An extended Nambu–Jona-Lasinio model for quark and nuclear matters

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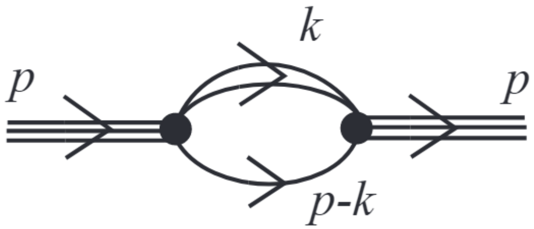
Motivation

- NJL model is a welcome QCD-like model with a lot of valuable features:
  - Showed a significant dynamical mass can be generated from the breaking of the approximate chiral symmetry
  - Gave a more physical picture on the quark-antiquark structure of mesons
  - Successful in reproducing the well-known GMOR relations of QCD
- Disadvantages of NJL model: no confinement, only quark degrees of freedom

Motivation

- How could nucleon emerges consistently from the quark NJL model?

In previous work, people extended the lagrangian including diquark channel in similar way as mesonic case, and consider nucleon as a bound state of diquarks and quarks.



However, in such a model at finite chemical potential, we might find a mixture of nucleons, quarks and diquark, which is not consistent with the picture of quarkyonic matter, since diquarks are colored and cannot be excited at color-confined phase.

The extended lagrangian

- For classical two flavor NJL model, the lagrangian reads

$$\mathcal{L}_0 = \bar{\psi}\left(\partial\!\!\!/ - m_0\right)\psi + G_2\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

- Now introduce a six-quark interaction term (for three quarks interacting through nucleon channels)

$$\begin{aligned}\mathcal{L} = &\ \bar{\psi}\left(i\partial\!\!\!/ + \gamma^0\frac{\mu_{\mathrm{B}}}{3} - m_0\right)\psi + G_2\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]\\ &- \left(\bar{\psi}\tau_2\epsilon_{\mathrm{c}}\gamma_5\psi_C\right)\bar{\psi}^{\mathrm{c}}\tilde{G}_3\left(i\partial\!\!\!/ + \gamma^0\mu_{\mathrm{B}}\right)\psi^{\mathrm{c}'}\left(\bar{\psi}_C\tau_2\epsilon_{\mathrm{c}'}\gamma_5\psi\right).\end{aligned}$$

- The form of six-quark interaction term is selected such way from QCD sum rule.

Mean field approximation

- After bosonization, the lagrangian becomes

$$\mathcal{L} = \bar{\psi}\left(\not{\partial} + \gamma^0 \frac{\mu_B}{3} - m_0 - \sigma - i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}\right)\psi - \frac{\sigma^2 + \boldsymbol{\pi}^2}{4G_2} - \tilde{G}_3 \left(\bar{\psi}\tau_2\epsilon_c\gamma_5\psi_C\right)\bar{\psi}^c \tilde{\not{P}}\psi^{c'}\left(\bar{\psi}_C\tau_2\epsilon_{c'}\gamma_5\psi\right)$$

where $\tilde{P}_\nu \equiv P_\nu + \mu_B\,\delta_{\nu 0}$ with $P = q_1 + q_2 + q_3$.

- To guarantee the confinement feature in QCD, we expect $\lim_{|\tilde{P}|\to 0} \tilde{G}_3 |\tilde{P}| \to \infty$, so we take $\tilde{G}_3 = G_3/|\tilde{P}^2|^d$, with $d = 1, 3/2$

- In the mean field approximation, the thermodynamics potential

$$\Omega_0 = \frac{(m - m_0)^2}{4G_2} - 2N_f N_c \int^{\Lambda} \frac{d^3k}{(2\pi)^3} E_{\mathbf{k}} - 2TN_f N_c \times \sum_{t=\pm} \int \frac{d^3k}{(2\pi)^3} \ln\left[1 + e^{-\frac{1}{T}\left(E_{\mathbf{k}} + t\,\frac{\mu_B}{3}\right)}\right]$$

- The dynamical quark mass is determined by the corresponding gap equation

Beyond mean field approximation

- The propagator of a quark with color c is

$$S_{\mathrm{q}}^{\mathrm{c}} \equiv i/(\not{q} + \gamma^0 \mu_{\mathrm{B}}/3 - m)$$

- The propagator of mesons is constructed through RPA method as

$$S_{\sigma/\boldsymbol{\pi}}^{-1}(p) = \frac{i}{2G} - \mathrm{Tr}\ S_{\mathrm{q}}^{\mathrm{c}}(q)\Gamma_{\sigma/\boldsymbol{\pi}} S_{\mathrm{q}}^{\mathrm{c}}(q-p)\Gamma_{\sigma/\boldsymbol{\pi}}$$

and meson masses can be calculated by solving the pole equation

- the contribution of pions to the thermodynamic potential can be evaluated in pole approximation as

$$\Omega_\pi = 3T \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \ln\left(1 - e^{-\frac{E_{\mathbf{k}}^\pi}{T}}\right)$$

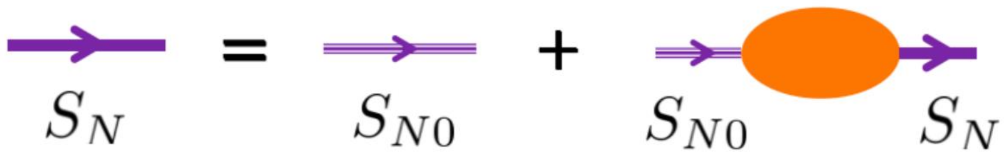
Beyond mean field approximation

- Here we introduce the nucleon degree of freedom as $N \equiv \sqrt{\tilde{G}_3}\psi^c \left(\psi^{\mathrm{T}} C\tau_2\epsilon_c\gamma_5\psi\right)$

$$\mathcal{L} = \bar{N}\, i\, S_{\mathrm{N0}}^{-1}\, N - \bar{N}\, i\,\Gamma\, \psi^c \left(\bar{\psi}_C \tau_2\epsilon_c\gamma_5\psi\right) - \left(\bar{\psi}\tau_2\epsilon_c\gamma_5\psi_C\right) \bar{\psi}^c\, i\,\Gamma\, N.$$

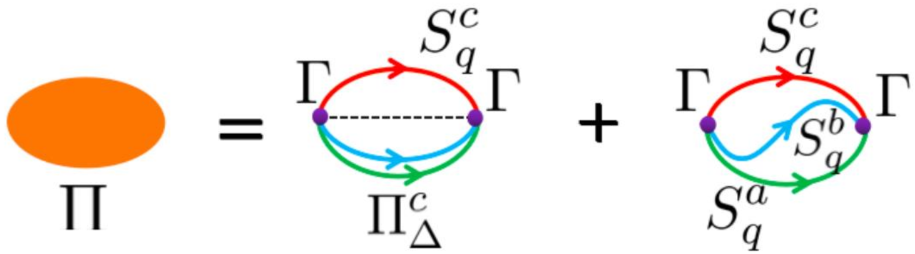
$$\Gamma(P,\mu_B) \equiv \sqrt{\tilde{G}_3} S_{\mathrm{N0}}^{-1}$$

- Propose an RPA-like scheme for the full propagator of a nucleon



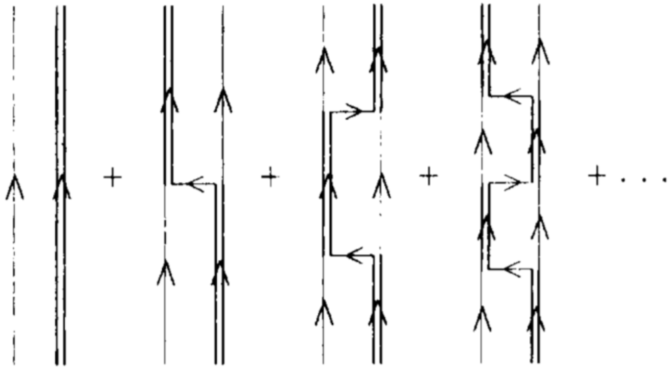
$$S_{\mathrm{N}} = S_{\mathrm{N0}}(1 + \Gamma\,\Pi\,\Gamma S_{\mathrm{N}}) = S_{\mathrm{N0}} + \bar{\Pi} S_{\mathrm{N0}}^{-1} S_{\mathrm{N}}$$

$$\bar{\Pi}(P) \equiv \Pi_{\mathrm{q}\Delta}(P) + \Pi_{\mathrm{qqq}}(P),$$



Color of diquark and quark unchanged

Color of diquark and quark changed



Beyond mean field approximation

- The propagator of nucleon can be derived as

$$\begin{aligned} S_{\mathrm{N}} &= \left[\mathbf{1} - \bar{\Pi}(P) S_{\mathrm{N0}}^{-1}\right]^{-1} S_{\mathrm{N0}} \\ &= i\, S_{\mathrm{N0}}^{-1} \left[i\, S_{\mathrm{N0}}^{-1} + i\, \bar{\Pi}(P) \tilde{P}^2\right]^{-1} S_{\mathrm{N0}} \end{aligned}$$

- The pole mass of nucleons can be obtained by solving

$$[m_{\mathrm{N}} - \bar{\Pi}_0(m_{\mathrm{N}}) m_{\mathrm{N}}^2]^2 - \bar{\Pi}_{\mathrm{s}}^2(m_{\mathrm{N}}) m_{\mathrm{N}}^4 = 0.$$

- And their contribution to the thermodynamic potential can also be evaluated as

$$\Omega_{\mathrm{N}} = -2T N_{\mathrm{f}} \sum_{t=\pm} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \ln\left[1 + e^{-\frac{1}{T}(E_{\mathbf{k}}^{\mathrm{N}} + t\, \mu_{\mathrm{B}})}\right] \qquad \Omega \equiv \Omega_0 + \Omega_\pi + \Omega_{\mathrm{N}}.$$

Regularization parameter

- The model parameter is fitted by $m_\pi = 0.138\ \mathrm{GeV}$ $f_\pi = 0.093\ \mathrm{GeV}$
- Propose two different sets of parameters

Small quark vacuum mass(SQVM)

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.25\ \mathrm{GeV})^3$$

$$m_0 = 5.29\ \mathrm{MeV},\ G_2 = 4.9316\ \mathrm{GeV}^{-2},$$

$$\Lambda = 0.65333\ \mathrm{GeV} \quad G_3 m_{\mathrm{N}}^{2-2d} = 2284\ \mathrm{GeV}^{-4}$$

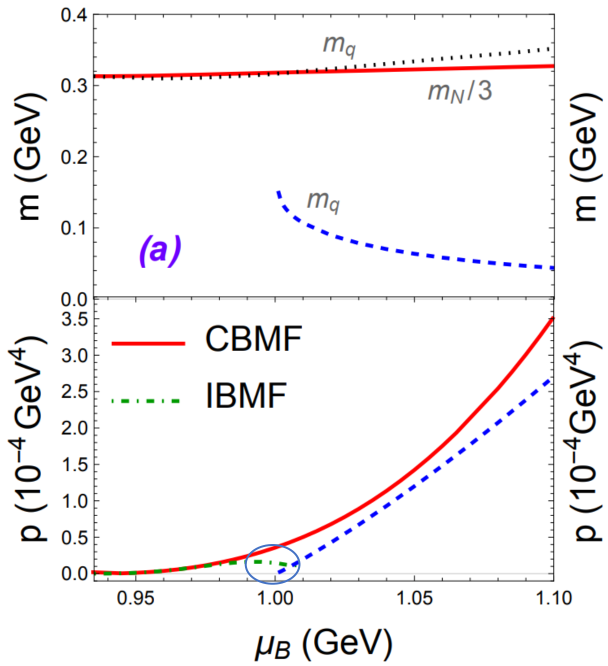
Large quark vacuum mass(LQVM)

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.249\ \mathrm{GeV})^3$$

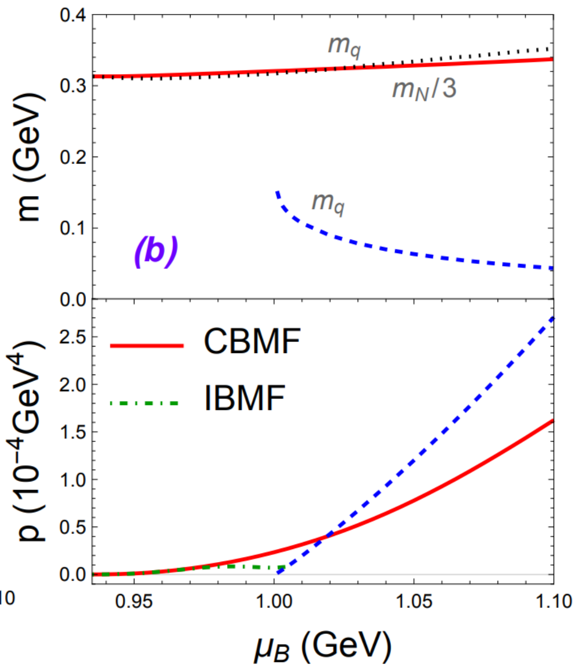
$$m_0 = 5.37\ \mathrm{MeV}, \qquad \Lambda = 0.6455\ \mathrm{GeV},$$

$$G_2 = 5.112\ \mathrm{GeV}^{-2},\ G_3 m_{\mathrm{N}}^{2-2n} = 2375\ \mathrm{GeV}^{-4}$$

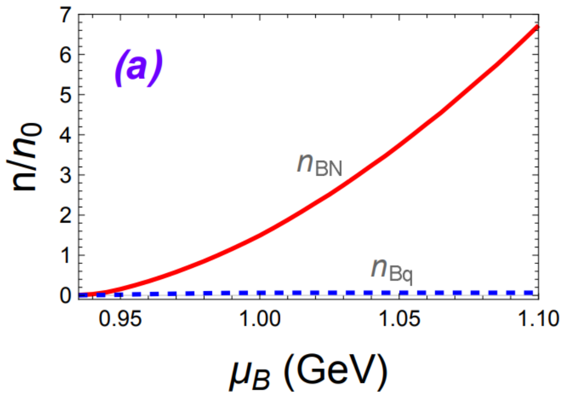
Results-Small quark vacuum mass

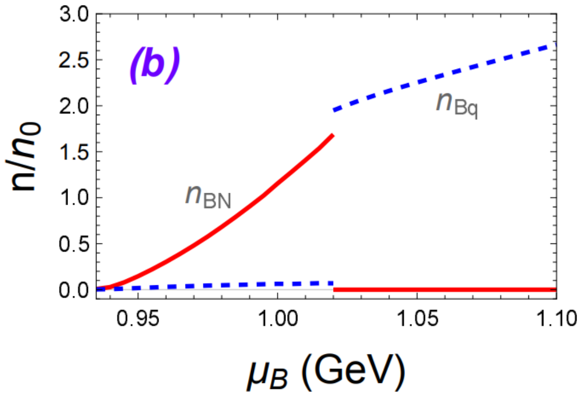


$d = 1$



$d = 3/2$



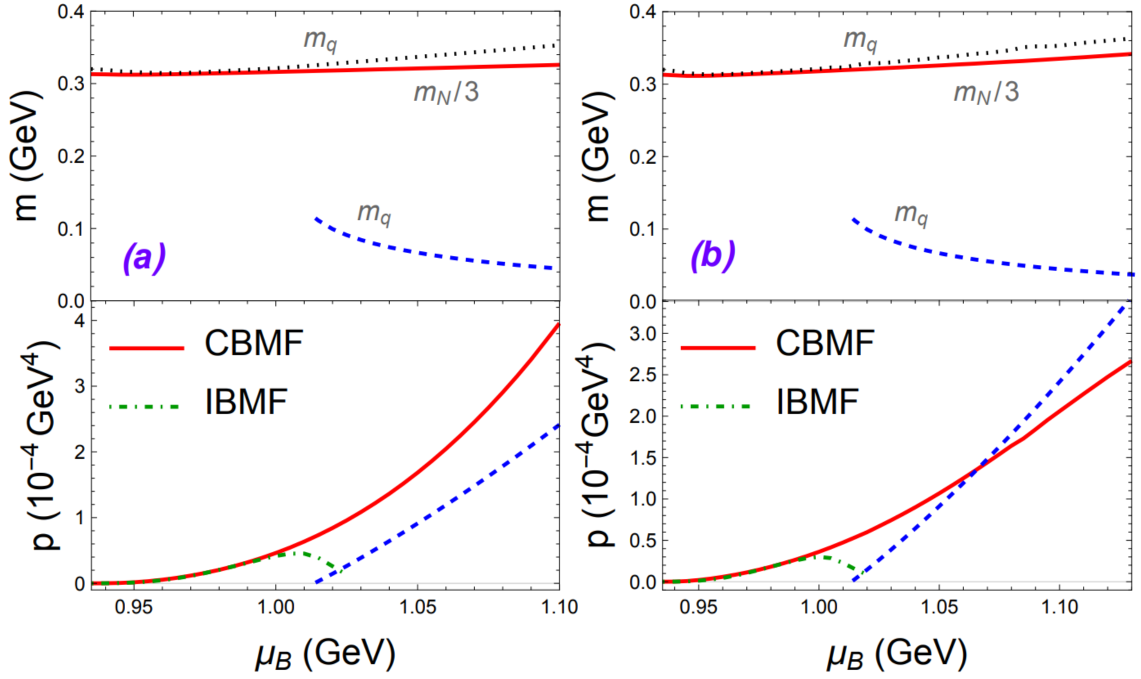


C(/I)BMF: (In)consistent beyond mean field approximation

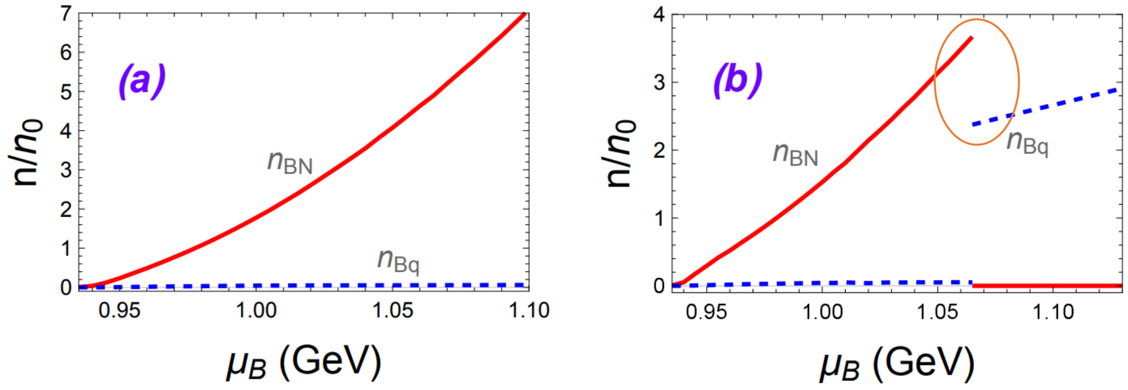
- Mean field result before first order phase transition is not shown in the figures
- For chiral symmetry restored scenario, all three cases shares a same quark mass.
- At zero temperature and finite chemical potential, the contribution for meson sector vanishes.
- For IBMF, negative derivative near phase transition point, unphysical.

- For $d = 1$ case, no phase transition
- For $d = 3/2$ case, first order phase transition (more preferable)

Results-Large quark vacuum mass



$d = 1$ $d = 3/2$



Why the drop in baryon density?
Pressure is the product of density and mean energy for free fermion gases. This can be understood by a larger mean energy for quark matter