Disentangling the structure of hidden and open charm tetraquark

Zhi Yang (杨智)

University of Electronic Science and
Technology of China, Chengdu (电子科技大学，成都)

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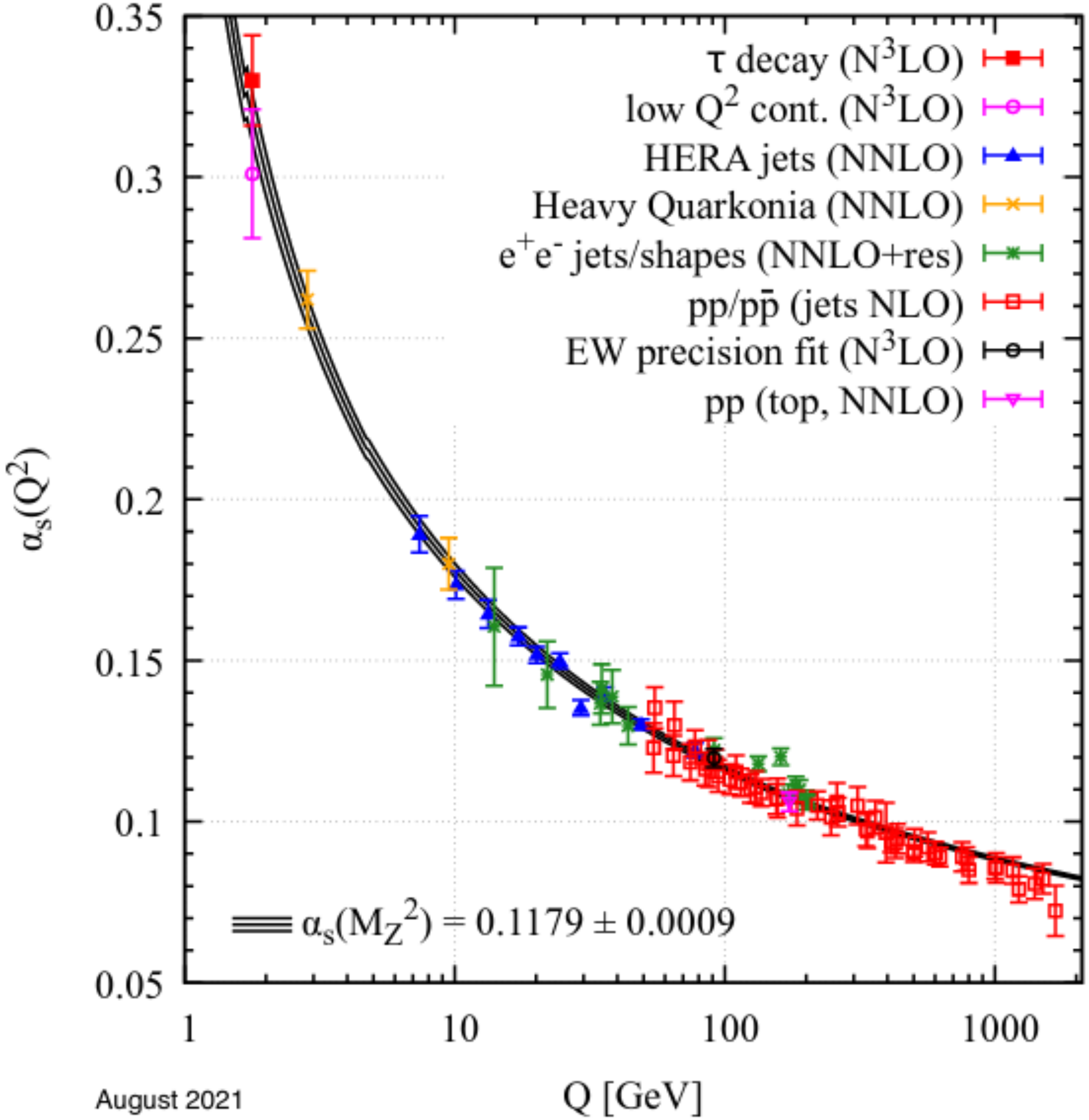
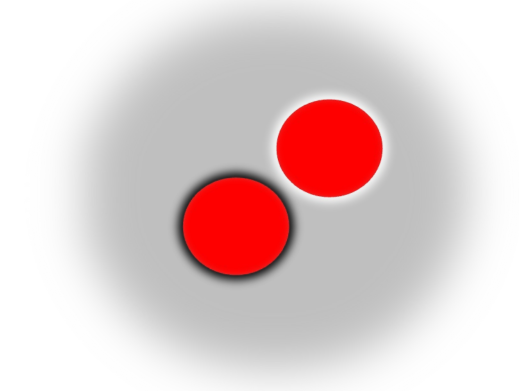
In collaboration with 王广娟, 吴佳俊, Makoto Oka, 朱世琳

第4届LHCb前沿物理研讨会 烟台 2024/7/28

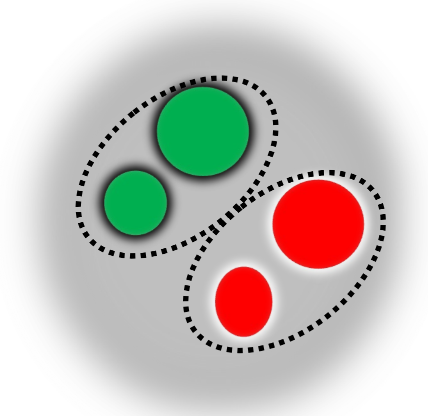
Hadron physics



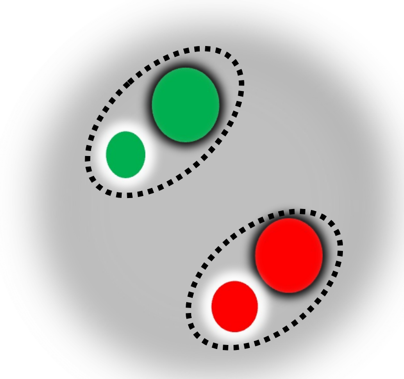
The running coupling constant



Multi-quark states



Compact multiquark



Hadronic molecule

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $AAAA\bar{A}$, $AAAAAA\bar{A}$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $A\bar{A}$, $AA\bar{A}\bar{A}$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $A\bar{A}$ and AAA, that is, "deuces and treys".

GI quark model

The relativized quark model:

$$H = H_0 + V$$

$$H_0 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

$$V = G_{\text{eff}}(r) + S_{\text{eff}}(r)$$

$$G_{\text{eff}}(r) = \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2} \tilde{G}(r) \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2} + \left[\frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{G}_{11}^{\text{so}(v)}}{\partial r} + \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial \tilde{G}_{22}^{\text{so}(v)}}{\partial r} + \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}}{m_1 m_2} \frac{1}{r} \frac{\partial \tilde{G}_{12}^{\text{so}(v)}}{\partial r}\right] + \frac{2\mathbf{S}_1 \cdot \mathbf{S}_2}{3 m_1 m_2} \nabla^2 \tilde{G}_{12}^{c} - \left[\frac{\mathbf{S}_1 \cdot \hat{r} \mathbf{S}_2 \cdot \hat{r} - \frac{1}{3} \mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\right] \left[\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right] \tilde{G}_{12}^{t}$$

$$S_{\text{eff}}(r) = \tilde{S}(r) - \frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{S}_{11}^{\text{so}(s)}}{\partial r} - \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial \mathbf{S}_{22}^{\text{so}(s)}}{\partial r}$$

$$H|\psi_\alpha\rangle = M_\alpha^0 |\psi_\alpha\rangle \longrightarrow$$ Mass & wave function

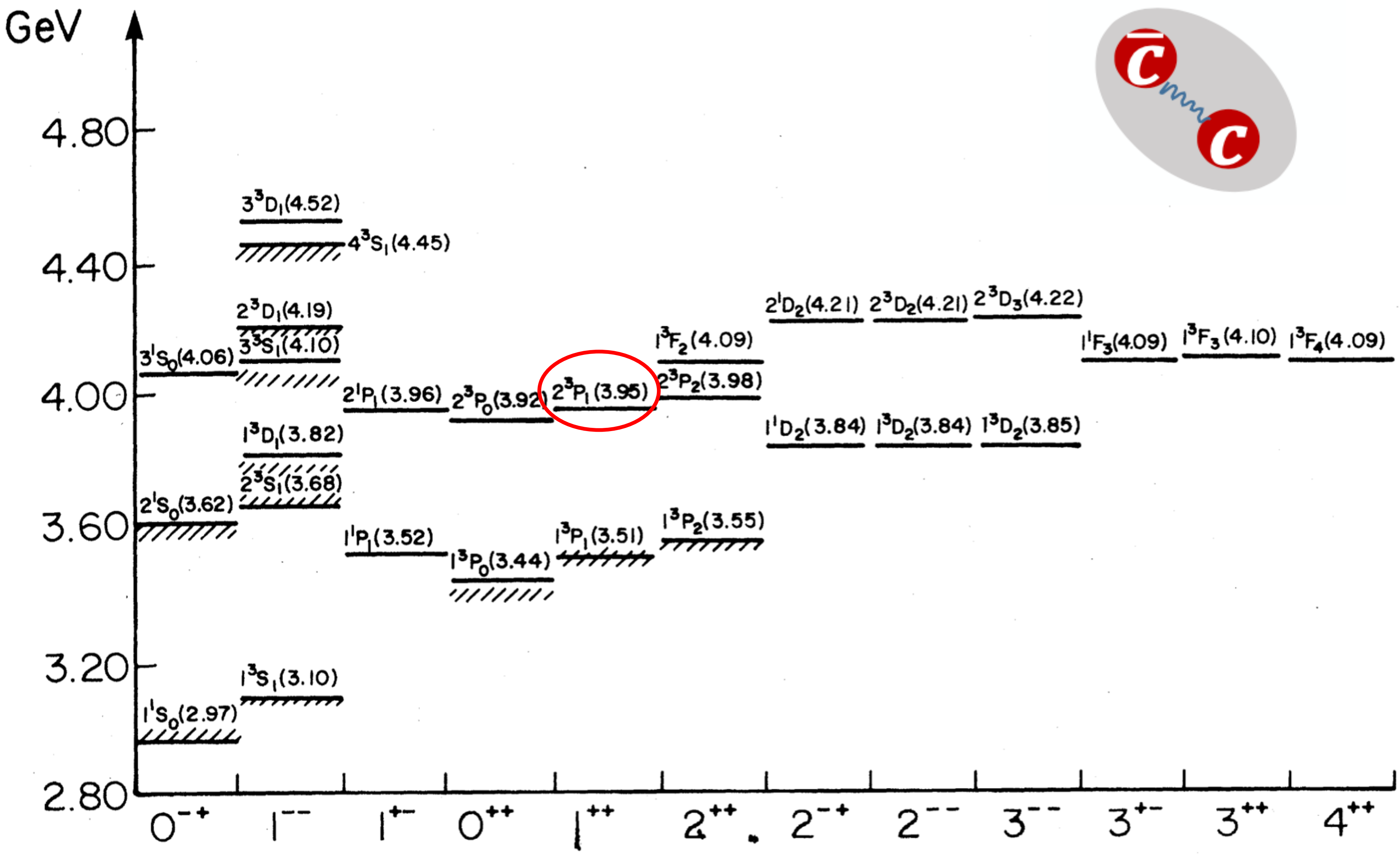
Mesons in a Relativized Quark Model with Chromodynamics #1

S. Godfrey (Toronto U.), Nathan Isgur (Toronto U.) (1985)

Published in: *Phys.Rev.D* 32 (1985) 189-231

DOI cite claim reference search 3,134 citations

Charmonium state in the GI quark model



Godfrey, Isgur, PRD32,189

X(3872)

Experiment	Mass [MeV]	Width [MeV]
Belle [63]	$3872 \pm 0.6 \pm 0.5$	< 2.3
Belle [75]	–	–
Belle [76]	$3875.4 \pm 0.7^{+0.4}_{-1.7} \pm 0.9$	–
Belle [77]	$3871.46 \pm 0.37 \pm 0.07$	–
Belle [78]	$3872.9^{+0.6+0.4}_{-0.4-0.5}$	$3.9^{+2.8+0.2}_{-1.4-1.1}$
Belle [79]	–	–
Belle [80]	$3871.84 \pm 0.27 \pm 0.19$	< 1.2
CDF [67]	$3871.3 \pm 0.7 \pm 0.4$	–
CDF [81]	–	–
CDF [82]	–	–
CDF [83]	$3871.61 \pm 0.16 \pm 0.19$	–
DØ [68]	$3871.8 \pm 3.1 \pm 3.0$	–
BaBar [84]	3873.4 ± 1.4	–
BaBar [85]	$3871.3 \pm 0.6 \pm 0.1$	< 4.1
	$3868.6 \pm 1.2 \pm 0.2$	–
BaBar [86]	–	–
BaBar [87]	$3875.1^{+0.7}_{-0.5} \pm 0.5$	$3.0^{+1.9}_{-1.4} \pm 0.9$
BaBar [88]	$3871.4 \pm 0.6 \pm 0.1$	< 3.3
	$3868.7 \pm 1.5 \pm 0.4$	–
BaBar [89]	–	–
BaBar [90]	$3873.0^{+1.8}_{-1.6} \pm 1.3$	–
LHCb [91]	$3871.95 \pm 0.48 \pm 0.12$	–
LHCb [70]	–	–
LHCb [92]	–	–
CMS [73]	–	–
BESIII [93]	$3871.9 \pm 0.7 \pm 0.2$	< 2.4

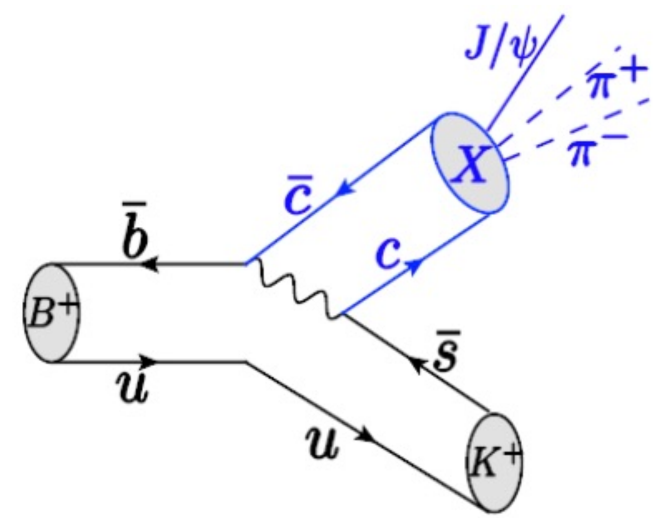
Phys. Rept. 639 (2016) 1-121

Observation of a narrow charmonium-like state in exclusive $B^\pm \to K^\pm \pi^+ \pi^- J/\psi$ decays

Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)

Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: hep-ex/0309032 [hep-ex]

pdf links DOI cite claim 2,295 citations



- The $D\bar{D}^*/D^*\bar{D}$ molecular state.

  Swanson, Wong, Guo, liu,....

Close to $D^0\bar{D}^{*0}/D^{*0}\bar{D}^0$ thresholds

$$\delta m = m_{D^0\bar{D}^{*0}} - m_{X(3872)}$$
$$= 0.00 \pm 0.18 \text{ MeV}$$

PDG 22

Theoretical interpretation of X(3872)

Where is the $\chi_{c1}(2P)$ in quark model?

- The mixing of the $\bar{c}c$ core with $D\bar{D}^*/D^*\bar{D}$ component.

  Chao, H. Q. Zheng, Yu. S. Kalashnikova, P. G. Ortega...

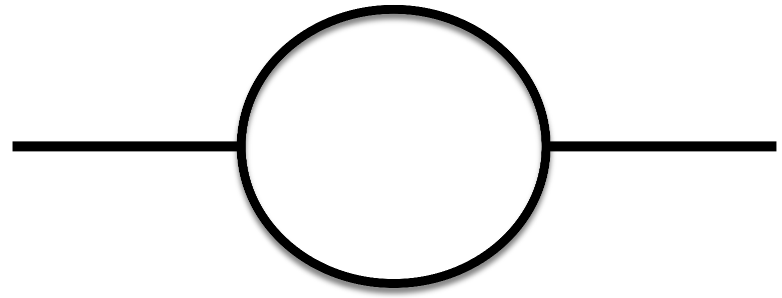
Close to charmonium $\chi_{c1}(2P)$: m=3953.5 MeV

$$\delta m = m_{\chi_{c1}(2P)} - m_{X(3872)} = 81.35 \text{ MeV}$$

→ *Complicated coupled-channel effect:* $\bar{c}c$ & $D\bar{D}^*/D^*\bar{D}$

Phys. Rev. D 32, 189 (1985)

Coupled-channel effect



1. Yu. S. Kalashnikova, Phys.Rev.D 72, 034010 (2005)

   ☞ Charmonium

2. F.-K. Guo, S. Krewald, and U.-G. Meißner, Phys.Lett.B 665,157 (2008)
   Z.-Y. Zhou and Z. Xiao, Phys. Rev. D 84, 034023 (2011)

   ☞ Charmed and charmed-strange spectra

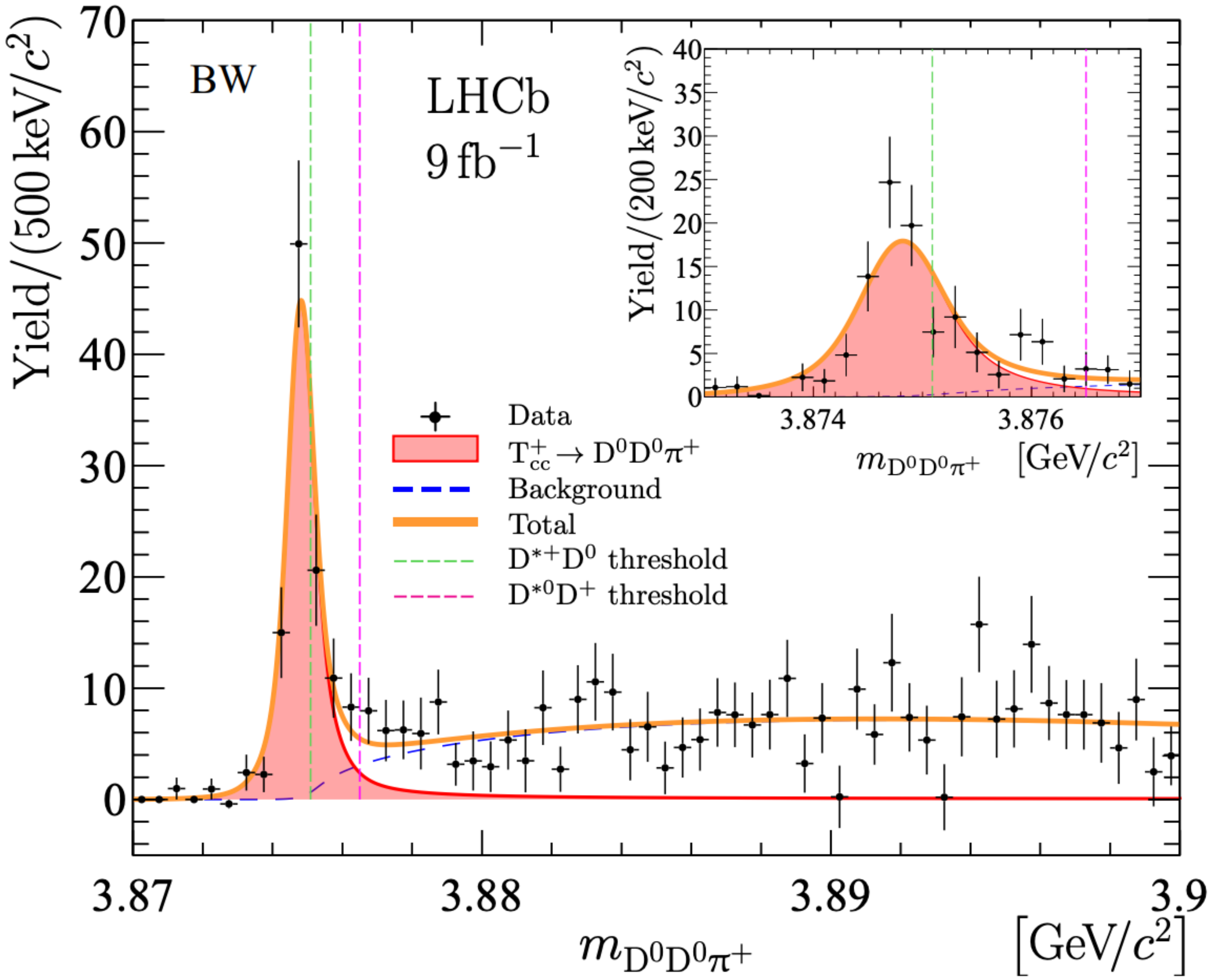
3. Y. Lu, M. N. Anwar, B. S. Zou, Phys.Rev.D 94, 034021 (2016)

   ☞ Bottomonium

……

- **Coupled-channel effect due to hadron loop could cause sizable mass shift on the state in quark model.**

How to determine the component in the X(3872): from Tcc



- $D^0D^0\pi^+$ channel
- Close to $D^{*+}D^0$ thresholds:

Conventional Breit-Wigner: assumed $J^P = 1^+$.

$$\delta m_{BW} = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -273 \pm 61 \text{ keV}$$
$$\Gamma_{BW} = 410 \pm 165\text{keV}$$

EPS-HEP conference, Ivan Polyakov's talk,29/07/2021; Nature Physics,22'

Unitarized Breit-Wigner:

$$\delta m_U = m_{T_{cc}} - m_{D^{*+}D^0}$$
$$= -361 \pm 40 \text{ keV}$$
$$\Gamma_U = 47.8 \pm 1.9 \text{ keV}$$

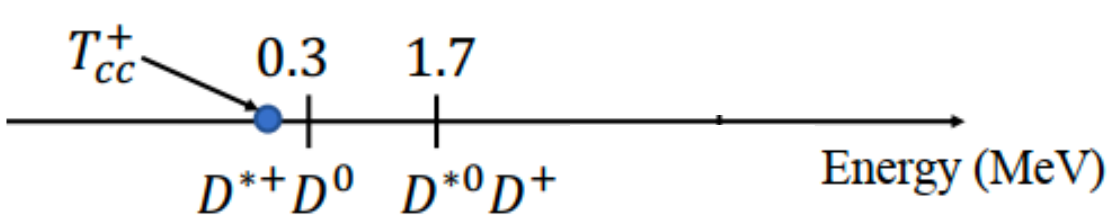
LHCb, Nature Commun. 13 (2022) 1, 3351

❖ Quark content: $cc\bar{u}\bar{d}$

❖ ***Only the D^*D coupled channel effect***

C-parity

$\overline{D^*}D$ / $\bar{D}D^*$ interaction



One-boson-exchange model

DD^*

$$H_a^{(Q)} = \frac{1+ \not{v}}{2}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]$$

$$\bar{H}_a^{(Q)} \equiv \gamma_0 H^{(Q)\dagger}\gamma_0 = \left[P_a^{*\dagger\mu}\gamma_\mu + P_a^\dagger\gamma_5\right]\frac{1+ \not{v}}{2}$$

$$P = \left(D^0, D^+, D_s^+\right) \& P^* = \left(D^{*0}, D^{*+}, D_s^{*+}\right)$$

$$\mathcal{L}_{MH^{(Q)}H^{(Q)}} = ig\,\mathrm{Tr}\left[H_b^{(Q)}\gamma_\mu\gamma_5 A_{ba}^\mu \bar{H}_a^{(Q)}\right]$$

$$\mathcal{L}_{VH^{(Q)}H^{(Q)}} = i\beta\,\mathrm{Tr}\left[H_b^{(Q)} v_\mu \left(V_{ba}^\mu - \rho_{ba}^\mu\right)\bar{H}_a^{(Q)}\right] + i\lambda\,\mathrm{Tr}\left[H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\right]$$

$D\bar{D}^*$

$$H_a^{(\bar{Q})} \equiv C\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T C^{-1} = \left[P_{a\mu}^{(\bar{Q})*}\gamma^\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1- \not{v}}{2}$$

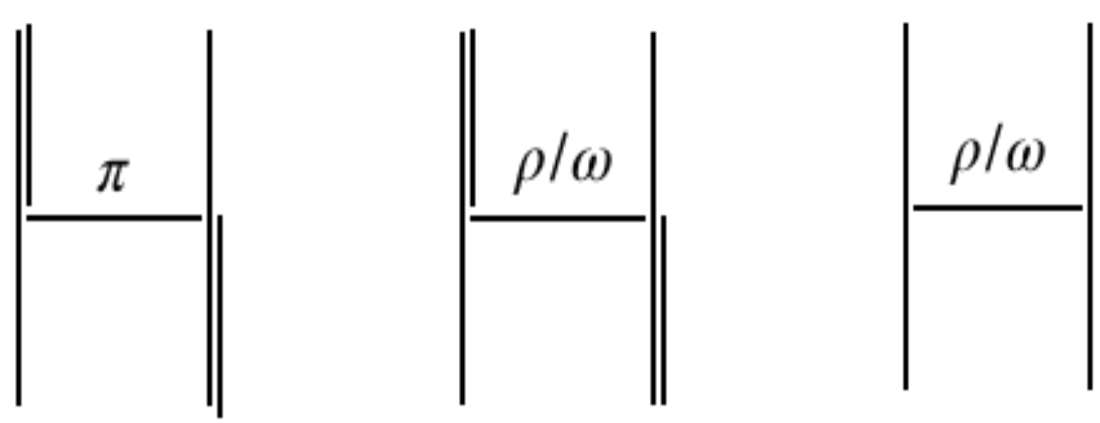
$$\bar{H}_a^{(\bar{Q})} \equiv \gamma_0 H_a^{(\bar{Q})\dagger}\gamma_0 = \frac{1- \not{v}}{2}\left[P_{a\mu}^{(\bar{Q})*\dagger}\gamma^\mu + P_a^{(\bar{Q})\dagger}\gamma_5\right]$$

$$\tilde{P} = \left(\bar{D}^0, D^-, D_s^-\right) \& \ \tilde{P}^* = \left(\bar{D}^{*0}, D^{*-}, D_s^{*-}\right)$$

$$\mathcal{L}_{MH^{(\bar{Q})}H^{(\bar{Q})}} = ig\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\gamma_\mu\gamma_5 A_{ab}^\mu H_b^{(\bar{Q})}\right]$$

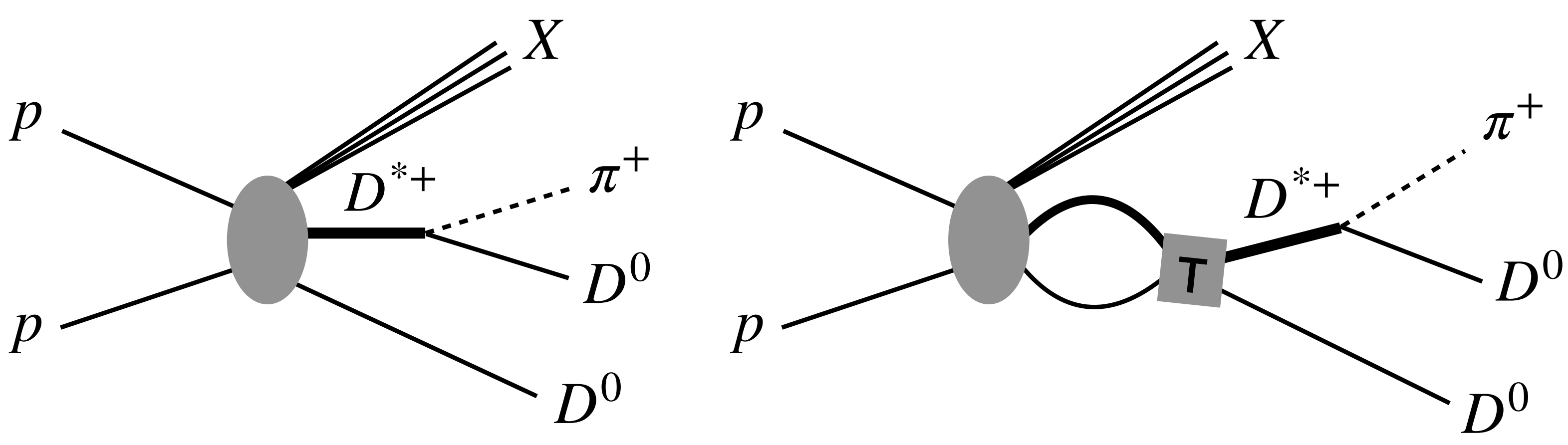
$$\mathcal{L}_{VH^{(\bar{Q})}H^{(\bar{Q})}} = -i\beta\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})} v_\mu \left(V_{ab}^\mu - \rho_{ab}^\mu\right) H_b^{(\bar{Q})}\right] + i\lambda\,\mathrm{Tr}\left[\bar{H}_a^{(\bar{Q})}\sigma_{\mu\nu}F_{ab}^{\prime\mu\nu}(\rho)H_b^{(\bar{Q})}\right]$$

- $g = 0.57$ is determined by the strong decays $D^* \to D\pi$.
- undetermined λ & β.



The inclusive production of the T_{cc}

$pp \to D^0(p_{D_1})D^0(p_{D_2})\pi^+(p_\pi)X$, X denotes all the other produced particles



The amplitude of the process

$$i\mathcal{M}_{pp\to DD\pi X} = \mathcal{A}^{\mu}_{pp\to DD^*X}\left\{g_{\mu\alpha} - \frac{i}{(2\pi)^4}\int d^4q_{D^*}G_{D^*\,\mu\nu}(q_{D^*})G_D(p_{D_1}+p_{D_2}+p_\pi-q_{D^*})T^{\nu}_{\alpha}(q_{D^*},p_{D_1}+p_\pi)\right\}$$
$$\times G^{\alpha\beta}_{D^*}(p_{D_2}+p_\pi)(g\,p_{\pi,\beta}) + (p_{D_1}\to p_{D_2}),$$

The iso-vector and iso-scalar assignment for the $\mathcal{A}$ with the production amplitudes satisfying

$$\mathcal{A}^{\mu}_{pp\to D^+D^{0*}X} = \pm\mathcal{A}^{\mu}_{pp\to D^0D^{*+}X}$$

- We can only find a satisfactory fit to the experimental data only in the iso-scalar case.

T-matrix

The T-matrix can be solved from the Lippmann-Schwinger equation

$$T(\vec{k}_{D^*},\vec{k}'_{D^*};E) = \mathcal{V}(\vec{k}_{D^*},\vec{k}'_{D^*};E) + \int d\,\vec{q}\,\frac{\mathcal{V}(\vec{k}_{D^*},\vec{q};E)T(\vec{q},\vec{k}'_{D^*};E)}{E-\sqrt{m_D^2+q^2}-\sqrt{m_{D^*}^2+q^2}+i\epsilon}$$

The effective potential is obtained with light-meson exchange potentials

$$\mathcal{V} = \left(V_\pi + V^t_{\rho/\omega} + V^u_{\rho/\omega}\right)\left(\frac{\Lambda^2}{\Lambda^2+p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2+p_i^2}\right)^2$$

with

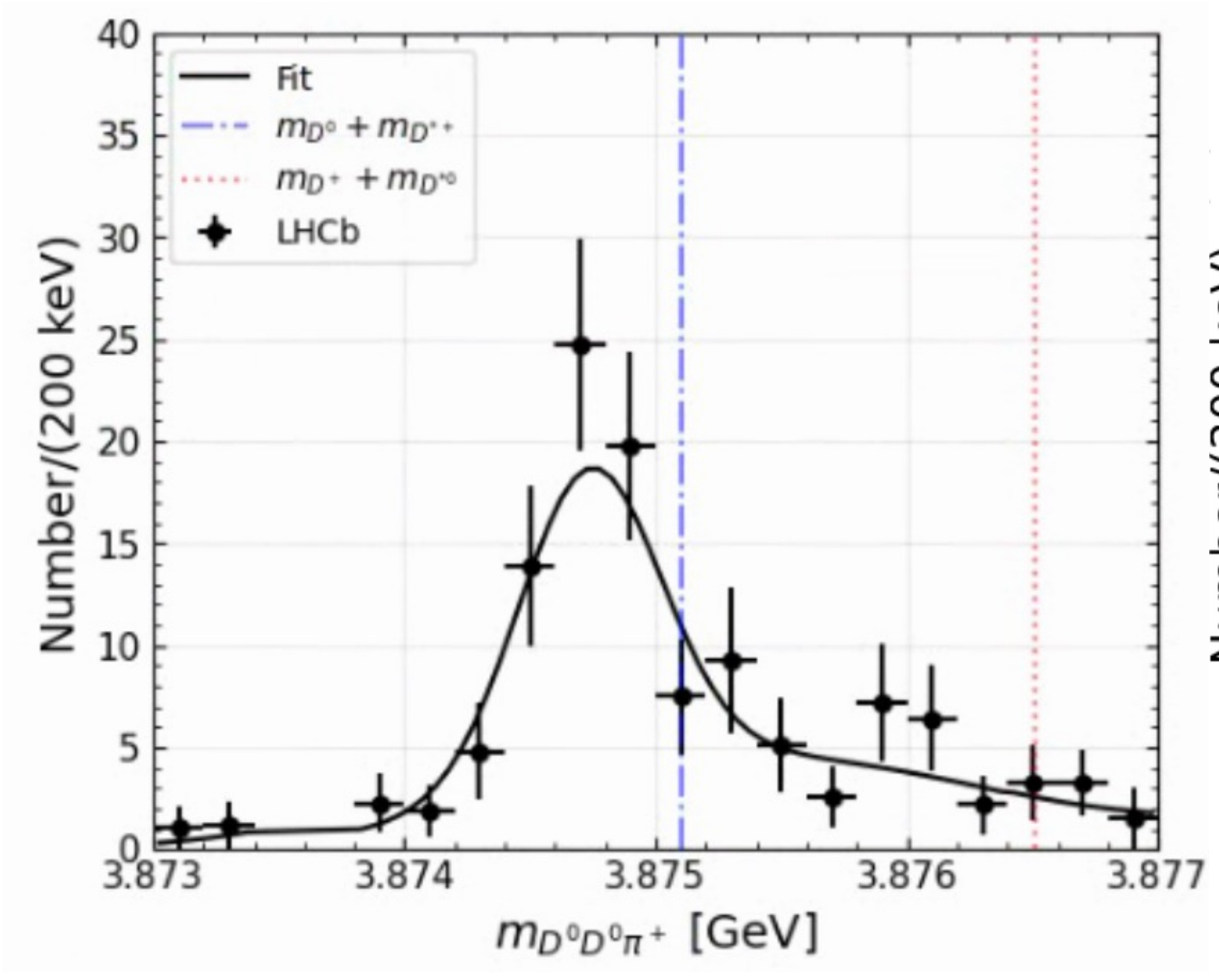
$$V_\pi = \frac{g^2}{f_\pi^2}\frac{(q\cdot\epsilon_\lambda)(q\cdot\epsilon^\dagger_{\lambda'})}{q^2-m_\pi^2},$$

$$\mathcal{V}^u_{\rho/\omega} = -2\lambda^2 g_V^2\,\frac{(\epsilon^\dagger_{\lambda'}\cdot q)(\epsilon_\lambda\cdot q)-q^2(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2-m^2_{\rho/\omega}},$$

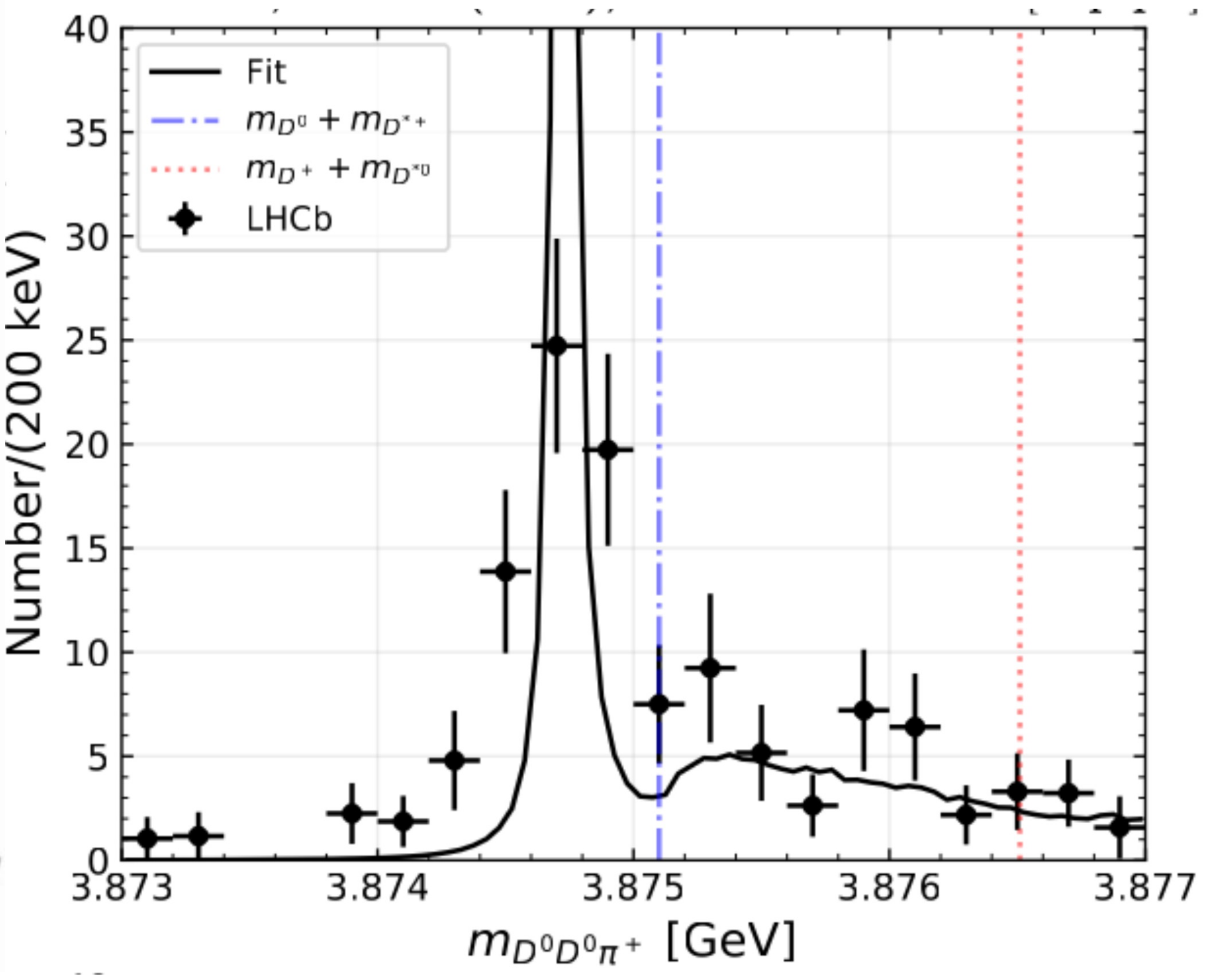
$$V^t_{\rho/\omega} = \frac{\beta^2 g_V^2}{2}\frac{(\epsilon_\lambda\cdot\epsilon^\dagger_{\lambda'})}{q^2-m^2_{\rho/\omega}}.$$

Fitting result

$\Lambda = 0.8$ GeV, χ^2/dof =0.76

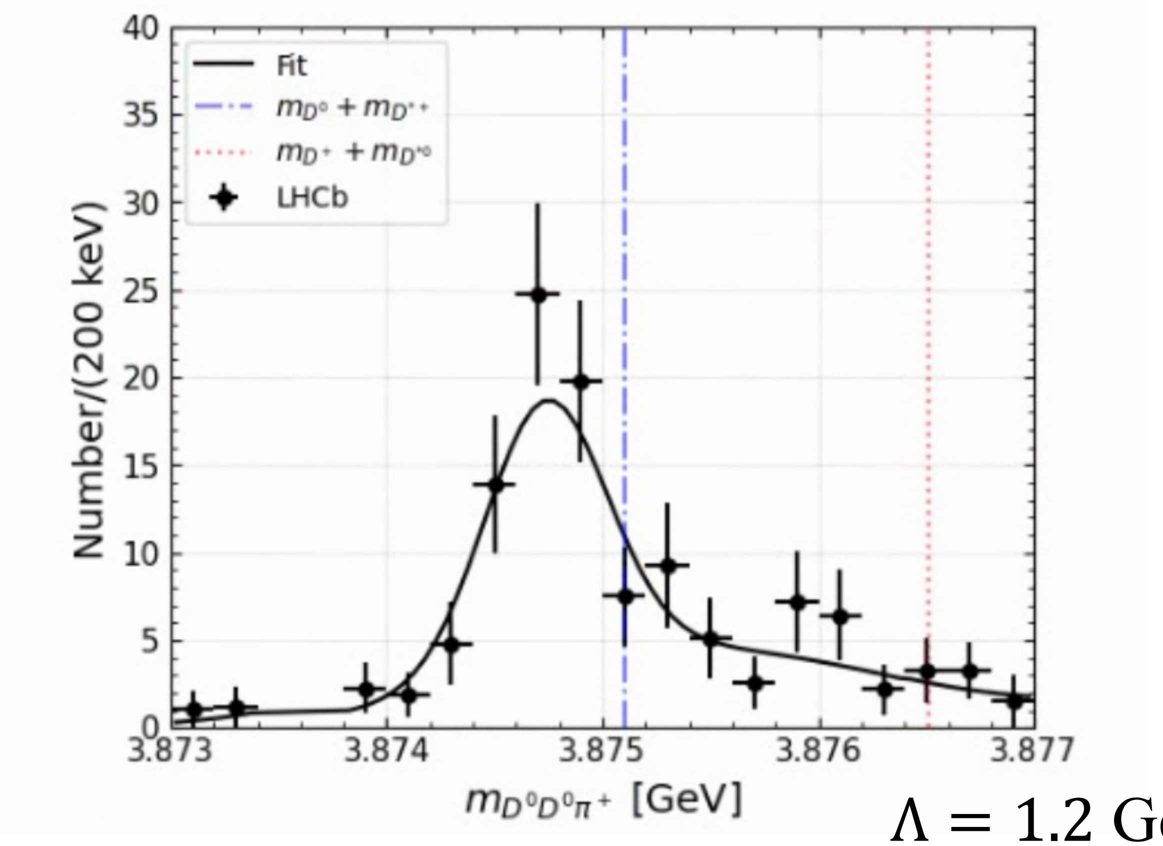


Without resolution function

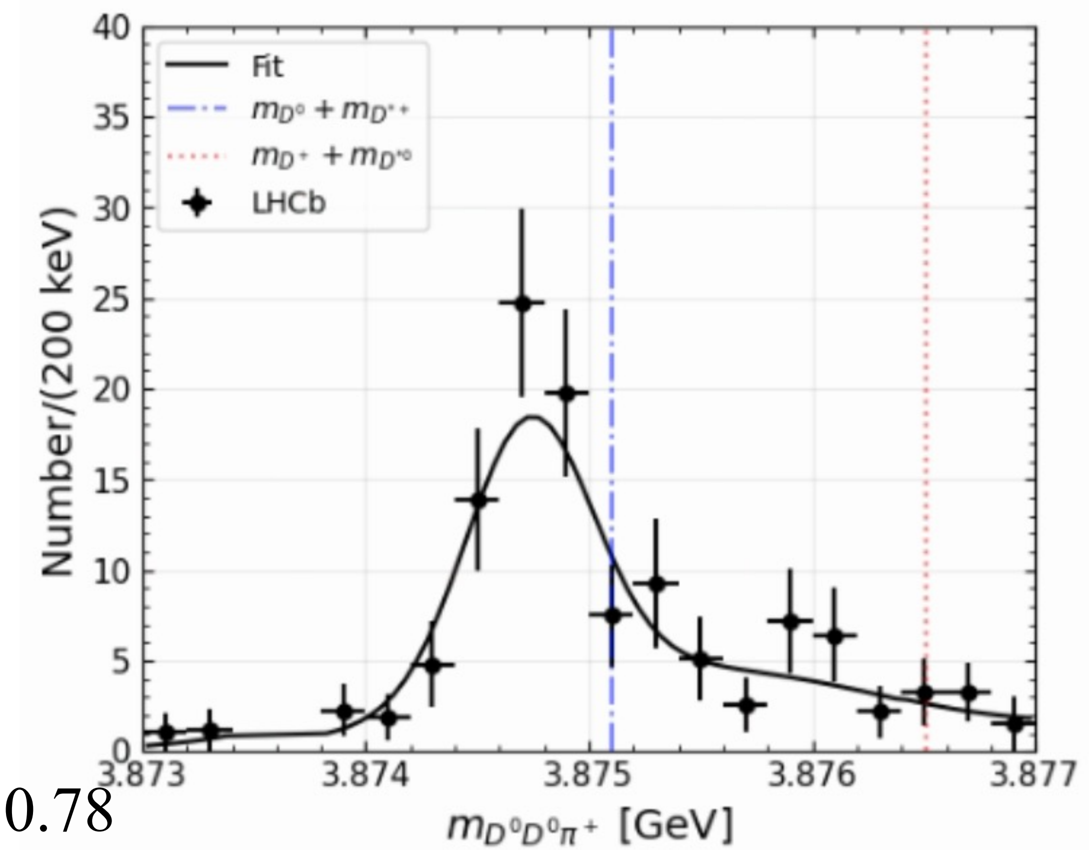


Fitting result

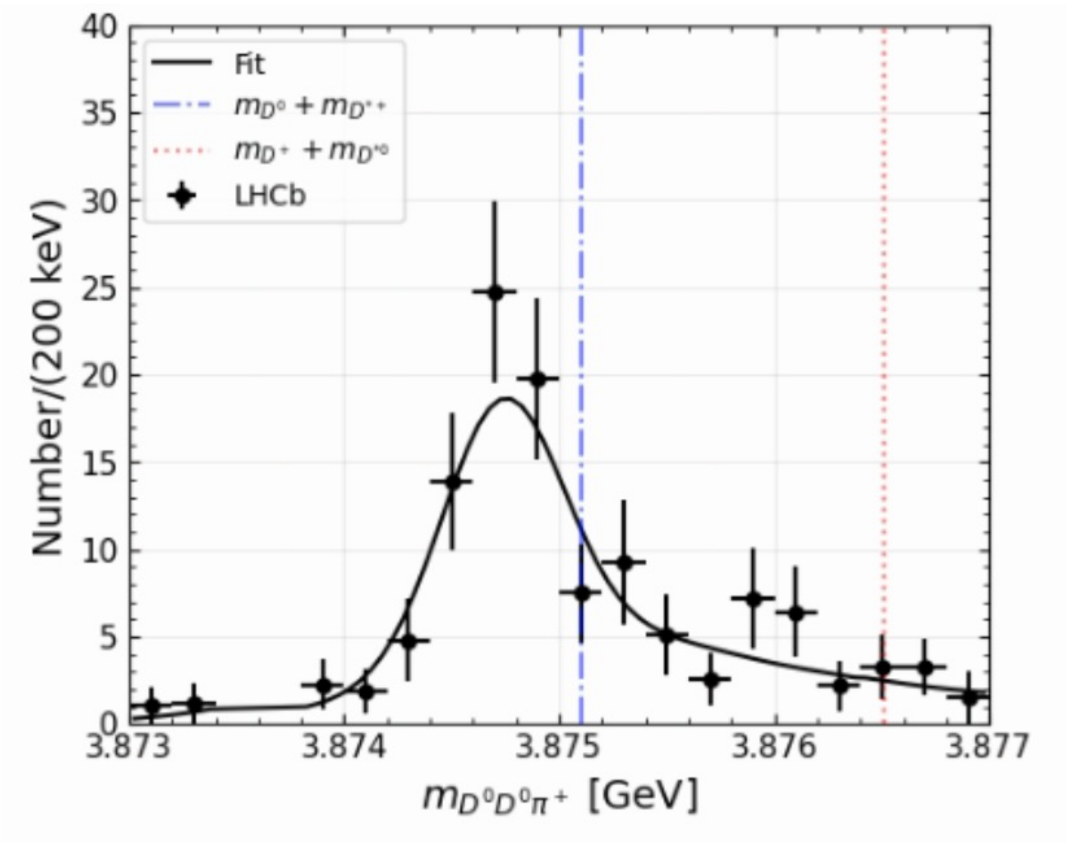
$\Lambda = 0.8$ GeV, χ^2/dof =0.76



$\Lambda = 1.0$ GeV, χ^2/dof =0.76



$\Lambda = 1.2$ GeV, χ^2/dof =0.78



Parameters

- Parameters consistent with those in one-boson-exchange model

Parameters	Λ(fixed)	λ	β
Best fit	0.8 GeV	0.890 ± 0.20	0.810 ± 0.11
Best fit	1 GeV	0.683 ± 0.025	0.687 ± 0.017
Best fit	1.2 GeV	0.587 ± 0.027	0.550 ± 0.027
Ref. [1]	1.17 GeV	0.56	0.9

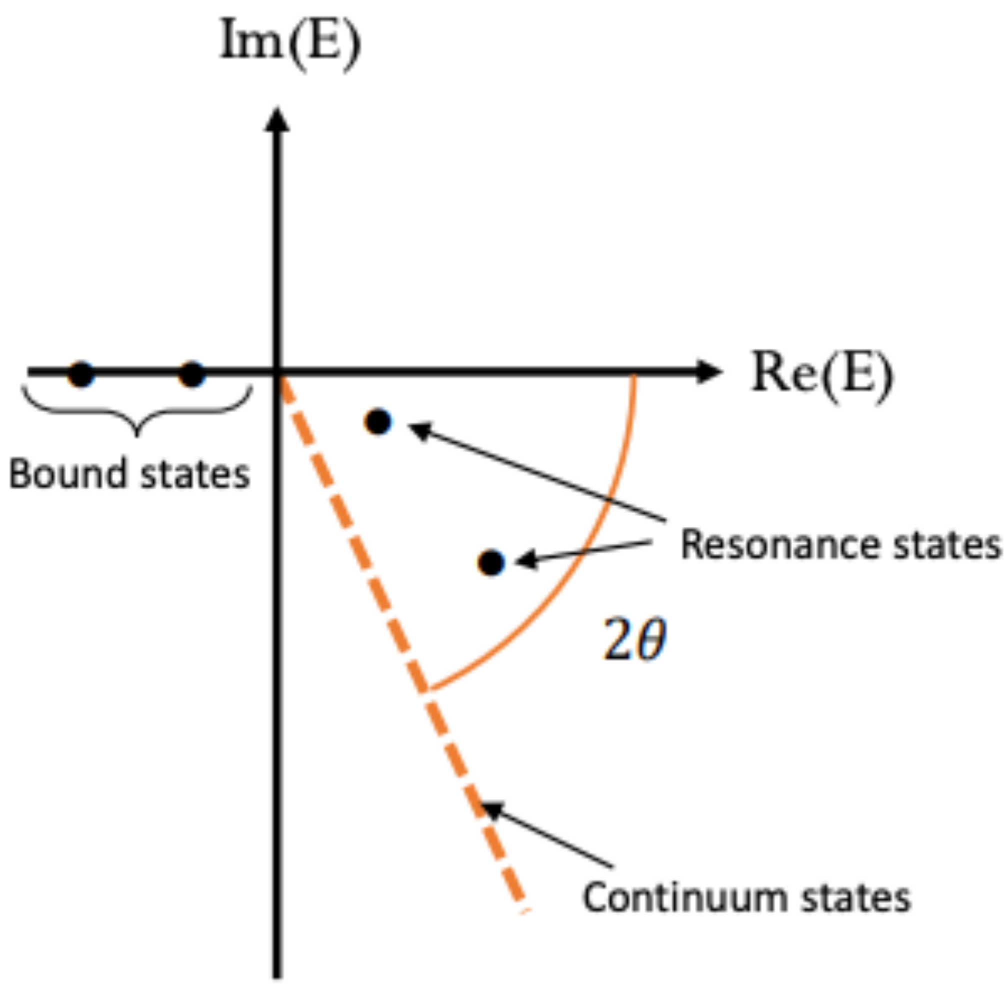
[1] Cheng, et al. Phys. Rev. D 106,016012 (2022).

Complex scaling method

The radius and momentum will rotate with an angle θ:

$$\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}, \quad \boldsymbol{q} \to \boldsymbol{q}e^{-i\theta}$$

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta, \qquad H_\theta = H(\mathbf{r}_\theta, \mathbf{q}_\theta) = \frac{q^2}{2u} e^{-2i\theta} + V(\mathbf{r}e^{i\theta}, \mathbf{q}e^{-i\theta})$$



S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, Physics reports 302, 212 (1998)

With the varying θ:

- the scattering states will rotate with 2θ
- while the bound and resonant states will stay stable

Results with $\Lambda = 0.8$ GeV

- Only one pole appears—bound states

  $m_{T_{cc}}$=3874.7 MeV, $\Delta E = -387.7$ keV
  $\Gamma_{T_{cc}} = 67.3$ keV

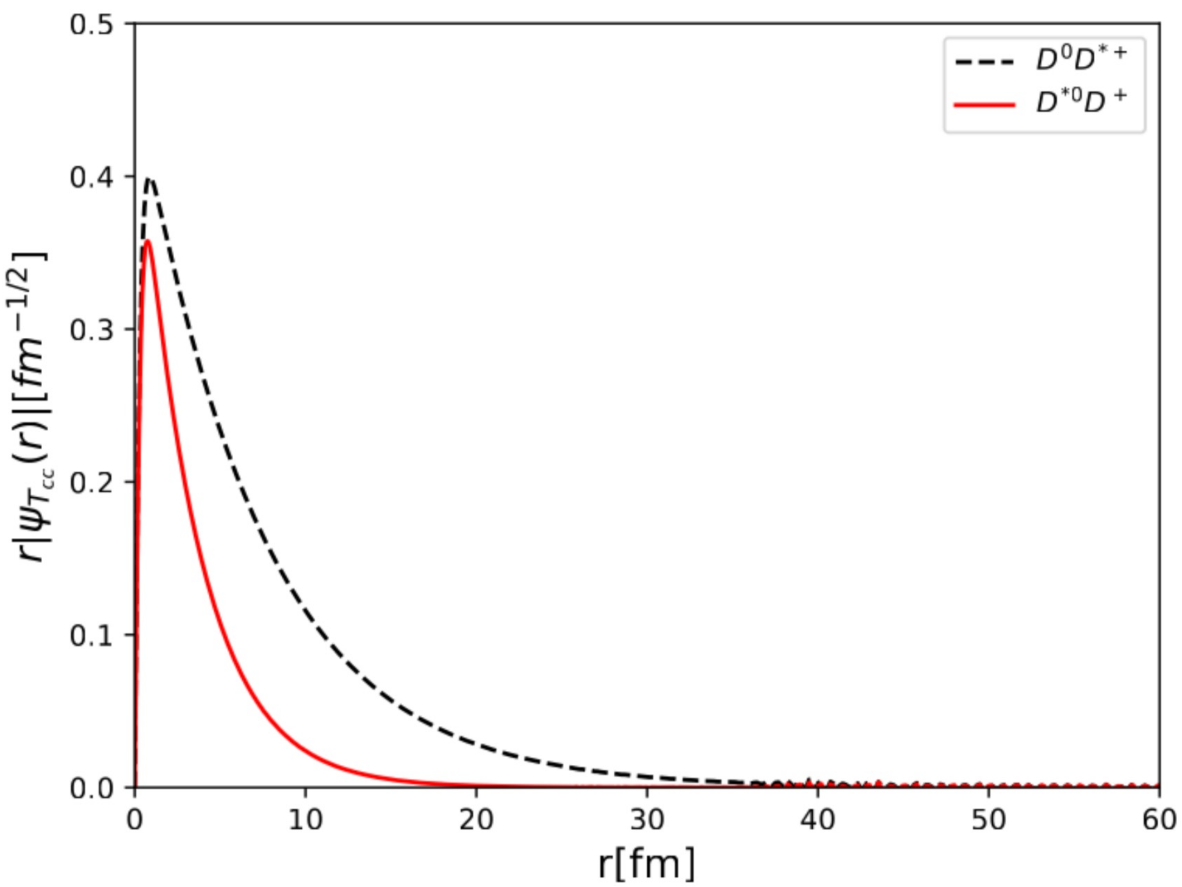
- $\sqrt{\langle r^2 \rangle} = 4.8\ fm$

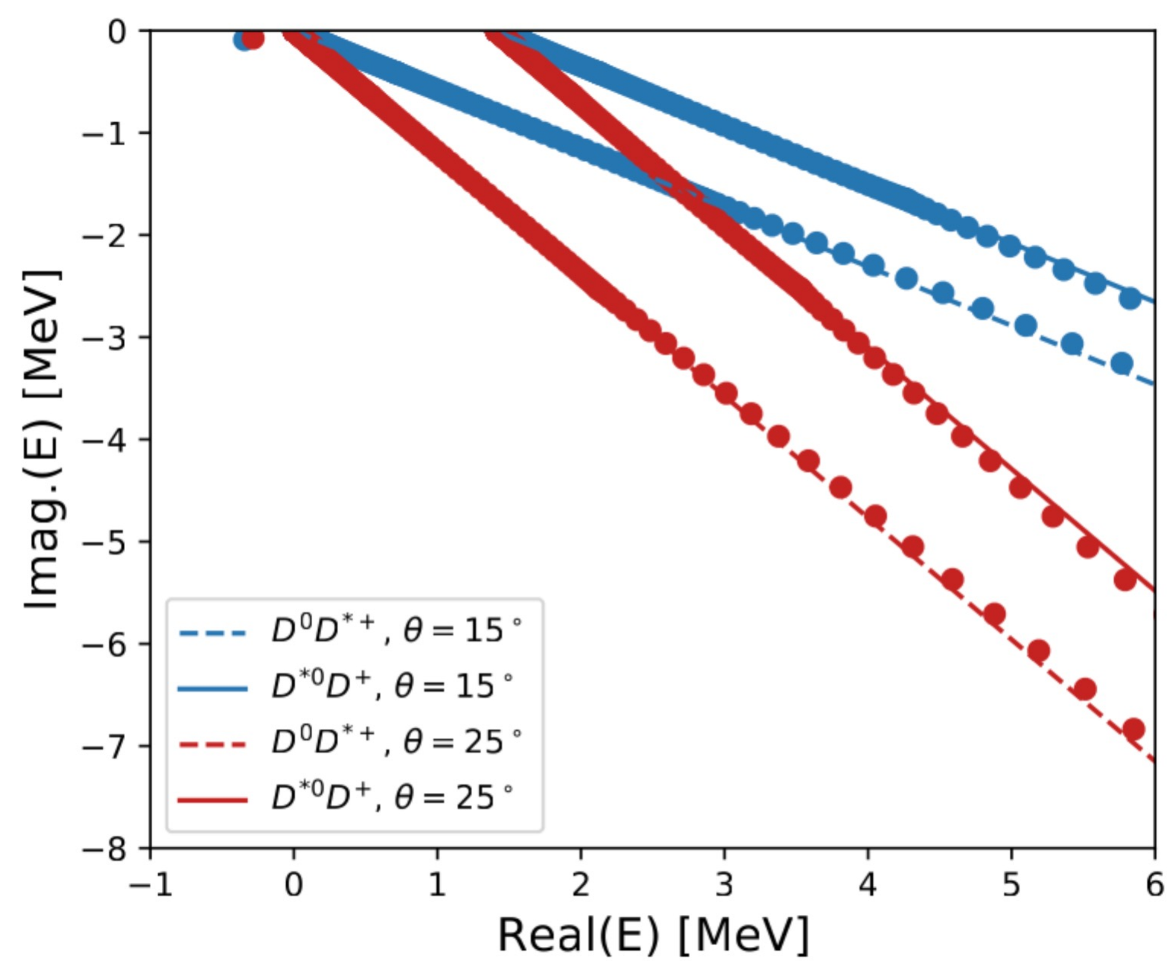
- 70.1% $D^{*+}D^0$, 30% D^+D^{*0} ⟺ 95.8%, DD$^*(I = 0)$ 4.2% DD$^*(I = 1)$

Mass differences of $D^{*+}D^0$ and D^+D^{*0}

$$[I=0] = \frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$[I=1] = \frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$





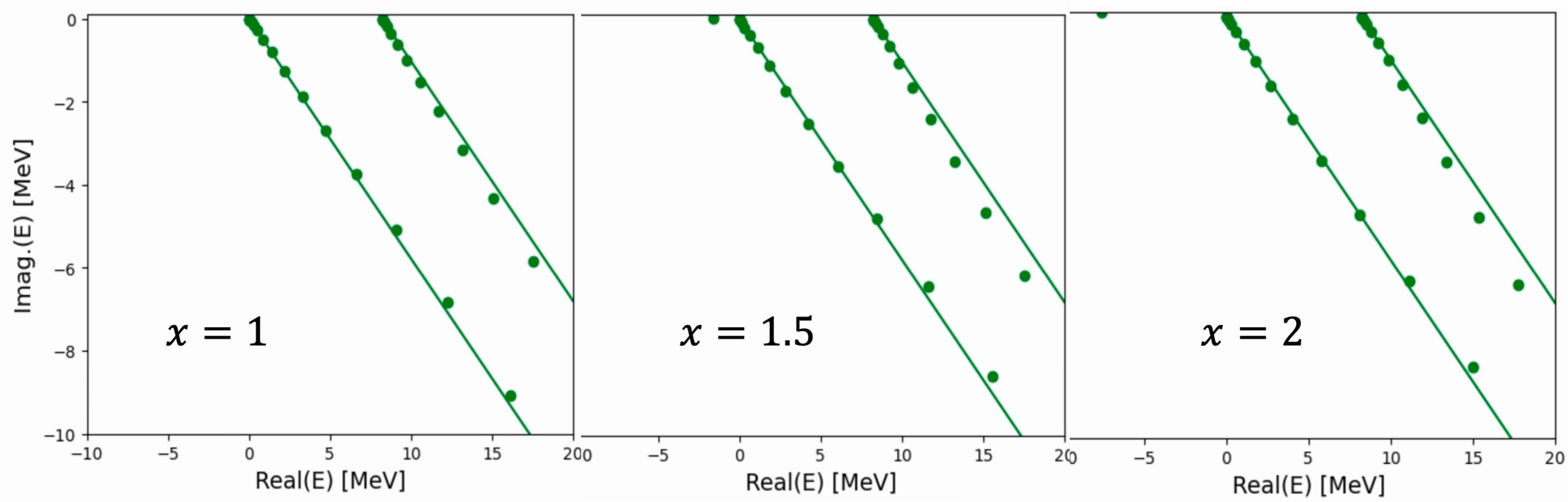
Results with three Λ

Λ (GeV)	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0D^{*+})$	$P(D^+D^{*0})$	$\frac{\mathrm{Res}(D^0D^{*+})}{\mathrm{Res}(D^+D^{*0})}$
0.8	-387.7	67.3	4.8 fm	95.8%	4.2%	70.0%	30.0%	$-1.063+0.001I$
1.0	-393.0	70.4	4.7 fm	95.8%	4.2%	70.0%	30.0%	$-1.055+0.001I$
1.2	-391.6	72.7	4.7 fm	95.7%	4.3%	70.3%	29.7%	$-1.052+0.001I$

- The conclusion remains the same using the three different cutoff values.
- The binding energy of the bound state is around $\Delta E \sim -390$keV, which is consistent with that of the measurement $(\Delta E_{\mathrm{exp}} = -360(40)\mathrm{keV})$. LHCb, Nature Commun. 13 (2022) 1, 3351

Direct application to $D\bar{D}^*$: $X(3872)$

- Without the $c\bar{c}$ core, there are no bound states.
- $V'_{D\bar{D}^*} = x * V_{D\bar{D}^*}$



$D\bar{D}^*$ interaction is attractive but not strong enough to form a bound state. ➡ ***Inclusion of $c\bar{c}$ core***

$X(3872): D\bar{D}^* + c\bar{c}$

- The $D\bar{D}^*$ system with quantum number $I(J^{PC}) = 0(1^{++})$ can couple with the $\chi_{c1}(2P)$.
- The coupled channel effect between them can be described by the quark-pair-creation model:

$$g_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|) = \gamma I_{D\bar{D}^*,c\bar{c}}(|\vec{k}_{D\bar{D}^*}|)$$

where $\vec{k}_{D\bar{D}^*}$ is the relative momentum in the $D\bar{D}^*$ channel.

$I_{D\bar{D}^*,cc}(|\vec{k}_{D\bar{D}^*}|)$ is the overlap of the meson wave functions ← GI quark model

- γ is determined to reproduce the $\psi(3770)$:

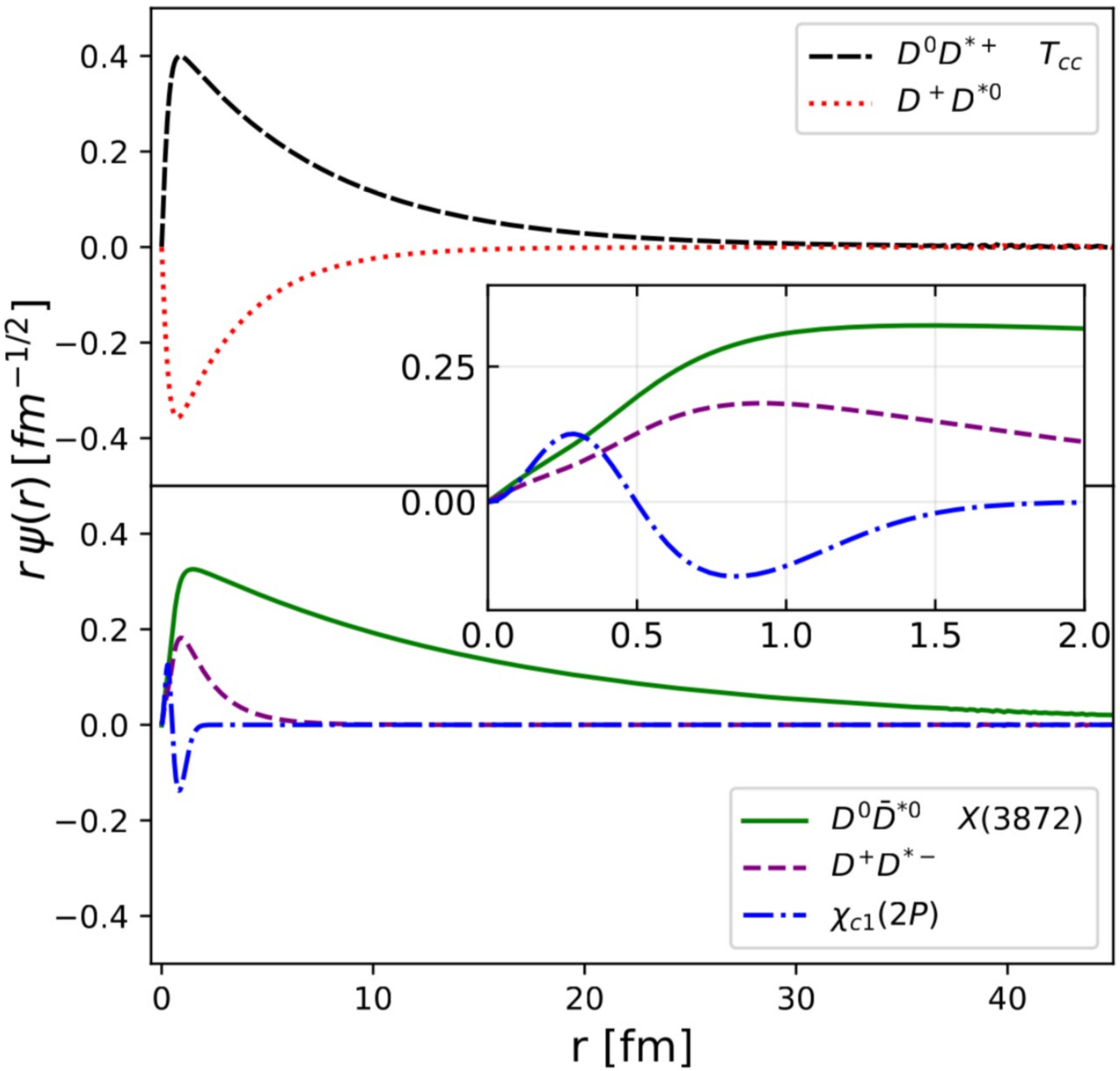
$$\gamma = 4.69$$

- The the X(3872) can be obtained:

$X(3872)$	BE (keV)	Γ (keV)	$\sqrt{\langle r^2 \rangle}$	$I=0$	$I=1$	$P(D^0\bar{D}^{*0})$	$P(D^+D^{*-})$	$P(c\bar{c})$
	-80.4	32.5	11.2 fm	71.9%	28.1%	94.0%	4.8%	1.2%

Direct application to $D\bar{D}^*$: X(3872)

Wave functions of Tcc and X(3872)



- Long tails for the radius distribution.
- $X(3872)$ has a even longer tails than T_{cc}

✓ $r < 2$ fm, $c\bar{c} + \bar{D}D^*$ are important.

✓ $r < 0.5$ fm, $c\bar{c}$ core dominates.

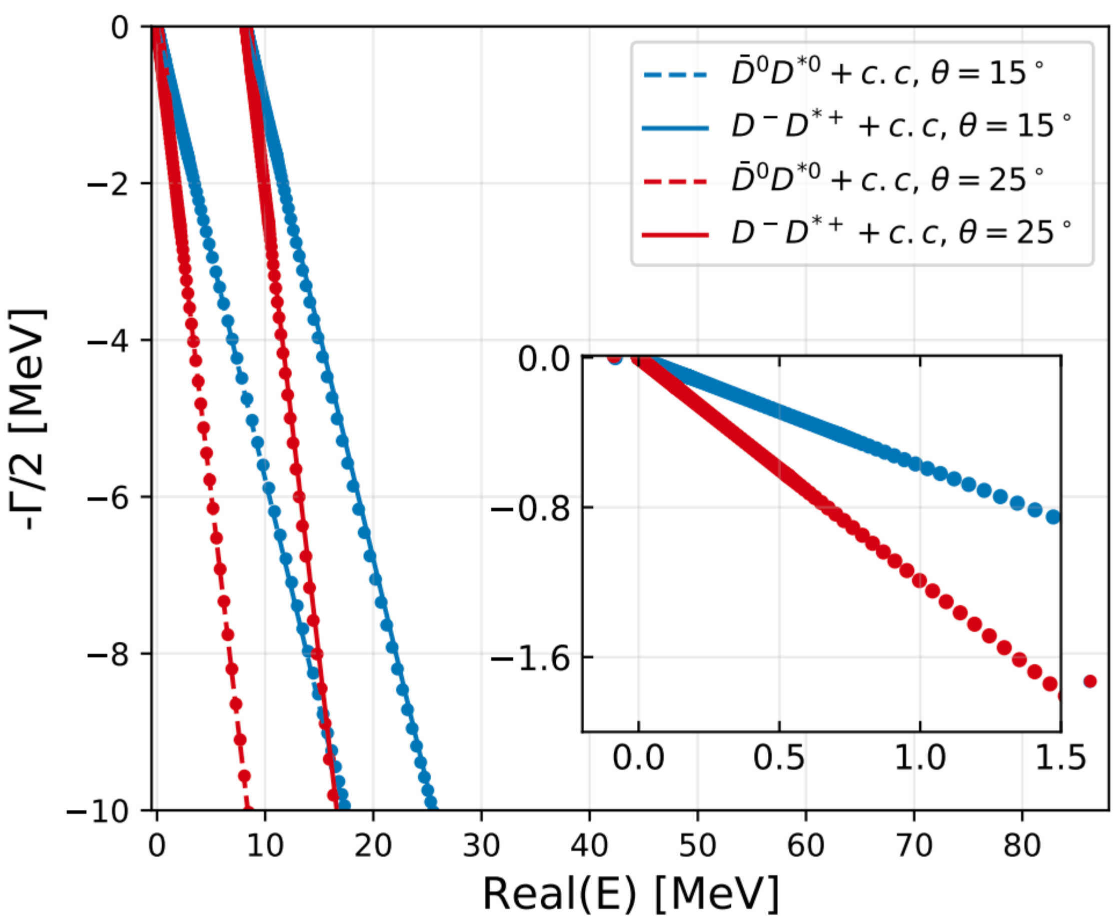
✓ $D\bar{D}^*$ plays the dominant role in the long-distance region, which contributes to $\sqrt{\langle r^2 \rangle}$.

Direct application to $D\bar{D}^*$: Candidate for X(3940)?

- Besides the $X(3872)$, we also find a signal of the resonant state $\chi_{c1}(2P)$ with

$$M = 3957.9\text{MeV},\ \Gamma = 16.7\text{MeV},$$

which might be related to the $X(3940)$ observed in the $D\bar{D}^*$ channel.



$X(3940)$ MASS	3942 ± 9 MeV
$X(3940)$ WIDTH	37^{+27}_{-17} MeV

$X(3940)$ **Decay Modes**

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D\overline{D}^*$ + c.c.	seen

Compare with the lattice results

Ours: $\chi_{c1}(2P) \rightarrow M = 3957.9\text{MeV}$

Haozheng Li et al, arXiv: 2402.14541

m_π(MeV)	250(3)	307(2)	362(1)	417(1)
m_R(MeV)	3924(5)	3926(6)	3969(4)	3995(4)
Γ_R(MeV)	63(23)	57(18)	37(13)	57(10)

$X \approx 1$ and indicates a predominant $D\bar{D}^*$ component. This state may correspond to $X(3872)$. On the other hand, our results of the finite volume energies also hint at the existence of a 1^{++} resonance below 4.0 GeV with a width around 60 MeV.

Compare with the experimental results

Ours: virtual state with 1^{+-} and $M = 3870.2$ MeV

COMPASS: $\tilde{X}(3872)$ with $M = 3860.0 \pm 10.4$ MeV   COMPASS, PLB783,334

Ours: $h_c(2P) \rightarrow \; M = 3961.3$MeV

$\chi_{c1}(2P) \rightarrow M = 3957.9$MeV

LHCb, arXiv:2406.03156

This work		Known states [6]		$c\bar{c}$ prediction [34]	
$\eta_c(3945)$	$J^{PC} = 0^{-+}$	$X(3940)$ [9, 10]	$J^{PC} = ?^{??}$	$\eta_c(3S)$	$J^{PC} = 0^{-+}$
$m_0 = 3945\,^{+28}_{-17}\,^{+37}_{-28}$	$\Gamma_0 = 130\,^{+92}_{-49}\,^{+101}_{-70}$	$m_0 = 3942 \pm 9$	$\Gamma_0 = 37\,^{+27}_{-17}$	$m_0 = 4064$	$\Gamma_0 = 80$
$h_c(4000)$	$J^{PC} = 1^{+-}$	$T_{c\bar{c}}(4020)^0$ [35]	$J^{PC} = ?^{?-}$	$h_c(2P)$	$J^{PC} = 1^{+-}$
$m_0 = 4000\,^{+17}_{-14}\,^{+29}_{-22}$	$\Gamma_0 = 184\,^{+71}_{-45}\,^{+97}_{-61}$	$m_0 = 4025.5\,^{+2.0}_{-4.7} \pm 3.1$	$\Gamma_0 = 23.0 \pm 6.0 \pm 1.0$	$m_0 = 3956$	$\Gamma_0 = 87$
$\chi_{c1}(4010)$	$J^{PC} = 1^{++}$			$\chi_{c1}(2P)$	$J^{PC} = 1^{++}$
$m_0 = 4012.5\,^{+3.6}_{-3.9}\,^{+4.1}_{-3.7}$	$\Gamma_0 = 62.7\,^{+7.0}_{-6.4}\,^{+6.4}_{-6.6}$			$m_0 = 3953$	$\Gamma_0 = 165$

Summary

- Tcc is used to fix the $\overline{\boldsymbol{D}^*}\boldsymbol{D}$ interactions in $X(3872)$.
- Short-range interactions and structures of $X(3872)$ should be studied by considering the $c\bar{c}$ core.
- What important role the $c\bar{c}$ core can play in the production and decay of the $X(3872)$?

Thank you !