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Global tensor polarizations of spin-3/2 hadrons and quark spin correlations in relativistic heavy ion collisions

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- **Zhe Zhang, Ji-peng Lv, Zi-han Yu, Zuo-tang Liang, Phys. Rev. D 110 (2024) 7, 074019**

Outline

1 Introduction

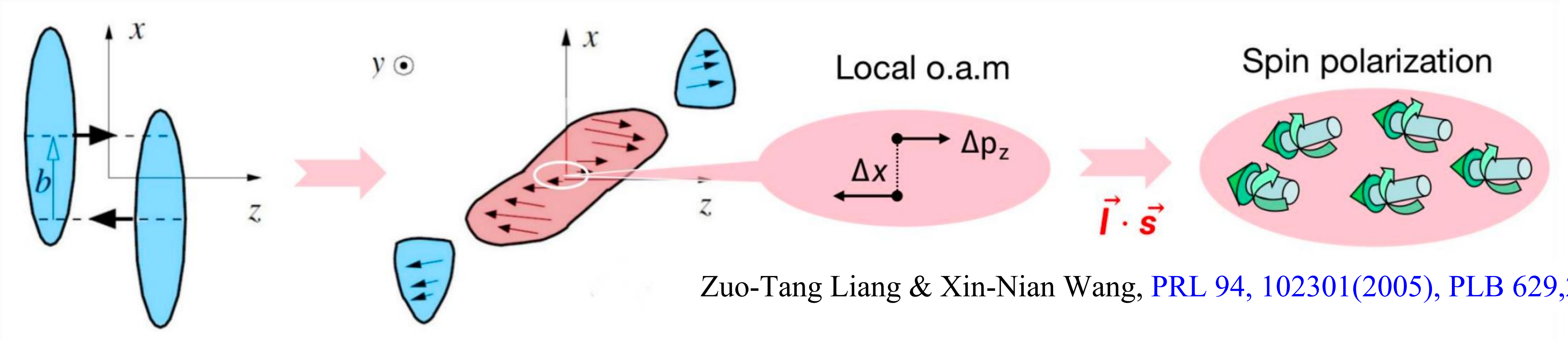
2 Description and measurment of the polarizations of spin-3/2 baryon

3 Spin density matrix of three quark system in HIC

4 Calculation with quark combination mechanism

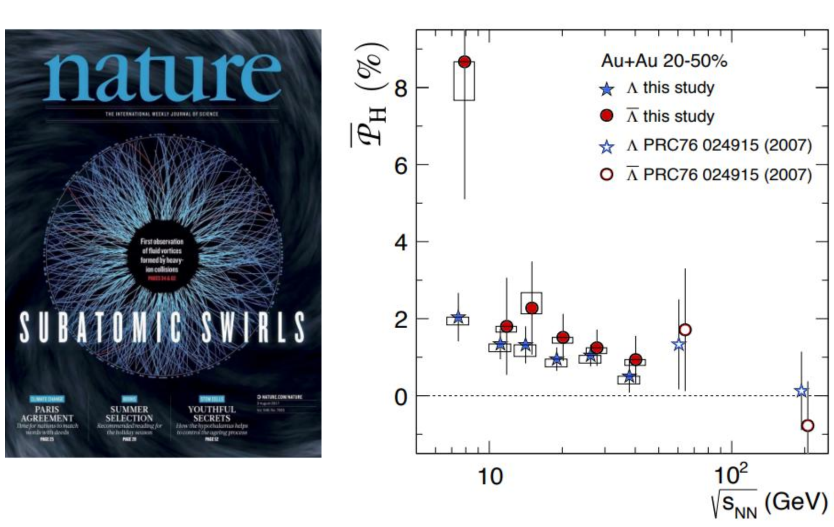
5 Summary and outlook

1 Introduction

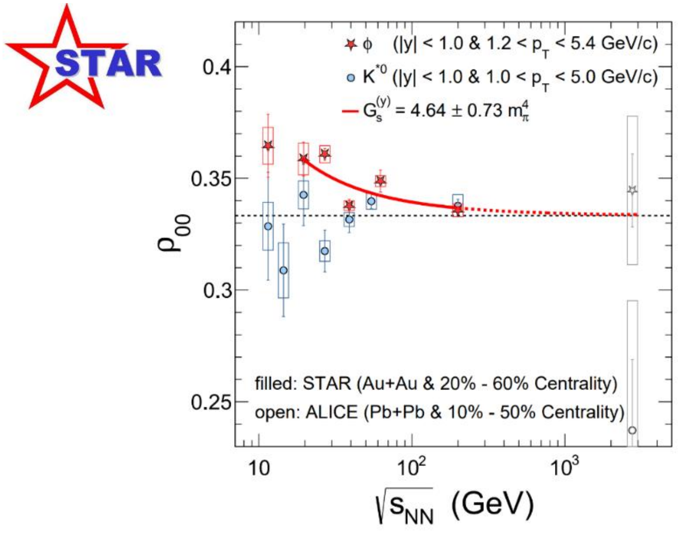


Zuo-Tang Liang & Xin-Nian Wang, PRL 94, 102301(2005), PLB 629,20(2005)

STAR, L. Adamczyk et al.,**Nature 548, 62-65 (2017).**



STAR, M.S. Abdallah et al., **Nature 614, 244 (2023).**



Confirmed:

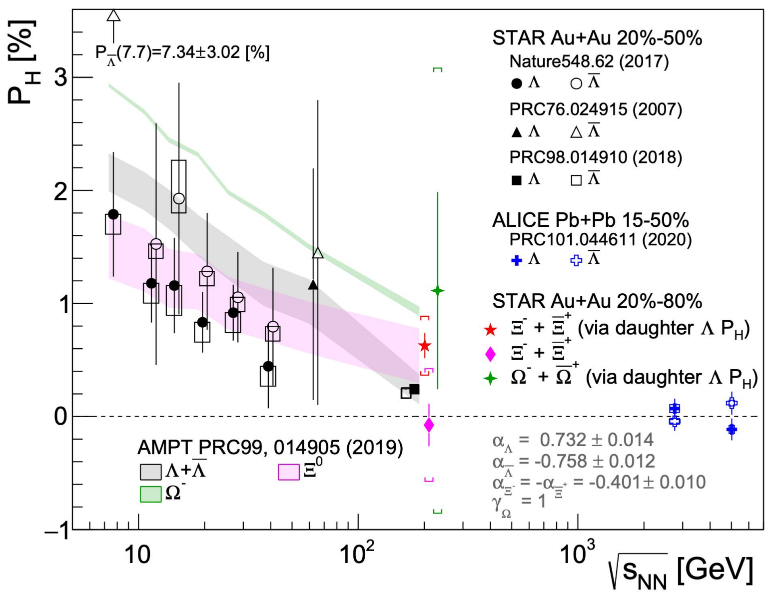
Global polarization effect

Revealed:

Strong spin correlations

1 Introduction

Completed and ongoing measurements of vector polarization of spin-3/2 particles



STAR, J. Adam et al., **PRL 126, 162301 (2021).**

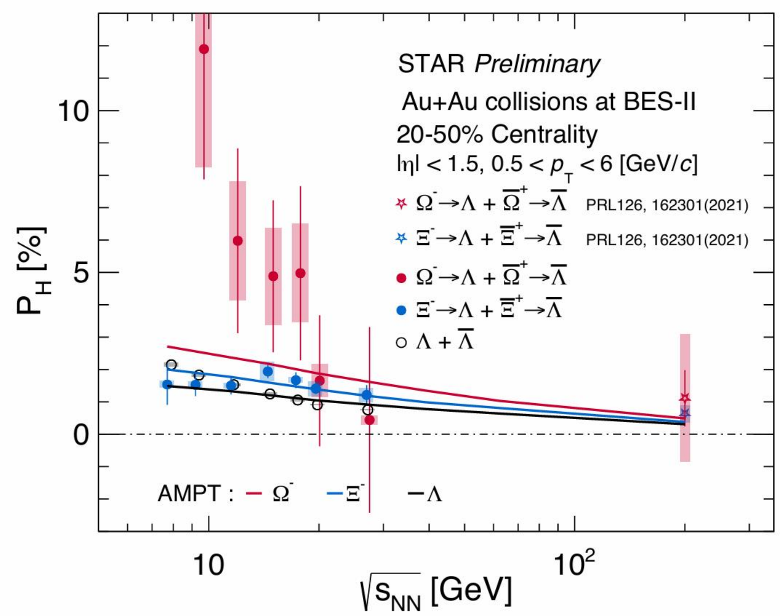
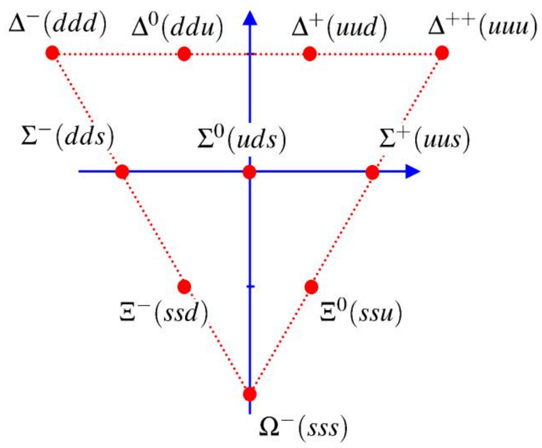


Figure from Xing-rui Gou in QM



A systematic study of spin-3/2 baryons should also include tensor polarizations.

Outline

2 Description of the polarization of spin-3/2 baryon

Spin 1/2 The spin density matrix (2×2): $\hat{\rho} = \frac{1}{2}(1 + S_i \sigma_i)$ Only vector polarization: $S^\mu = (0, \vec{S}_T, S_L)$

Spin 1 See e.g. A.Bacchetta, & P.J. Mulders, PRD62, 114004 (2000)

The spin density matrix (3×3): $\hat{\rho} = \frac{1}{3}(1 + \frac{3}{2}S^i\Sigma^i + 3T^{ij}\Sigma^{ij})$

Vector polarization: $S^\mu = (0, \vec{S}_T, S_L)$

Tensor polarization: $S_{LL}, S_{LT}^i = (S_{LT}^x, S_{LT}^y),\ S_{ij}^{TT} = \begin{pmatrix} S_{TT}^{xx} & S_{TT}^{xy} \\ S_{TT}^{xy} & -S_{TT}^{xx} \end{pmatrix}$

Spin alignment is studied by Tensor polarization component S_{LL}

$$S_{LL} = \frac{1}{2}(1 - 3\rho_{00})$$

Spin 3/2 See e.g. Jing Zhao, Zhe Zhang, Zuo-Tang Liang, Tianbo Liu, Ya-jin Zhou, PRD106, 094006 (2022)

The spin density matrix (4×4): $\hat{\rho} = \frac{1}{4}(1 + \frac{4}{5}S^i\Sigma^i + \frac{2}{3}T^{ij}\Sigma^{ij} + \frac{8}{9}R^{ijk}\Sigma^{ijk})$

Vector polarization: $S^\mu = (0, \vec{S}_T, S_L)$

Rank 2 Tensor polarization: $S_{LL}, S_{LT}^i = (S_{LT}^x, S_{LT}^y), S_{TT}^{ij} = \begin{pmatrix} S_{TT}^{xx} & S_{TT}^{xy} \\ S_{TT}^{xy} & -S_{TT}^{xx} \end{pmatrix}$

Rank 3 Tensor polarization: $S_{LLL}, S_{LLT}^i = (S_{LLT}^x, S_{LLT}^y),\ S_{LTT}^{ij} = \begin{pmatrix} S_{LLT}^{xx} & S_{LLT}^{xy} \\ S_{LTT}^{xy} & -S_{LTT}^{xx} \end{pmatrix},\ S_{TTT}^{ijx} = \begin{pmatrix} S_{TTT}^{xxx} & S_{TTT}^{yxx} \\ S_{TTT}^{yxx} & -S_{TTT}^{xxx} \end{pmatrix}$

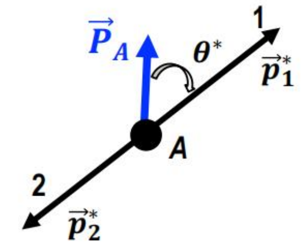
2 Mearment of the polarization of spin-3/2 baryon

For the strong decay $A \to 1+2$ such as $\Delta \to N + \pi$

$$W(\theta_N,\phi_N) \sim 2 + S_{LL}(1-3cos^2\theta_N)$$
$$-(S^x_{LT}cos\phi + S^y_{LT}sin\phi)sin2\theta_N - (S^x_{LTT}cos2\phi + S^{xy}_{LTT}sin2\phi)sin^2\theta_N$$

$$W(\theta_N) \sim 2 + S_{LL}(1-3cos^2\theta_N)$$

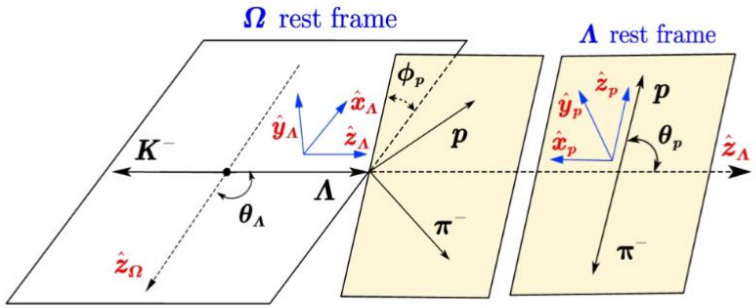
$A \to 1 + 2$



For the strong decay , followed by the weak decay

such as $\Sigma^* \to \Lambda + \pi^-, \Lambda \to p + \pi^-$

$$W(\theta_\Lambda,\theta_p) \sim 1 + \frac{2}{5}\alpha_\Lambda S_L cos\theta_\Lambda cos\theta_p - \frac{1}{4}S_{LL}(1+3cos2\theta_\Lambda)$$
$$-\frac{1}{4}\alpha_\Lambda S_{LLL}(3cos\theta_\Lambda + 5cos3\theta_\Lambda)cos\theta_p$$



For the weak decay , followed by the weak decay

such as $\Omega^- \to \Lambda + K^-, \Lambda \to p + \pi^-$

$$W(\theta_\Lambda,\theta_p) \sim (1+\alpha_\Omega\alpha_\Lambda cos\theta_p)[1-\frac{1}{4}S_{LL}(1+3cos2\theta_\Lambda)]$$
$$+[\frac{2}{5}S_L cos\theta_\Lambda - \frac{1}{4}S_{LLL}(3cos\theta_\Lambda + 5cos3\theta_\Lambda)](\alpha_\Omega + \alpha_\Lambda cos\theta_p)$$

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Outline

Spin density matrix of three quark system in HIC

We now try to build spin density matrix for multi-quark system

- Single particle： basis $\{\mathbb{I}, \hat{\sigma}_{1i}\}$ $\quad \hat{\rho}^{(1)} = \frac{1}{2}[\mathbb{I} + P_{1i}\hat{\sigma}_{1i}]$

- 2 particle system： basis $\{\mathbb{I}_1, \hat{\sigma}_{1i}\} \otimes \{\mathbb{I}_2, \hat{\sigma}_{2j}\}$

Correlation term

$$\hat{\rho}^{(12)} = \frac{1}{2^2}[\mathbb{I}_1 \otimes \mathbb{I}_2 + P_{1i}\hat{\sigma}_{1i} \otimes \mathbb{I}_2 + P_{2j}\mathbb{I}_1 \otimes \hat{\sigma}_{2j} + t_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}]$$

No spin correlation, $\hat{\rho}^{(12)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}$, $t_{ij}^{(12)} = P_{1i}P_{2j} \neq 0$ Define： $\longrightarrow$ $c_{ij}^{(12)} = t_{ij}^{(12)} - P_{1i}P_{2j}$

$$\hat{\rho}^{(12)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)} + \frac{1}{2^2}c_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

- 3 particle system spin density matrix：

$$\begin{aligned}\hat{\rho}^{(123)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)} \otimes \hat{\rho}^{(3)} &+ \frac{1}{2^2}[c_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\rho}^{(3)} \\ &+ c_{jk}^{(23)}\hat{\rho}^{(1)} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k} + c_{ik}^{(13)}\hat{\sigma}_{1i} \otimes \hat{\rho}^{(2)} \otimes \hat{\sigma}_{3k}] \\ &+ \frac{1}{2^3}c_{ijk}^{(123)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k}\end{aligned}$$

Spin density matrix of three quark system in HIC

We now consider an additional degree of freedom α

- Single particle：

$$\hat{\rho}^{(1)}(\alpha) = \frac{1}{2}[\mathbb{I} + P_{1i}(\alpha_1)\hat{\sigma}_{1i}]$$

- 2 particle system：

$$\hat{\rho}^{(12)}(\alpha_1,\alpha_2) = \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2) + \frac{1}{2^2}c_{ij}^{(12)}(\alpha_1,\alpha_2)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

- 3 particle system spin density matrix：

$$\begin{aligned}\hat{\rho}^{(123)}(\alpha_1,\alpha_2,\alpha_3) = {} & \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2) \otimes \hat{\rho}^{(3)}(\alpha_3) + \frac{1}{2^2}[c_{ij}^{(12)}(\alpha_1,\alpha_2)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\rho}^{(3)}(\alpha_3) \\ & + c_{jk}^{(23)}(\alpha_2,\alpha_3)\hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k} + c_{ik}^{(13)}(\alpha_1,\alpha_3)\hat{\sigma}_{1i} \otimes \hat{\rho}^{(2)}(\alpha_2) \otimes \hat{\sigma}_{3k}] \\ & + \frac{1}{2^3}c_{ijk}^{(123)}(\alpha_1,\alpha_2,\alpha_3)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k}\end{aligned}$$

Spin density matrix of three quark system in HIC

Measure directly quark α ✖ Average quark α at fixed system α ✔

- Assume the final state at $|\alpha_f\rangle$

$$\hat{\bar\rho}^{(12)}(\alpha_f) \equiv \langle \alpha_f|\hat\rho^{(12)}(\alpha_1,\alpha_2)|\alpha_f\rangle = \langle \hat\rho^{(12)}(\alpha_1,\alpha_2)\rangle$$
$$= \underline{\langle \hat\rho^{(1)}(\alpha_1)\otimes\hat\rho^{(2)}(\alpha_2)\rangle} + \frac{1}{2^2}\langle c_{ij}^{(12)}(\alpha_1,\alpha_2)\rangle\hat\sigma_{1i}\otimes\hat\sigma_{2j}$$

Average inside the final state particle

If we decompose in the same way before

$$\hat{\bar\rho}^{(12)}(\alpha_f) = \underline{\hat{\bar\rho}^{(1)}(\alpha_f)\otimes\hat{\bar\rho}^{(2)}(\alpha_f)} + \frac{1}{2^2}\bar c_{ij}^{(12)}(\alpha_f)\hat\sigma_{1i}\otimes\hat\sigma_{2j}\ \text{, where } \hat{\bar\rho} = \frac{1}{2}(\mathbb{I}+\bar P_i\hat\sigma_i)$$

Inconsistency $\langle \hat\rho^{(1)}(\alpha_1)\otimes\hat\rho^{(2)}(\alpha_2)\rangle \neq \hat{\bar\rho}^{(1)}(\alpha_{12})\otimes\hat{\bar\rho}^{(2)}(\alpha_{12})$

Consequence

$$\bar c_{ij}^{(12)} = \langle c_{ij}^{(12)}\rangle + \bar c_{ij}^{(12;0)}(\alpha_{12})$$

"effective correlation" = "genuine correlation" + "induced correlation"

the observed   dynamical process   due to average over α_i

$$\bar c_{ij}^{(12;0)} \equiv \langle \hat\rho^{(1)}(\alpha_1)\otimes\hat\rho^{(2)}(\alpha_2)\rangle - \hat{\bar\rho}^{(1)}(\alpha_{12})\otimes\hat{\bar\rho}^{(2)}(\alpha_{12}) = \langle P_{1i}(\alpha_1)P_{2j}(\alpha_2)\rangle - \langle P_{1i}(\alpha_1)\rangle\langle P_{2j}(\alpha_2)\rangle$$

Spin density matrix of three quark system in HIC

Same partern for 3 particle system spin density matrix

$$\hat{\bar{\rho}}^{(123)} = \hat{\bar{\rho}}^{(1)} \otimes \hat{\bar{\rho}}^{(2)} \otimes \hat{\bar{\rho}}^{(3)} + \frac{1}{2^2}[\bar{c}_{ij}^{(12)} \hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\bar{\rho}}^{(3)}$$
$$+\bar{c}_{ij}^{(23)} \hat{\bar{\rho}}^{(1)} \otimes \hat{\sigma_{2j}} \otimes \hat{\sigma}_{3k} + \bar{c}_{ik}^{(13)} \hat{\sigma}_{1i} \otimes \hat{\bar{\rho}}^{(2)} \otimes \hat{\sigma}_{3k}]$$
$$+\frac{1}{2^3} \bar{c}_{ijk}^{(123)} \hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k}$$

Where $\bar{c}_{ijk}^{(123)} = \langle c_{ijk}^{(123)} \rangle + \bar{c}_{ijk}^{(123;1)}$

More terms raising from the average

And $$\bar{c}_{ijk}^{(123;1)} = \langle P_{1i} P_{2j} P_{3k} + c_{ij}^{(12)} P_{3k} + c_{ik}^{(13)} P_{2j} + c_{jk}^{(23)} P_{1i} \rangle$$
$$-\bar{P}_{1i} \bar{P}_{2j} \bar{P}_{3k} - \bar{c}_{ij}^{(12)} \bar{P}_{3k} - \bar{c}_{ik}^{(13)} \bar{P}_{2j} - \bar{c}_{jk}^{(23)} \bar{P}_{1i}$$

Outline

Calculation with quark combination mechanism

For combination of $q_1q_2q_3 \to B$, we have $\hat{\rho}^B = \hat{M}\hat{\rho}^{(q_1q_2q_3)}\hat{M}^\dagger$ $\hat{M}$: transition operator

The initial 3 quark state ($\hat{\rho}^{(q_1q_2q_3)}$) evolve ($\hat{M}$) into final state spin-3/2 baryon ($\hat{\rho}^B$).

Then for matrix element of the spin-3/2 baryon

$$\rho^B_{mm'}(\alpha_B) = \langle j_B m, \alpha_B|\hat{M}\hat{\rho}^{(q_1q_2q_3)}(\alpha_n)\hat{M}^\dagger|j_B m', \alpha_B\rangle$$

Apply α_B to $\hat{\rho}^{(q_1q_2q_3)}$ to obtain $\hat{\bar{\rho}}^{(q_1q_2q_3)}$ if $|j_B m', \alpha_B\rangle = |j_B m'\rangle \otimes |\alpha_B\rangle$

$$= \langle j_B m|\hat{M}\hat{\bar{\rho}}^{(q_1q_2q_3)}\hat{M}^\dagger|j_B m'\rangle$$

Assert quark basis

$$= \sum_{m_{qi}m'_{qi}} \langle j_B m|\hat{M}|m_{q_1}m_{q_2}m_{q_3}\rangle\langle m_{q_1}m_{q_2}m_{q_3}|\hat{\bar{\rho}}^{(q_1q_2q_3)}|m'_{q_1}m'_{q_2}m'_{q_3}\rangle\langle m'_{q_1}m'_{q_2}m'_{q_3}|\hat{M}^\dagger|j_B m'\rangle$$

The $\hat{M}$ dependent part

The matrix elements of $\hat{\bar{\rho}}^{(q_1q_2q_3)}$

$$\langle j_B m|\hat{M}|m_i\rangle = \sum_{j'm'}\langle j_B m|\hat{M}|j'm'\rangle\langle j'm'|m_i\rangle$$

$$= \langle j_B m|\hat{M}|j_B m\rangle\langle j_B m|m_i\rangle$$

$\hat{M}$ do not change j_B and m

Wigner-Eckart theorem

$\langle j_B m|\hat{M}|j_B m\rangle = \langle j_B||\hat{M}||j_B\rangle$

Constant absorbed into the normallization

Clebsch–Gordan coefficients

Transition operator do not contributs

Only C-G coefficient left

$$\rho^B_{mm'} = \sum_{m_i m'_i}\langle j_B m|m_i\rangle\langle m'_i|j_B m'\rangle\rho^{(q_1q_2q_3)}_{m_i m'_i}$$

Calculation with quark combination mechanism

Density matrix ⟶ Polarization components: Components = $Tr[\rho\cdot\text{Basis}]$

Vector polarization: $S_L = \langle \Sigma^z \rangle = \frac{1}{2\bar{C}_3}(5\sum_{j=1}^{3}\bar{P}_{q_j z} + \bar{t}_{zii}^{\{q_1 q_2 q_3\}})$

Rank 2 tensor polarization: $S_{LL} = \langle \Sigma^{zz} \rangle = \frac{1}{\bar{C}_3}[(3\bar{t}_{zz}^{(q_1 q_2)} - \bar{t}_{ii}^{(q_1 q_2)}) + c(123)]$

Rank 3 tensor polarization: $S_{LLL} = \langle \Sigma^{zzz} \rangle = \frac{9}{10\bar{C}_3}(5\bar{t}_{zzz}^{(q_1 q_2 q_3)} - \bar{t}_{zii}^{\{q_1 q_2 q_3\}})$

Conventions for abbreviations: $c(123)$ represents the cyclic exchange terms

$$\bar{t}_{ijk}^{\{q_1 q_2 q_3\}} \equiv \bar{t}_{ijk}^{(q_1 q_2 q_3)} + \bar{t}_{ijk}^{(q_3 q_1 q_2)} + \bar{t}_{ijk}^{(q_2 q_3 q_1)}, \quad \bar{t}_{ij}^{(q_1 q_2)} \equiv \bar{c}_{ij}^{(q_1 q_2)} + \bar{P}_{q_1 i}\bar{P}_{q_2 j}$$

$$\bar{t}_{ijk}^{(q_1 q_2 q_3)} \equiv \bar{c}_{ijk}^{(q_1 q_2 q_3)} + \bar{c}_{ij}^{(q_1 q_2)}\bar{P}_{q_3 k} + \bar{c}_{jk}^{(q_2 q_3)}\bar{P}_{q_1 i} + \bar{c}_{ki}^{(q_3 q_1)}\bar{P}_{q_2 j} + \bar{P}_{q_1 i}\bar{P}_{q_2 j}\bar{P}_{q_3 k}$$

$$\bar{C}_3 = Tr\hat{\bar{\rho}}^B = 3 + \bar{t}_{ii}^{(q_1 q_2)} + \bar{t}_{ii}^{(q_2 q_3)} + \bar{t}_{ii}^{(q_3 q_1)}$$

Calculation with quark combination mechanism

- Vector polarization

$$S_L = \frac{5}{2}\bar{P}_q + \frac{3}{\bar{C}_3}\left[\bar{P}_q(\bar{c}_{zz}^{(qq)} - 2\bar{c}_{ii}^{(qq)} - 2\bar{P}_q^2) + \frac{1}{2}\bar{c}_{zii}^{(qqq)}\right]$$

The leading term is quark polarization

- Rank-2 tensor polarization

$$S_{LL} = \frac{3}{\bar{C}_3}\left(3\bar{c}_{zz}^{(qq)} - \bar{c}_{ii}^{(qq)} + 2\bar{P}_q^2\right)$$

The leading term is 2 quark correlation

- Rank-3 tensor polarization

$$S_{LLL} = \frac{9}{10\bar{C}_3}\left[5\bar{c}_{zzz}^{(qqq)} - 3\bar{c}_{zii}^{(qqq)} + 3\bar{P}_q(3\bar{c}_{zz}^{(qq)} - \bar{c}_{ii}^{(qq)}) + 2\bar{P}_q^3\right]$$

The leading term is 3 quark correlation

Different rank of polarization, different local spin correlations!

The systematic picture

Hadron	Measurables	sensitive quantities
spin-1/2 (hyperon H)	Hyperon polarization P_H	**average** quark polarization $\langle P_q \rangle$
	Hyperon spin correlation $c_{H_1H_2}, c_{H_1\bar{H}_2}$	**long range** spin correlations $c_{qq}, c_{q\bar{q}}$
spin-1 (Vector mesons)	Spin alignment ρ_{00}	**local** spin correlations $c_{q\bar{q}}$
	off-diagnal elements $\rho_{m'm}$	**local** spin correlations $c_{q\bar{q}}$
spin-3/2 $J^P = \left(\frac{3}{2}\right)^+$ baryons	Hyperon polarization S_L	**average** quark polarization $\langle P_q \rangle$
	Rank 2 tensor polarization S_{LL}	**local** spin correlations c_{qq}
	Rank 3 tensor polarization S_{LLL}	**local** spin correlations c_{qqq}

⇨ **Systematic studies of quark spin correlations in QGP!**

5 Summary and outlook

- We extend the study on global polarizations and spin correlations to spin-3/2 hadrons and present the results for their polarizations components.

- The results show that the vector polarizations are mainly determined by the quark polarization, the second rank tensor polarizations are determined by the local quark-quark spin correlations and the third rank tensor polarizations are determined by the local spin correlations of three quarks.

- These different components of polarizations can measured in the joint distribution of the decay products in spin-3/2 baryon successive decays.

Thank you for your attention!

Back up

$$\rho = \frac{1}{4}\left(1 + \frac{4}{5}S^i\Sigma^i + \frac{2}{3}T^{ij}\Sigma^{ij} + \frac{8}{9}R^{ijk}\Sigma^{ijk}\right) \tag{3.13}$$

对于矢量极化 $\hat{s} = \Sigma^x, \Sigma^y, \Sigma^z$，在 $(\hat{s}^2, \Sigma_z)$ 表象，我们有

$$\Sigma^x = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}, \quad \Sigma^y = \begin{pmatrix} 0 & -\frac{i\sqrt{3}}{2} & 0 & 0 \\ \frac{i\sqrt{3}}{2} & 0 & -i & 0 \\ 0 & i & 0 & -\frac{i\sqrt{3}}{2} \\ 0 & 0 & \frac{i\sqrt{3}}{2} & 0 \end{pmatrix}, \quad \Sigma^z = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}. \tag{3.14}$$

Back up

$$\Sigma^{ij} = \frac{1}{2}\left(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i\right) - \frac{5}{4}\delta^{ij}\mathbf{1}$$

$$\Sigma^{ijk} = \frac{1}{6}\Sigma^{\{i}\Sigma^j\Sigma^{k\}} - \frac{41}{60}\left(\delta^{ij}\Sigma^k + \delta^{jk}\Sigma^i + \delta^{ki}\Sigma^j\right) \quad (3.15)$$

$$= \frac{1}{3}(\Sigma^{ij}\Sigma^k + \Sigma^{jk}\Sigma^i + \Sigma^{ki}\Sigma^j) - \frac{4}{15}(\Sigma^i\delta_{jk} + \Sigma^j\delta_{ik} + \Sigma^k\delta_{ij})$$

自旋 3/2 密度矩阵有 15 个独立的分量，3 个独立矢量极化分量 S^i,5 个独立的二阶张量极化 T^{ij},7 个独立的三阶张量极化

$$S^i : S_L, S_T^x, S_T^y$$

$$T^{ij} : S_{LL}, S_{LT}^x, S_{LT}^y, S_{TT}^{xx}, S_{TT}^{xy} \quad (3.16)$$

$$R^{ijk} : S_{LLL}, S_{LLT}^x, S_{LLT}^y, S_{LTT}^{xx}, S_{LTT}^{xy}, S_{TTT}^{xxx}, S_{TTT}^{yxx}$$

对于矢量极化

$$S_L = \langle\Sigma^z\rangle, \quad S_T^x = \langle\Sigma^x\rangle, \quad S_T^y = \langle\Sigma^y\rangle \quad (3.17)$$

对于二阶张量极化

$$S_{LL} = \langle\Sigma^{zz}\rangle, \quad S_{LT}^x = 2\langle\Sigma^{xz}\rangle, \quad S_{LT}^y = 2\langle\Sigma^{yz}\rangle,$$

$$S_{TT}^{xy} = 2\langle\Sigma^{xy}\rangle, \quad S_{TT}^{xx} = \langle\Sigma^{xx} - \Sigma^{yy}\rangle \quad (3.18)$$

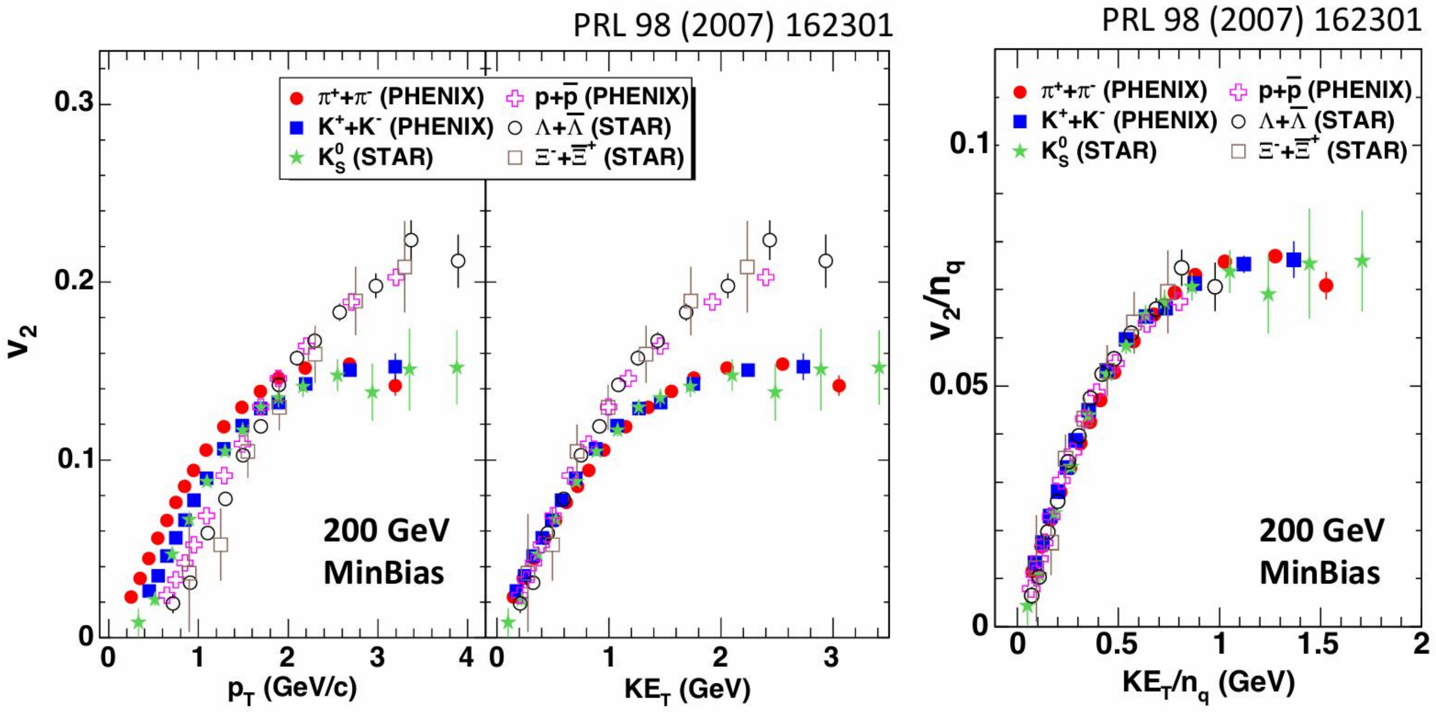
对于三阶张量极化

$$S_{LLL} = \langle\Sigma^{zzz}\rangle, \quad S_{LLT}^x = \langle\Sigma^{xzz}\rangle, \quad S_{LLT}^y = \langle\Sigma^{yzz}\rangle, \quad S_{LTT}^{xy} = 4\langle\Sigma^{xyz}\rangle,$$

$$S_{LTT}^{xx} = 2\langle\Sigma^{xxz} - \Sigma^{yyz}\rangle, \quad S_{TTT}^{xxx} = \langle\Sigma^{xxx} - 3\Sigma^{xyy}\rangle, S_{TTT}^{yxx} = \langle 3\Sigma^{yxx} - \Sigma^{yyy}\rangle \quad (3.19)$$

Back up

1:



$$q+\bar{q}\rightarrow V \qquad q_1+q_2+q_3\rightarrow B$$

$$p_{\perp 1}=p_{\perp 2}=\frac{p_T}{2} \qquad p_{\perp 1}=p_{\perp 2}=p_{\perp 3}=\frac{p_T}{3}$$

$$f_M(p_T,\varphi)=f_q^2\left(\frac{p_T}{2},\varphi\right)$$

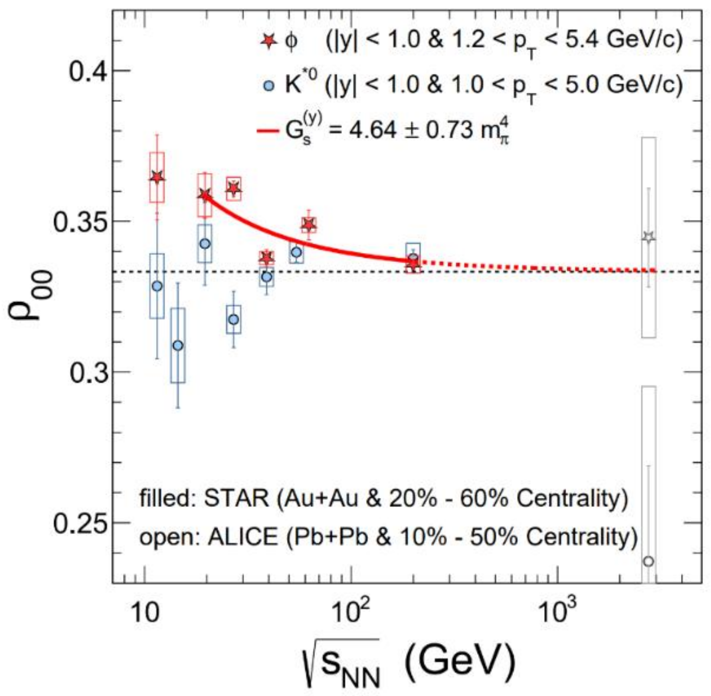
$$f_B(p_T,\varphi)=f_q^3\left(\frac{p_T}{3},\varphi\right)$$

2: $|Baryon\rangle = f_0|qqq\rangle + f_1|qqqg\rangle + f_2|qqq\bar{q}q\rangle + \cdots + f_n|qqq\bar{q}\bar{q}\bar{q}\bar{q}qqqqgggggg\ldots\rangle + \ldots$

Dominate term in combination
Because f_0 is big

Although f_n should be small,
there are too many partons inside

Back up



$$c^{(q_1 q_2)} > P_{q_1} P_{q_2}$$

Assumption:

Less the particle, larger the quantity

Same particle correlation > polarization

$$P_q > c^{(q_1 q_2)} > P_{q_1} P_{q_2} > c^{(q_1 q_2 q_3)} > P_{q_1} P_{q_2} P_{q_3}$$

Back up

Differences between p+p and A+A

- Initial-state effects
- Influence of QGP evolution
- Hardronization: Fragmentation vs Combination