

TMD Theory

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SPIN2025@Qingdao, 09/25/2025

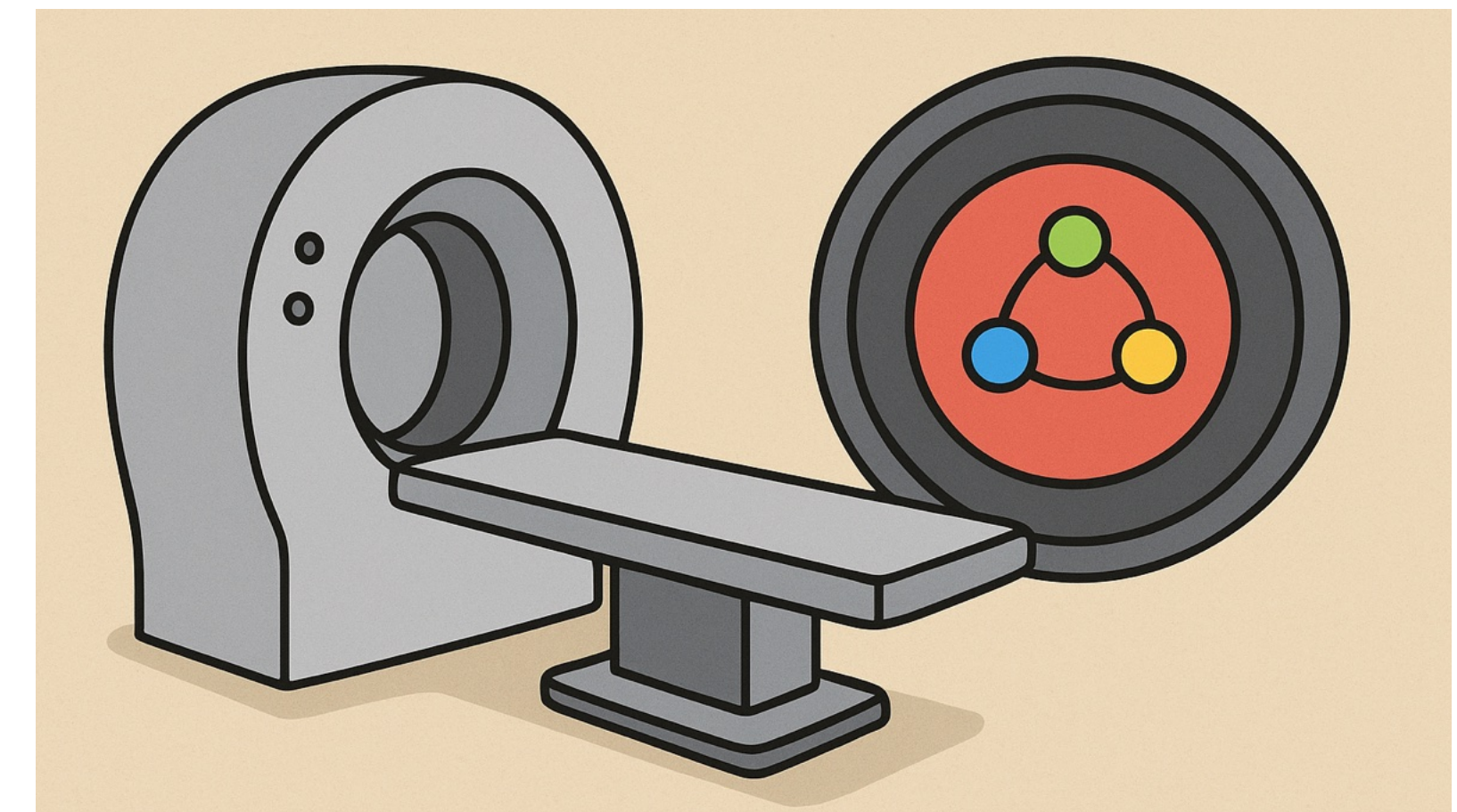
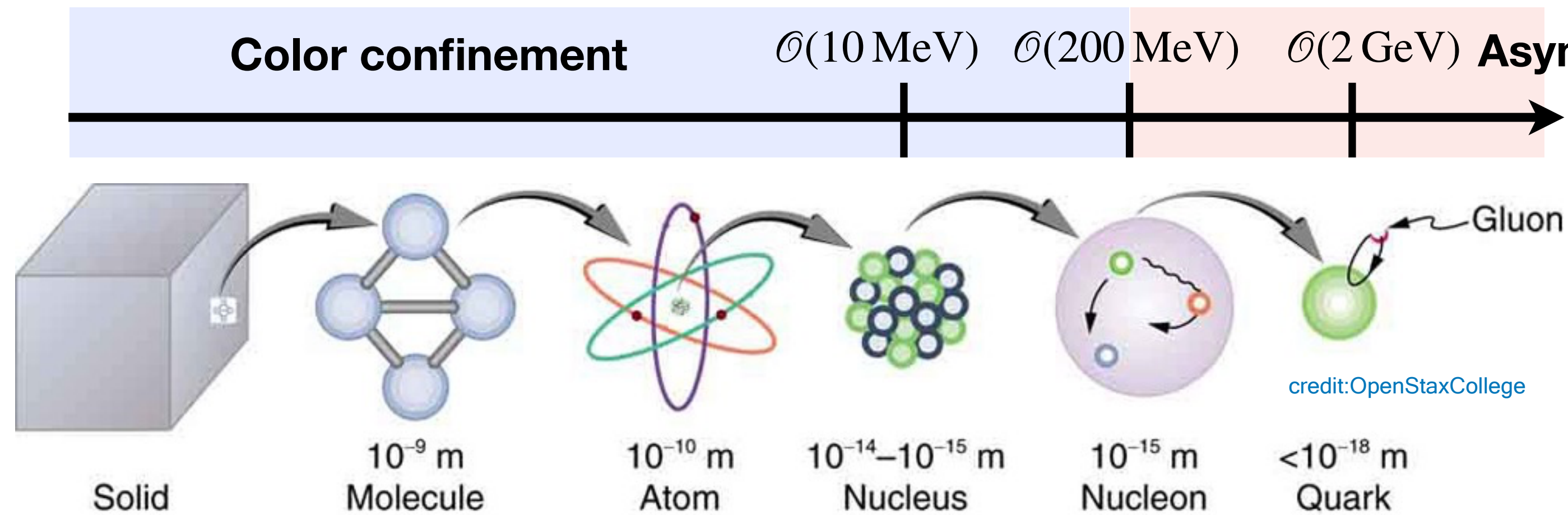
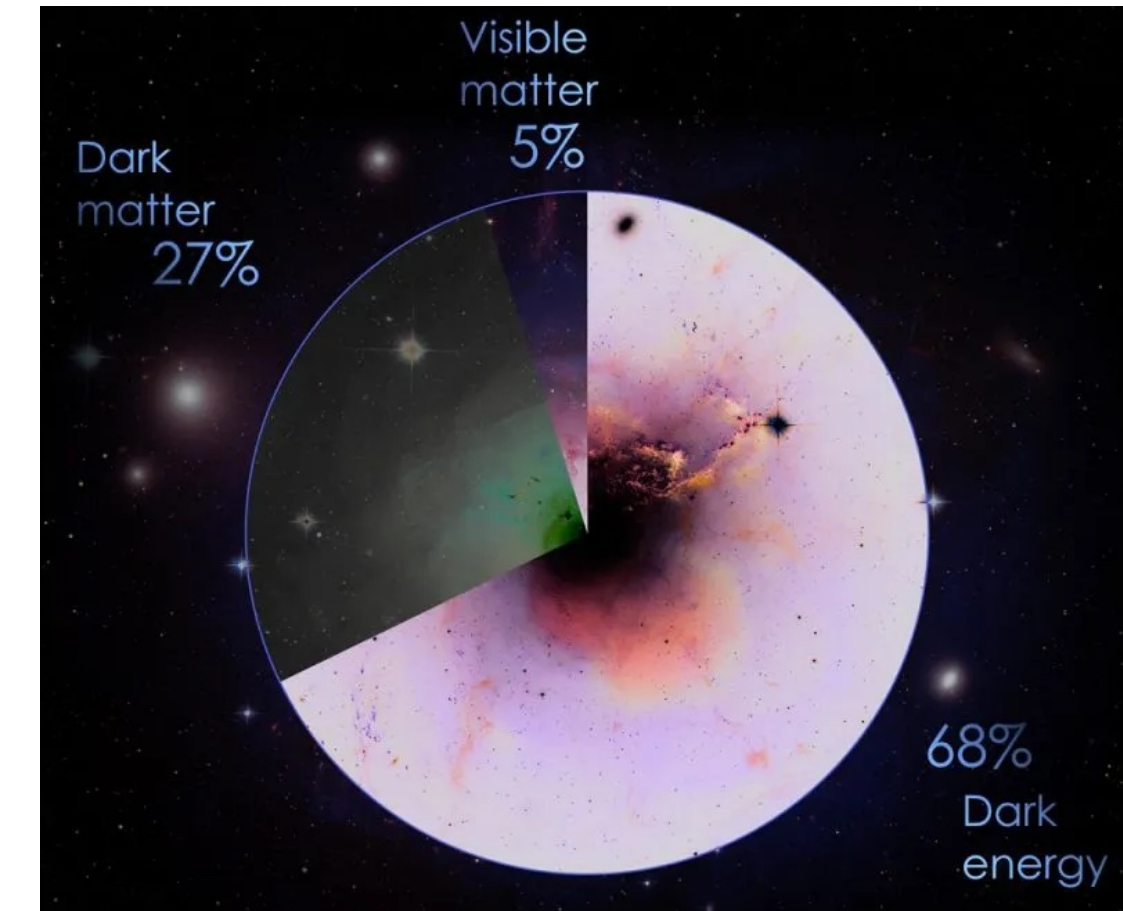
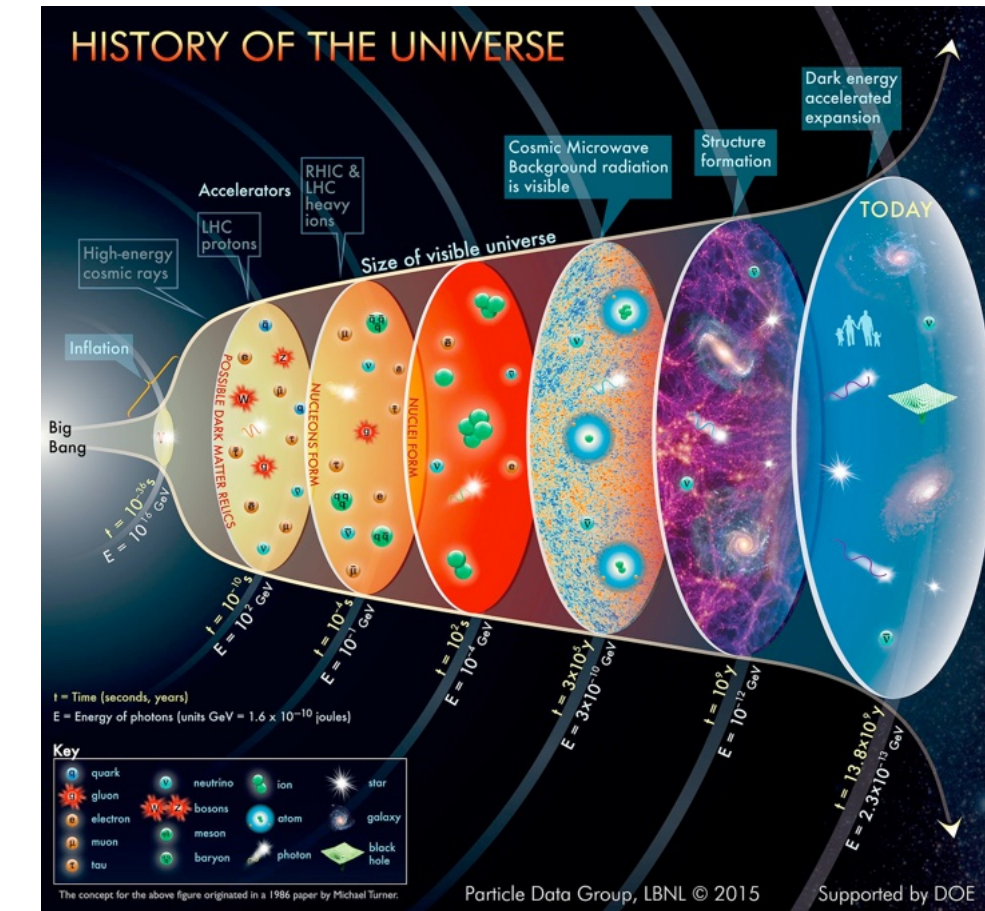


26th International
Symposium on Spin Physics
A Century of Spin

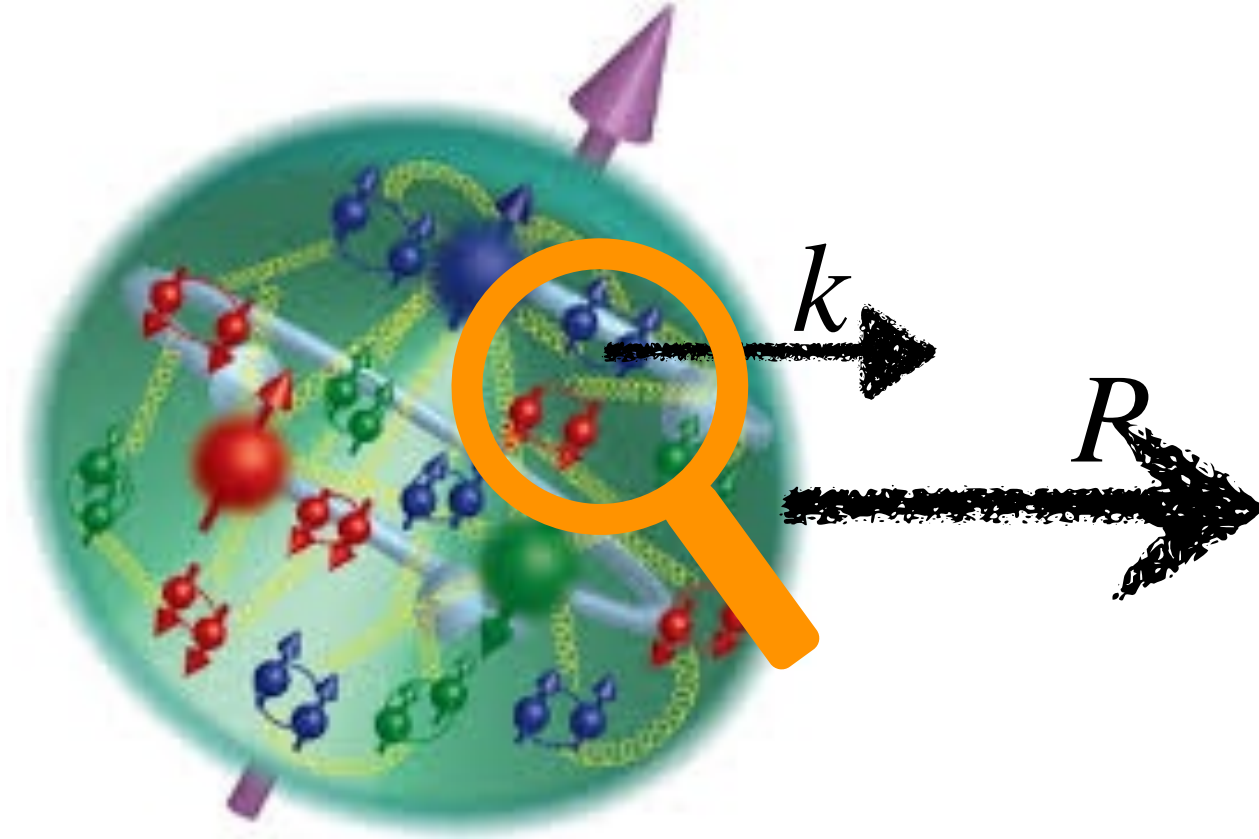


Fundamental structures of matter

- ❖ What is the origin of this universe?
 - ❑ QCD at high temperature, emergent hadrons
- ❖ What are the building blocks of visible matter?
 - ❑ Internal structure of hadrons and nuclei
- ❖ What does bind us all?
 - ❑ Strong interaction between quarks and gluons



Internal structure of hadrons



- ❖ Hadron structure $\langle P, S | \mathcal{O}(\bar{\psi}, \psi, A) | P, S \rangle$ is a quantum probability.
- ❖ Parton Distribution Functions (PDFs) $f_{i/p}(x, Q^2)$ can be extracted from data using a controllable approximation.

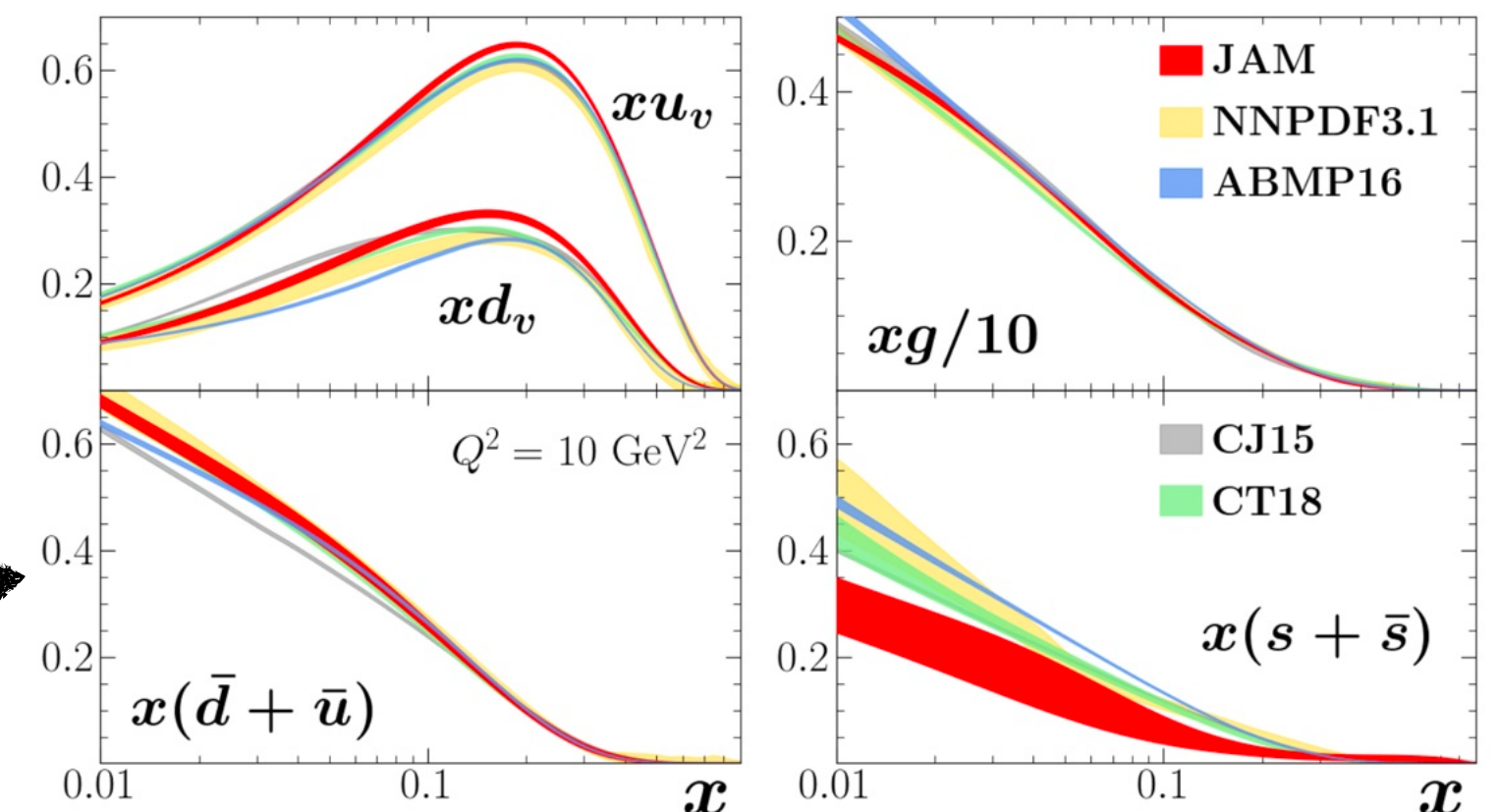
Factorization: approximation

$$\sigma_{\text{DIS}} \propto \left| \text{Diagram 1} \right|^2 \approx \left| \text{Diagram 2} \right|^2 \otimes \left| \text{Diagram 3} \right|^2 + \mathcal{O}\left(\frac{1}{Q^n}\right)$$

Power corrections: Approximation

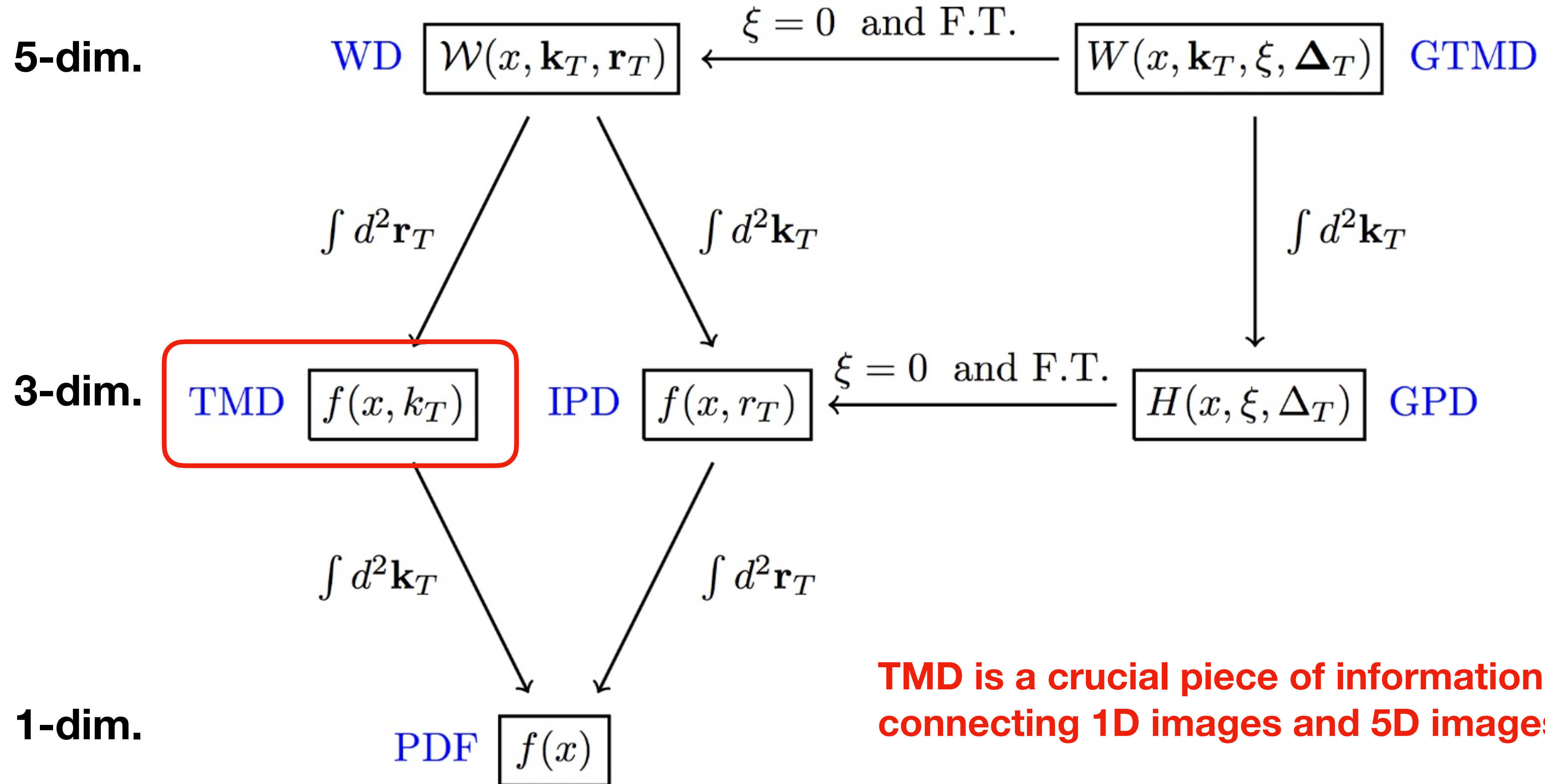
Nonperturbative PDFs:
Hadron structure

Hard part:
Probe



Multi-dimensional quantum images

R. Boussarie, *et al.* [arXiv:2304.03302 [hep-ph]].



Nucleon/Nuclear Femtography

Origin of nucleon's spin

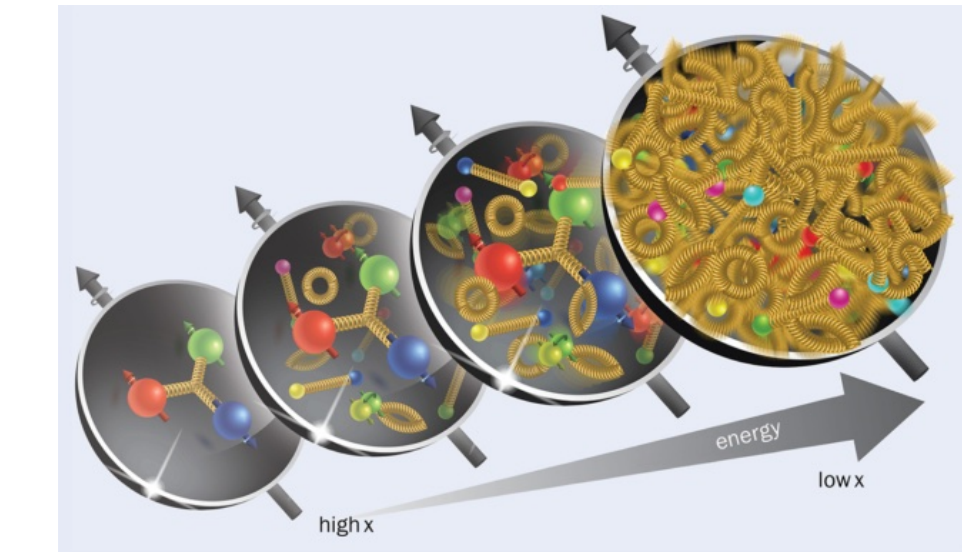
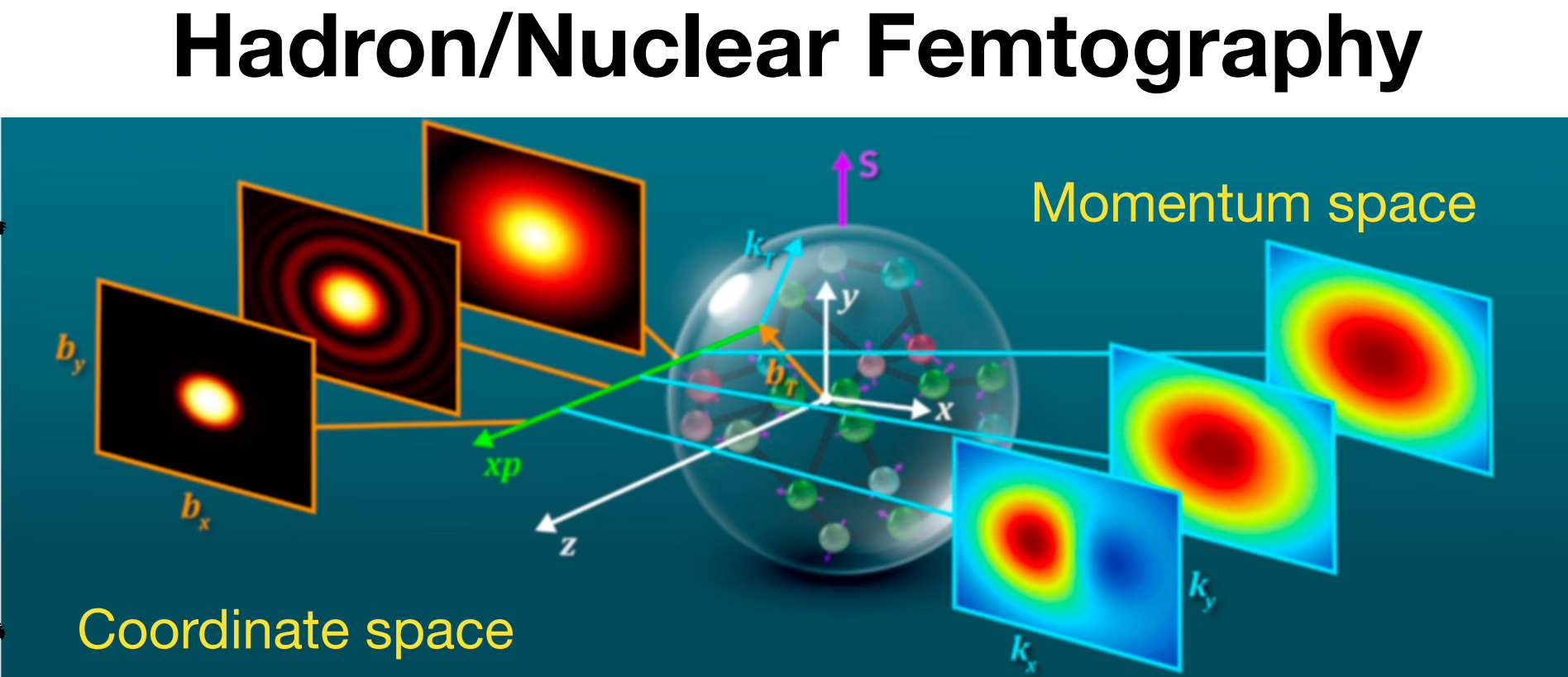


Quark and gluon internal motion

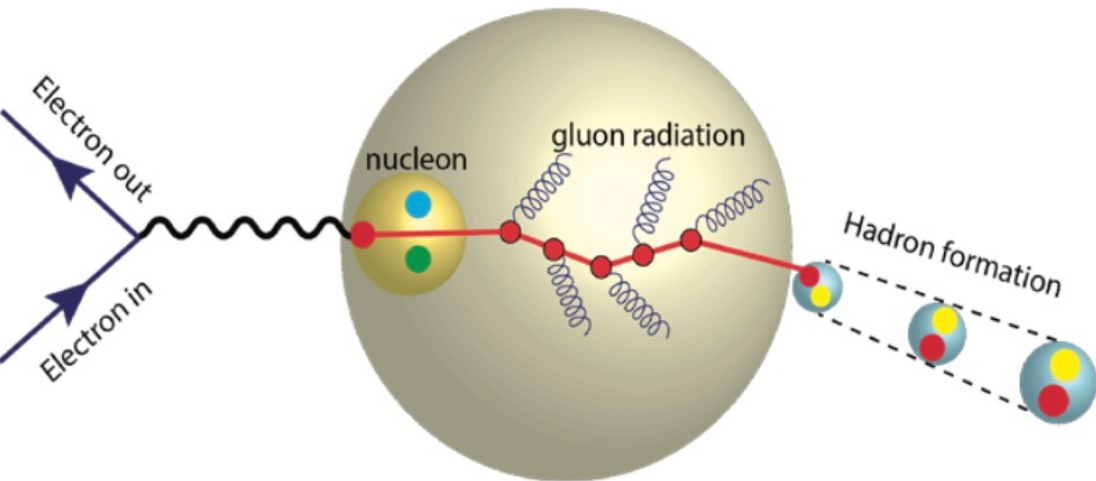
Quark spin

Gluon spin

Origin of nucleon's mass

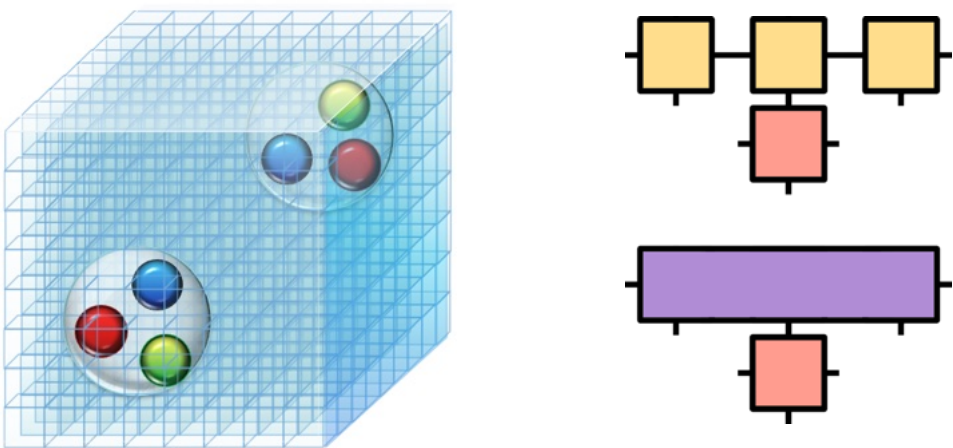


Dense gluonic matter

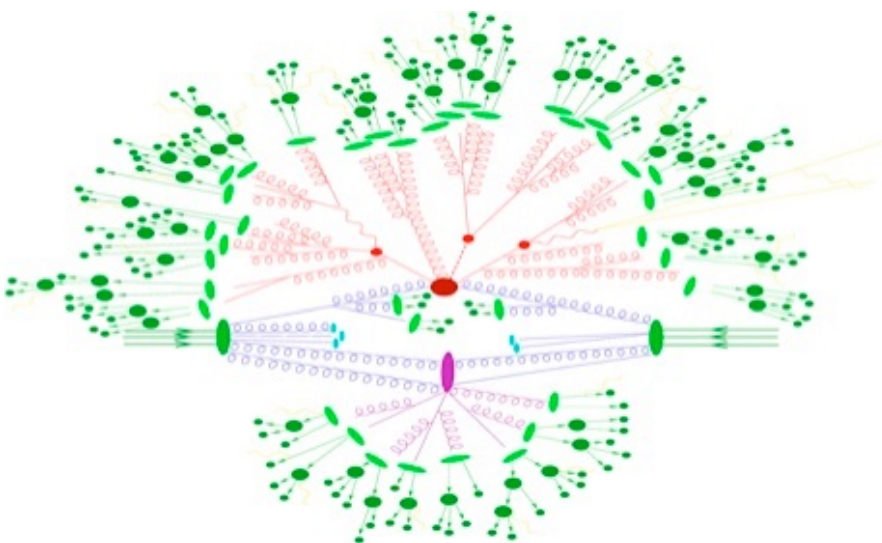
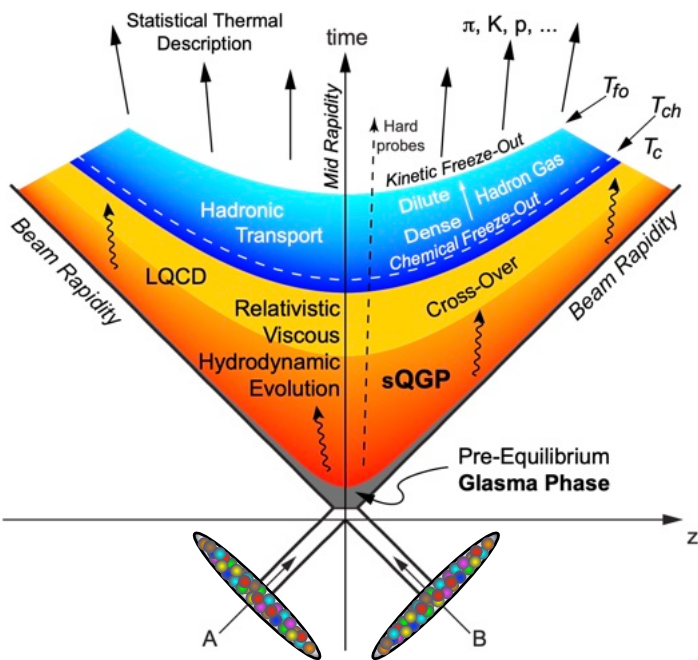


Hadronization

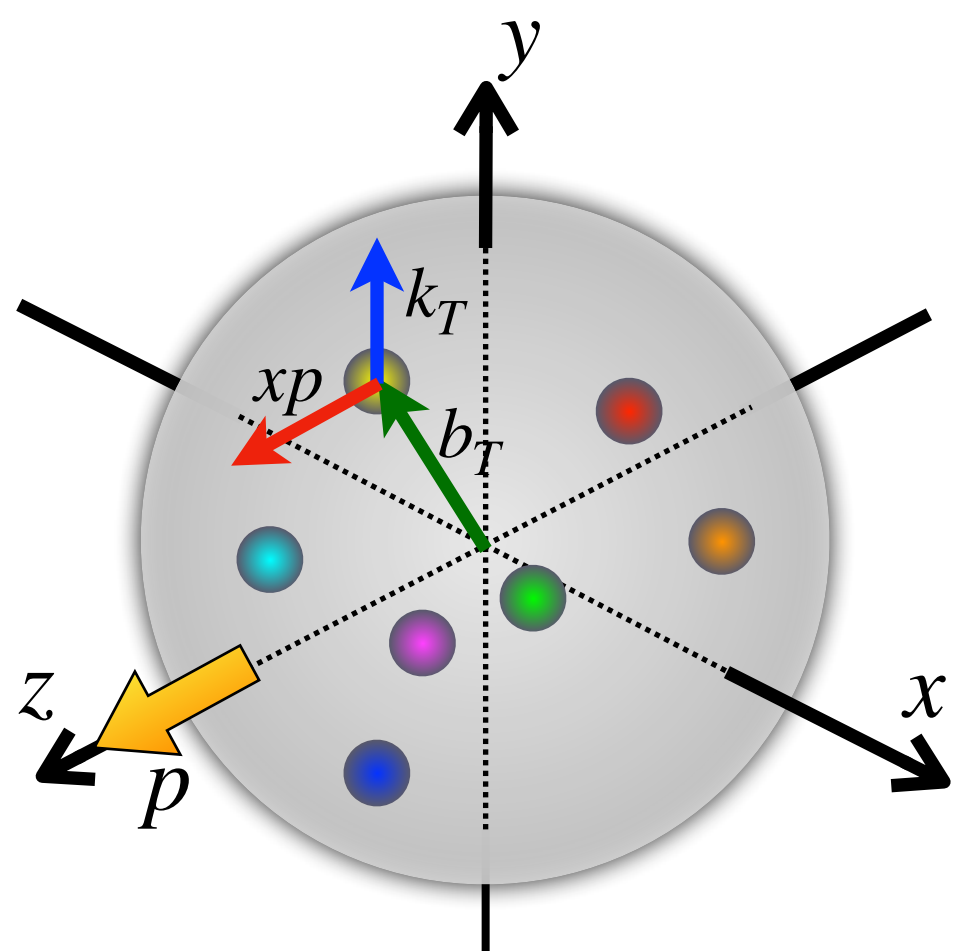
Development of numerical simulations of quantum many-body systems: lattice QCD, Tensor Network

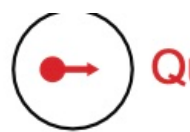


Phenomenology in heavy-ion physics: initial and final state interactions



3D parton structure of hadrons



Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

- ❖ More sensitive to the QCD dynamics binding quarks and gluons into a hadron.
- ❖ Parton's confined motion: $k_T \sim 1/\text{fm} \ll Q$
- ❖ Hard probe Q does not clarify the details of the nucleon structure.

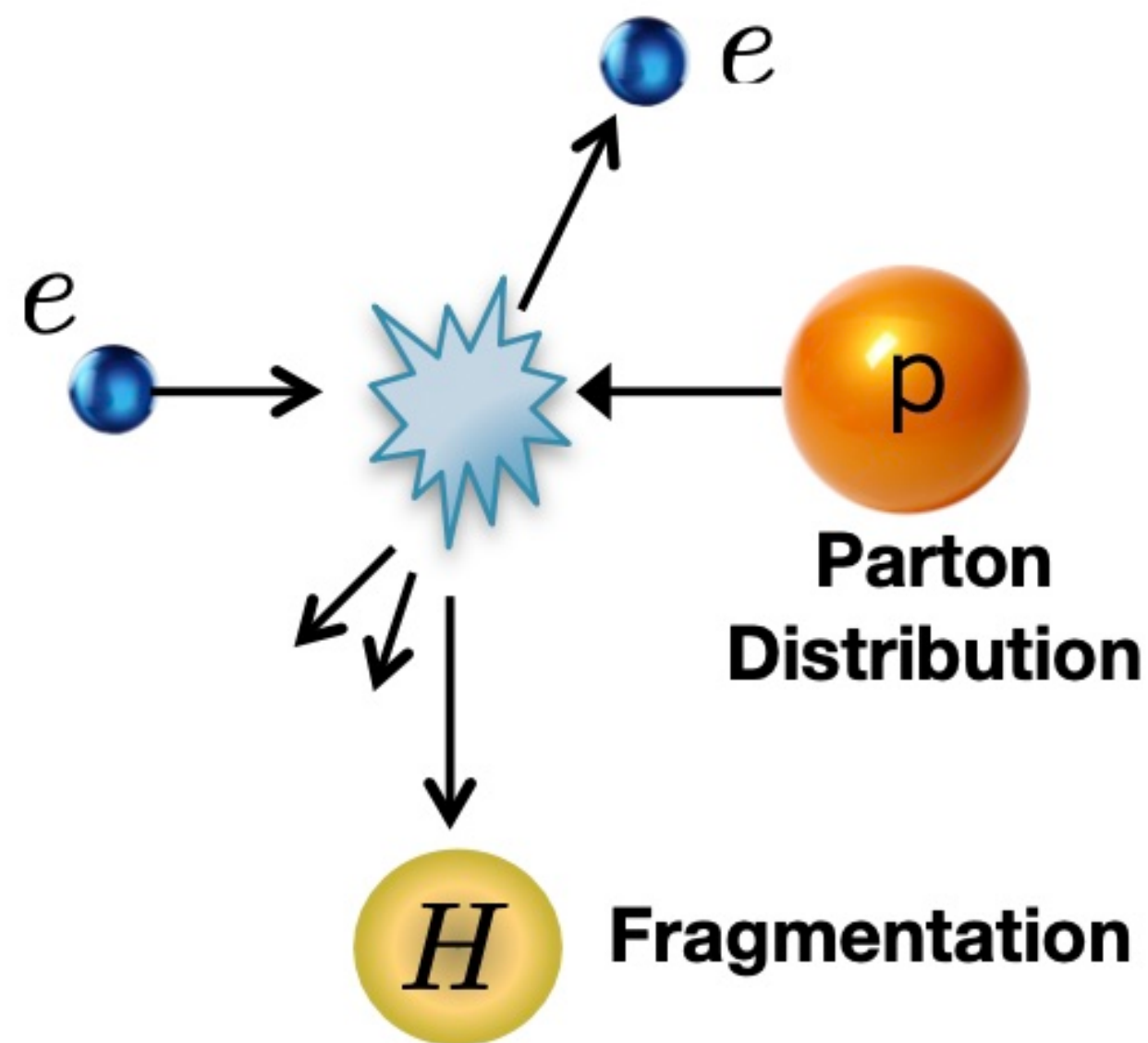
Observables with $Q_1 \gg Q_2 \gg \Lambda_{\text{QCD}}$ are required!

- ✓ Hard scale Q_1 localizes the probe to see the particle nature of quarks and gluons.
- ✓ Hard but "softer" scale Q_2 is sensitive to the emergent regime of nucleon structure.

* Analogous tables for gluon, fragmentation functions

Primary observables for TMD studies

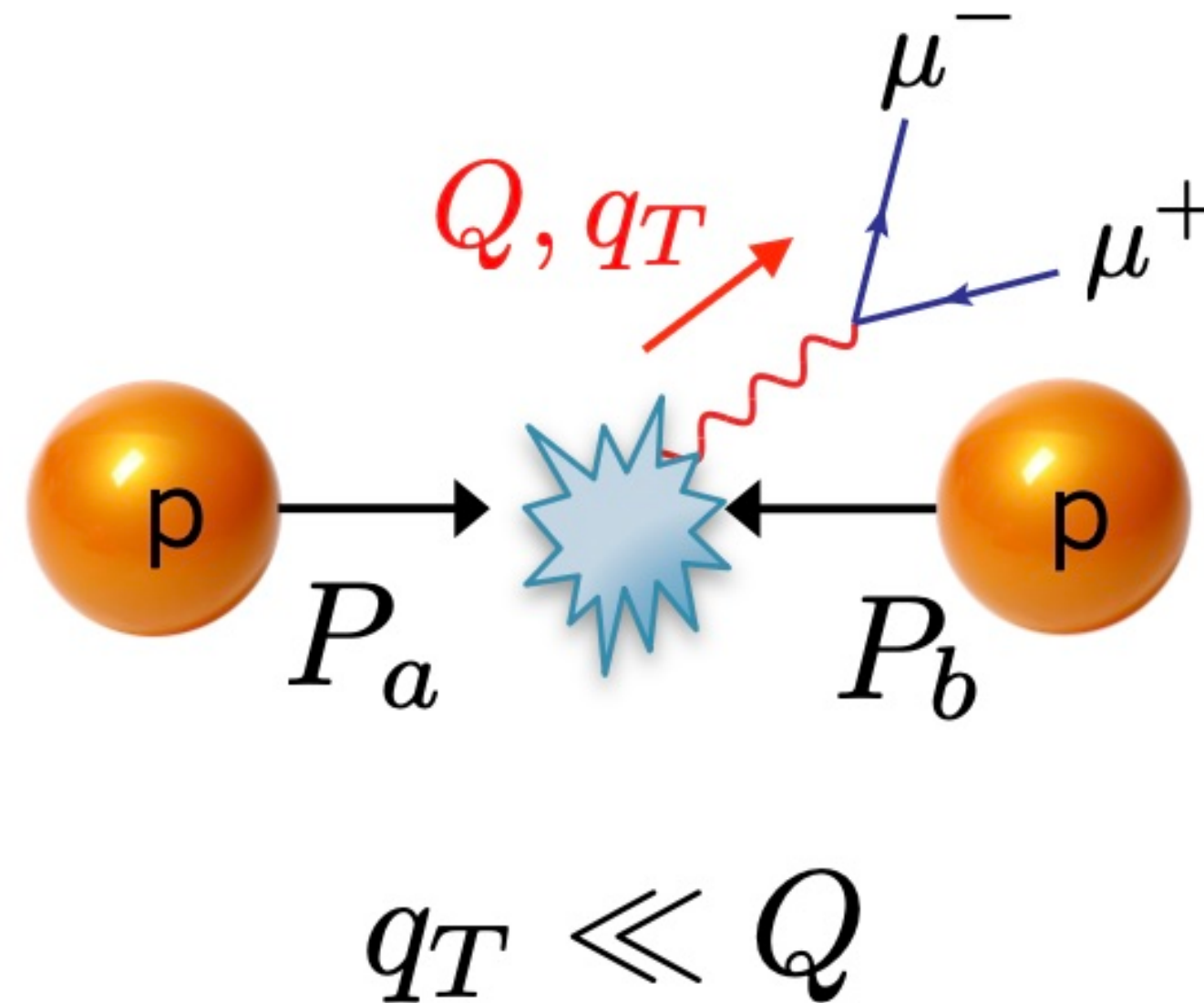
Semi-Inclusive DIS



$$d\sigma \sim f_{q/p}(x, k_T) \otimes D_q^h(z, k_T)$$

The transverse momentum of a produced hadron in the photon-hadron frame is smaller than the momentum transfer Q .

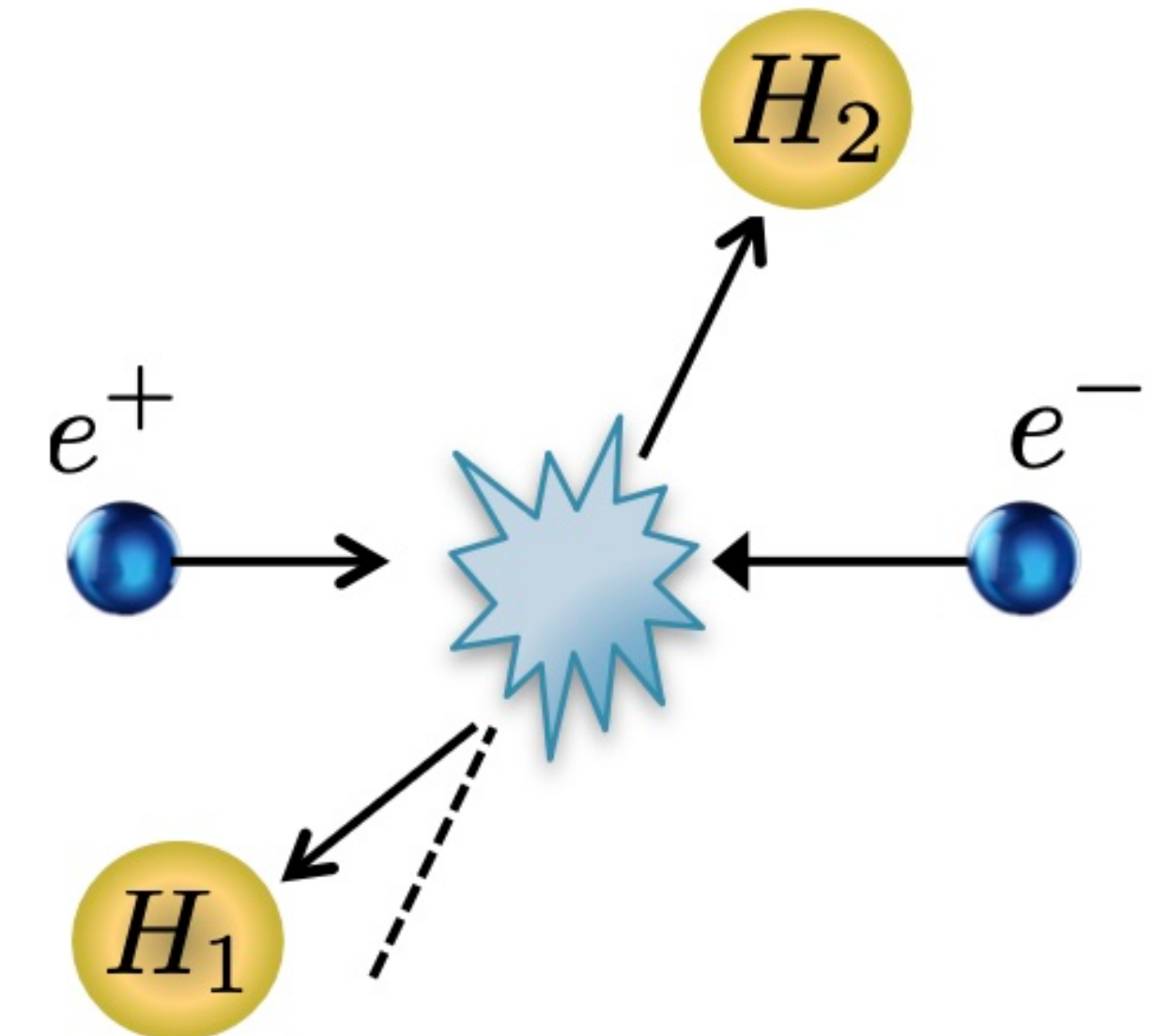
Drell-Yan



$$d\sigma \sim f_{q/p}(x_a, k_T) \otimes f_{\bar{q}/p}(x_b, k_T)$$

The transverse momentum of the dilepton is smaller than the c.o.m energy Q .

Dihadron in e^+e^-



$$d\sigma \sim D_q^{H_1}(z_1, k_T) \otimes D_q^{H_2}(z_2, k_T)$$

The transverse momentum of the dihadron system is smaller than the c.o.m energy Q .

An overview of TMD factorization

Semi-Inclusive DIS

$$\frac{d\sigma}{dx dz dQ^2 dp_T^2} \propto \sum_f e_f^2 H(Q, \mu) \int d^2 b_T e^{iq_T \cdot b_T} f_1(x, b_T; \mu, \zeta) D_1(z, b_T; \mu, \zeta') + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

Drell-Yan

$$\frac{d\sigma}{dy dQ dq_T} \propto H(Q, \mu) \int d^2 b_T e^{iq_T \cdot b_T} f_1(x_a, b_T; \mu, \zeta_a) f_1(x_b, b_T; \mu, \zeta_b) + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

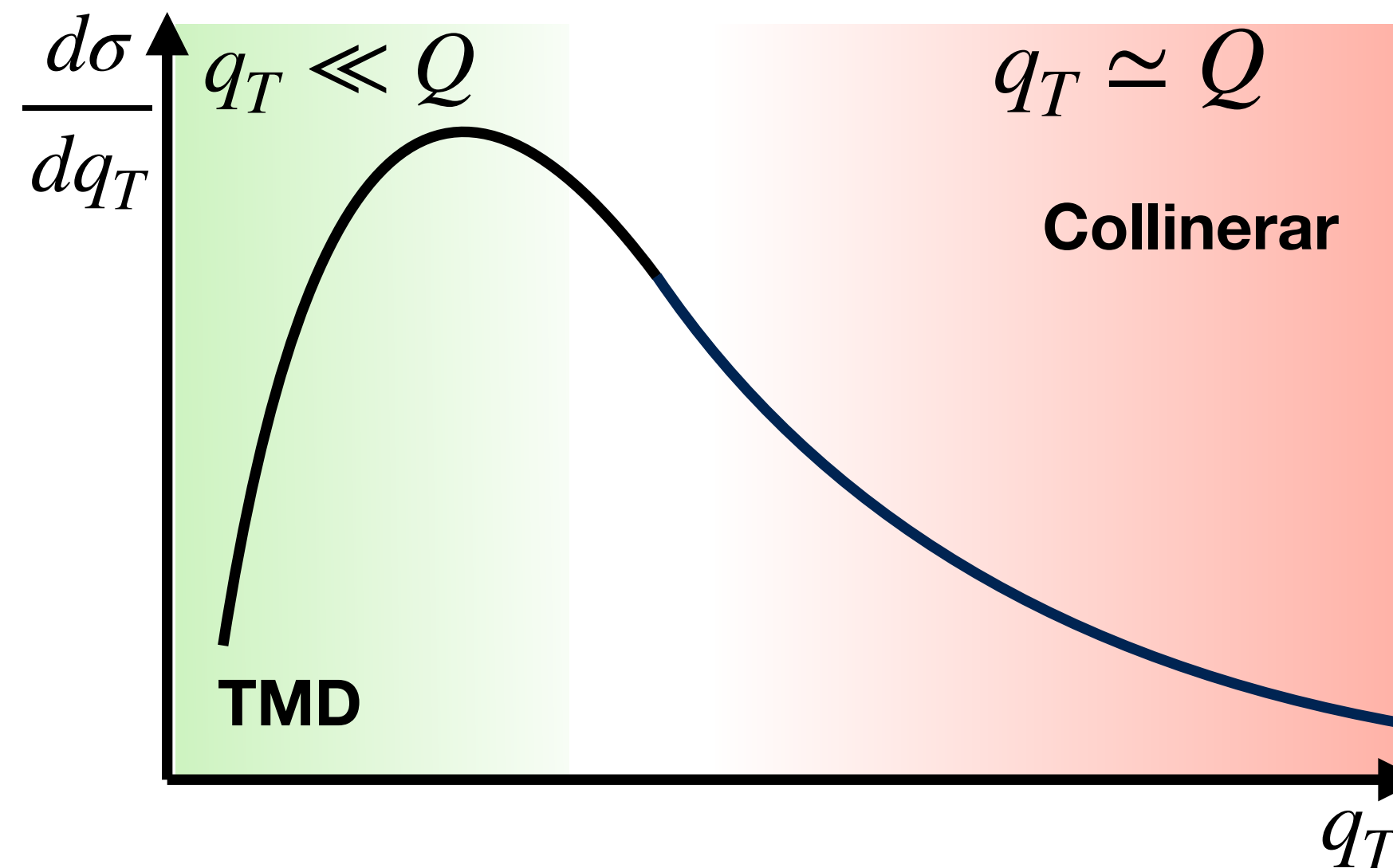
Hard parts
Fourier Transform of $f_1(x, k_T)$ or $D_1(z, k_T)$

Power correction (higher twist)

I. Balitsky and A. Tarasov, JHEP05, 150 (2018)
 A. Arroyo-Castro, I. Scimemi and A. Vladimirov, JHEP06, 202 (2025)
 ...

TMD factorization is applicable
 at **large b_T (small q_T)**.

$$q_T^2/Q^2 \ll 1 \rightarrow b_T \lesssim \Lambda_{\text{QCD}}^{-1}$$



$$k_T \sim b_T^{-1} \sim Q \gg \Lambda_{\text{QCD}}$$

$f_1(x, k_T)$ is matched to a fixed order result (will be mentioned later).

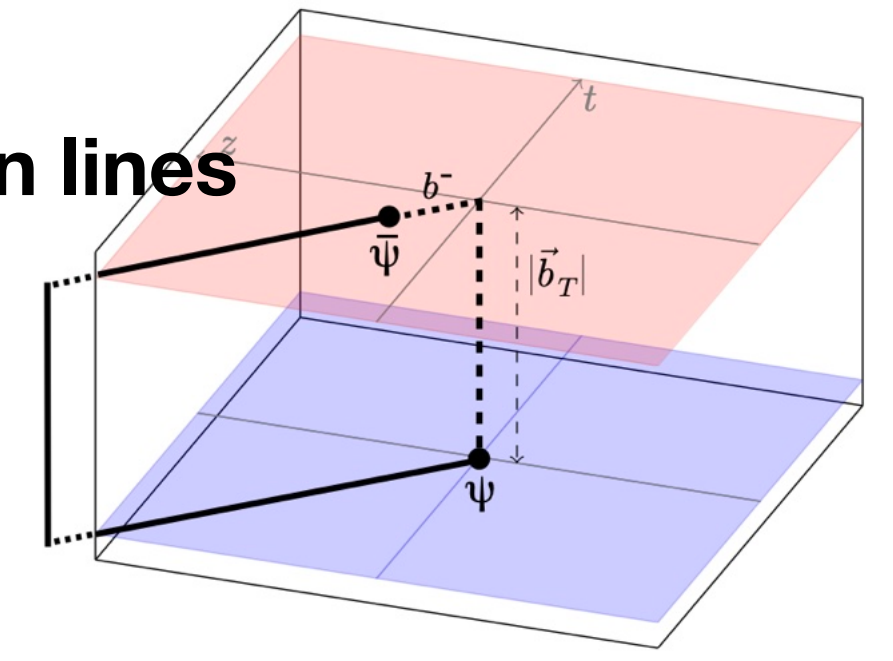
Operator definition of unpolarized TMD PDFs

Bare beam function:

$$\text{F.T. } B_{i/p}(x, k_{\perp}) = \int \frac{d^2 b_{\perp}}{(2\pi)^2} e^{+ik_{\perp} \cdot b_{\perp}} \tilde{B}_{i/p}(x, b_{\perp})$$

$$\tilde{B}_{i/p}(x, b_{\perp}) = \frac{1}{2} \int \frac{db^{-}}{2\pi} e^{-ixP^{+}b^{-}} \langle P | \bar{\psi}(b^{-}, b_{\perp}) \mathcal{W}(b^{-}, b_{\perp})^{\dagger} \gamma^{+} \mathcal{W}(0) \psi(0) | P \rangle$$

Staple-shaped Wilson lines



❖ The precise direction of $\mathcal{W}$ depends on processes: e.g., future-pointing in SIDIS (FSI), past-pointing in Drell-Yan (ISI). The sign of Sivers is not universal: $f_{1T}^{\perp \text{DY}} = -f_{1T}^{\perp \text{SIDIS}}$.

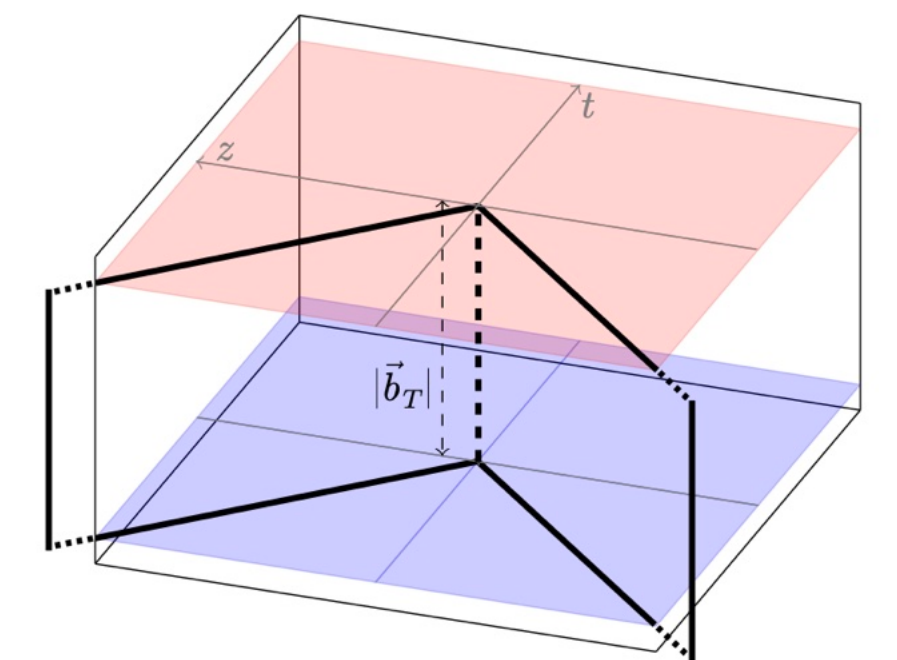
❖ Rapidity divergence can be subtracted by a soft factor: $\tilde{B} / \tilde{S}^{\text{subt}}$.

J. Collins, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.32, 1-624 (2011)

A generic form of TMD PDF for Drell-Yan:

$$\tilde{f}_{i/p}(x, b_{\perp}, \mu, \zeta) = \lim_{\epsilon \rightarrow 0, \tau \rightarrow 0} \underbrace{Z_{\text{UV}}(\epsilon, \mu, \zeta)}_{\text{Renormalization}} \underbrace{\tilde{B}_{i/p}(x, b_{\perp}, \epsilon, \tau, \zeta)}_{\text{Collinear mode}} \underbrace{\sqrt{\tilde{S}(b_{\perp}, \epsilon, \tau)}}_{\text{Soft mode}}$$

ϵ : UV regulator, τ : Rapidity regulator



Rectangle-shaped Soft Wilson lines

TMD evolution

CSS: Collins-Soper-Sterman
SCET: Soft Collinear Effective Theory

$$\frac{\partial \ln f(x, b_T; \zeta, \mu)}{\partial \ln \sqrt{\zeta}} \stackrel{\text{CSS}}{=} K(b_T; \mu) \stackrel{\text{SCET}}{=} -2D(b_T; \mu)$$

Collins-Soper evolution eq.
known up to 4-loop

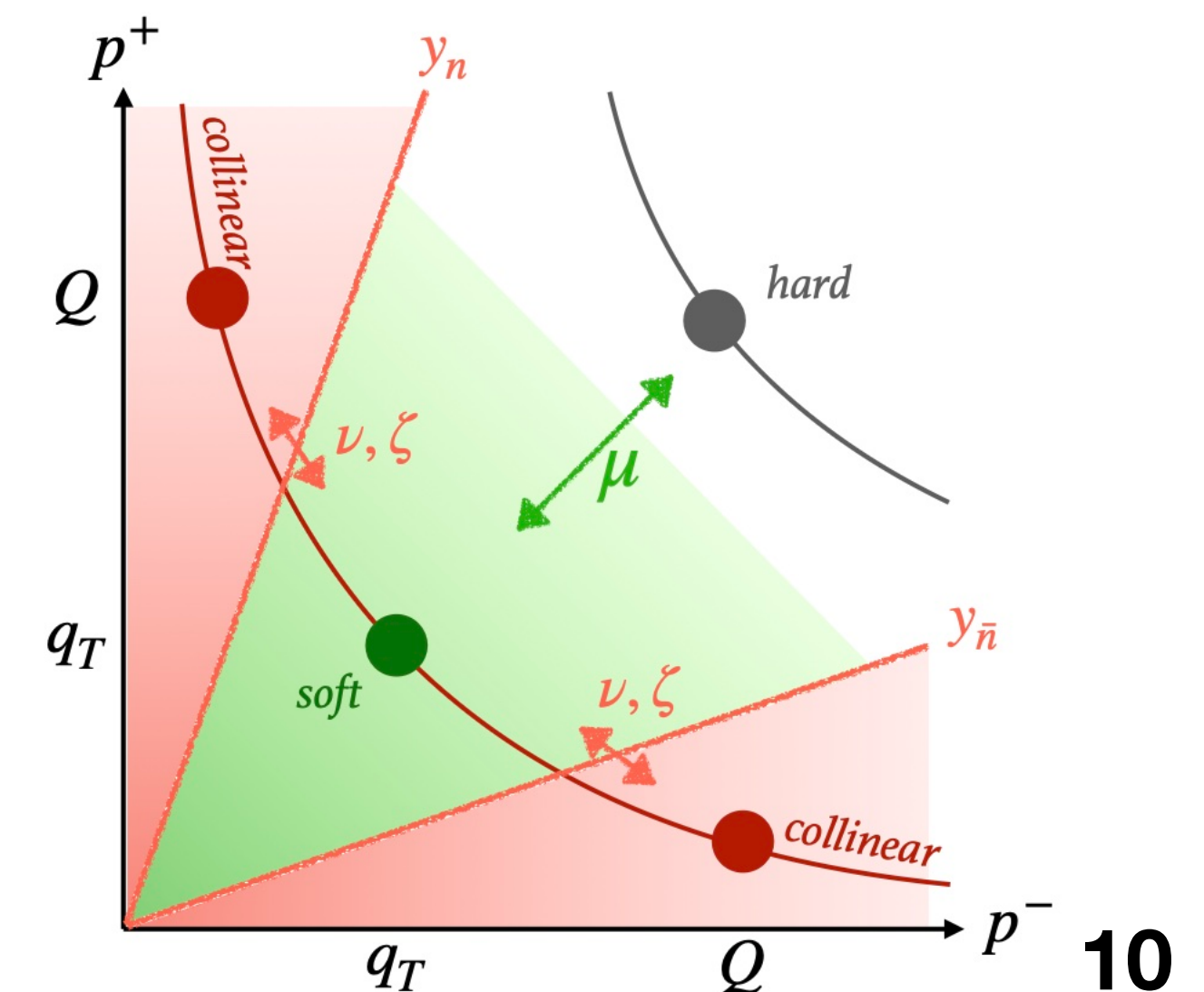
$$\frac{\partial f(x, b_T; \zeta, \mu)}{\partial \ln \mu} = \gamma(\mu, \zeta)$$

Renormalization group eq.

$$\frac{dK(b_T; \mu)}{d \ln \mu} = \frac{d\gamma(\mu, \zeta)}{d \ln \sqrt{\zeta}} = -\gamma_K(\alpha_s(\mu))$$

Cusp anomalous dimension

- ❖ The standard renormalization handles UV divergence.
- ❖ A rapidity (Collins-Soper) scale $\sqrt{\zeta}$ separates collinear modes from anti-collinear/soft modes.
- ❖ There is a scheme dependence on the choice of $\sqrt{\zeta}$ (or rapidity regulator). The beam function and the soft function have scheme dependence.



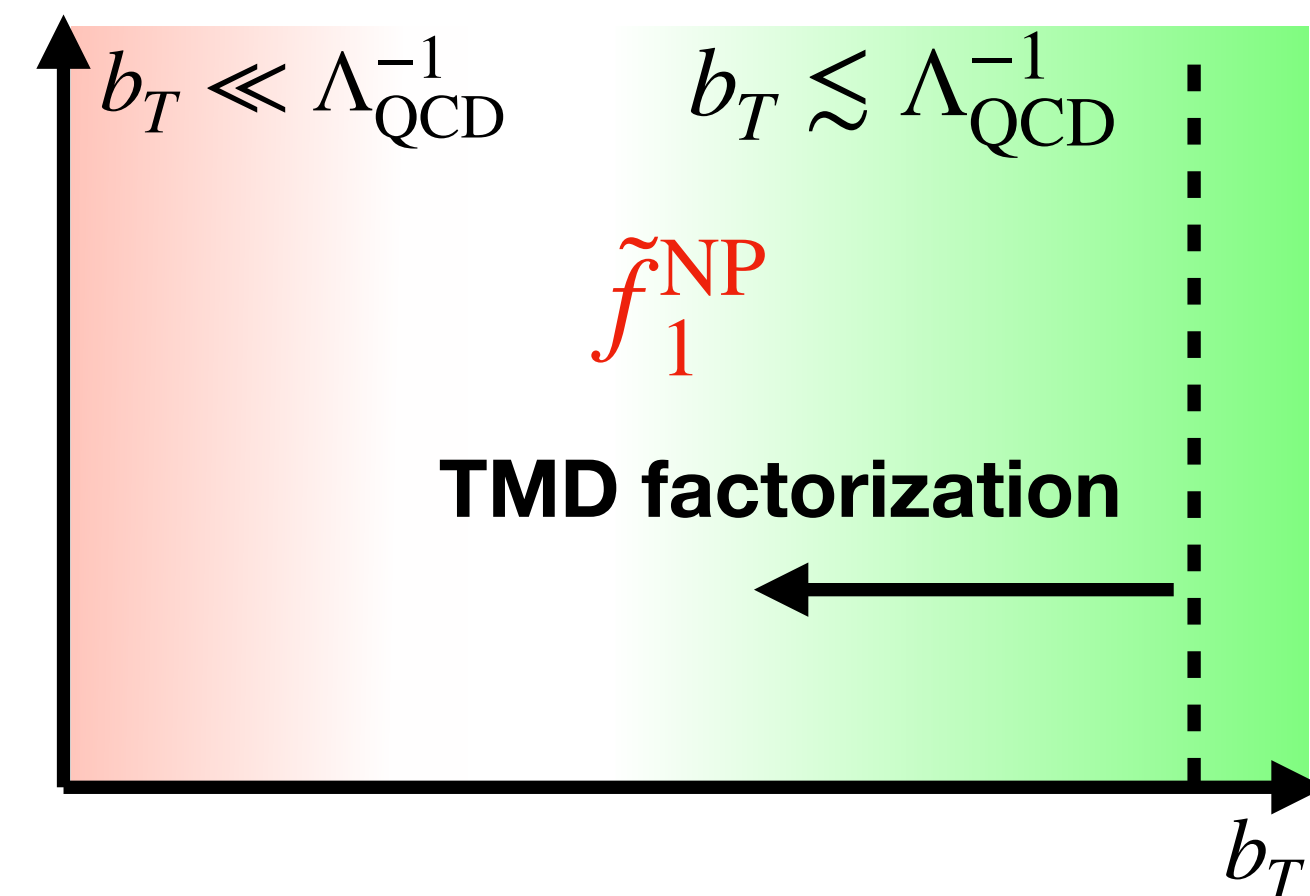
Collinear Matching

- ❖ Many perturbative calculations have been performed for $b_T \ll \Lambda_{\text{QCD}}^{-1}$, while it is challenging to calculate TMD PDFs at $b_T \lesssim \Lambda_{\text{QCD}}^{-1}$.
- ❖ Collinear matching in the small- b_T expansion:

I. Scimemi, A. Tarasov and A. Vladimirov, JHEP 05, 125 (2019)

$$\tilde{f}_{i/p}(b_\perp, \mu) = \overset{\text{Perturbative matching coefficients}}{C_1(b_T, L)} \otimes \overset{\text{Collinear PDFs Leading, Subleading}}{\tilde{f}_1} + b_T^2 \overset{\text{Perturbative matching coefficients}}{C_2(b_T, L)} \otimes \tilde{f}_2 + \dots$$

$L \propto \ln(\mu^2 b_T^2)$



It is challenging to calculate higher twist corrections in TMD factorization extensively, so we need to introduce a nonperturbative function to connect $b_T \ll \Lambda_{\text{QCD}}^{-1}$ to $b_T \lesssim \Lambda_{\text{QCD}}^{-1}$.

$$b_T^* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{\text{max}}^2}}}$$

$$\mu_{b^*} \sim 1/b_T^*$$

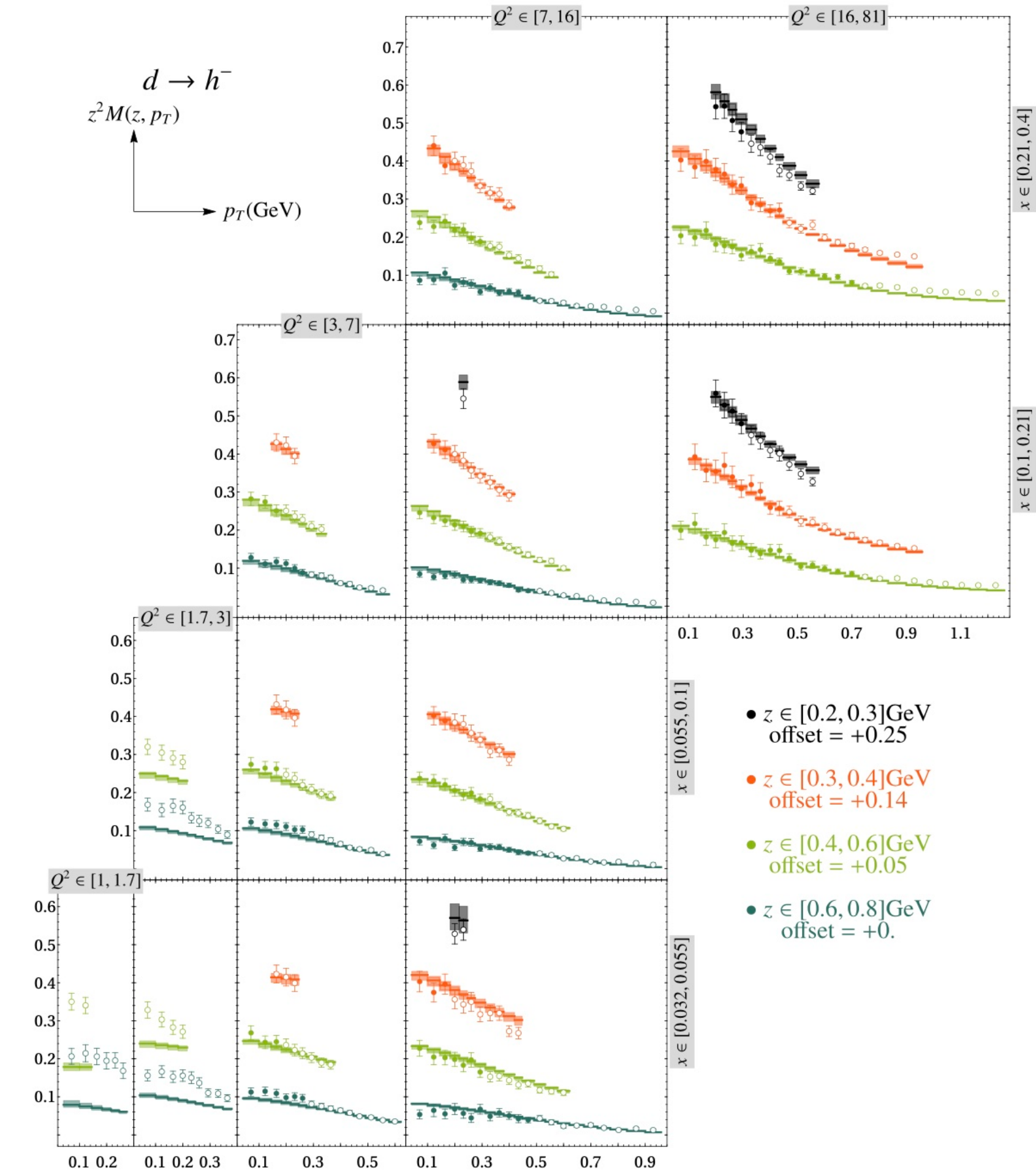
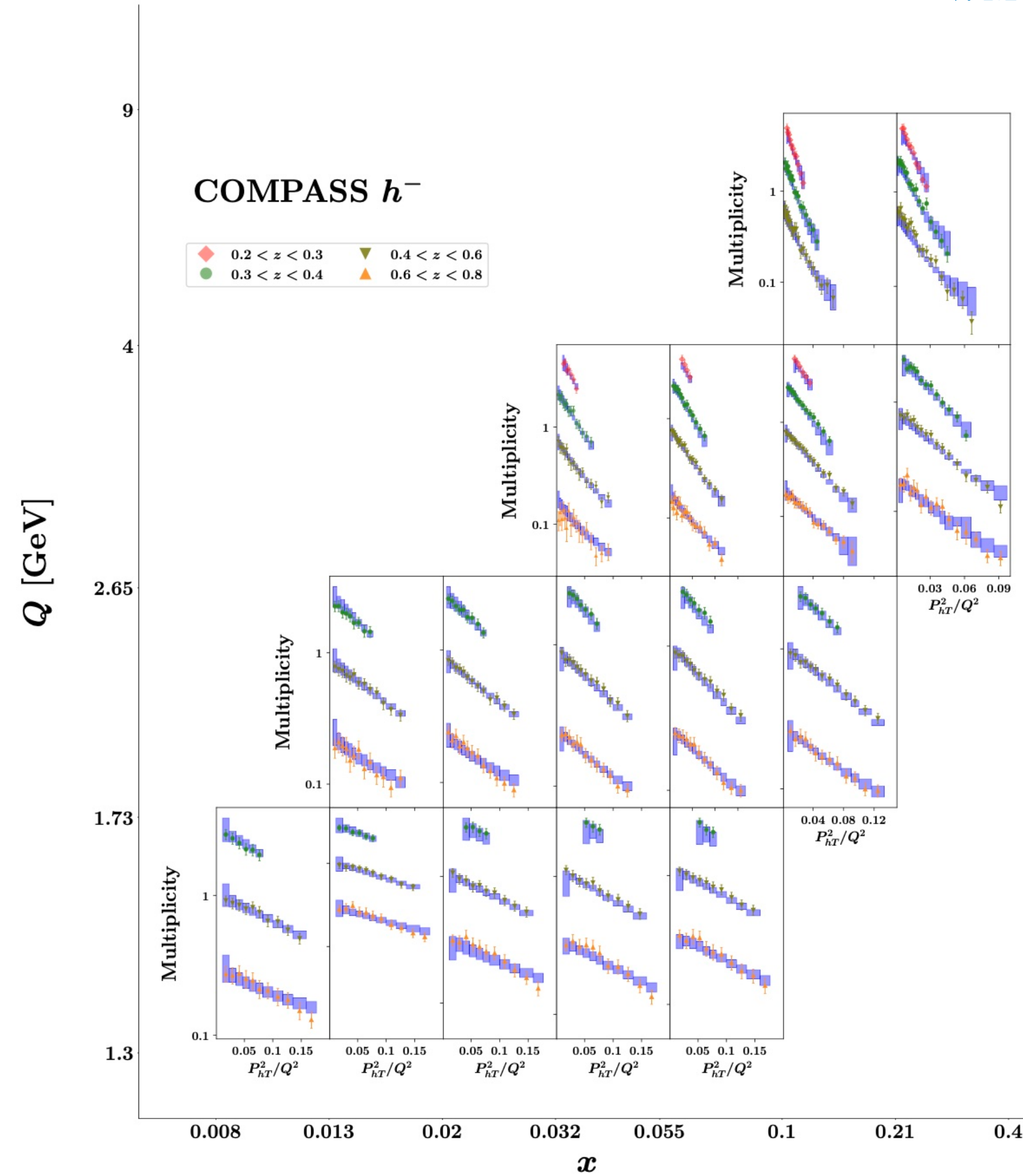
$$\tilde{f}_{i/p}(b_\perp, \mu) = \underset{\text{Collinear matching at LT}}{[C_1 \otimes \tilde{f}_1](b_T^*, \mu_{b^*})} \exp \left[\int_{\mu_*}^{\mu_f} \frac{d\mu}{\mu} \gamma(\mu, \zeta_f) \right] \left(\frac{\zeta_f}{\mu_{b^*}^2} \right)^{-D(b_*, \mu_{b^*})} \underset{\text{Model-dependent nonperturbative function}}{\tilde{f}_1^{\text{NP}}(b_T, \zeta_f)}$$

Collins-Soper-Sterman evolution

Global data fit: unpolarized TMDs

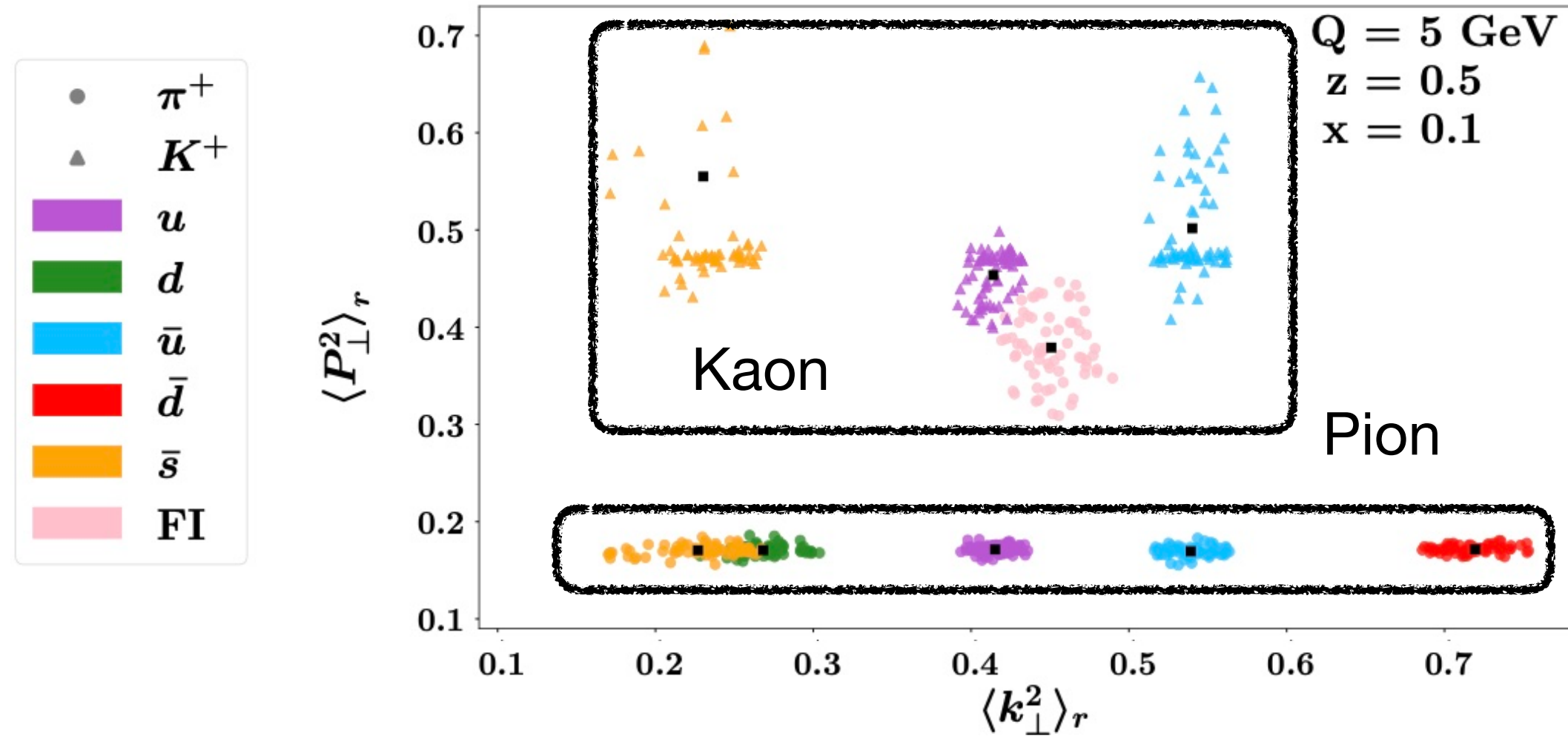
A. Bacchetta *et al.* [MAP], JHEP08, 232 (2024)

V. Moos, I. Scimemi, A. Vladimirov and P. Zurita, [arXiv:2503.11201 [hep-ph]].



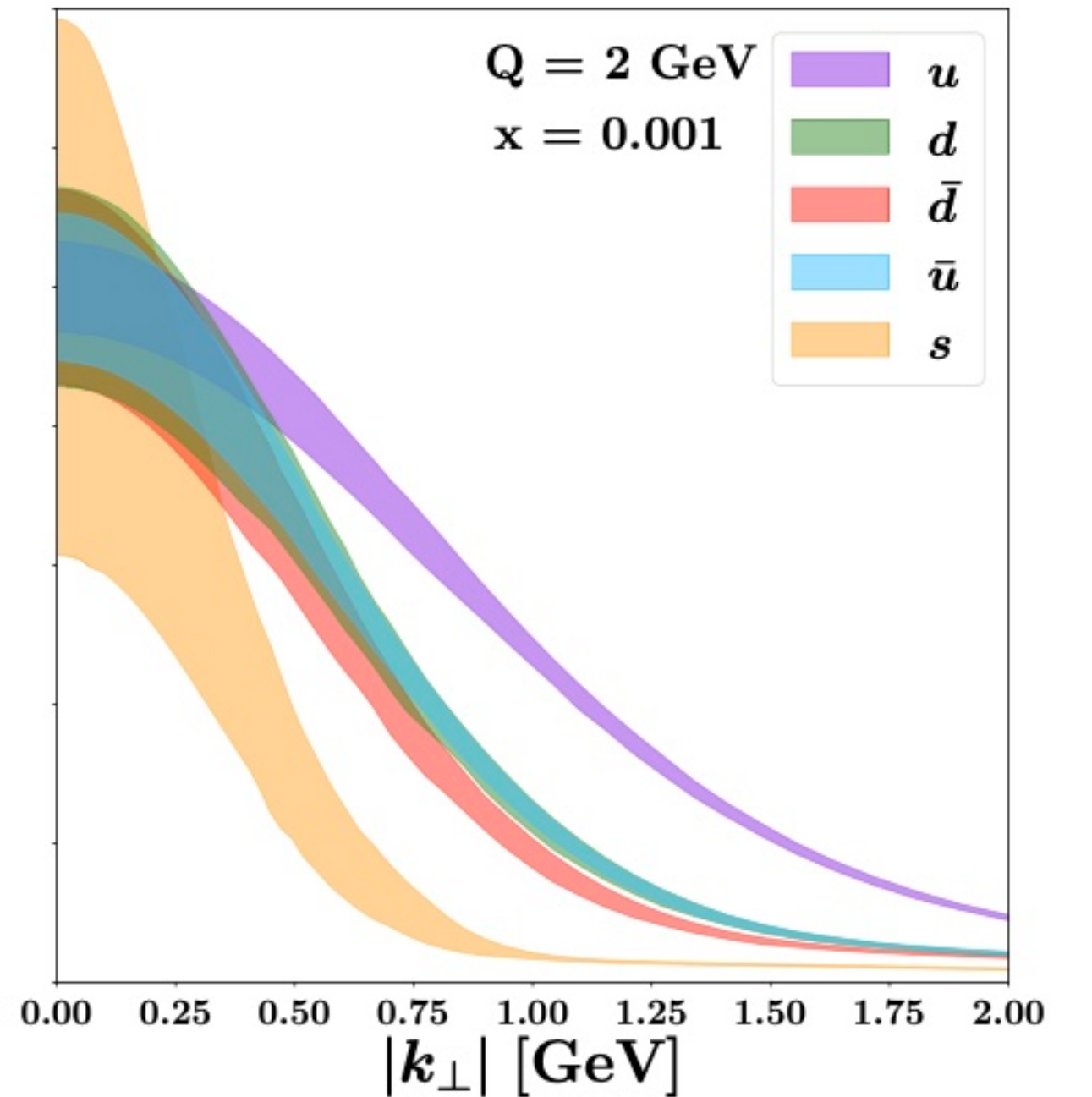
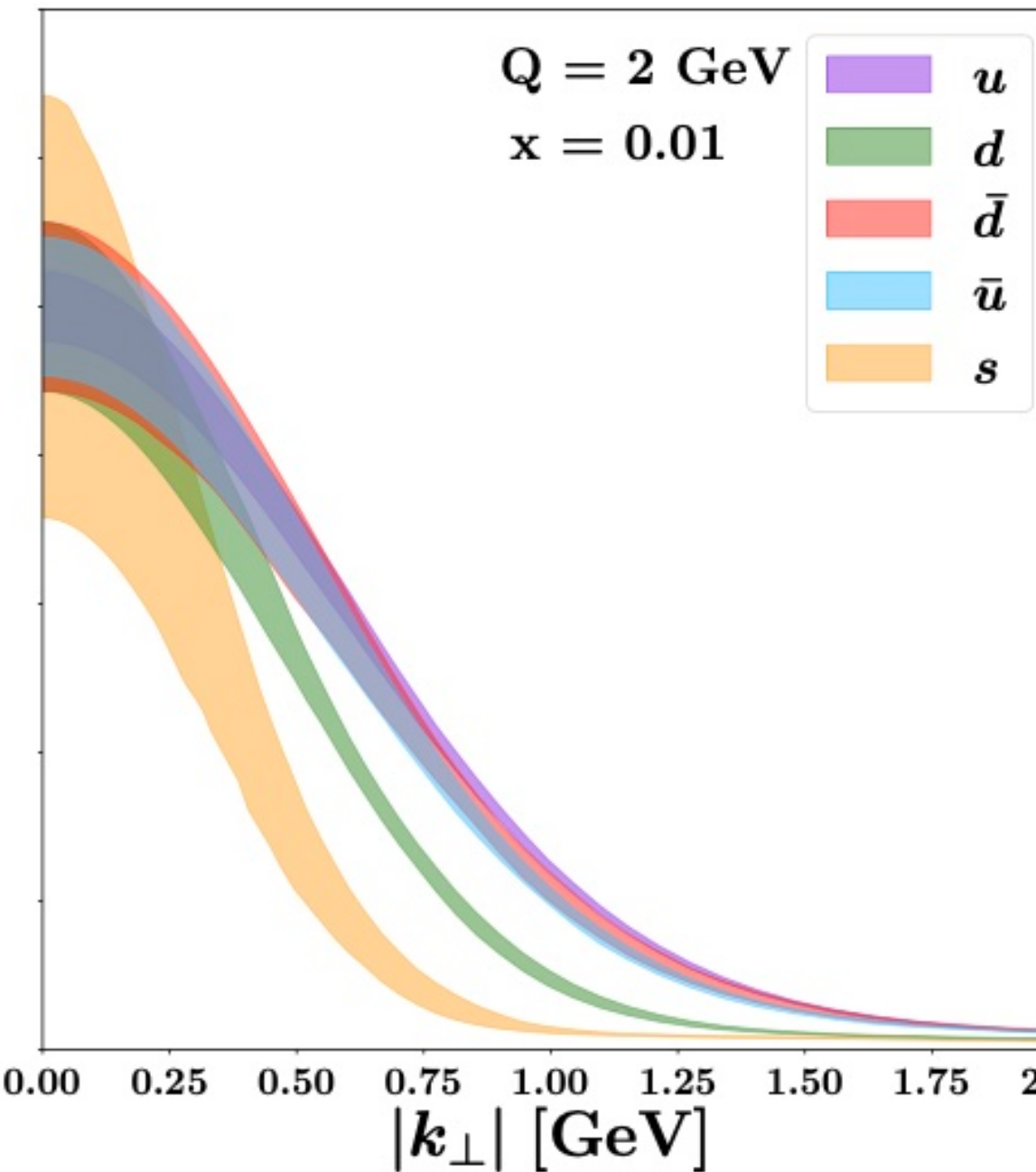
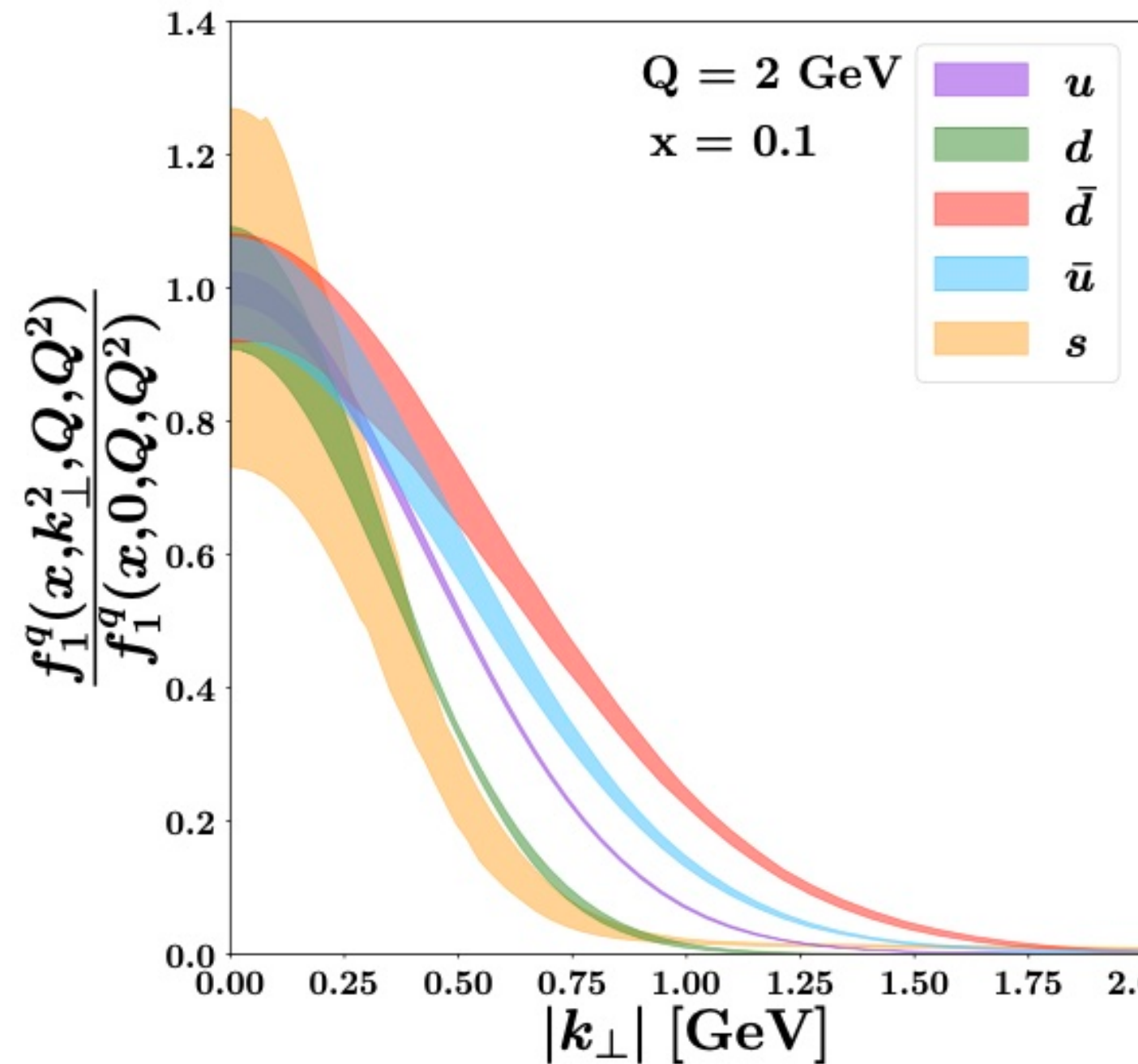
See back-to-back talks by A. Bacchetta and V. Moos, Tuesday at 10:20AM, 10:50AM

Unpolarized quark TMD PDF: Flavor dependence

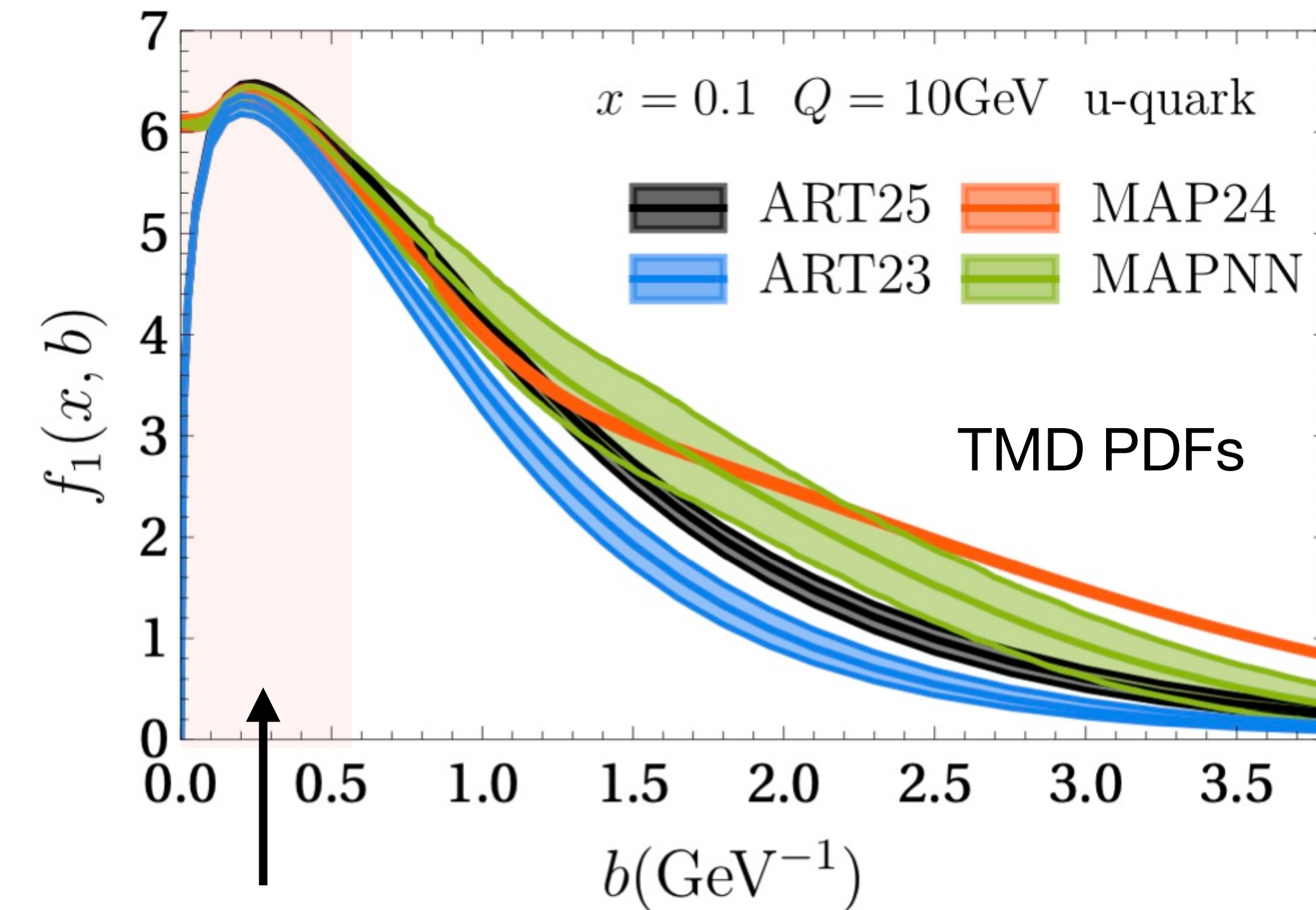


Flavor independent (FI) extraction
vs.
Hadron dependent extraction

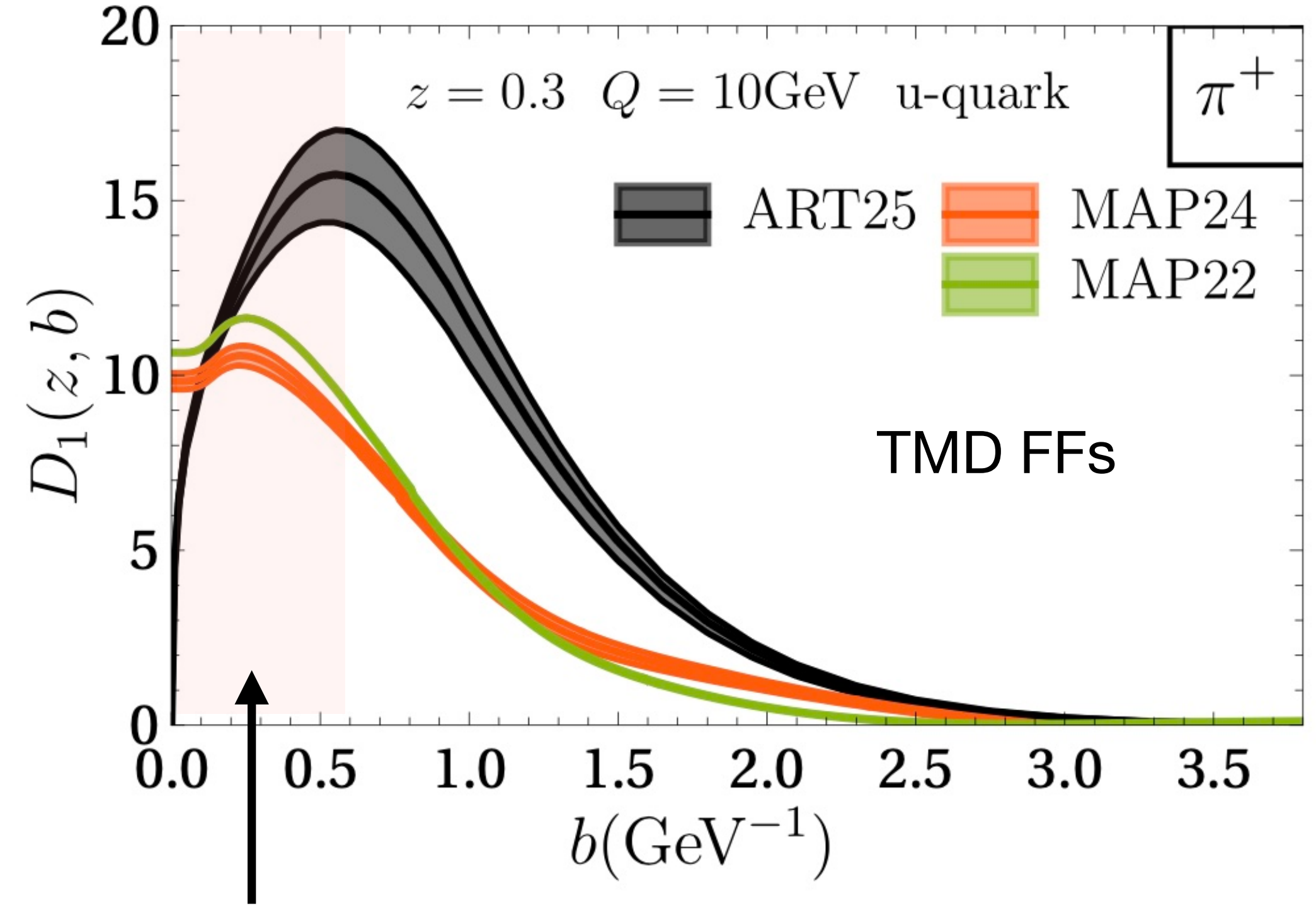
$$f_1 = \text{Unpolarized}$$



NP Model dependence



Collinear factorization at
Leading Twist

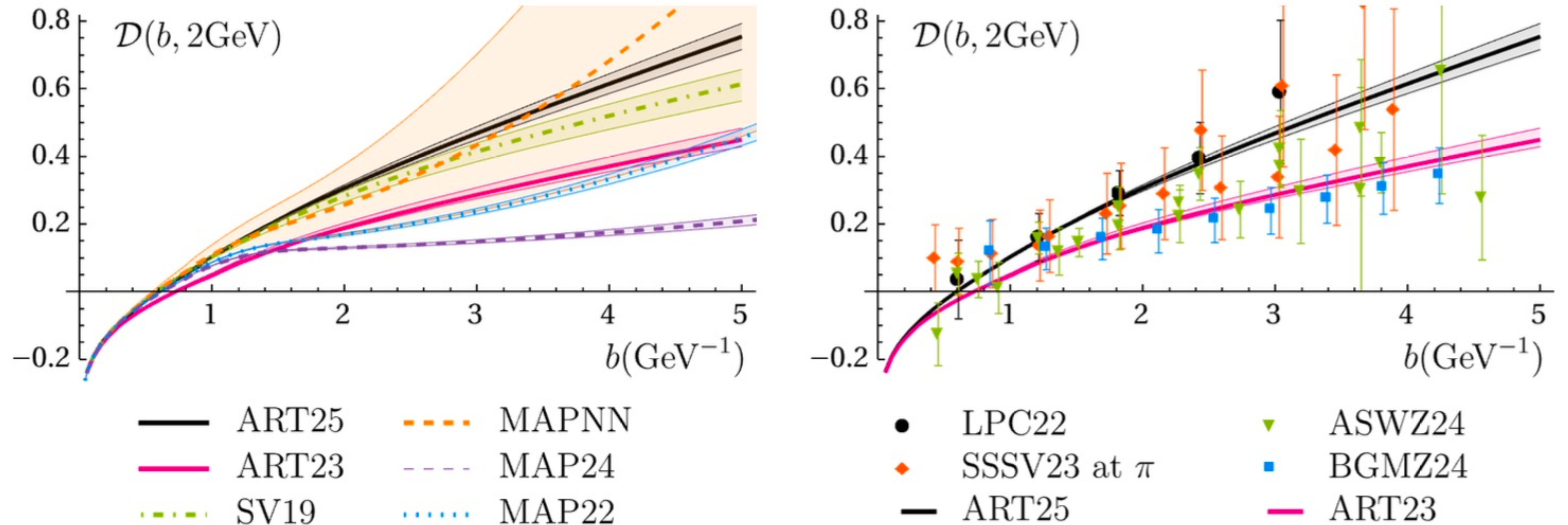


Collinear factorization at
Leading Twist

- ❖ Significant dependence on the NP function's model.
- ❖ b_{min} -prescription is adopted for MAP, giving the difference between MAP and ART at $b_T \lesssim 0.2 \text{ GeV}^{-1}$.
- ❖ NN approach: less bias $\leftrightarrow$ sizable band [A. Bacchetta *et al.* \[MAP\], PRL135, no.2, 021904 \(2025\)](#)

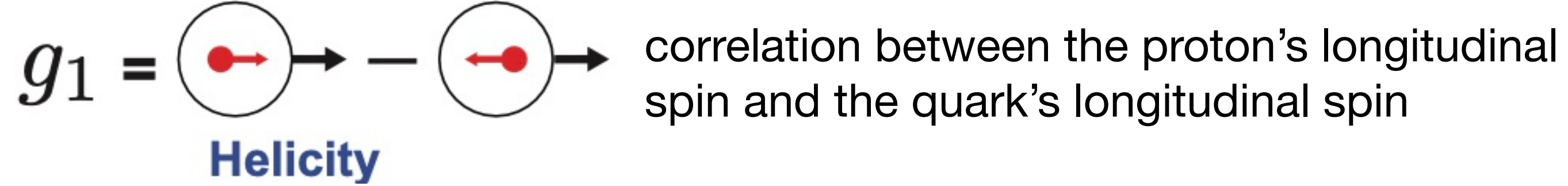
Collins-Soper Kernel: Data extraction

V. Moos, I. Scimemi, A. Vladimirov and P. Zurita, [arXiv:2503.11201 [hep-ph]].

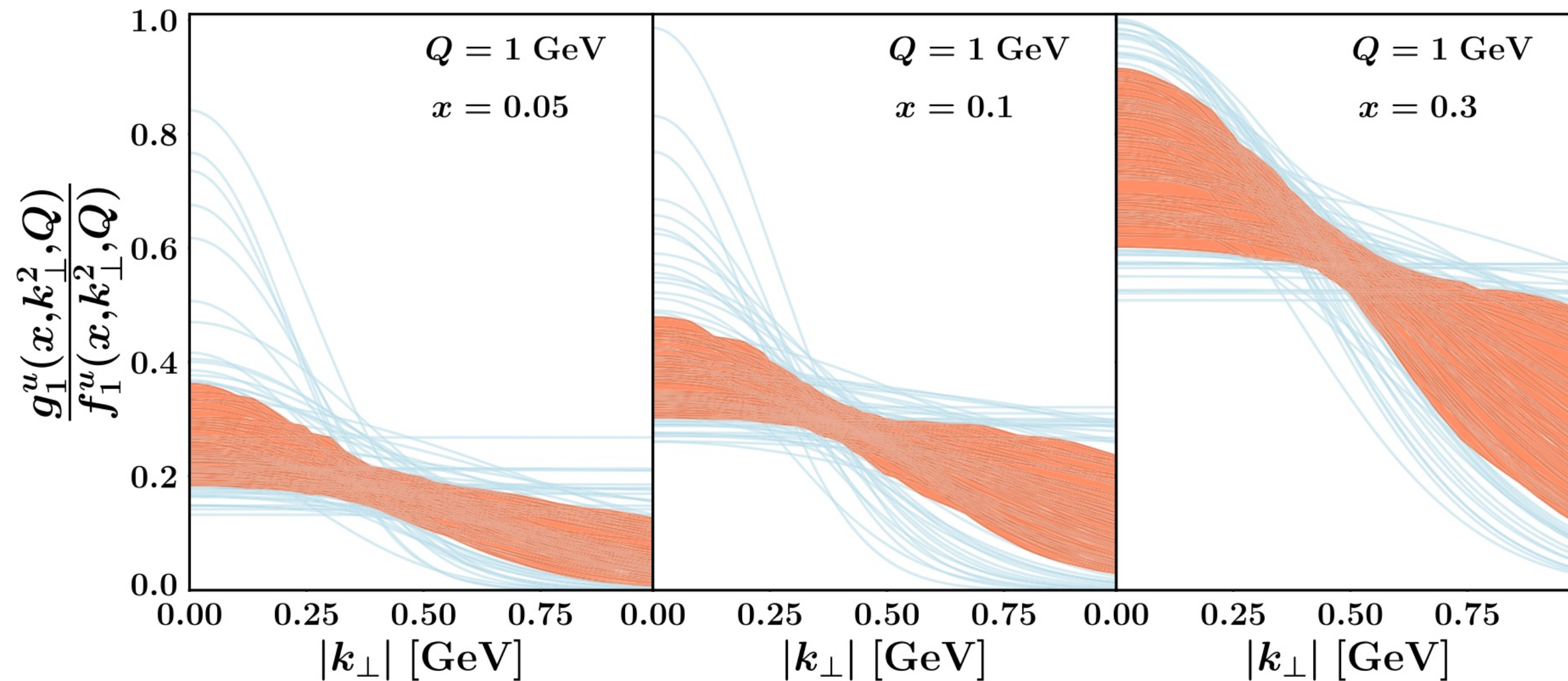


Data analysis and Lattice QCD are compatible up to large b_T .

Global data fit: Helicity distribution

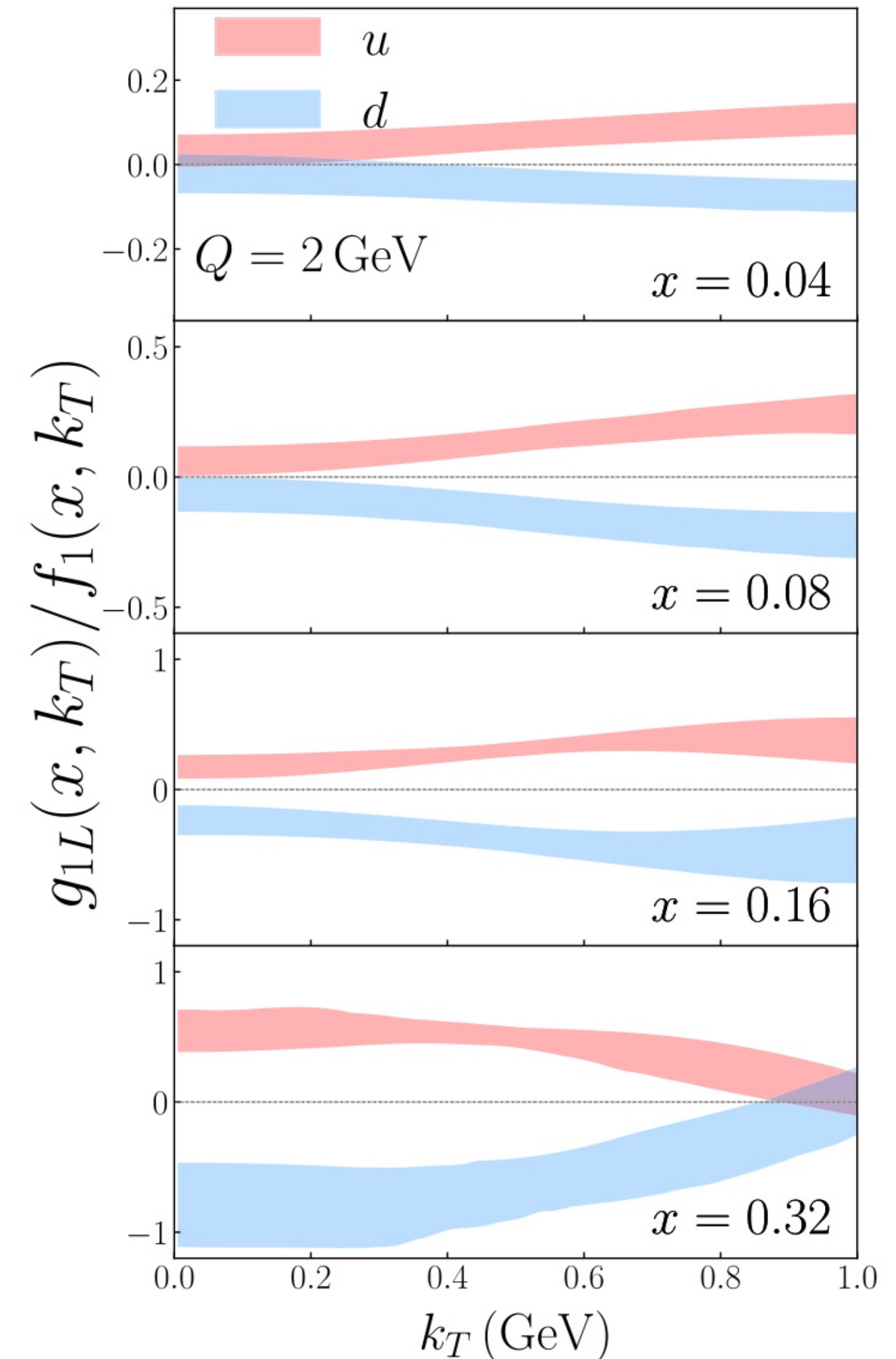


g_1/f_1 : polarization rate of quarks.



Some differences between the two approaches, including the scale dependence of TMDs, data set, positivity constraint $|g_1| \leq f_1$, etc.

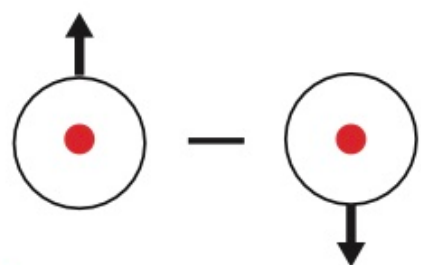
A. Bacchetta *et al.* [MAP], PRL134, no.12, 121901 (2025)
K. Yang *et al.* [Transverse Nucleon Tomography], PRL134, no.12, 121902 (2025)



See Ke Yang on Tuesday at 11:20 AM

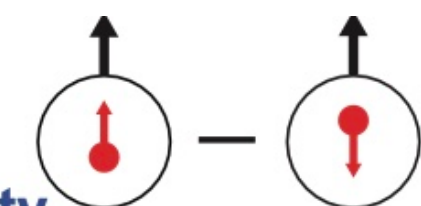
Extraction of polarized TMDs

C. Zeng, H. Dong, T. Liu, P. Sun and Y. Zhao, [arXiv:2412.18324 [hep-ph]]

$$f_{1T}^\perp = \text{Sivers}$$


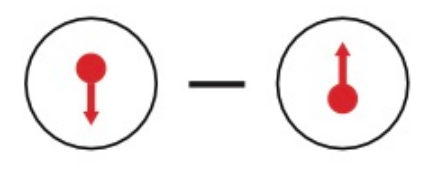
Sivers

correlation between proton's transverse spin S_T and quark's transverse momentum k_T

$$h_1 = \text{Transversity}$$


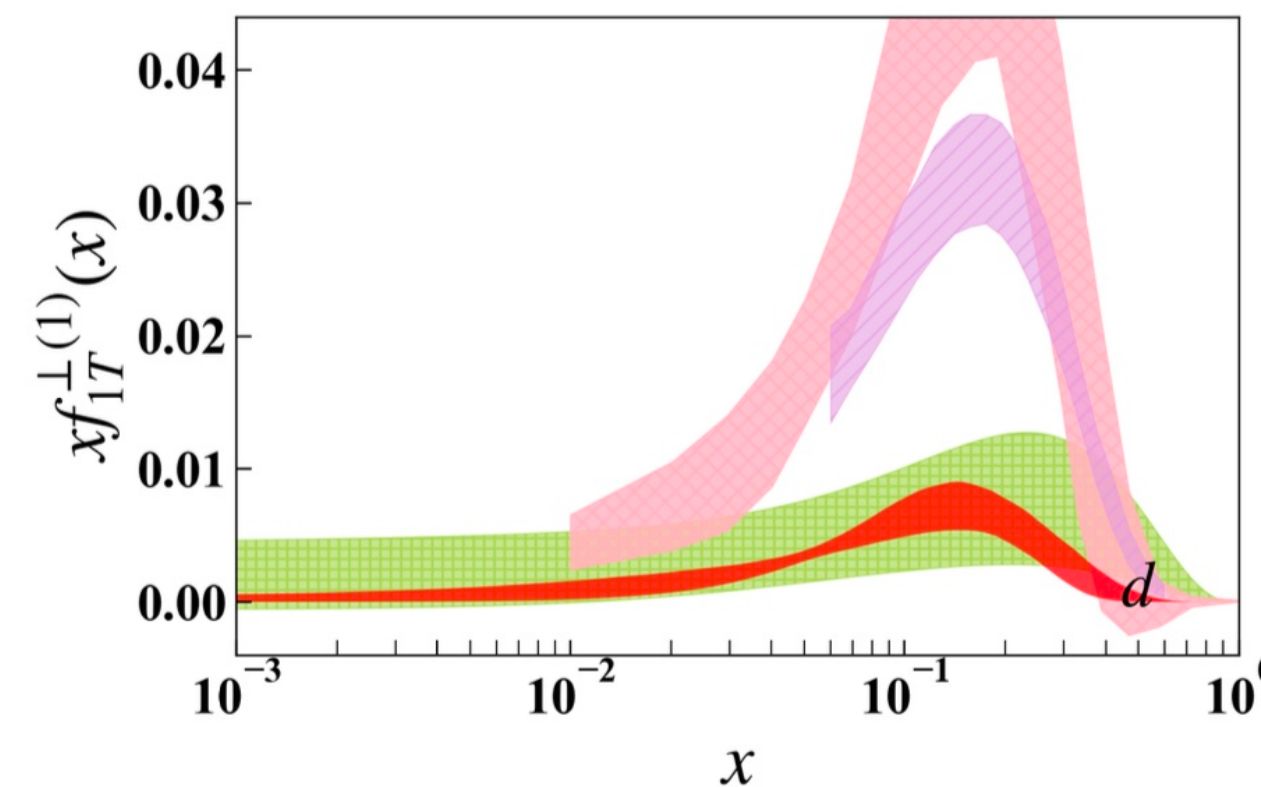
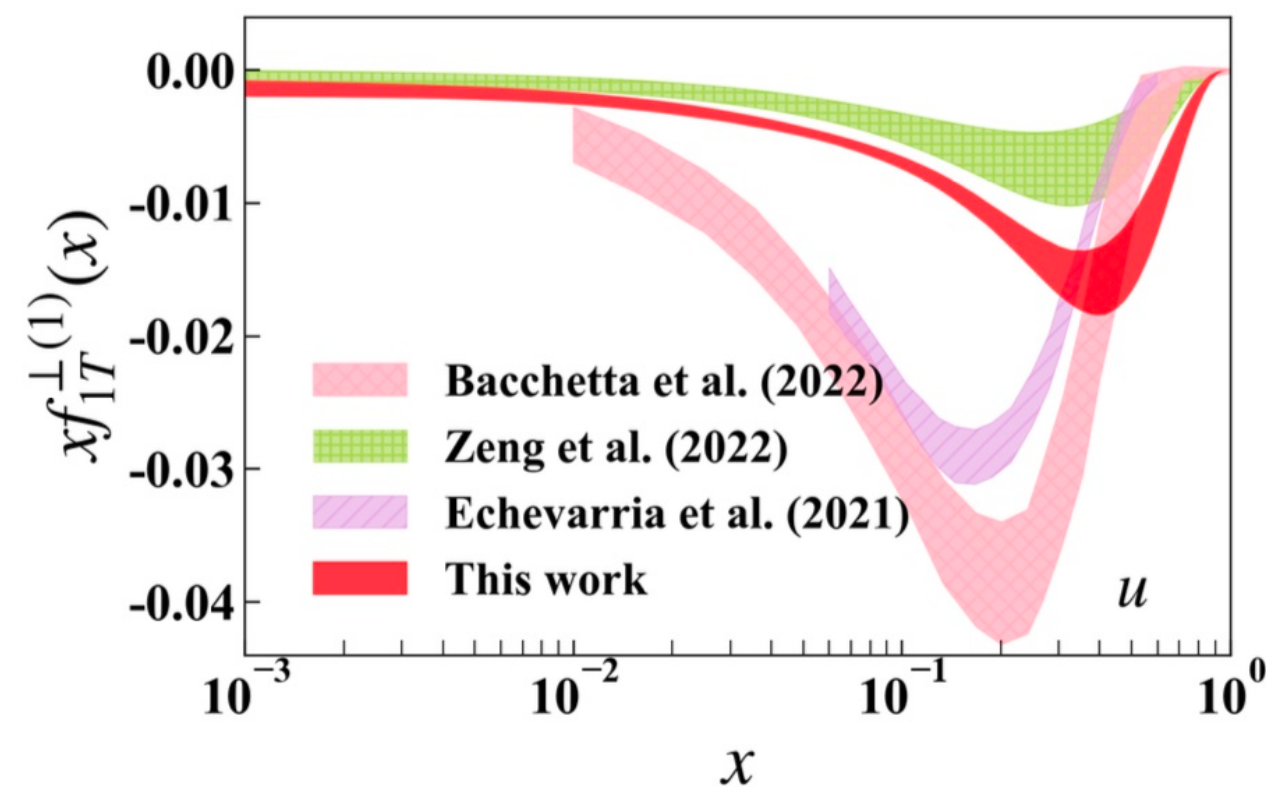
Transversity

correlation between proton's transverse spin S_T and quark's transverse spin s_T

$$H_1^\perp = \text{Collins}$$


Collins

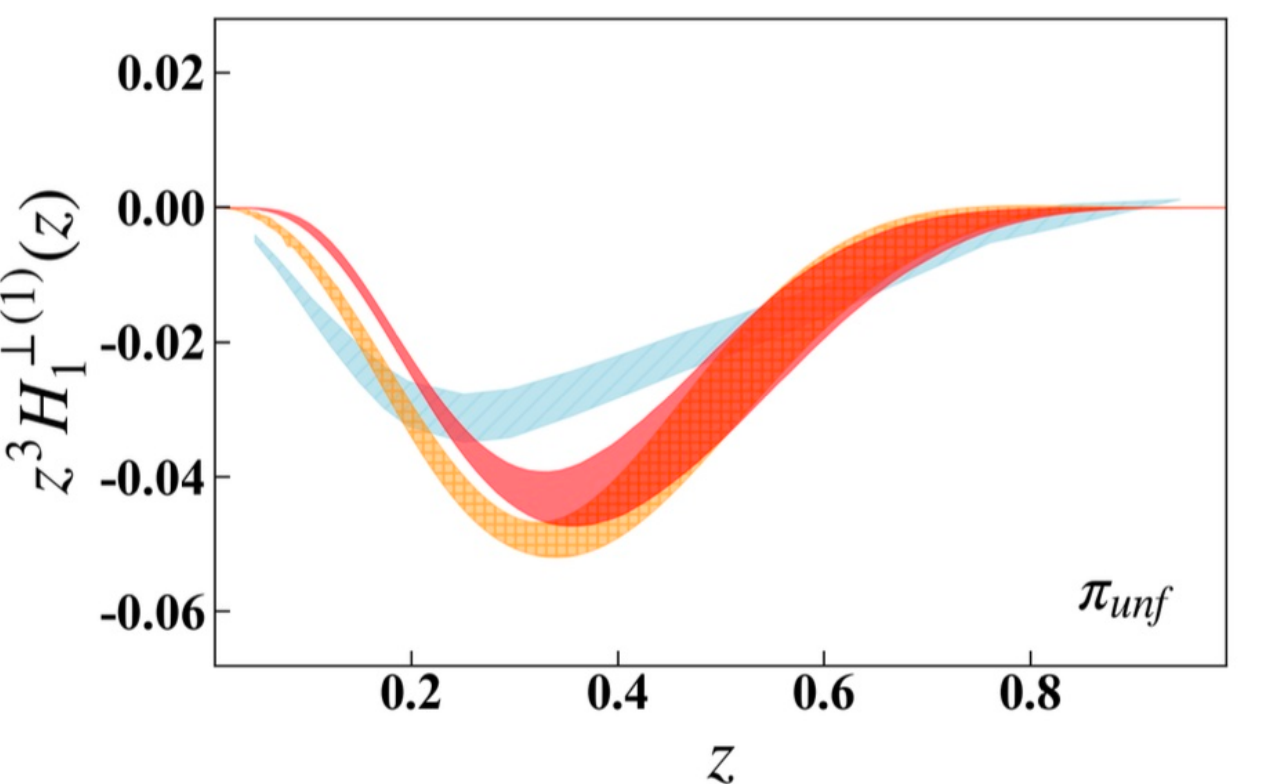
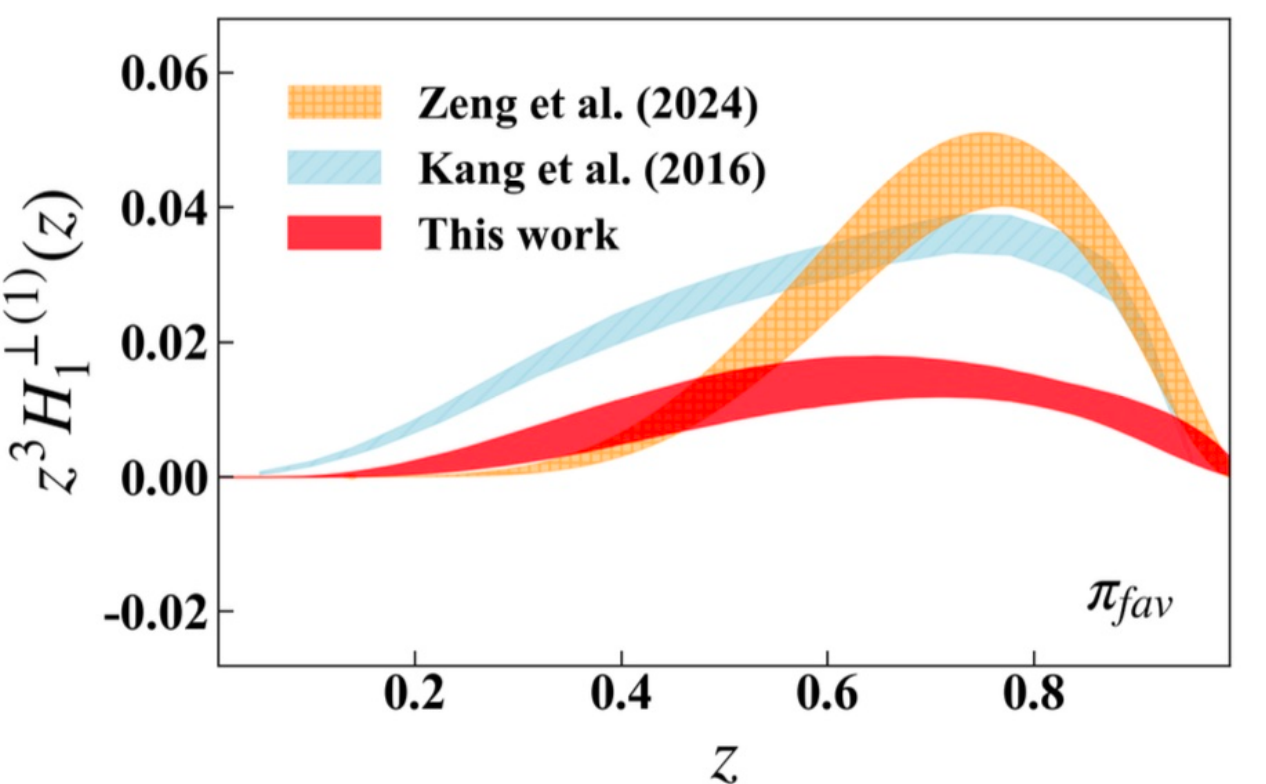
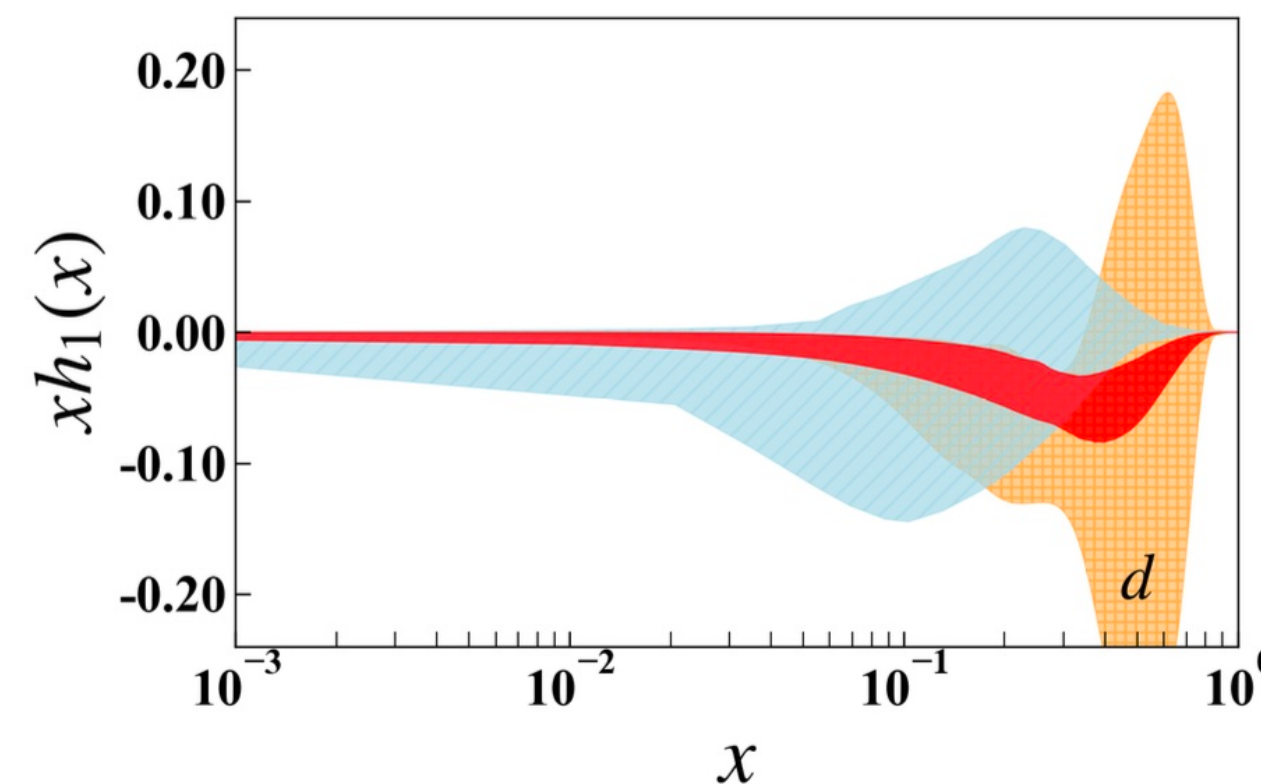
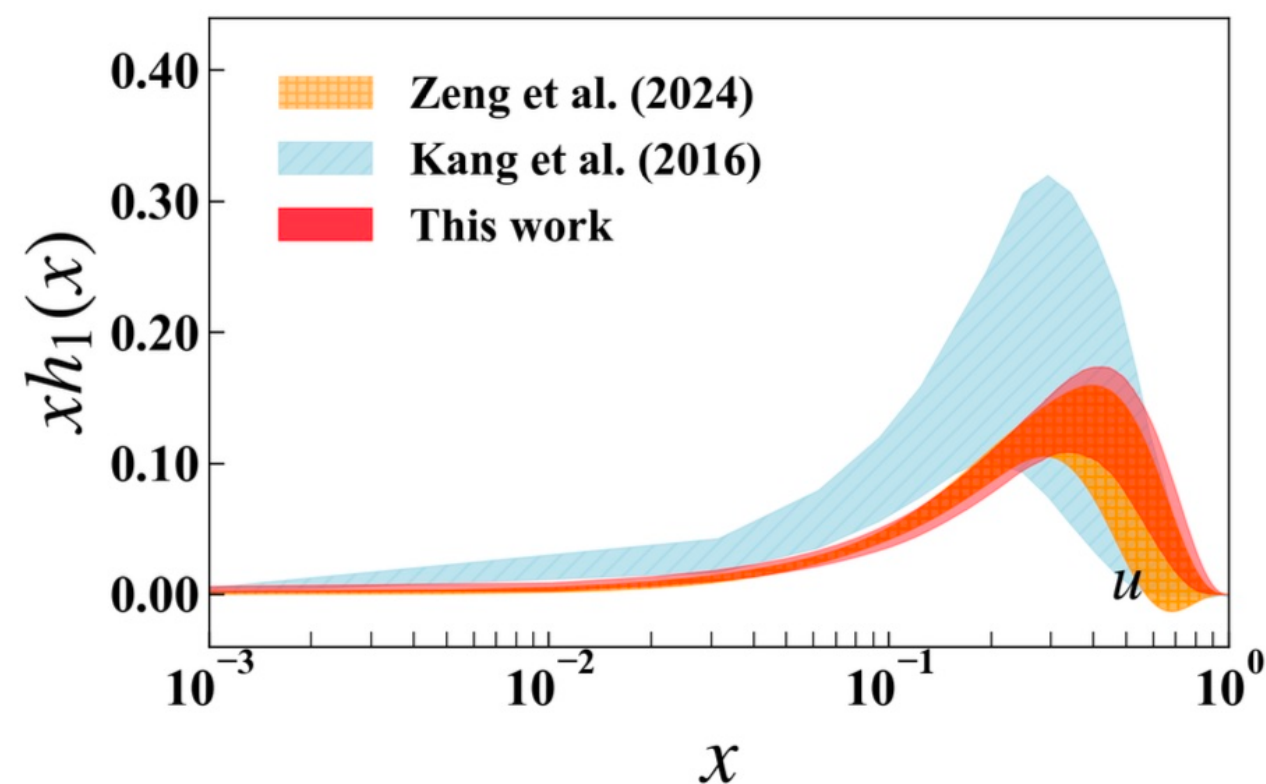
correlation between the transverse spin of a fragmenting quark and the transverse momentum of the hadron relative to the quark jet



$$f_{1T}^{\perp(1)}(x) = \int_0^{k_T^{\text{cut}}} d^2 \mathbf{k}_T \frac{k_T^2}{2M^2} f_{1T}^\perp(x, k_T),$$

$$H_1^{\perp(1)}(z) = \int_0^{p_T^{\text{cut}}} d^2 \mathbf{p}_T \frac{p_T^2}{2z^2 M_h^2} H_1^\perp(z, p_T),$$

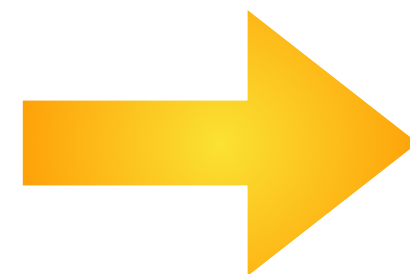
$$h_1(x) = \int_0^{k_T^{\text{cut}}} d^2 \mathbf{k}_T h_1(x, k_T).$$



Tensor charges

$$\delta q_v = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

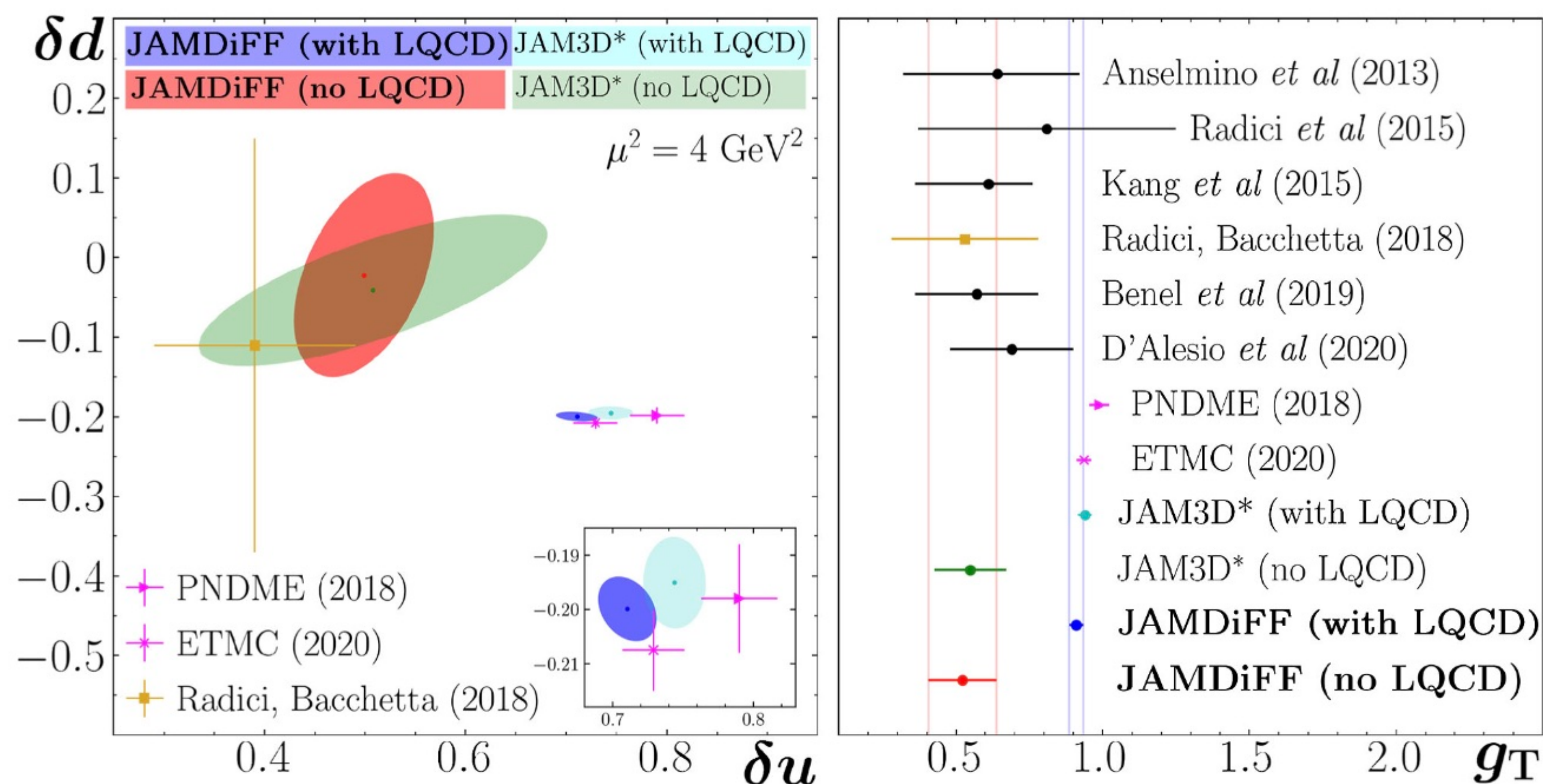
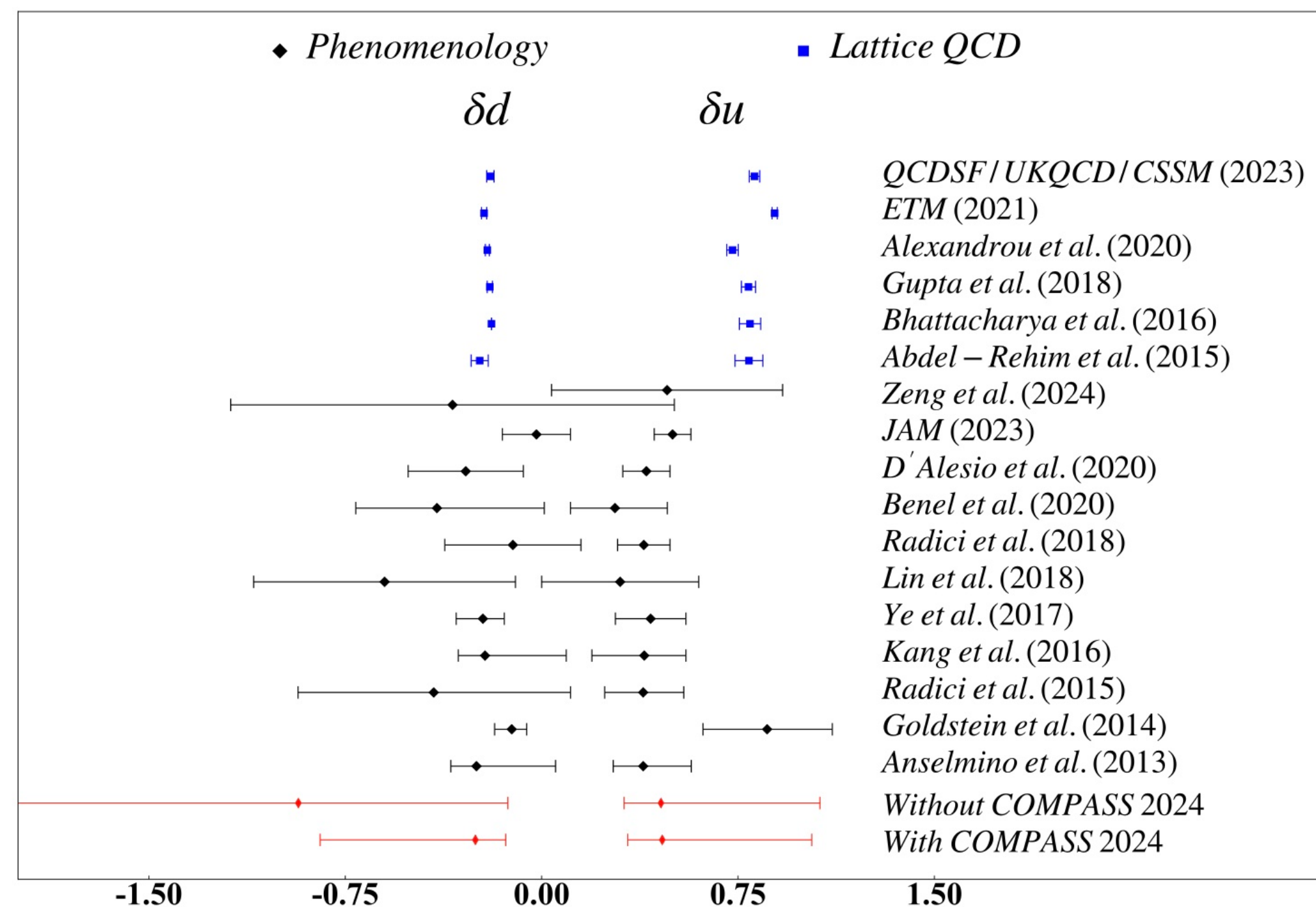
Important for BSM physics



Nucleon's EDM $d_N \propto \sum_q \text{quark's EDM} \times \text{Tensor charge} \delta q$

C. Zeng, H. Dong, T. Liu, P. Sun and Y. Zhao, [arXiv:2412.18324 [hep-ph]]

C. Cocuzza *et al.* [JAM], PRL132, no.9, 091901 (2024)



Isovector combination:

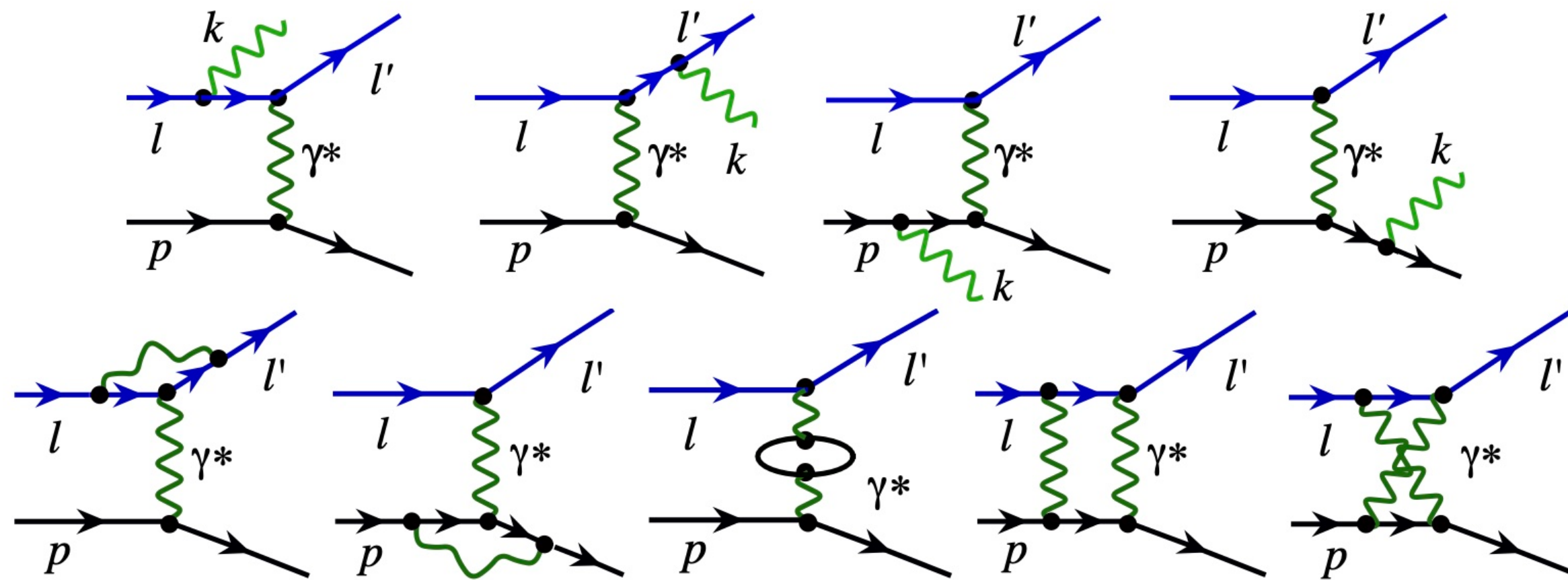
$$g_T = \delta u - \delta d = 0.80^{+1.69}_{-0.31}$$

❖ By using data on di-hadron measurements, both Dihadron FFs and transversity PDFs are extracted simultaneously.

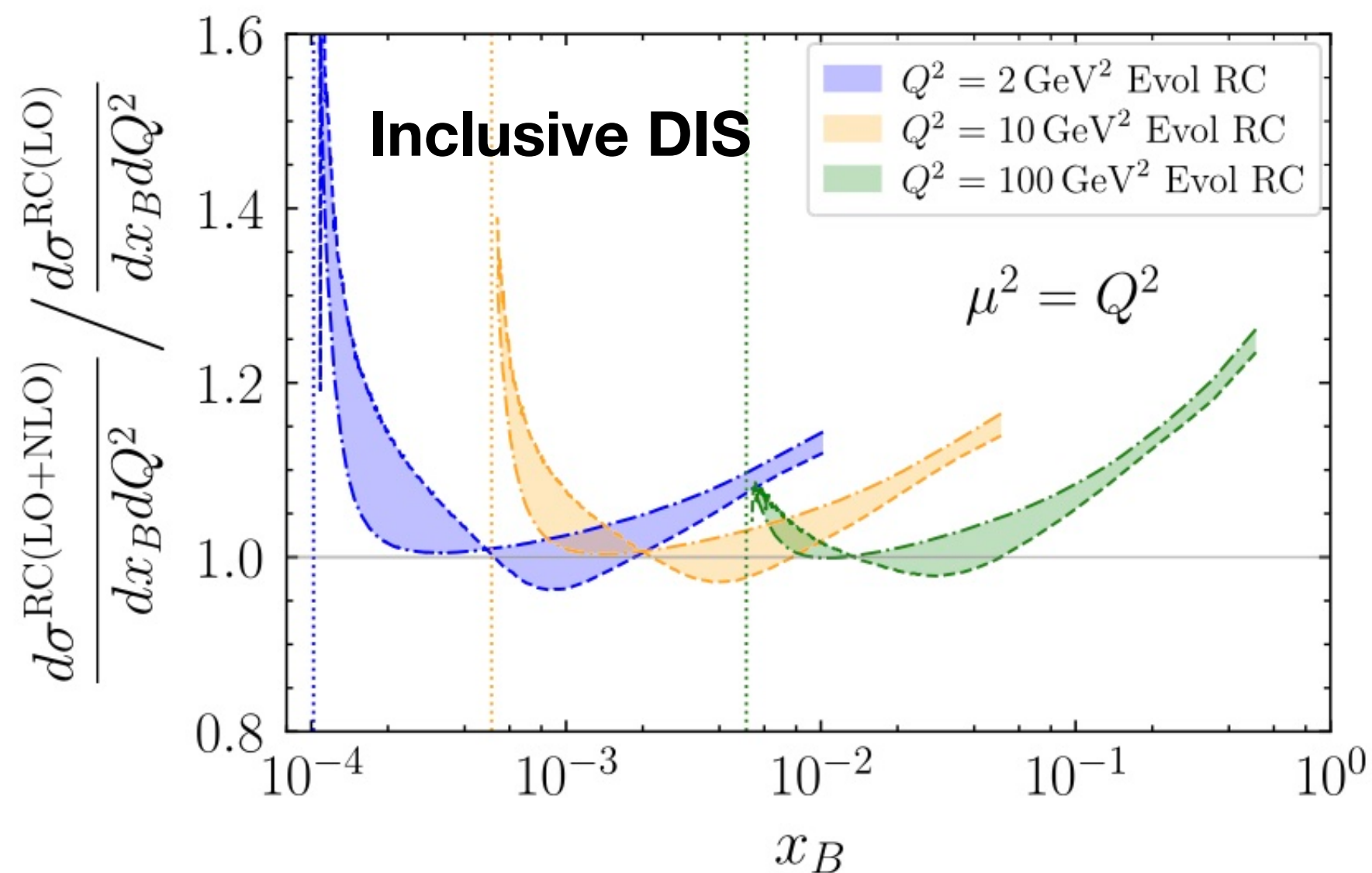
❖ JAMDiFF and JAM3D are compatible with Lattice Data.

Remarks on QED radiative corrections

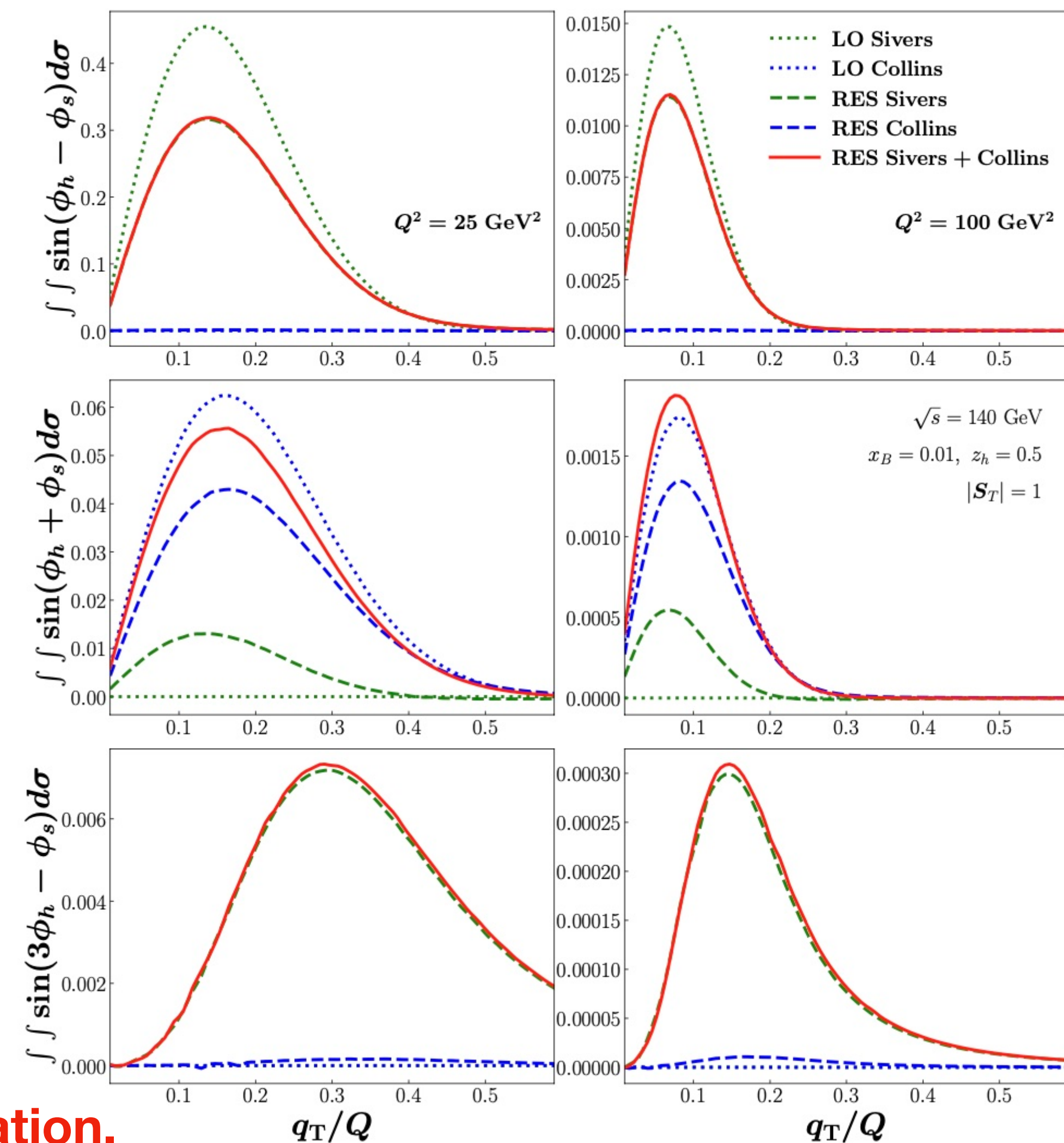
Liu, Melnitchouk, Qiu, Sato, PRD104, no.9, 094033 (2021), JHEP11, 157 (2021)



J. Cammarota, J.-W. Qiu, KW and J.-Y. Zhang, PRD112, no.5, 056007 (2025)



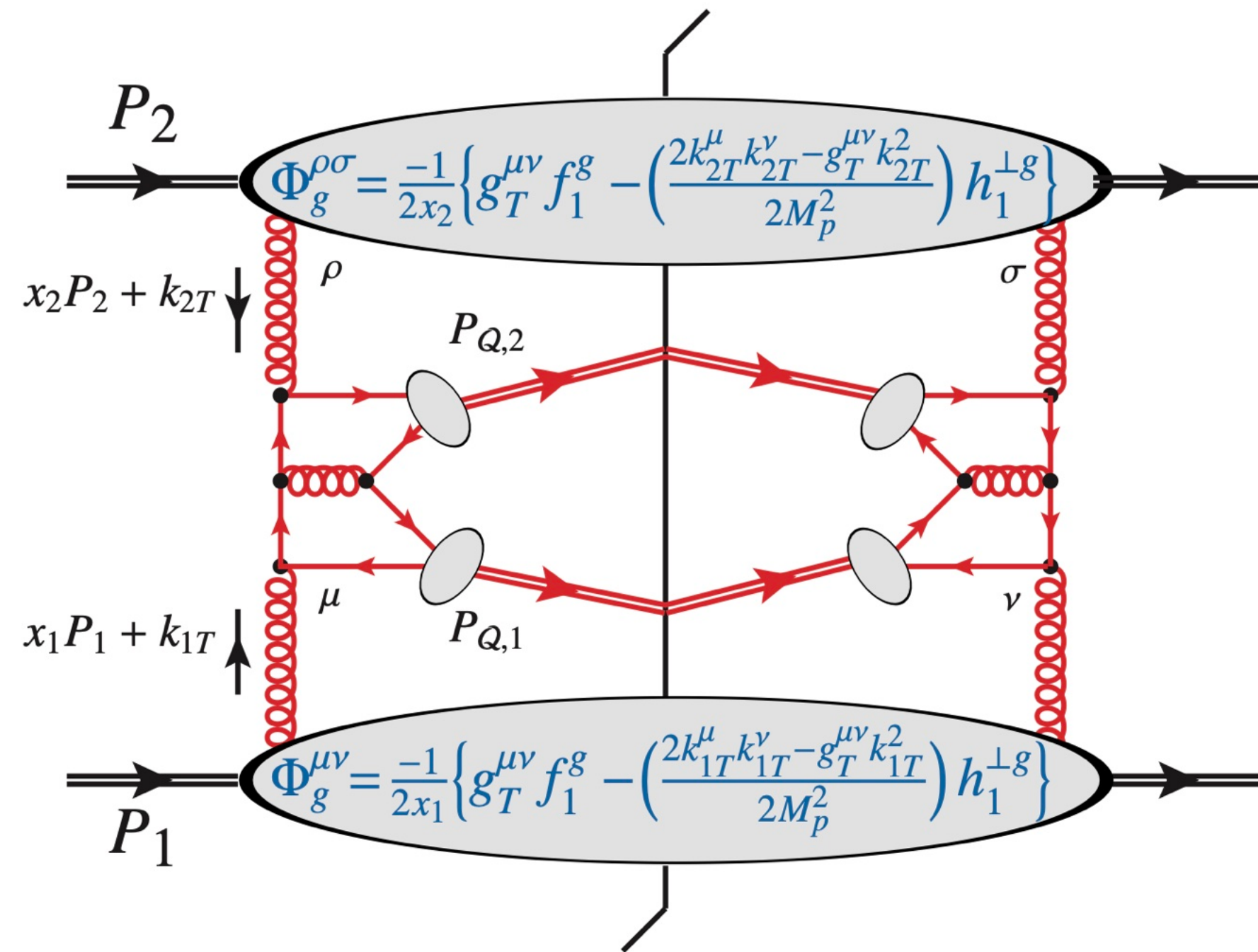
QED RCs could cause a modification of angular modulations between the lepton plane and the hadron plane.



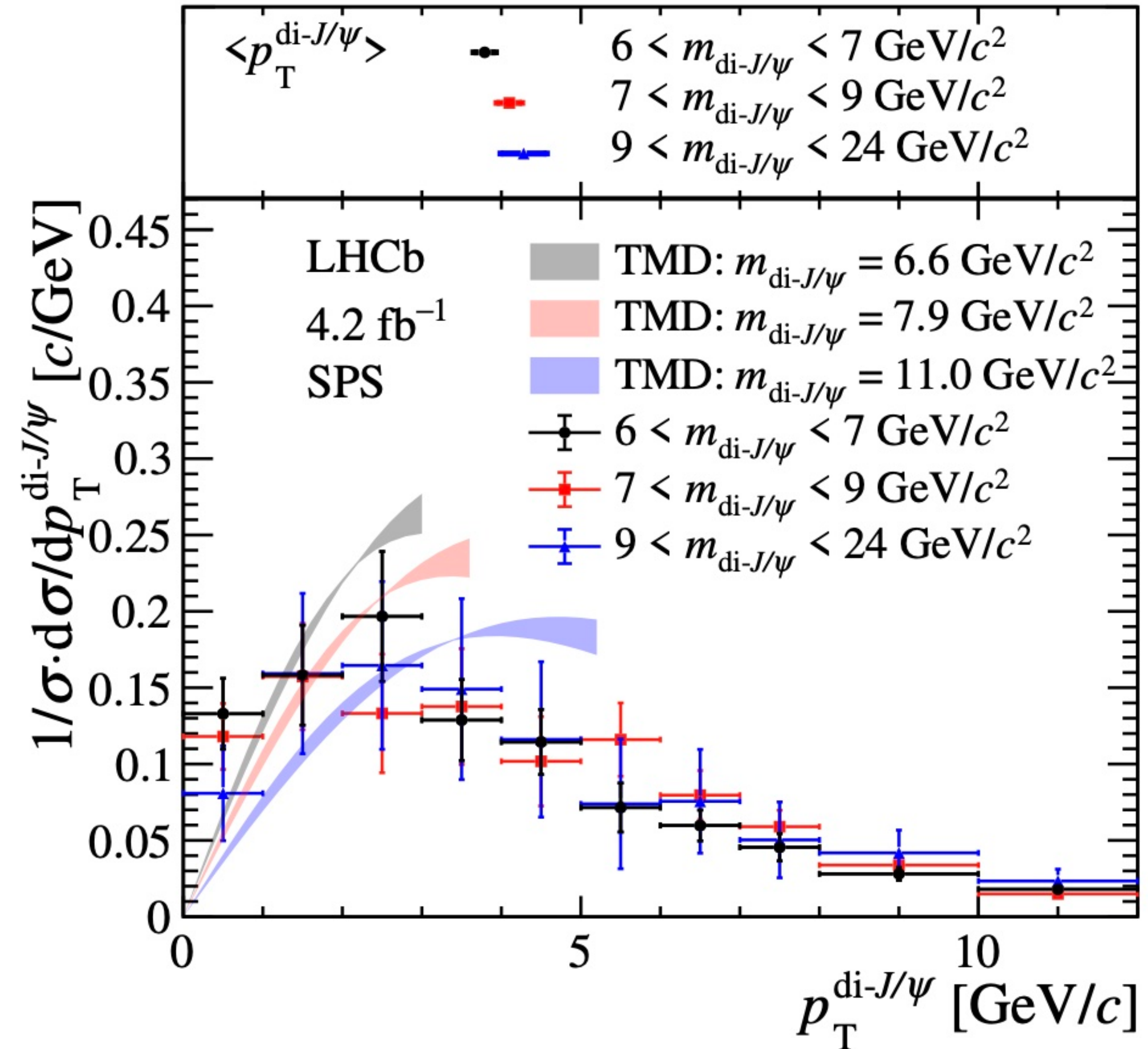
The impact of QED RCs on other observables is under investigation.

Probing gluon TMD PDFs

R. Aaij et al. [LHCb], JHEP03, 088 (2024)

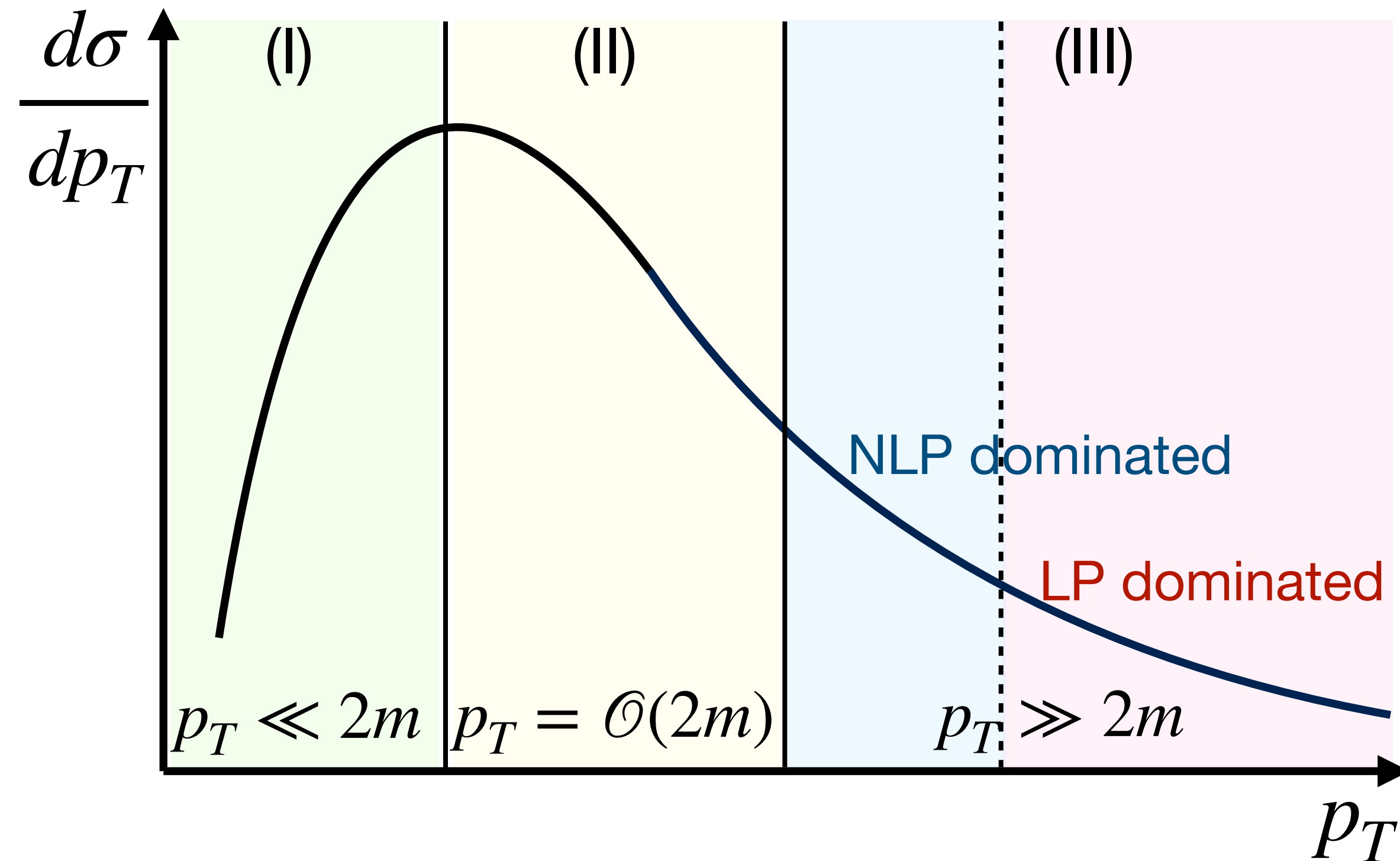


F. Scarpa, D. Boer, M. G. Echevarria, J. P. Lansberg, C. Pisano and M. Schlegel, Eur. Phys. J. C 80, no.2, 87 (2020)



- ❖ TMD Factorization may not hold for hadron-hadron scattering processes with explicitly TMD correlators. J.Collins and J.W.Qiu, PRD75, 114014 (2007)
- ❖ J/ψ production in the color singlet model: TMD factorization is not violated.

Single quarkonium production mechanism



(I) TMD factorization + CEM or NRQCD

Berger, Qiu and Wang, PRD 71, 034007 (2005)

Sun, Yuan and Yuan, PRD88, 054008 (2013)

CGC framework + CEM or NRQCD (forward)

Ma, Venugopalan, PRL113, 19, 192301 (2014)

KW, Xiao, PRD92, 11, 111502 (2015)

(II) NRQCD or pNRQCD factorization

Butenschoen, Kniehl, PRD84, 051501 (2011)

Chao, Ma, Shao, Wang, Zhang, PRL108, 242004 (2012)

Gong, Wan, Wang, Zhang, PRL110, 042002 (2013)

(III) QCD factorization w/ Fragmentation Functions

Kang, Qiu and Sterman, PRL108, 102002 (2012)

Bodwin, Chung, Kim, Lee, PRL113, 022001 (2014)

Ma, Qiu, Sterman, Zhang, PRL113, 14, 142002 (2014)

Lee, Qiu, Sterman, KW, SciPost Phys. Proc.8, 143 (2022)

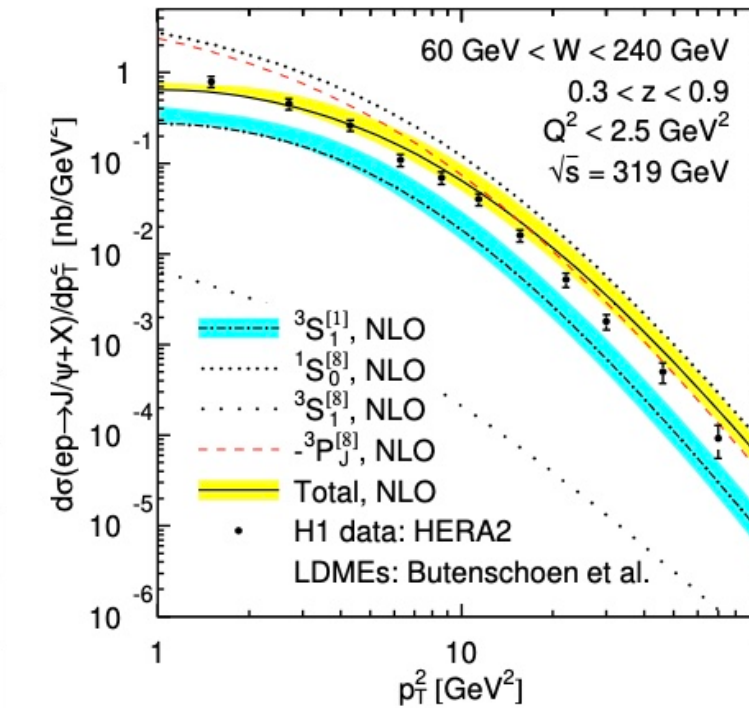
Matching condition btw (I) and (II) (similar to W- and Y-term in TMD factorization):

$$\frac{d\sigma_{A+B \rightarrow \psi+X}(m \neq 0)}{d^2p_T dy} = \underbrace{\frac{d\sigma_{A+B \rightarrow \psi+X}^{\text{Resum}}(m \neq 0)}{d^2p_T dy}}_{p_T \ll M} + \underbrace{\frac{d\sigma_{A+B \rightarrow \psi+X}^{\text{NRQCD-(n)}}(m \neq 0)}{d^2p_T dy}}_{p_T \gtrsim \mathcal{O}(M)} - \underbrace{\frac{d\sigma_{A+B \rightarrow \psi+X}^{\text{Asym-(n)}}(m \neq 0)}{d^2p_T dy}}_{\text{Subtraction of double counting}}$$

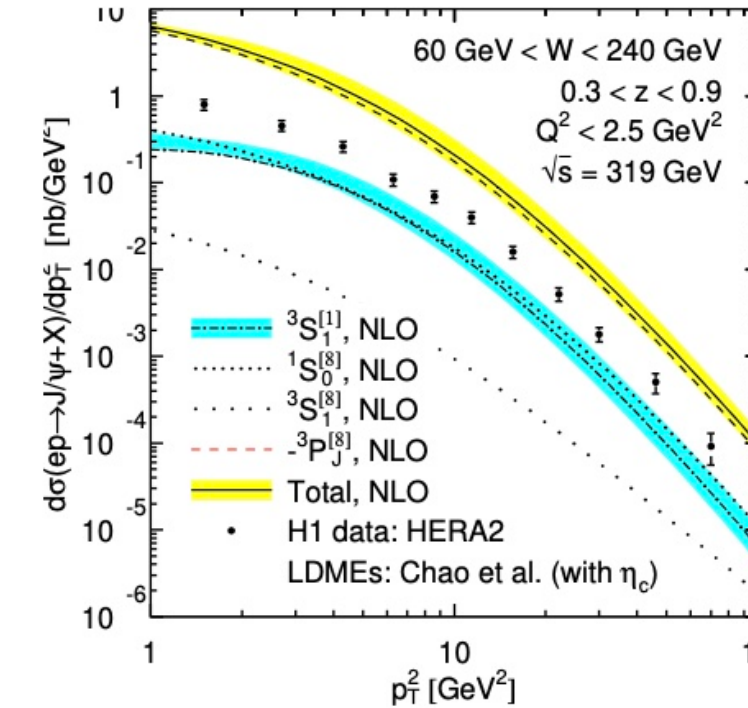
A longstanding issue on LDMEs

D. Boer et al., Prog. Part. Nucl. Phys. 142, 104162 (2025)

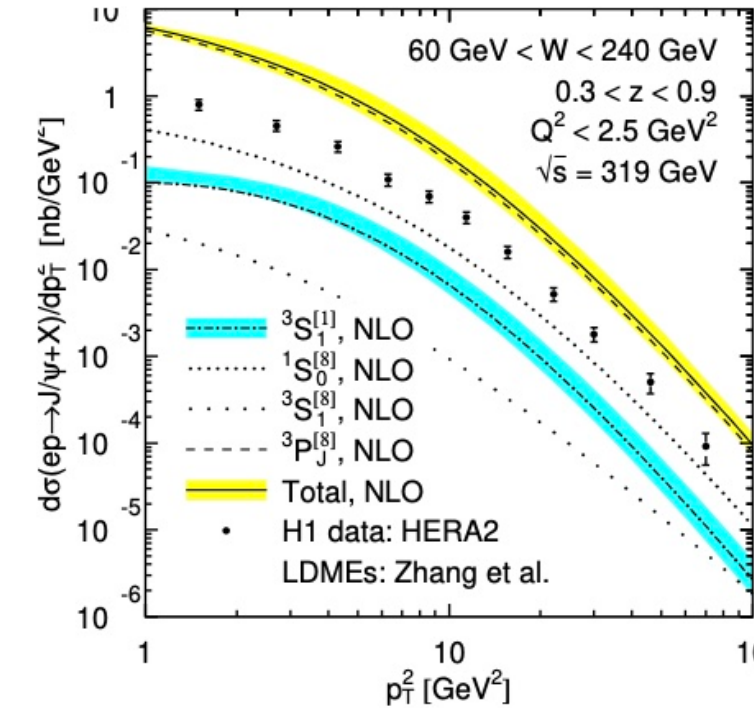
Acronym	Reference	J/ψ hadropr.	J/ψ photopr. and e^+e^-	J/ψ polar. in hadropr.	η_c hadropr. ($P_T > 6.5$ GeV)
BK11	Butenschön et al. [104, 105, 106, 107]	✓ ($P_T > 3$ GeV)	✓	✗	✗
H14	Chao et al. + η_c [114]	✓ ($P_T > 6.5$ GeV)	✗	✓	✓
Z14	Zhang et al. [115]	✓ ($P_T > 6.5$ GeV)	✗	✓	✓
G13	Gong et al. [109]	✓ ($P_T > 7$ GeV)	✗	✓	✗
C12	Chao et al. [108]	✓ ($P_T > 7$ GeV)	✗	✓	✗
B14	Bodwin et al. [80]	✓ ($P_T > 10$ GeV)	✗	✓	✗
pNRQCD	Brambilla et al. [110, 116]	✓ ($P_T > 15$ GeV)	✗	✓	✗✓



(a) BK11



(b) H14



(c) Z14

$$d\sigma_{Q=\psi} \approx \sum_i d\hat{\sigma}_{Q\bar{Q}[i]} \langle \mathcal{O}^\psi[i] \rangle$$

Collinear Long-Distance Matrix Elements (LDMEs)

↔ Gluonic correlators in pNRQCD factorization

$$\langle \mathcal{O}^Q[i = {}^{2S+1}L_J] \rangle \propto \sum_{X_s=q,g,\dots} \langle 0 | (\mathcal{O}_i^\dagger \mathcal{Y}_n^\dagger)^{ab}(0) | Q + X_s \rangle \langle Q + X_s | (\mathcal{Y}_n \mathcal{O}_i)^{ba}(0) | 0 \rangle$$

color indices

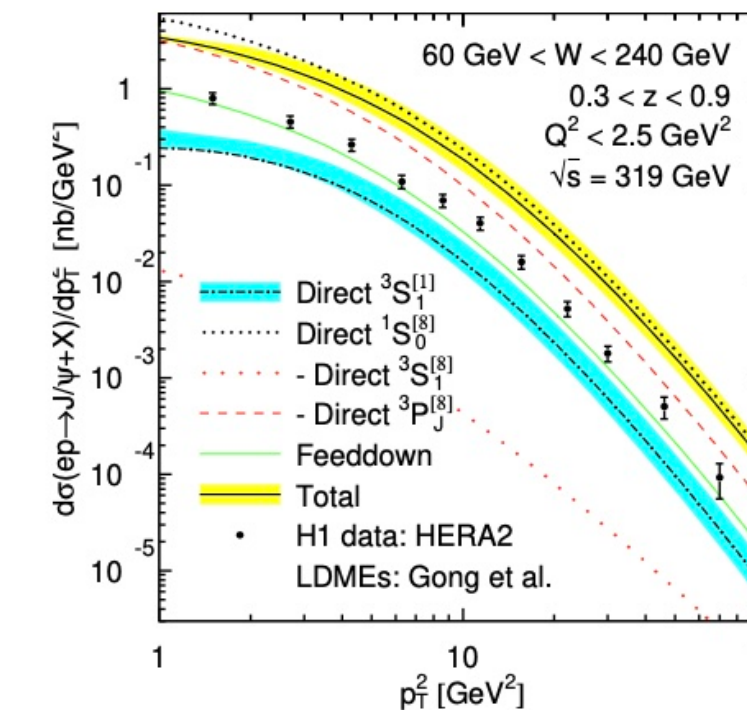
$Q = J/\psi, \eta_c, \dots$

Heavy
quark pair

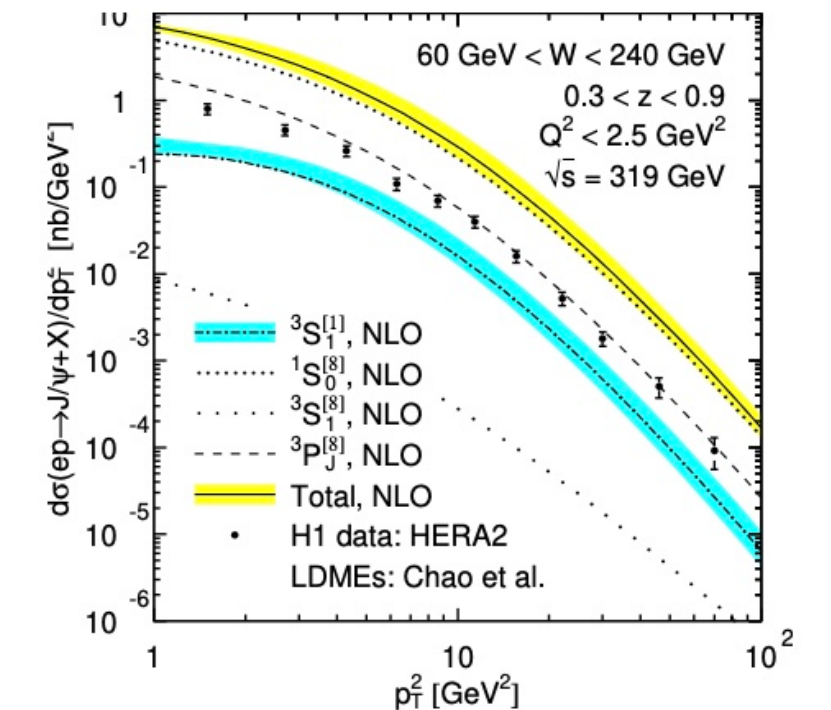
Wilson lines along the
light-like direction n

Nonperturbative LDMEs should be universality, however:

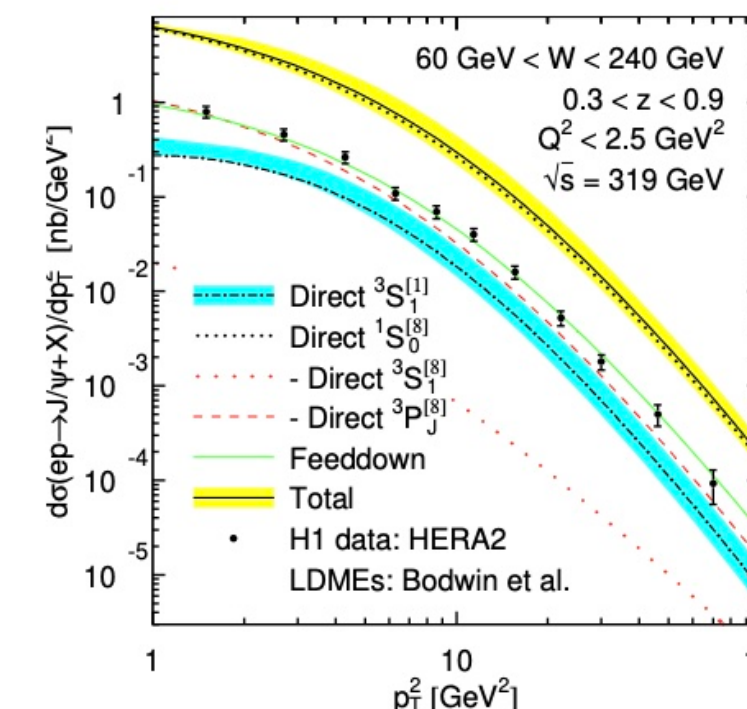
- Numbers are not the same.
- Not even the sign.



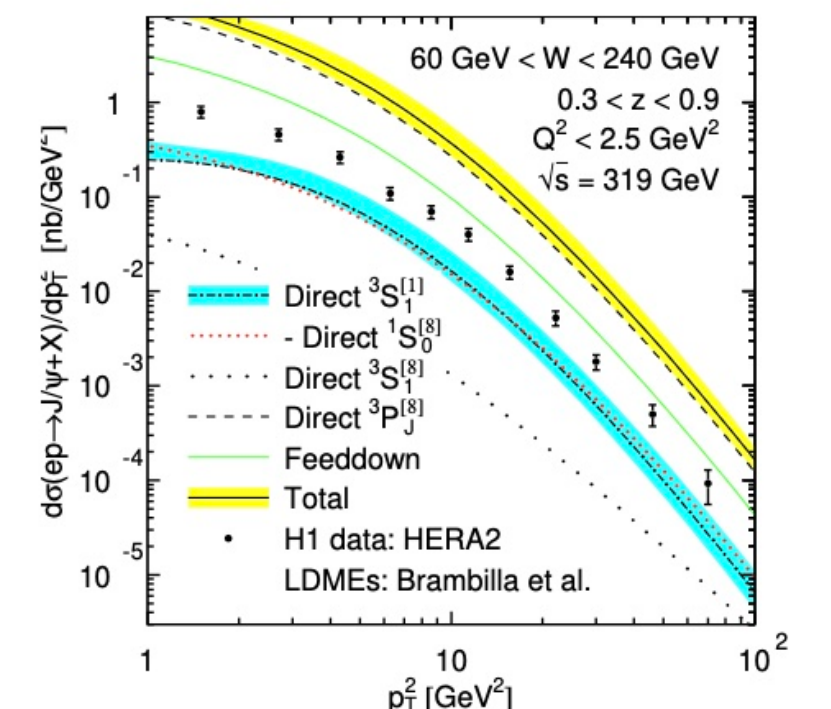
(d) G13



(e) C12



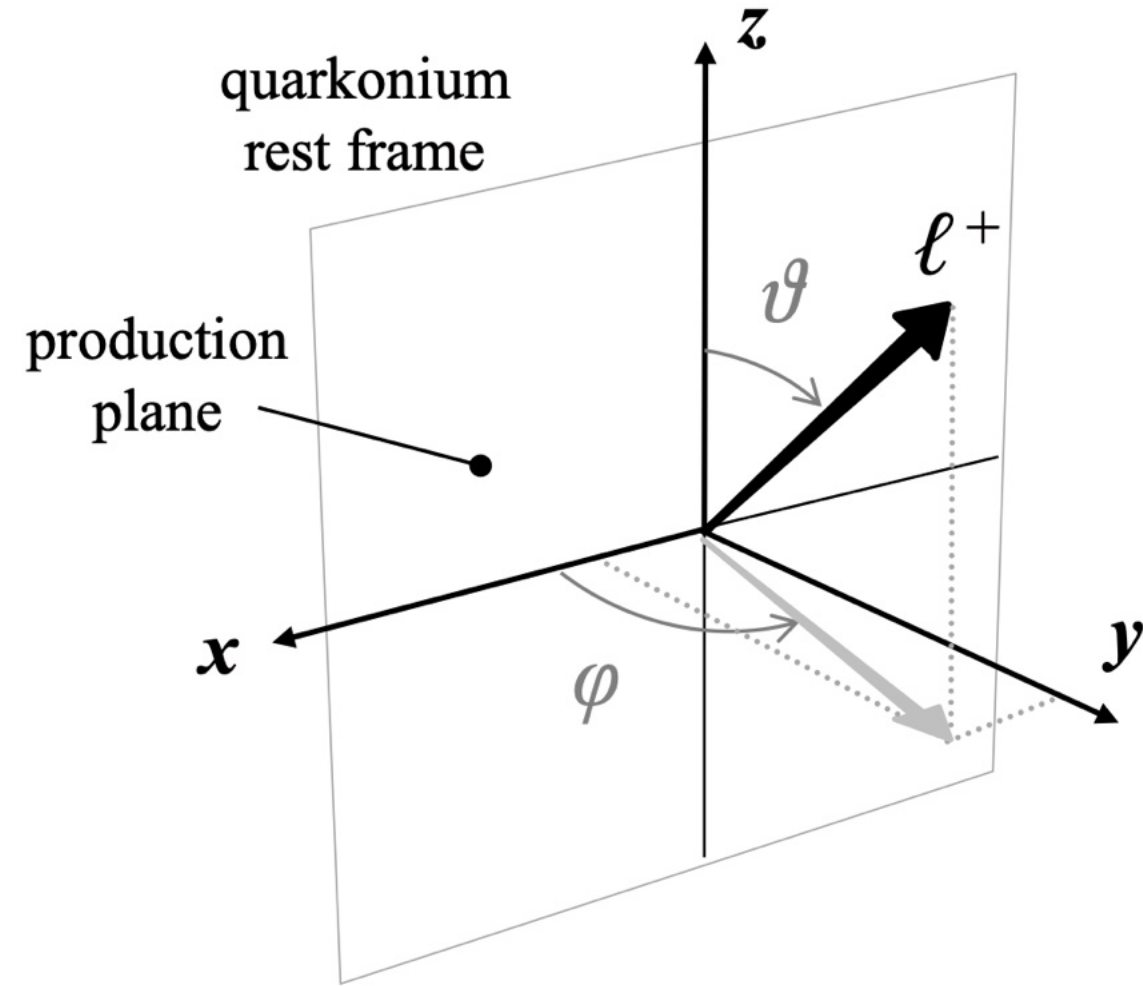
(f) B14



(g) pNRQCD

Recent updates: Xiangpeng Wang, Tuesday at 9:15 AM

Polarization of quarkonium



θ : polar angle
 ϕ : azimuthal angle

“production plane” contains the momenta of the beams

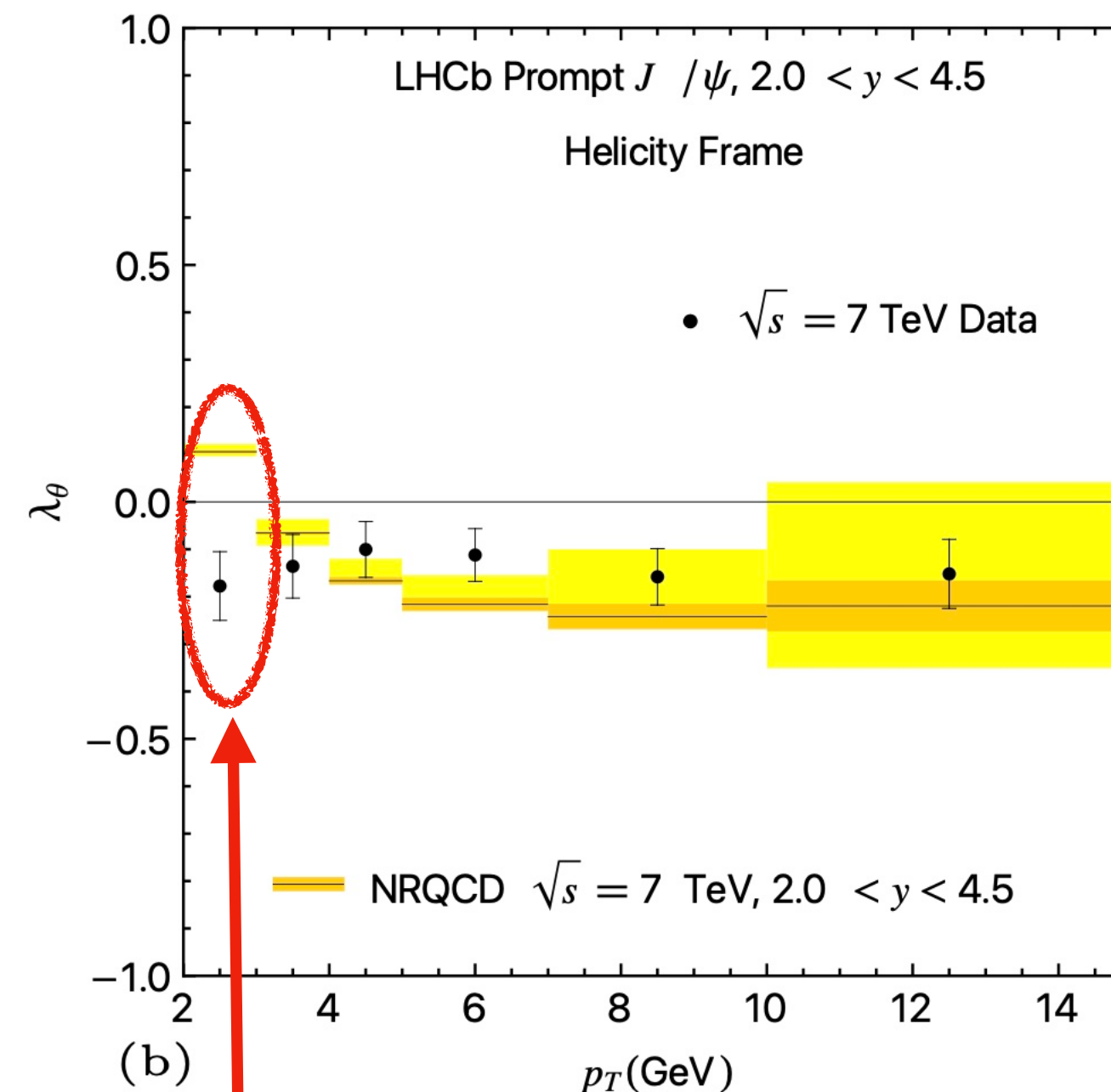
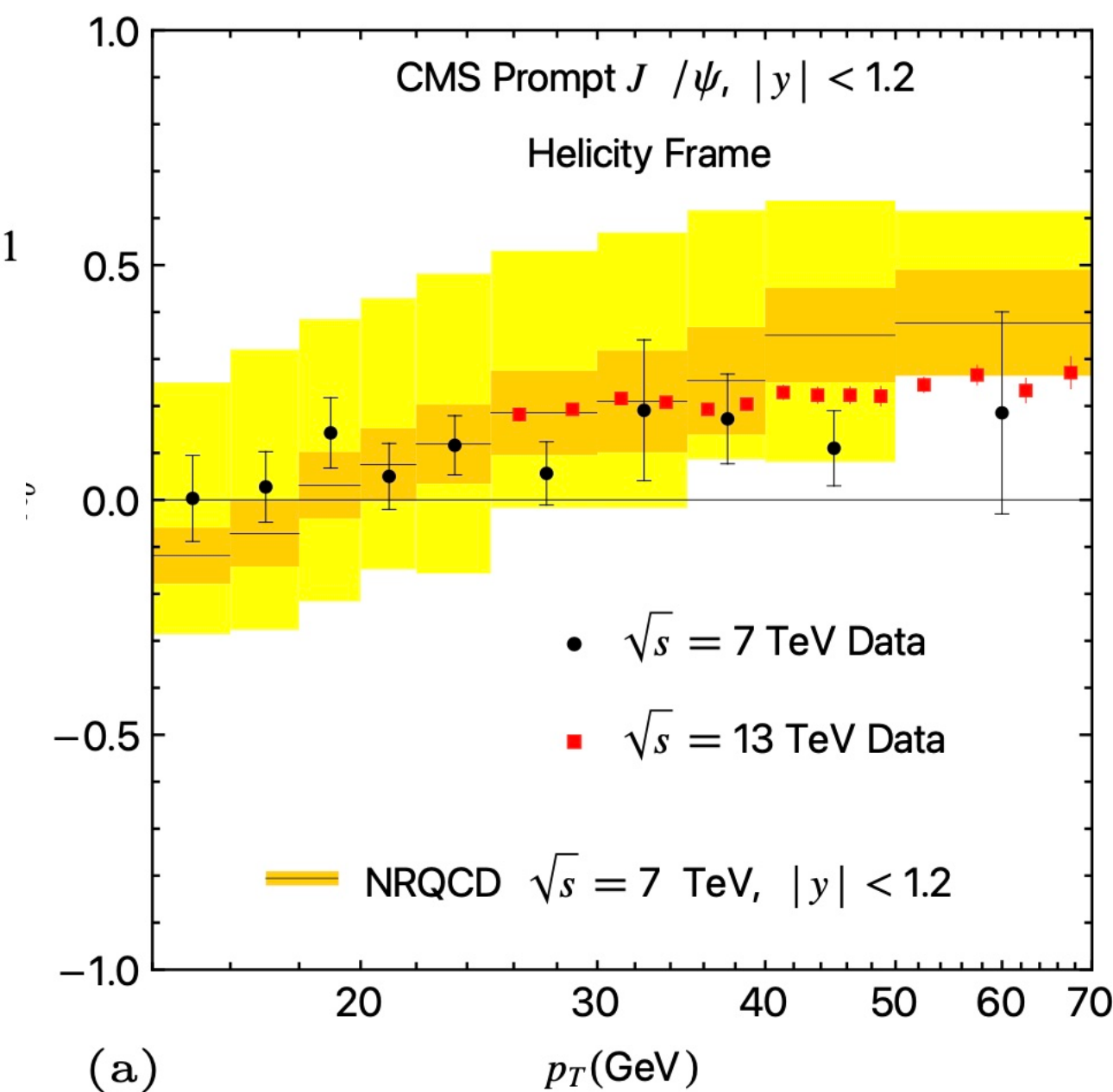
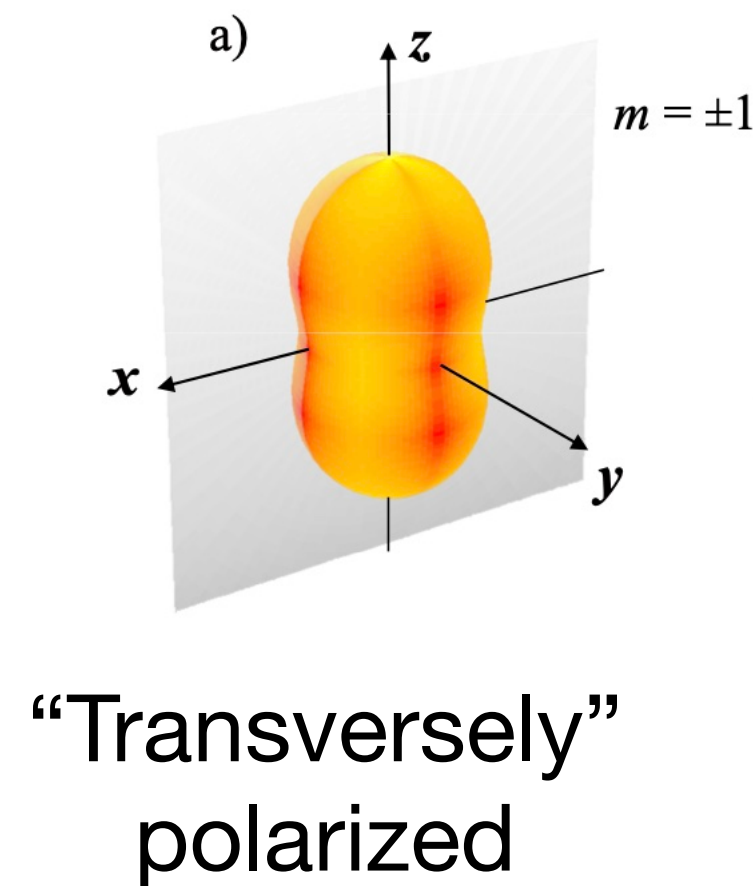
figures from [Faccioli, Lourenco, Seixas and Wohri, Eur. Phys. J. C69, 657-673 (2010)]

$$\frac{d\sigma^{J/\psi(\rightarrow l^+l^-)}}{d\Omega} \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos \phi$$

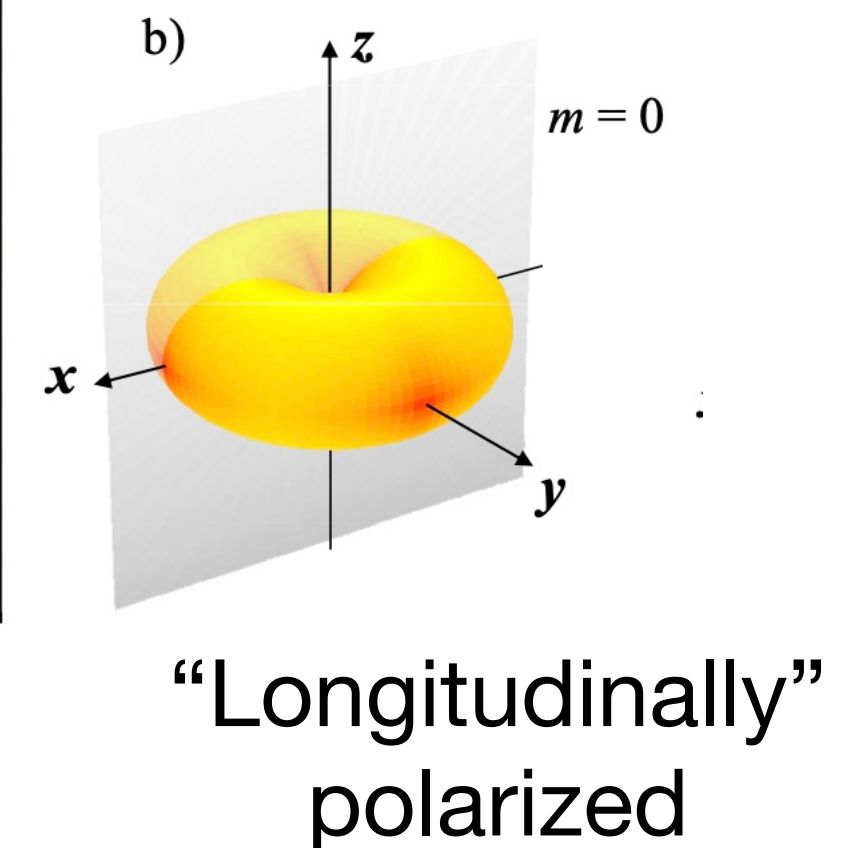
Transverse pol.: $\lambda_\theta = +1$ (photon-like)

Longitudinal pol.: $\lambda_\theta = -1$

Unpolarized: $\lambda_\theta = 0$



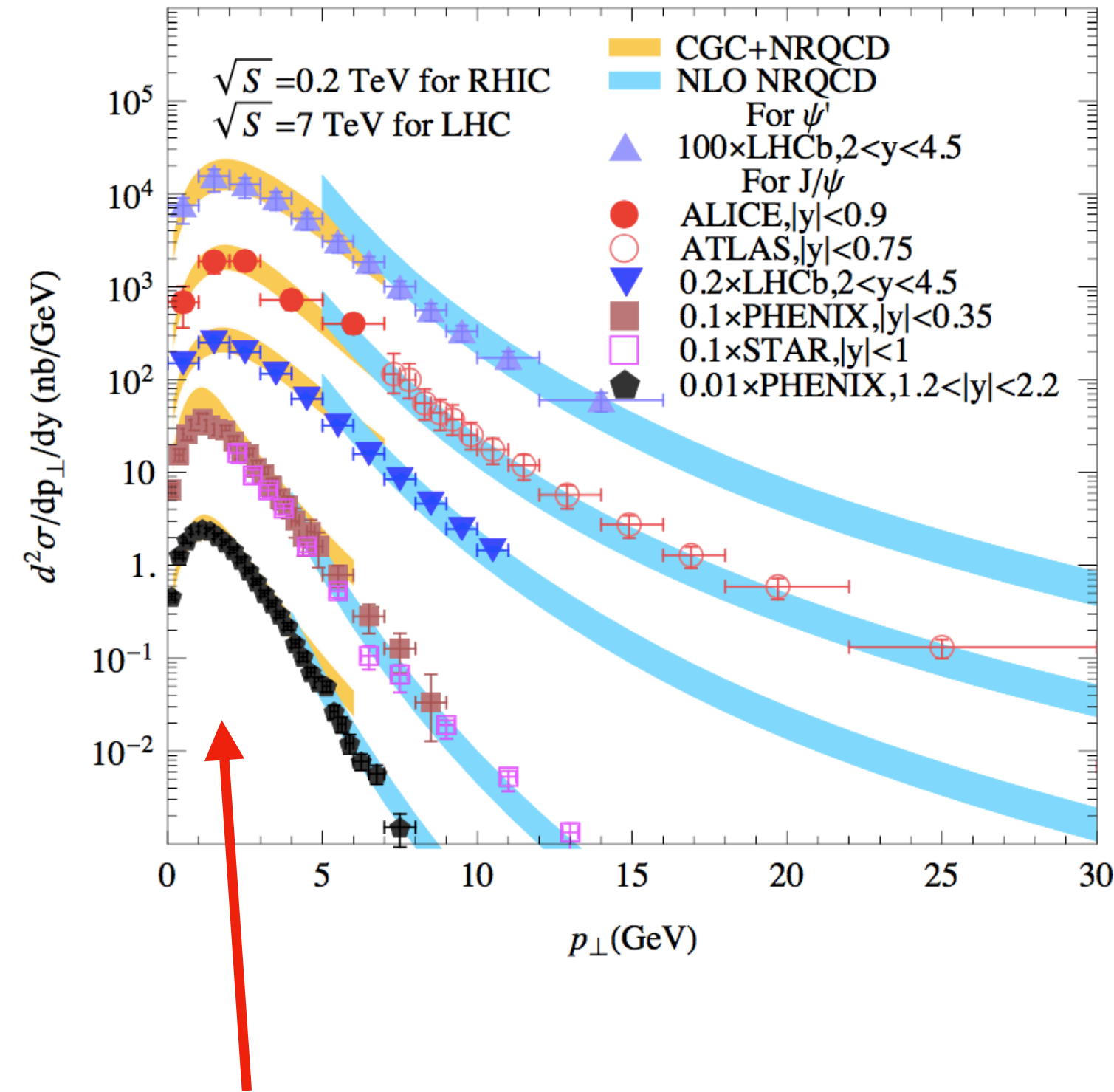
N. Brambilla, M. Butenschoen and X. P. Wang, PRD112, no.1, 1 (2025)



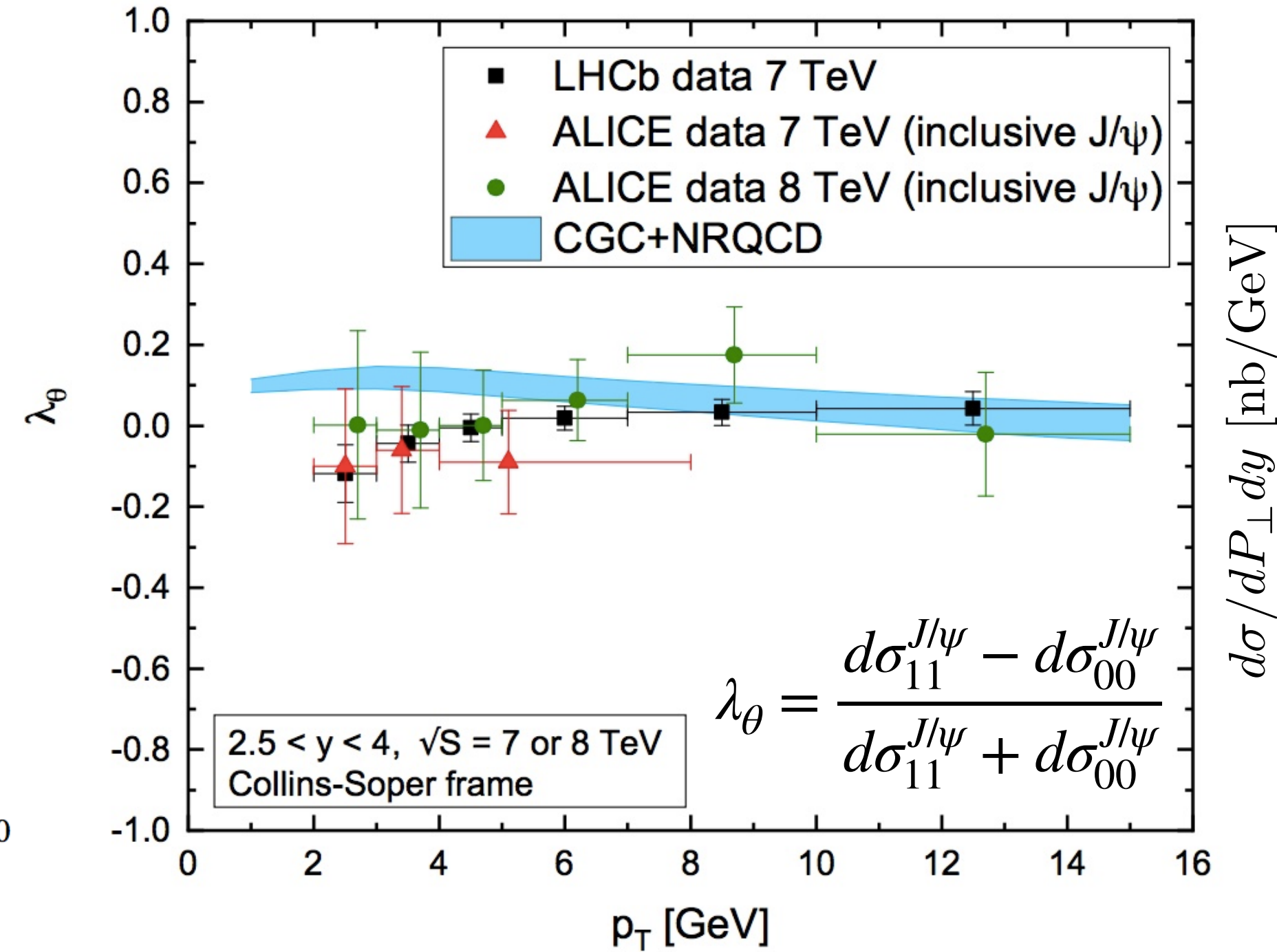
Need small-x resummation?

Forward quarkonium production: TMD vs. CGC

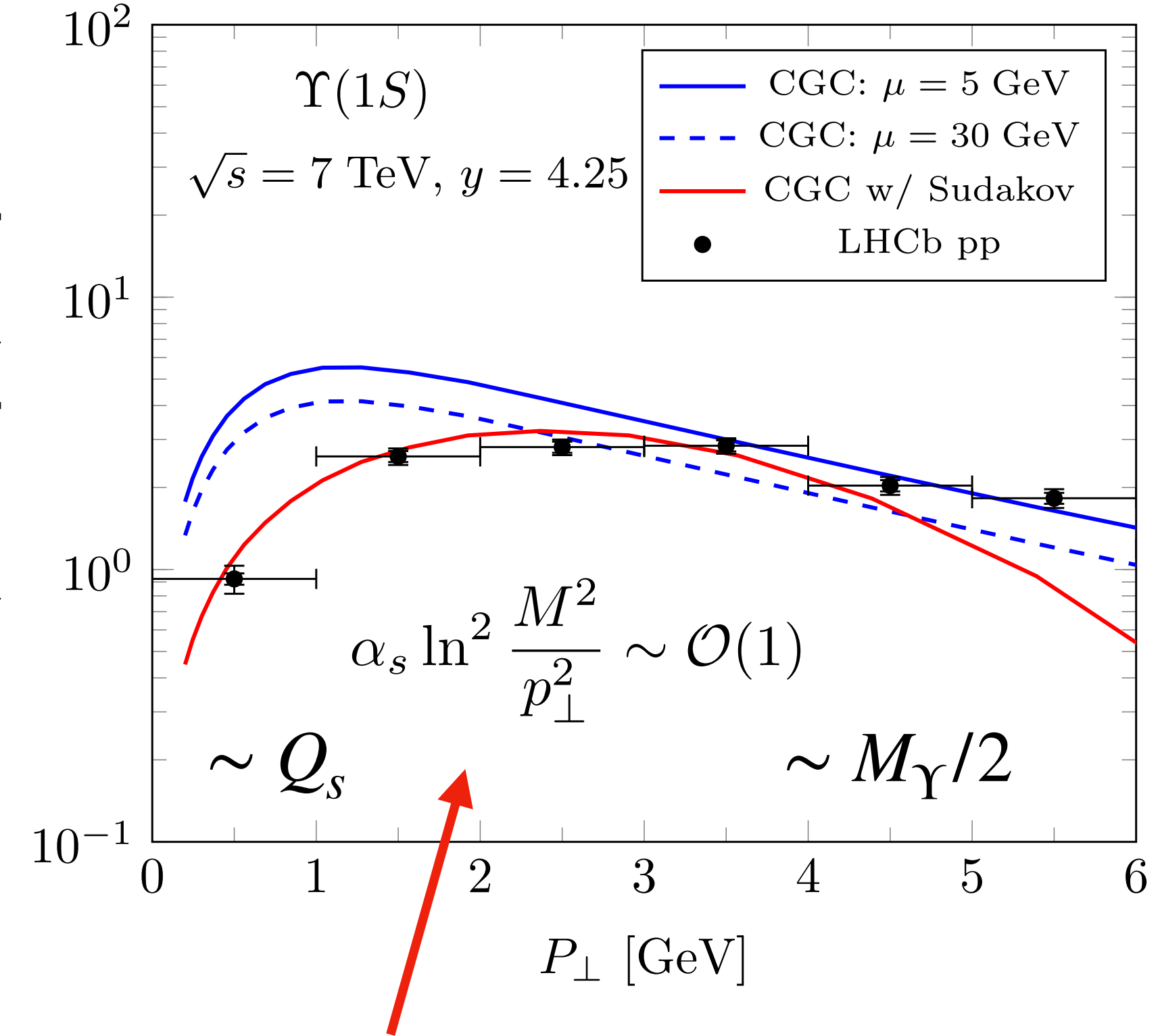
Ma, Venugopalan, PRL113, 19, 192301 (2014)



Ma, Stebel and Venugopalan, JHEP12, 057 (2018)



KW, Xiao, PRD92 (2015) 11, 111502



- ❖ The MV-model + JIMWLK evolution gives a good parametrization of the unpolarized gluon TMD PDF at small- x .
- ❖ Note: Positive definite LDMEs are used.

- ❖ Forward bottomonium probes $x \sim 10^{-4} - 10^{-5}$, giving $Q_s^2 \sim 1 \text{ GeV}^2$.
- ❖ Initial state soft-collinear parton shower effect (Sudakov: CSS evolution) is important.

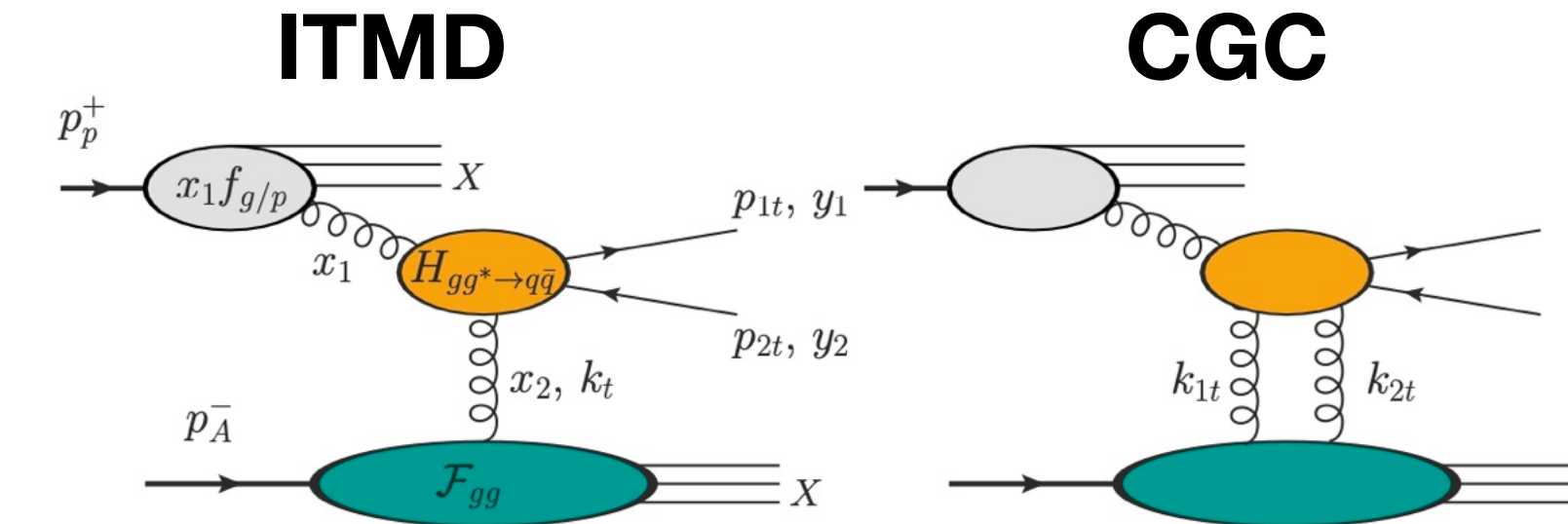
Remarks: from TMD to CGC

k_t : off-shellness in short distance parts
 Q : a hard scale.

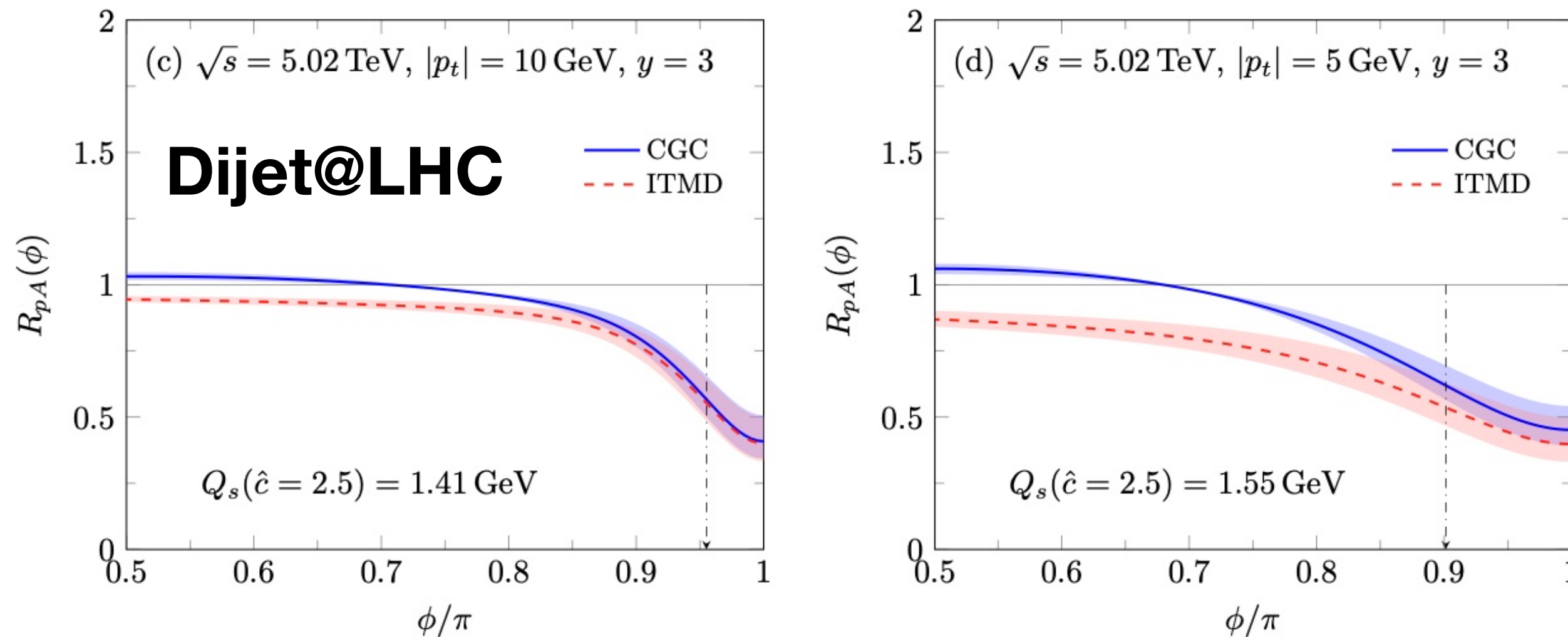
$\sigma_{\text{TMD}} + \text{kinematic twist } \mathcal{O}\left(\frac{k_t}{Q}\right) = \text{Improved TMD (ITMD)}$

$\sigma_{\text{ITMD}} + \text{higher-body genuine twist } \mathcal{O}\left(\frac{Q_s}{Q}\right) = \text{Color-Glass-Condensate (CGC)}$

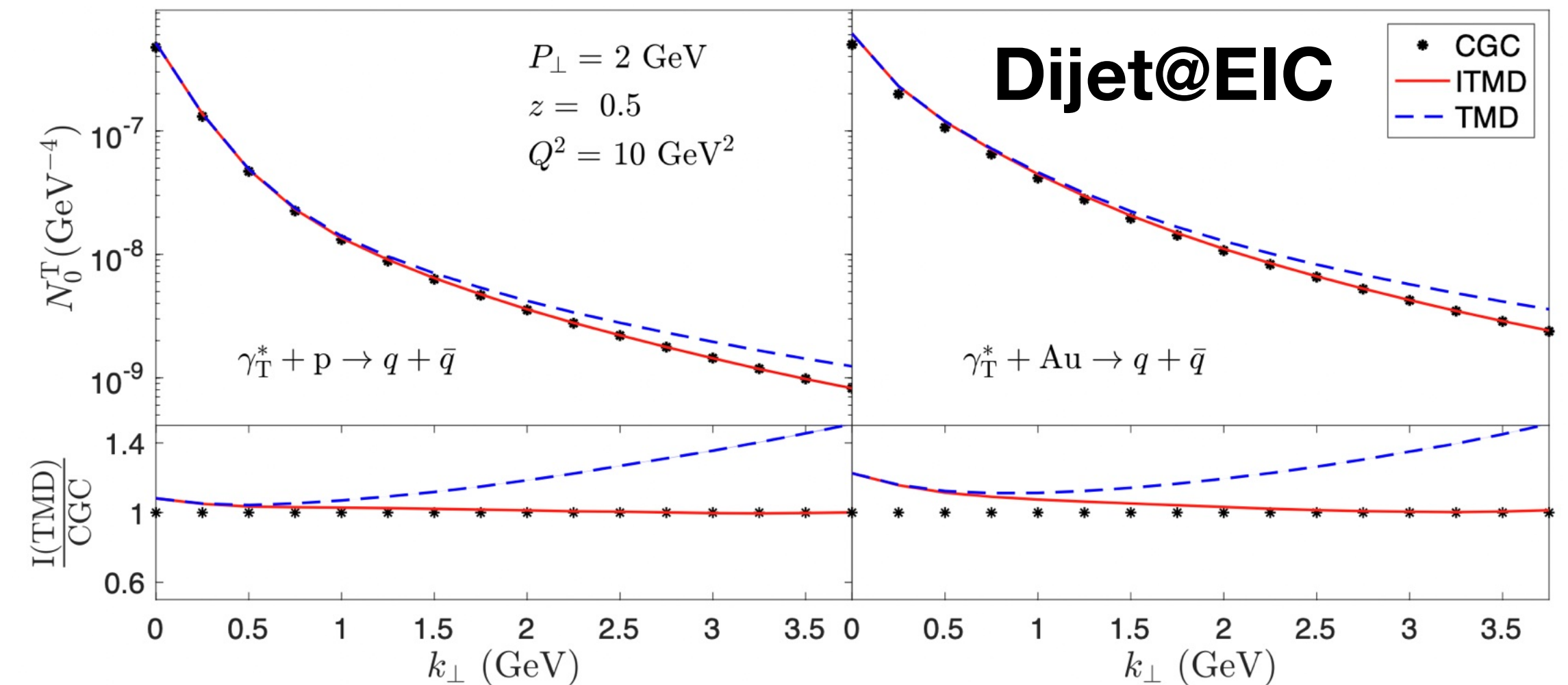
Saturation scale $Q_s^2 \sim A^{1/3} x^{-0.3}$



H. Fujii, C. Marquet and KW, JHEP12, 181 (2020).



R. Boussarie, H. Mantysaari, F. Salazar and B. Schenke, JHEP 09, 178 (2021)



Altinoluk, Boussarie and Kotko, JHEP05, 156 (2019).

Mantysaari, Mueller, Salazar and Schenke, PRL124, no.11, 112301 (2020).

Altinoluk, Marquet and Tael, JHEP06, 085 (2021).

Boussarie, Mantysaari, Salazar and Schenke, [arXiv:2106.11301 [hep-ph]].

...

TMD Shape Functions (ShFs)

M. G. Echevarria, JHEP10, 144 (2019)

S. Fleming, Y. Makris, and T. Mehen, JHEP04, 122 (2020)

$$d\sigma^{e+p \rightarrow Q+X} \sim F_{g/P}(b_T; \mu, \zeta) \sum_{i=1} S_0^{[8]}, \dots H^{[i]}(M_Q, Q; \mu) \Delta^{[i]}(b_T; \mu, \zeta)$$

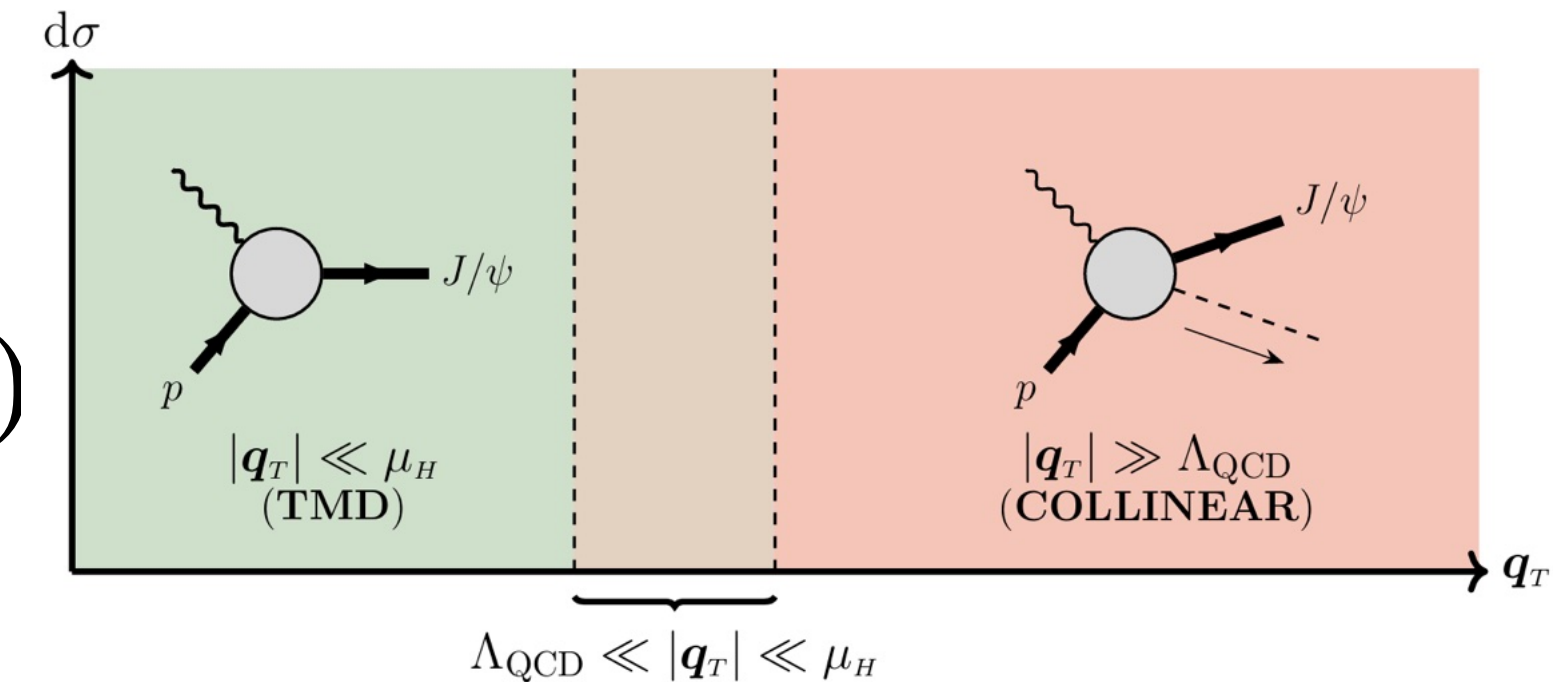
8-leading twist gluon TMDs

TMD ShFs: TMD generalization of the LDME operator definition.

$$\Delta^{[i]}(b_T; \mu, \zeta) \propto \sum_{X_s} \langle 0 | (\mathcal{O}_i^\dagger \mathcal{Y}_n^\dagger)^{ab}(\mathbf{b}_T) | Q + X_s \rangle \langle Q + X_s | (\mathcal{Y}_n \mathcal{O}_i)^{ba}(0) | 0 \rangle$$

OPE: $b_T \rightarrow 0 \rightarrow \sim \sum_n C_n^{[i]}(b_T; \mu, \zeta) \times \langle \mathcal{O}^Q[n] \rangle(\mu) + \mathcal{O}(b_T)$

$$\Rightarrow \Delta_{\text{NP}}^{[i]}(b_T; \mu, \zeta) \sum_n C_n^{[i]}(b_T^*; \mu, \zeta) \langle \mathcal{O}^Q[n] \rangle(\mu)$$



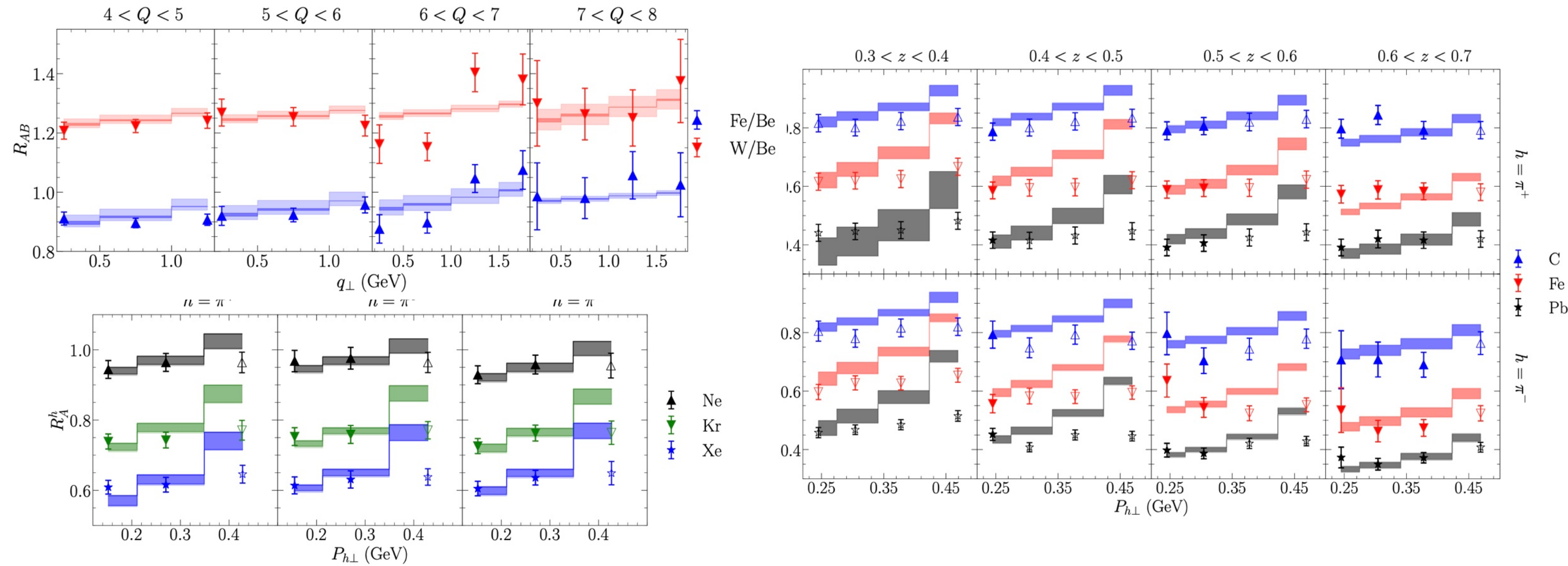
We need to extract both information on gluon TMD PDFs and the TMD ShFs, as we do for SIDIS (TMD PDFs + TMD FFs), in the future.

D. Boer, J. Bor, L. Maxia, C. Pisano and F. Yuan, JHEP08, 105 (2023)

Nuclear TMDs

M. Alrashed, D. Anderle, Z.B. Kang, J. Terry and H. Xing, PRL129, no.24, 242001 (2022)
M. Alrashed, Z. B. Kang, J. Terry, H. Xing and C. Zhang, [arXiv:2312.09226 [hep-ph]].

QCD global fit using TMD QCD factorization

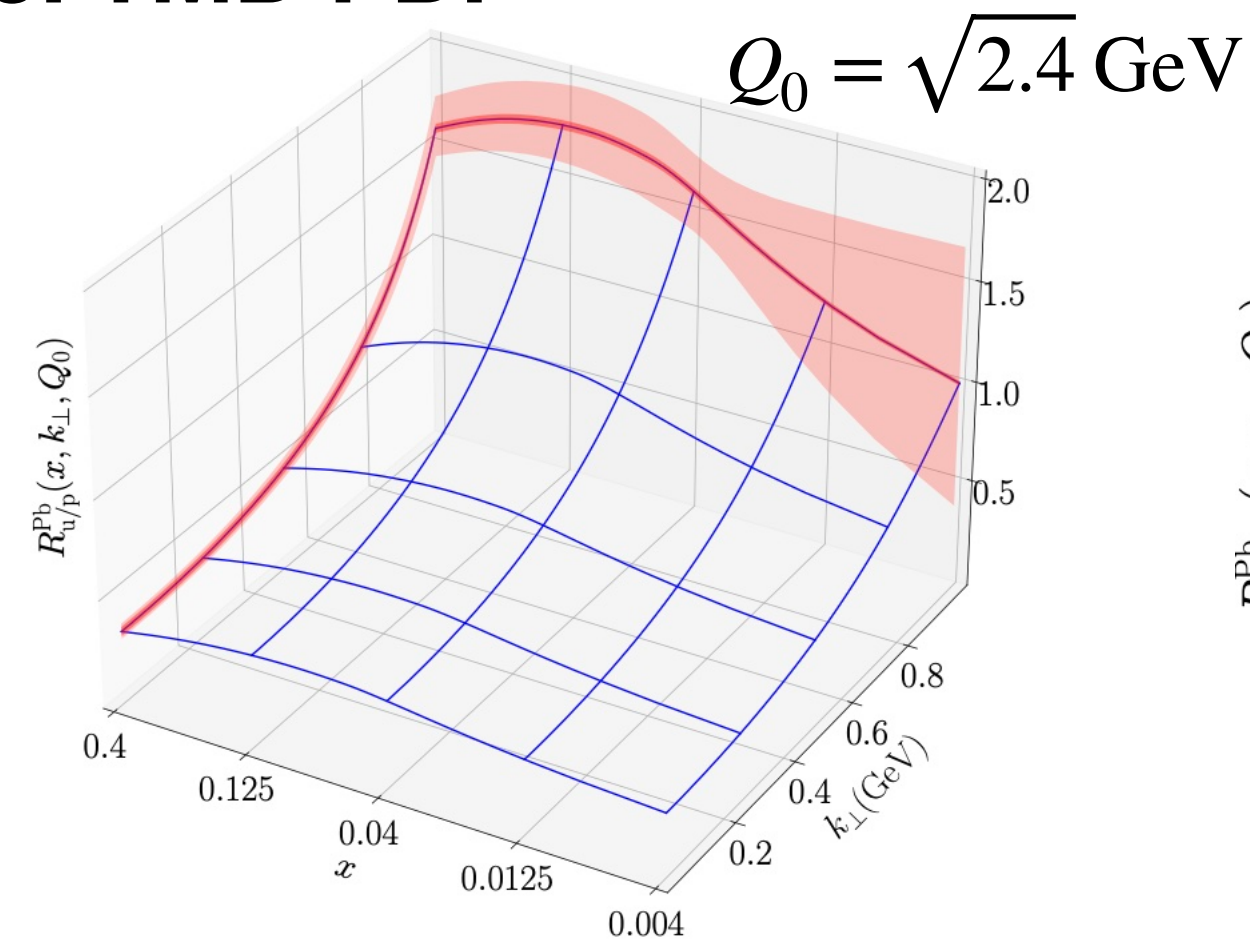


Collaboration	Process	Baseline	Nuclei	N _{data}	χ^2
JLAB [49]	SIDIS(π)	D	C, Fe, Pb	36	41.7
HERMES [40]	SIDIS(π)	D	Ne, Kr, Xe	18	10.2
RHIC [43]	DY	p	Au	4	1.3
E772 [41]	DY	D	C, Fe, W	16	40.2
E866 [42]	DY	Be	Fe, W	28	20.6
CMS [63]	γ^*/Z	N/A	Pb	8	10.4
ATLAS [83]	γ^*/Z	N/A	Pb	7	13.3
Total				117	137.8

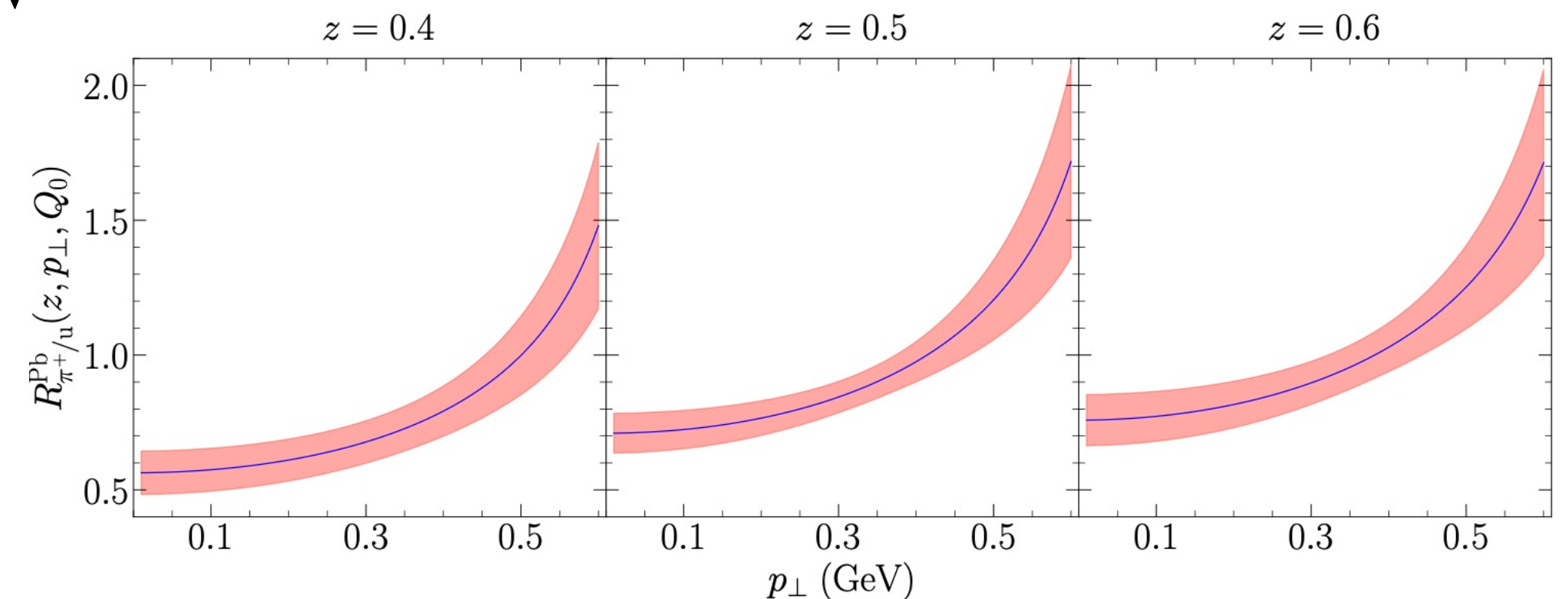
$$\chi^2/\text{d.o.f} = 1.275$$

Nuclear ratio: TMD PDF

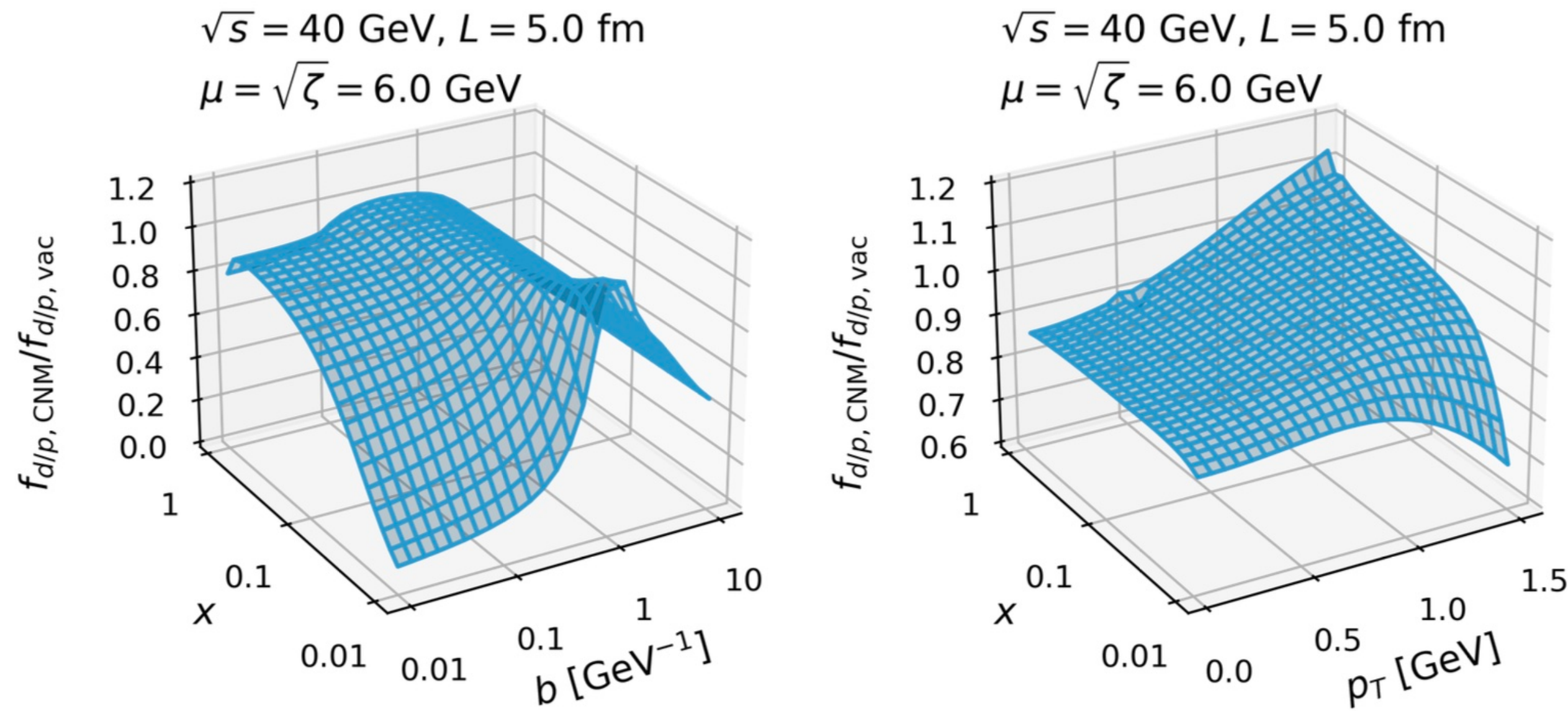
- ❖ Broadening in medium.
- ❖ Shadowing, antishadowing, EMC effects from collinear PDFs are seen.



Nuclear ratio: TMD FF

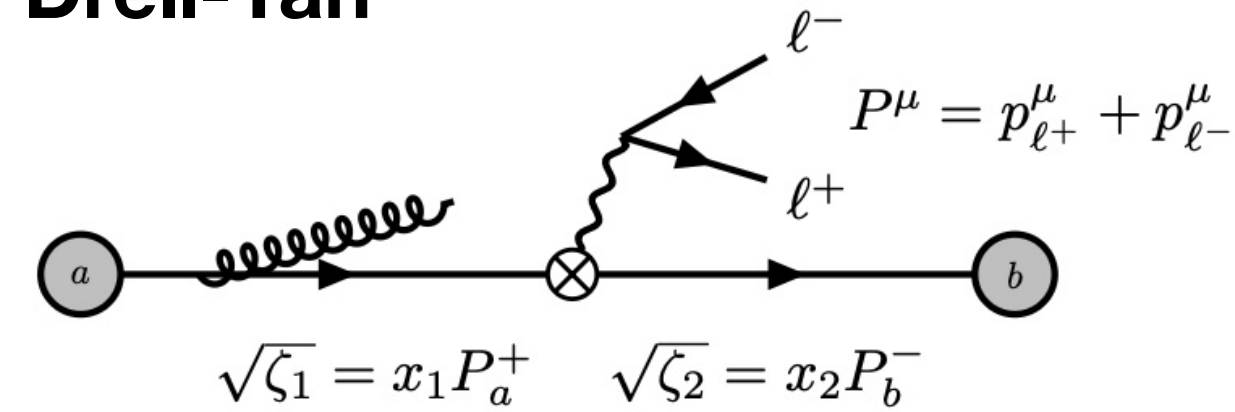


Cold Nuclear Matter Effects

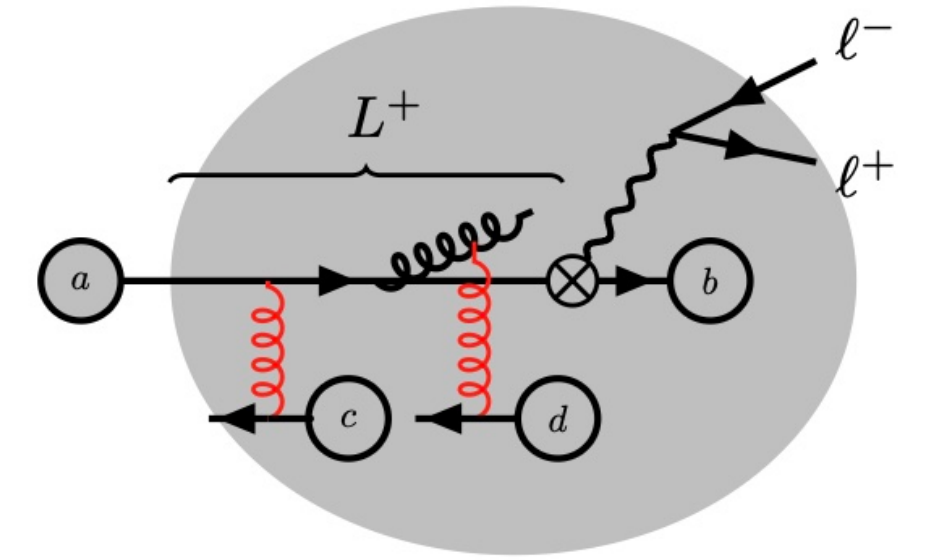


W. Ke, J. Terry and I. Vitev, JHEP02, 102 (2025)

Drell-Yan

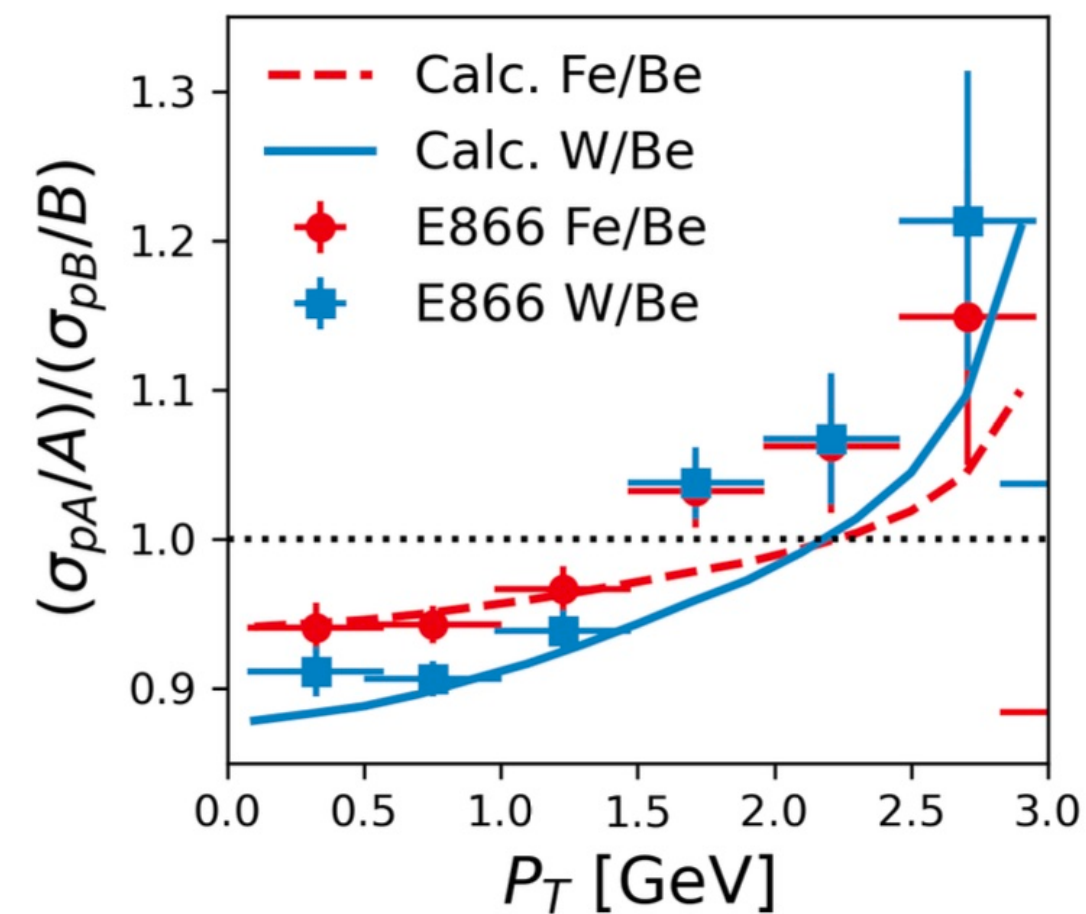
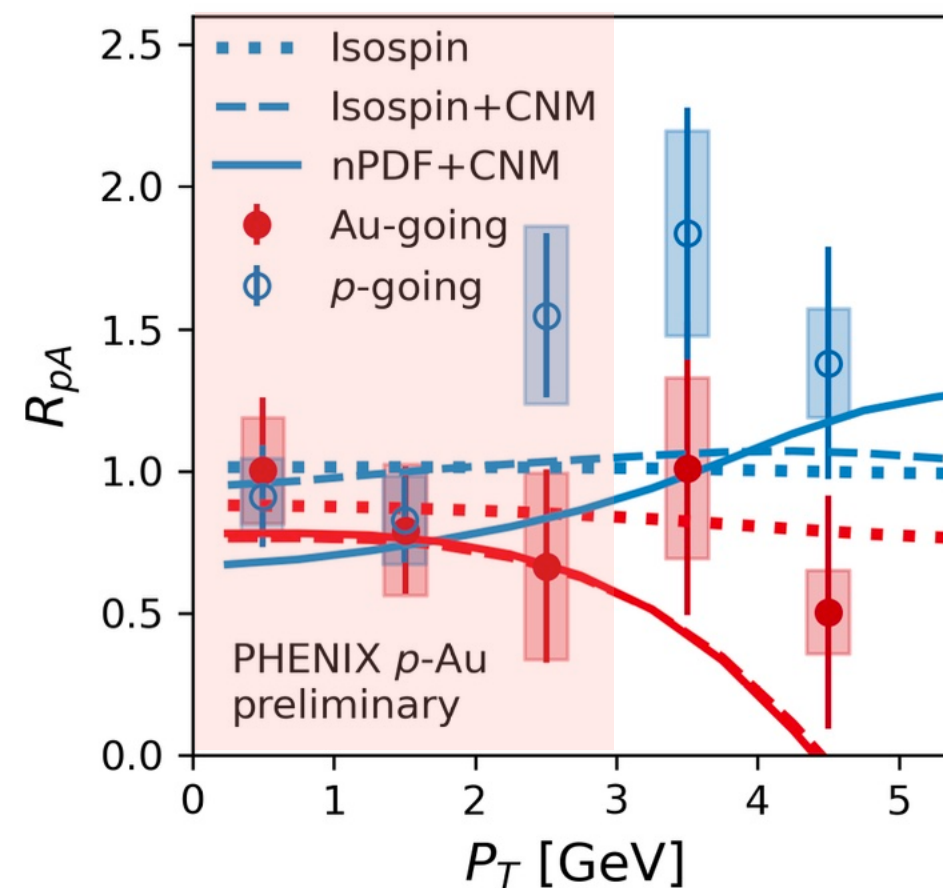
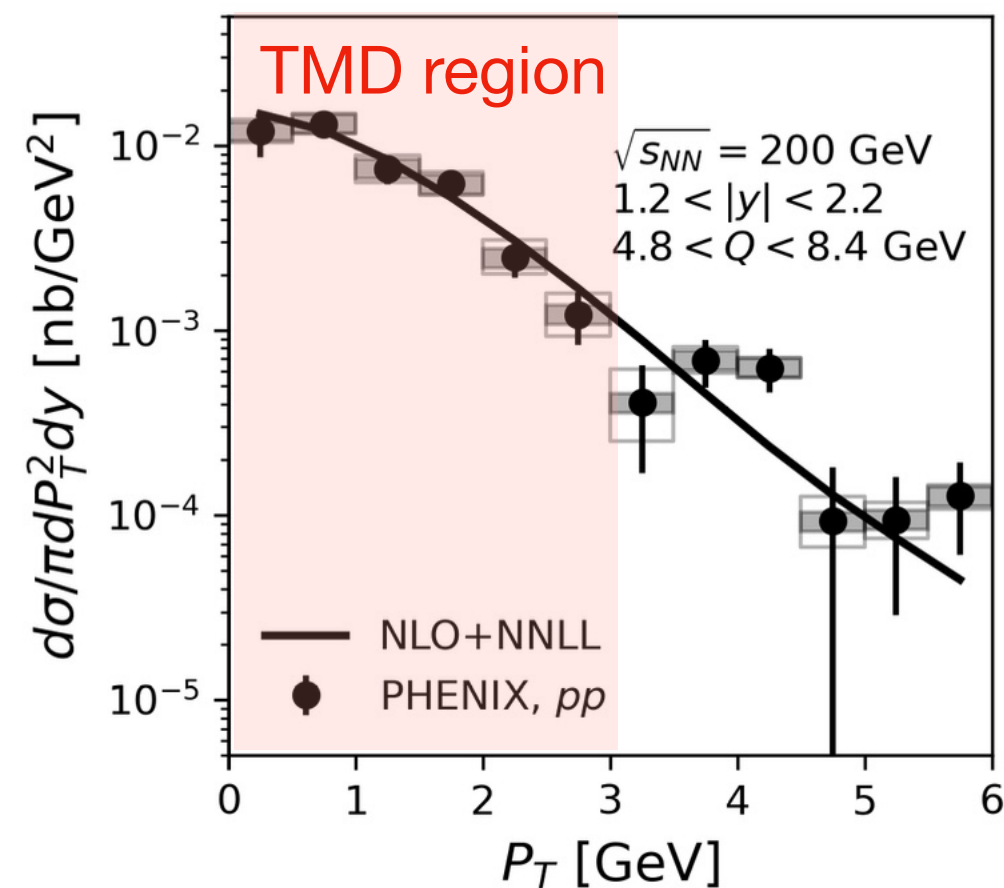


Nuclear Drell-Yan



- Multiple scattering in the medium (LPM effect) modifies the beam function of the projectile.
- If there are both ISI&FSI (e.g., π , J/ψ production), fully coherent energy loss effect is significant.

G. Jackson, S. Peigné and KW, JHEP05, 207 (2024)



A systematic extraction of the information on CNM using nuclear SIDIS and Drell-Yan data is feasible.

Concluding remarks

- ❖ TMD PDFs and FFs can be studied using two-scale observables, such as SIDIS, Drell-Yan, Di-hadron production in e^+e^- , and heavy hadron production in hadronic collisions.
- ❖ There have been rapid developments in TMD Theory, in conjunction with experiments and Lattice QCD; Global data fits of TMDs with higher precision are promising.
- ❖ Deeper insights into TMD physics would not only unravel the internal structure of hadrons but also impact many other physics, such as heavy-ion physics.
- ❖ Many other interesting topics, not covered in this talk, include subleading TMDs, Jet production, EE correlators, and so forth.

Thank you!

Appendix