Fragmentation Energy Correlators

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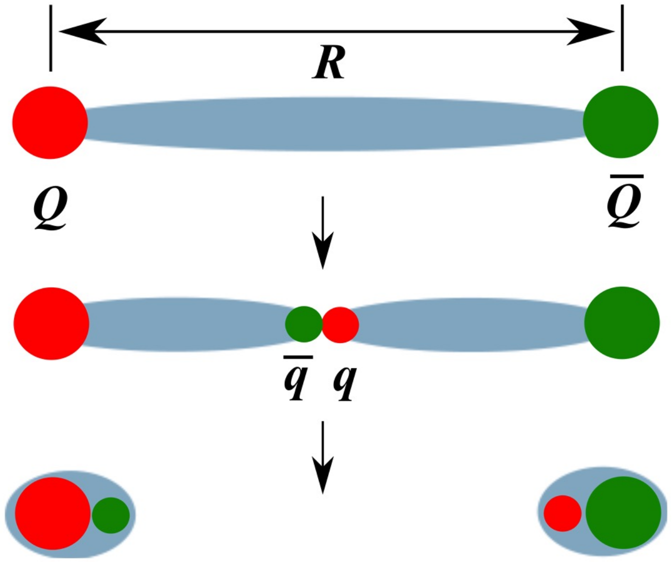
Unveiling Confinement and Spin Structure



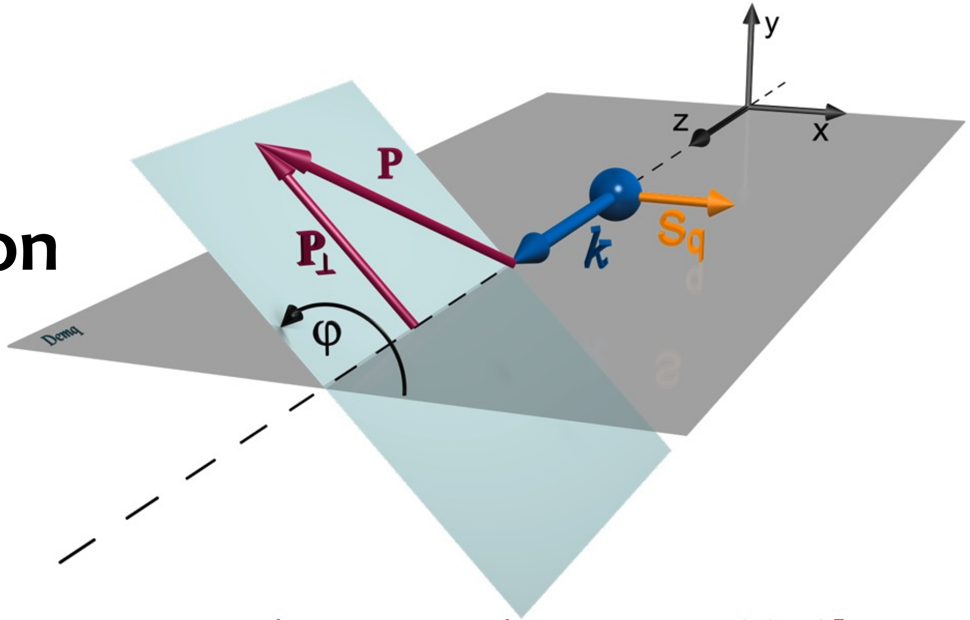
[M. N. Chernodub, 2010]

The mystery of confinement

- Still not calculable
- Learn by experiments

Spin structure influences Fragmentation

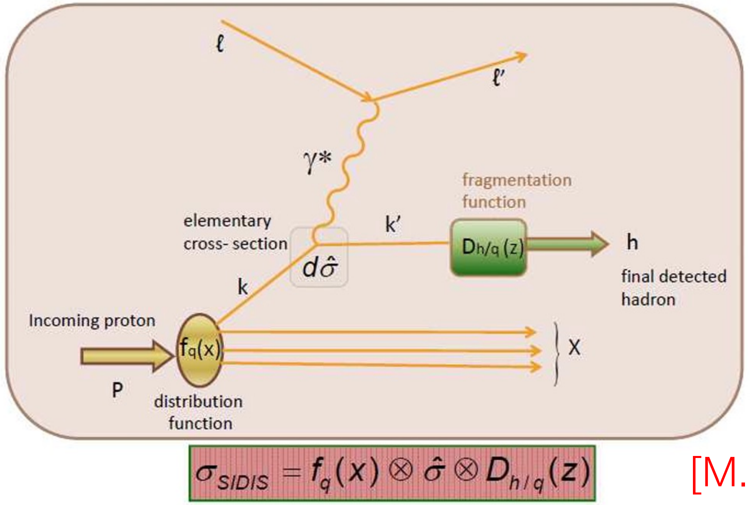
- Collins effect
- Inner dynamics in fragmenting
- 3D imaging



[H. H. Matevosyan, A. W. Thomas, and W. Bentz, 2012]

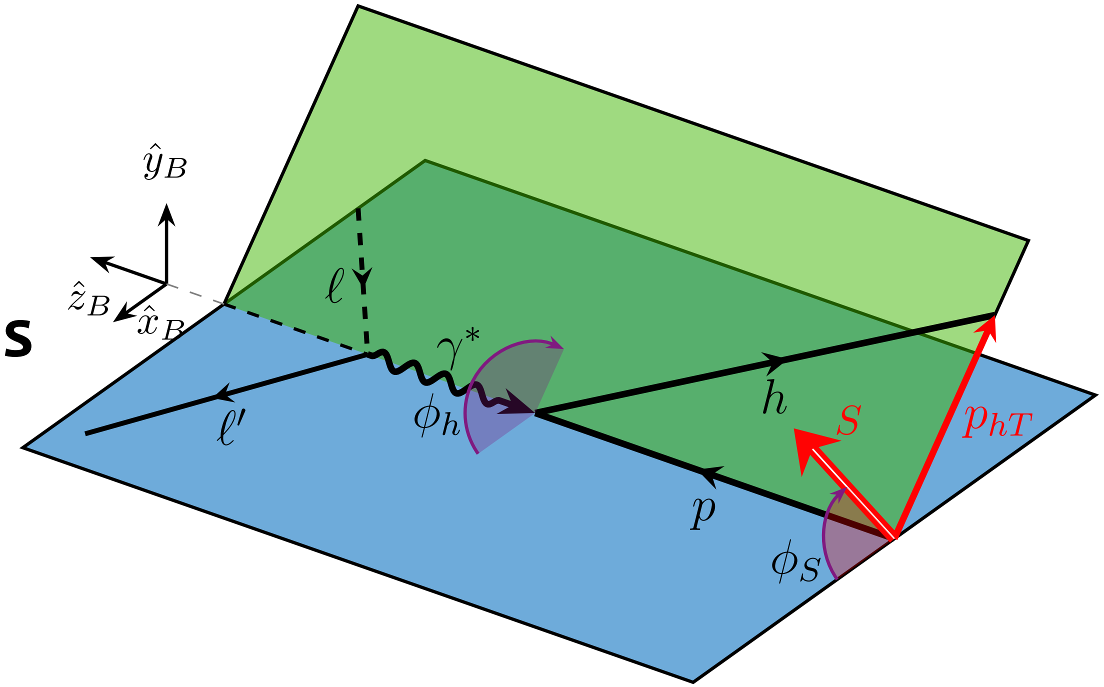
Experimental Tool: Semi-Inclusive DIS

Semi-Inclusive Deep Inelastic Scattering (SIDIS)



[M. Boglione, 2011]

- $e + p \to e' + h + X.$
- Non-perturbative in PDF and fragmentation.



Transverse momentum dependent functions

Small p_{hT} leads to:

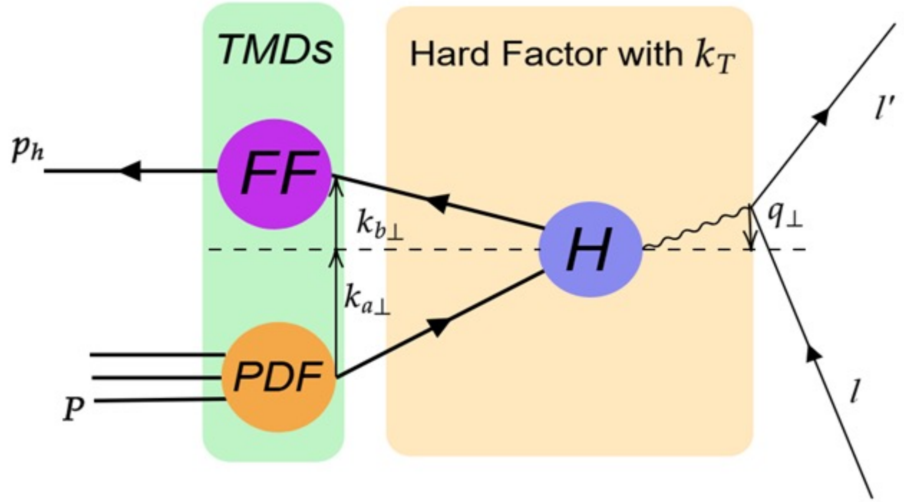
- New TMD functions
- 3D imaging
- ϕ_h dependence from spin structure

3D Imaging: TMDs

John Collins, 1993

Extraction Problem:

Entangled with TMD PDF.



$$E' E_B \frac{d\sigma}{d^3 l'\, d^3 p_B} = \sum_a \int d\xi \int \frac{d\zeta}{\zeta} \boxed{\int d^2 k_{a\perp} \int d^2 k_{b\perp}} \boxed{\hat{f}_{a/A}(\xi, k_{a\perp})}\, E' E_{k_b} \frac{d\hat{\sigma}}{d^3 l'\, d^3 k_b} \boxed{\hat{D}_{B/a}(\zeta, k_{b\perp})}$$

$$+ Y(x_{\rm Bj}, Q, z, q_\perp/Q).$$

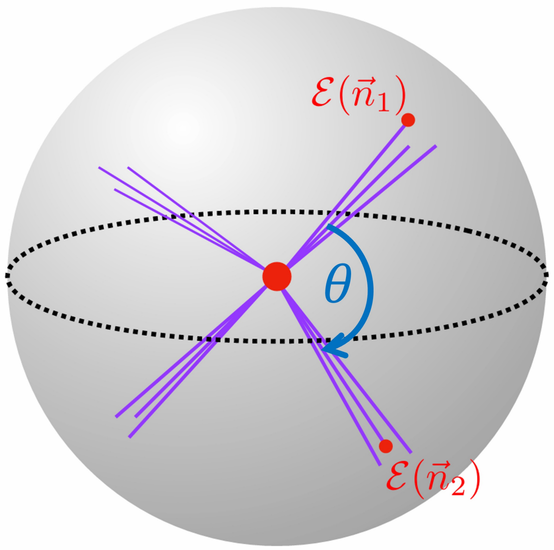
New Opportunities: The Energy-Energy Correlator

Basham, Brown, Ellis and Love, 1978

Definition of Energy-Energy Correlator(EEC)

Probe two energy flux coincidently

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta^{(2)}(\Omega_a - \Omega_{p_i}) \delta^{(2)}(\Omega_b - \Omega_{p_j})$$

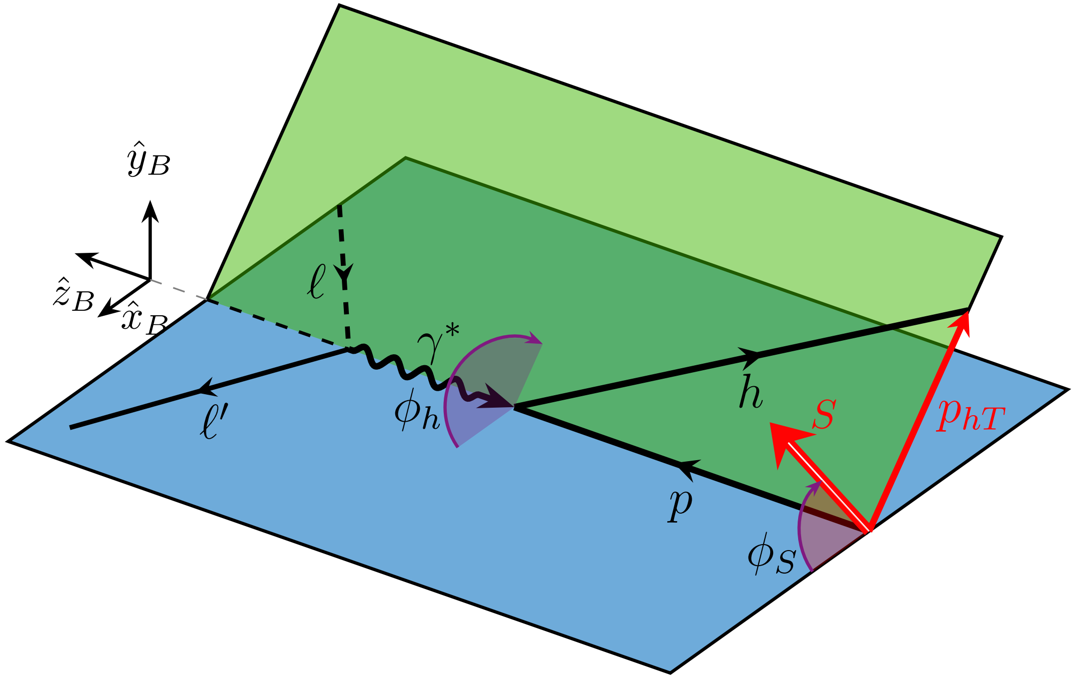


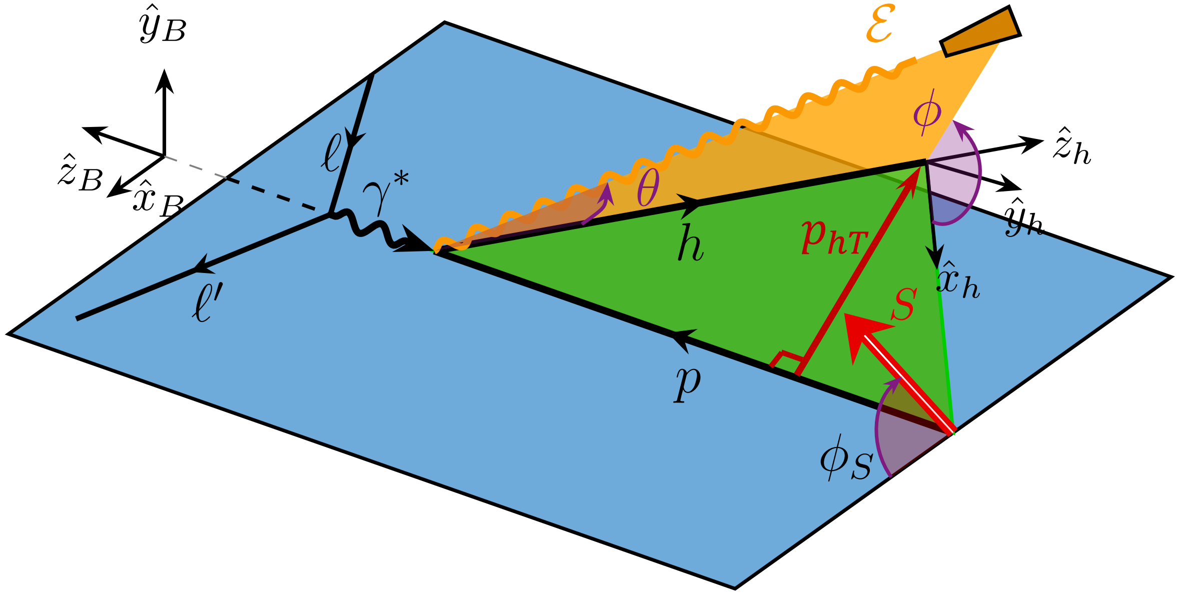
Widely used for probing microscopic details

1. **Strong Coupling Constant**
   The CMS Collaboration, 2402.13846 ···
2. **Nucleon Structure**
   X.H. Liu, H.X. Zhu, 2209.02080 ···
3. **Many Other Applications**
   ···

Observable Definition

Observe final state hadron, with energy flux around it



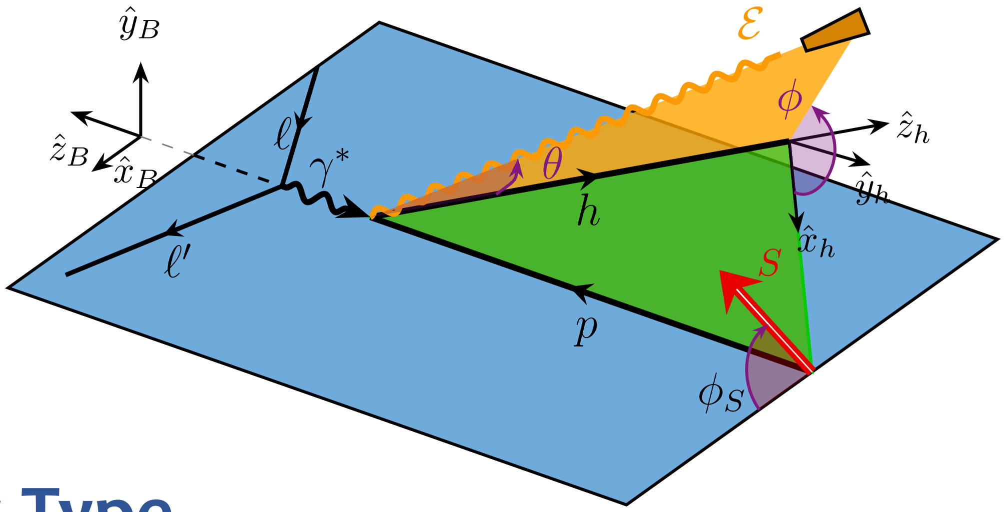


Fragmentation Energy Correlator (FEC)

Factorization: Collinear and TMD

Inclusively observe the p_{hT}, leaving the collinear factorization

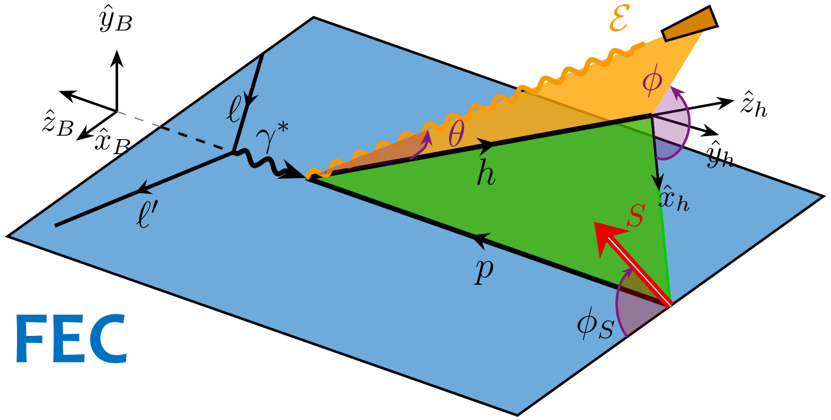
- *Extra handle for the non-perturbative effects*
- *Collinear factorization*
- *Angles replace p_{hT}*
- *Similar structure functions to TMD FFs*



	Unpolarized	Collins Type
FEC	$D_{1,q}(\xi_2, q_F)$	$\frac{(\hat{p}_h \times \boldsymbol{n}_T)^i}{\lvert\boldsymbol{n}_T\rvert} H^{\perp}_{1,q}(\xi_2, q_F)$
TMD FF	$D_{1,h/q}(z, zk_T)$	$H^{\perp}_{1,h/q}(z, zk_T)\frac{\epsilon^{ij}_{\perp} k^j_T}{M_h}$

$$q_F = m_\pi e^{y_\pi - \eta} \approx \frac{P_h^+ \theta}{2}$$

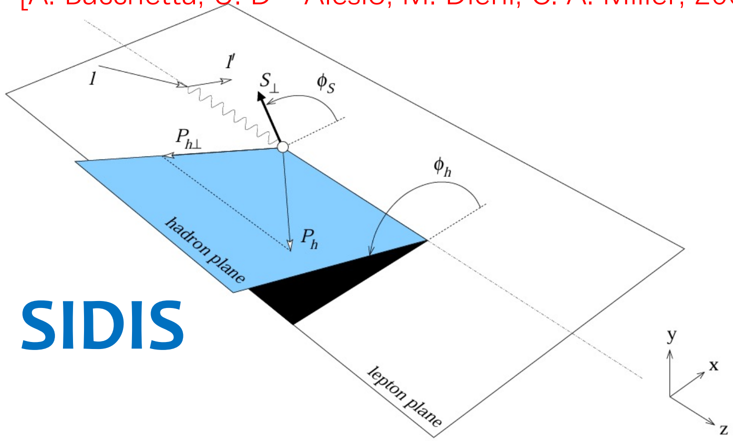
Factorization: Collinear and TMD



FEC

$$\overline{|\mathcal{M}_{\mathcal{E}}|^2} \simeq \sum_{a,b} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2^2} \Big\{ \mathcal{D}_{1,h/b}(\xi_2, q_F) f_{a/p}(\xi_1)\, C_{ab} + \mathcal{H}^{\perp}_{1,h/b}(\xi_2, q_F)\, h_{a/p}(\xi_1) \sum_{i,j=1}^{2} \frac{(\hat{p}_h \times \boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|} T_{ab}^{ij} s_T^j \Big\}.$$

[A. Bacchetta, U. D' Alesio, M. Diehl, C. A. Miller, 2004]



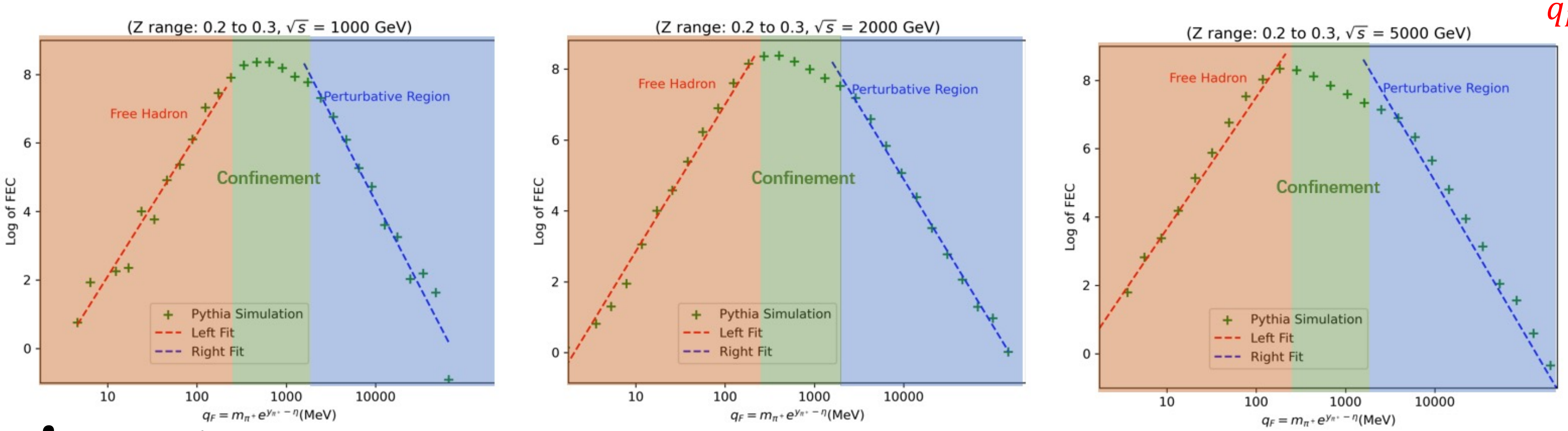
SIDIS

$$\overline{|\mathcal{M}|^2} \simeq \sum_{a,b} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2^2} \int d^2\boldsymbol{k}_{aT} \int d^2\boldsymbol{k}_{bT} \Big\{ D_{1,h/b}(\xi_2, \boldsymbol{k}_{bT}) f_{a/p}(\xi_1)\, C_{ab} + H^{\perp}_{1,h/b}(\xi_2, \boldsymbol{k}_{aT})\, h_{a/p}(\xi_1) \sum_{i,j=1}^{2} \frac{\epsilon^{ik} p_{hT}^k}{m_h} T_{ab}^{ij} s_T^j \Big\}.$$

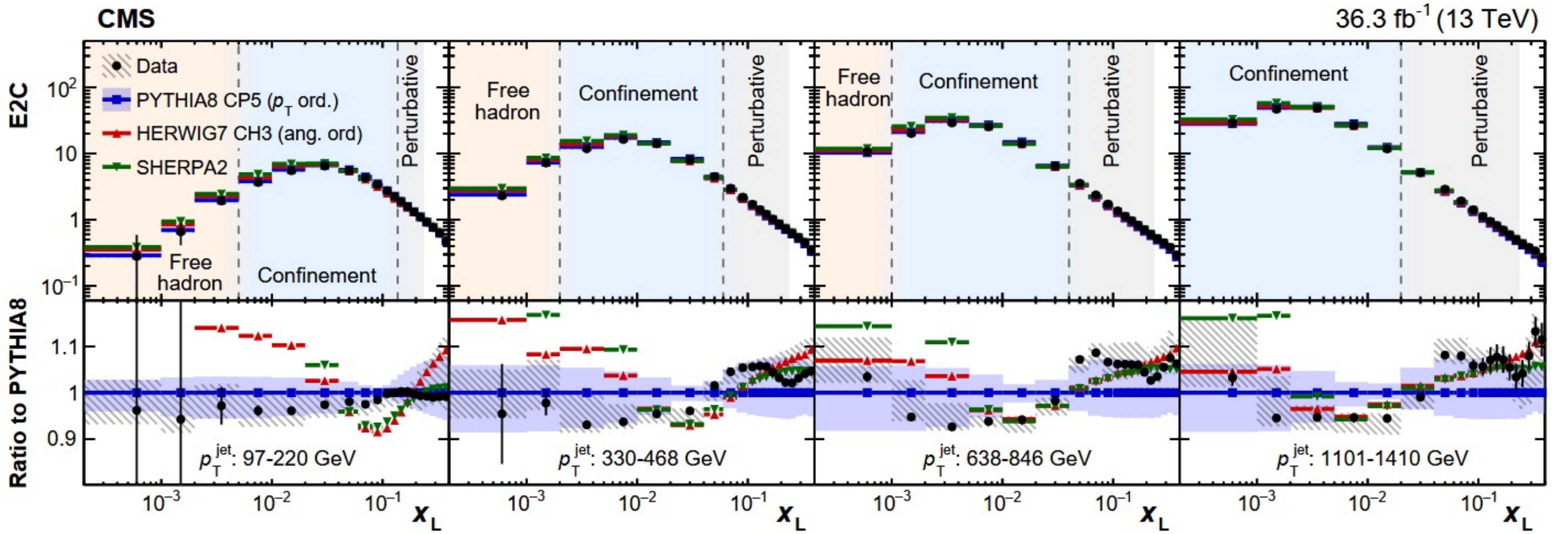
Scale Invariance of Variable q_F

Use Pythia to show confinement with FEC (Unpol.).

$$q_F \approx \frac{P_h^+\theta}{2}$$



Scaling law in experiments



$$\chi_L = \langle \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2 \rangle$$

[CMS, 2024]

NLO Calculation for IR cancellation

During the integral of p_{hT}, subtlety comes from the phase space at $zk_{2T}/\hat{z} = p_{hT} = 0$.

$$[\Delta_1^R W]_{\lambda\lambda'}^{\lambda_\gamma\lambda'_\gamma}(\hat{x},\hat{z};Q^2/\mu^2) = (4\pi^2\mu^2)^\epsilon \int d^{d-2}\boldsymbol{k}_{2T} \left(8\pi^2 e_q^2 \alpha_s C_F\right) \hat{z}\,\delta\left(k_{2T}^2 - \hat{z}(1-\hat{z})\hat{s}\right) \times e^{i[(\lambda-\lambda')-(\lambda_\gamma-\lambda'_\gamma)]\phi_h} \cdot \bar{w}_{\lambda\lambda'}^{\lambda_\gamma\lambda'_\gamma}(\hat{x},\hat{z})$$

$$= (2\pi)^4 \hat{z}\, e_q^2 \frac{\alpha_s C_F}{2\pi} \frac{e^{\epsilon(L+\gamma_E)}}{\Gamma(1-\epsilon)} \int_0^{2\pi} \frac{d\phi_h}{2\pi} \left[e^{i[(\lambda-\lambda')-(\lambda_\gamma-\lambda'_\gamma)]\phi_h} \frac{\hat{x}^\epsilon\, \hat{z}^{-\epsilon}\, \bar{w}_{\lambda\lambda'}^{\lambda_\gamma\lambda'_\gamma}(\hat{x},\hat{z})}{(1-\hat{x})^\epsilon (1-\hat{z})^\epsilon} \right],$$

⬇

May not separately integrate ϕ_h, k_{2T}^2, need regularization at $k_{2T} = 0$.

NLO Calculation for IR cancellation

Cancel the $1/\epsilon^2$ poles successfully, take $q \to q$ Non-singlet channel, a specific helicity arrangement as an example.

$$
\begin{aligned}
[\Delta_1 W]^{++}_{++} &= [\Delta_1^V W]^{++}_{++} + [\Delta_1^R W]^{++}_{++} \\
&= 4(2\pi)^4 \hat{z}\, e_q^2 \,\frac{\alpha_s C_F}{2\pi} \frac{e^{\epsilon(L+\gamma_{\rm E})}}{\Gamma(1-\epsilon)} \Bigg\{ -8\,\delta(1-\hat{x})\delta(1-\hat{z}) + \frac{1+\hat{x}^2\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} \\
&\quad - \delta(1-\hat{z}) \left[\frac{P_{qq}(\hat{x})}{\epsilon} + (1+\hat{x}^2) \left(\frac{\ln \hat{x}}{1-\hat{x}} - \left(\frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ \right) \right] \\
&\quad - \delta(1-\hat{x}) \left[\frac{K_{qq}(\hat{z})}{\epsilon} - (1+\hat{z}^2) \left(\frac{\ln \hat{z}}{1-\hat{z}} + \left(\frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ \right) \right] + \mathcal{O}(\epsilon) \Bigg\},
\end{aligned}
$$

where the $1/\epsilon$ poles should be absorbed by perturbative PDF and FF.

NLO Calculation for IR cancellation

Extract the collinear divergence.

$$[\Delta_1^{\rm col} W]^{++}_{++} = 4(2\pi)^4\, e_q^2\, \hat{z}\, \frac{\alpha_s C_F}{2\pi} \left[-\frac{S_\epsilon}{\epsilon}\right] \left[P_{qq}(\hat{x})\, \delta(1-\hat{z}) + \delta(1-\hat{x})\, K_{qq}(\hat{z})\right],$$

The finite hard coefficient:

$$\begin{aligned}
[\Delta_1^{\rm sub} W]^{++}_{++} &= [\Delta_1 W]^{++}_{++} - [\Delta_1^{\rm col} W]^{++}_{++} \\
&= 4(2\pi)^4 \hat{z}\, e_q^2\, \frac{\alpha_s C_F}{2\pi} \Bigg[-8\,\delta(1-\hat{x})\delta(1-\hat{z}) + \frac{1+\hat{x}^2\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} \\
&\qquad - A(\hat{x})\,\delta(1-\hat{z}) - \delta(1-\hat{x})\, B(\hat{z}) \Bigg],
\end{aligned}$$

with

$$A(\hat{x}) = P_{qq}(\hat{x}) \ln\frac{\mu^2}{Q^2} + (1+\hat{x}^2)\left[\frac{\ln\hat{x}}{1-\hat{x}} - \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+\right],$$

$$B(\hat{z}) = K_{qq}(\hat{z}) \ln\frac{\mu^2}{Q^2} - (1+\hat{z}^2)\left[\frac{\ln\hat{z}}{1-\hat{z}} + \left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+\right],$$

TMD FFs from FECs

X.H. Liu and H.X. Zhu, 2024

- Physical picture: transverse momentum conservation.

TMD FF **FEC**

$$\frac{1}{zN_c}\sum_X\int\frac{dy^-}{2\pi}e^{iP_h^+y^-/z}\mathrm{Tr}\left[\Gamma\langle 0|W(\infty,y^-;w)\psi(y^-)\delta^{(2)}\left(\boldsymbol{k}_T-\int d\Omega\,\mathcal{E}(\boldsymbol{n})\boldsymbol{n}_T\right)|h,X\rangle\times\left\langle h,X|\bar{\psi}(0)W^\dagger(\infty,0;w)|0\right\rangle\right]$$

- Very Interesting **Moment Relation** to TMD FF.

$$z^2\int d^2\boldsymbol{k}_T\,k_T^{i_1}\cdots k_T^{i_N}\,d_{h/q}^{[\Gamma]}(z,-z\boldsymbol{k}_T)=\int d\Omega_1\cdots d\Omega_N\,n_{1T}^{i_1}\cdots n_{NT}^{i_N}\,\mathcal{D}_{h/q,N}^{[\Gamma]}(z,\{\boldsymbol{n}_i\};p_h),$$

- N points energy correlators recover different moments of TMD FFs.

Magnitude Estimation

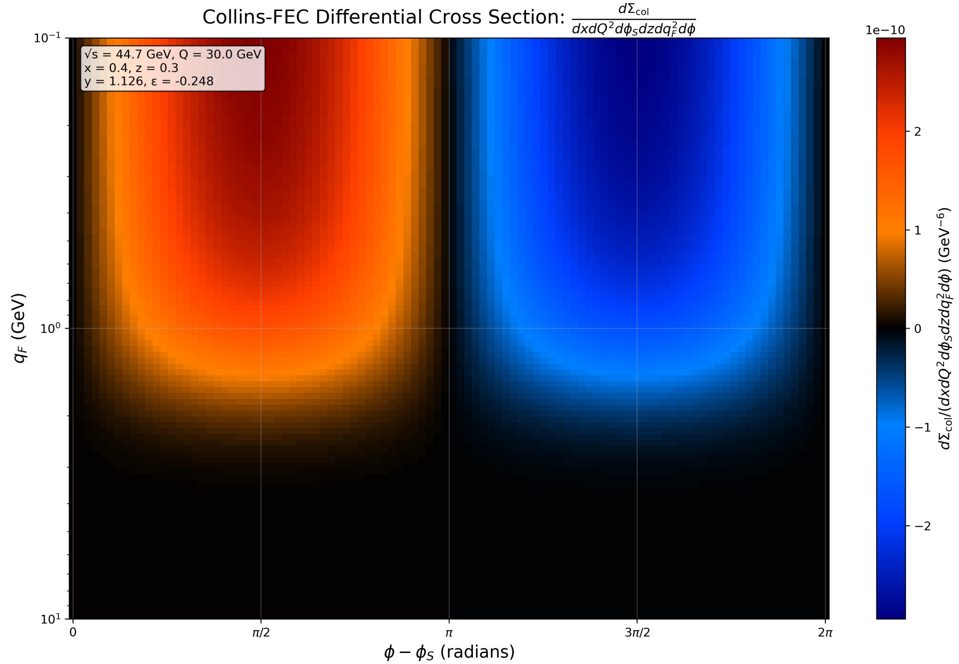
X.H. Liu and H.X. Zhu, 2024

- Momentum Relation gives the magnitude estimation.

$$\int dq_F \, \mathcal{H}^{\perp}_{1,q}(z, q_F) = -z^2 \int \frac{dk_T^2}{2M_h} k_T^2 H^{\perp}_{1,h/q}(z, z k_T).$$

- Easily Decide the Experimental Signal Magnitude for Collins-Type FEC.

- Partially integrate Q^2 and q_F^2, the FEC is in the unit of cross section.

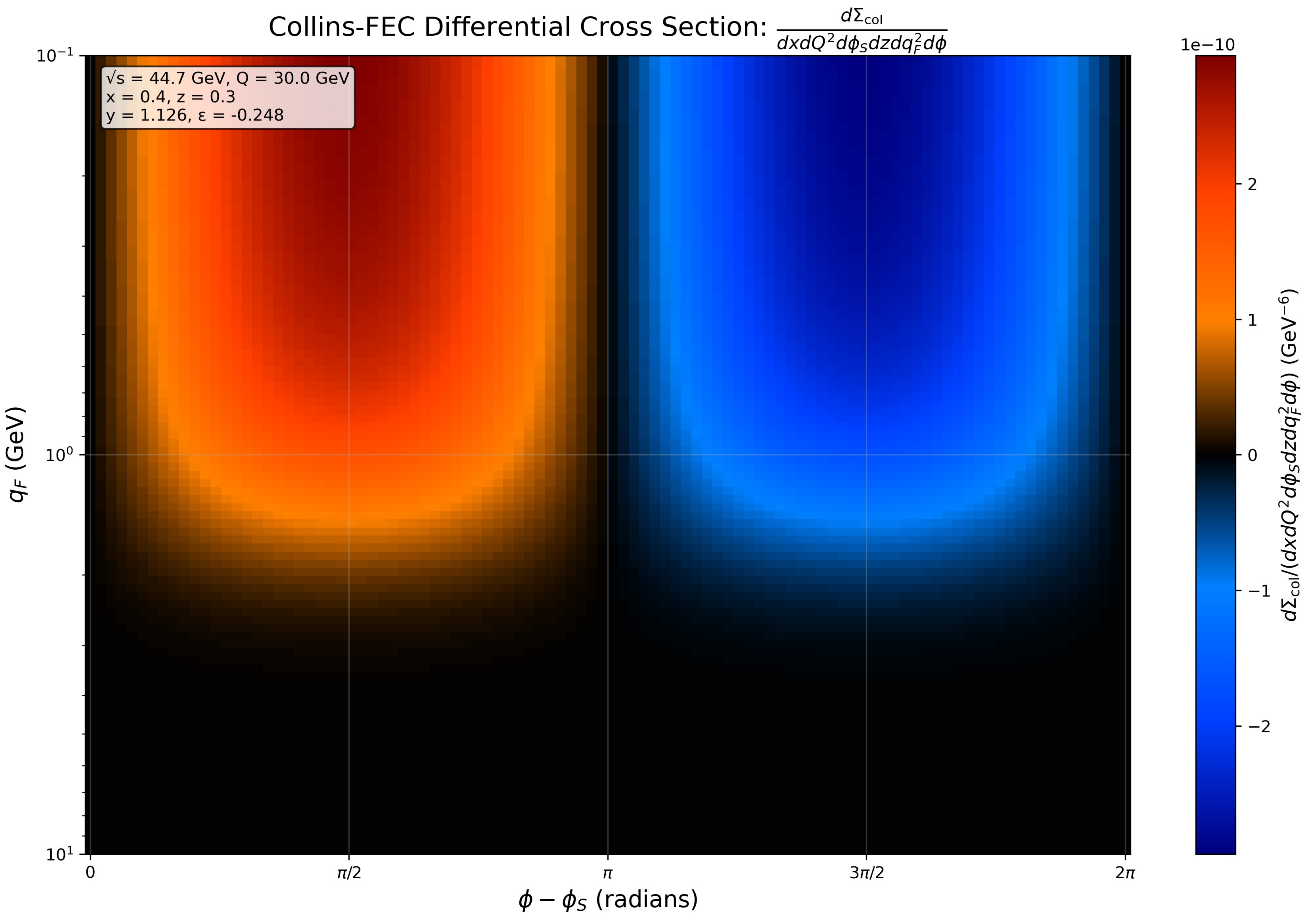


Magnitude Estimation

[H1, 2014]

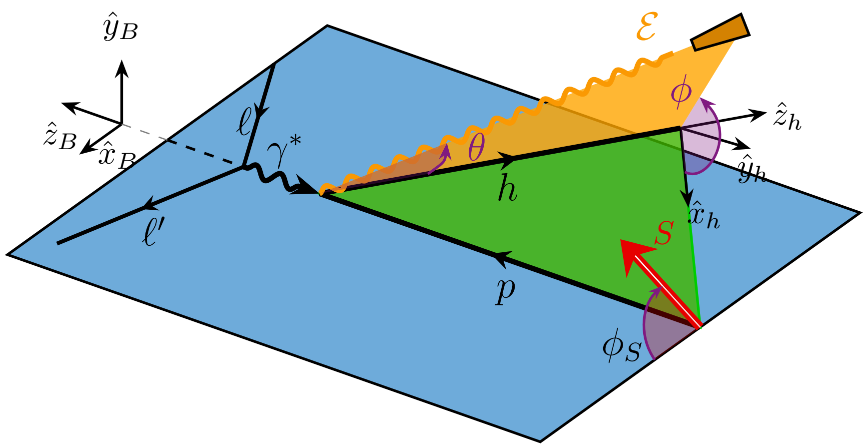
- According to previous measurement, the SIDIS cross section is of order 10pb.

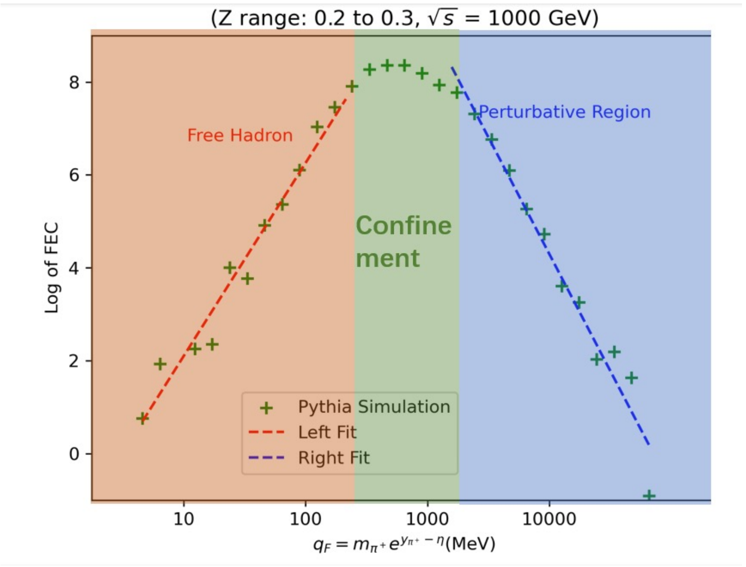
- For the present estimation, partially integrated cross section ~ 1 pb, giving an appropriate estimation of asymmetry of order 0.1.

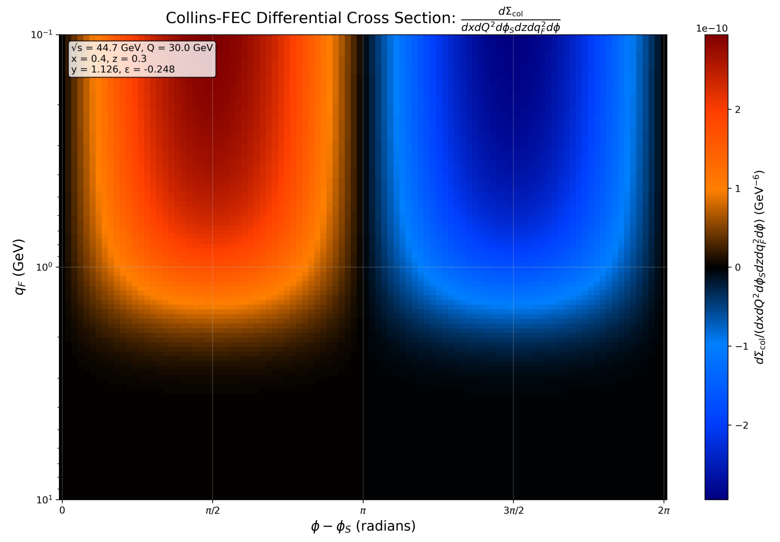


Summary

- Introduce the boost invariant Fragmentation Energy Correlators as a probe for transverse structure of fragmenting hadrons.
- NLO calculation to show the *consistence* of the observable, IR cancellation.
- Naturally connecting to TMD moments, giving magnitude estimation.







Thank You!

Back Up

Splitting kernels, hard coefficients and other auxiliary functions:

$$P_{qq}(x) = K_{qq}(x) = -1 - x + \frac{2}{(1-x)_+} + \frac{3}{2}\delta(1-x).$$

$$\delta_T P_{qq}(x) = \delta_T K_{qq}(x) = -2 + \frac{2}{(1-x)_+} + \frac{3}{2}\delta(1-x).$$

$$S_\epsilon = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} = 1 + \epsilon \ln\big(4\pi e^{-\gamma_{\rm E}}\big) + \mathcal{O}\big(\epsilon^2\big).$$

$$\begin{aligned}
[\Delta_1^{\rm sub} W]^{+-}_{+-} &= [\Delta_1 W]^{+-}_{+-} - [\Delta_1^{\rm col} W]^{+-}_{+-} \\
&= 4(2\pi)^4 \hat{z}\, e_q^2 \, \frac{\alpha_s C_F}{2\pi} \Big[-8\,\delta(1-\hat{x})\delta(1-\hat{z}) \\
&\qquad - A'(\hat{x})\,\delta(1-\hat{z}) - \delta(1-\hat{x})\,B'(\hat{z}) \Big],
\end{aligned}$$

$$A'(\hat{x}) = \delta_T P_{qq}(\hat{x}) \ln\frac{\mu^2}{Q^2} + 2\hat{x}\left[\frac{\ln\hat{x}}{1-\hat{x}} - \left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+\right],$$

$$B'(\hat{z}) = \delta_T K_{qq}(\hat{z}) \ln\frac{\mu^2}{Q^2} - 2\hat{z}\left[\frac{\ln\hat{z}}{1-\hat{z}} + \left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+\right].$$

Back Up

Operator definition, and Dirac structure projections

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\boldsymbol{n};p_h) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi} e^{ip_h^+ y^-/z}\ \mathrm{Tr}\Big[\Gamma\langle 0|\, W(\infty,y^-;w)\,\psi(y^-)\,\mathcal{E}(\boldsymbol{n})\,|h,X;\mathrm{out}\rangle \times \langle h,X;\mathrm{out}|\,\bar{\psi}(0)\,W^\dagger(\infty,0;w)\,|0\rangle\Big],$$

$$\mathcal{D}^{[\gamma^+/2]}_{h/q,1}(z,\boldsymbol{n};p_h) = \mathcal{D}_{1,h/q}(z,\theta;p_h),$$

$$\mathcal{D}^{[\gamma^+\gamma^i\gamma_5/2]}_{h/q,1}(z,\boldsymbol{n};p_h) = \frac{(\hat{z}\times\boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|}\mathcal{H}^{\perp}_{1,h/q}(z,\theta;p_h) = (-\sin\phi,\cos\phi)^i\mathcal{H}^{\perp}_{1,h/q}(z,\theta;p_h),$$

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\Lambda_F,\phi) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi p_h^+} e^{ip_h^+ y^-/z}\ \mathrm{Tr}\Big[\Gamma\langle 0|\, W(\infty,y^-;w)\,\psi(y^-) \times \mathcal{E}_L(\eta,\phi)\,|h,X;\mathrm{out}\rangle\langle h,X;\mathrm{out}|\,\bar{\psi}(0)\,W^\dagger(\infty,0;w)\,|0\rangle\Big].$$

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\Lambda_F,\phi) = \frac{1+\tanh\eta}{\sqrt{2}\,p_h^+\cosh^2\eta}\,\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\boldsymbol{n};p_h).$$