26th International Symposium on Spin Physics

A Century of Spin

@青島!!

Physics-informed neural networks for angular momentum conservation in computational relativistic spin hydrodynamics

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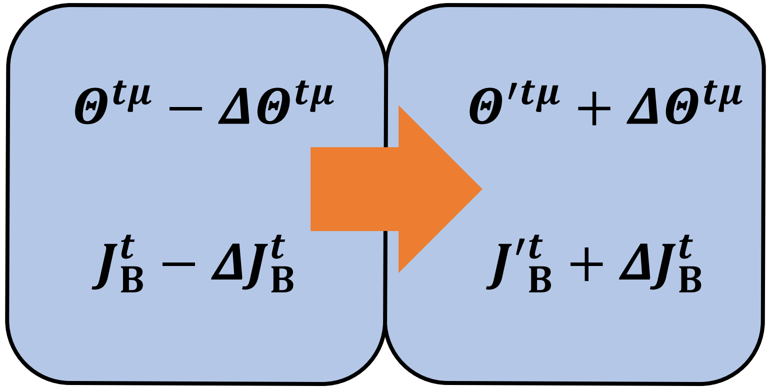
Numerical Challenges in Angular Momentum Conservation

- Finite Volume Method with Godunov's scheme is widely used in computational hydrodynamics, which enables solving continuity equations ($\partial_\mu \Theta^{\mu\nu} = 0, \partial_\mu J_{\mathrm{B}}^{\mu} = 0$) while preserving associated conservation laws within numerical accuracy.



- ***However***, angular momentum conservation is generally not guaranteed in this scheme…

→ Exploration of complementary numerical strategies

Physics-Informed Neural Networks (PINNs)

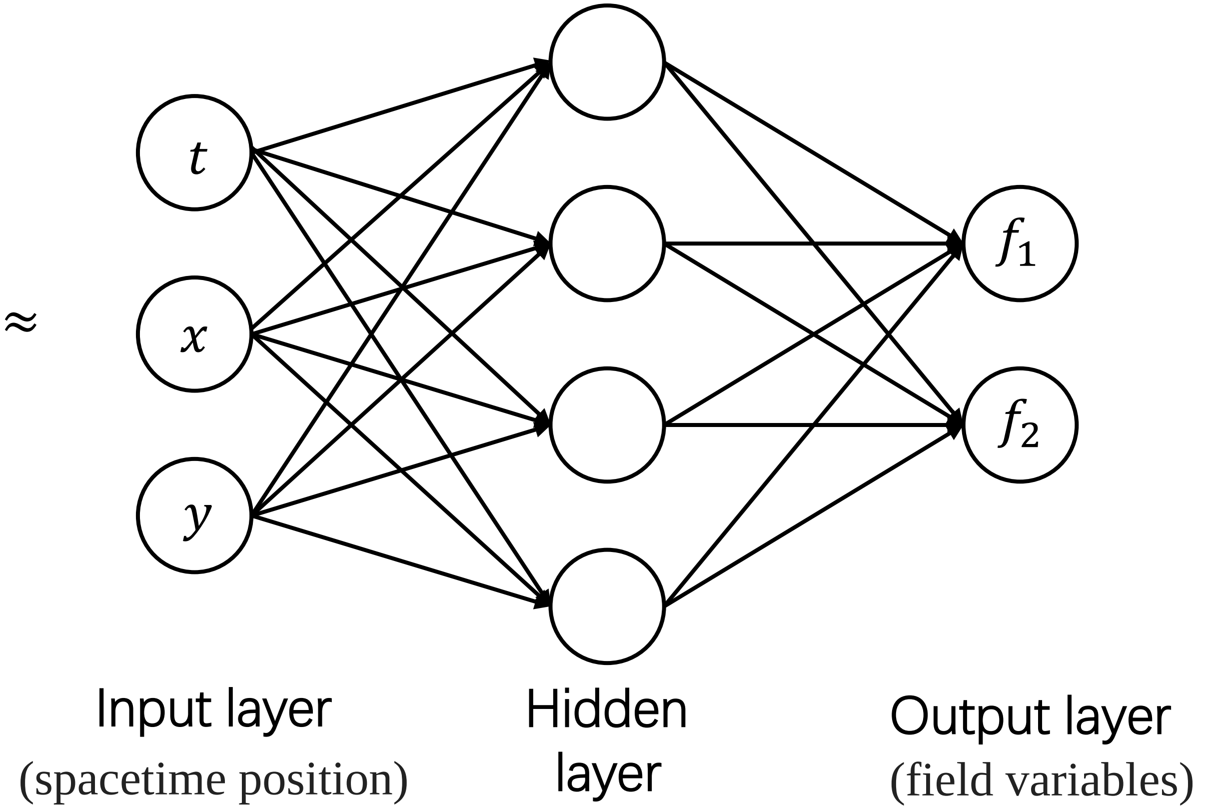
M. Raissi, et.al. (2019).

- Neural network approximate solutions of given partial differential equation (PDE): $\boldsymbol{\mathcal{F}}[f(t,\vec{x})] = 0$
- Loss function = PDE + boundary condition (B.C.) + optional constraint
- Active development of applications in hydrodynamics

M. Raissi, et.al. (2020); Z. Mao, et.al. (2020); X. Jin, et.al. (2021); S. Cai, et. al. (2021).

$$f_1(t,x,y),f_2(t,x,y) \quad \approx$$

(hydrodynamic fields)



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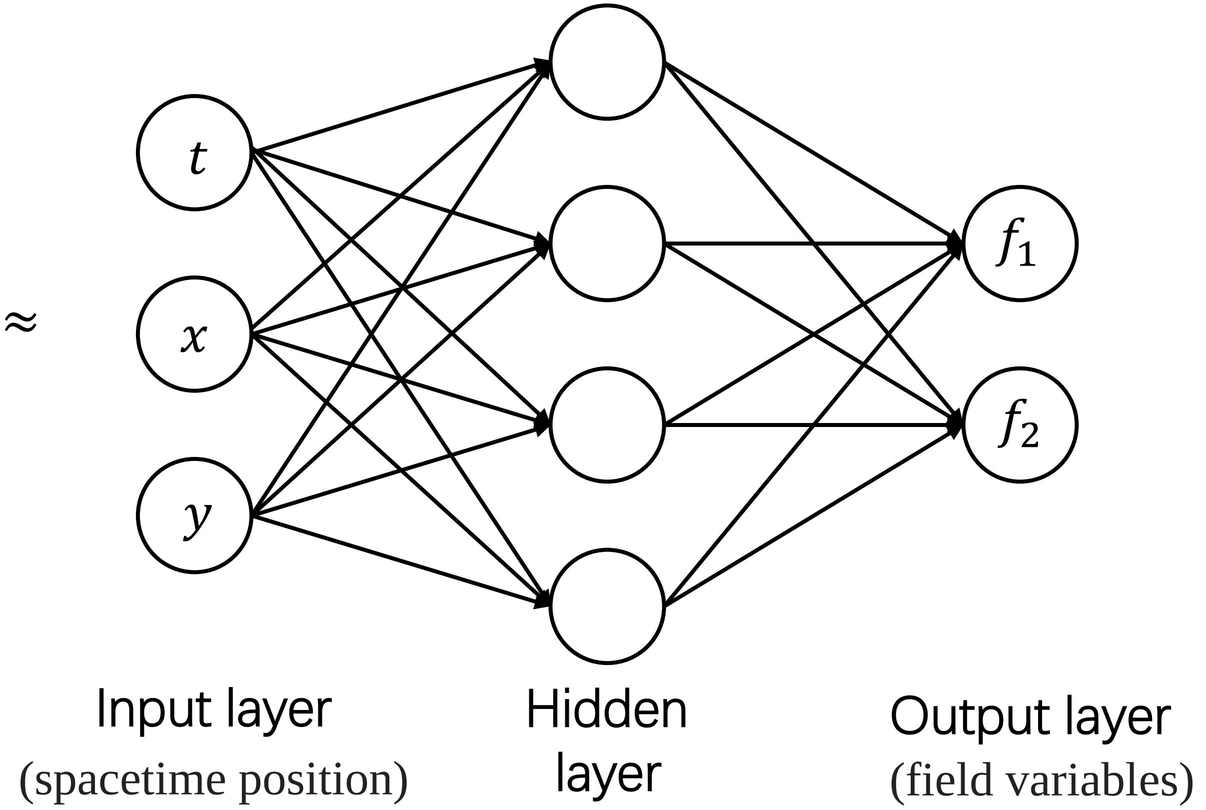

Physics-Informed Neural Networks (PINNs)

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- Neural network approximate solutions of given partial differential equation (PDE): $\boldsymbol{\mathcal{F}}[f(t,\vec{x})] = 0$
- Loss function = PDE + boundary condition (B.C.) + Angular momentum (AM) conservation
- Active development of applications in hydrodynamics

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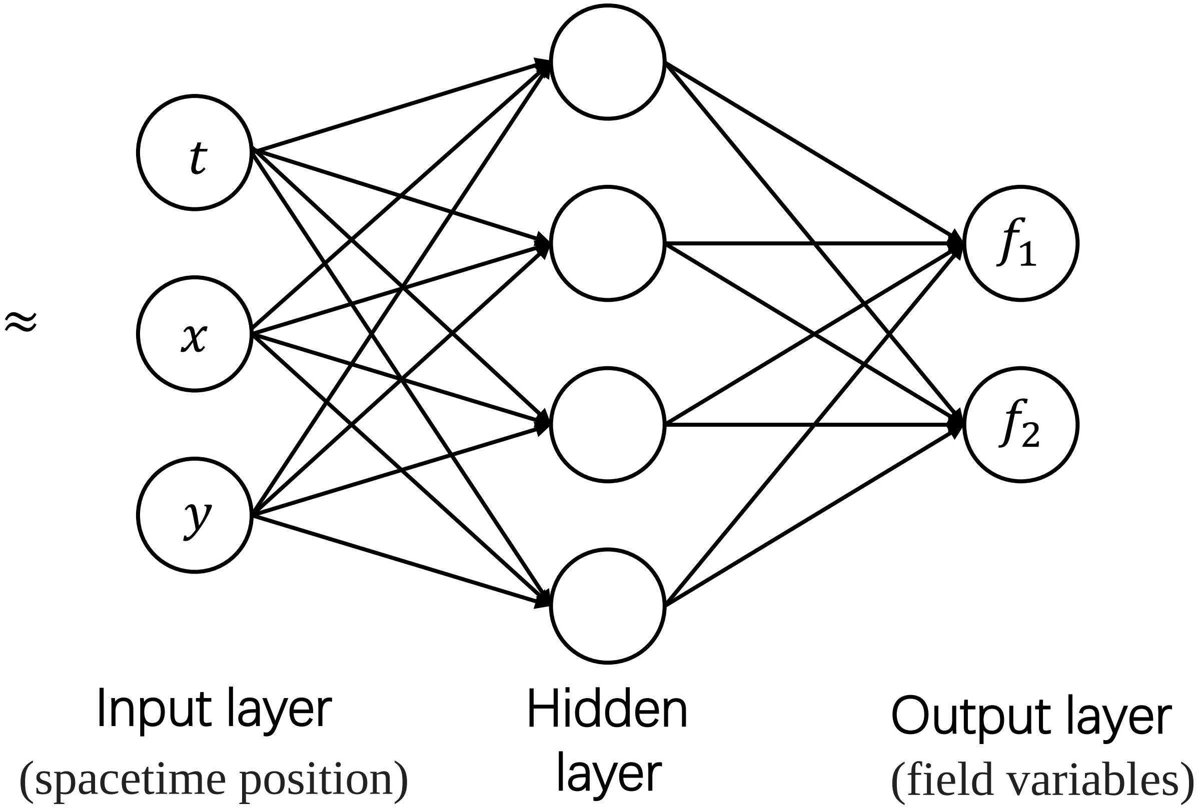


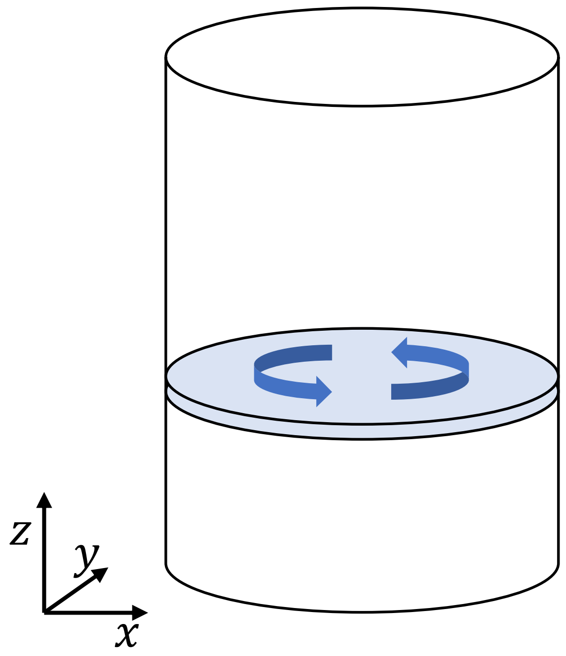
This Study

- ***In this study***, we build a PINNs-based framework for relativistic spin hydrodynamics
- As a proof of concept, we consider a simple setup: 2nd order relativistic spin fluid confined in a cylinder, with symmetry assumptions that reduce the problem to two dimensions (t, r).

✔ Cylindrical symmetry

✔ Translational symmetry along the *z*-direction

✔ Parity symmetry with respect to the *z*-axis



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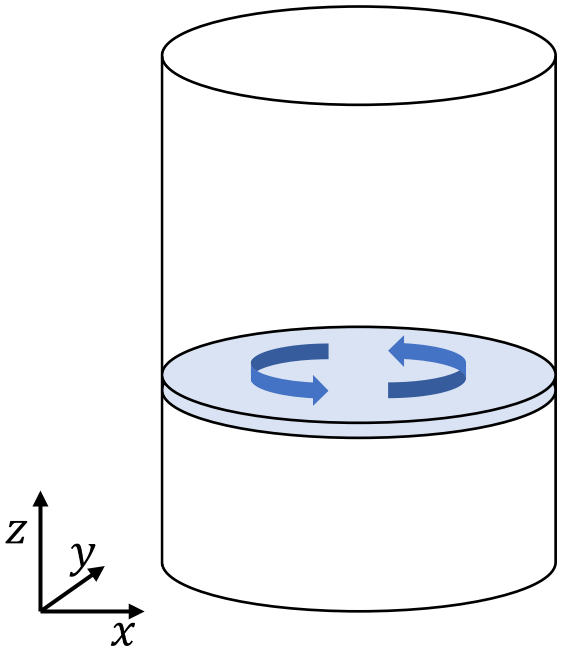
Relativistic Spin Hydrodynamic Equation

W. Florkowski, et.al. (2018).
X.-G. Huang (2024). [Review]

Hydrodynamic Variables

$$e(t,r), \quad u^r(t,r), \quad u^\theta(t,r), \quad S^z(t,r)(\equiv \Sigma^{txy}), \quad \phi^{r\theta}(t,r)$$



Tensor Decompositions (Landau frame and Totally antisymmetric pseudo gauge)

$$\Theta^{\mu\nu} = eu^\mu u^\nu + P\Delta^{\mu\nu} + \phi^{\mu\nu}$$

$$J^{\mu\nu\xi} = (x^\nu \Theta^{\mu\xi} - x^\xi \Theta^{\mu\nu}) + \Sigma^{\mu\nu\xi}$$

Hydrodynamic Equations (Continuity Equations and Constitutive Relations)

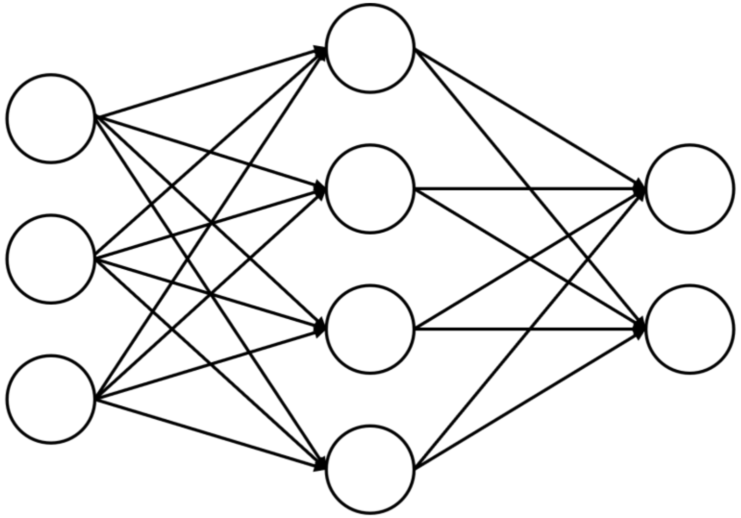
$$\nabla_\mu \Theta^{\mu t} = 0, \quad \nabla_\mu \Theta^{\mu x} = 0, \quad \nabla_\mu \Theta^{\mu y} = 0, \quad \nabla_\mu J^{\mu xy} = 0$$

$$\tau_\phi \Delta^r{}_\alpha \Delta^\theta{}_\beta D\phi^{xy} = -2\gamma\rho^{xy} - \phi^{xy} - \frac{2\tau_\phi}{3}(\nabla \cdot u)\phi^{xy}$$

(Muller-Israel-Stewart Relaxation)

I. Muller (1967); W. Israel (1976); W. Israel and J. M. Stewart (1976,1979)

Specific Design of PINNs



Input (2 neurons) : $t, \; r$

Output (5 neurons) : $NN_1, NN_2, \cdots, NN_5$

e. g. $\quad e(t,r) \approx e(t=0,r) + NN_1(t,r) - NN_1(t=0,r)$

Loss Function : $L(\psi) = L_{\mathrm{Hydro}}(\psi) + L_{\mathrm{B.C.}}(\psi) + L_{AM}(\psi)$

e. g. simplest loss form

$$L(\psi) = \sum_i \langle R_{\mathrm{Hydro},i} \rangle^2 + \sum_i \langle R_{\mathrm{B.}C.,i} \rangle^2 + \sum_i \langle R_{\mathrm{AM.},i} \rangle^2$$

$\langle \quad \rangle$: spacetime average

Residual

- $R_{\mathrm{Hydro},i}$: hydrodynamic equations, e. g. $\nabla_\mu \Theta^{\mu\nu}(t,r)$
- $R_{\mathrm{B.C.},i}$: Dirichlet boundary condition, e. g. $\partial_r u^r(t, r=R)$
- $R_{\mathrm{AM},i}$: local AM conservation, $\nabla_\mu J^{\mu xy}(t,r)$

  global AM conservation, $\int dr\, r\, J^{txy}(t,r) - \int dr\, r\, J^{txy}(t=0,r)$

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Verification: Setup

Calculation Setup

- Hidden layer: 1
- Neuron (hidden layer): 250
- Collocation points: 75,000
- Activation function: tanh
- Optimizer: Adam

Physics Setup

- Initial condition

$$e(t=0,r)=\bar{e}\ ,$$
$$u^r(t=0,r)=0\ ,$$
$$u^\theta(t=0,r)=\delta_1\cdot\sin^4\left(\frac{\pi r}{R}\right)\ ,$$
$$S^z(t=0,r)=0\ ,$$
$$\phi^{r\theta}(t=0,r)=0\ ,$$

- Boundary condition

Dirichlet type:
All variables for $r=0$ and $u^r, s^z, \phi^{r\theta}$ for $r=R$

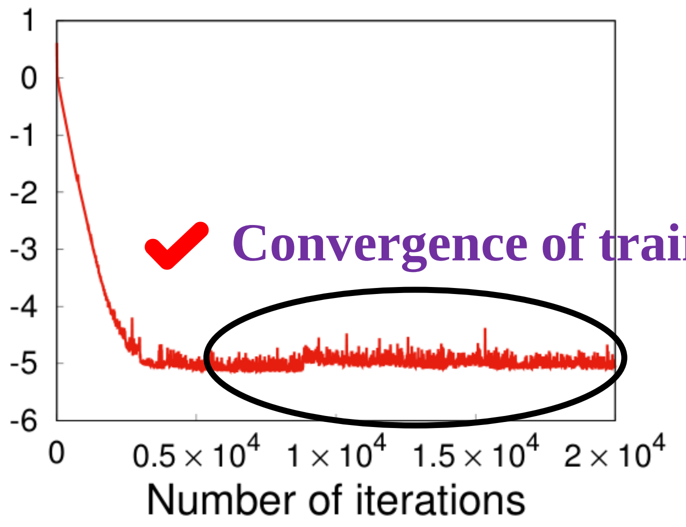
Neumann: $\partial_r u^r, \partial_r \phi^{r\theta}$ for $r=R$

- Parameters

$\bar{e}$	1
R	1
$t_{\max}$	0.15
γ_ϕ	2
τ_ϕ	2
δ_1	0.2

Verification: Results

Loss function
$L(\psi)$



✔ Convergence of training

$$e(t=0,r)=\bar{e}\,,$$
$$u^r(t=0,r)=0\,,$$
$$u^\theta(t=0,r)=\delta_1\cdot\sin^4\left(\frac{\pi r}{R}\right)\,,$$
$$S^z(t=0,r)=0\,,$$
$$\phi^{r\theta}(t=0,r)=0\,,$$

$\bar{e}$	1
R	1
$t_{\max}$	0.15
γ_ϕ	2
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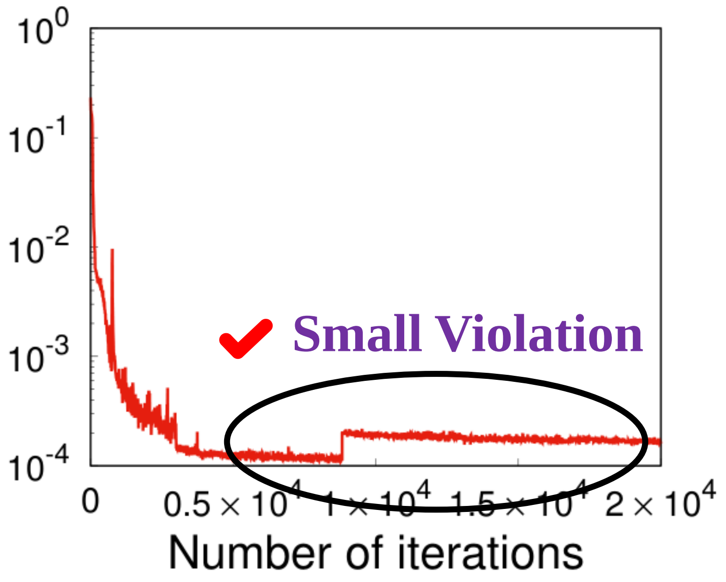
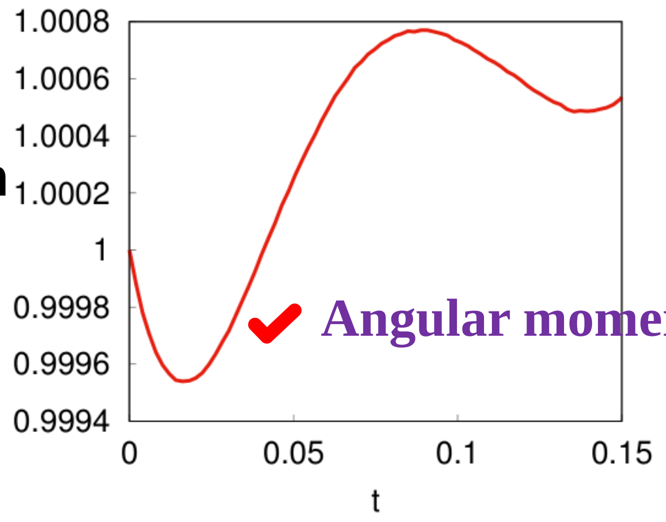
δ_1	0.2

Residual for Hydro eq.
$$\sum_i \frac{\int dr(2\pi r)\left|R_{\mathrm{Hydro},i}\right|^2}{\mathrm{Volume}}$$



✔ Small Violation

Net angular momentum
$$\frac{\int dr\, r\, J^{txy}(t,r)}{\int dr\, r\, J^{txy}(t=0,r)}$$



✔ Angular momentum conserved within 0.1%

Application: Setup

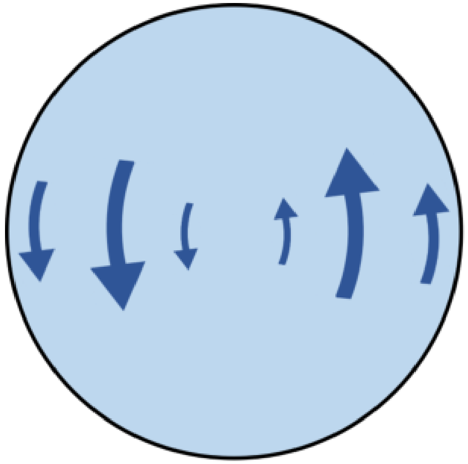
Case 1. Orbital Initial Condition

✔ Motivated by QGP in high-energy heavy-ion collisions

Z.-T. Liang and X.-N. Wang (2005).

$$e(t=0,r)=\bar{e}\,,$$
$$u^r(t=0,r)=0\,,$$
$$u^\theta(t=0,r)=\delta_1\cdot\sin^4\left(\frac{\pi r}{R}\right)\,,$$
$$S^z(t=0,r)=0\,,$$
$$\phi^{r\theta}(t=0,r)=0\,.$$

$\bar{e}$	1
R	1
$t_{\max}$	0.4
γ_ϕ	0, 2
τ_ϕ	γ_ϕ
δ_1	0.2



$t=0$

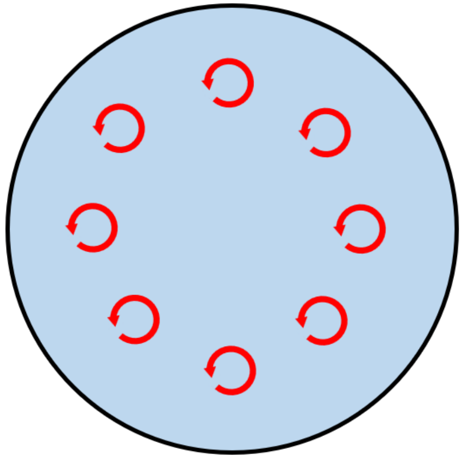
Case 2. Spin Initial Condition

✔ Motivated by Einstein-de Haas effect

A. Einstein and W. De Haas (1915).

$$e(t=0,r)=\bar{e}\,,$$
$$u^r(t=0,r)=0\,,$$
$$u^\theta(t=0,r)=0\,,$$
$$S^z(t=0,r)=\delta_2\cdot\sin^4\left(\frac{\pi r}{R}\right)\,,$$
$$\phi^{r\theta}(t=0,r)=0\,.$$

$\bar{e}$	1
R	1
$t_{\max}$	0.4
γ_ϕ	2
τ_ϕ	2
δ_2	0.2

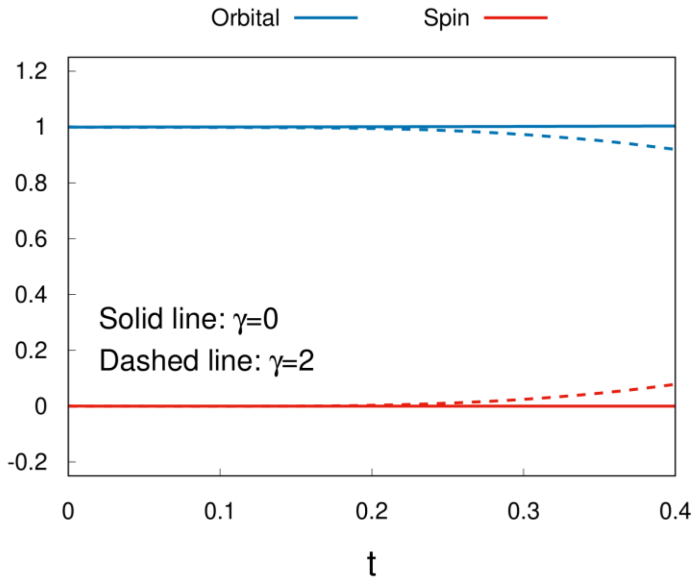


$t=0$

Application: Net AM

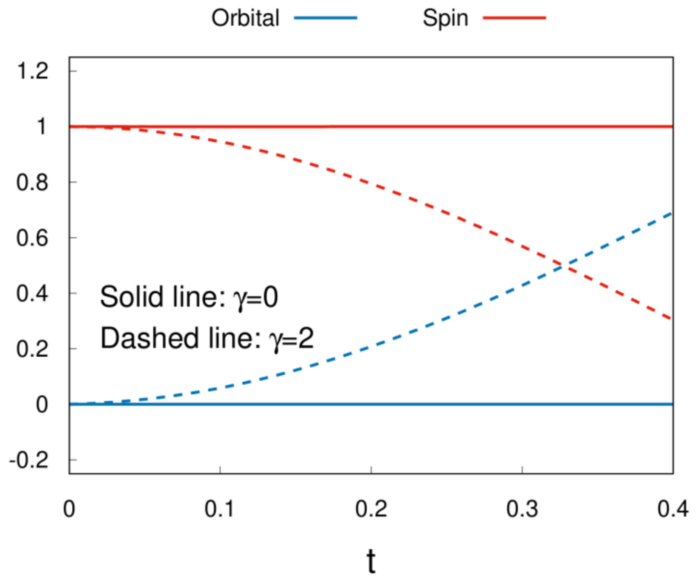
Case 1. Orbital Initial Condition

- ✔ Orbital-to-Spin AM conversion at $\gamma = 2$



Case 2. Spin Initial Condition

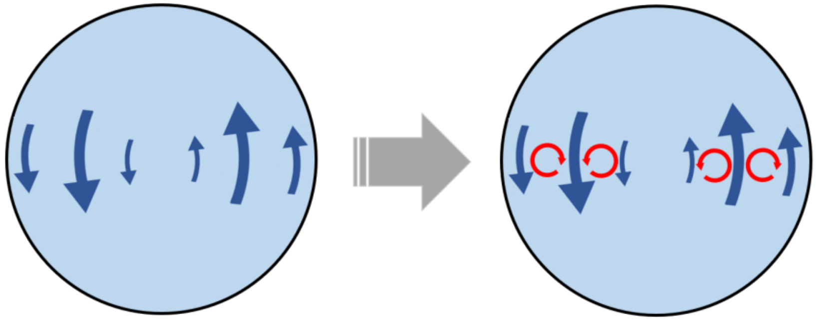
- ✔ Spin-to-Orbital AM conversion at $\gamma = 2$

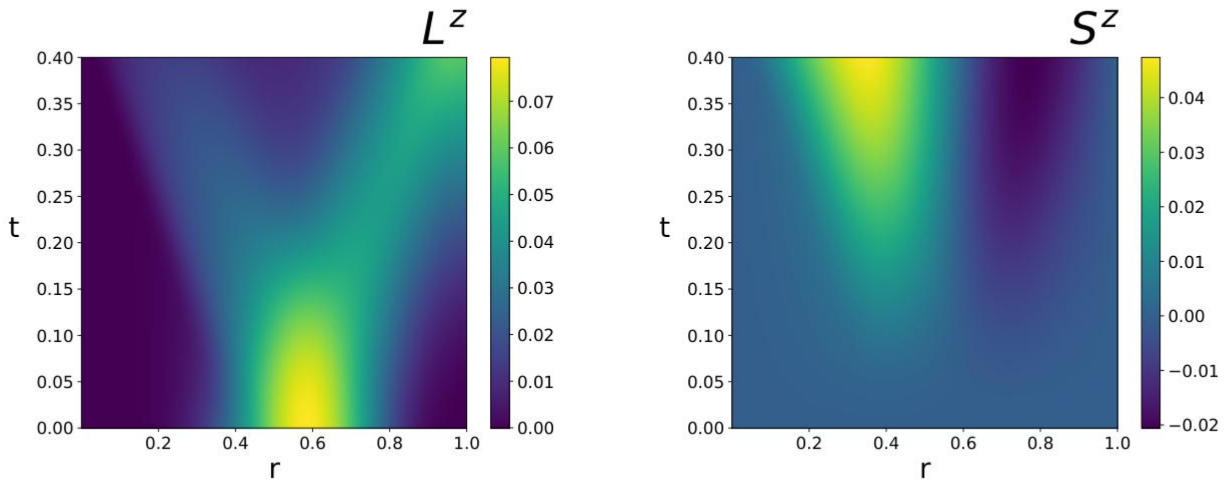


Application: Heatmap of AM

Case 1. Orbital Initial Condition

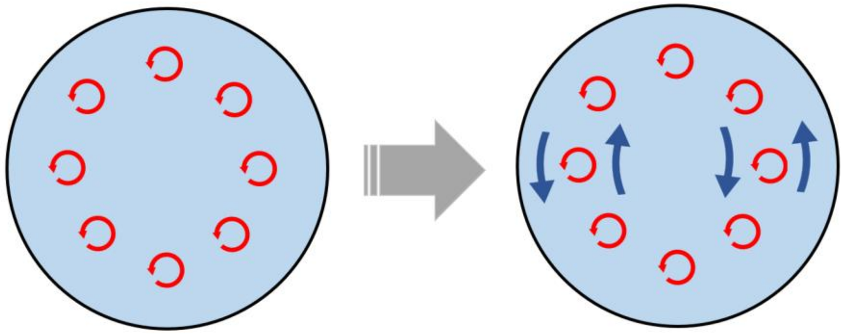
- ✔ Generated spin has opposite signs for $r > 0.5R$ and $r < 0.5R$

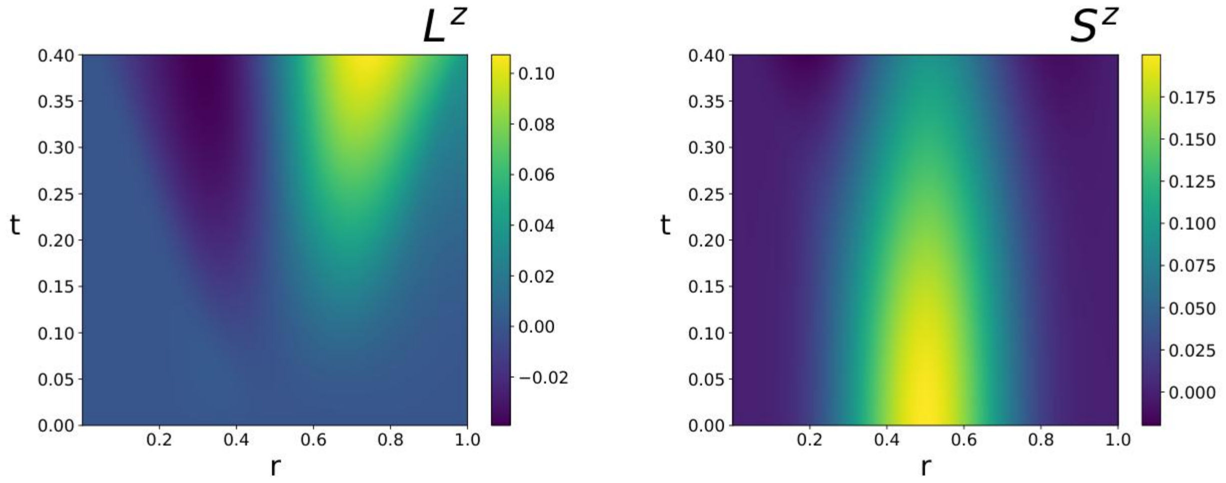




Case 2. Spin Initial Condition

- ✔ Generated orbital spin has opposite signs for $r > 0.5R$ and $r < 0.5R$





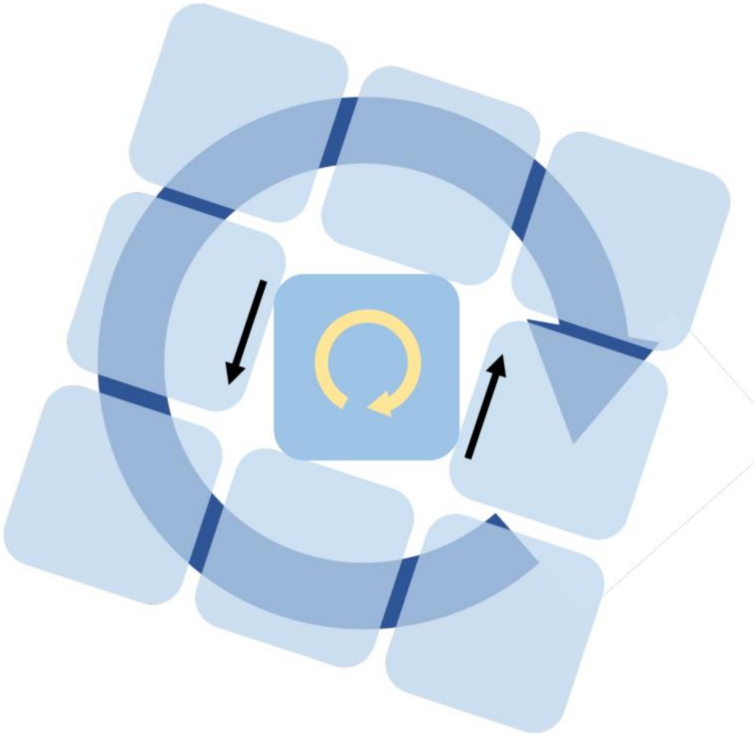
Application: Discussion

- **Rotational viscous correction** is generated by the difference between **transversely projected thermal vorticity** ($\varpi_\perp$) and **spin potential** ($\omega^{\mu\nu}$)

$$\tau_\phi \Delta^r{}_\alpha \Delta^\theta{}_\beta D\phi^{\alpha\beta} = -2\gamma\rho^{r\theta} - \phi^{r\theta} - \frac{2\tau_\phi}{3}\theta\phi^{r\theta}$$

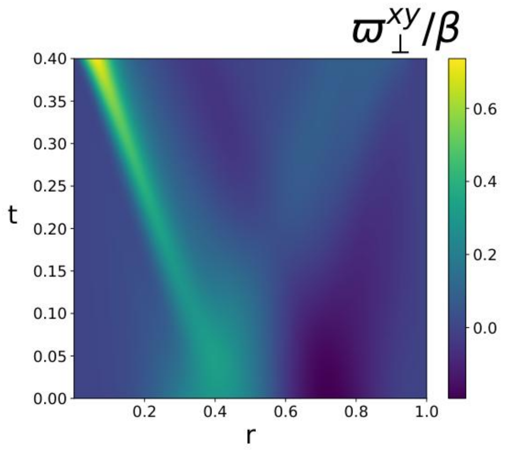
source term

$$\rho^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_{[\alpha}\beta_{\beta]} - 2\omega_{\alpha\beta}\right) = \beta^{-1}\varpi_\perp^{\mu\nu} - 2\omega^{\mu\nu}$$

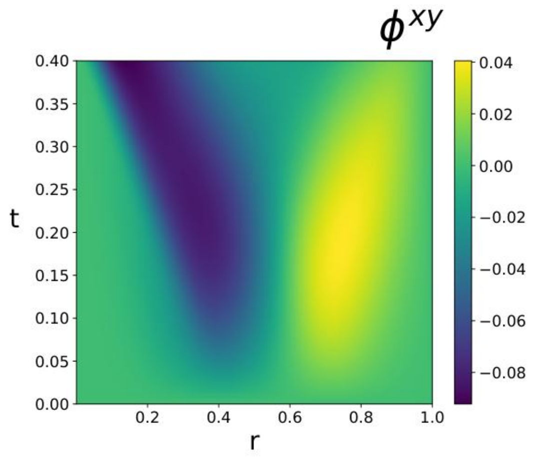


Application: Conversion Process

Case 1. Orbital Initial Condition

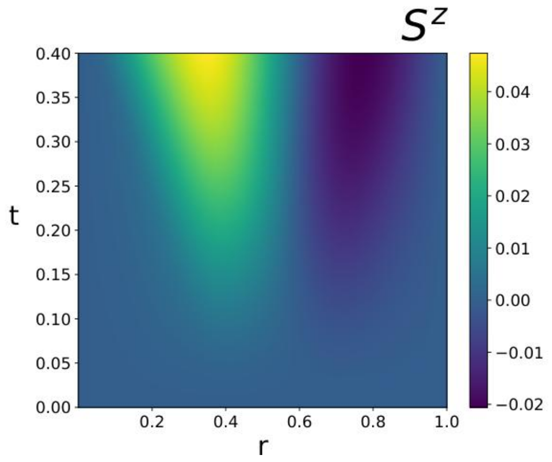


Initial ϖ generate ϕ

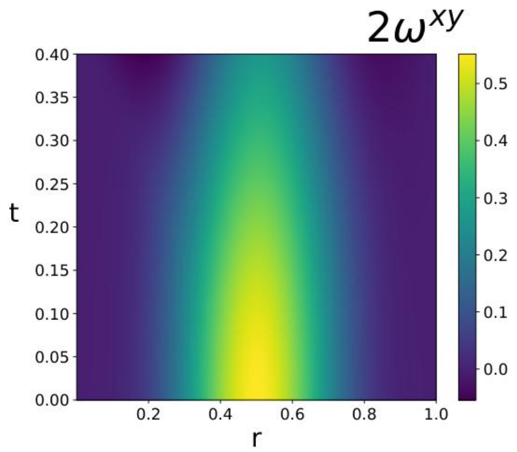


ϕ generate S^z

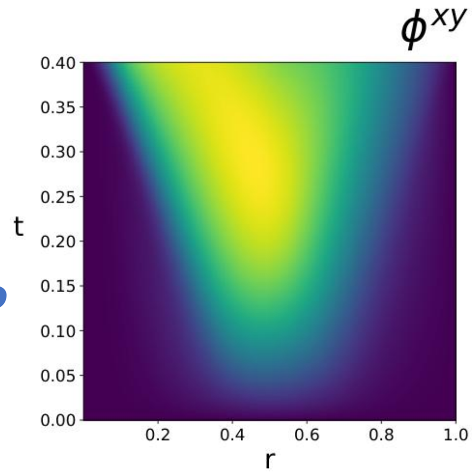
$$\left(\partial_t S^z = -2\phi^{xy}\right)$$



Case 2. Spin Initial Condition

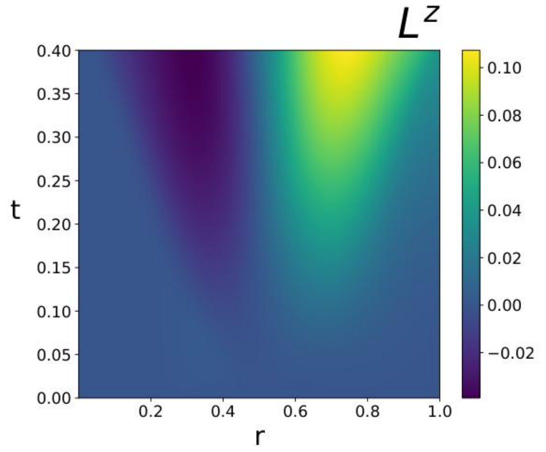


Initial ω generate ϕ



ϕ generate L^z

$$\left(\frac{4}{3}e\partial_t r u^\theta \sim -\partial_r \phi^{xy}\right)$$



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Summary & Outlook

Summary

- We have developed a PINNs-based framework for relativistic spin hydrodynamics.
- Angular momentum conservation is found to be successfully enforced in the training process.
- Our proof-of-concept simulations have demonstrated orbital–spin angular momentum conversion, which is driven by the rotational effect.
- Our demonstration has illustrated that the rotational viscous effect in second-order viscous hydrodynamics can be interpreted as a friction between a rotating fluid cell and the surrounding rotational flow.

Outlook

- Compare with linear analyses to deepen the understanding of nonlinear effects
- Examine system behavior when treating spin potential and thermal vorticity at zeroth order, or when incorporating second-order effects such as those induced by a boosted heat current
- Apply the framework to the analysis of high-energy heavy-ion collisions (long-term goal)