Spin physics and a solvable model for spin polarizations in HIC

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Outline

- **Introduction: rotation and spin in HIC**
- **A microscopic model for the emergence of spin-vorticity coupling from spin-orbit coupling in parton-parton scatterings with non-local collisions**
- **Vortex formation in collisions of BEC as topological realization of global spin polarization**
- **A solvable blast wave model for spin polarizations with flow-momentum correspondence**
- **Summary**

Rotation and Spin in HIC

Rotation effects in HIC

- **Huge global orbital angular momenta are produced**

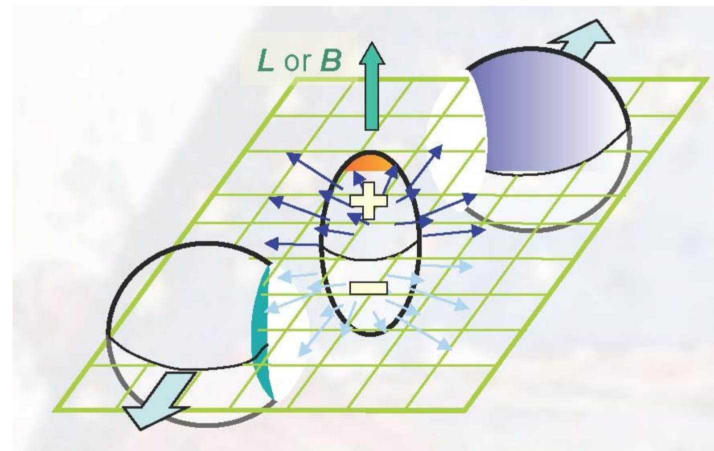
$$\mathrm{L} \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$\mathrm{B} \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **How do orbital angular momenta be transferred to the matter in HIC?**

- **How is spin coupled to the local vorticity in the fluid?**



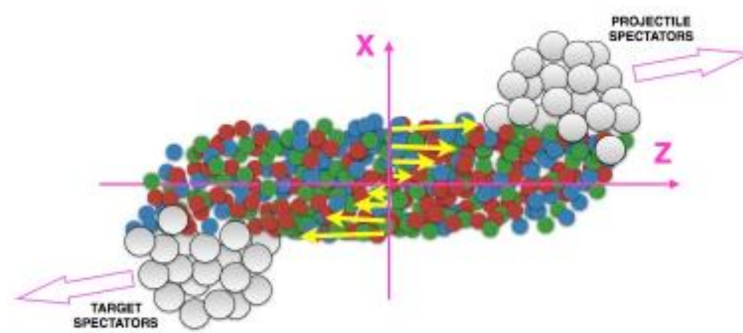
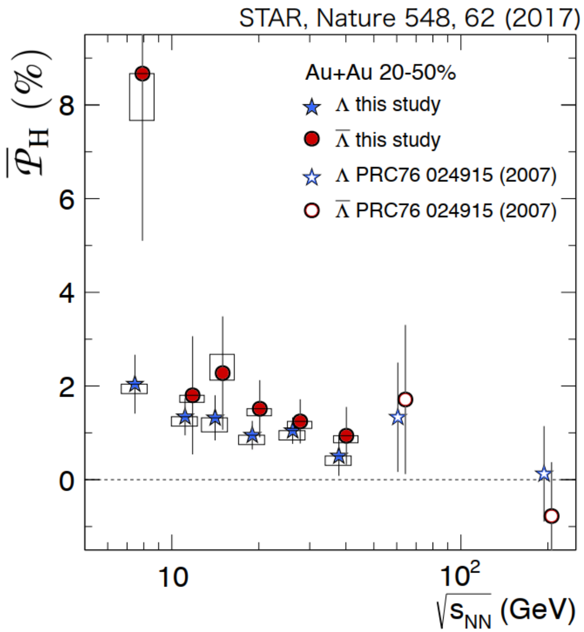


Figure taken from Becattini et al, 1610.02506

STAR: global polarization of Λ hyperon



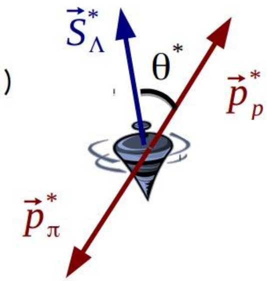
parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

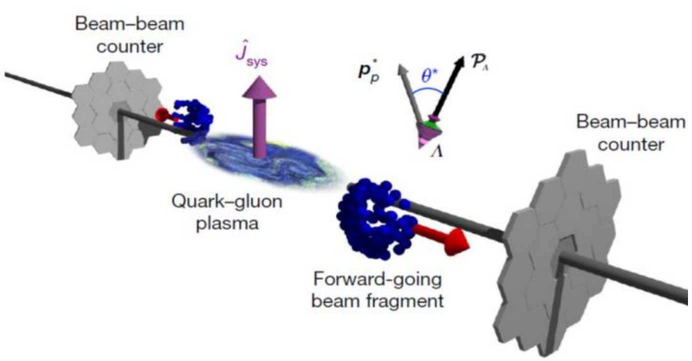
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_\Lambda \cdot \mathbf{p}_p^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

Updated by BES III, PRL129, 131801 (2022)



$\Lambda \to p + \pi^+$
(BR: 63.9%, cτ~7.9 cm)

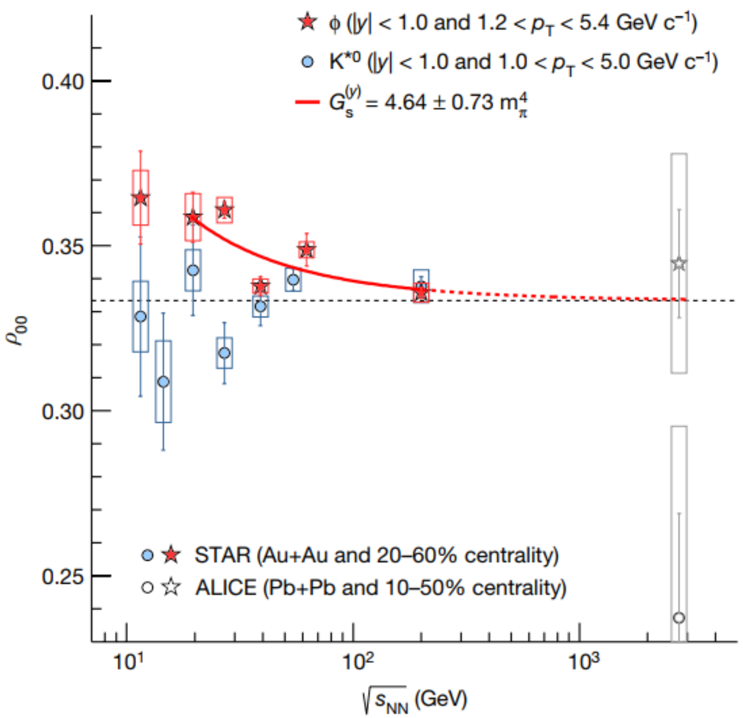


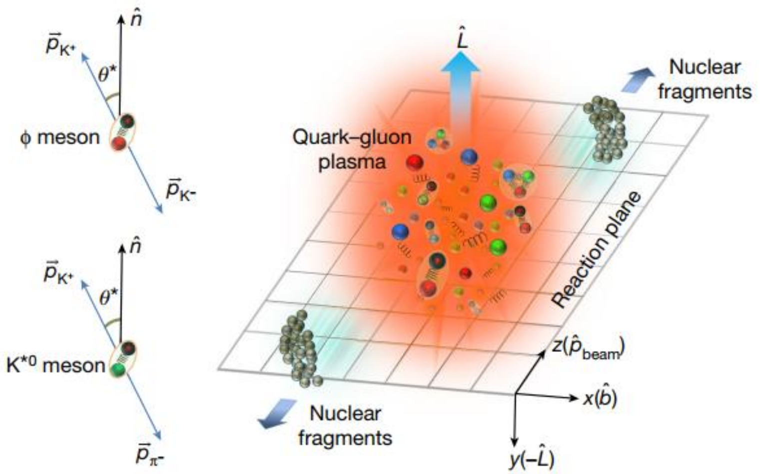
ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

STAR: global spin alignments of vector mesons

STAR, Nature 614, 244 (2023);





Implication of correlation or fluctuation in polarization s and s-bar

Theory prediction:

Liang, Wang, PLB(2005);
Sheng, Oliva, QW, PRD(2020);
Sheng, Oliva, Liang, QW, Wang, PRL(2022).

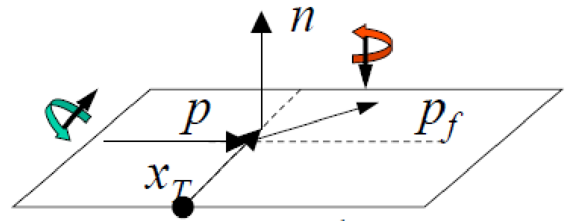
$$P_\Lambda \sim \langle P_s \rangle, \quad P_{\bar{\Lambda}} \sim \langle P_{\bar{s}} \rangle$$

$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle \sim P_\Lambda P_{\bar{\Lambda}}$$

Emergence of spin-vorticity coupling from spin-orbit coupling

Quark polarization in potential scatterings

- **Quark scatterings at small angle in static potential at impact parameter x_T**
- **Unpolarized and polarized cross sections**



$$\frac{d\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} + \frac{d\sigma_-}{d^2\vec{x}_T} = 4C_T\alpha_s^2 K_0(\mu x_T)$$

$$\frac{d\Delta\sigma}{d^2\vec{x}_T} = \frac{d\sigma_+}{d^2\vec{x}_T} - \frac{d\sigma_-}{d^2\vec{x}_T} \propto \vec{n}\cdot(\vec{x}_T \times \vec{p})$$

$$A^0(q_T) = \frac{1}{q_T^2 + \mu^2}$$

screening mass

$$\mu \sim T\sqrt{\alpha_S}$$

Spin quantization direction **OAM** **Spin-orbit coupling**

- **Polarization for small angle scattering and** $m_q \gg p, \mu$

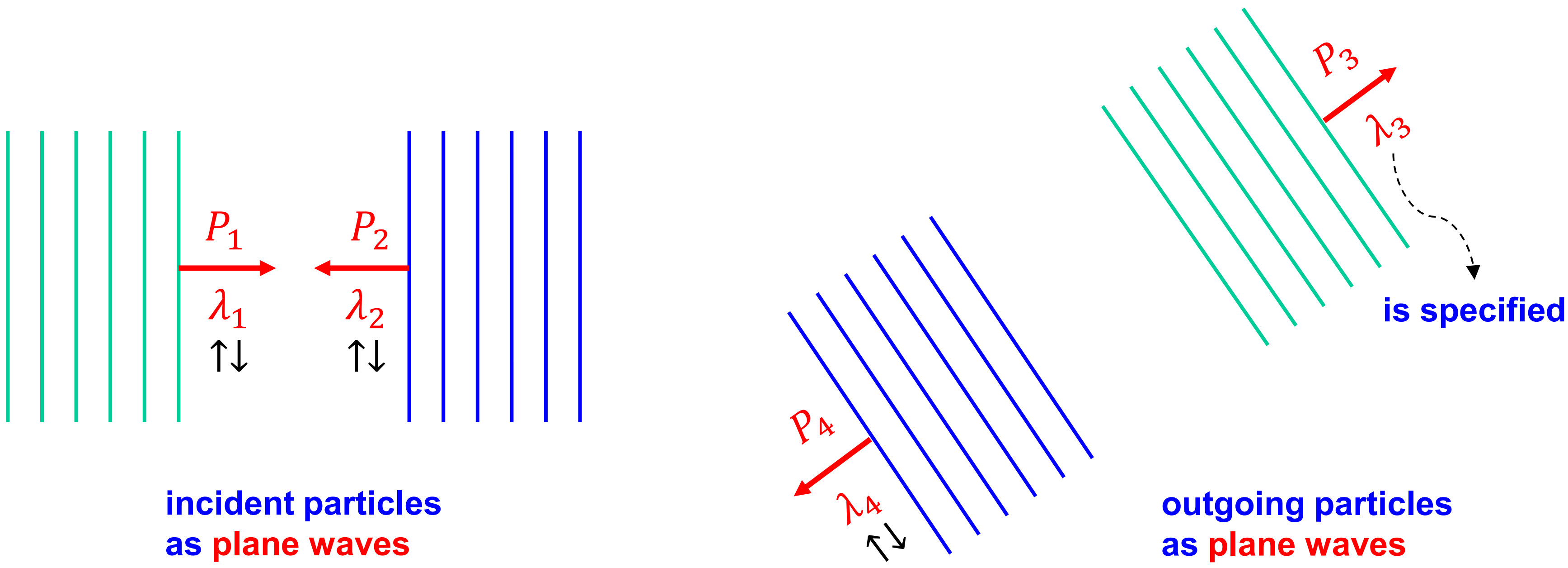
$$P_q \approx -\pi\frac{\mu p}{4m_q^2} \sim -\frac{\Delta E_{LS}}{E_0}$$

Liang, Wang, PRL 94, 102301(2005)

- **With initial polarization P_i, the final polarization P_f after one scattering is** $P_f = P_i - \frac{(1-P_i^2)\pi\mu p}{2E(E+m) - P_i\pi\mu p}.$

Huang, Huovinen, Wang, PRC84, 054910(2011)

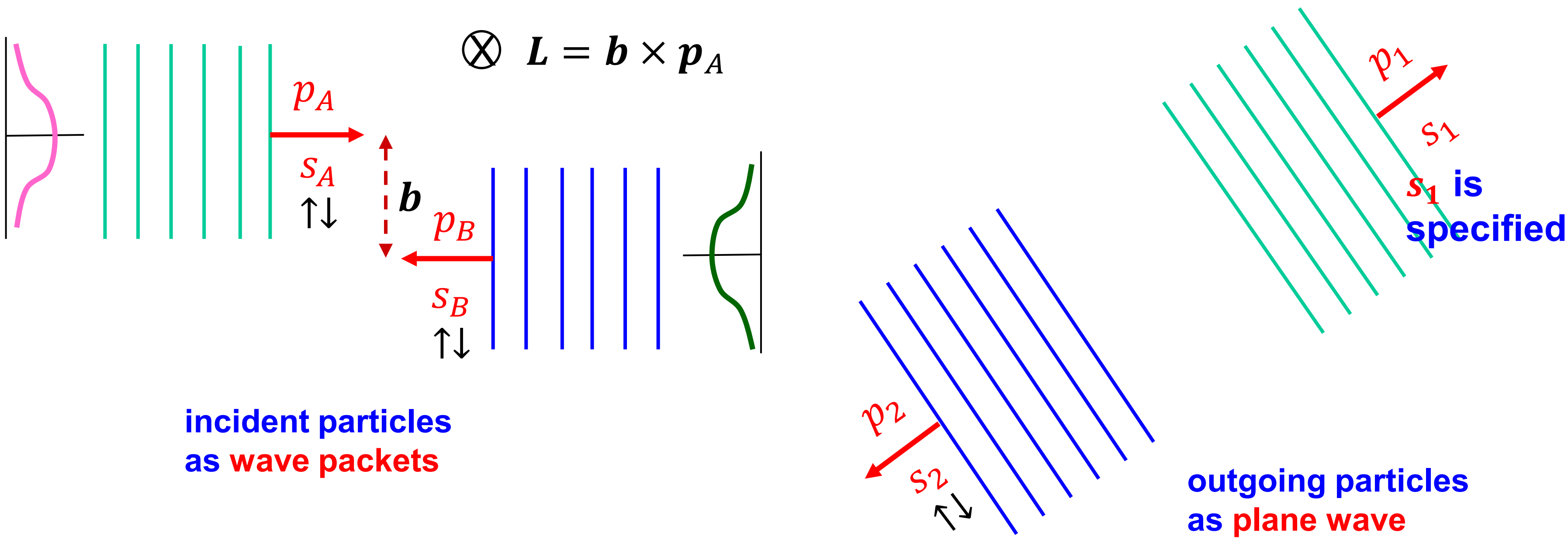
Collisions of particles as plane waves



Particle collisions as plane waves:
since there is no favored position for particles, so the OAM vanishing

$$\langle \hat{x} \times \hat{p} \rangle = 0 \longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{\lambda_3=\downarrow}$$

Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\downarrow}$$

Quark-quark scattering at fixed impact parameter

For the quark-quark scattering of spin-momentum states

$$q_1(P_1,\lambda_1)+q_2(P_2,\lambda_2)\rightarrow q_1(P_3,\lambda_3)+q_2(P_4,\lambda_4)$$

where $P_i=(E_i,\vec{p}_i)$ and λ_i denote spin states, the difference cross section (λ_3 is specified)

$c_{qq}=2/9$ (color factor)

$$d\sigma_{\lambda_3}=\frac{c_{qq}}{4F}\sum_{\lambda_1\lambda_2\lambda_4}\mathcal{M}(Q)\mathcal{M}^*(Q)(2\pi)^4\delta^{(4)}(P_1+P_2-P_3-P_4)\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

fixed

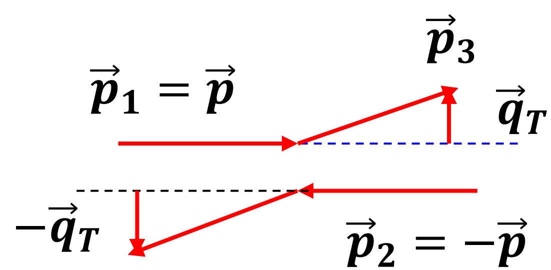
sum over ↑↓

$Q=P_3-P_1=P_2-P_4$ (momentum transfer)

$F=4\sqrt{(P_1\cdot P_2)^2-m_1^2m_2^2}$ (flux factor)

Integrate $\vec{p}_4$ and $p_{3z}^{\pm}=\pm\sqrt{p^2-q_T^2}$

to remove $\delta^{(4)}(P_1+P_2-P_3-P_4)$



Quark-quark scattering at fixed impact parameter

We obtain $d\sigma_{\lambda_3}$ for scattered quark with spin state λ_3

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1\lambda_2\lambda_4} \sum_{i=+,-} \frac{1}{(E_1+E_2)|p_{3z}^i|} \mathcal{M}(Q_i)\mathcal{M}^*(Q_i) \frac{d^2\vec{q}_T}{(2\pi)^2}$$

for small angle scattering, only $i = +$ is relevant

Jacobian

momentum transfer in small angle scattering

Then we can introduce impact parameter $\vec{x}_T = (x_T, \phi)$

$$d\sigma_{\lambda_3} = \frac{c_{qq}}{16F} \sum_{\lambda_1,\lambda_2,\lambda_3} \int d^2\vec{x}_T \int \frac{d^2\vec{q}_T}{(2\pi)^2} \int \frac{d^2\vec{k}_T}{(2\pi)^2} \exp\left[i\left(\vec{k}_T - \vec{q}_T\right)\cdot\vec{x}_T\right] \frac{\mathcal{M}(\vec{q}_T)\mathcal{M}^*(\vec{k}_T)}{\Lambda(\vec{q}_T)\Lambda^*(\vec{k}_T)}$$

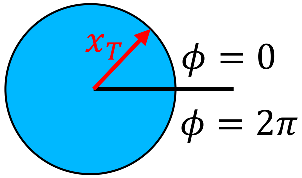
$$\Lambda(\vec{q}_T)\Lambda^*(\vec{k}_T) = \sqrt{(E_1+E_2)|p_{3z}^+(q_T)|}$$

$\Rightarrow d^2\sigma_{\lambda_3}/d^2\vec{x}_T$

1

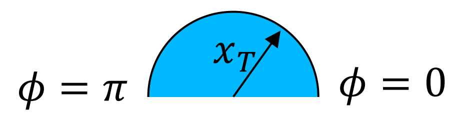
If we integrate over $\vec{x}_T$ in whole space we obtain

$$\sigma_{\lambda_3} = \int_0^\infty dx_T\, x_T \int_0^{2\pi} d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_\uparrow = \sigma_\downarrow$$



Quark-quark scattering at fixed impact parameter

If we integrate over $\vec{x}_T$ in half-space we obtain



$$\sigma_{\lambda_3} = \int_0^\infty dx_T\, x_T \int_0^\pi d\phi \frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} \Longrightarrow \sigma_\uparrow \neq \sigma_\downarrow$$

The differential cross section for spin-independent and spin-dependent part

$$\frac{d^2\sigma_{\lambda_3}}{d^2\vec{x}_T} = \frac{d^2\sigma}{d^2\vec{x}_T} + \lambda_3 \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$\frac{d^2\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_\uparrow}{d^2\vec{x}_T} + \frac{d^2\sigma_\downarrow}{d^2\vec{x}_T}\right) = F(x_T)$$

$$\frac{d^2\Delta\sigma}{d^2\vec{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_\uparrow}{d^2\vec{x}_T} - \frac{d^2\sigma_\downarrow}{d^2\vec{x}_T}\right) = \underline{\vec{n}\cdot(\vec{x}_T \times \vec{p})}\,\Delta F(x_T)$$

spin-orbit coupling

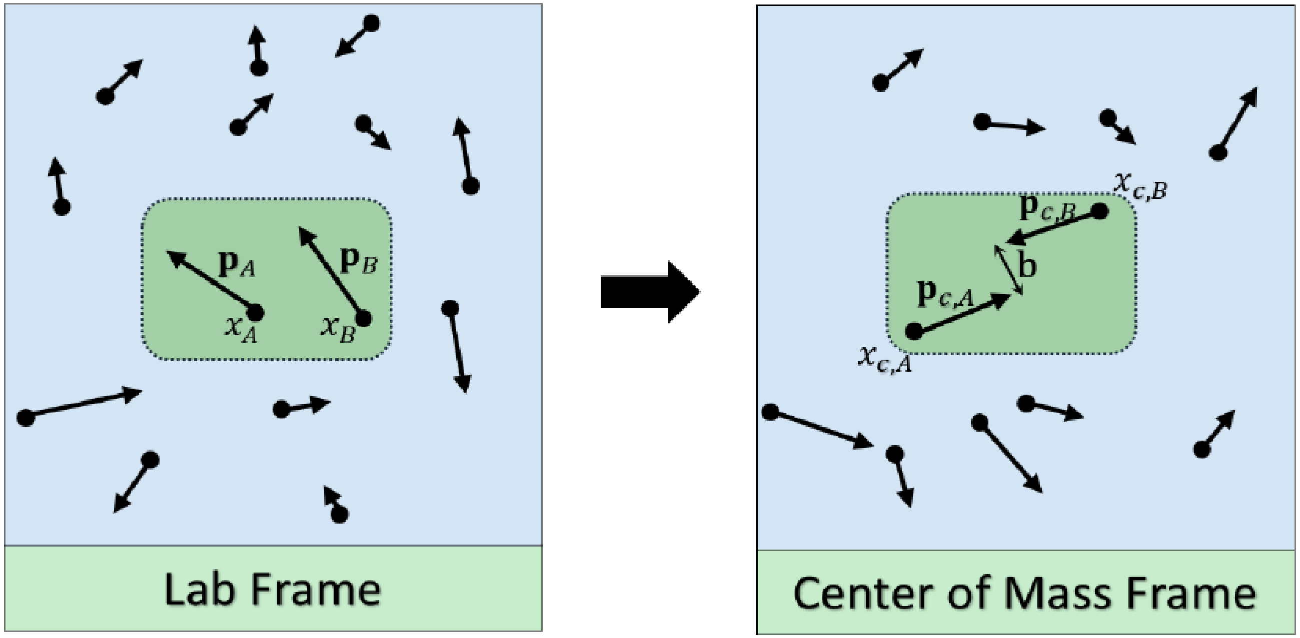
$$\Delta\sigma = \int_0^\infty dx_T x_T \int_0^\pi d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$\sigma = \int_0^\infty dx_T x_T \int_0^{2\pi} d\phi \frac{d^2\Delta\sigma}{d^2\vec{x}_T}$$

$$P_q = \frac{\Delta\sigma}{\sigma}$$

Gao, Chen, Deng, et al., PRC 77, 044902 (2008)

Ensemble average in thermal QGP for global polarization through spin-orbit couplings in parton scatterings



Zhang, Fang, QW, Wang, PRC 100, 064904 (2019)

From spin-orbit coupling to spin-vorticity coupling: ensemble average

- **Quark polarization rate per unit volume: 10D + 6D integration**

$$\frac{d^4\mathrm{P}_{AB\to 12}(X)}{dX^4} = \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B} \times |v_{c,A}-v_{c,B}|[\Lambda^{-1}]^\nu_{\ j}\mathrm{e}_{c,i}\epsilon_{ikh}\hat{\mathrm{p}}^h_{c,A} \times f_A(X,p_A)f_B(X,p_B)(p^\rho_A - p^\rho_B)\Theta_{jk}(\mathrm{p}_{c,A})$$

$$\equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\mathrm{W}^{\rho\nu}$$

6D integral

Lorentz boost

10D integral

16D integral !!

3-vector form

$$\frac{d^4\mathrm{P}_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta \mathbf{u})$$

- **Numerical challenge !!!** **We have developed ZMCintegral-3.0, a Monte Carlo integration package that runs on multi-GPUs** [Wu, Zhang, Pang, QW, Comp. Phys. Comm. (2020) (1902.07916)]

- **Another challenge:** **there are more than 5000 terms in polarized amplitude squared for 2-to-2 parton scatterings**

$$I_M^{q_a q_b\to q_a q_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}(\{s_A,k_A;s_B,k_B\}\to\{s_1,p_1;s_2,p_2\})\mathcal{M}^*(\{s_A,k'_A;s_B,k'_B\}\to\{s_1,p_1;s_2,p_2\})$$

Nonlocal collisions in Boltzmann equations

Extend phase space by introducing a classical spin variable s^{μ}

$$f(x,p) \Longrightarrow f(x,p,s) \qquad \int dS(p) = \frac{1}{\kappa(p)} \int d^4 s \delta(s^2+3)\delta(s\cdot p)$$

Boltzmann equation with non-local collisions

$$p\cdot\partial f(x,p,s) = C[f]$$

$$C[f] = \underline{C_{p+s}[f]} + \underline{C_s[f]}$$

$$= \int d\Gamma_1 d\Gamma_2 d\Gamma' \mathcal{W}$$

$$\times [f(x+\underline{\Delta_1},p_1,s_1)f(x+\underline{\Delta_2},p_2,s_2) - f(x+\underline{\Delta},p,s)f(x+\underline{\Delta'},p',s')]$$

$$+ \int d\Gamma_2 dS_1(p)\mathcal{M} f(x+\underline{\Delta_1},p,s_1)f(x+\underline{\Delta_2},p_2,s_2)$$

$$d\Gamma = \underline{\int d^4 p \delta(p^2-m^2)\int dS(p)}$$

spin-momentum integral

Δ : space-shift
Side-jump for chiral fermions: Chen, Son, Stephanov (2015)

Weickgenannt, Speranza, Sheng, QW, Rischke (2021); Wagner, Weickgenannt, Rischke (2022)
Enrico Speranza's talk

Numerical results and comparison with data

AMPT transport model

-- Li, Pang, QW, Xia, PRC96, 054908(2017)

-- Wei, Deng, Huang, PRC99, 014905(2019)

UrQMD + vHLLE hydro

-- Karpenko, Becattini, EPJC 77, 213(2017)

PICR hydro

-- Xie, Wang, Csernai, PRC 95,031901(2017)

Chiral Kinetic Equation + Collisions

-- Sun, Ko, PRC96, 024906(2017)

-- Liu, Sun, Ko, PRL125, 062301(2020)

AVE+3FD

-- Ivanov, 2006.14328

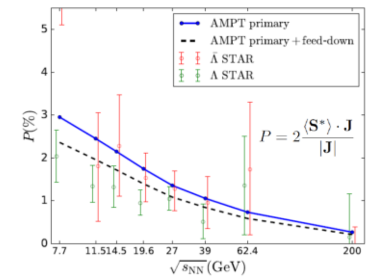
Reviews:

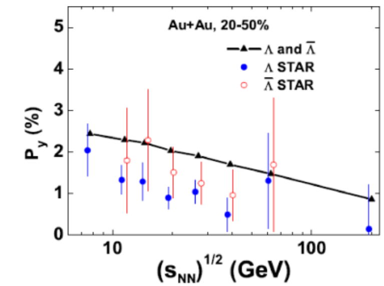
-- Huang, Liao, QW, Xia (2021)

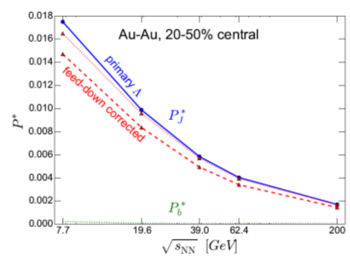
-- Becattini, Buzzegoli, Niida, Pu, Tang, QW (2024)

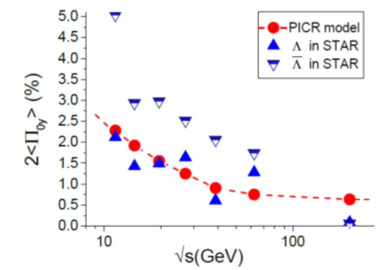
vorticity $\omega \Longrightarrow S$ spin on freeze-out hypersurface

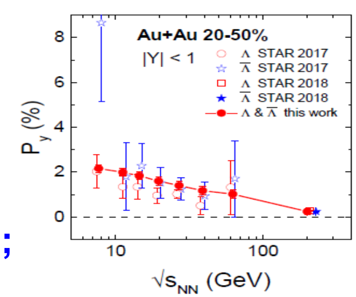
Becattini et al. (2013); Fang et al. (2016)

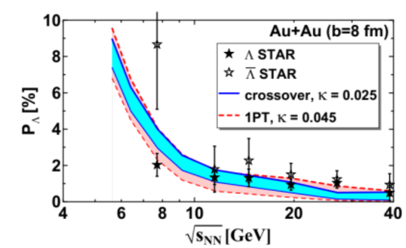












Vortex formation in collisions of BEC as topological realization of global spin polarization

J. Deng, S. Schlichting, R. Venugopalan, QW, PRA (2018)
J. Deng, QW, H. Zhang, POF (2022)

Relativistic vs non-relativistic fields

Relativistic real scalar (ϕ^4 int.)

(heavy ion collision)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

$$(\partial_\mu \partial^\mu + m^2)\phi + \frac{1}{6}\lambda\phi^3 = 0$$

Klein-Gordon equation (non-linear)
No U(1) global symmetry: particle number is not conserved

Non-relativistic complex scalar

(cold bosonic atoms)

$$\mathcal{H} = -\frac{1}{2m}\psi^*\nabla^2\psi + V|\psi|^2 + \frac{1}{2}g|\psi|^4$$

$$\mathcal{L} = \frac{1}{2}i\left(\psi^*\dot{\psi} - \psi\dot{\psi}^*\right) - \mathcal{H}$$

$$i\frac{\partial\psi}{\partial t} = \left(-\frac{1}{2m}\nabla^2 + V + g|\psi|^2\right)\psi$$

Gross-Pitaevskii equation
(Non-linear Schroedinger equation)
U(1) global symmetry: particle number conservation

NR complex scalar field theory (large mass effective theory) can be derived from relativistic real scalar FT by classical canonical transformation

Deng, Schlichting, Venugopalan, QW (2018)

Canonical transformation: derive H_{GP}

Relativistic real scalar (heavy ion collision)

Non-relativistic complex scalar (cold bosonic atoms)

$$H_0(a,a^*) = \int [d^3\mathbf{k}] E_k a_\mathbf{k} a^*_\mathbf{k},$$

$$H_{\rm int}(a,a^*) = \frac{\lambda}{24}\int \frac{[d^3\mathbf{k}][d^3\mathbf{k}_1][d^3\mathbf{k}_2][d^3\mathbf{k}_3]}{\sqrt{16 E_k E_{k1} E_{k2} E_{k3}}}(2\pi)^3$$
$$\times [\, a_\mathbf{k} a_{\mathbf{k}1} a_{\mathbf{k}2} a_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$
$$+4\, a_\mathbf{k} a_{\mathbf{k}1} a_{\mathbf{k}2} a^*_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)$$
$$+6\, a_\mathbf{k} a_{\mathbf{k}1} a^*_{\mathbf{k}2} a^*_{\mathbf{k}3}\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)$$
$$+4\, a_\mathbf{k} a^*_{\mathbf{k}1} a^*_{\mathbf{k}2} a^*_{\mathbf{k}3}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)$$
$$+\, a^*_\mathbf{k} a^*_{\mathbf{k}1} a^*_{\mathbf{k}2} a^*_{\mathbf{k}3}\delta(-\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)]$$

particle number changing terms (not conserved)

canonical transformation: $a_k \Longrightarrow b_k$

$$H_{GP}(b,b^*) \approx \int [d^3\mathbf{p}]\, E_p b_\mathbf{p} b^*_\mathbf{p}$$
$$+\frac{\lambda}{16}\int \prod_{i=1}^{4}[d^3\mathbf{k}_i]\,\frac{1}{\sqrt{E_{k1}E_{k2}E_{k3}E_{k4}}}$$
$$\times\, b_{\mathbf{k}1} b_{\mathbf{k}2} b^*_{\mathbf{k}3} b^*_{\mathbf{k}4}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)$$
$$+\cdots\cdots$$

There are only particle number conserving terms with particle number changing terms suppressed

H_{GP} as non-relativistic effective theory of relativistic real scalar field

Deng, Schlichting, Venugopalan, QW (2018)

Canonical transformation: derive H_{GP}

New canonical variable by time shift through H_{aux}

initial auxiliary time z=0

$$b_{\mathbf{p}} \equiv b_{\mathbf{p}}(0)$$
$$b^*_{\mathbf{p}} \equiv b^*_{\mathbf{p}}(0)$$

$$H_{\mathrm{aux}}(b, b^*)$$

at auxiliary time z

$$a_{\mathbf{p}} \equiv b_{\mathbf{p}}(z)$$
$$a^*_{\mathbf{p}} \equiv b^*_{\mathbf{p}}(z)$$

$$\begin{aligned} a_{\mathrm{p}} &= b_{\mathrm{p}} - iz\{b_{\mathrm{p}}, H_{\mathrm{aux}}\}_b \\ &+ \frac{1}{2}(-iz)^2\{\{b_{\mathrm{p}}, H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b \\ &+ \cdots \end{aligned}$$

substitute

$$H(a, a^*)$$

$$H'(b, b^*)$$

$$\begin{aligned} H'(b,b^*) &= H(b,b^*) + (-iz)\{H(b,b^*), H_{\mathrm{aux}}\}_b \\ &+ \frac{1}{2}(-iz)^2\{\{H(b,b^*), H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b \\ &+ \frac{1}{6}(-iz)^3\{\{\{H(b,b^*), H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b, H_{\mathrm{aux}}\}_b \\ &+ \cdots \end{aligned}$$

with only resonant terms

$$H_{\mathrm{GP}}(b, b^*)$$

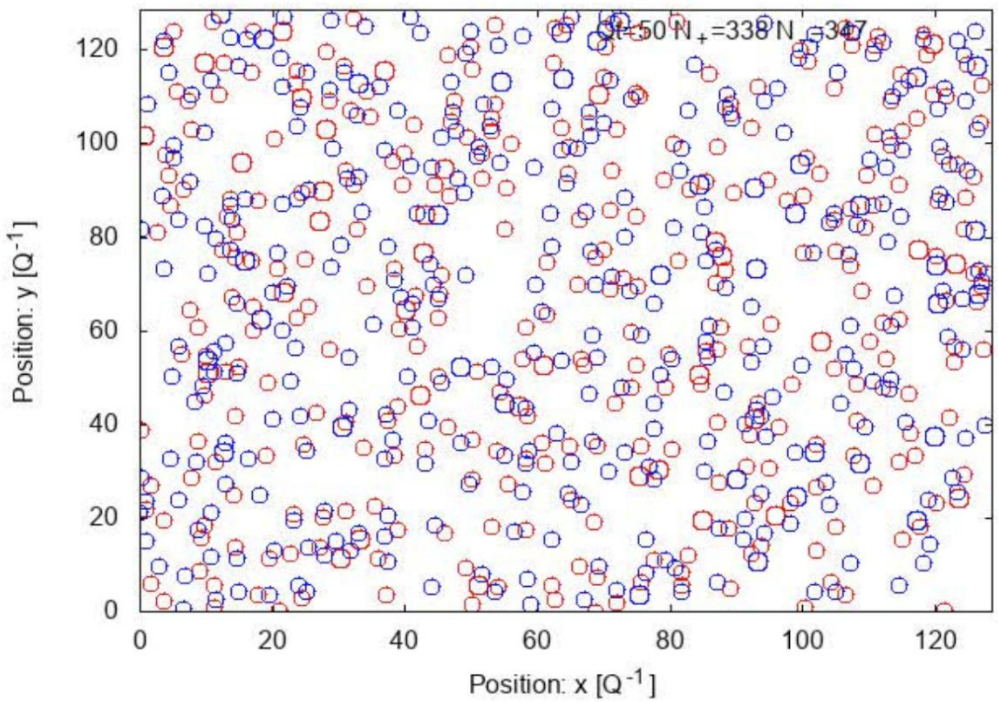
vanishing of non-resonant terms fix B_1 and B_2 in H_{aux}

Deng, Schlichting, Venugopalan, QW (2018)

Vortex-anti-vortex pairs (topological realization of spin doublet)

Striking similarities with previous observations in simulations of Gross-Pitaevski equation

Slide from J. Deng



vortex

anti-vortex

Deng, Schlichting, Venugopalan, QW (2018)

Scaling regime (Qt>1000) features individual vortices as well as loosely bound vortex— anti-vortex pairs

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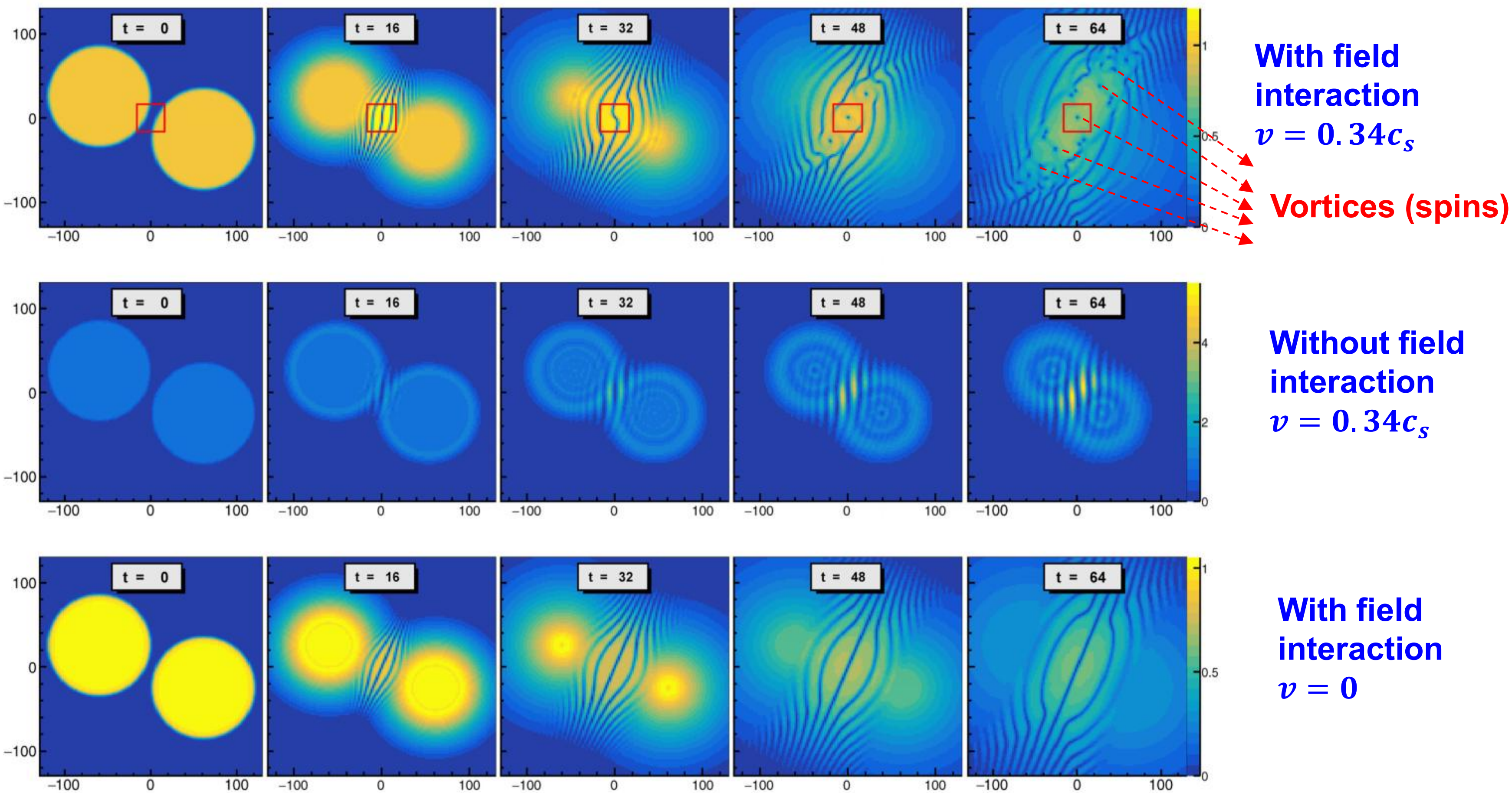
Vortex formation in collisions of BEC as topological realization of global spin polarization



Deng, QW, Zhang (2022)

A solvable blast wave model for spin polarization with flow-momentum correspondence

A. Aslan, W.B. Dong, G.L. Ma, S.Pu, QW, PRC(2025)
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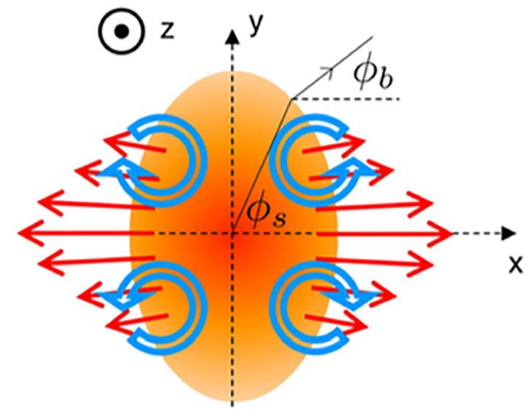
The idea: how did it come up?

- **The longitudinal spin polarization was once a puzzle since the hydro-simulation using the thermal vorticity contradicts the data, but after including the shear contribution, the agreement between theory and data can be realized.** **[Fu, Liu, Pang, Song, Yin (2021); Becattini, Buzzegoli, Inghirami, Karpenko, Palermo (2021)]**
- **On the other hand, a very simple picture works based on transverse flow profile in blast wave model** **[STAR Collab., PRL 123, 132301(2019)]**



transverse flow velocity profile

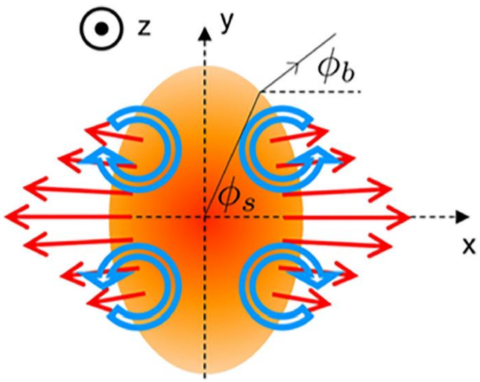
$$\boldsymbol{v} = \boldsymbol{e}_r v_r \left[1 + v_2 \cos(2\phi)\right]$$
$$v^x = v_r \cos\phi \left[1 + v_2 \cos(2\phi)\right]$$
$$v^y = v_r \sin\phi \left[1 + v_2 \cos(2\phi)\right]$$

$$\omega^z \sim (\nabla \times \boldsymbol{v})_z = \frac{\partial v^y}{\partial x} - \frac{\partial v^x}{\partial y}$$
$$\sim \frac{\partial r}{\partial x}\partial_r v^y + \frac{\partial \phi}{\partial x}\partial_\phi v^y - \left(\frac{\partial r}{\partial y}\partial_r v^x + \frac{\partial \phi}{\partial y}\partial_\phi v^x\right)$$
$$\approx -\frac{\sin\phi}{r}\partial_\phi v^y - \frac{\cos\phi}{r}\partial_\phi v^x = \boxed{2v_2 \frac{v_r}{r}\sin(2\phi)}$$

→ **correct sign !**

Blast wave model: kinematic variables

- **The QGP can be approximated as a longitudinally boost-invariant system, thus it is natural to use the proper time and the space-time rapidity (τ, η) as variables to replace (t, z). Accordingly, the particle's momentum in longitudinal direction can also be described by the transverse mass and momentum rapidity (m_T, Y) or (p_T, Y).**



$$\tau = \sqrt{t^2 - z^2},\ \eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

$$m_T = \sqrt{m^2 + p_T^2},\ Y = \frac{1}{2}\ln\frac{E_p + p_z}{E_p - p_z}$$

$$\tan\phi_b = \frac{R_x^2}{R_y^2}\tan\phi_s \approx (1-2\epsilon)\tan\phi_s$$

ϕ_b: emission angle
ϕ_s: position angle

- **Therefore the flow four-velocity and the particle's four-momentum can be parametrized as**

ρ: transverse rapidity

$$u^\mu(x) = (\cosh\eta\cosh\rho, \sinh\rho\cos\phi_b, \sinh\rho\sin\phi_b, \sinh\eta\cosh\rho)$$

$$p^\mu = (m_T\cosh Y, p_T\cos\phi_p, p_T\sin\phi_p, m_T\sinh Y)$$

Lee, Heinz, Schnedermann (1990); Schnedermann, Sollfrank, Heinz (1993); Retiere, Lisa (2004)

Modified blast wave model: flow-momentum correspondence

- Transverse expansion of the fireball is described by **transverse rapidity**

$$\rho(r,\phi_s,\eta) = \tilde{r}\left[\rho_0 + \underline{\rho_1(\eta)}\cos(\phi_b) + \underline{\rho_2}\cos(2\phi_b)\right]$$

v1 flow — v2 flow

$$\tilde{r} = \sqrt{\frac{(r\cos\phi_s)^2}{R_x^2} + \frac{(r\sin\phi_s)^2}{R_y^2}} \approx \frac{r}{R}\left[1 + \frac{1}{2}\underline{\epsilon}\cos(2\phi_s)\right]$$

radial flow or radial expansion

ellipticity parameter

- We will use the ordering of parameters $\alpha_1 \sim \rho_2 \sim \epsilon \ll \rho_0 \sim O(1)$
- The particle's distribution function in phase space $f(x,p)$ is assumed to follow the Boltzmann distribution under the condition $\beta p \cdot u \gg 1$

$$\begin{aligned} f(x,p) \equiv & f(p\cdot u) = \exp(-\beta p \cdot u) \\ & = \exp\left\{-\beta\left[m_T\cosh\rho\cosh(\eta - Y) - p_T\sinh\rho\cos(\phi_b - \phi_p)\right]\right\} \end{aligned}$$

- **Flow-momentum correspondence: $f(x,p)$ reaches a maximum at**

$$\eta \approx Y, \quad \phi_b \approx \phi_p, \quad \frac{p_T}{m_T} = \tanh\rho$$

Observables: average over phase space variables

- **Physical observables can be computed on the freeze-out hyper-surface**

$$\langle O(p)\rangle = \frac{\int d^4x O(x,p) S(x,p)}{\int d^4x S(x,p)}$$

Emission function

- **The emission function $S(x,p)$ represents the probability of emitting a particle with the momentum p at the space-time x and thus defines a freeze-out hyper-surface for particle emission at the freeze-out temperature T_f**

$$S(x,p) = m_T \cosh(\eta - Y)\delta(\tau - \tau_f)\Theta(R - r) f(x,p)$$

- **The observables**

$$\langle O\rangle (p_T) = \frac{\int d^4x dY d\phi_p O(x,p) S(x,p)}{\int d^4x dY d\phi_p S(x,p)} \qquad d^4x = \tau r d\tau d\eta dr d\phi_s$$

$$\langle O\rangle (\phi_p) = \frac{\int d^4x dp_T dY p_T O(x,p) S(x,p)}{\int d^4x dp_T dY p_T S(x,p)} \qquad \frac{d^3\mathbf{p}}{E_p} = p_T dp_T dY d\phi_p$$

Collective flows: analytical results

The directed flow v_1 as a function of Y and the elliptic flow v_2 as a function of p_T as

$$v_1(Y) = \frac{\int d^4x \int dp_T d\phi_p p_T \ \cos(\phi_p) S(x,p)}{\int d^4x \int dp_T d\phi_p p_T \ S(x,p)} = \alpha_1 Y \frac{\beta}{2R} \frac{N_{v1}}{N_0}$$

$$v_2(p_T) = \frac{\int d^4x \int dY d\phi_p \ \cos(2\phi_p) S(x,p)}{\int d^4x \int dY d\phi_p \ S(x,p)}$$

$$= \left(\rho_2 + \frac{1}{2}\epsilon\rho_0\right) \frac{\beta}{2R} \frac{N_{v2}(p_T)}{N_0(p_T)} + \epsilon \frac{N_{v2}^{\epsilon}(p_T)}{N_0(p_T)}$$

where

$$N_0 = \int_{p_T^{\min}}^{p_T^{\max}} dp_T p_T \ N_0(p_T)$$

$$N_{v1} = \int_{p_T^{\min}}^{p_T^{\max}} dp_T p_T N_{v1}(p_T)$$

$$N_0(p_T) = \int_0^R dr \ r m_T \ K_1(\beta m_T \cosh\bar{\rho}) I_0(\beta p_T \sinh\bar{\rho})$$

$$N_{v1}(p_T) = \int_0^R dr \ r^2 m_T \left[m_T \sinh\bar{\rho} K_1'(\beta m_T \cosh\bar{\rho}) I_1(\beta p_T \sinh\bar{\rho}) \right.$$
$$\left. + p_T \cosh\bar{\rho} K_1(\beta m_T \cosh\bar{\rho}) I_1'(\beta p_T \sinh\bar{\rho})\right]$$

$$N_{v2}(p_T) = \int_0^R dr \ r^2 m_T \left[m_T \sinh\bar{\rho} K_1'(\beta m_T \cosh\bar{\rho}) I_2(\beta p_T \sinh\bar{\rho}) \right.$$
$$\left. + p_T \cosh\bar{\rho} K_1(\beta m_T \cosh\bar{\rho}) I_2'(\beta p_T \sinh\bar{\rho})\right]$$

$$N_{v2}^{\epsilon}(p_T) = \int_0^R dr \ r m_T \ K_1(\beta m_T \cosh\bar{\rho}) I_2(\beta p_T \sinh\bar{\rho})$$

Spin polarizations: analytical results

The spin vectors are defined as

$$\hat{P}^{\mu}_{\omega} = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}(1-f)\omega_{\nu\sigma}p_{\tau}$$

$$\hat{P}^{\mu}_{\xi} = -\frac{1}{2m}\epsilon^{\mu\nu\sigma\tau}(1-f)\frac{p_{\tau}p^{\rho}}{E_p}\hat{t}_{\nu}\xi_{\rho\sigma}$$

vorticity and shear tensors

$$\omega^{\mu\nu} = -\frac{1}{2}\left[\partial^{\mu}\left(\beta u^{\nu}\right) - \partial^{\nu}\left(\beta u^{\mu}\right)\right]$$

$$\xi^{\mu\nu} = \frac{1}{2}\left[\partial^{\mu}\left(\beta u^{\nu}\right) + \partial^{\nu}\left(\beta u^{\mu}\right)\right]$$

The spin polarization as functions of ϕ_p

$$\begin{aligned} P^{y}(\phi_p) &= \left\langle \hat{P}^{y}_{\omega} + \hat{P}^{y}_{\xi} \right\rangle(\phi_p) \\ &\approx \alpha_1 \frac{1}{4mT_f\tau_f R}\frac{1}{N_0}\left[N_1(2,1,2) + N_1(2,3,0) - 2N_2(2,2,1)\right]\cos^2\phi_p \\ P^{z}(\phi_p) &= \left\langle \hat{P}^{z}_{\omega} + \hat{P}^{z}_{\xi} \right\rangle(\phi_p) \\ &\approx \rho_2 \frac{1}{2mT_f R}\frac{1}{N_0}\left[N_1(1,1,2) + N_1(1,3,0) - 2N_2(1,2,1)\right]\sin(2\phi_p) \\ &\quad - \epsilon\frac{1}{4mT_f}\frac{1}{N_0}\left[N_2(0,1,2) - N_2(0,3,0)\right]\sin(2\phi_p) \end{aligned}$$

where $N_{1,2}(n_1, n_2, n_3)$ are integrals over p_T and r .

Spin polarizations: analytical results

The spin polarization as functions of p_T

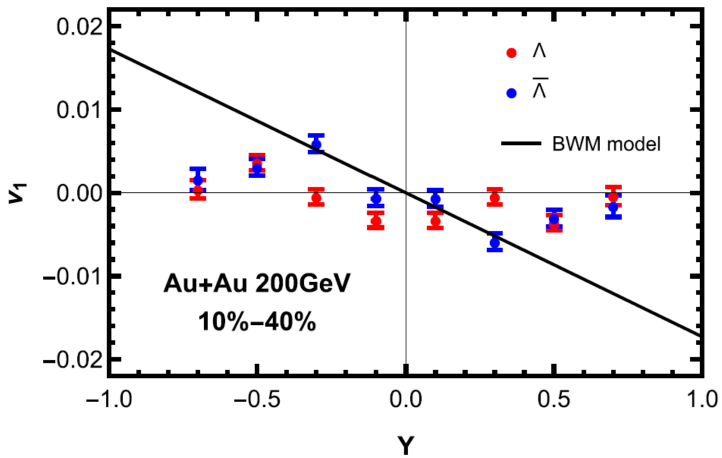
$$\begin{aligned} P^y(p_T) &= \left\langle \hat{P}^y_\omega + \hat{P}^y_\xi \right\rangle (p_T) \\ &\approx \alpha_1 \frac{1}{8mT_f R\tau_f} \frac{1}{N_0(p_T)} \left[N_{p1}(2,0,2) + N_{p1}(2,2,0) - 2N_{p2}(2,1,1) \right] \\ P^z_{\sin(2\phi)}(p_T) &\equiv \left\langle \left(\hat{P}^z_\omega + \hat{P}^z_\xi \right) \sin(2\phi_p) \right\rangle (p_T) \\ &\approx \rho_2 \frac{1}{4mT_f R} \frac{1}{N_0(p_T)} \left[N_{p1}(1,0,2) + N_{p1}(1,2,0) - 2N_{p2}(1,1,1) \right] \\ &\quad - \epsilon \frac{1}{8mT_f} \frac{1}{N_0(p_T)} \left[N_{p2}(0,0,2) - N_{p2}(0,2,0) \right] \end{aligned}$$

where $N_{p1,p2}(n_1, n_2, n_3)$ are integrals over r and functions of p_T.

The centrality dependence:

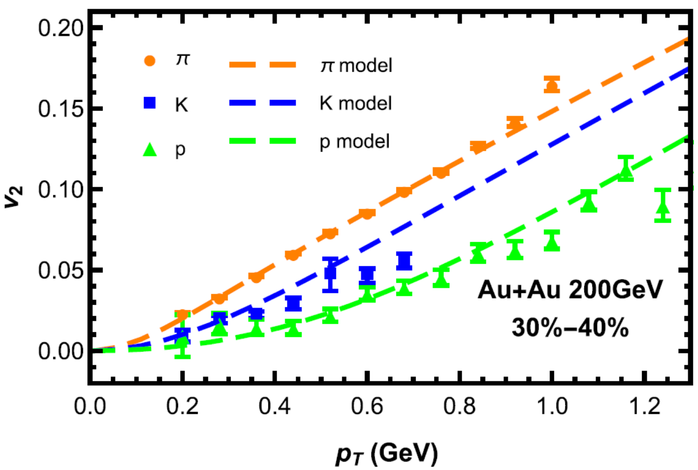
$$\begin{aligned} P^y &\approx \alpha_1 \frac{1}{8mT_f R\tau_f} \frac{1}{N_0} \left[N_1(2,1,2) + N_1(2,3,0) - 2N_2(2,2,1) \right] \\ P^z_{\sin(2\phi)} &\approx \rho_2 \frac{1}{4mT_f R} \frac{1}{N_0} \left[N_1(1,1,2) + N_1(1,3,0) - 2N_2(1,2,1) \right] \\ &\quad - \epsilon \frac{1}{8mT_f} \frac{1}{N_0} \left[N_2(0,1,2) - N_2(0,3,0) \right] \end{aligned}$$

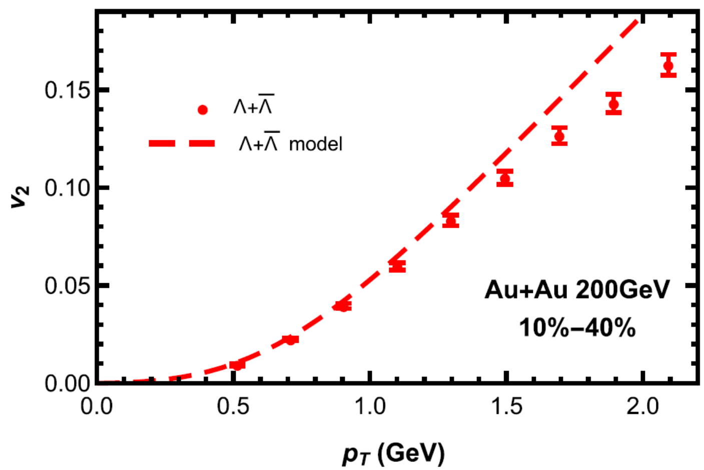
Comparison with data: v_1 and v_2



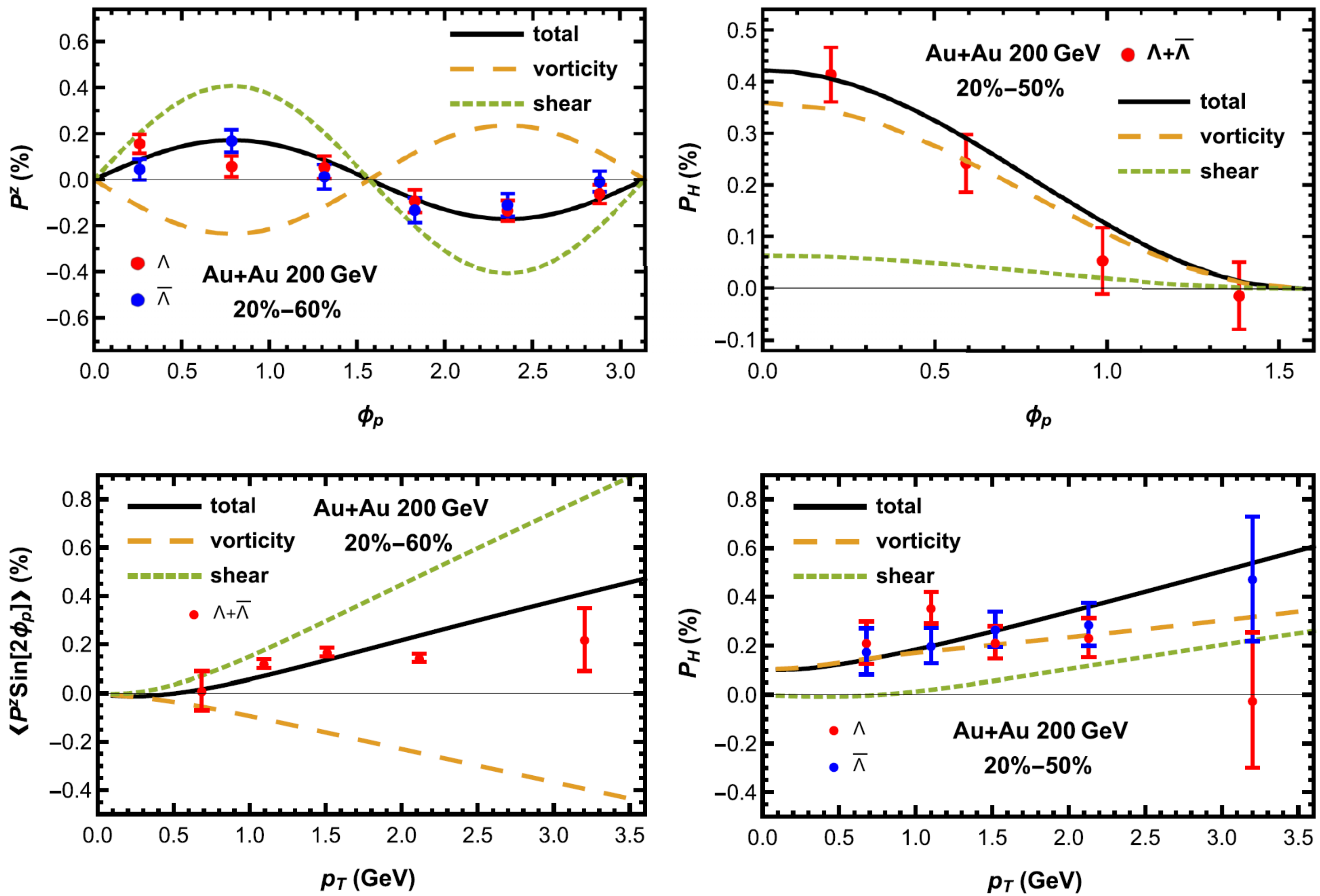
Upper-Left panel: The results for v_1 of Λ and $\bar{\Lambda}$ in Au+Au collisions at 200 GeV. The p_T range is [0.2,5.0] GeV. The experimental data [STAR Collab., PRL 120, 062301 (2018)].

Lower panel: The results for v_1 of $\pi, K, p, \Lambda, \bar{\Lambda}$. The data for light particles are from [STAR Collab., PRC 72, 014904 (2005)] and those for Λ and $\bar{\Lambda}$ are from [STAR Collab., PRC 77, 054901 (2008)].

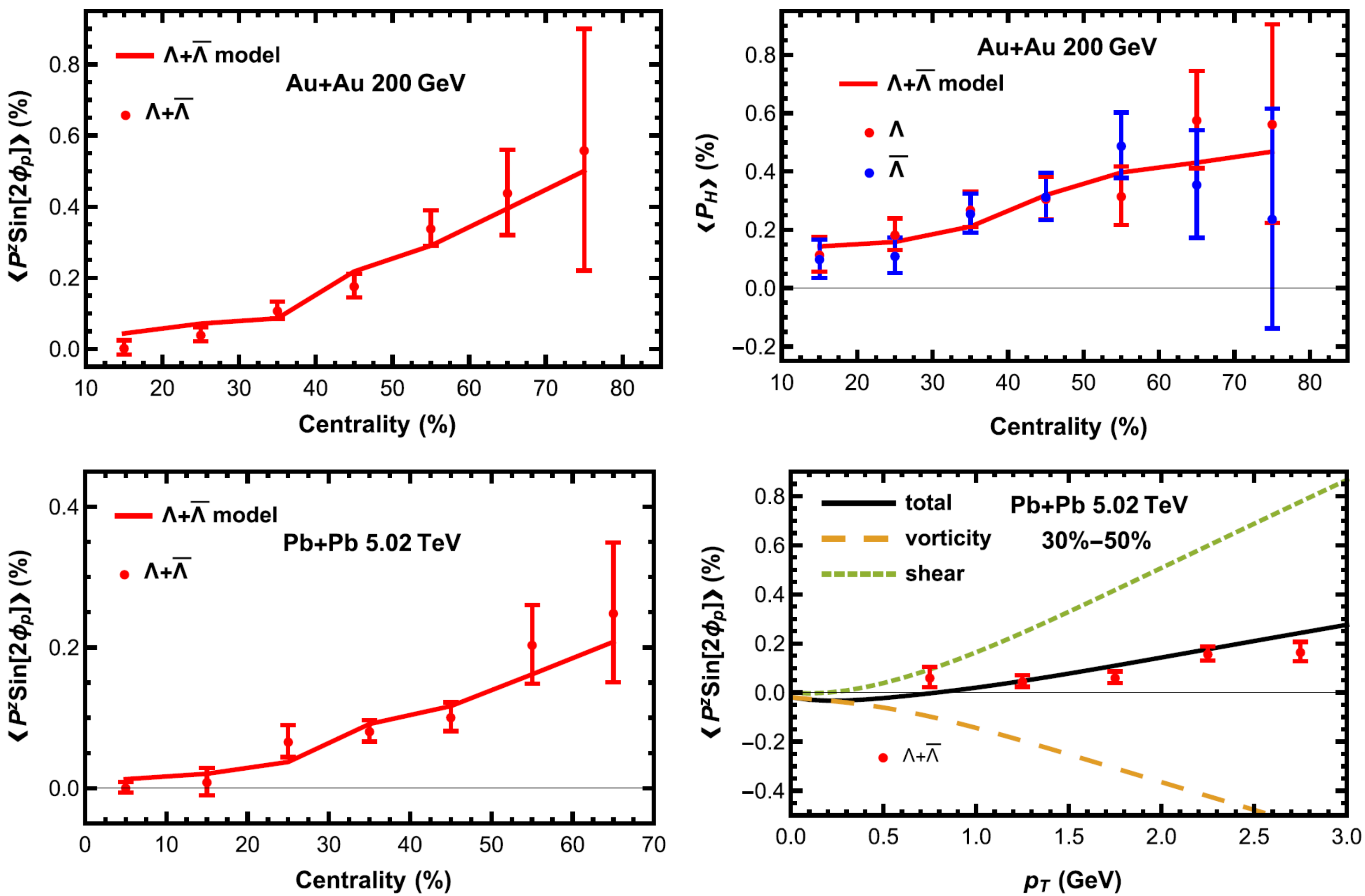




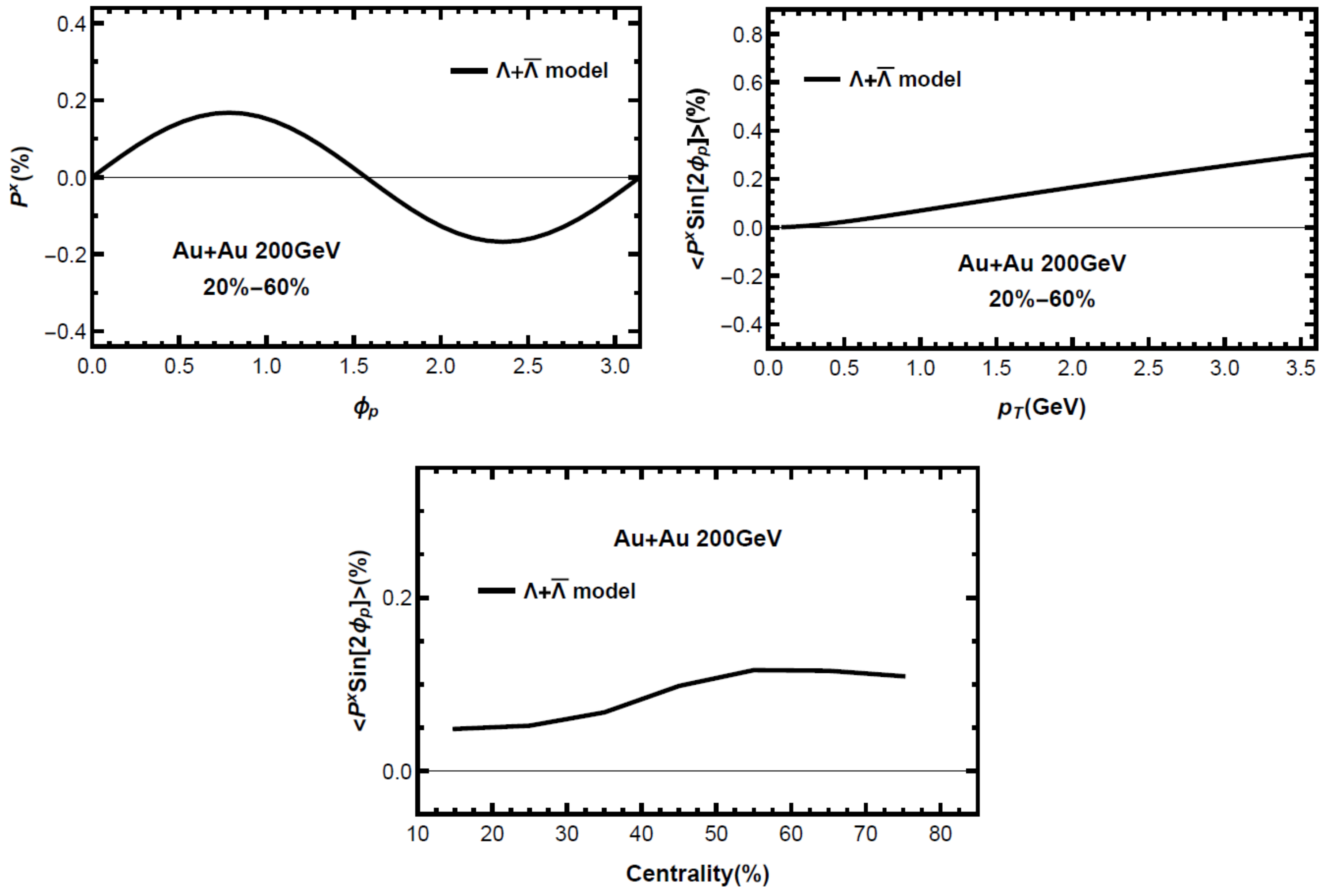
Comparison with data (200 GeV)



Comparison with data: 200 GeV and 5.02 TeV



Prediction of new observable: P^x



Summary

Directed flow v_1 → Polarization in OAM direction P_y

Directed flow v_1 → In-plane transverse polarization P_x (prediction for new observable)

Elliptic flow v_2 and ellipticity ϵ → Longtitudinal polarization P_z

Blast wave model provides a simple and comprehensive parameterization for spin polarization phenomena in HIC