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26th International Symposium on Spin Physics

A Century of Spin

6D Light-front Wigner distributions of the proton

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Introduction–Motivation

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Structure of hadrons

- **Hadron Physics:** Nucleons exhibit a complex and dynamic internal structure → Dynamics of quarks and gluons → Structure of hadrons
  - **1D:** The parton distribution function (PDF).
  - **3D:** Generalized Parton Distributions (GPDs) and Transverse Momentum Distributions (TMDs).

Motivation

- **A more complete description:** Utilizing the distribution function in phase space, which is a joint distribution in coordinate space and momentum space.

History of Wigner distribution

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- **Foundation:** Dirac and Heisenberg provide a theoretical foundation for the connection between quantum mechanics and classical phase space. [1]
- **Introduction:** The Wigner distribution has been introduced by E.Wigner as a quantum phase space distribution.[2]
- **6D Non-relativistic:** The first attempt to transpose this formalism to the framework of QFT by Ji et al..[3]
- **5D Relativistic:** C. Lorcé and B. Pasquini introduce a 5D Wigner distribution function in the infinite momentum frame. [4]
- **6D Relativistic:** Y. Han et al. defines the boost-invariant 6D Wigner distribution function, [5] based on the boost-invariant space coordinates.

[1] P. A. M. Dirac, Math. Proc. Cambridge Philos. Soc. 26, 376 (1930); W. Heisenberg, Original Sci. Papers 1985, 627.

[2] E. P. Wigner, Phys. Rev. 40, 749 (1932).

[3] X. D. Ji, Phys. Rev. Lett. 91, 062001 (2003); A. V. Belitsky et al., Phys. Rev. D 69, 074014 (2004).

[4] C. Lorcé, B. Pasquini, Phys. Rev. D 84, 014015 (2011); Phys. Rev. D 93, 034040 (2016).

[5] Y. Han, T. Liu, B. Q. Ma, Phys. Lett. B 830, 137127 (2022).

5D & 6D Wigner Distributions Related Topics

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5D Wigner Distributions[1]

1. Different polarization Wigner distribution:
   - Quark Wigner distribution of pion meson (Spin-0).[2]
   - Quark Wigner distribution of proton (Spin-1/2).[3]
2. The relationship with intrinsic orbital angular momentum.[4]

6D Wigner Distributions[5]

1. Different polarization Wigner distribution:
   - Quark Wigner distribution of pion meson (Spin-0).[6]
   - Quark Wigner distribution of proton (Spin-1/2). [7]

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[1] C. Lorcé, B. Pasquini, Phys. Rev. D 84, 014015 (2011); Phys. Rev. D 93, 034040 (2016).

[2] Z. L. Ma, Z. Lun, Phys. Rev. D 98, 054024 (2018).

[3] T. Liu, B. Q. Ma, Phys. Rev. D 91, 034019 (2015).

[4] C. Lorcé et al., Phys. Rev. D 85, 114006 (2012).

[5] Y. Han, T. Liu, B. Q. Ma, Phys. Lett. B 830, 137127 (2022).

[6] Y. Han, T. Liu, B. Q. Ma, Nucl. Phys. A 1040, 122757 (2023).

[7] **Y. Yang, T. Liu, B. Q. Ma, Eur. Phys. J. C 85(5), 1-39 (2025).**

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6D Light-front Wigner Distributions

5D Wigner Distributions + Boost invariant longitudinal coordinates

6D Wigner operator[1]

$$\widehat{W}^{[\Gamma]}\left(b^-, \boldsymbol{b}_\perp, k^+, \boldsymbol{k}_\perp\right) = \int \frac{\mathrm{d}y^- \mathrm{d}^2\boldsymbol{y}_\perp}{2(2\pi)^3} e^{ik^+y^- - i\boldsymbol{k}_\perp\cdot\boldsymbol{y}_\perp} \times \bar{\psi}\left(b - \frac{y}{2}\right)\Gamma\mathcal{L}_{[b-y/2, b+y/2]}\Psi\left(b + \frac{y}{2}\right)\Big|_{y^+=0}, \tag{1}$$

6D Wigner distributions

$$\rho^{[\Gamma]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S\right) = \int \frac{\mathrm{d}\xi \mathrm{d}^2\boldsymbol{\Delta}_\perp}{4\pi^3} \times \left\langle P + \frac{\Delta}{2}, S \middle| \widehat{W}^{[\Gamma]}\left(b^-, \boldsymbol{b}_\perp, k^+, \boldsymbol{k}_\perp\right) \middle| P - \frac{\Delta}{2}, S \right\rangle, \tag{2}$$

Integrating over $\tilde{z}$ leads back to the 5D Wigner distribution function.

[1] Y. Han, T. Liu, B. Q. Ma, Phys. Lett. B 830, 137127 (2022).

All Polarization States

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		Quark Polarization		
		Unpolarized (U)	Longitudinal Polarized (L)	Transverse Polarized (T)
Nucleon Polarization	U	ρ_{UU} Unpolarized $\int \rho_{UU} = N_q$	ρ_{UL} Quark helicity in unpolarized proton $\int \rho_{UL} = 0$	ρ^j_{UT} "Boer-Mulders" type $\int \rho^j_{UT} = 0$
	L	ρ_{LU} OAM-sensitive $\int \rho_{LU} = 0$	ρ_{LL} Helicity distribution $\int \rho_{LL} = \Delta q$	ρ_{LT} Longitudinal-transverse $\int \rho_{LT} = 0$
	T	ρ^i_{TU} "Sivers" type $\int \rho^i_{TU} = 0$	ρ^i_{TL} Transverse-longitudinal $\int \rho^i_{TL} = 0$	$\rho_{TT}, \rho^{\perp}_{TT}$ Transverse and pretzelous $\int \rho^{(\perp)}_{TT} = \Delta_{Tq}/0$

Spectator Model

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Spectator Model

- In the spectator model, the hadron is viewed as a struck quark and a spectator which contains the remaining constituents, and a portion of the nonperturbative effects can be absorbed in its mass.
- The amplitudes of the quark-spectator states in a hadron are described by the light-cone wave functions.

Spectator Model of Proton

Considering the leading term in the Fock states expansion for the proton is the valence three quark state $|uud\rangle$. The light-cone Fock expansion for the pion is

$$
\begin{aligned}
&\left|P^+, \boldsymbol{P}_\perp, \lambda_N\right\rangle \\
&= \sum_{q,D,\lambda_q,\lambda_D} \int \frac{\mathrm{d}x \mathrm{d}^2\boldsymbol{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Psi_{qD}(x, \boldsymbol{k}_\perp) \left|qD; x, \boldsymbol{k}_\perp, \lambda_q, \lambda_D\right\rangle
\end{aligned} \tag{3}
$$

Constrained by the quantum numbers of the proton and the quark, the spectator can only be either a scalar or an axial vector, and we express the proton state as

$$
|p\rangle = \sin\theta\phi_S\,|qS\rangle + \cos\theta\phi_V\,|qV\rangle \tag{4}
$$

where S and V stand for the scalar and the axial-vector spectators.

Spectator Model of Proton

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The instant wave function of the proton

$$|p\rangle^{\uparrow,\downarrow} = \frac{1}{\sqrt{2}}|uS(ud)\rangle_T^{\uparrow,\downarrow} - \frac{1}{\sqrt{6}}|uV(ud)\rangle_T^{\uparrow,\downarrow} + \frac{1}{\sqrt{3}}|\mathrm{d}V(uu)\rangle_T^{\uparrow,\downarrow}. \quad (5)$$

The transformation of the spinor of the quark from the instant form to the light-front form

$$\begin{bmatrix} q_T^{\uparrow} \\ q_T^{\downarrow} \end{bmatrix} = W_q \begin{bmatrix} q_F^{\uparrow} \\ q_F^{\downarrow} \end{bmatrix}, \quad (6)$$

W_q is the Melosh-Wigner rotation matrix of spinors

$$W_q = \omega \begin{bmatrix} k^+ + m & -k^R \\ k^L & k^+ + m \end{bmatrix}, \quad (7)$$

where $\omega = 1/\sqrt{(k^+ + m)^2 + k^L k^R}$ is a normalization factor.

Spectator Model of Proton

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1. **Scalar di-quark:** transformation remains invariant
2. **Axial-vector di-quark:** Melosh-Wigner transformation

$$\begin{bmatrix} V_T^1 \\ V_T^0 \\ V_T^{-1} \end{bmatrix} = W_V \begin{bmatrix} V_F^1 \\ V_F^0 \\ V_F^{-1} \end{bmatrix} \tag{8}$$

The corresponding Melosh–Wigner transformation matrix is

$$W_V = \omega_V^2 \begin{bmatrix} \left(k_V^+ + m_V\right)^2 & -\sqrt{2}\left(k_V^+ + m_V\right) k_V^R & \left(k_V^R\right)^2 \\ \sqrt{2}\left(k_V^+ + m_V\right) k_V^L & \left(k_V^+ + m_V\right)^2 - k_V^L k_V^R & -\sqrt{2}\left(k_V^+ + m_V\right) k_V^R \\ \left(k_V^L\right)^2 & \sqrt{2}\left(k_V^+ + m_V\right) k^L & \left(k_V^+ + m_V\right)^2 \end{bmatrix} \tag{9}$$

where m_V is the mass of the di-quark and $\omega_V = 1/\sqrt{(k_V^+ + m_V)^2 + k_V^L k_V^R}$ is a normalization factor.

BHL wave function

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The Brodsky–Huang–Lepage (BHL) form of the light-front wave function

$$\varphi_{qD}\left(x, \boldsymbol{k}_\perp\right) = A_{qD} \exp\left(-\frac{\mathcal{M}^2}{8\beta_D^2}\right), \tag{10}$$

where D represents the spectator, with $D = S$ for the scalar spectator and $D = V$ for the axial-vector spectator. Besides, β_D is the scale parameter of the harmonic oscillator, A_{qD} is the normalization factor and $\mathcal{M}$ is the invariant mass as

$$\mathcal{M} = \sqrt{\frac{m_q^2 + \boldsymbol{k}_\perp^2}{x} + \frac{m_D^2 + \boldsymbol{k}_\perp^2}{1-x}}, \tag{11}$$

where m_q and m_D are the mass of quarks and spectators, respectively.

Spectator Model of Proton

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The quark-parton state can be expressed as

$$|qS\rangle_T^\lambda = \int \frac{\mathrm{d}x\mathrm{d}^2\boldsymbol{k}_\perp}{16\pi^3} \sum_{i=\{\uparrow,\downarrow\}} A_{qS}^{i,\lambda} \left|q^i S\right\rangle_F , \tag{12}$$

$$|qV\rangle_T^\lambda = \int \frac{\mathrm{d}x\mathrm{d}^2\boldsymbol{k}_\perp}{16\pi^3} \sum_{i=\{\uparrow,\downarrow\}} \sum_{j=\{-1,0,1\}} B_{qV}^{ij,\lambda} \left|q^i V^j\right\rangle_F , \tag{13}$$

where $\lambda =\uparrow$ or $\downarrow$, and the coefficients $A_{qS}^{i,\lambda}$ and $B_{qV}^{ij,\lambda}$ are defined as

$$A_{qS}^{i,\lambda} = W_q^{\lambda,i}\varphi_S, \tag{14}$$

$$B_{qV}^{ij,\uparrow} = \left(-\sqrt{\frac{1}{3}}W_q^{\uparrow,i}W_V^{0,j} + \sqrt{\frac{2}{3}}W_q^{\downarrow,i}W_V^{1,j}\right)\varphi_V, \tag{15}$$

$$B_{qV}^{ij,\downarrow} = \left(-\sqrt{\frac{1}{3}}W_q^{\downarrow,i}W_V^{0,j} + \sqrt{\frac{2}{3}}W_q^{\uparrow,i}W_V^{-1,j}\right)\varphi_V. \tag{16}$$

Spectator Model of Proton

The quark-parton state can be expressed as

$$|qS\rangle_T^\lambda = \int \frac{\mathrm{d}x\mathrm{d}^2\boldsymbol{k}_\perp}{16\pi^3} \sum_{i=\{\uparrow,\downarrow\}} A_{qS}^{i,\lambda} \left|q^i S\right\rangle_F , \tag{17}$$

$$|qV\rangle_T^\lambda = \int \frac{\mathrm{d}x\mathrm{d}^2\boldsymbol{k}_\perp}{16\pi^3} \sum_{i=\{\uparrow,\downarrow\}} \sum_{j=\{-1,0,1\}} B_{qV}^{ij,\lambda} \left|q^i V^j\right\rangle_F , \tag{18}$$

The light-front wave functions of the proton can be derived as

$$\psi_{uS(ud)}^\lambda (x, \boldsymbol{k}_\perp, \lambda_u = i) = \frac{1}{\sqrt{2}} A_{uS(ud)}^{i,\lambda}, \tag{19}$$

$$\psi_{uV(ud)}^\lambda (x, \boldsymbol{k}_\perp, \lambda_u = i, \lambda_V = j) = -\frac{1}{\sqrt{6}} B_{uV(ud)}^{ij,\lambda}, \tag{20}$$

$$\psi_{dV(uu)}^\lambda (x, \boldsymbol{k}_\perp, \lambda_d = i, \lambda_V = j) = \frac{1}{\sqrt{3}} B_{dV(uu)}^{ij,\lambda}. \tag{21}$$

6D Wigner Distribution

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Use $\lambda_{q,\bar{q}}$ to denote the helicities of quark and spectator diquark

$$\rho^{[\gamma^+]}(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp)=\int\frac{\mathrm{d}\xi\mathrm{d}^2\boldsymbol{\Delta}_\perp}{4\pi^3}e^{-2i\xi\tilde{z}-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\frac{1}{16\pi^3}\sum_{\lambda_{\bar{q}}}\Big(\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},+,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},+,\lambda_{\bar{q}}\right)+\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},-,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},-,\lambda_{\bar{q}}\right)\Big)$$

$$\rho^{[\gamma^+\gamma_5]}(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp)=\int\frac{\mathrm{d}\xi\mathrm{d}^2\boldsymbol{\Delta}_\perp}{4\pi^3}e^{-2i\xi\tilde{z}-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\frac{1}{16\pi^3}\sum_{\lambda_{\bar{q}}}\Big(\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},+,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},+,\lambda_{\bar{q}}\right)-\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},-,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},-,\lambda_{\bar{q}}\right)\Big)$$

$$\rho^{[i\sigma^{j+}\gamma_5]}(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp)=\int\frac{\mathrm{d}\xi\mathrm{d}^2\boldsymbol{\Delta}_\perp}{4\pi^3}e^{-2i\xi\tilde{z}-i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\frac{1}{16\pi^3}\sum_{\lambda_{\bar{q}}}\Big(\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},\uparrow,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},\uparrow,\lambda_{\bar{q}}\right)-\psi^*\left(x^{\rm out},\boldsymbol{k}_\perp^{\rm out},\downarrow,\lambda_{\bar{q}}\right)\psi\left(x^{\rm in},\boldsymbol{k}_\perp^{\rm in},\downarrow,\lambda_{\bar{q}}\right)\Big)$$

where $x^{\rm out}=\frac{x-\xi}{1-\xi}, x^{\rm in}=\frac{x+\xi}{1+\xi}, \boldsymbol{k}_\perp^{\rm out}=\boldsymbol{k}_\perp+\frac{(1-x)\boldsymbol{\Delta}_\perp}{2(1-\xi)}, \boldsymbol{k}_\perp^{\rm in}=\boldsymbol{k}_\perp-\frac{(1-x)\boldsymbol{\Delta}_\perp}{2(1+\xi)},$

$\boldsymbol{p}_\perp^{\rm out}=\boldsymbol{k}_\perp^{\rm out}+x^{\rm out}\frac{\boldsymbol{\Delta}_\perp}{2}=\boldsymbol{k}_\perp+\frac{\boldsymbol{\Delta}_\perp}{2}, \boldsymbol{p}_\perp^{\rm in}=\boldsymbol{k}_\perp^{\rm in}-x^{\rm in}\frac{\boldsymbol{\Delta}_\perp}{2}=\boldsymbol{k}_\perp-\frac{\boldsymbol{\Delta}_\perp}{2}, \boldsymbol{p}_\perp^{\rm out}-\boldsymbol{p}_\perp^{\rm in}=\boldsymbol{\Delta}_\perp.$

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Physical Significance

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The skewness variable ξ is defined as

$$\xi = -\frac{\Delta^+}{2P^+}, \tag{22}$$

The longitudinal coordinate $\tilde{z}$ is defined as

$$\tilde{z} = b^- P^+, \tag{23}$$

The distribution function with respect to the longitudinal coordinates $\tilde{z}$

$$\rho^{[\Gamma]}(\tilde{z}, S) = \int \mathrm{d}^2\boldsymbol{k}_\perp \, \mathrm{d}^2\boldsymbol{b}_\perp \, \mathrm{d}x \, \rho^{[\Gamma]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S\right). \tag{24}$$

The expression for GPDs

$$H(x, \xi, t, S) = \int \mathrm{d}^2\boldsymbol{k}_\perp \int \mathrm{d}\tilde{z} \, \mathrm{d}^2\boldsymbol{b}_\perp \, e^{2i\xi\tilde{z} + i\boldsymbol{b}_\perp \cdot \boldsymbol{\Delta}_\perp} \rho(\tilde{z}, x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp, S), \tag{25}$$

where $t = \boldsymbol{\Delta}_\perp^2$.

6D Light-front Wigner Distributions

The integration over $\tilde{z}$ leads back to the 5D Wigner distribution function

$$\begin{aligned}
&\int \mathrm{d}\tilde{z}\rho^{[\Gamma]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S\right) \\
&= -\int \mathrm{d}b^- \int \frac{\mathrm{d}\Delta^+ \mathrm{d}^2\Delta_\perp}{(2\pi)^3} e^{ib^-\Delta^+} \left\langle P + \frac{\Delta}{2}, S \middle| \widehat{W}^{[\Gamma]}\left(0, \boldsymbol{b}_\perp, k^+, \boldsymbol{k}_\perp\right) \middle| P - \frac{\Delta}{2}, S \right\rangle \quad (26) \\
&= \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \left\langle P + \frac{\Delta_\perp}{2}, S \middle| \widehat{W}^{[\Gamma]}\left(0, \boldsymbol{b}_\perp, k^+, \boldsymbol{k}_\perp\right) \middle| P - \frac{\Delta_\perp}{2}, S \right\rangle
\end{aligned}$$

The 6D Wigner distribution is the 3D Fourier transform of the GTMD correlation function.

$$\rho^{[\Gamma]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S\right) = \int \frac{\mathrm{d}\xi \mathrm{d}^2\boldsymbol{\Delta}_\perp}{4\pi^3} e^{-2i\xi\boldsymbol{z} - i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp} W^{[\Gamma]}\left(\xi, x, \boldsymbol{\Delta}_\perp, \boldsymbol{k}_\perp, S\right) \quad (27)$$

The 6D Wigner distribution contains the coordinate space information, which is the most comprehensive partial distribution function known at present.

6D Light-front Wigner Distributions

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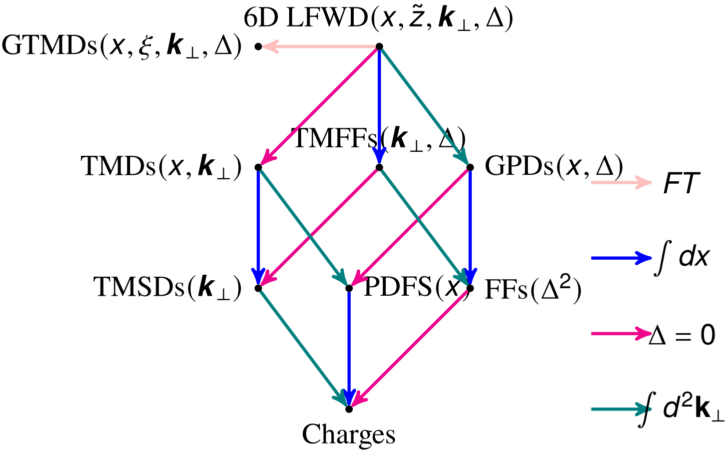

Figure 1: Conversion from the 6D LF Wigner function to other distribution functions

6D Light-front Wigner Distributions

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Integrating the function defined above yields a new distribution function. The specific integration relations are shown in the table below:

Integration Dimensions (Integrated Variables)	Final Distribution Function
$\boldsymbol{k}_\perp, \boldsymbol{b}_\perp$	$\rho^{[\Gamma]}(\tilde{z}, x, S)$
$x, \boldsymbol{k}_\perp$	$\rho^{[\Gamma]}(\tilde{z}, \boldsymbol{b}_\perp, S)$
$x, \boldsymbol{b}_\perp$	$\rho^{[\Gamma]}(\tilde{z}, \boldsymbol{k}_\perp, S)$
x, b_y, k_x	$\rho^{[\Gamma]}(\tilde{z}, b_x, k_y, S)$
x, b_x, k_y	$\rho^{[\Gamma]}(\tilde{z}, b_y, k_x, S)$
x, b_x, k_x	$\rho^{[\Gamma]}(\tilde{z}, b_y, k_y, S)$
x, b_y, k_y	$\rho^{[\Gamma]}(\tilde{z}, b_x, k_x, S)$

Note: $\boldsymbol{b}_\perp = (b_x, b_y)$ = transverse coordinate vector; $\boldsymbol{k}_\perp = (k_x, k_y)$ = transverse momentum vector; x = longitudinal momentum fraction.

Intrinsic Orbital Angular Momentum

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Physical Interpretation:

- Both quantities probe the coupling between spatial and momentum degrees of freedom in the nucleon.
- ℓ_z^q: Measures the **intrinsic orbital angular momentum** carried by quarks.
- C_z^q: Encodes the **correlation between quark spin and orbital motion**.

Mathematical Structure:

$$\ell_z^q = \int \mathrm{d}\tilde{z}\,\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp\,\mathrm{d}^2\boldsymbol{b}_\perp \underbrace{(\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z}_{\text{Orbital angular momentum operator}} \rho_{LU}^q(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S),$$

$$C_z^q = \int \mathrm{d}\tilde{z}\,\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp\,\mathrm{d}^2\boldsymbol{b}_\perp\,(\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z \quad \rho_{UL}^q(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, S).$$

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$\rho_{\mathrm{UU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x$ plane

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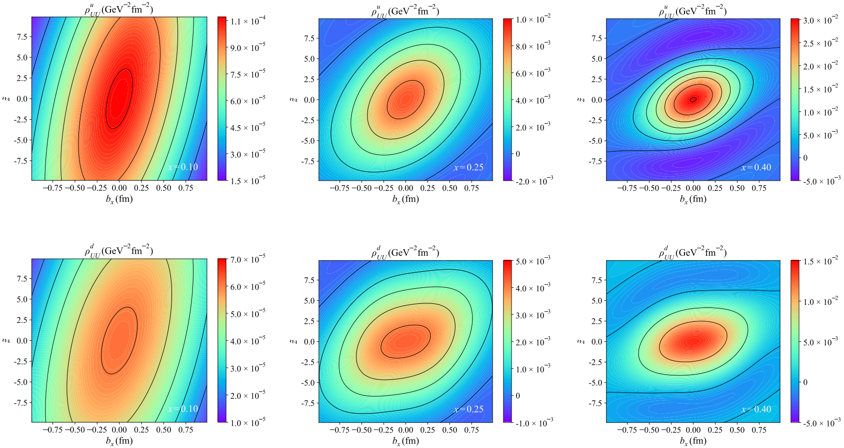

Figure 2: $\rho_{\mathrm{UU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ with the transverse momentum fixed at $\boldsymbol{k}_\perp = 0.3\,\mathrm{GeV}\hat{\boldsymbol{e}}_x$ (where $\hat{\boldsymbol{e}}_x$ is the unit vector in the x-direction) and the transverse coordinate component fixed at $b_y = 0.4\,\mathrm{GeV}^{-1}$. The three columns correspond to $x = 0.10$, $x = 0.25$, and $x = 0.40$.

$\rho_{\rm UU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_y$ plane

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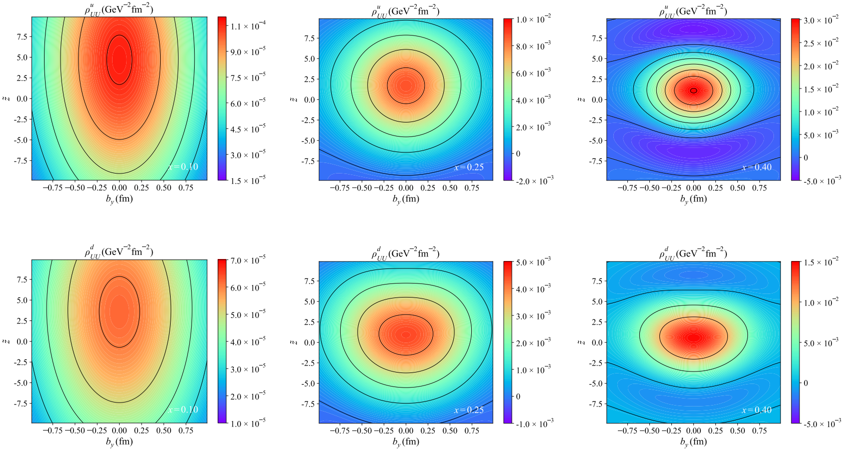

Figure 3: $\rho_{\rm UU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ with the transverse momentum fixed at $\boldsymbol{k}_\perp = 0.3\,\mathrm{GeV}\hat{\boldsymbol{e}}_x$ (where $\hat{\boldsymbol{e}}_x$ is the unit vector in the x-direction) and the transverse coordinate component fixed at $b_x = 0.4\,\mathrm{GeV}^{-1}$. The three columns correspond to $x = 0.10$, $x = 0.25$, and $x = 0.40$.

$\rho_{\rm UU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - k_x$ plane

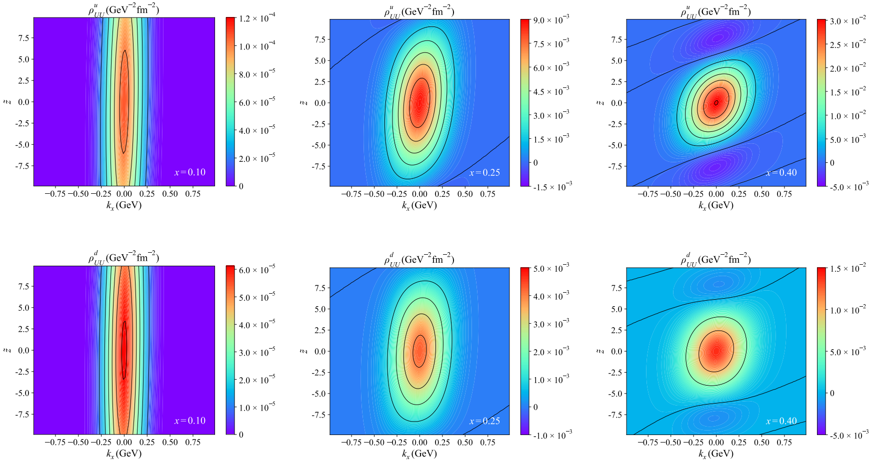

Figure 4: $\rho_{\rm UU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ with the transverse coordinate fixed at $\boldsymbol{b}_\perp = 0.4\,\text{GeV}^{-1}\hat{\boldsymbol{e}}_x$ (where $\hat{\boldsymbol{e}}_x$ is the unit vector along the x-axis) and the transverse momentum component fixed at $k_y = 0.3\,\text{GeV}$. The three columns correspond to $x = 0.10$, $x = 0.25$, and $x = 0.40$.

$\rho_{\text{UU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - k_y$ plane

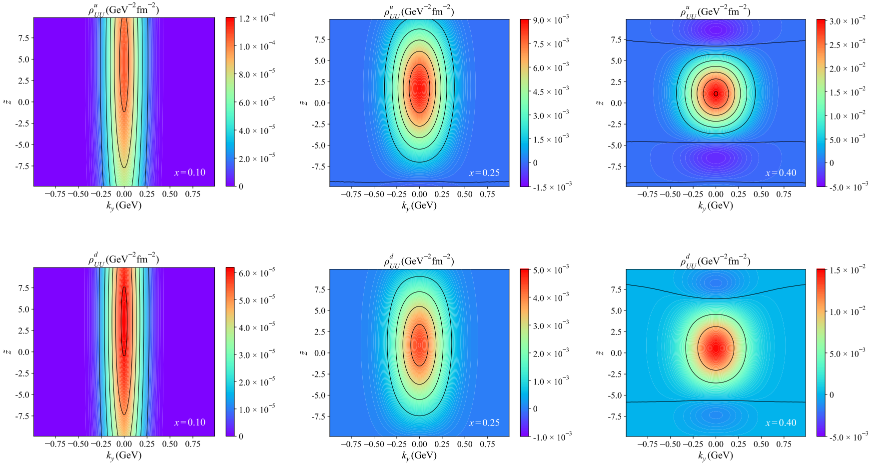

Figure 5: $\rho_{\text{UU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ with the transverse coordinate fixed at $\boldsymbol{b}_\perp = 0.4\,\text{GeV}^{-1}\hat{\boldsymbol{e}}_x$ (where $\hat{\boldsymbol{e}}_x$ is the unit vector along the x-axis) and the transverse momentum component fixed at $k_x = 0.3\,\text{GeV}$. The three columns correspond to $x = 0.10$, $x = 0.25$, and $x = 0.40$.

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Summary

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Current Progress

1. We investigate six-dimensional quark Wigner distributions of the proton in a light-front quark spectator-diquark model.
2. We define 16 independent distribution functions in according to different combinations of quark and proton polarizations at the leading twist.
3. We present the numerical results of all these Wigner distributions and explore the connection to experimental observables.

Research Significance

1. The 6D light-front Wigner distributions provide complete information of one parton distribution in the phase space and the correlation with spins.
2. The 6D Wigner distribution can be transformed into other distribution functions.
3. The 6D Wigner distribution can be associated with important physical quantities such as orbital angular momentum.

Summary-Complete description

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- How to fully depict the three-dimensional coordinate-momentum correlation of protons?——Boost invariant longitudinal coordinates $\tilde{z}$

Table 1: Comparison between Traditional Distribution Function and 6D Light-Front Wigner Distribution

Distribution Type	Dimensions	Coordinate Space	Momentum Space	Polarization Correlation
PDF	1D	✗	✓	Partial (longitudinal only)
TMD	3D	✗	✓	Partial (includes transverse)
GPD	4D	partial ✓	✓	Partial (cross terms)
6D LFWD	6D	✓	✓	Complete

Summary

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Physical Significance

1. Visualization provides a complete spin-phase-space map of the proton $(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$.
2. The 6D Wigner distributions exhibit widespread non-positive definiteness directly confirming quantum phase-space interference effects.
3. Provided new insights into the proton structure: Spin-orbit coupling, quark flavor asymmetry and longitudinal spatial dynamics.

Experimental Observables

1. Indirect measurements using GTMDs
2. Transform into the distribution functions of known observables, such as TMDs, GPDs and PDFs
3. Define and observe the distribution function with respect to the $\tilde{z}$-dependence of the longitudinal invariant coordinates
4. Observation of the New Distribution Function

Summary

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- [1] **Yang Y**, Liu T, Ma B Q. Six-dimensional light-front Wigner distributions of the proton[J]. **The European Physical Journal C, 2025, 85(5): 1-39**. ✓
- [2] Y. Han, T. Liu and B. Q. Ma, Six-dimensional light-front Wigner distribution of hadrons, Phys. Lett. B 830, 137127 (2022).
- [3] Han Y, Liu T, Ma B Q. Six-dimensional light-front Wigner distribution of hadrons[J]. Physics Letters B, 2022, 830: 137127.

Thank you !

6D Light-front Wigner Distributions

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By combining different polarization states of hadrons and quarks.

1. unpolarized case

$$\rho_{\mathrm{UU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[\gamma^+]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) + \rho^{[\gamma^+]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (28)$$

2. unpolarized-longitudinal case

$$\rho_{\mathrm{UL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[\gamma^+\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) + \rho^{[\gamma^+\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (29)$$

3. unpolarized-transverse case

$$\rho^j_{\mathrm{UT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) + \rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (30)$$

6D Light-front Wigner Distributions

4. longitudinal-unpolarized case

$$\rho_{\mathrm{LU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[\gamma^+]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) - \rho^{[\gamma^+]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (31)$$

5. longitudinal case

$$\rho_{\mathrm{LL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[\gamma^+\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) - \rho^{[\gamma^+\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (32)$$

6. longitudinal-transverse case

$$\rho^j_{\mathrm{LT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \frac{1}{2}\left[\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, \hat{\boldsymbol{e}}_z\right) - \rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp, -\hat{\boldsymbol{e}}_z\right)\right], \quad (33)$$

6D Light-front Wigner Distributions

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7. transverse-unpolarized case

$$\rho^{i}_{\mathrm{TU}}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp}\right)=\frac{1}{2}\left[\rho^{[\gamma^{+}]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},\hat{\boldsymbol{e}}_{i}\right)-\rho^{[\gamma^{+}]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},-\hat{\boldsymbol{e}}_{i}\right)\right], \quad (34)$$

8. transverse-longitudinal case

$$\rho^{i}_{\mathrm{TL}}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp}\right)=\frac{1}{2}\left[\rho^{[\gamma^{+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},\hat{\boldsymbol{e}}_{i}\right)-\rho^{[\gamma^{+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},-\hat{\boldsymbol{e}}_{i}\right)\right], \quad (35)$$

9. transverse case

$$\rho_{\mathrm{TT}}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp}\right)=\frac{1}{2}\delta_{ij}\left[\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},\hat{\boldsymbol{e}}_{i}\right)-\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},-\hat{\boldsymbol{e}}_{i}\right)\right], \quad (36)$$

10. pretzelous case

$$\rho^{\perp}_{\mathrm{TT}}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp}\right)=\frac{1}{2}\epsilon_{ij}\left[\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},\hat{\boldsymbol{e}}_{i}\right)-\rho^{[i\sigma^{j+}\gamma_5]}\left(\tilde{z},x,\boldsymbol{b}_{\perp},\boldsymbol{k}_{\perp},-\hat{\boldsymbol{e}}_{i}\right)\right], \quad (37)$$

6D Light-front Wigner Distributions

By integrating over all relevant phase-space variables ($\tilde{z}$, x, $\boldsymbol{b}_\perp$, and $\boldsymbol{k}_\perp$)

1. unpolarized case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho_{\mathrm{UU}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=N_q, \tag{38}$$

2. unpolarized-longitudinal case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho_{\mathrm{UL}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=0, \tag{39}$$

3. unpolarized-transverse case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho^j_{\mathrm{UT}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=0, \tag{40}$$

6D Light-front Wigner Distributions

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4. longitudinal-unpolarized case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \rho_{\mathrm{LU}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = 0, \tag{41}$$

5. longitudinal case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \rho_{\mathrm{LL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = \Delta_q, \tag{42}$$

6. longitudinal-transverse case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{k}_\perp \rho^j_{\mathrm{LT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right) = 0, \tag{43}$$

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7. transverse-unpolarized case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho^i_{\mathrm{TU}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=0, \tag{44}$$

8. transverse-longitudinal case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho^i_{\mathrm{TL}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=0, \tag{45}$$

9. transverse case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho_{\mathrm{TT}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=\Delta_{Tq}, \tag{46}$$

10. pretzelous case

$$\int \mathrm{d}\tilde{z}\mathrm{d}x\mathrm{d}^2\boldsymbol{b}_\perp\mathrm{d}^2\boldsymbol{k}_\perp\rho^\perp_{\mathrm{TT}}\left(\tilde{z},x,\boldsymbol{b}_\perp,\boldsymbol{k}_\perp\right)=0, \tag{47}$$

$\rho_{UU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z}-b_x, \tilde{z}-b_y, \tilde{z}-k_x, \tilde{z}-k_y$ plane

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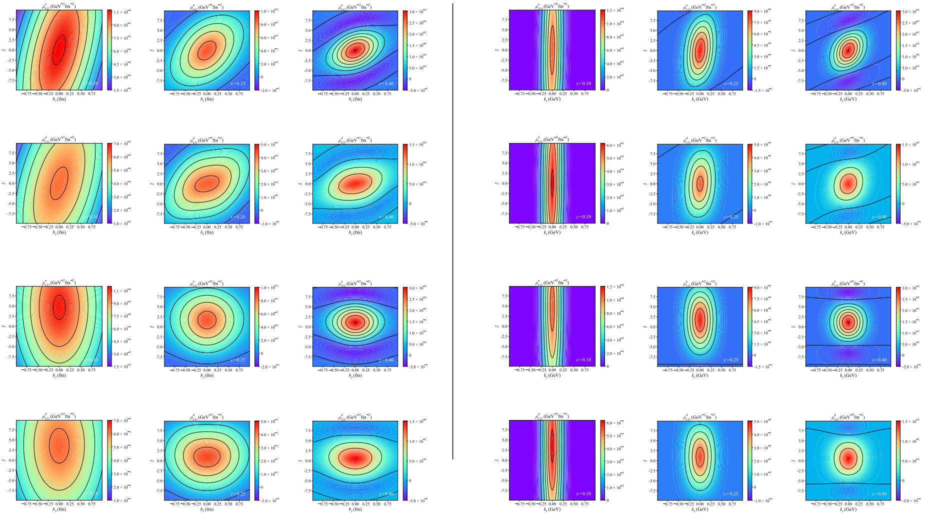

$\rho_{\mathrm{UL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z}-b_x, \tilde{z}-b_y, \tilde{z}-k_x, \tilde{z}-k_y$ plane

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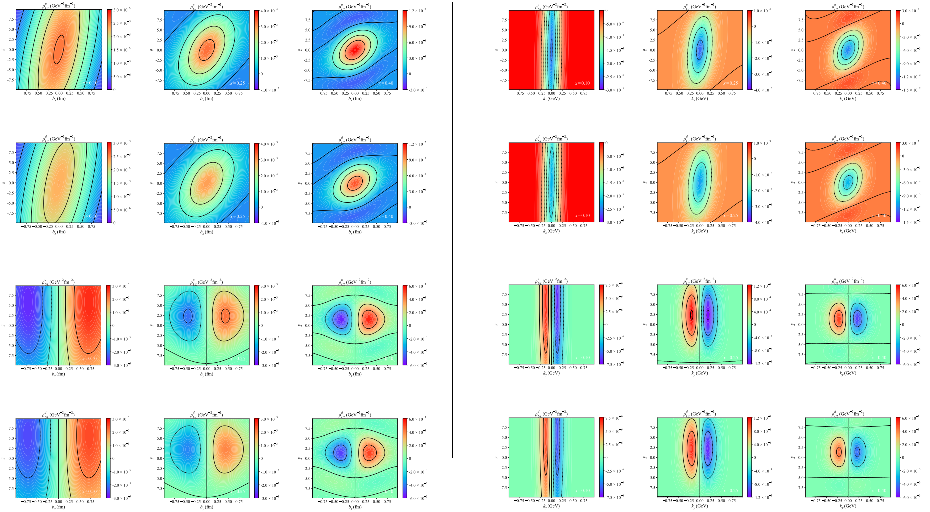
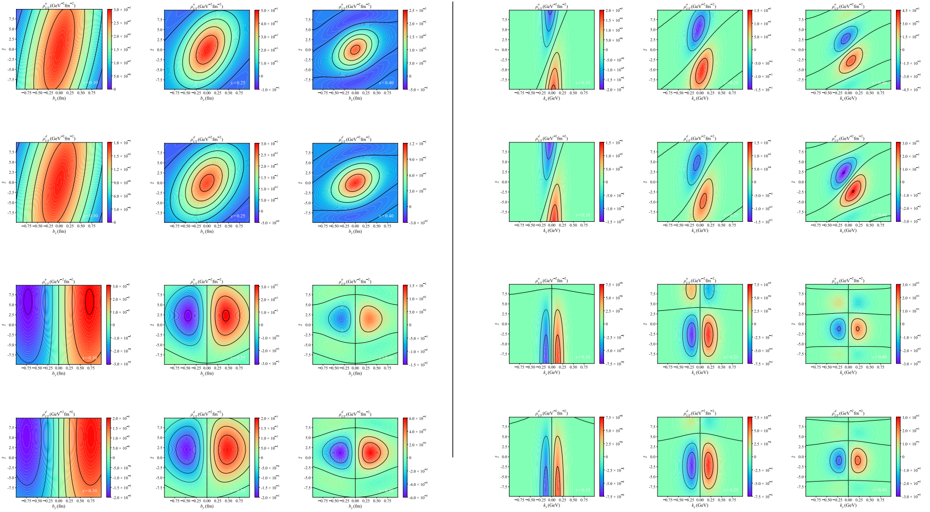


$\rho_{\mathrm{UT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x, \tilde{z} - b_y, \tilde{z} - k_x, \tilde{z} - k_y$ plane



$\rho_{\rm LU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z}-b_x, \tilde{z}-b_y, \tilde{z}-k_x, \tilde{z}-k_y$ plane

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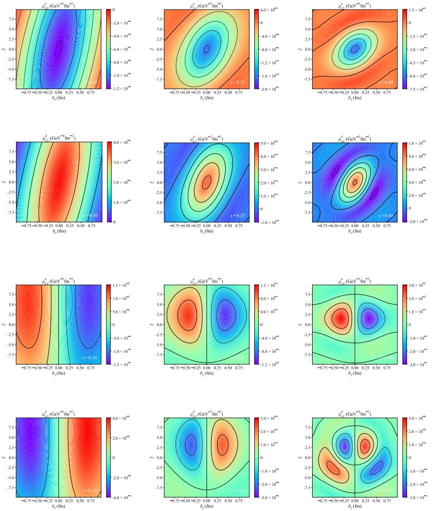
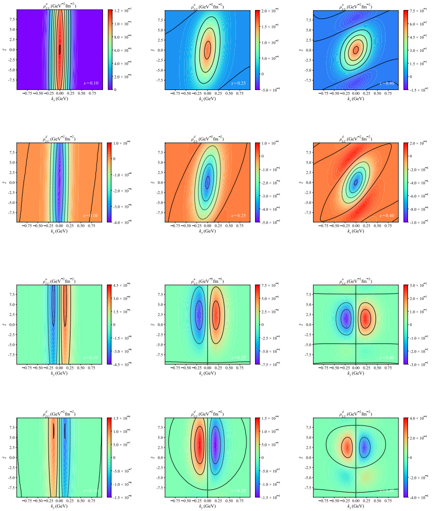
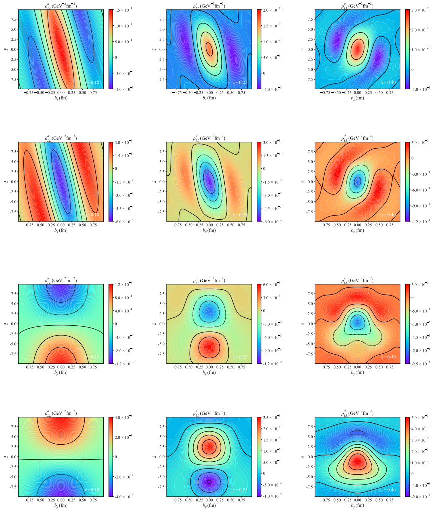
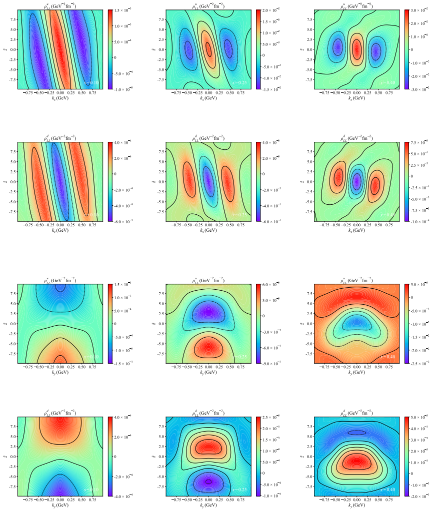


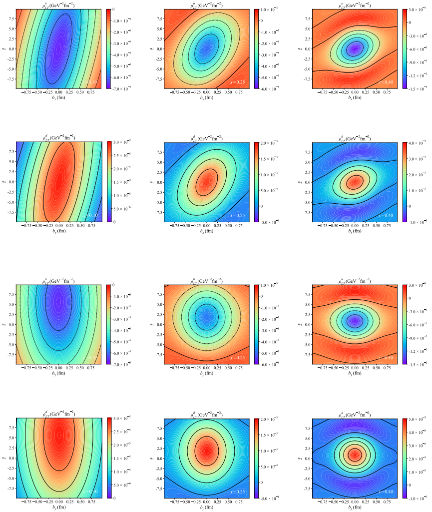


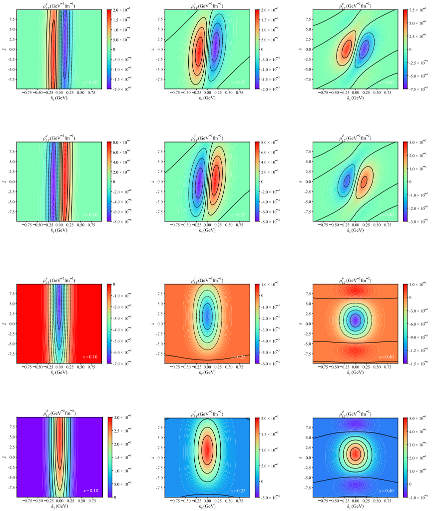
$\rho_{\mathrm{LL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x, \tilde{z} - b_y, \tilde{z} - k_x, \tilde{z} - k_y$ plane





$\rho_{\mathrm{LT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z}-b_x, \tilde{z}-b_y, \tilde{z}-k_x, \tilde{z}-k_y$ plane





$\rho_{TU}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x, \tilde{z} - b_y, \tilde{z} - k_x, \tilde{z} - k_y$ plane

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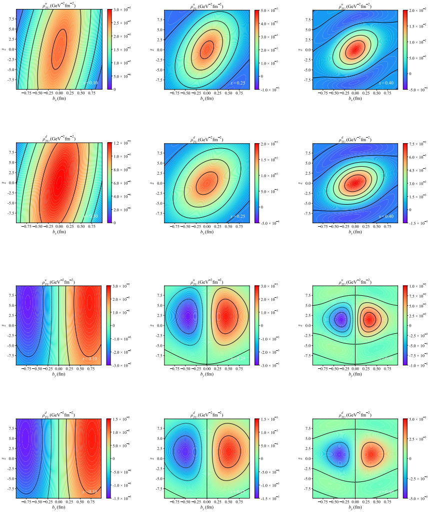
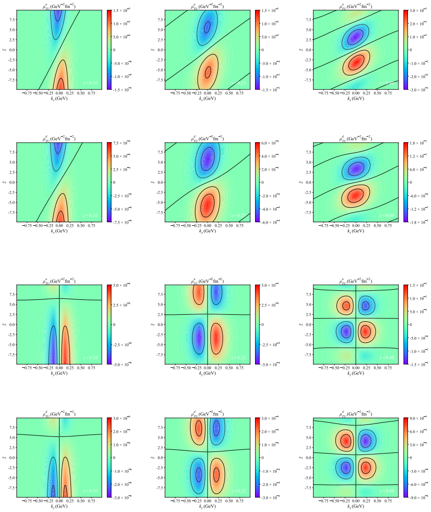
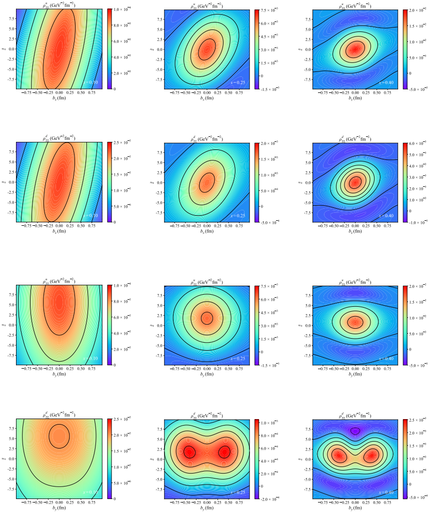
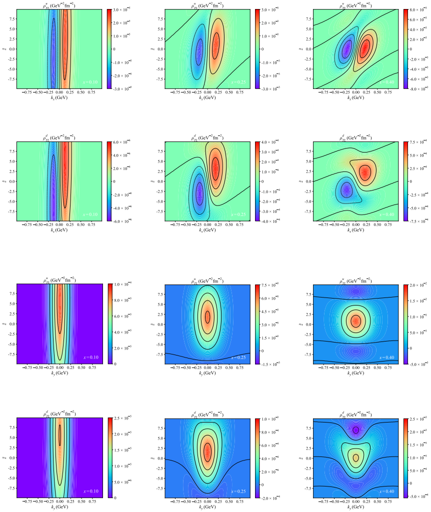




$\rho_{\mathrm{TL}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x, \tilde{z} - b_y, \tilde{z} - k_x, \tilde{z} - k_y$ plane





$\rho_{\mathrm{TT}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z}-b_x, \tilde{z}-b_y, \tilde{z}-k_x, \tilde{z}-k_y$ plane

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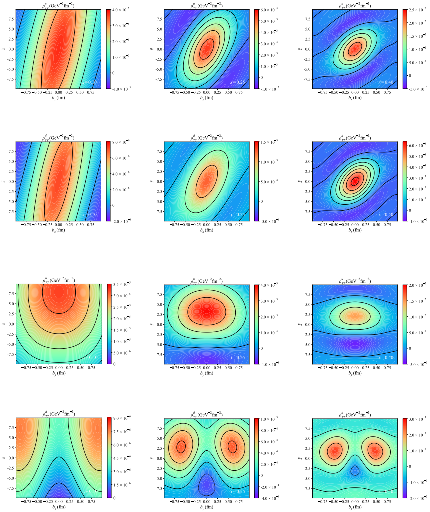

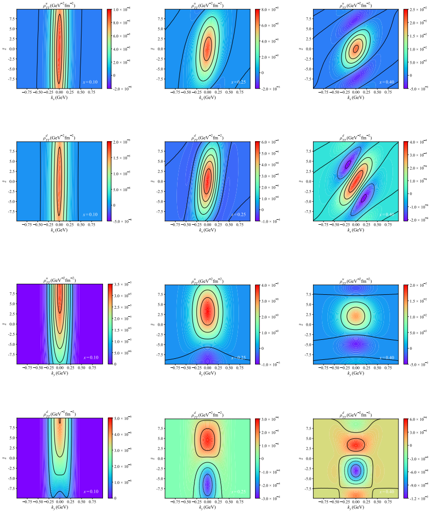

$\rho_{\mathrm{TTt}}\left(\tilde{z}, x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp\right)$ in $\tilde{z} - b_x, \tilde{z} - b_y, \tilde{z} - k_x, \tilde{z} - k_y$ plane

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