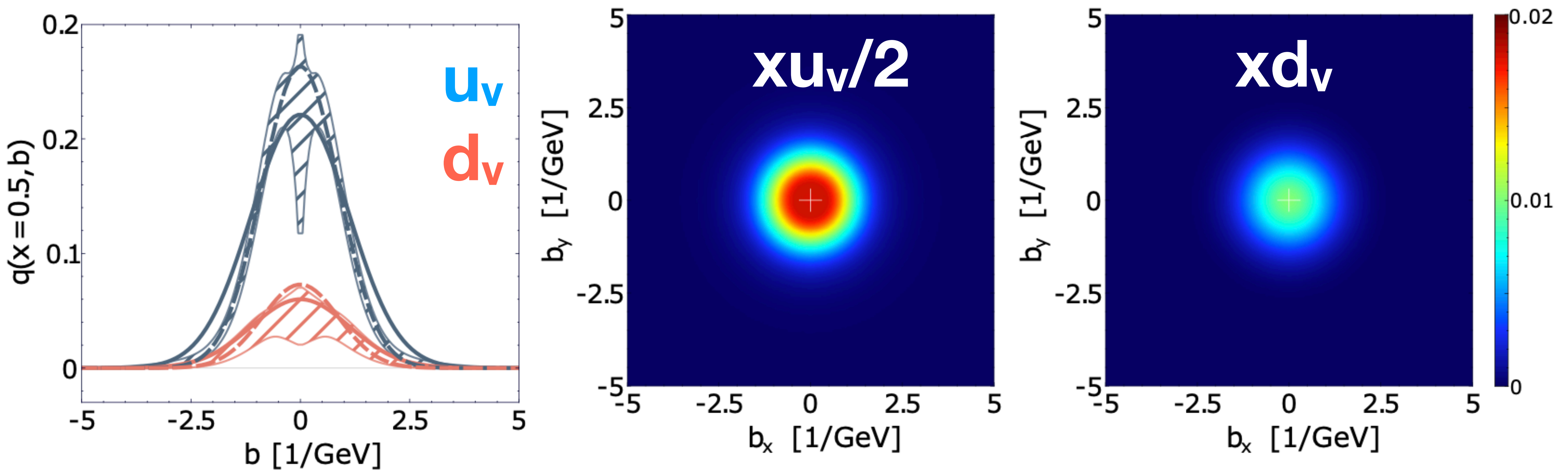
Results - nucleon tomography

K. Cichy et al.,
PRD 110 (2024) 11, 114025

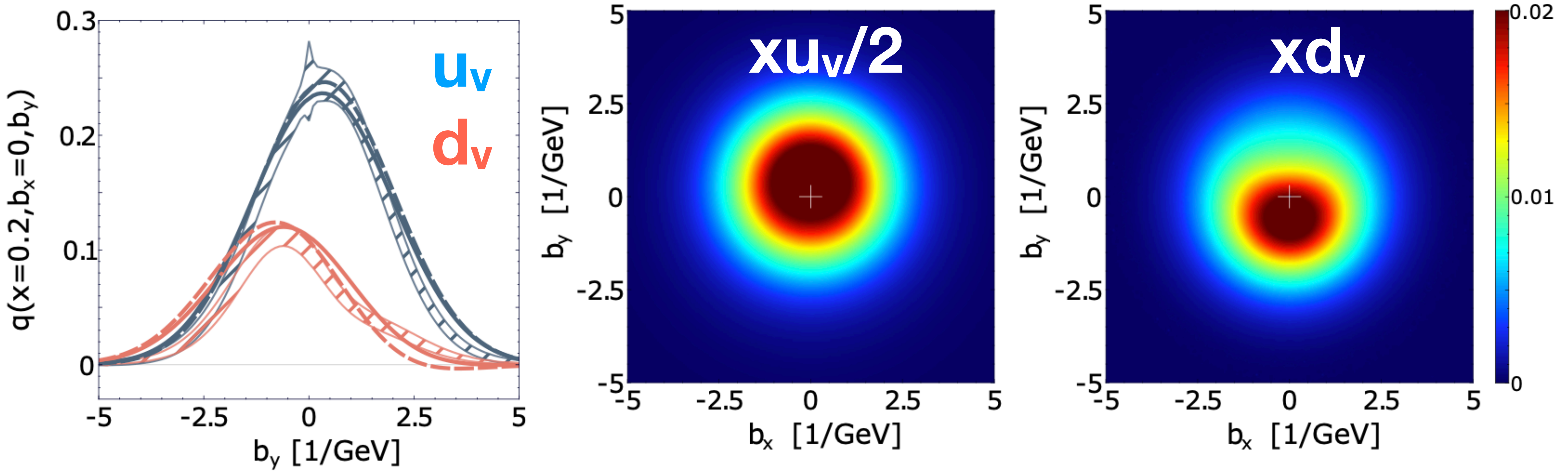
x = 0.2 (unpolarised proton)



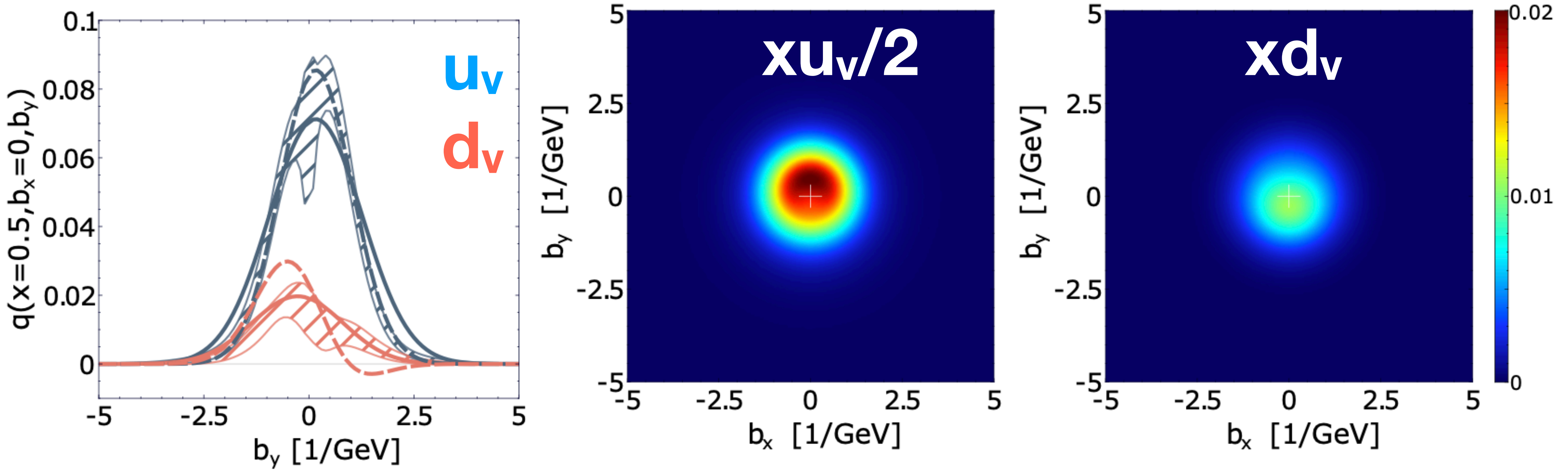
x = 0.5 (unpolarised proton)



x = 0.2 (transversely polarised proton)

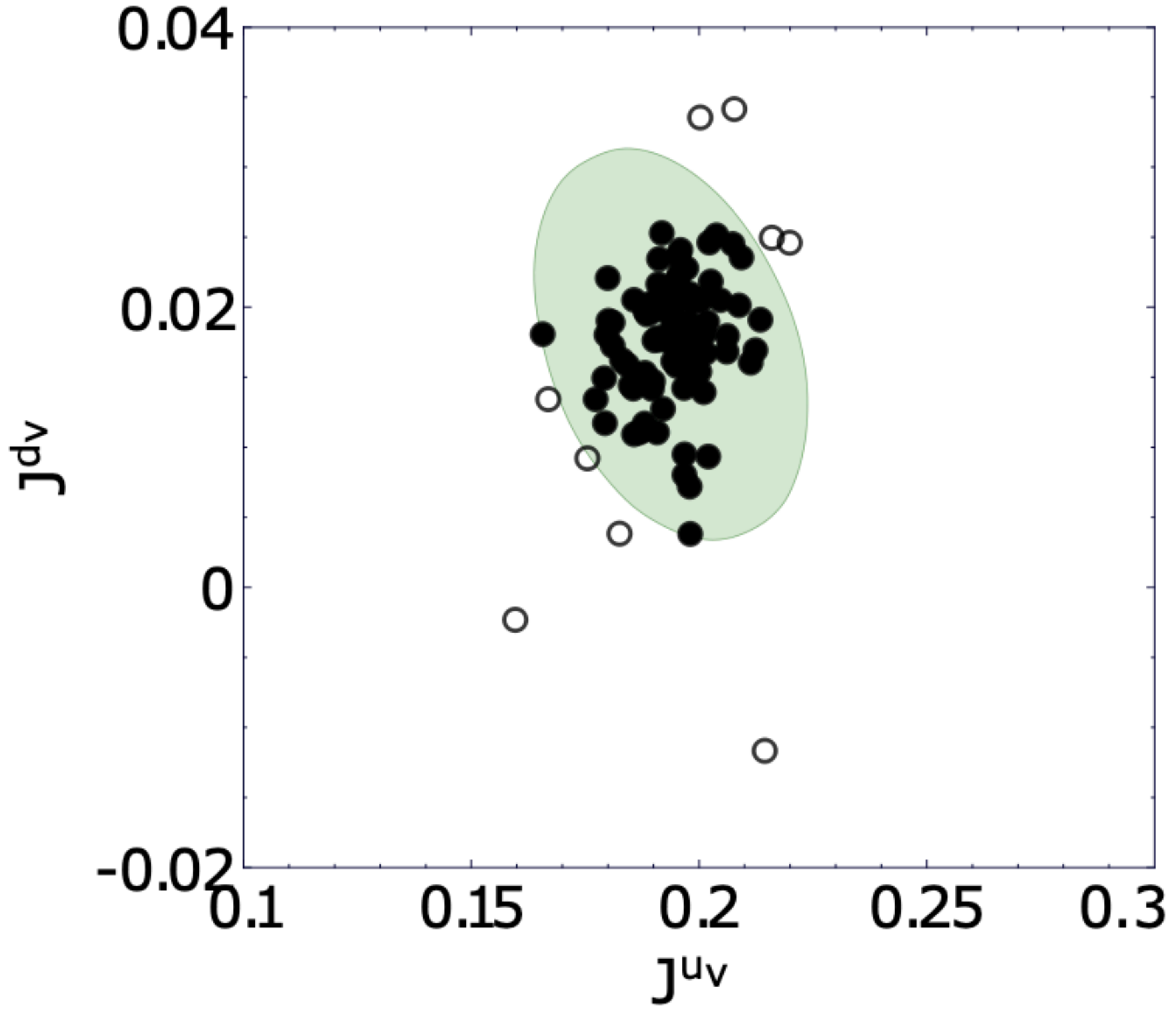


x = 0.5 (transversely polarised proton)



Results - total angular momentum

K. Cichy et al.,
PRD 110 (2024) 11, 114025



3σ ellipse
● replicas giving values inside ellipse
○ replicas giving values outside ellipse

This result
(elastic and lattice-QCD data):

$$J^{u_v} = 0.195 \pm 0.010$$
$$J^{d_v} = 0.0173 \pm 0.0046$$

Diehl-Kroll / EPJC 73, 2397 (2013)
(elastic data):

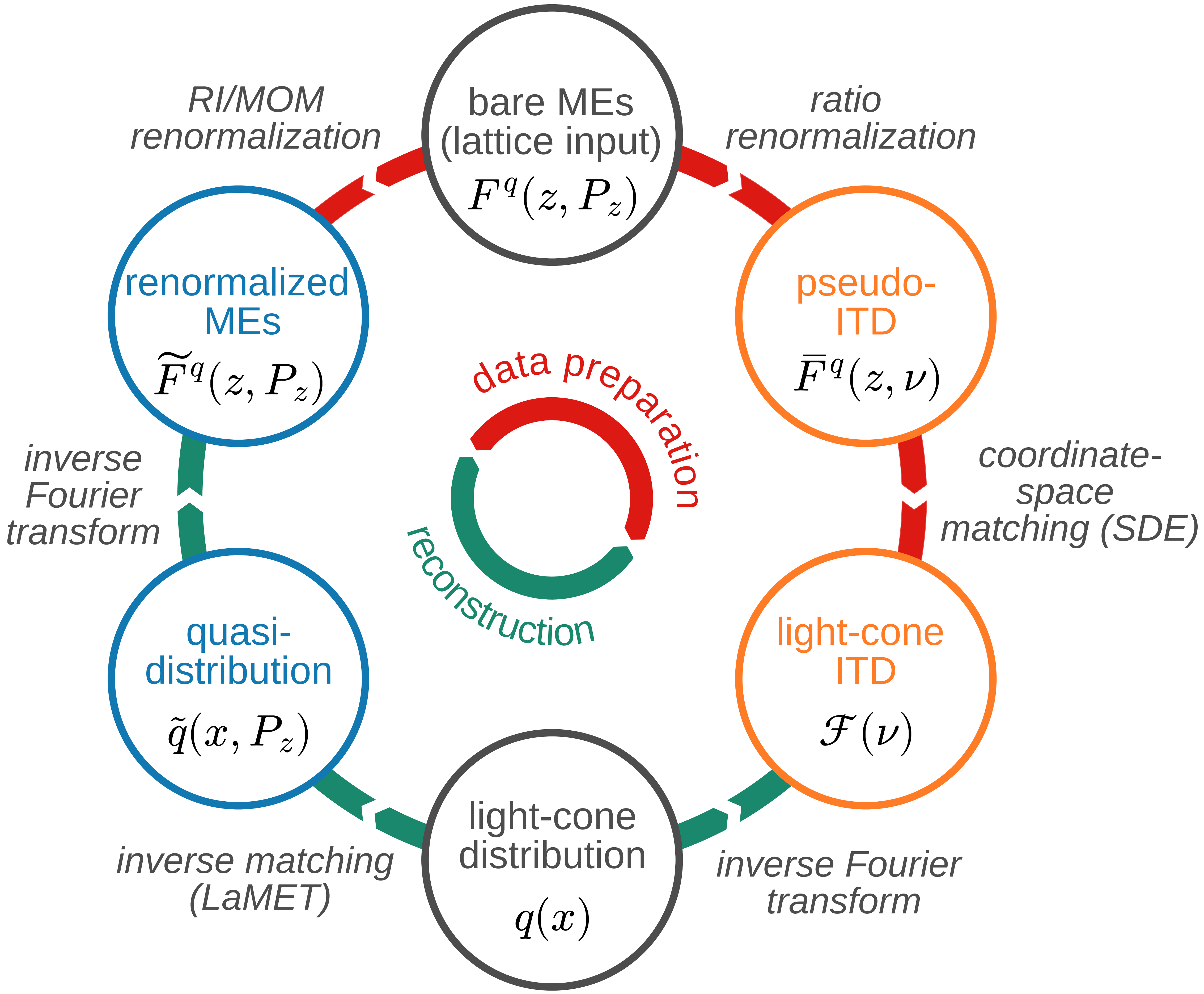
$$J^{u_v} = 0.230^{+0.009}_{-0.024}$$
$$J^{d_v} = -0.004^{+0.010}_{-0.016}$$

Bacchetta-Radici / PRL 107, 212001 (2011)
(SIDIS data, Sivers function related to GPD E via Burkardt's "lensing function")

$$J^u = 0.229 \pm 0.002^{+0.008}_{-0.012}, \quad J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000}$$
$$J^d = -0.007 \pm 0.003^{+0.020}_{-0.005}, \quad J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000}$$

(all estimates given at μ = 2 GeV)

M.-H. Chu et al.,
arXiv: hep-lat/2509.15931



Crucial steps in both procedures:

LaMET (quasi-distributions)

$$\widetilde{q}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{x^2 P_z^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-x)^2 P_z^2}\right)$$

usable range: $x \in [x_{\mathrm{min}} \sim \frac{\Lambda_{\mathrm{QCD}}}{P^z},\ x_{\mathrm{max}} \sim 1 - x_{\mathrm{min}}]$

SDE (pseudo-distributions)

$$\bar{F}^q(z, \nu) = \int_{-1}^{1} dy\, C_\nu(y)\, \mathcal{F}^q(y\nu) + \mathcal{O}\left(z^2 \Lambda^2_{\mathrm{QCD}}\right)$$

usable range: $[0,\ \nu_{\mathrm{max}} \sim z_{\mathrm{max}} P_z]$

Unified framework for LaMET and SDE

Method outlined in
X. Ji, Research 8 (2025) 0695

M.-H. Chu et al.,
arXiv: hep-lat/2509.15931

Distribution used to fit LaMET quantities

$$q^{(Q)}(x) = \begin{cases} q^{(0)}(x) + q^{(1)}(x)\,, & 0 < x < x_{\min}\,, \\ q^{(0)}(x) + q^{(2)}(x)\,, & x_{\max} < x < 1\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

Distribution used to fit SDE quantities

$$q^{(P)}(x) = \begin{cases} q^{(0)}(x) + q^{(3)}(x)\,, & x_{\min} < x < x_{\max}\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

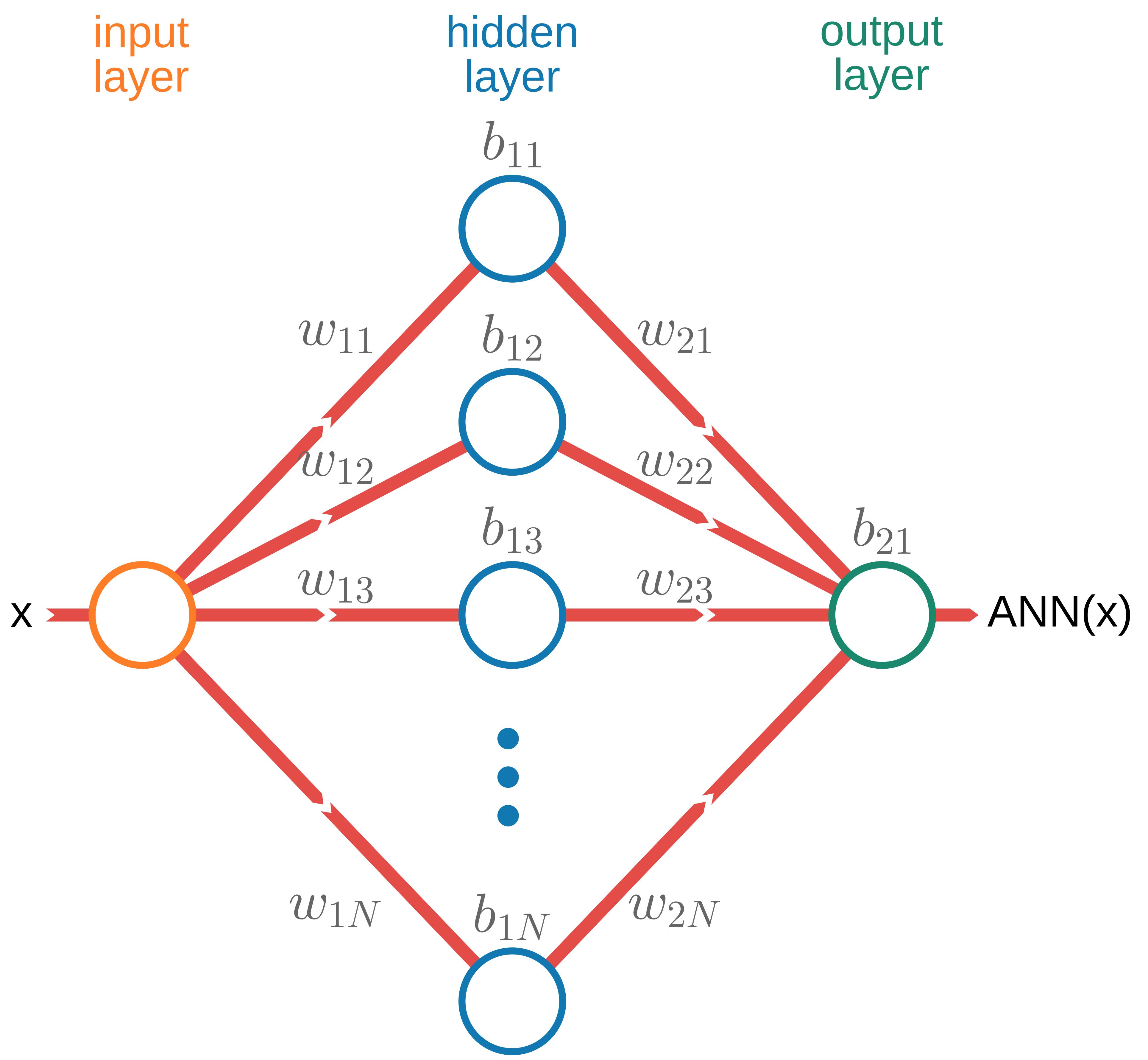
where

$$q^{(0)}(x) = x^{\delta^{(0)}}(1-x)^{\rho^{(0)}}\,\mathrm{ANN}^{(0)}(x)$$

$$q^{(1)}(x) = x^{\delta^{(1)}}(x_{\min}-x)^{\rho^{(1)}}\,\mathrm{ANN}^{(1)}(x)\,,$$

$$q^{(2)}(x) = (x-x_{\max})^{\delta^{(2)}}(1-x)^{\rho^{(2)}}\,\mathrm{ANN}^{(2)}(x)\,,$$

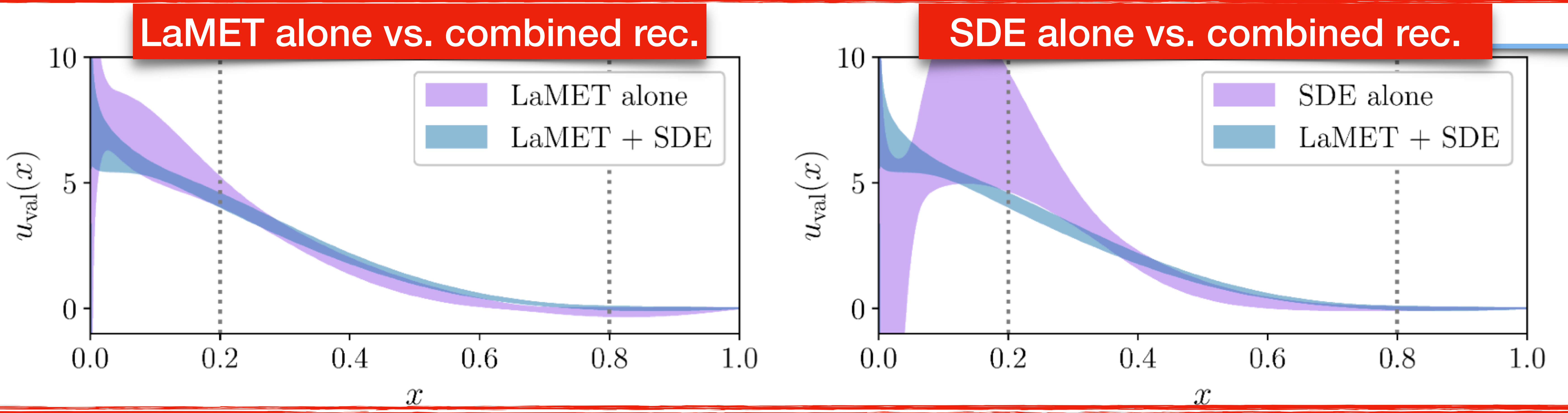
$$q^{(3)}(x) = (x-x_{\min})^{\delta^{(3)}}(x_{\max}-x)^{\rho^{(3)}}\,\mathrm{ANN}^{(3)}(x)$$



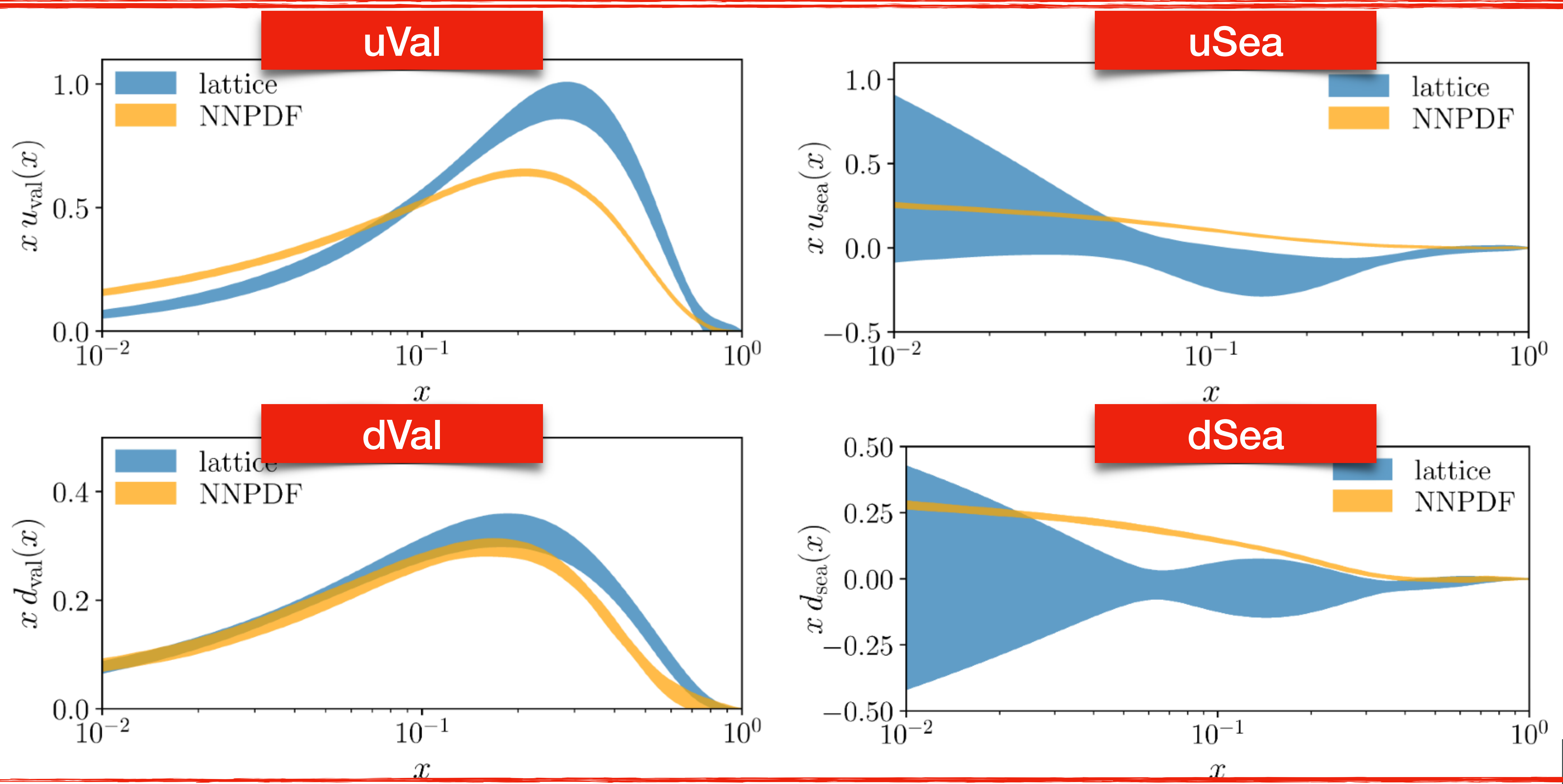
Key results for PDFs

M.-H. Chu et al.,
arXiv: hep-lat/2509.15931

Combined reconstruction vs. LaMET/SDE alone

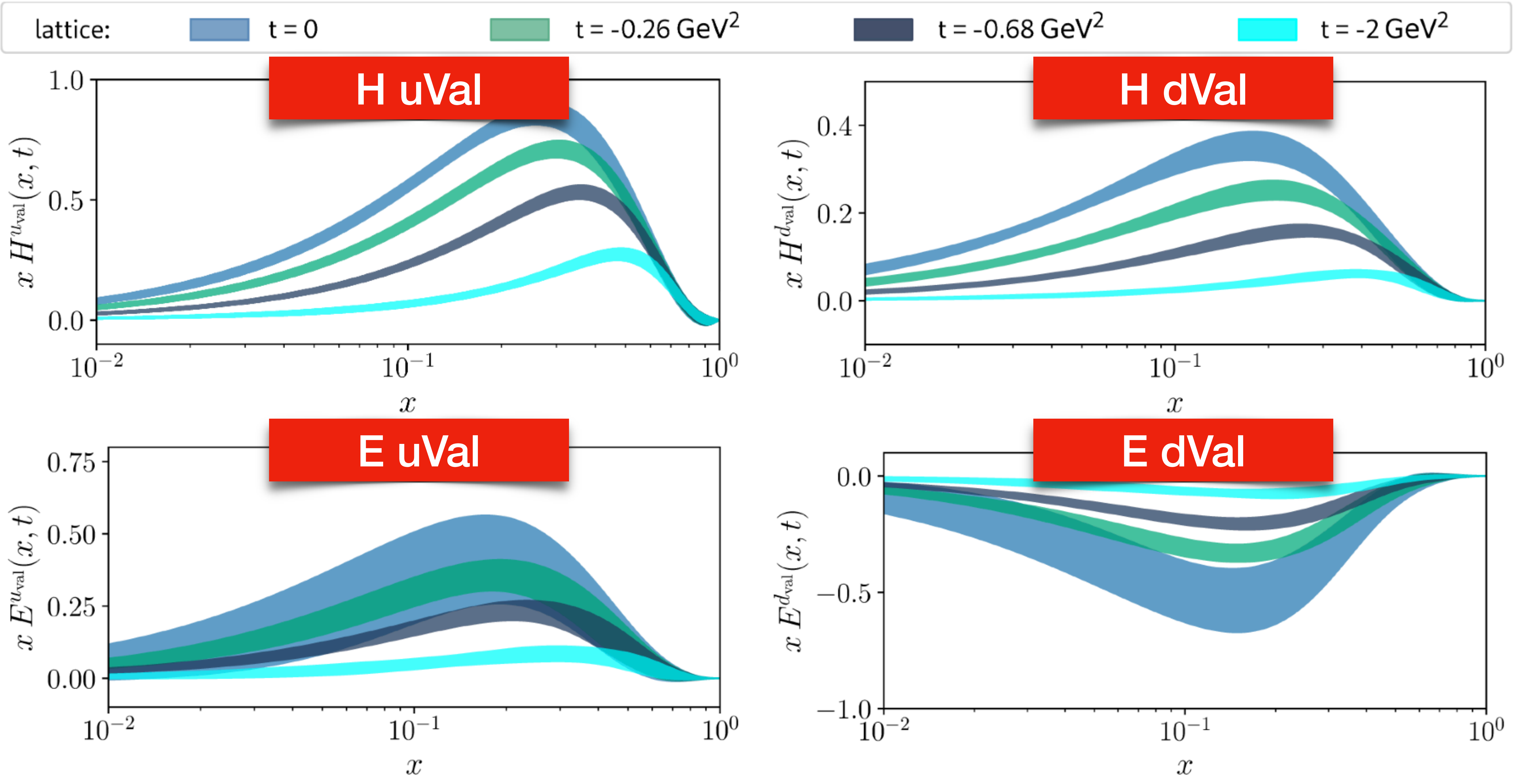


Reconstructed distributions vs. phono. parameterizations

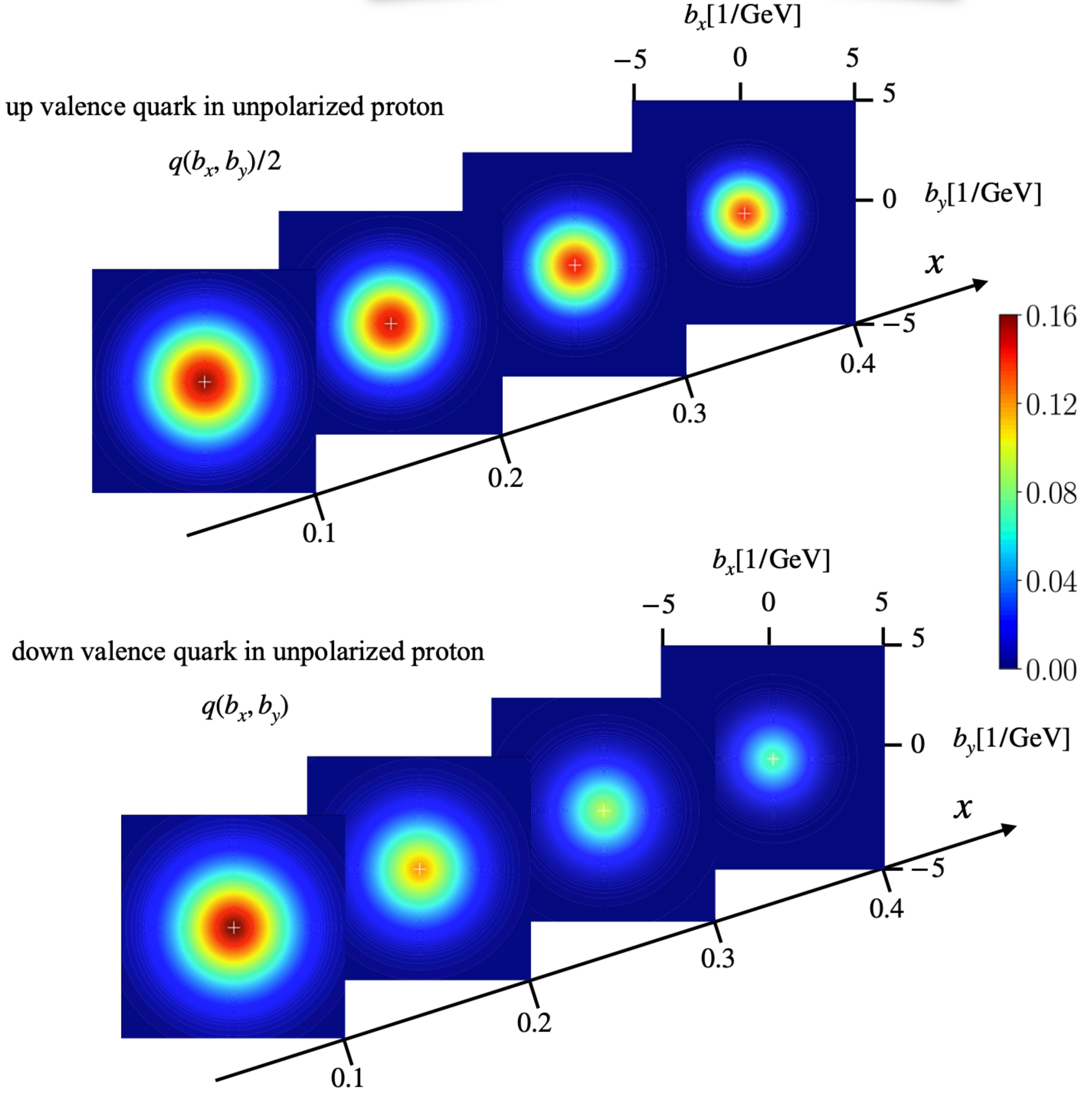


Key results for GPDs (xi = 0)

M.-H. Chu et al.,
arXiv: hep-lat/2509.15931



nucleon tomography
(unpolarised nucleon)



Summary

- Substantial progress in:
  - description of exclusive processes
  - understanding fundamental problems, like deconvolution of CFFs and analysis methods
  - preparation of new measurements
  - incorporation of lattice QCD results

This progress is important for the precision era of GPD extraction allowed by the new generation of experiments