Hadronization and SIDIS

26th International
Symposium on Spin Physics
A Century of Spin

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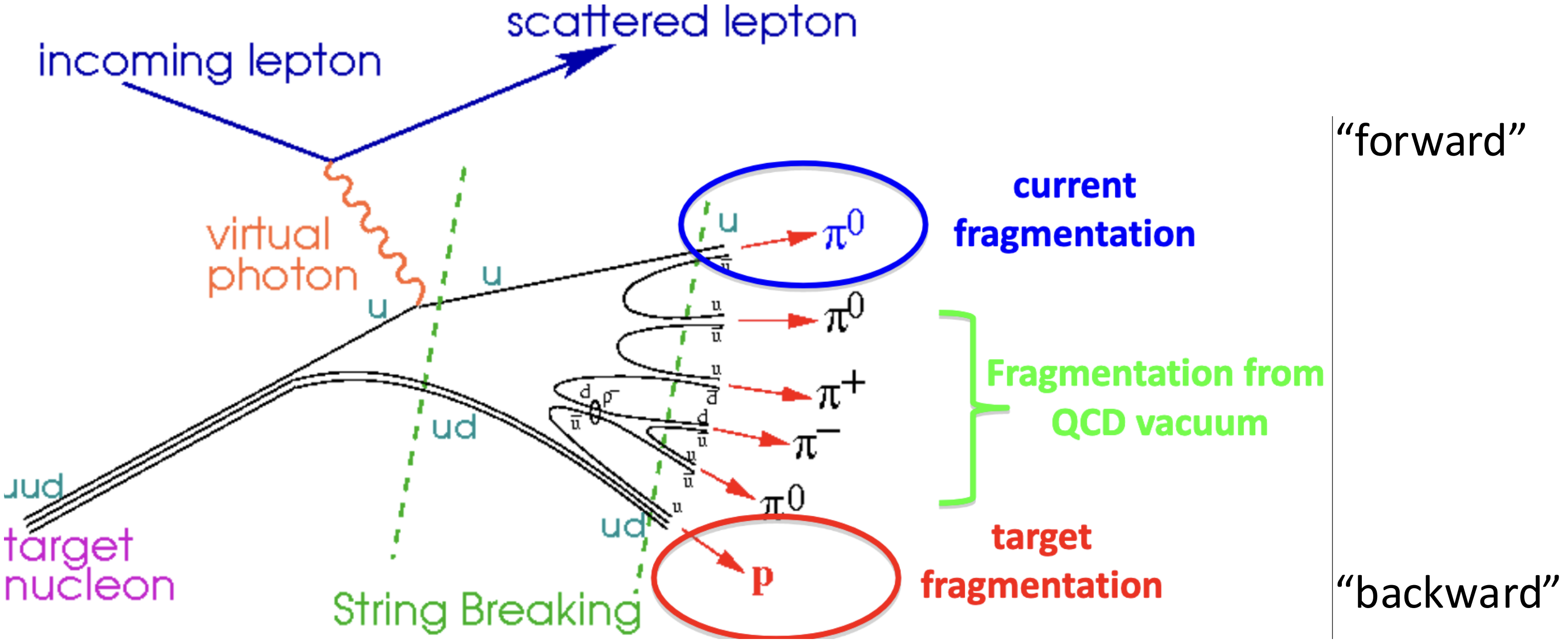
Anselm Vossen

Duke UNIVERSITY

Deep Inelastic Scattering and Hadronization



current fragmentation

Fragmentation from QCD vacuum

target fragmentation

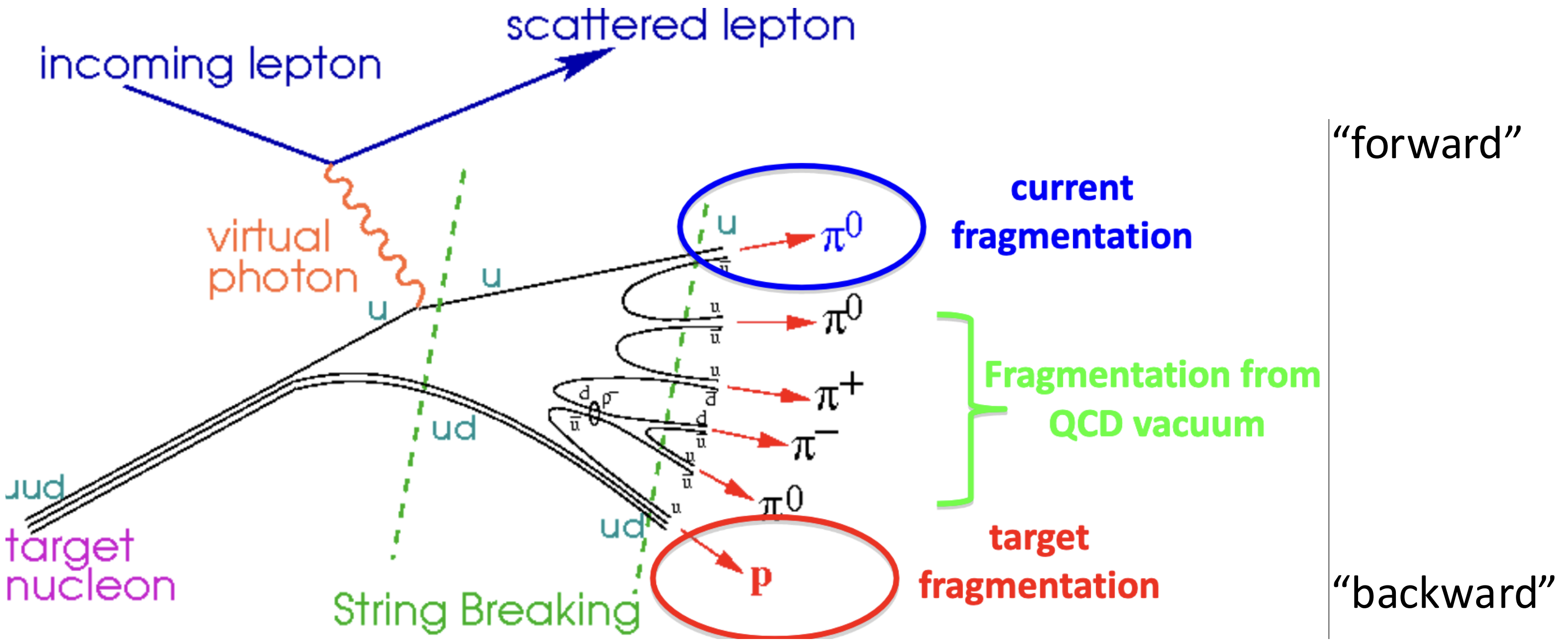
"forward"

"backward"

- Factorized QCD $ep \to e'hX$

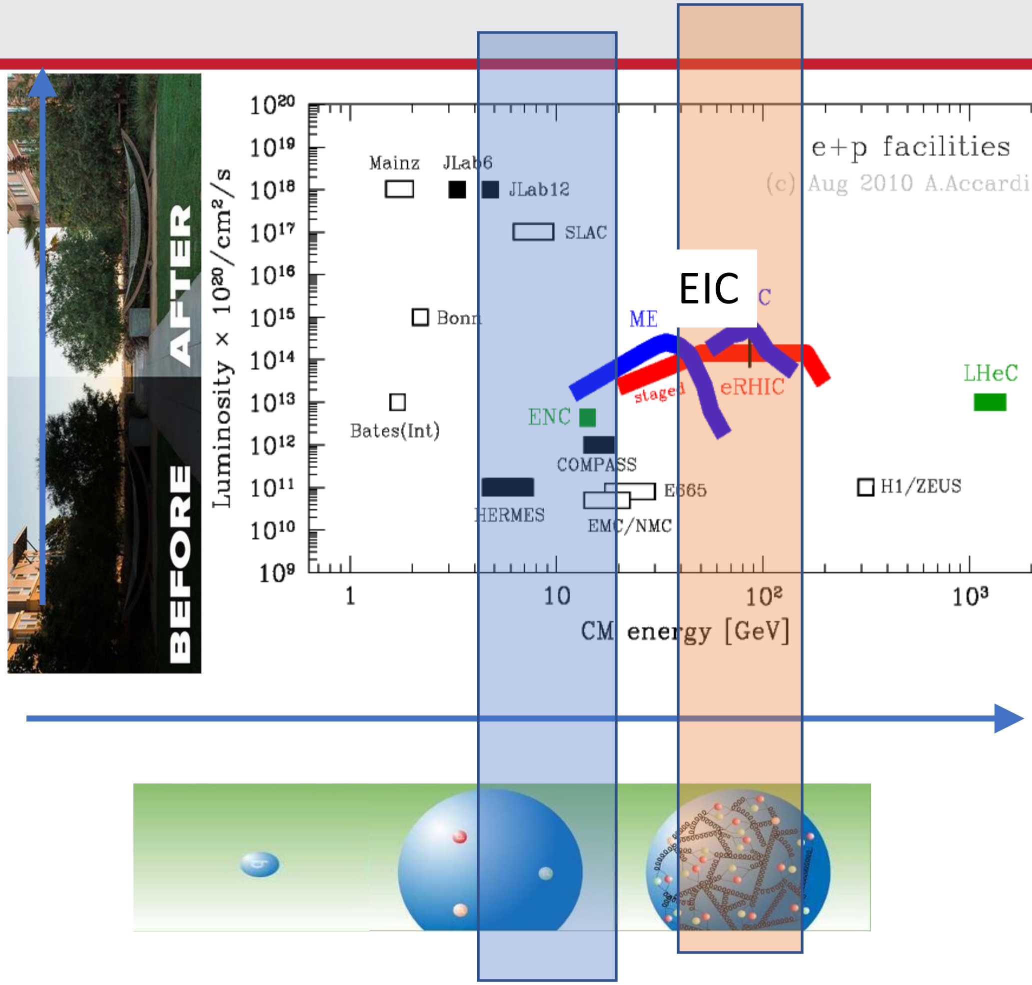
$$\sigma_{ep\to e'hX} \propto PDF \otimes \hat{\sigma} \otimes Fragmentation\ Function$$

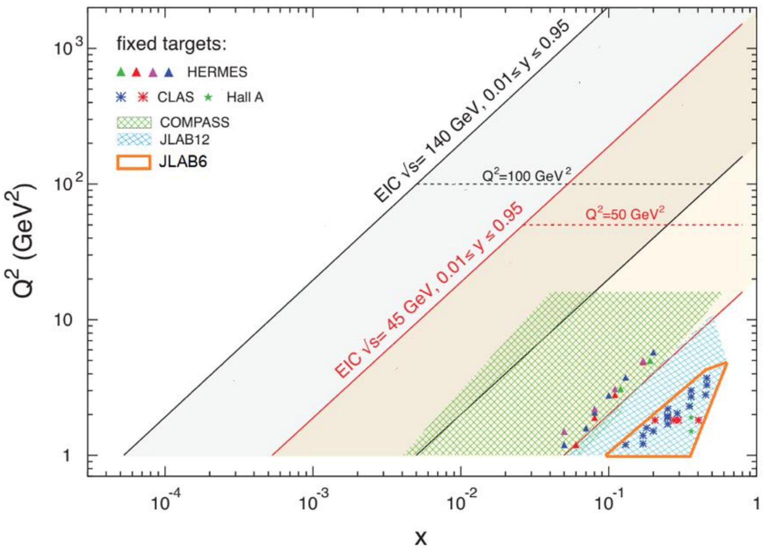
Deep Inelastic Scattering and Hadronization



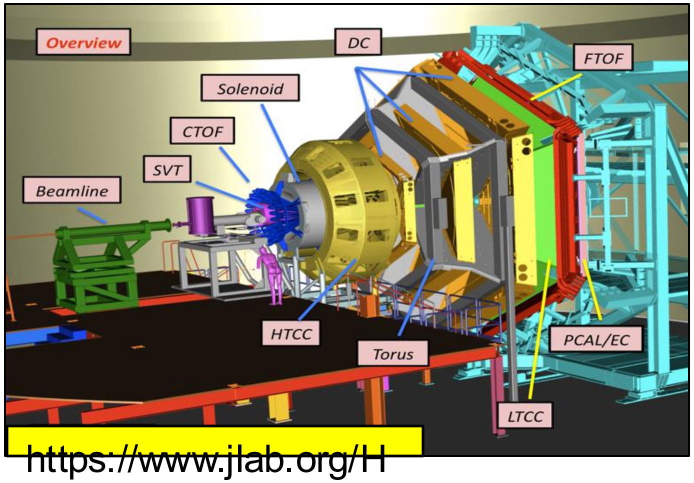
- Factorized QCD: $\sigma_{ep\rightarrow e'hX} \propto PDF \otimes \hat{\sigma} \otimes Fragmentation\ Function$
- More Information by using complex final states carrying additional Info
  - Hadron Pairs → relative angular momentum
  - Polarized hadron → polarization direction
- Hadronization Studies Complementary to Nucleon Structure
  - Confinement
  - Not accessible on the lattice
  - Connection to Model

Present and Future: Jlab and EIC





JLab: Valence quark distribution
Compass: Sea quark distribution
EIC: Gluon distribution

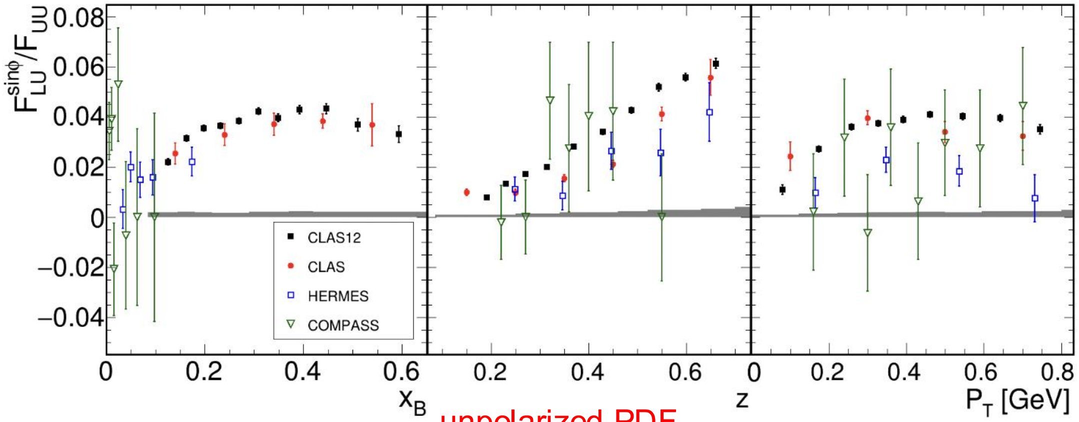


https://www.jlab.org/Hall-B/clas12-web/

Concentrate on CLAS12 @Jlab
→Full acceptance
EIC: See B. Surrow's talk on Friday

Case in point: Pion Beam Spin Asymmetries

Phys.Rev.Lett. 128 (2022) 6, 062005



twist-3 chiral odd pdf

unpolarized PDF

twist-3 t-odd PDF

Boer-Mulders

$$F_{LU}^{\sin\phi} \propto C^{[k_T]}\left[xeH_1^{\perp} + \frac{M_h}{M} f_1 \frac{\tilde{G}^{\perp}}{z}\right] + C^{[p_T]}\left[xg^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp} \tilde{E}\right]$$

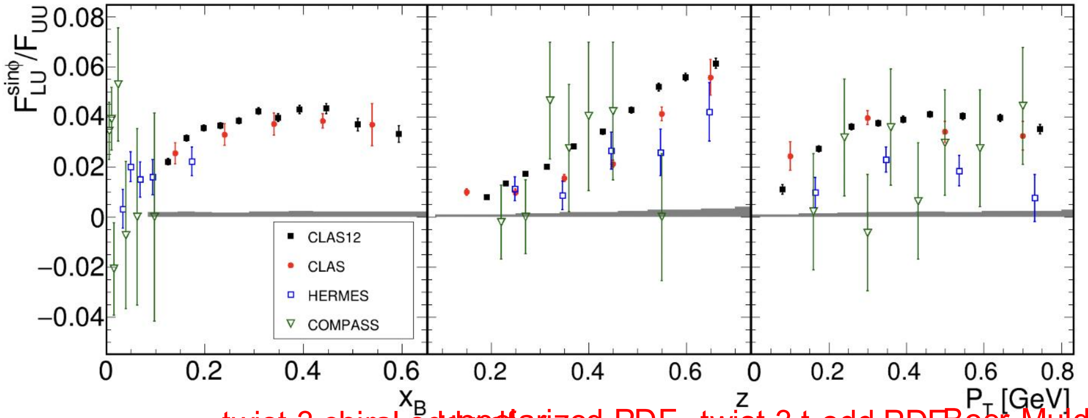
Collins FF

twist-3 FF

unpolarized FF

Case in point:
Single Pion Beam Spin Asymmetries

Phys.Rev.Lett. 128 (2022) 6, 062005



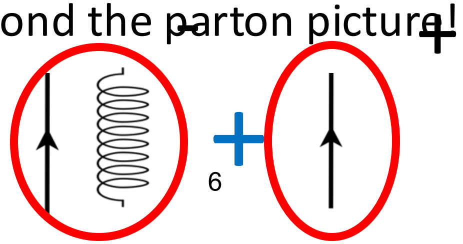
twist-3 chiral odd pdf unpolarized PDF twist-3 t-odd PDF Boer-Mulders

$$F_{LU}^{\sin\phi} \propto C^{[k_T]}\left[\boldsymbol{xeH_1^{\perp}} + \frac{M_h}{M} f_1 \frac{\tilde{G}^{\perp}}{z}\right] + C^{[p_T]}\left[x g^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp} \tilde{E}\right]$$

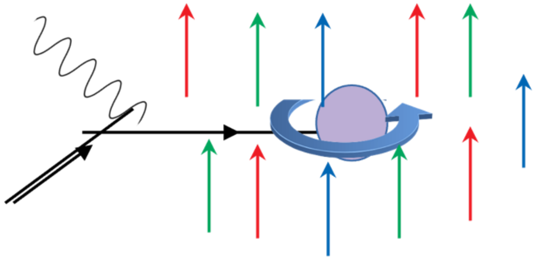
Collins FF twist-3 FF unpolarized FF

“pure” Twist3 PDF e(x) : Interference of quark-gluon amplitudes

Go beyond the parton picture!



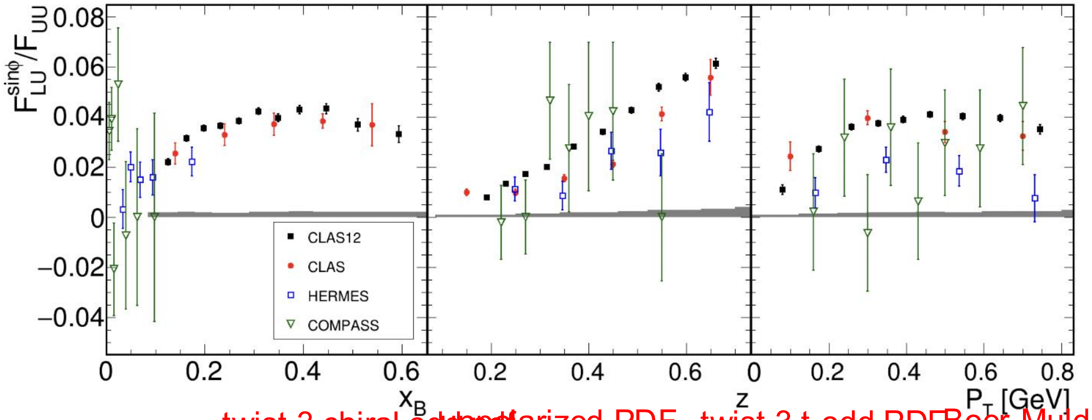
Phys.Rev.D 88 (2013) 114502



- First moment: “Boer-Mulders Force”: Transverse force exerted by color field on q↑ after scattering, in an unpolarized nucleon

Case in point:
Single Pion Beam Spin Asymmetries

Phys.Rev.Lett. 128 (2022) 6, 062005



twist-3 chiral odd pdf

unpolarized PDF

twist-3 t-odd PDF

Boer-Mulders

$$F_{LU}^{\sin\phi} \propto C^{[k_T]}\left[\boldsymbol{xeH_1^\perp} + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z}\right] + C^{[p_T]}\left[x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \tilde{E}\right]$$

Collins FF

twist-3 FF

unpolarized FF

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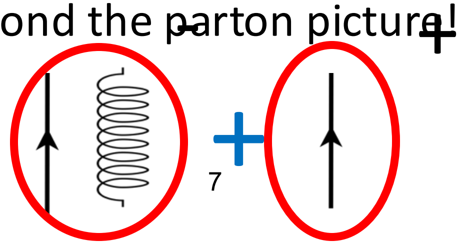
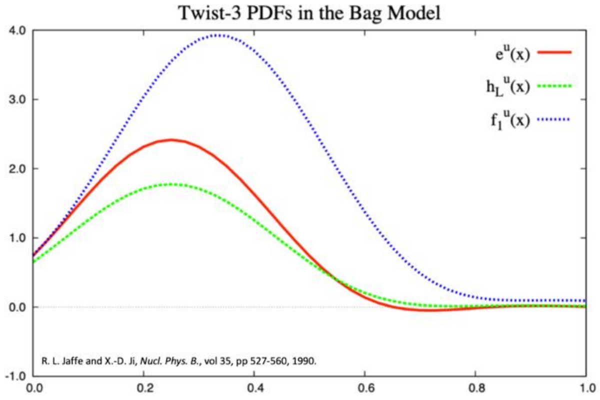




Jaffe, Ji, Nucl. Phys.B 35,557,1990

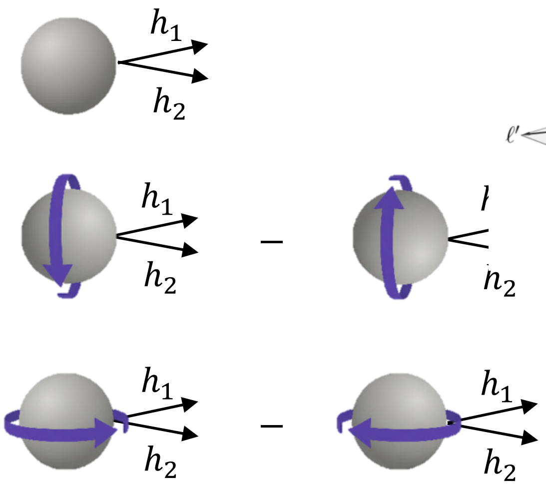
Better:
Dihadron Fragmentation Functions

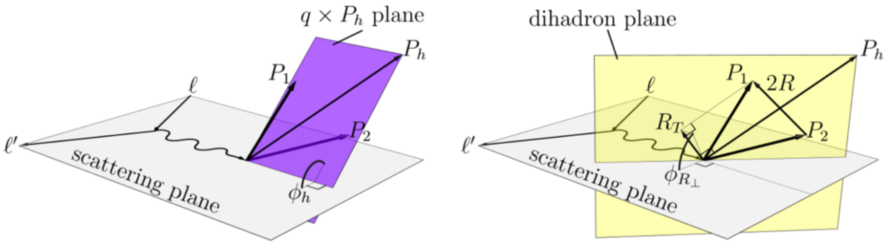
- Correlations in ϕ_R, ϕ_h access quark polarization

Unpolarized
$D_1(z, M_h, p_T^2)$

Helicity-dependent
$G_1^\perp(z, M_h, p_T^2)$

S_{qT}/k_T-dependent
$H_1^\perp(z, M_h, p_T^2)$



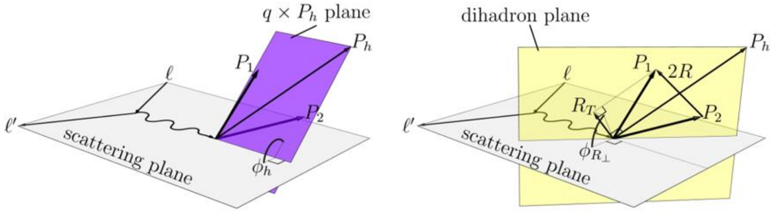


Better:
Dihadron Fragmentation Functions

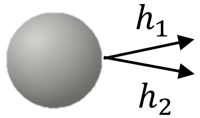
- DiFF's expand into *partial waves* $|\ell, m\rangle$ w.r.t θ →Connect to MCEGs

→Complete set of spin orbit correlations in hadronization

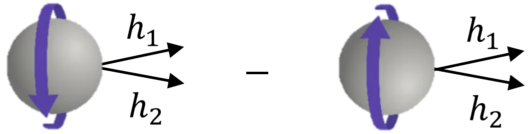
- Different Partial Wave contribute to different Interference terms on the QCD amplitude level



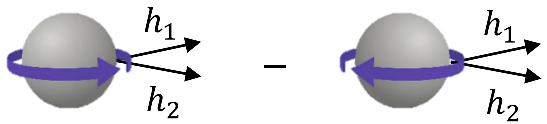
Unpolarized
$D_1(z, M_h, p_T^2)$



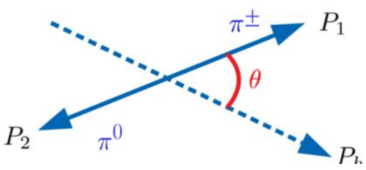
Helicity-dependent
$G_1^{\perp}(z, M_h, p_T^2)$

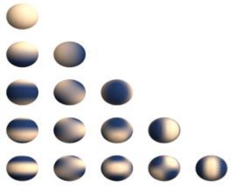


S_{qT}/k_T-dependent
$H_1^{\perp}(z, M_h, p_T^2)$



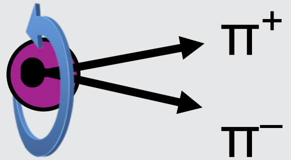
Dihadron CoM frame

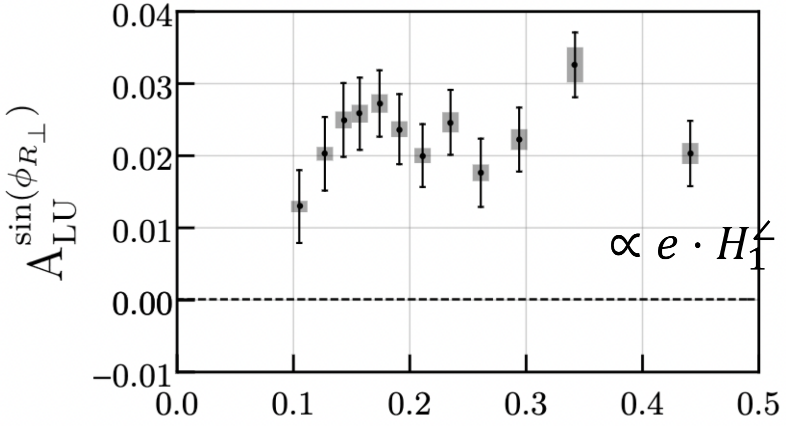




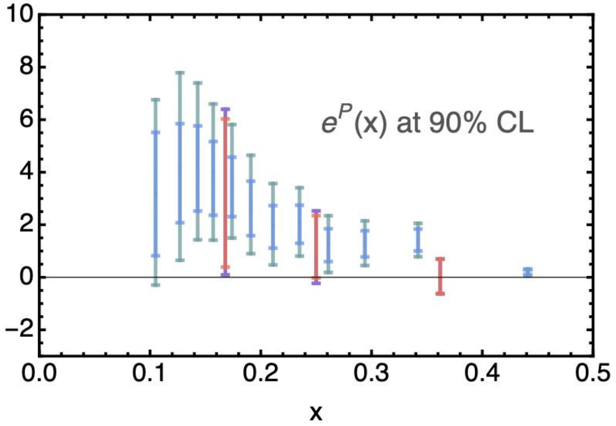
2-Point Correlations
In phase space
→insight into hadronization
→connection to models

Better: di-hadrons





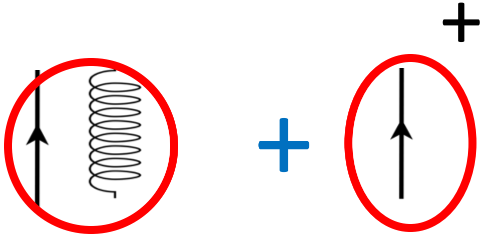
Phys.Rev.Lett. 126 (2021) 152501



Phys.Rev.D 106 (2022) 1, 014027

$$F_{LU}^{\sin\phi_R} \propto \frac{M}{m_{hh}} \boldsymbol{xe(x)H^{\angle}}(z, cos\theta, m_{hh}) + \frac{1}{z} f_1 \tilde{G}^{\angle}(z, cos\theta, m_{hh})$$

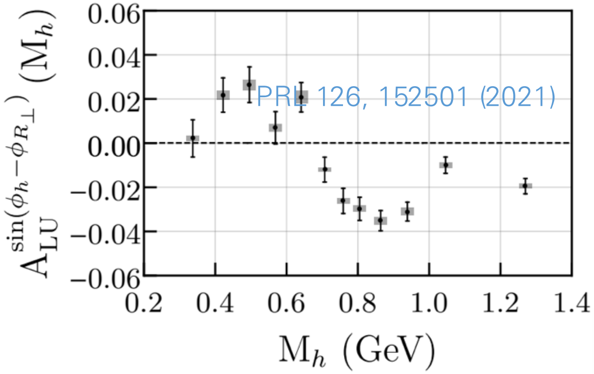
- First extraction of chiral odd $e(x)$ →point-by-point!   –



First Measurement of $G_1^\perp$ →no 1h equivalent

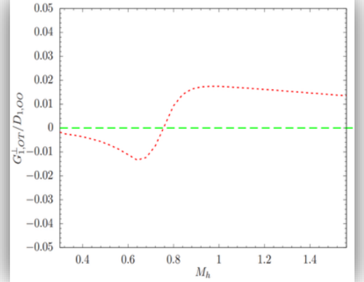
- $d\sigma_{LU} \propto f_1 \otimes G_1^\perp$
- Fragmentation sensitive to quark helicity

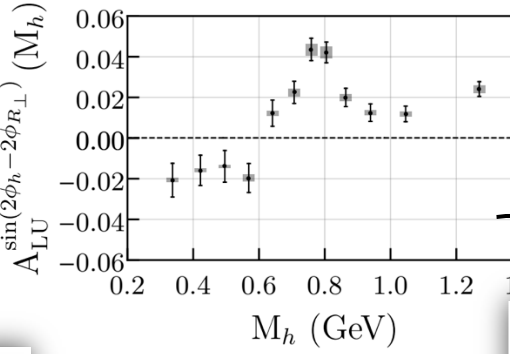
$d\sigma_{LU}$



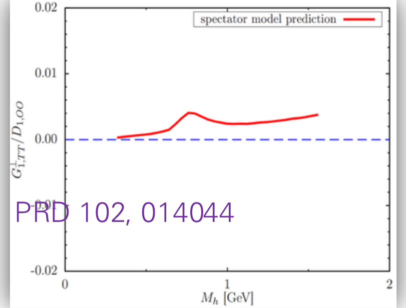
PRL 126, 152501 (2021)

$G^\perp_{1,OT+LT}$





$G^\perp_{1,TT}$

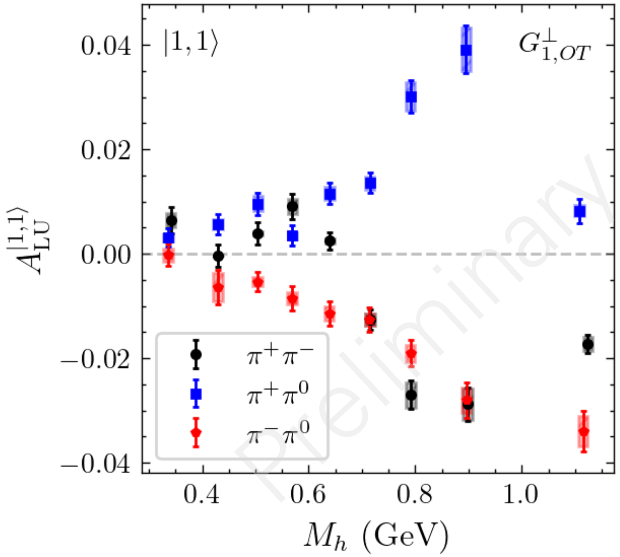


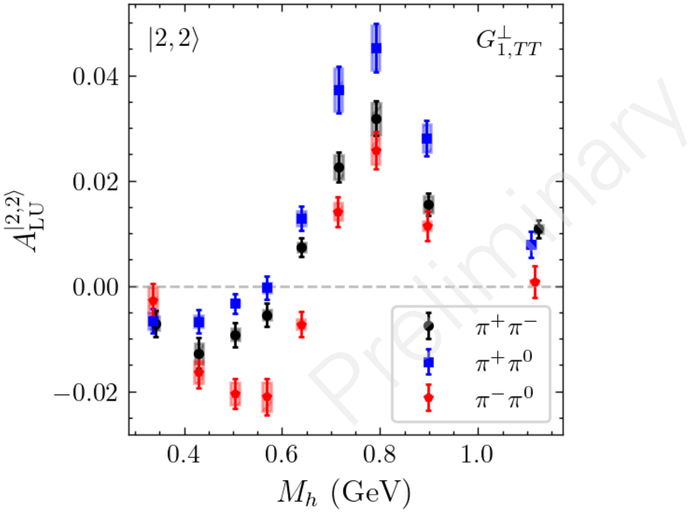
PRD 102, 014044

- Firt nonzero $G_1^\perp$ signal
- Resonance structure **predicted** by spectator models and **verified** at CLAS12

PRD 101, 054020

Flavor dependence of Different Partial Waves





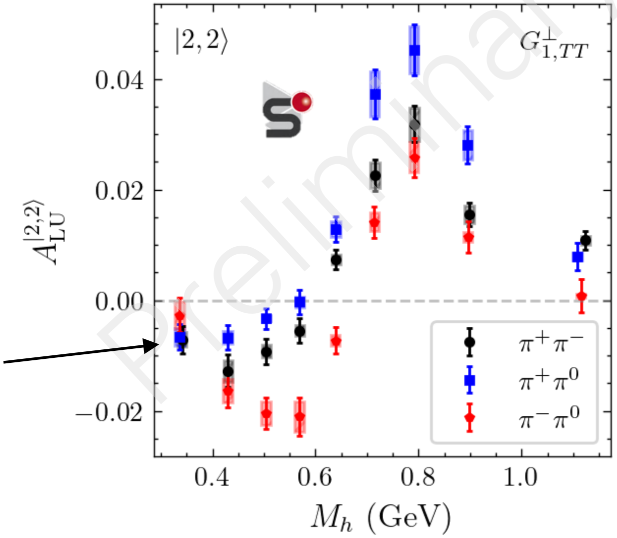
$A^{|1,1\rangle}_{LU} \propto f_1(x) \otimes G^{\perp}_{1,OT}$

$A^{|2,2\rangle}_{LU} \propto f_1(x) \otimes G^{\perp}_{1,TT}$

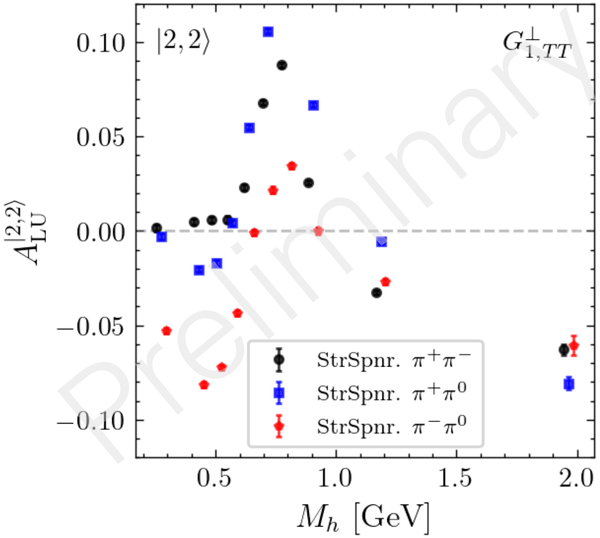
- Partial Waves are Flavor dependent
  - Different Decay Channels contribute to different PWs

Reproduction in Event Generator Models

- **StringSpinner** introduces *spin-dependent* fragmentation into PYTHIA8 based on the string+$^3P_0$ model of hadronization ($q\bar{q}$ produced in $L = S = 1, J = 0$ quantum state)
- Pythia allows to extrapolate beyond validity of factorized QCD framework
- With parameter tuning, StringSpinner can replicate several of our dihadron beam-spin asymmetries across multiple $\pi\pi$ channels → <u>huge step for phenomenology</u>!



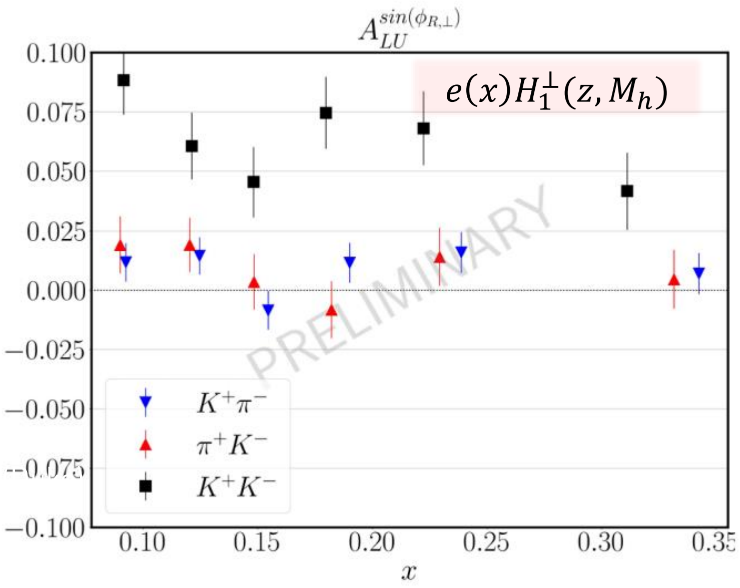
Possible dip from $\omega \to \pi\pi X$

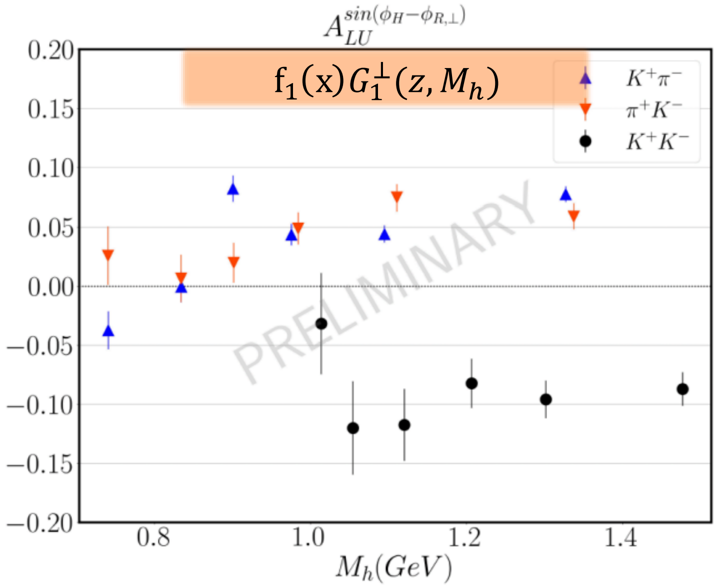


StringSpinner Modeling

Large Kaon Asymmetries Currently not understood

- Large asymmetries observed in K^+K^- dihadrons → larger than $\pi^+\pi^-$ dihadrons! (compare to large A_C at BaBar)
- Correlations of πK appear absent in $H_1^\perp$ DiFF (left plot) yet large in $G_1^\perp$ (right plot)
- Access to K^* and ϕ vector meson contributions to single hadron SIDIS

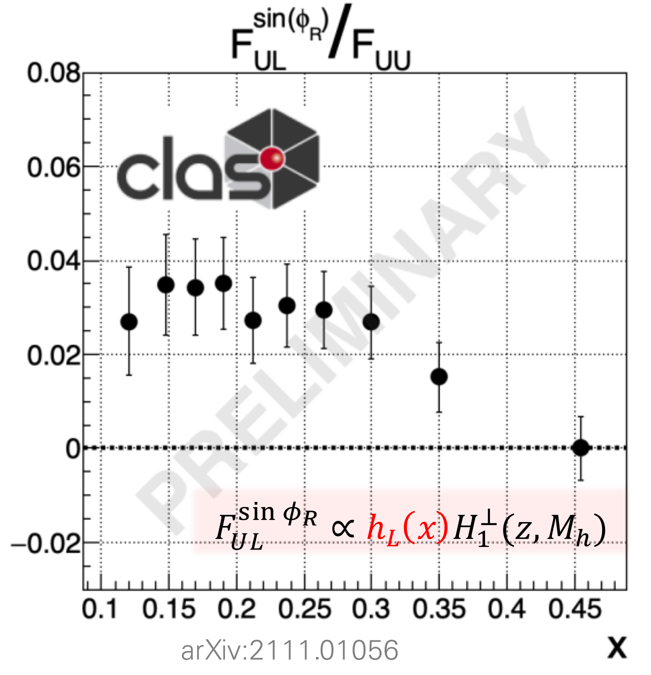




New: Longitudinal Target SSAs

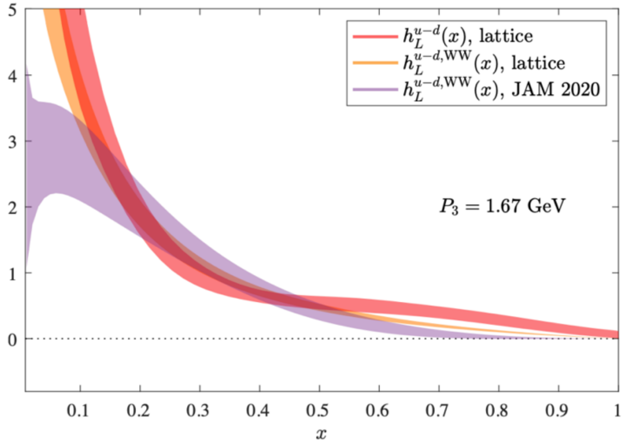
$d\sigma_{UL}$

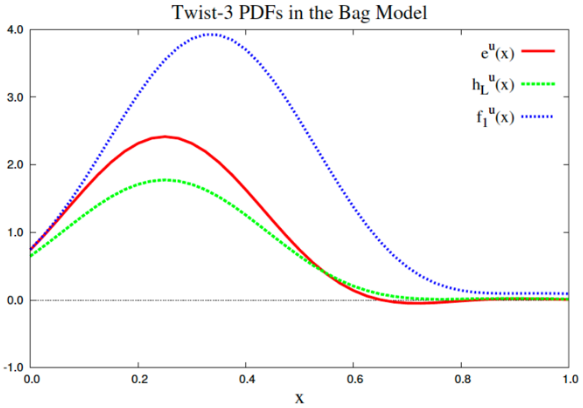
- No published results on chiral odd twist-3 $h_L(x)$, only lattice and phenomenological models
  - x^3 moment connected to average longitudinal gradient of the transverse force on T. pol. Quarks in L proton Nucl. Phys. B375, 527 (1992)
  - Sign of gradient will help to study correlations of nucleon spin and color magnetic field



$F_{UL}^{\sin\phi_R} \propto h_L(x) H_1^{\perp}(z, M_h)$

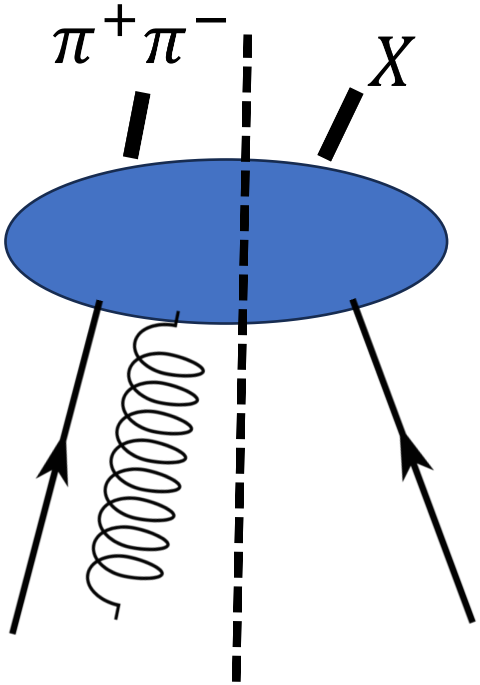
arXiv:2111.01056

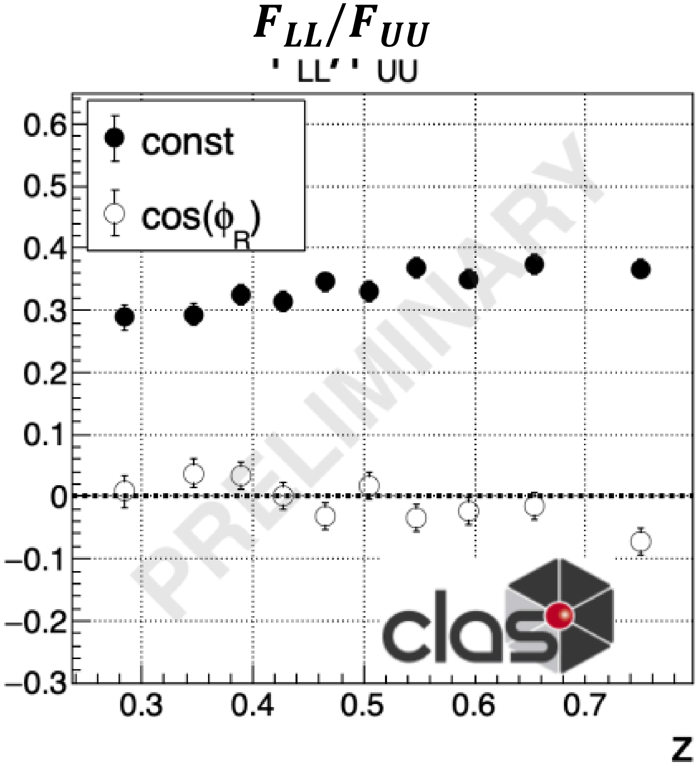




Access to Twist3 FFs in Longitudinal Double Spin Asymmetries

$d\sigma_{UL}$





- $F_{LL}^{\text{const}}/F_{LL}^{\cos\phi_R}$ sensitive to $g_{1L}D_1/g_{1L}\widetilde{D}$
  →probe of twist-3 DiFF $\widetilde{D}$ size

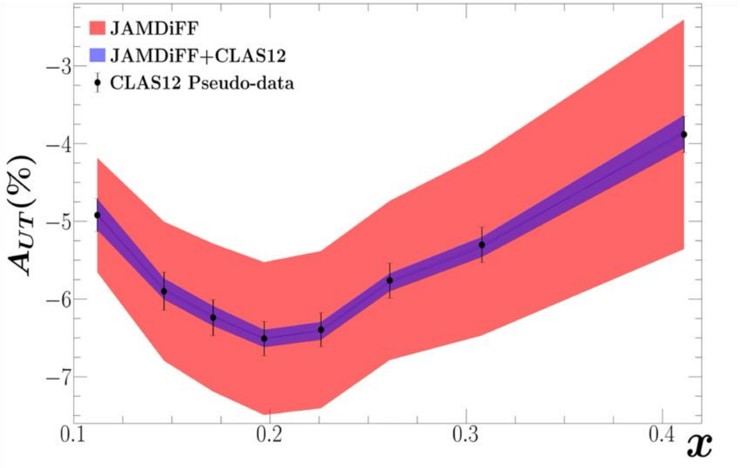
➢ Preliminary results indicate $\widetilde{D}$ is roughly one order of magnitude smaller than D_1

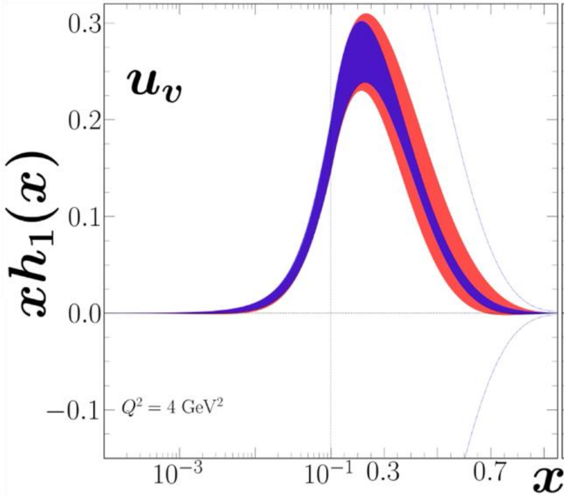
  ➢ Different than PDFs!

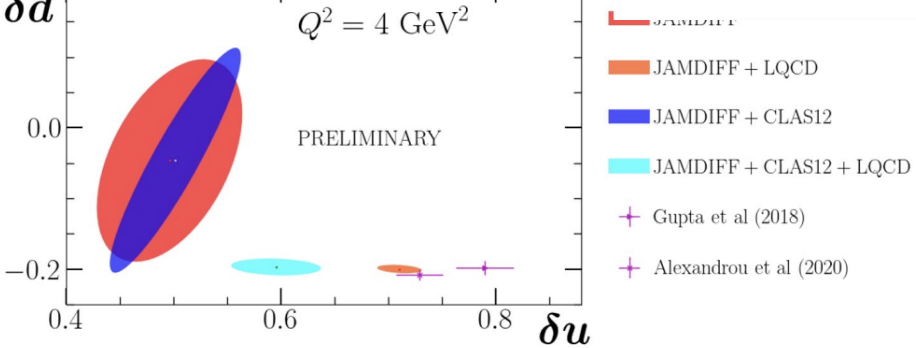
$d\sigma_{UT}$

Tranversely Polarized Target ≈ 2029

- **Run Group H:** 1st transversely polarized target experiment at CLAS12
- Significant impact in valence region
- Projected to be able to test Phenomenology – Lattice Tension

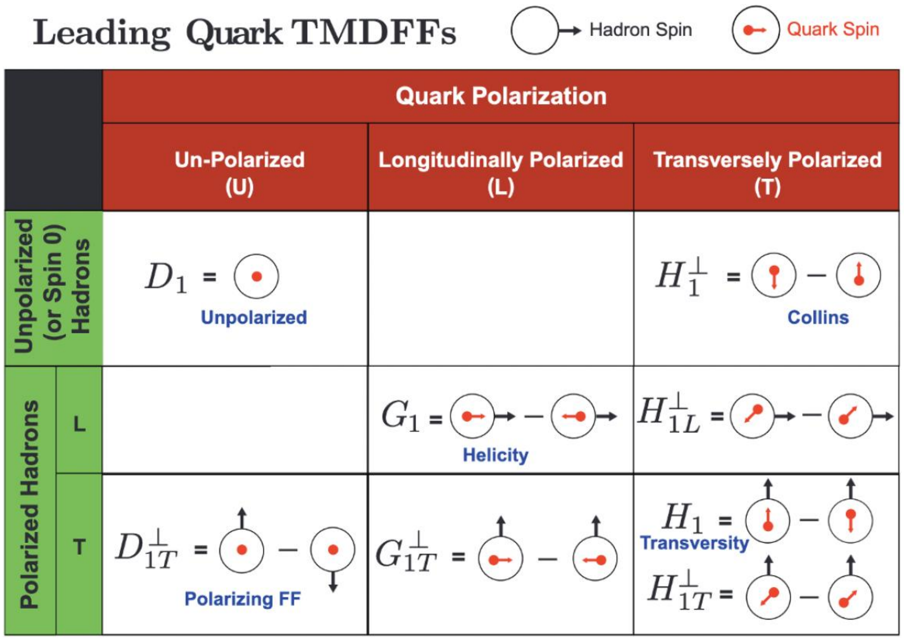






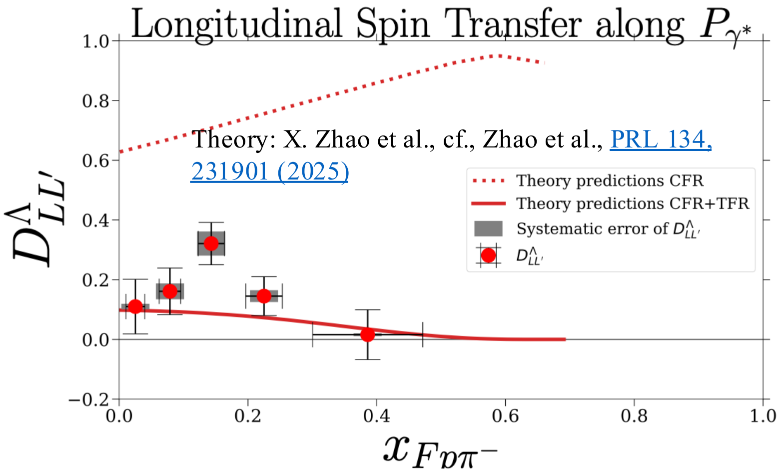
See M. Contalbrigo's talk Thursday morning

Polarized Λ's as a tool to Learn about Spin Structure



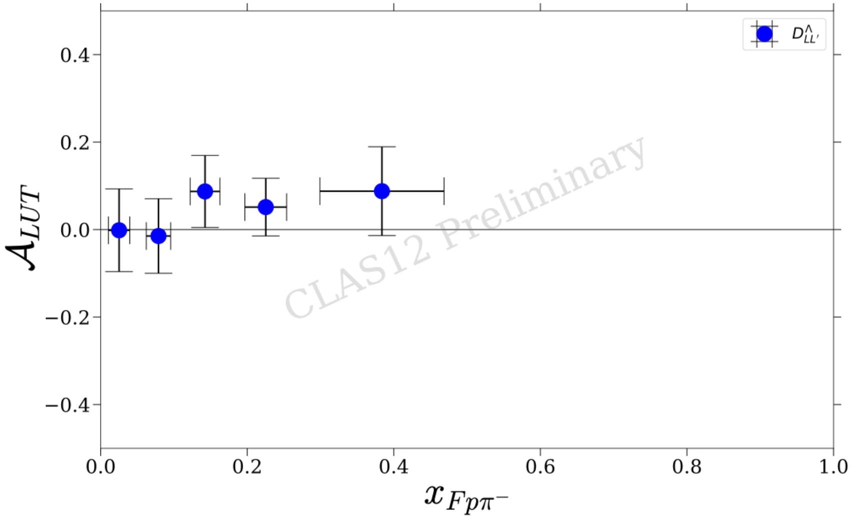
Leading Quark TMDFFs — Hadron Spin, Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 =$ Unpolarized		$H_1^\perp =$ Collins
Polarized Hadrons	L		$G_1 =$ Helicity	$H_{1L}^\perp =$
	T	$D_{1T}^\perp =$ Polarizing FF	$G_{1T}^\perp =$	$H_1 =$ Transversity $H_{1T}^\perp =$



Longitudinal Spin Transfer along P_{γ^*}

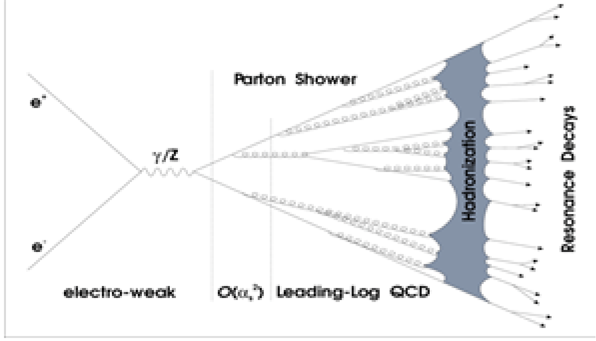
Theory: X. Zhao et al., cf., Zhao et al., PRL 134, 231901 (2025)



- First measurement of A_{LUT}
- Future:
  - Precision Λ program at CLAS12
  - Comparison with MC models

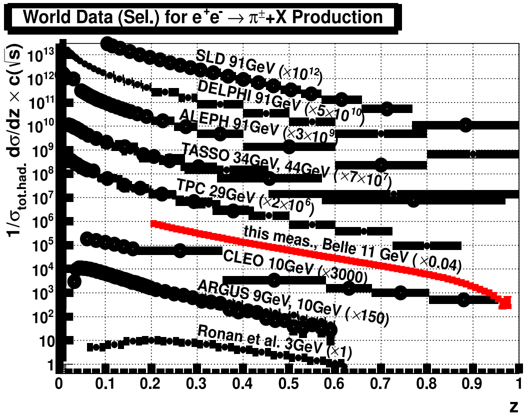
$$A_{LUT}^{\cos\phi_{S_\Lambda}} = \frac{F_{LUT}^{\cos\phi_{S_\Lambda}}}{F_{UUU}} = -\frac{\sum_a e_a^2\left[\frac{Mx}{Q}e^a(x)H_1^a(z_h)+\frac{M_h}{zQ}f_1^a(x)\tilde{G}^a(z_h)\right]}{\sum_a e_a^2 f_1^a(x)D_1^a(z_h)}$$

Studying Hadronization in e^+e^-

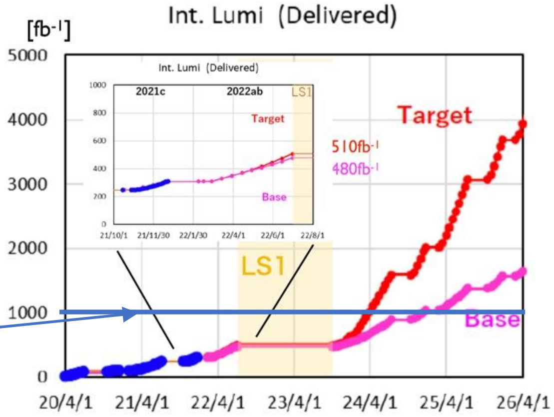


- Dominated by B factories Belle, BaBar
  →For BES III see Hai-Bo Li ‘s talk

- Belle II aims to have significantly higher luminosity, current record: 4.7x10^{34}cm^{-2}s^{-1}.

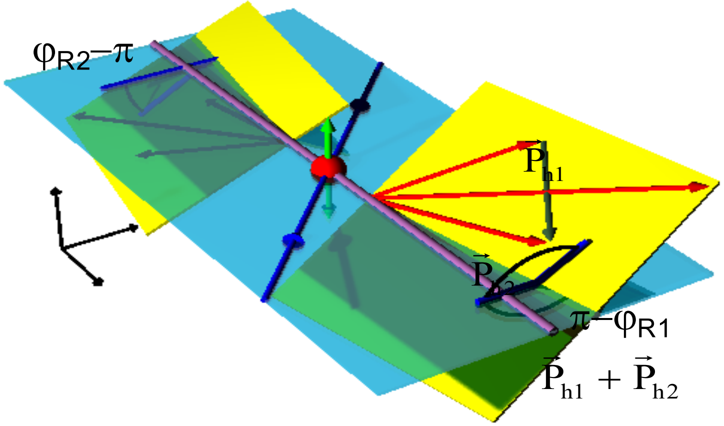


- Whitepaper on QCD physics: 2204.02280 [hep-ex]
  - E.g. Use high statistics for complex final states
  - EE correlators
  - g-2
  - α_S
  - …



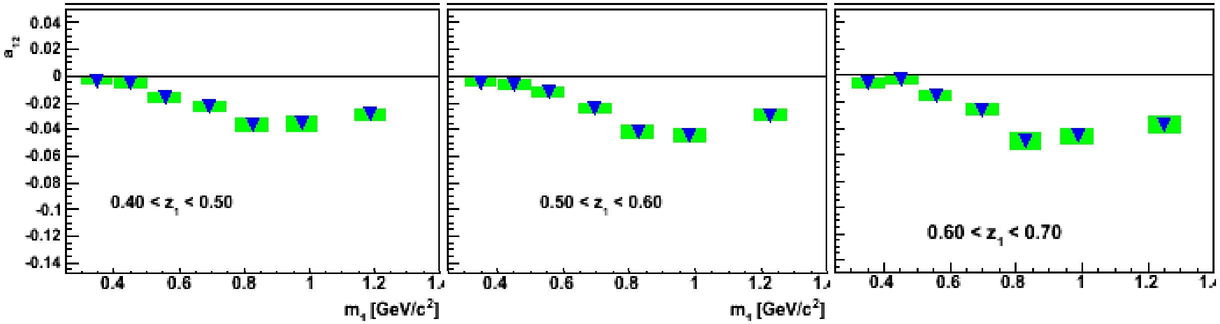
Belle

Polarized FFs from Belle



Cross-section $e^+e^- \to (h_1h_2)(\overline{h_1}\,\overline{h_2}) + X$

$$\propto D_1^\perp\,\overline{D_1^\perp} + H_1^\perp\,\overline{H_1^\perp}\cos(\phi_{R1}+\phi_{R2})$$



PRL **107,** 072004(2011)

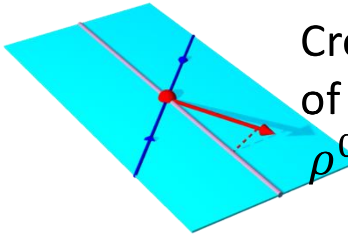
Cross section: Phys. Rev. D **96**, 032005

Single Hadrons

PRL 96 (2006) 232002
PRD 78 (2008) 032011
*PRD*100 (2019) 9, 092008 (p_T, π^0, η)

- Statistics Hungry, only possible at B-factories

Examples: Single/Di-hadron Multiplicities, Polarization and back-to-back Correlations to access Fragmentation Studies

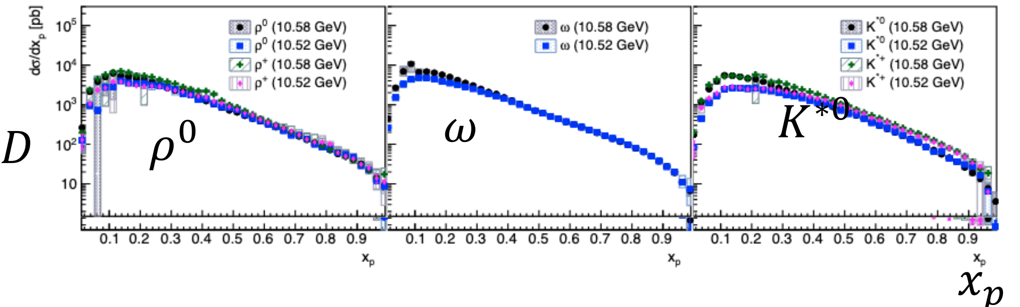


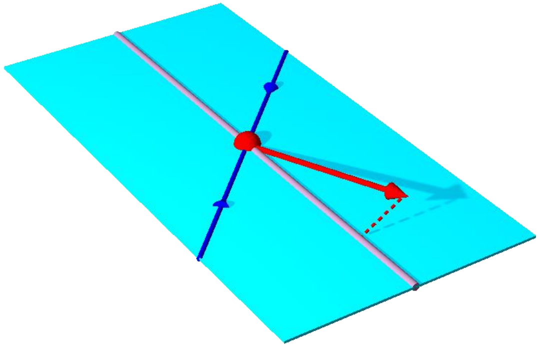
$D_{1,q}^{h}(z, k_T, Q^2)$

Cross-sections

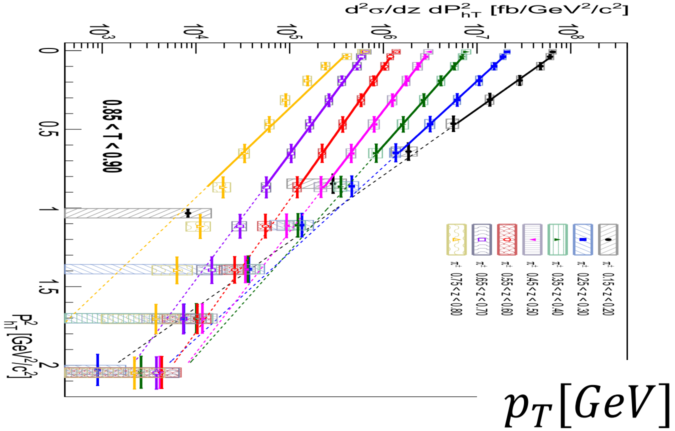
of VMs and unstable particles

$\rho^0, \omega, K^{*0}, \phi, \eta, K_S^0, f_0(980), D$



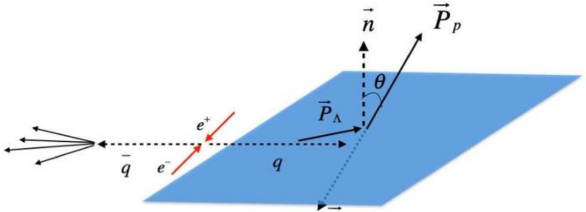


p_T dependence of single hadron fragmentation



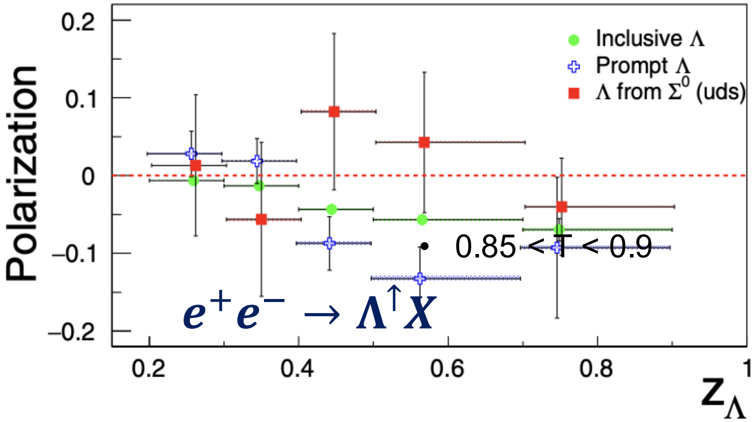
: *Phys.Rev*

Phys.Rev.D 99 (2019) 11, 112006



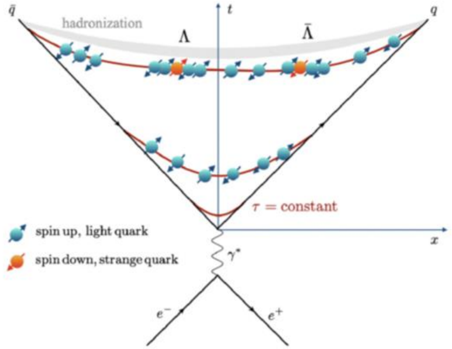
Phys. Rev. Lett. 122, 042001 (2019)

First measurement of spontaneous Λ polarization, first prompt Λ



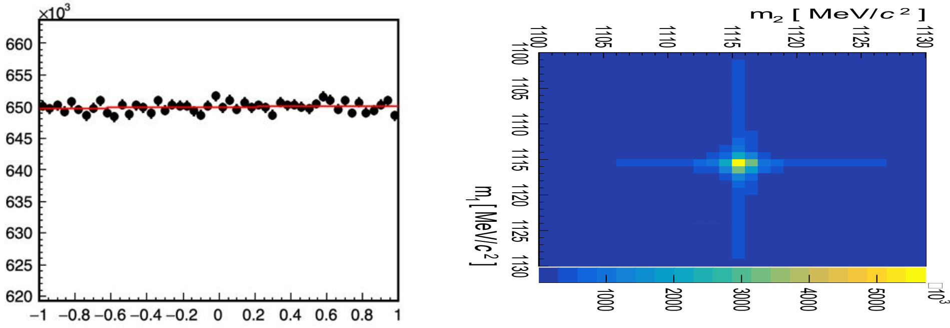
Current Program at Belle II – Λ

- Spin Entanglement in $\Lambda^{\uparrow} - \Lambda^{\uparrow}$ correlations
    - Compare with calculations in Schwinger model and Quantum Computer
        - Also di-hadrons
    - Long/Trans spin transfer to Λ in $\Lambda - \Lambda$ and $\Lambda - h_1 h_2$ correlations



From Phys. Rev. D **109**, 116003,Phys. Rev. Lett. **131**, 021902 (di-hadrons)

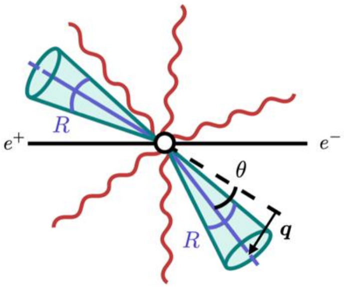
See also Lin, Liu, Shao, Wei arXiv:2507.15387

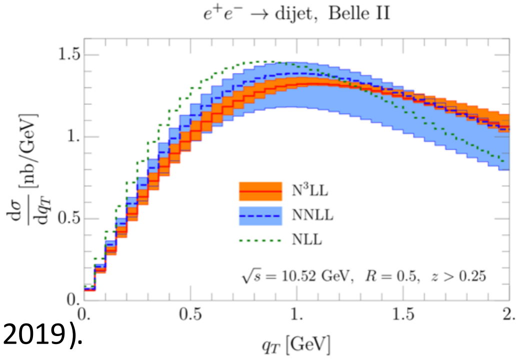


Belle II off-resonance simulation
for $\Lambda - \Lambda$ polarization correlations
$e^+e^- \to q\bar{q}, q \in u, d, s, c$
$P_{\Lambda\bar{\Lambda}} = 0.0006 \pm 0.0005$ (fit)

Brand New Opportunities at Belle II: Precision Jet Physics in e^+e^-

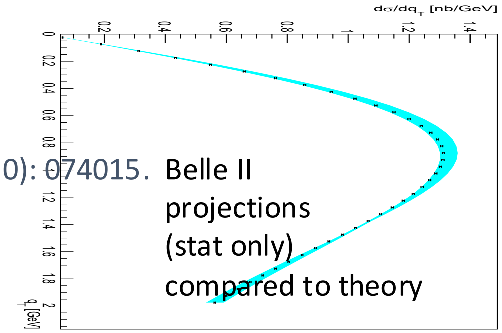
- Jet physics (will) play an important role at the EIC and LHC
- Precision measurements in e^+e^- annihilation will test current theoretical understanding (N^3LL)
- Lower energies like Belle in particular sensitive to hadronization effects
- Example: Transverse Momentum Imbalance ← → TMD framework





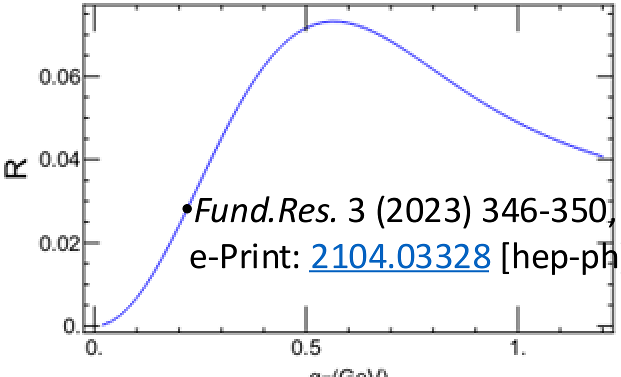
JHEP. **2019,** 31 (2019).

Gutierrez-Reyes, D., Scimemi, I., Waalewijn, W.J. *et al.* Transverse momentum dependent distributions i e^+e^- and semi-inclusive deep-inelastic scattering using jets. *J. High Energ. Phys.* **2019,** 31 (2019).



PRD102.7 (2020): 074015. Belle II projections (stat only) compared to theory

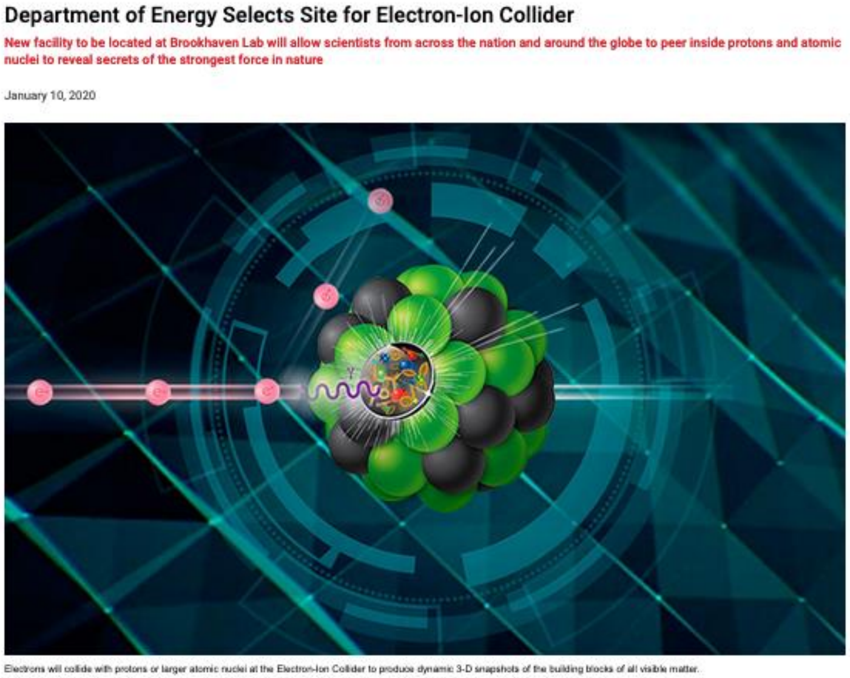
- New suggestion: Measure Colllins-like back-to-back azimuthal correlations for jets
- Di-hadron FFs of pions and kaons around jets



- *Fund.Res.* 3 (2023) 346-350, e-Print: 2104.03328 [hep-ph]

➔**Sensitive to transversity**

SIDIS at the EIC

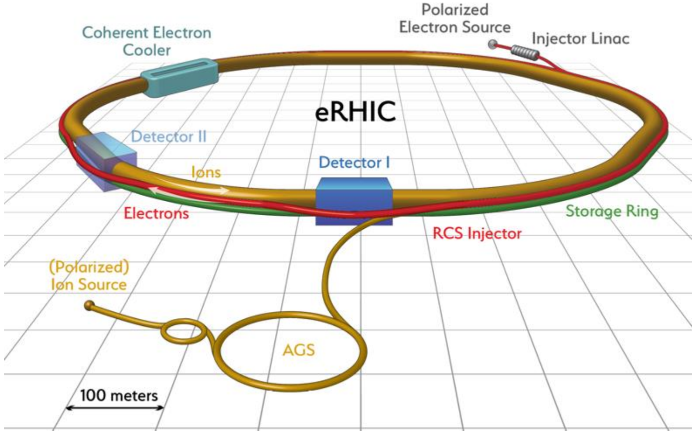


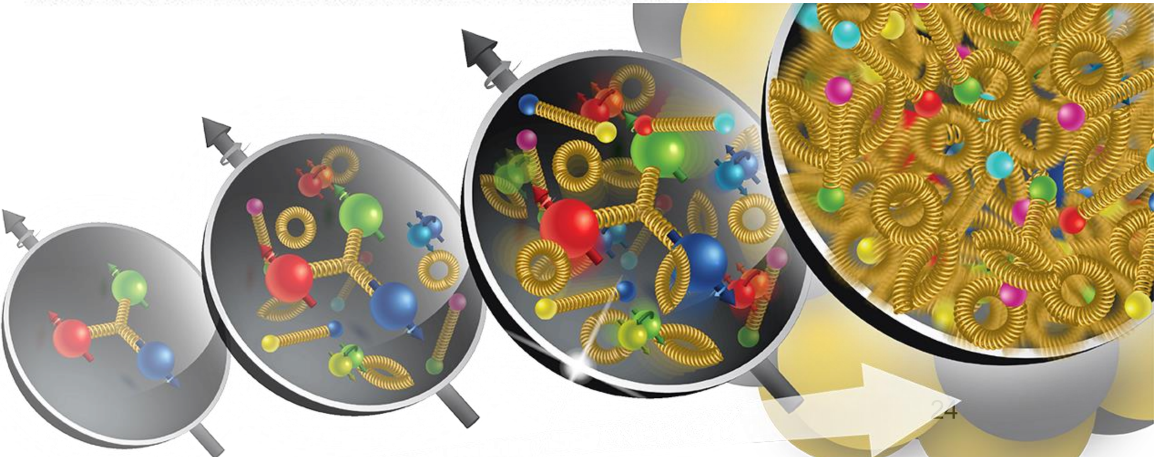
Department of Energy Selects Site for Electron-Ion Collider

New facility to be located at Brookhaven Lab will allow scientists from across the nation and around the globe to peer inside protons and atomic nuclei to reveal secrets of the strongest force in nature

January 10, 2020

Electrons will collide with protons or larger atomic nuclei at the Electron-Ion Collider to produce dynamic 3-D snapshots of the building blocks of all visible matter.

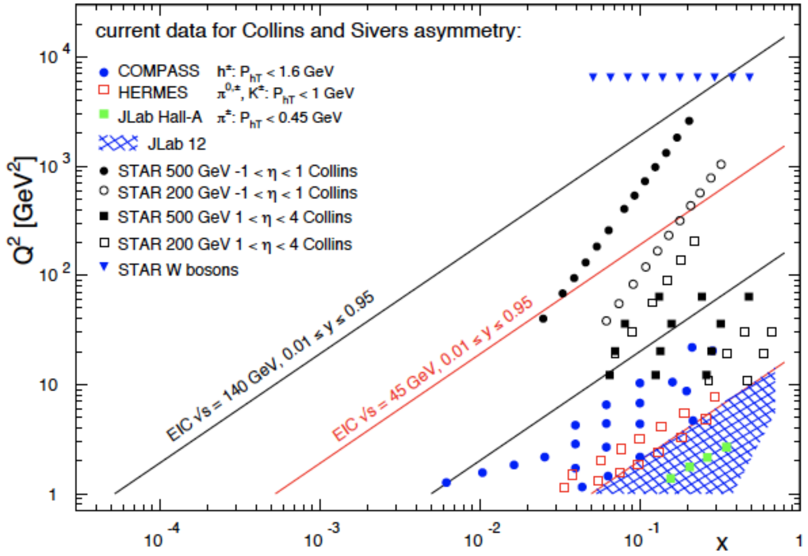




SIDIS physics at an EIC: Coverage

- Common theme on EIC impact
  - Extended kinematic coverage and precision, along with polarization and possible beam charge degrees of freedom allow multi-pronged approach →needed to extract multidimensional objects
  - TMD factorization is valid

Large Q^2 lever arm: probe evolution, disentangle contributions to σ

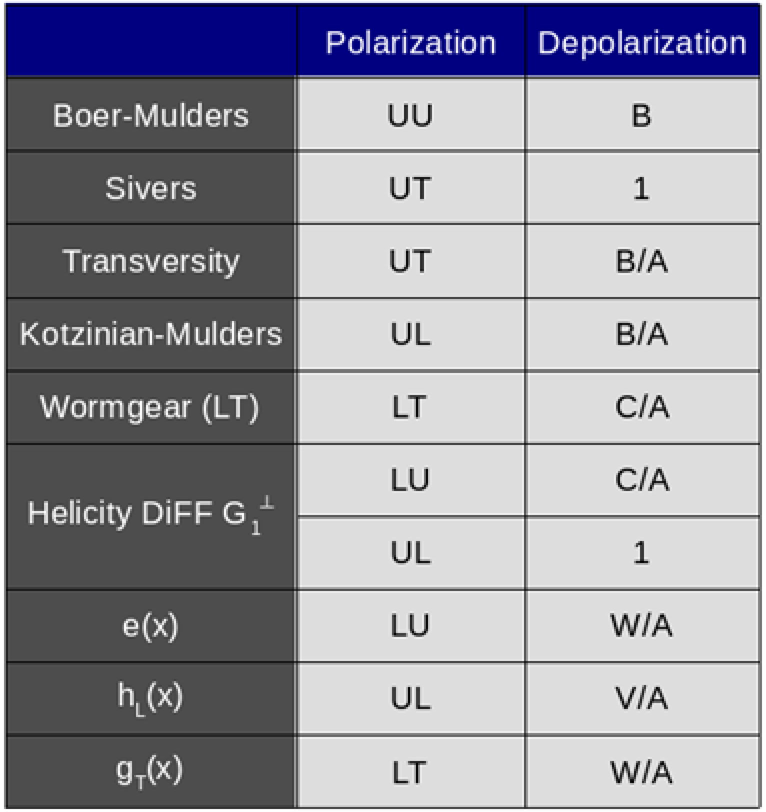


Coverage to low x: access sea and gluon distributions

Depolarization Factors

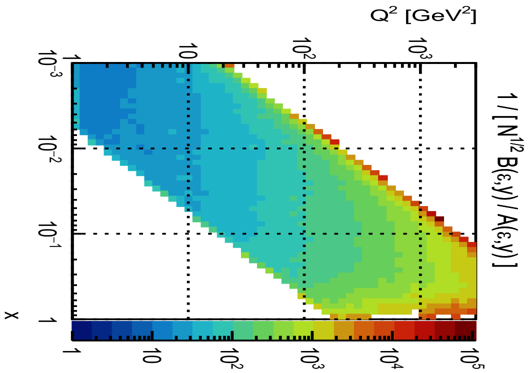
Twist 2

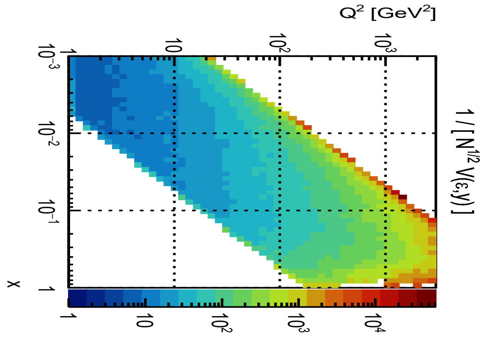
Twist 3



	Polarization	Depolarization
Boer-Mulders	UU	B
Sivers	UT	1
Transversity	UT	B/A
Kotzinian-Mulders	UL	B/A
Wormgear (LT)	LT	C/A
Helicity DiFF $G_1^{\perp}$	LU	C/A
	UL	1
e(x)	LU	W/A
$h_L(x)$	UL	V/A
$g_T(x)$	LT	W/A

Suppressed at EIC

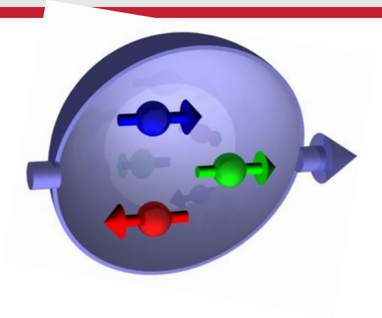


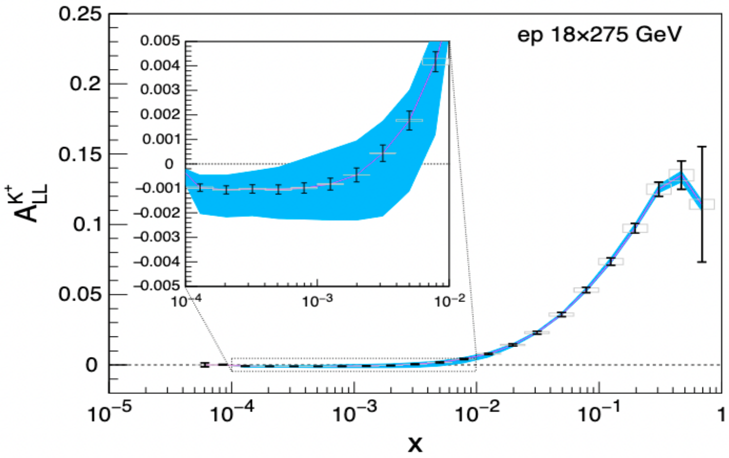


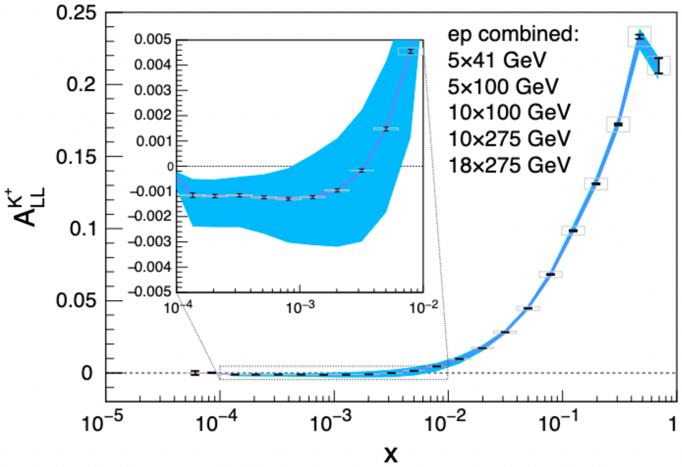
Slide from C. Dilks

Longitudinal double spin asymmetries

- $A_{LL} = \frac{\sigma^{\uparrow\Uparrow} - \sigma^{\uparrow\Downarrow}}{\sigma^{\uparrow\Uparrow} + \sigma^{\uparrow\Downarrow}} \propto g_1$







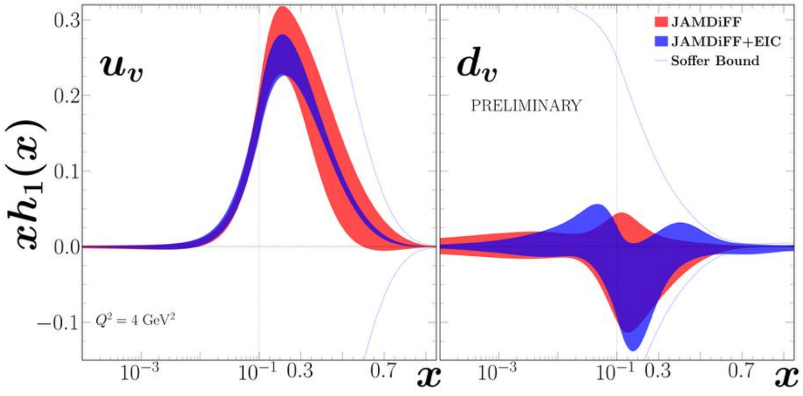
- Projections for Athena (2022 *JINST* **17** P10019)
- 3% point-to-point, 2% scale uncertainties (from Hera experience)
- $z > 0.2$
- $15.5\ fb^{-1}$ at $18x275$, other datasets scaled accordingly

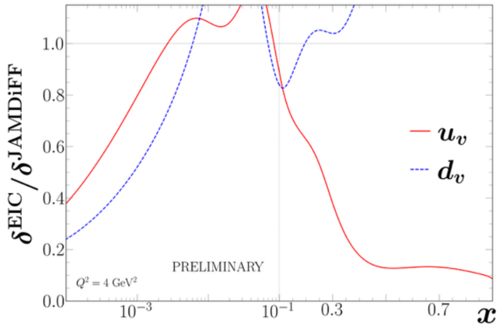
➔See also double tagged A_1

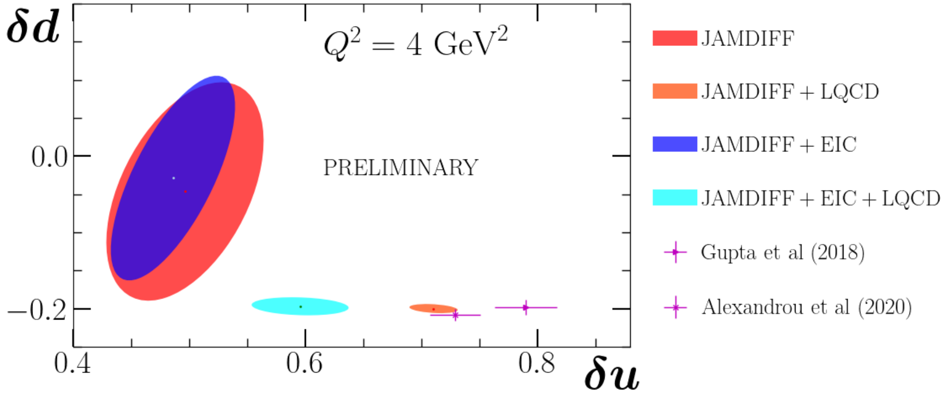
Example: Transversity Extraction from Di-hadrons

- Only proton data
- Test small$-x$ constraint
- See also *Phys.Lett.B* 816 (2021) 136255 for single hadrons
- Projected to be able to distinguish between lattice and phenomenology

(Similar to CLAS)

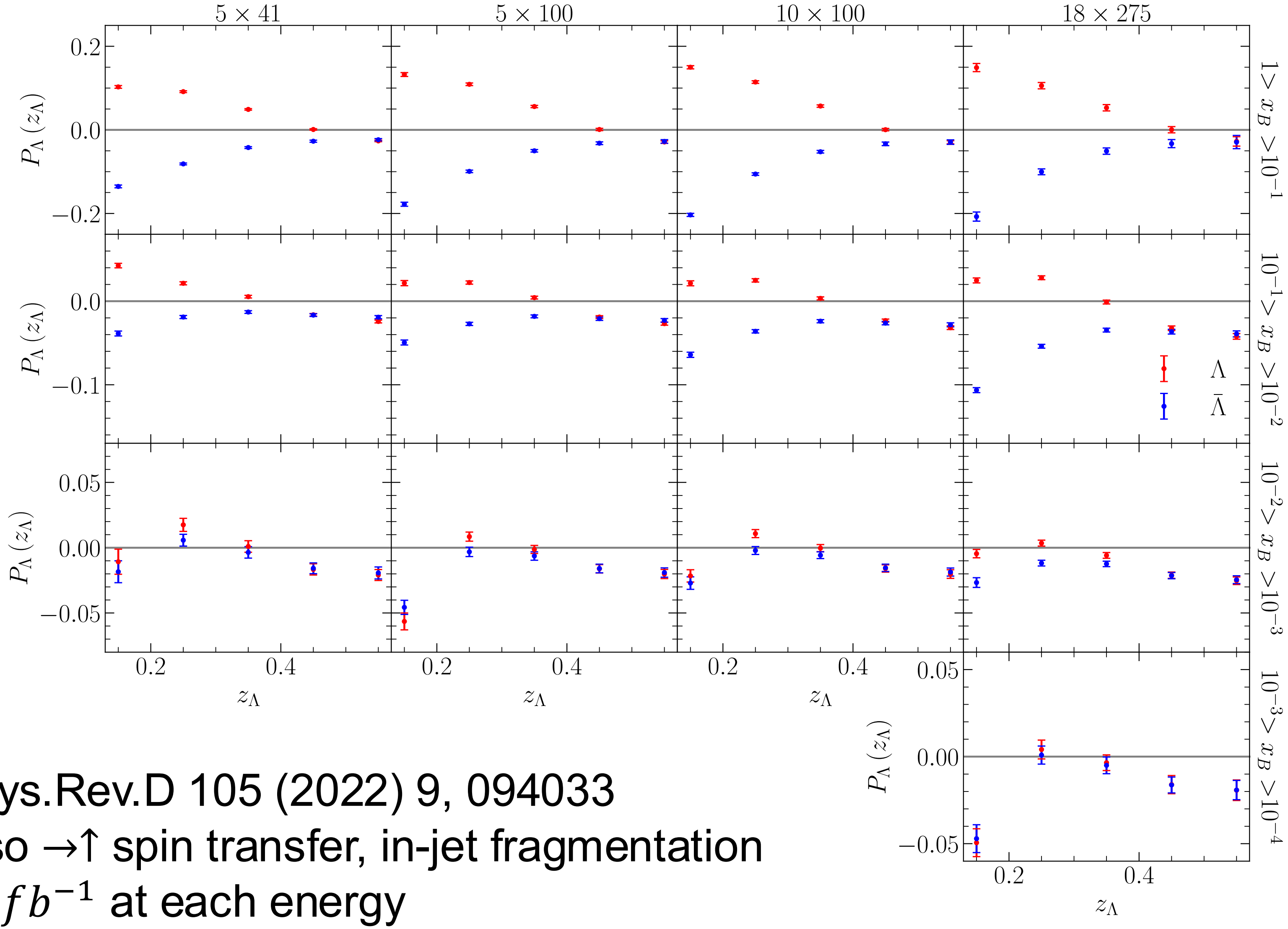






5×41: 2.857 fb^{-1}
10×100: 51.298 fb^{-1}
18×275: 10 fb^{-1}

Precision Λ physics at the EIC



- Phys.Rev.D 105 (2022) 9, 094033
- Also →↑ spin transfer, in-jet fragmentation
- 40 fb^{-1} at each energy
- Significant impact of low $\sqrt{s}$ data

Early Running Constraints

Year	Species	Energy	Luminosity (fb^{-1})	p/A polarization
Year 1	$e + Ru/Cu$	10×115	1	N/A
Year 2	$e + d$	10×130	10	N/A
Year 2	$e + p$	10×130	1	trans
Year 3	$e + p$	10×130	5	trans&long
Year 4	$e + Au$	10×100	0.5	N/A
Year 4	$e + p$	10×250	4	trans&long
Year 5	$e + Au$	10	0.5	N/A

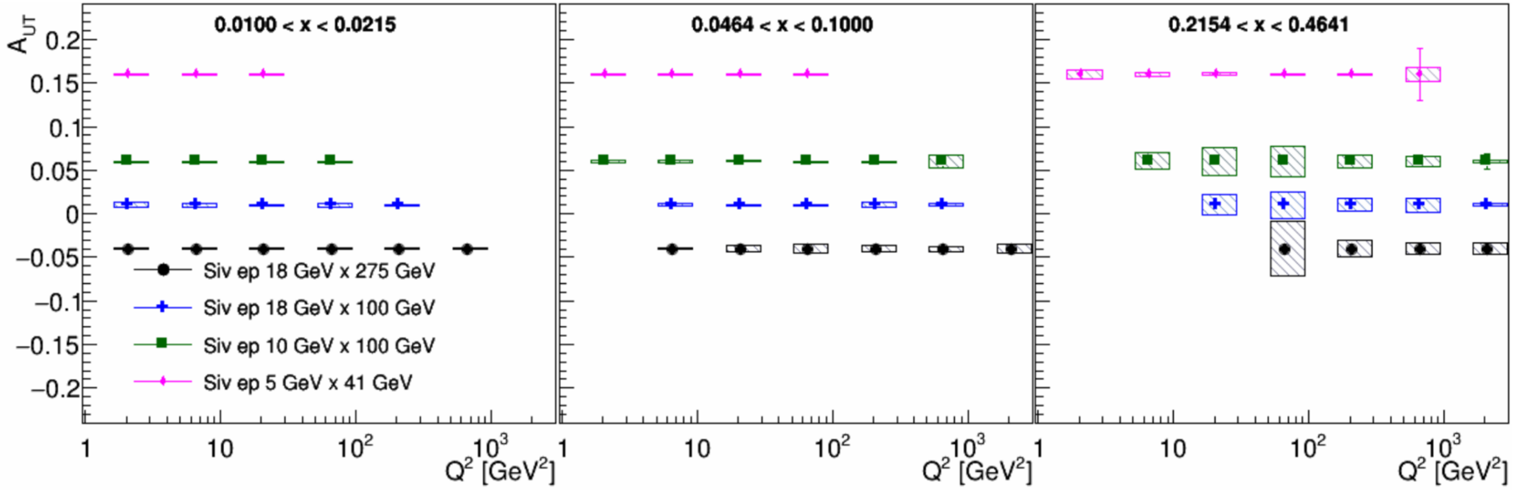
- $O(10\%)$ of YR projection data
- Early HI data
- top energy in Year 4

- Cross-section measurements ((n)PDF, (n)FFs, are not luminosity hungry but need good understanding of detector

- BSAs a good start but kinematically suppressed
- Also consider asymmetries in species, final state, charge..

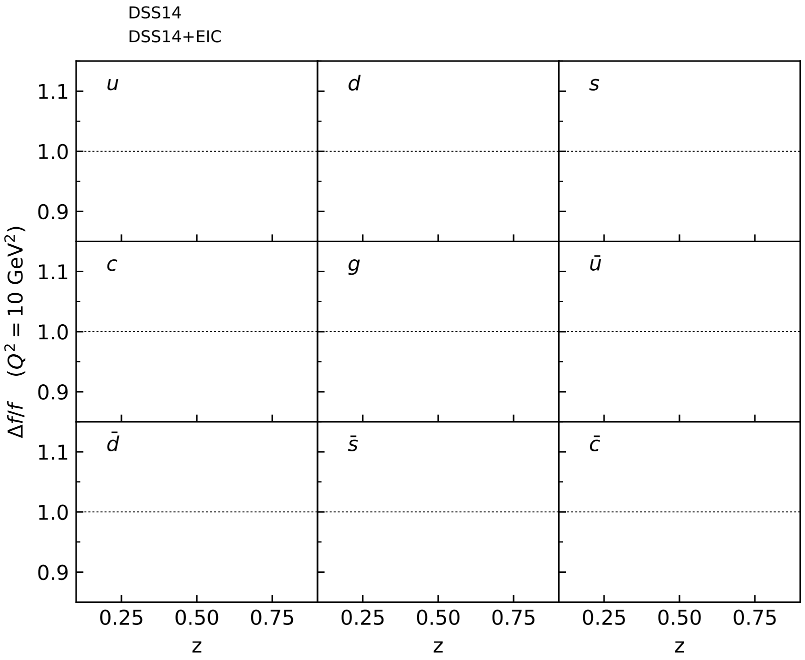
Transverse Single Spin Asymmetries at $10\ fb^{-1}$

- Uncertainties ≪ then expected asymmetries
- Insights into TMD framework

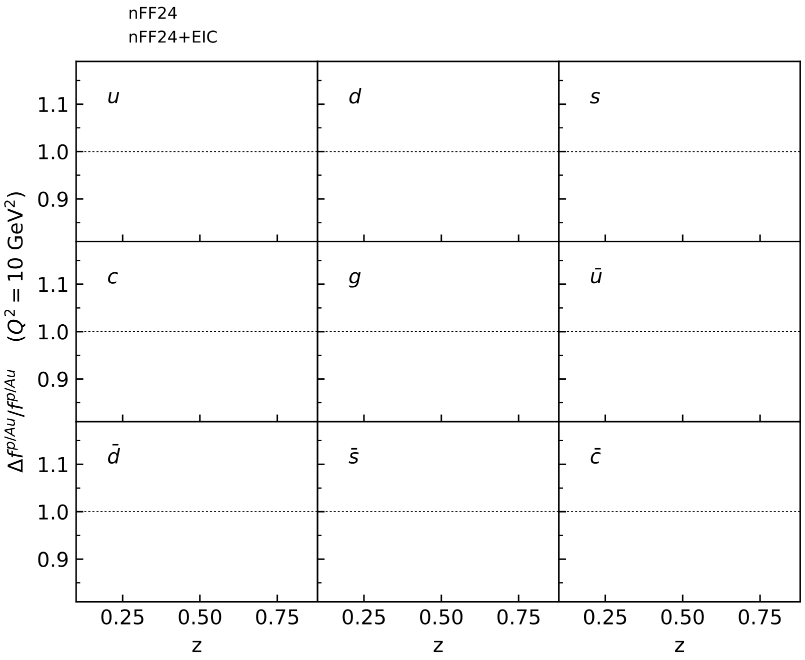


Ecce projections for Sivers asymmetries
(NIMA 1049 (2023) 168017)

(n)FFs



DSS14

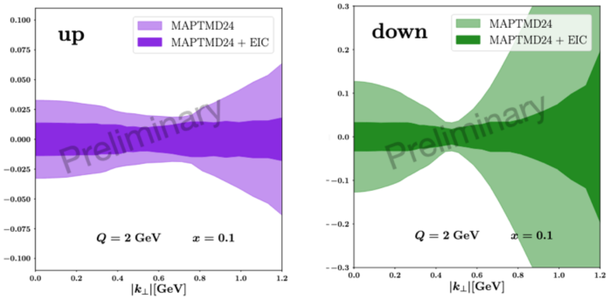


nFF24

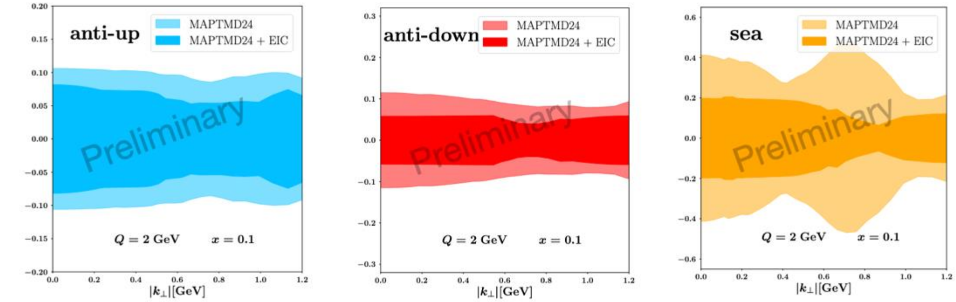
- Plots by P. Zurita with $10/\ 1\ fb^{-1}$ for p/A
- Larger impact on nFFs, remaining questions with interpolation to intermediate A
- Beyond impact on individual FF set, need to probe consistency and compatibility with other datasets

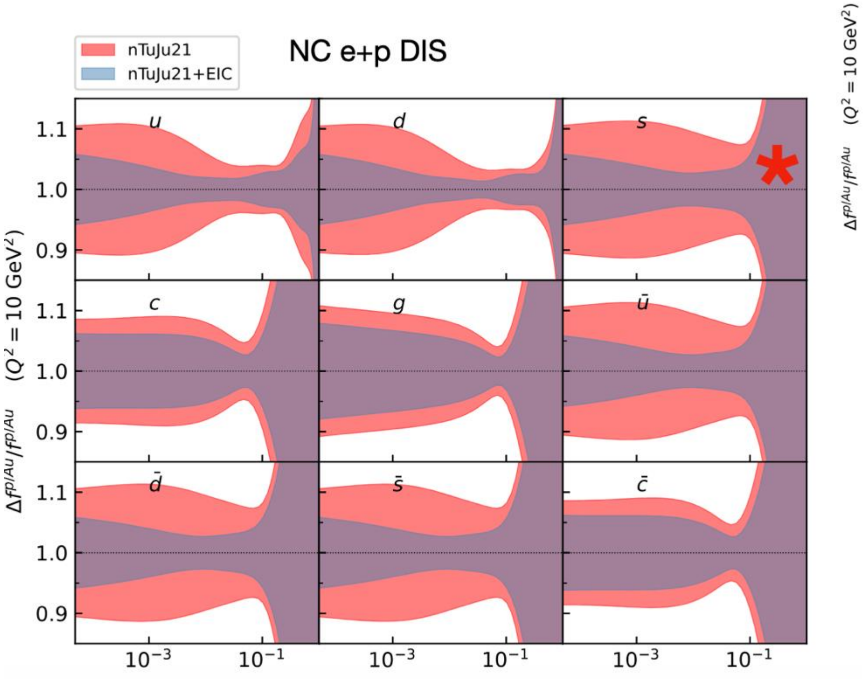
Unpolarized (n)(TMD) PDFs

$x = 0.1$



Energy	Lumi
5x41	2.85
10x100	5.13
18x275	10





- MAP analysis (Bacchetta at ePIC collaboration meeting),
- Simulated data by G. Matousek (Duke)

Impact of $1 fb^{-1}$ DIS on nTuJu21 nPDFs (P. Zurita)

SIDIS Impact?

- Significant impact, even with limited data
- Absolute cross-sections will be challenging in the beginning
- Nuclear PDFs/FF can also be measured as ratios to p/d

Summary & Conclusion

- Final states beyond single hadron fragmentation essential for precision studies in SIDIS
    - Di-hadrons
    - Λ Hyperons

- Study of Hadronization allows complementary insights into QCD
    - Fragmentation mechanisms
    - Spin-orbit correlations in fragmentation
    - (Target Fragmentation)

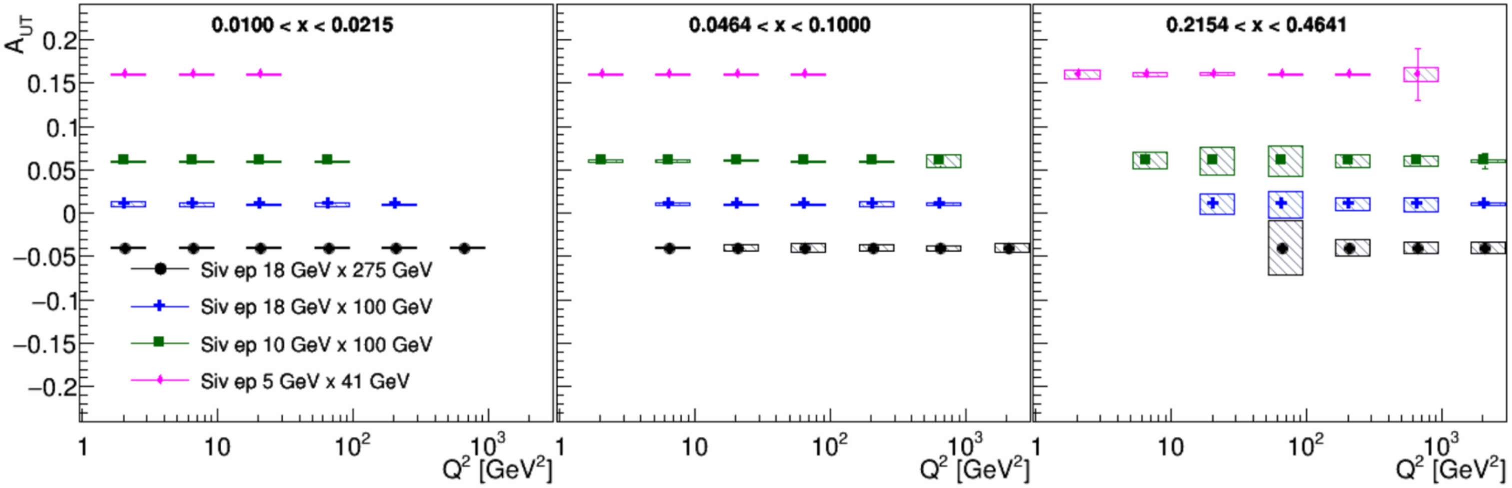
- CLAS12 is on the way to a precision program in SIDIS
    - Unpolarized, longitudinally, transversely polarized targets
    - Proton, Deuterium, He3 targets
    - Multidimensional studies

- Belle/Belle II essential for precision studies of hadronization

- EIC will open new opportunities to study SIDIS over a wide range of Q^2

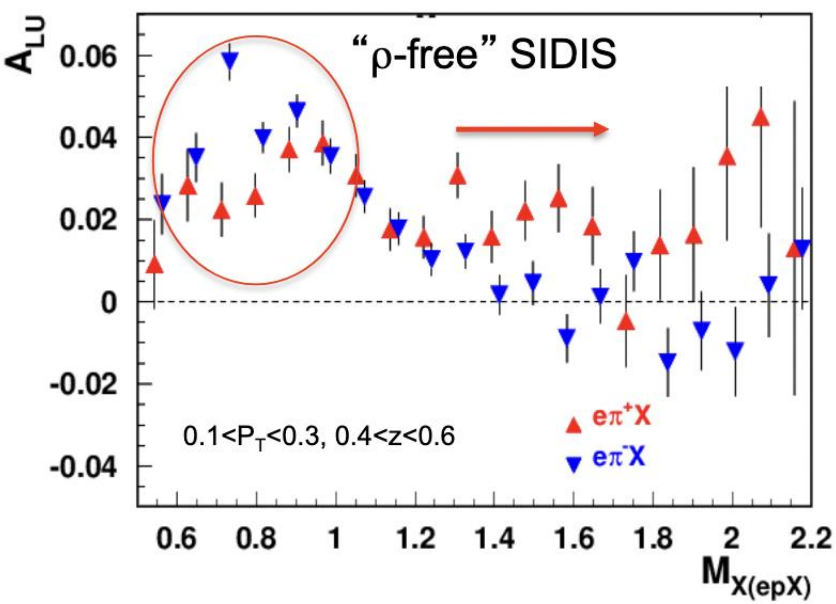
Transverse Single Spin Asymmetries at $10\ fb^{-1}$

- Previous Λ projections assume $40\ fb^{-1}$
- TSSAs: Uncertainties ≪ then expected asymmetries
- Insights into TMD framework



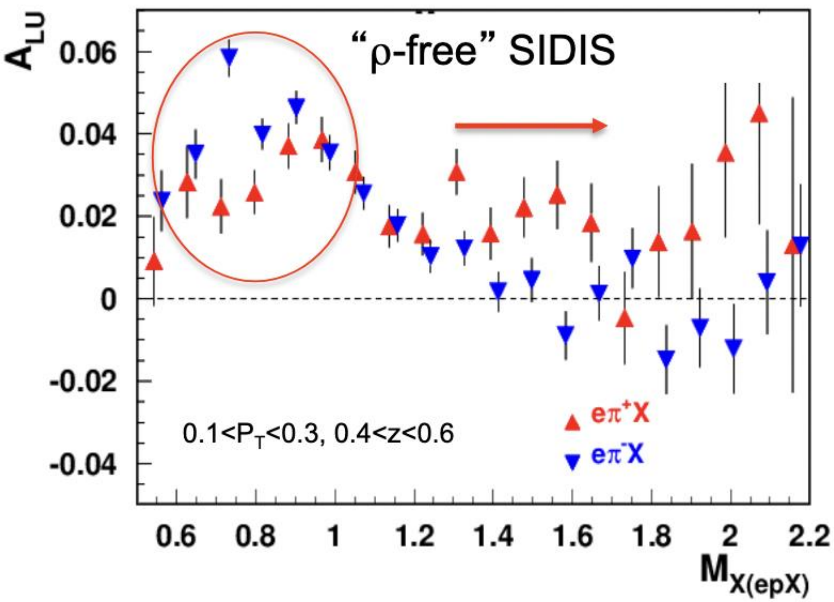
Ecce projections for Sivers asymmetries
(NIMA 1049 (2023) 168017)

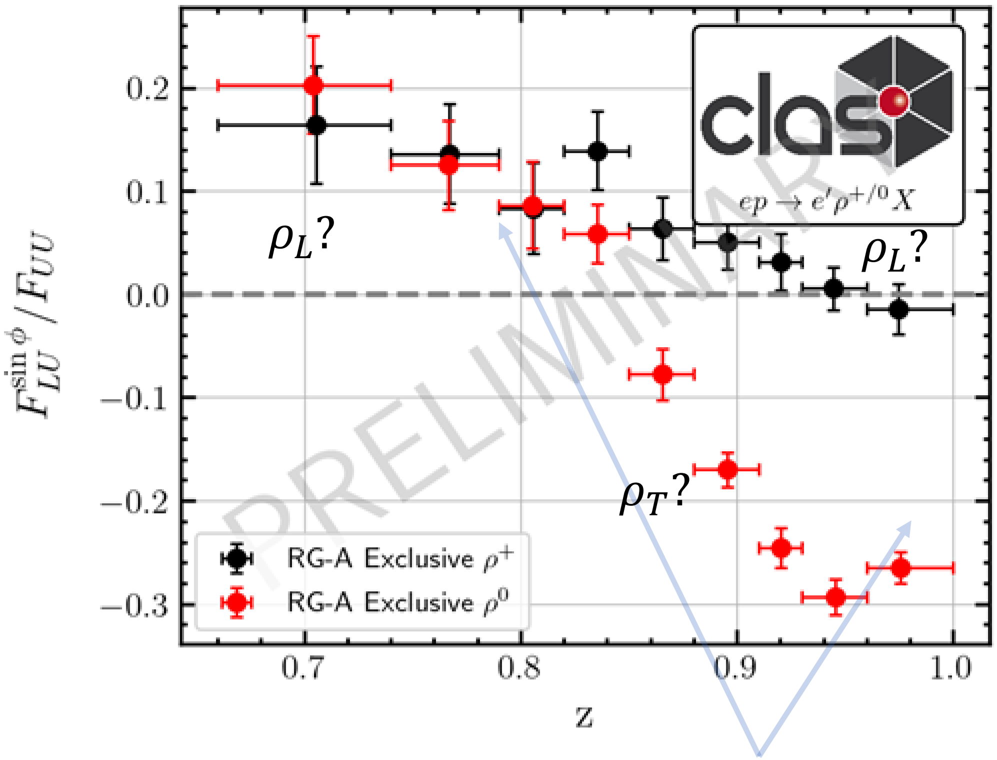
Background from Exclusive Vector Meson Production



- ρ contribution has complex kinematical dependence
- SDMEs depend on θ ➔complex interplay with acceptance
- Different for ρ^0 and ρ^+ in part due to different polarization states

Background from Exclusive Vector Meson Production

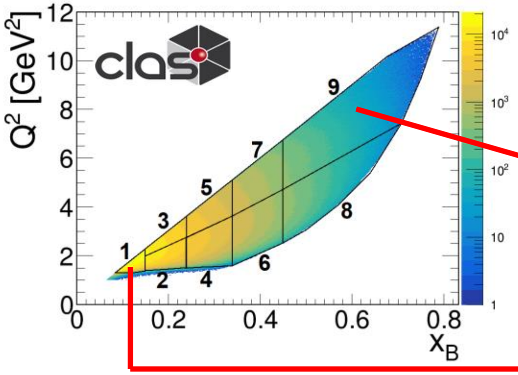


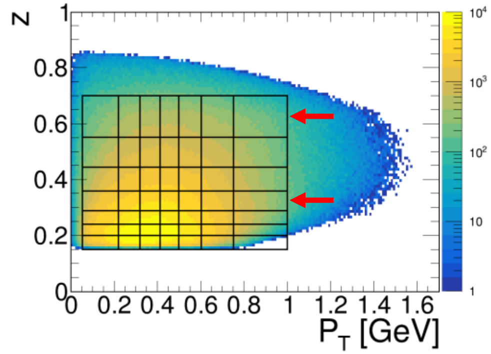


- ρ contribution depends on SDMEs
- Highly dependent on experimental acceptance

Comparison to ρ^0 indicates where the "diffractive" events are appearing. There are separate dynamical contributions with wildly different azimuthal moments that complicate the picture. Which kinematic regions are contributing to the measurements in single pion observables?

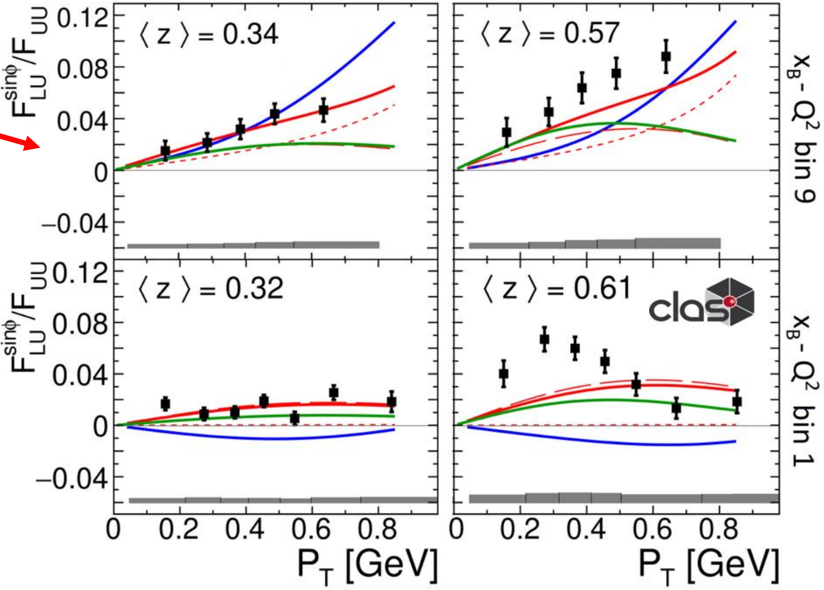
Theoretical Analysis Inconclusive: Too many unknown ingredients





in total: 344 bins x 12 bins in ϕ
~ 4130 BSA bins

S. Diehl et. al (CLAS collab.), Phys. Rev. Lett. 128, 062005 (2022)



— model 1 (Mao et al., EPJ C 73)

— model 2 (Mao et al., EPJ C 74)

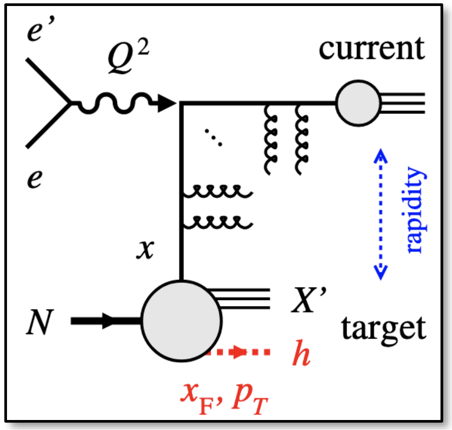
– – eH$_1^\perp$ - - - g$^\perp$D$_1$

— model 3 eH$_1^\perp$ (Schweitzer et al.)

➔ Proton = active quark plus spectator scalar and axial-vector diquarks

➔ e(x) based on the chiral quark soliton model

Fracture Functions to describe Target Region



- probability for the target (p/n) remnant to form a hadron *given* ejected quark
  No hard/soft energy scale separation

- Direct relationship to traditional **PDFs** by integrating over fractional longitudinal nucleon momentum ζ

$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z} = \sum_a e_a^2\,(1-x_B)\boxed{M_a(x_B,(1-x_B)}z)\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y}$$

$$\boxed{\sum_h \int_0^{1-x} d\zeta\,\zeta\,\hat{\boldsymbol{u}}_1(\boldsymbol{x},\boldsymbol{\zeta}) \;=(1-x)\,\boldsymbol{f}_1(\boldsymbol{x})}$$

$$\sum_h \int_0^{1-x} d\zeta\zeta M_a(x,\zeta) = (1-x) f_a(x)$$

$$\boxed{\sum_h \int_0^{1-x} d\zeta\,\zeta\,\hat{\boldsymbol{l}}_{1L}(\boldsymbol{x},\boldsymbol{\zeta}) \;=(1-x)\,\boldsymbol{g}_{1L}(\boldsymbol{x})}$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-ph] 1102.4214

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Unpolarized PDF analog

helicity PDF analog

etc. etc.

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

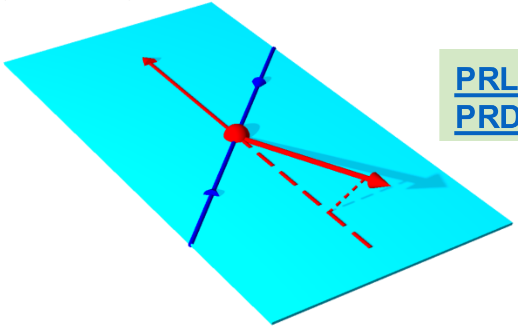
CFR ← → TFR

Precision x-sections available from Belle for π, K, p and more

Single hadron cross sections:

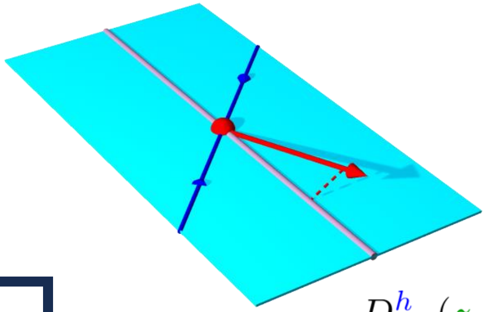
$D^h_{1,q}(z, Q^2)$

$e^+e^- \rightarrow hX$



PRL111 (2013) 062002
PRD101(2020) 092004

Transverse momentum dependent asymmetries
$e^+e^- \rightarrow (h)(h)X,$



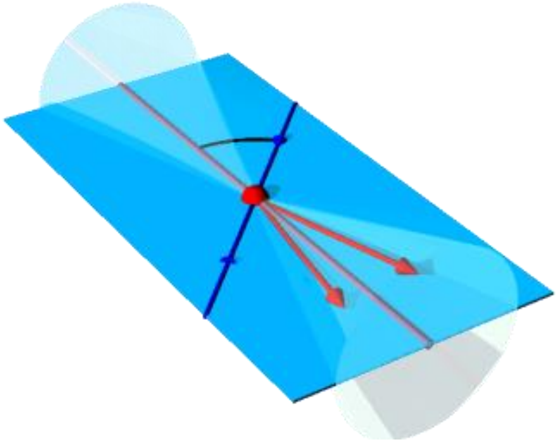
$D^h_{1,q}(z, k_T, Q^2)$

PRD 99 (2019) 112006

Polarizing Λ fragmentation
$D^{\perp}_{1T}(z, k_t, Q^2)$

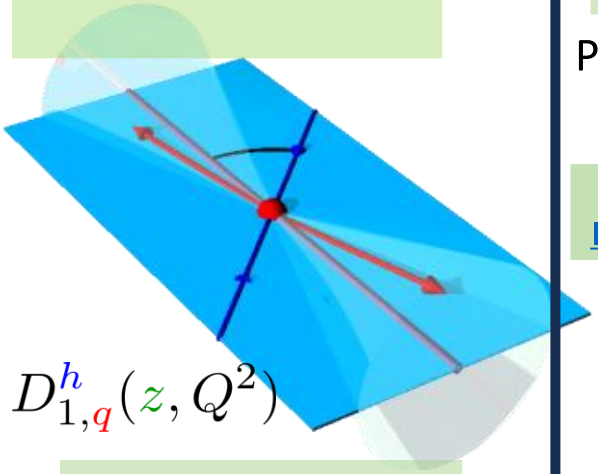
PRL 122 (2019), 042001

Unpol SIDIS, pp: $\frac{d\sigma}{dz}$
$e^+e^- \rightarrow (h)(h)X$
and scale dependence



$D^{h_1 h_2}_{1,q}(z, m, Q^2)$

PRD96 (2017) 032005

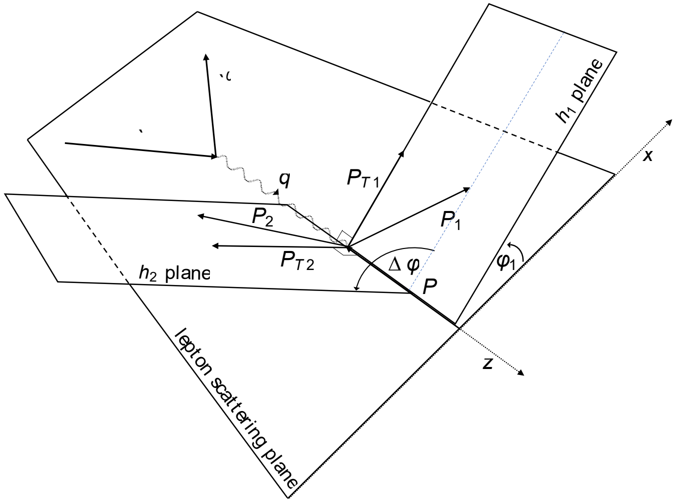


$D^h_{1,q}(z, Q^2)$

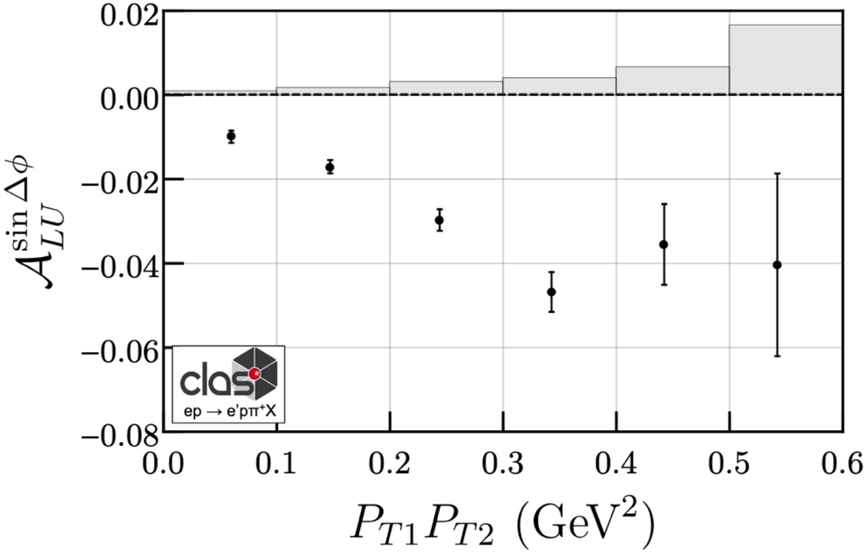
PRD92 (2015) 092007
PRD101(2020) 092004

Transverse momentum dependence underway

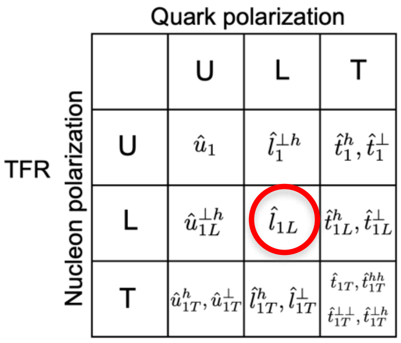
First observation of correlations between Current and target region



- “back-to-back” configuration h_1 in the CFR h_2 in TFR →convolution of a fracture function and a fragmentation function.
- Unique access to longitudinally polarized quarks in unpolarized nucleons… no corresponding PDF!
- Make use of different asymmetries in CFR and TFR to distinguish regions



$$A_{LU} = -k(\epsilon)\frac{P_{T1}P_{T2}}{m_1 m_2}\frac{\mathcal{C}\left[w_5 \hat{l}_1^{\perp h}(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}\sin(\Delta\phi)$$

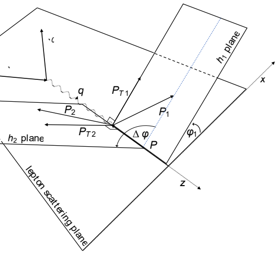


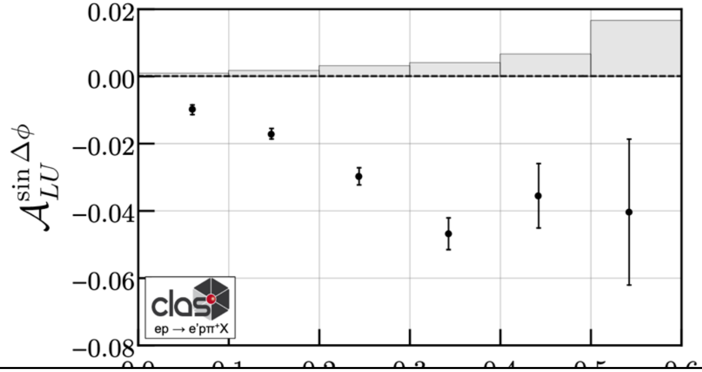
		Quark polarization		
TFR	Nucleon polarization	U	L	T
	U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
	L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
	T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

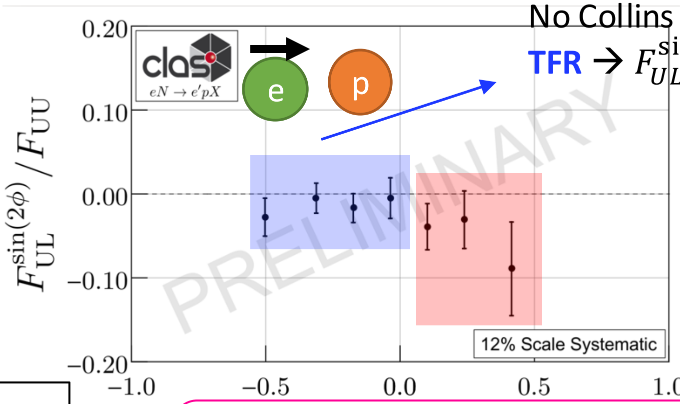
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Fracture Functions

- First observation of correlations between Current and target region
- Visible separation between TFR ($x_F < 0$) and CFR ($x_F > 0$)







No Collins mechanism in **TFR** → $F_{UL}^{\sin 2\phi} \approx 0$

$$A_{LU} = -k(\epsilon)\frac{P_{T1}P_{T2}}{m_1 m_2}\frac{\mathcal{C}\left[w_5 \hat{l}_1^{\perp h}(x,\zeta_2,P_{T2}) D_1(z_1,P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x,\zeta_2,P_{T2}) D_1(z_1,P_{T1})\right]}\sin(\Delta\phi)$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$

s are produced
with one in the CFR
R the structure
a convolution of a
nd a fragmentation

m(target)-spin

- Leading twist beam(target)-spin asymmetry.

Quark polarization

TFR Nucleon polarization	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

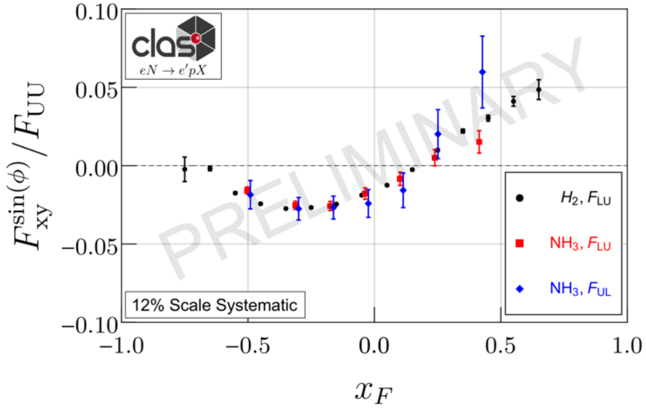
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

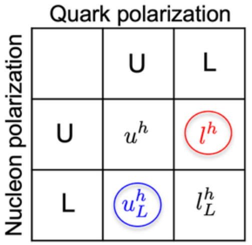
Unique access to longitudinally polarized quarks in unpolarized nucleons… no corresponding PDF! Reverse situation in target-spin asymmetry (which uniquely has no depolarization, similar to Sivers)

First measurement sensitive to TMD Fract

Preliminary Analysis: Fracture Functions

- First observation of correlations between Current and target region
- Visible separation between TFR ($x_F < 0$) and CFR ($x_F > 0$)

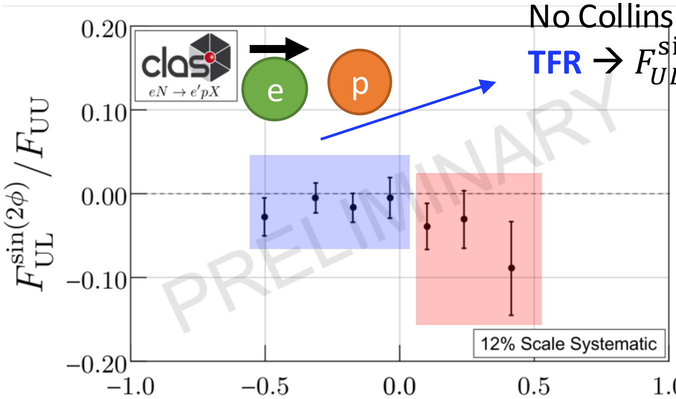



Quark polarization

Nucleon polarization

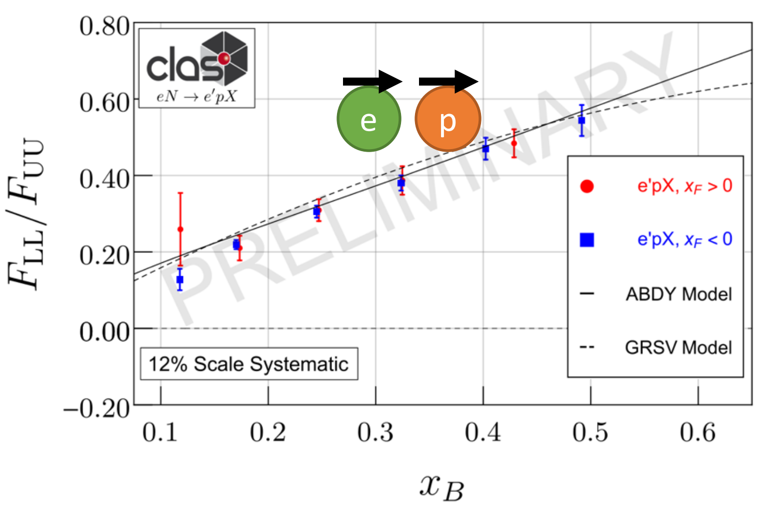
	U	L
U	u^h	l^h
L	u_L^h	l_L^h

Twist-3 Collinear terms;
Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2308.11251



No Collins mechanism in **TFR** → $F_{UL}^{\sin 2\phi} \approx 0$

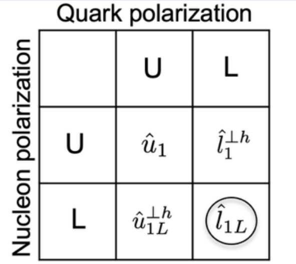
$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$



TFR Access to helicity distribution g_{1L}

$$A_{LL} = \lambda_\ell S_L \frac{\sqrt{1-\epsilon^2}F_{LL}}{F_{UU,T}}$$

Integral relation holds!


Quark polarization

Nucleon polarization

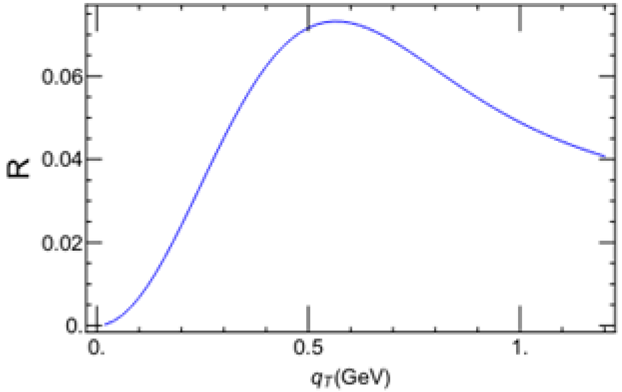
	U	L
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Azimuthal Asymmetries in back-to-back jets

- New suggestion: Measure Colllins-like back-to-back azimuthal correlations for jets

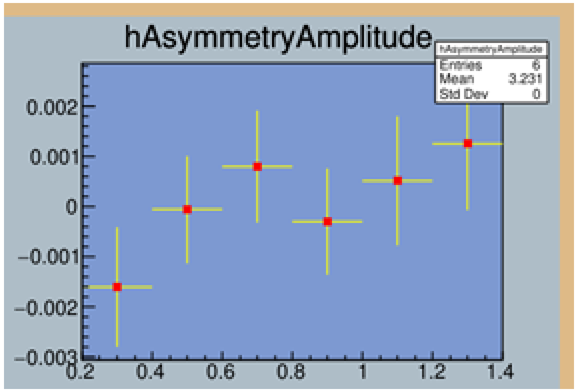
➔**Sensitive to transversity**



The time-reversal odd side of a jet

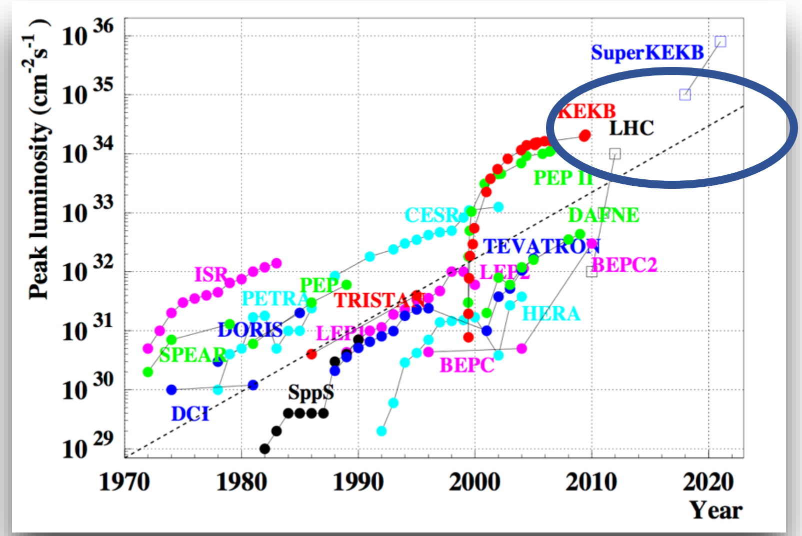
•*Fund.Res.* 3 (2023) 346-350, e-Print: 2104.03328 [hep-ph]

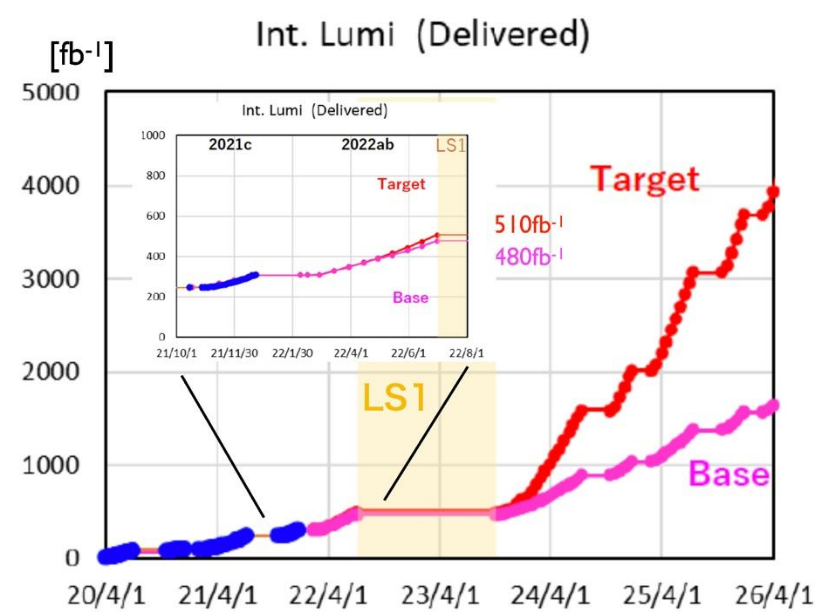
- Current Belle II projections for acceptance effects encouraging
- Charm contributions will be important
- Analysis about to enter Collaboration Review



S. Schneider

Next Generation B factory SuperKEKB





Beam currents *only* a factor of two higher than KEKB (~PEPII)

“nano-beams” are the key; vertical beam size is 50nm at the IP

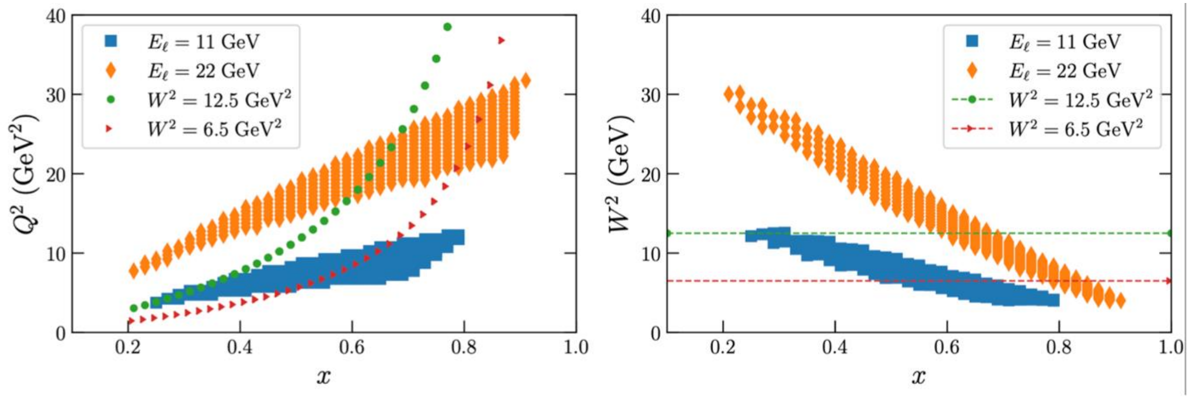
Belle II already delivered world record luminosity

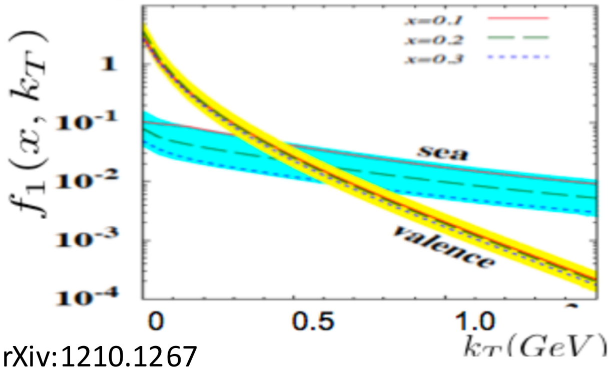
Belle II aims to have significantly higher luminosity, current record: $4.7 \times 10^{34} cm^{-2}s^{-1}$.

Physics see:
“Opportunities for precision QCD physics in hadronization at Belle II -- a snowmass whitepaper”

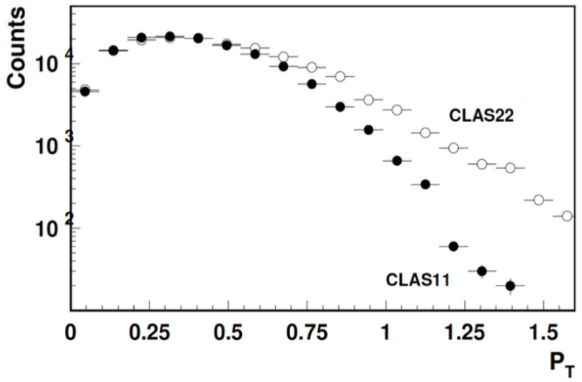
- Hadronization studies
    - FFs of complex states, Model indendent measurements

High x at Jlab 22





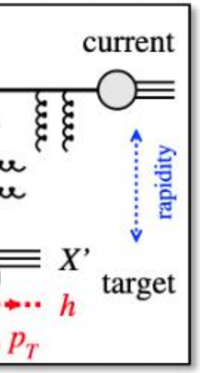
P.Schweitzer et al. arXiv:1210.1267



Strong Interaction Physics at the Luminosity Frontier
with 22 GeV Electrons at Jefferson Lab

t: 2306.09360 [nucl-ex]

- ≈doubling beam energy significantly increases phase space
- Pin down valence structure of the proton
- Integration in global analyses (e.g, strange distributions, CS Kernel)



- probability for the target (p/n) remnant to form a hadron *given* ejected quar
  No hard/soft energy scale separation
- Direct relationship to traditional **PDFs** by integrating over fractional longitudinal nucleon momentum ζ

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\,dy\,dz} = \sum_a e_a^2\,(1-x_B)\boxed{M_a(x_B,(1-x_B)z)}\,\frac{d\hat\sigma}{dy}$$

$$\Sigma_h \int_0^{1-x} d\zeta\,\zeta\,\hat{u}_1($$

$$\Sigma_h \int_0^{1-x} d\zeta\,\zeta\,\hat{l}_{1L}$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-p

$$^{1-x} d\zeta\zeta M_a(x,\zeta) = (1-x)f_a(x)$$

rization

L	T
	$h_1^\perp$
$_{1L}$	$h_{1L}^\perp$
$_{1T}$	$h_1, h_{1T}^\perp$

Unpolarized PDF analog

helicity PDF analog

etc. etc.

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
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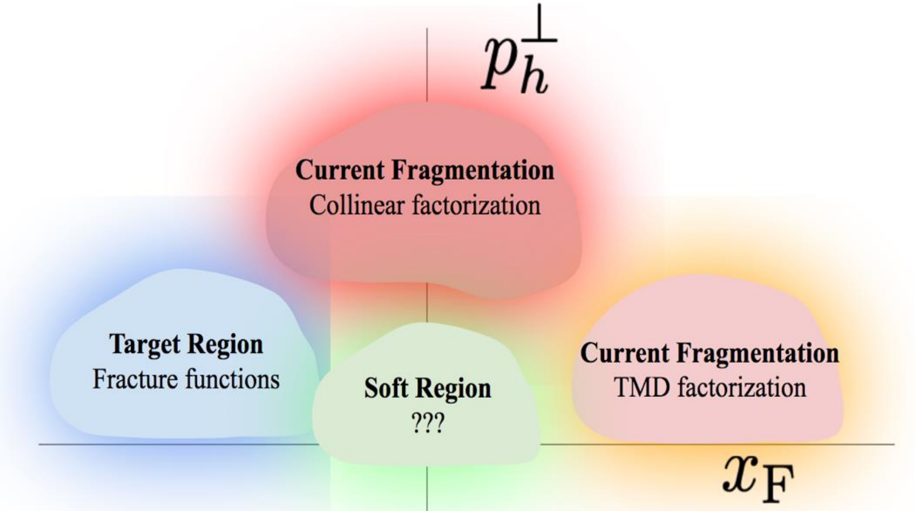
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132



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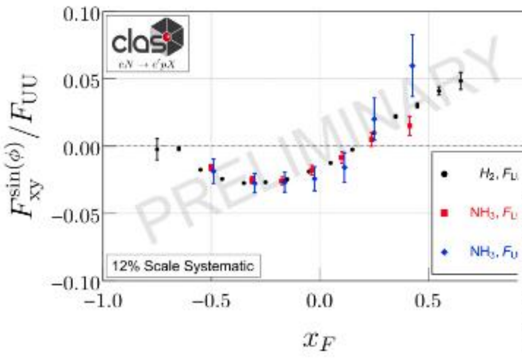
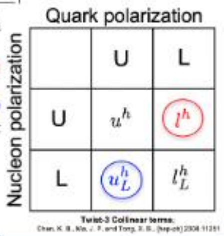
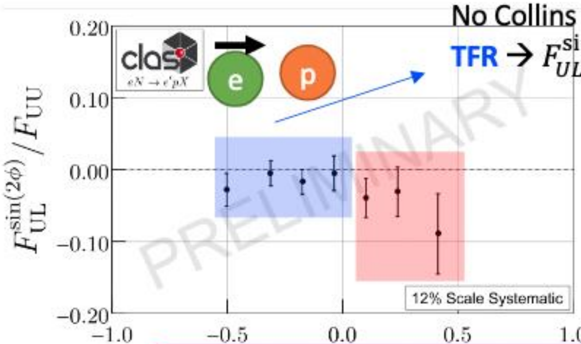




Quark polarization

Nucleon polarization	U	L
U	u^h	l^h
L	u^h_L	l^h_L

Twist-3 Collinear terms:
Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2008.11251



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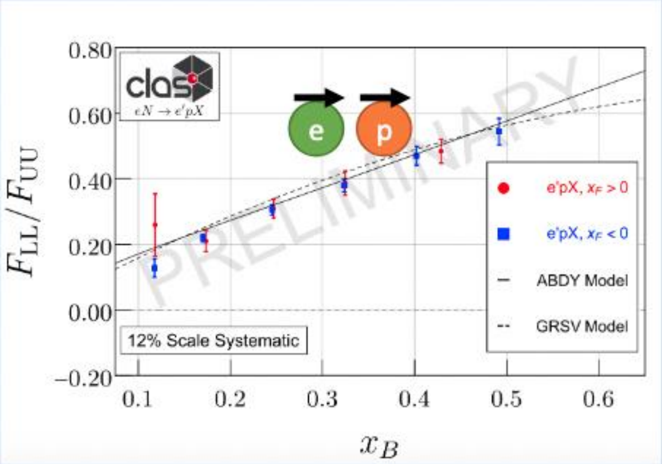

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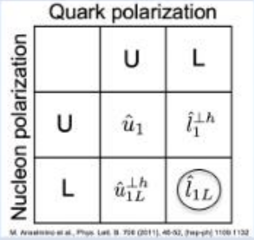

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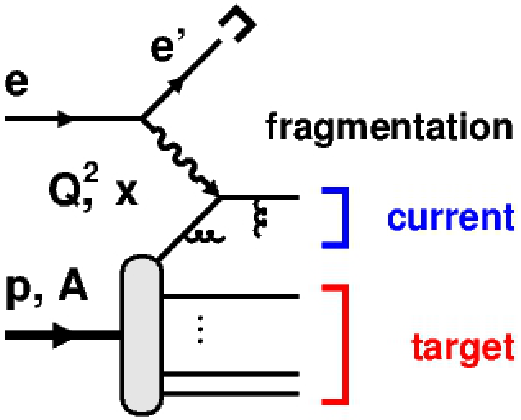
M. Anselmino et al., Phys. Lett. B 706 (2011), 46-52, [hep-ph] 1109.1132

Integral relation holds!

Target Fragmentation → Fracture Functions



- Correlations between CFR-TFR allows to extract fracture functions coupled to FFs

Quark polarization

Nucleon polarization	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
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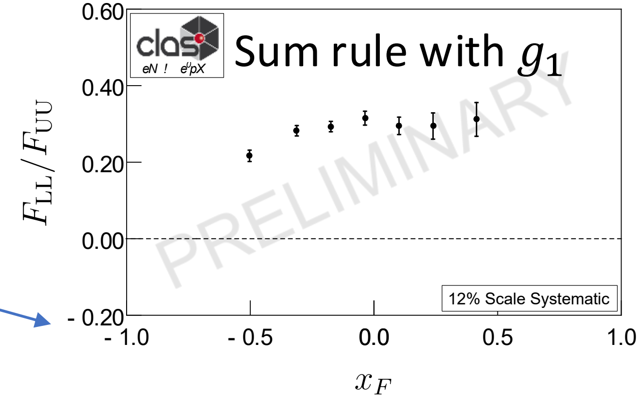
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132



Quark polarization

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Twist-3 Collinear terms;
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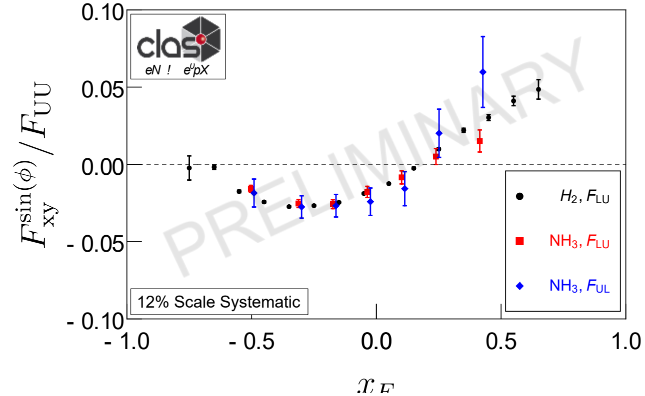
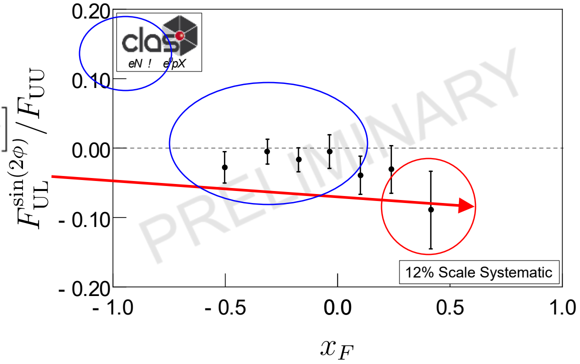


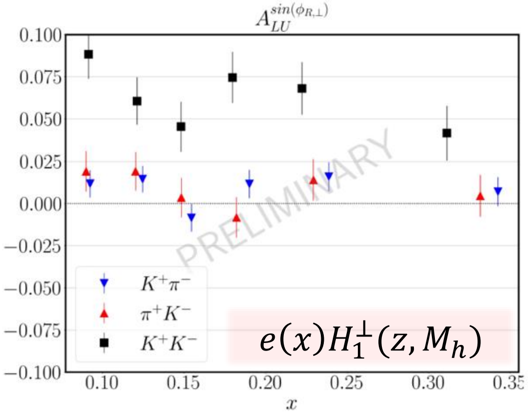
$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right]$$



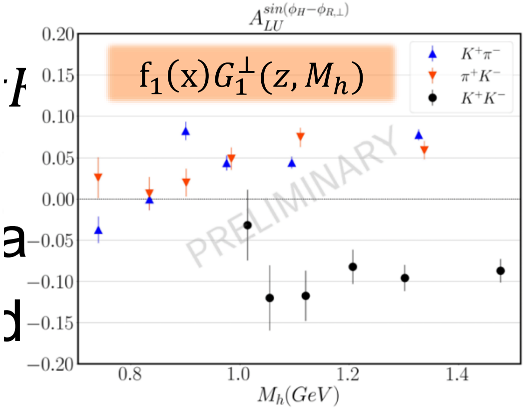
$e^- p \to e^- K\pi X$
$e^- p \to e^- K^+K^- X$

$d\sigma_{LU}$

- Large asymmetries observed in K^+K^- dihadrons → larger than



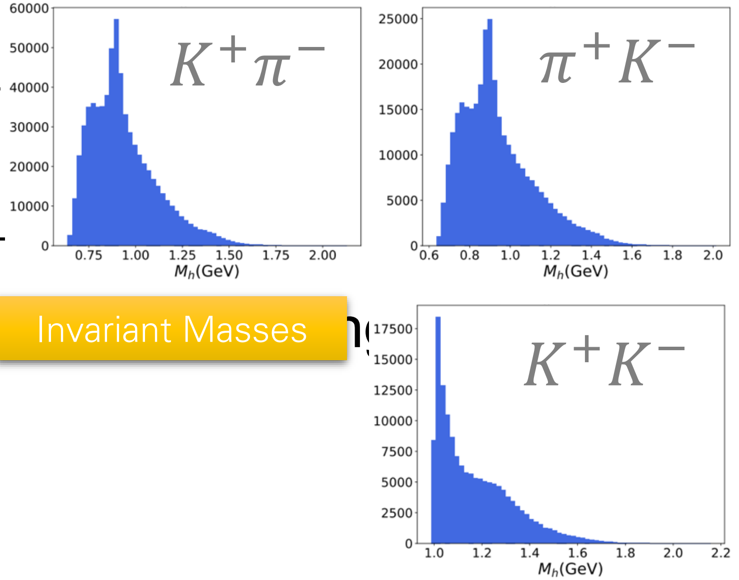
$e(x)H_1^\perp(z,M_h)$



$f_1(x)G_1^\perp(z,M_h)$

$H_1^\perp$

$G_1^\perp$

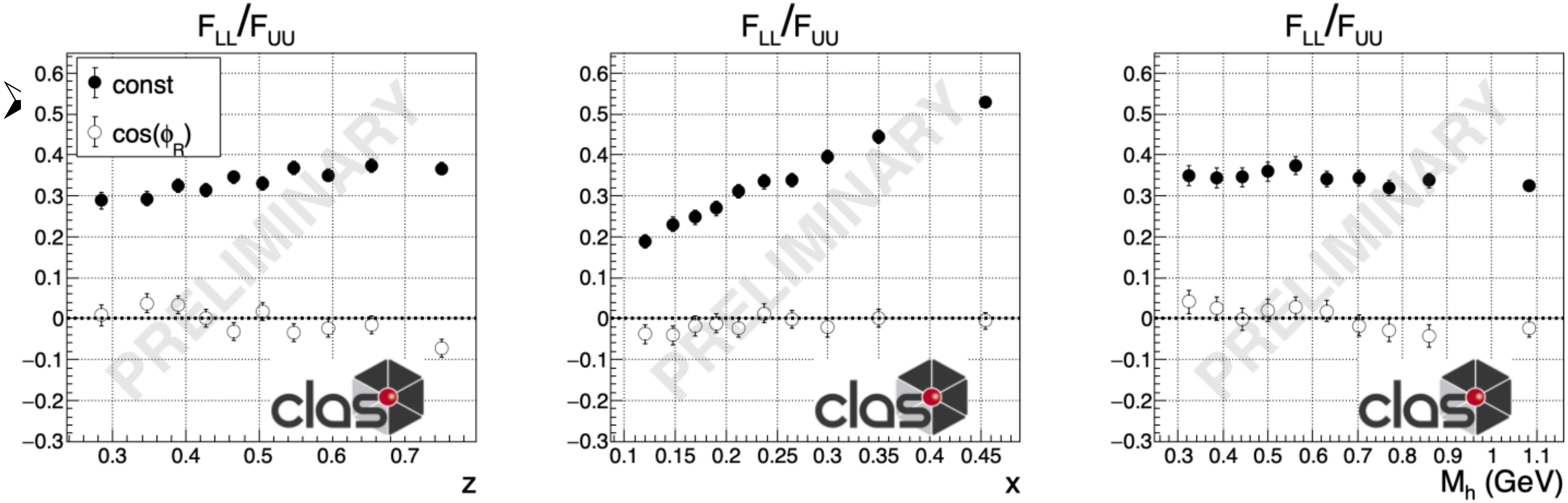


Invariant Masses

$e^- p \to e^- \pi^+ \pi^- X$

$d\sigma_{LL}$

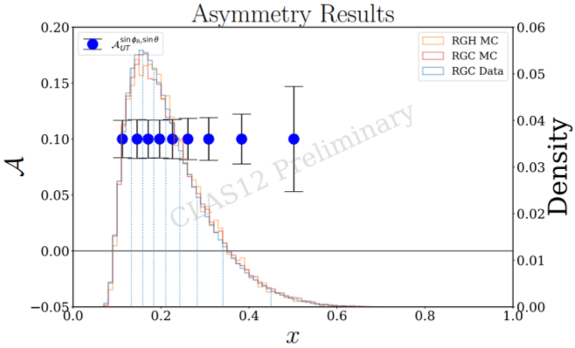
- $F_{LL}^{\text{const}} / F_{LL}^{\cos\phi_R}$ sensitive to $g_{1L} D_1 / g_{1L} \widetilde{D}$ → probe of twist-3 DiFF $\widetilde{D}$



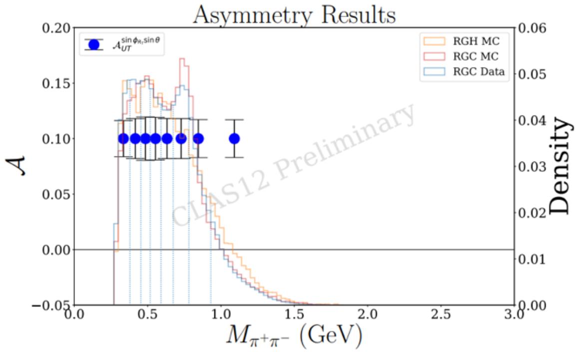
$e^- p \to e^- \pi^+ \pi^- X$

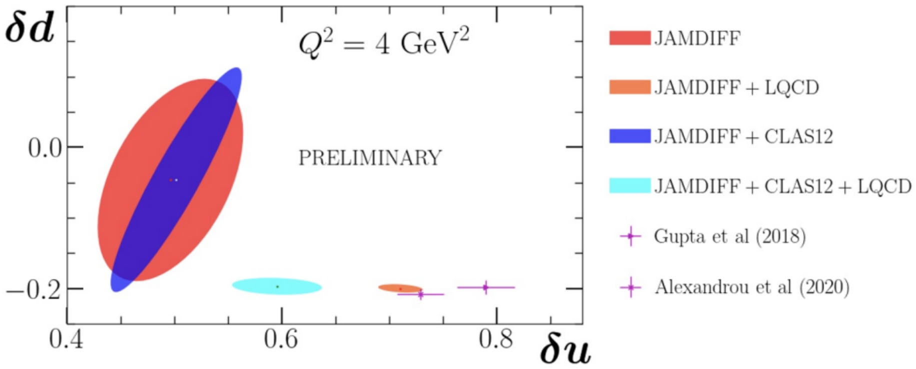
$d\sigma_{UT}$

- **Run Group H:** 1st transversely polarized target experiment at Cl

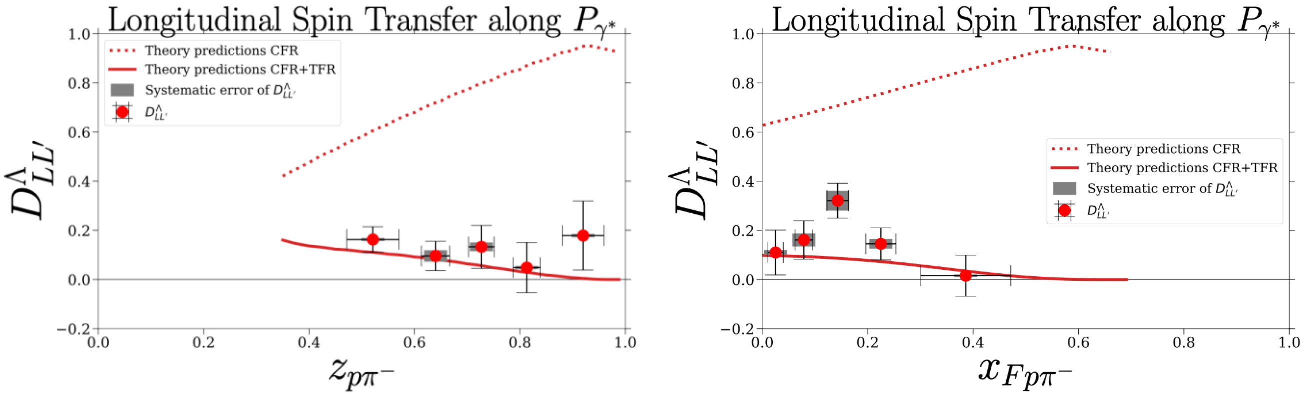


- R rating **A** by the Program Advisory Co iting beam-time
- R the JAM group show **RG-H** will be *cc* efforts to constrain the tensor charge [8]





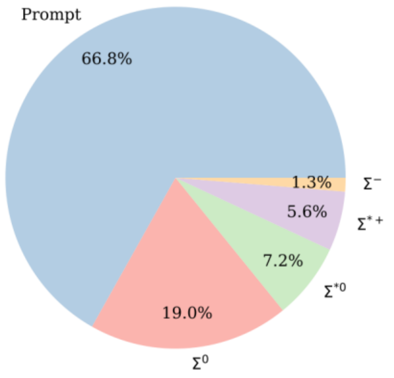
Comparison with Theory



Predictions by X. Zhao et al., cf., Zhao et al., PRL 134, 231901 (2025)

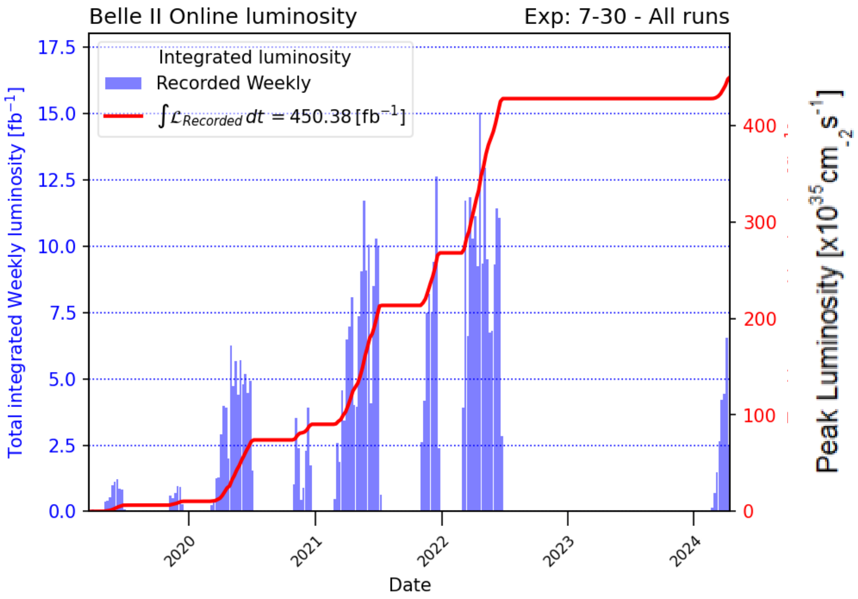
- CFR predictions come from pQCD fits to e^+e^- data
- TFR predictions come from a spectator quark-diquark model calculation

- Lambda also Spin transfer to Lambda
- Also additional vector in final state
- →azimuthal asymmetires
- Future: More lambda program at CLAS…

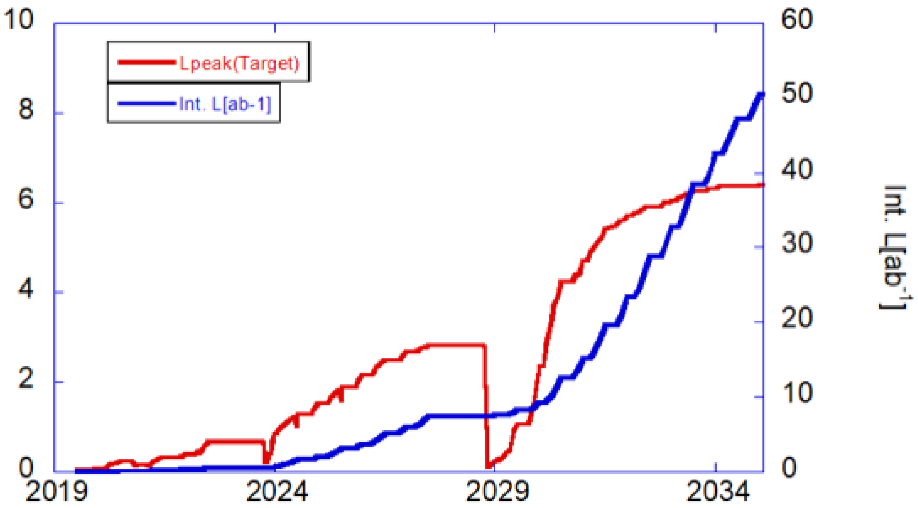


Also Stringspinner

Short and long term goals



Updated on 2024/04/11 19:55 JST

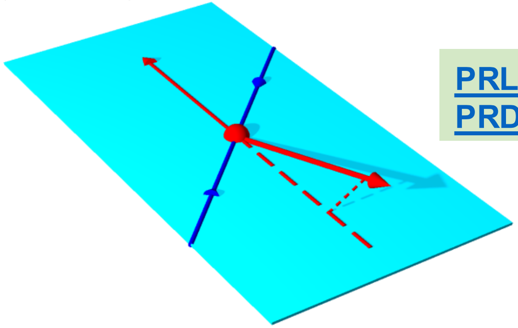


Single Hadron FFs from Belle

Single hadron cross sections:

$e^+e^- \to hX$

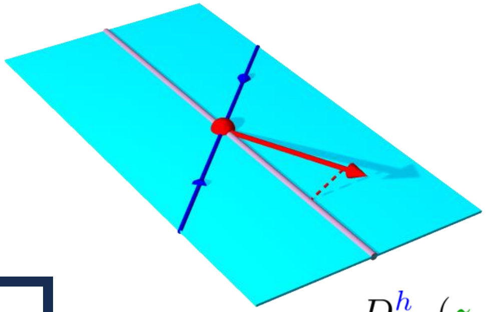
$D^h_{1,q}(z, Q^2)$



PRL111 (2013) 062002
PRD101(2020) 092004

Transverse momentum dependent asymmetries

$e^+e^- \to (h)(h)X,$



$D^h_{1,q}(z, k_T, Q^2)$

PRD 99 (2019) 112006

Polarizing Λ fragmentation

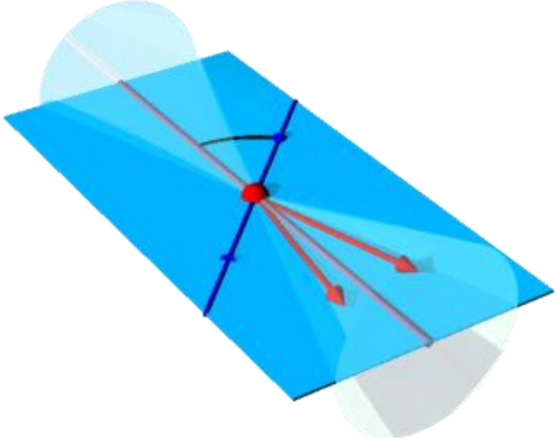
$D^{\perp}_{1T}(z, k_t, Q^2)$

PRL 122 (2019), 042001

Unpol SIDIS, pp: $\frac{d\sigma}{dz}$

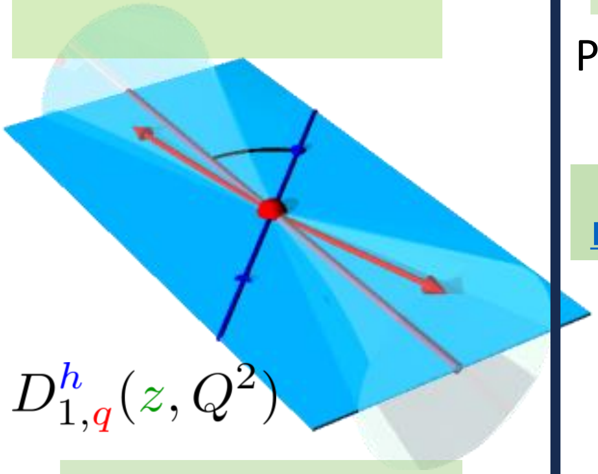
$e^+e^- \to (h)(h)X$

and scale dependence



$D^{h_1 h_2}_{1,q}(z, m, Q^2)$

PRD96 (2017) 032005

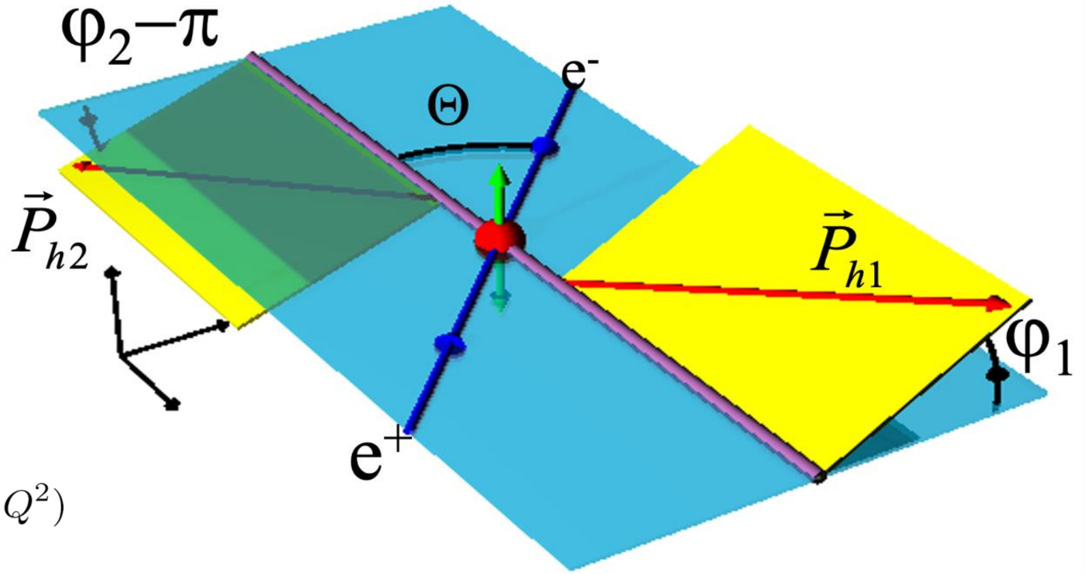


$D^h_{1,q}(z, Q^2)$

PRD92 (2015) 092007
PRD101(2020) 092004

Transverse momentum dependence underway

Polarized FFs from Belle



$H_{1,q}^{\perp(1)h}(z,Q^2)$

Azimuthal asymmetries:

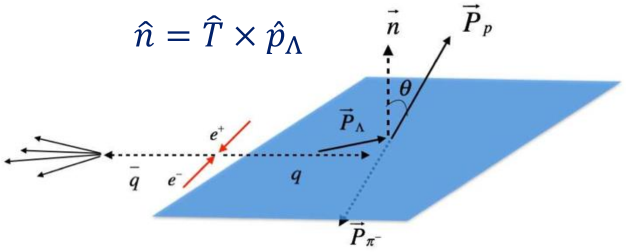
$e^+e^- \to (h)(h)X,$

$\sigma \propto \mathrm{D}_1\overline{\mathrm{D}_1} + H_1\overline{H_1}\cos(\phi_1+\phi_2)$

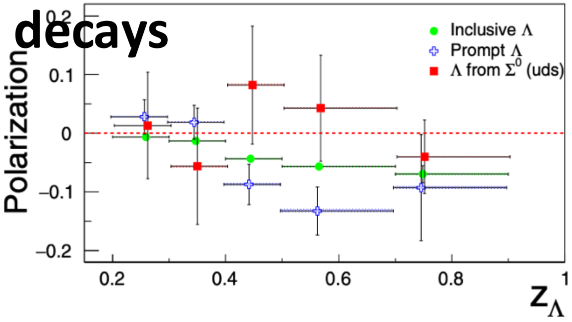
PRL 96 (2006) 232002
PRD 78 (2008) 032011
*PRD*100 (2019) 9, 092008 (p_T, π^0, η)

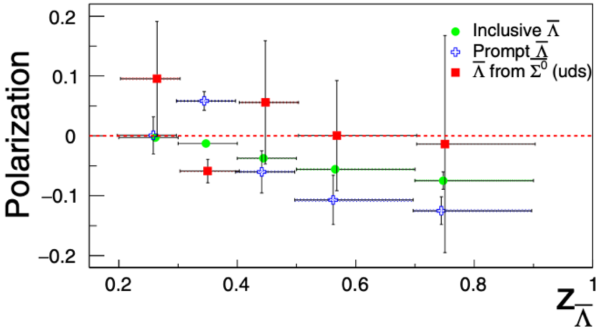
- Statistics Hungry, only possible at B-factories

Measurement of Λ polarization



- Observed non-zero polarization in
  $e^+e^- \to \Lambda X$ at Belle
  $\to$ **hadronization effect**
- **Nonzero transverse polarization observed for Λ and $\bar{\Lambda}$ as function of $z_h$and p_T**
- **Investigate feed-down contributions from Σ^0 and charm decays**

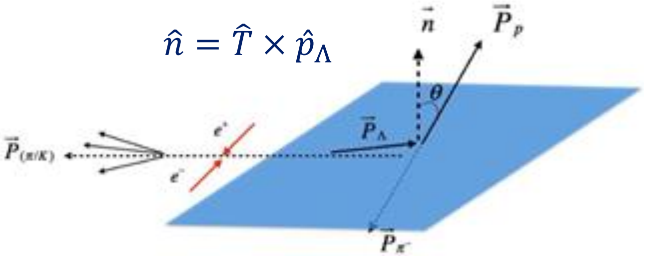


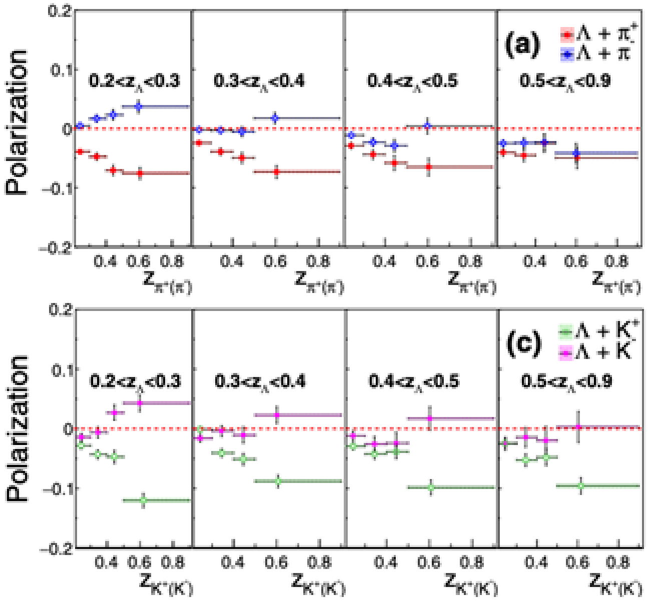


Measurement of Λ polarization

- Observed non-zero polarization in
  $e^+e^- \to \Lambda X$ at Belle
  $\to$ **hadronization effect**
- Nonzero transverse polarization observed for Λ and $\bar{\Lambda}$ as function of $z_h$and p_T
- Investigate feed-down contributions from Σ^0 and charm decays
- **Polarization measurement also with respect to hadron in opposite hemisphere**

$\hat{n} = \hat{T} \times \hat{p}_\Lambda$





Entanglement via $\Lambda\Lambda$ spin correlations

Phys. Rev. D 106, L031501 (2022)
Phys. Rev. D 109, 116003 (2024)

- **Entanglement as a probe to hadronization**
  - Experimentally track entangled $s\bar{s}$ quark into hadrons
  - Theoretical framework:
    - Quantum simulations to validate entanglement observable
    - Real time dynamics modeled via 1+1D four-flavor Schwinger model with string-breaking dynamics
  - **Experimental:**
    - Spin correlation extracted from the correlation of relative spin projections

$$N \propto 1 + \alpha^2 P_{\Lambda,\Lambda} \cos(n\theta_{ab})$$

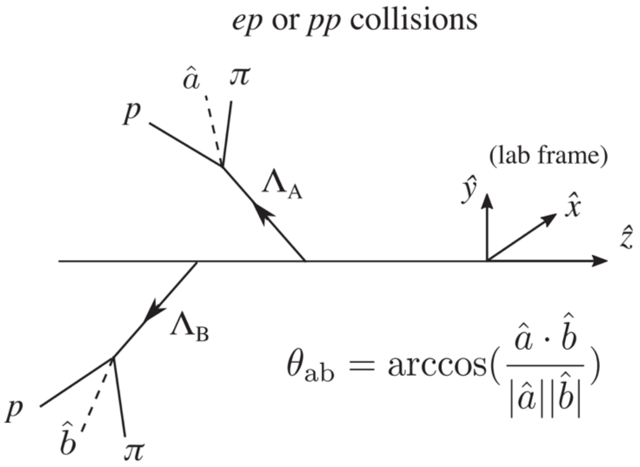


FIG. 3. Illustration of double Λ polarization; here $\hat{a}$ ($\hat{b}$) denotes the momentum direction of Λ_A (Λ_B) daughter particle in the Λ_A (Λ_B) rest frame.

Entanglement via $\Lambda\Lambda$ spin correlations

- Past particle correlation measurements have been carried out at a wide variety of collisions
- Limited by low statistics for spin analyses
- Recently, Λ hyperon pair spin-spin correlation in *pp* collisions measurement at STAR

Λ particle correlation measurement examples:

DELPHI Collaboration, Phys. Lett. B 318 249-262 (1993)

OPAL Collaboration, Phys. Lett. B 384 377-387 (1996)

ALEPH Collaboration, Phys. Lett. B. 475 395-406 (1999)

NA49 Collaboration, Nucl. Phys. A 715 55-64 (2002)

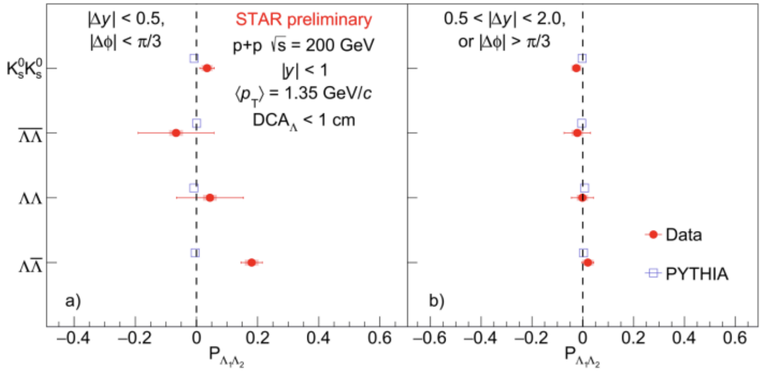
SELEX Experiment, J. Phys.: Conf. Ser. 295 012089 (2011)

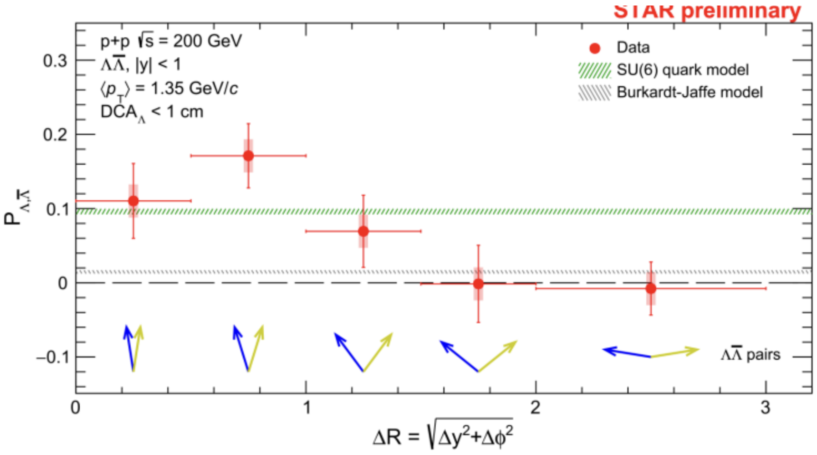
STAR Collaboration, Phys. Rev. Lett. 114 022301 (2015)

BESIII Collaboration. Nat Commun 16, 4948 (2025)

. . .

Preliminary results from Quark Matter 2025, Jan Vanek





Longitudinal spin transfer via dihadron polarization

- Helicity correlation of two produced partons
- Alternative approach to traditional methods using polarized beams and targets

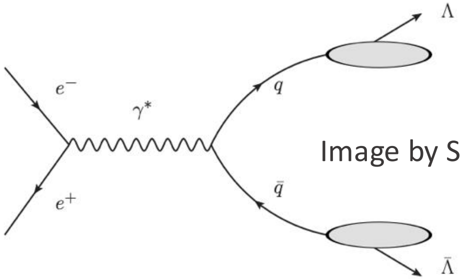
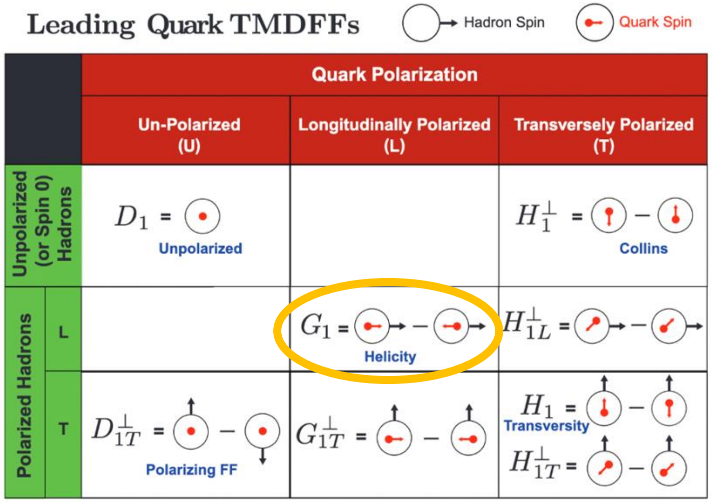


Image by S.Y. Wei, DIS24

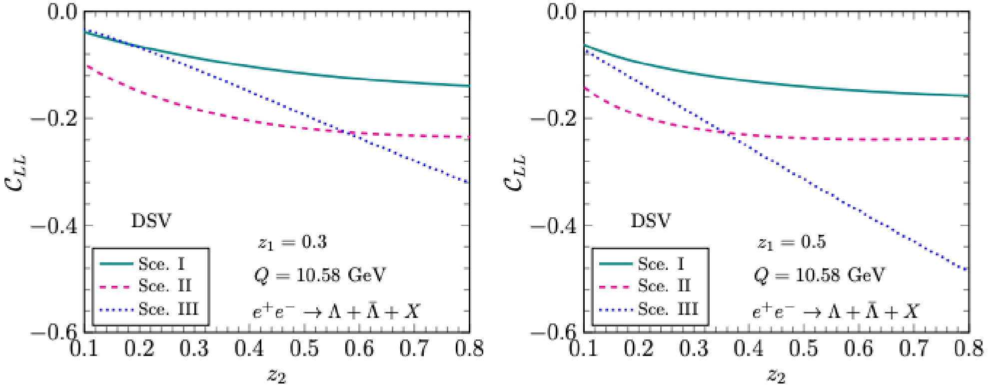


Leading Quark TMDFFs Hadron Spin Quark Spin

		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Unpolarized (or Spin 0) Hadrons		D_1 = **Unpolarized**		$H_1^{\perp}$ = − **Collins**
Polarized Hadrons	**L**		G_1 = − **Helicity**	$H_{1L}^{\perp}$ = −
	T	$D_{1T}^{\perp}$ = − **Polarizing FF**	$G_{1T}^{\perp}$ = −	H_1 = − **Transversity** $H_{1T}^{\perp}$ = −

Image from arXiv:2304.03302v1

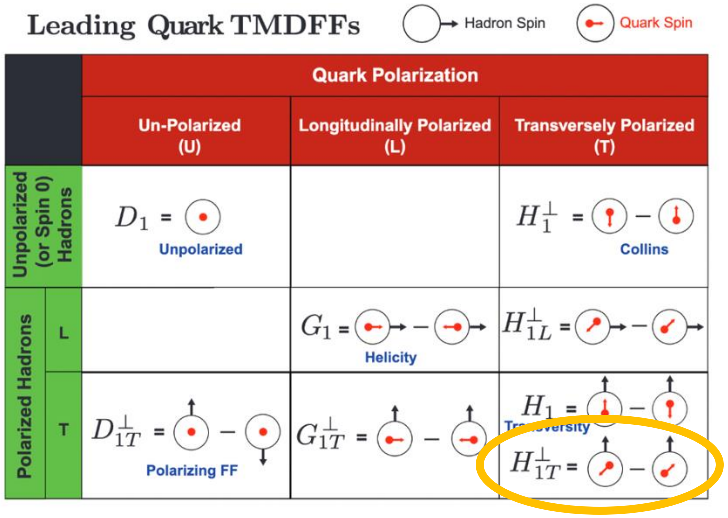
Phys.Lett.B 839, 137821 (2023)



$$\frac{1}{N}\frac{dN}{d\cos\theta_1^*\, d\cos\theta_2^*} = \frac{1}{4} + P_L^{\Lambda}\frac{1}{4}\alpha\cos\theta_1^* + P_L^{\bar{\Lambda}}\frac{1}{4}\alpha\cos\theta_2^* + \mathcal{C}_{LL}\frac{1}{4}\alpha^2\cos\theta_1^*\cos\theta_2^*,$$

Longitudinal spin transfer via dihadron polarization

- **Experimental considerations:**
  - Contributions from longitudinal polarization not exactly zero (but expected to be small)
- **Other possible future measurement** 2410.20917
  - Transverse spin correlation of two Λ hyperons sensitive to H_{1T}
  - Measurement of transverse spin correlation of two Λ hyperons normal to the hadron production plane (defined by thrust axis)



Leading Quark TMDFFs

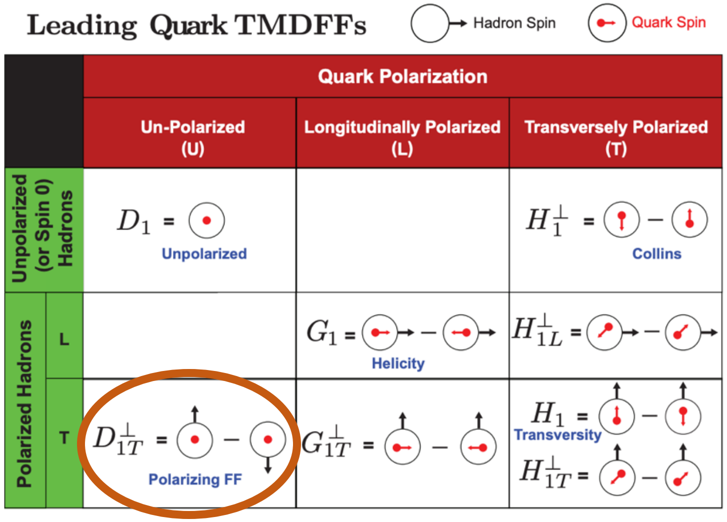
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = − Collins
Polarized Hadrons	L		G_1 = − Helicity	$H_{1L}^\perp$ = −
	T	$D_{1T}^\perp$ = − Polarizing FF	$G_{1T}^\perp$ = −	H_1 = − Transversity $H_{1T}^\perp$ = −

Image from arXiv:2304.03302v1

Transverse Λ polarization

- Measurement sensitive to **polarizing transverse-momentum dependent (TMD) fragmentation functions (FF) $D_{1T}^{\perp\Lambda/q}\left(z,k_{\perp}^{2}\right)$**
- Belle measurement data accurate enough for **phenomenological studies to extract FF**

  Phys. Rev. D 102, 054001 (2020)
  Phys. Rev. D 102, 096007 (2020)
  Phys. Lett. B 809, 135756 (2020) + . . .

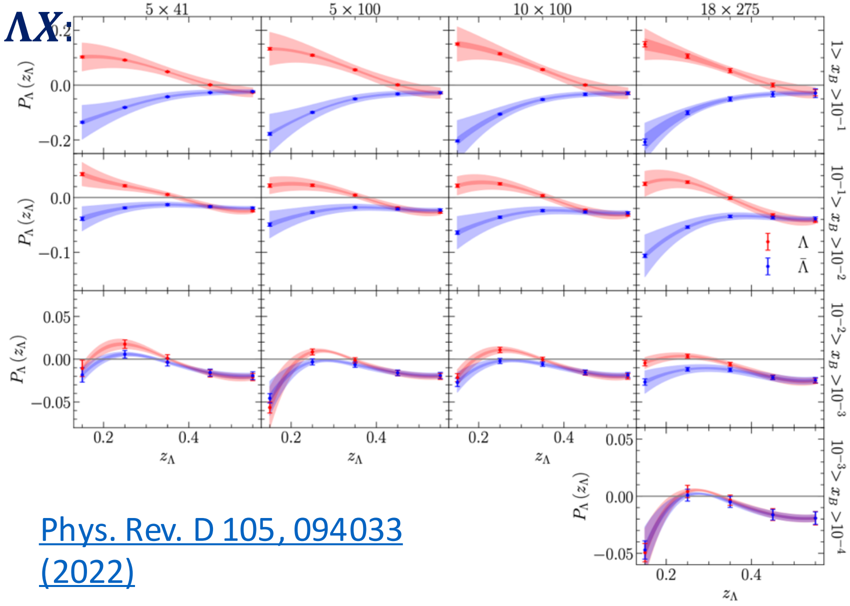


Leading Quark TMDFFs ○→ Hadron Spin ⊙ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = ⊙ Unpolarized		$H_1^{\perp}$ = ⊙ − ⊙ Collins
Polarized Hadrons	L		G_1 = ⊙→ − ⊙→ Helicity	$H_{1L}^{\perp}$ = ⊙→ − ⊙→
	T	$D_{1T}^{\perp}$ = ⊙ − ⊙ Polarizing FF	$G_{1T}^{\perp}$ = ⊙ − ⊙	H_1 = ⊙ − ⊙ Transversity $H_{1T}^{\perp}$ = ⊙ − ⊙

Image from arXiv:2304.03302v1

- Used for ***Λ* polarization predictions in *ep* → *ΛX*;**

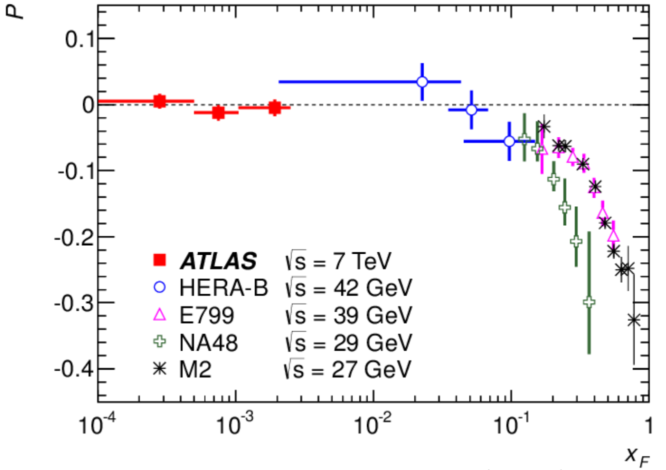


Phys. Rev. D 105, 094033 (2022)

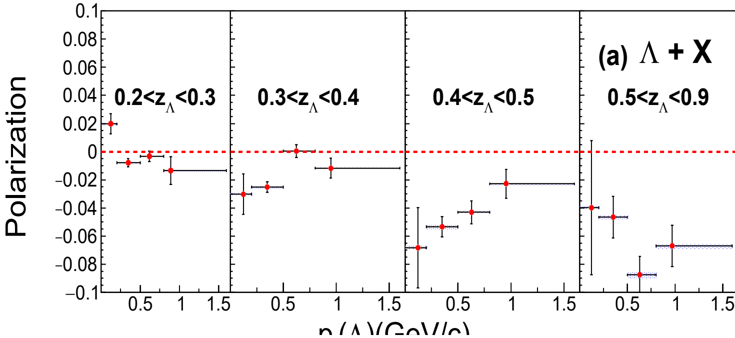
Polarized Hyperon Production

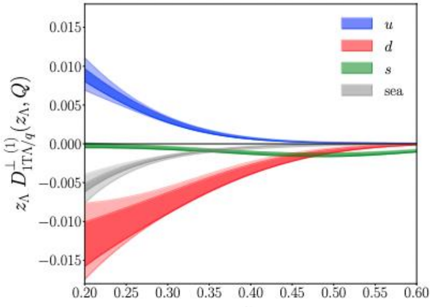
- Large Λ transverse polarization in unpolarized pp collision PRL36, 1113 (1976); PRL41, 607 (1978)
- Caused by polarizing FF $D_{1T}^{\perp}(z, p_{\perp}^{2})$?
- Polarizing FF is chiral-even, has been proposed PRL105,202001 (2010)
  as a test of universality.
- FF counterpart of the Sivers function.
- OPAL experiment at LEP has studied transverse Λ
  polarization, no significant signal was observed. Eur. Phys. J. C2, 49 (1998)
- First Observation at Belle !

➔Extraction of PFF (Cagliari, UCLA)



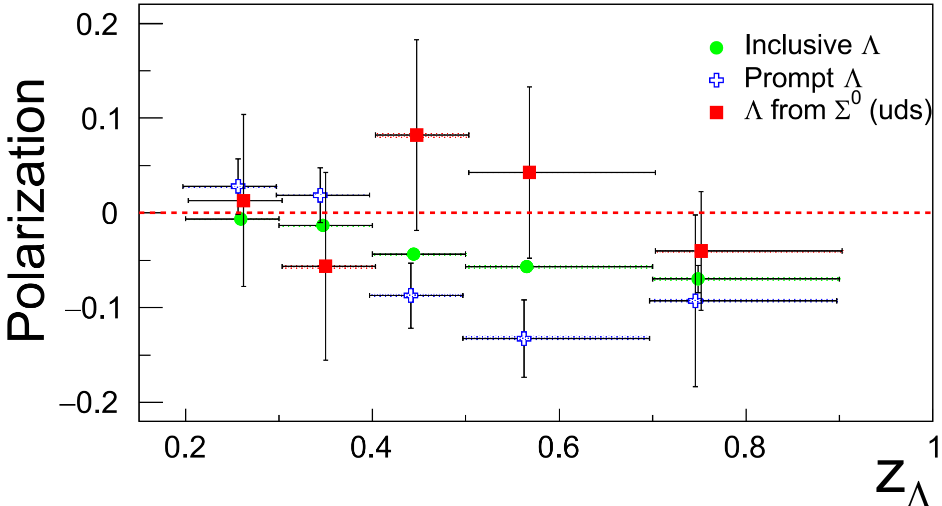
PRD 91, 032004 (2015)



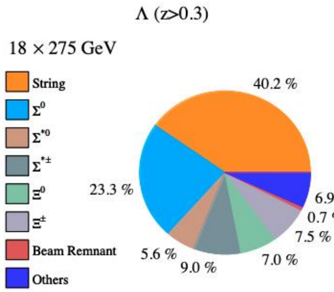


PRD 105 (2022) 9, 094033

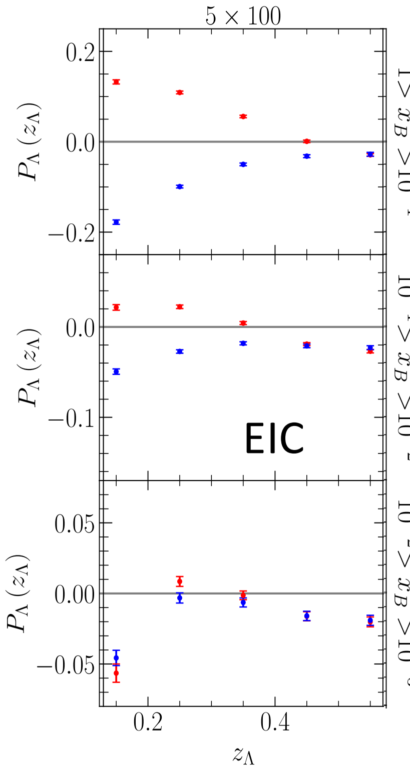
Belle II Makes Precision Λ program possible!



PRL122 (2019) 4, 042001



Example,
Λ feeddown at the EIC

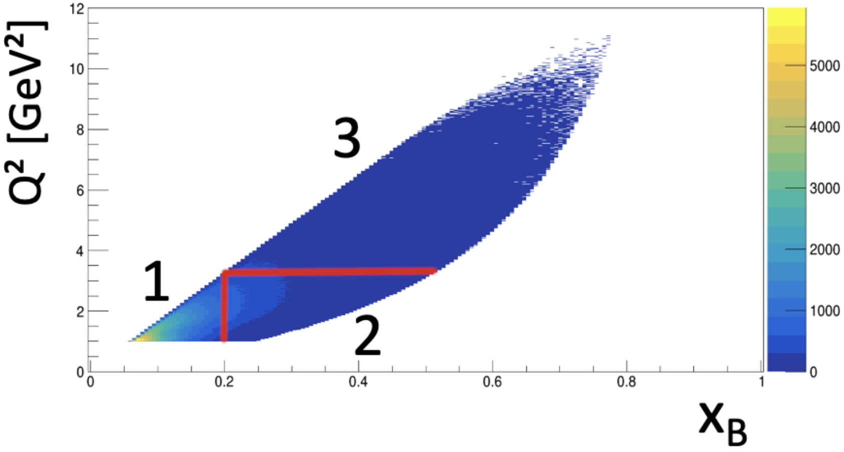


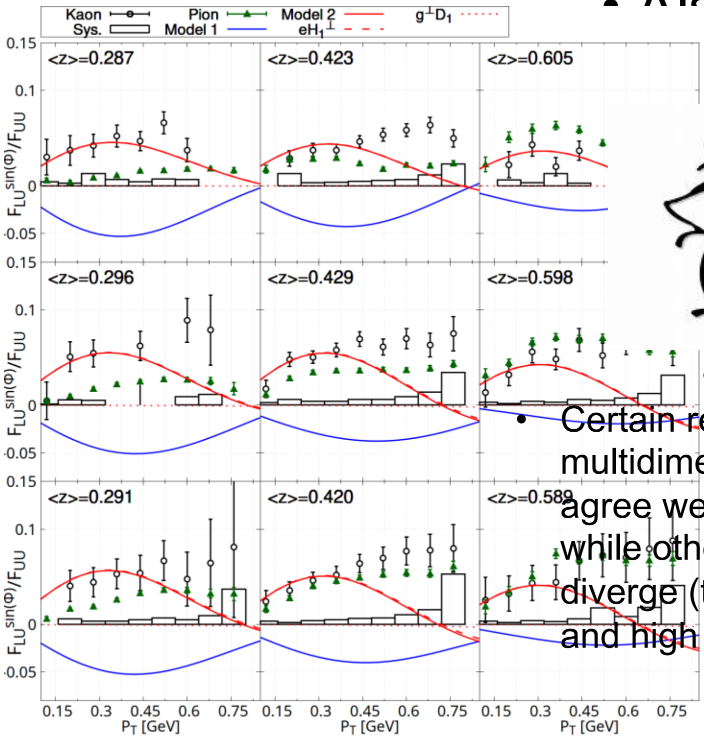
- Opportunities at Belle II:
  - Feed down correction for p_T dependence and associated production
    - (currently only for z dependence, introduces large uncertainties)
    - $\Lambda^\uparrow - \Lambda^\uparrow$ correlations →Entanglement studies
    - Extension to tensor polarized FFs: e-Print: 2206.11742 [hep-ph]
    - ….
  - Explore low p_T region (not shown here) with higher statistics and better tracking resolution

- Complementary Statistics to Λ program at the EIC
  - Universality test
  - Λ FFs to extract polarized PDFs
  - Flavor separation
  - …

CLAS12 pion BSAs

Phys.Rev.Lett. 128 (2022) 6, 062005





☐ K^+ 🟩 π^+

model 1 (Mao et al., EPJ C 73)

model 2 (Mao et al., EPJ C 74)

- Also

tiD

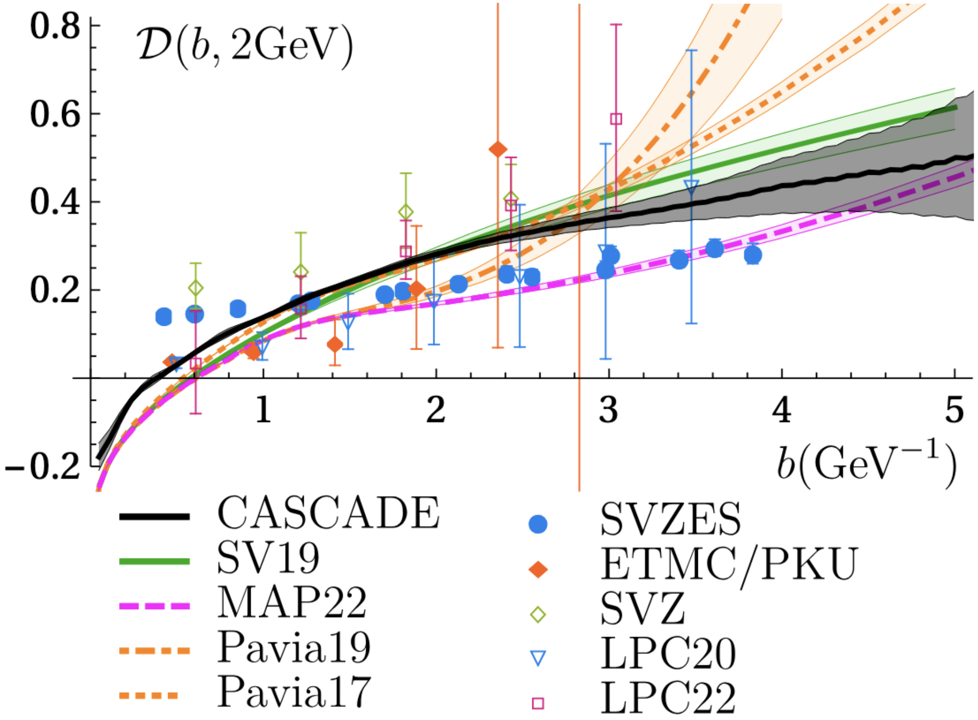
–

- Certain regions of multidimensional analysis agree well with predictions while others clearly diverge (typically low P_T and high z).

- Issues in the interpretation
  - Four different terms contributing (all poorly known)
  - Background processes from exclusive VMs and target FF
- Use final state with more degrees of freedom → more targeted

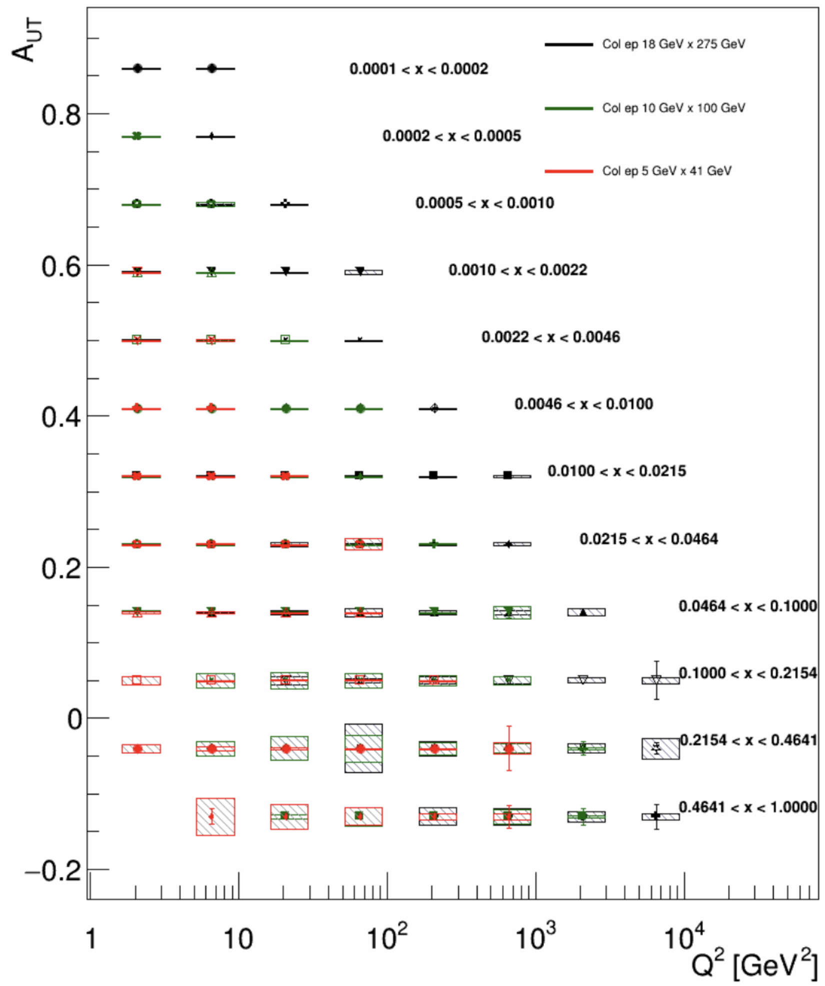
EIC kinematic leverarm provides Insight into Evolution

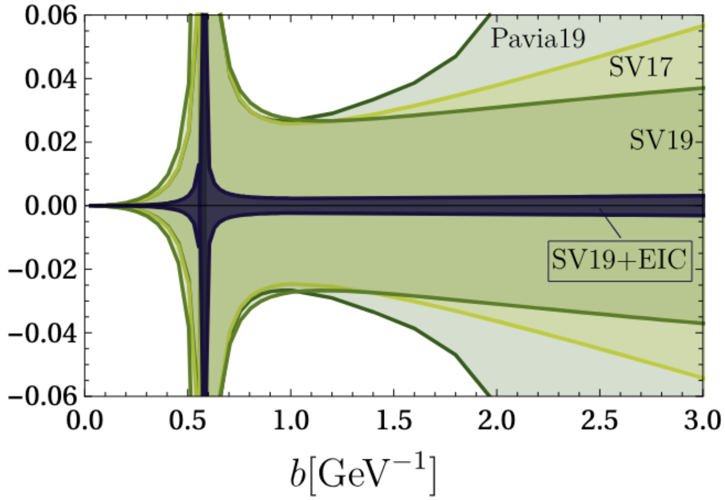
- CS kernel sensitive to vacuum structure [Vladimirov 2020]
- Significant uncertainties on extractions
- Disagreement in different extractions and with lattice[4]



From Bermudez-Martinez, Vladimirov , arxiv:2206.01105

EIC kinematic leverarm provides Insight into Evolution

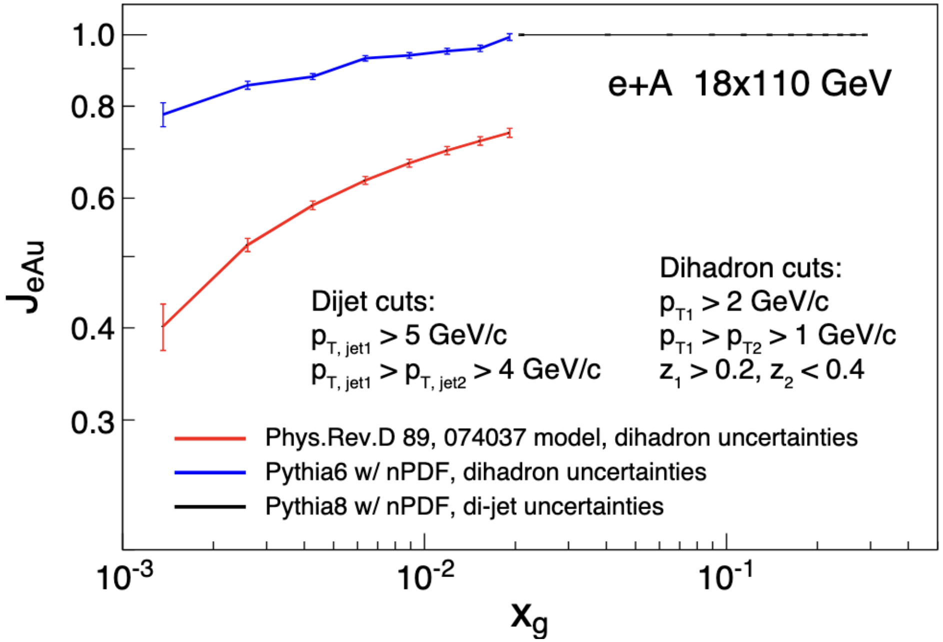


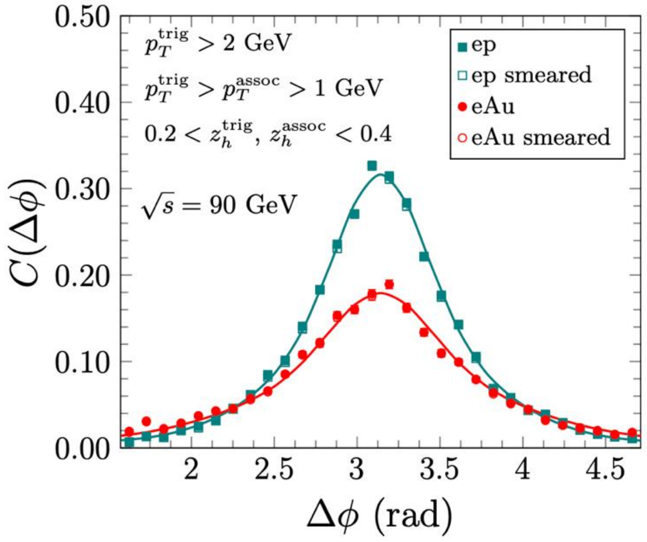


EIC Yellow Report

Ecce projections for Collins asymmetries for 10 fb^{-1} at each energy
(NIMA 1049 (2023) 168017)

Di-hadrons to access saturation

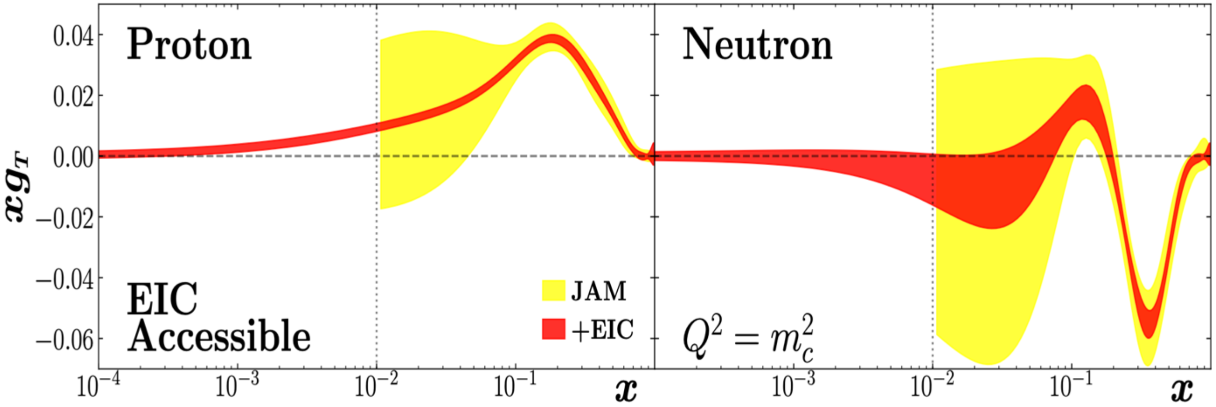




- Signals in di-hadron correlation and broadening large

→First hint with limited datasets? (projections here for $10\ fb^{-1}$)

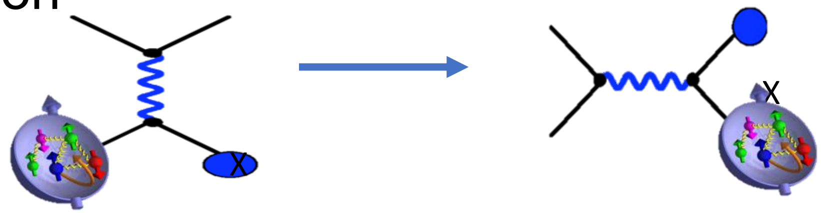
Impact Twist-3 PDF g_T



- A_{LT} (here projection with $100\ fb^{-1}$)

PDF in SIDIS ⇔ FF in e^+e^-

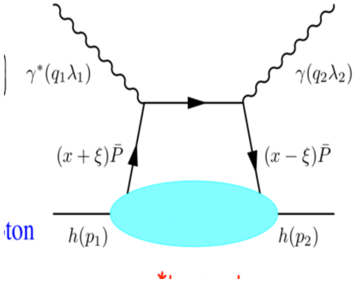
- E.g. Sivers ⇔ $\Lambda^\uparrow$ production
-

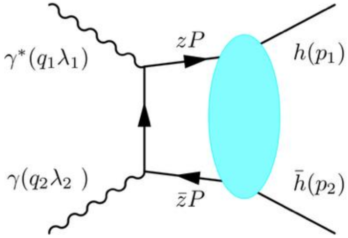


Spacelike SIDIS

Timelike SIA

  - GPDs ⇔ GDAs





The process γ*γ →h bar{h}

72

Tentative Timelines relevant for TMD program shown here

CLAS12 in Hall B

- 2018-2020: unpolarized proton/deuterium target – long. polarized beam
- 2022/23 longitudinally polarized proton/deuterium long target → ≈ 5 % produced
- Future Polarized He3 (long/trans)

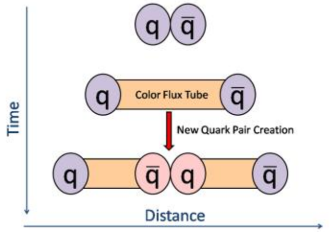
Hall A

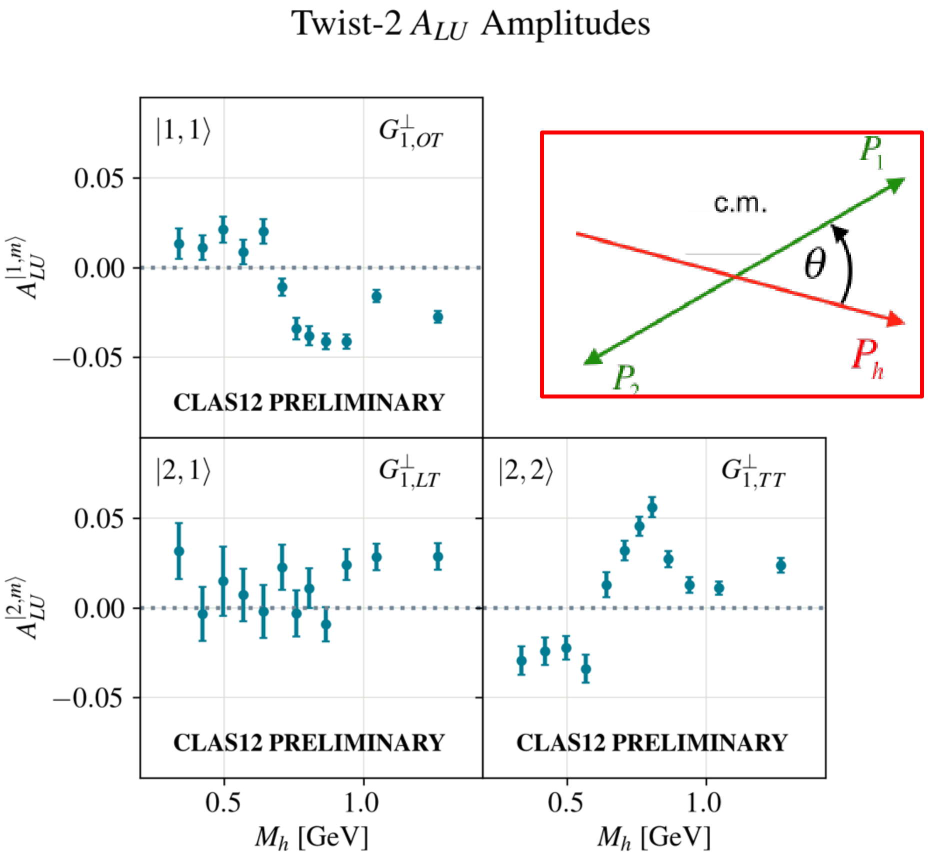
- 2028 SoLID with He3/proton target (long/transverse)

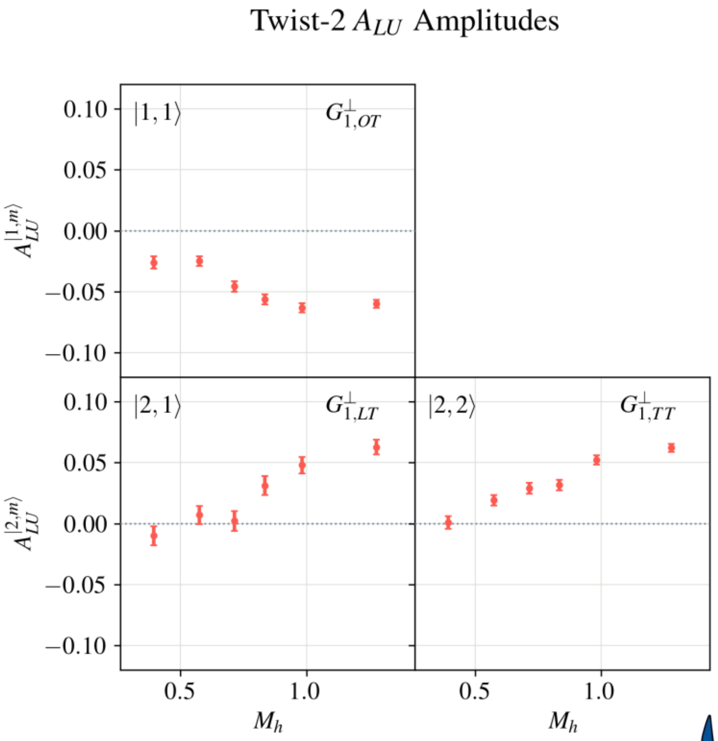
See talk by Chao Peng

Compare Partial Wave Decomposition in MC and Data

- Comparing to Polarized Lund model here (StringSpinner, A. al, *Comput.Phys.Commun.* 272 (2022))



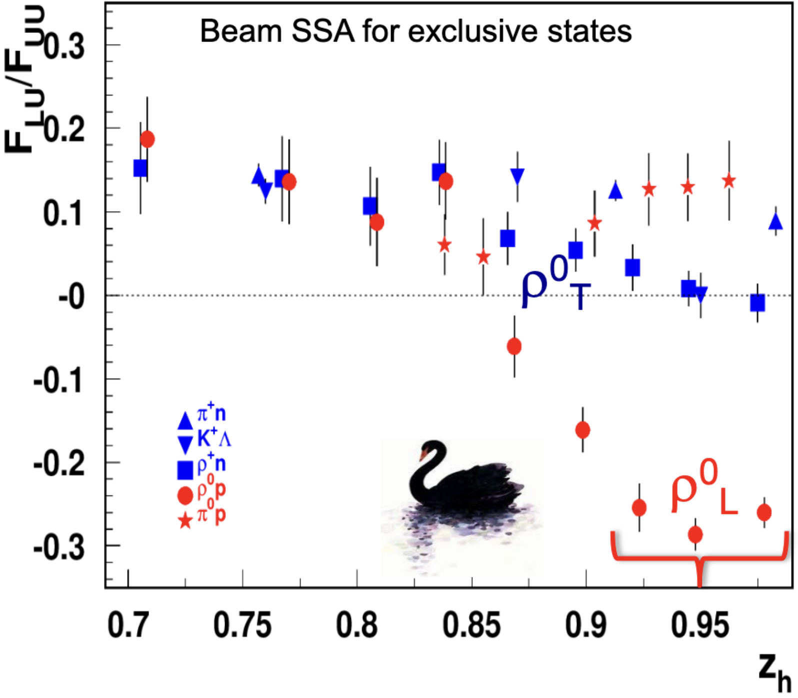


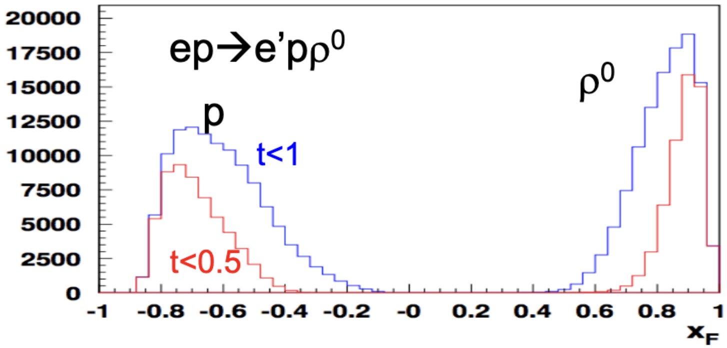


- New version of string+$3P_0$ model shown at SPIN (Kerbizi) with improved VM treatment

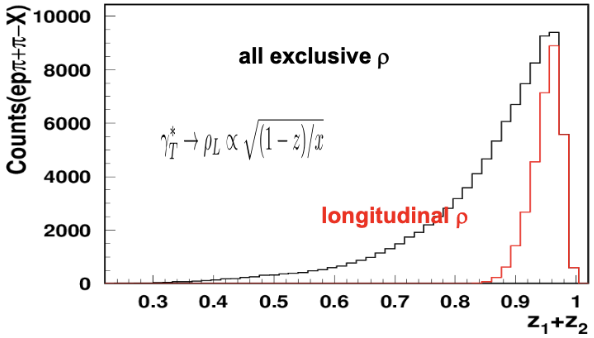
“diffractive” VMs: rapidity gap

G.Matousek (Duke) & N.Trotta (UCONN)





Significant rapidity gap between protons (backward) and rho (forward)

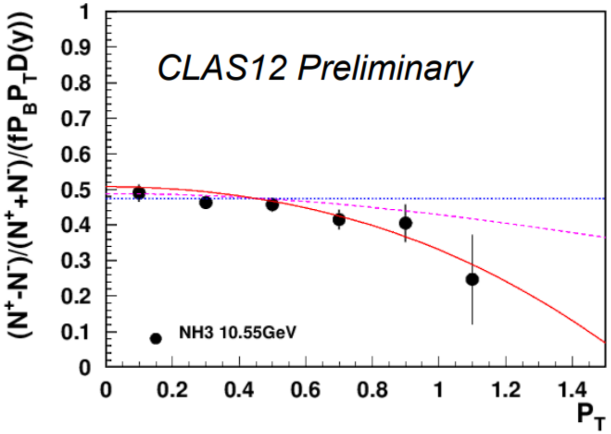


- All exclusive hadrons dominated by u-quarks have the same SSA
- ρ^0_L and ρ^0_T →2 different particles!!!

Identify kinematics of “diffractive” ρ^0 by comparison with ρ^+

Longitudinal target results

- Results represent 5% proton target



Harut Avakian at

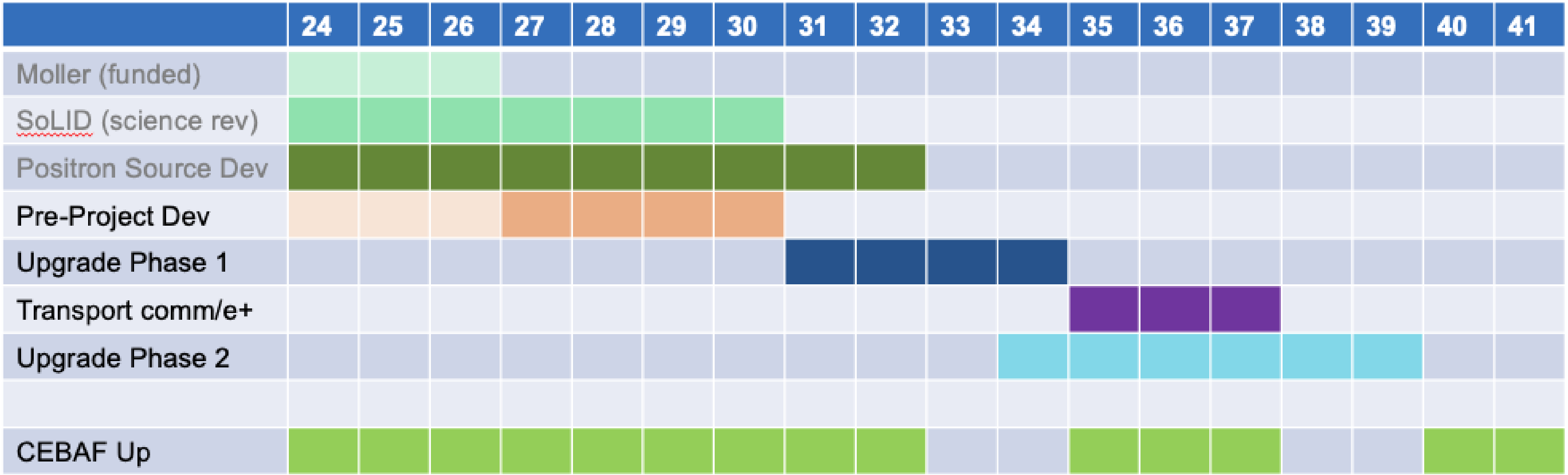
$$F_{LL} \propto g_1(x, k_T) \otimes D_1(z, p_T)$$

Convolution over transverse momentum space

- Rich program underway

Notional CEBAF & upgrade schedule (FY24 – FY42)

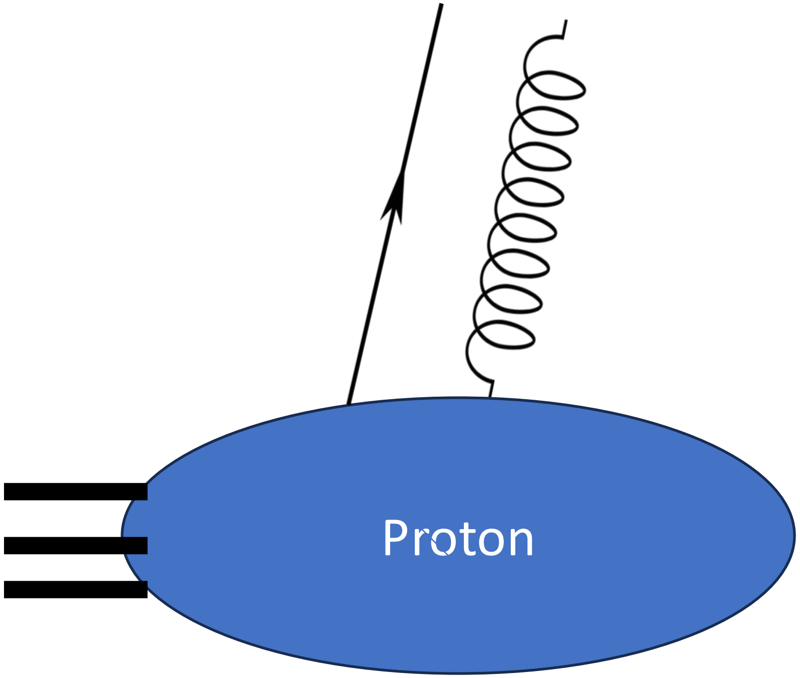
- Accelerator and engineering team have worked up an early schedule and cost estimate
  - Schedule assumptions based on a notional timing of when funds might be available (near EIC ramp down based on EIC V3 profile)
  - For completeness, Moller and SoLID (part of 12 GeV program) are shown; positron source development also shown



	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
Moller (funded)	🟩	🟩	🟩															
SoLID (science rev)	🟩	🟩	🟩	🟩	🟩	🟩	🟩											
Positron Source Dev	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩									
Pre-Project Dev	🟧	🟧	🟧	🟧	🟧	🟧	🟧											
Upgrade Phase 1								🟦	🟦	🟦	🟦							
Transport comm/e+												🟪	🟪	🟪				
Upgrade Phase 2											🟦	🟦	🟦	🟦	🟦	🟦		
CEBAF Up	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩			🟩	🟩	🟩			🟩	🟩

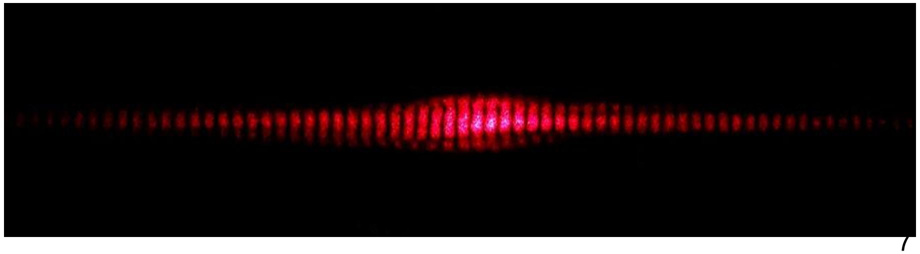
Tia Keppel at

What about quark-gluon interactions?



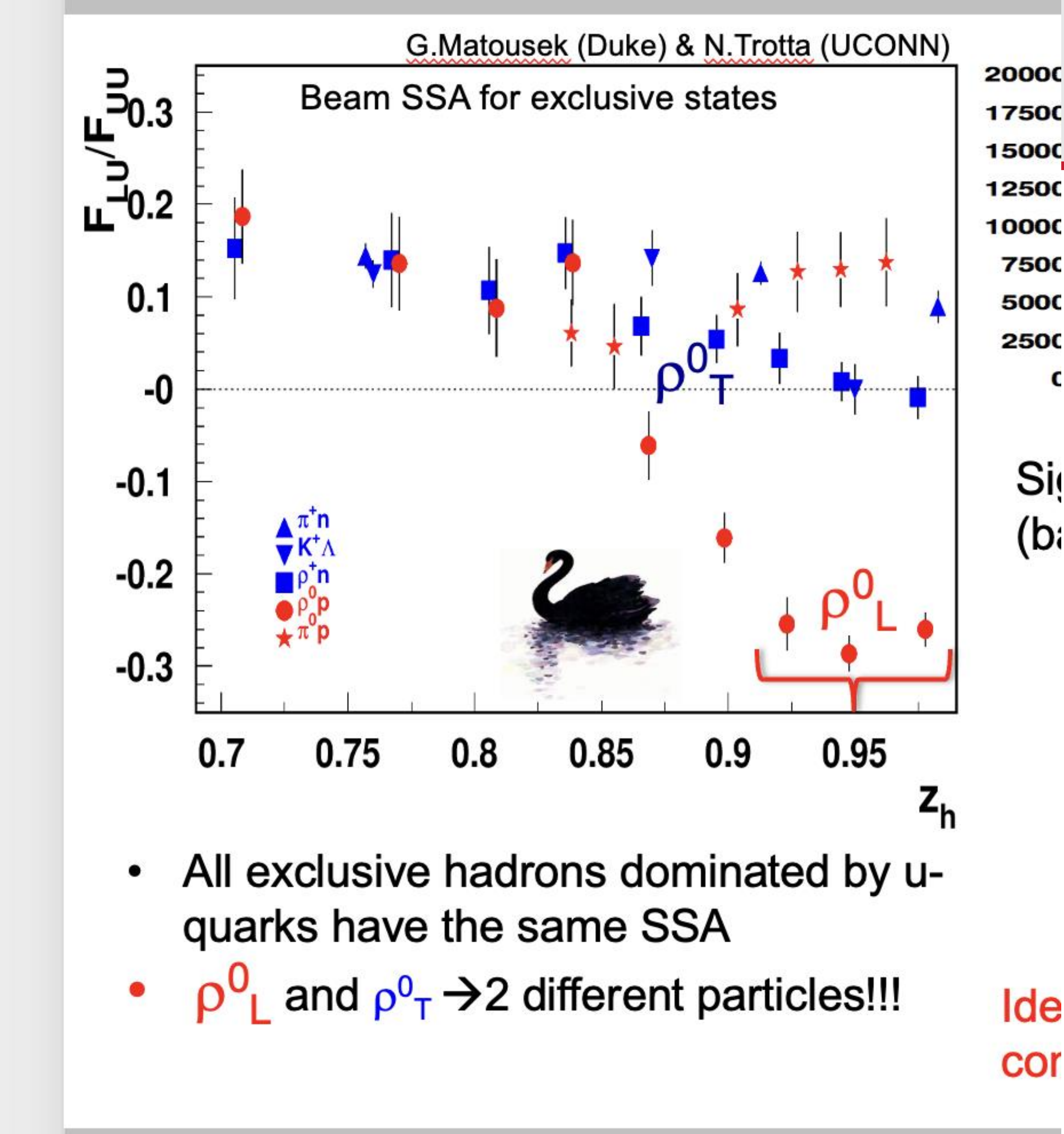
- Consider coherent quark gluon pair
- Cross-section suppressed with $1/Q^2$ →very hard to measure due to uncertainty of overall cross-section

- But: (→QED) What about polarized interference pattern?



Di-ha

- More t
- More c
    - Co



- All exclusive hadrons dominated by u-quarks have the same SSA
- ρ^0_L and ρ^0_T →2 different particles!!!

20000
17500
15000
12500
10000
7500
5000
2500
0

Si
(ba

Ide
cor

Regions & complications? Role of MC in hadronization

- Maybe more complicated, in particular at low energy

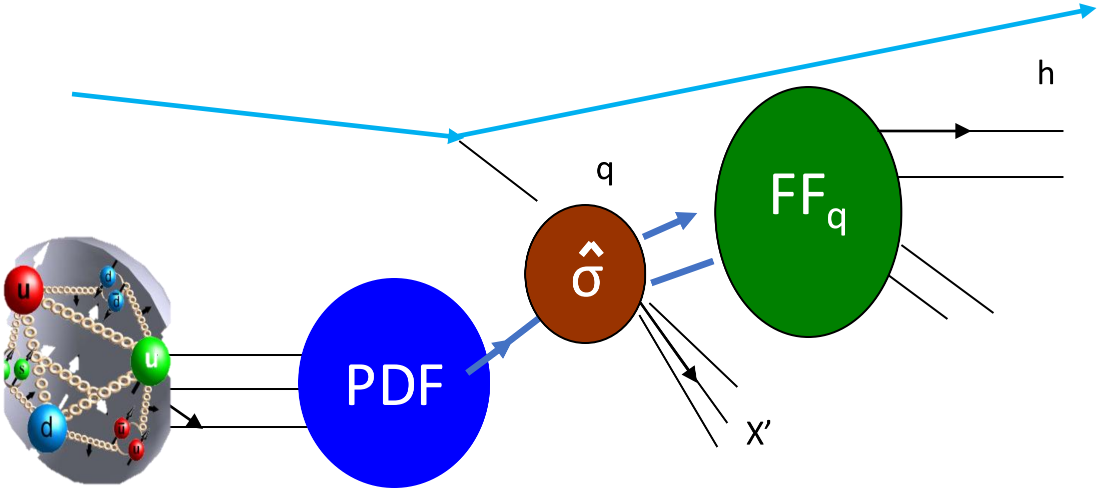
- Hadronization studies necessary to separate effects, correct for acceptance
- (i.e. first show raw process then with acceptance), ISR?

- Role of MC

- Jlab (now) (compass →See Fulvio's talk) EIC
- (e.g. show transversity impact as interplay between EIC and Jlab)

- Timeline from now to EIC…

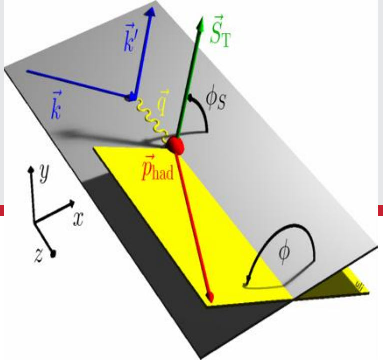
SIDIS is a premier tool to probe the quark and gluon degrees of freedom



- 3D Spin-Momentum Structure
- Sea Quark Polarization
- Saturation Effects

- Fragmentation Functions
- Passage of color through nuclear matter (nFFs)

SIDIS cross-section



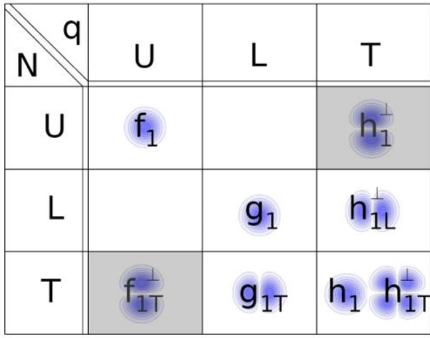
$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

- Disentangling the different contributions is not trivial
- Ratio of T to L flux
  - At fixed x e.g. change Q

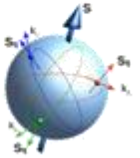
$$
\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}, \qquad \gamma = \frac{2Mx}{Q}.
$$

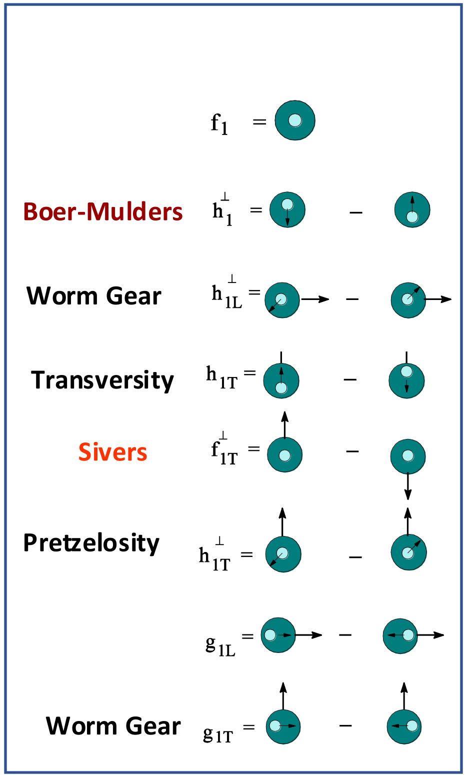
SIDIS X-section in the Parton Model

- Massive simplification, Nontrivial assumptions for validity
  - Power corrections, decays, targetFF, …
  - Open questions
    - transverse momentum spectrum
    - TMD evolution
    - …



N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$





$$d^6\sigma = \frac{4\pi\alpha^2 sx}{Q^4}\times$$

$$\{[1+(1-y)^2]\sum_{q,\bar{q}} e_q^2 f_1^q(x) D_1^q(z,P_{h\perp}^2)$$
$$+(1-y)\frac{P_{h\perp}^2}{4z^2 M_N M_h}\cos(2\phi_h^l)\sum_{q,\bar{q}} e_q^2 h_1^{\perp(1)q}(x) H_1^{\perp q}(z,P_{h\perp}^2)$$

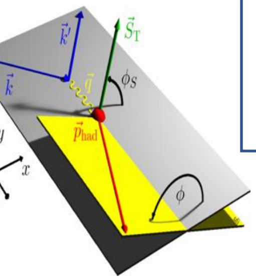
Unpolarized

$$-|S_L|(1-y)\frac{P_{h\perp}^2}{4z^2 M_N M_h}\sin(2\phi_h^l)\sum_{q,\bar{q}} e_q^2 h_{1L}^{\perp(1)q}(x) H_1^{\perp q}(z,P_{h\perp}^2)$$
$$+|S_T|(1-y)\frac{P_{h\perp}}{zM_h}\sin(\phi_h^l+\phi_S^l)\sum_{q,\bar{q}} e_q^2 h_1^q(x) H_1^{\perp q}(z,P_{h\perp}^2)$$
$$+|S_T|(1-y+\frac{1}{2}y^2)\frac{P_{h\perp}}{zM_N}\sin(\phi_h^l-\phi_S^l)\sum_{q,\bar{q}} e_q^2 f_{1T}^{\perp(1)q}(x) D_1^q(z,P_{h\perp}^2)$$
$$+|S_T|(1-y)\frac{P_{h\perp}^3}{6z^3 M_N^2 M_h}\sin(3\phi_h^l-\phi_S^l)\sum_{q,\bar{q}} e_q^2 h_{1T}^{\perp(2)q}(x) H_1^{\perp q}(z,P_{h\perp}^2)$$
$$+\lambda_e|S_L|y(1-\frac{1}{2}y)\sum_{q,\bar{q}} e_q^2 g_1^q(x) D_1^q(z,P_{h\perp}^2)$$

Polarized target

$$+\lambda_e|S_T|y(1-\frac{1}{2}y)\frac{P_{h\perp}}{zM_N}\cos(\phi_h^l-\phi_S^l)\sum_{q,\bar{q}} e_q^2 g_{1T}^{(1)q}(x) D_1^q(z,P_{h\perp}^2)\}$$

Polarzied beam and target



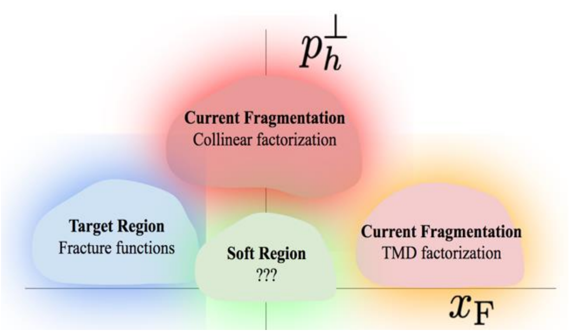
S_L and S_T: Target Polarizations; λe: Beam Polarization

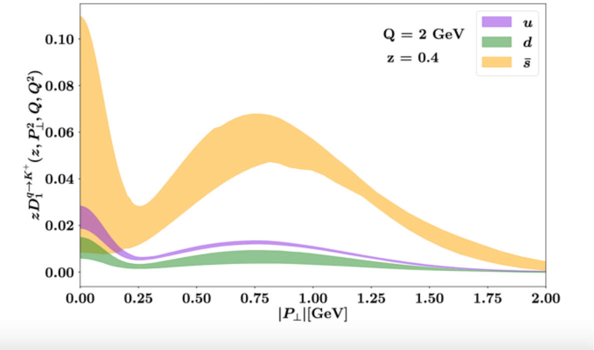
x: momentum fraction carried by struck quark, z: fractional energy of hadron

Validations of Theory Framework

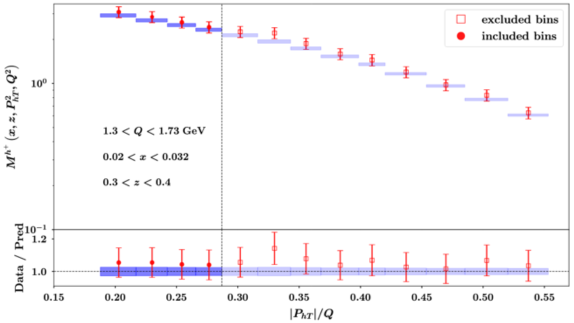
- TMD extraction is non-trivial
- Higher Twist Contributions
- Overlap of regions that are not captured by factorized TMD picture
- VM Meson decays
- Radiative corrections
- Evolution (CS Kernel)

➔EIC will be critical test of our understanding (high lumi, leverarm in kinematics to disentangle various contributions)





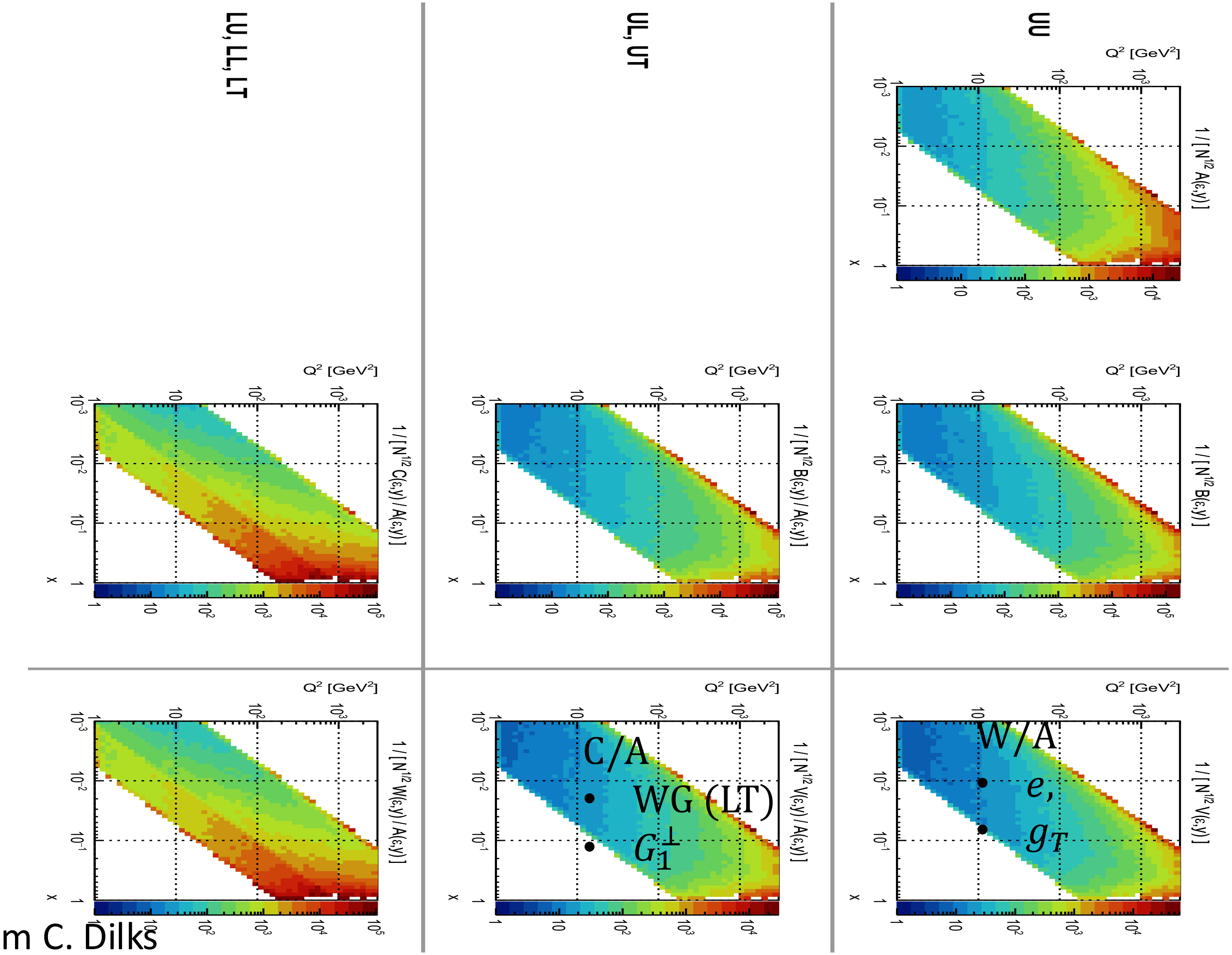
MAP extraction of FFs



Expect corrections in powers of$\delta \sim$ PhT /z/Q

Plots from A. Bacchetta’s talk at ePIC Collaboration meeting

Statistical uncertainty scaling factor for $18x275$



W/A

e, g_T

C/A

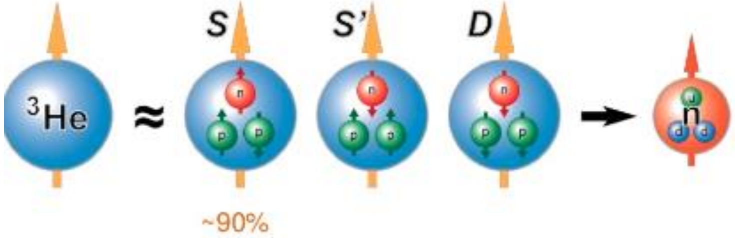
WG (LT)

$G_1^\perp$

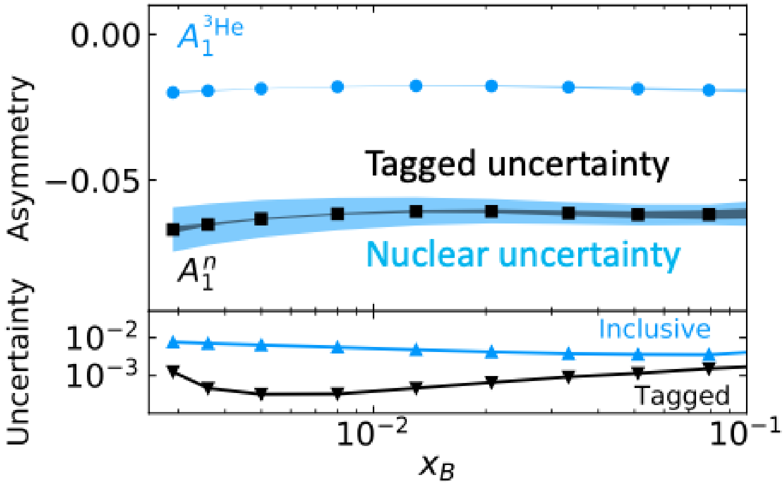
Slide from C. Dilks

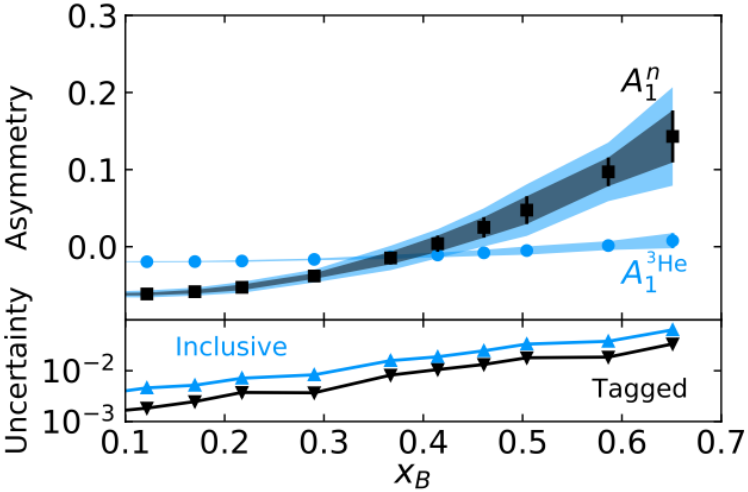
He^3 Double Tagging at the EIC allows clean neutron measurement

- Neutron is to 87% polarized
- Double tagged events thus provide access to polarized neutron beam



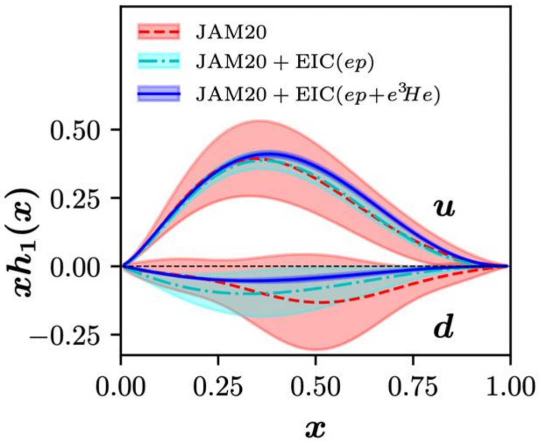
- Reconstruction of initial neutron momentum from tagged protons allows reduction of uncertainties from nuclear corrections

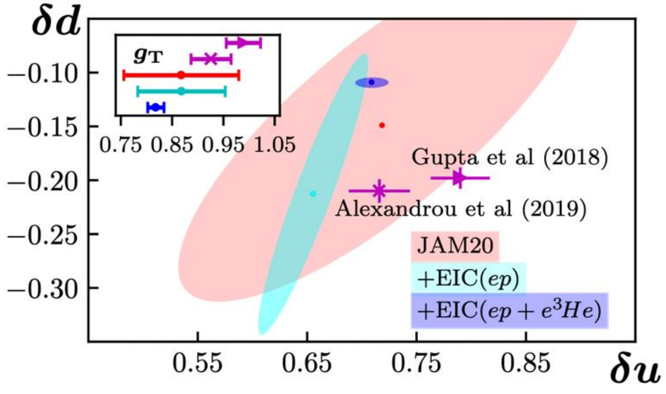


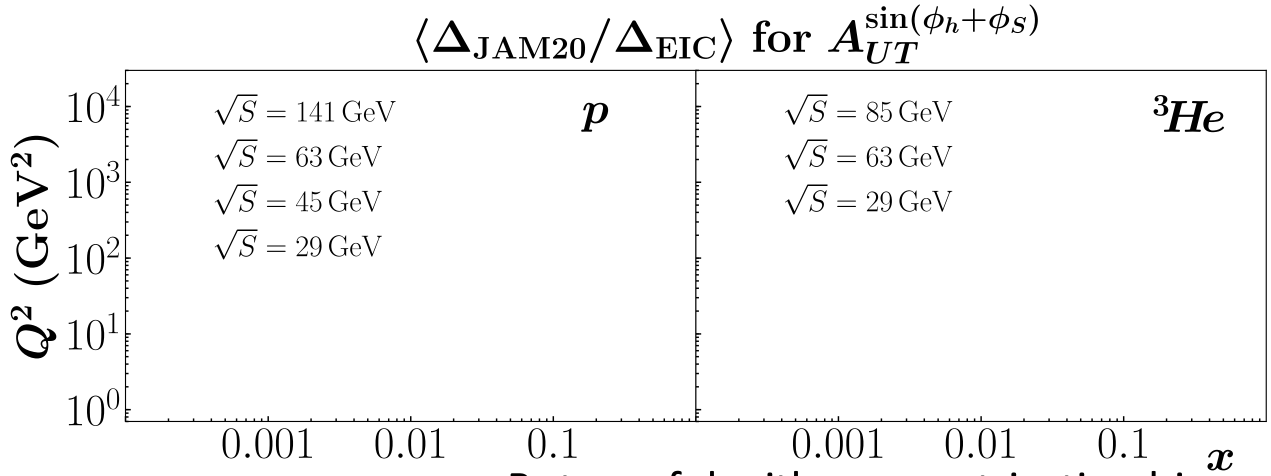


Friščić, I, Nguyen, D, Pybus, JR, Jentsch et al

Example: transversity extraction from Jlab and the EIC







But careful with parametrization bias…

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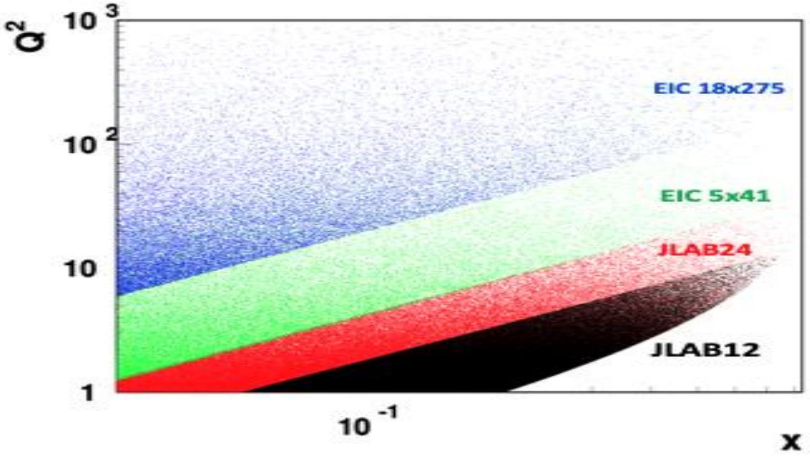
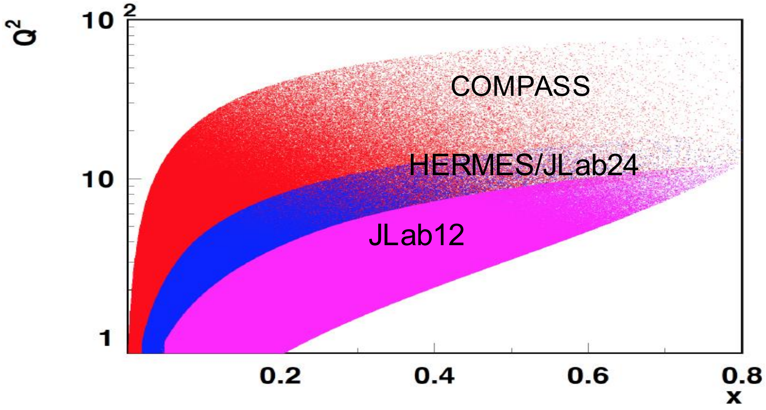
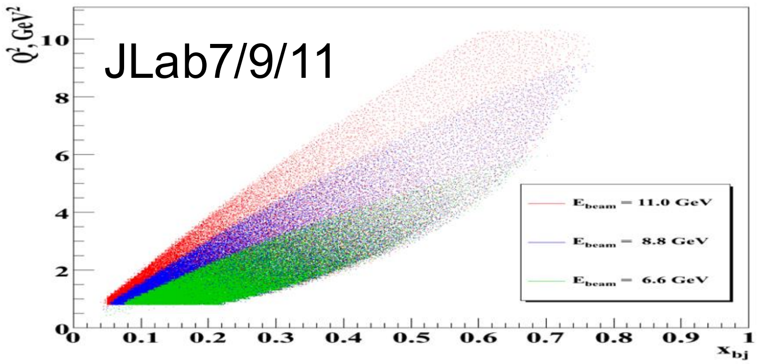
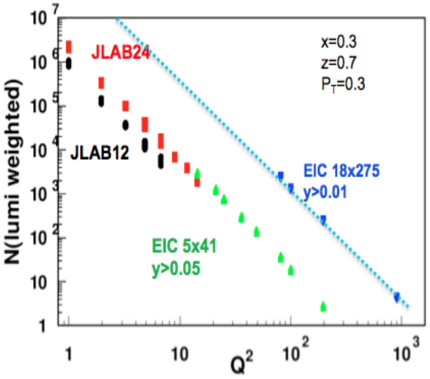
Kinematic comparisons



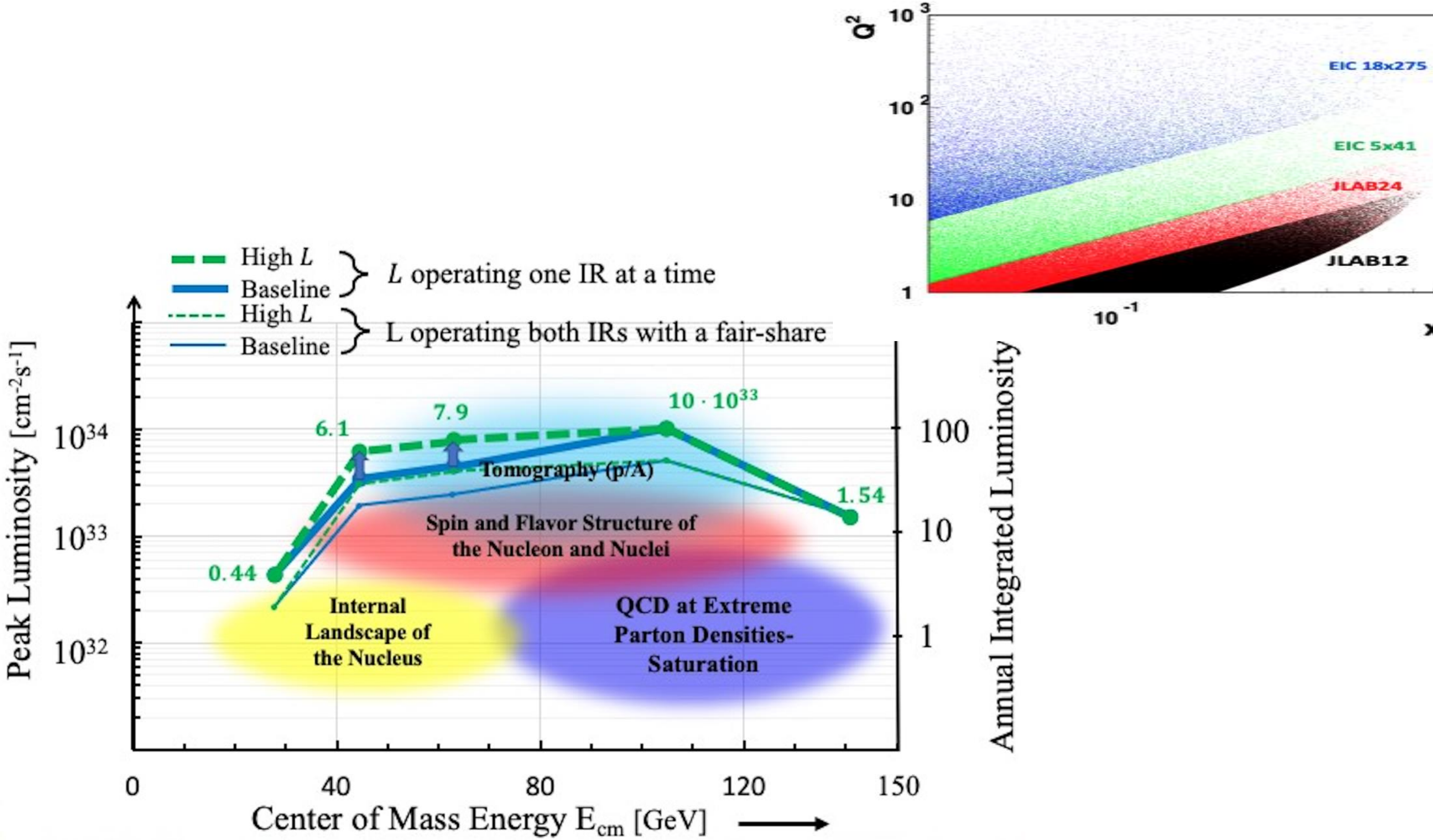






NB: Kinematic slice heavily biased towards Jlab

Order of magnitude in luminosity depending on $\sqrt{s}$ (beware of projections with fixed $\int L$)



Wide Coverage