Is the shear induced spin polarization non-dissipative?

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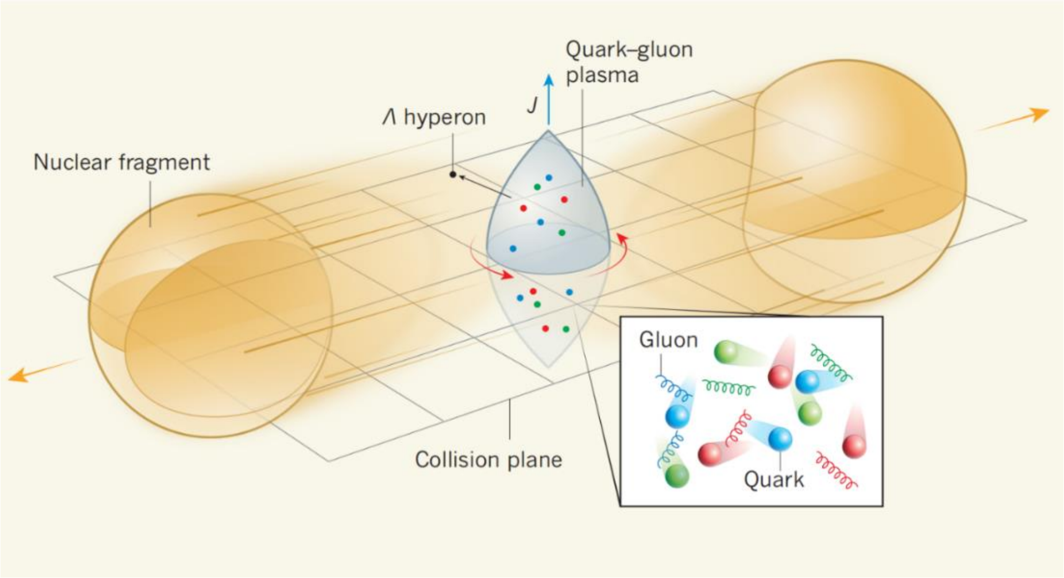
Based on JRW, Shuo Fang, Di-Lun Yang, Shi Pu, arXiv:2507.15238

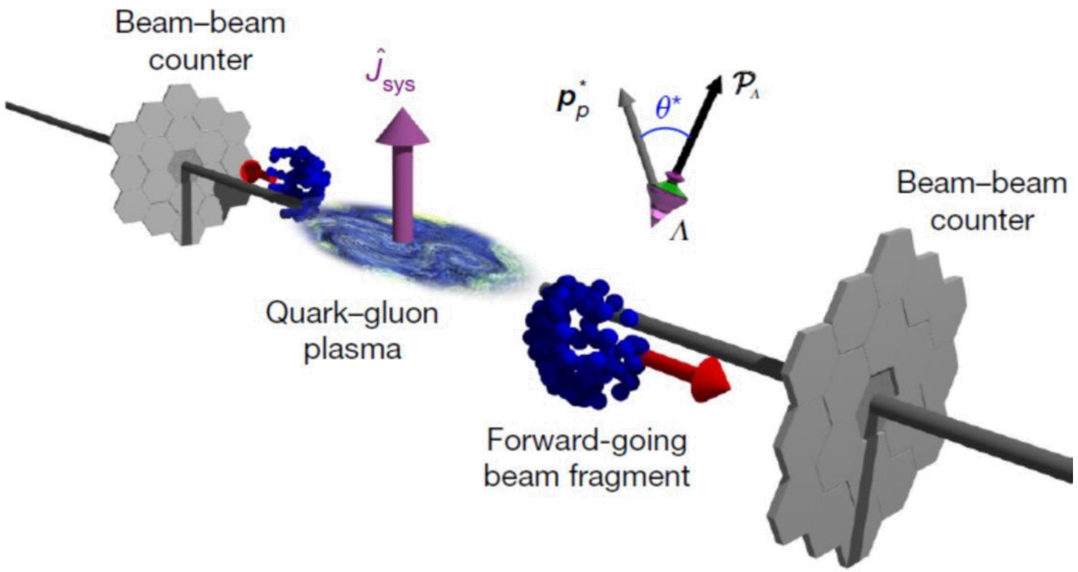
Outline

- **Introduction**
- **Entropy analysis for shear-induced polarization**
  - ✓ Entropy in classical kinetic theory
  - ✓ Extension to quantum kinetic theory
  - ✓ Zubarev's approaches
- **Summary**

Introduction

Relativistic heavy-ion collisions



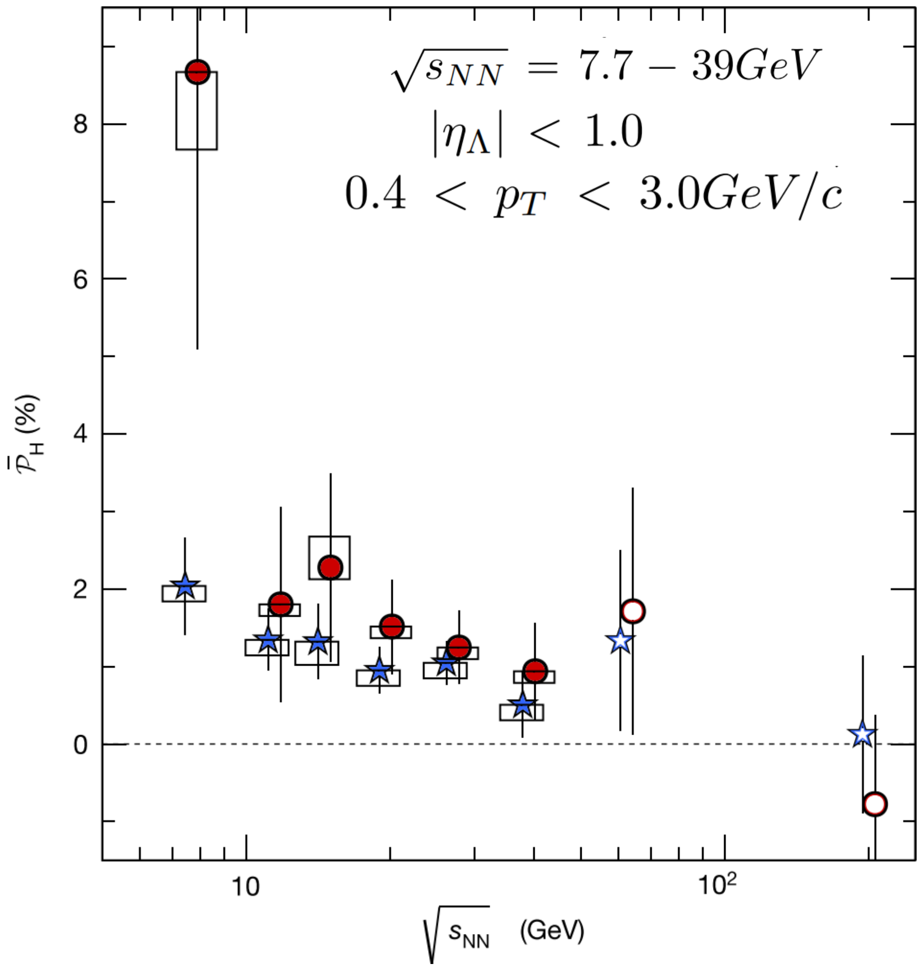


Hannah Petersen, Nature 548, 34–35 (2017)

- **QCD matter formed in HIC at finite impact parameter carries a large orbital angular momentum**
- **Final state products could be globally polarized through spin-orbit coupling.**

Zuo-Tang Liang, Xin-Nian Wang, PRL 94, 102301 (2005)

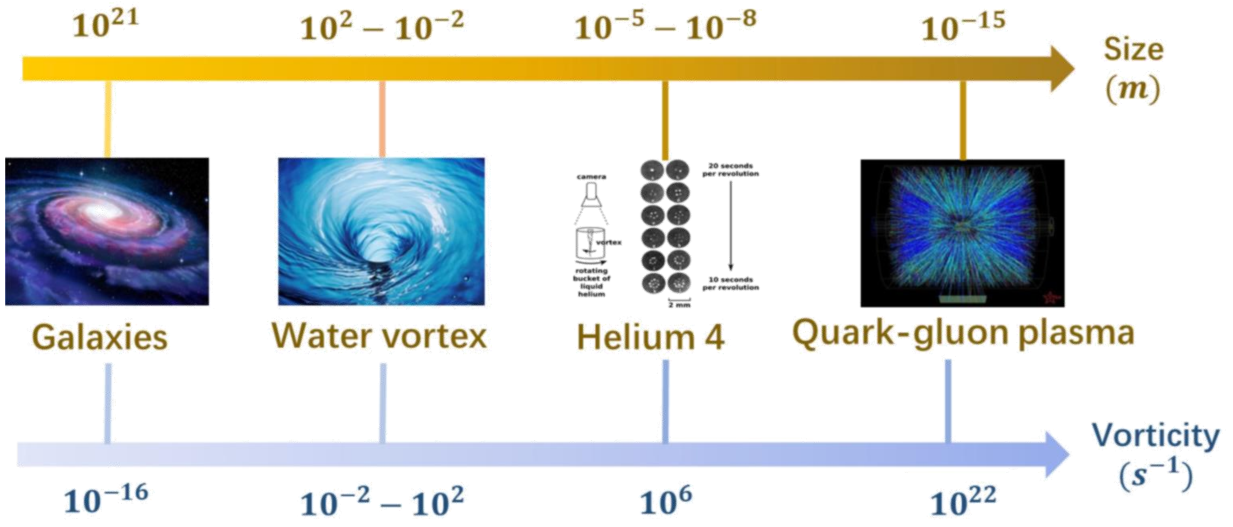
Global polarization of Λ hyperons



STAR, Nature 548, 62 (2017)

- **Vorticity estimating from the polarization**

$$\omega \approx \frac{1}{\hbar} k_B T \left(\bar{P}_\Lambda + \bar{P}_{\bar{\Lambda}}\right) \approx (9 \pm 1) \times 10^{21} s^{-1}$$

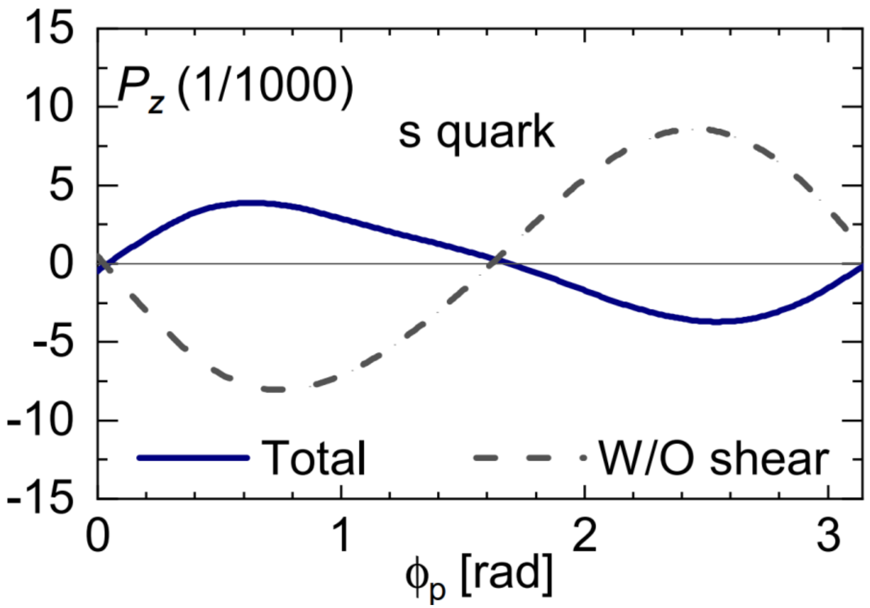


- **QGP is most vortical fluid so far.**

Becattini, et. al, PRC95, 054902 (2017)

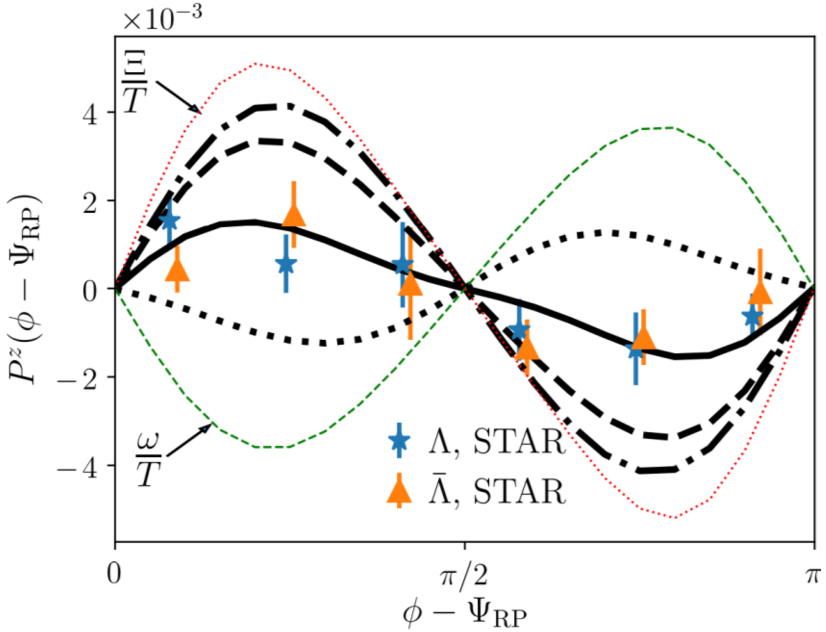
Shear induced polarization in local polarization

Shear-induced polarization plays a crucial role in understanding local spin polarization.



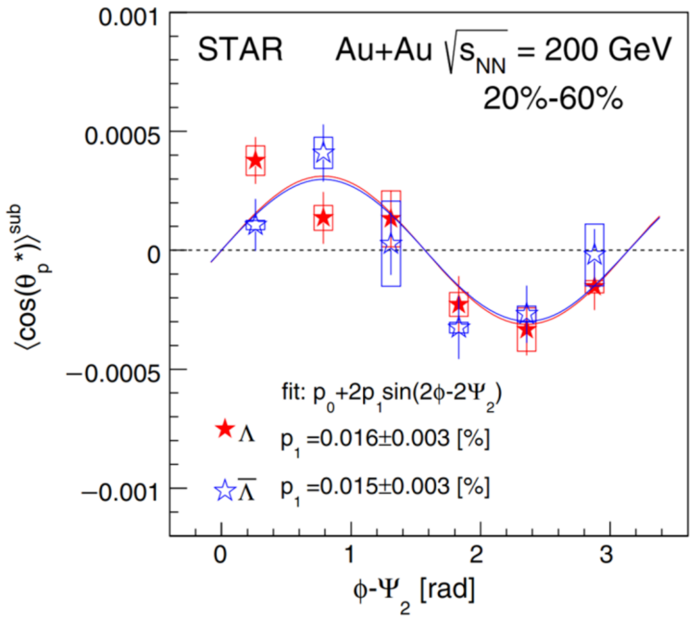
Thermal + Shear, s quark equilibrium

Fu, Liu, Pang, Song, Yin, PRL 2021



Thermal + Shear, isothermal equilibrium

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, Phys. Rev. Lett. 127, 272302



Experimental data

STAR, Phys. Rev. Lett. 123, 132301

Puzzle: is shear induced polarization dissipative?

- **Commonly used: T-odd/even ⟷ dissipative/non-dissipative**

Example: Ohm law

$$\boldsymbol{j} = \sigma \boldsymbol{E} \xrightarrow{\text{Time reversal}} \boldsymbol{j} \to -\boldsymbol{j} \quad \boldsymbol{E} \to \boldsymbol{E}$$

σ: T-odd → dissipative

- **Shear-induced polarization**

$$s^i \sim C^{ijk} \partial_{\langle j} u_{k\rangle} \xrightarrow{\text{Time reversal}} s^i \to -s^i \quad \partial_{\langle j} u_{k\rangle} \to -\partial_{\langle j} u_{k\rangle}$$

C: T-even → non-dissipative?

↕ ?

- **But, shear viscous tensor in hydrodynamics is dissipative!**

Our strategy

- **Our strategy:** using H theorem and entropy production rate

Classical kinetic theory:

H theorem,

$$\frac{dH}{dt} \geq 0$$

Entropy production rate

$$\partial_\mu s^\mu \geq 0$$

Well-established

→

Quantum kinetic theory:

H theorem,

?

Entropy production rate

?

Unknown

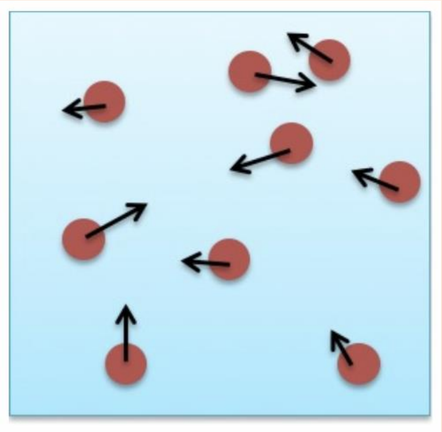
Revisit classical kinetic theory

Relativistic kinetic equation

➢ **Relativistic Boltzmann equation:**

$$\frac{d}{dt} f_p \equiv \frac{\partial}{\partial t} f_p + \dot{\boldsymbol{x}} \cdot \boldsymbol{\nabla}_x f_p + \dot{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_p f_p = C[f_p]$$

distribution function



Distribution function:
Probability to find particles in a small volume of phase space

Assumptions:
Mean free path >> collision length scaling

Particle's velocity:

$$\dot{\boldsymbol{x}} = \boldsymbol{\nabla}_p \varepsilon$$

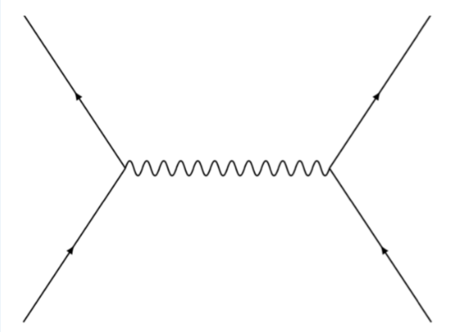
ε : **Particle's energy**

Effective force:

$$\dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

Collision term

Describe the interaction between particles



Boltzmann H theorem in classical kinetic theory

➢ **Conventional Boltzmann's H function**

$$H[f_p] = \int \frac{d^3p}{(2\pi)^3} \mathcal{H}[f_p]$$

For fermions: $\mathcal{H}[f] = -f \ln f - (1-f)\ln(1-f)$

H function is the entropy density well-known in statistic physics.

H-theorem $$\frac{dH}{dt} \geq 0$$

Second law of thermodynamics in the theoretical framework of classical kinetic theory

Entropy current in a relativistic system

Velocity × distribution function

Conserved current

$$j^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f_p$$

Entropy current

$$s^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \mathcal{H}[f_p]$$

Velocity × H function

G. S. Rocha, D. Wagner, G. S. Denicol, J. Noronha, and D. H. Rischke, Entropy 26, 189 (2024)

Test the entropy current (I)

- **It is straightforward to find that, up to $O(\partial)$**

$$s^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu}{T}j^\mu$$

Current:

$$j^\mu = \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu f_p$$

Energy-momentum:

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu p^\nu f_p$$

✓ **It agrees with the results from relativistic hydrodynamics.**

✓ **It is a relativistic extension of thermodynamic relation.** $s = (P + \varepsilon - \mu n)/T$

W. Israel and J. M. Stewart, Annals Phys. 118, 341 (1979)

Test the entropy current (II)

- **One can reproduce the second law of thermodynamics by using relativistic entropy current in classical kinetic theory.**

$$\partial_\mu s^\mu = \frac{1}{4}\int \frac{d^3p}{2E_p} d\Gamma_{kk'\to pp'} f_k f_{k'}$$

$$(1-f_p)(1-f_{p'})\left[(1-X)\ln X\right] \geq 0$$

$$X = \frac{f_p f_{p'}(1-f_k)(1-f_{k'})}{(1-f_p)(1-f_{p'})f_k f_{k'}}$$

Shear viscous tensor in classical kinetic theory

- **Consider a perturbation around the equilibrium**

$$f_p = f_p^{(0)} + \delta f_p$$

Shear viscous tensor

$$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle} = \int \frac{d^3p}{(2\pi)^3 E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p$$

- **Inserting it to entropy current yields,**

$$\partial_\mu s^\mu_{(1)} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \delta f_p \partial_\mu \ln\left(\frac{1 - f_p^{(0)}}{f_p^{(0)}}\right)$$

$$= \frac{\partial_{\langle\mu} u_{\nu\rangle}}{T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p \geq 0$$

shear viscous tensor

The shear viscous tensor contributes to an increase in entropy → dissipative

Brief summary in classical kinetic theory

Step 1: Construct entropy current

Conserved current
$$j^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f_p$$

Entropy current
$$s^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} \mathcal{H}[f_p]$$

Step 2: Test two physical conditions:

(a) Is it satisfied the relativistic hydrodynamics?

$$s^\mu = \frac{P}{T} u^\mu + \frac{1}{T} u_\nu T^{\mu\nu} - \frac{\mu}{T} j^\mu$$

(b) Is entropy production rate positive-define?

$$\partial_\mu s^\mu \geq 0$$

Step 3: Consider perturbation related to shear tensor and compute the entropy production rate

shear viscous tensor

$$\partial_\mu s^\mu_{(1)} = \frac{\partial_{\langle\mu} u_{\nu\rangle}}{T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} p^{\langle\mu} p^{\nu\rangle} \delta f_p \geq 0$$

Extension to quantum kinetic theory

Which kinds of quantum kinetic theory we adopted?

$$s^\mu = \frac{P}{T}u^\mu + \frac{1}{T}u_\nu T^{\mu\nu} - \frac{\mu}{T}j^\mu$$

- ✓ Chiral kinetic theory (massless fermions, right handed fermions only)

  Polarization vector in phase space → Wigner function for right handed fermions
  → Current after momentum integration
  will contribute to entropy production rate directly

- Quantum kinetic theory for massive fermions

  Polarization vector in phase space → Axial Wigner function in phase space
  → NOT a conserved current after momentum integration
  will NOT contribute to entropy production rate directly

What is chiral kinetic theory?

Chiral kinetic theory

- **Boltzmann equation** ⟷

$$\frac{\partial}{\partial t} f_p + \dot{\boldsymbol{x}} \cdot \boldsymbol{\nabla}_x f_p + \dot{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_p f_p = C\left[f_p\right]$$

- **Massless fermions**
- **Quantum corrections** ⟷

$$\begin{aligned} \varepsilon &= |\boldsymbol{p}| + \hbar \ldots \\ \dot{\boldsymbol{x}} &= \boldsymbol{\nabla}_p \varepsilon + \hbar \ldots \\ \dot{\boldsymbol{p}} &= \boldsymbol{E} + (\boldsymbol{\nabla}_p \varepsilon) \times \boldsymbol{B} + \hbar \ldots \end{aligned}$$

Chiral kinetic equation in relaxation-time approximation

- **Chiral kinetic equation**

$$\left[p \cdot \partial + \hbar \left(\partial_\mu S^{\mu\nu}_{(n)}\right) \partial_\nu\right] f^{(n)}_p = \bar{\mathcal{C}}\left[f^{(n)}_p\right] \qquad S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta}{2\,(p \cdot n)}$$

relaxation-time approximation

$$\bar{\mathcal{C}}\left[f^{(n)}_p\right] = -\frac{1}{\tau_R}\left(p \cdot u + \hbar \frac{p \cdot \mathcal{R}}{(p \cdot u)^2}\right) \delta f_p$$

- **Power counting:**

$$\bar{\mathcal{C}} \sim \mathcal{O}\left(\partial^1\right), \ \delta f_p \sim \mathcal{O}\left(\partial^1\right) \to \mathcal{R}^\mu \sim \mathcal{O}(1)$$

- **Parameterization:**

$$\mathcal{R}^\mu = c_1 u^\mu + c_2 \bar{p}^\mu + c_3 \bar{R}^\mu \qquad \bar{p}^\mu = p_\nu \Theta^{\mu\nu}, \ \bar{R}^\mu = \left(\Theta^{\mu\nu} - \frac{\bar{p}^\mu \bar{p}^\nu}{\bar{p}^2}\right) R_\nu \qquad c_{i=1,2,3} \sim \mathcal{O}(1)$$

Entropy flow in chiral kinetic theory

- **Entropy current**: replace the f in current by H function

For simplicity, we neglect the electromagnetic fields.

$$J^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left\{\left(p^{\mu}+\hbar S^{\mu\nu}_{(u)}\partial_{\nu}\right)f^{(u)}_p-\hbar S^{\mu\nu}_{(u)}\mathcal{C}_{\nu}\left(f^{(u)}_p\right)\right\}$$

$$s^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left\{\left(p^{\mu}+\hbar S^{\mu\nu}_{(u)}\partial_{\nu}\right)\mathcal{H}\left[f^{(u)}_p\right]-\hbar S^{\mu\nu}_{(u)}\mathcal{C}_{\nu}\left(\mathcal{H}\left[f^{(u)}_p\right]\right)\right\}$$

collision kernel

Also see early discussion in Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

H theorem in the chiral kinetic theory

- **Using the similar relaxation-time approach to the collision kernel in entropy, we compute the entropy production rate as:**

$$\partial_\mu s^\mu = 2\int \frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\ln\left(\frac{1-f_p^{(u)}}{f_p^{(u)}}\right)\bar{\mathcal{C}}\left[f_p^{(u)}\right]$$

$$\sim \frac{2\hbar}{\tau_R}\int \frac{d^4p}{(2\pi)^3}\frac{\bar{\epsilon}_{(u)}}{p\cdot u}\delta\left(p^2\right) c_1 \frac{\left(\delta f_p\right)^2}{f_p^{eq}\left(1-f_p^{eq}\right)} + \text{classical terms}$$

$$c_1 \geq 0 \quad \longrightarrow \quad \partial_\mu s^\mu \geq 0$$

physical condition

Brief summary in chiral kinetic theory

✓ **Step 1: Construct entropy current**

Conserved current → **Entropy current**

$$j_R^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^{\mu}+\hbar S_{(u)}^{\mu\nu}\mathcal{D}_{\nu}\right)f_p^{(u)} \quad \Rightarrow \quad s_R^{\mu}=2\int\frac{d^4p}{(2\pi)^3}\bar{\epsilon}_{(u)}\delta\left(p^2\right)\left(p^{\mu}+\hbar S_{(u)}^{\mu\nu}\mathcal{D}_{\nu}\right)\mathcal{H}\left[f_p^{(u)}\right]$$

✓ **Step 2: Test physical condition: Is entropy production rate positive-define?**

$$\partial_{\mu}s^{\mu}\sim\frac{2\hbar}{\tau_R}\int\frac{d^4p}{(2\pi)^3}\frac{\bar{\epsilon}_{(u)}}{p\cdot u}\delta\left(p^2\right)c_1\frac{\left(\delta f_p\right)^2}{f_p^{eq}\left(1-f_p^{eq}\right)} \qquad c_1\geq 0 \quad \Rightarrow \quad \partial_{\mu}s^{\mu}\geq 0$$

Step 3: Compute the entropy production rate related to the shear-induced polarization

Anomalous Hall effect

➢ **Interaction corrections to the axial current:**

$$\delta S_R^{<,\mu} = 2\pi p^\mu \delta\left(p^2\right) \frac{\tau_R}{u\cdot p} f_p^{(0)}\left(1-f_p^{(0)}\right)\beta p^\rho p^\sigma \partial_{\langle\rho}u_{\sigma\rangle}$$ → **Classical term**

$$-\frac{4\pi\hbar}{u\cdot p}\frac{\tau_R}{\tau_R'}\delta\left(p^2\right) S_{(u)}^{\mu\nu} f_p^{(0)}\left(1-f_p^{(0)}\right) c_3 \beta p^\sigma \partial_{\langle\nu}u_{\sigma\rangle}$$

Anomalous Hall effect ←

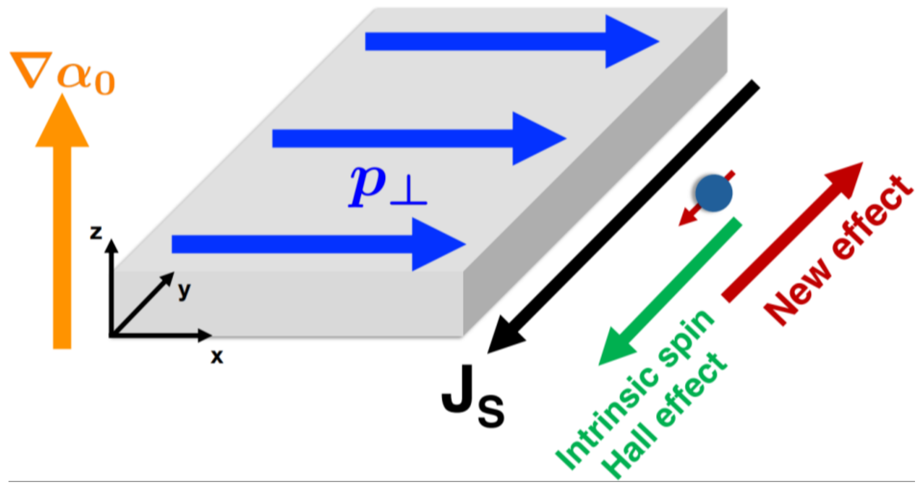


Figure from C.Yi, et al,,arXiv:2509.00377

Also see the results from QED in HTL:
Shuo Fang, Shi Pu, Phys. Rev. D 111, 034015

- **Coupling constant independence originated from the "side jump" effect, which is known in condensed matter physics.**

Naoto Nagaosa, et al, Rev. Mod. Phys. 82, 1539

Dilemma in entropy production rate

- **Entropy production rate:**

$$\partial_\mu s^\mu = \frac{4}{15}\tau_R\beta^2 \partial_{\langle\mu}u_{\nu\rangle}\partial^{\langle\mu}u^{\nu\rangle}\int \frac{d^4p}{(2\pi)^3}\delta\left(p^2\right)(p\cdot u)^3 f_p^{(0)}\left(1-f_p^{(0)}\right)\geq 0$$

Dilemma:

➢ **The shear-induced polarization and anomalous Hall effect do not contribute to entropy production, suggesting they are non-dissipative.**

↕

➢ **Total entropy increases due to the classical shear viscous tensor. Dissipative?**

Linear response theory in Zubarev's approach

- **Consider an operator O(x) in a system out of global equilibrium (GE),**

$$O(x) - \left\langle \hat{O}(x) \right\rangle_{GE} \longrightarrow \text{non-dissipative}$$

$$= \lim_{k\to 0} \mathrm{Im} \frac{i}{\beta(x) k_0} \int_{t_0}^{t} d^4x' e^{-ik\cdot(x'-x)}$$

$$\times \left\langle \left[\hat{O}(x), \hat{j}^{\mu}(x')\right] \partial_\mu \alpha(x) - \left[\hat{O}(x), \hat{T}^{\mu\nu}(x')\right] \partial_\mu \beta_\nu(x) \right\rangle$$

$$+\mathcal{O}\left(\partial^2\right)$$

dissipative

Textbook: Zubarev, Nonequilibrium statistical thermodynamics, 1973

Review: F. Becattini, M. Buzzegoli, and E. Grossi, Particles 2, 197 (2019)

Polarization out of GE in the Zubarev's approaches

- **We derive the shear induced polarization in Zubarev's approaches**

$$\delta\mathcal{A}^{<,\mu}_{\mathrm{LE},\xi}(q,X) = -q^{\beta}\xi_{\alpha\beta}(X) 2\pi\delta(q^2-m^2)\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{2|q_0|} f^{(0)}_q(1-f^{(0)}_q)$$

Thermal shear tensor $\xi_{\alpha\beta} = \frac{1}{2}\partial_{(\alpha}\beta_{\beta)}$

$$\delta\mathcal{A}^{<,\mu}_{\mathrm{LE},\alpha}(q,X) = (\partial_\nu\alpha) 2\pi\delta\left(q^2-m^2\right)\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{2\left|q_0\right|} f^{(0)}_p\left(1-f^{(0)}_p\right)$$

chemical potential gradient

- **Coming from off-equilibrium corrections, the polarization induced by the shear tensor and chemical potential gradient could be dissipative.**

Summary

- **We introduce entropy flow and the H-theorem in the chiral kinetic theory.**

- **Dilemma: Shear induced polarization does not contribute to the entropy production rate but the total entropy increases.**

- **Linear response theory in Zubarev's approaches: shear-induced polarization can be dissipative.**

Thank you for your listening!

Any comments or suggestions are welcome!

Backup

H theorem in classical kinetic theory

- **Positive entropy production rate:**

$$\partial_\mu s^\mu = \frac{1}{4}\int \frac{d^3p}{2E_p} d\Gamma_{kk'\to pp'} f_k f_{k'}$$

$$(1-f_p)(1-f_{p'})\left[(1-X)\ln X\right] \geq 0$$

$$X = \frac{f_p f_{p'}(1-f_k)(1-f_{k'})}{(1-f_p)(1-f_{p'})f_k f_{k'}}$$

Calculation of the entropy production rate

$$\delta f_p = f_p^{(0)}(1 - f_p^{(0)})\chi_p$$

- **Insert into** $\frac{d}{dt} f_p^{(0)} = C[\delta f_p]$

$$\chi_p = A(\mathbf{p}) p^i p^j \partial_{\langle i} u_{j\rangle} = A(\mathbf{p}) p^i p^j \frac{\pi_{ij}}{2\eta}$$

With the constraint:

$$f_p^{(0)}(1 - f_p^{(0)}) p^{\langle i} p^{j\rangle} = \frac{T}{2E_p} \int d\Gamma_{kk' \to pp'} f_k^{(0)} f_{k'}^{(0)} f_p^{(0)} f_{p'}^{(0)} \left[B^{ij}(\mathbf{p}) + B^{ij}(\mathbf{p}') - B^{ij}(\mathbf{k}) - B^{ij}(\mathbf{k}')\right]$$

$$B^{ij}(\mathbf{k}) = A(\mathbf{k}) k^{\langle i} k^{j\rangle}$$

Calculation of the entropy production rate

➢ **The correction to entropy flow:**

$$s^{\mu}_{(1)} = \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}}{E_p} \ln\left(\frac{1 - f_p^{(0)}}{f_p^{(0)}}\right) \delta f_p$$

$$\begin{aligned}
\partial_{\mu} s^{\mu}_{(1)} &= \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}}{E_p} \delta f_p \partial_{\mu} \ln\left(\frac{1 - f_p^{(0)}}{f_p^{(0)}}\right) \\
&= \frac{\partial_{\mu} u_{\nu}}{T} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} p^{\mu} p^{\nu} \delta f_p = \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta T} \geq 0
\end{aligned}$$

Review on the chiral kinetic theory

➢ **Kinetic equation for distribution function:**

on-shell condition

$$\delta\left(p^2-\hbar\frac{B\cdot p}{p\cdot u}\right)\left[p\cdot\Delta+\hbar\frac{S^{\mu\nu}_{(u)}E_\mu}{p\cdot u}\Delta_\nu+\hbar S^{\mu\nu}_{(u)}\left(\partial_\mu F_{\rho\nu}\right)\partial^\rho_p+\hbar\left(\partial_\mu S^{\mu\nu}_{(u)}\right)\Delta_\nu\right]f^{(u)}_p=\delta\left(p^2-\hbar\frac{B\cdot p}{p\cdot u}\right)\bar{\mathcal{C}}$$

collision term

Yoshimasa Hidaka et al. Phys. Rev. D 95, 091901 (2017)

Yoshimasa Hidaka et al. Phys. Rev. D 97, 016004 (2018)

Review on the chiral kinetic theory

- **The canonical energy momentum tensor and current:**

$$T^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4} \left(p^\mu \grave{S}^{<\nu} + p^\nu \grave{S}^{<\mu} \right) = u^\mu u^\nu \epsilon - P\Theta^{\mu\nu} + \Pi^{\mu\nu}_{non} + \Pi^{\mu\nu}_{dis}$$

$$J^\mu = 2\int \frac{d^4p}{(2\pi)^4} \grave{S}^{<\mu} = nu^\mu + \nu^\mu_{non} + \nu^\mu_{dis},$$

Yoshimasa Hidaka et al. Phys. Rev. D 97, 016004 (2018)

- **The conservation equations with chiral anomaly:**

$$\partial_\mu J^\mu = -\frac{\hbar}{4\pi^2} E_\mu B^\mu, \; \partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)

Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

Entropy flow in chiral kinetic theory

- Consistent with the entropy flow in the chiral anomalous hydrodynamics:

$$s^{\mu}_{R,leq} = \beta\left(u^{\mu}P + T^{\mu\nu}_{leq}u_{\nu} - \mu_R J^{\mu}_{leq} + \xi_B B^{\mu} + 2\xi_{\omega}\omega^{\mu}\right)$$

- No entropy production in local equilibrium

$$\partial_{\mu}s^{\mu}_{R,leq} = 0$$

D. T. Son and P. Surowka, Phys. Rev. Lett. 103, 191601 (2009)

Di-Lun Yang, Phys. Rev. D 98, 076019 (2018)

Relaxation time approach

$$\mathcal{C}_\nu[\mathcal{H}(f_p^{(n)})] = \mathcal{C}_\nu^{leq} + \mathcal{C}_\nu^{\delta}$$

$$\mathcal{C}_\nu^{leq} = (b_1 u_\nu + b_2 p_\nu + b_3 \partial_\nu^p) f_p^{eq} + b_{4,\nu} f_p^{(0)}$$

$$\mathcal{C}_\nu^{\delta} = \frac{1}{\tau_R''}(a_1 u_\nu + a_2 p_\nu + a_3 \partial_\nu^p)\delta f_p$$

$$f_p^{eq} = \frac{1}{e^g + 1}, \quad g = \beta p \cdot u - \beta \mu_R + \frac{\hbar S_{(u)}^{\mu\nu}}{2} \partial_\mu (\beta u_\nu)$$

$$f_p^{(0)} = \frac{1}{\exp(\beta p \cdot u - \beta \mu_R) + 1}$$

- $\mathcal{C}_\nu[\mathcal{H}(f_p^{(n)})]$ **is linear to** $f_p^{(n)}$;
- $s^\mu \sim \mathcal{O}(\partial^1) \rightarrow b_i \sim \mathcal{O}(\partial^0),\ b_{4,\nu} \sim \mathcal{O}(\partial^1)$; $a_i \sim \mathcal{O}(\partial^0)$;
- **After momentum integration, the terms related to collision vanish.**

Relaxation time approach

➢ **Insert δf_p to the axial current:**

$$\delta f_p = \frac{\tau_R}{u\cdot p} f_p^{(0)}(1-f_p^{(0)})\beta p^\mu p^\nu \partial_{\langle\mu} u_{\nu\rangle} + \ldots.$$

$$\delta S_R^{<,\mu} = 2\pi p^\mu \delta\left(p^2\right)\delta f_p + 2\pi\hbar\delta\left(p^2\right) S_{(u)}^{\mu\alpha}\left(\partial_\alpha \delta f_p - \mathcal{C}_\alpha\left[\delta f_p\right]\right)$$

$$\mathcal{C}_\nu[\delta f_p] = \frac{1}{\tau_R'}(d_1 u_\nu + d_2 p_\nu + d_3 \partial_\nu^p)\delta f_p$$

Entropy production rate

- **Entropy flow related to the shear tensor**

$$\delta s^{\mu} = \beta u_{\nu} \delta T^{\mu\nu} - \beta \mu_R \delta J^{\mu}$$

$$\delta T^{\mu\nu} = \frac{4}{15} \tau_R \beta \partial^{\langle\mu} u^{\nu\rangle} \int \frac{d^4 p}{(2\pi)^3} \delta\left(p^2\right) (p \cdot u)^3 f_p^{(0)} \left(1 - f_p^{(0)}\right)$$

$$\delta J^{\mu} = 0$$

shear induced polarization vanish after momentum integration

The linear response theory in the Zubarev's approaches

- **The non-equilibrium density operator**

$$\hat{\rho} = \hat{\rho}_{\mathrm{GE}} + \delta\hat{\rho}_{\mathrm{GE}}$$

equilibrium density operator

$$\hat{\rho}_{\mathrm{GE}} = \frac{1}{Z}\exp\left[-\beta_\nu \hat{P}^\nu + \alpha\hat{Q} + \frac{1}{2}\hat{\mathcal{J}}^{\mu\nu}\varpi_{\mu\nu}\right]$$

corrections

$$\delta\hat{\rho}_{\mathrm{GE}}(\tau) = \lim_{\varepsilon\to 0}\int_{-\infty}^{\tau} \mathrm{d}\tau' e^{\varepsilon(\tau'-\tau)} \int_{\Sigma_{\tau'}} \mathrm{d}\Sigma_\lambda(x) n^\lambda \left[\int_0^1 \mathrm{d}z \hat{\rho}_{\mathrm{GE}}^{z} \left(\hat{T}^{\nu\mu}(x)\partial_\nu\beta_\mu(x) - \hat{j}^\nu(x)\partial_\nu\alpha(x)\right)\hat{\rho}_{\mathrm{GE}}^{1-z} - \left(\langle\hat{T}^{\nu\mu}(x)\rangle_{\mathrm{GE}}\partial_\nu\beta_\mu(x) - \langle\hat{j}^\nu(x)\rangle_{\mathrm{GE}}\partial_\nu\alpha(x)\right)\hat{\rho}_{\mathrm{GE}}\right]$$

F. Becattini, M. Buzzegoli, and E. Grossi, Particles 2, 197 (2019)