Holographic spin alignment of vector mesons in heavy ion collisions

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Based on: JHEP 08 (2024) 070 ; PRD 110 (2024) 5, 056047

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Experiment results

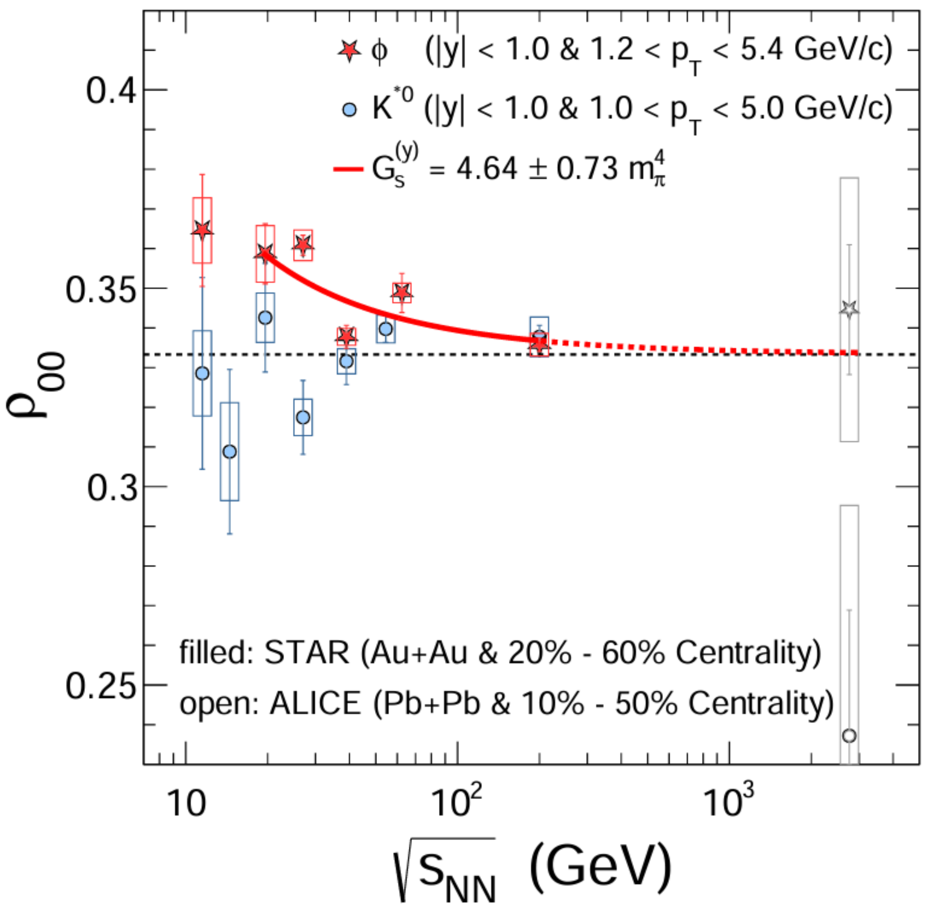
STAR, Nature 614 (7947) (2023) 244–248.

Global spin alignment of ϕ and K_0^* mesons in Au-Au collisions.

ϕ Meson

$\rho_{00} > 1/3$

$$\lambda_\theta = \frac{3\rho_{00} - 1}{1 - \rho_{00}}$$



- ϕ mesons are preferably to be longitudinally polarized especially for low collision energy.
- The spin alignment of a meson K_0^* mesons are almost zero across all beam energy scans.
- The spin alignment is strongly dependent on the vector meson species.

Experiment results

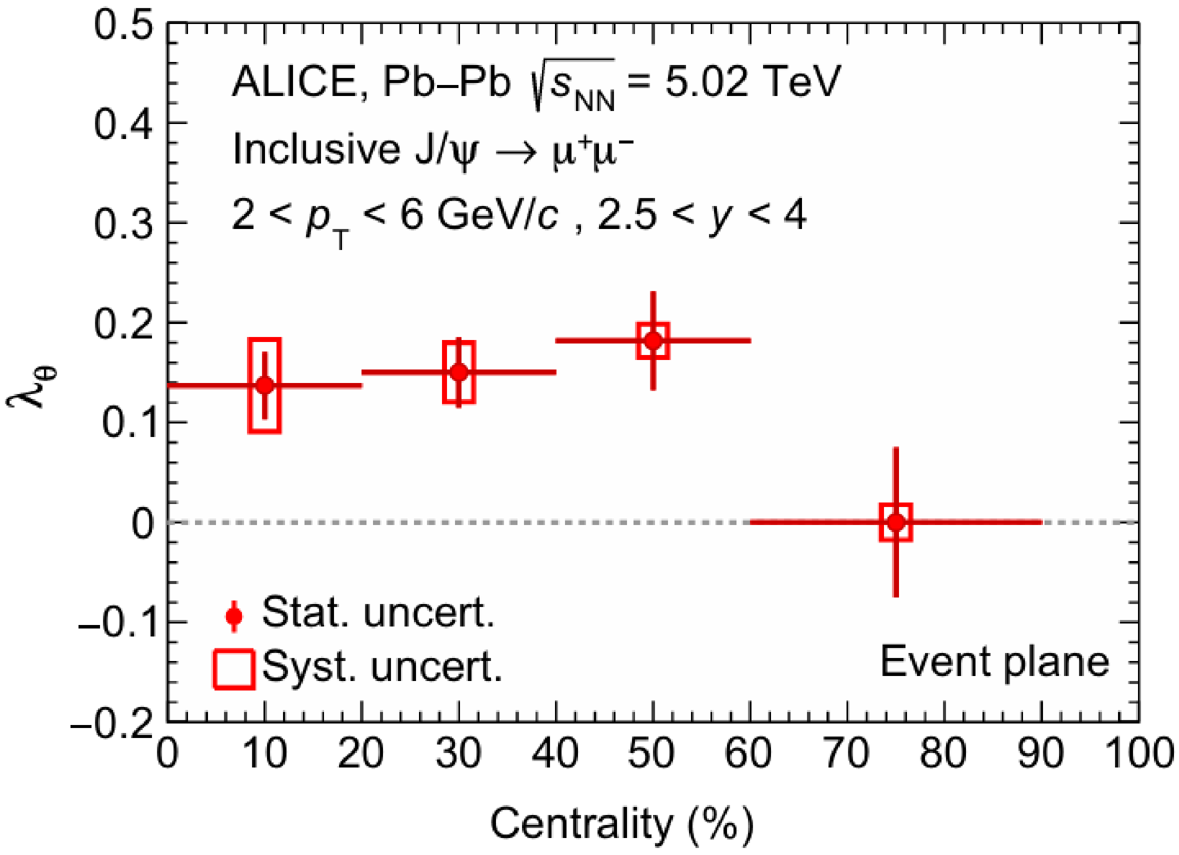
ALICE, Phys. Rev. Lett. 131 (4) (2023) 042303.

Global spin alignment of J/ψ vector mesons in Pb-Pb collisions.

***J/ψ* Meson**

$$\rho_{00} < 1/3$$

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}}$$



J/ψ **are preferably to be transversely polarized.**

Theoretical approaches

➢ Coalescence model with spin

- Quark/antiquark polarized by external fields
- Non-equilibrium process described by kinetic theory

➢ Spin kinetic equation and linear response theory

➢ Spectral function method

In this talk ! ! !

- Splitting between spectral functions of longitudinal and transverse modes due to external fields or motion relative to a thermal background, calculated by QFT, NJL model, holographic model...
- Meson at thermodynamical equilibrium

Theoretical approaches

Smaller deviation from 1/3 — Larger deviation from 1/3 ?

$$\rho_{00} \approx \frac{1}{3} + c_{hadro} + c_{EM} + c_F + c_A + c_h + c_{strong} + \cdots$$

- c_{strong}: Anisotropic strong force [4,15,18-21]
- c_h: Helicity Polarization[17]
- c_A: Anomalous color field [16]
- c_F: Fragmentation [1]
- c_{EM}: Electromagnetic fields [3,4,14,15]
- c_{hadro}: Hydrodynamic gradient (vorticity; Acceleraction; shear tensor; second order;) [1-13,22]

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2. F. Becattini, L. Csernai, D.-J. Wang, PRC 88, 034905 (2013)
3. Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)
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17. J.-H. Gao, PRD 104, 076016 (2021)
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22. E. Grossi, A. Palermo, I. Zahed, e-Print: 2507.19228.

Outline

- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- Magnetic field induced spin alignment
- Summary

Spin alignment

- S-matrix element($J/\psi \to l+\bar{l}$ and $\phi \to l+\bar{l}.$):

C. Gale and J. I. Kapusta, Nucl. Phys. B 357 (1991) 65–89

$$S_{fi} = \int d^4x d^4y \langle f, l\bar{l} | J_\mu(y) G_R^{\mu\nu}(x-y) J_\nu^l(x) | i \rangle$$

where

J_μ is the current that couples to V.M. ;

J_ν^l is the leptonic current

$$J_\nu^l(x) = g_{Ml\bar{l}} \bar{\psi}_l(x) \Gamma_\nu \psi_l(x)$$

- Propagators (vacuum): Coordinate space

$$G_{R/A}^{\mu\nu} = -\frac{\eta^{\mu\nu} + p^\mu p^\nu / p^2}{p^2 + m_V^2 \pm i m_V \Gamma}$$

- Retarded current-current correlation:

$$D^{\mu\nu}(x,p) \equiv \int d^4y \theta(y^0) \langle [J^\mu(y), J^\nu(0)] \rangle_{T(x)} e^{-ip\cdot y}$$

- Spectral function:

$$\varrho_{\alpha\beta}(x,p) \equiv -\mathrm{Im} D_{\alpha\beta}(x,p)$$

- Differential production rate:

$$n(x,p) = -\frac{2g_{Ml\bar{l}}^2}{3(2\pi)^5}\left(1-\frac{2m_l^2}{p^2}\right)\sqrt{1+\frac{4m_l^2}{p^2}}\, p^2 n_B(x,\omega) \times \left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) G_A^{\mu\alpha}(p) \varrho_{\alpha\beta}(x,p) G_R^{\beta\nu}(p),$$

Y. Burnier, M. Laine, M. Vepsalainen,, JHEP 02 (2009) 008.

L. D. McLerran, T. Toimela, Phys. Rev. D 31 (1985) 545.

H. A. Weldon, Phys. Rev. D 42 (1990) 2384–2387.

Spin alignment

- Spectral function:

$$\varrho^{\mu\nu}(x,p) = \sum_{\lambda,\lambda'=0,\pm1} v^{\mu}(\lambda,p) v^{*\nu}(\lambda',p) \tilde{\varrho}_{\lambda\lambda'}(x,p)$$

- Polarization vectors:

$$v^{\mu}(\lambda,p) = \left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M}, \boldsymbol{\epsilon}_{\lambda} + \frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M(\omega+M)}\mathbf{p} \right)$$

Orthonormality conditions:

$$\eta_{\mu\nu} v^{\mu}(\lambda,p) v^{*\nu}(\lambda',p) = \delta_{\lambda\lambda'}$$

Completeness condition:

$$\sum_{\lambda} v^{\mu}(\lambda,p) v^{*\nu}(\lambda,p) = (\eta^{\mu\nu} + p^{\mu}p^{\nu}/p^2)$$

- Dilepton production rate:

$$n_{\lambda}(x,p) = -\frac{2g^2_{Ml\bar{l}}}{3(2\pi)^5}\left(1-\frac{2m_l^2}{p^2}\right) \times \sqrt{1+\frac{4m_l^2}{p^2}}\,\frac{p^2 n_B(x,\omega)\tilde{\varrho}_{\lambda\lambda}(x,p)}{(p^2+m_V^2)^2+m_V^2\Gamma^2}.$$

- Spin alignment:

$$\rho_{00}(x,\mathbf{p}) \equiv \frac{\int d\omega\, n_0(x,p)}{\sum_{\lambda=0,\pm1}\int d\omega\, n_{\lambda}(x,p)}$$

Spin space

Correlation function

- **AdS/CFT dictionary:**

$$Z_{\rm QFT}[A_\mu^{(0)}] = Z_{\rm gravity}[A_\mu]$$

where

$$Z_{\rm QFT}[A_\mu^{(0)}] = \left\langle \exp\left\{ \int_{\partial\mathcal{M}} J^\mu A_\mu^{(0)} d^4x \right\} \right\rangle,$$

$$Z_{\rm gravity}[A_\mu] = \exp\{-S_{\rm bulk}[A_\mu]\}$$

- **Bulk geometry:** $0 \le \zeta \le \zeta_h$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{\zeta\zeta}d\zeta^2$$

- **Two-point correlators**

$$\langle [J^\mu, J^\nu] \rangle = D^{\mu\nu} \propto \frac{\delta^2 Z_{\rm QFT}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}} = \frac{\delta^2 Z_{\rm gravity}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}}$$

- **Bulk mesonic action:**

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843882.
T. Sakai, S. Sugimoto, Prog. Theor. Phys. 114 (2005) 1083–1118.
A. Karch, E. Katz, D. T. Son, M. A. Stephanov, Phys. Rev. D 74(2006) 015005.

$$S_{\rm bulk} = -\int d^4x d\zeta Q(\zeta) F_{MN}F^{MN}$$

- **Equation of motion:** $\partial_M[Q(\zeta)F^{MN}] = 0$ **Radial gauge condition:** $A_\zeta = 0$
- **Fourier transformation:** $A_\mu(x,\zeta) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} A_\mu(p,\zeta)$
- **Electric fields:** $E_i(p,\zeta) \equiv -p_0 A_i(p,\zeta) + p_i A_0(p,\zeta)$
- **Boundary condition:** $\lim_{\zeta\to 0} \tilde{E}_i(j,p,\zeta) = \delta_{ij}$
- **Equation of motion :**

$$\partial_\zeta^2 E_i(p,\zeta) + \frac{[\partial_\zeta Q(\zeta) g^{\zeta\zeta}]}{Q(\zeta) g^{\zeta\zeta}}[\partial_\zeta E_i(p,\zeta)] - \frac{p^2}{g^{\zeta\zeta}} E_i(p,\zeta)$$
$$+ (-p_0 g_{i\mu} + p_i g_{0\mu})(\partial_\zeta g^{\mu\nu})[\partial_\zeta A_\nu(p,\zeta)] = 0,$$

Correlation function

- Correlation function:

D. T. Son and A. O. Starinets, JHEP 09 (2002) 042

$$D^{\mu\nu}(p) = \lim_{\zeta\to 0} g^{\zeta\zeta} g^{\mu\alpha} Q(\zeta) \frac{\delta[\partial_\zeta A_\alpha(p,\zeta)]}{\delta A_\nu(p,\zeta)}\bigg|_{A_\mu(p,0)=0}$$

$$D^{\mu 0}(p) = -\lim_{\zeta\to 0} \frac{p_j}{p_0}\left(g^{\mu k} - \frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(j,p,\zeta),$$

$$D^{\mu i}(p) = \lim_{\zeta\to 0}\left(g^{\mu k} - \frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(i,p,\zeta).$$

- Ward identity:

$$p_\mu D^{\mu\nu} = p_\mu D^{\nu\mu} = 0$$

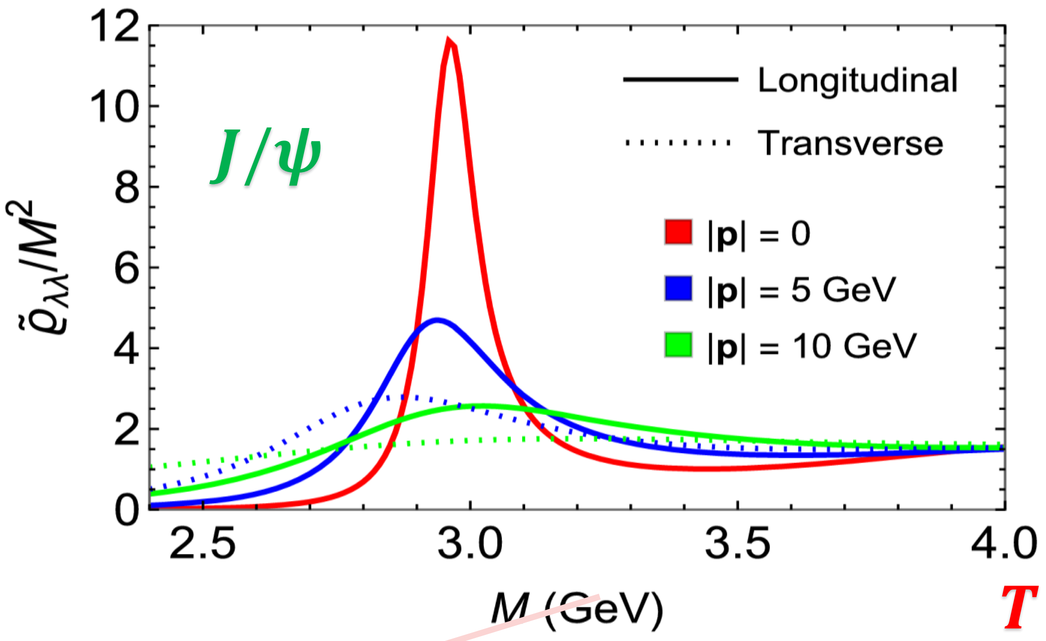
- Spectral function:

$$\tilde{\varrho}_{\lambda\lambda}(p) = v_\mu^*(\lambda,p) v_\nu(\lambda,p) \mathrm{Im} D^{\mu\nu}(p)$$

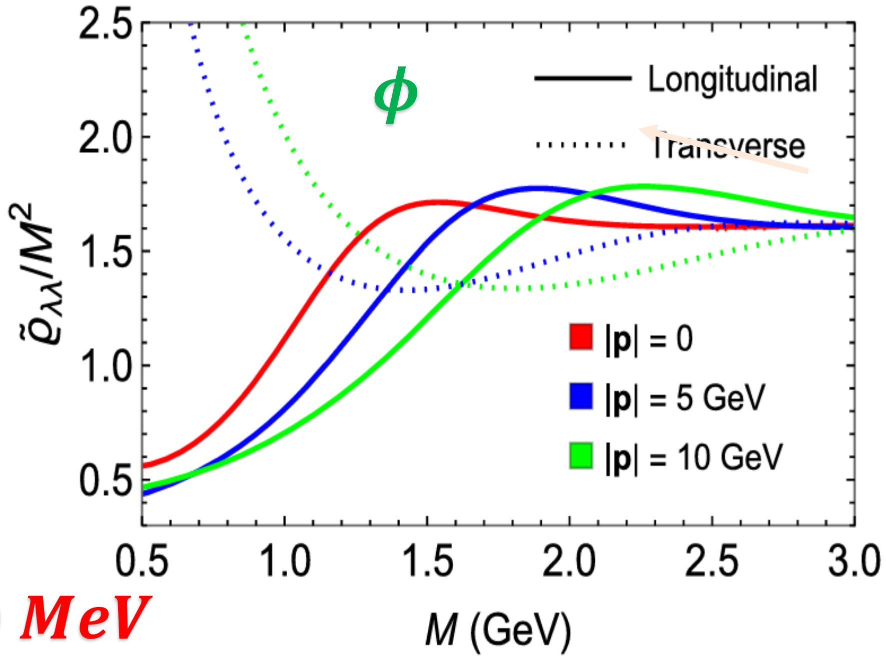
Momentum induced spin alignment

Motion of J/ψ relative to a thermal background breaks symmetry between longitudinally polarized state and transversely polarized state

Spectral function



$T = 150\ MeV$



At zero momentum, spectral functions for all spin states are degenerate because of the rotation symmetry.

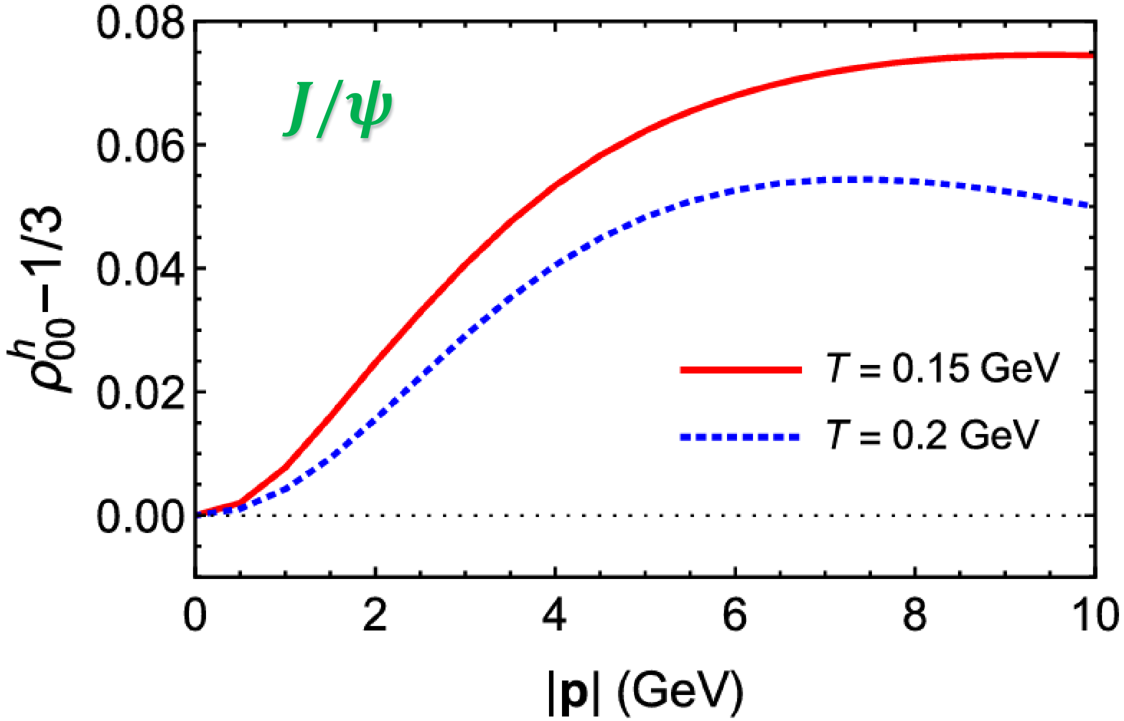
XL Sheng,YQ Zhao SW Li , F Becattini, DF Hou PRD 110 (2024) 056047

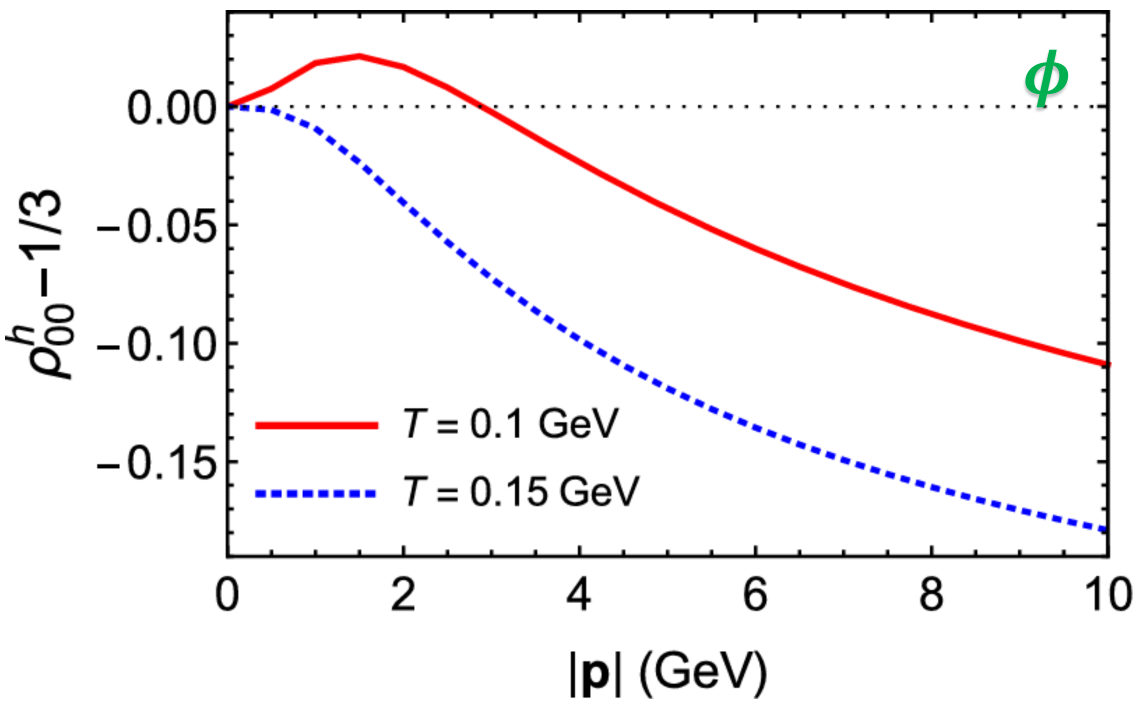
Momentum induced spin alignment

Spin alignment in the helicity frame

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},$$

$$\lambda_\theta = \frac{3\rho_{00}-1}{1-\rho_{00}}$$



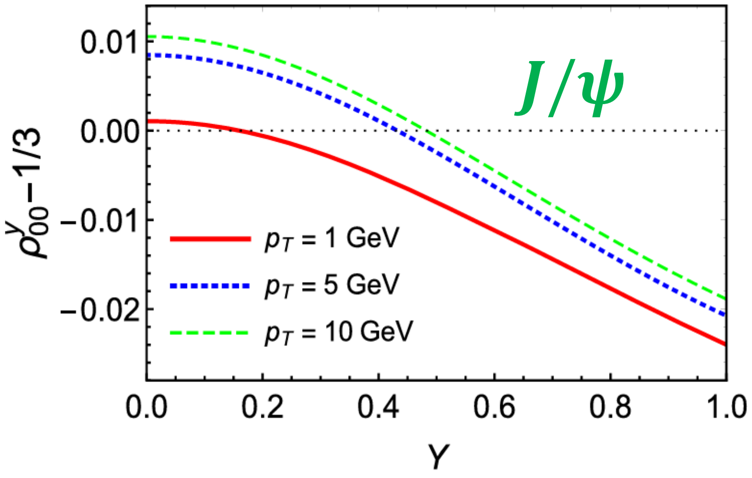


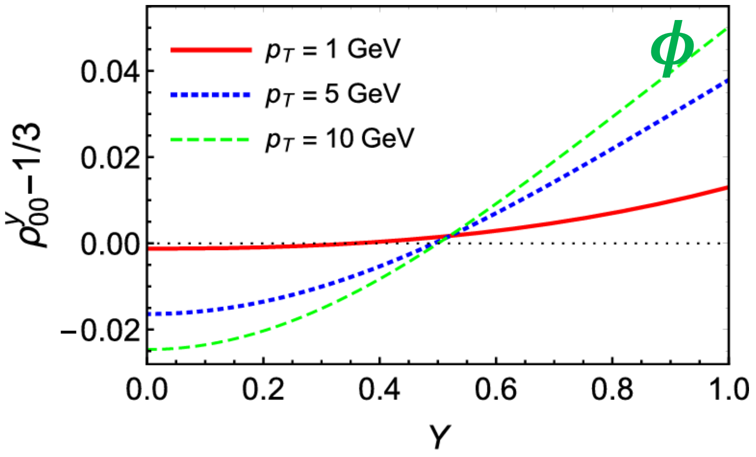
X.-L. S, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, PRD 110 (2024) 5, 056047

Momentum induced spin alignment

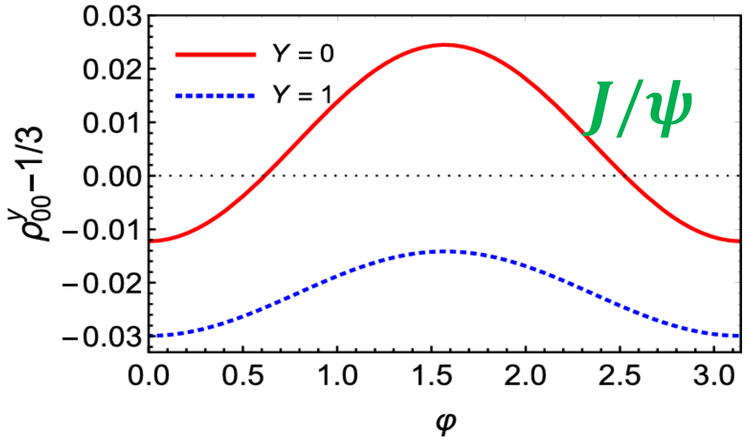
Global Spin alignment: setting the spin quantization direction as y-direction, i.e., the direction of global OAM in heavy-ion collisions.

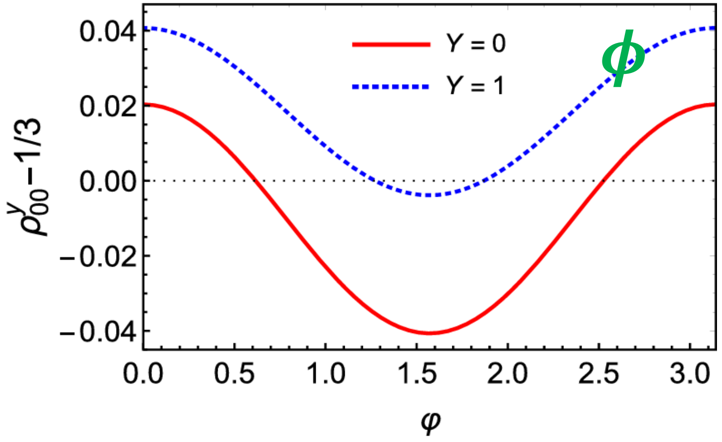
$$\epsilon_0 = \epsilon_0^y = (0, 1, 0)$$





$T = 150\ MeV$





$p_T = 2\ GeV,$
$T = 150\ MeV$

X.-L. S, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, PRD 110 (2024) 5, 056047

Magnetic field induced spin alignment

Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070

- Action: Only considering J/ψ mesons.

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}\right)+\frac{1}{8\pi G_5}\int d^4x\sqrt{-\gamma}\left(K-\frac{3}{L}\right)+S_{\mathrm{f}},$$
$$S_{\mathrm{f}} = -\frac{N_c}{16\pi^2}\int d^4x\int_0^{\zeta_h} d\zeta Q(\zeta)\mathrm{Tr}\left(F_L^2+F_R^2\right),$$

- Holographic background metric:

$$ds^2=\frac{L^2}{\zeta^2}\left(-f(\zeta)dt^2+h_T(\zeta)(dx^2+dy^2)+h_P(\zeta)dz^2+\frac{d\zeta^2}{f(\zeta)}\right).$$
$$f(\zeta)=1-\frac{\zeta^4}{\zeta_h^4}+\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta^4\ln\frac{\zeta}{\zeta_h}+\mathcal{O}(e^4B^4),$$
$$h_T(\zeta)=1-\frac{4}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$
$$h_P(\zeta)=1+\frac{8}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$

- Hawking temperature:

$$T=\frac{1}{4\pi}\left|\frac{4}{\zeta_h}-\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta_h^3\right|.$$

- Magnetic field constraint conditions

$$eB<\sqrt{\frac{3}{2}}\frac{1.6}{\zeta_h^2}\approx\frac{1.96}{\zeta_h^2}$$

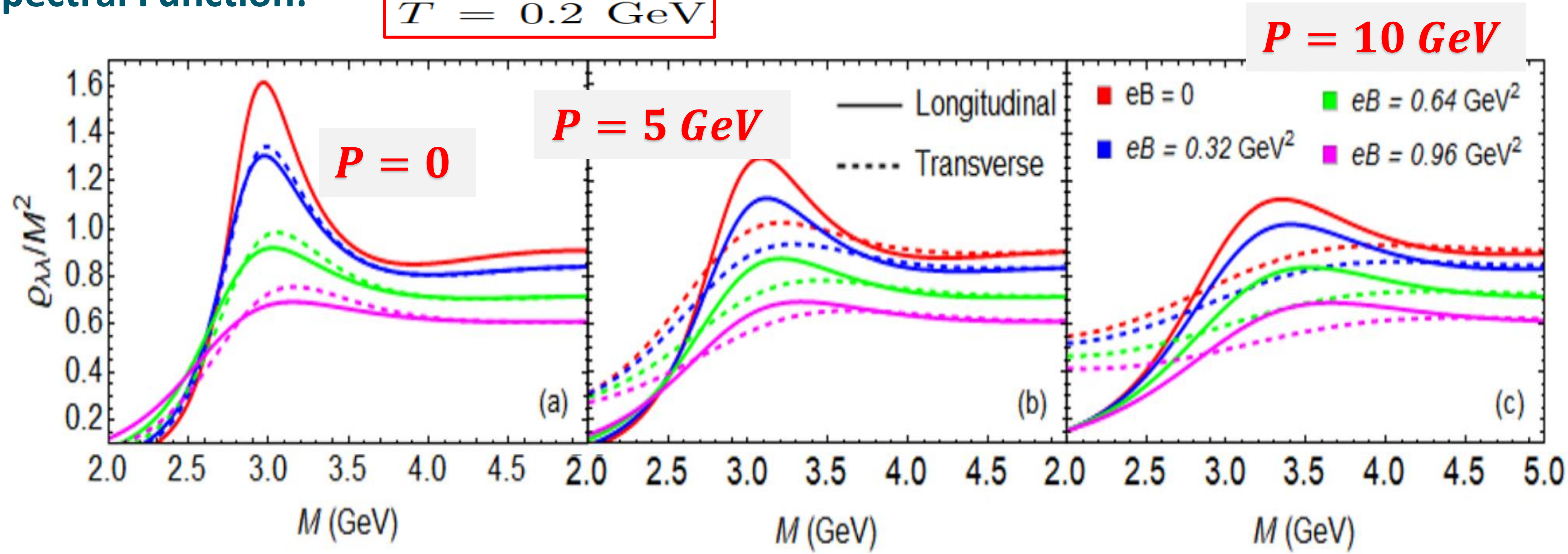
J/ψ meson in magnetized plasma

- Magnetic field parallel to momentum

Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070

- Spectral Function:

$T = 0.2$ GeV

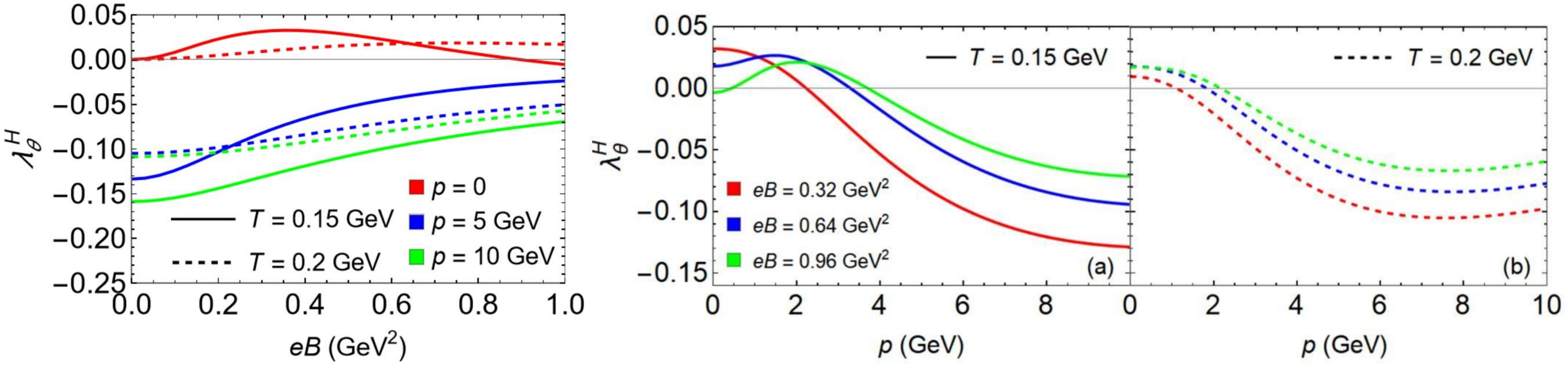


A nonzero magnetic field or a nonzero momentum will induce a separation between longitud. transversely polarized modes.

Magnetic field induced spin alignment

➢ Magnetic field parallel to momentum $\mathbf{p} = (0, 0, p)$

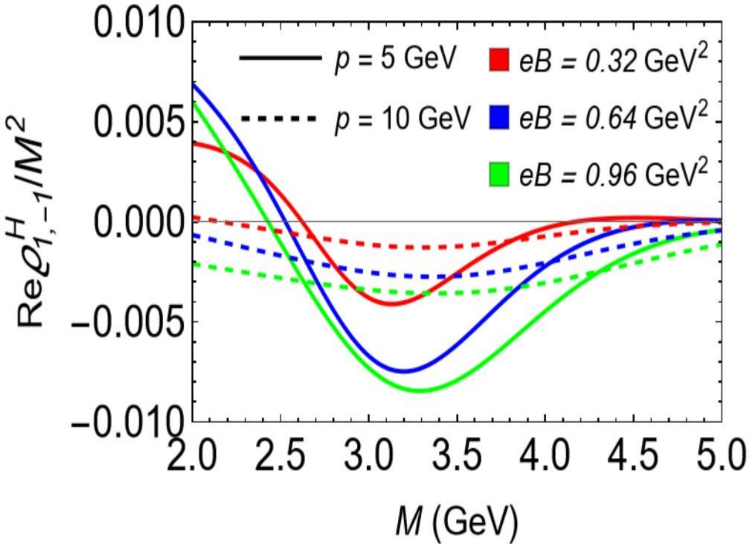
Magnetic Field: Momentum:



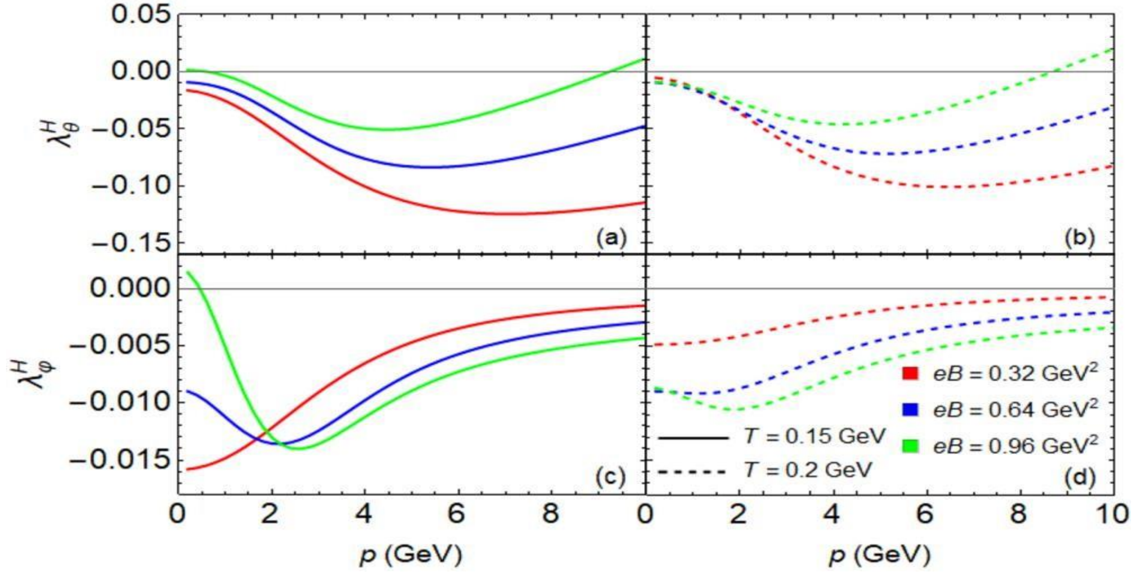
Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070

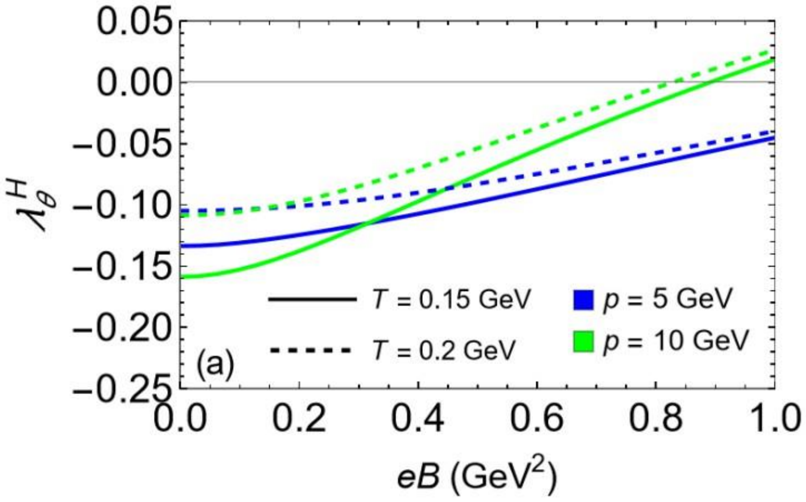
➢ Magnetic field perpendicular to momentum

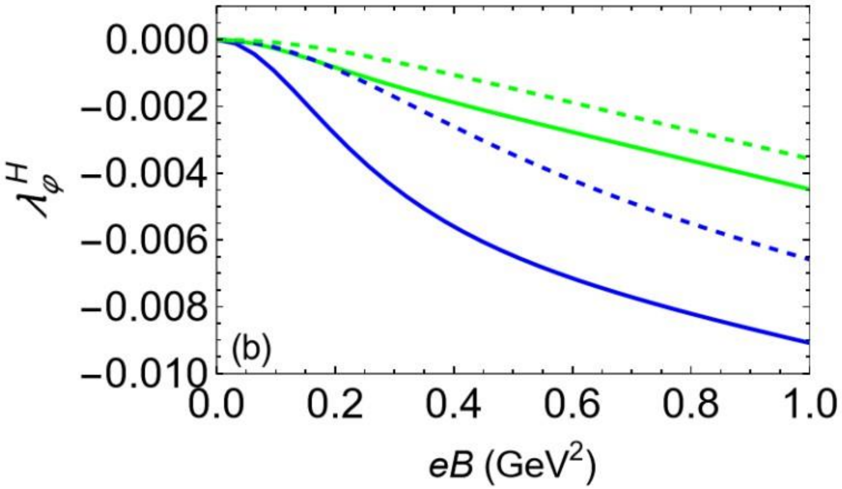
Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070



$$\mathbf{p} = (p, 0, 0)$$







$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}},$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1 + \rho_{00}},$$

➢ Application to heavy-ion collisions(the magnetic field alongs the y- direction)

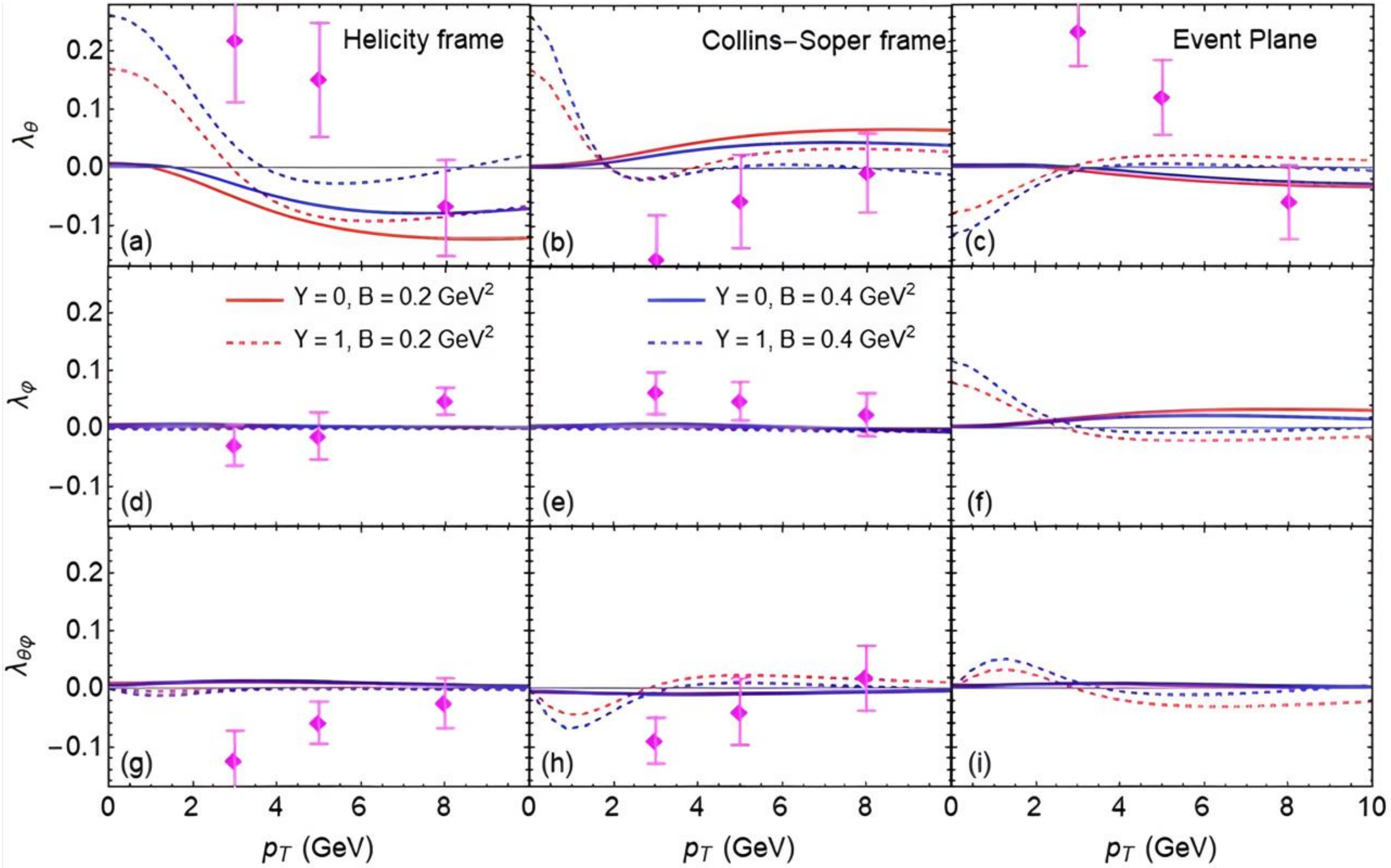
Yan-Qing Zhao ,Xin-Li Sheng ,Si-Wen Li DF Hou , JHEP08 (2024)070

$$T = 0.15\ \mathrm{GeV}$$

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1+\rho_{00}},$$

$$\lambda_{\theta\varphi} = \frac{\sqrt{2}\mathrm{Re}(\rho_{01}-\rho_{0,-1})}{1+\rho_{00}},$$



ALICE , PRL 131 (2023) 042303; ALICE , PLB 815 (2021)136146

the Collins-Soper frame, we find that the λ_θ parameter is dominant

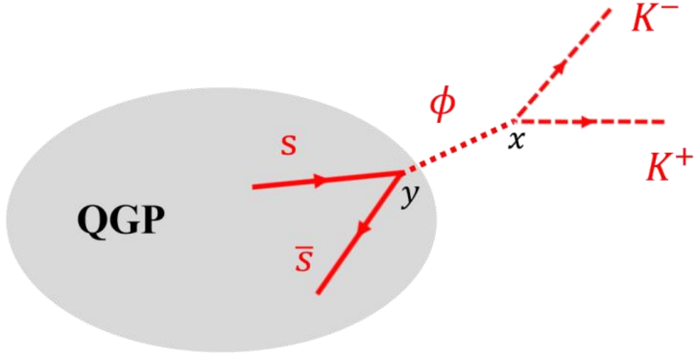
when measuring along the event plane direction, all three parameters $\lambda_\theta^{\mathrm{EP}}$, $\lambda_\varphi^{\mathrm{EP}}$, and $\lambda_{\theta\varphi}^{\mathrm{EP}}$ are of the same order.

Summary

- The spin alignment can be purely induced by the motion of vector meson relative to the background.
- The holographic prediction shows that J/ψ and ϕ have opposite behaviours. $J/\psi(\phi)$ are preferably to be transversely(longitudinally) polarized. This phenomenon is fully consistent with experiments' observation.
- The vector meson's spin alignment is a non-perturbative property in the strongly interacting matter.
- Magnetic field induces $\lambda_\theta^H > 0$ when the meson's momentum p is very small, while $\lambda_\theta^H < 0$ when p is large enough.
- Comparisons with experimental data show qualitative agreement for spin parameters λ_θ and λ_φ in the helicity and Collins-Soper frames.
- We also find significant differences between our results for $\lambda_{\theta\varphi}^H$ and λ_θ^{EP} with experiments .

Thanks!

$\phi \to K^+K^-$: Pair Production Rate and Spectral Function



S-matrix element:

Kaon's current

$$S_{fi} = \int d^4x \int d^4y \langle f, K^+K^- | J_\mu(y) D^{\mu\nu}_{\mathrm{R,vac}}(x-y) J^K_\nu(x) | i \rangle$$

Production rate:

$$n_{K^+K^-} = \sum_f \sum_i \frac{1}{Z} e^{-E_i/T} R_{fi}$$

Transition rate:

$$R_{fi} \equiv \lim_{\tau V \to \infty} \frac{|S_{fi}|^2}{\tau V}$$

$$n_{K^+K^-} = -2 \int \frac{d^3\mathbf{p}_+}{(2\pi)^3 2E_+} \frac{d^3\mathbf{p}_-}{(2\pi)^3 2E_-} n_B(\omega)$$
$$\times \tilde{\Gamma}^{\mathrm{vac}\,*}_\mu(p_+, p_-) \rho^{\mu\nu}(p) \tilde{\Gamma}^{\mathrm{vac}}_\nu(p_+, p_-)$$

Spectral function:

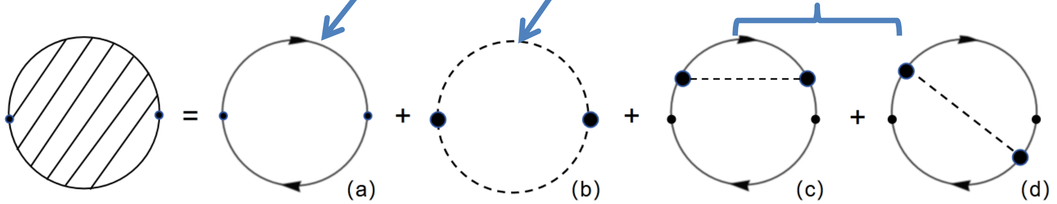
$$\rho^{\mu\nu}(p) = -\left[D^{\alpha\mu}_{\mathrm{R,vac}}(p)\right]^* \left[\mathrm{Im}\Pi^{\mathrm{R}}_{\alpha\beta}(p)\right] D^{\beta\nu}_{\mathrm{R,vac}}(p)$$

✦ ϕ meson's propagator and self-energy

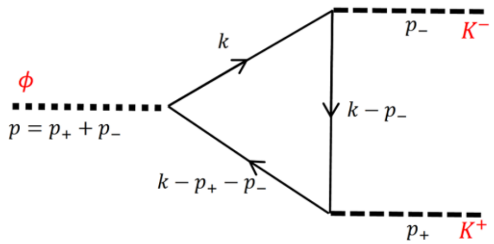
$$D_{L/T}(p) = \frac{4G_V}{1 + 4G_V \Pi^{\mathrm{tot}}_{L/T}(p)}$$

next-to-leading

$$\Pi^{\mathrm{tot}}_{\mu\nu} = \Pi^{\text{Q-loop}}_{\mu\nu} + \Pi^{\text{K-loop}}_{\mu\nu} + \Pi^{\text{K-tad}}_{\mu\nu}$$



✦ $\phi - -$ vertex



$$\tilde{\Gamma}^\mu(p_+, p_-) \approx (p^\mu_+ - p^\mu_-)\Gamma_{\mathrm{on}}(M_\phi, |\mathbf{p}|)$$

✦Spectral function

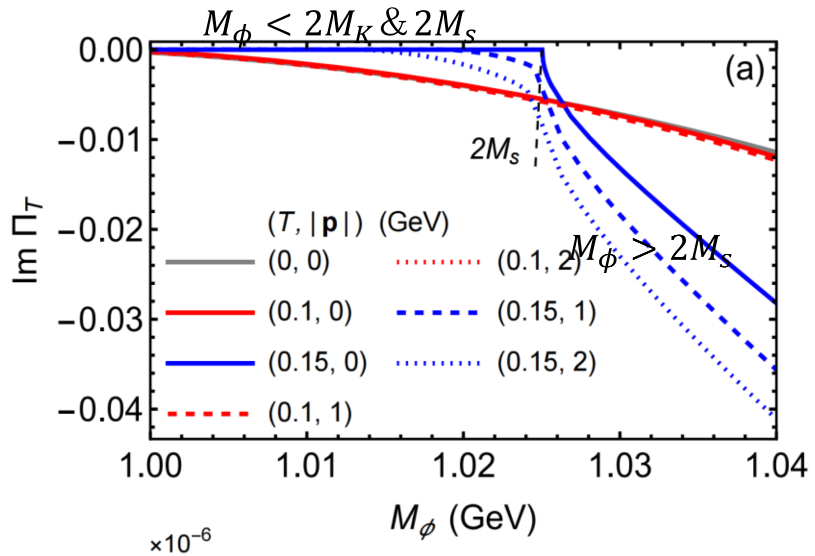
$$\rho_{L/T}(p) = -\left|\frac{4G_V}{1 + 4G_V \Pi^{\mathrm{tot}}_{\mathrm{vac}}(p)}\right|^2 \mathrm{Im}\,\Pi^{\mathrm{tot}}_{L/T}(p)$$

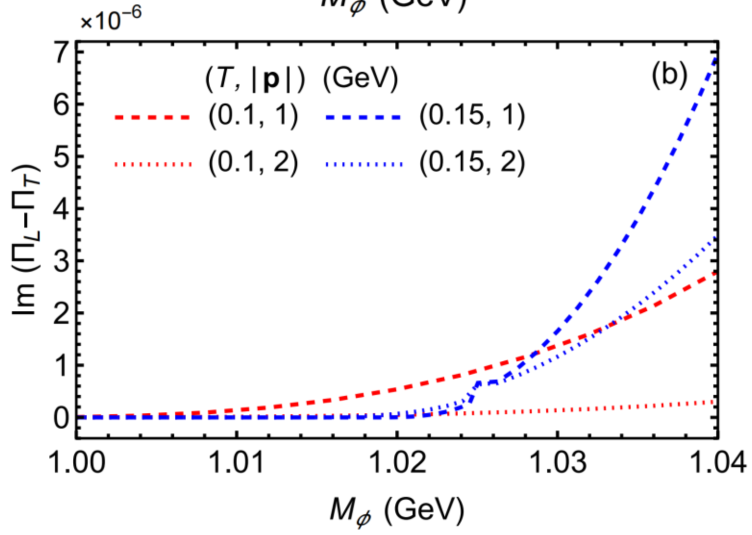
C. Gale , J. I. Kapusta , Nucl.Phys.B 357 (1991), 65-89, Nucl.Phys.B 357 (1991), 65-89.

X. N. Zhu, X. L. Sheng, Defu Hou, Phys. Rev. D 112, 056011

Meson’s Self-Energy and Mass Spectrum

$$\Pi^{\mu v}_{\text{tot}}(p) = -\epsilon^{\mu}_{H}\epsilon^{v}_{H}\Pi_L(p) + \left(g^{\mu v} - p^{\mu}p^{v}/p^2 + \epsilon^{\mu}_{H}\epsilon^{v}_{H}\right)\Pi_T(p)$$



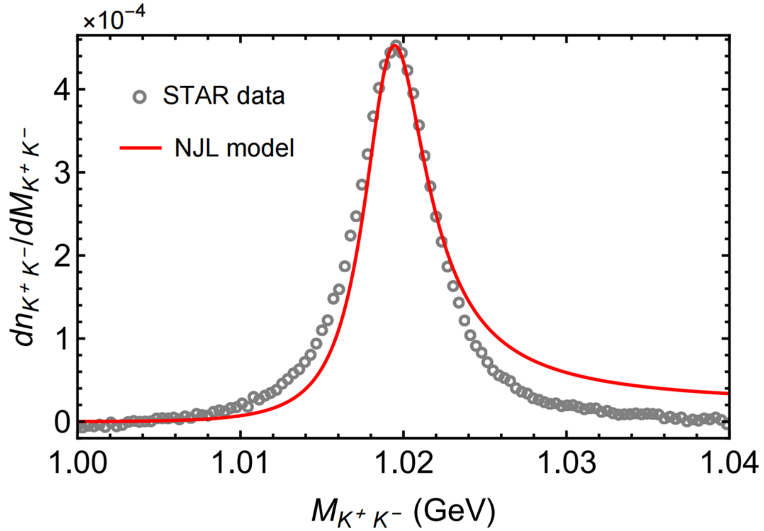


At T ≤ 0.1 GeV, M_ϕ is always larger than $2M_s$, quark loop dominates.

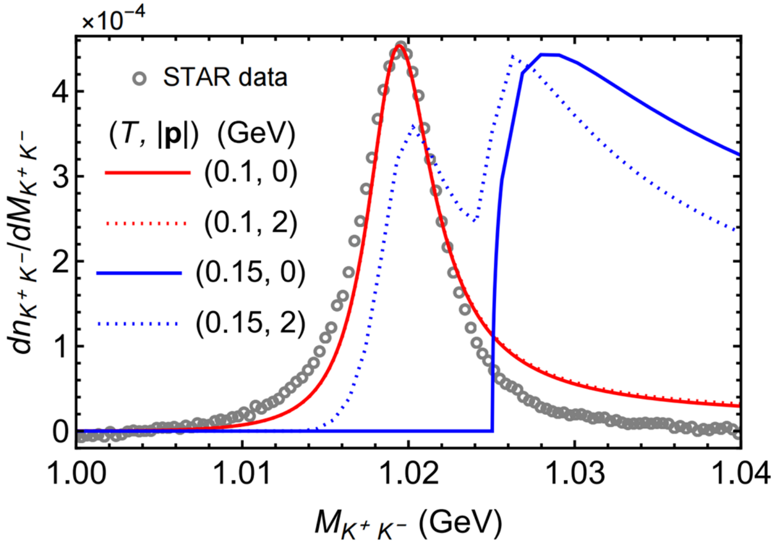
At T = 0.15 GeV, the value turns to be nonzero as M_ϕ increases over $2M_s$, kaon loop dominating.

The deviation between the two part is only the order of 10^{-4} compared with the above.

$$\frac{dn_{K^+K^-}}{dM_\phi} \propto \frac{1}{M_\phi}\left(M_\phi^2 - 4M_{K,\text{vac}}^2\right)^{3/2} \left|\Gamma^{\text{vac}}_{\text{on}}(M_\phi)\right|^2 \rho(p)$$



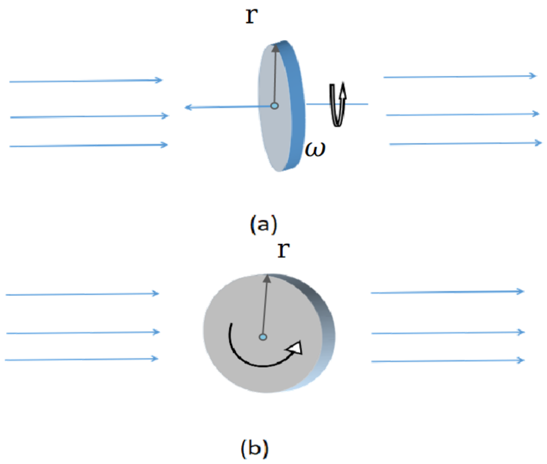
In vacuum, the K^+K^-spectrum is well reproduced. The spectrum is underestimated below the peak and overestimated above it.



At T ≤ 0.1 GeV, the spectrum remains almost unchanged; at T = 0.15 GeV, the peak shifts to higher mass and a double-peak structure appears at finite momentum.

ϕ Meson's Global Spin Alignment

Disc in the water flow



Motion leads to mesons in different spin states having different energy.

Spin alignment

$$\rho_{00} = \frac{\bar{\rho}_{00}}{\sum_{\lambda=0,\pm1} \bar{\rho}_{\lambda\lambda}}$$

The probability of mesons in the spin-0 states

Diagonal spin density matrix of meson in bound state

$$\rho_\lambda \sim \frac{1}{\exp(M_{\phi,\lambda}/T) - 1}$$

$$\bar{M}_\phi = \frac{1}{3}\sum_{\lambda=0,\pm1} M_{\phi,\lambda} \qquad M_{\phi,0} = \bar{M}_\phi + \Delta, \quad M_{\phi,\pm1} = \bar{M}_\phi - \frac{\Delta}{2}.$$

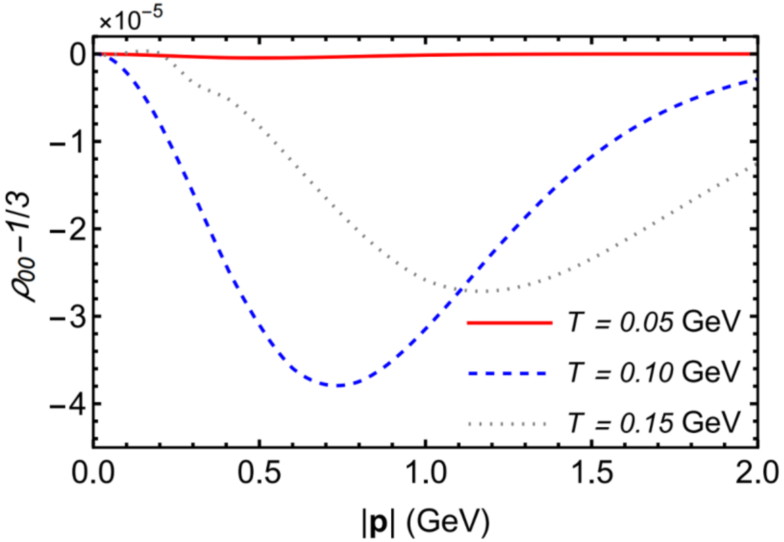
$$\rho_{00} \equiv \frac{f_0}{f_1 + f_0 + f_{-1}} \simeq \frac{1}{3} - \frac{\Delta}{3T}\left[1 + \frac{1}{\exp(\bar{M}_\Phi/T) - 1}\right] + O\left[\left(\frac{\Delta}{T}\right)^2\right]$$

Spin alignment of ϕ meson

$$\bar{\rho}_{00}(\mathbf{p}) - \frac{1}{3} = \frac{2\int_{M_{\min}}^{M_{\max}} dM_\phi \delta f(p)}{3\int_{M_{\min}}^{M_{\max}} dM_\phi \left[3 f_T(p) + \delta f(p)\right]}$$

Auxiliary function

$$\begin{pmatrix} f_T(p) \\ \delta f(p) \end{pmatrix} \equiv \frac{1}{\omega} n_B(\omega)(M_\phi^2 - M_{K,\text{vac}}^2)^{3/2} \left|\Gamma_{\text{on}}^{\text{vac}}(M_\phi)\right|^2 \begin{pmatrix} \rho_T(p) \\ \rho_L(p) - \rho_T(p) \end{pmatrix}$$



At T = 0.1 GeV, $\bar{\rho}_{00}$ reaches its minimum value at |p| = 0.72 GeV, while at T = 0.15 GeV the minimum value corresponds to a larger |p|.
The magnitude of the deviations is just 10^{-5} at T = 0.1 or 0.15 GeV.