26th International Symposium on Spin Physics
A Century of Spin

Transverse Momentum Dependent Distributions from Lattice QCD

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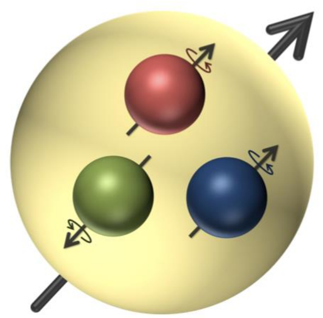
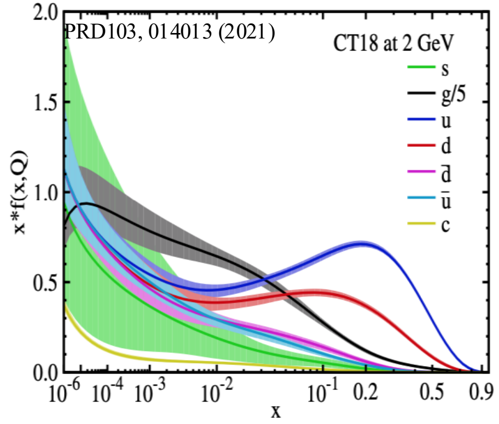
Shanghai Jiao Tong University

On behalf of Lattice Parton Collaboration

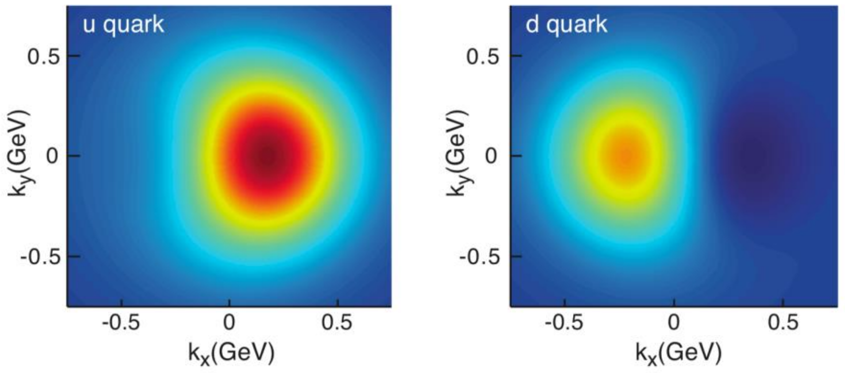
The 26th international symposium on spin physics (SPIN2025)

TMDs: 3D hadron Structure





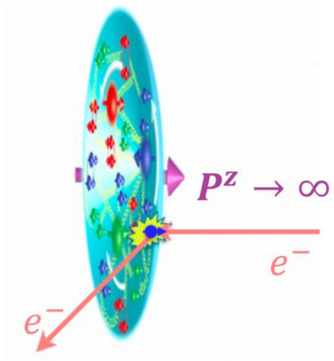
EIC and EicC

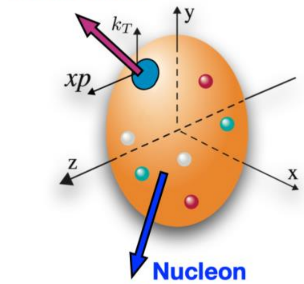


0D: spin and mass

1D: PDF

3D: TMDs



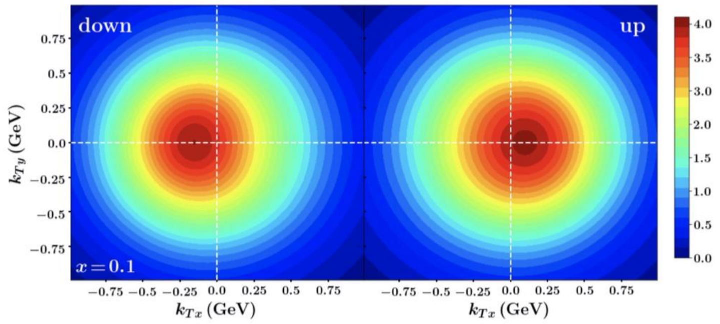


TMDs: 3D hadron Structure

Wigner distributions
(Generalized TMDs)
$W(x, \vec{k}_T, \vec{b}_T)$

$\int d^2\vec{b}_T$

Transvers momentum distributions (TMDs)



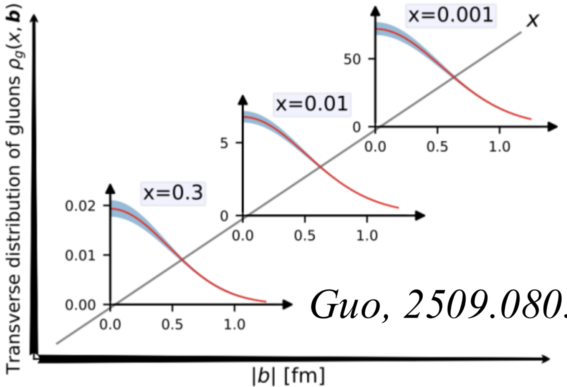
JAM, PRD102, 054002 (2020)

$\int d^2\vec{k}_T$

$\int d^2\vec{k}_T$

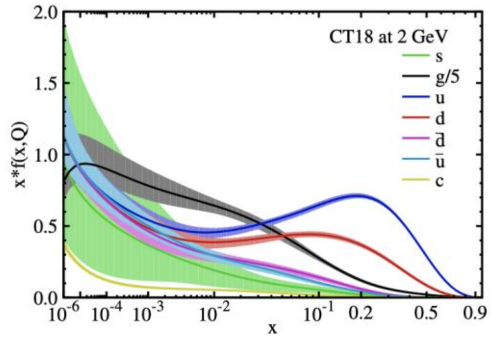
Generalized parton distributions (GPDs)

Parton distribution functions (PDFs)



Guo, 2509.08037

$\int d^2\vec{b}_T$



CTEQ, PRD103, 014013(2021)

TMDs: 3D hadron Structure

Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_{1L} = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

Spin-dependent TMDPDFs

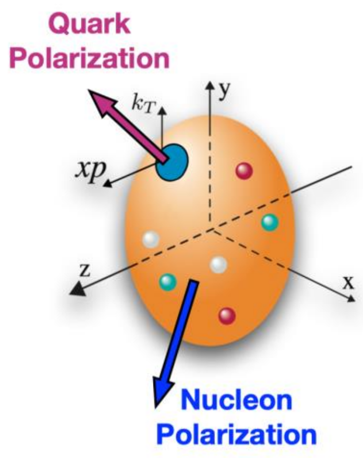
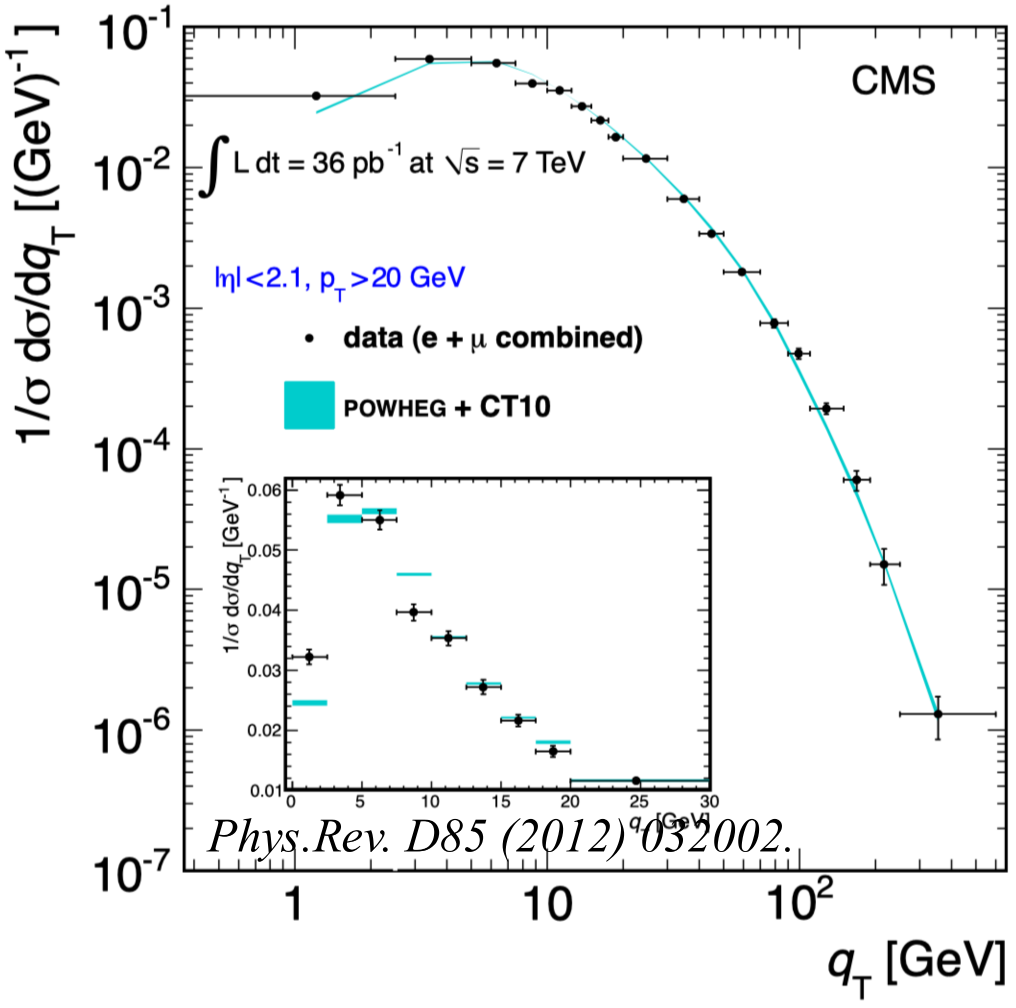


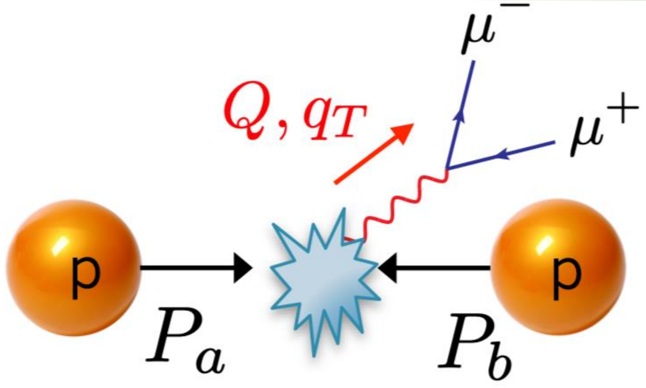
Figure Credit:

TMD Handbook, 2304.03302

[For theory overview, see the talk by Kazuhiro Watanabe]

TMDPDF

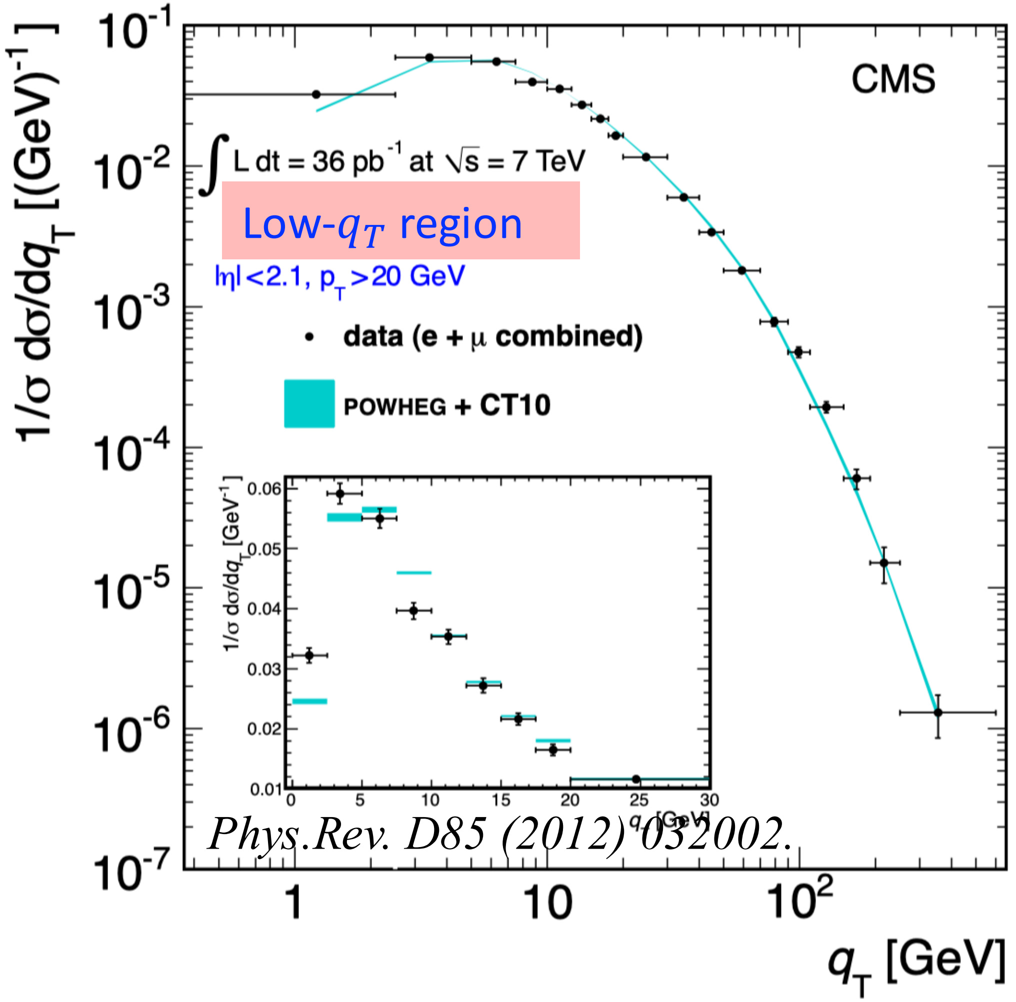


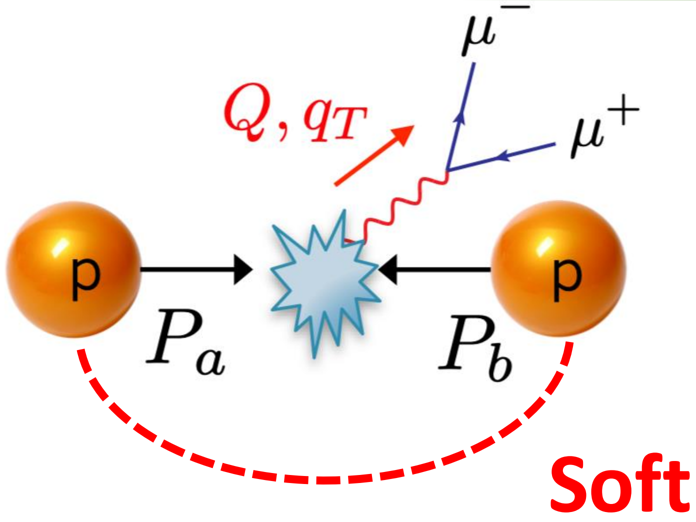


- **High-q_T region:**

  Collinear factorization => PDF

TMDPDF





- **High-q_T region:**

  Collinear factorization => PDF

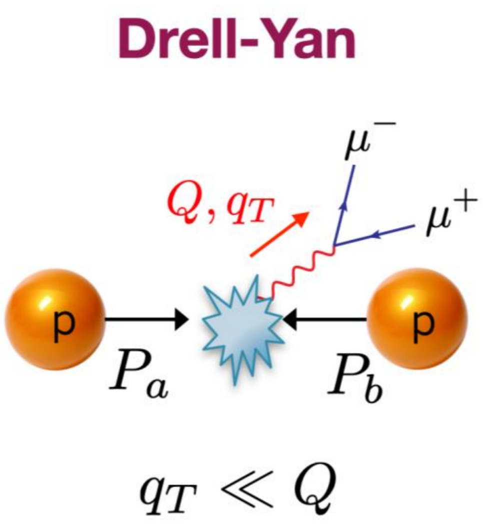
- **Low-q_T region:**

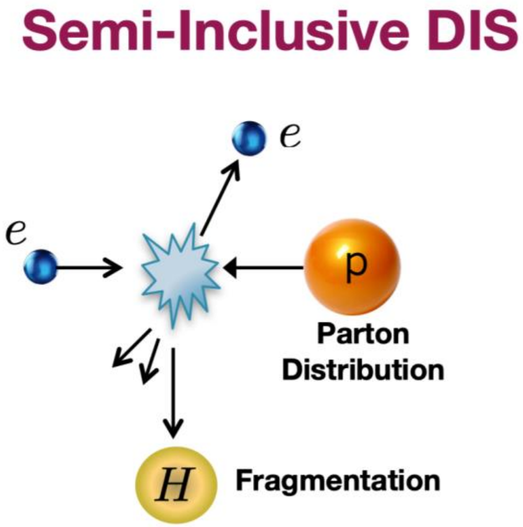
  TMD factorization

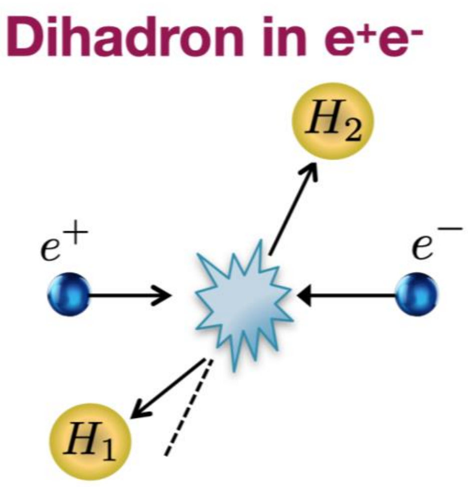
  => Generalize to TMDPDFs

TMDPDF and TMDFFs are mandatory in hadron processes

TMDPDF







LHC, FermiLab, RHIC, …

HERMES, COMPASS, JLab, EIC, …

BESIII, Babar, Belle, …

$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

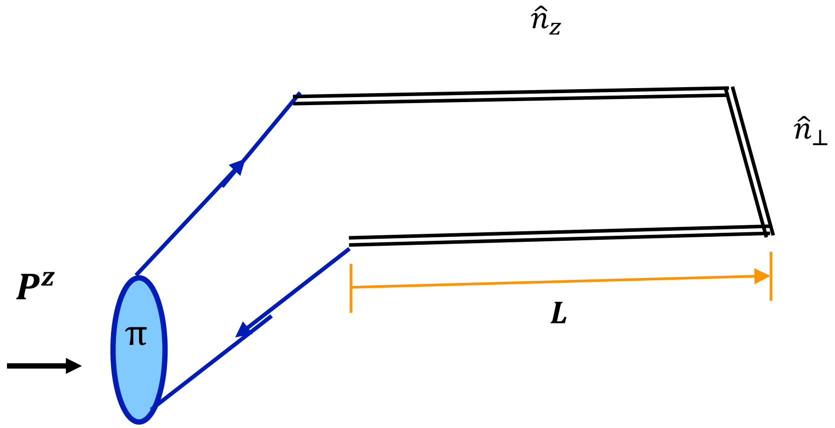
Figure Credit: TMD Handbook, 2304.03302

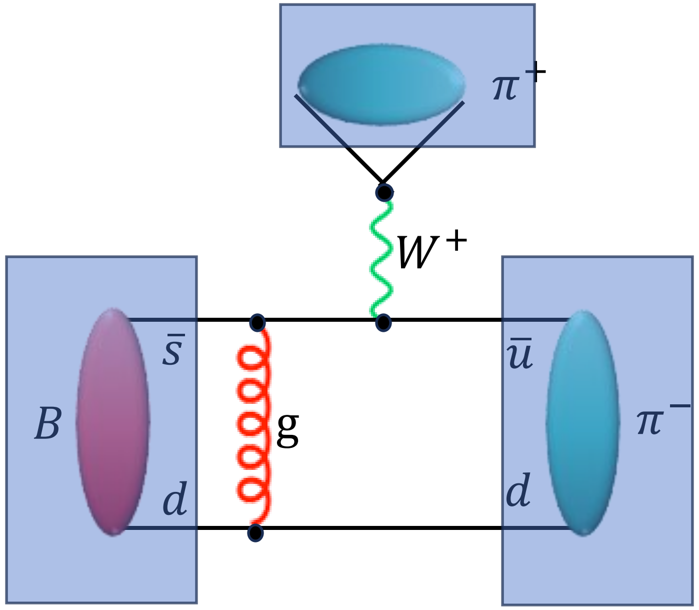
TMDPDF and TMDFFs are mandatory in hadron processes

TMD wavefunctions

- **PQCD for B-meson decays:**

$$C(t)\otimes H(t)\otimes \phi(x,b_\perp)\otimes \exp[-s(P,b_\perp)-2\int_{1/b_\perp}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}))]$$





Keum, Li, Sanda, PRD,63,054008(2001)
Lu, Ukai, Yang, PRD, 63, 074009,2001

TMDWFs are necessary inputs for heavy meson decays.

Progress in the Study of TMDPDFs

- **Theoretical analysis**
  - **TMD factorization, evolution and resummation:**

    Boussarie et al., TMD Handbook, 2304.03302;

    Collins, Foundations of perturbative QCD; ……

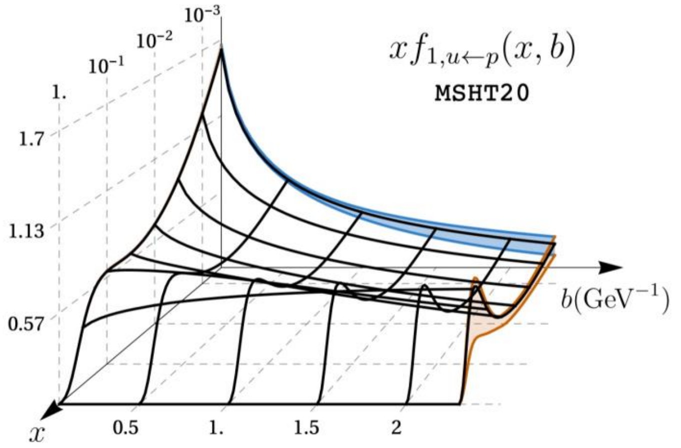
- **Phenomenological parametrizations and extractions**
  - **Unpolarized:**

    Moos, JHEP05 (2024); Bacchetta, JHEP10 (2022);

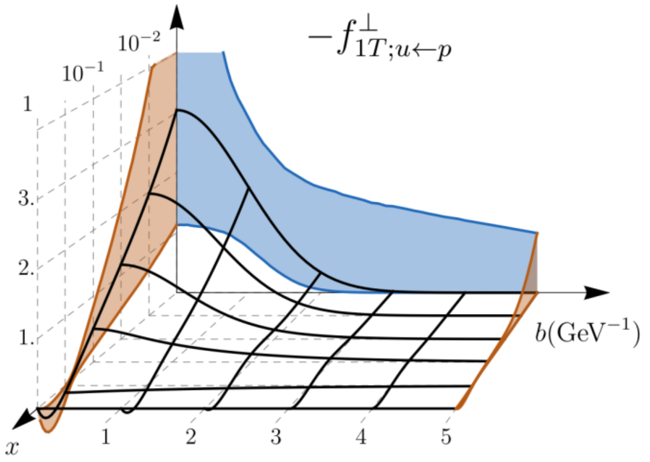
    Scimem, JHEP06 (2020); Bacchetta, JHEP06 (2017); ……
  - **Sivers, Boer-Mulders：**

    Bury, PRL126 (2021), JHEP05 (2021) ; Cammarota, PRD102(2020);

    Zhang, PRD77 (2008), Lu, PRD81 (2010) ; ……
  - **Others: worm-gear, gluon TMDs, ……**

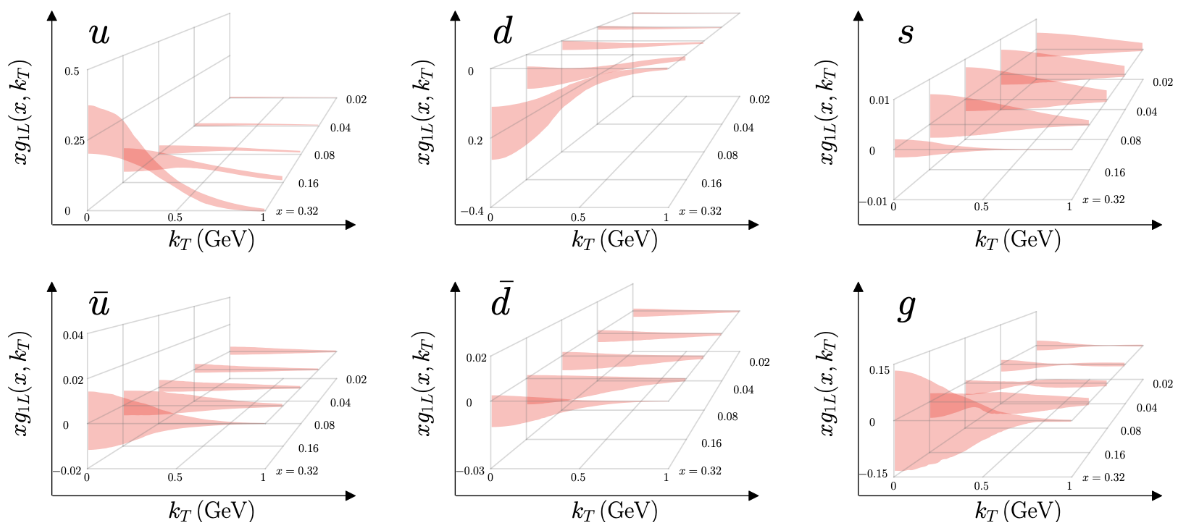


u-quark unpolarized TMDPDF, 2201.07114

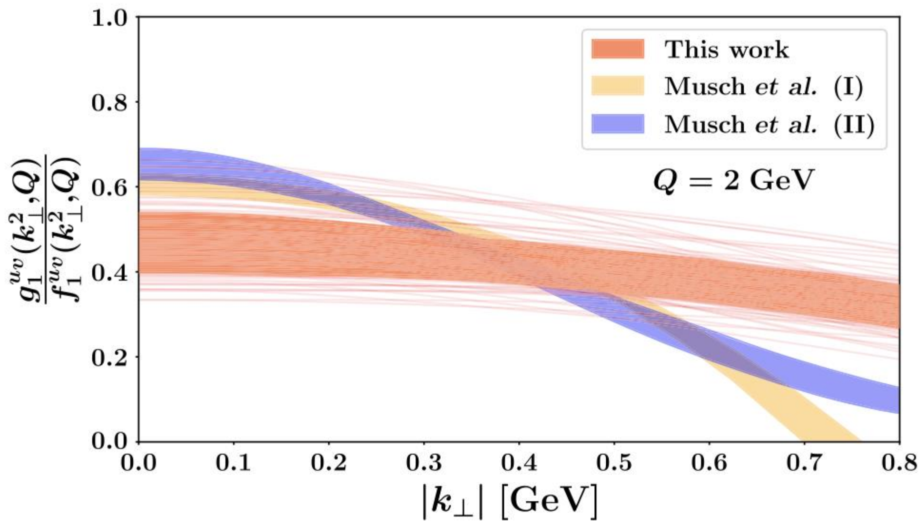


u-quark Sivers function, PRL126 (2021)

Experimental Extraction of TMDPDFs



Yang, Liu, Sun, Zhao, Ma, PRL 134, 121902 (2025)



MAP, PRL 134,121901 (2025)

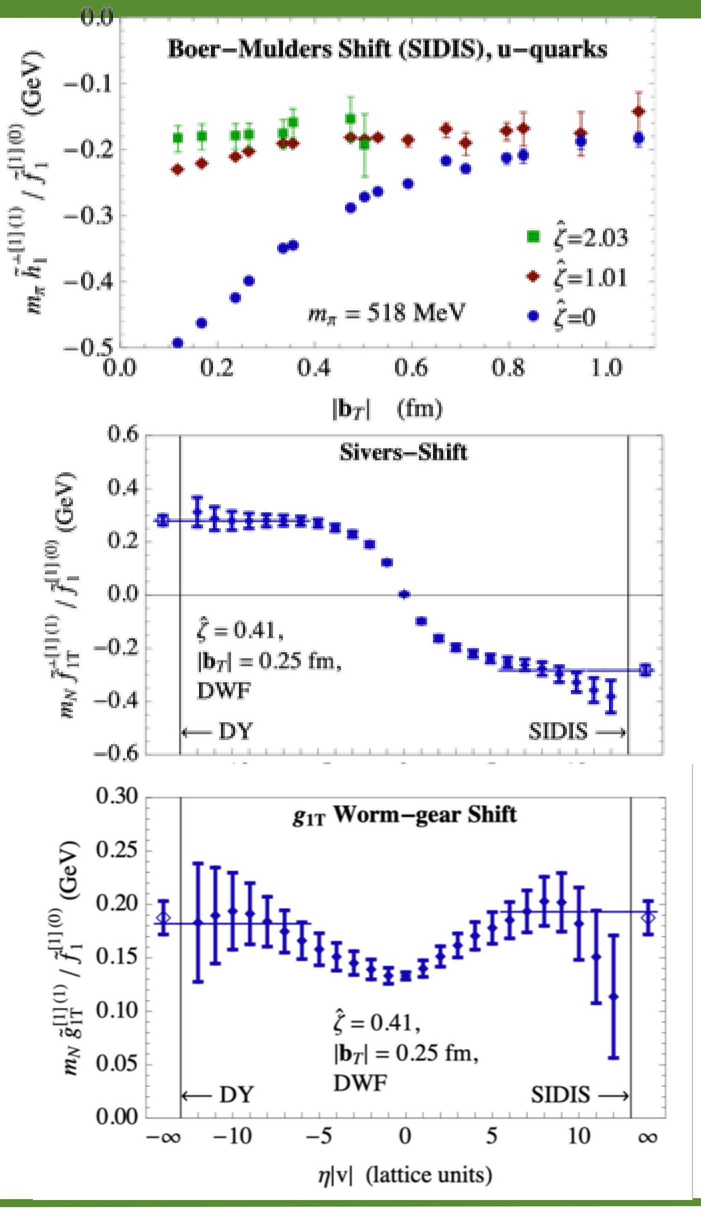
[see talk by Ke Yang on Tuesday]

[see talk by Alessandro Bacchetta on Tuesday]

TMD from Lattice: Ratios of Mellin Moments

- Through the **OPE**, TMDs can be converted into **local moments** that are straightforward to compute on the lattice.
- The **ratio** eliminates the soft function.
- One can extract the **Boer-Mulders shift**, **Sivers shift**, **worm-gear shift**, etc. from the ratio of lowest order moments:

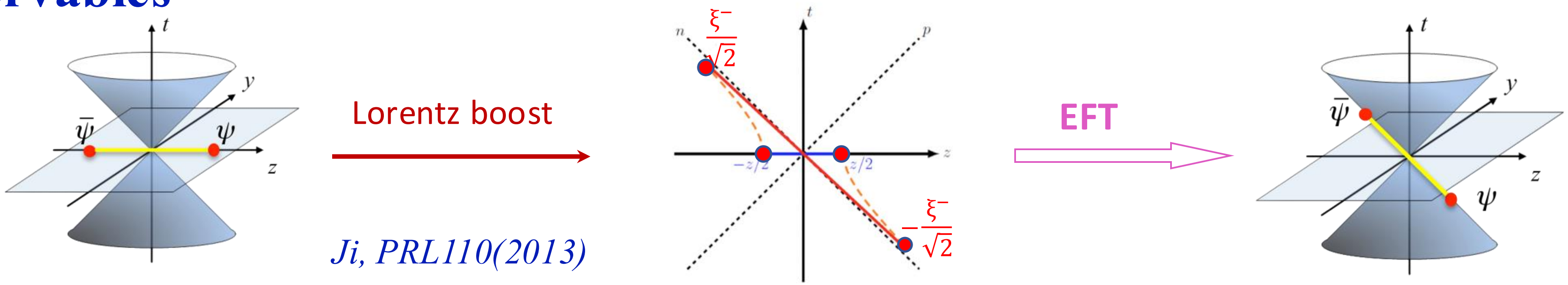
$$\text{Sivers shift} \propto \frac{\tilde{f}_{1T}^{[1](1)}}{\tilde{f}_{1T}^{[1](0)}},\ \text{BM shift} \propto \frac{\tilde{h}_{1}^{[1](1)}}{\tilde{f}_{1}^{[1](0)}},\ \text{WG shift} \propto \frac{\tilde{g}_{1T}^{[1](1)}}{\tilde{f}_{1}^{[1](0)}}$$



Engelhardt, et.al.. PRD93, 054501 (2016)
Yoon, et.al., PRD96, 094508 (2017)

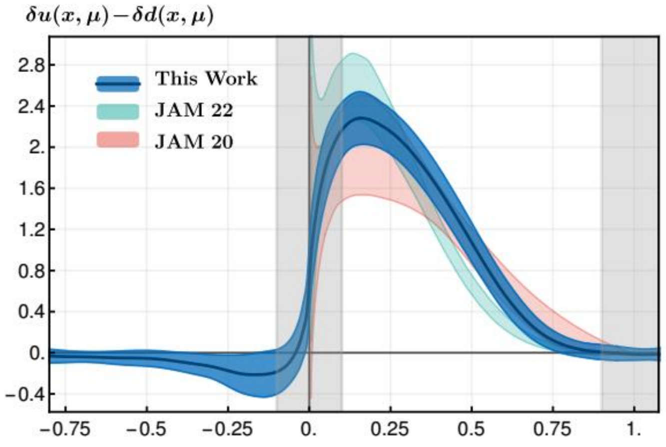
LaMET: PDFs at Lattice QCD

- **Large-momentum effective theory: connecting Euclidean lattice and physical observables**

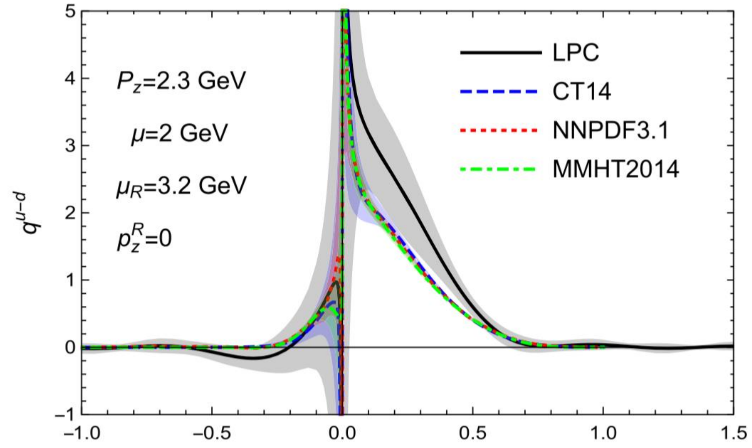


Ji, PRL110(2013)

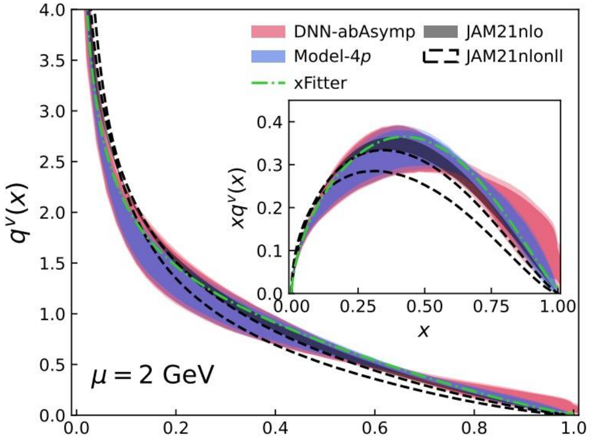
- **Achieved great success in the studies of PDFs and LCDA:**



Proton transversity PDF, PRL131(2023)



Unpolarized isovector quark PDF PRD 101 (2020)



Pion valance PDF, PRD106(2022)

Recent Progress in LaMET

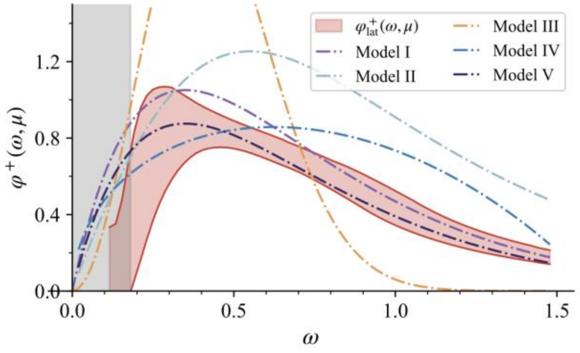
- ✓ **PDFs:**
  - ➢ **Quark: unpolarized, helicity, transversity**
  - ➢ **Gluon:**

  [Talks by L. Liu/D.Zhao on Tuesday]
- ✓ **GPDs:**

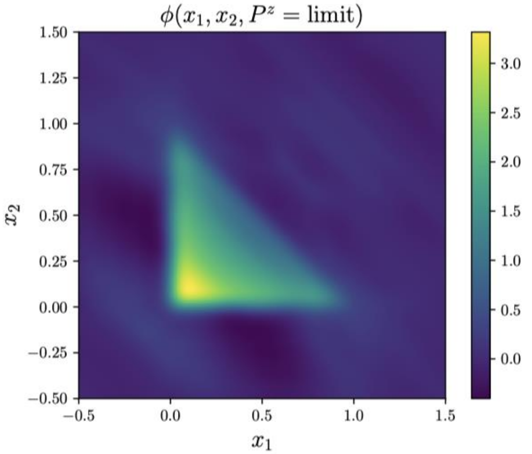
  [See plenary talk by Huey-Wen Lin on Thursday]
- ✓ **TMDPDFs:**
- ✓ **Deuteron PDFs:**

- ✓ **LCDAs:**
  - ✓ **Pion, K, K***
  - ✓ **Heavy meson LCDA:**
  - ✓ **Baryon LCDA:**

  [Talk by M.H. Zhang on Tuesday]



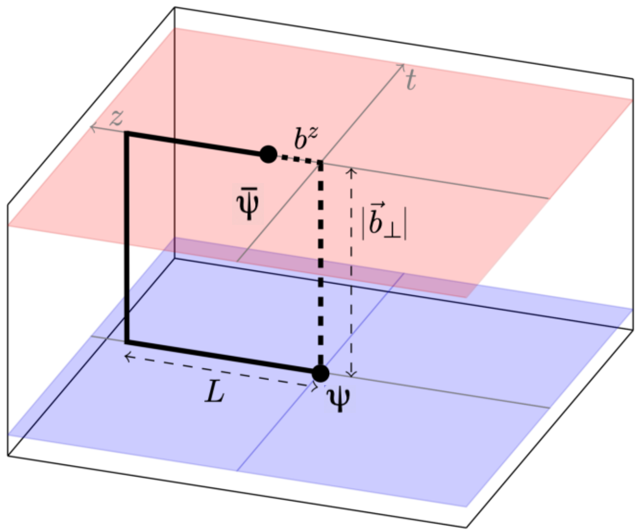
PRD 111, L111503 & 034503(2025)



PRD 111, 034510 (2025)

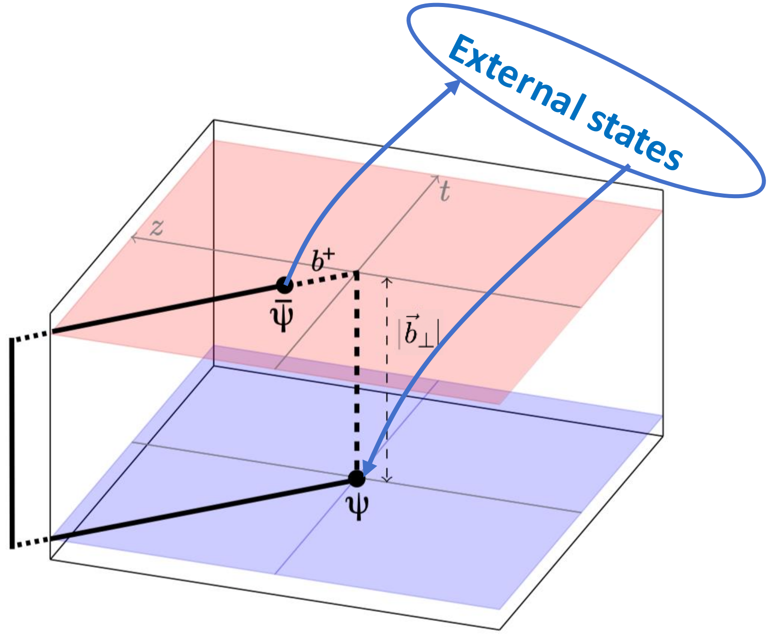
Apologize for unable list all relevant references and many results can be found in:
Ji, Liu, Liu, Zhang, Zhao,Rev. Mod. Phys. 93, 035005 (2021)

TMDs on Lattice QCD -- LaMET



Lorentz boost

$L \to \infty$
$P^z \to \infty$



Equal-time correlators,
directly calculable on lattice

Space-like correlators,
NO effective method for directly calculation

Ji, PLB811(2020); Ebert, JHEP 09 (2019), JHEP04(2022)

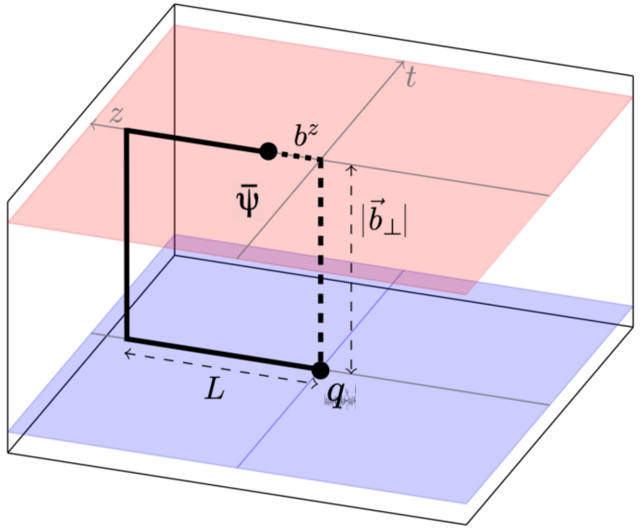
Connected at large-momentum limit

Perturbative matching kernel

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z) S_I^{\frac{1}{2}}(b_\perp, \mu) = H_\Gamma(\zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\zeta^z - i\epsilon}{\zeta}\right] f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta^z}\right)$$

Quasi TMDs **Intrinsic soft function** **Collins-Soper kernel** **TMDs**

TMDs on Lattice QCD -- LaMET



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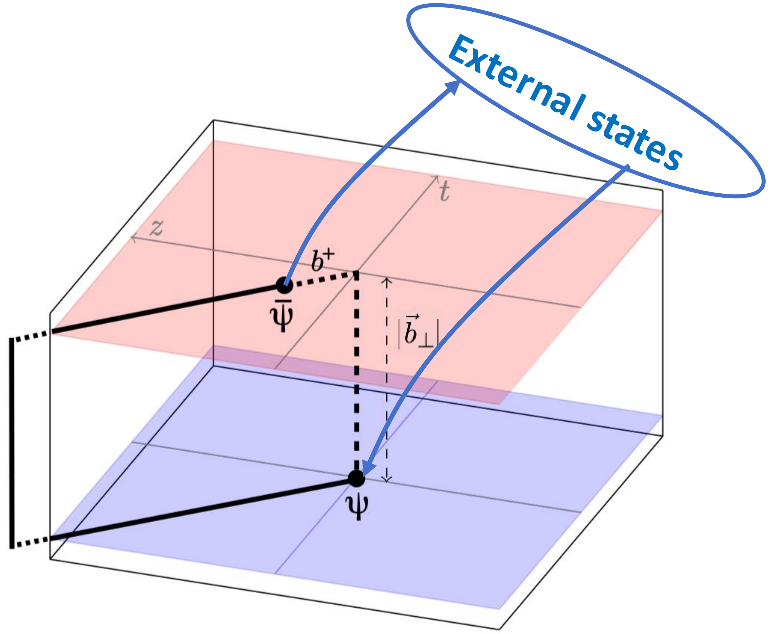
$L \to \infty$
$P^z \to \infty$



Equal-time correlators,
directly calculable on lattice

Space-like correlators,
NO effective method for directly calculation

Ji, PLB811(2020); Ebert, JHEP 09 (2019), JHEP04(2022)

Connected at large-momentum limit

LQCD LQCD **Perturbative matching kernel** LQCD

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z) S_I^{\frac{1}{2}}(b_\perp, \mu) = H_\Gamma(\zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln\frac{\zeta^z - i\epsilon}{\zeta}\right] f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta^z}\right)$$

Quasi TMDs **Intrinsic soft function** **Collins-Soper kernel** **TMDs**

TMDs on Lattice QCD: CS kernel

➤ Taking ratio, S_I and f cancelled each other:

$$K(b_\perp,\mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left(\frac{H_\Gamma(\zeta_2^z,\mu)\tilde{f}_\Gamma(x,b_\perp,\zeta_1^z,\mu)}{H_\Gamma(\zeta_1^z,\mu)\tilde{f}_\Gamma(x,b_\perp,\zeta_2^z,\mu)}\right)$$

$\tilde{f}_\Gamma$ could be either unsubtracted quasi TMD wave function or quasi TMDPDF.

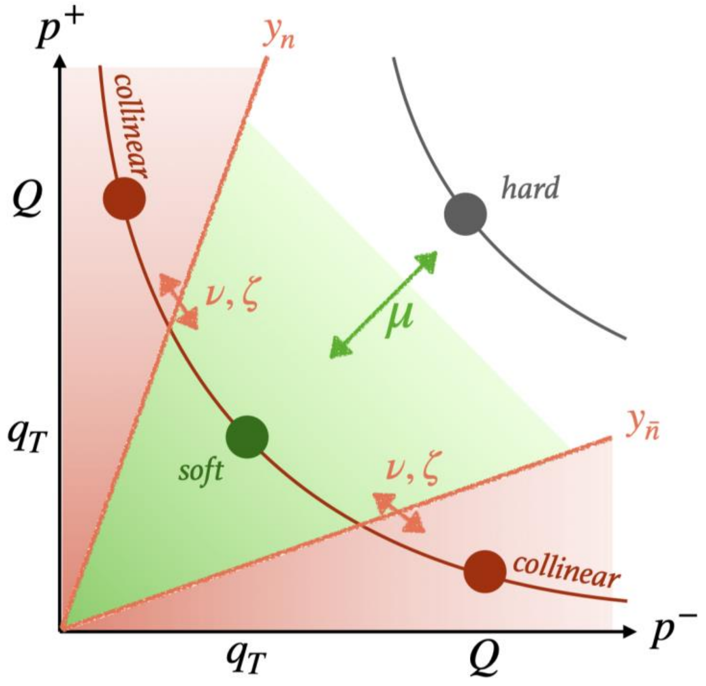


Figure: TMD Handbook, 2304.03302

$$\tilde{f}_\Gamma(x,b_\perp,\mu,\zeta^z)S_I^{\frac{1}{2}}(b_\perp,\mu) = H_\Gamma(\zeta^z,\mu)\exp\left[\frac{1}{2}K(b_\perp,\mu)\ln\frac{\zeta^z - i\epsilon}{\zeta}\right] f(x,b_\perp,\mu,\zeta) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{\zeta^z},\frac{M^2}{(P^z)^2},\frac{1}{b_\perp^2\zeta^z}\right)$$

TMDs on Lattice QCD: CS kernel

	Pion mass	Renormalization	Fourier transform	Matching	*x*-plateau	Continuum	$b_\perp$ separation (*)
SWZ20 PRD 102, 014511 (2020) Quenched	1.2 GeV	Yes	Yes	LO	Yes	No	0.8fm
LPC20 PRL 125,192001 (2020)	547 MeV	N/A	N/A	LO	N/A	No	0.3fm
SVZES JHEP08 (2021), 2302.06502	422 MeV	N/A	N/A	NLO	N/A	No	0.5fm
PKU/ETMC 21 PRL128, 062002 (2022)	827 MeV	N/A	N/A	LO	N/A	No	0.3fm
SWZ21 PRD 104,114502 (2022)	580 MeV	Yes	Yes	NLO	Yes	No	0.5fm
LPC22 PRD 106, 034509 (2022)	670 MeV	Yes	Yes	NLO	Yes	No	0.6fm
LPC23 JHEP 08,172 (2023)	220 MeV	Yes	Yes	NLO	Yes	No	0.6fm
ASWZ23 PRD108, 114505 (2023)	148.8 MeV	Yes	Yes	NNLL	Yes	No	0.8fm
ASWZ24 PRL132, 231901 (2024)	148.8 MeV	Yes	Yes	NNLL	Yes	Model (0.15,0.12,0.09) fm	0.9fm
LPC 25 (preliminary)	135.5 MeV	Yes	Yes	NLO	Yes	Yes	~1.0fm

Table adapted/updated from Yong Zhao's collection

(*) Only the results that deviate from 0 by more than 2σ are considered.

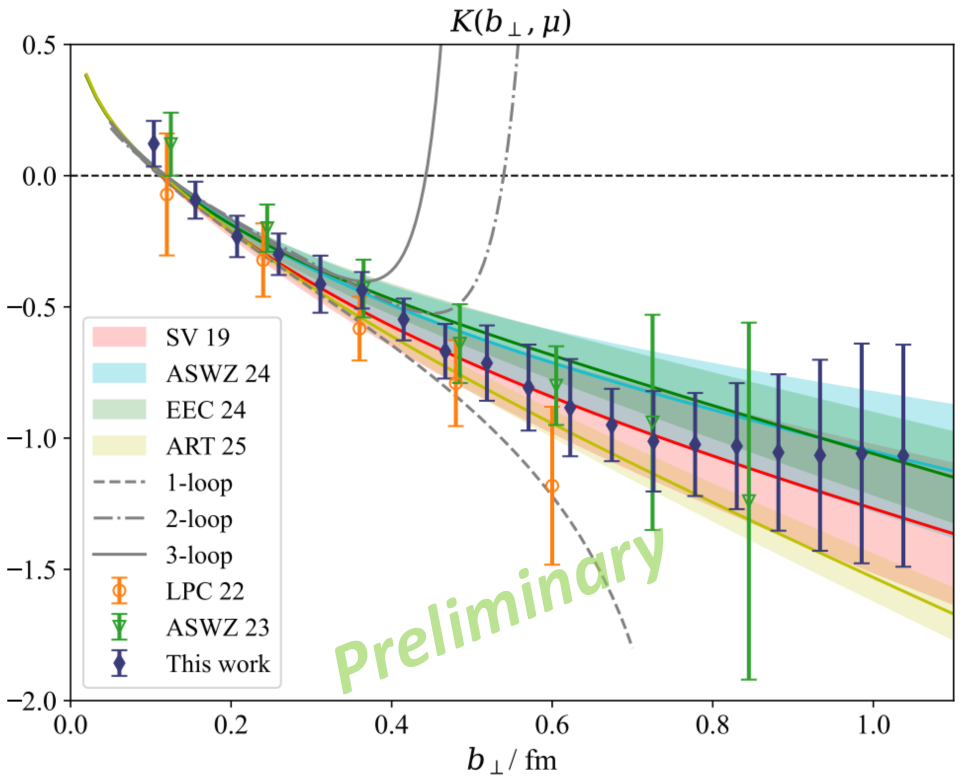
TMDs on Lattice QCD: CS kernel

- **CLQCD ensembles, $a = \{0.105, 0.077, 0.052\}$ fm, $m_\pi \simeq \{300, 135\}$ MeV;**

✓ **The state-of-art self renormalization;**

- **uNLO matching kernel;**

✓ **Infinity momentum, continuum and physical mass extrapolation**

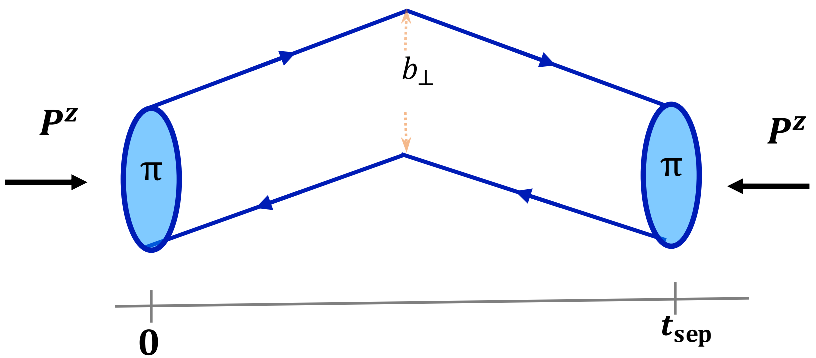


[More details in the talk Jin-Xin Tan on Wednesday]

TMDs on Lattice QCD: Intrinsic Soft Function

- **Four-quark form factor with large P^z:**

$$F(b_\perp, P^z) = \langle \pi(P^z)|(\bar{q}_1\Gamma q_1)(b_\perp)(\bar{q}_2\Gamma q_2)(0)|\pi(P^z)\rangle$$



Ji, Nucl.Phys.B 955, 115054 (2020)

- **Factorization of form factor in the framework of LaMET:**

$$F(b_\perp, P^z) = \int dx\, dx'\, H(x, x', P^z) \frac{\tilde{\phi}(x, \bar{Y}, P^z, b_\perp)}{S(Y, \bar{Y}, b_\perp)} \frac{\tilde{\phi}^\dagger(x', \bar{Y}', P^z, b_\perp)}{S(Y', \bar{Y}', b_\perp)} S(Y, Y', b_\perp)$$

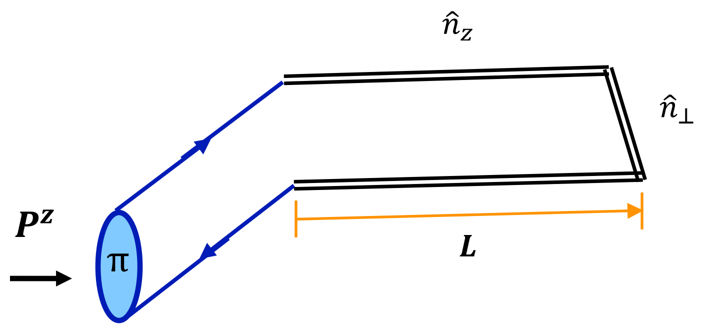
perturbative matching kernel

subtracted quasi-TMD wave function

TMDs on Lattice QCD: Intrinsic Soft Function

- **Subtracted quasi-TMD wave function:**

$$\tilde{\phi}(z, P^z, b_\perp) = \lim_{L\to\infty} \frac{\langle 0|\bar{q}(z\hat{n}_z + b_\perp\hat{n}_\perp)\gamma^t\gamma_5 U_\sqsupset(z\hat{n}_z + b_\perp\hat{n}_\perp, 0; L)q(0)|\pi(P^z)\rangle}{\sqrt{Z_E(2L+z, b_\perp)}}$$



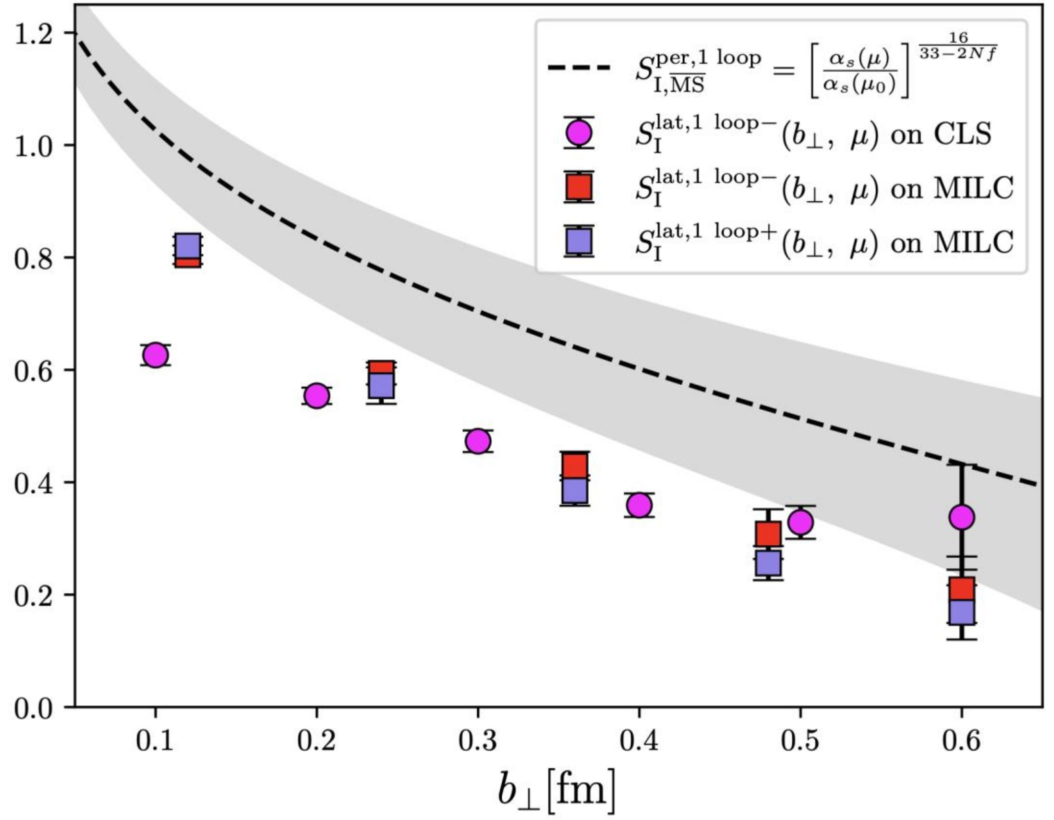
Ji, Nucl.Phys.B 955, 115054 (2020)

- **Intrinsic soft function:**

$$S_I(b_\perp, \mu) = \frac{S(b_\perp, \mu, Y, Y')}{S(b_\perp, \mu, Y, 0)S(b_\perp, \mu, 0, Y')}$$
$$= \frac{F(b_\perp, P^z)}{\int dx\, dx'\, H(x, x', P^z)\tilde{\phi}(x, \bar{Y}, P^z, b_\perp)\tilde{\phi}^\dagger(x', \bar{Y}', P^z, b_\perp)}$$

TMDs on Lattice QCD: Intrinsic Soft Function

- ✓ **MILC + CLS ensembles, a = (0.098, 0.12) fm, m_π = 670 MeV;**
- **Largest P^z up to 2.58 GeV;**
- **Eliminated partial high-twist contaminations;**
- ✓ **NLO matching kernel**
- ✓ **Verified the robustness of the approach from different ensembles.**



LPC, PRD109, L091503 (2024), JHEP08, 172 (2023)

TMDs on Lattice QCD: Intrinsic Soft Function

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LPC20 PRL 125,192001 (2020)	547 MeV	N/A	N/A	LO	N/A	No	0.3fm
PKU/ETMC 21 PRL128, 062002 (2022)	827 MeV	N/A	N/A	LO	N/A	No	0.3fm
LPC22 PRD 106, 034509 (2022)	670 MeV	Yes	Yes	NLO	Yes	No	0.6fm
LPC23 JHEP 08,172 (2023)	220 MeV	Yes	Yes	NLO	Yes	No	0.6fm
LPC 25 (preliminary)	300MeV 135MeV	Yes	Yes	NNLL	Yes	Yes	0.9fm

(*) Only the results that deviate from 0 by more than 2σ are considered.

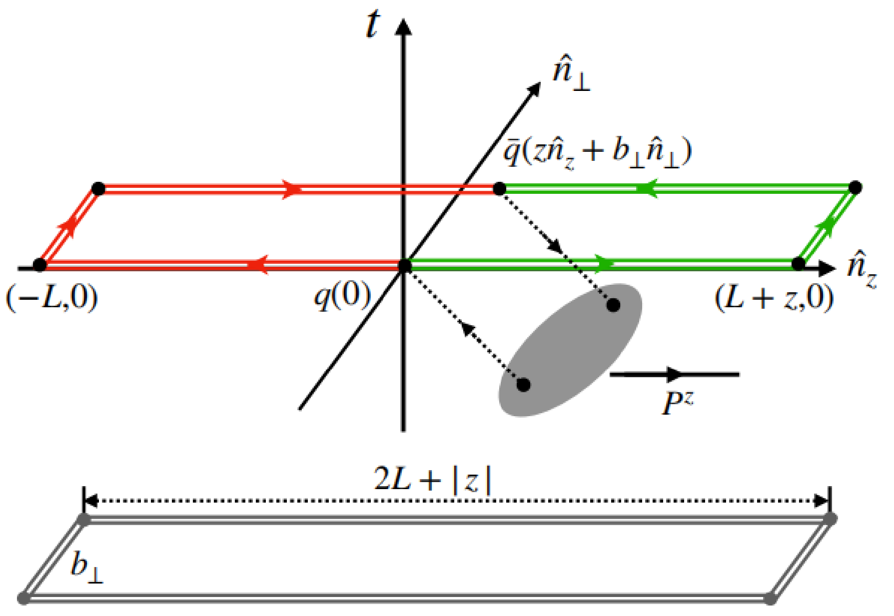
TMDs on Lattice QCD: TMD wave functions

- **The quasi-TMD wave functions can be defined as:**

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta_z) = \lim_{L\to\infty} \int \frac{dzP^z}{2\pi} e^{ixzP^z} \frac{\tilde{\Phi}^{\pm 0}(z, b_\perp, P^z, a, L)}{Z_O(\mu, a)\sqrt{Z_E(2L+z, b_\perp, \mu, a)}},$$

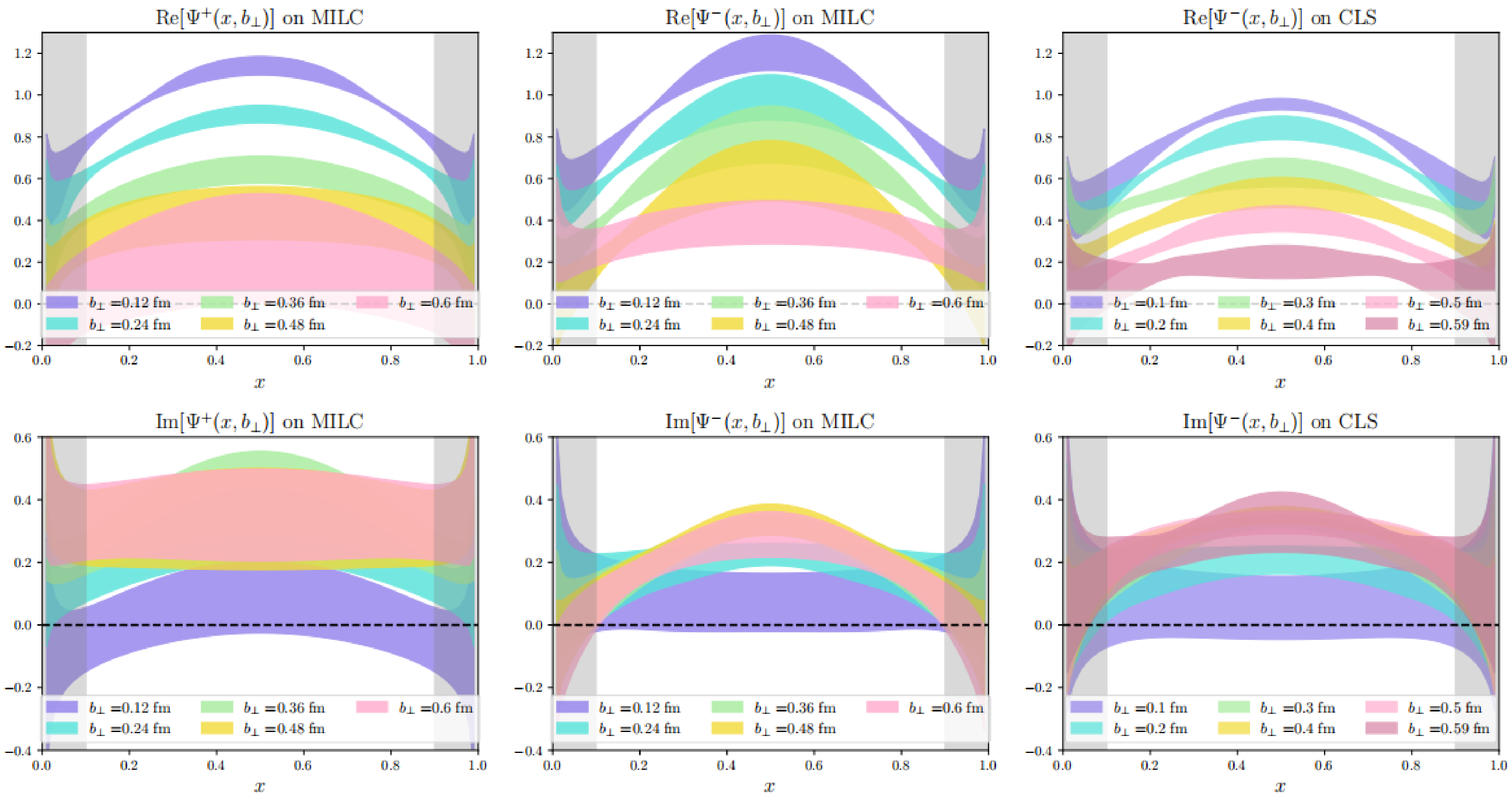
where the bare quasi-TMD wave function can be constructed as :

$$\tilde{\Phi}^{\pm 0}(z, b_\perp, P^z, a, L) = \langle 0|\bar{q}(z\hat{n}_z + b_\perp \hat{n}_\perp)\Gamma U_{\sqsupset,\pm}(L, z, b_\perp) q(0)|P^z\rangle.$$



LPC, PRD109(2024); LPC, PRD 106(2022)

TMDs on Lattice QCD: TMD wave functions



Chu et al., PRD109(2024)

TMDs on Lattice QCD: TMDPDFs

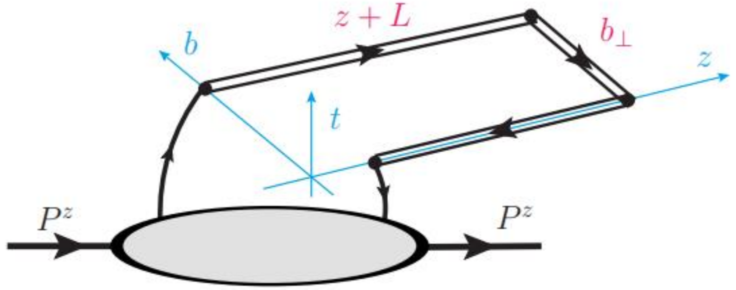
- The quasi-TMD PDFs can be constructed as:

$$\tilde{f}_{\Gamma}(x,b_{\perp},P^{z},\mu)\equiv\lim_{\substack{a\to0\\L\to\infty}}\int\frac{dz}{2\pi}e^{-iz(xP^{z})}\frac{\tilde{h}^{0}_{\Gamma}(z,b_{\perp},P^{z},a,L)}{\sqrt{Z_{E}(2L+z,b_{\perp},a)}Z_{O}(1/a,\mu,\Gamma)}$$

where the bare quasi-TMDPDF matrix element is:

$$\tilde{h}^{0}_{\Gamma}(z,b_{\perp},P^{z},a,L)=\langle P^{z}|\tilde{O}^{0}_{\Gamma,\square}(z,b_{\perp},L;P^{z})|P^{z}\rangle$$

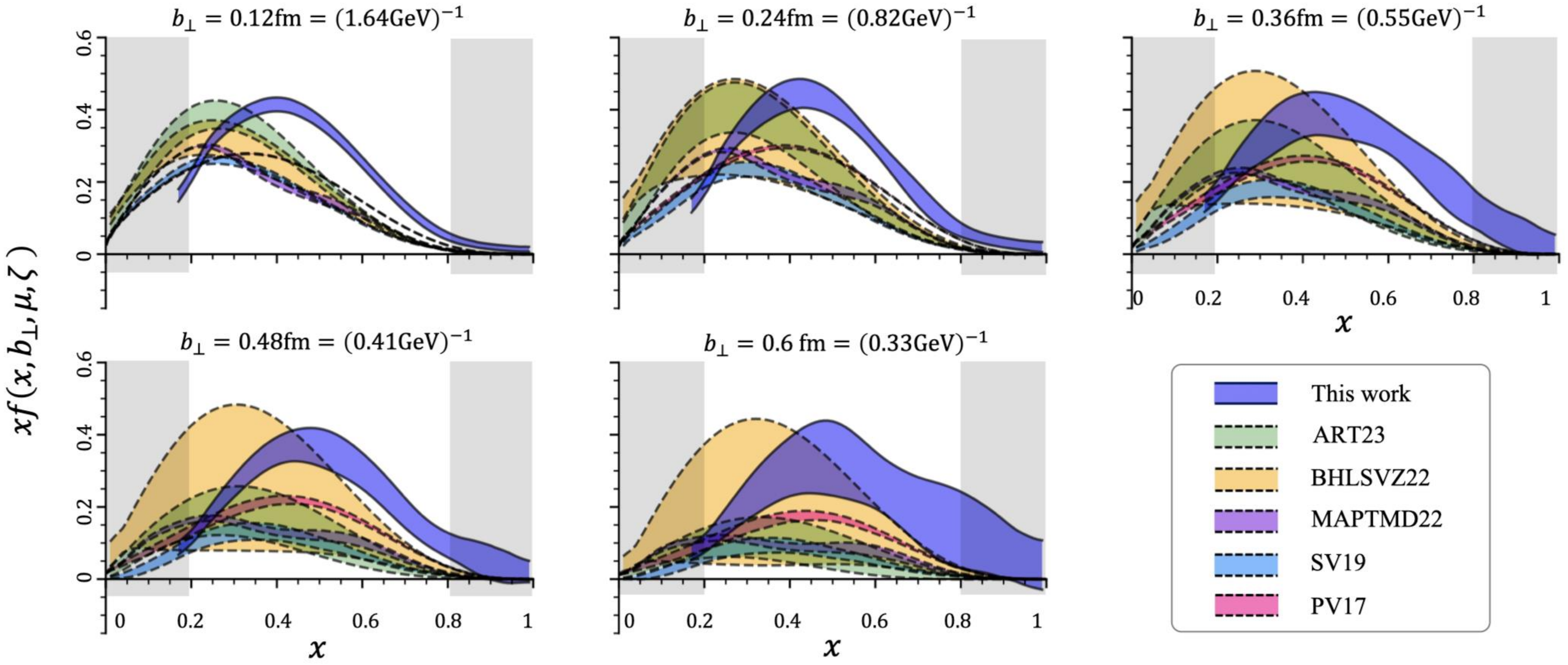
$$\tilde{O}^{0}_{\Gamma,\square}(z,b_{\perp},L)\equiv\bar{\psi}(b_{\perp}\hat{n}_{\perp})\Gamma U_{\square,L}(b_{\perp}\hat{n}_{\perp},z\hat{n}_{z})\psi(z\hat{n}_{z})$$



LPC. PRD 109 (2024) 11, 114513

TMDs on Lattice QCD: TMDPDFs

➤ **The first lattice QCD calculation of nucleon unpolarized TMDPDF:**

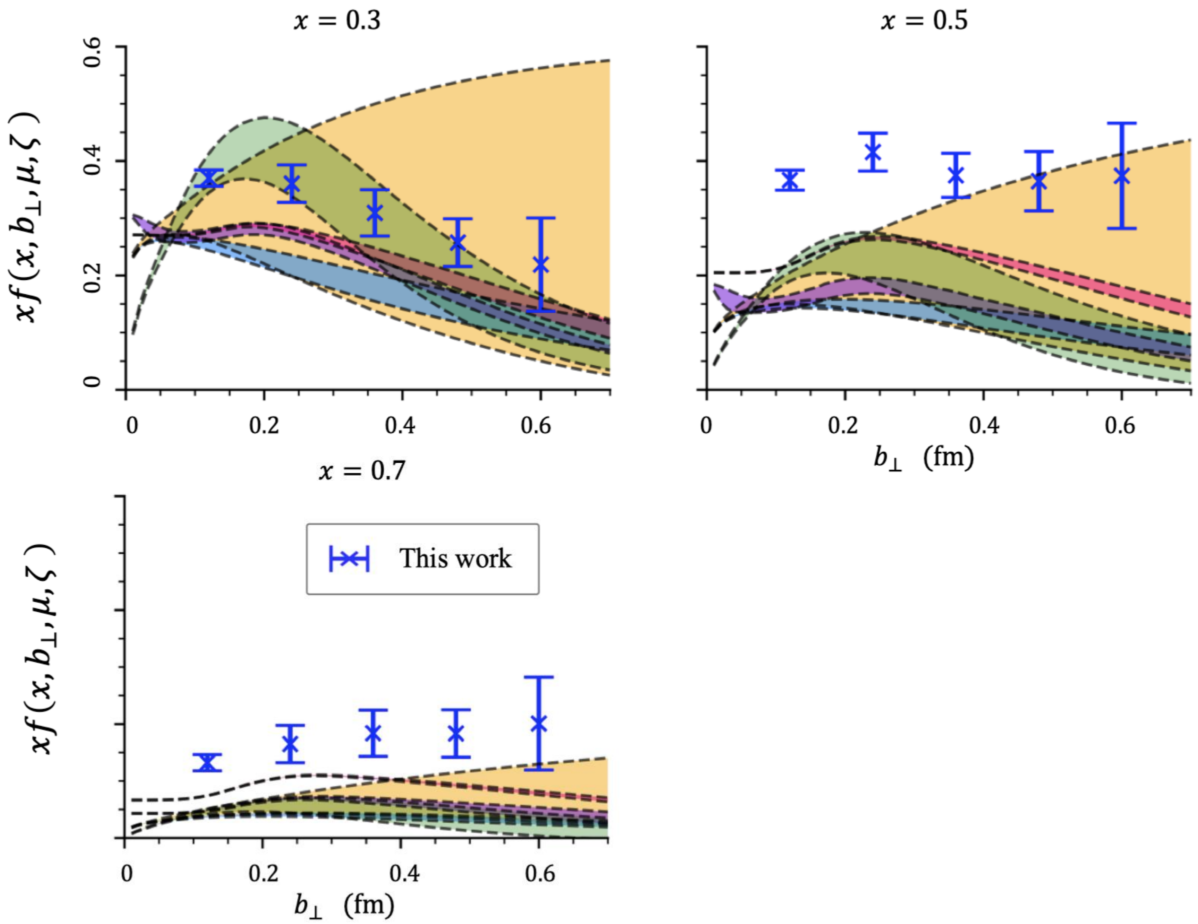


[ART 25: see talk by Valentin Moos on Tuesday]

LPC. PRD 109 (2024) 114513

TMDs on Lattice QCD: TMDPDFs

- MILC ensemble, $a = 0.12$ fm; $m_\pi = (220, 310)$ MeV, largest P^z up to 2.58 GeV;
- ✓ Chiral and infinity momentum extrapolation;
- ✓ NNLO + RGR matching kernel
- ✓ The systematic uncertainty from the operator mixing has been taken into account
- ✓ More comprehensive analysis is under progress



LPC. PRD 109 (2024) 114513

TMDs on Lattice QCD: Boer-Mulders Functions

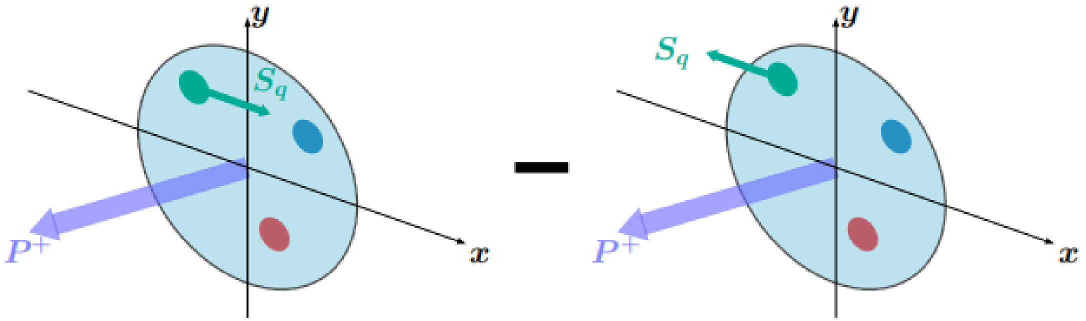
➢ **The quasi-Boer-Mulders Functions can be constructed as:**

$$\tilde{f}(z,b_\perp,P^z,a) \equiv \lim_{L\to\infty} \frac{\langle P|\tilde{O}(z,L,b_\perp)|P\rangle}{\sqrt{Z_E(2L+z,b_\perp,a)}}$$

where the bare matrix element is:

$$\tilde{O}(z,L,b_\perp) \equiv \bar{\psi}(b_\perp\hat{n}_\perp)i\sigma^{yt}\gamma_5 U_{\sqsubset,L}(b_\perp\hat{n}_\perp,z\hat{n}_z,L)\psi(z\hat{n}_z)$$

Boer-Mulders: transversely-polarized quarks

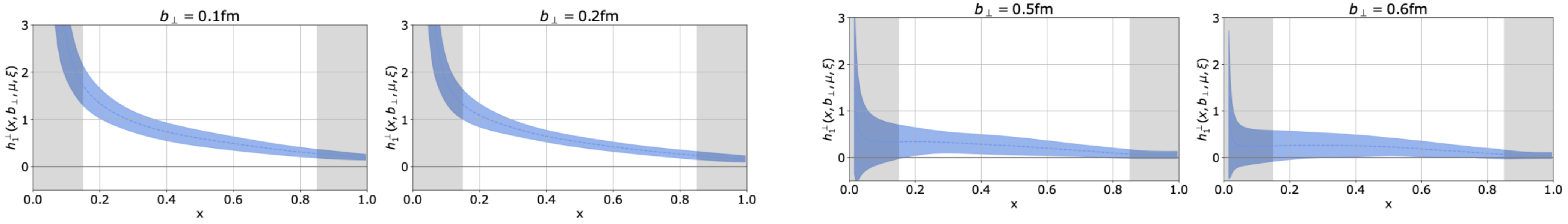


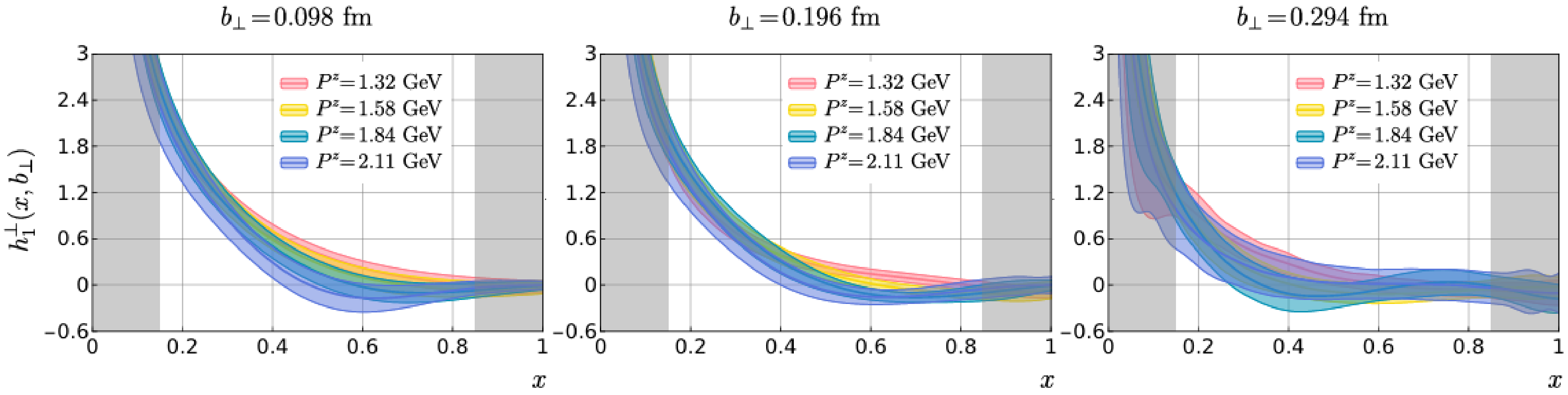
LPC, PRD 111, 094507 (2025); JHEP 08 (2025) 086

TMDs on Lattice QCD: Boer-Mulders Functions

- **Comparison of physical Boer-Mulders TMDPDFs at different P^z and $b_\perp$:**

LPC, PRD 111, 094507 (2025)





LPC, JHEP 08 (2025) 086

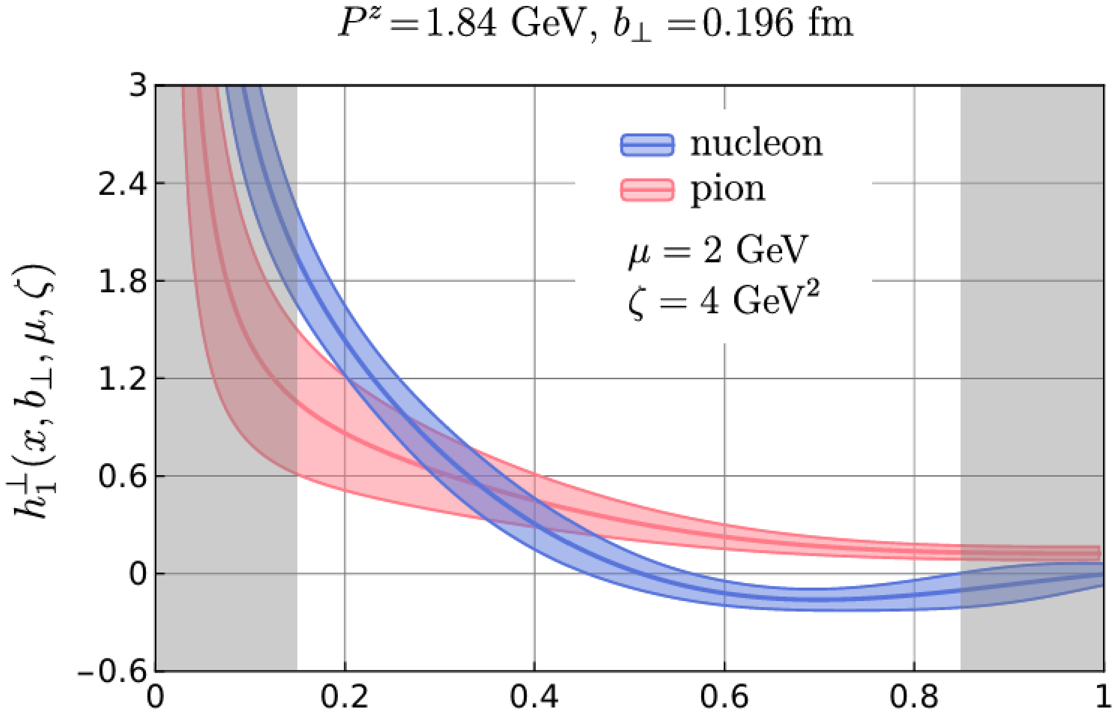
TMDs on Lattice QCD: Boer-Mulders Functions

- **CLS ensemble, $a = 0.098$ fm;**
- **$m_\pi = 338$ MeV, largest P^z up to 2.11 GeV;**

✓ **Infinity momentum extrapolation;**

✓ **NNLO + RGR matching kernel;**

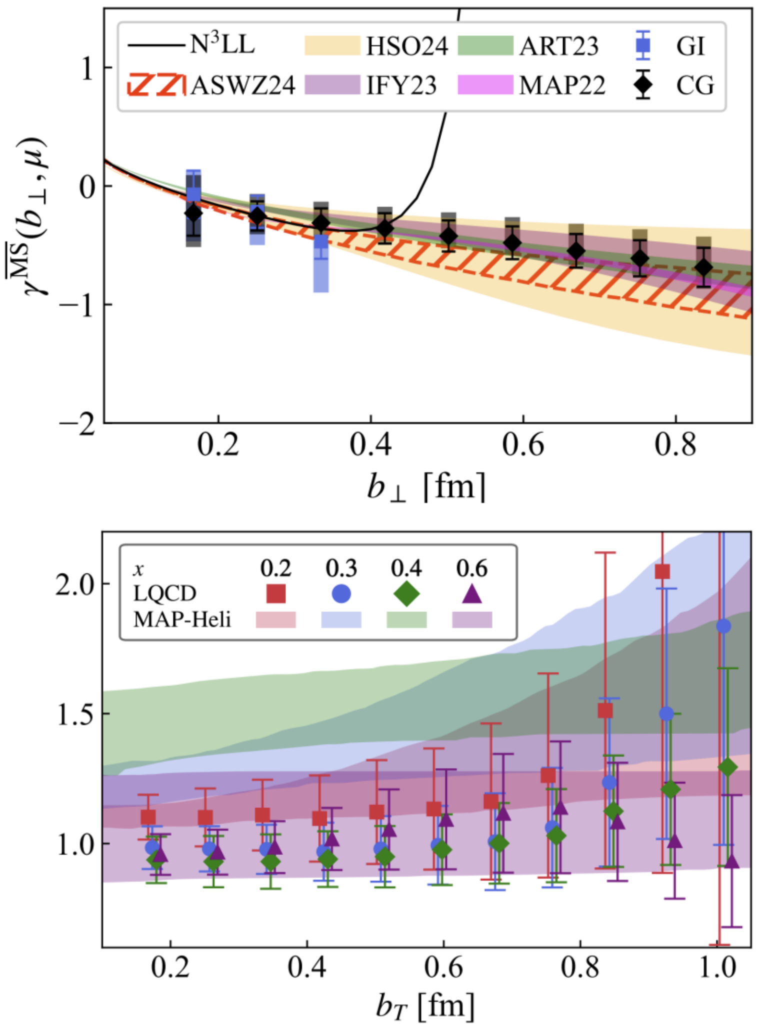
✓ **compare the results in the nucleon and pion.**



LPC, JHEP 08 (2025) 086

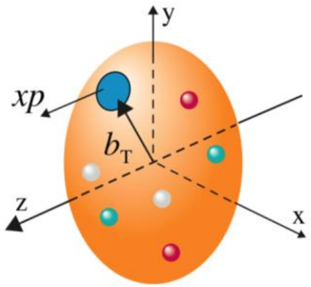
TMDs on Lattice QCD: CG method

- **Under Coulomb gauge fixing, Wilson lines vanish, signal-to-noise ratio can be significantly improved.** *Yong Zhao, Phys.Rev.Lett. 133 , 241904 (2024)*
- **Calculations of CS kernel have demonstrated this improvement, and its accuracy remains to be validated for larger separation**
- **Preliminary result for the ratio of helicity to unpolarized TMDPDFs has been performed.**



Bollweg, et.al.. PLB 852 (2024), 138617; 2505.18430

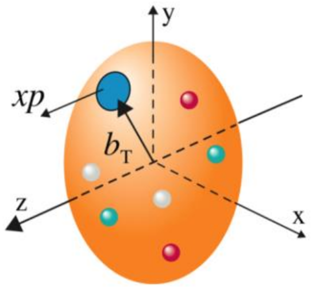
Summary and Outlook



Lattice QCD can investigate the 3D structures of nucleons from the first-principle:

- ✓ **Rapid progress in recent years include:**
  - ✓ **Collins-Soper kernel and intrinsic soft function: relatively mature, yet certain theoretical issues remain to be addressed**
  - ✓ **TMDWF and TMDPDFs: preliminary; further theoretical and lattice QCD efforts are required.**

Summary and Outlook



In the future, more are anticipated:

- **precise calculations of unpolarized PDFs: multiple lattice spacings, multiple pion masses, extrapolation to physical point, b~1fm(hadron size);**
- **Enable computations of spin-dependent TMDPDFs;**
- **CG, enhanced operators, grading flow…**
- **…**

Thank you for your attention!

Backup