Spin physics at small x [1]

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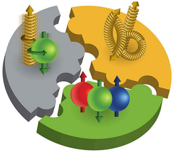
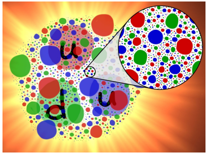
A Century of Spin

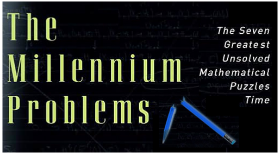
[1] Special thanks to J. Zhou and T. Liu for inputs.

Three pillars of EIC Physics

To understand our physical world, we have to understand QCD!



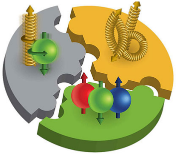


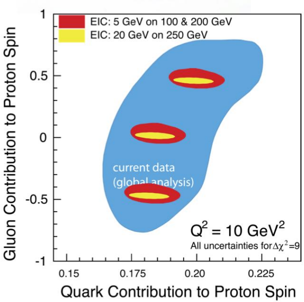


- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

EICs: keys to unlocking these mysteries! Many opportunities will be in front of us!

Understanding Nucleon Spin



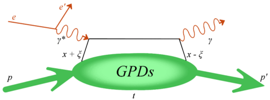


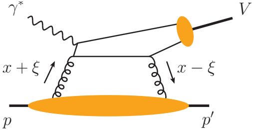
Jaffe-Manohar decomposition

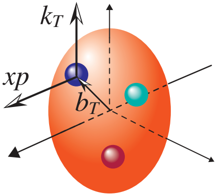
$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma + L_q}_{\text{Quark}} + \underbrace{\Delta G + L_g}_{\text{Gluon}}$$

- Quark spin $\Delta\Sigma$ is only 30% of proton spin.
  $g_1(x, Q^2) = \frac{1}{2}\sum e_q^2\,[\Delta q + \Delta\bar{q}]$ [EMC, 1988]
- Spin Puzzle: contradicts with the quark model.
- The rest of the proton spin must come from the gluon spin ΔG, quark and gluon OAM $L_{q,g}$.
- EIC will help determine ΔG precisely.

Exclusive DIS (DVCS and DVVP)







Ji Sum Rule:

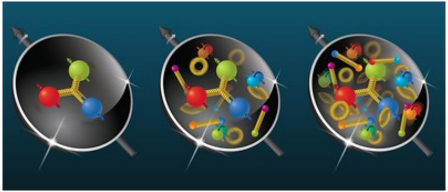
$$\frac{1}{2} = J_q + J_g$$

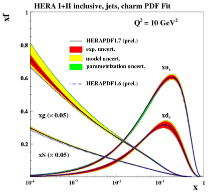
$$J_q = \frac{1}{2}\Delta\Sigma + L_q = \frac{1}{2}\int dxx\,(H_q + E_q)\ ,$$

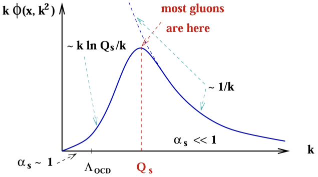
$$J_g = \frac{1}{4}\int dx\,(H_g + E_g)\,.$$

- Allows us to access to spacial distributions (related to GPDs via FT) of quarks and gluon in the nucleon.
- Obtain the information about the orbital motions indirectly.

Saturation Physics (Color Glass Condensate) at High Energy

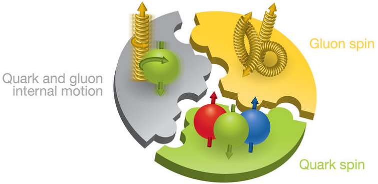


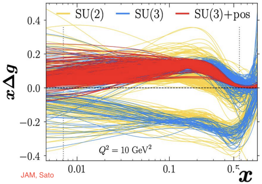




- Gluon density grows rapidly as x gets small.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK/JIMWLK) $\Rightarrow$ ultra-dense gluonic matter
- Multiple Scattering (MV model) + Small-x (high energy) evolution
- Small-x UGD $\simeq$ Gluon TMD: [Bomhof, Mulders, Pijlman, 06] Link and [Dominguez, Marquet, Xiao, Yuan, 11] Link; Small-x evolution + Sudakov [Mueller, Xiao, Yuan, 13]

Quark and Gluon Helicity at Small x





Why Small-x Helicity Matters

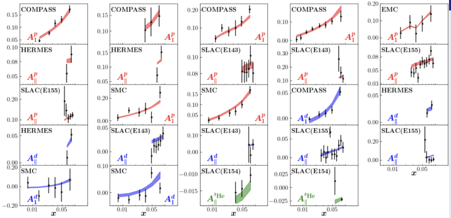
- Small-x: $\Delta q(x)$, $\Delta G(x)$ are expected to increase, yet they are not well determined.
- Do gluons carry significant spin at small x? **Need theory for small-x helicity**
- BER papers and Infrared Evolution Equations (IREE) [Bartels, Ermolaev, Ryskin, 1996; etc] Helicity evolution resummation parameter is double-logarithmic $\alpha \ln^2 \frac{1}{x}$ instead of $\alpha \ln \frac{1}{x}$. (The 2nd $\ln x$ arises due to transverse momentum integral).

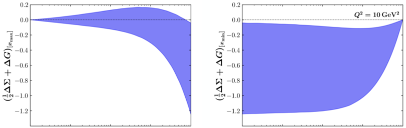
Light-Cone Operator Treatment (LCOT) by Kovchegov et al

Recent Progress: New small x evolution equation for helicity

- Identify sub-Eikonal line operator def. for the quark and gluon helicity distribution.
- **Formulation of helicity evolution equations** including $q \to g$ and $g \to q$ transitions
- **Exact analytic solution** in large-N_c and N_f limit, Resum Double-log: $\alpha_s \ln^2(1/x)$
- [Kovchegov, Pitonyak, Sievert, Cougoulic, Santiago, Tawabutr, Tarasov, Adamiak, Melnitchouk, Sato, Borden, Li, Manley, Baldonado, Becker, 15-25]
- Power-law growth: $\Delta\Sigma \sim \Delta G \sim g_1(x) \sim x^{-\lambda}$ with $\lambda = 3.661\sqrt{\frac{\alpha_s N_c}{2\pi}}$.
- Compared with BER results: $\lambda = 3.664\sqrt{\frac{\alpha_s N_c}{2\pi}}$, which differs in the 3rd decimal point. At 4-loop: $\Delta\gamma_{GG}^{BER/Kov}(\omega) = \frac{4\bar{\alpha}_s}{\omega} + \frac{8\bar{\alpha}_s^2}{\omega^3} + \frac{56\bar{\alpha}_s^3}{\omega^5} + \frac{504(496)\bar{\alpha}_s^4}{\omega^7} + \cdots$

JAMsmallx Results: Experimental Evidence for Small-x Spin





$$5 \times 10^{-3} < x < 0.1,$$
$$1.69 < Q^2 < 10.4\ \mathrm{GeV}^2$$

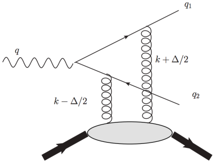
How much spin is there at small x?

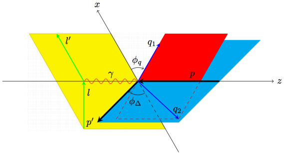
- [JAMsmallx Collaboration, Adamiak, et al, 23]
- 226 polarized DIS and SIDIS data points from COMPASS, HERMES, etc with $\chi^2/N_{pts} = 1.03$
- Running-coupling large-N_c&N_f evolution
- Total helicity contribution becomes negative at small-x. Large uncertainties, need better constraints

$$\int_{10^{-5}}^{0.1} dx \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = -0.64 \pm 0.60$$

Gluon and Quark OAMs at the EIC

Wigner Distribution [Hatta, Xiao, Yuan, 16] Directly probing the gluon OAM at the EIC. [Ji, Yuan, Zhao, 16; Hatta, Nakagawa, Xiao, Yuan, Zhao, 16]





Measurement of single longitudinal target-spin asymmetry (two minijets diffractive production)

$$A_{\sin(\phi_q-\phi_\Delta)} \propto L_g$$

- Double spin asymmetry: [Bhattacharya, Boussarie, Hatta, 22]
- First measurement of the gluon OAM in the proton Jaffe-Manohar spin sum rule!
- Quark OAM studies: [Bhatttacharya, Metz, Zhou, 17; Engelhardt, 17] [Bhattacharya, Zheng, Zhou, 24] $ep \to ep\pi^0$ exclusive.

Small-x Evolution of Gluon GPD E_g

Evolution Equation for spin flip GPD $E_g(x)$ from **dipole S-matrix** approach

$$\partial_Y \mathcal{E}(k_\perp) = \frac{\bar{\alpha}_s}{\pi} \int \frac{d^2k'_\perp}{(k_\perp - k'_\perp)^2} \left[\mathcal{E}(k'_\perp) - \frac{k_\perp^2}{2k'^2_\perp} \mathcal{E}(k_\perp) \right] - 4\pi^2 \alpha_s^2 \mathcal{F}_{1,1}(k_\perp) \mathcal{E}(k_\perp).$$

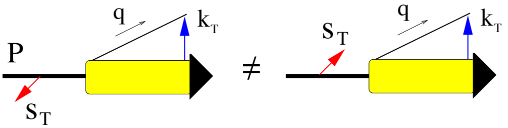
Results and Impact

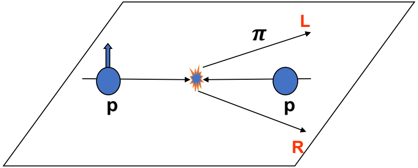
- Ji sum rule: $J_g = \frac{1}{2} \int_0^1 dx\, x[H_g + E_g]$ with $xE_g = \int d^2k_\perp \mathcal{E}(k_\perp)$.
- [Hatta & Zhou, 22] E_g has identical small-x behavior to BFKL pomeron: $xE_g(x) \sim xH_g(x) \sim xG(x) \propto \left(\frac{1}{x}\right)^{\bar{\alpha}_s 4 \ln 2}$.
- **Saturation**: E_g/H_g ratio is expected to saturate to a constant at very small-x.
- **Impact**: Possible significant small-x contribution to nucleon spin sum rule from E_g.

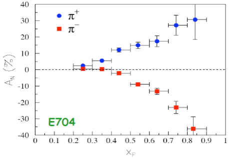
Sivers function and Single Spin Asymmetry in $p^{\uparrow}p \to \pi + X$

[Sivers, 90] correlation between proton $\vec{S}_T$ and parton $\vec{k}_\perp$

$$\hat{f}_{a/p^{\uparrow}}(x, k_\perp) = f_{a/p}(x, k_\perp) - \frac{k_\perp}{M_p} f_{1T}^{\perp a}(x, k_\perp) \vec{S}_T \cdot (\hat{\vec{P}} \times \hat{k}_\perp).$$







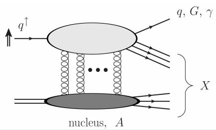
- Need novel mechanism beyond collinear framework: Sivers function describes the transverse momentum distribution correlated with the transverse polarization vector of the nucleon.
- $A_N = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$ implies u and d anisotropy.

Sivers effect in pp/pA collisions and Forward Single Spin Asymmetry A_N

Single spin asymmetry

- [STAR Collaboration, 20] A_N for forward π^0 in pp, pAl, and pAu at $\sqrt{s_{NN}} = 200$ GeV.
- [Boer, Dumitru, Hayashigaki, 06]
  - Single transverse-spin asymmetries in forward pion production
  - Application to RHIC processes
- [Hatta, Xiao, Yoshida, Yuan, 16] SSA in forward pA collisions at small-x
- [Kovchegov, Santiago, 20] Lensing mechanism + Small-x mechanism (Multiple scattering and Evolution)

Small-x Effects



- [Kovchegov, Sievert, 12] Odderon and New mechanism for SSA
- [Zhou, 14; Boer, Echevarria, Mulders, Zhou, 2017; Yao, Hagiwara, Hatta, 18] Gluon Sivers and the Odderon
- [Dong, Zheng, Zhou, 18; Kovchegov, Santiago, 21] Quark Sivers, odderon and evolution

The Einstein-Podolsky-Rosen Paradox

- **The EPR Paper** [Phys. Rev. 47, 777 (1935)]
  - Can Quantum-Mechanical Description of Physical Reality be Considered Complete?
  - Challenge to Copenhagen orthodox interpretation
- **Quantum Entanglement**
  - The quintessential phenomenon of QM introduced by Schrödinger in response to the EPR paper.
  - Non-local correlations between particles
  - Violates local realism assumptions
- Einstein's famous phrase: **"God does not play dice"**
  - To which Bohr replied: **"Einstein, stop telling God what to do"**
- **The EPR paradox revealed the profound nature of quantum entanglement!**

Einstein, Podolsky, and Rosen
"Spooky action at a distance"

Schrödinger, Bohr and Einstein in 1925

Bell's Theorem

- **Quantum Indeterminacy**
  - **Realism:** Quantum indeterminacy reflects our ignorance of hidden variables; outcomes are determined but unknown.
  - **Copenhagen:** Indeterminacy is fundamental; outcomes are truly probabilistic until measured.
  - **Agnosticism:** The reality behind quantum events is unknowable; only predictive power of the theory matters.
- **Bell Nonlocality** [Bell, 1964]
  - Bell inequality: It makes an observable difference for Realism vs Copenhagen, and eliminates Agnostic view.
  - Decisive evidence supporting QM (Copenhagen).
- **CHSH Inequality** [Clauser et al., 1969]
  - Generalized Bell inequality
  - Foundation for quantum information theory

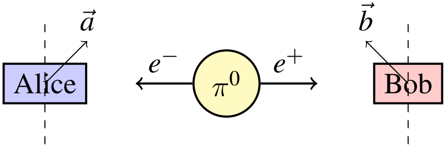
Entangled State

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

John Stewart Bell

EPRB Experiment: Testing Bell Nonlocality

Einstein-Podolsky-Rosen-Bohm Experiment



Spin-singlet: $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Correlation: $E(\vec{a}, \vec{b}) = \langle A(\vec{a}) \cdot B(\vec{b}) \rangle$

Bell CHSH Inequality:

$$\mathbb{B}_{HT} = |E(a,b) - E(a,b') + E(a',b) + E(a',b')| \leq 2$$

$$\mathbb{B}_{QM} = |\cos\theta_{ab} - \cos\theta_{ab'} + \cos\theta_{a'b} + \cos\theta_{a'b'}| \leq 2\sqrt{2}$$

Local Hidden Variable Theory

- Pre-existing density $P(\lambda)$ for λ
- $E(\vec{a}, \vec{b}) = \int P(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda$
- Local realism: $\mathbb{B}_{HT} \leq 2$

Quantum Mechanics

- No predetermined values
- $E(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} = -\cos\theta_{ab}$
- Nonlocality: $2 < \mathbb{B}_{QM} \leq 2\sqrt{2}$

  Proof: $\alpha\cos\theta + \beta\sin\theta \leq \sqrt{\alpha^2 + \beta^2}$

QM violates Bell inequality ⇒ Nature is nonlocal!

Concurrence: Measuring the Degree of Entanglement

Time Reversal Operation flips spins:

- $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
- $\mathcal{C}(|\psi\rangle) \equiv |\langle\tilde{\psi}|\psi\rangle|$, $|\tilde{\psi}\rangle = -\sigma_y \otimes \sigma_y|\psi^*\rangle$
- [Wootters, 98] flip spins with $\hat{T} = -i\sigma^y\hat{K}$ (Anti-Unitary)
- $|\tilde{\psi}\rangle = \delta^*|00\rangle - \gamma^*|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle$, the spin-flipped complex conjugate.
- $\mathcal{C}(|\psi\rangle) = 2|\alpha\delta - \beta\gamma|$ measures overlap with time-reversed state.
- $\mathcal{C} = 0$: Separable (no entanglement)
- $0 < \mathcal{C} < 1$: Partially entangled
- $\mathcal{C} = 1$: Maximally entangled

$\mathcal{C}$ = invariance under time reversal

Concurrence in general:

[Hill, Wootters, 97; Wootters, 98]

- Define: $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$
- Compute: $\mathcal{R} = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$
- Eigenvalues of $\mathcal{R}$: $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ (descending order)

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

Example - Werner State:

$$\rho_W = p|\Psi^-\rangle\langle\Psi^-| + \frac{1-p}{4}\mathbb{1}_4$$

- $p = 1$: Pure Bell state
- $\mathcal{C}(\rho_W) = \max\{0, \frac{3p-1}{2}\}$
- Entangled when $p > 1/3$

Spin Density Matrix: Physical Interpretation

The most general two-qubit density matrix:

$$\rho = \frac{1}{4}\left(\mathbb{1}_4 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_j^- \mathbb{1}_2 \otimes \sigma^j + C_{ij}\sigma^i \otimes \sigma^j\right)$$

Physical Quantities:

- $B_i^+ = \mathrm{Tr}\,\rho(\sigma_i \otimes \mathbb{1}_2)$
- $B_j^- = \mathrm{Tr}\,\rho(\mathbb{1}_2 \otimes \sigma_j)$
- $C_{ij} = \mathrm{Tr}\,\rho(\sigma_i \otimes \sigma_j)$
  Spin correlation /NB: Not [$\mathcal{C}$]

Special Case:

For Bell states: $B_i^+ = B_j^- = 0$
(No individual spin polarization)

Bell States & Correlation Matrices:

State	Correlation Matrix
$\lvert\Psi^-\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle)$	$C_{ij} = \mathrm{diag}(-1,-1,-1)$
$\lvert\Psi^+\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle)$	$C_{ij} = \mathrm{diag}(1,1,-1)$
$\lvert\Phi^+\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle + \lvert\downarrow\downarrow\rangle)$	$C_{ij} = \mathrm{diag}(1,-1,1)$
$\lvert\Phi^-\rangle = \frac{1}{\sqrt{2}}(\lvert\uparrow\uparrow\rangle - \lvert\downarrow\downarrow\rangle)$	$C_{ij} = \mathrm{diag}(-1,1,1)$

For singlet state:

$C_{ij} = -\delta_{ij}$ means spins are always anti-parallel.

Correlation matrix C_{ij} fully characterizes entanglement structure for Bell states

Entanglement and Bell Nonlocality Conditions

$$\rho_{\alpha\alpha',\beta\beta'} = \frac{1}{4}\left(\mathbb{1}_{\alpha\alpha',\beta\beta'} + C_{ii}\sigma^i_{\alpha\beta} \otimes \sigma^i_{\alpha'\beta'}\right)$$ with Anti-correlation: $C_{xx}, C_{yy}, C_{zz} < 0$

- Def: $D = (C_{xx} + C_{yy} + C_{zz})/3 = \mathrm{tr}C/3$
- $D = -1$: Perfect anti-correlation

Four eigen values of $\mathcal{R} = \rho$ (since $\tilde{\rho} = \rho$)

$$\begin{aligned}
\lambda_1 &= \frac{1}{4}(1 - C_{xx} - C_{yy} - C_{zz}),\\
\lambda_2 &= \frac{1}{4}(1 + C_{xx} + C_{yy} - C_{zz}),\\
\lambda_3 &= \frac{1}{4}(1 + C_{xx} - C_{yy} + C_{zz}),\\
\lambda_4 &= \frac{1}{4}(1 - C_{xx} + C_{yy} + C_{zz}).
\end{aligned}$$

Entanglement Condition

- Concurrence $\mathcal{C}[\rho] = \frac{1}{2}(-3D - 1) > 0$:

$$\boxed{D < -\frac{1}{3}}$$

Bell Nonlocality Condition

- For CHSH violation $\mathbb{B} > 2$: [Horodecki, et al, 95]

$$\boxed{D < -\frac{1}{\sqrt{2}} \approx -0.707}$$

Hierarchy: Bell Nonlocality $\subset$ Entanglement $\subset$ All Quantum States

First Observation of Quark Entanglement at the LHC

[ATLAS (Nature 2024):] First observation of entanglement in quarks at the highest-energy.

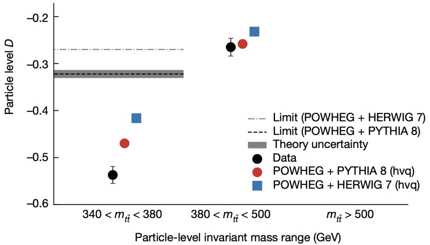
Entanglement Marker:

$$D = \mathrm{tr}[C]/3 = -3\langle \cos\phi \rangle$$

where ϕ is the angle between charged leptons in their parent top/antitop rest frames

Key Features:

- Spin transferred to decay products
- Measured near $t\bar{t}$ threshold
- From atomic physics to high-energy collisions: **A new frontier!**
- **CMS, STAR, BES-III** more to come.



$\sqrt{s} = 13$ TeV, 140 fb^{-1} data (2015-2018)

Measured: $D < -1/3$ (Entanglement criterion)
$D = -0.547 \pm 0.002$ (stat.) ± 0.021 (syst.)

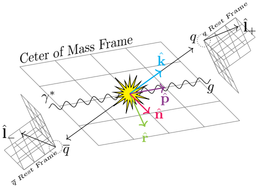
- Observed: $> 5\sigma$ from no entanglement
- **Yet, Bell Nonlocality**: $D < -1/\sqrt{2}$

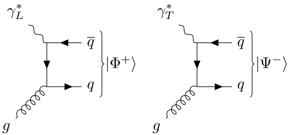
Quark Pair Production in Photon-Gluon Fusion: Longitudinal case

[Qi, Guo, Xiao] arXiv:2506.12889v1 [hep-ph]

Photon-Gluon Fusion Process

$$\gamma^*_{\lambda=\pm,0} + g \to q + \bar{q}$$





For $q\bar{q}$ with $\beta \to 0$ and $\theta = \frac{\pi}{2}$

Longitudinal photons contribution:

$$C_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\chi_1 & -\chi_2 \\ 0 & -\chi_2 & \chi_1 \end{pmatrix}$$

with $$\chi_1 = \frac{1 - 2z^2 + z^2\beta^2}{1 - z^2\beta^2}, \quad \chi_2 = \sqrt{1 - \chi_1^2}.$$

- ρ_L is given by a pure state $= |\Psi\rangle\langle\Psi|$, with

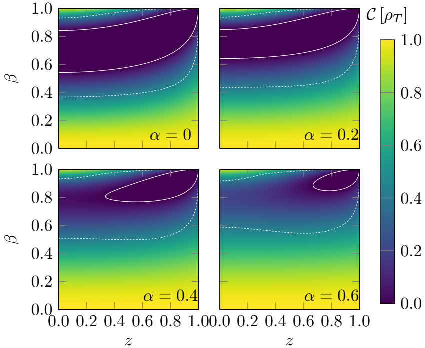
$$|\Psi\rangle = \frac{1}{2}\left(\sqrt{1+\chi_1}, \mathrm{i}\sqrt{1-\chi_1}, \mathrm{i}\sqrt{1-\chi_1}, \sqrt{1+\chi_1}\right).$$

- **Near Threshold** : $|\Phi^+\rangle$; **High Energy**: $|\Phi^+\rangle$.
- $q\bar{q}$ has spin 1 with nonzero OAM and $\mathcal{C}[\rho_L] = 1$!

Always Maximally Entangled! Very Special!

Quark Pair Production in Photon-Gluon Fusion: Transverse case

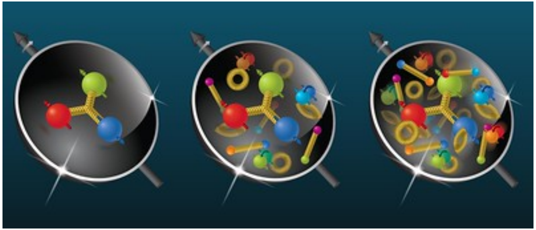
[Qi, Guo, Xiao] arXiv:2506.12889v1 [hep-ph] **Transverse photons**: similar to $gg \to q\bar{q}$ channel.

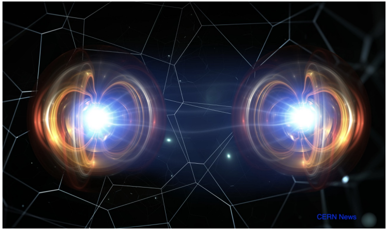


- Density plots of the concurrence for transverse photon as functions of β and $z = \cos\theta$ at given $\alpha \equiv Q^2/\hat{s}$.
- Solid lines (entanglement ($\mathcal{C}[\rho_T] = 0$)) and dashed lines (Bell nonlocality).
- **Near Threshold ($\beta \to 0$):** Maximally entangled singlet Ψ^-
- **High Energy ($\beta \to 1$) with $\theta = \pi/2$:** Maximally entangled triplet Φ^-.

- Low background and Maximal signal. Better to have LT separation! (Also UPC)
- Diffractive production also see [Fucilla and Hatta, 2509.05267].
- Possible measurements: $b\bar{b}$ or $c\bar{c}$ or hyperon $\Lambda\bar{\Lambda}$.

Summary and Outlook





- Spin physics at small x has become the high energy frontier of nucleon structure study.
- Small-x limit allows us to better understand spin puzzle and other spin effects.
- EIC offers the ideal facility for measuring Entanglement and Bell Nonlocality.
- **New opportunities** to explore the interplay of quantum information phenomena and high energy and hadronic physics in the years to come.

Top Quark Weak Decay and Spin Transfer (Backup 1)

Top Quark Decay: Choose its rest frame

$$t \to W^+ b \to \ell^+ \nu_\ell b,\ \bar{t} \to W^- \bar{b} \to \ell^- \bar{\nu}_\ell \bar{b}$$

Decay Spin Density Matrix:

$$\Gamma_\pm = \frac{\mathbb{1}_2 + \kappa_\pm \vec{\sigma}_t \cdot \hat{l}_\pm}{2}$$

Parity Violating Angular Distribution:

$$\frac{d\Gamma}{d\cos\theta} \propto 1 + \kappa_\pm \cos\theta$$

- **Spin-momentum correlation**
- $\kappa_\pm = \pm 1$ ($t\bar{t}$) **spin analyzing power**
- $\sigma_{l_+ l_-} \propto \mathrm{tr}[\Gamma_+ \otimes \Gamma_- \rho]$ NB $\mathrm{tr}[\sigma^i \sigma^j] = 2\delta^{ij}$

Entanglement: Lepton Correlation

$$\frac{d^2\sigma}{\sigma d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left[1 - \hat{l}_+ \cdot C \cdot \hat{l}_-\right]$$

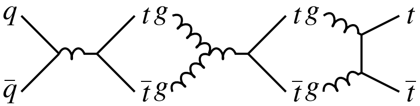
$$\boxed{\langle \cos\varphi \rangle = -\frac{1}{3}D = -\frac{1}{9}\mathrm{Tr}(C)}$$

- Extract $D = \mathrm{Tr}(C)/3$ parameter directly
- **Quantum Tomography**: all elements of ρ can be measured. [Bernreuther, Heisler, Si, 15; ATLAS, 1612.07004; CMS, 1907.03729]

Theory vs Experiment: Top Quark Entanglement (Backup 2)

Quantum State Tomography $\rho_{\alpha\alpha',\beta\beta'} = R_{\alpha\alpha',\beta\beta'}/\mathrm{tr}R$ [Afik, de Nova, 2022]

Top quark pair production



$$R_{\alpha\alpha',\beta\beta'} = \frac{1}{N}\sum \mathcal{M}^*_{t_\alpha \bar{t}_{\alpha'}} \mathcal{M}_{t_\beta \bar{t}_{\beta'}}$$

- **Measured $D \approx -0.54$** near threshold
- **Gluon fusion dominance** at LHC
- **Angular momentum conservation**
- **Statistical mixture** of $q\bar{q}$ and gg

Near Threshold ($\beta \to 0$):

- $q\bar{q}$: Separable state ($\mathcal{C} = 0$), since $t\bar{t}$ spin (± 1) is equally mixed along beam.
- gg: Maximally entangled singlet Ψ^-

High Energy ($\beta \to 1$) with $\theta = \pi/2$:

- Both channels: Maximally entangled triplet Ψ^+ along $\hat{n}$ with nonzero OAM.

Mixed State at LHC

$$\rho = w_{q\bar{q}}\rho_{q\bar{q}} + w_{gg}\rho_{gg}$$

"Observation of Entanglement but not Bell Nonlocality due to Quark channel mixture"