

ALESSANDRO BACCHETTA, PAVIA U. AND INFN

A visualization of the cosmic web, showing a complex network of dark matter filaments and galaxy clusters. The background is dark, with numerous small, distant galaxies visible. In the center, there are several bright, glowing clusters of galaxies, with one cluster in the upper right being particularly prominent and glowing with a blue and white light. The overall structure is a dense, interconnected web of matter.

RECENT RESULTS ON TMDs FROM THE MAP COLLABORATION



BY THE MAP COLLABORATION

<https://github.com/MapCollaboration>

MAINLY BASED ON RESULTS OBTAINED WITH

3

Valerio Bertone



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Giuseppe Bozzi



Matteo Cerutti



Filippo Delcarro



Fulvio Piacenza



Marco Radici



Simone Rodini



Lorenzo Rossi



Andrea Signori



**Finanziato
dall'Unione europea**
NextGenerationEU



NEW SINCE SPIN 2023

NEW SINCE SPIN 2023

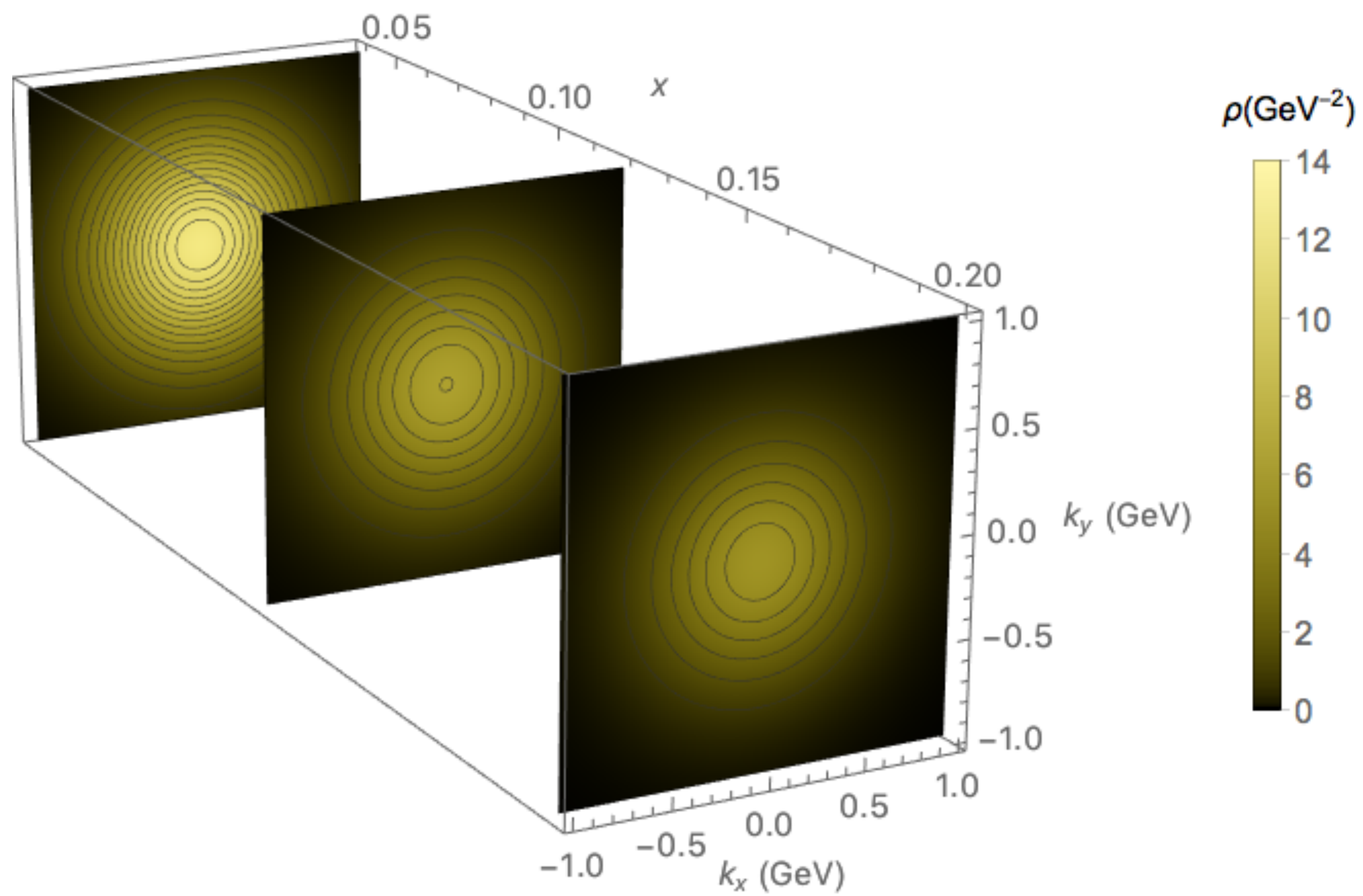
- ▶ MAP24: extraction of unpolarized TMDs with flavor dependence ([arXiv:2405.13833](https://arxiv.org/abs/2405.13833))

NEW SINCE SPIN 2023

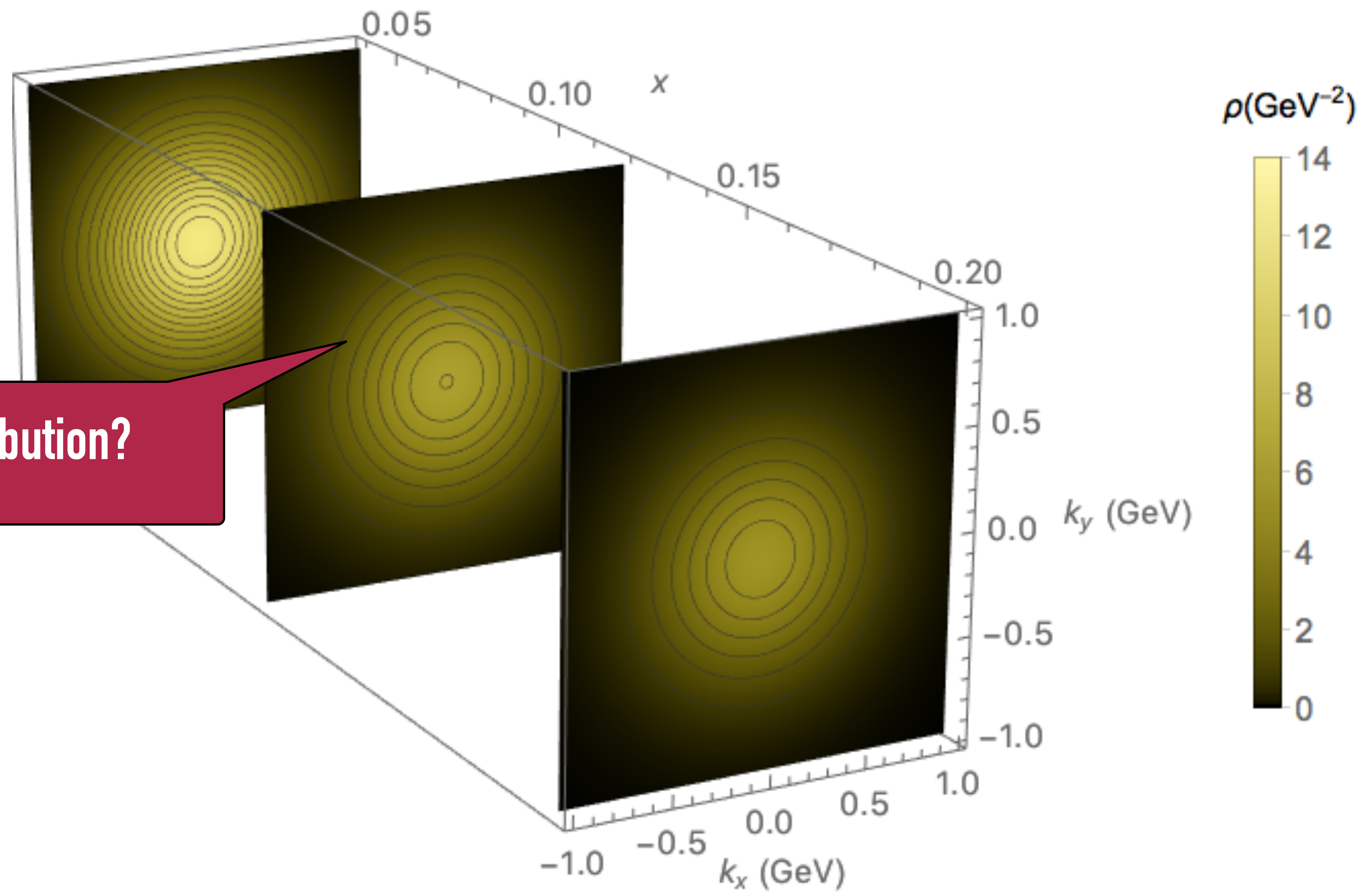
- ▶ MAP24: extraction of unpolarized TMDs with flavor dependence ([arXiv:2405.13833](#))
- ▶ MAPTMDNN: proof-of-concept extraction of unpolarized TMDs with NN ([arXiv:2502.04166](#))

NEW SINCE SPIN 2023

- ▶ MAP24: extraction of unpolarized TMDs with flavor dependence ([arXiv:2405.13833](#))
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- ▶ MAPTMDpol: extraction of helicity TMDs ([arXiv:2409.18078](#))

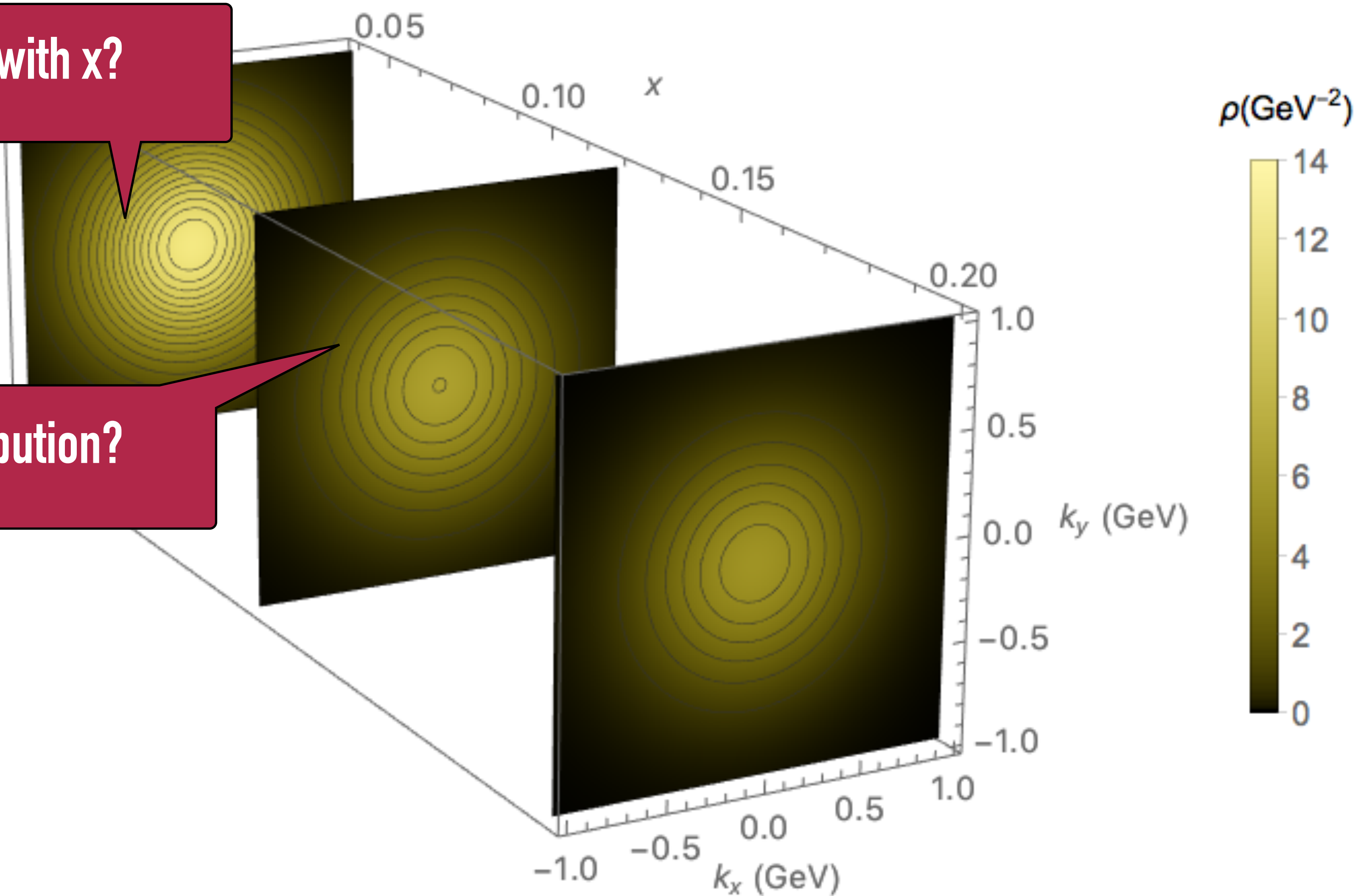


How “wide” is the distribution?



How does it change with x ?

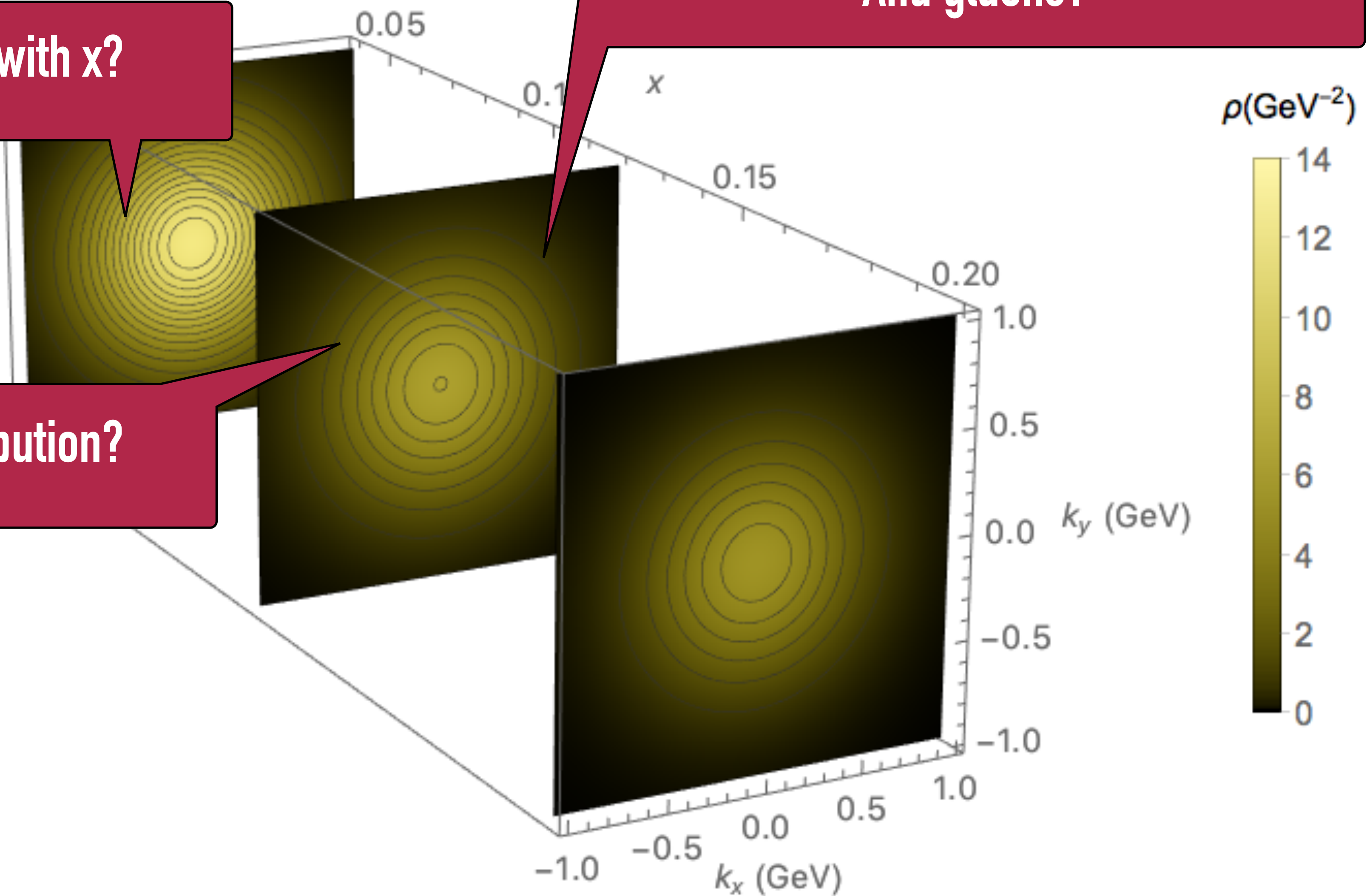
How “wide” is the distribution?



How does it change with x ?

Is there a difference between flavors?
And gluons?

How “wide” is the distribution?

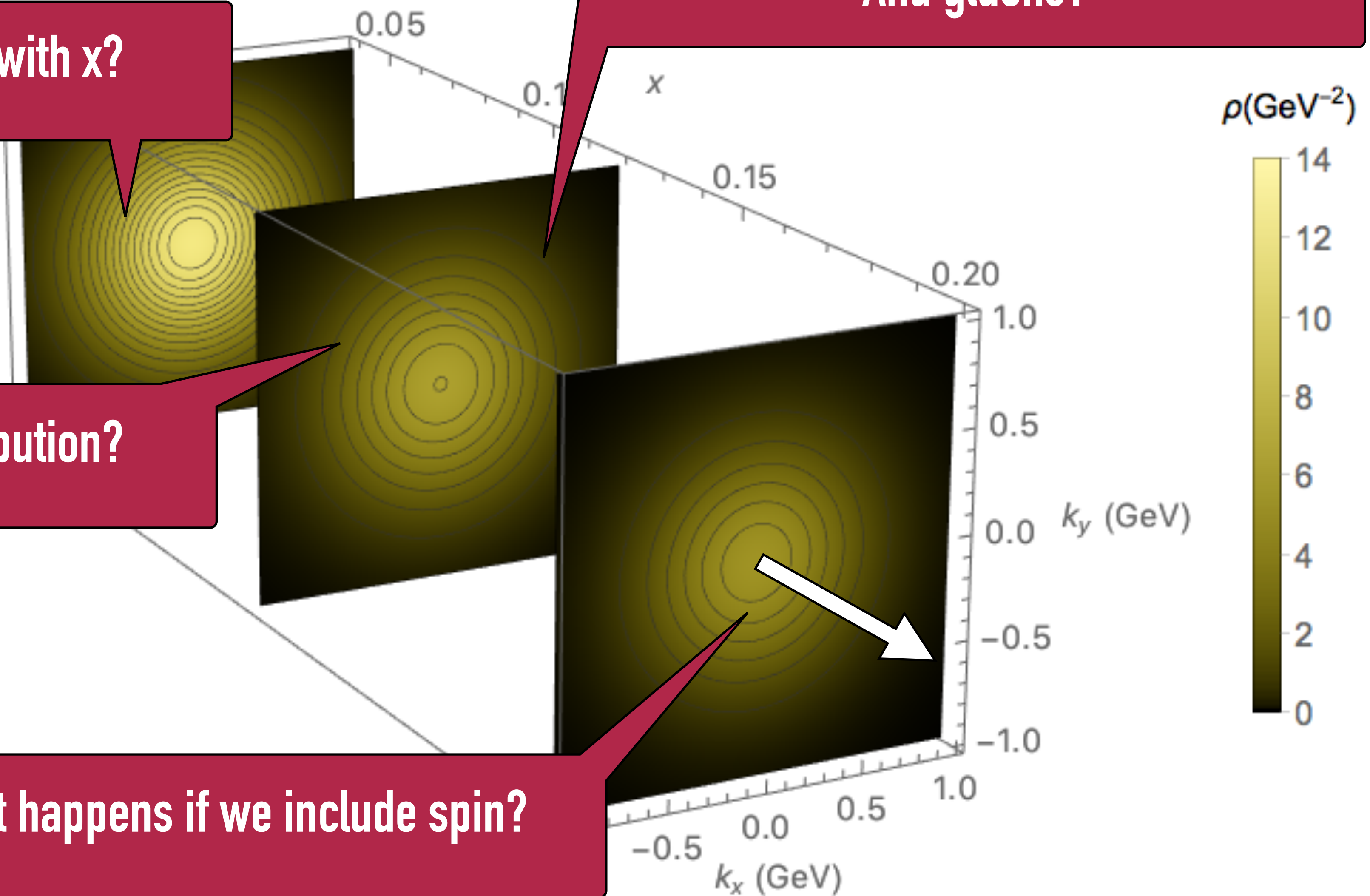


How does it change with x ?

Is there a difference between flavors?
And gluons?

How “wide” is the distribution?

What happens if we include spin?





UNPOLARIZED QUARK TMDs

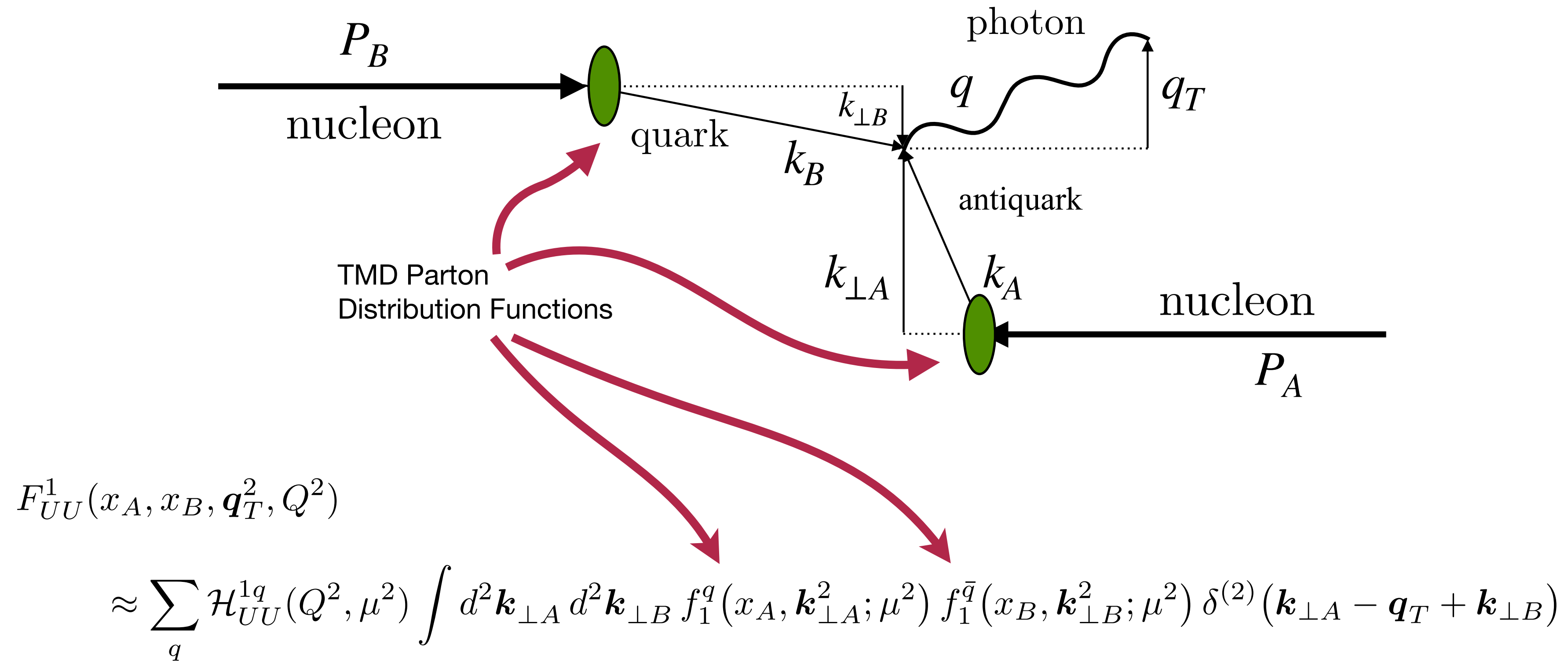


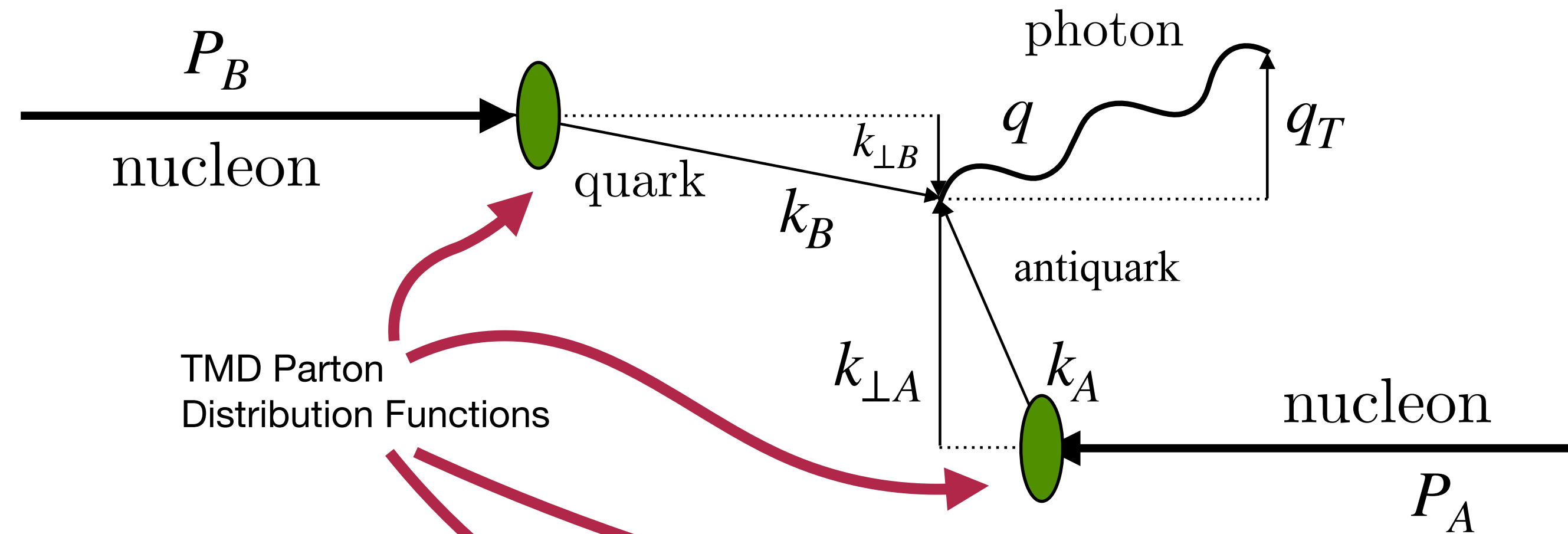
Diagram illustrating the Drell-Yan process. Two nucleons, labeled P_B and P_A , collide. A quark from the P_B nucleon and an antiquark from the P_A nucleon annihilate via a photon q to produce a lepton pair. The quark and antiquark have transverse momenta $k_{\perp B}$ and $k_{\perp A}$ respectively. The photon has transverse momentum q_T . The diagram is linked by red arrows to the mathematical expression for the first moments of the parton distribution functions.

$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

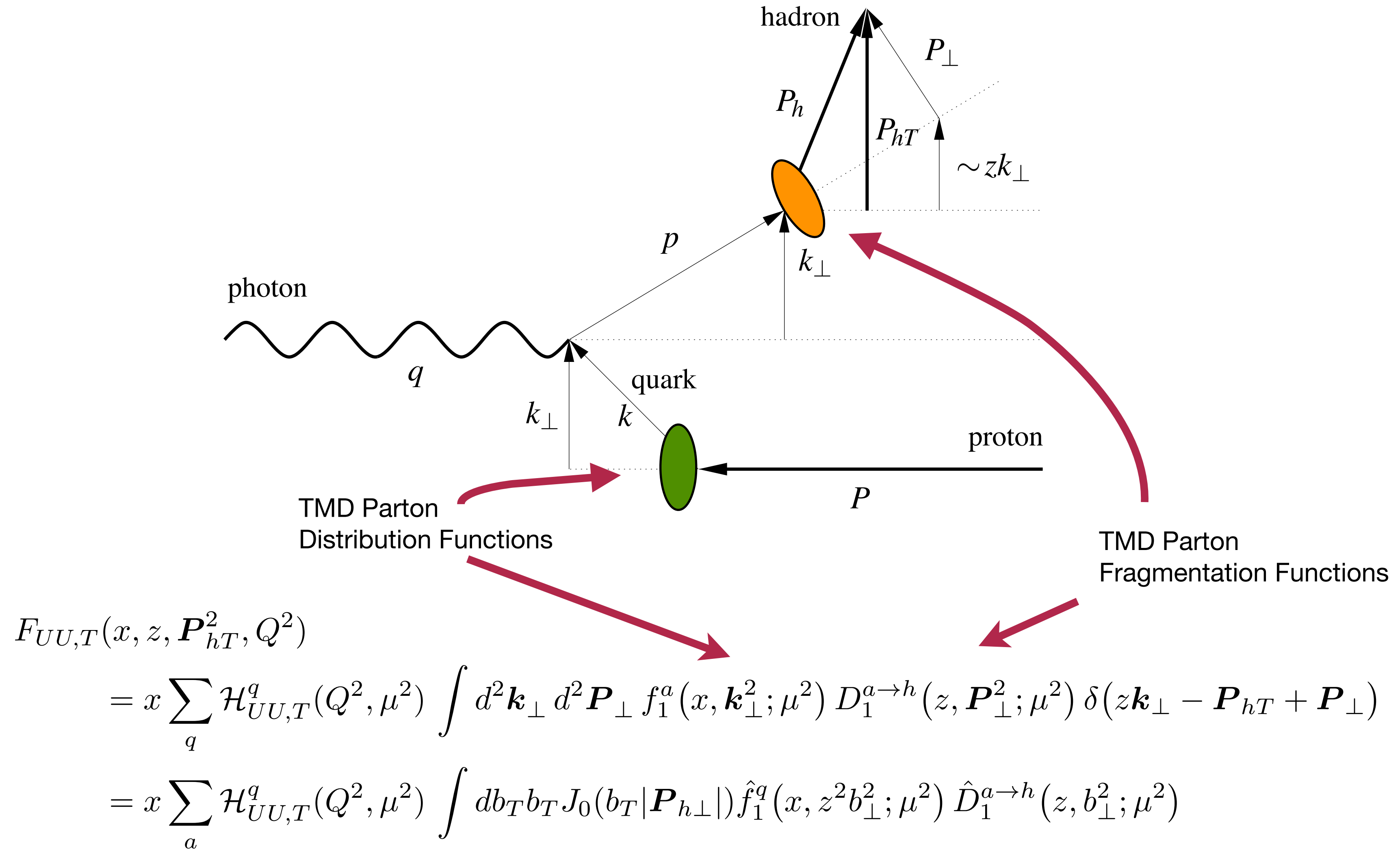
$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space

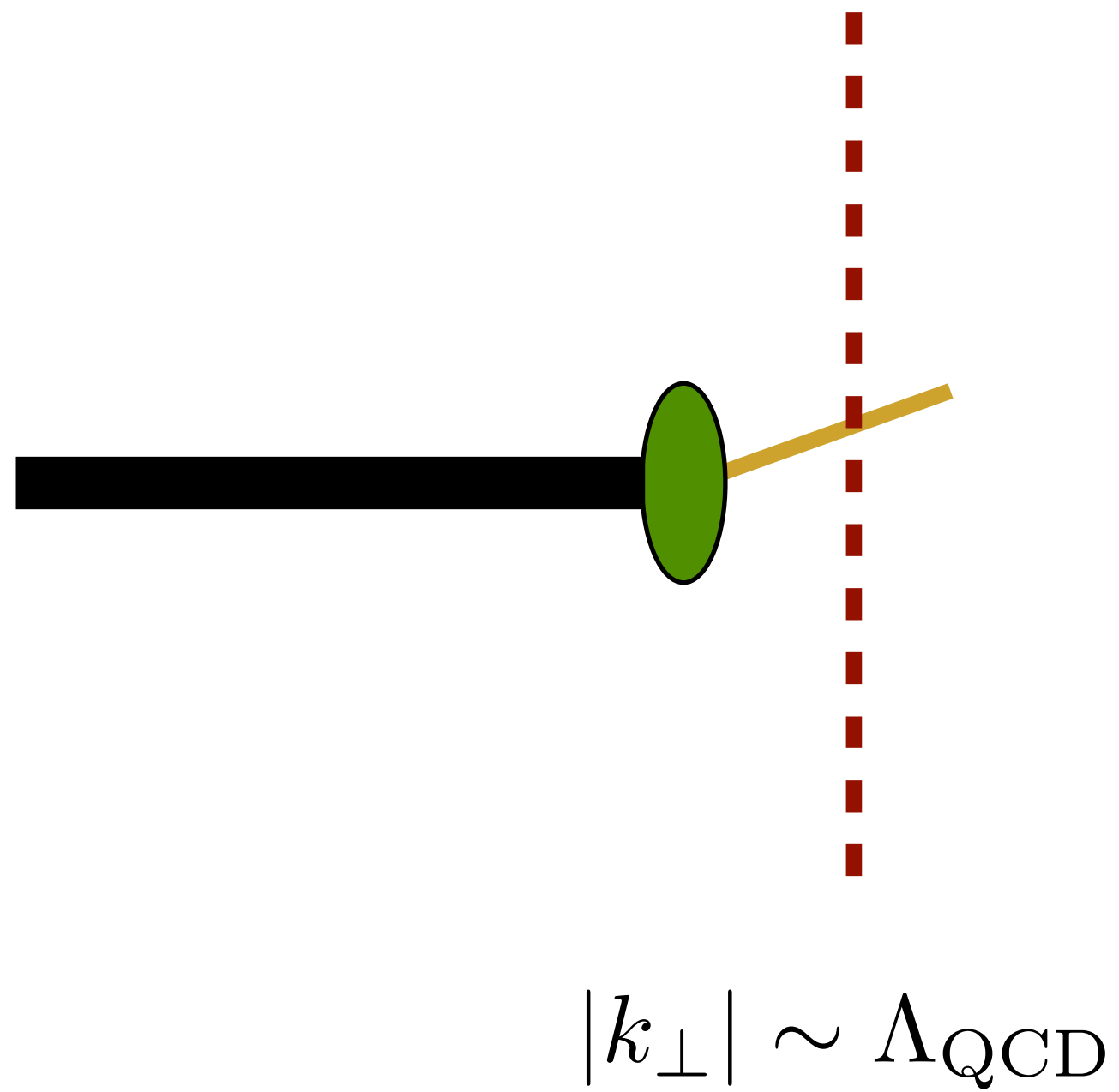


$$\begin{aligned}
 F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2) & \\
 &\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B}) \\
 &= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)
 \end{aligned}$$

The analysis is usually done in Fourier-transformed space
 TMDs formally depend on two scales, but we set them equal.

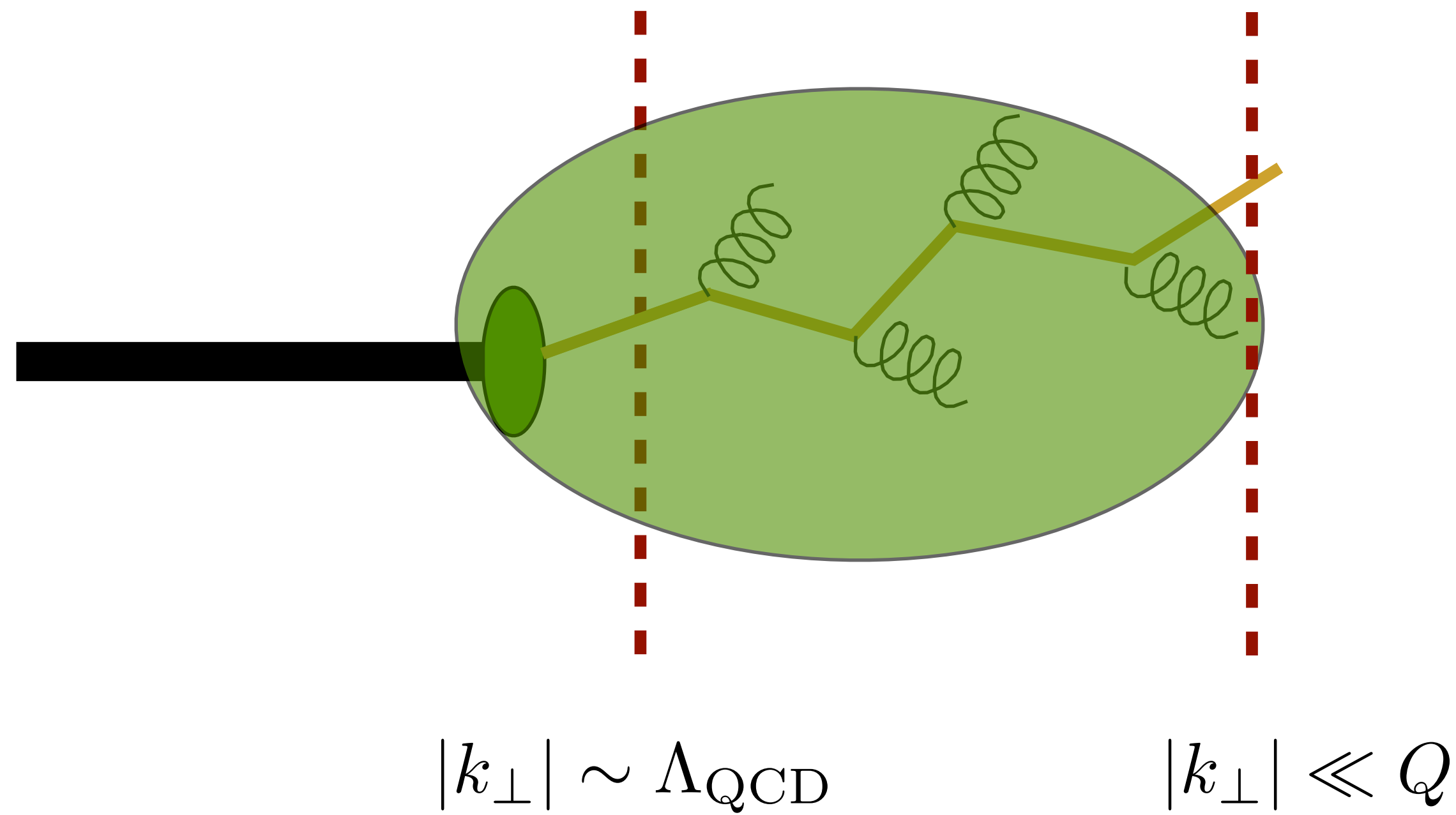


“intrinsic”
transverse
momentum



"intrinsic"
transverse
momentum

soft and collinear
gluon radiation



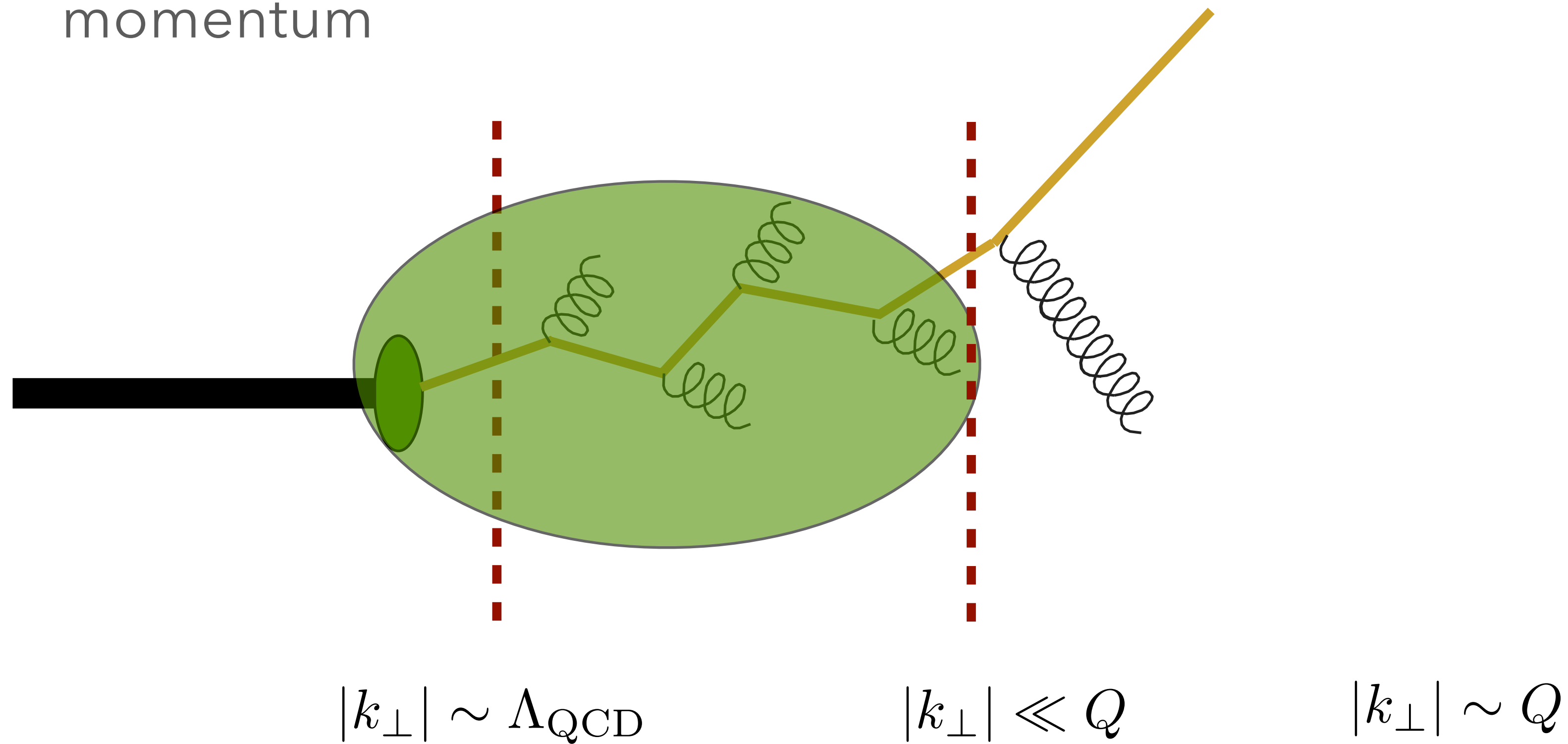
CONTRIBUTIONS TO TRANSVERSE MOMENTUM

9

"intrinsic"
transverse
momentum

soft and collinear
gluon radiation

hard gluon
radiation
(not in TMD
region)



$$\hat{f}_1^a(x, |\mathbf{b}_T|; \mu, \zeta) = \int d^2\mathbf{k}_\perp e^{i\mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^a(x, \mathbf{k}_\perp^2; \mu, \zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

$$\hat{f}_1^a(x, |\mathbf{b}_T|; \mu, \zeta) = \int d^2 \mathbf{k}_\perp e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^a(x, \mathbf{k}_\perp^2; \mu, \zeta)$$

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perturbative Sudakov
form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}}}$$

collinear PDF

Collins-Soper kernel

matching coefficients
(perturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

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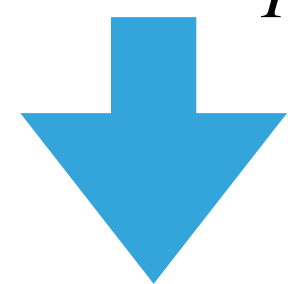
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collinear PDF

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perturbative Sudakov
form factor

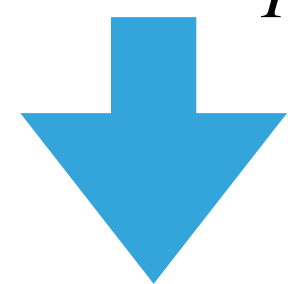
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collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

matching coefficients
(perturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$



$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

[see, e.g., Collins, "Foundations of Perturbative QCD" \(11\)](#)
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perturbative Sudakov
form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

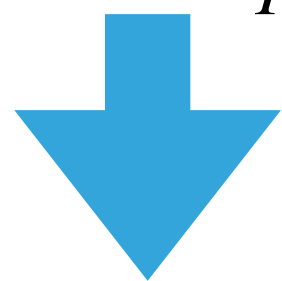
collinear PDF

matching coefficients
(perturbative)

Collins-Soper kernel
(perturbative and
nonperturbative)

nonperturbative part
of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$



$$\mu_{b_*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

[see, e.g., Collins, "Foundations of Perturbative QCD" \(11\)](#)
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	Accuracy	PDF uncertainty	flavor dependence	SIDIS	DY	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✗	✗	✓	✓	8059	1.55

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SV 2019 arXiv:1912.06532	N ³ LL [−]	✗	✗	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N ³ LL [−]	✗	✗	✓	✓	2031	1.06

– not all ingredients are available

	Accuracy	PDF uncertainty	flavor dependence	SIDIS	DY	N of points	χ^2/N_{points}
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MAP22 arXiv:2206.07598	N ³ LL [−]	✗	✗	✓	✓	2031	1.06
MAP24 arXiv:2405.13833	N ³ LL	✓	✓	✓	✓	2031	1.08

– not all ingredients are available

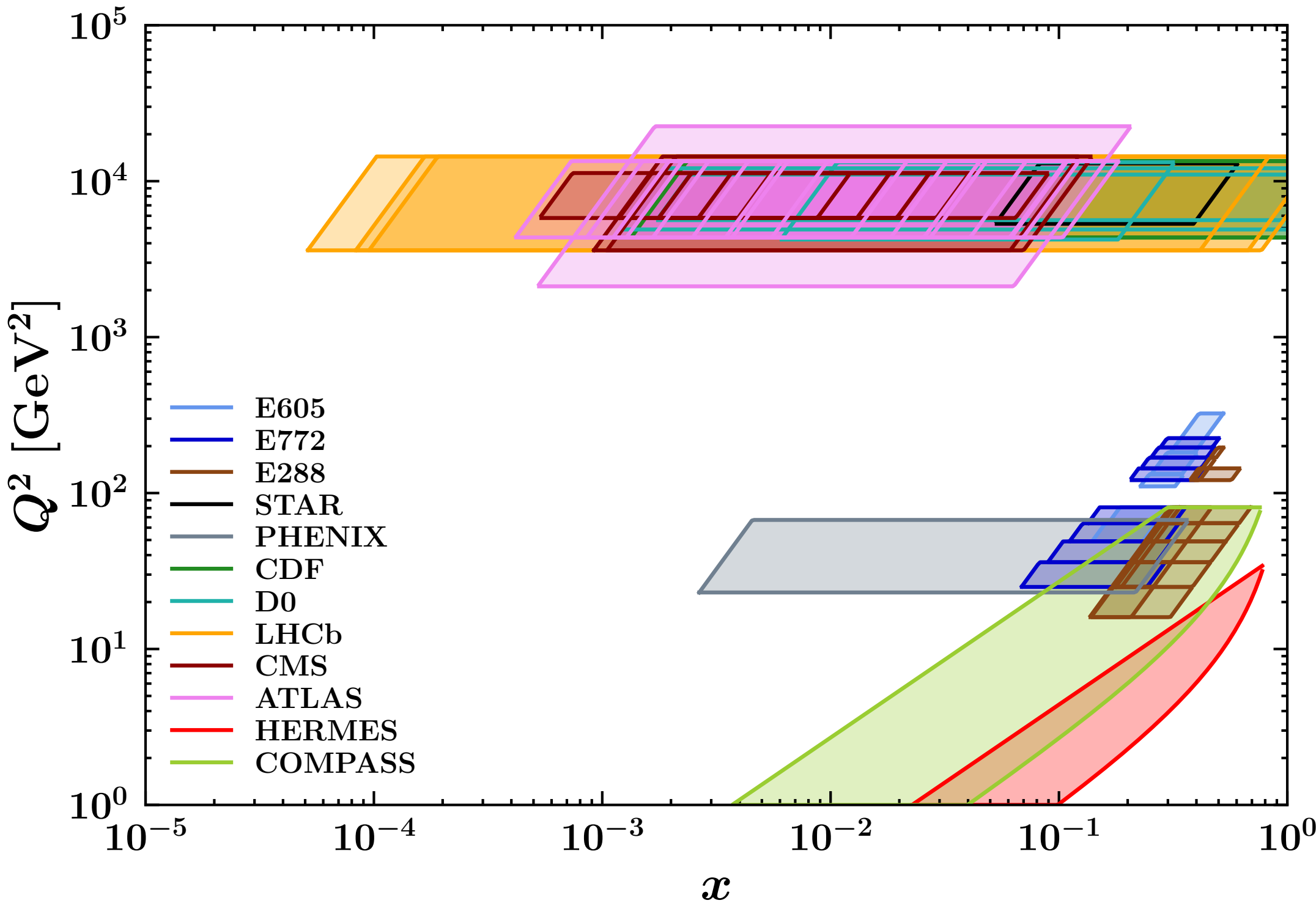
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MAP24 arXiv:2405.13833	N ³ LL	✓	✓	✓	✓	2031	1.08
ART25 arXiv:2503.11201	N ⁴ LL ⁻	✓	✓	✓	✓	1209	1.05

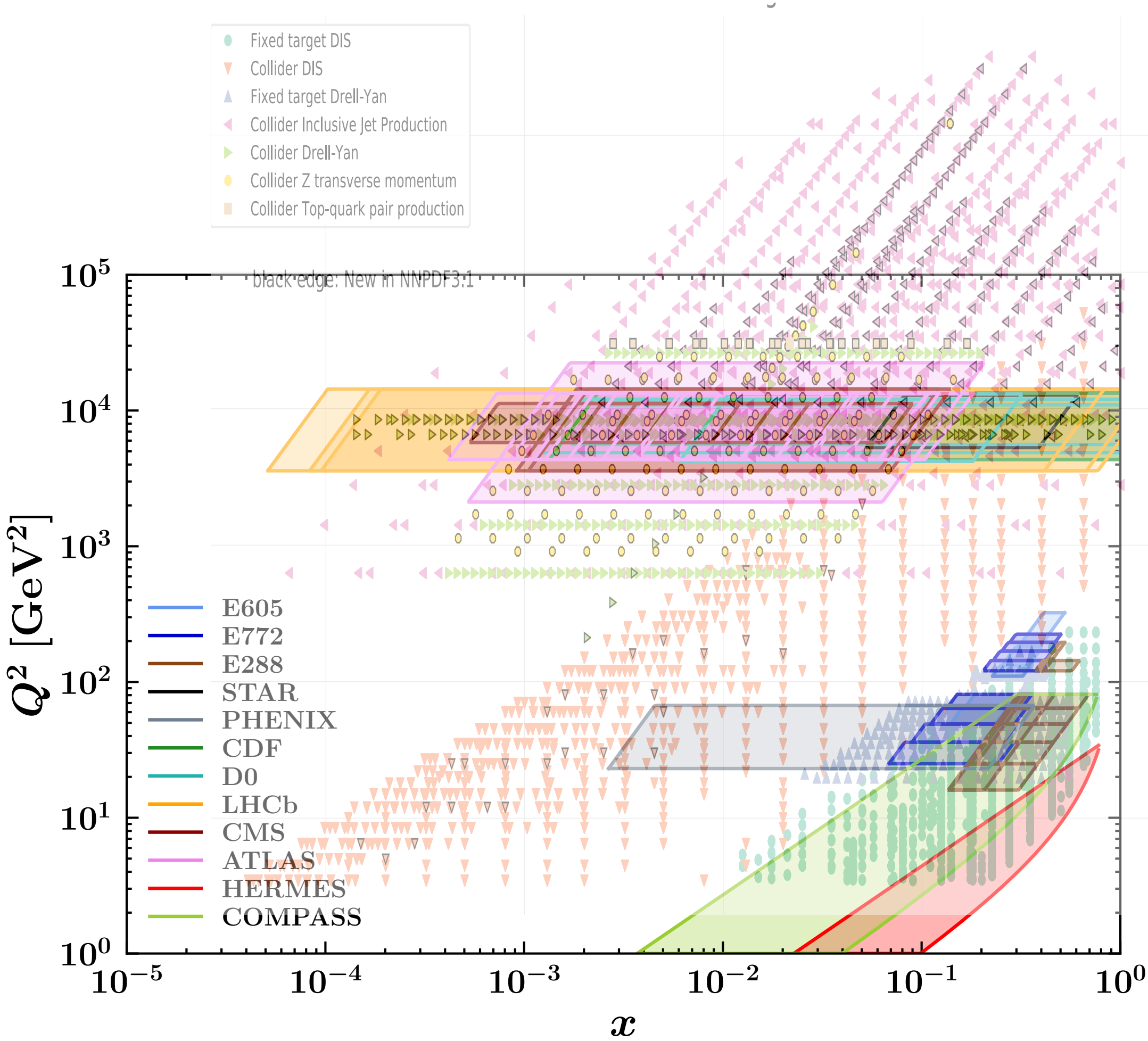
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ART25 arXiv:2503.11201	N ⁴ LL ⁻	✓	✓	✓	✓	1209	1.05

– not all ingredients are available

[see next talk by V. Moos](#)

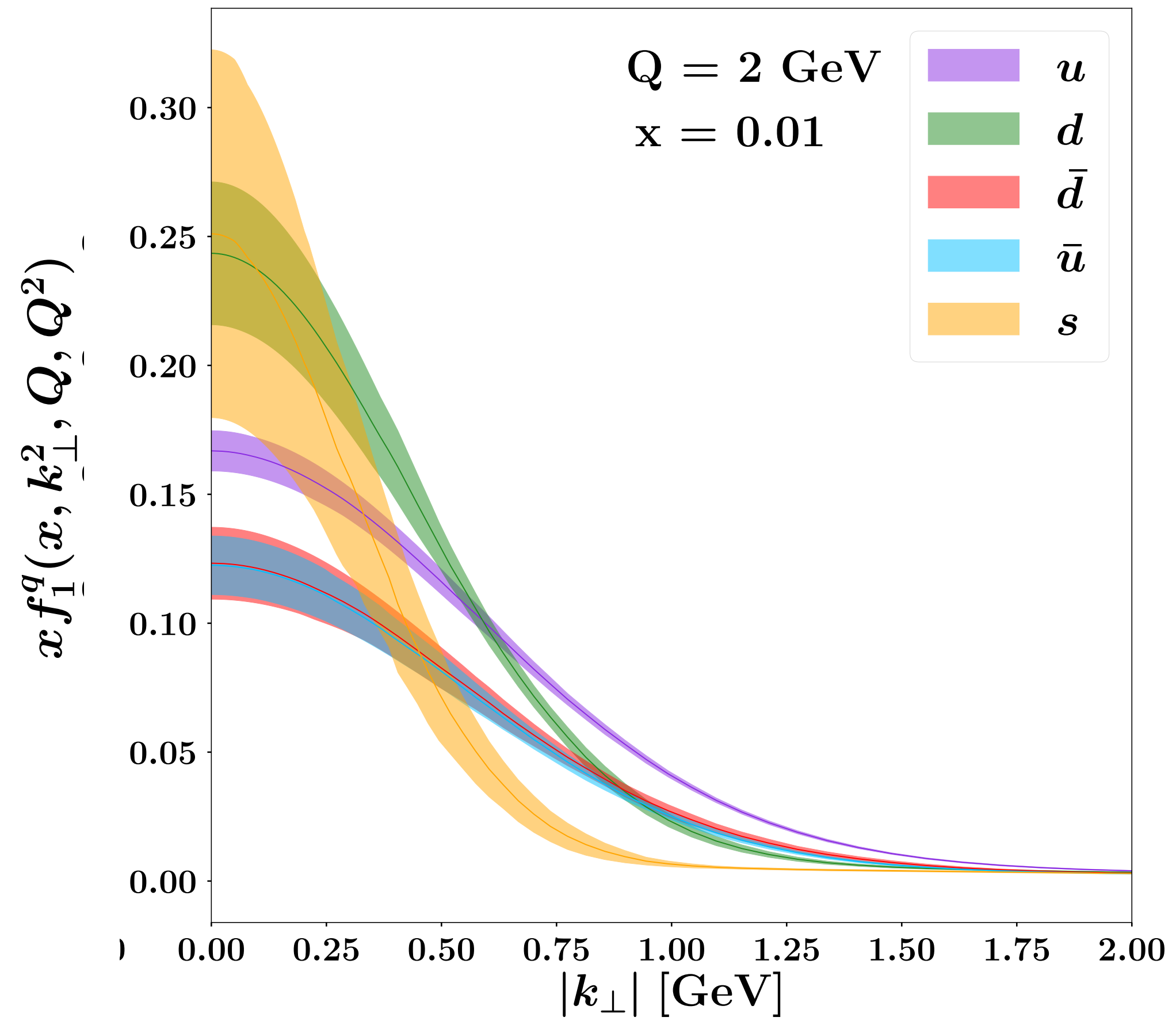


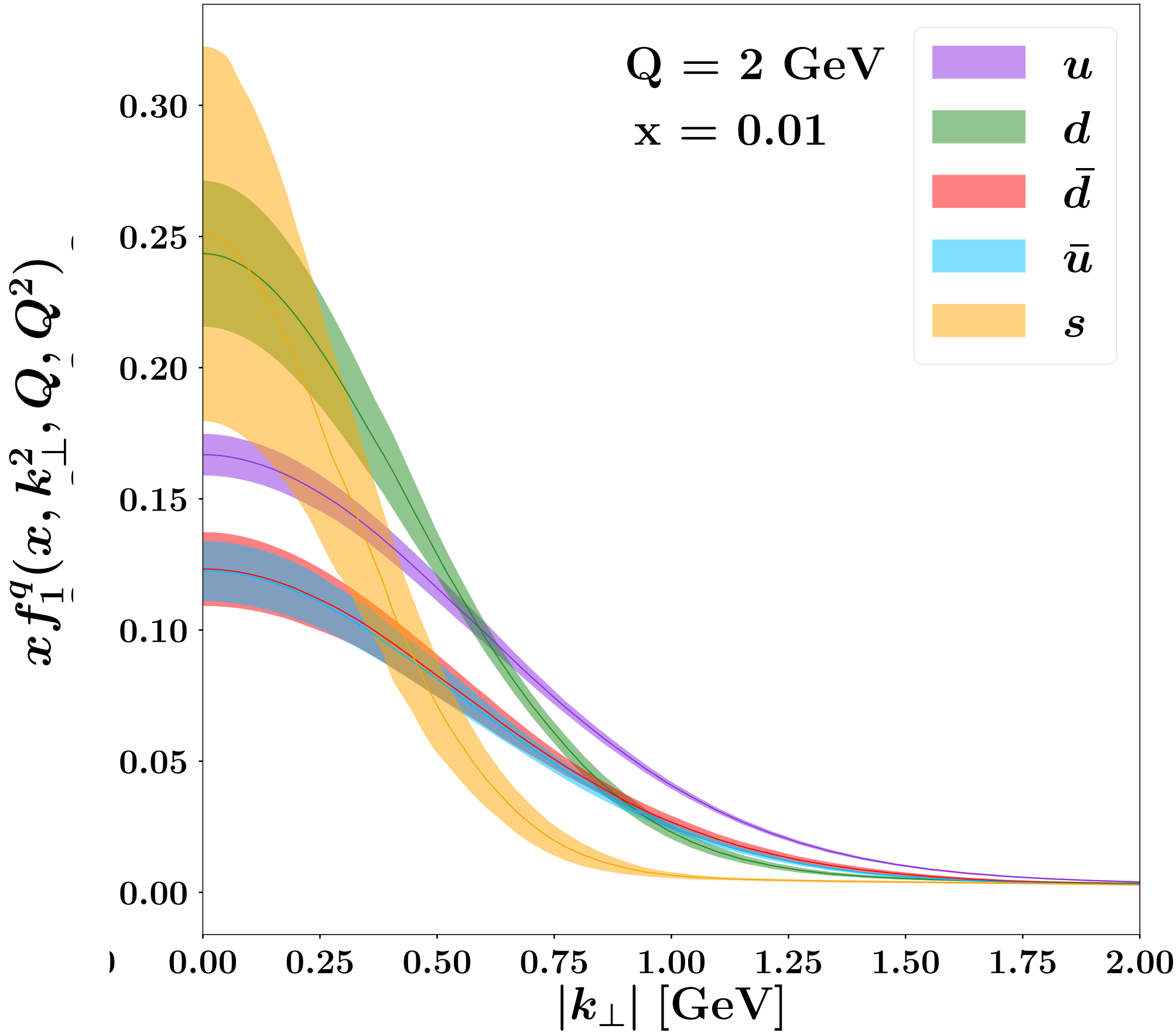


MAP24 [arXiv:2405.13833](https://arxiv.org/abs/2405.13833)

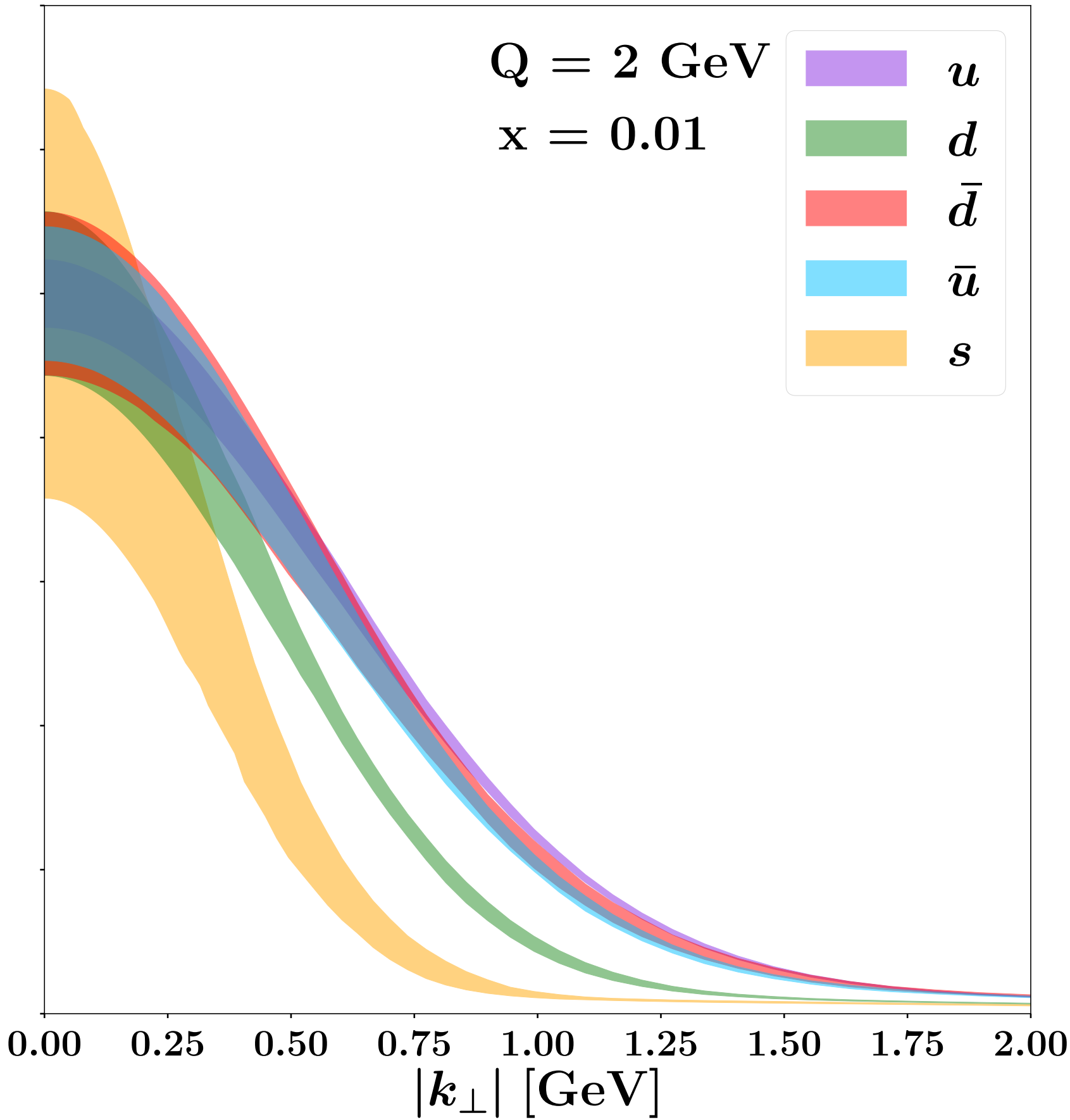
	N ³ LL			
Data set	N_{dat}	χ_D^2	χ_λ^2	χ_0^2
<i>Tevatron total</i>	71	1.10	0.07	1.17
<i>LHCb total</i>	21	3.56	0.96	4.52
<i>ATLAS total</i>	72	3.54	0.82	4.36
<i>CMS total</i>	78	0.38	0.05	0.43
PHENIX 200	2	2.76	1.04	3.80
STAR 510	7	1.12	0.26	1.38
DY collider total	251	1.37	0.28	1.65
E288 200 GeV	30	0.13	0.40	0.53
E288 300 GeV	39	0.16	0.26	0.42
E288 400 GeV	61	0.11	0.08	0.19
E772	53	0.88	0.20	1.08
E605	50	0.70	0.22	0.92
DY fixed-target total	233	0.63	0.31	0.94
<i>HERMES total</i>	344	0.81	0.24	1.05
<i>COMPASS total</i>	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	2031	0.81	0.27	1.08

EXAMPLE OF RESULTING TMDS

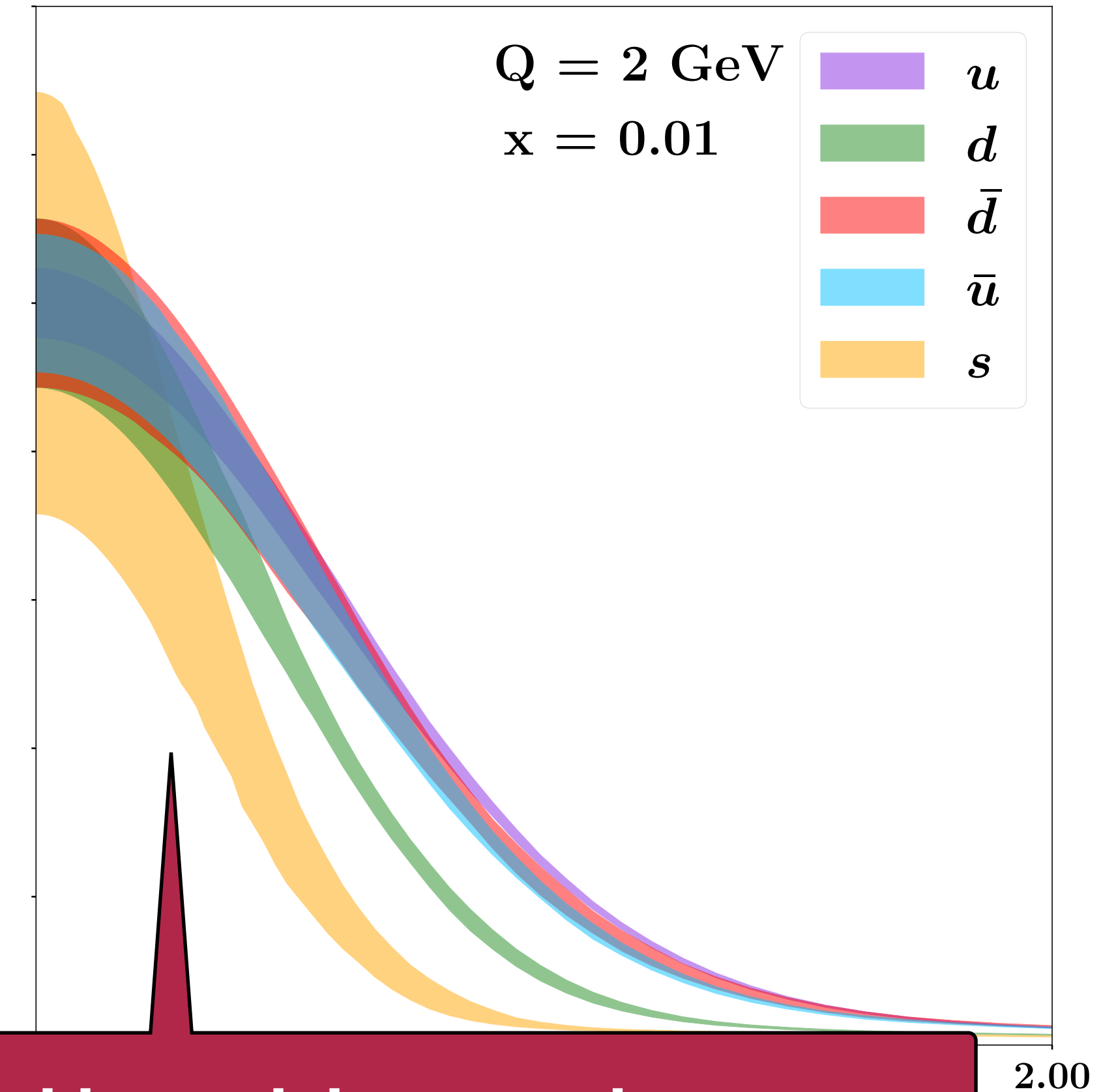
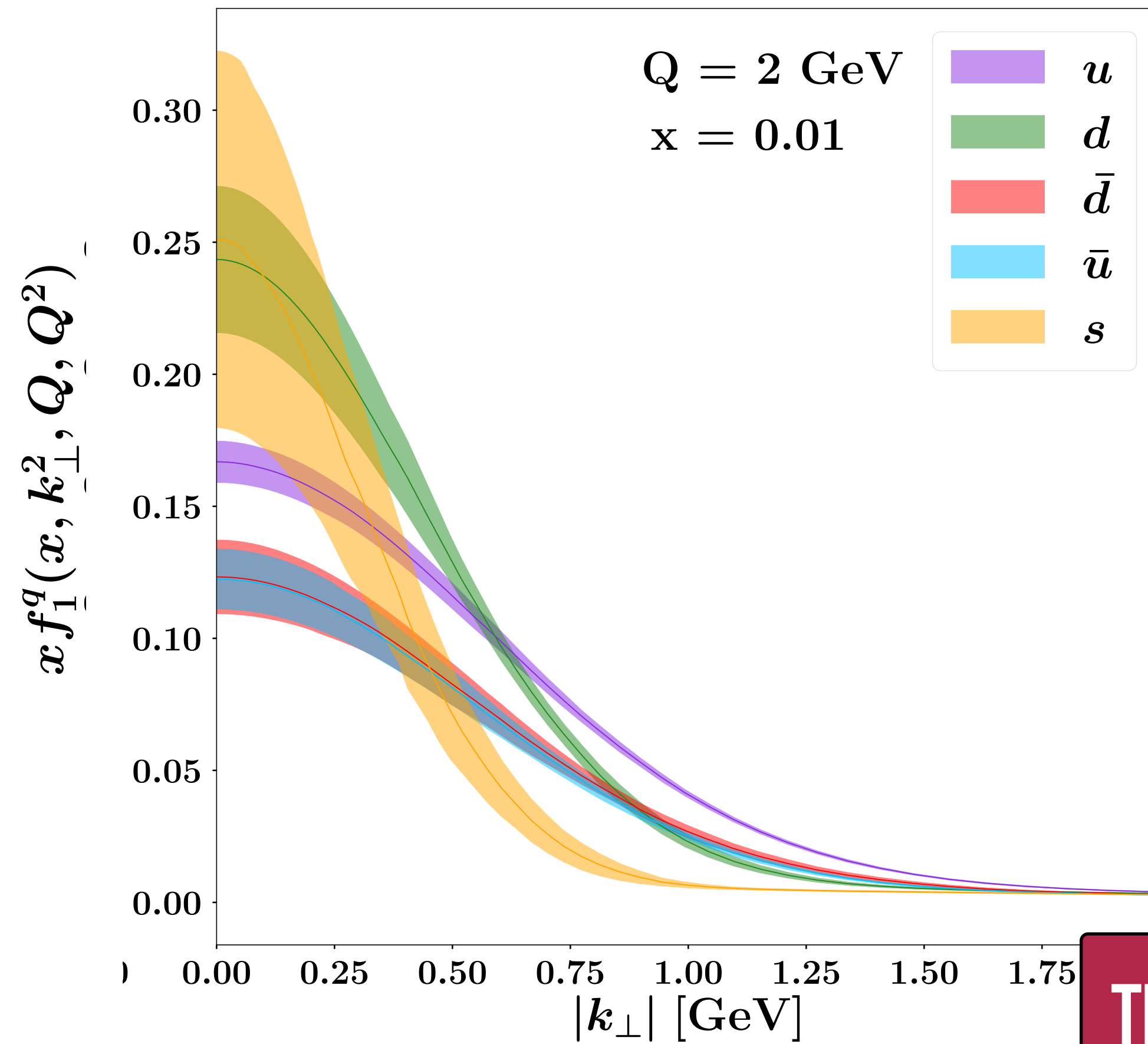




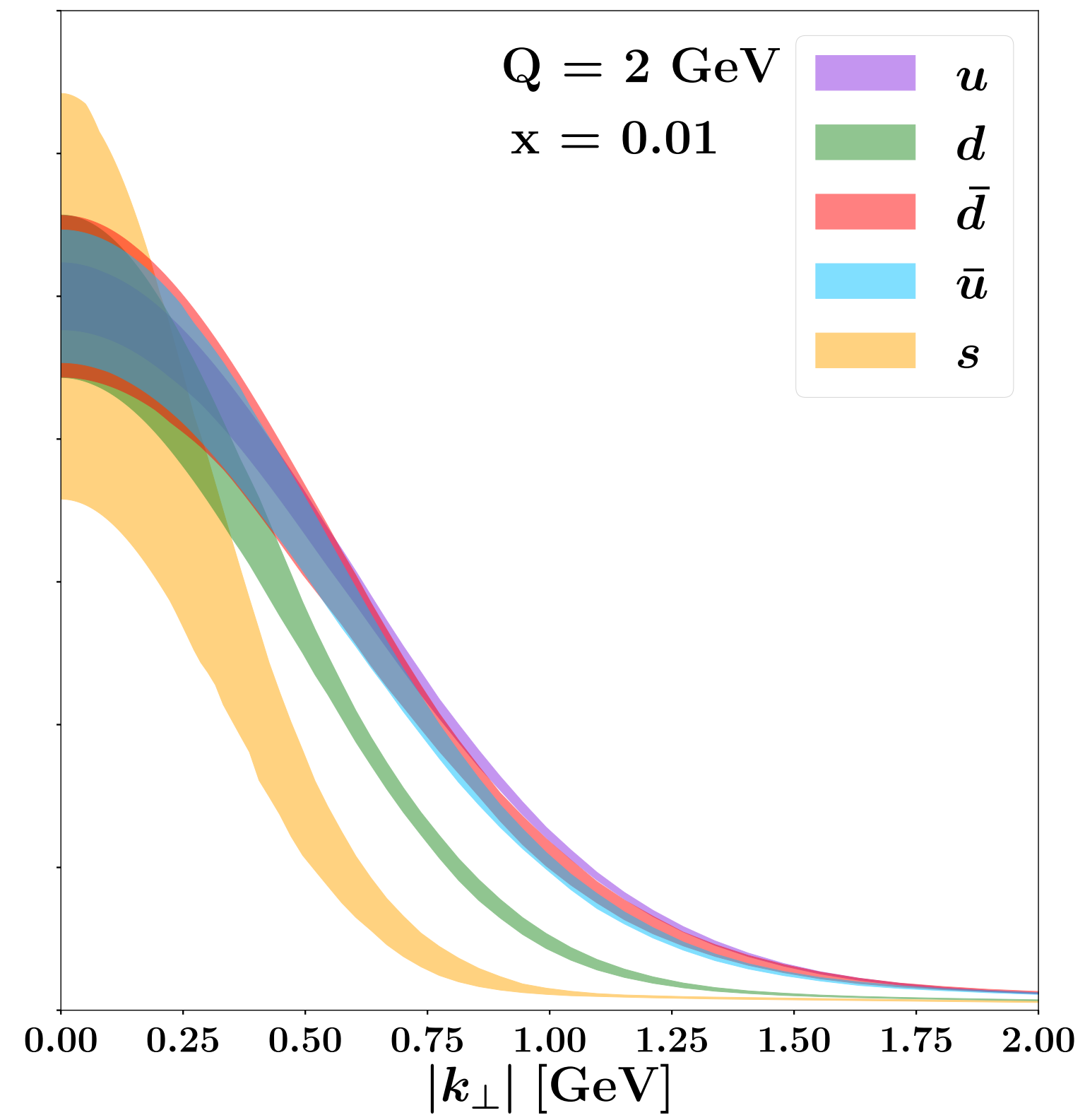
normalized to the same value at $k_\perp = 0$

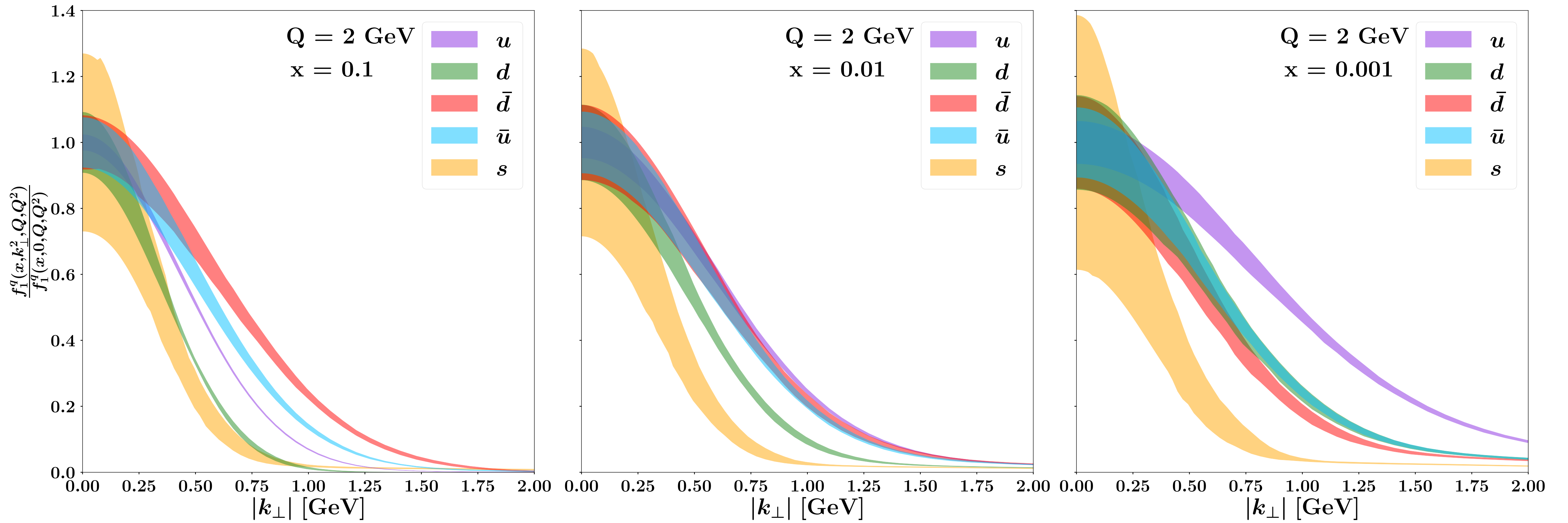


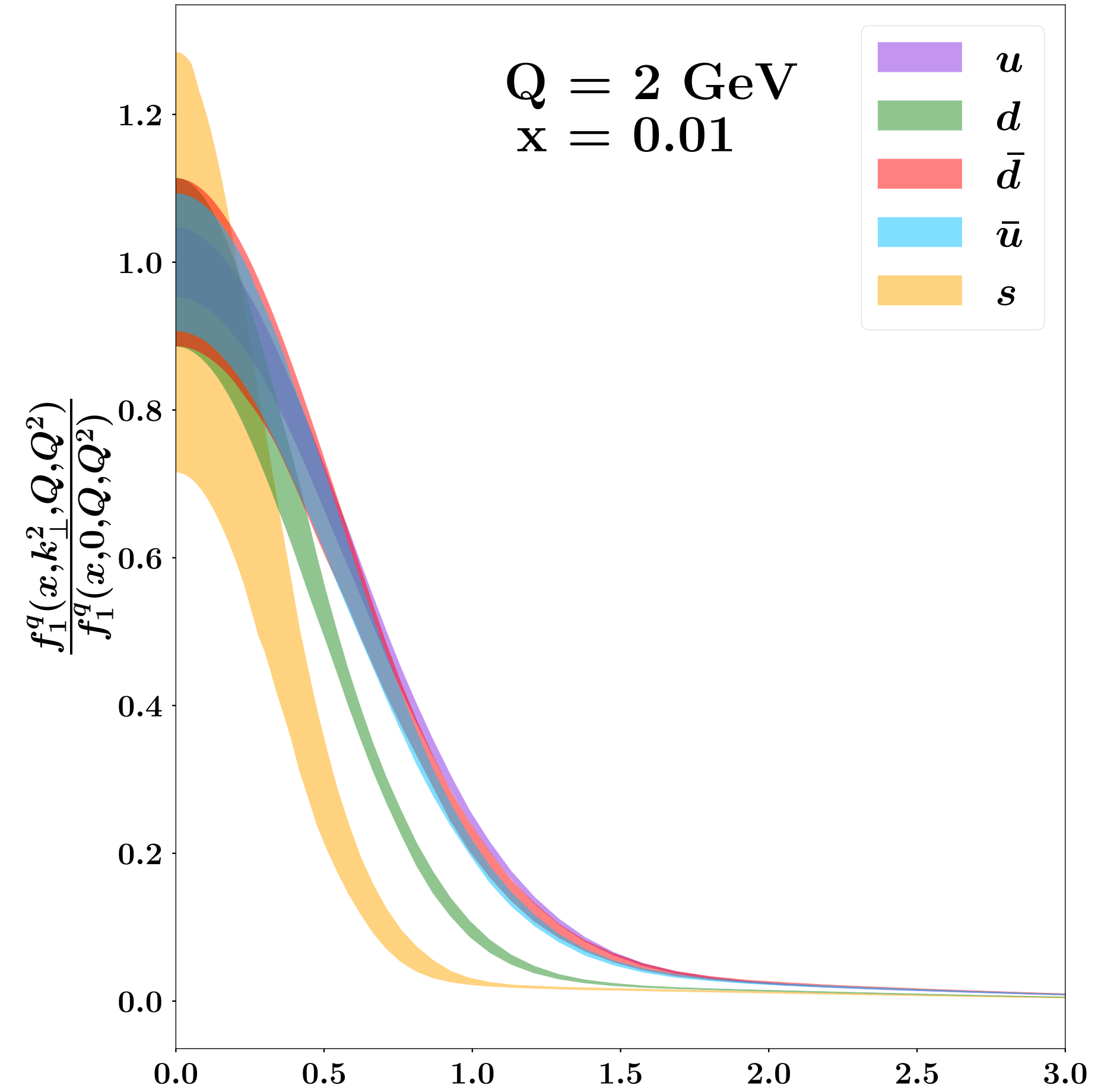
normalized to the same value at $k_{\perp} = 0$

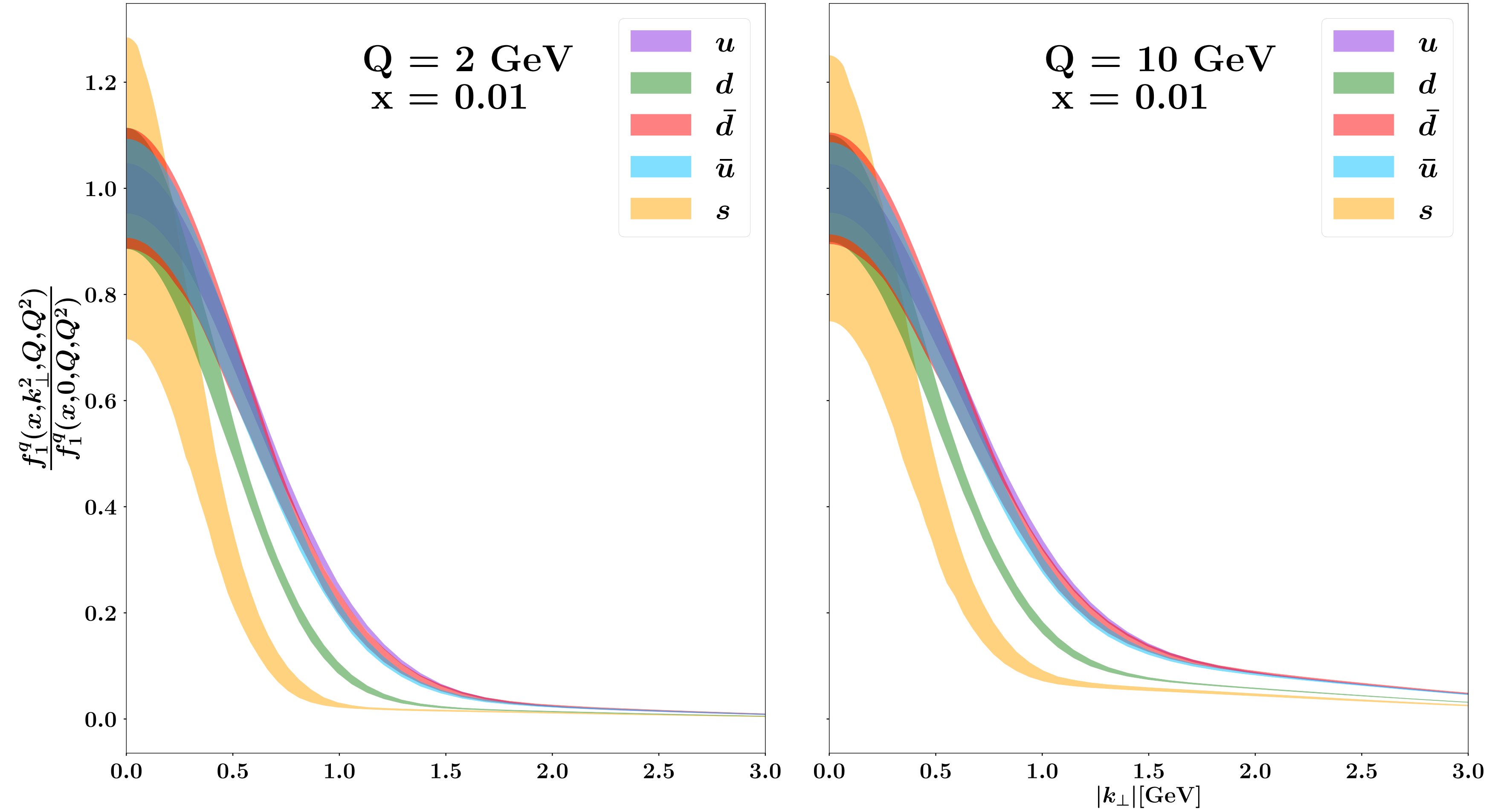


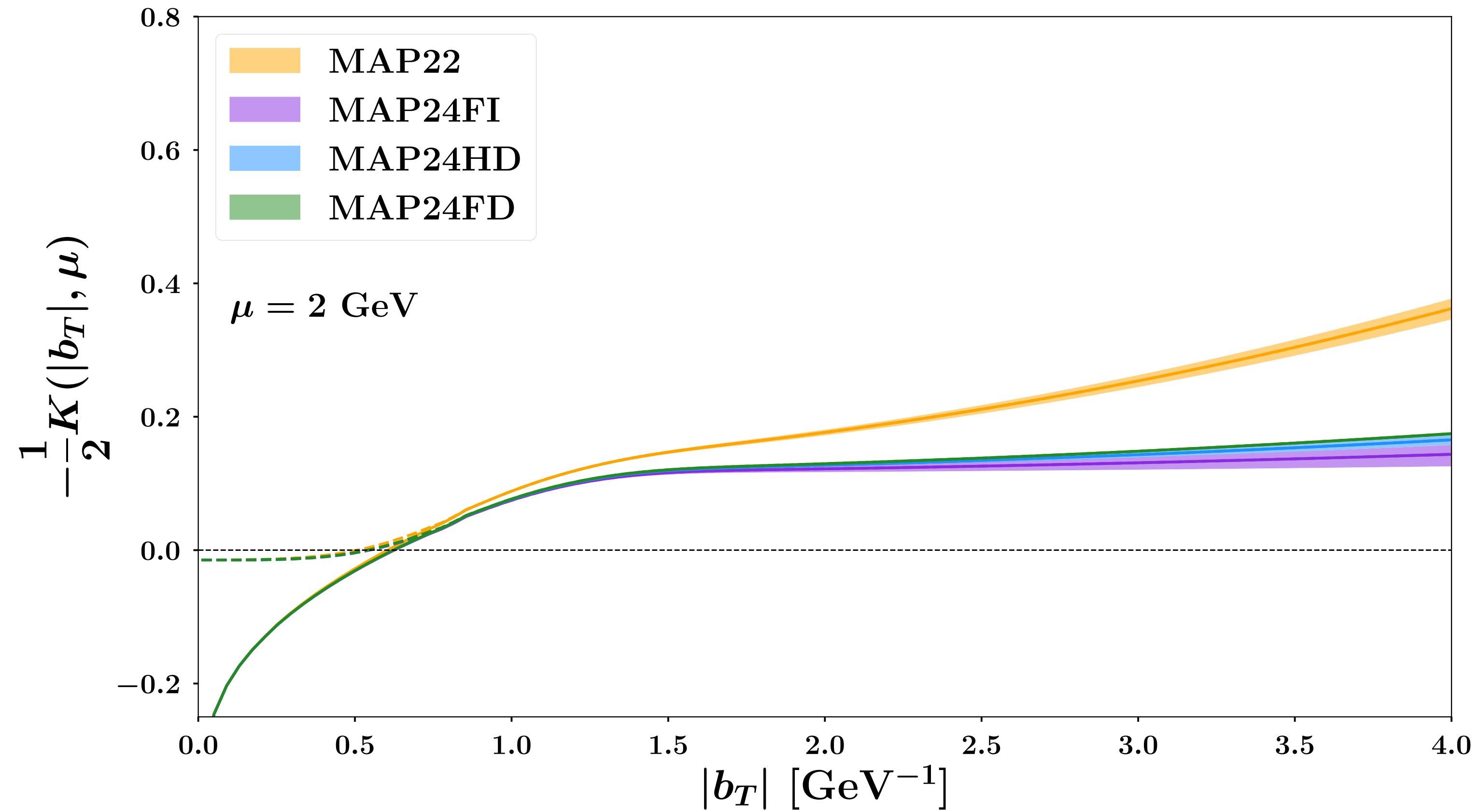
The u quark is wider and the s quark narrower

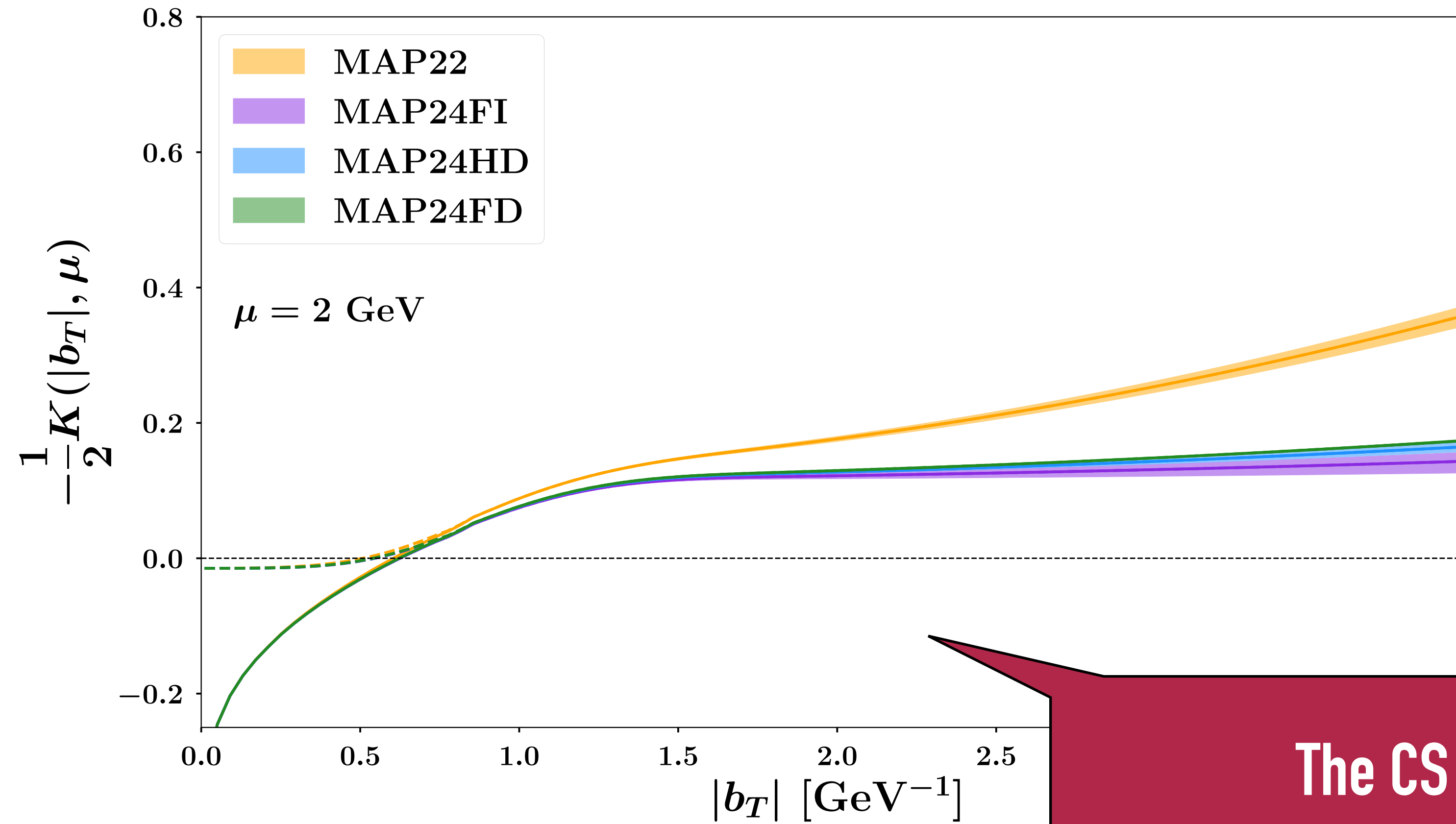








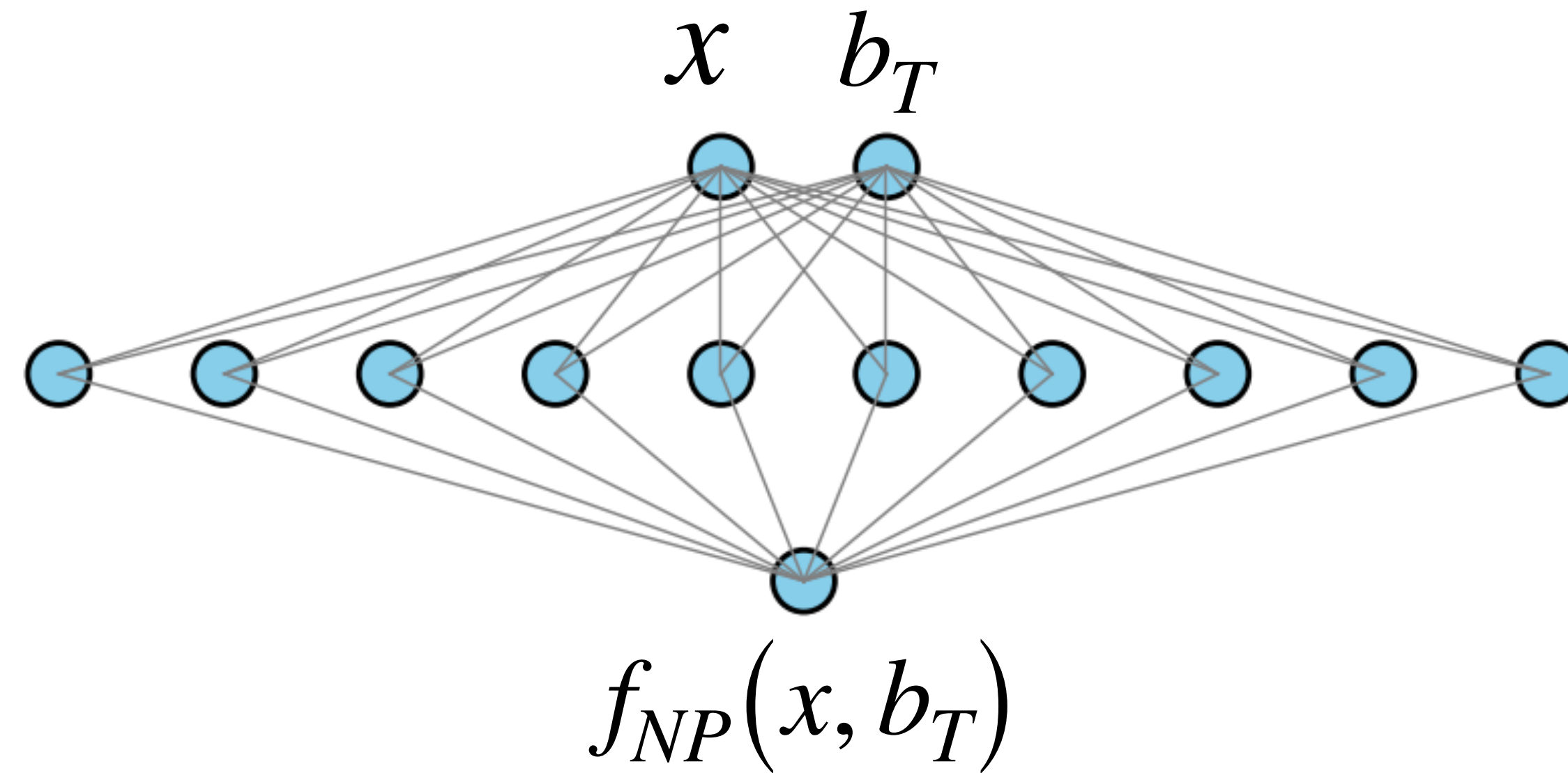




The CS kernel is “universal”
(no flavor, no x , no hadron dependence)



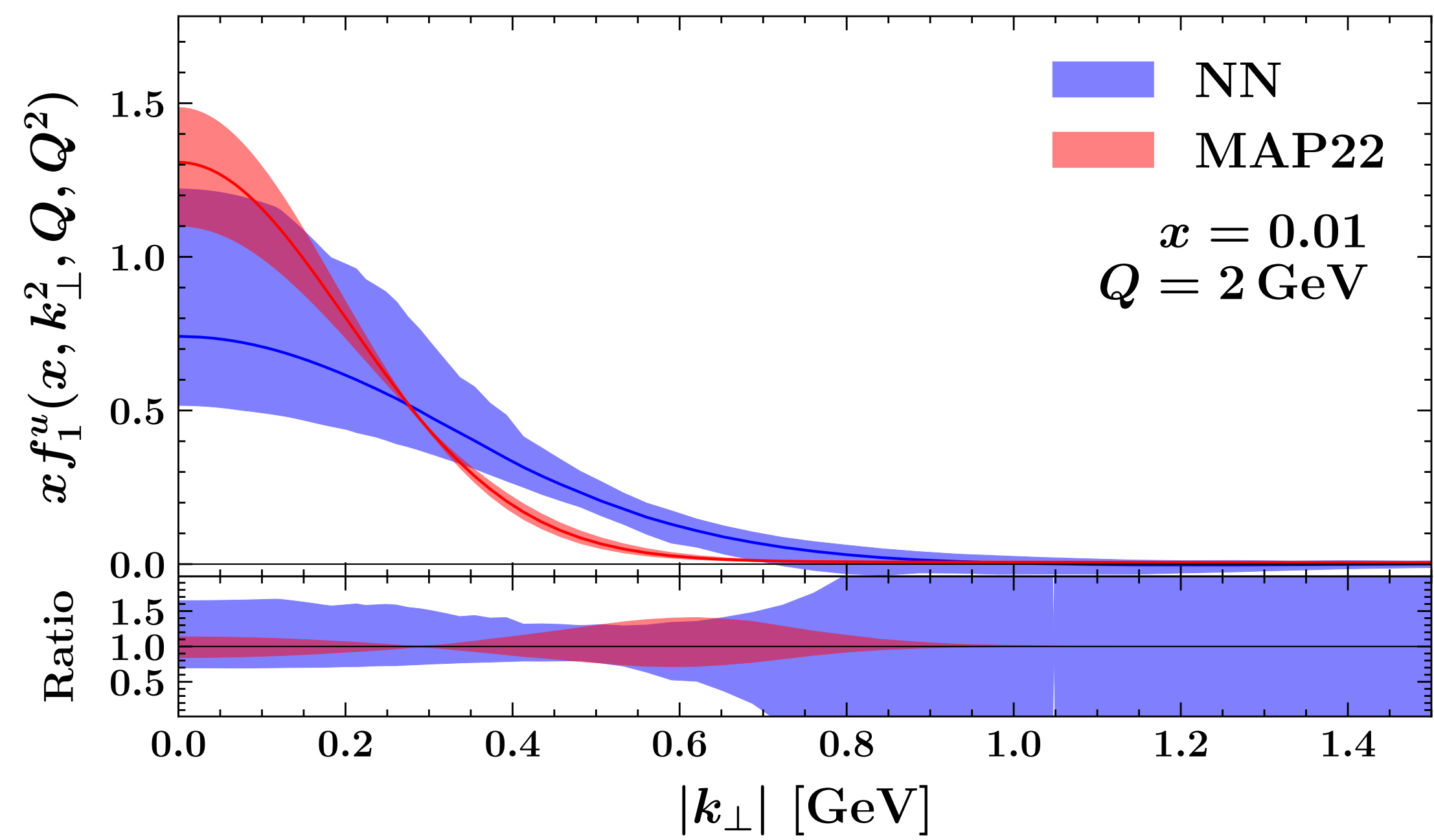
NEURAL NETWORK EXTRACTION



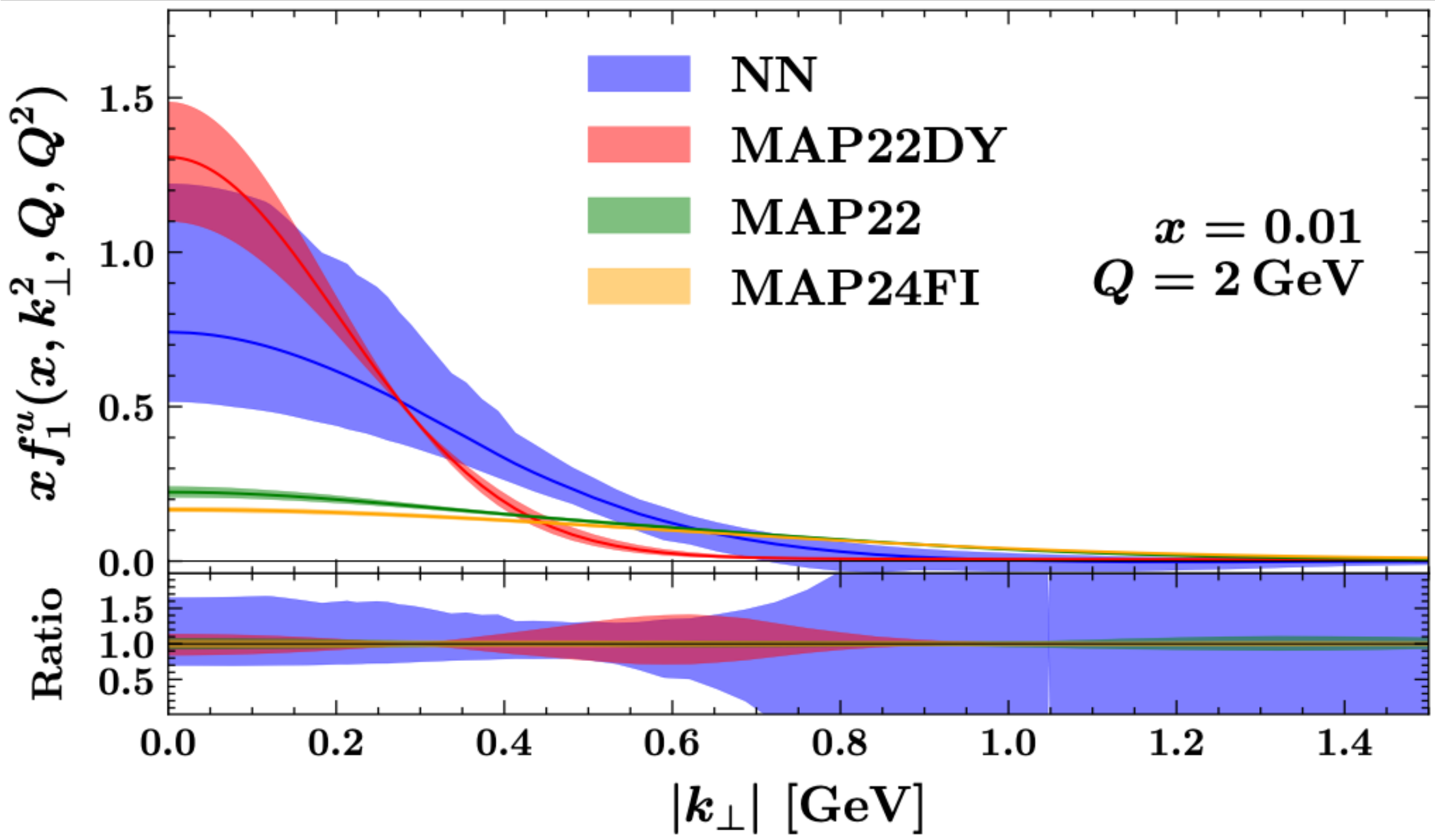
$$f_{NP}(x, b_T; \zeta) = \frac{\text{NN}(x, b_T)}{\text{NN}(x, 0)} \exp \left[-g_2^2 b_T^2 \log \left(\frac{\zeta}{Q_0^2} \right) \right]$$

41 parameters + 1 parameter for CS kernel

Experiment	N_{dat}	$\bar{\chi}^2 \ (\bar{\chi}_D^2 + \bar{\chi}_\lambda^2)$	
		NN	MAP22
Fixed-target	233	1.08 (0.98 + 0.10)	0.91 (0.70 + 0.21)
RHIC	7	1.11 (1.03 + 0.07)	1.45 (1.37 + 0.08)
Tevatron	71	0.80 (0.73 + 0.06)	1.20 (1.17 + 0.04)
LHCb	21	0.98 (0.88 + 0.10)	1.25 (1.05 + 0.20)
CMS	78	0.40 (0.38 + 0.02)	0.41 (0.35 + 0.06)
ATLAS	72	1.38 (1.09 + 0.29)	3.51 (3.03 + 0.49)
Total	482	0.97 (0.86 + 0.11)	1.28 (1.09 + 0.20)

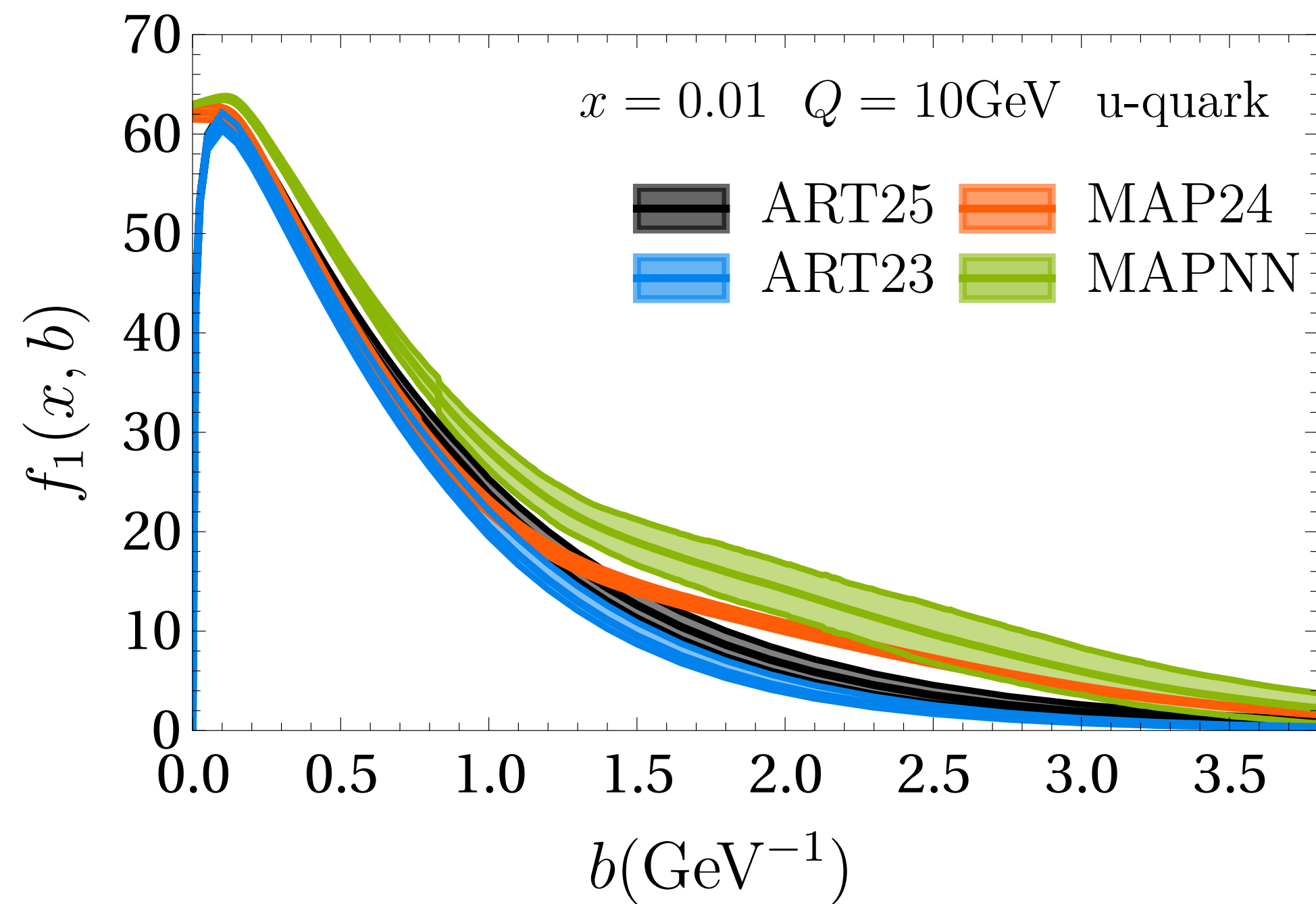


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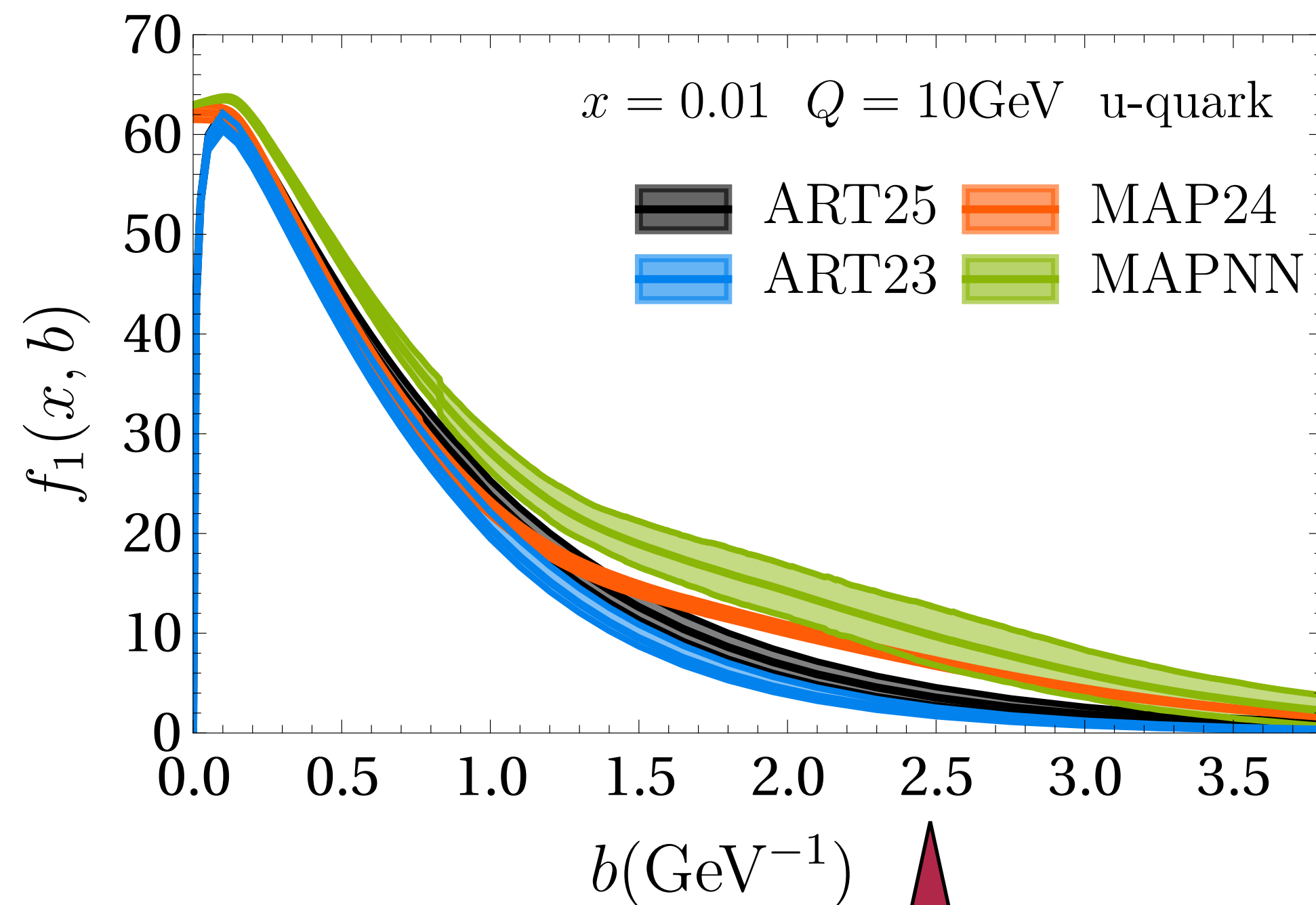
ART25 [arXiv:2503.112021](https://arxiv.org/abs/2503.112021), see talk by V. Moos

comparison in b_T space



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comparison in b_T space

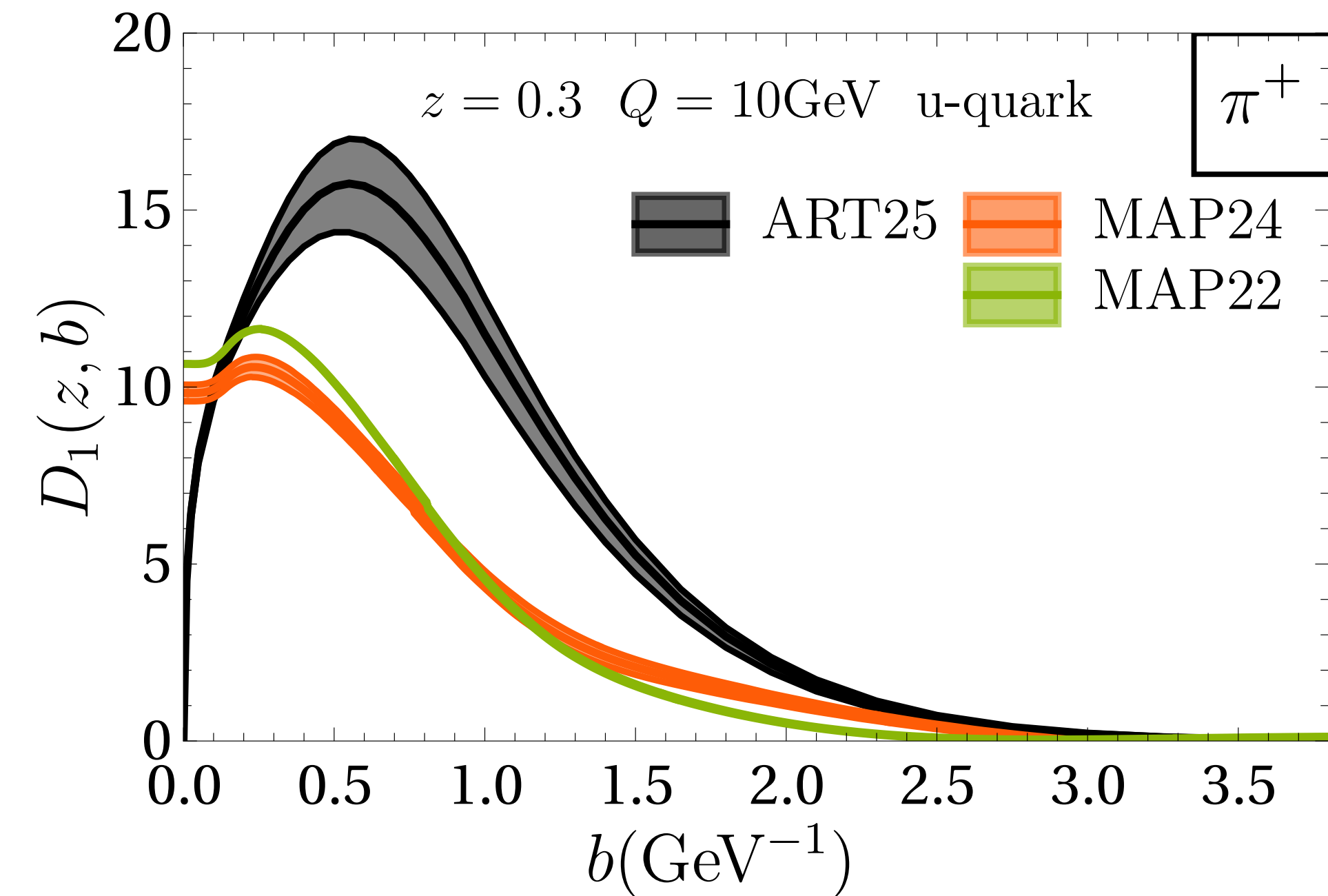
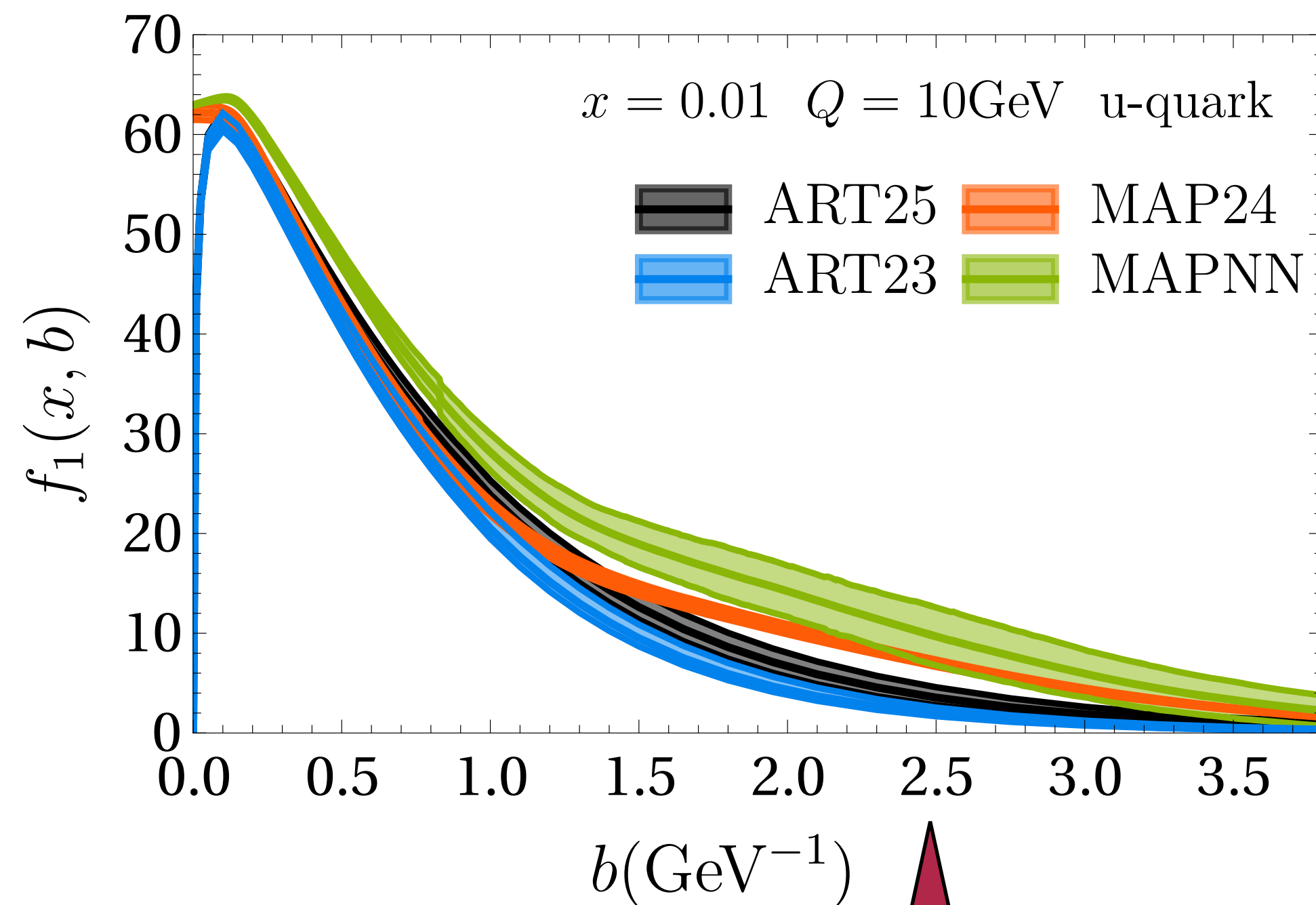


There are significant differences between different extractions.

The error bands are probably underestimated

ART25 [arXiv:2503.112021](https://arxiv.org/abs/2503.112021), see talk by V. Moos

comparison in b_T space

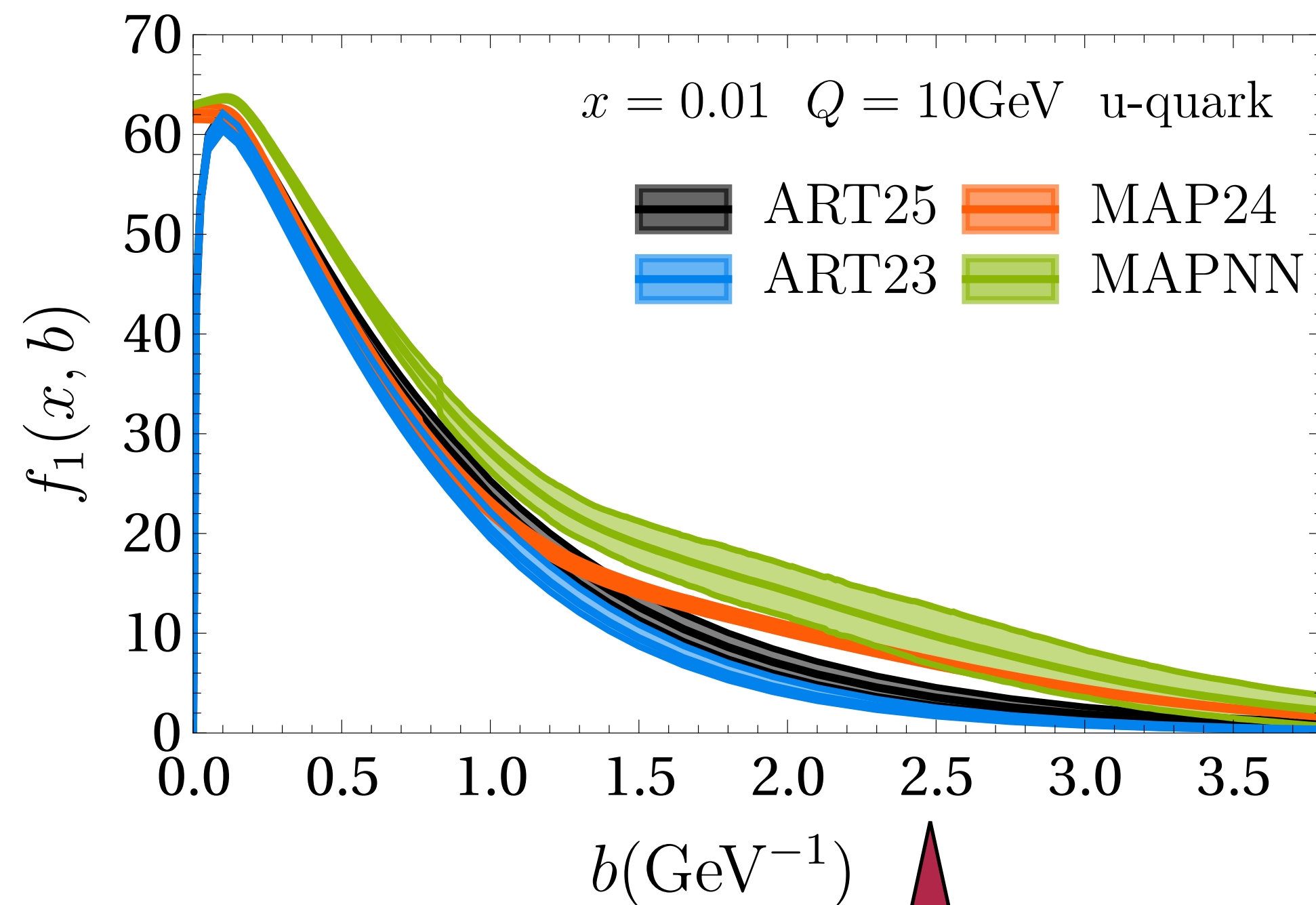


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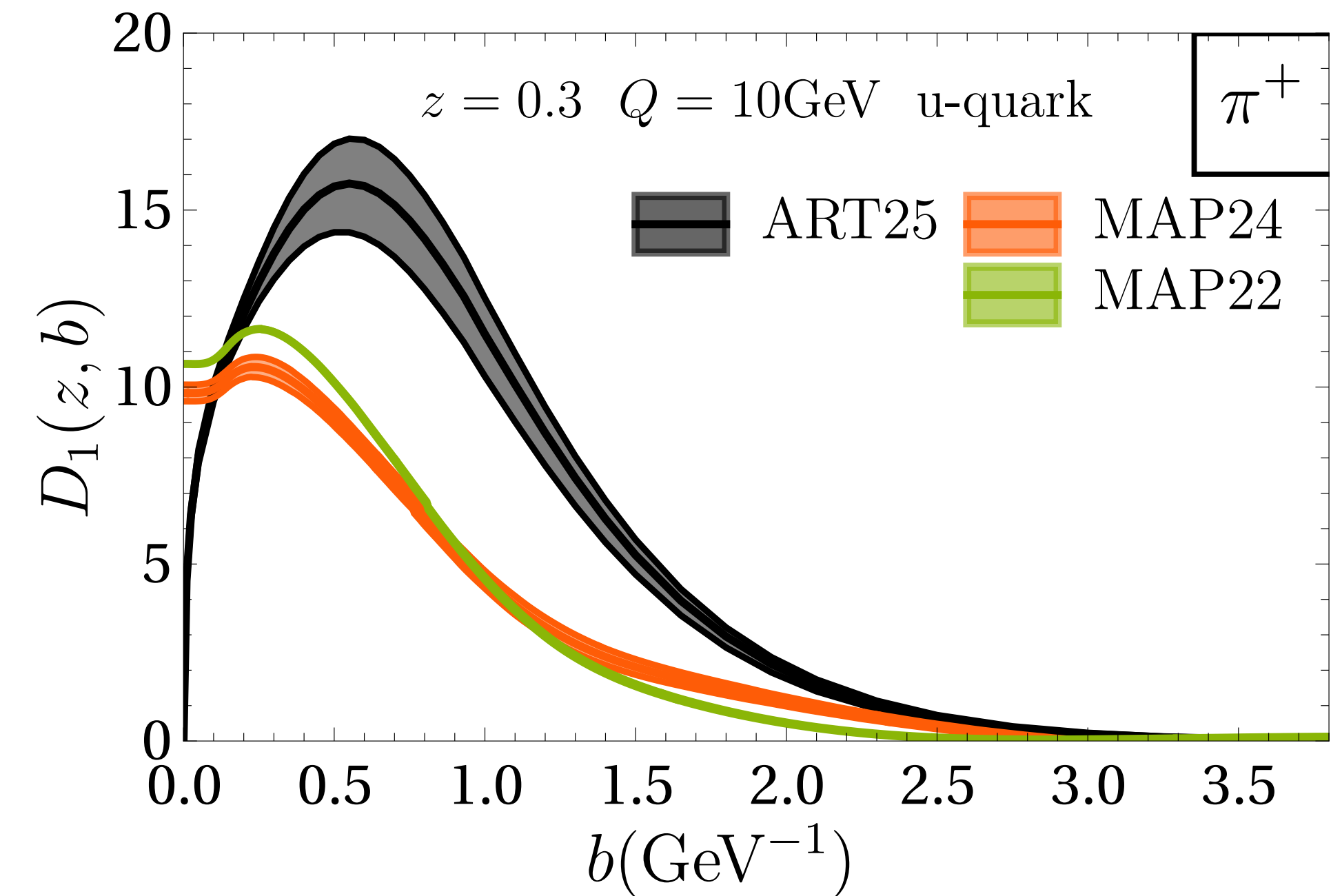
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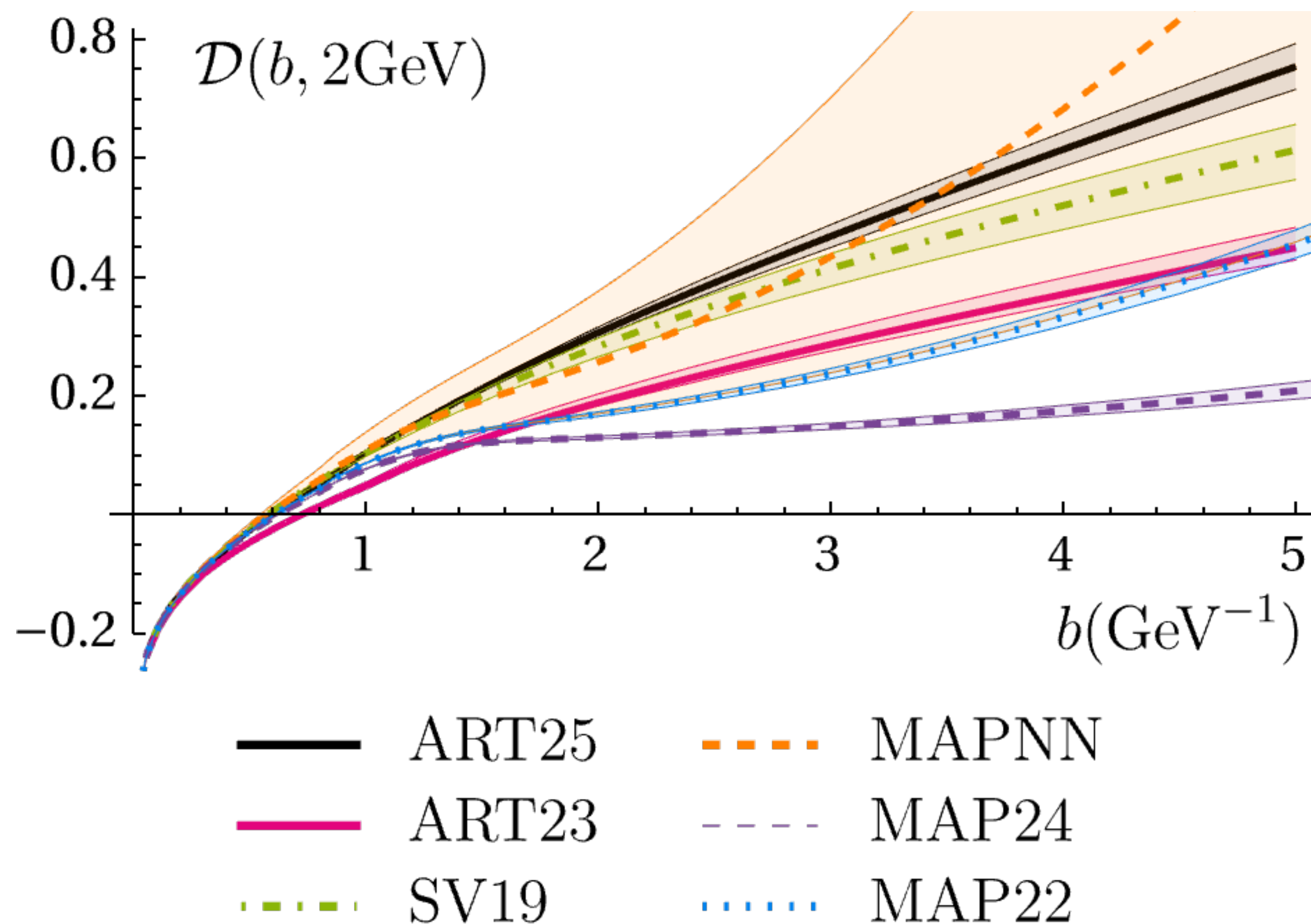
comparison in b_T space

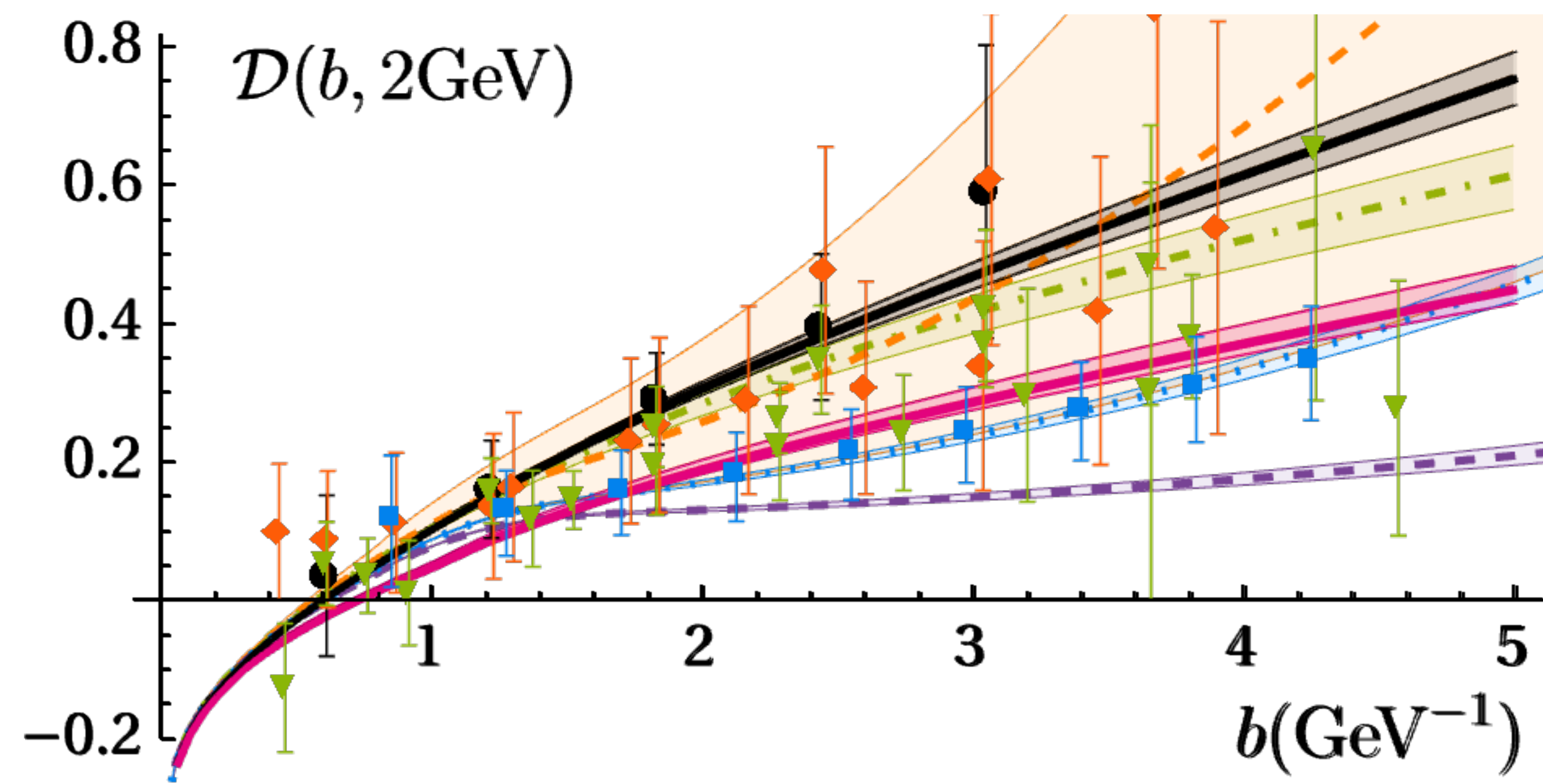


There are significant differences between different extractions.
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Even larger differences in the Fragmentation Functions

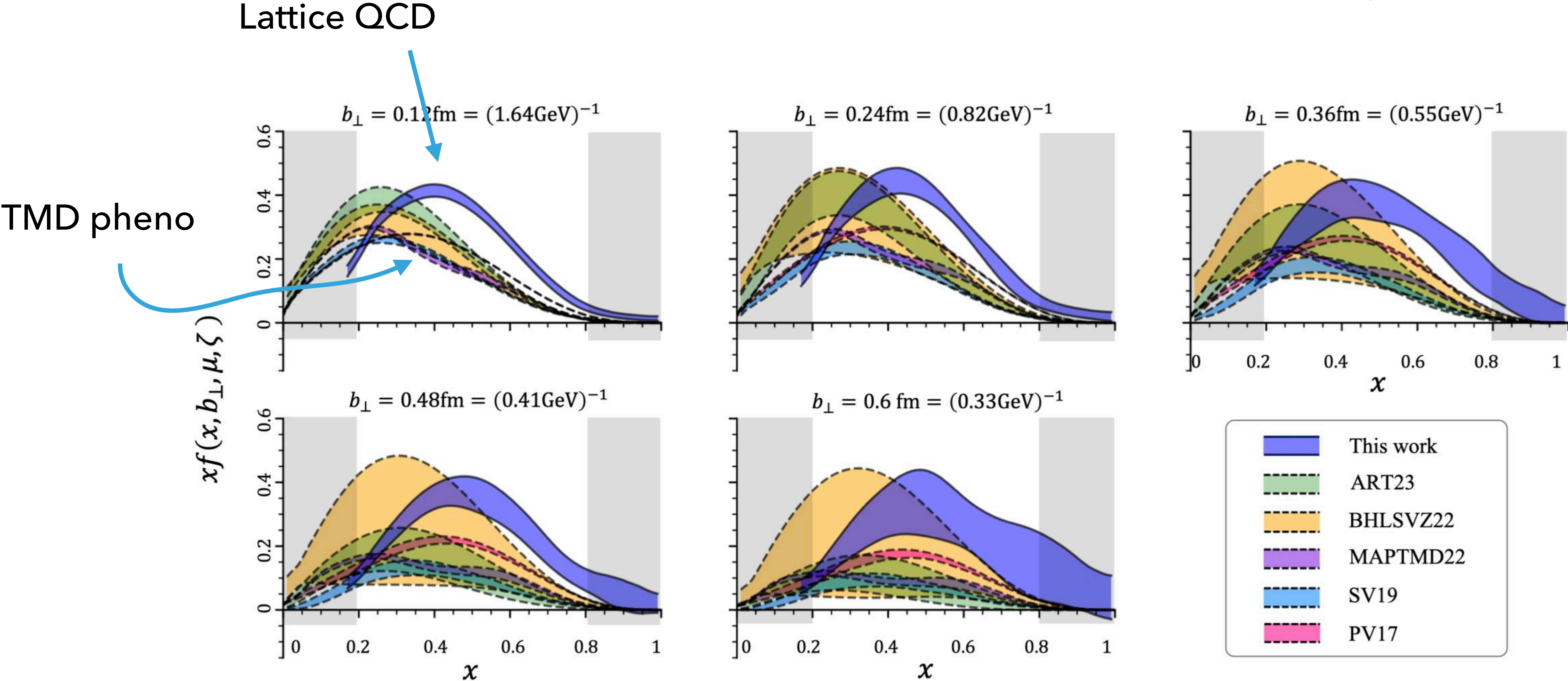




TMD phenomenology

Lattice QCD

[LPC collaboration, arxiv:2211.02340](#)

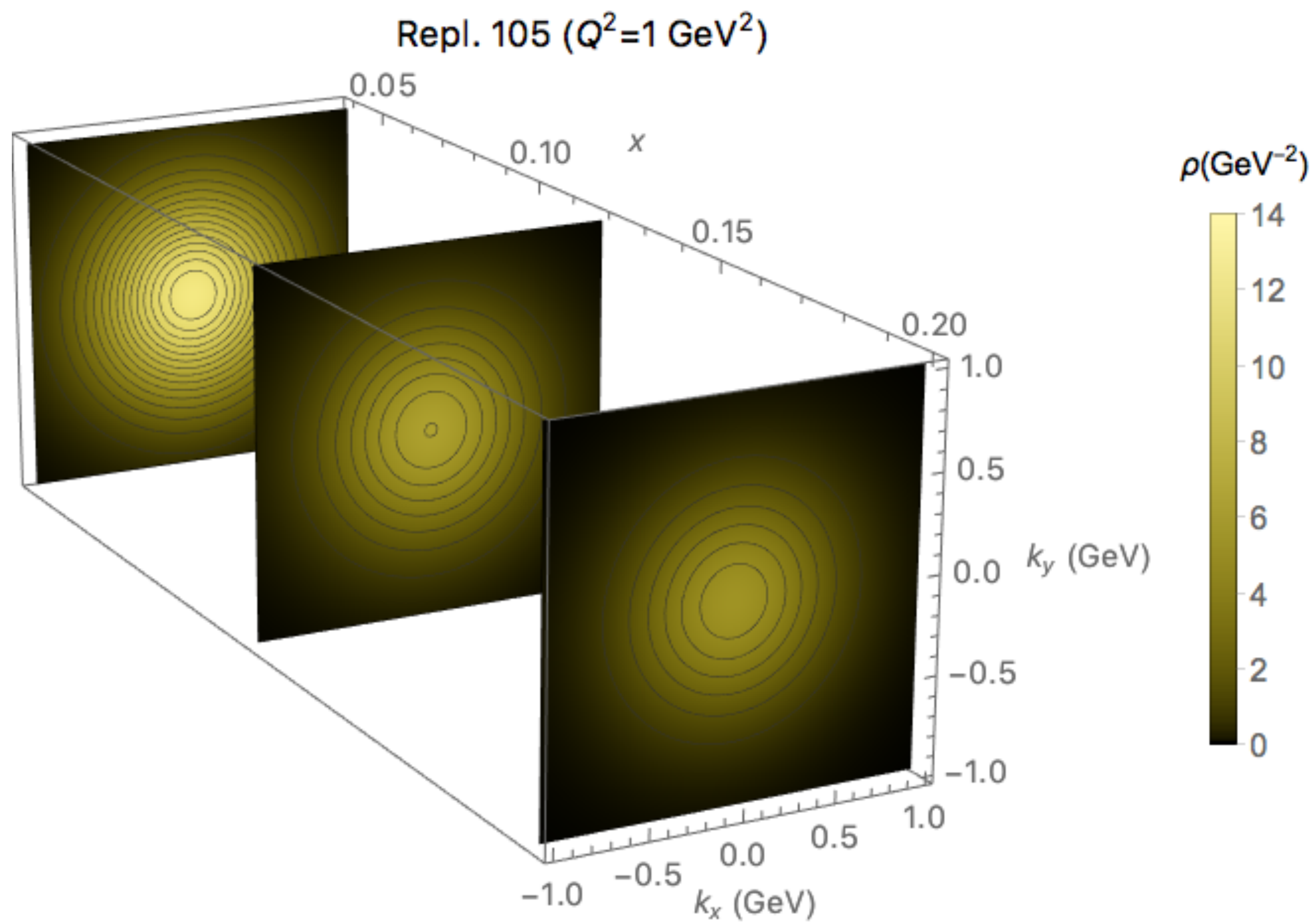


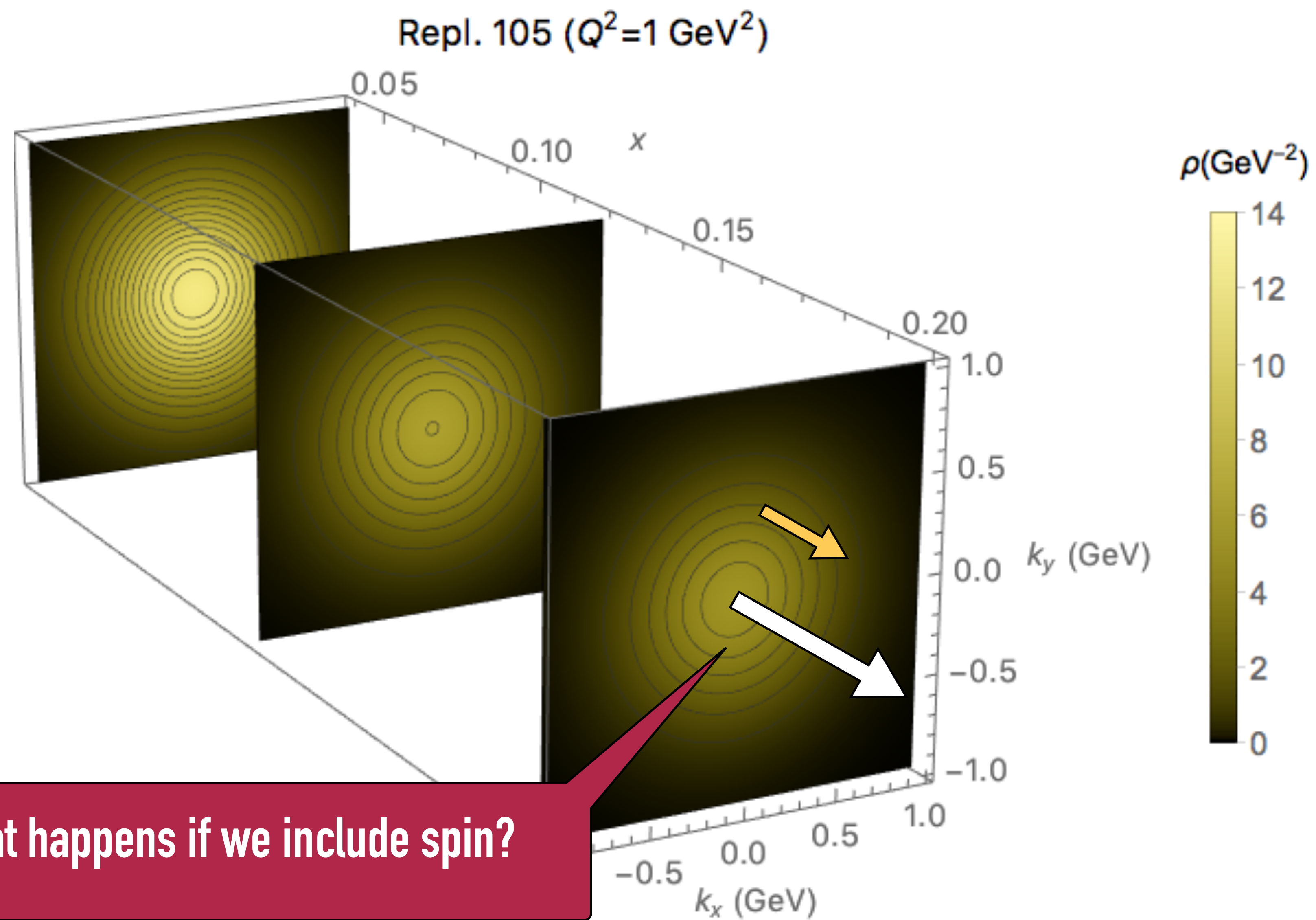


HELICITY QUARK TMD



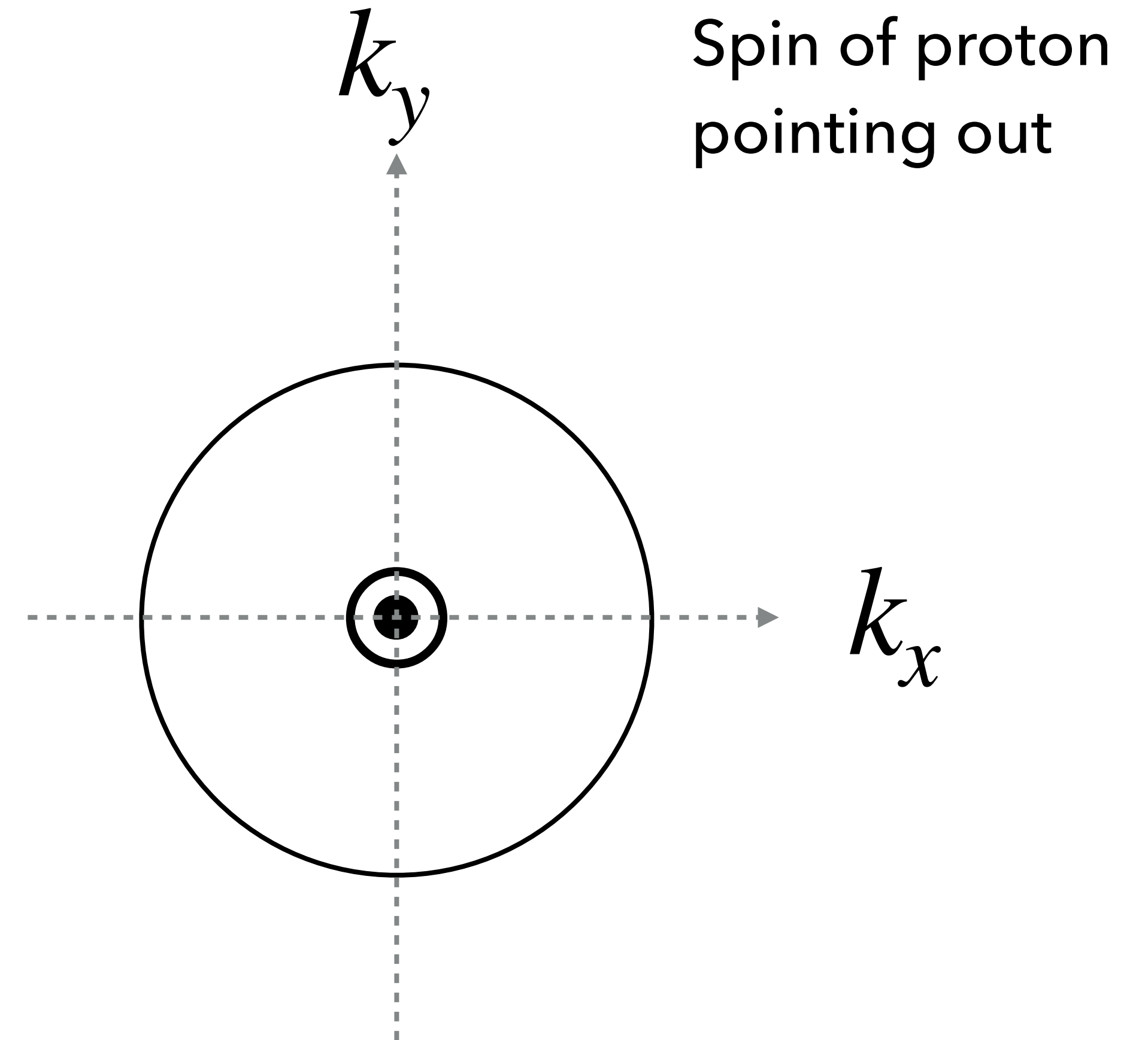
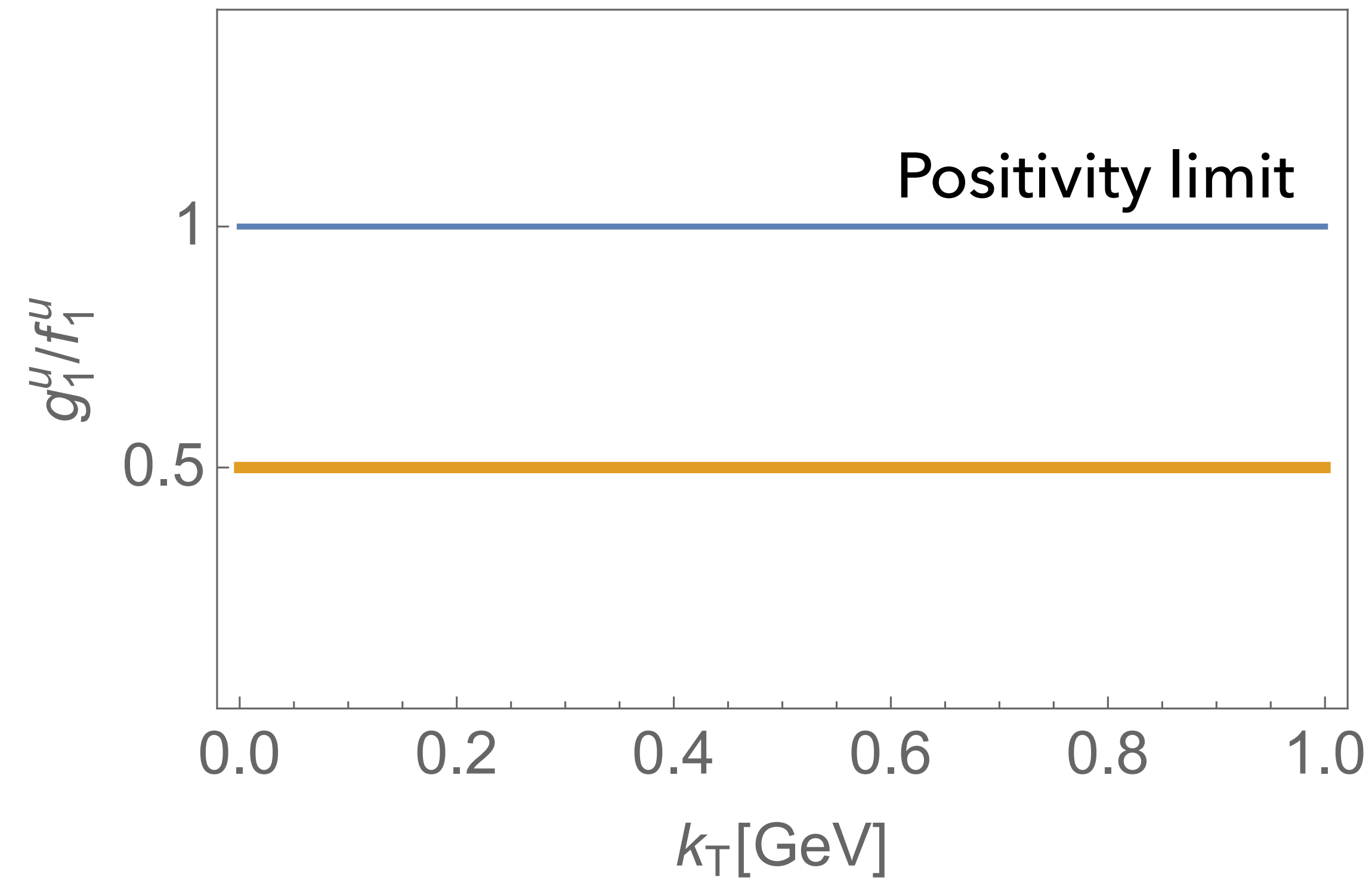
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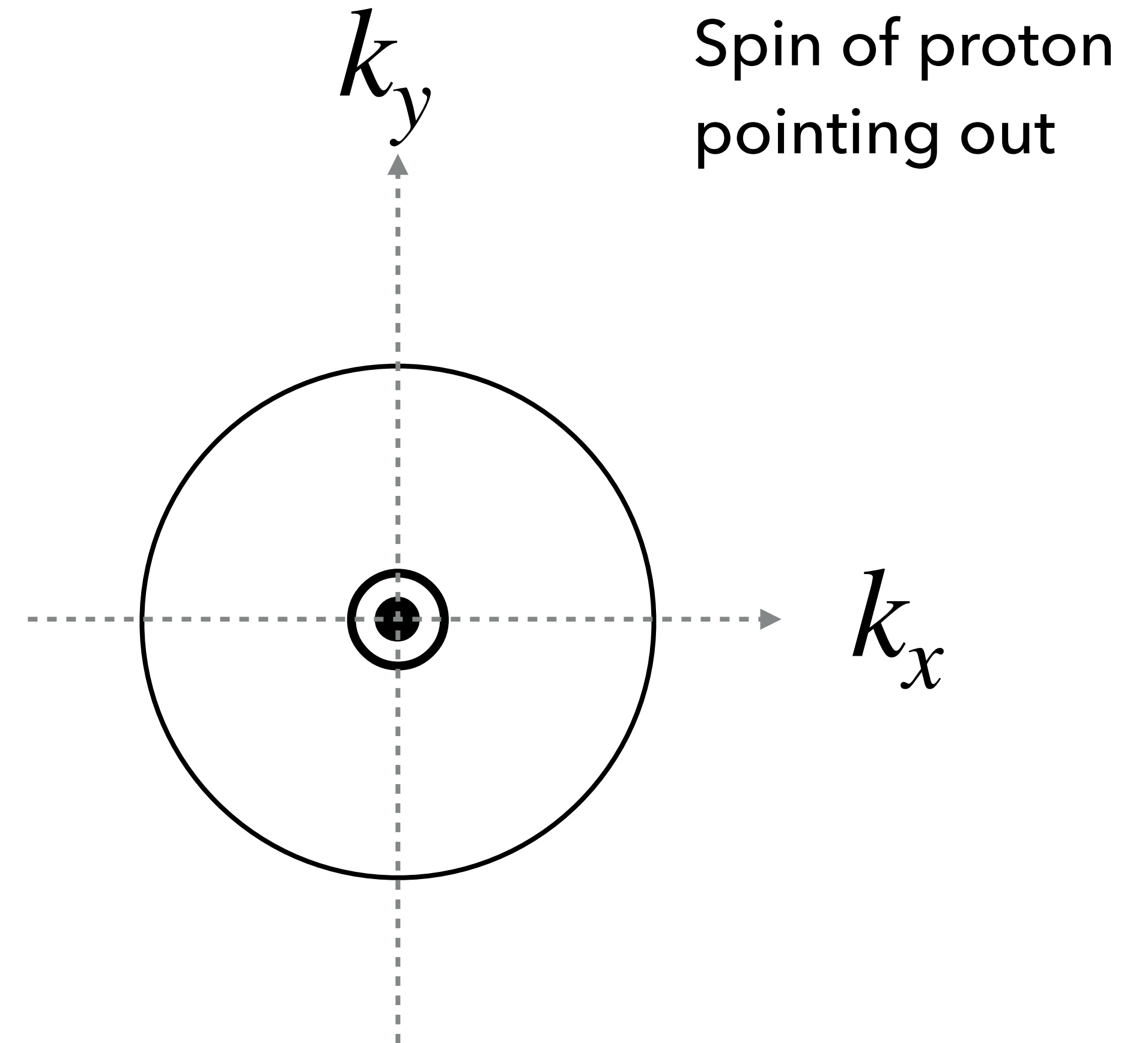
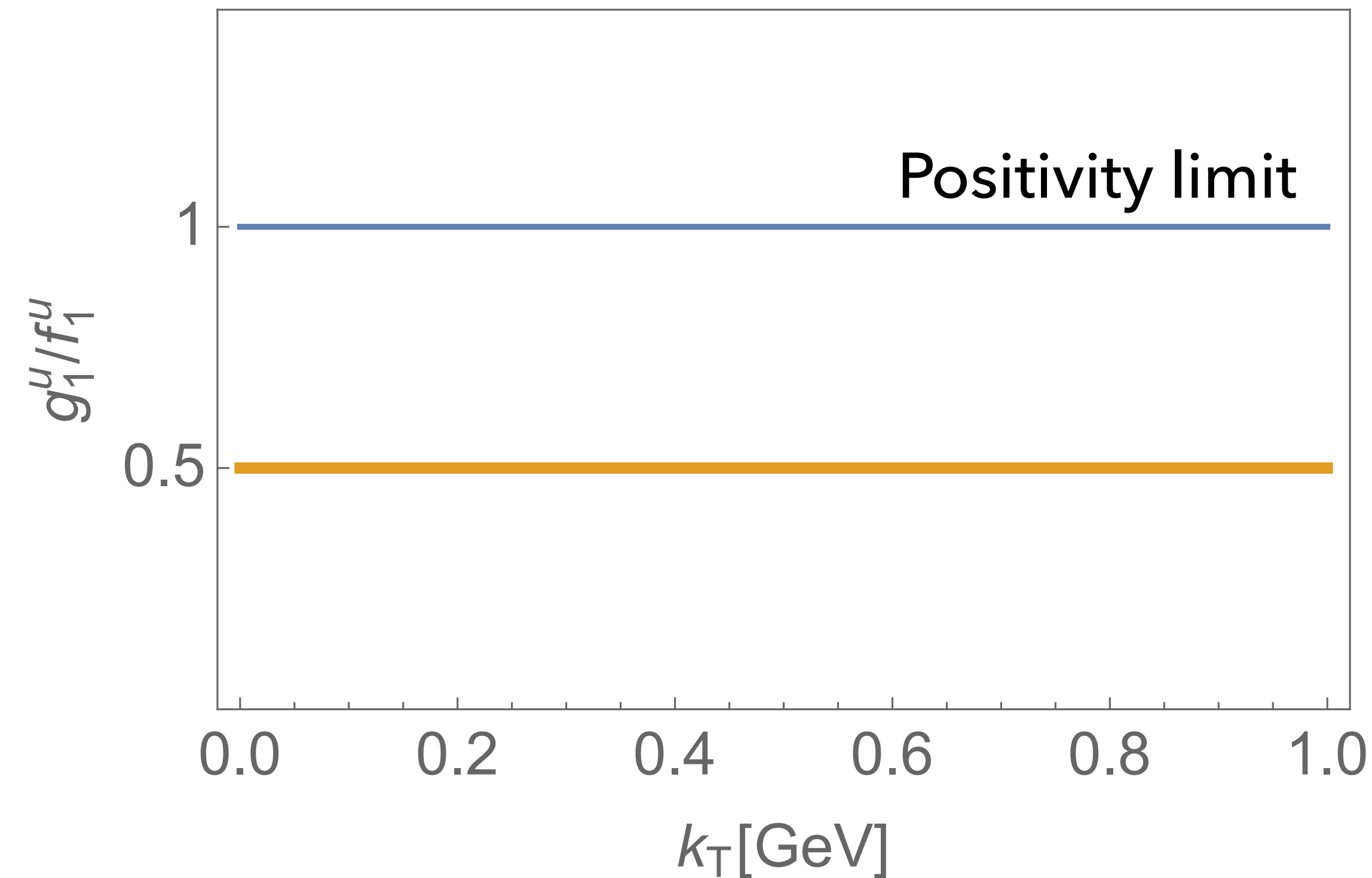


What happens if we include spin?

At a fixed value of x

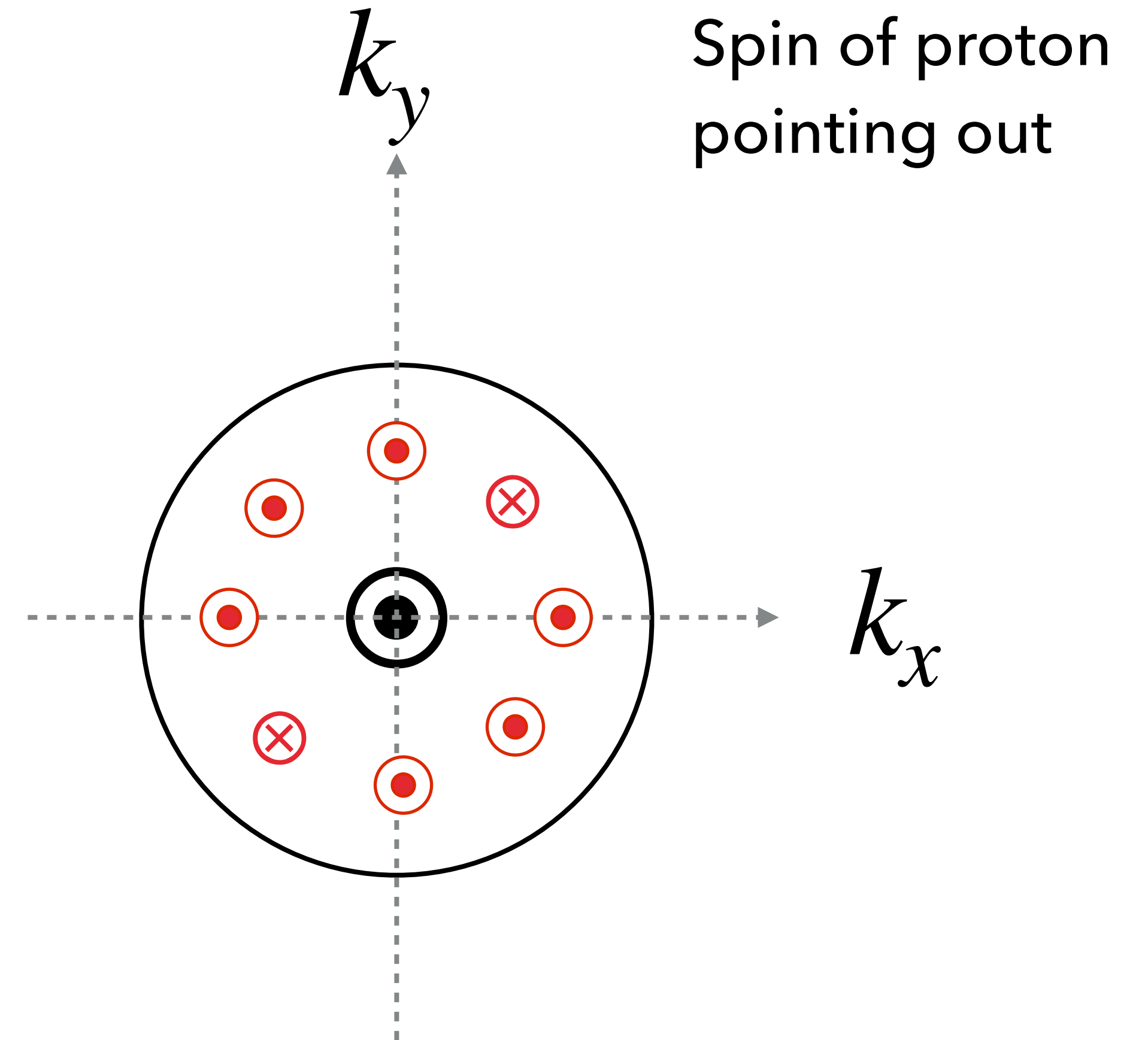
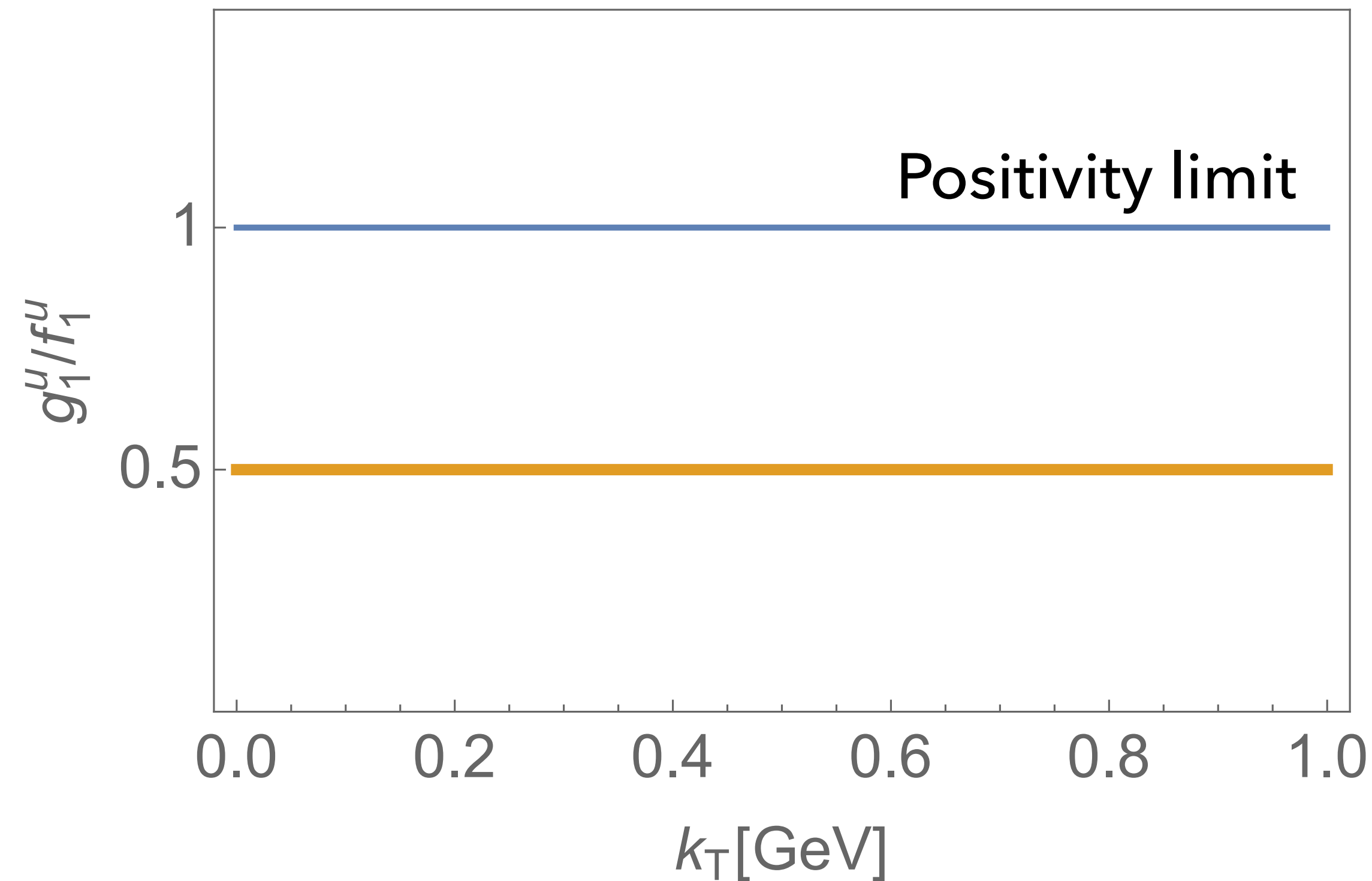


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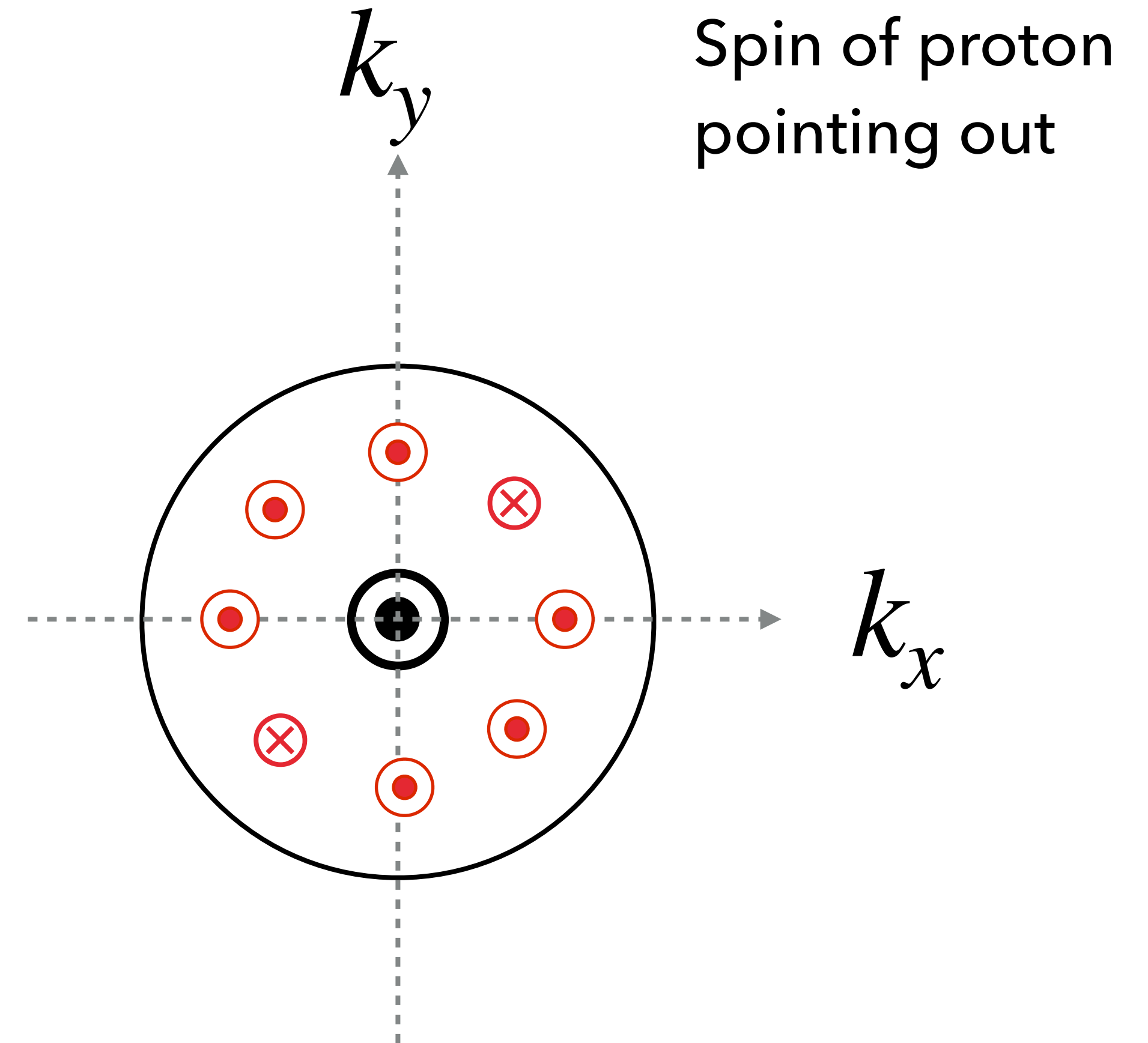
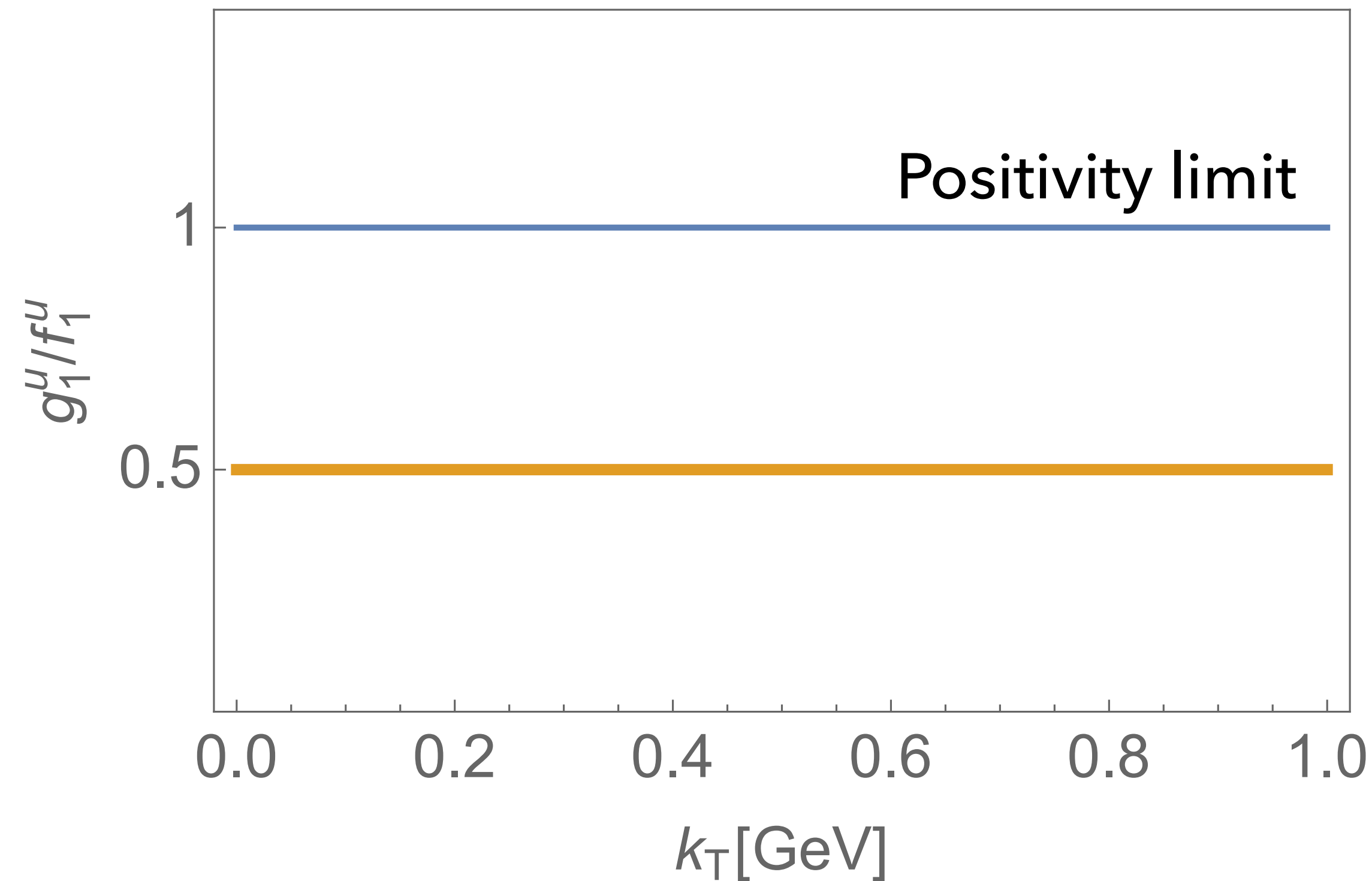
We try to always impose positivity limits ([hep-ph/9912490](https://arxiv.org/abs/hep-ph/9912490)).
We prefer rigid and physical to flexible and unphysical

At a fixed value of x



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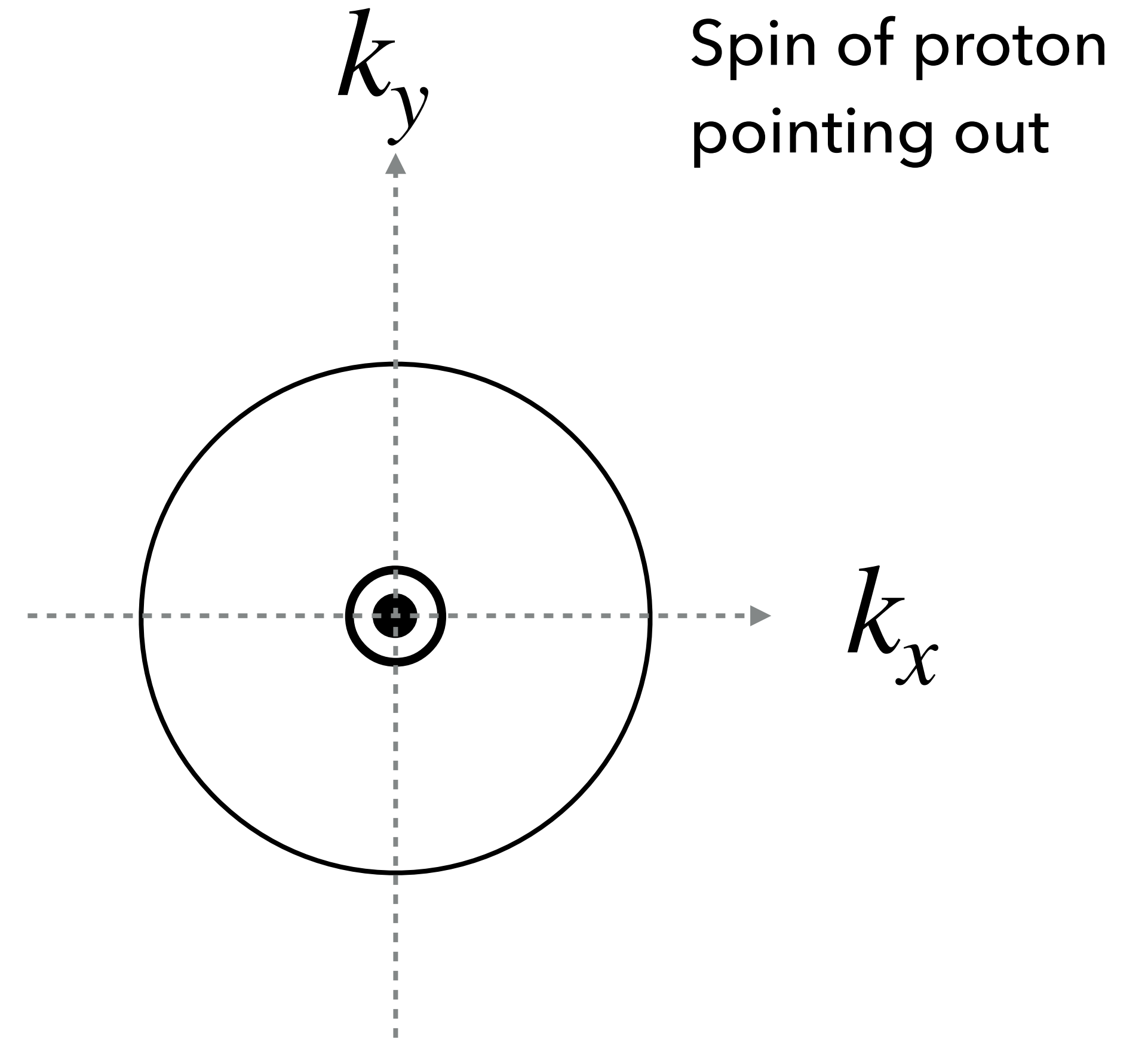
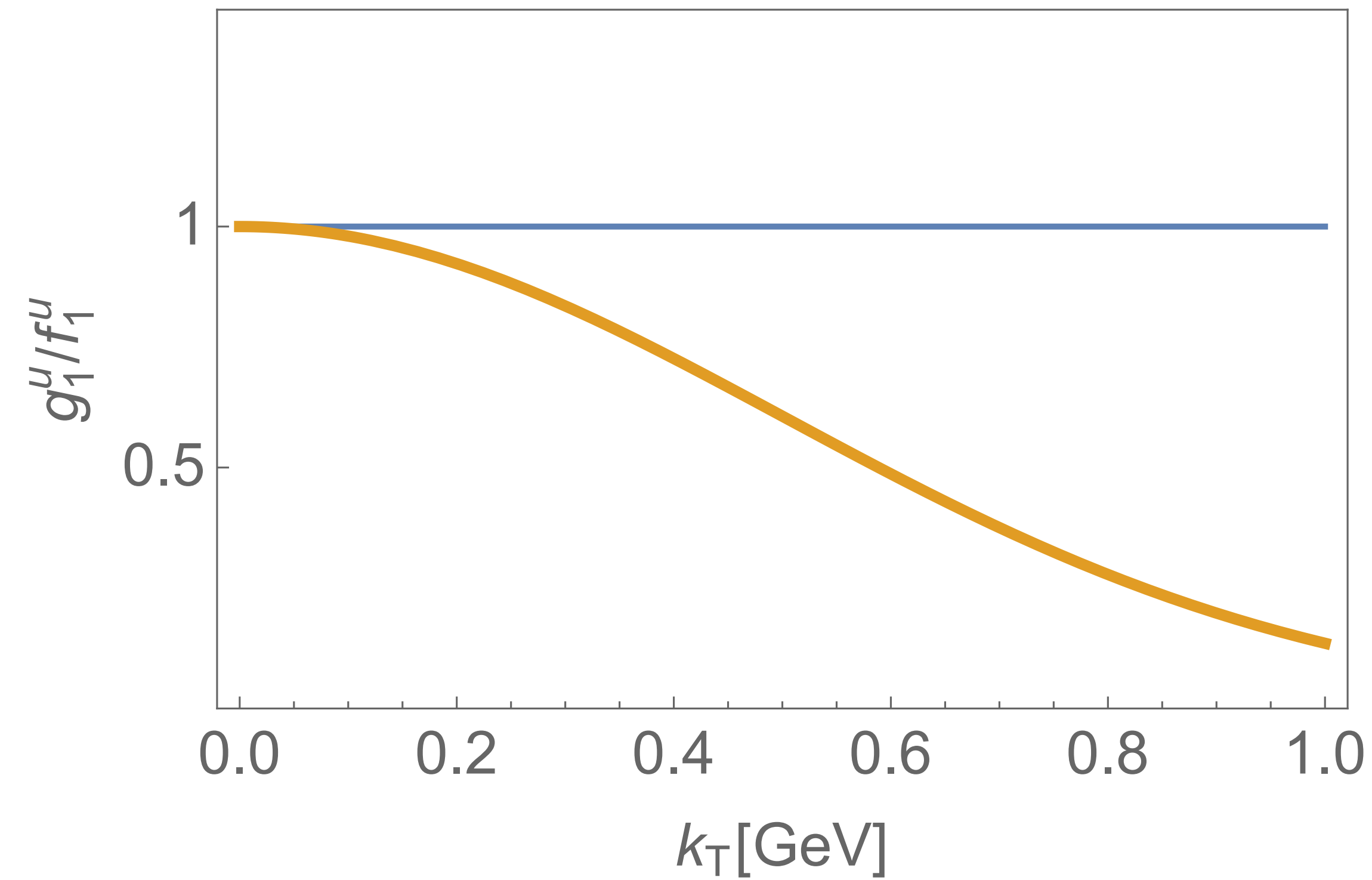


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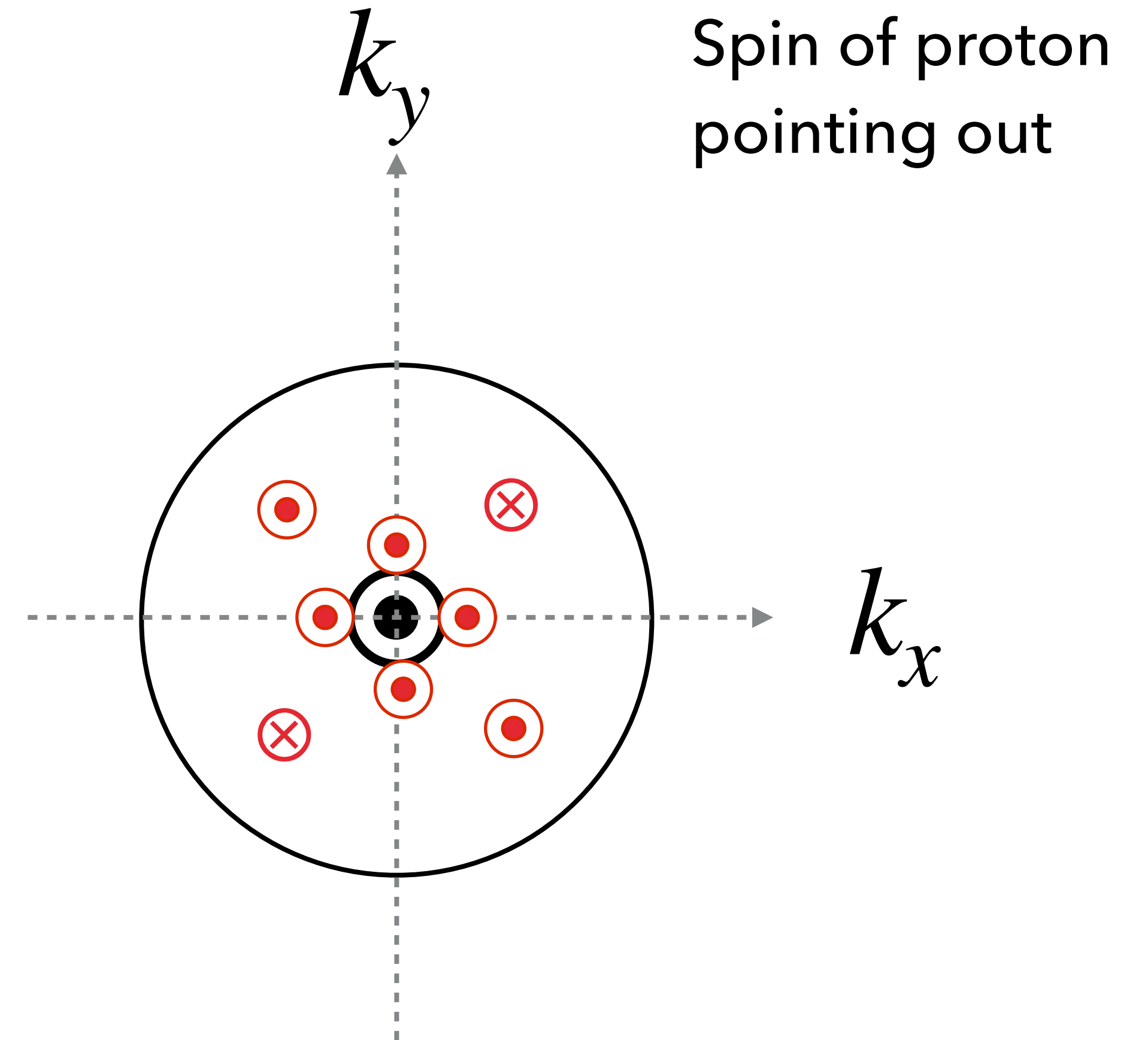
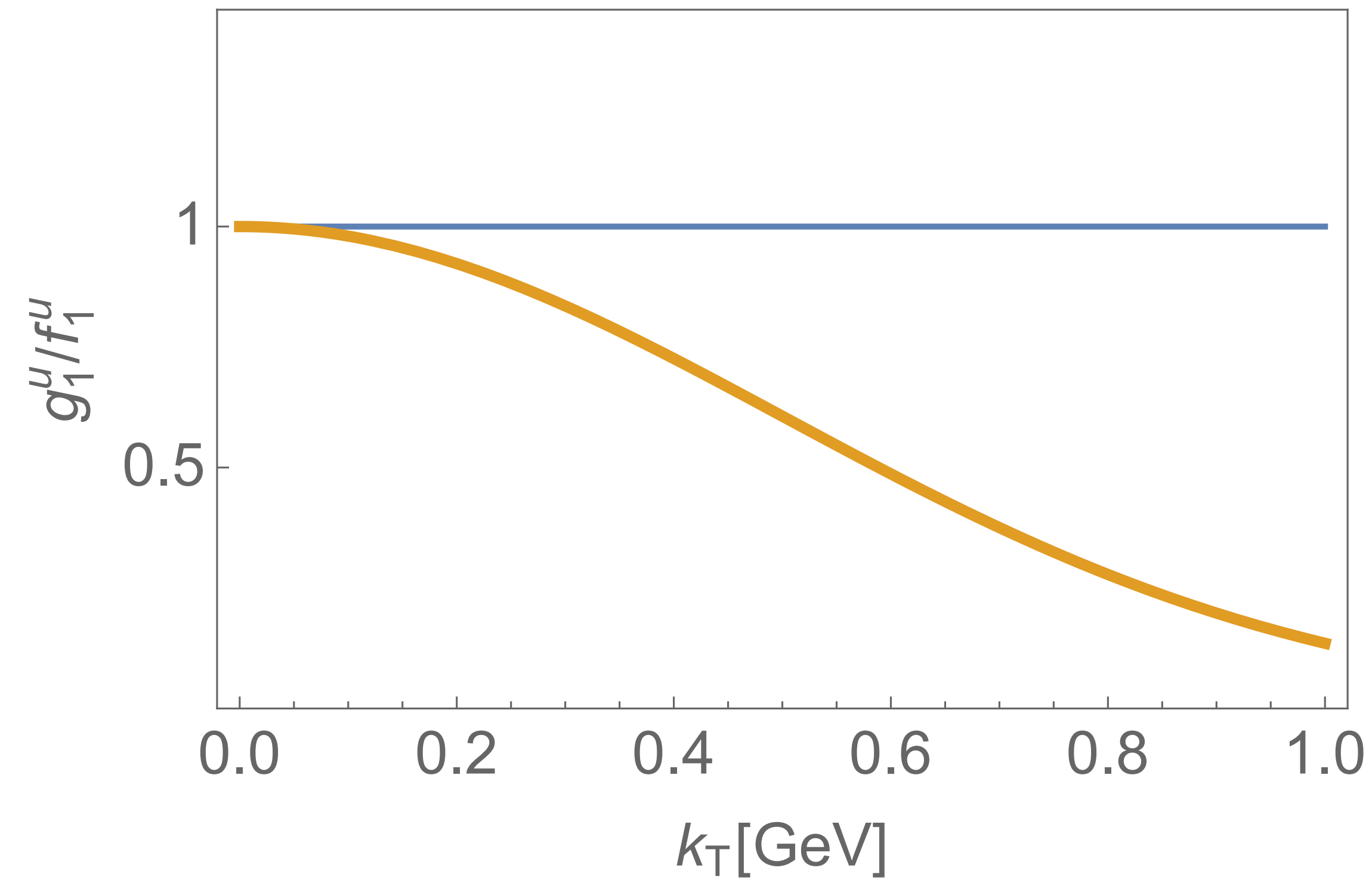
We prefer rigid and physical to flexible and unphysical

The fraction of same/opposite helicities is the same at any transverse momentum

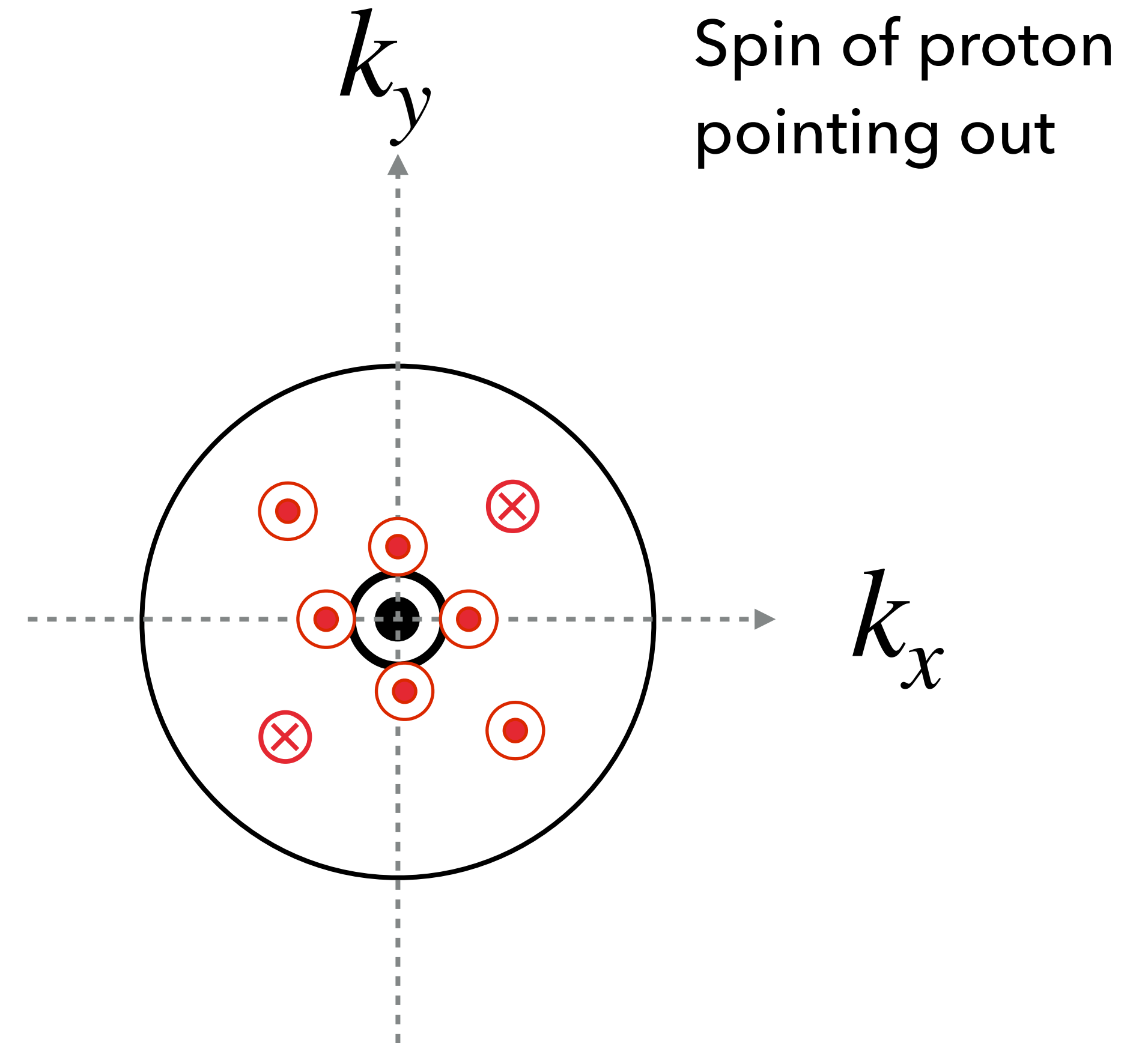
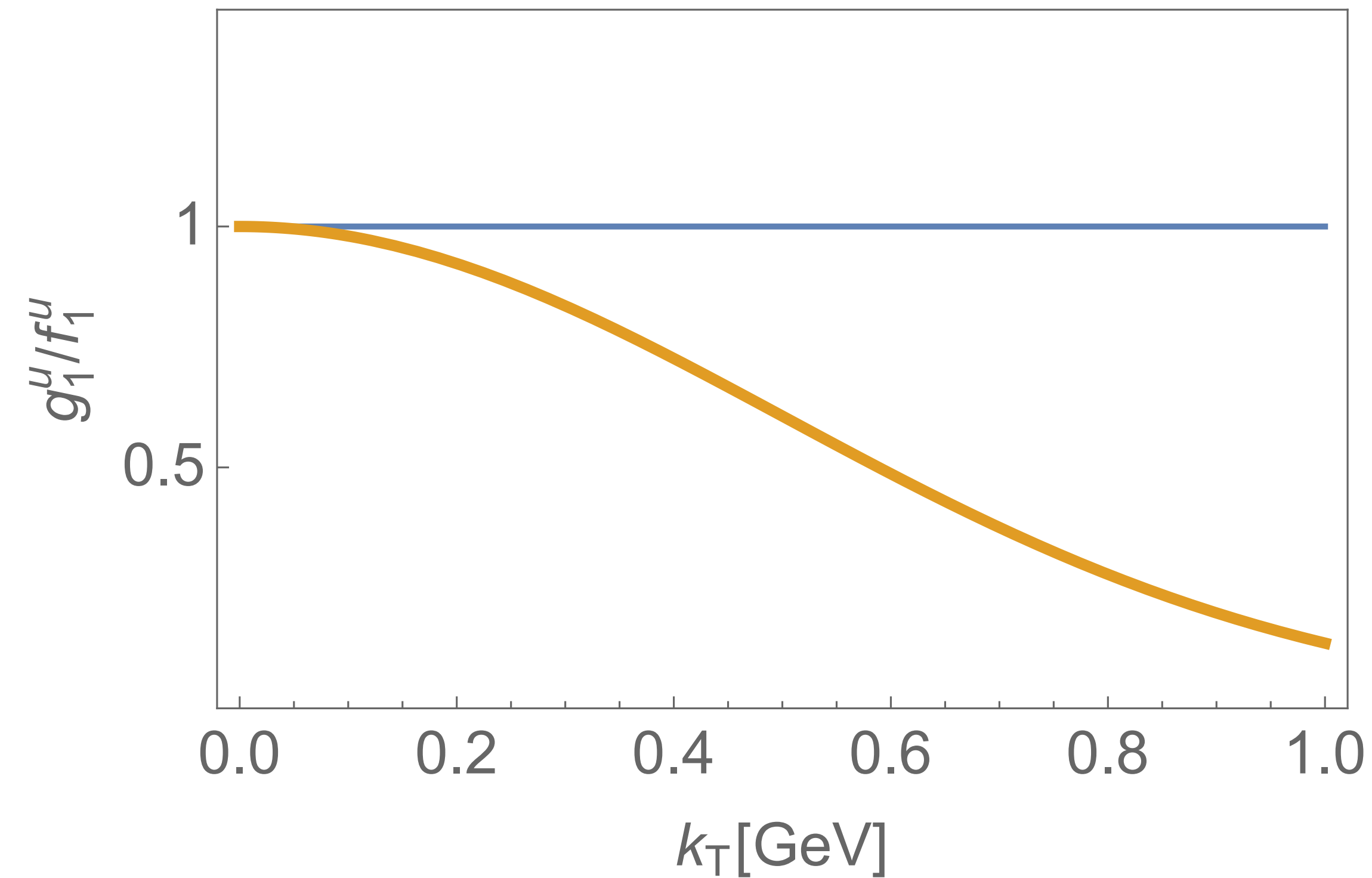
At a certain value of x



At a certain value of x

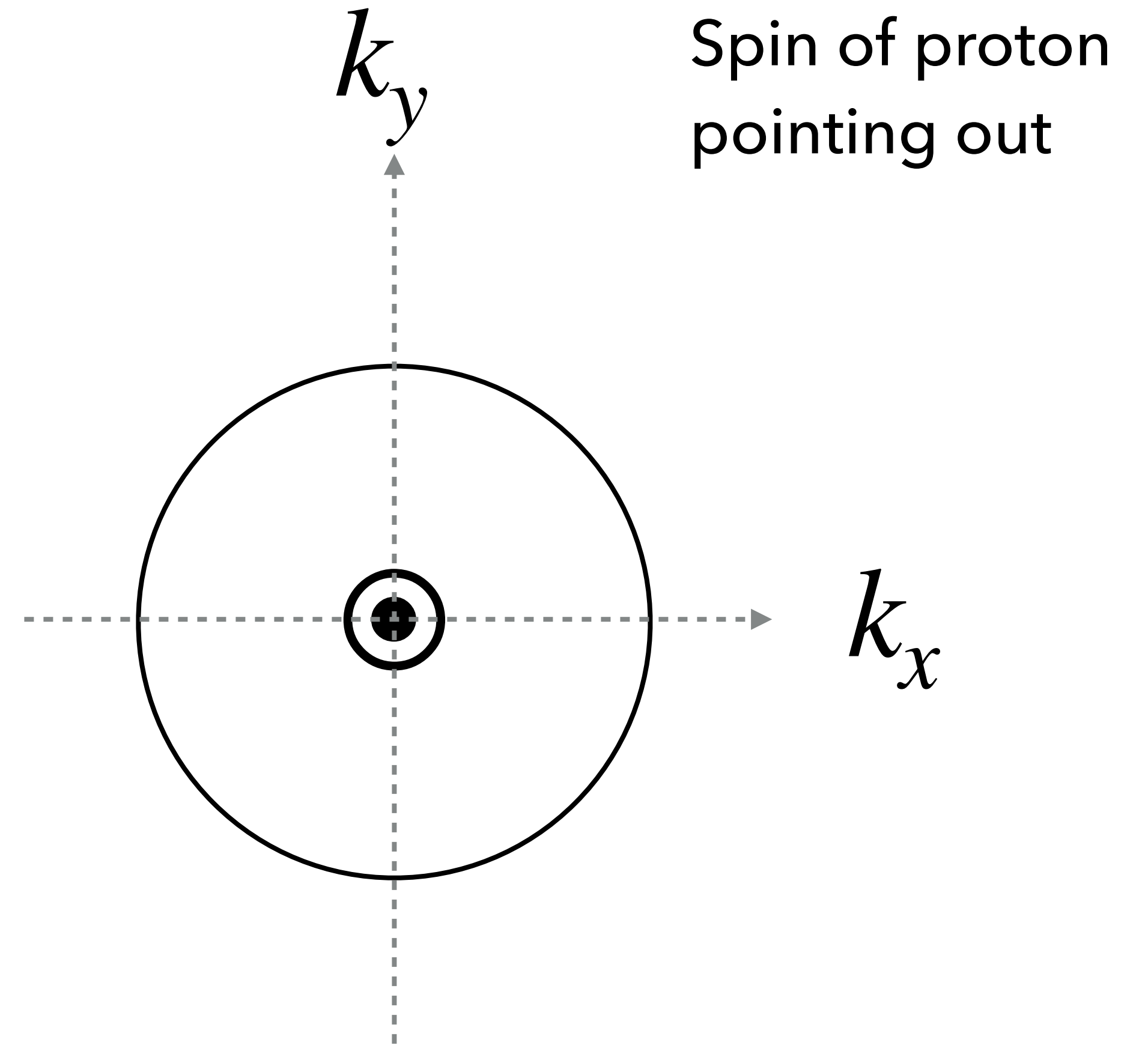
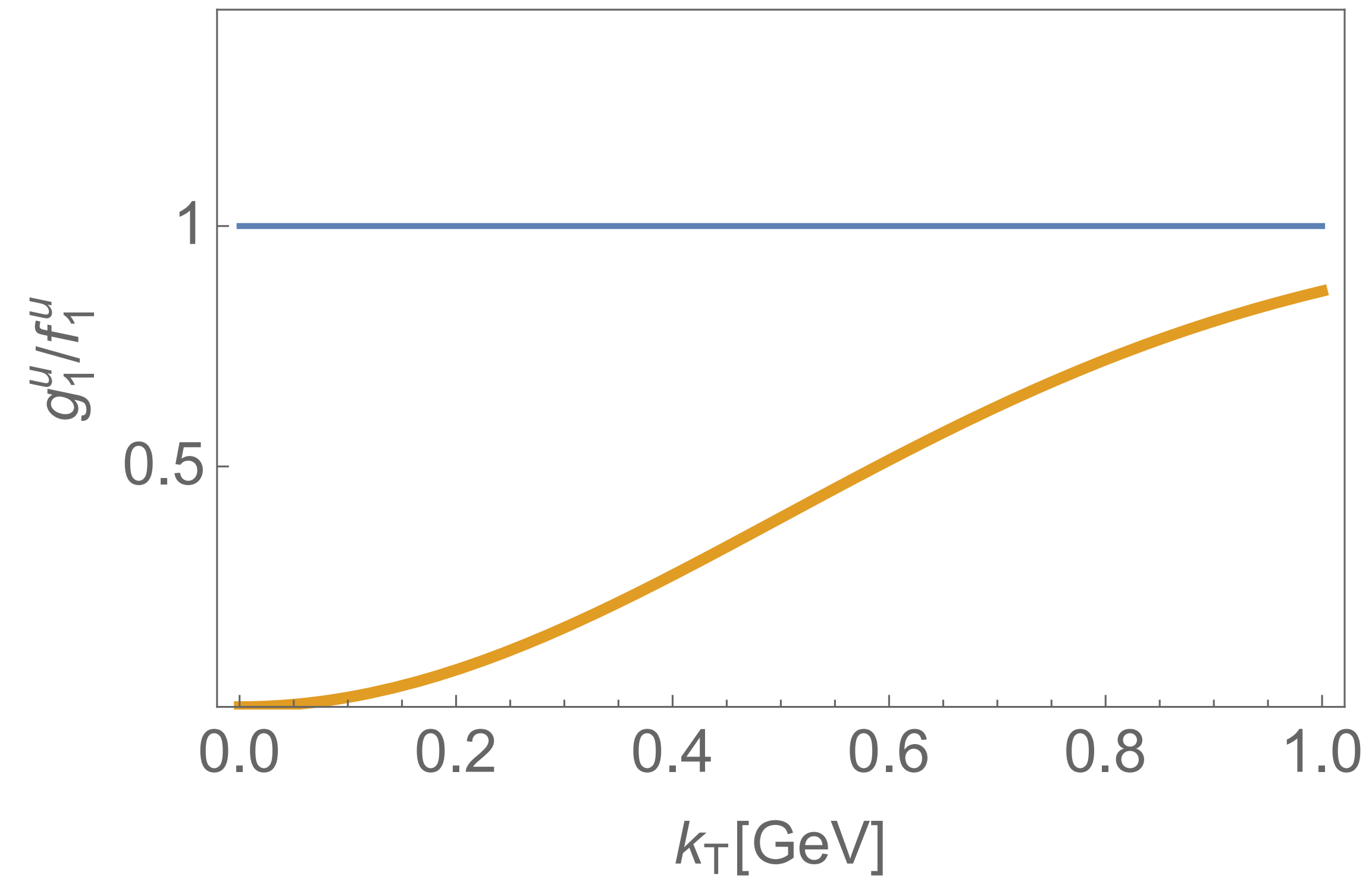


At a certain value of x

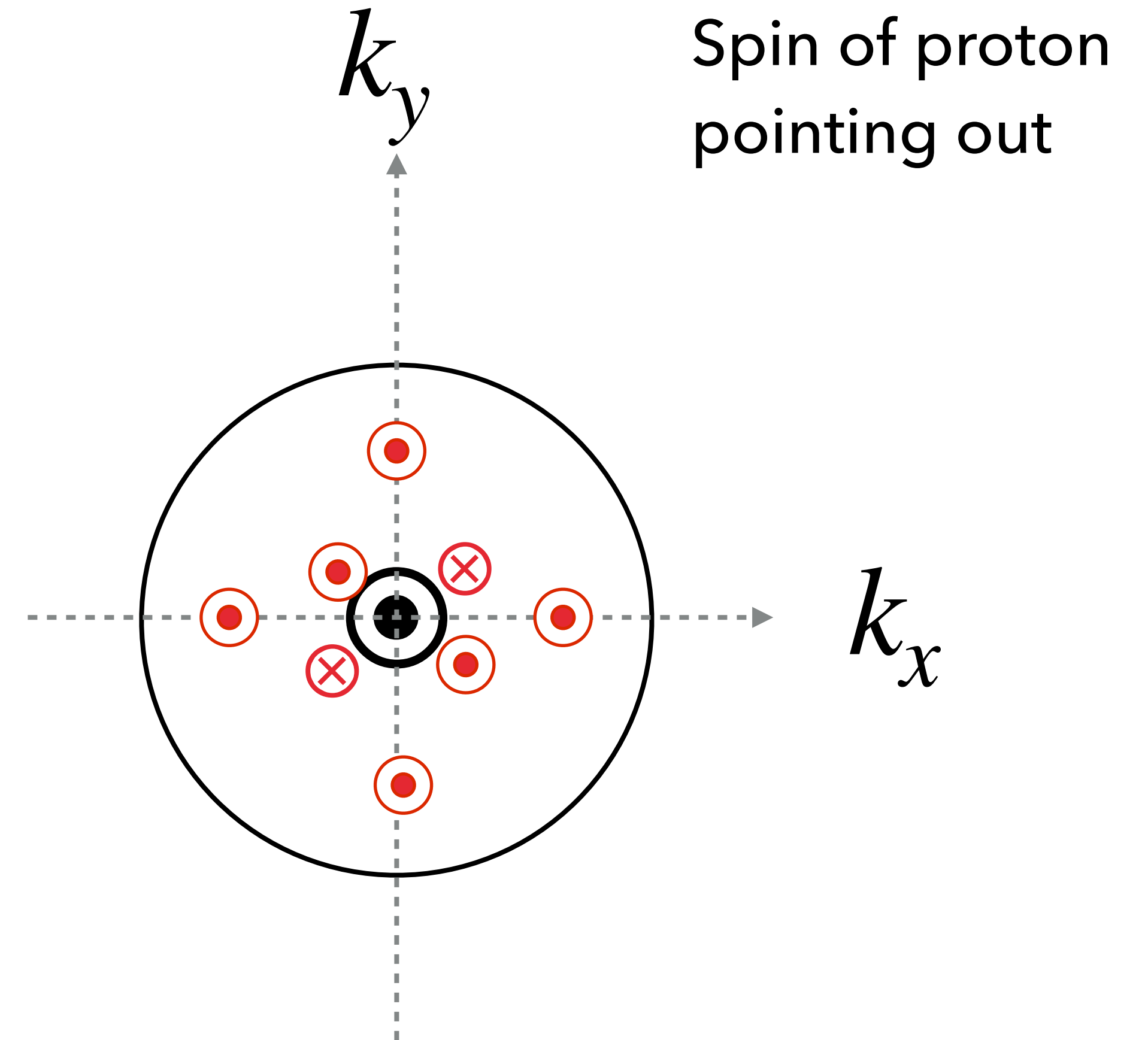
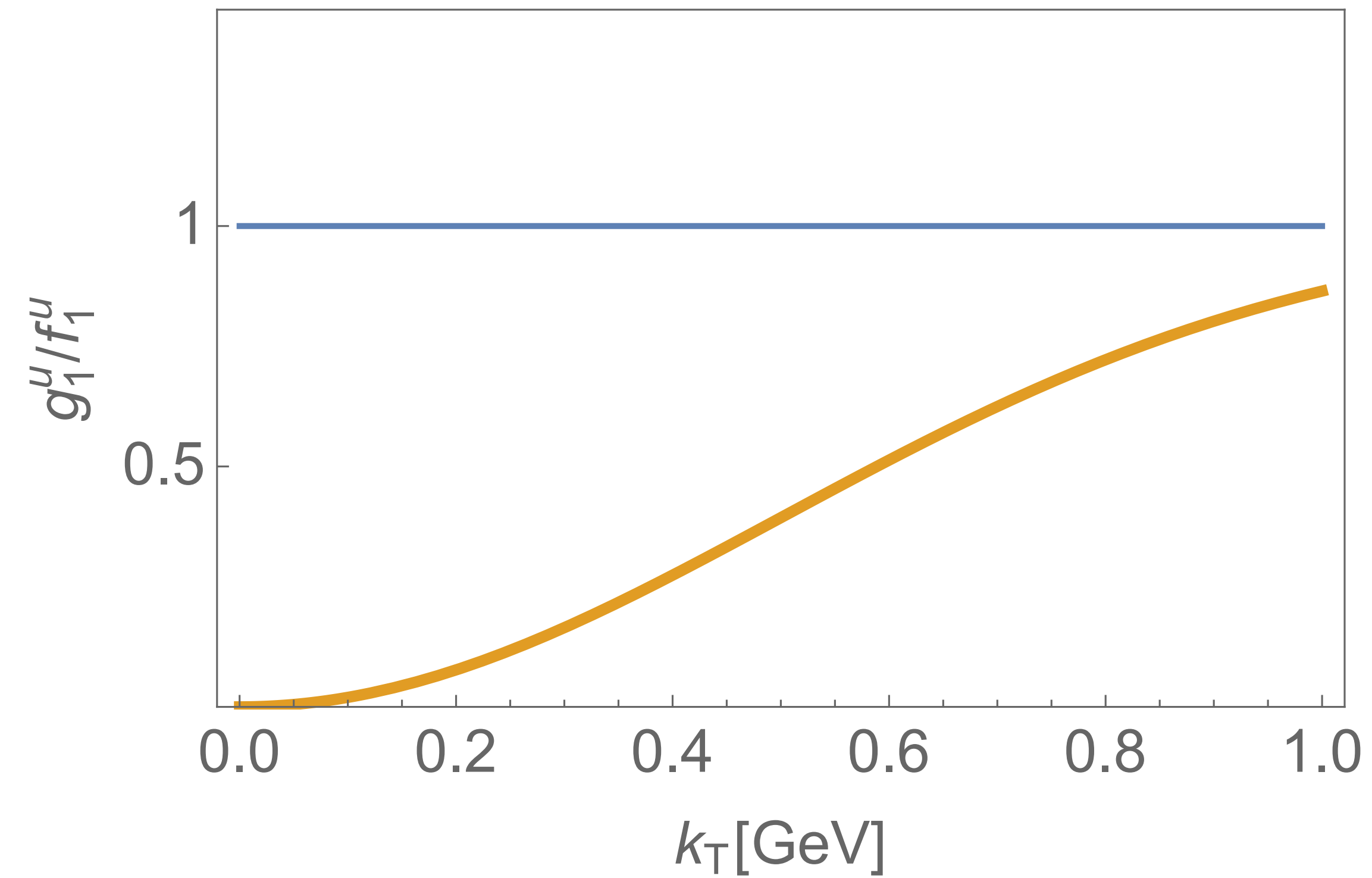


The quarks with the same helicity as the proton's have less transverse momentum

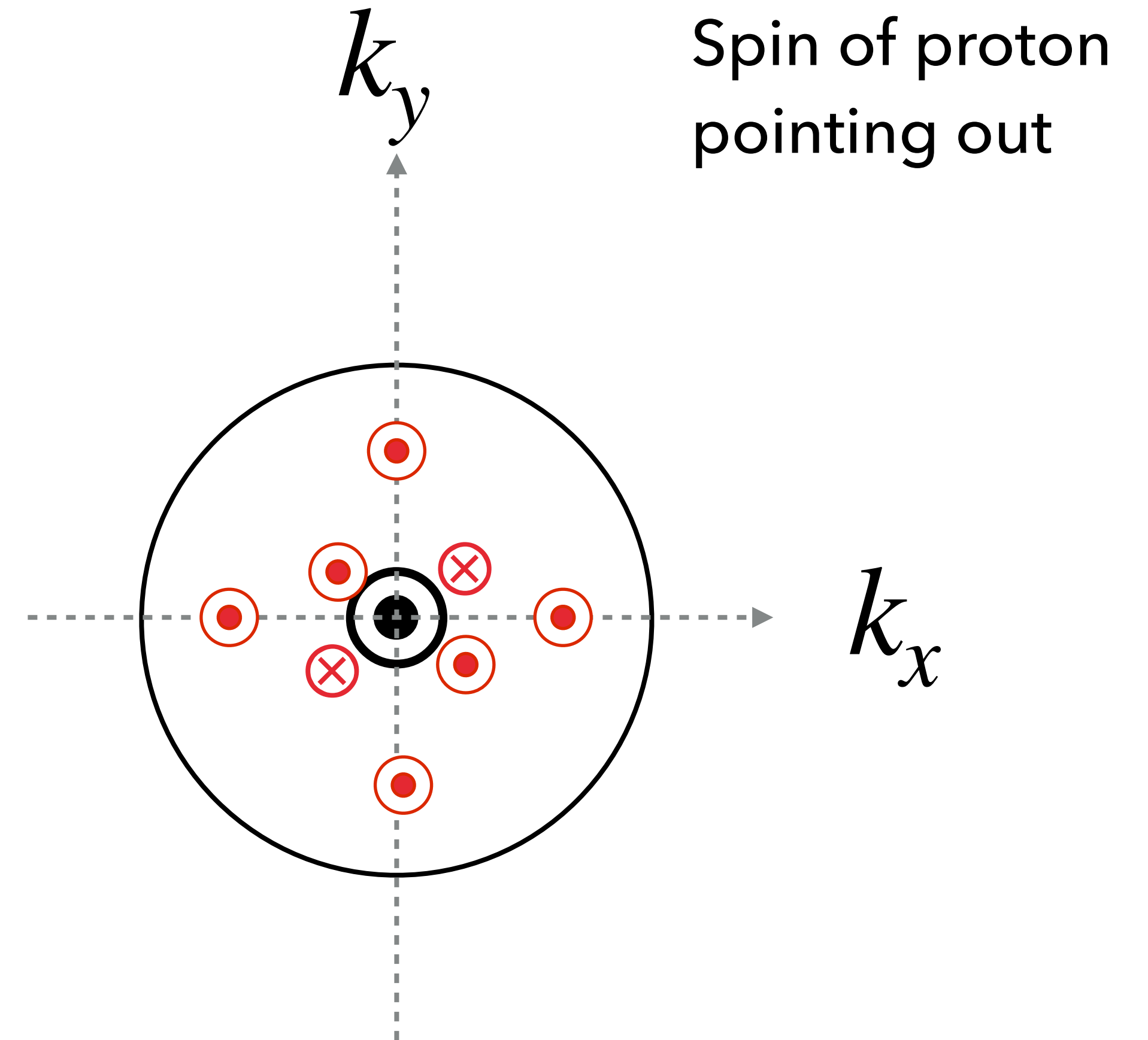
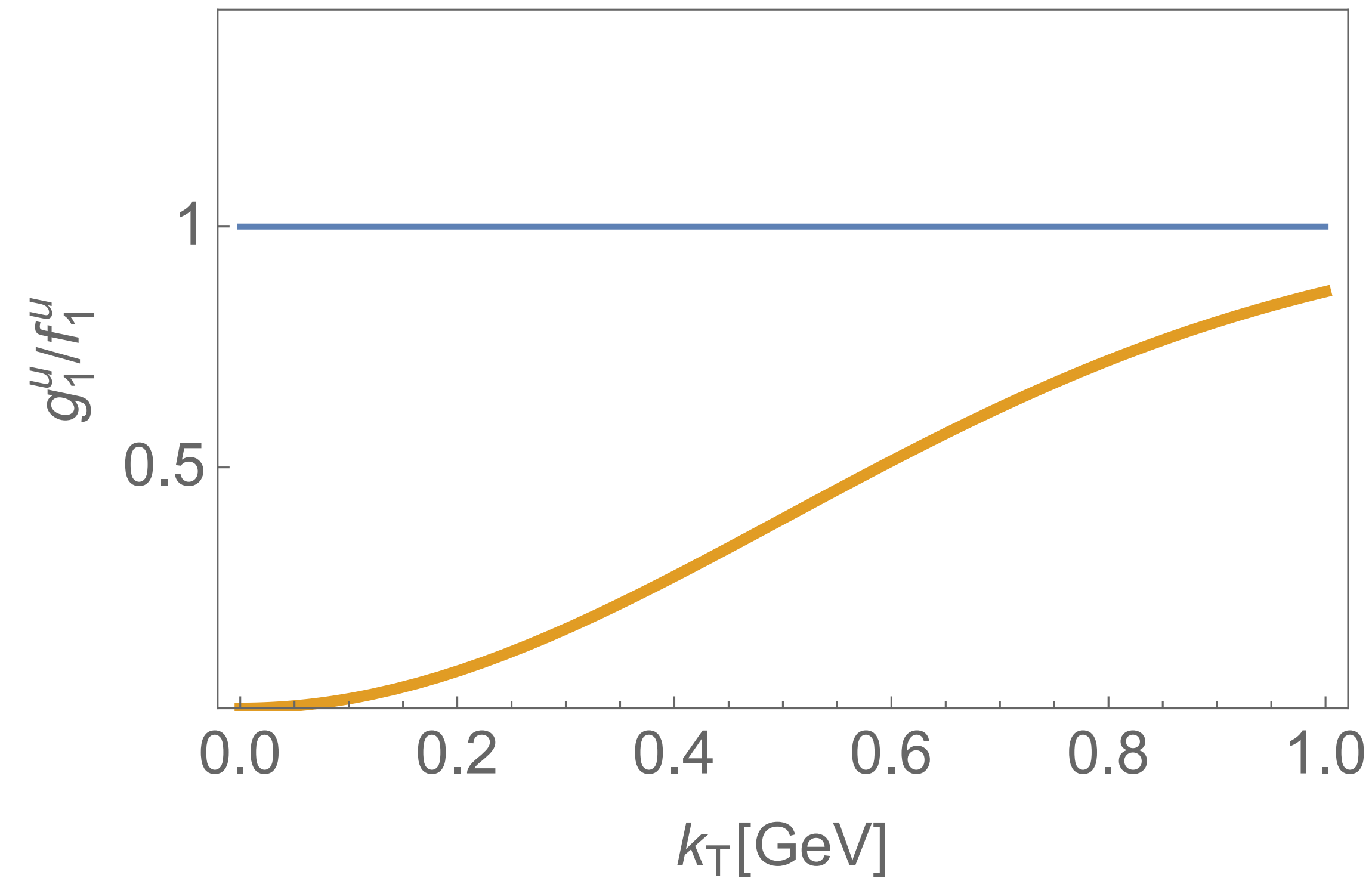
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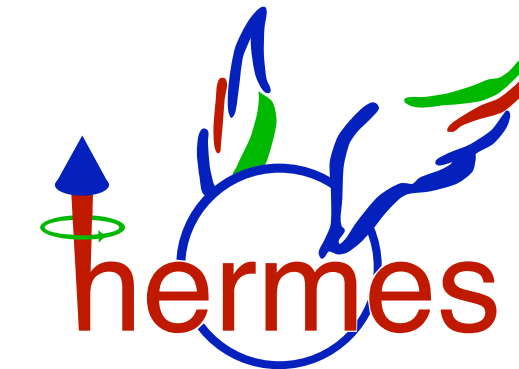
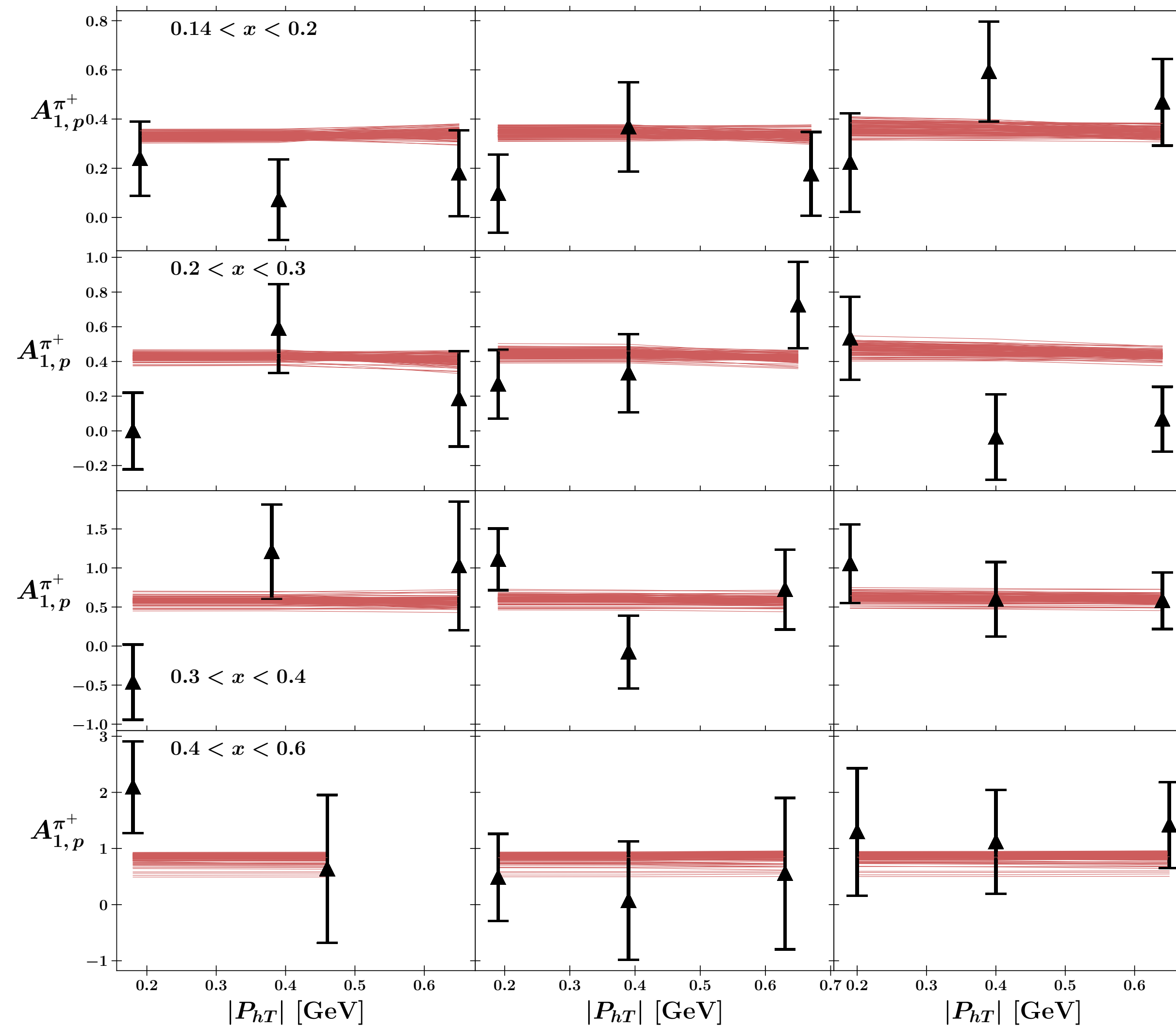
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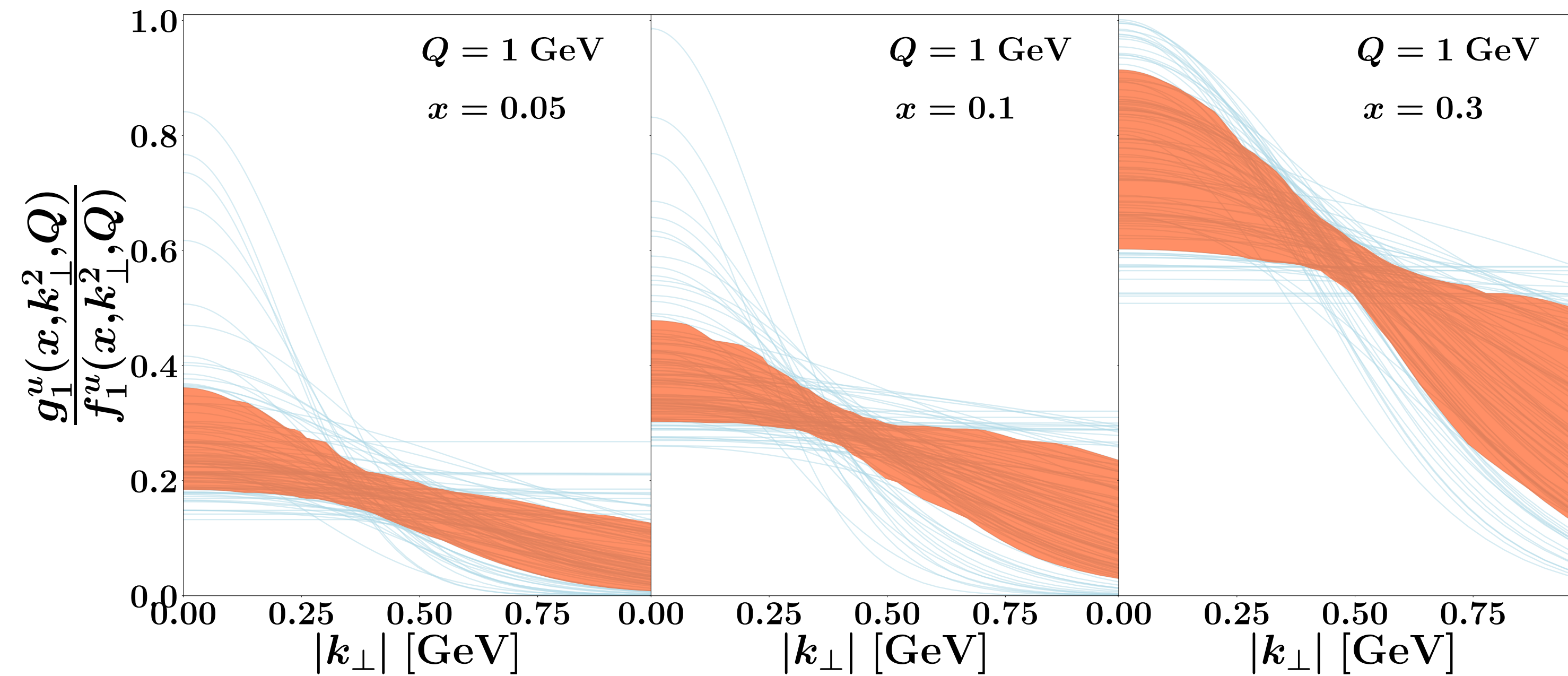


The quarks with the same helicity as the proton's have more transverse momentum



EXTRACTION OF TMD DEPENDENCE OF HELICITY

[MAP collaboration, arXiv:2409.18078](#)

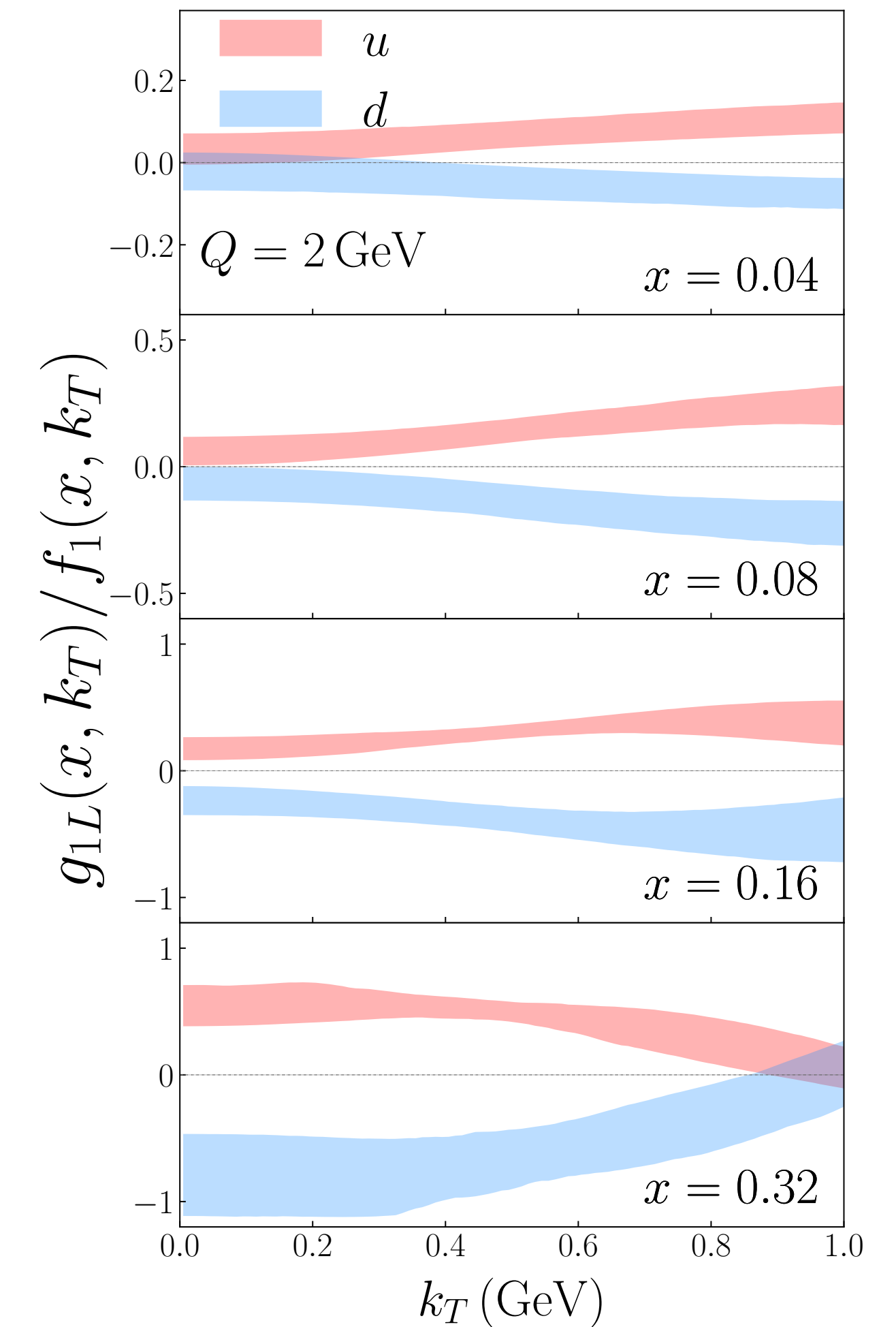
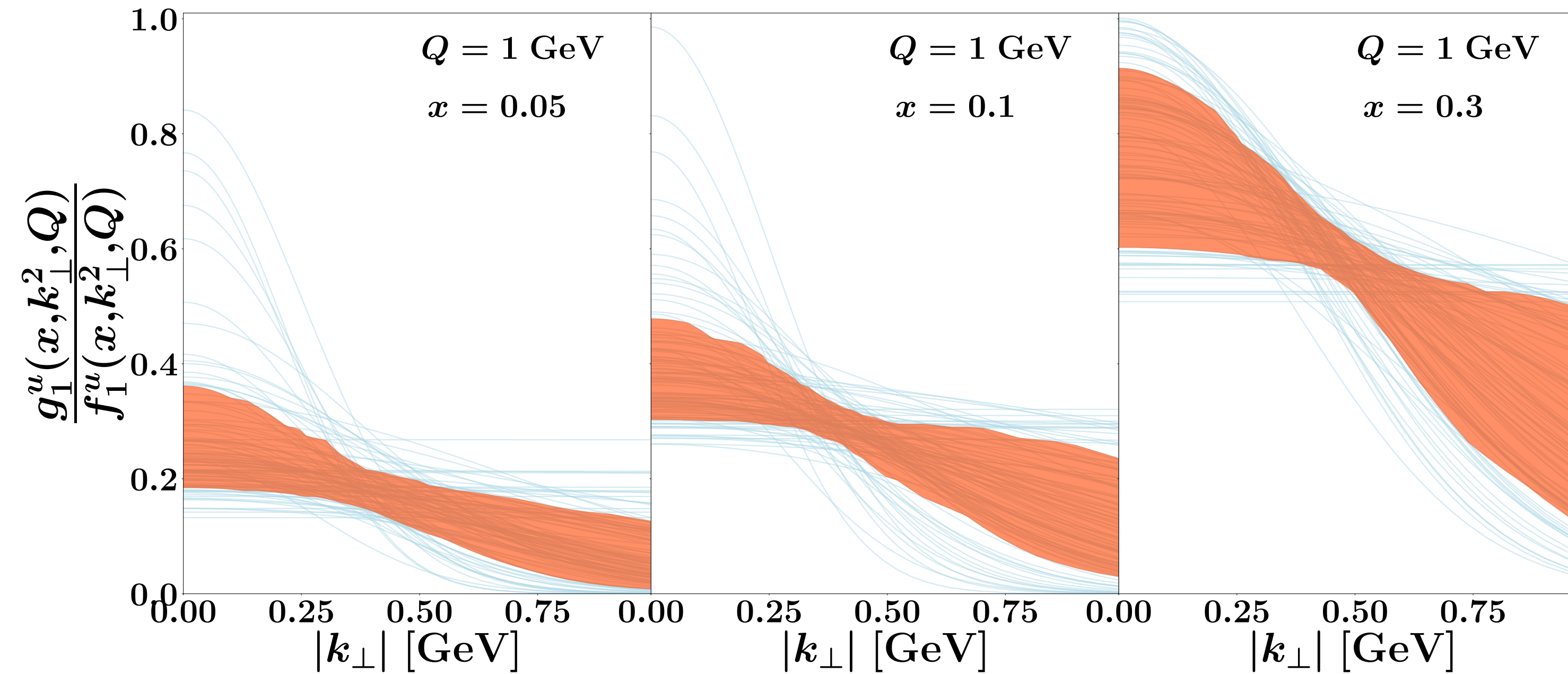


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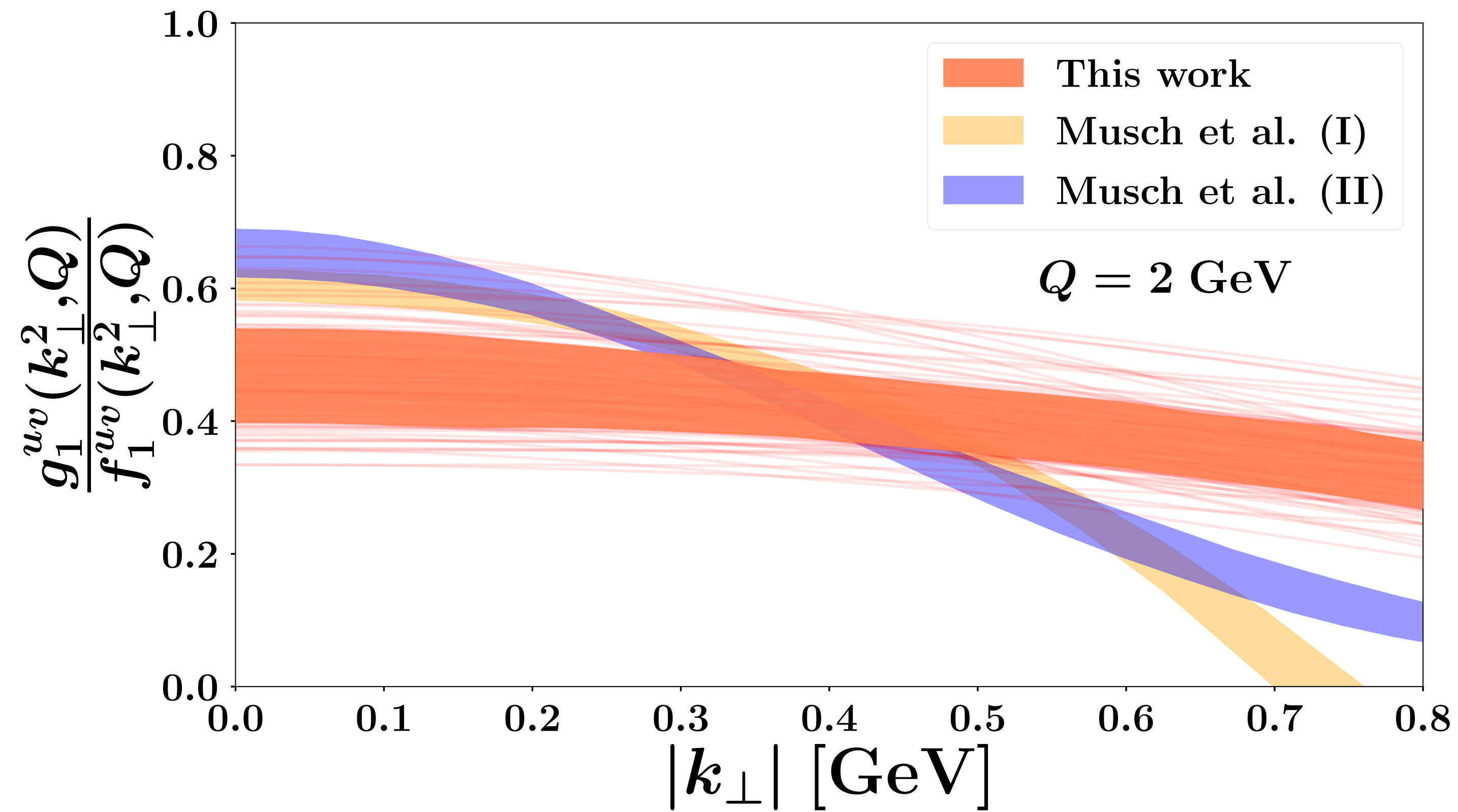
32

[MAP collaboration, arXiv:2409.18078](#)

[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#), see talk by K. Yang

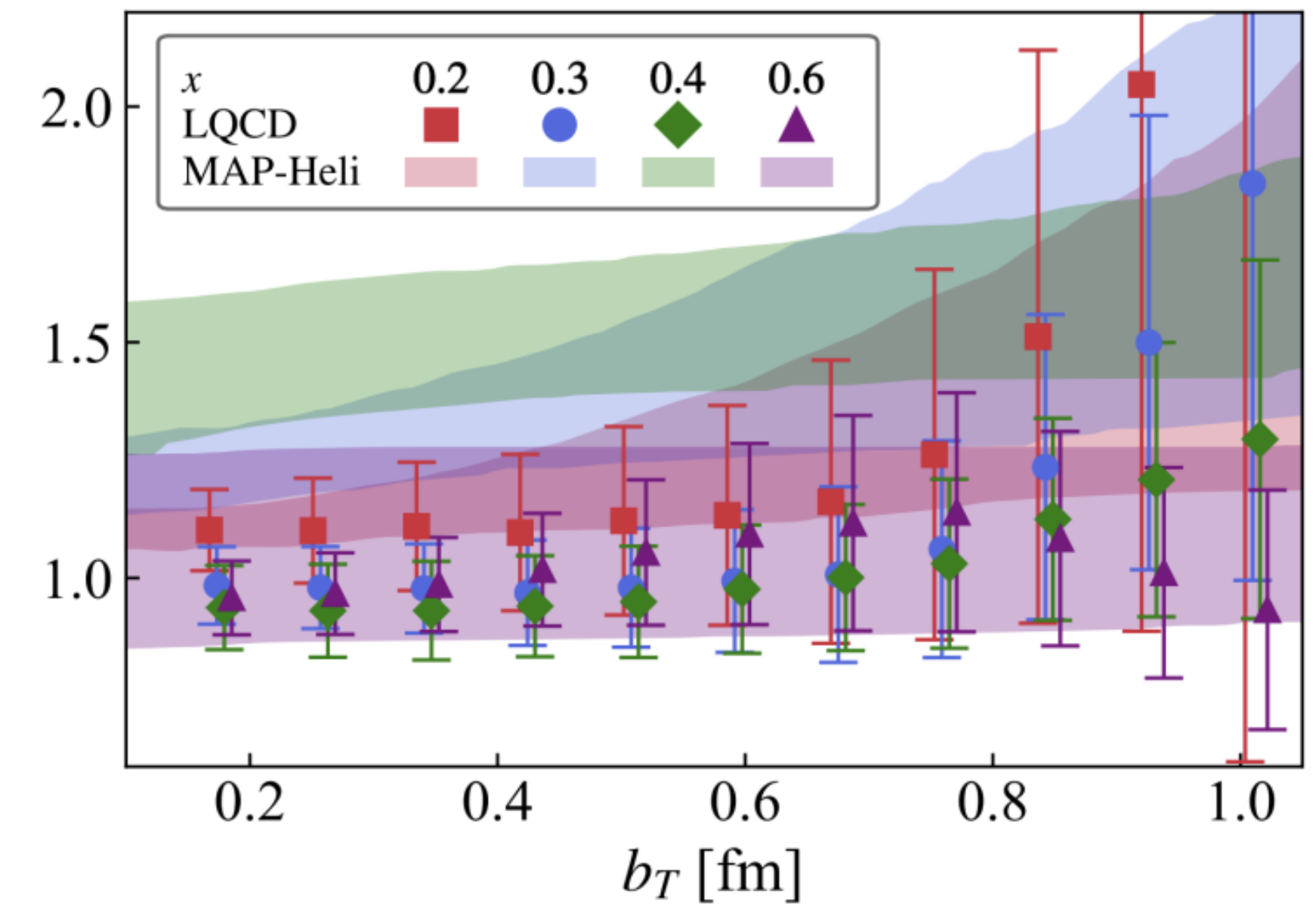


[MAP collaboration, arXiv:2409.18078](#)



[Bollweg et al., arXiv:2505.18430](#)

see talk by Wei Wang



[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#)

[MAP collaboration, arXiv:2409.18078](#)

Experiment	Process	Data points	χ^2/N
HERMES[82]	$e^\pm p \rightarrow e^\pm hX$	84 (160)	0.72
HERMES[82]	$e^\pm d \rightarrow e^\pm hX$	160 (317)	0.71
CLAS[83]	$e^- p \rightarrow e^- \pi^0 X$	9 (21)	1.43
Total		253 (498)	0.74

Experiment	N_{dat}	$\chi^2_{\text{NLL}}/N_{\text{dat}}$	$\chi^2_{\text{NNLL}}/N_{\text{dat}}$
HERMES ($d \rightarrow \pi^+$)	47	1.34	1.30
HERMES ($d \rightarrow \pi^-$)	47	1.10	1.08
HERMES ($d \rightarrow K^+$)	46	1.26	1.25
HERMES ($d \rightarrow K^-$)	45	0.93	0.89
HERMES ($p \rightarrow \pi^+$)	53	1.17	1.21
HERMES ($p \rightarrow \pi^-$)	53	0.86	0.86
Total	291	1.11	1.09

[Yang, Liu, Sun, Zhao, Ma, arXiv:2409.08110](#)

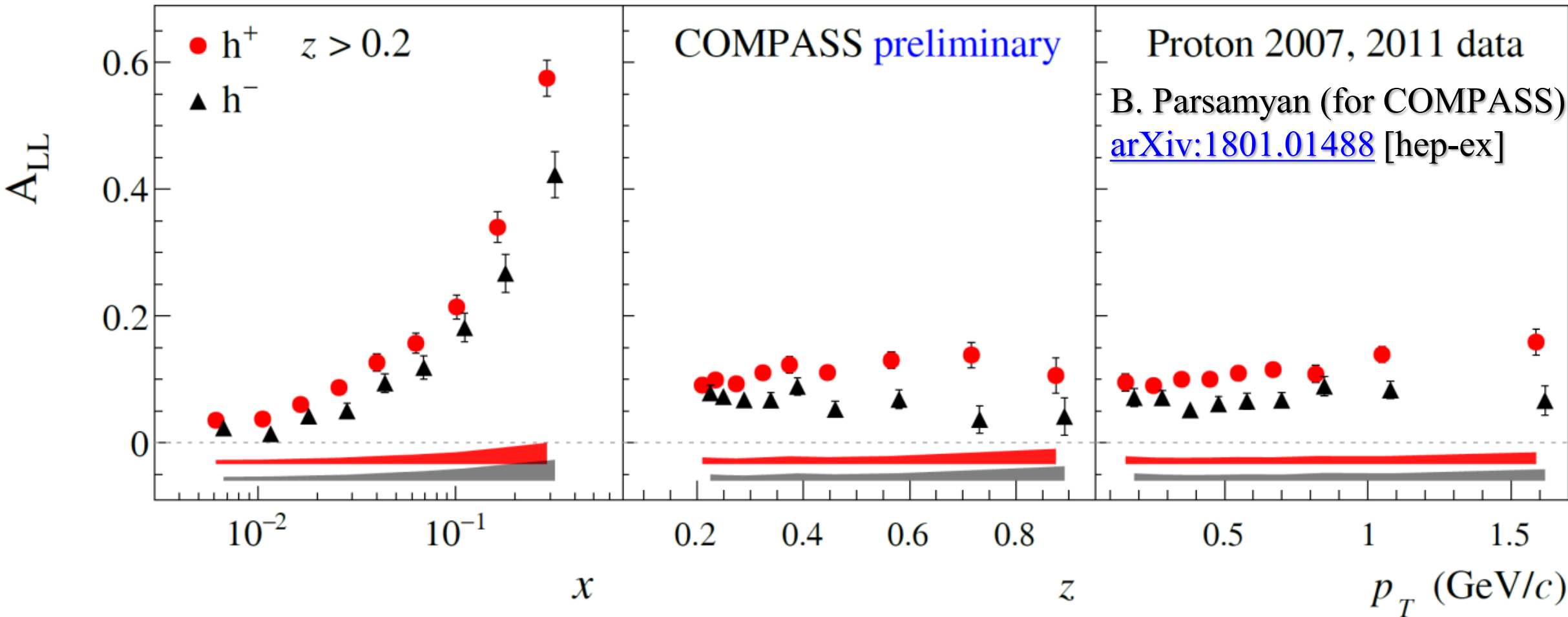
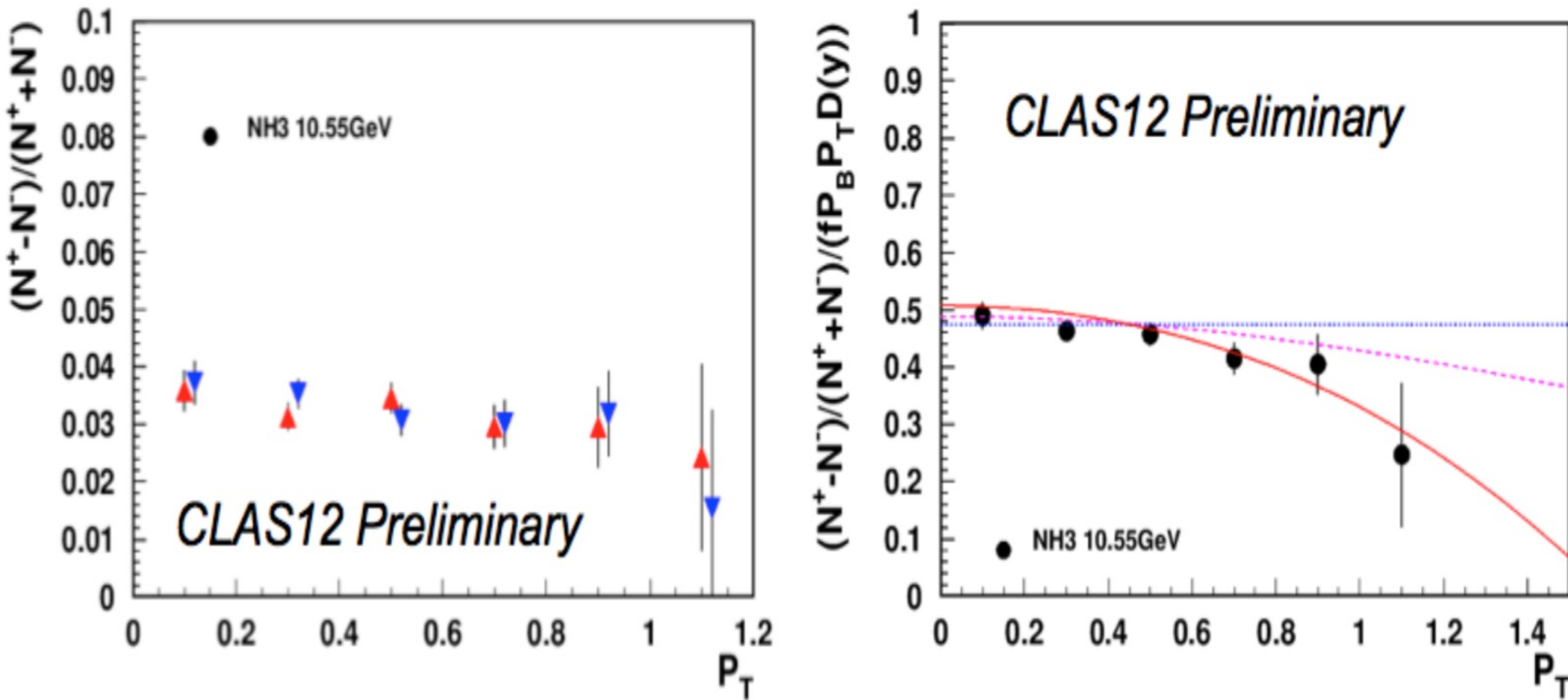
[MAP collaboration, arXiv:2409.18078](#)

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Total	291	1.11	1.09

More data needed!

Multidimensional binning needed



CONCLUSIONS

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- ▶ Data are better described by flavor-dependent TMD PDFs and FFs
- ▶ There are still several sources of uncertainty to be carefully studied
- ▶ A first-ever TMD fit based on Neural Networks is available
- ▶ The transverse-momentum dependence of the helicity quark distribution has been investigated

BACKUP

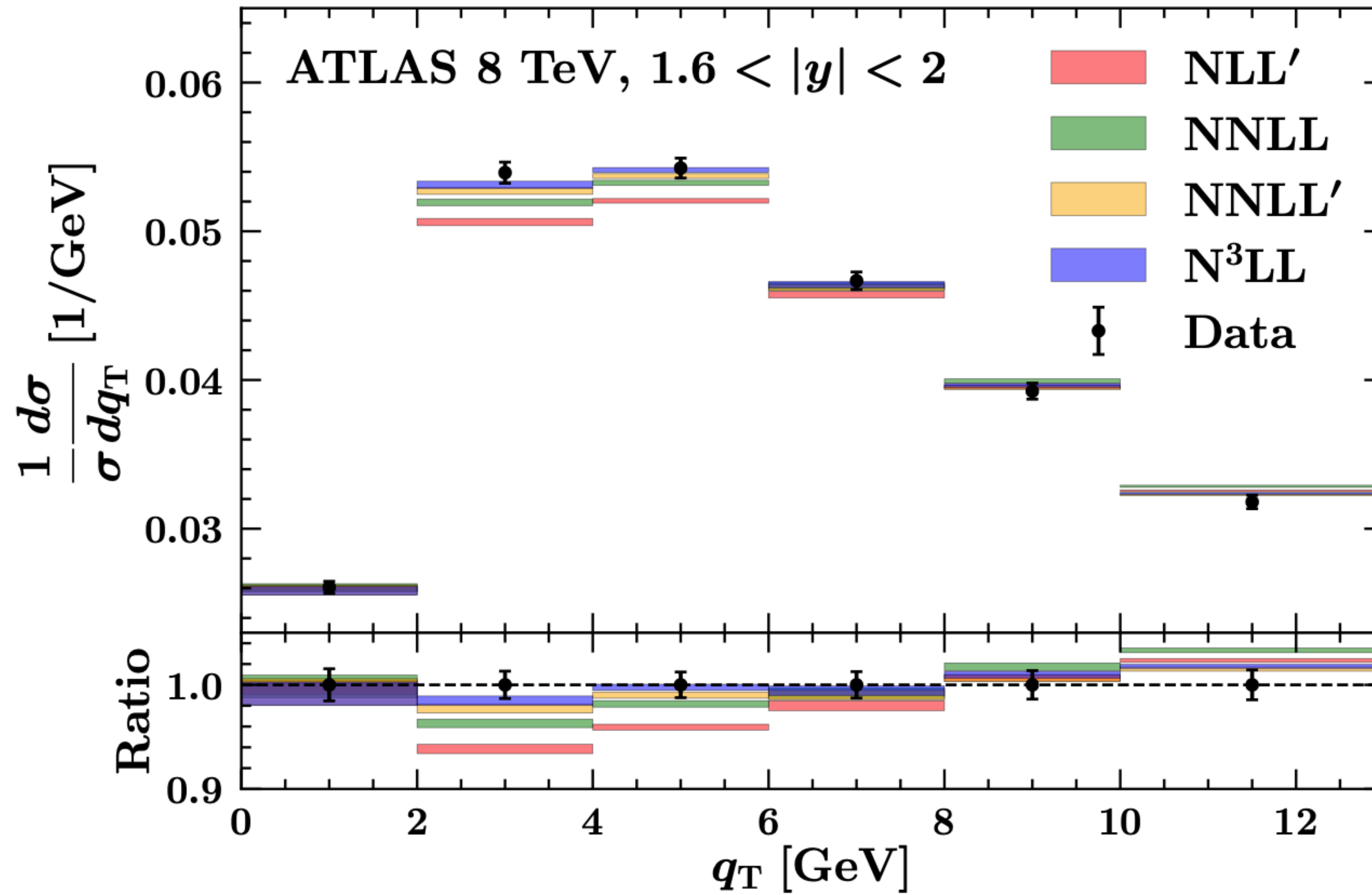
MAP24: same as MAP22, but 5 different flavors for PDFs and 5 different fragmentation function

MAP22

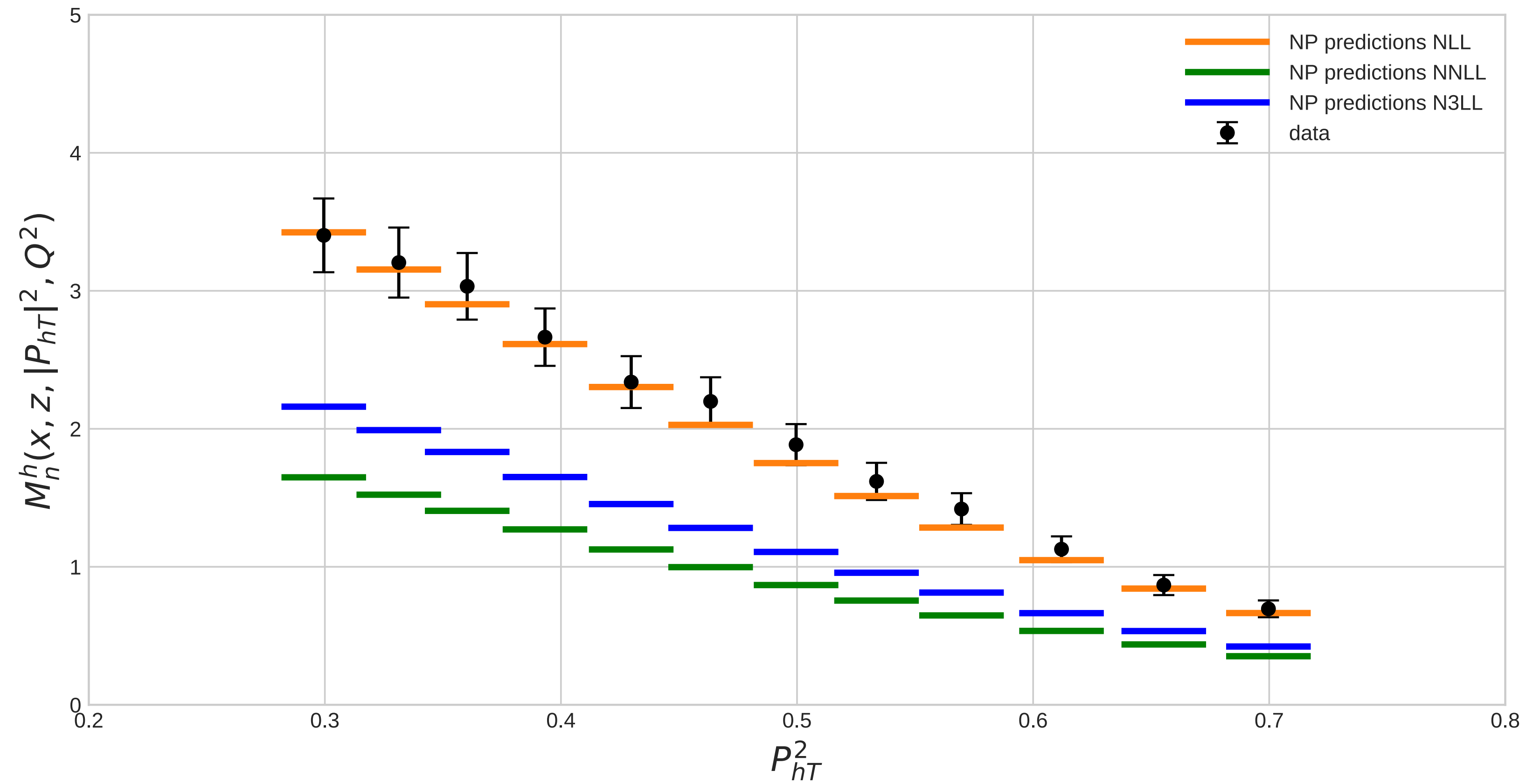
$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_T^2}{g^1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g^1 B}} + \lambda_2^2 e^{-\frac{k_T^2}{g^1 C}} \right) \quad g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$g_K(b_T^2) = -\frac{g_2^2}{2} b_T^2$$

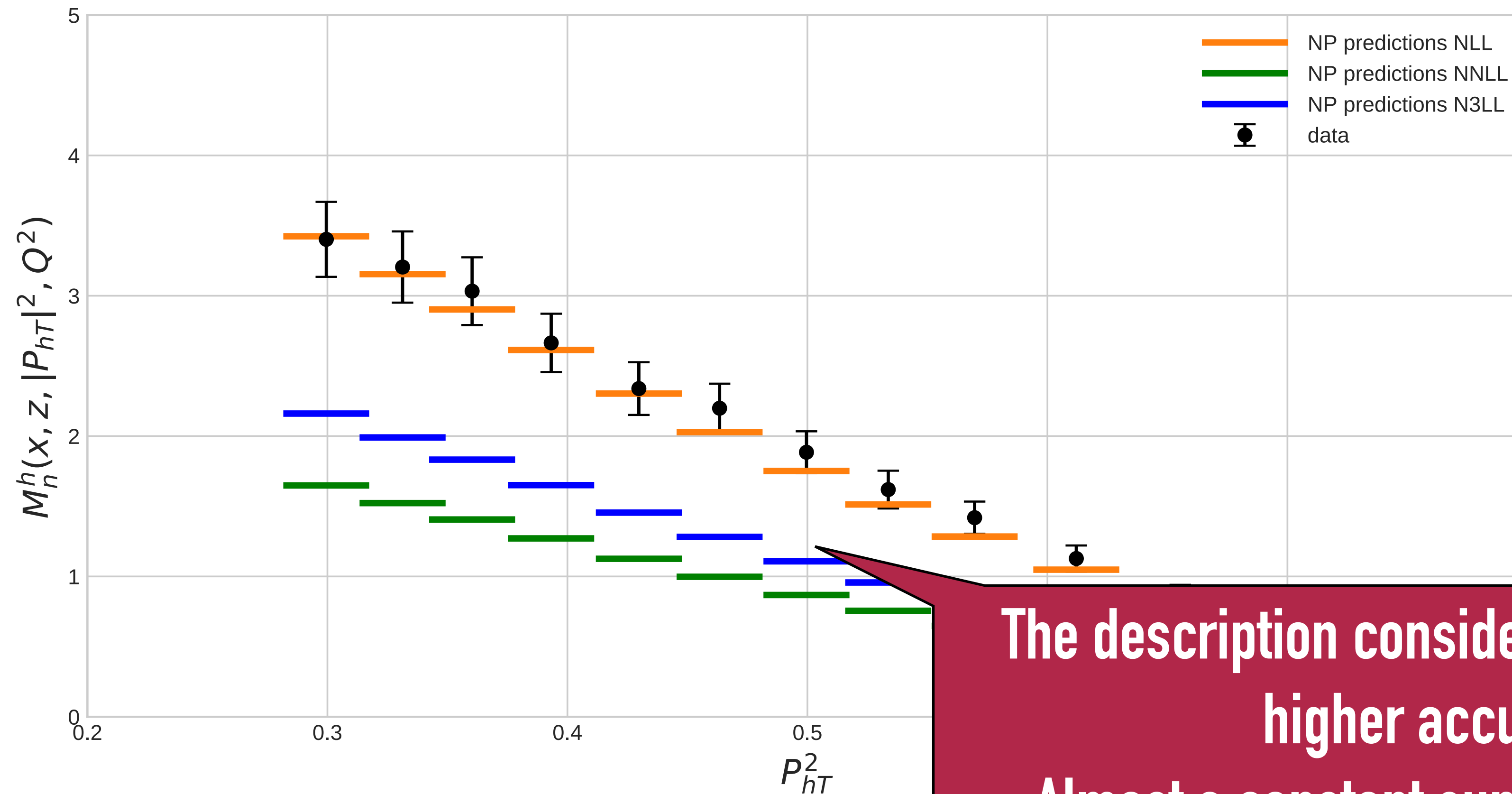
11 parameters for TMD PDF
+ 1 for NP evolution +9 for FF
= 21 free parameters



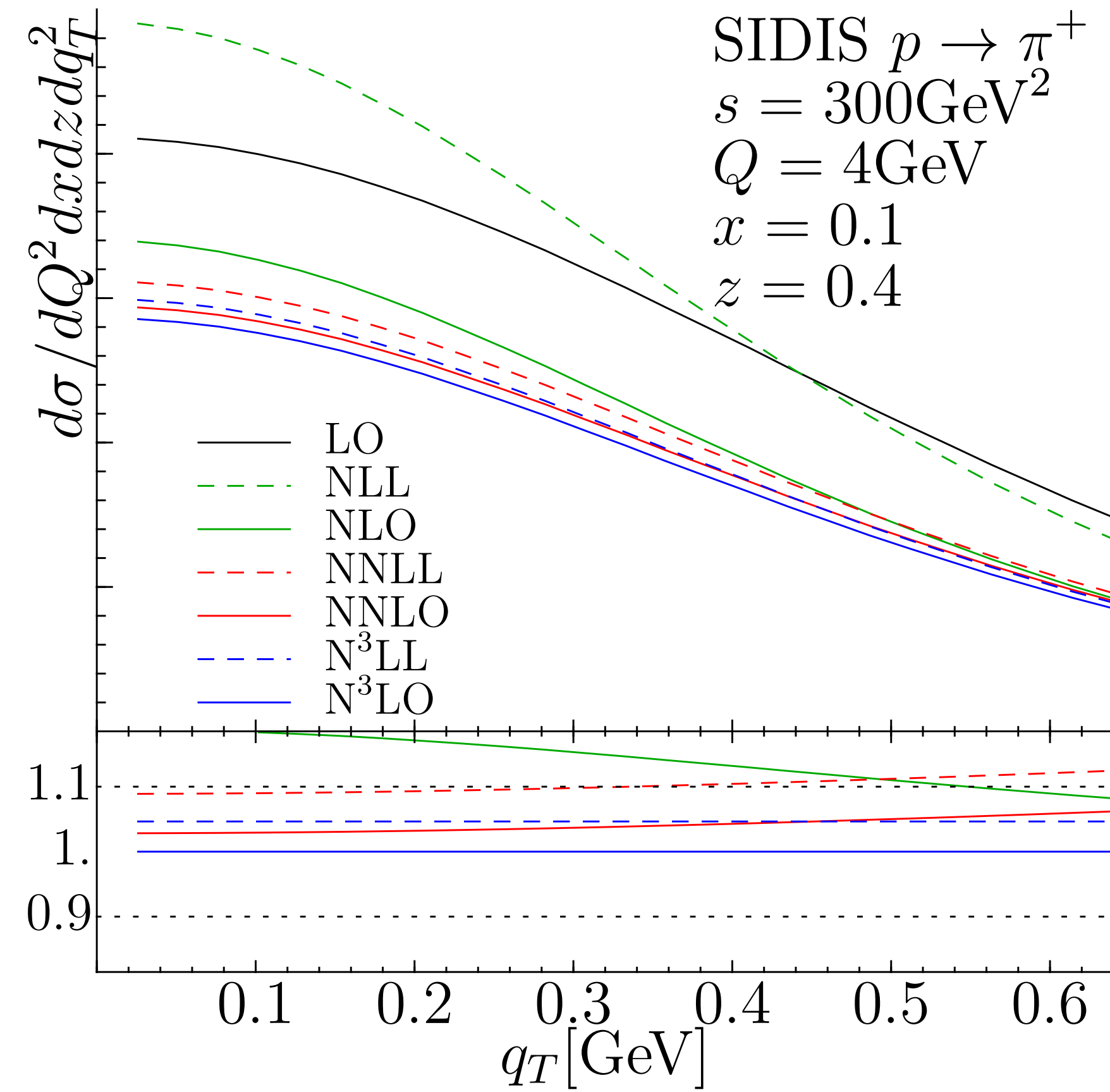
COMPASS multiplicities (one of many bins)

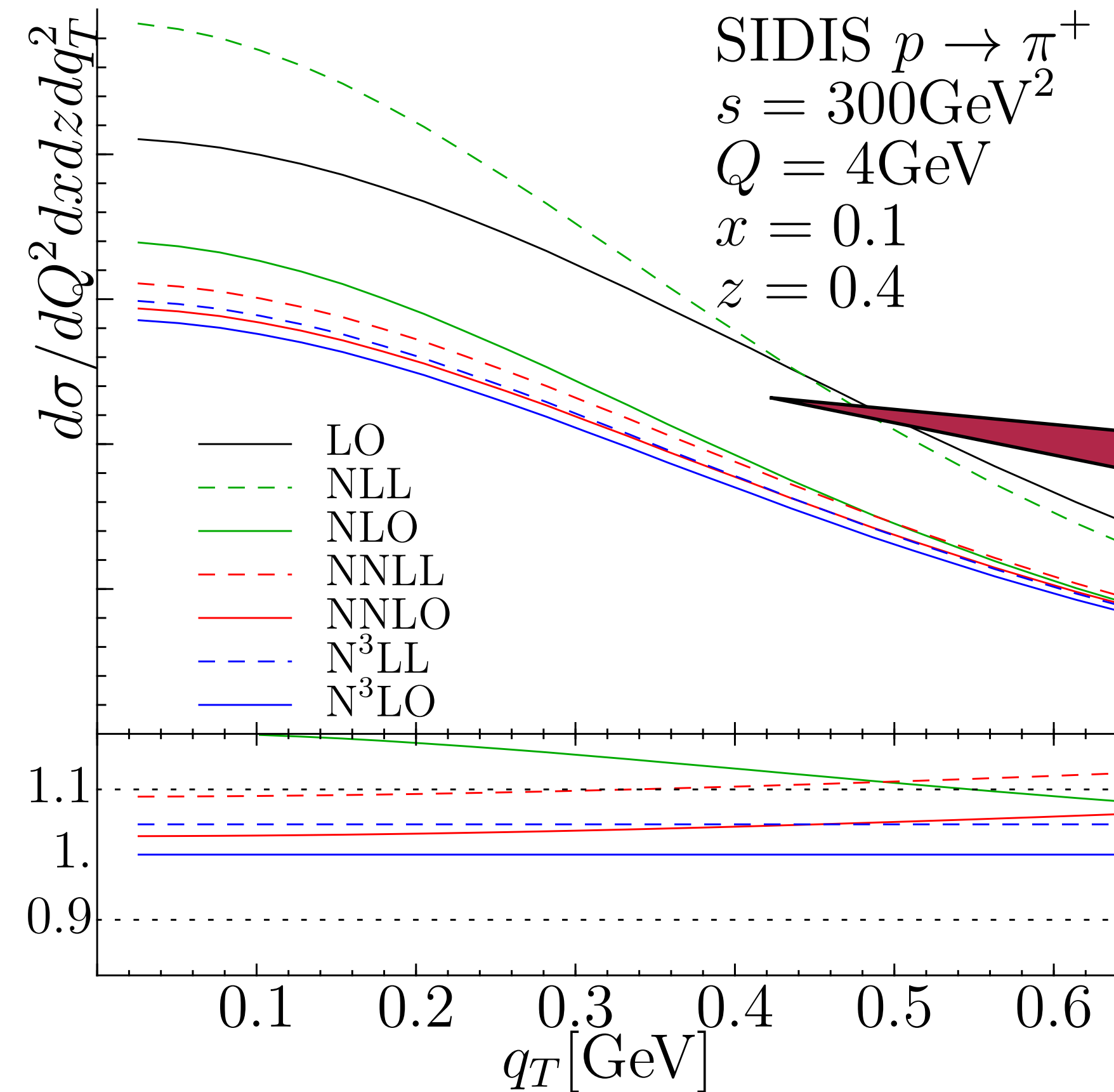


COMPASS multiplicities (one of many bins)



The description considerably worsens at higher accuracy.
Almost a constant suppression factor.





Also in the SV19 study, the overall decrease is evident, but they did not report problems with the data



**ENHANCEMENT
PREFACTOR**

$$= \frac{\left. \frac{d\sigma}{dx dz dQ^2} \right|_{\text{nonmix.}}}{\int \text{TMD } d^2 P_{hT}}$$



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PREFACTOR** $= \frac{\left. \frac{d\sigma}{dx dz dQ^2} \right|_{\text{nonmix.}}}{\int \text{TMD } d^2 P_{hT}}$

The prefactor is independent of the fitting parameters

Higher-order corrections decrease the
role of the TMD region.
We need to enhance it with a prefactor.

Sudakov form factor

$$\text{LL} \quad \alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$$

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Sudakov form factor

matching coeff.

LL $\alpha_S^n \ln^{2n} \left(\frac{Q^2}{\mu_b^2} \right)$

C^0

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$$\left(C^0 + \alpha_S C^1 \right)$$

Sudakov form factor

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$$\left(C^0 + \alpha_S C^1 \right)$$

the difference between the two is formally NNLL $\alpha_S^n \ln^{2n-2} \left(\frac{Q^2}{\mu_b^2} \right)$

$$\log(Q^2 b_T^2) \rightarrow \log(Q^2 b_T^2 + 1)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$\log(Q^2 b_T^2) \rightarrow \log(Q^2 b_T^2 + 1)$$

*[see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104](#)*

$$b_*(b_c(b_T)) = \sqrt{\frac{b_T^2 + b_0^2/(C_5^2 Q^2)}{1 + b_T^2/b_{\max}^2 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

$$b_{\min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q} \sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_{\max}^2)}}$$

*[Collins et al.
arXiv:1605.00671](#)*

$$\hat{f}_1^q(x, b_T; \mu^2) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\text{NP}}^q(x, b_T)$$

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

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$$\mu_b = 2e^{-\gamma_E}/b_* \quad \bar{b}_* \equiv b_{\text{max}} \left(\frac{1 - e^{-b_T^4/b_{\text{max}}^4}}{1 - e^{-b_T^4/b_{\text{min}}^4}} \right)^{1/4} \quad b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = \frac{2e^{-\gamma_E}}{Q}$$

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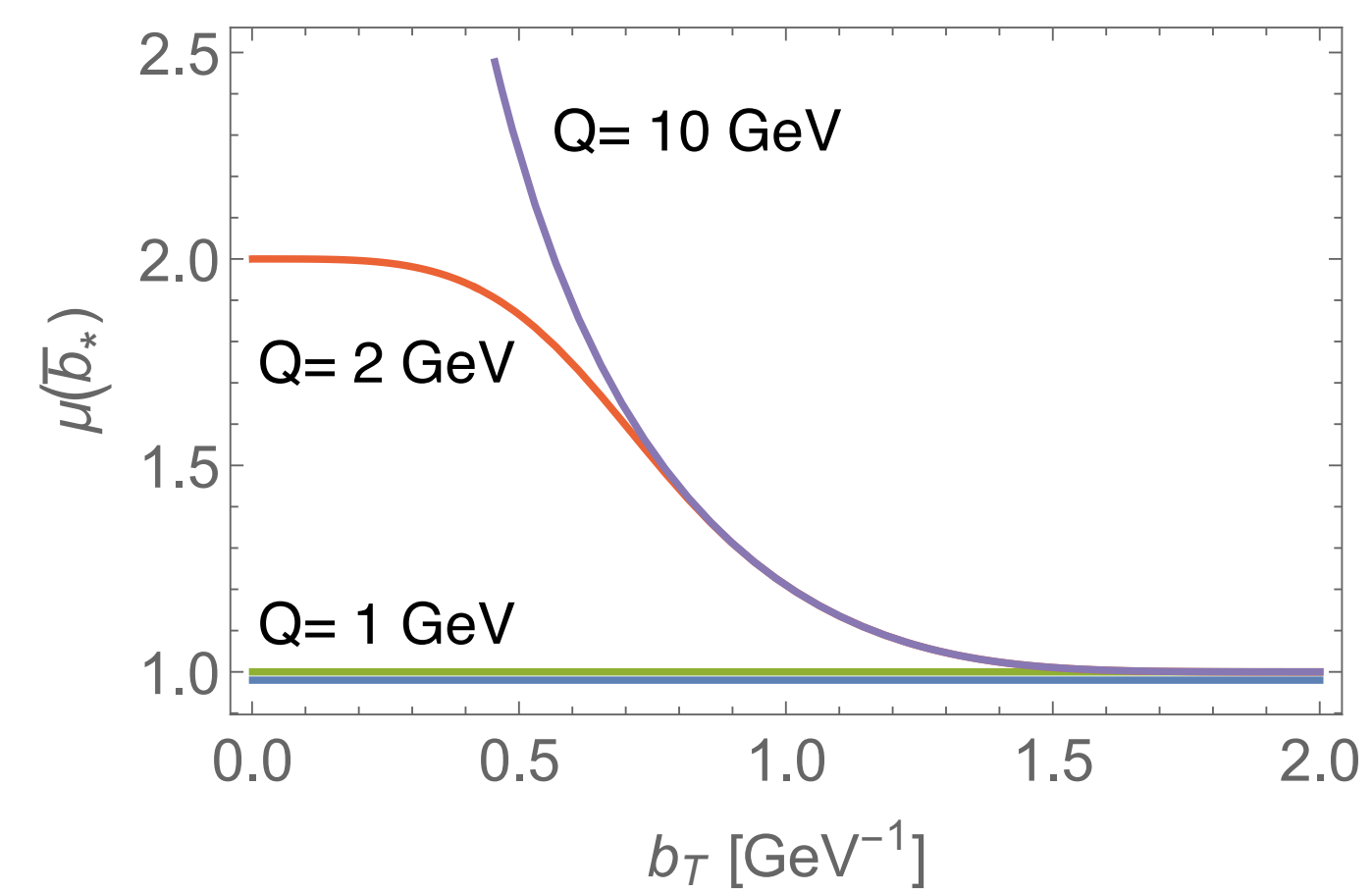
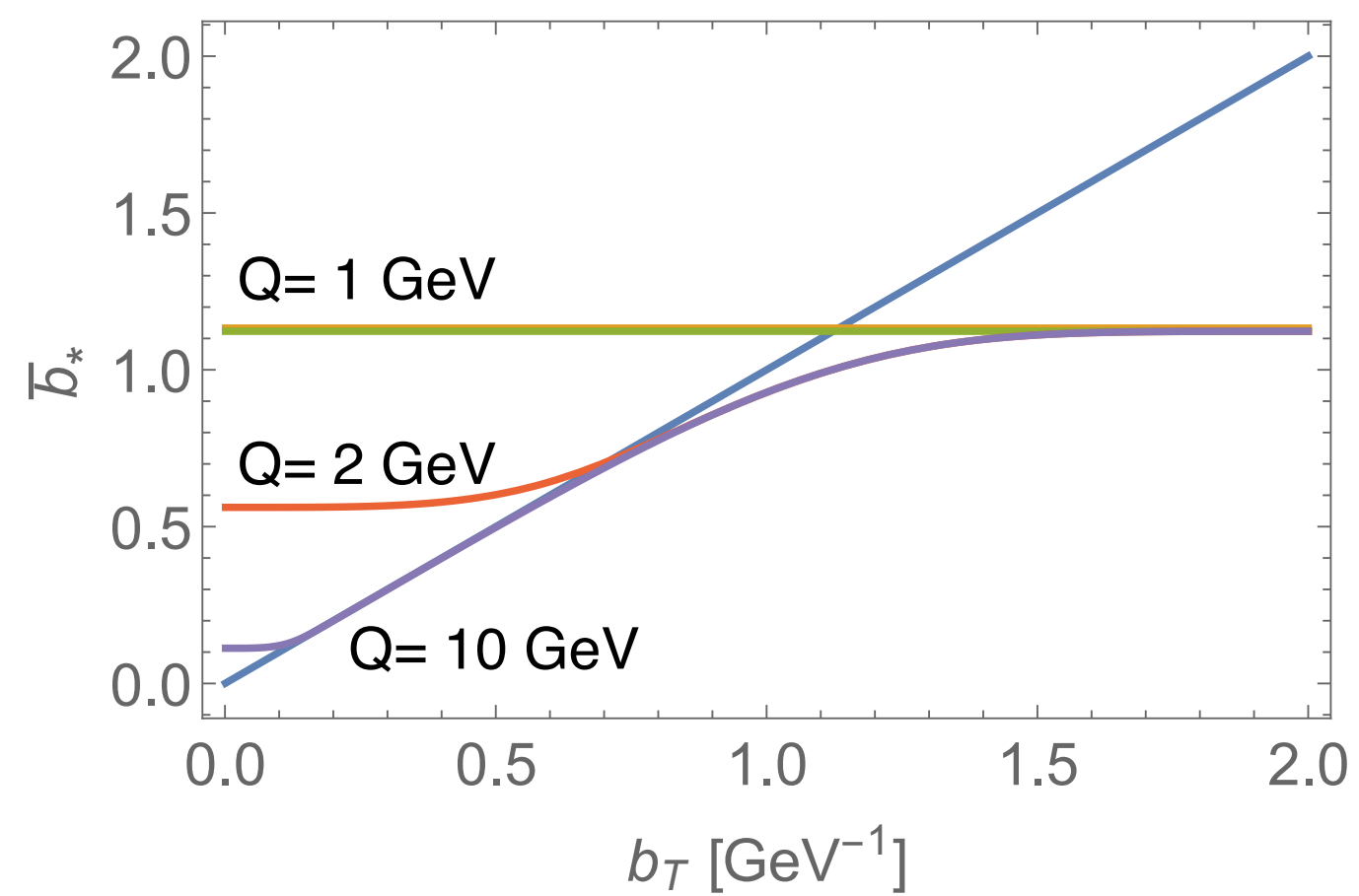
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These are all choices that should be at some point checked/challenged

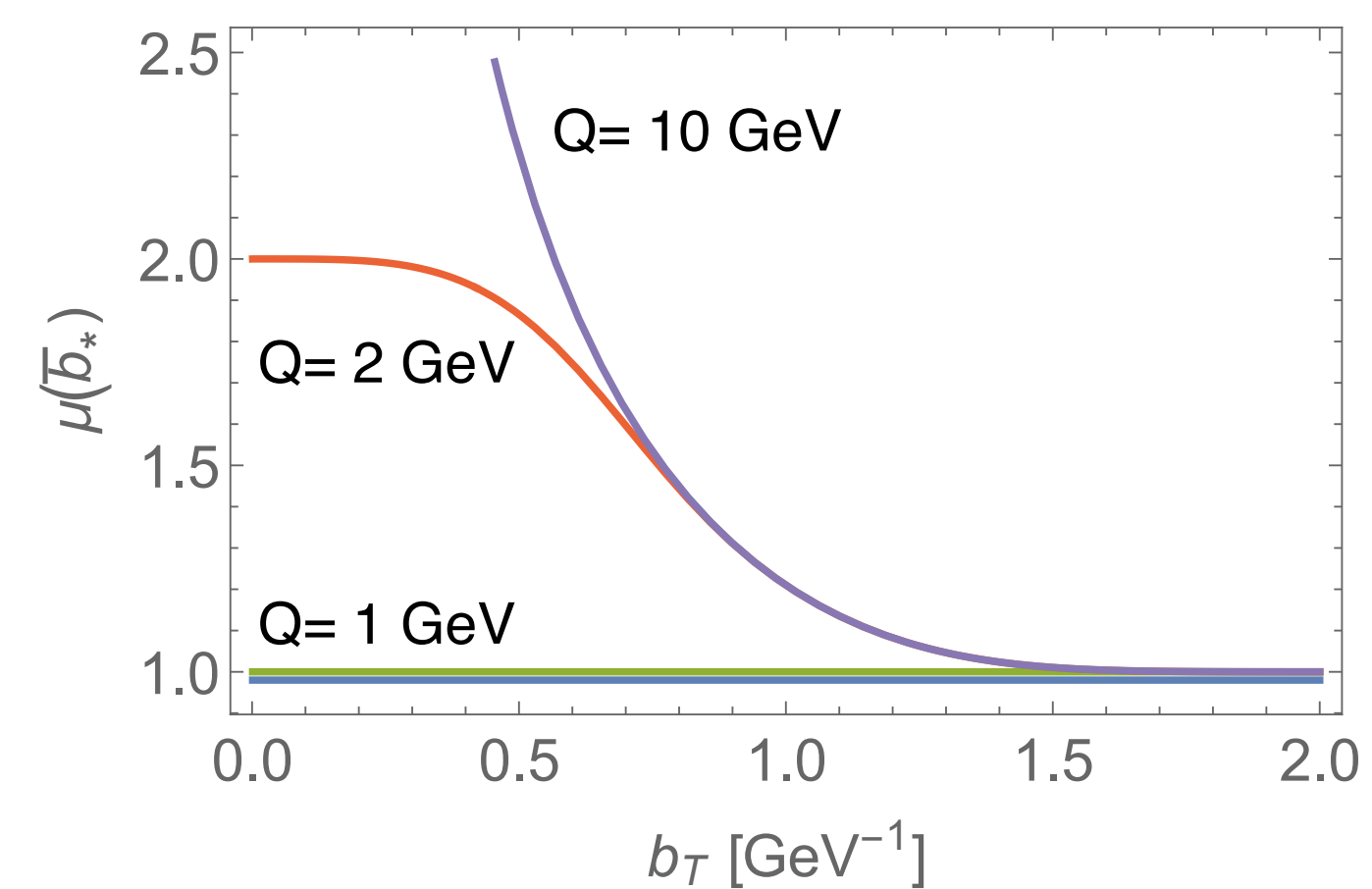
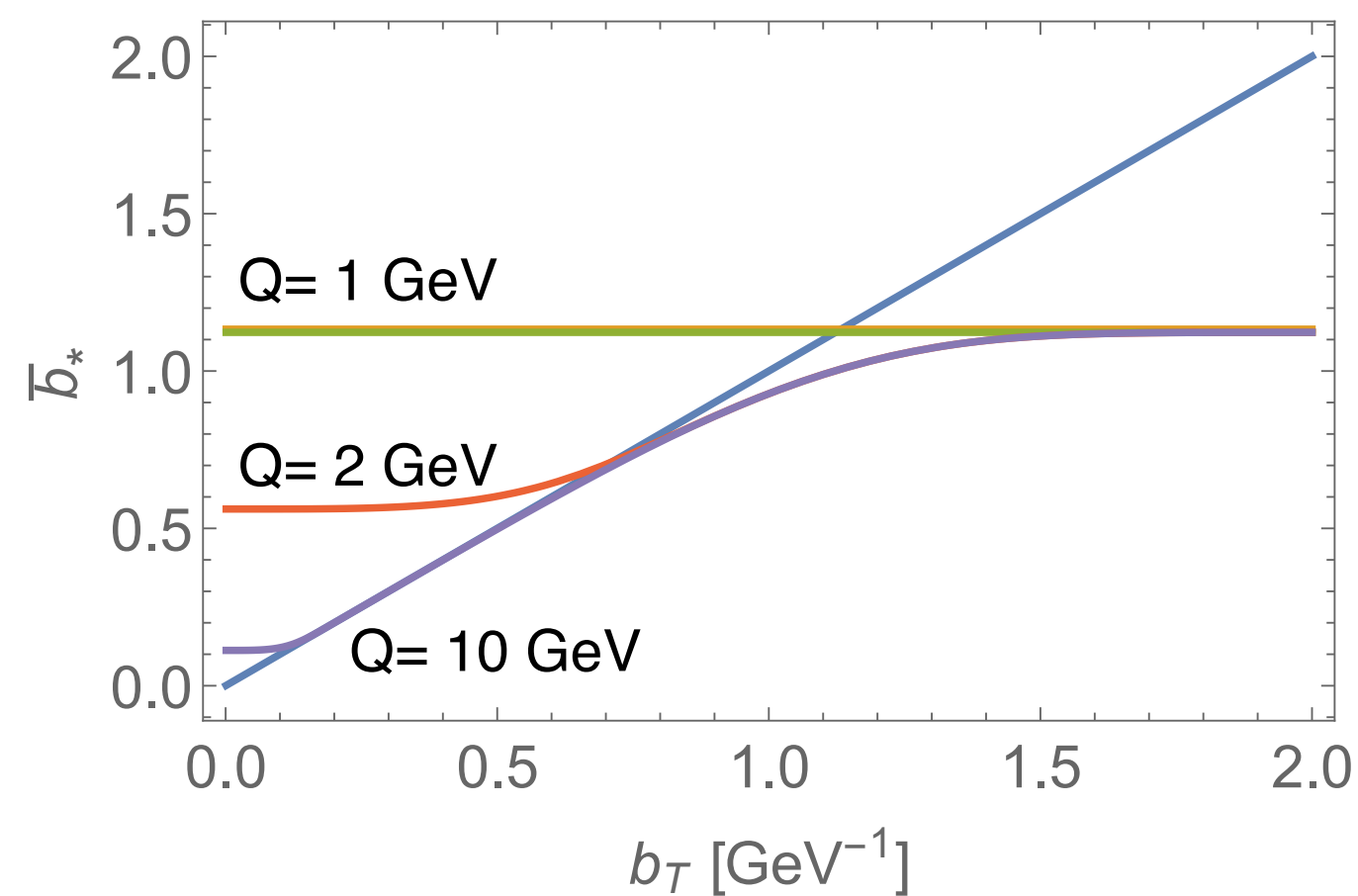
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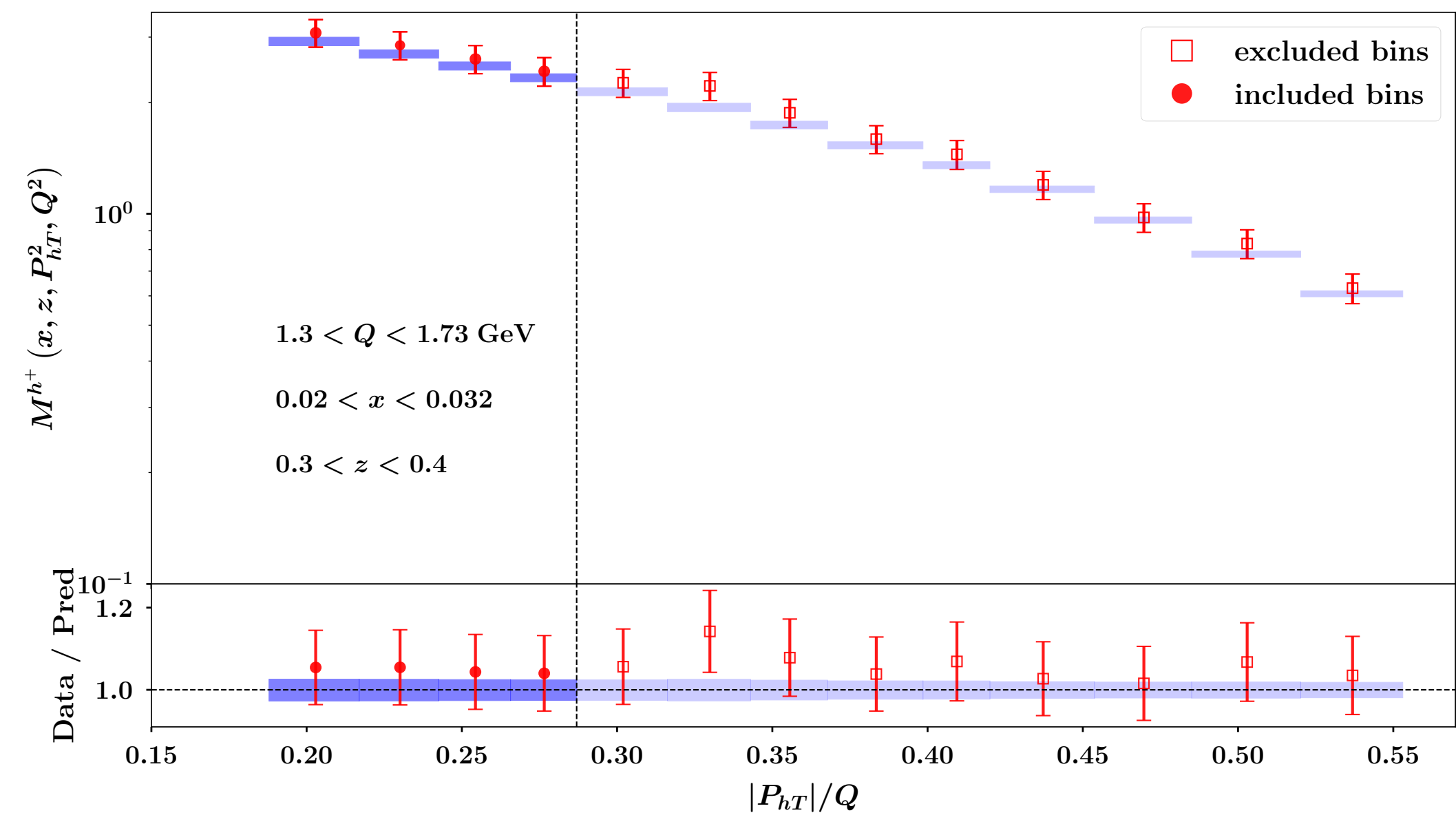
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$$b_{\min} = \frac{2e^{-\gamma_E}}{Q}$$

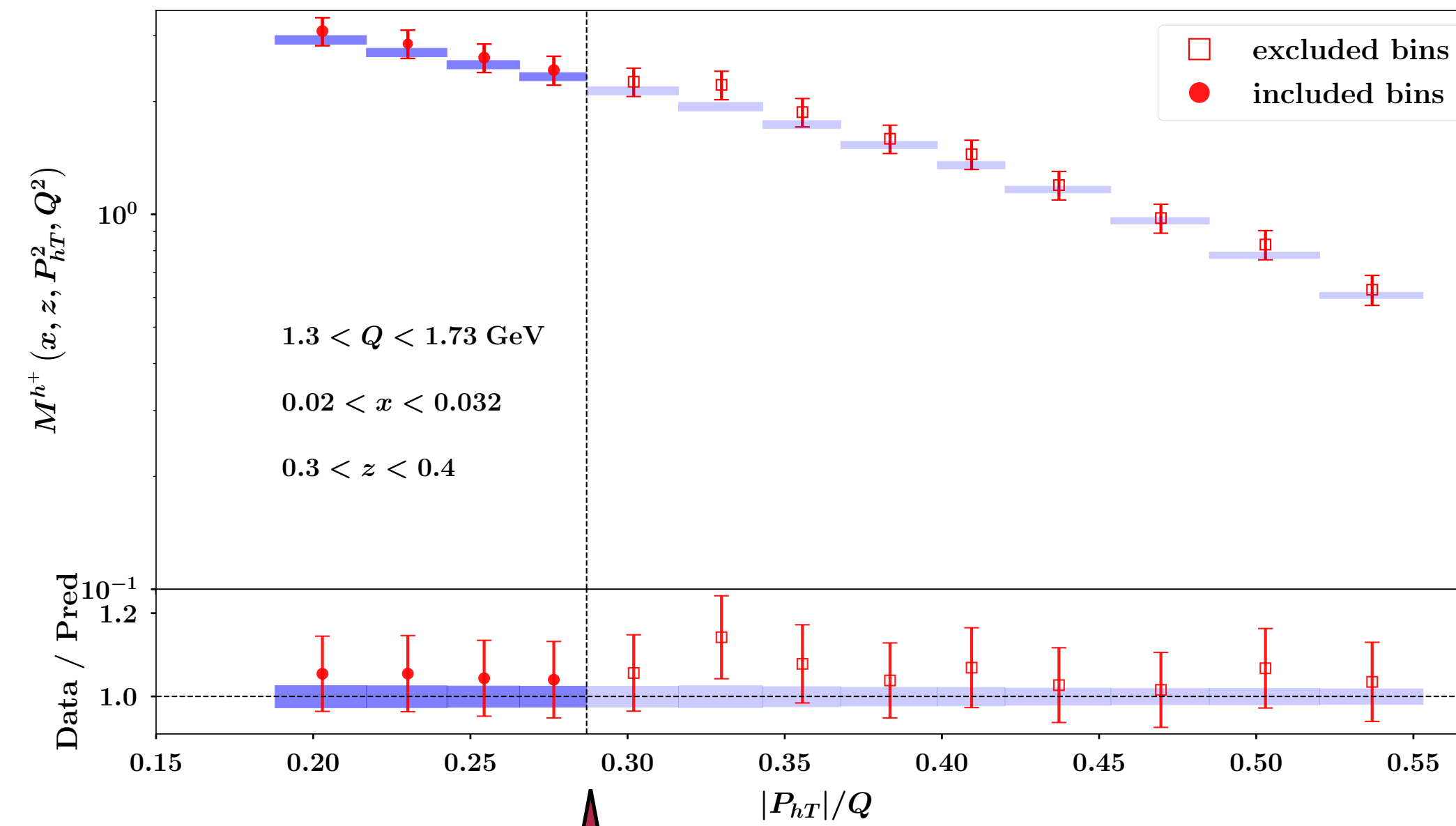


No significant effect at high Q , but large effect at low Q
(inhibits perturbative contribution)

$$|q_T| = |P_{hT}|/z \ll Q$$

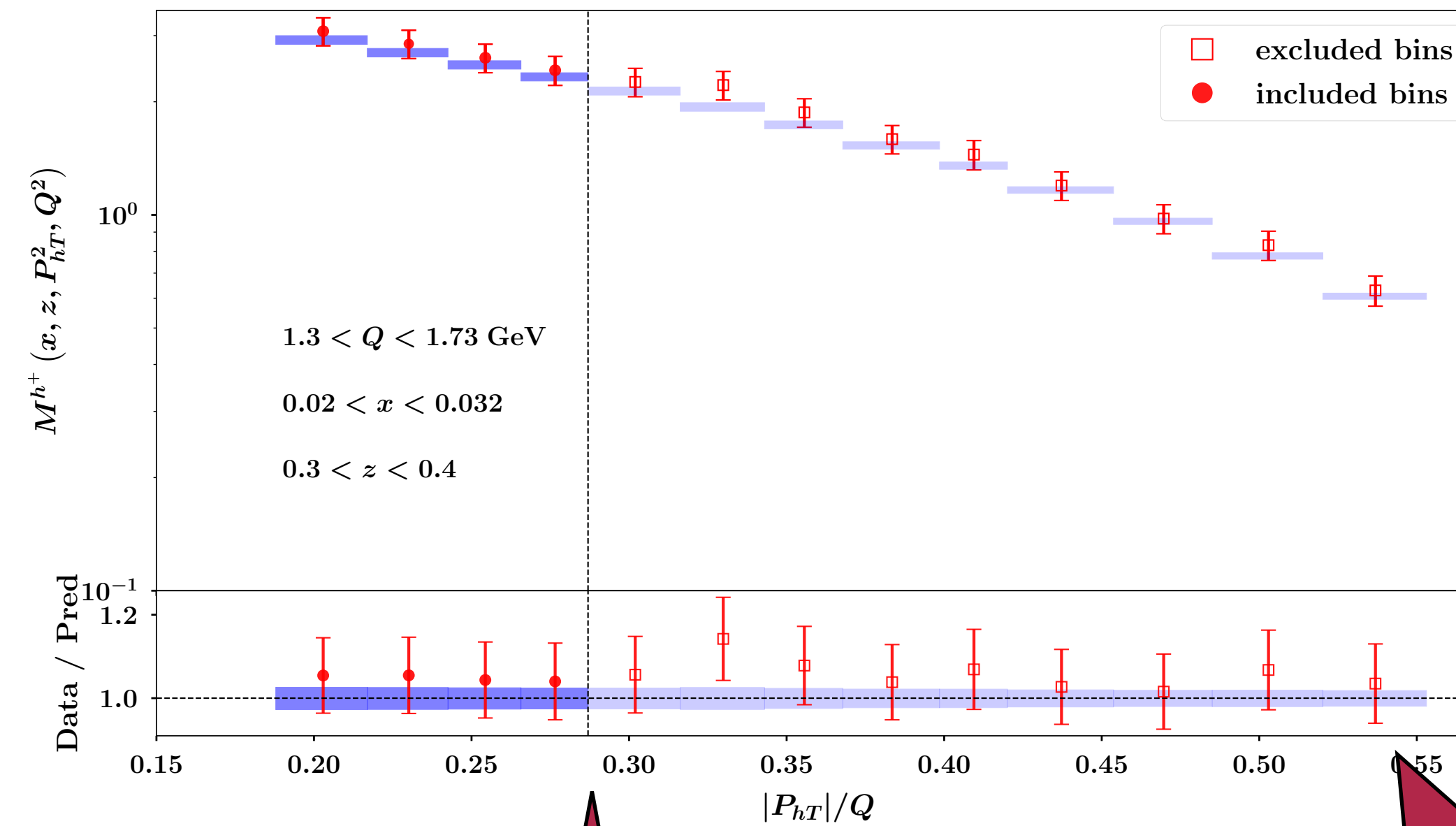


$$|q_T| = |P_{hT}|/z \ll Q$$



MAP22 cut

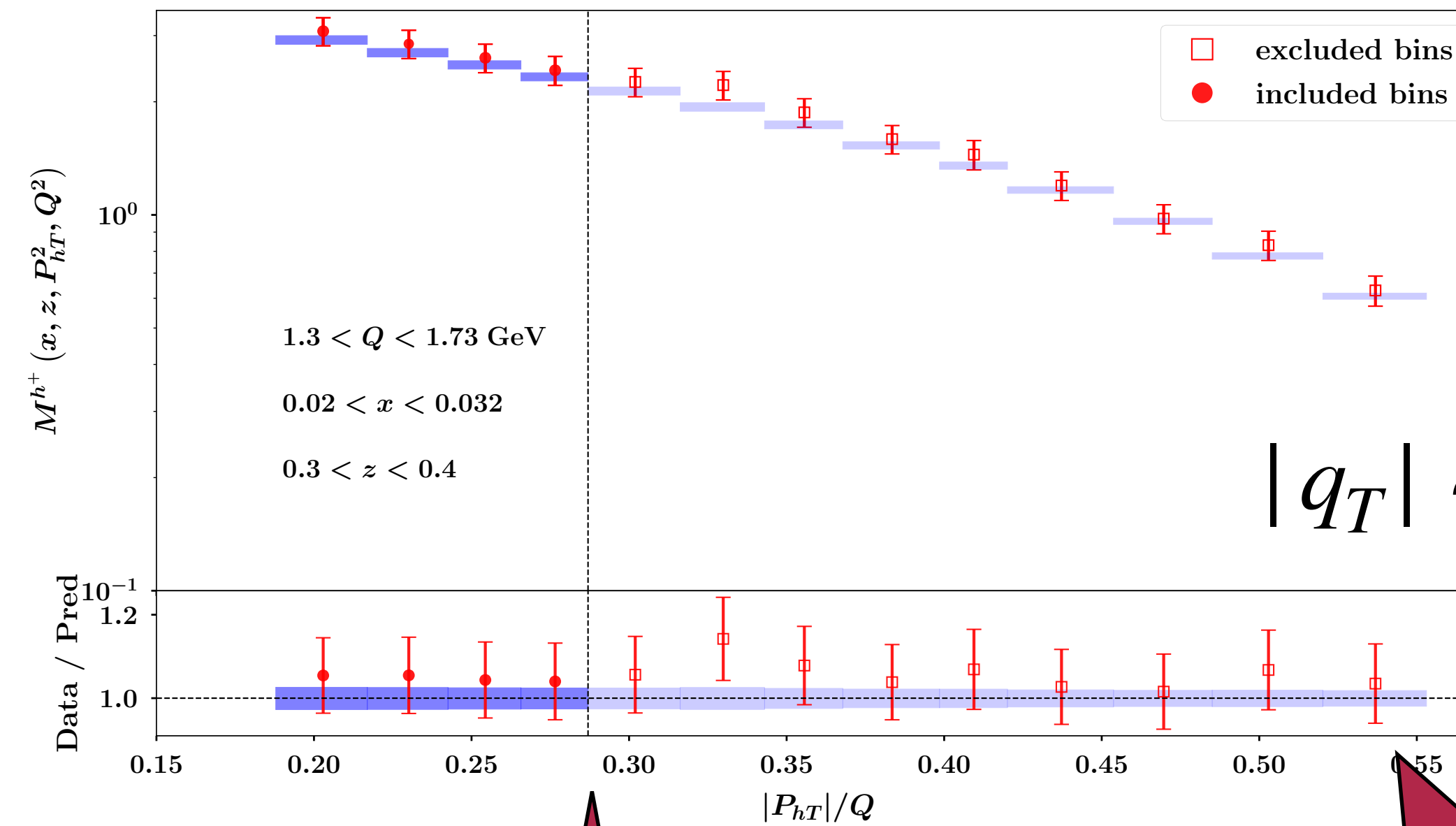
$$|q_T| = |P_{hT}|/z \ll Q$$



MAP22 cut

MAP22
extrapolation

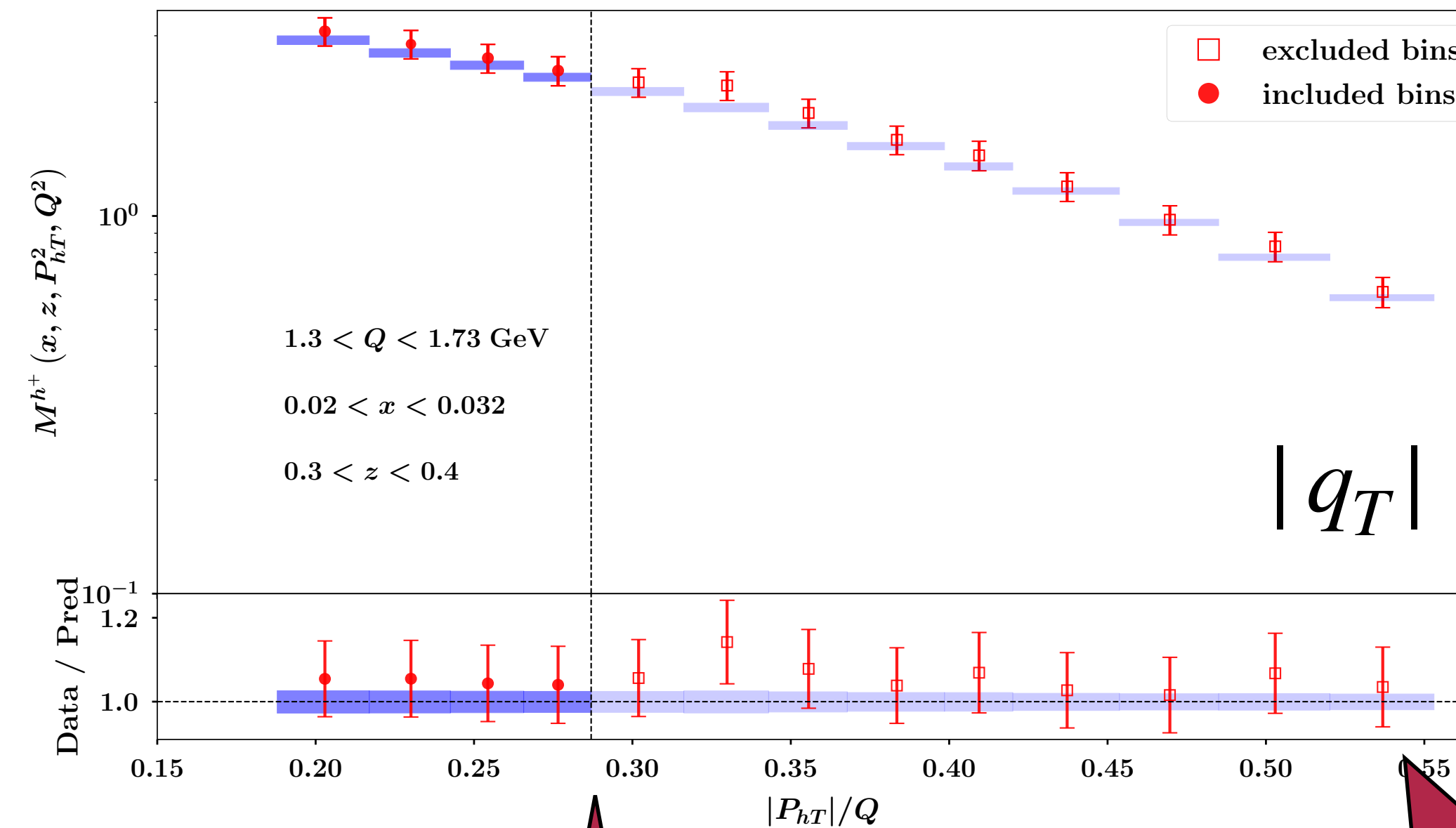
$$|q_T| = |P_{hT}|/z \ll Q$$



MAP22 cut

MAP22
extrapolation

$$|q_T| = |P_{hT}|/z \ll Q$$



$$|q_T| \sim 1.5 Q !$$

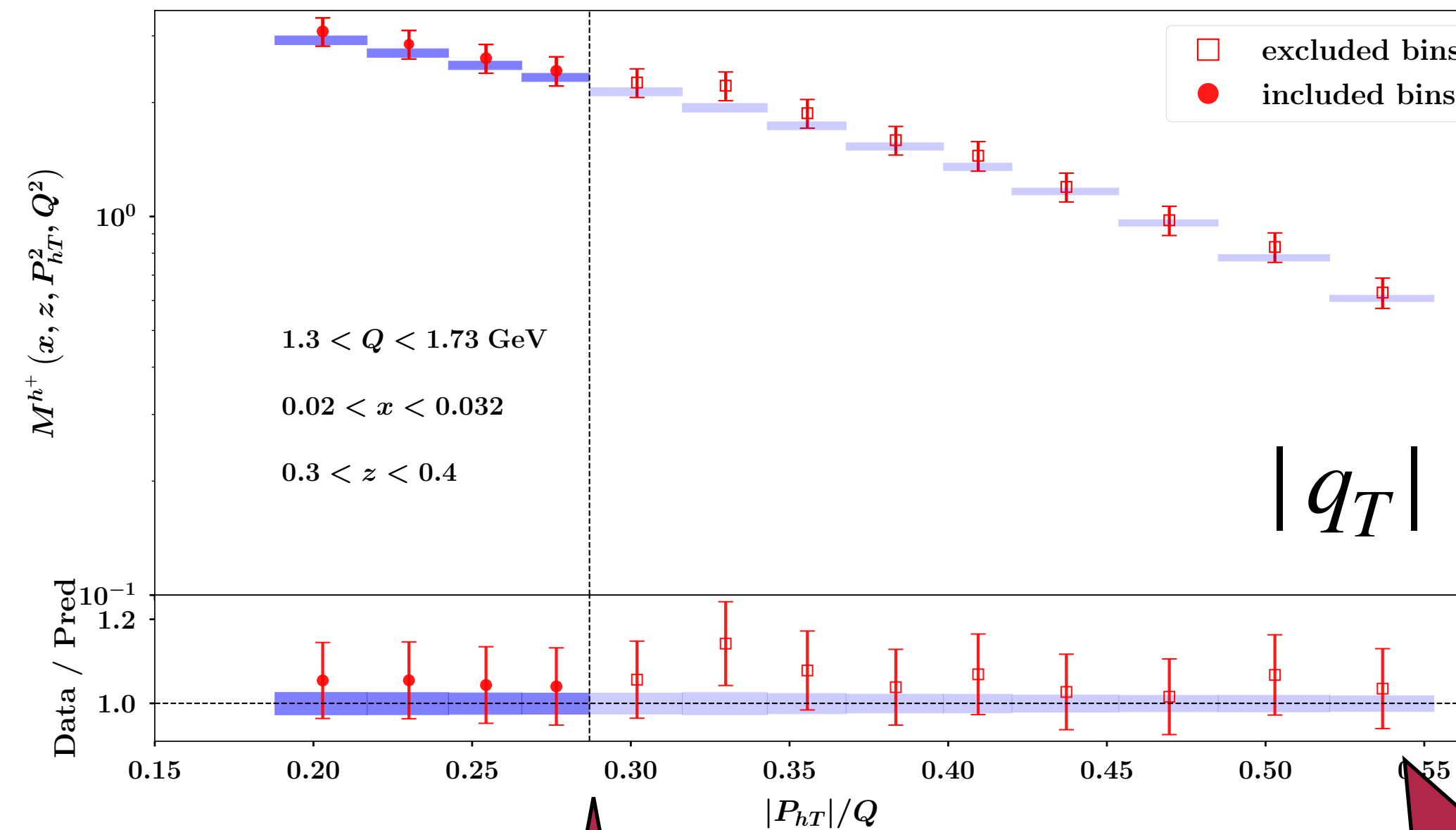
MAP22 cut

MAP22
extrapolation

The MAP22 cut is already considered to be “generous”,
but the physics seems to be the same for a much wider transverse momentum

$$|q_T| = |P_{hT}|/z \ll Q$$

?



$$|q_T| \sim 1.5 Q !$$

MAP22 cut

MAP22
extrapolation

The MAP22 cut is already considered to be “generous”,
but the physics seems to be the same for a much wider transverse momentum