



# 轻子普适性理论研究

袁兴博

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see also PeiLian Li, Pei-Rong Li, Chengping Shen, Liang Sun's talks

# Introduction

## Lepton Flavour Universality Test

$\pi, K, \tau$  system,  $W, Z$  boson,  $R_{D^{(*)}}$  and  $R_{K^{(*)}}$

## Implications for New Physics

Connections to other anomalies  $\left\{ \begin{array}{l} (g-2)_\mu \text{ anomaly} \\ B^+ \rightarrow K^+ \nu \bar{\nu} \text{ excess at Belle II} \\ \text{Cabibbo angle anomaly} \end{array} \right.$

Origin of LFU violation

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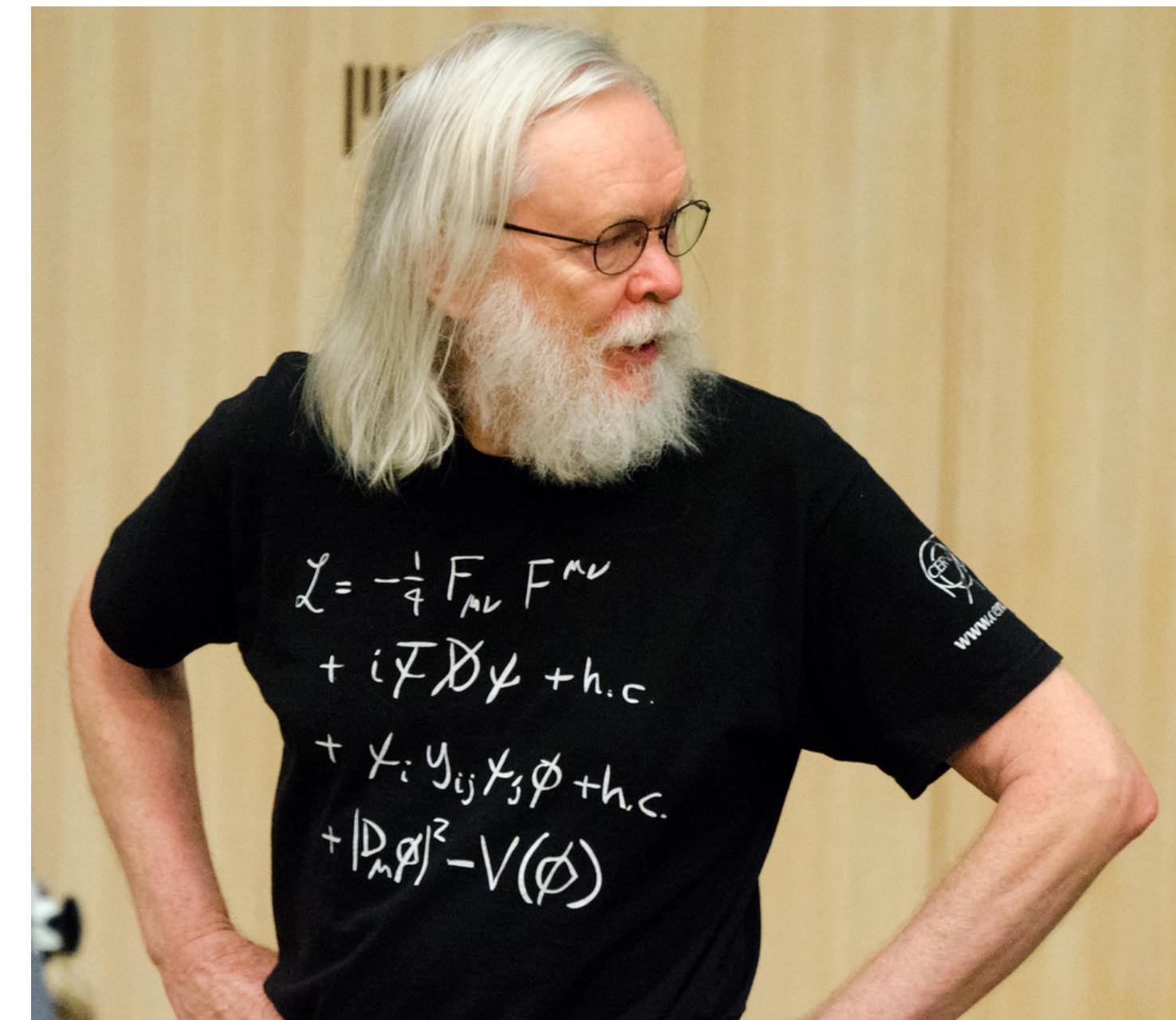
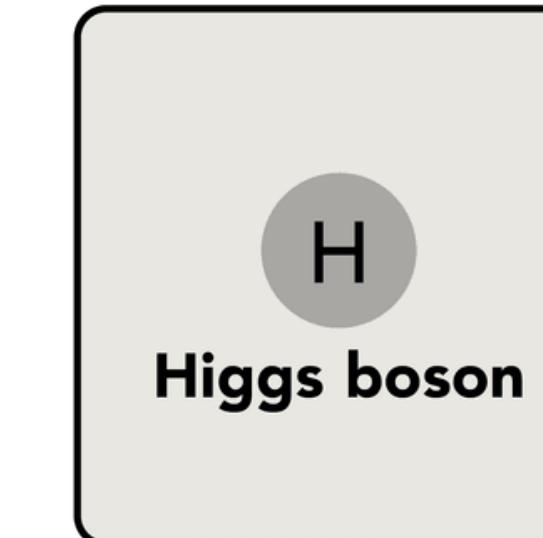
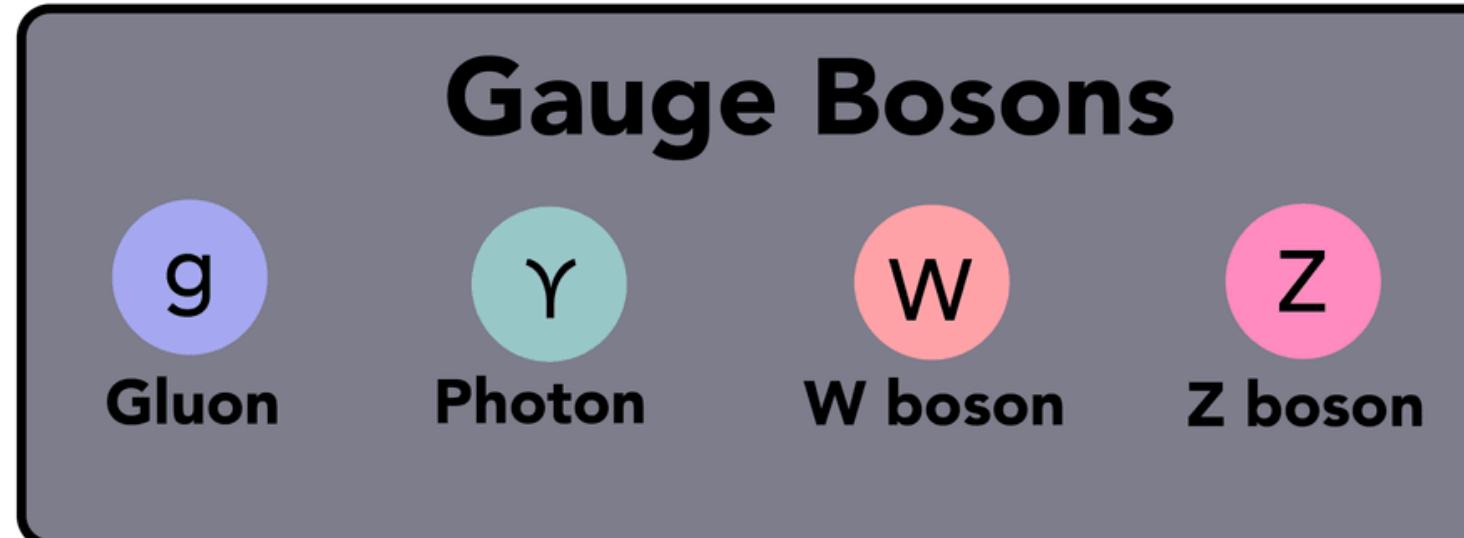
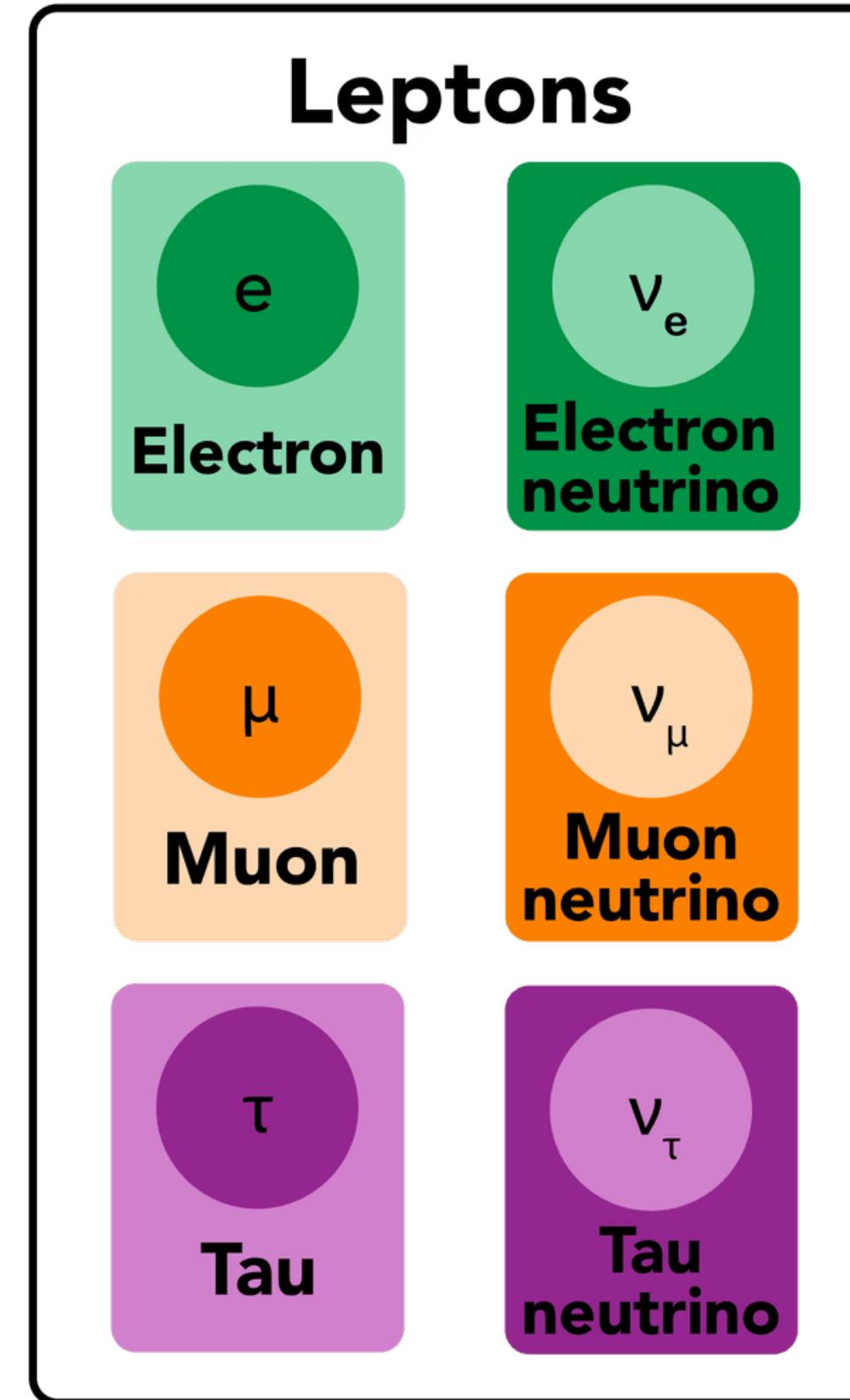
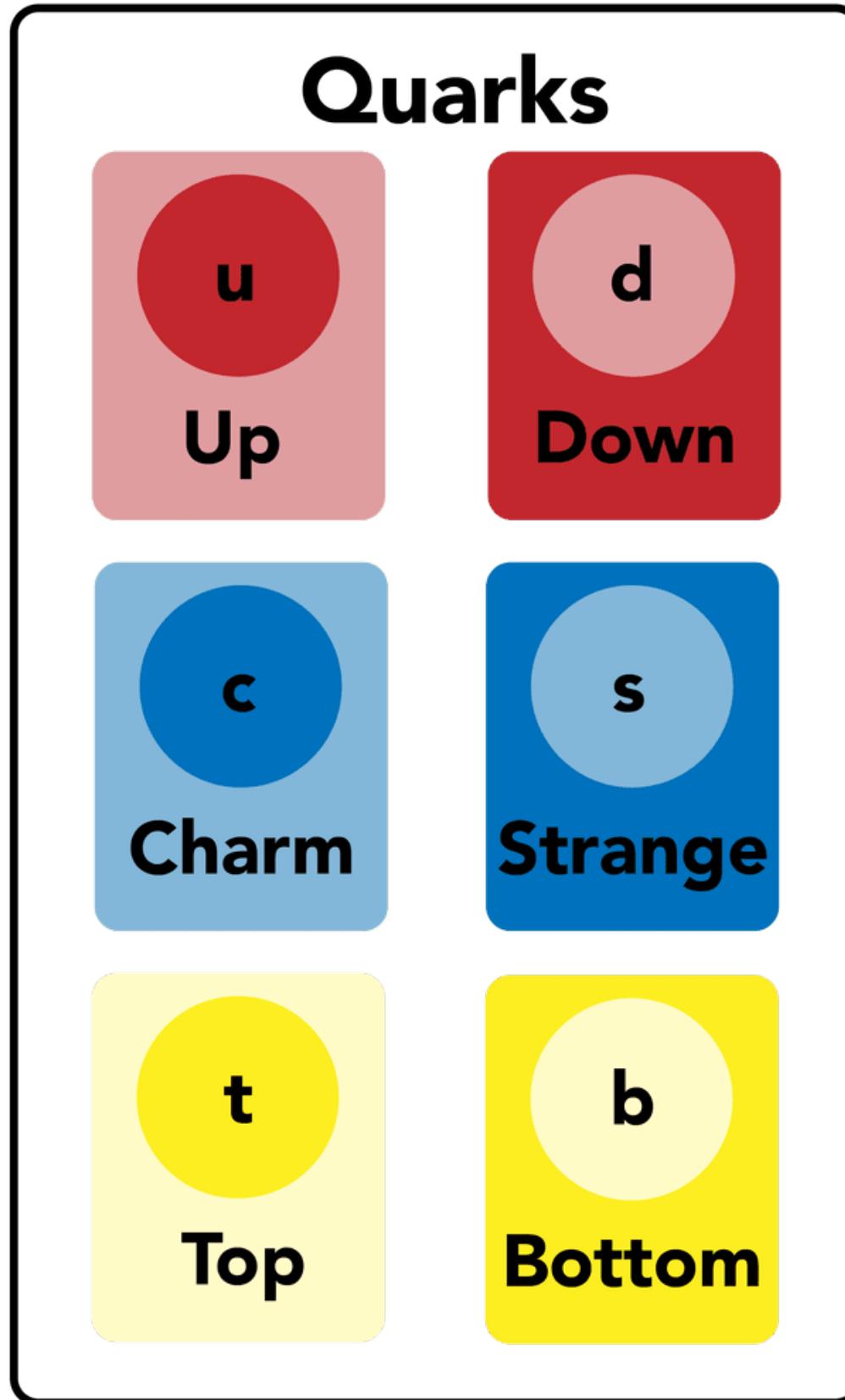
Connections to other anomalies  $\left\{ \begin{array}{l} (g-2)_\mu \text{ anomaly} \\ B^+ \rightarrow K^+ \nu \bar{\nu} \text{ excess at Belle II} \\ \text{Cabibbo angle anomaly} \end{array} \right.$

Origin of LFU violation

## Summary

# Standard Model of Particle Physics

Gauge theory based on  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$



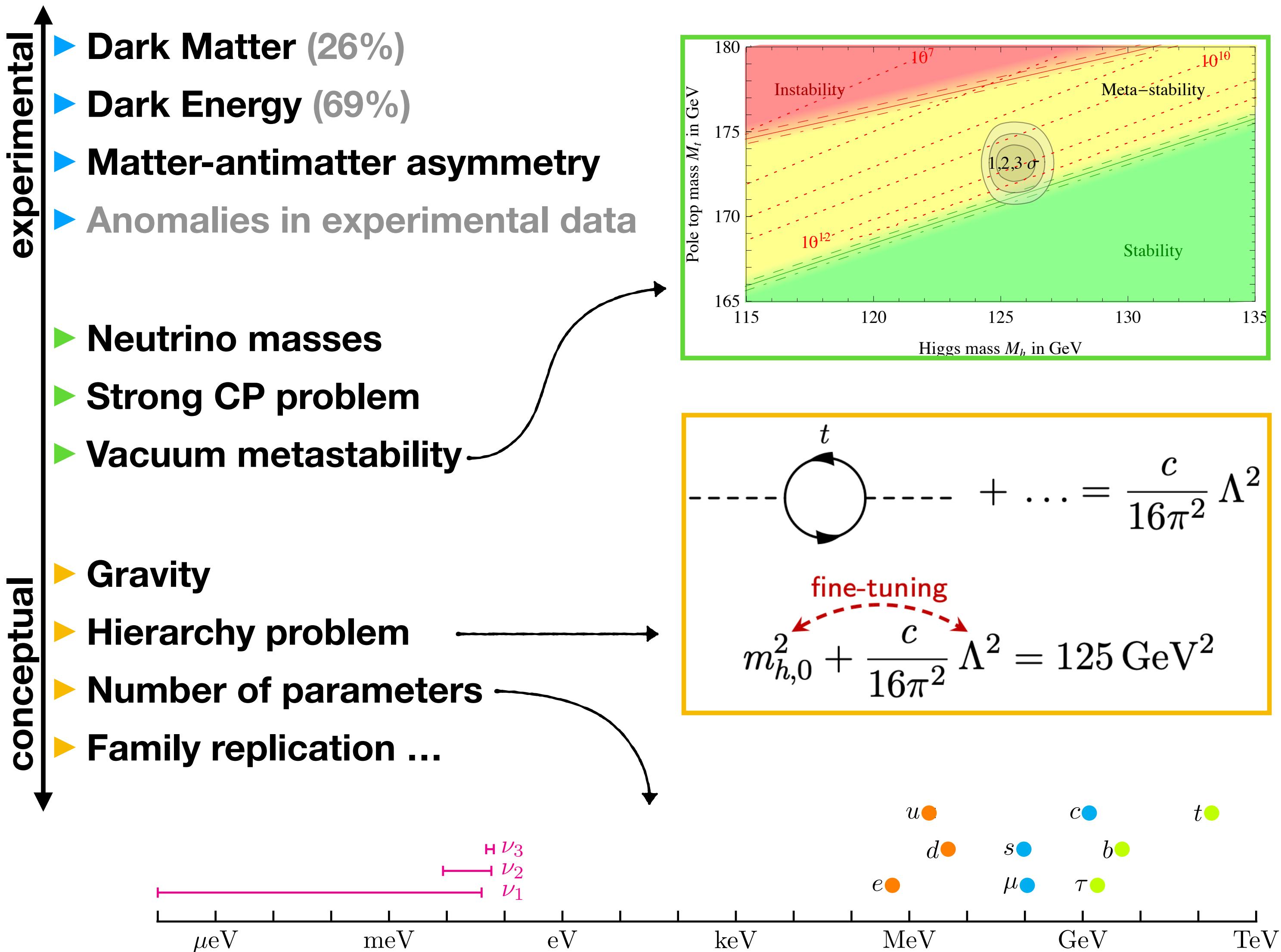
John Ellis



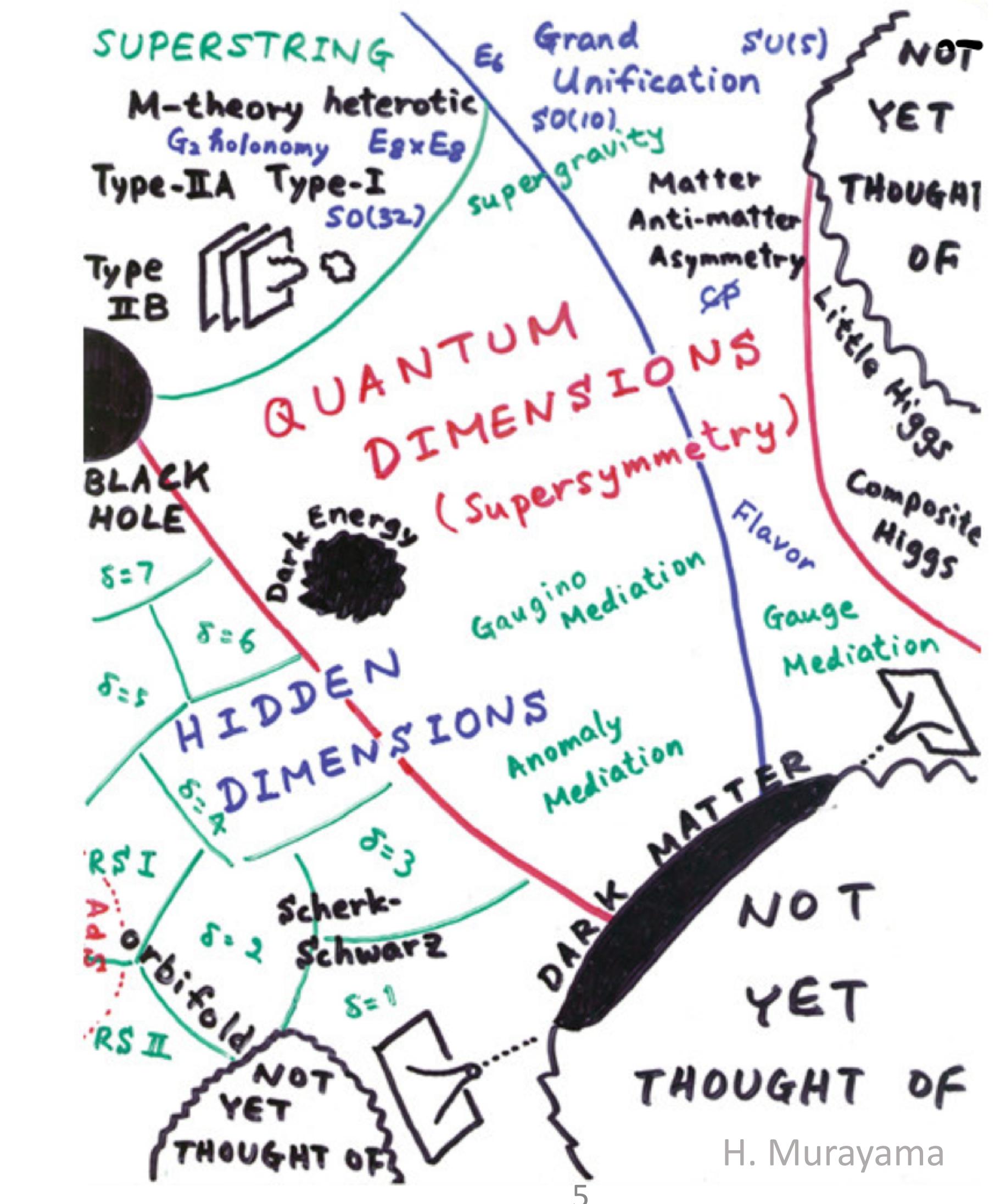
François Englert and Peter Higgs  
2013 Nobel

# Physics beyond the Standard Model

Problems of the SM  $\implies$  Physics beyond the SM

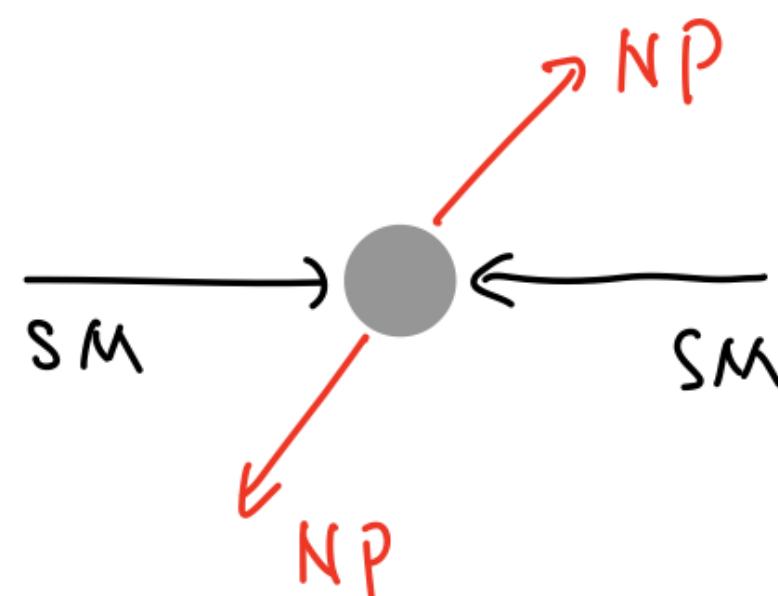


Theories beyond the SM



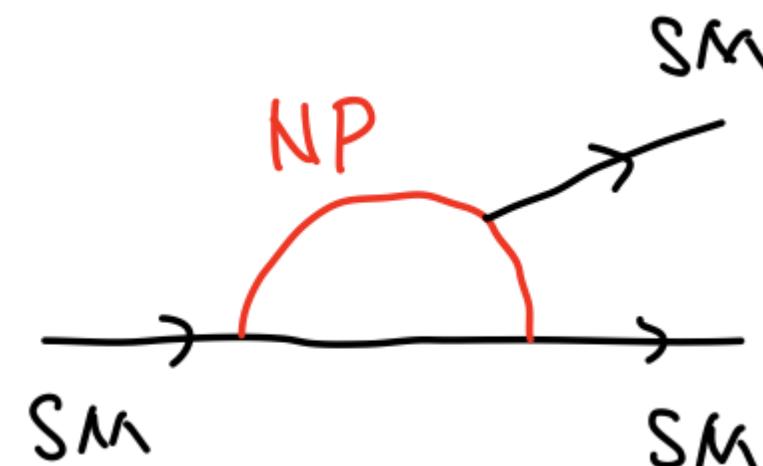
# Experimental Searches

# Direct Searches



- ▶ **energy frontier**
  - ▶ **almost model independent**
  - ▶ **probe  $m_{\text{NP}} < E_{\text{collider}}$**

# Indirect Searches

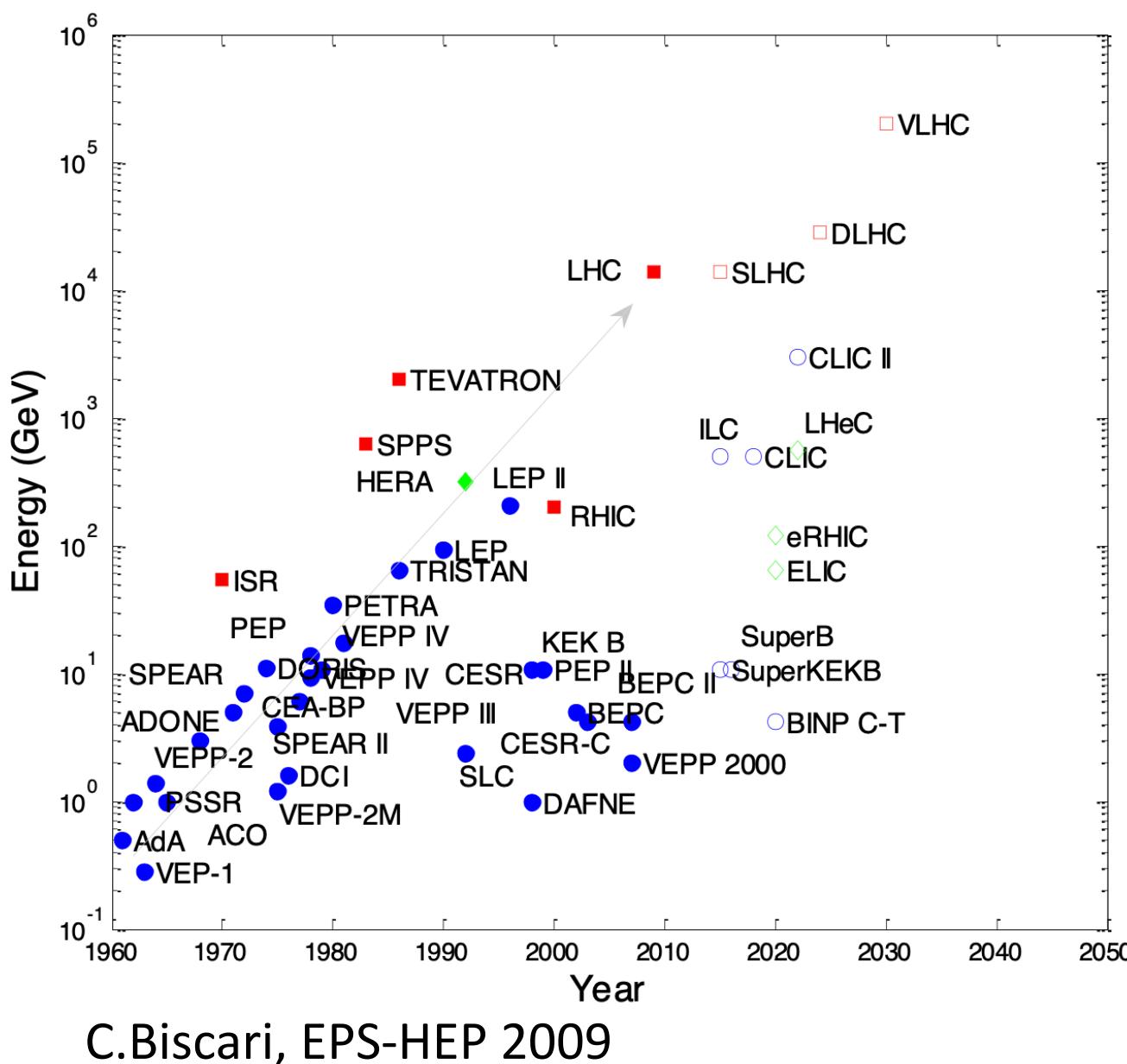


- ▶ precision (intensity) frontier
  - ▶ model dependent
  - ▶ probe very large  $m_{\text{NP}}$

# 上一代对撞机(1983–2011)



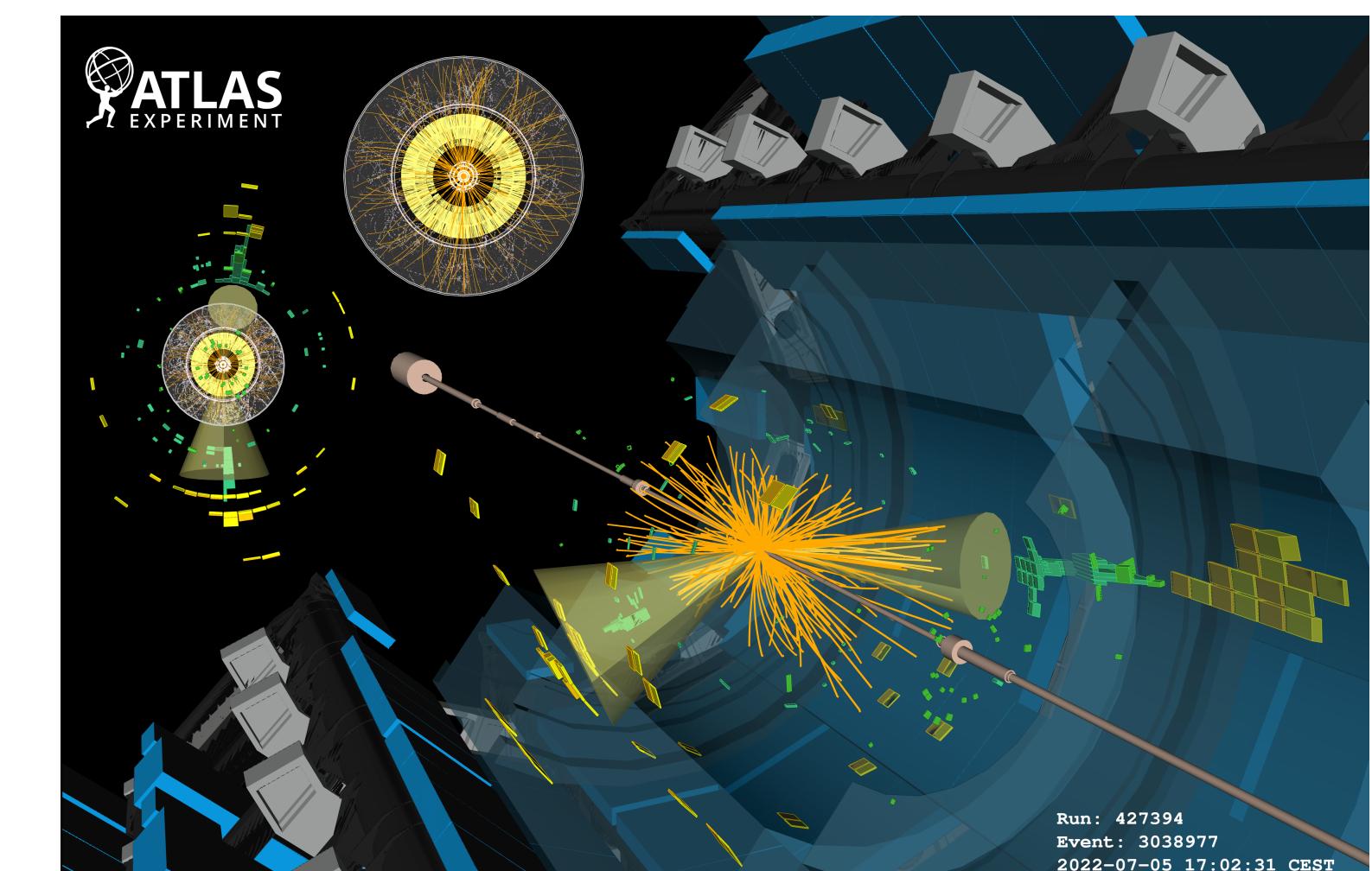
# Fermilab的Tevatron (1983–2011, 2TeV)



## 当前的对撞机



# CERN的大型强子对撞机(LHC, 14TeV)

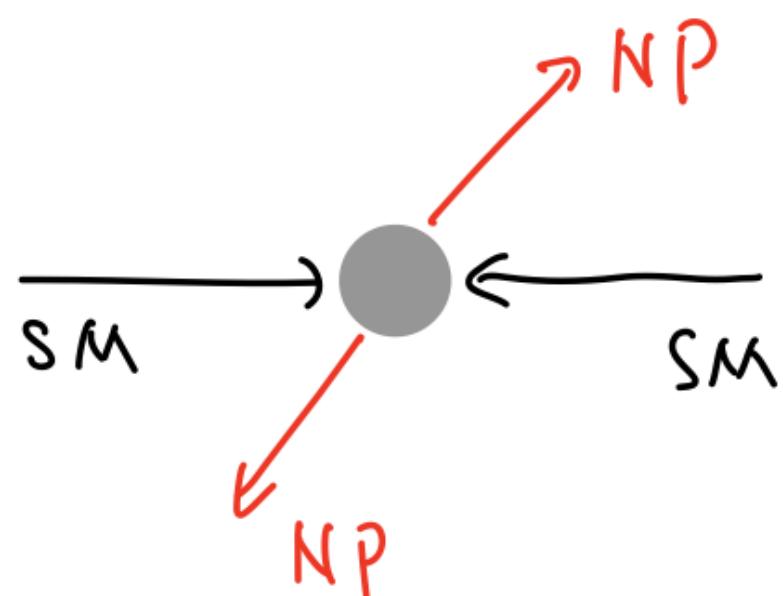


22年7月5日，ATLAS探测器中的1个 $pp$ 对撞事例

# Experimental Searches

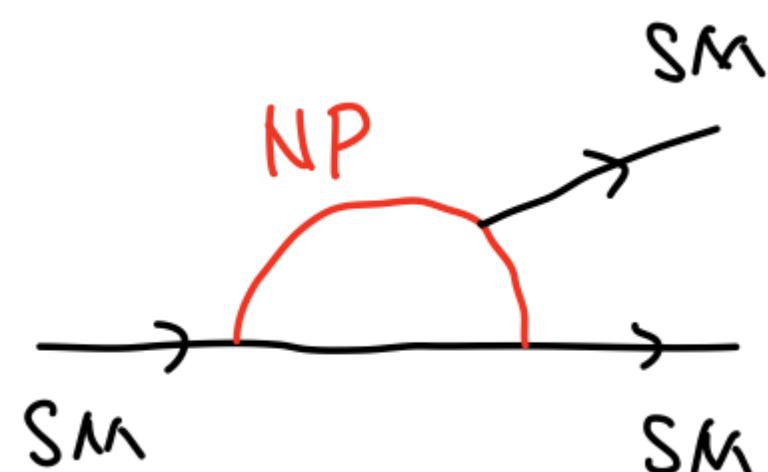
上一代对撞机

## Direct Searches

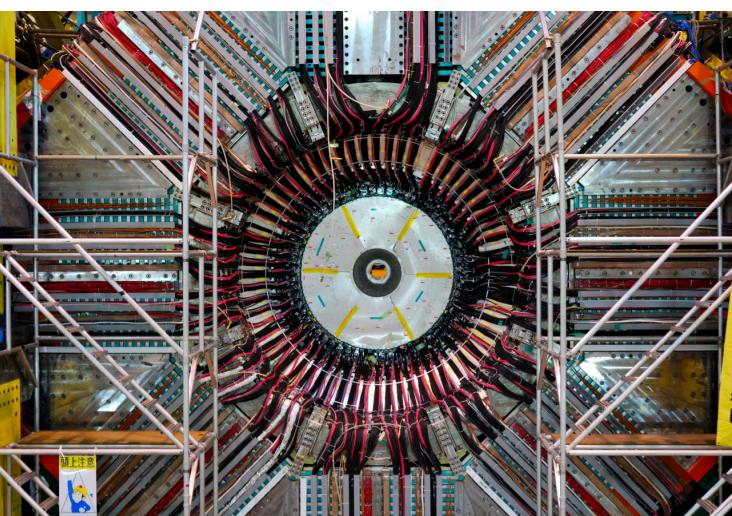


- ▶ energy frontier
- ▶ almost model independent
- ▶ probe  $m_{\text{NP}} < E_{\text{collider}}$

## Indirect Searches



- ▶ precision (intensity) frontier
- ▶ model dependent
- ▶ probe very large  $m_{\text{NP}}$
- ▶ flavour physics



日本高能加速器上的Belle (1999–2010)



美国斯坦福直线加速器上的BaBar (1999–2008)

- ▶ 共产生约 $10^9 B\bar{B}$  事例
- ▶ 证实了SM中CP破缺的KM机制

Nobel Prize 2008 for

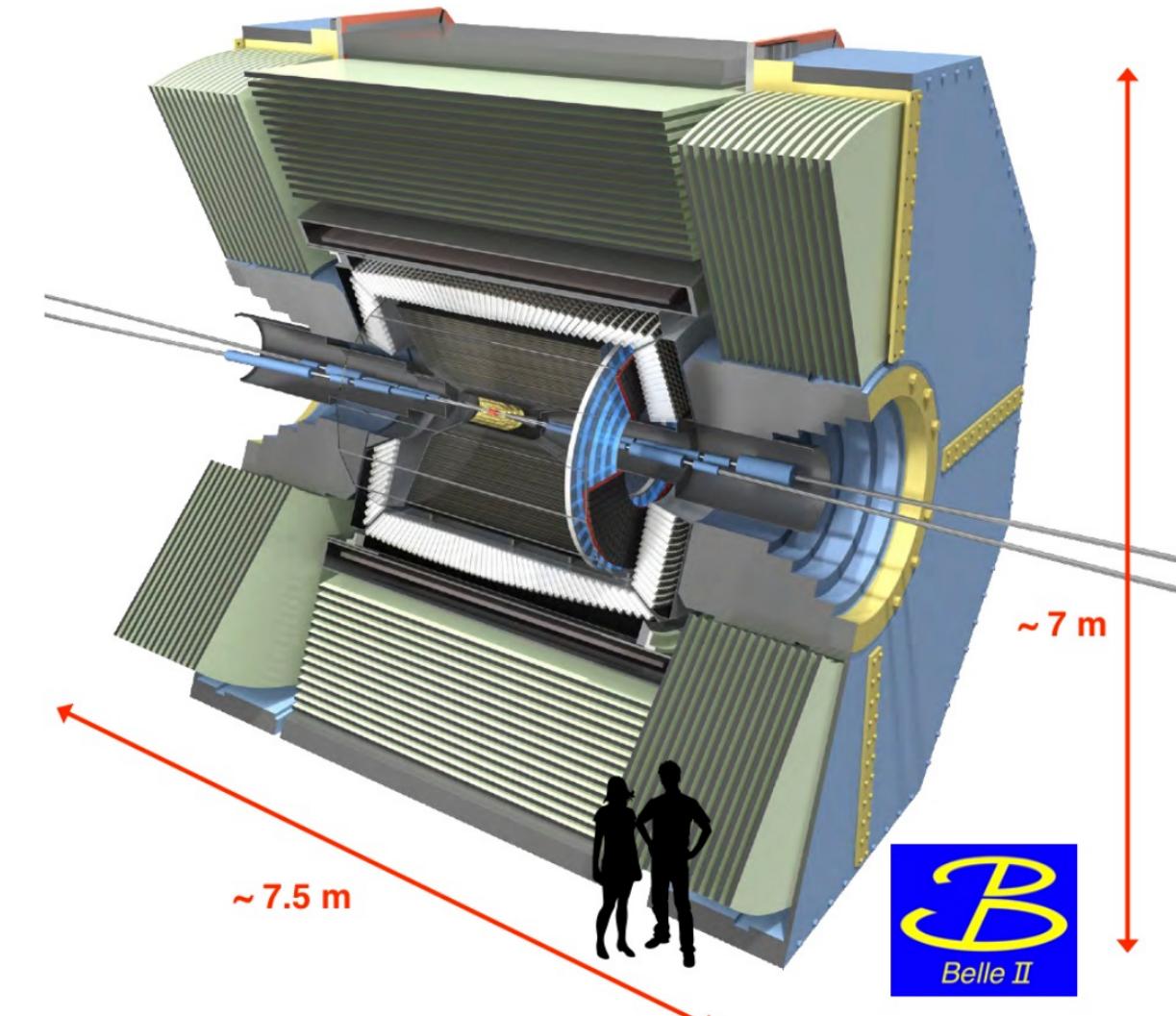


Makoto  
Kobayashi

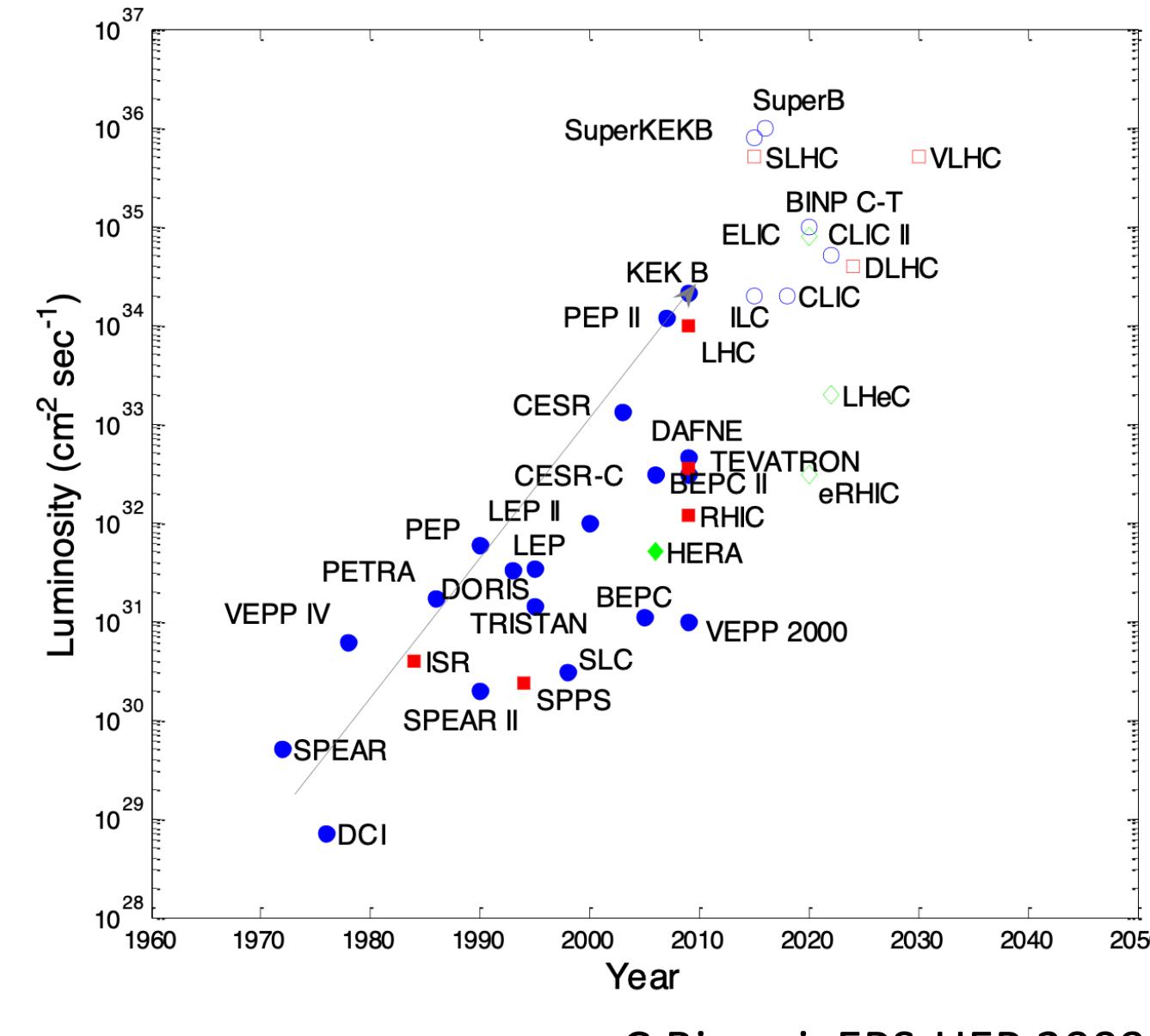


Toshihide  
Maskawa

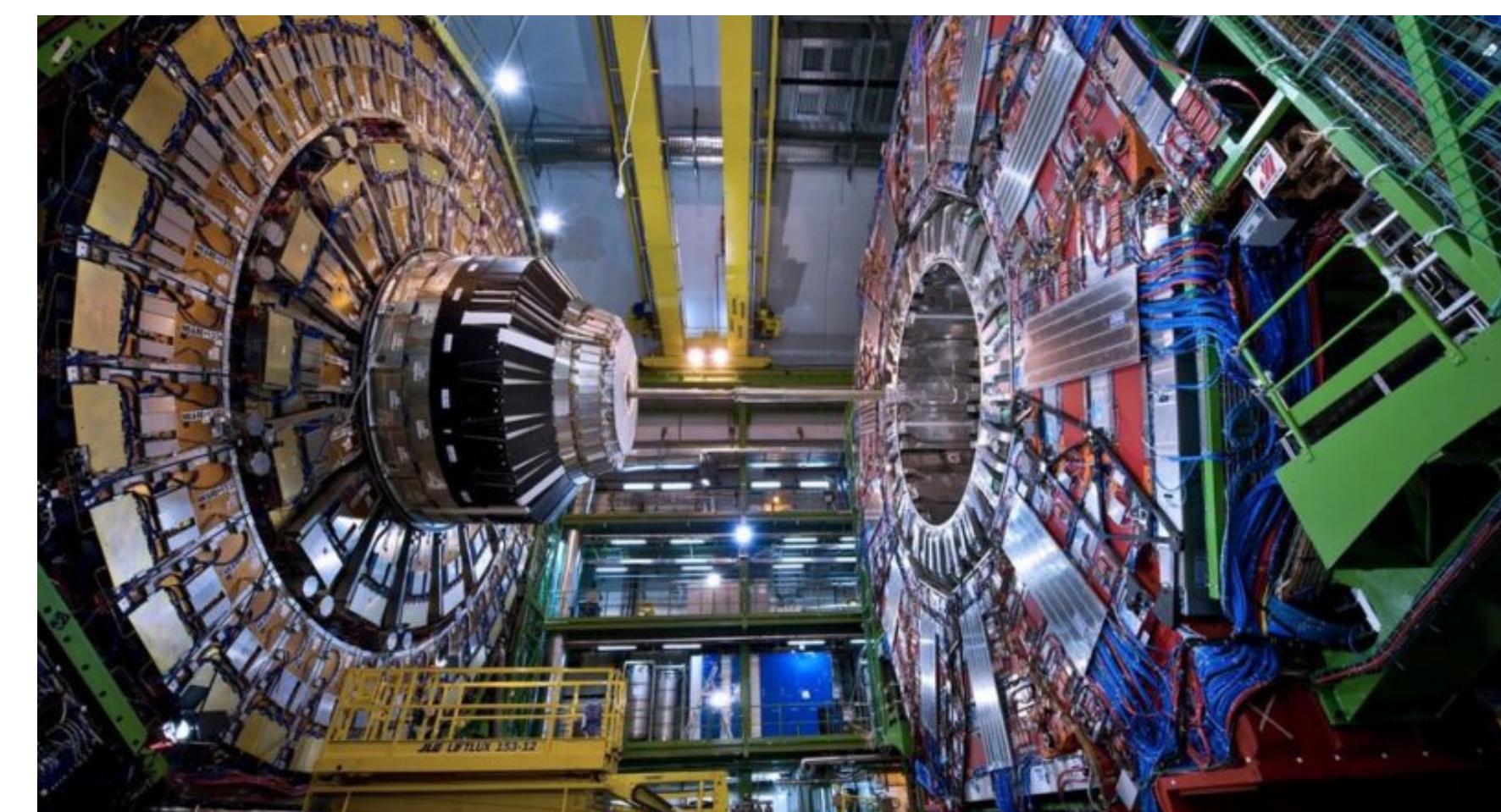
当前的对撞机



位于日本的超级B工厂



C.Biscari, EPS-HEP 2009



LHC上的LHCb实验

# Flavour Physics

## ► Flavour universal

- ▶ couplings  $\propto \delta_{ij}$  in flavour space
- ▶ example: strong and electromagnetic interactions
- ▶ consequence of gauge invariance

## ► Flavour diagonal

- ▶ couplings  $\propto \lambda_i \delta_{ij}$  (diagonal, but not necessarily universal)
- ▶ example: Yukawa interactions

## ► Flavour violation (changing)

- ▶ couplings involve different quarks
- ▶ no flavour violation in lepton sector ( $m_\nu = 0$ )
- ▶ example:  $W^\pm$  interactions in quark section

## ► Flavour Changing Neutral Current (FCNC)

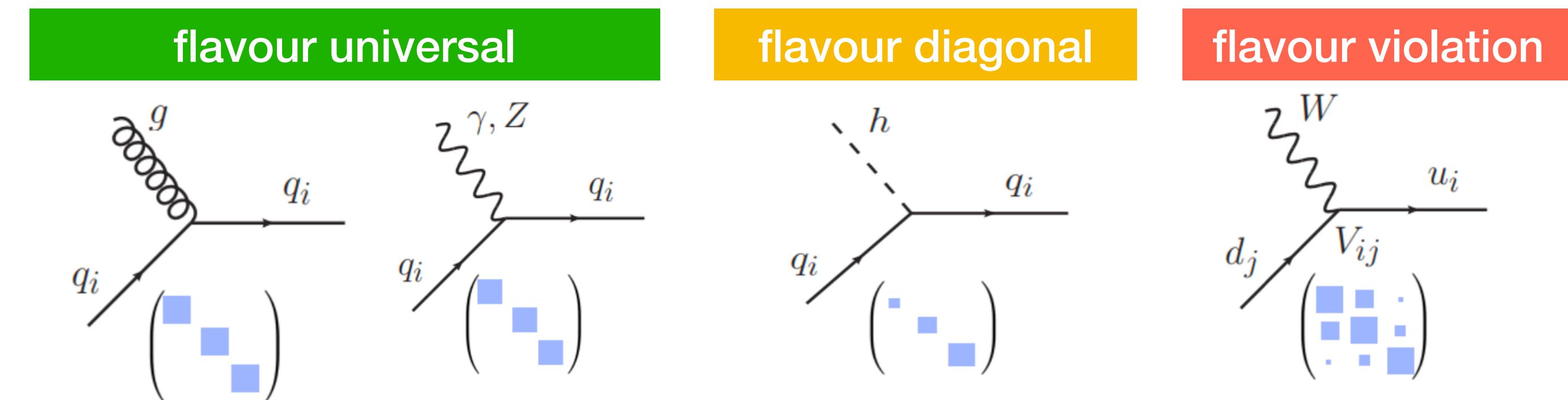
- ▶ absent at the tree-level
- ▶ arise at the one-loop, but suppressed by GIM mechanism

## ► Why flavour physics

- ▶ New physics  $\Leftarrow \mathcal{O}(10^9) B\bar{B}$  events at BaBar and Belle
- ▶ structure of CKM and mass
- ▶ CP violation
- ▶ strong interaction

experimental status  
no evidence of NP  
but, anomalies

$> 5\sigma$   
 $2 \sim 4\sigma$



## CKM matrix

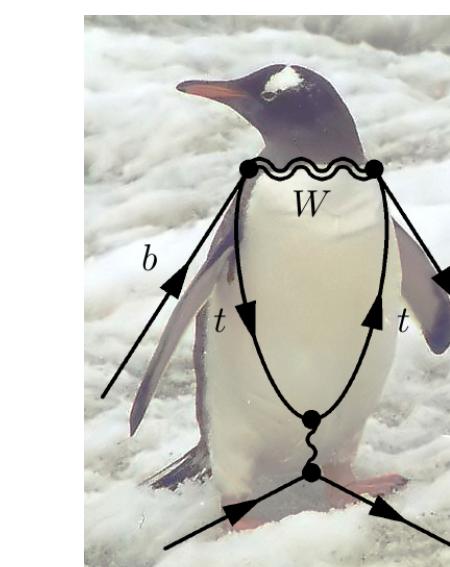
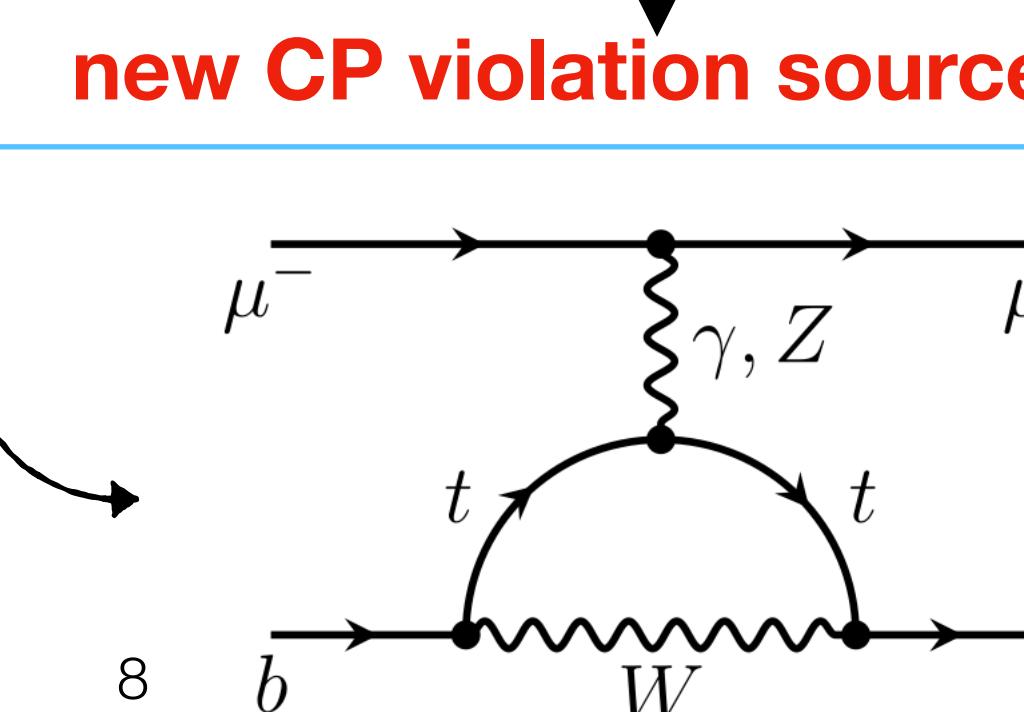
- ▶ Cabibbo–Kobayashi–Maskawa matrix

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

- ▶ 3 mixing angles and 1 CP phase
- ▶ CP violation in the Standard Model

not enough to explain the baryon asymmetry in our universe

new CP violation sources



penguin  
diagram

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## Summary

# LFU test

## ► Z boson decay

$$\Gamma(\mu^+\mu^-)/\Gamma(e^+e^-)$$

VALUE	DOCUMENT ID	TECN	COMMENT	$\Gamma_2/\Gamma_1$
<b>1.0001±0.0024 OUR AVERAGE</b>				
0.9974±0.0050	1 AABOUD	17Q ATLAS	$E_{cm}^{pp} = 7 \text{ TeV}$	
1.0009±0.0028	2 LEP-SLC	06	$E_{cm}^{ee} = 88\text{--}94 \text{ GeV}$	

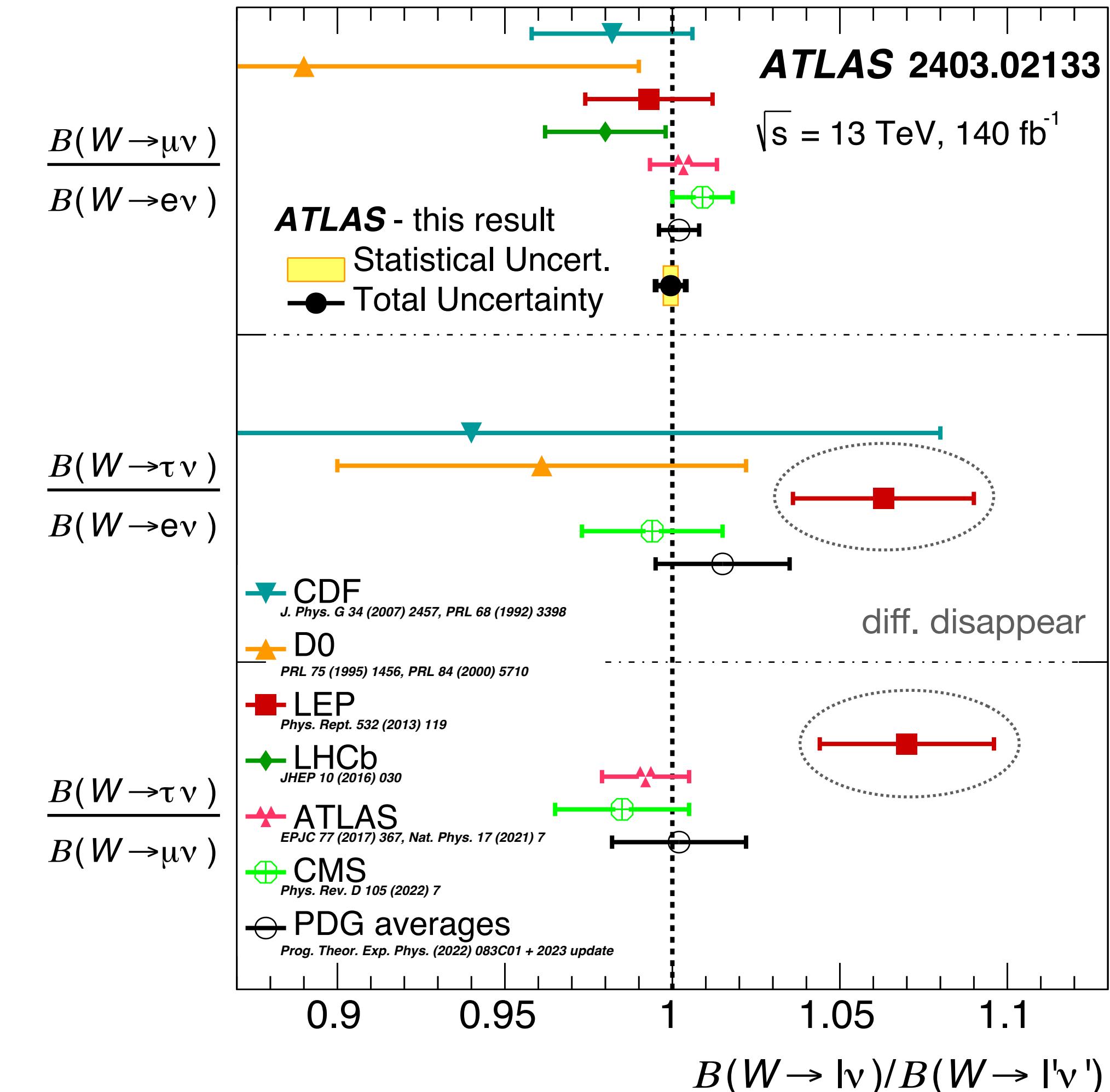
$$\Gamma(\tau^+\tau^-)/\Gamma(e^+e^-)$$

VALUE	DOCUMENT ID	TECN	COMMENT	$\Gamma_3/\Gamma_1$
<b>1.0020±0.0032 OUR AVERAGE</b>				
1.02 ± 0.06	1 AAIJ	18AR LHCb	$E_{cm}^{pp} = 8 \text{ TeV}$	
1.0019±0.0032	2 LEP-SLC	06	$E_{cm}^{ee} = 88\text{--}94 \text{ GeV}$	

$$\Gamma(\tau^+\tau^-)/\Gamma(\mu^+\mu^-)$$

VALUE	DOCUMENT ID	TECN	COMMENT	$\Gamma_3/\Gamma_2$
<b>1.0010±0.0026 OUR AVERAGE</b>				
1.01 ± 0.05	1 AAIJ	18AR LHCb	$E_{cm}^{pp} = 8 \text{ TeV}$	
1.0010±0.0026	2 LEP-SLC	06	$E_{cm}^{ee} = 88\text{--}94 \text{ GeV}$	

## ► W boson decay



# LFU test

A. Pich, Prog. Part. Nucl. Phys. 75 (2014) 41

B. Bryman, Cirigliano, Crivellin, Inguglia, Annu. Rev. Nucl. Part. Sci. 2022. 72:69-91

## ► $\pi$ decay

$$R_{e/\mu}^P = \frac{\Gamma[P \rightarrow e\bar{\nu}_e(\gamma)]}{\Gamma[P \rightarrow \mu\bar{\nu}_\mu(\gamma)]}$$

$$\left(\frac{A_\mu}{A_e}\right)_{R_{e/\mu}^\pi} = 1.0010 \pm 0.0009$$

## ► $K$ decay

$$R_{e/\mu}^K = \frac{\Gamma[K^+ \rightarrow e^+\nu(\gamma)]}{\Gamma[K^+ \rightarrow \mu^+\nu(\gamma)]}$$

$$\left(\frac{A_\mu}{A_e}\right)_{R_{e/\mu}^K} = 0.9978 \pm 0.0018$$

$$R_{e/\mu}^{K \rightarrow \pi} = \frac{\Gamma[K \rightarrow \pi e\nu(\gamma)]}{\Gamma[K \rightarrow \pi \mu\nu(\gamma)]}$$

$$\left(\frac{A_\mu}{A_e}\right)_{R_{e/\mu}^{K_L \rightarrow \pi}} = 1.0022 \pm 0.0024$$

$$\left(\frac{A_\mu}{A_e}\right)_{R_{e/\mu}^{K^\pm \rightarrow \pi^\pm}} = 0.9995 \pm 0.0026$$

## ► $\tau$ decay

$$R_{\tau/e}^\tau = \frac{\text{Br}(\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau)}{\text{Br}(\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu)}$$

$$\left(\frac{A_\tau}{A_e}\right)_\tau = 1.0029 \pm 0.0014$$

$$R_{\tau/\mu}^\tau = \frac{\text{Br}(\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau)}{\text{Br}(\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu)}$$

$$\left(\frac{A_\tau}{A_\mu}\right)_\tau = 1.0010 \pm 0.0014$$

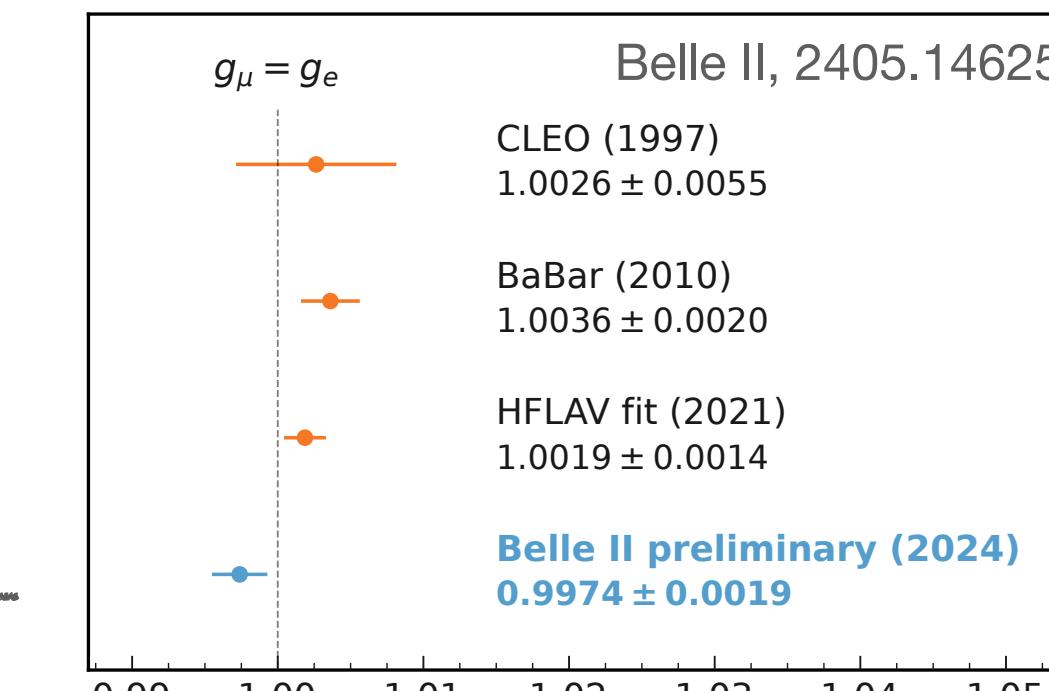
$$R_{\mu/e}^\tau = \frac{\text{Br}(\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau)}{\text{Br}(\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau)}$$

$$\left(\frac{A_\mu}{A_e}\right)_\tau = 1.0018 \pm 0.0014$$

$$R_{\tau/\mu}^{\tau\pi(K)} = \frac{\text{Br}[\tau \rightarrow \pi(K)\nu_\tau]}{\text{Br}[\pi(K) \rightarrow \mu\nu_\mu]}$$

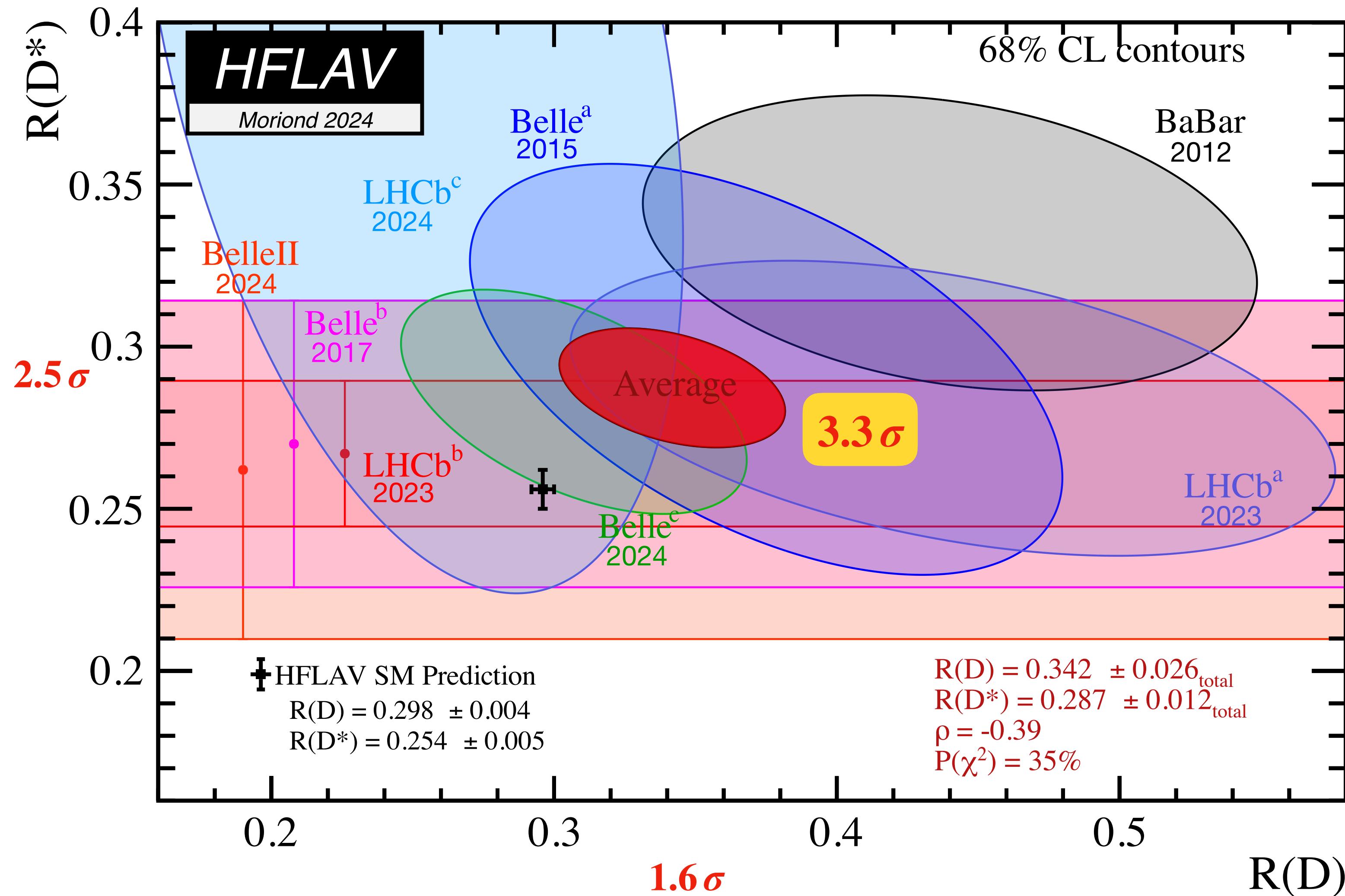
$$\left(\frac{A_\tau}{A_\mu}\right)_\pi = 0.9964 \pm 0.0038$$

recent updates



$$\left(\frac{A_\tau}{A_\mu}\right)_K = 0.9857 \pm 0.0078$$

# $R_D$ and $R_{D^*}$ anomalies: exp



► **LFU Violation ratio:**

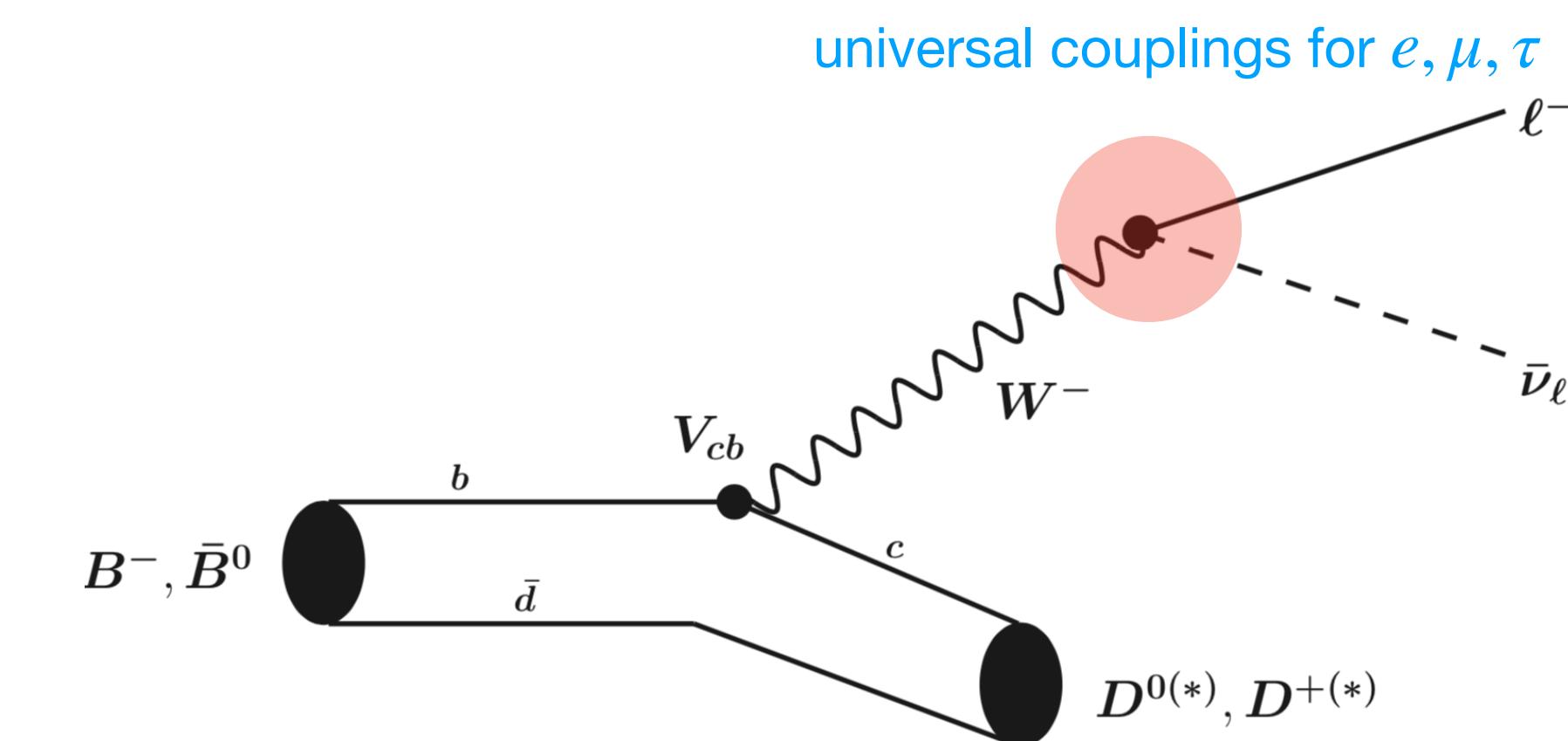
$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu)} \quad (\ell = e, \mu)$$

► **LFUV:  $\tau$  v.s.  $e, \mu$**

► **QCD and EW contributions factorized**

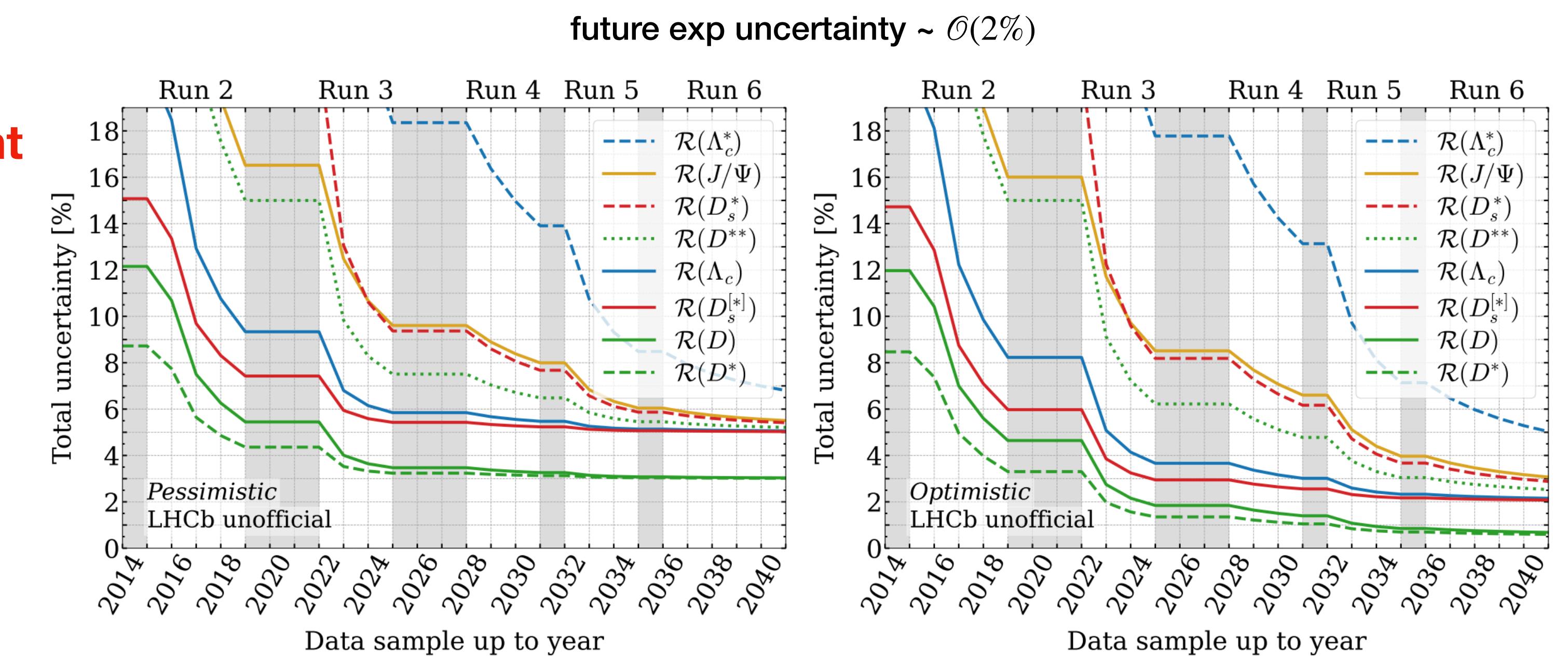
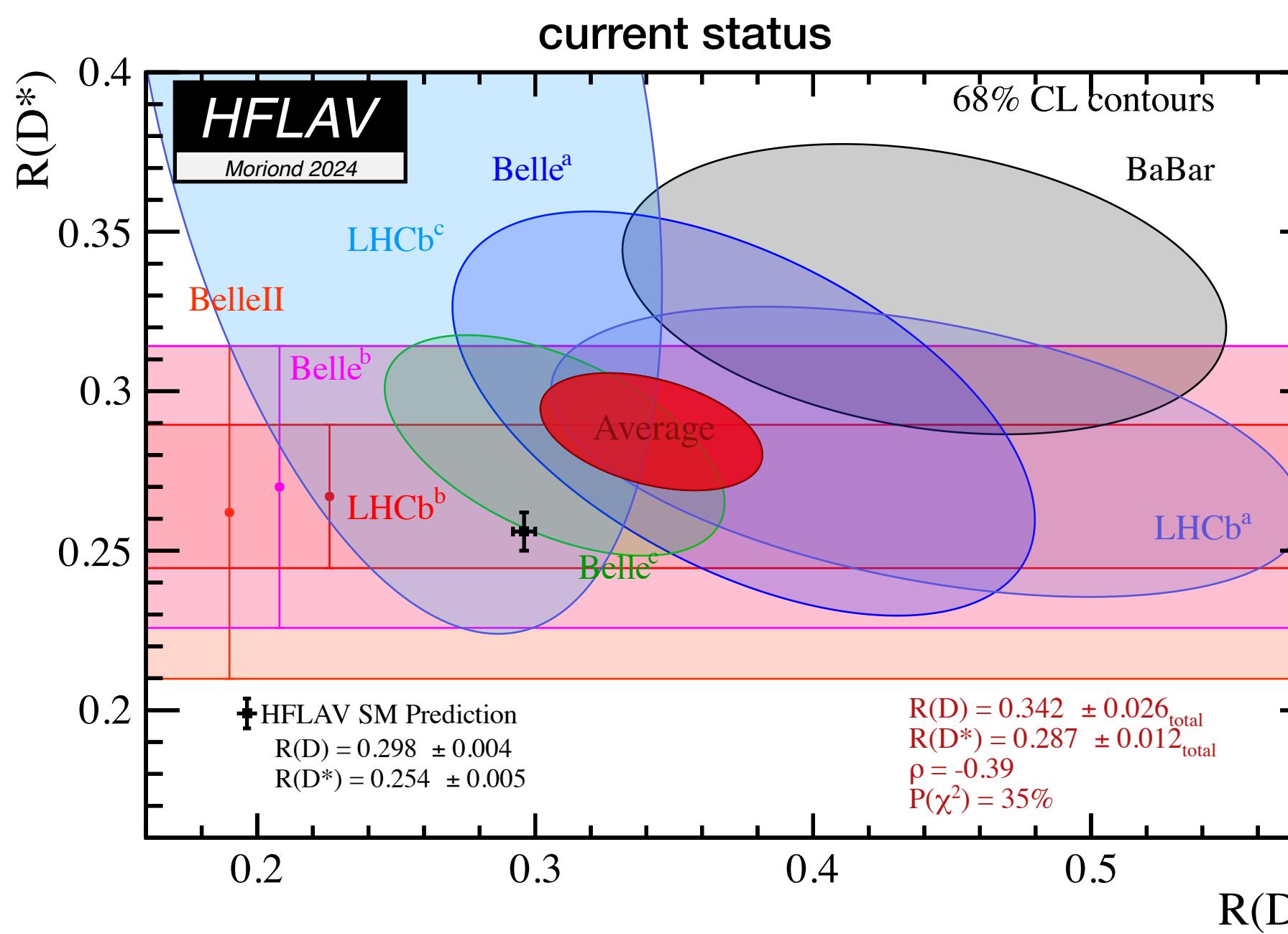
► **hadronic and experimental uncertainties cancelled**

► **tiny theoretical uncertainties**

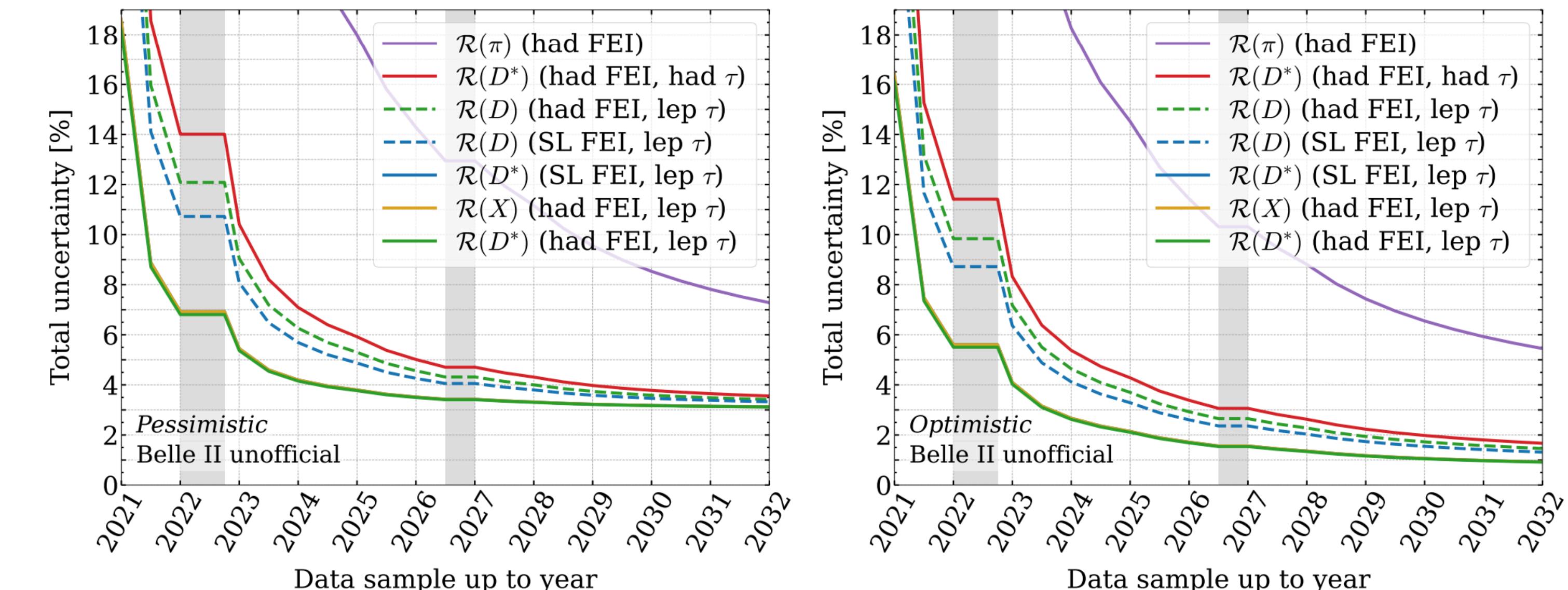


# $R_D$ and $R_{D^*}$ anomalies

- ▶ Anomalies mainly from BaBar measurement
- ▶ LHCb 2022: muonic tau,  $3 \text{ fb}^{-1}$  @Run I
- ▶ LHCb 2023: hadronic tau,  $2 \text{ fb}^{-1}$  @Run II
- ▶ Outlook
  - ▶ Analysis based on Run I + II ( $9 \text{ fb}^{-1}$ )
  - ▶  $R_{\Lambda_c}, R_{D_s}, R_D$
  - ▶ Belle II

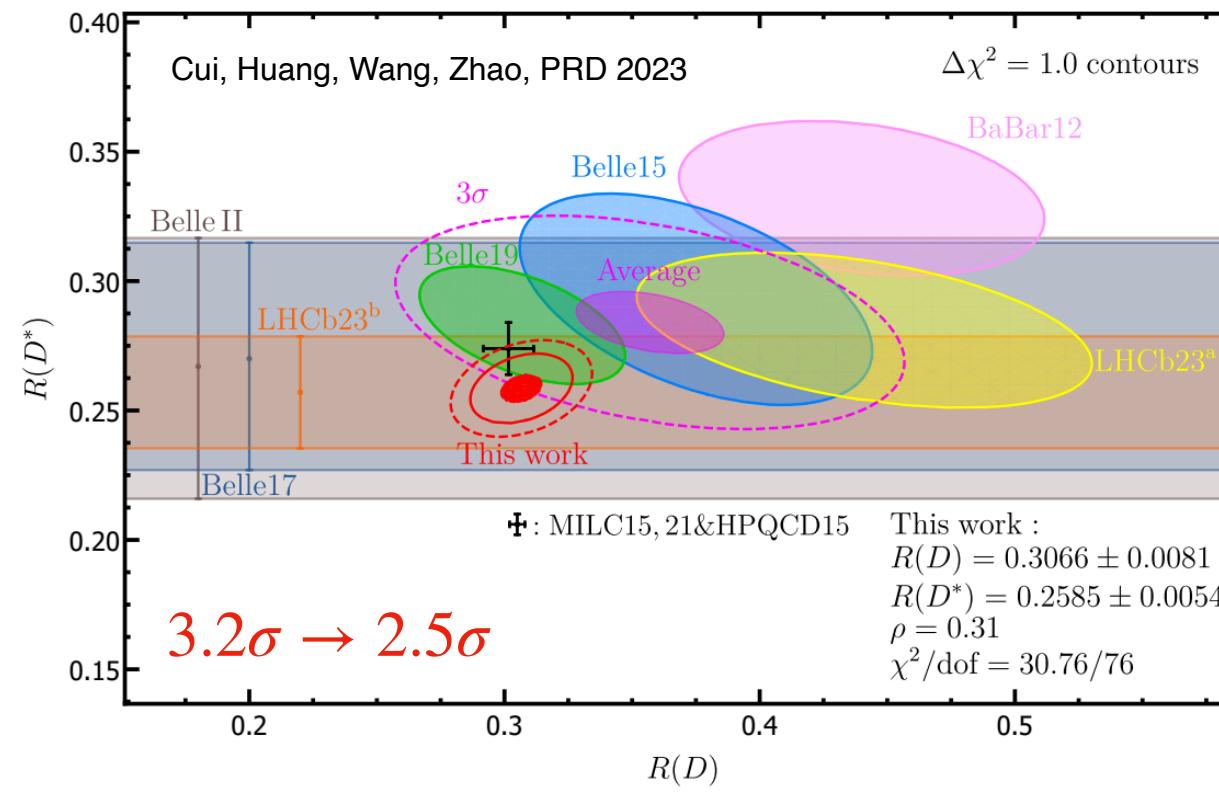
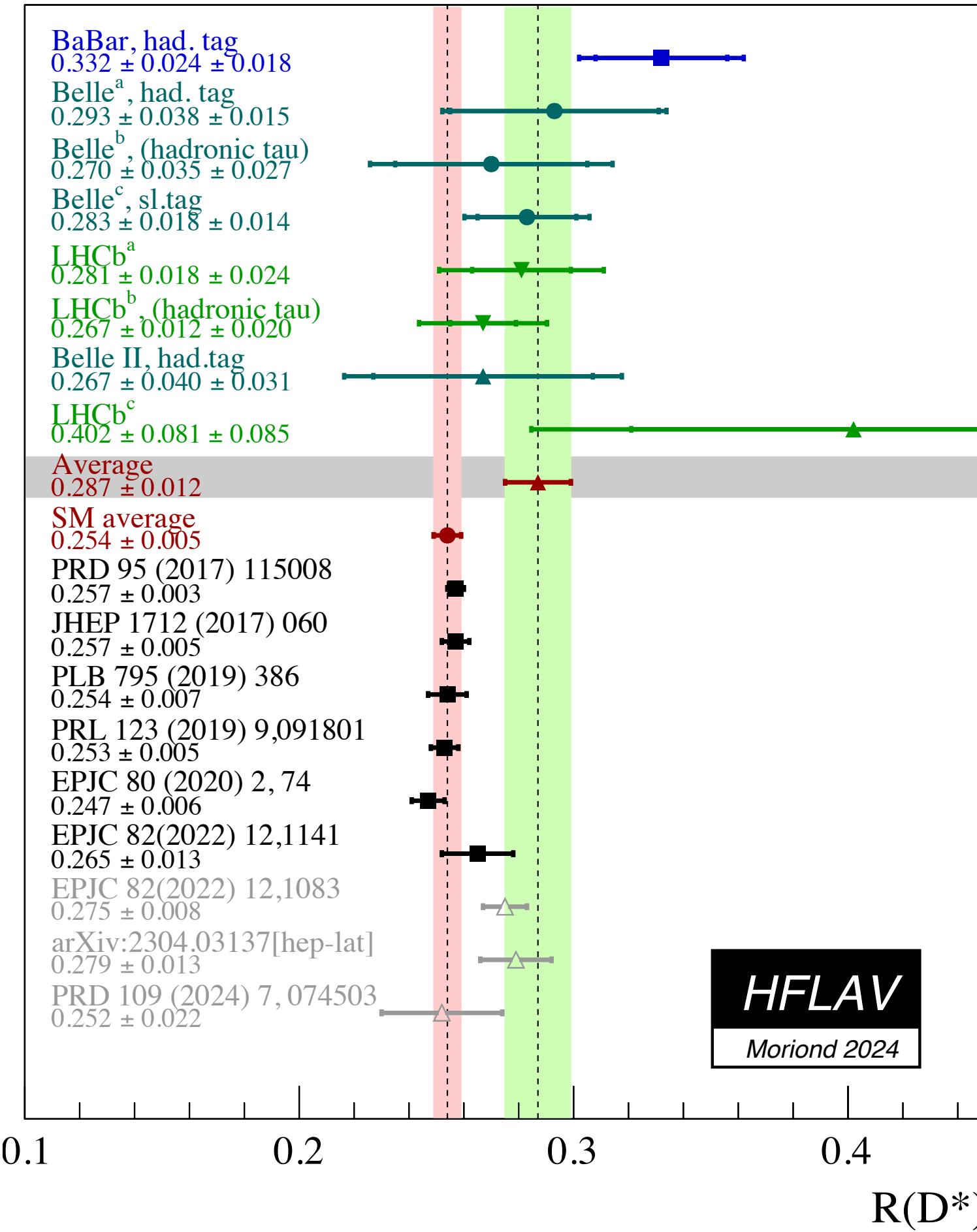
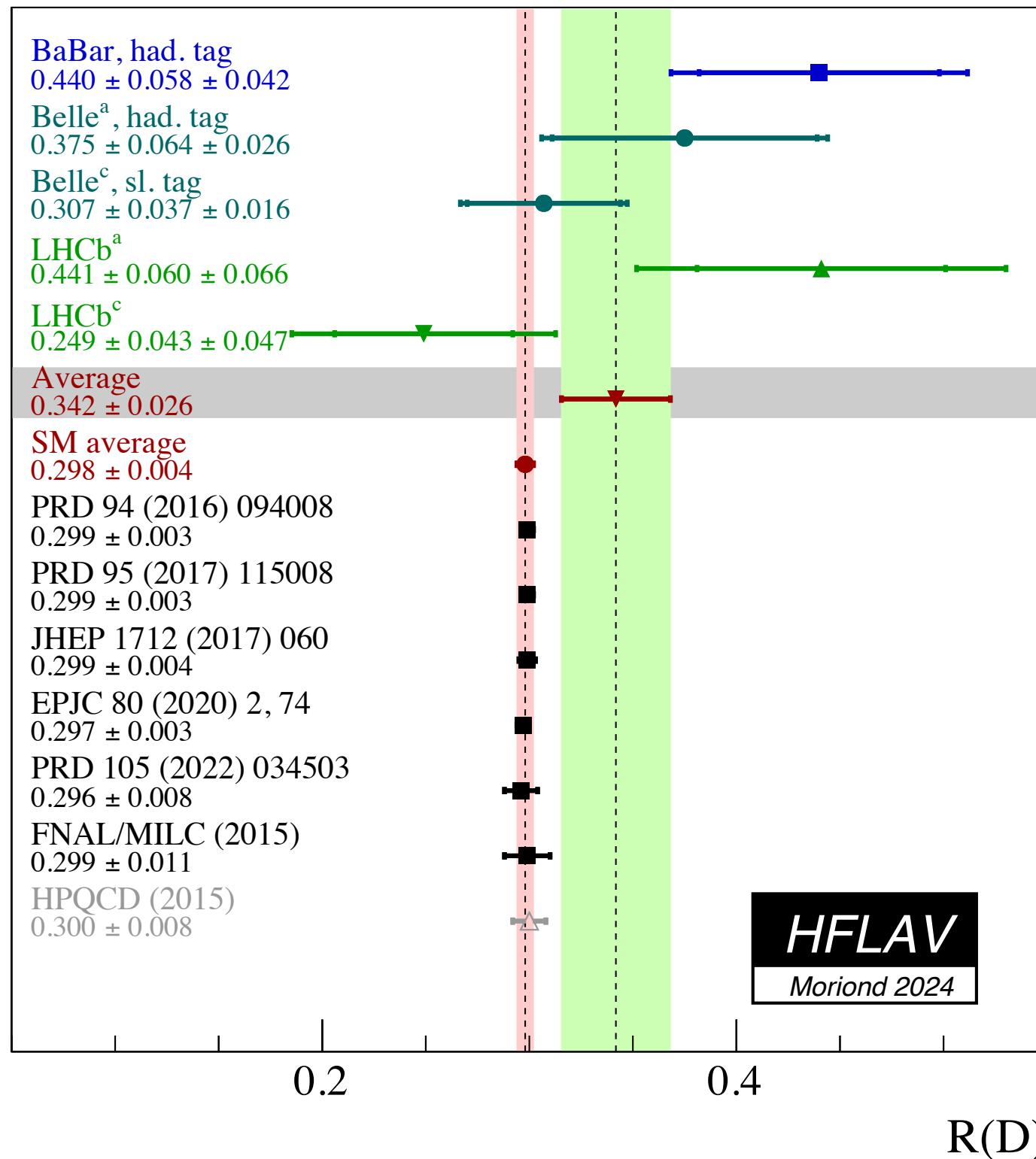


Bernlochner, Sevilla, Robinson, Wormser, 2101.08326



# $R_D$ and $R_{D^*}$ anomalies: theory

uncertainty: current theo  $\approx$  exp@2040



$$R_{D^{(*)}} = \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}\ell\nu)}$$

$$R_D = \frac{\int_{m_\tau^2}^{q_{\max}^2} \frac{d\Gamma(B \rightarrow D\tau\nu)}{dq^2}}{\int_{m_\ell^2}^{q_{\max}^2} \frac{d\Gamma(B \rightarrow D\ell\nu)}{dq^2}}$$

form factor in  $q^2 \in (m_\ell^2, m_\tau^2)$  not cancelled

$$\tilde{R}_D = \frac{\int_{m_\tau^2}^{q_{\max}^2} \frac{d\Gamma(B \rightarrow D\tau\nu)}{dq^2}}{\int_{m_\tau^2}^{q_{\max}^2} \frac{d\Gamma(B \rightarrow D\ell\nu)}{dq^2}}$$

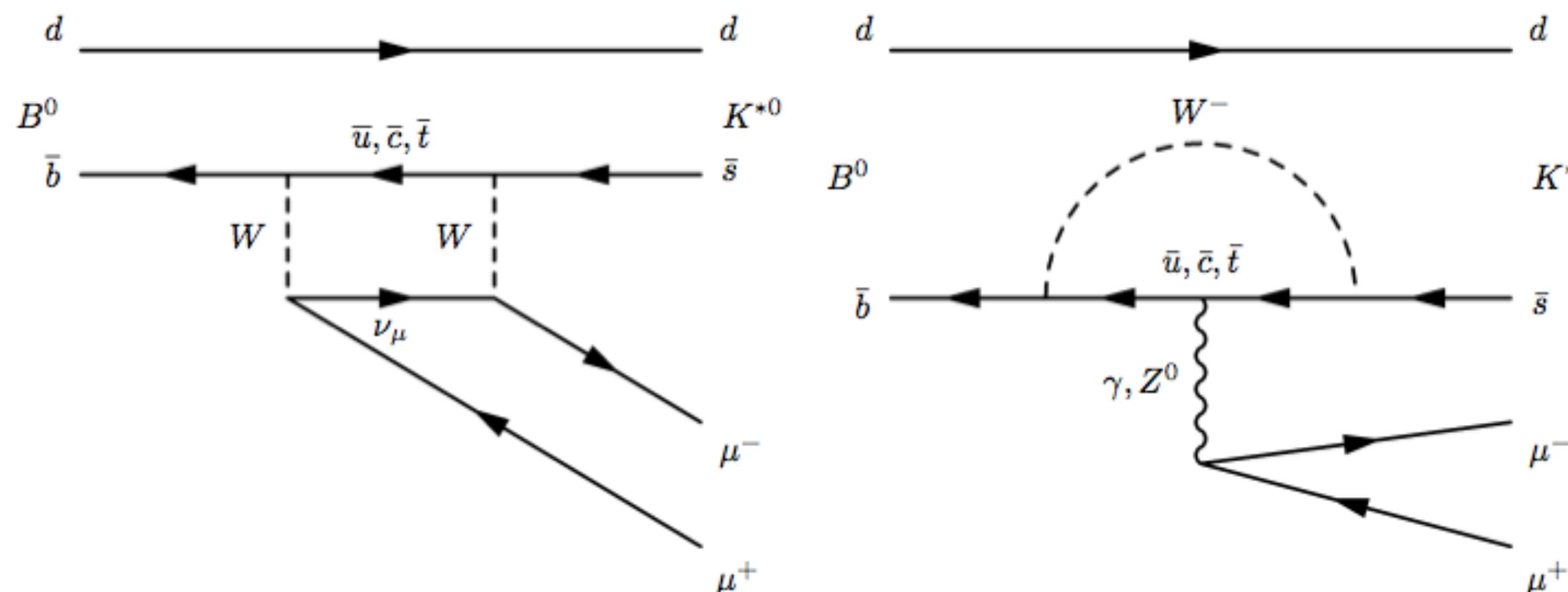
form factors in the same region

$$r_D(q^2) = \frac{\frac{d\Gamma(B \rightarrow D\tau\nu)}{dq^2}}{\frac{d\Gamma(B \rightarrow D\ell\nu)}{dq^2}}$$

Form factors could not be a problem for  $R_{D^{(*)}}$  in the future, but still important for angular observables e.g.,  $P_\tau(D^*)$  and  $F_{L,\tau}(D^*)$ . form factor fully cancelled, for large data samples

# $b \rightarrow s\ell^+\ell^-$ decays

- ▶  $B_s \rightarrow \ell^+\ell^-$
- ▶  $B \rightarrow X_s \ell^+\ell^-$
- ▶  $B \rightarrow K\ell^+\ell^-$
- ▶  $B \rightarrow K^*\ell^+\ell^-$
- ▶  $B_s \rightarrow \phi\ell^+\ell^-$
- ▶  $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$



## ▶ Flavour-Changing Neutral Current (FCNC)

- ▶ Tree-level: forbidden
- ▶ Loop-level: suppressed by GIM,  $\mathcal{B} \lesssim \mathcal{O}(10^{-6})$
- ⇒ **Sensitive to New Physics**
- ▶ Many observables: branching ratio, angular distribution, LFV ratio
- ▶ NP effects can be sizable compared to the SM amplitude
- ▶ This transition is LFU in the SM

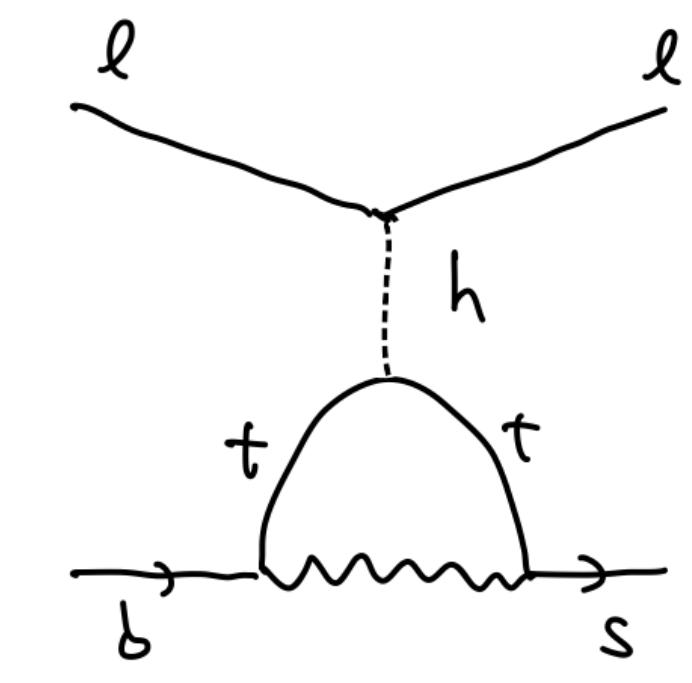
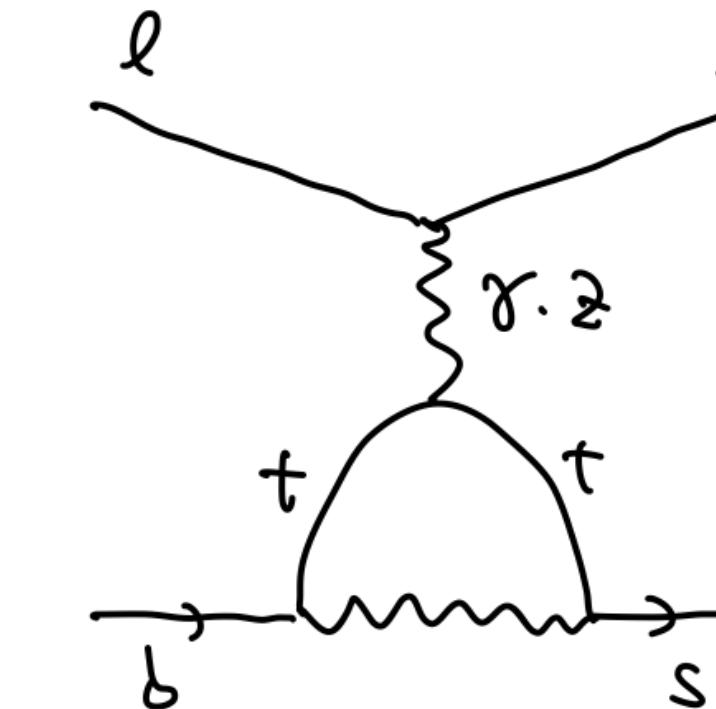
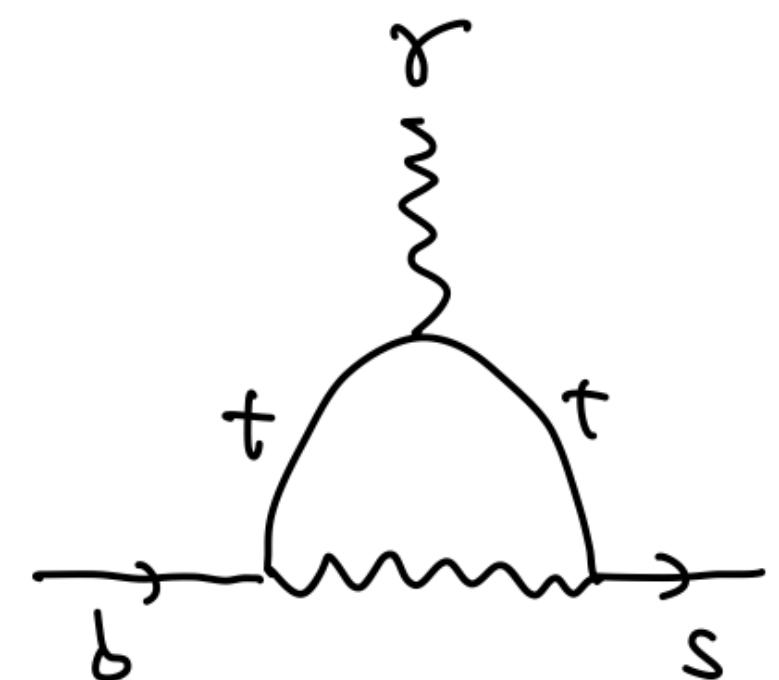
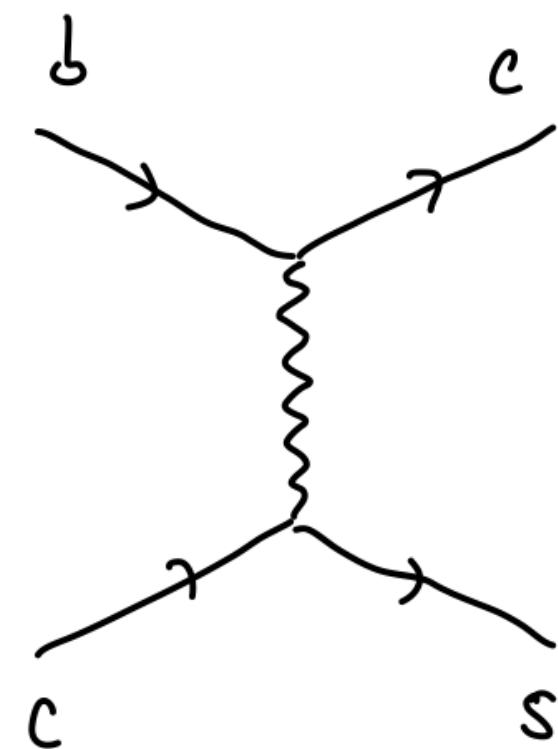
# $b \rightarrow s \ell^+ \ell^-$ : theory

## ► Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \left( \sum_{i=1,\dots,6} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \sum_{\ell} \sum_{i=9,10,P,S} (C_i^\ell O_i^\ell + C_i^{\prime\ell} O_i^{\prime\ell}) \right)$$

## ► Effective operator

$$O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b) \quad O_7^{(\prime)} = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}, \quad O_9^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell), \quad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \quad O_S^{(\prime\ell)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\ell) \\ O_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b) \quad C_7^{\text{SM}} \simeq -0.3, \quad C_9^{\text{SM}} \simeq 4, \quad C_{10}^{\text{SM}} \simeq -4. \quad O_P^{(\prime\ell)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\gamma_5 \ell)$$



## ► Feynman Diagram

# $b \rightarrow s \ell^+ \ell^-$ : theory

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## ► Effective operator

$$O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b) \quad O_7^{(\prime)} = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}, \quad O_9^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell), \quad O_{10}^{(\prime)\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \quad O_S^{(\ell)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\ell) \\ O_2 = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b) \quad C_7^{\text{SM}} \simeq -0.3, \quad C_9^{\text{SM}} \simeq 4, \quad C_{10}^{\text{SM}} \simeq -4. \quad O_P^{(\ell)} = (\bar{s}P_{R(L)} b)(\bar{\ell}\gamma_5 \ell)$$

## ► Amplitude:

 $\mathcal{M}(B \rightarrow M\ell\ell) = \langle M\ell\ell | \mathcal{H}_{\text{eff}} | B \rangle = \mathcal{N} \left[ (\mathcal{A}_V^\mu + \mathcal{H}^\mu) \bar{u}_\ell \gamma_\mu v_\ell + \mathcal{A}_A^\mu \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell + \mathcal{A}_S \bar{u}_\ell v_\ell + \mathcal{A}_P \bar{u}_\ell \gamma_5 v_\ell \right]$ 

**Local:**

$$\mathcal{A}_V^\mu = -\frac{2im_b}{q^2} C_7 \langle M | \bar{s} \sigma^{\mu\nu} q_\nu P_R b | B \rangle + C_9 \langle M | \bar{s} \gamma^\mu P_L b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i)$$

$$\mathcal{A}_A^\mu = C_{10} \langle M | \bar{s} \gamma^\mu P_L b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i)$$

$$\mathcal{A}_{S,P} = C_{S,P} \langle M | \bar{s} P_R b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i)$$

**Non-Local:**

$$\mathcal{H}^\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1,\dots,6,8} C_i \int dx^4 e^{iq \cdot x} \langle M | T \{ j_{\text{em}}^\mu(x), O_i(0) \} | B \rangle$$

$$j_{\text{em}}^\mu = \sum_q Q_q \bar{q} \gamma^\mu q$$

From talk by B. Capdevila, M. Fedele, S. Neshatpour, P. Stang

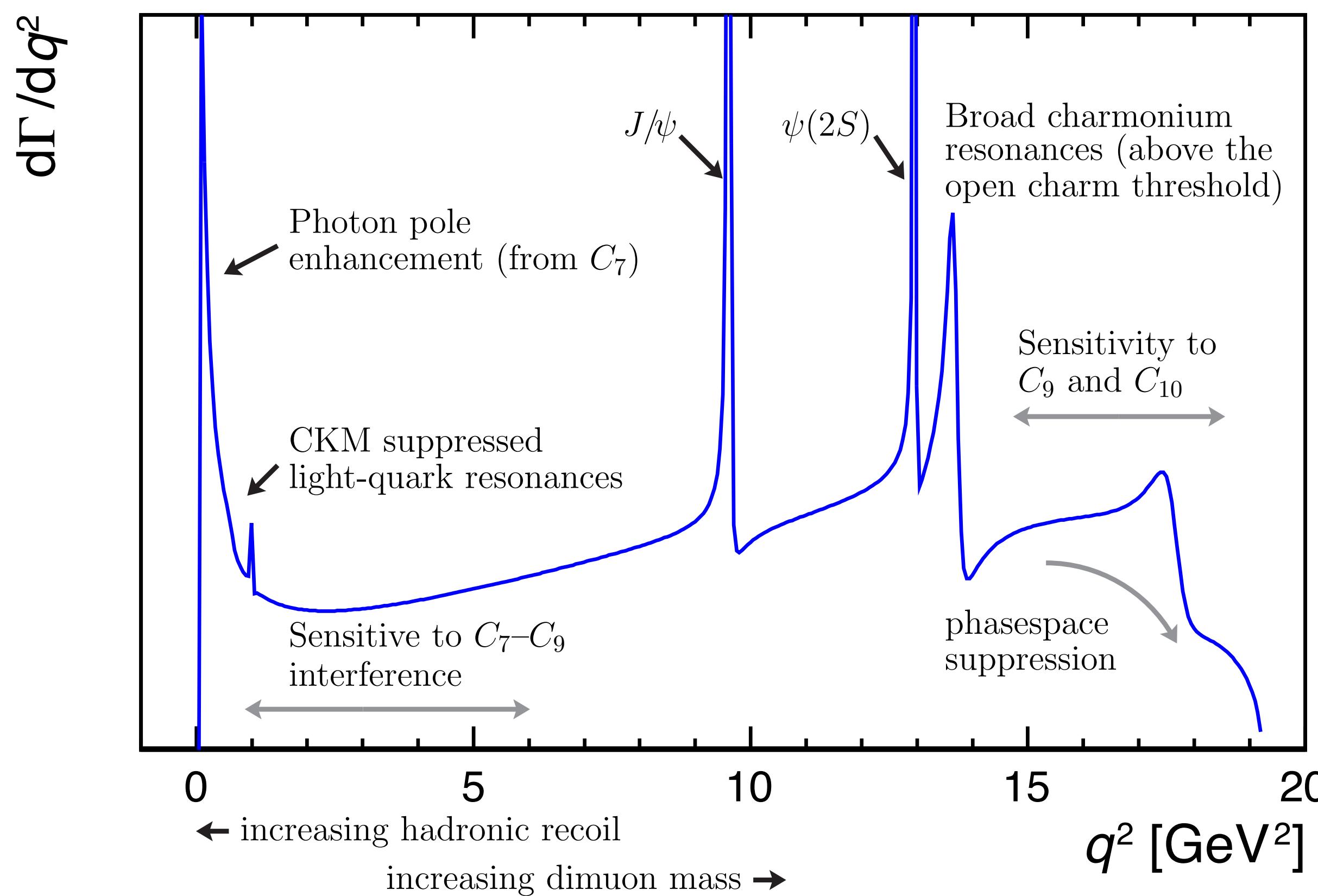
## ► Wilson Coefficient

- perturbative
- short-distance physics
- $q^2$  independent
- NNLO QCD + NLO EW@SM
- parameterization of heavy NP
- $C_i = C_i^{\text{SM}} + C_i^{\text{NP}}$

## ► Matrix Element

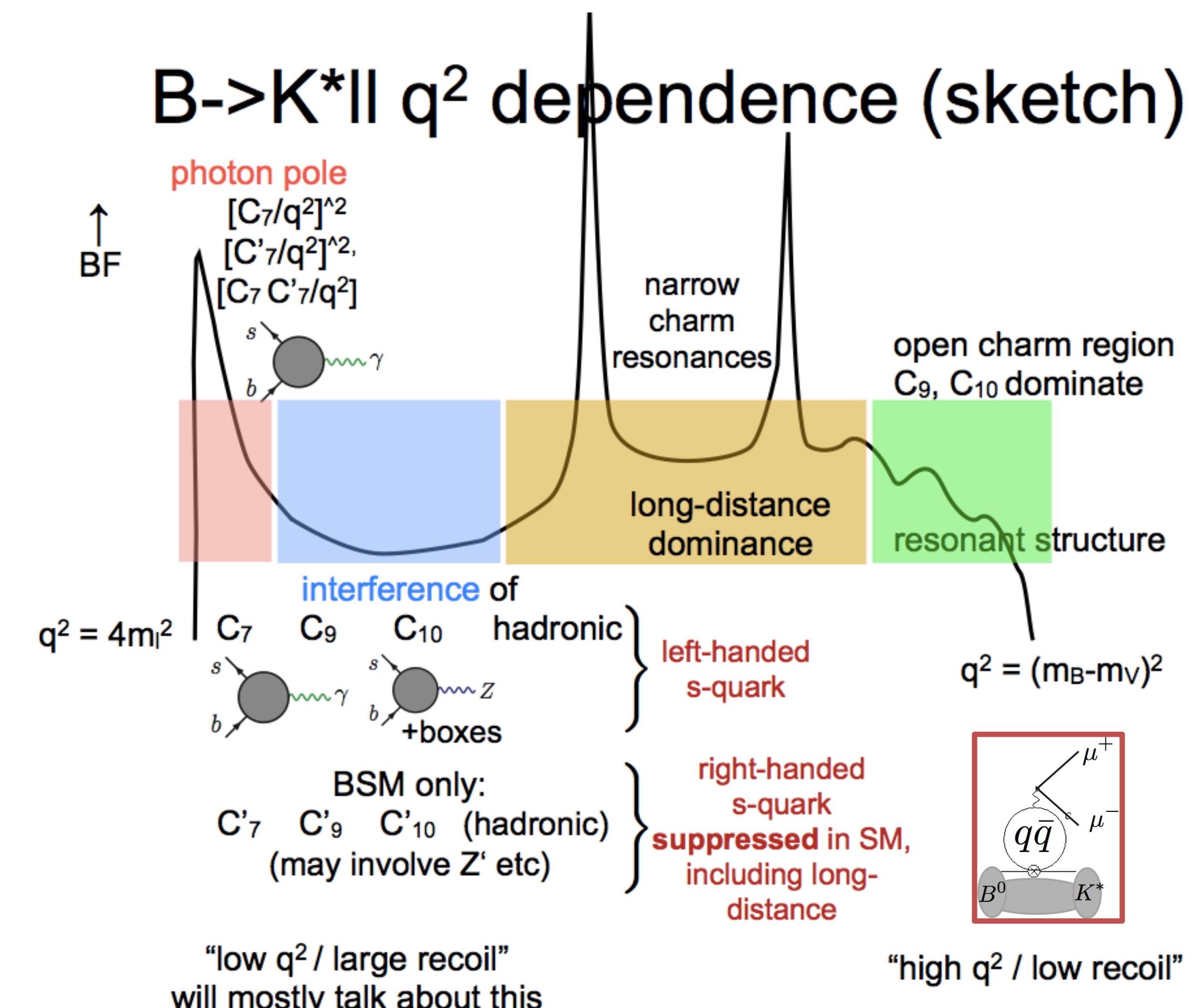
- non-perturbative
- long-distance physics
- $q^2$  dependent
- theoretically challenging
- main source of uncertainties

# $b \rightarrow s \ell^+ \ell^-$ : theory



T.Blake, G.Lanfranchi, D.Straub, 1606.00916

## $B \rightarrow K^* ll$ $q^2$ dependence (sketch)



From S.Jager's talk

# $b \rightarrow s\ell^+\ell^-$ : observables

- ▶  $B_s \rightarrow \ell^+\ell^-$
- ▶  $B \rightarrow X_s\ell^+\ell^-$
- ▶  $B \rightarrow K\ell^+\ell^-$
- ▶  $B \rightarrow K^*\ell^+\ell^-$
- ▶  $B_s \rightarrow \phi\ell^+\ell^-$
- ▶  $\Lambda_b \rightarrow \Lambda\ell^+\ell^-$

theoretical cleanliness

- ▶ Branching Ratio
- ▶ Angular Distribution
- ▶ Lepton Flavour Universality (LFU) ratio

function of  $(C_{7\gamma}, C_9, C_{10})$

LFU ratio in  $B \rightarrow K\ell^+\ell^-$

$$R_K = \frac{\mathcal{B}(B \rightarrow K\mu^+\mu^-)}{\mathcal{B}(B \rightarrow Ke^+e^-)}$$

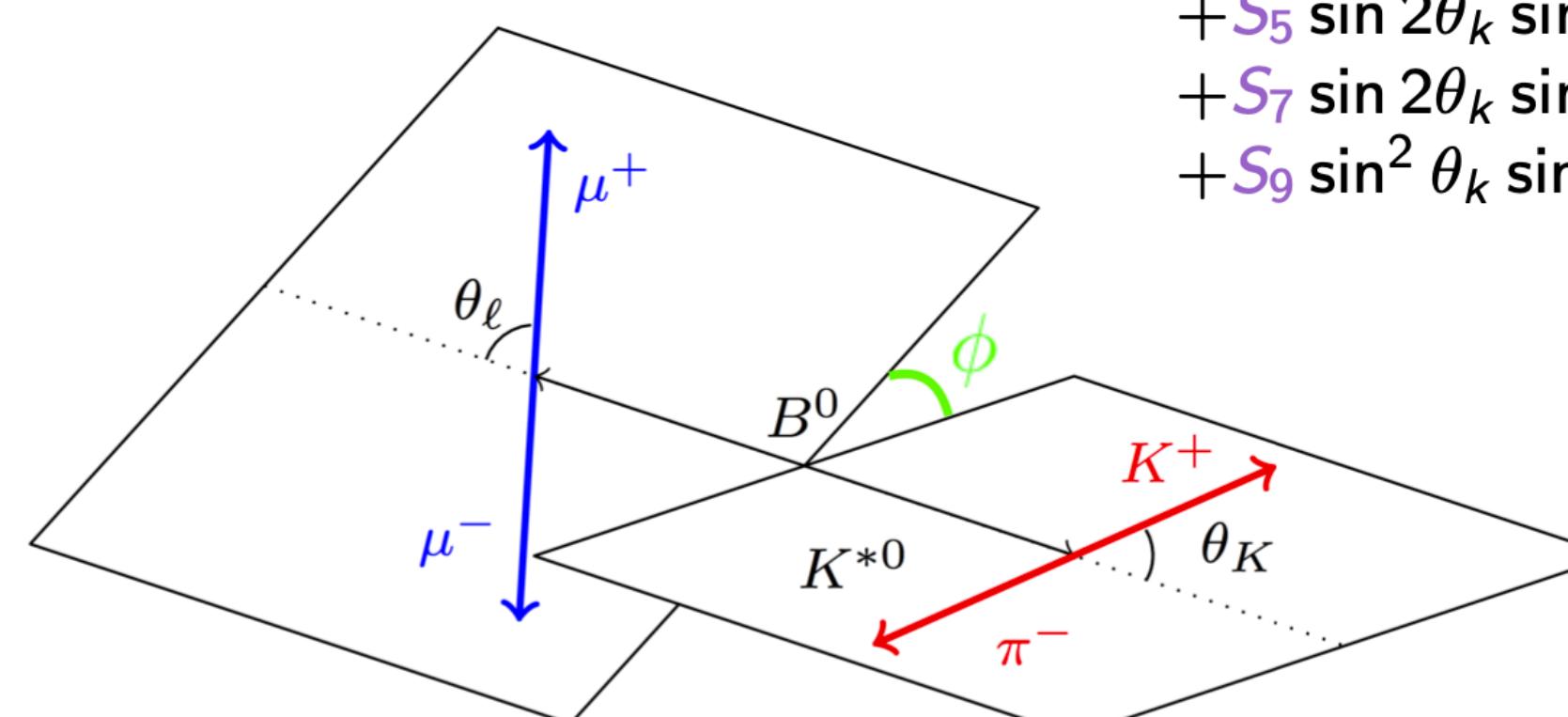
- ▶  $R_K^{\text{SM}} \approx 1$
- ▶ Hadronic uncertainties cancel
- ▶  $\mathcal{O}(10^{-2})$  QED correction
- deviation from unity



Physics beyond the SM

Angular distribution of  
 $B \rightarrow K^*( \rightarrow K\pi)\mu^+\mu^-$

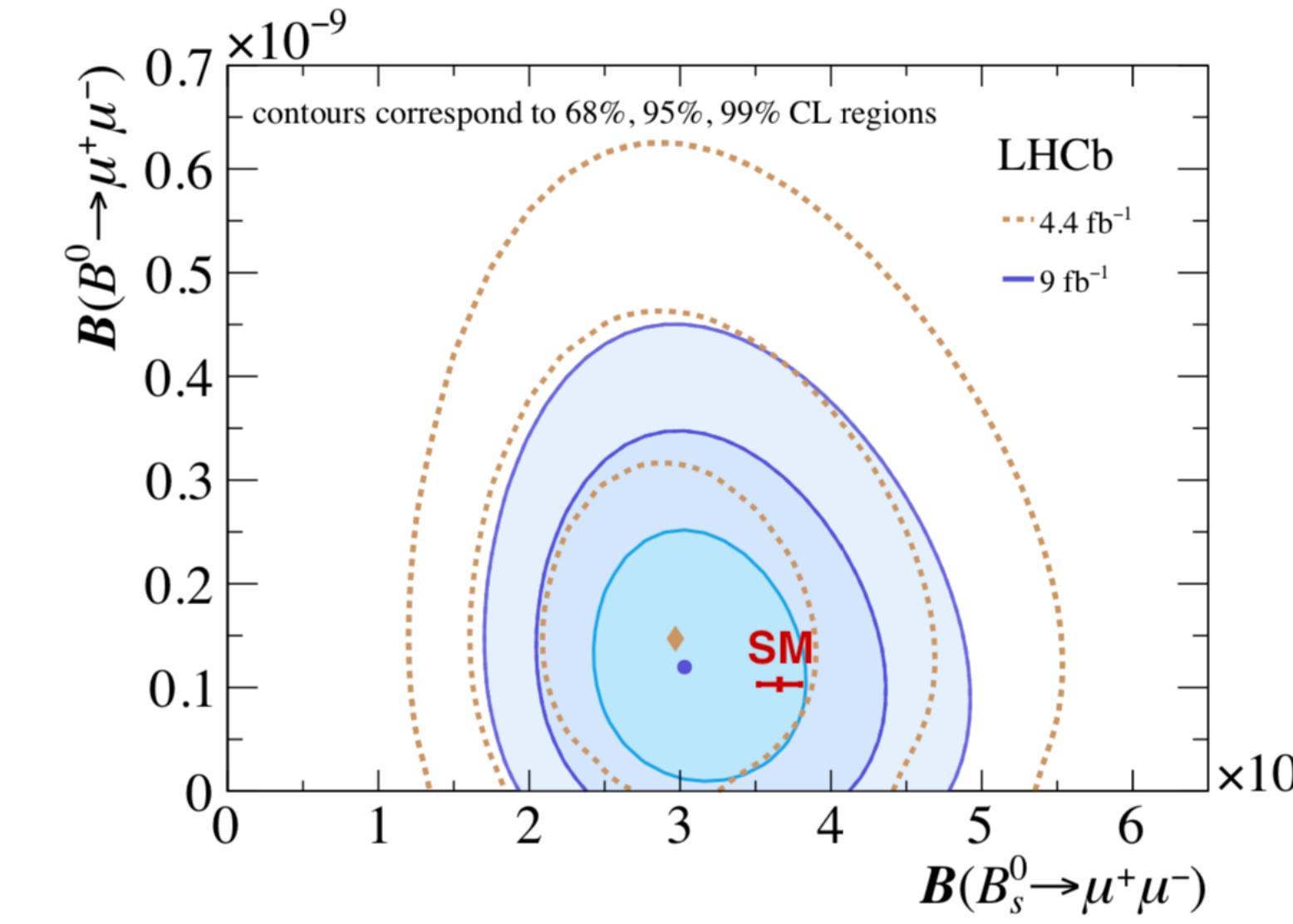
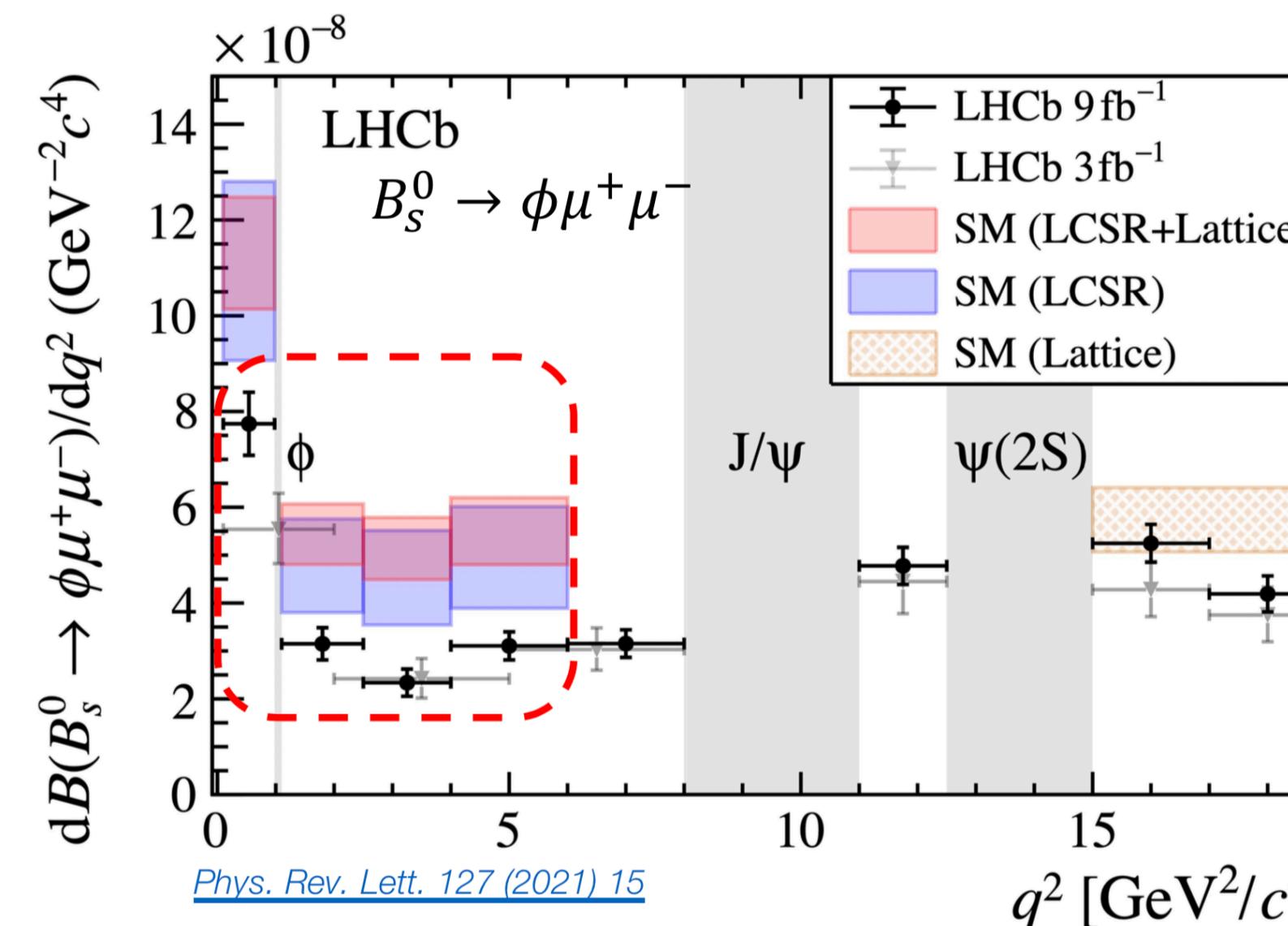
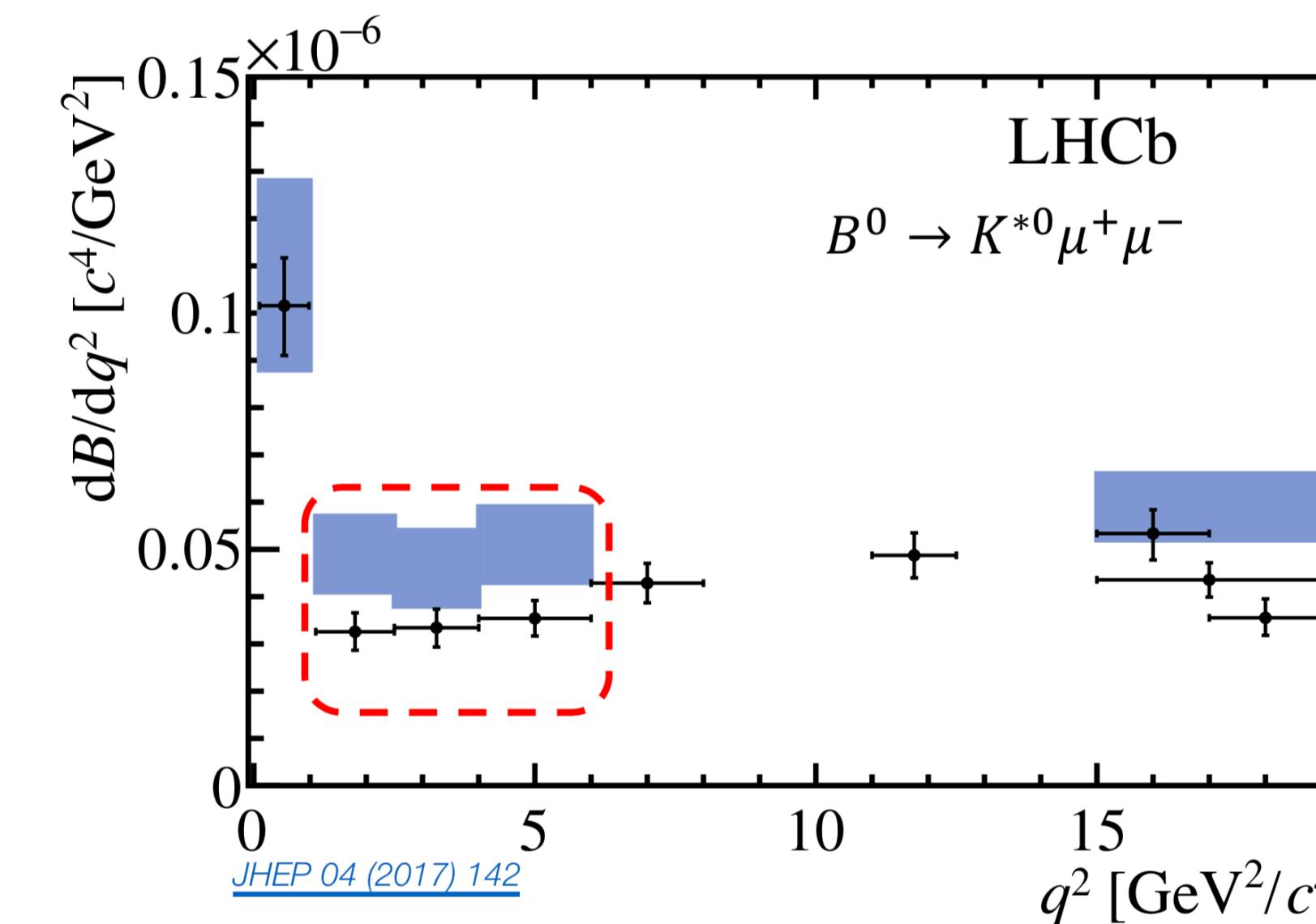
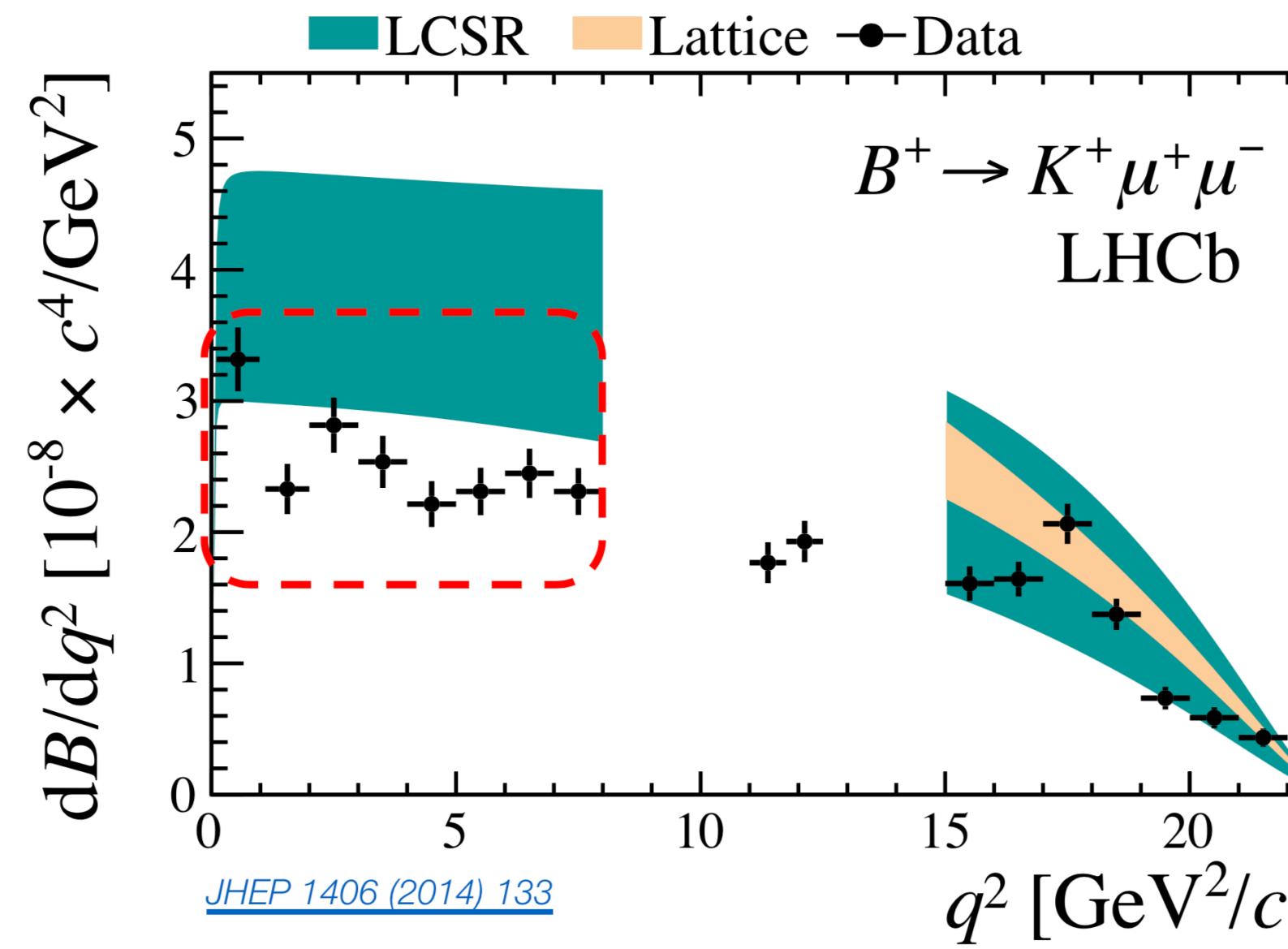
$$\begin{aligned} \frac{1}{d(\Gamma + \bar{\Gamma})/dq^2} \frac{d^4(\Gamma + \bar{\Gamma})}{d\vec{\Omega} dq^2} &= \frac{9}{32\pi} [\frac{3}{4}(1 - F_L) \sin^2 \theta_k + F_L \cos^2 \theta_k \\ &+ \frac{1}{4}(1 - F_L) \sin^2 \theta_k \cos 2\theta_\ell - F_L \cos^2 \theta_k \cos 2\theta_\ell \\ &+ S_3 \sin^2 \theta_k \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_k \sin 2\theta_\ell \cos \phi \\ &+ S_5 \sin 2\theta_k \sin \theta_\ell \cos \phi + \frac{4}{3} A_{FB} \sin^2 \theta_k \cos \theta_\ell \\ &+ S_7 \sin 2\theta_k \sin \theta_\ell \sin \phi + S_8 \sin 2\theta_k \sin 2\theta_\ell \sin \phi \\ &+ S_9 \sin^2 \theta_k \sin^2 \theta_\ell \sin 2\phi], \end{aligned}$$



angular observables  
 $F_L, A_{FB}, S_i = f(C_7, C_9, C_{10}),$   
combinations of  $K^{*0}$  decay amplitudes

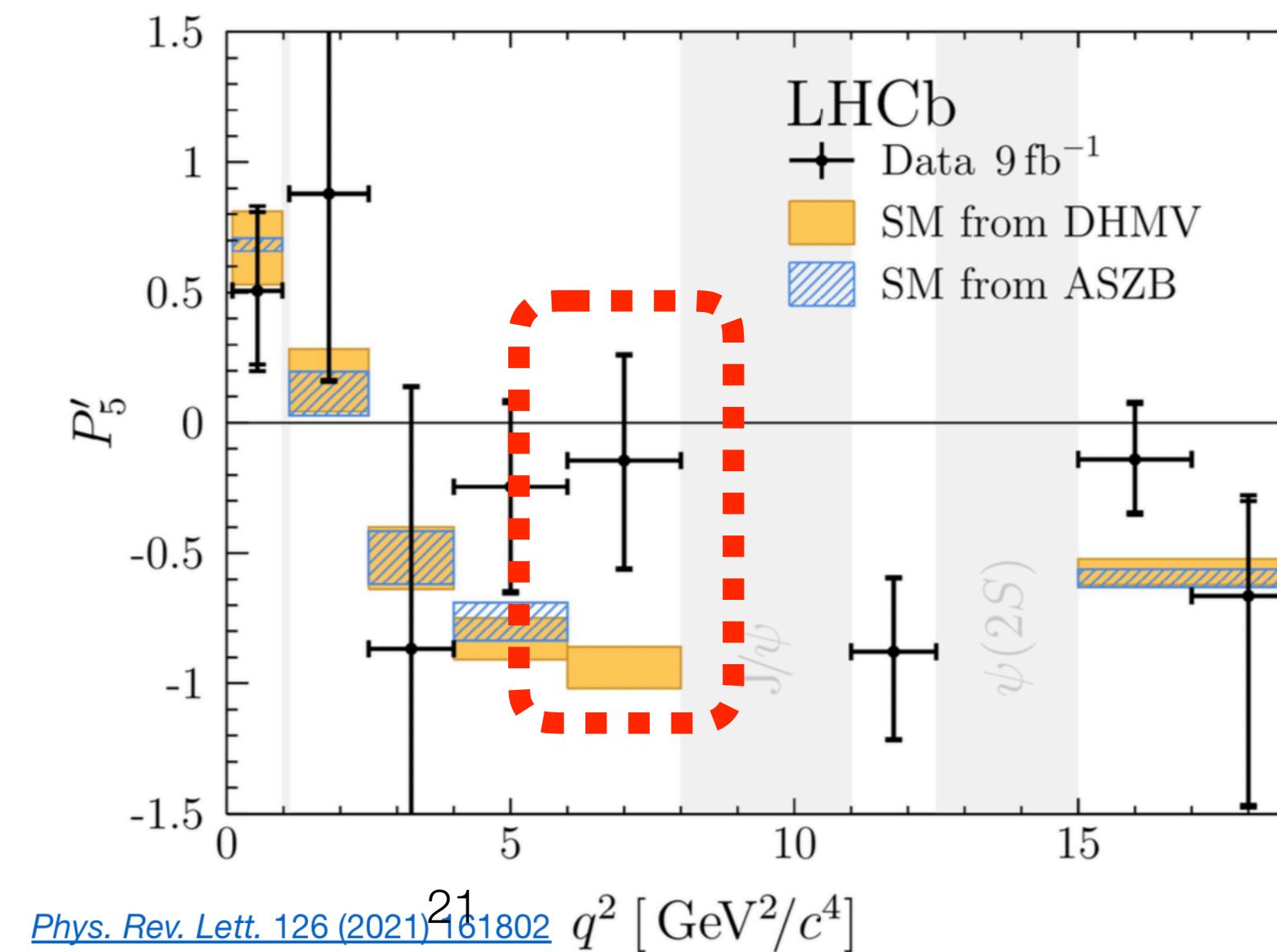
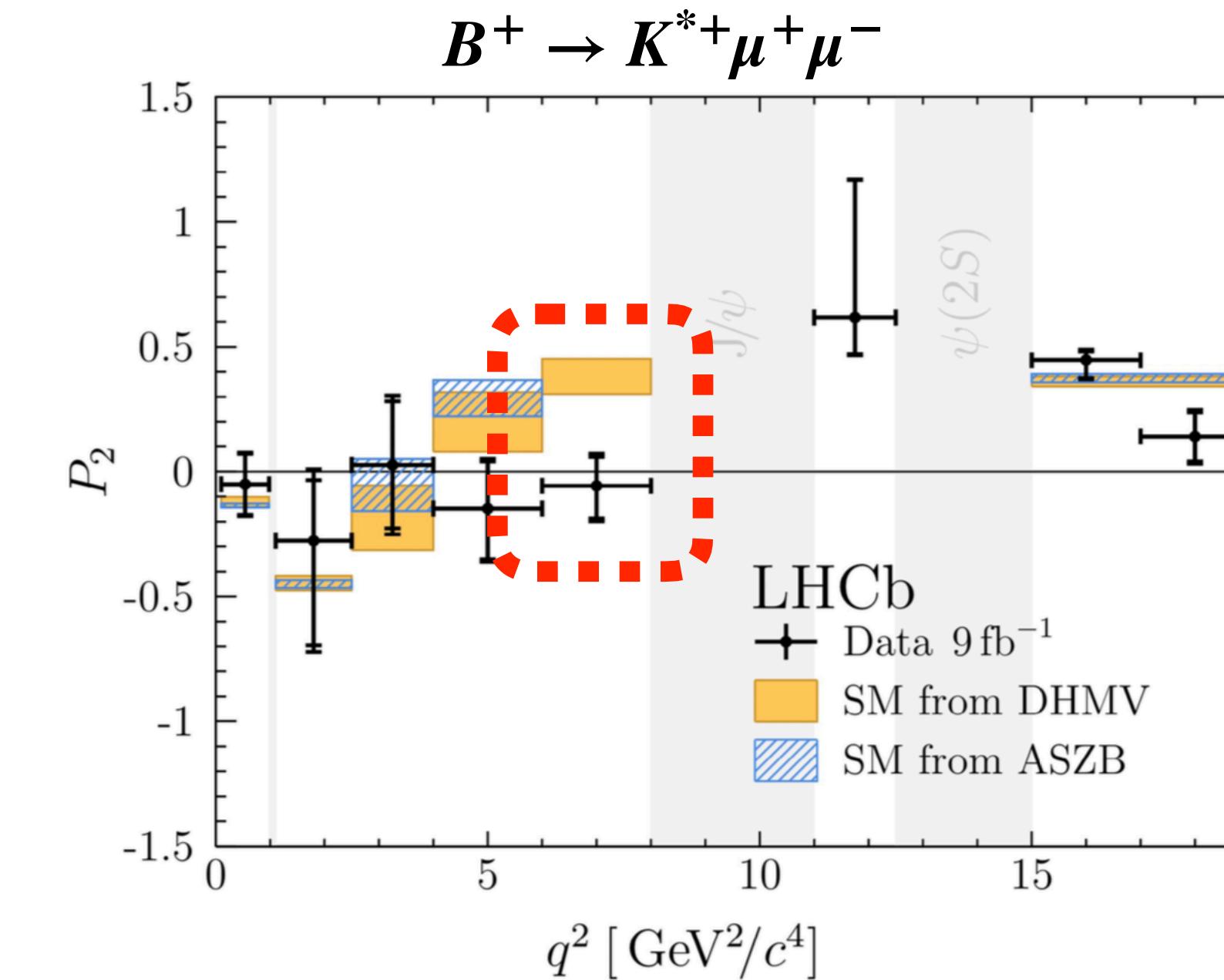
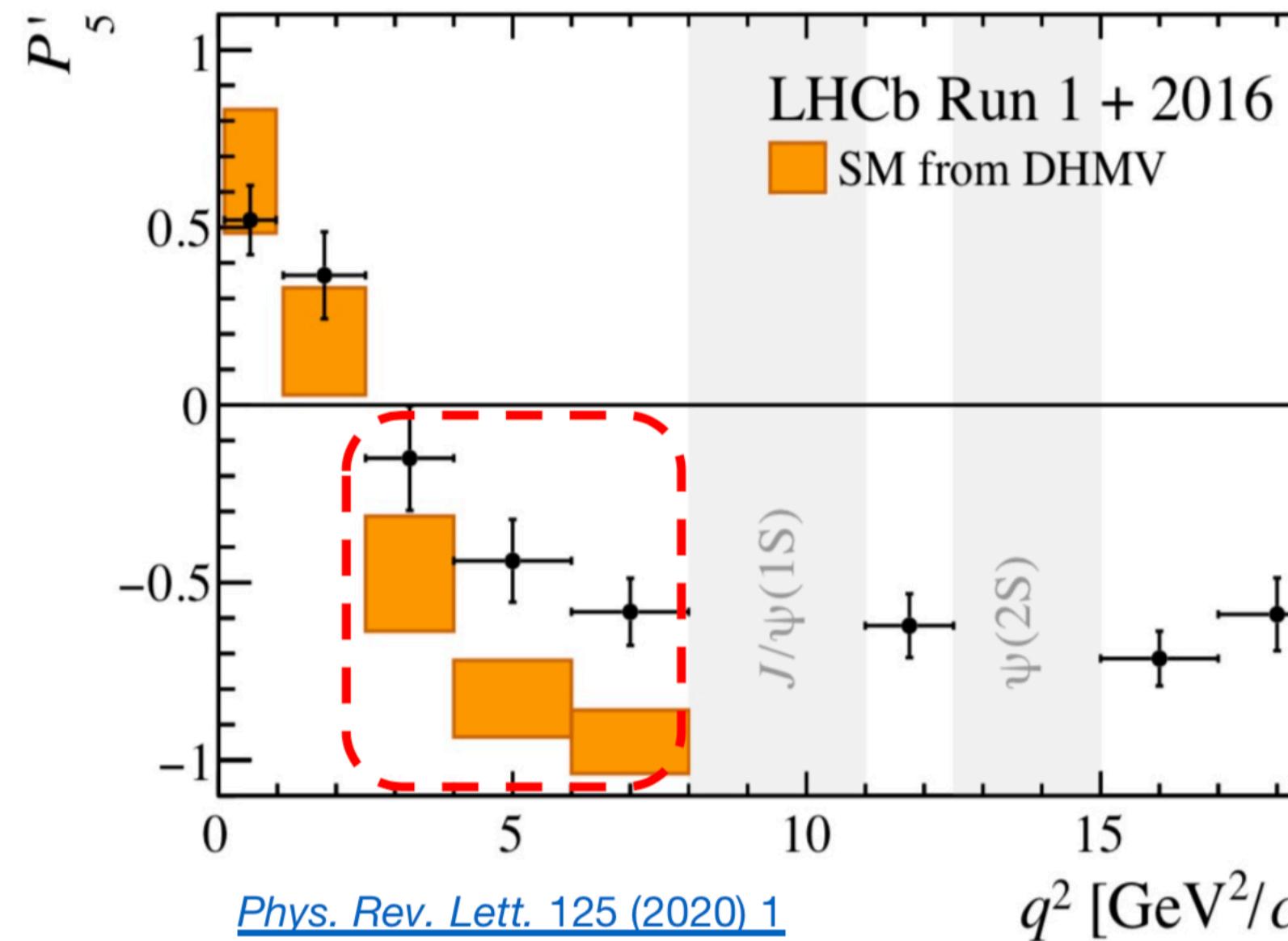
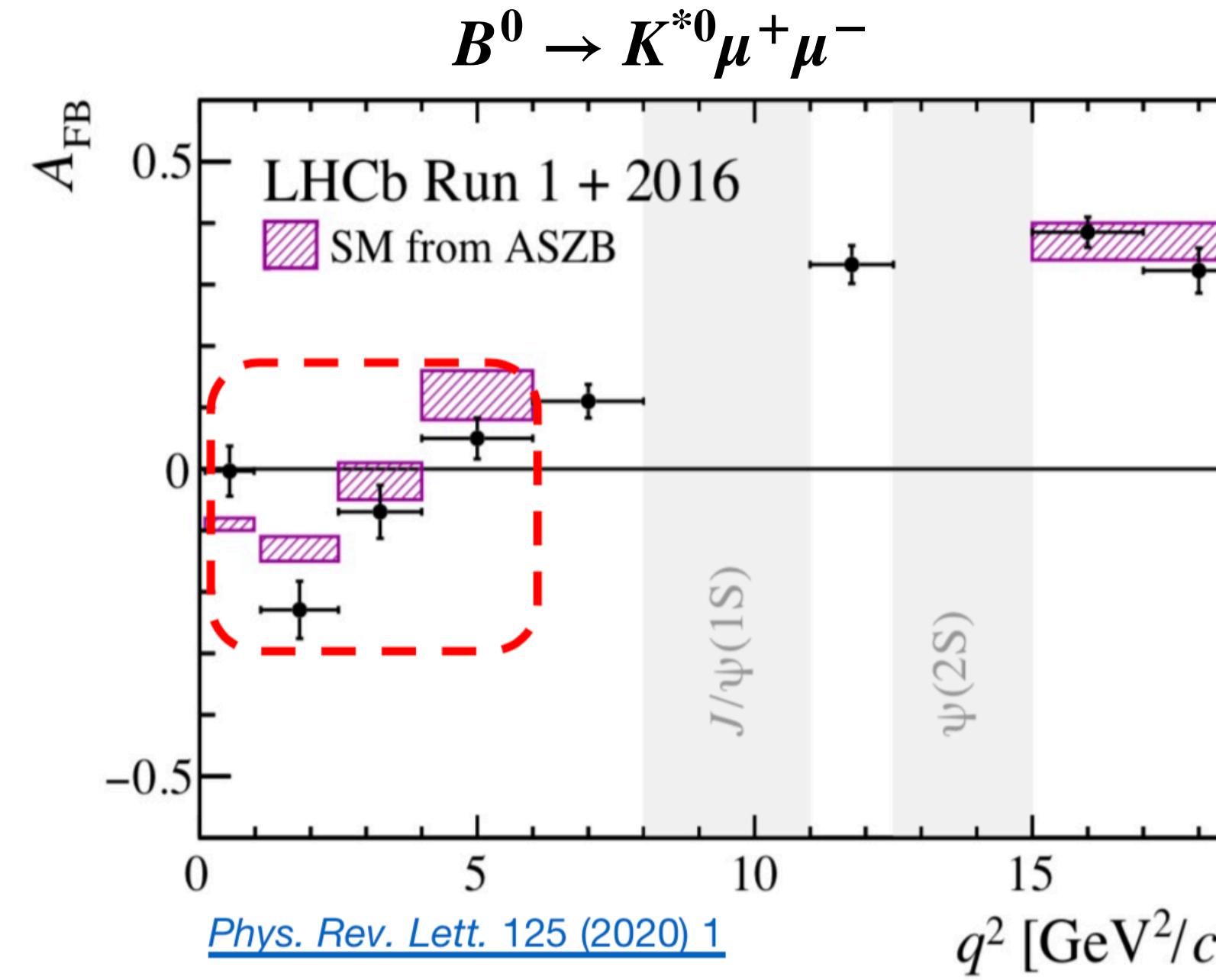
$$\begin{aligned} P_1 &= \frac{2S_3}{1 - F_L} \\ P_2 &= \frac{2}{3} \frac{A_{FB}}{1 - F_L} \\ P_3 &= -\frac{S_9}{1 - F_L} \\ P'_{i=4,5,6,8} &= \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1 - F_L)}}. \end{aligned}$$

# $b \rightarrow s\ell\ell$ anomalies@mid.2022: branching ratio



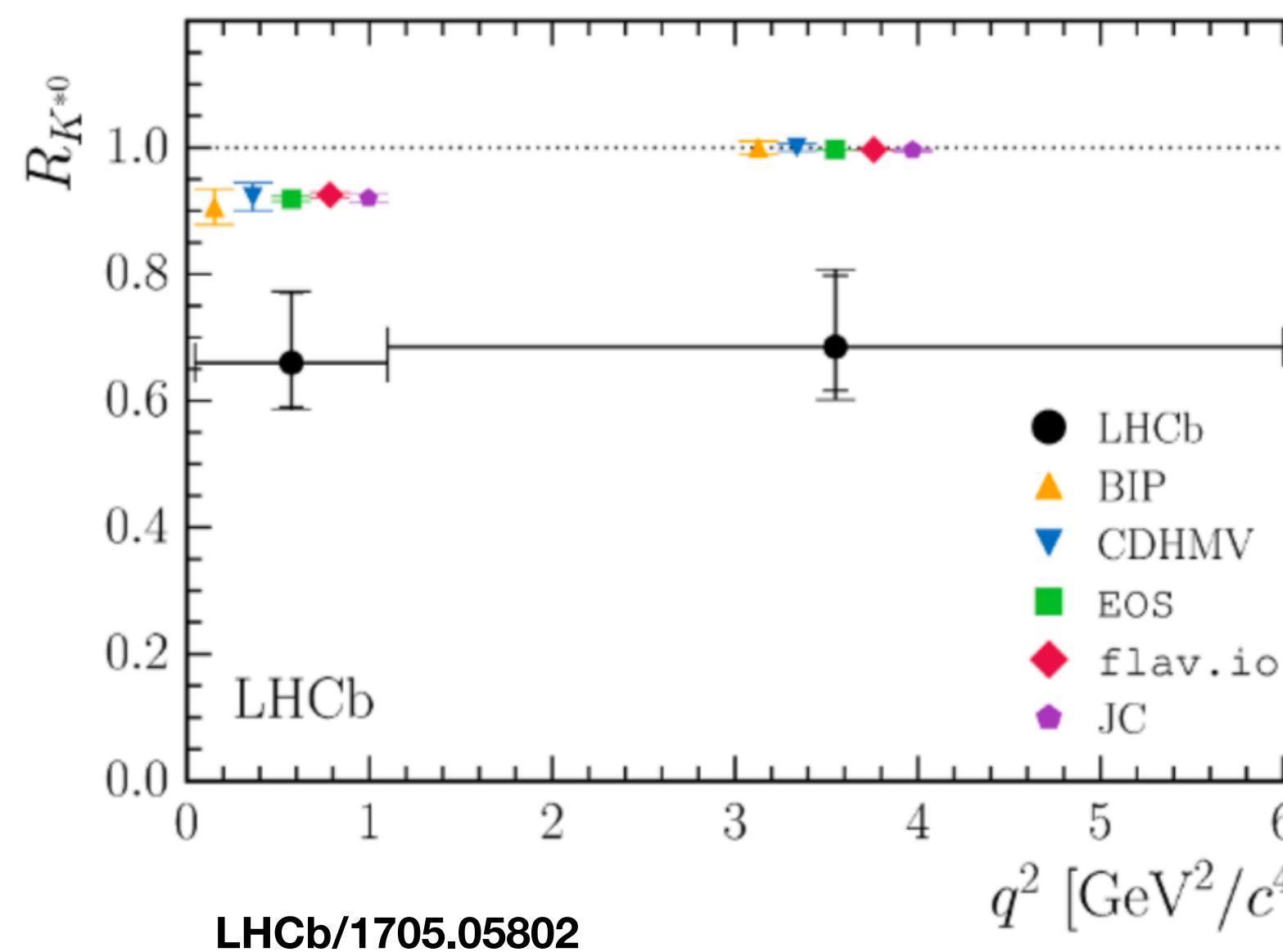
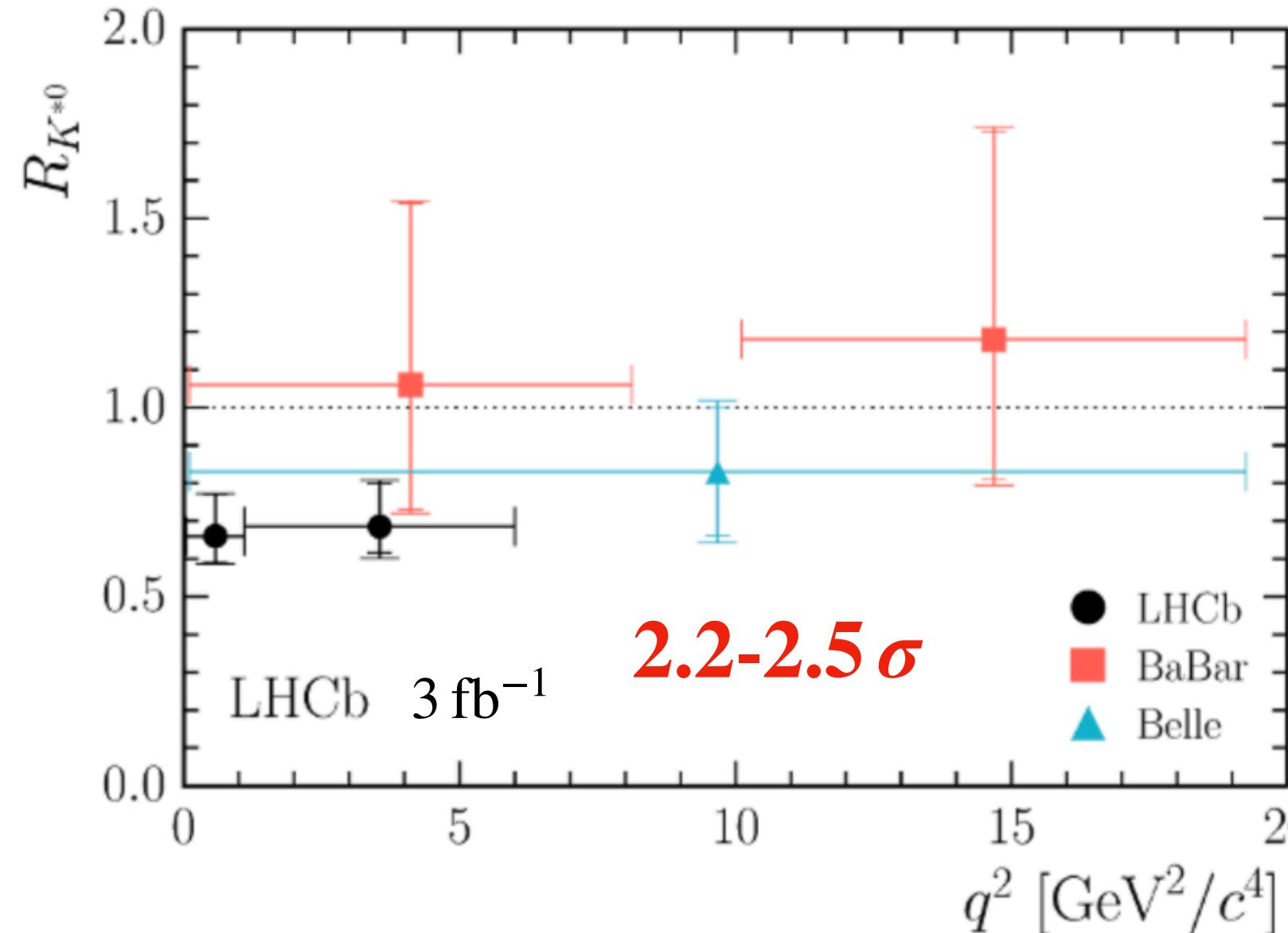
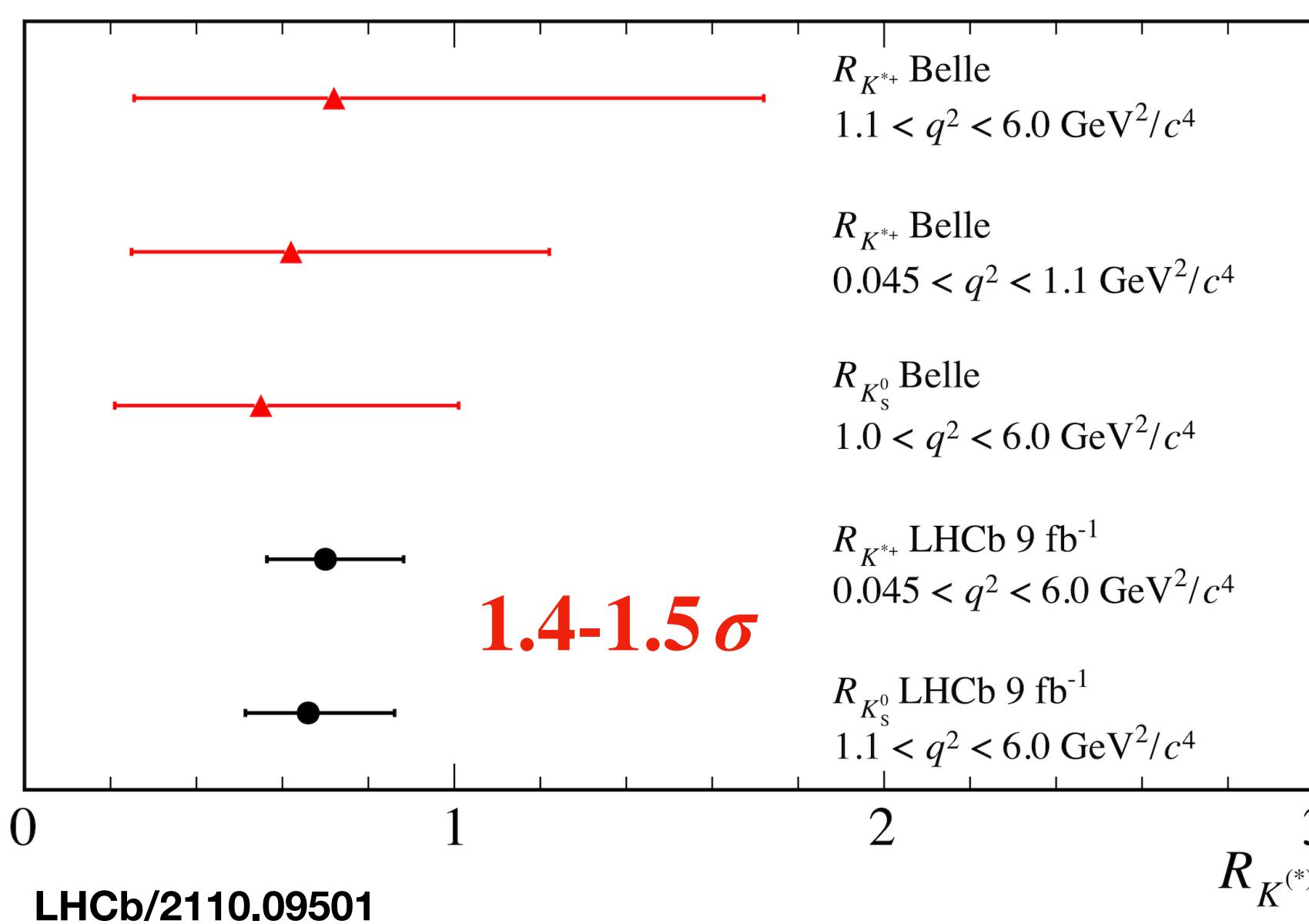
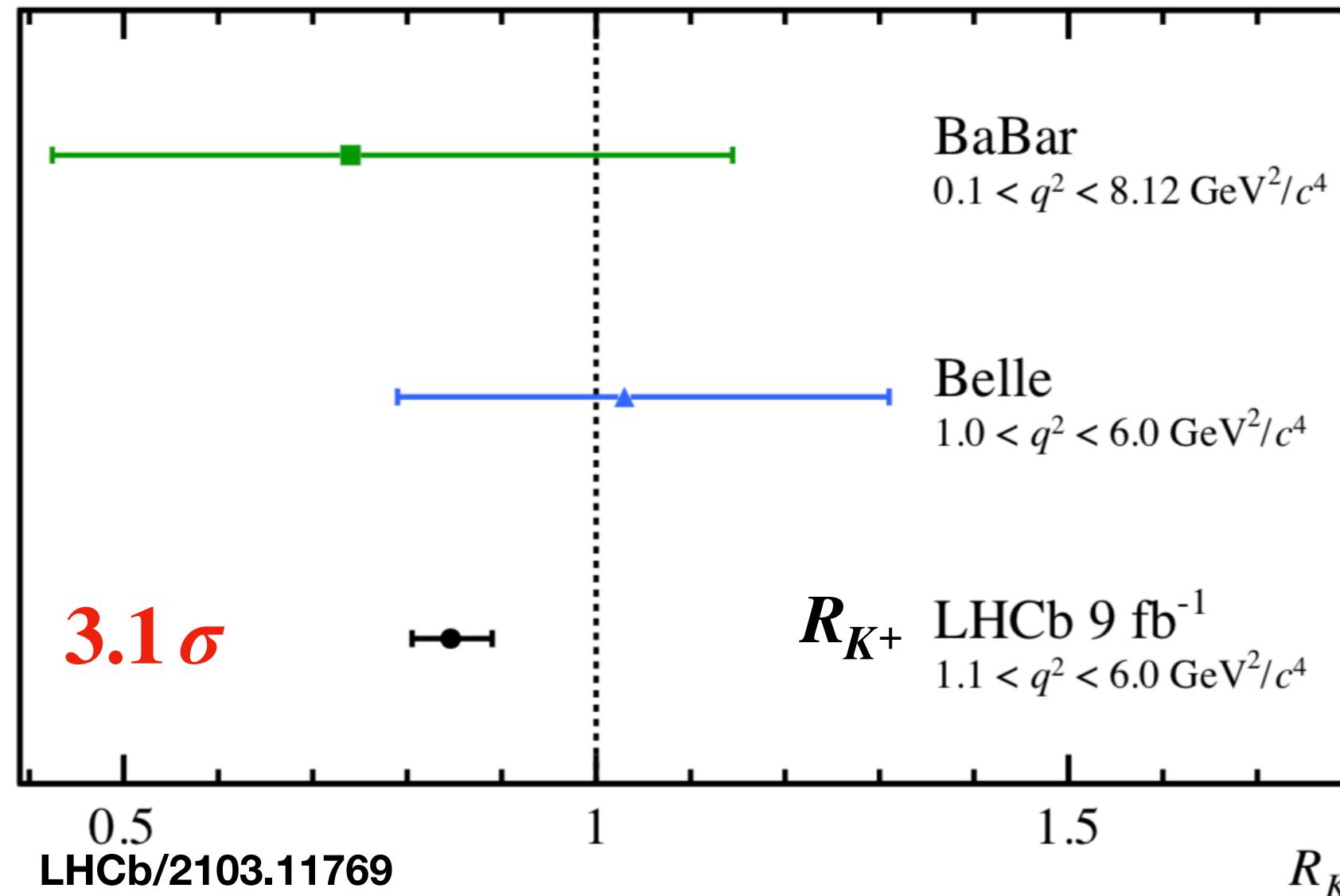
- ▶ EXP below SM
- ▶ Low  $q^2$
- ▶ Theoretical Uncertainties: 😢

# $b \rightarrow s\ell\ell$ anomalies@mid.2022: angular distribution



- ▶ Similar deviations in the 2 modes
- ▶ Theoretical Uncertainties:
  - branching ratio: 😭
  - angular distribution: 😢

# $b \rightarrow s\ell\ell$ anomalies@mid.2022: lepton flavour universality ratio



$$R_{K^+} = \frac{\mathfrak{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathfrak{B}(B^+ \rightarrow K^+ e^+ e^-)}$$

- ▶  $R_H^{\text{SM}} \approx 1$
- ▶ Hadronic uncertainties cancel
- ▶  $\mathcal{O}(10^{-2})$  QED correction

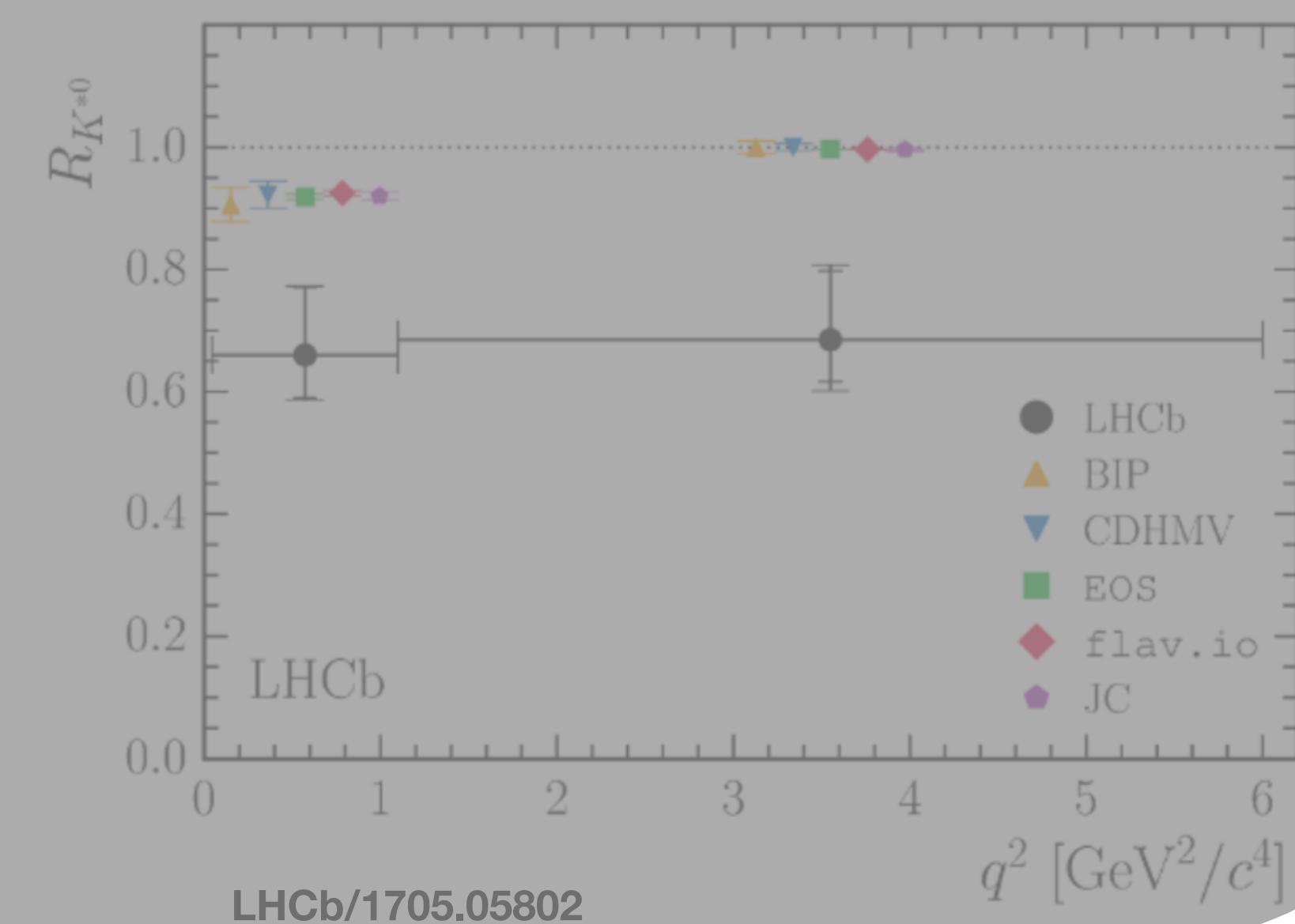
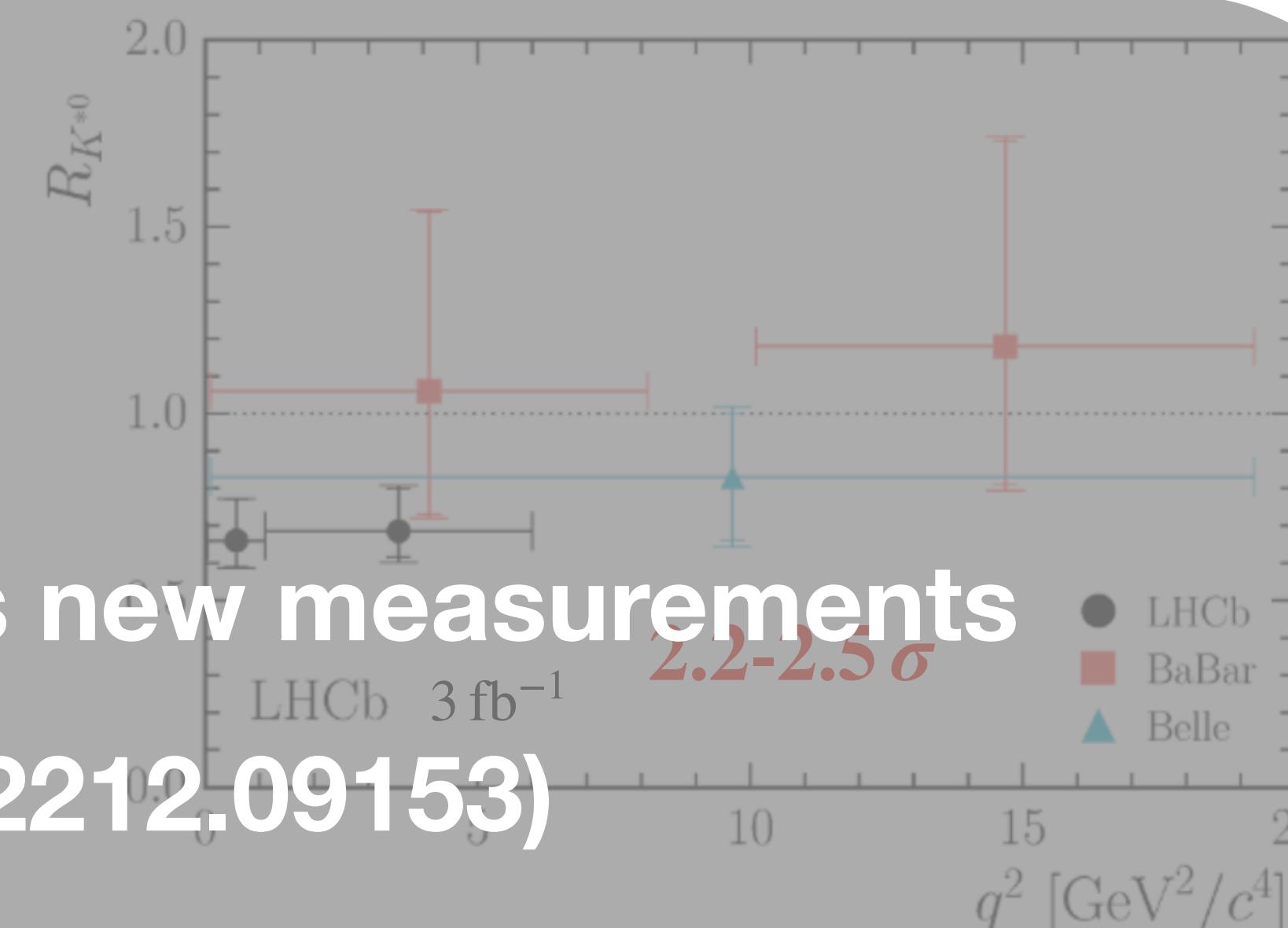
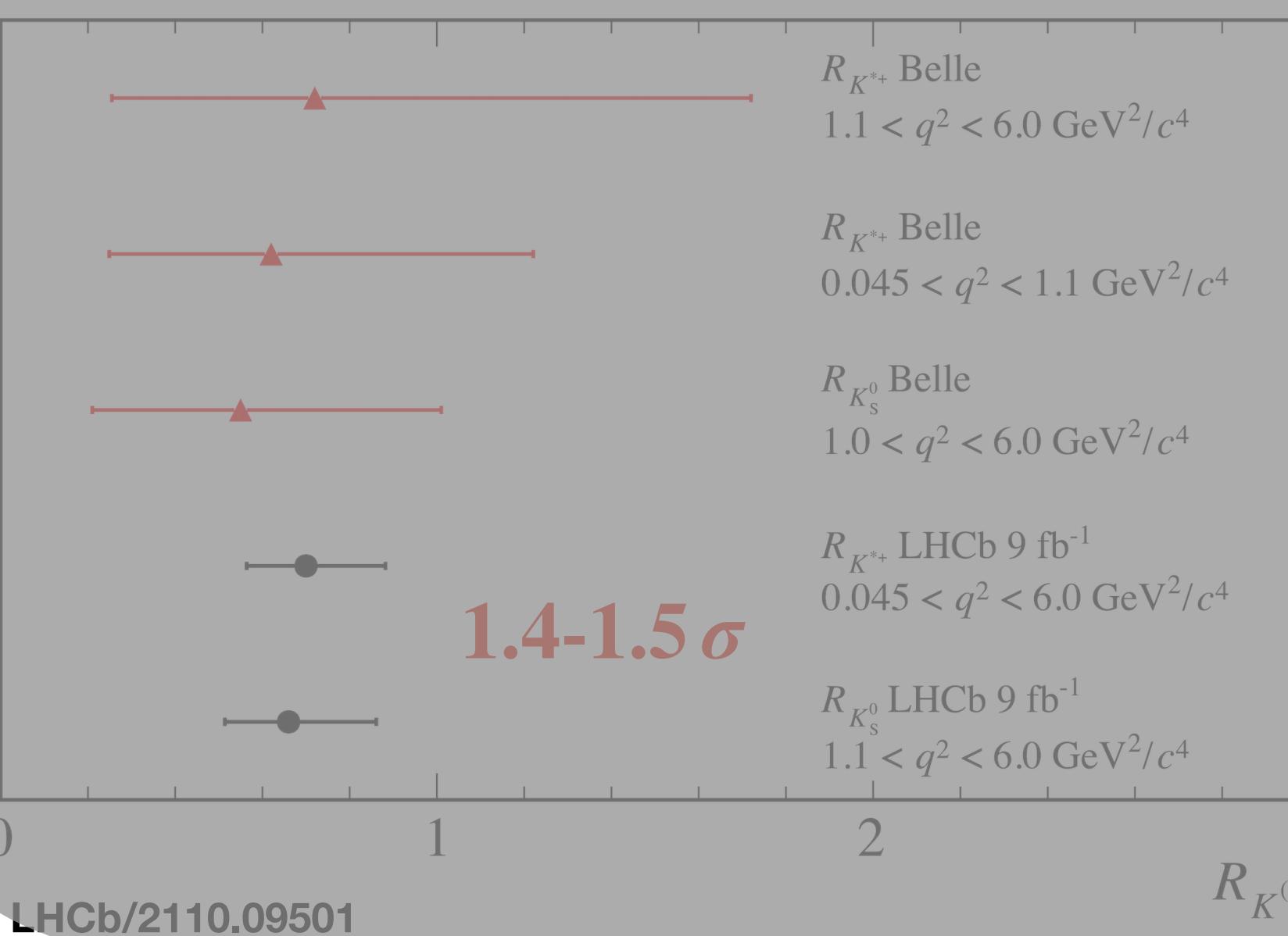
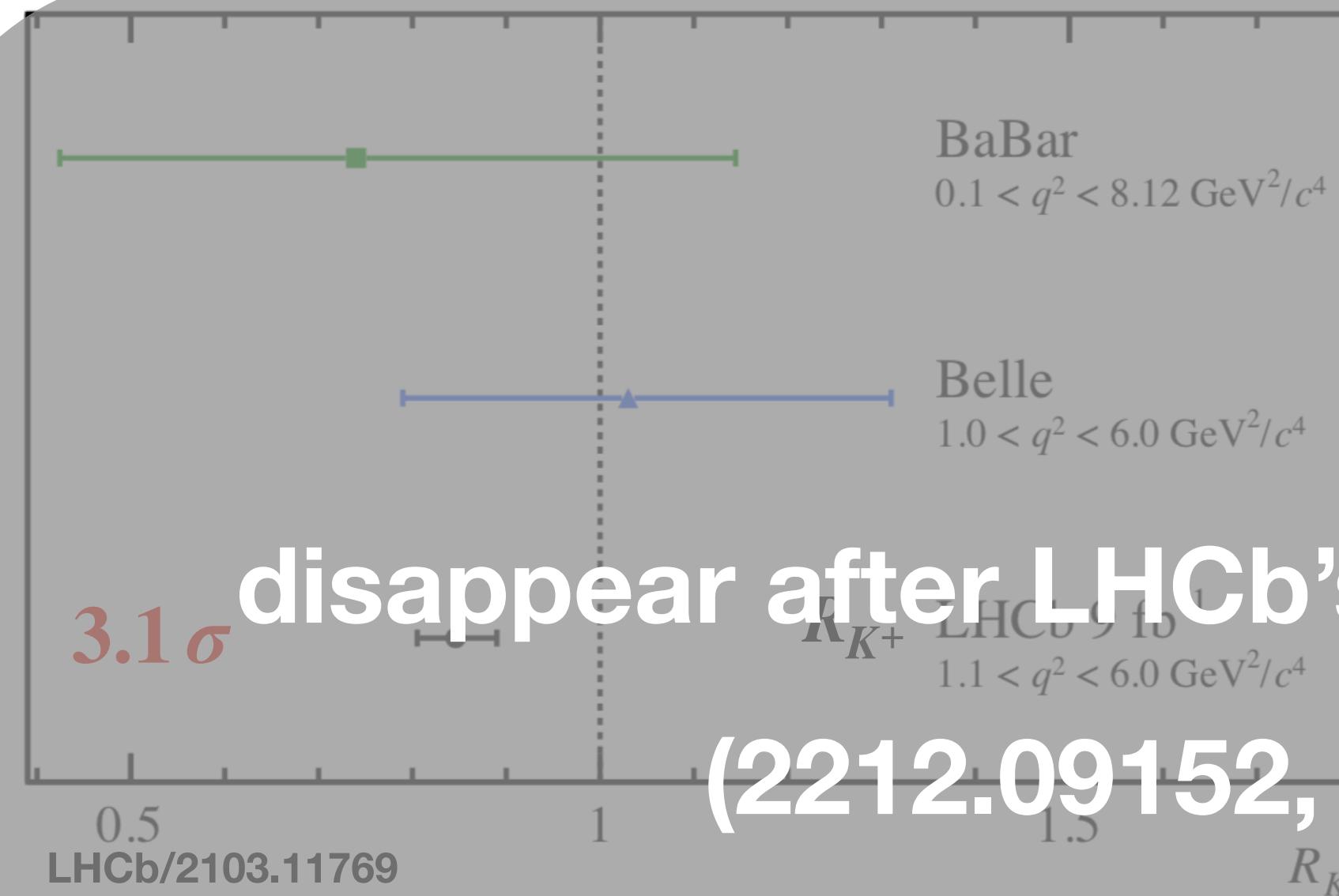
## Theoretical Uncertainties:

- branching ratio: 😢
- angular distribution: 😢
- LFV ratio: 😊

deviation from unity

Physics beyond the SM

# $b \rightarrow s\ell\ell$ anomalies@mid.2022: lepton flavour universality ratio



$$R_{K^+} = \frac{\mathfrak{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathfrak{B}(B^+ \rightarrow K^+ e^+ e^-)}$$

- ▶  $R_H^{\text{SM}} \approx 1$
- ▶ Hadronic uncertainties cancel
- ▶  $\mathcal{O}(10^{-2})$  QED correction

## Theoretical Uncertainties:

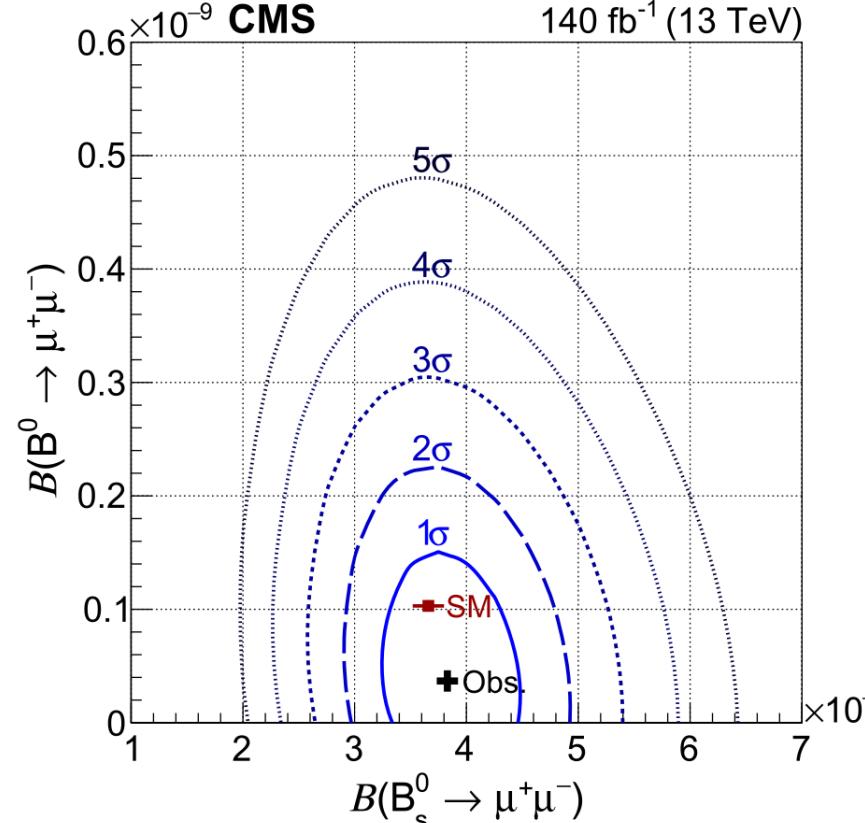
- branching ratio: 😢
- angular distribution: 😢
- LFV ratio: 😊

deviation from unity

Physics beyond the SM

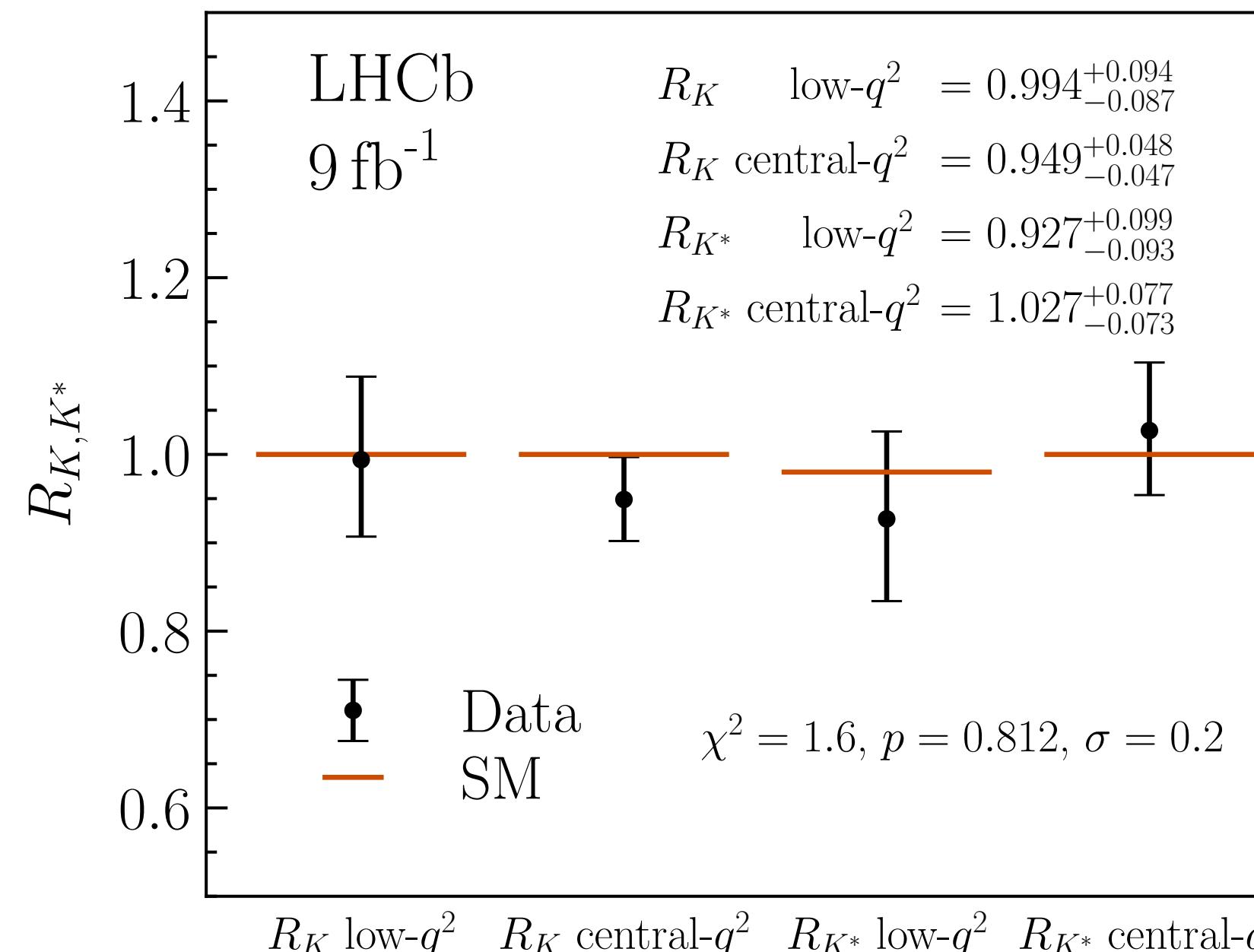
# $b \rightarrow s\ell\ell$ anomalies@2023

## ► New CMS measurements on $B_s \rightarrow \mu^+\mu^-$ (arXiv: 2212.10311)

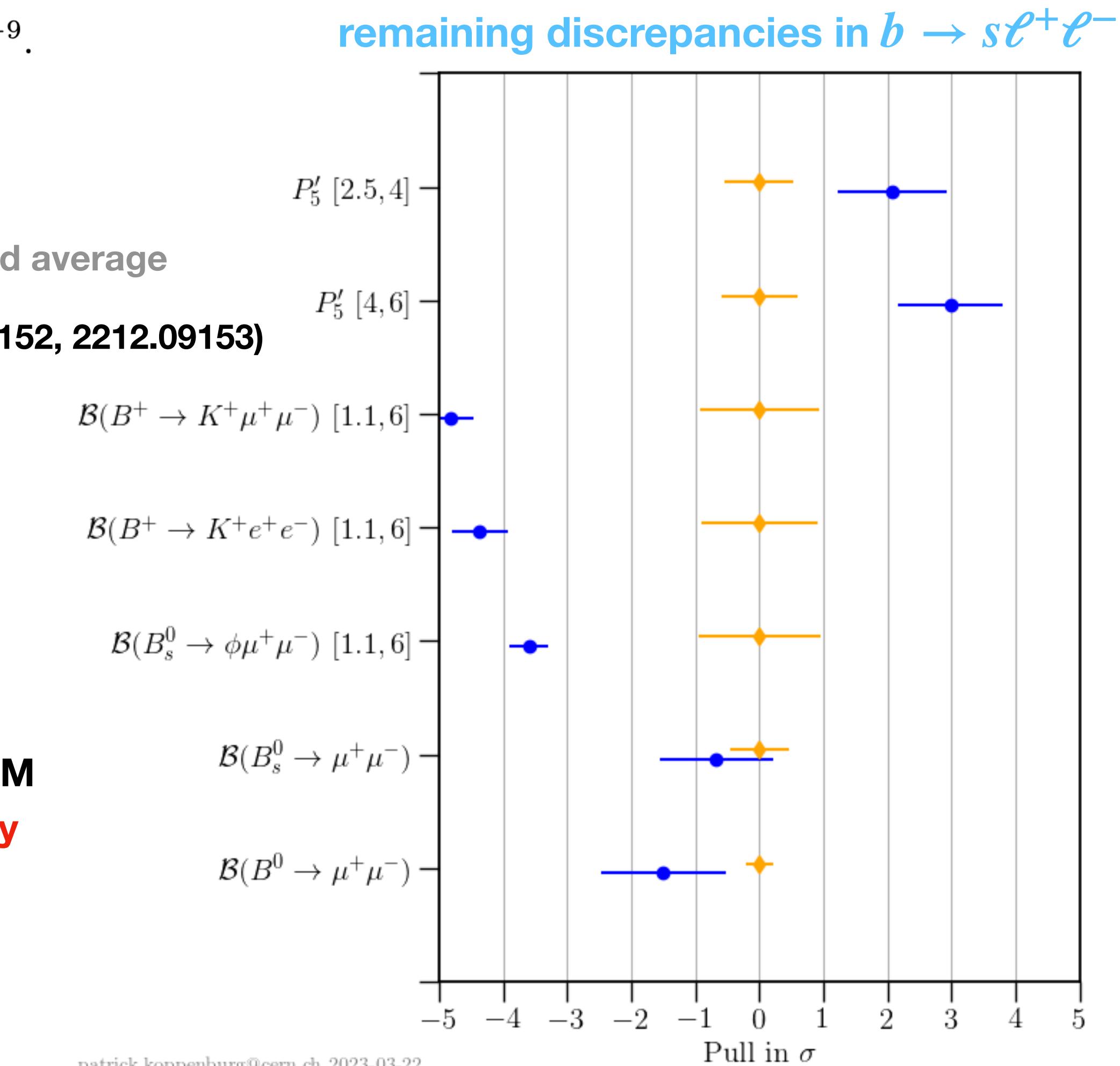


$$\begin{aligned}\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{ATLAS}} &= (2.8^{+0.8}_{-0.7}) \times 10^{-9}, \\ \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{LHCb}} &= (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}, \\ \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{CMS}} &= (3.83^{+0.38+0.19+0.14}_{-0.36-0.16-0.13}) \times 10^{-9}. \\ \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{avg}} &= (3.52^{+0.32}_{-0.30}) \times 10^{-9} \\ \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} &= (3.66 \pm 0.14) \times 10^{-9} \\ \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{avg}} &= (2.93 \pm 0.35) \times 10^{-9}.\end{aligned}$$

## ► New LHCb measurements on $R_K$ and $R_{K^*}$ (arXiv: 2212.09152, 2212.09153)



all consistent with SM  
 $R_K$  and  $R_{K^*}$  anomaly  
 disappear

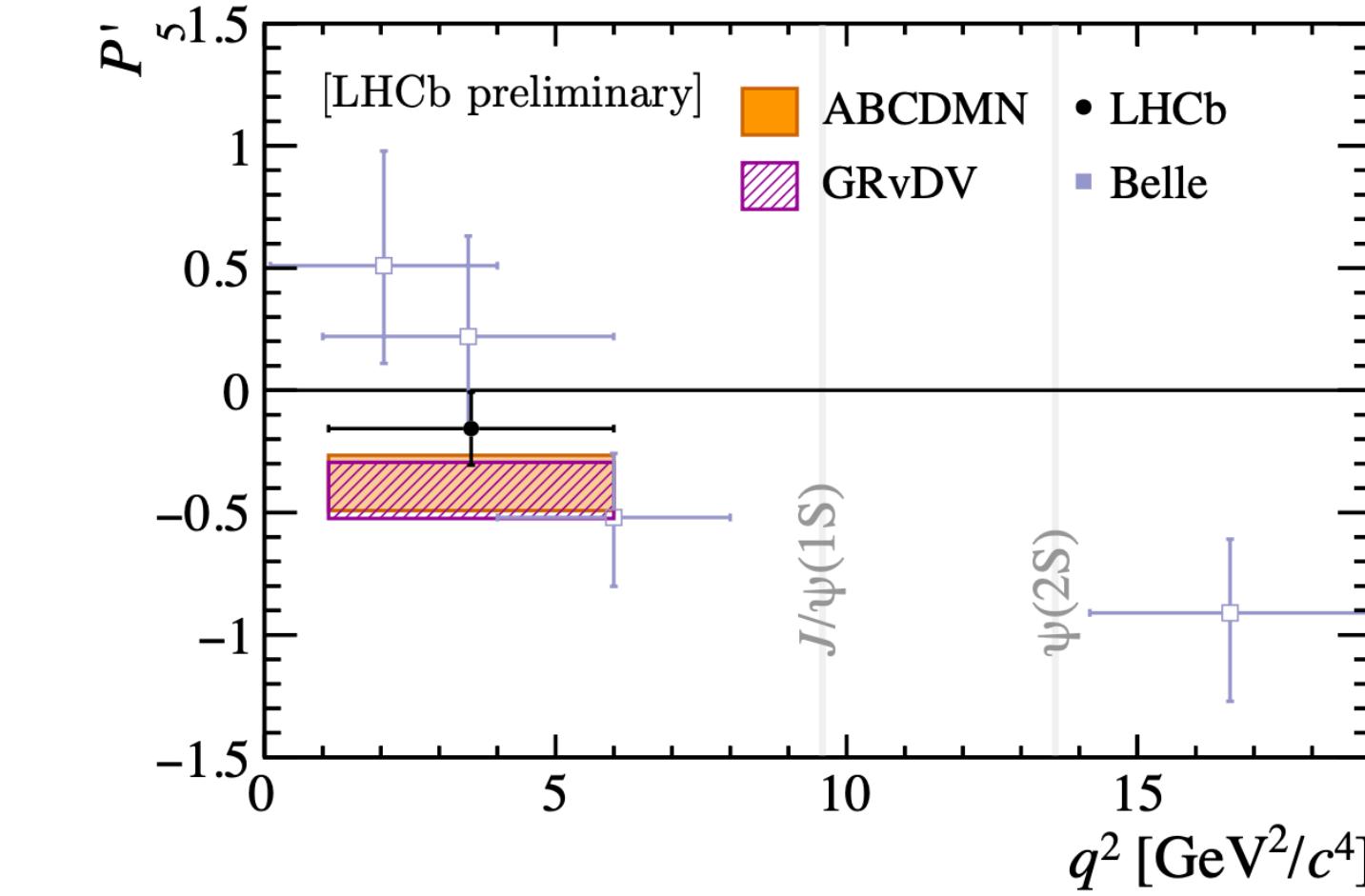
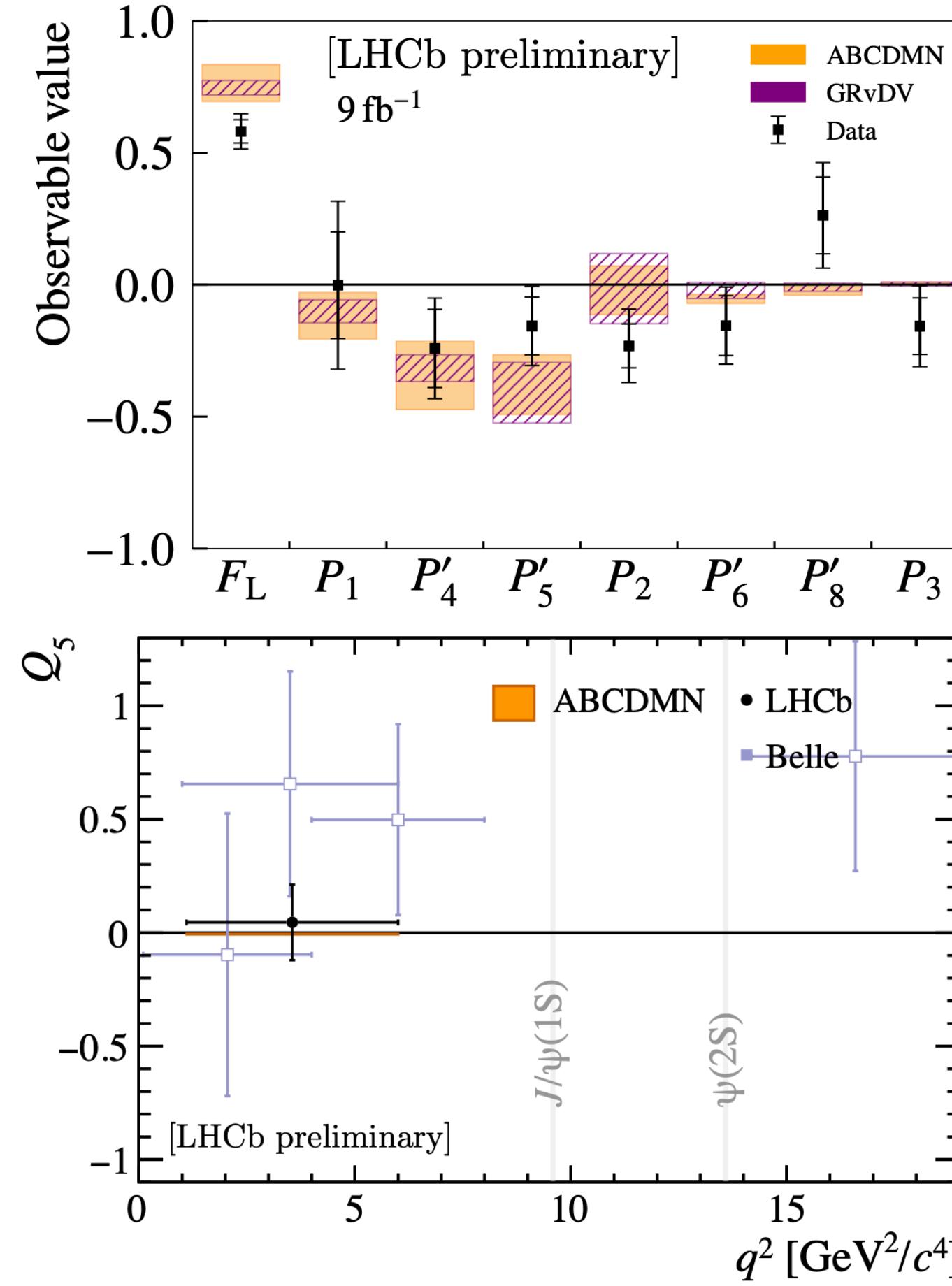
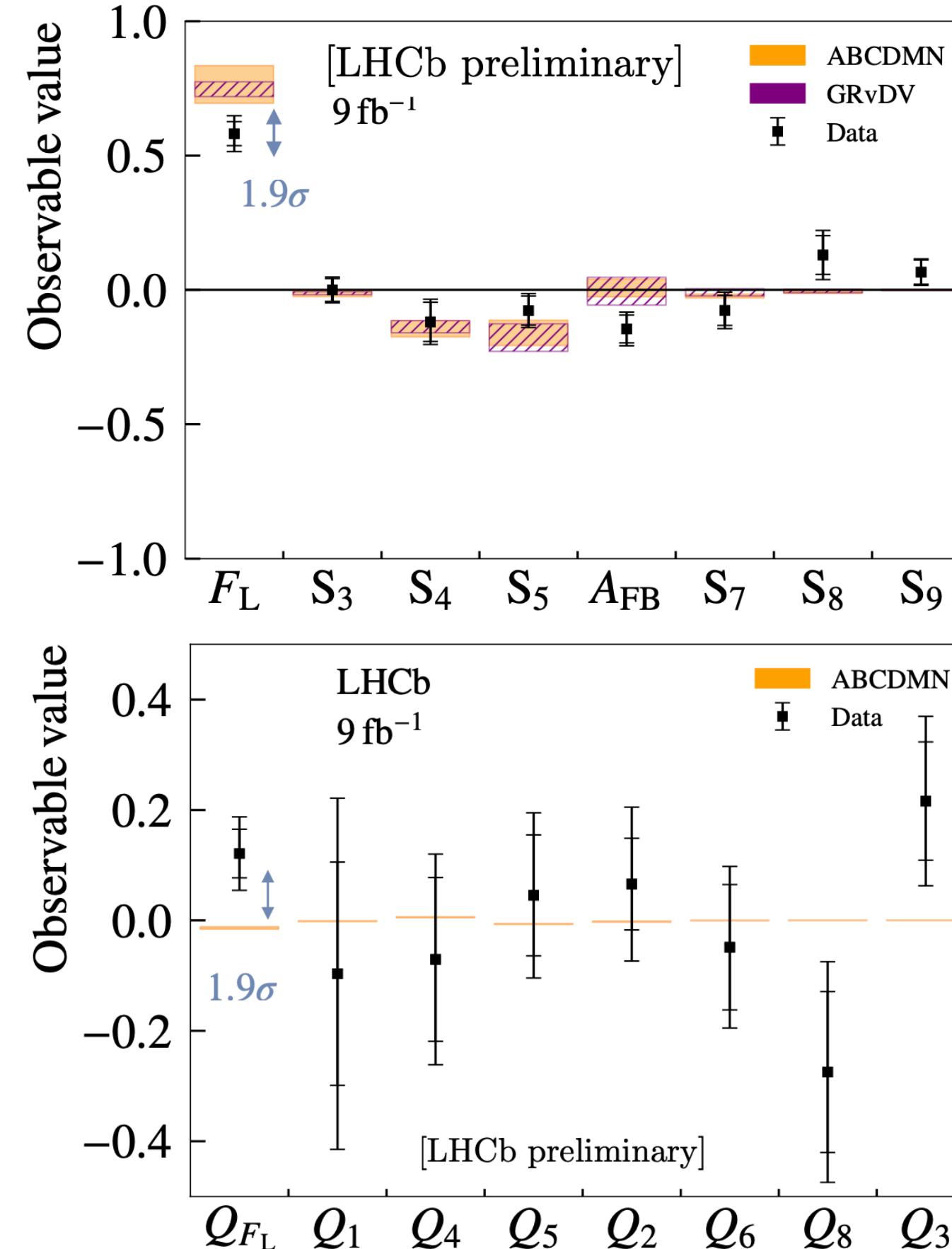


# $b \rightarrow s\ell\ell$ anomalies@mid.2024

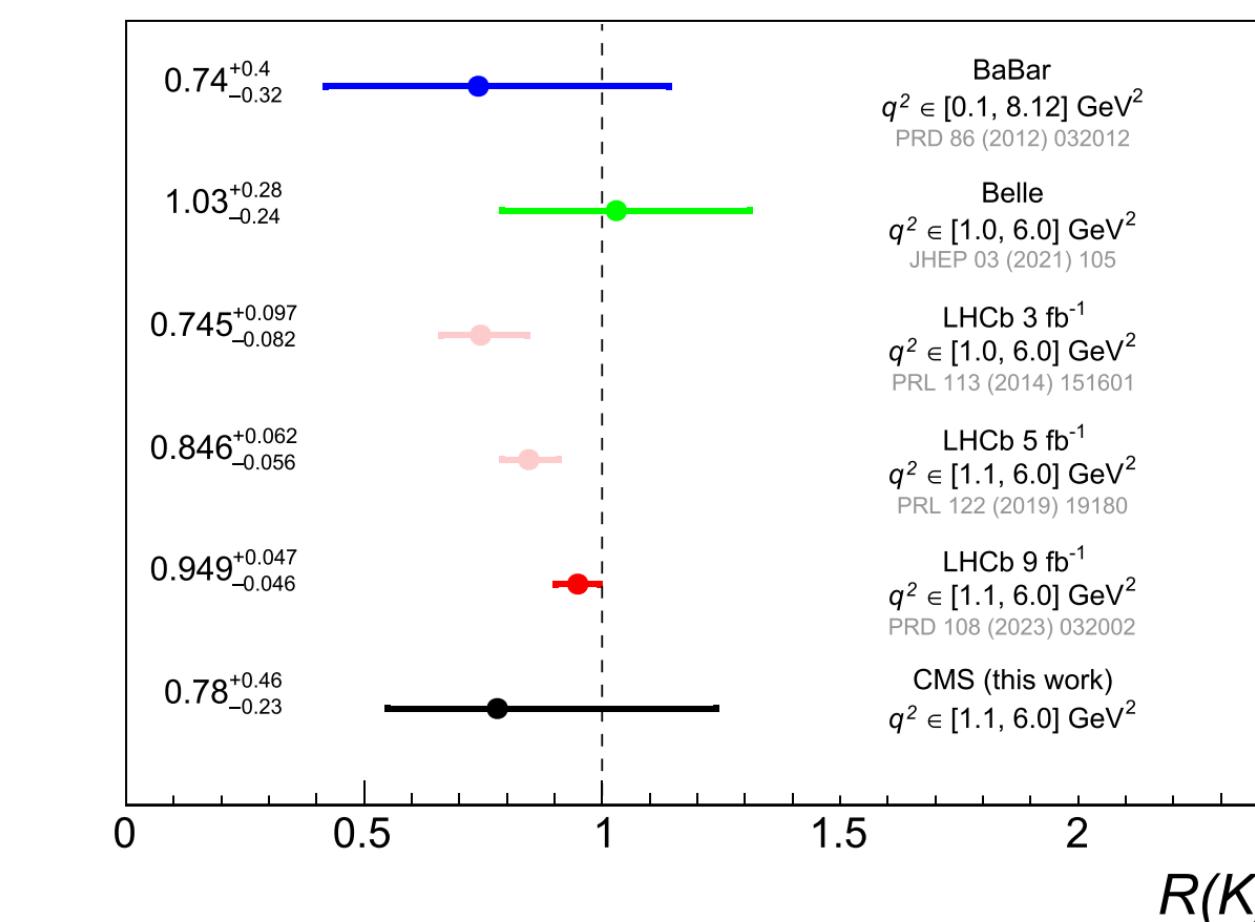
## ► Angular analysis of $B^0 \rightarrow K^{*0}e^+e^-$ at LHCb

LHCb-Paper-2024-022  
R.S.Coutinho's talk@ICHEP

- Based on full Run I + II data
- Performed in  $[1.1, 6.0 (7.0)] \text{ GeV}^2$  region
- $Q_i = P_i^{(\mu)} - P_i^{(e)}$
- All consistent with SM



►  **$R_K$  at CMS** Rep. Prog. Phys. 87 (2024) 077802



## ► Search for $B^0 \rightarrow K^{*0}\tau^+\tau^-$ at Belle II (Meihong Liu's talk@ICHEP)

$\mathcal{B}_{\text{exp}} < 1.8 \times 10^{-3}$  is still far from the SM prediction  $1.0 \times 10^{-7}$

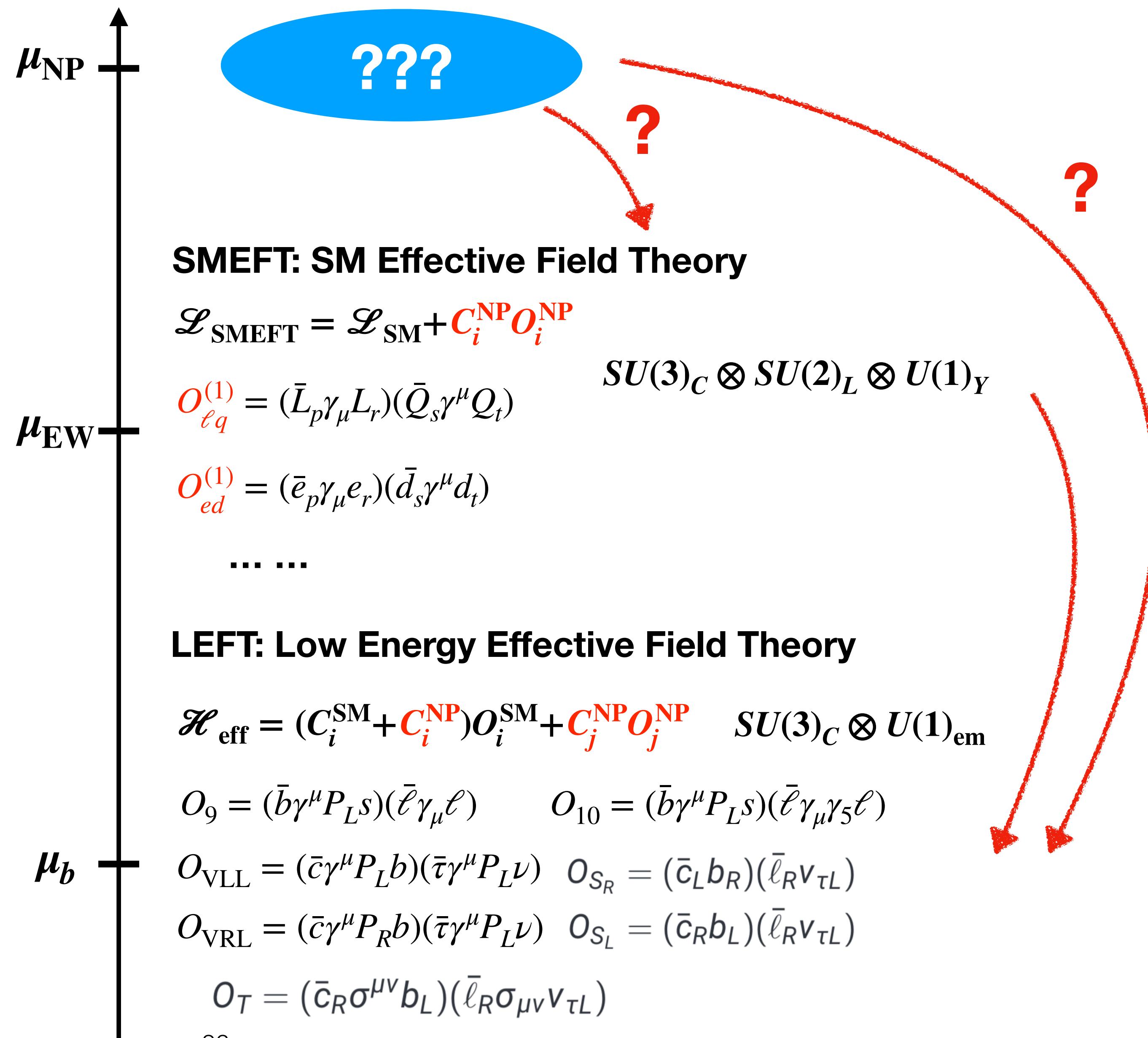
# Flavour anomalies: New Physics interpretation

## ► $b \rightarrow s\ell^+\ell^-$ anomalies

- branching ratio:  $\mathfrak{B}(B_s \rightarrow \phi\mu^+\mu^-)$ , ...
- angular distribution:  $P'_5$  in  $B \rightarrow K^*\mu^+\mu^-$ , ...

## ► $b \rightarrow c\tau\nu$ anomalies

- LFUV ratios (  $\tau$  vs.  $\mu, e$  ) in  $B \rightarrow D^{(*)}\tau\nu$ , ...

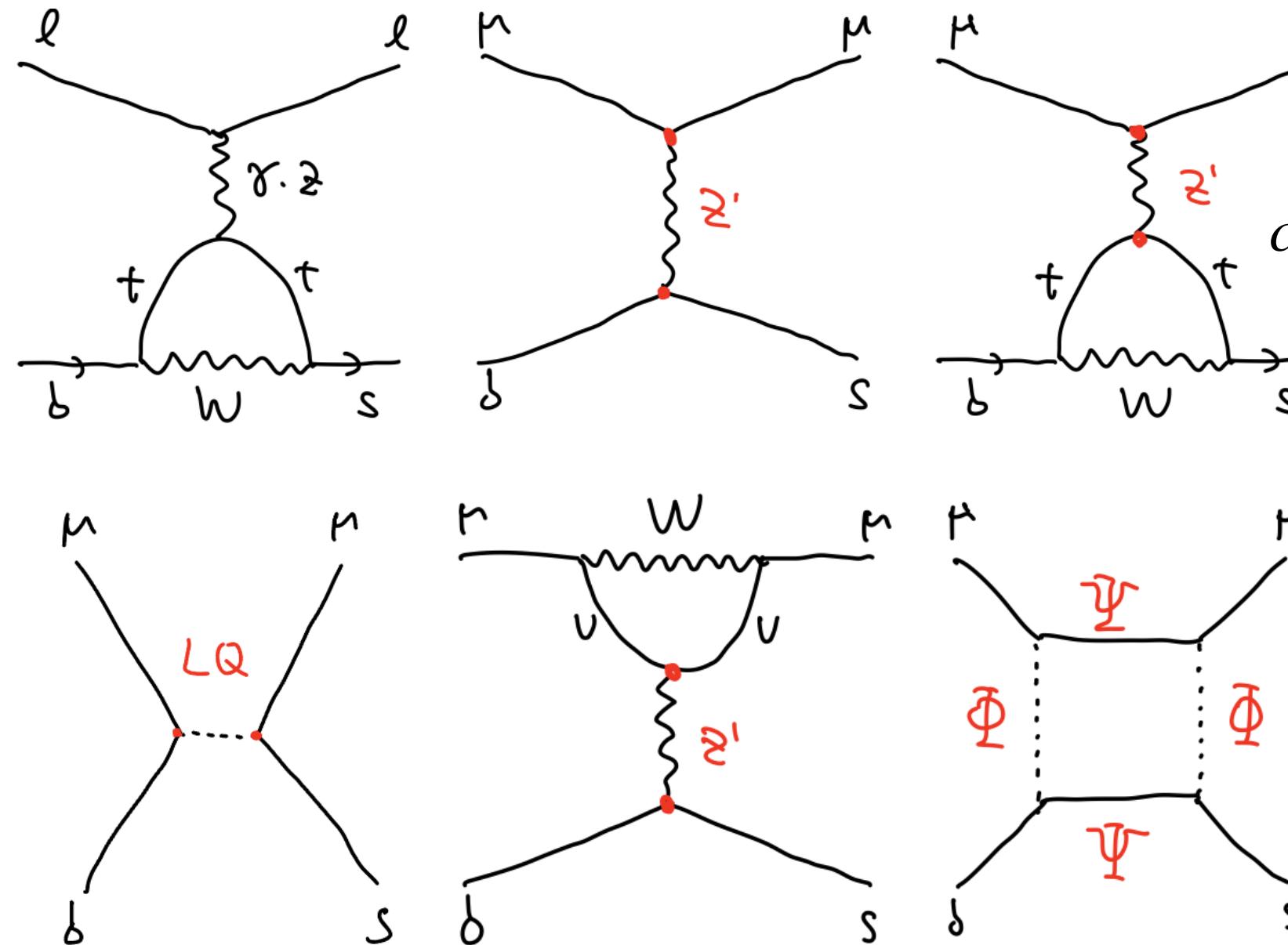


# Flavour anomalies: New Physics interpretation

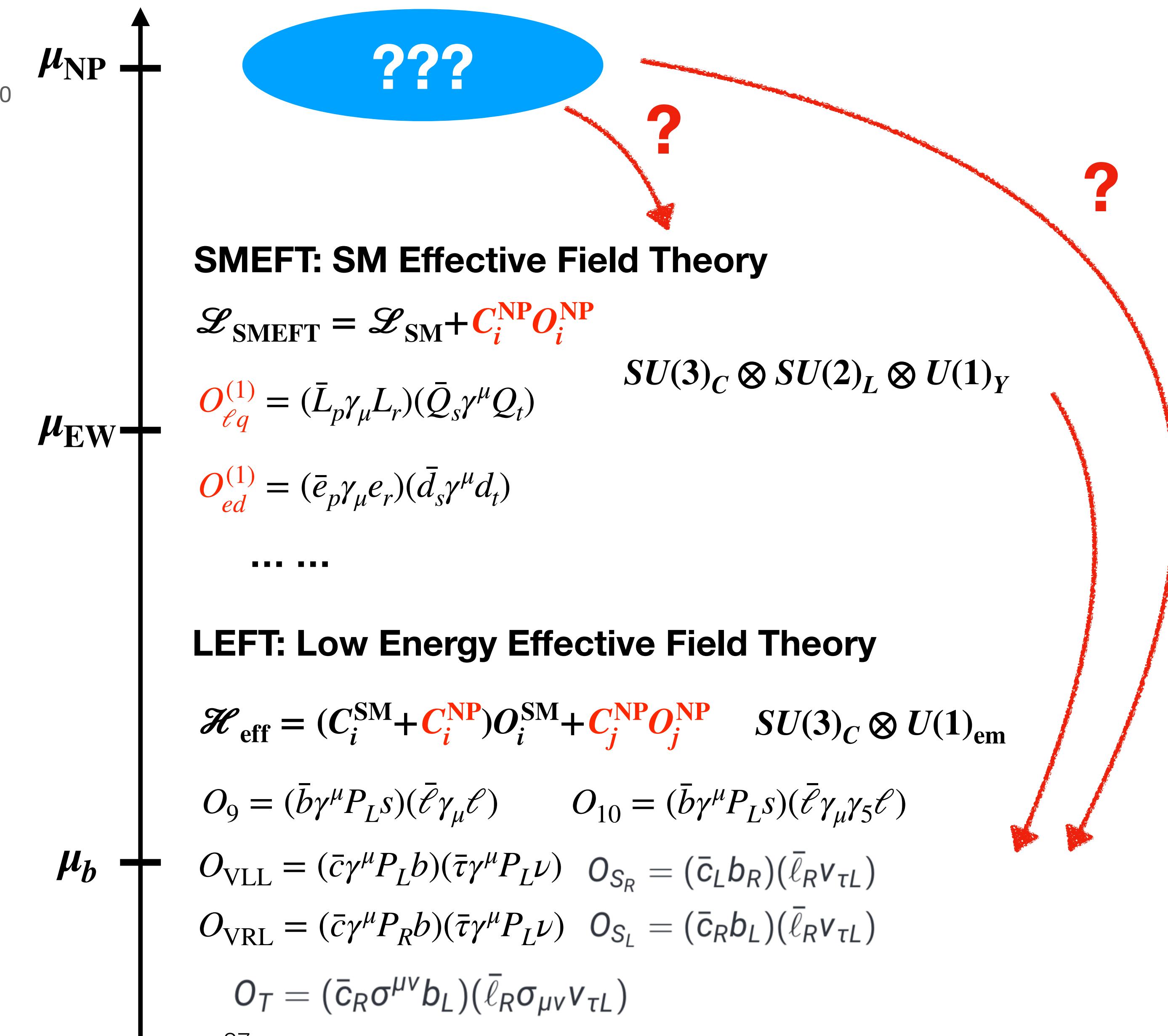
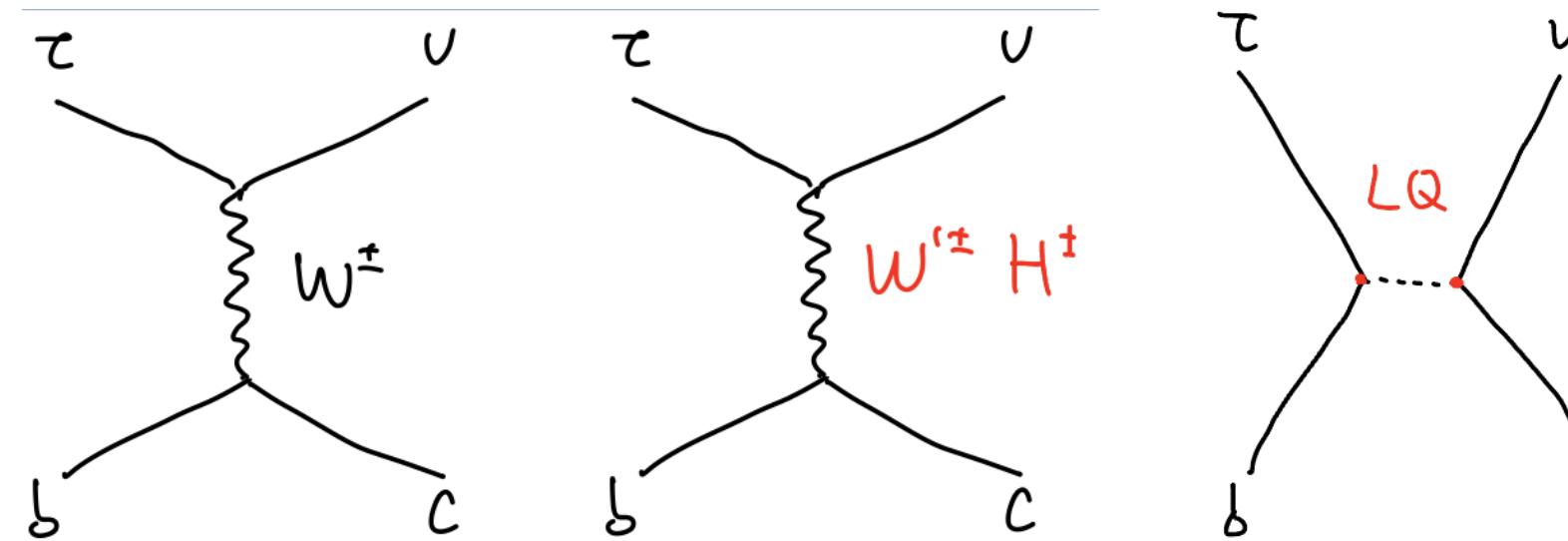
Ying Li, Cai-Dian Lu, 1808.02990

## ► $b \rightarrow s \ell^+ \ell^-$ anomalies

X.Q.Li, Y.D.Yang, XBY, et al, 2112.14215, 2205.02205, 2307.05290



## ► $b \rightarrow c \tau \nu$ anomalies



# $b \rightarrow s\ell\ell$ global fit@mid.2023

## Recent Global Fit

1D Hyp.	All			
	Best fit	$1\sigma/2\sigma$	$\text{Pull}_{\text{SM}}$	p-value
$C_{9\mu}^{\text{NP}}$	-0.67	$[-0.82, -0.52]$ $[-0.98, -0.37]$	4.5	20.2 %
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.19	$[-0.25, -0.13]$ $[-0.32, -0.07]$	3.1	9.9 %

2D Hyp.	All			
	Best fit	$\text{Pull}_{\text{SM}}$	p-value	
$(C_{9\mu}^{\text{NP}}, C_{10\mu}^{\text{NP}})$	$(-0.82, -0.17)$	4.4	21.9%	
$(C_{9\mu}^{\text{NP}}, C_{7'})$	$(-0.68, +0.01)$	4.2	19.4%	
$(C_{9\mu}^{\text{NP}}, C_{9'\mu})$	$(-0.78, +0.21)$	4.3	20.7%	
$(C_{9\mu}^{\text{NP}}, C_{10'\mu})$	$(-0.76, -0.12)$	4.3	20.5%	
$(C_{9\mu}^{\text{NP}}, C_{9e}^{\text{NP}})$	$(-1.17, -0.97)$	5.6	40.3%	

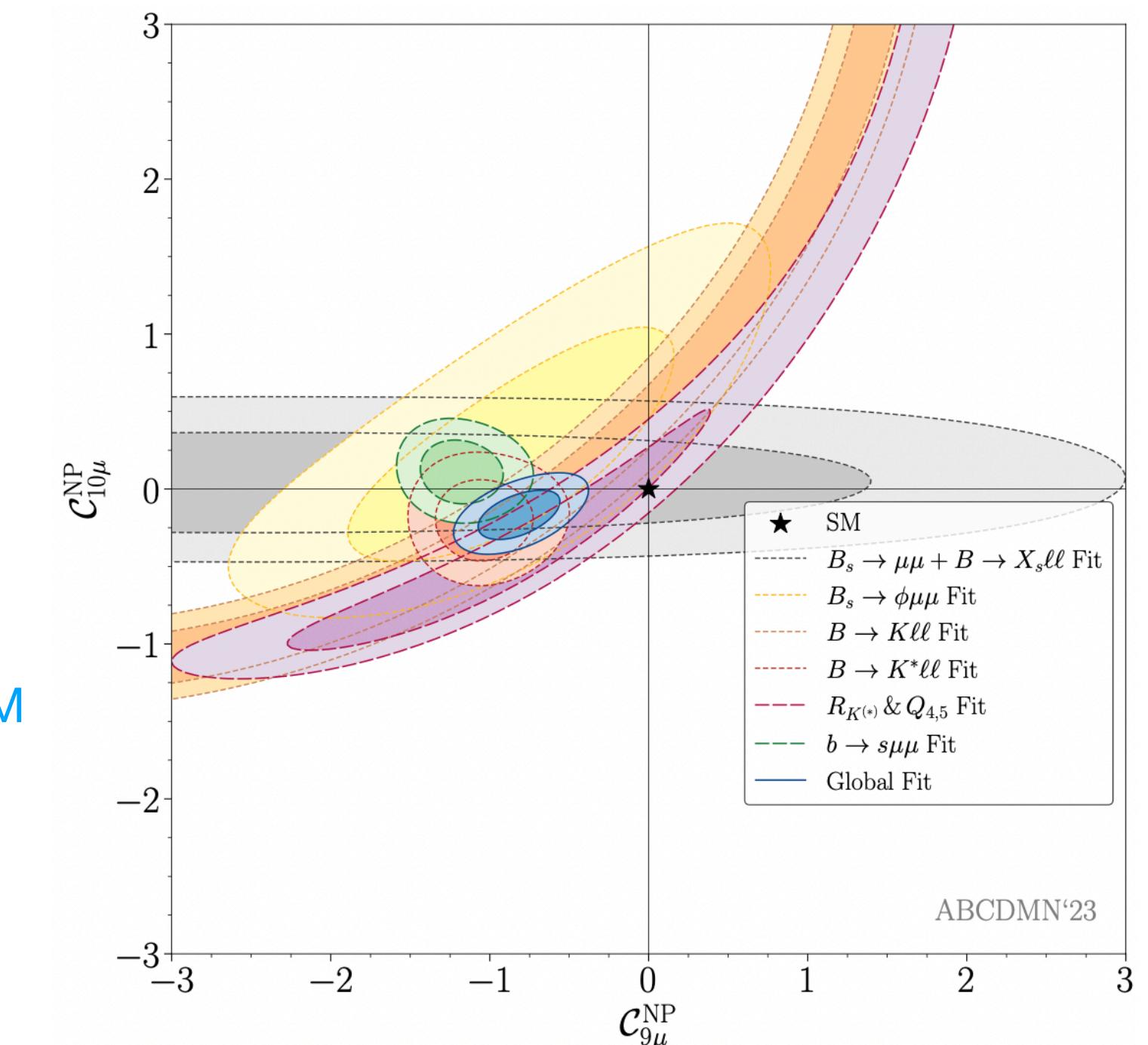
Scenario	Best-fit point	$1\sigma$	$\text{Pull}_{\text{SM}}$	p-value
Scenario 0 $C_{9\mu}^{\text{NP}} = C_{9e}^{\text{NP}} = C_9^U$	-1.17	$[-1.33, -1.00]$	5.8	39.9 %
Scenario 5 $C_{9\mu}^V$	-1.02	$[-1.43, -0.61]$		
Scenario 5 $C_{10\mu}^V$	-0.35	$[-0.75, -0.00]$	4.1	21.0 %
Scenario 5 $C_9^U = C_{10}^U$	+0.19	$[-0.16, +0.58]$		
Scenario 6 $C_{9\mu}^V = -C_{10\mu}^V$	-0.27	$[-0.34, -0.20]$	4.0	18.0 %
Scenario 6 $C_9^U = C_{10}^U$	-0.41	$[-0.53, -0.29]$		
Scenario 7 $C_{9\mu}^V$	-0.21	$[-0.39, -0.02]$	5.6	40.3 %
Scenario 7 $C_9^U$	-0.97	$[-1.21, -0.72]$		
Scenario 8 $C_{9\mu}^V = -C_{10\mu}^V$	-0.08	$[-0.14, -0.02]$	5.6	41.1 %
Scenario 8 $C_9^U$	-1.10	$[-1.27, -0.91]$		

Ciuchini et al 2212.10516  
Alguero et al 2304.07330  
Qiaoyi Wen, Fanrong Xu 2305.19038

$$O_9 = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \ell)$$

$$O_{10} = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  consistent with SM  
( $C_{10}$  can't be too large)

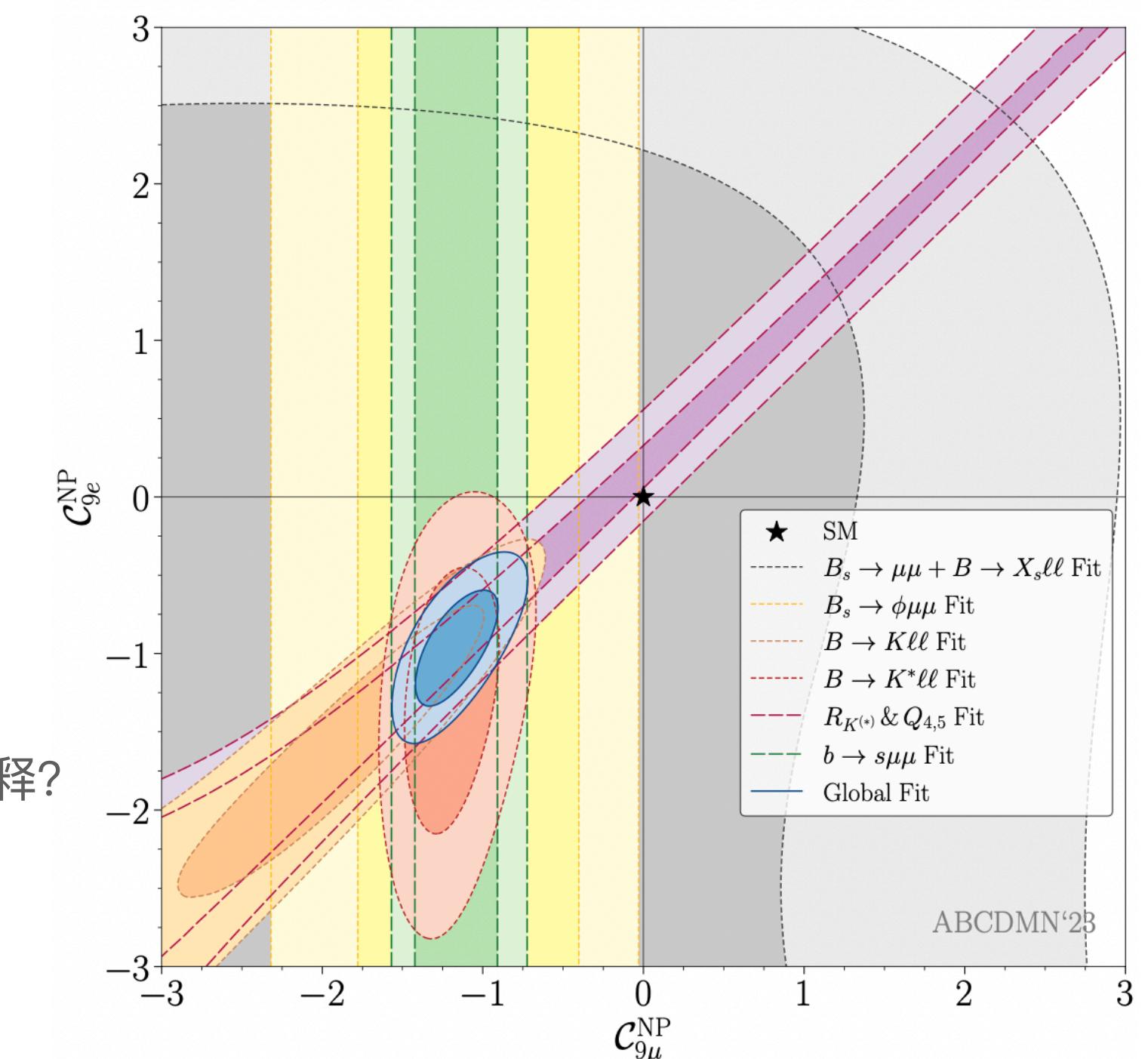


No  $R_K, R_{K^*}$  anomalies now !

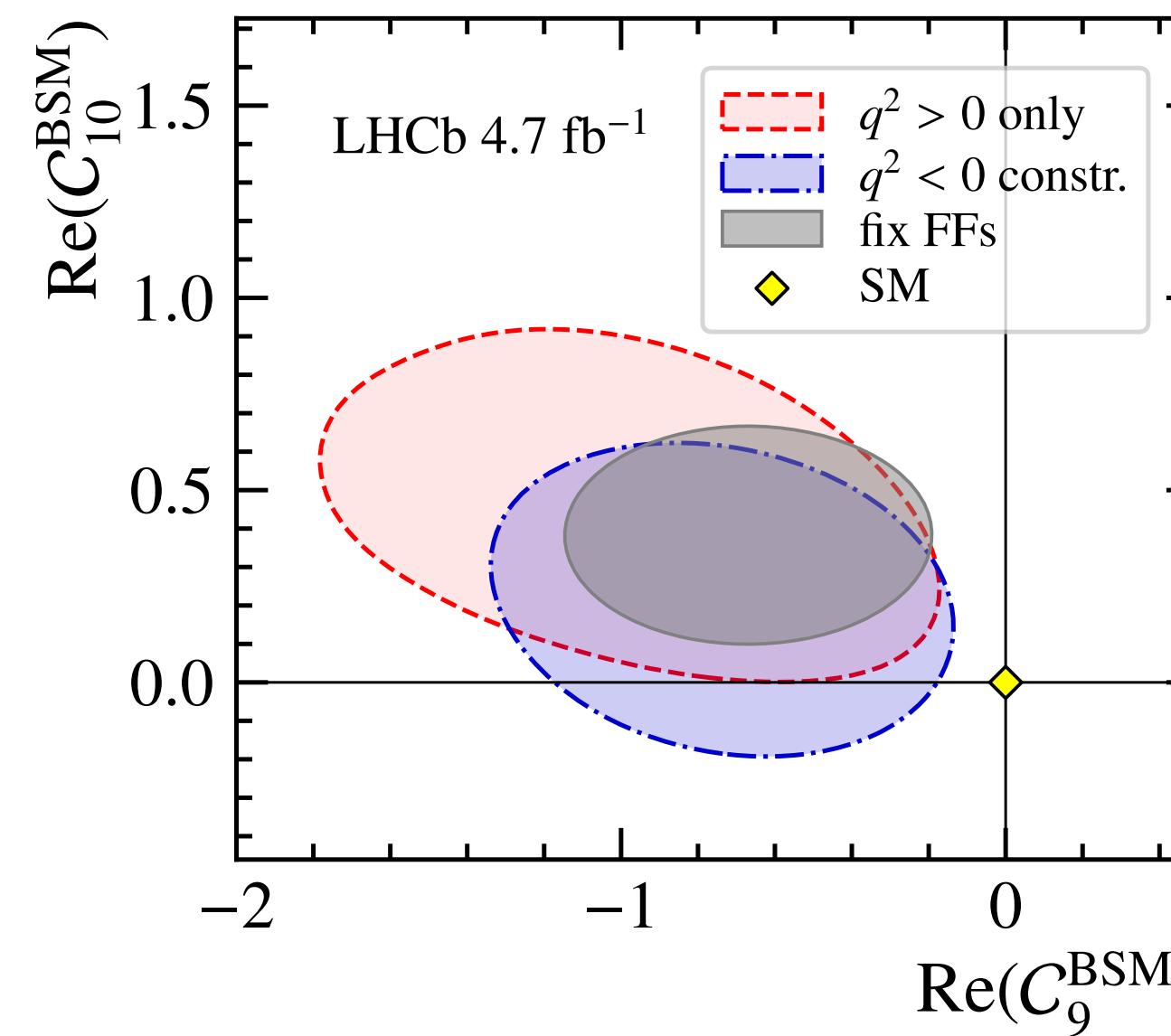
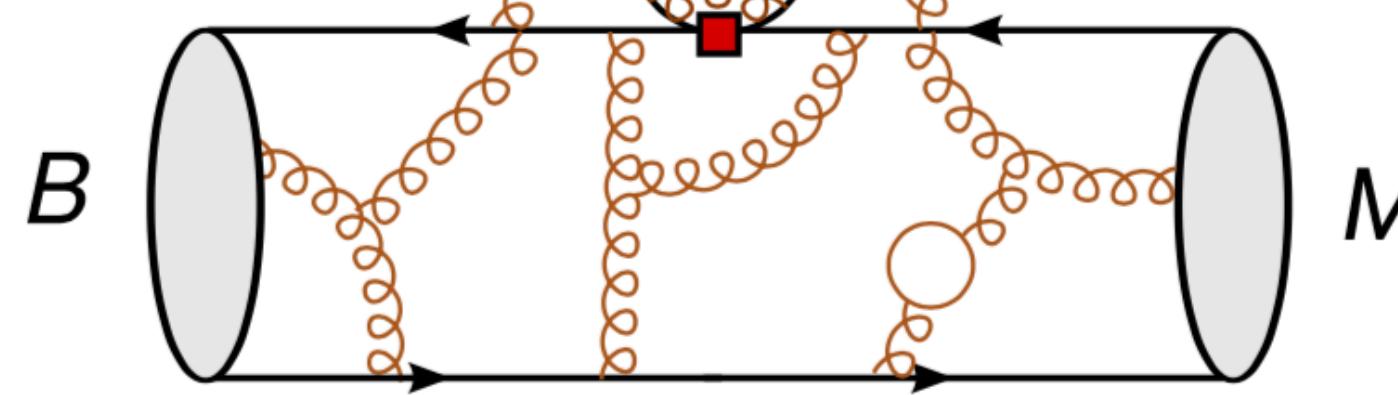
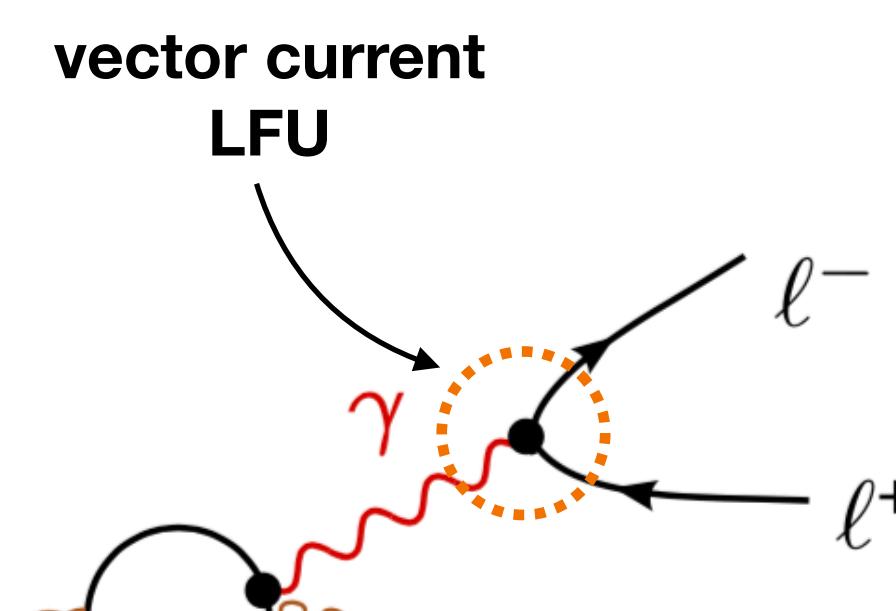
$$C_{9e} = C_9^U$$

$$C_{9\mu} = C_9^U + C_9^V$$

所有的good fit 都不包含大的C10  
不包含C10意味着可以用non-local matrix解释?



# Charm-loop contribution



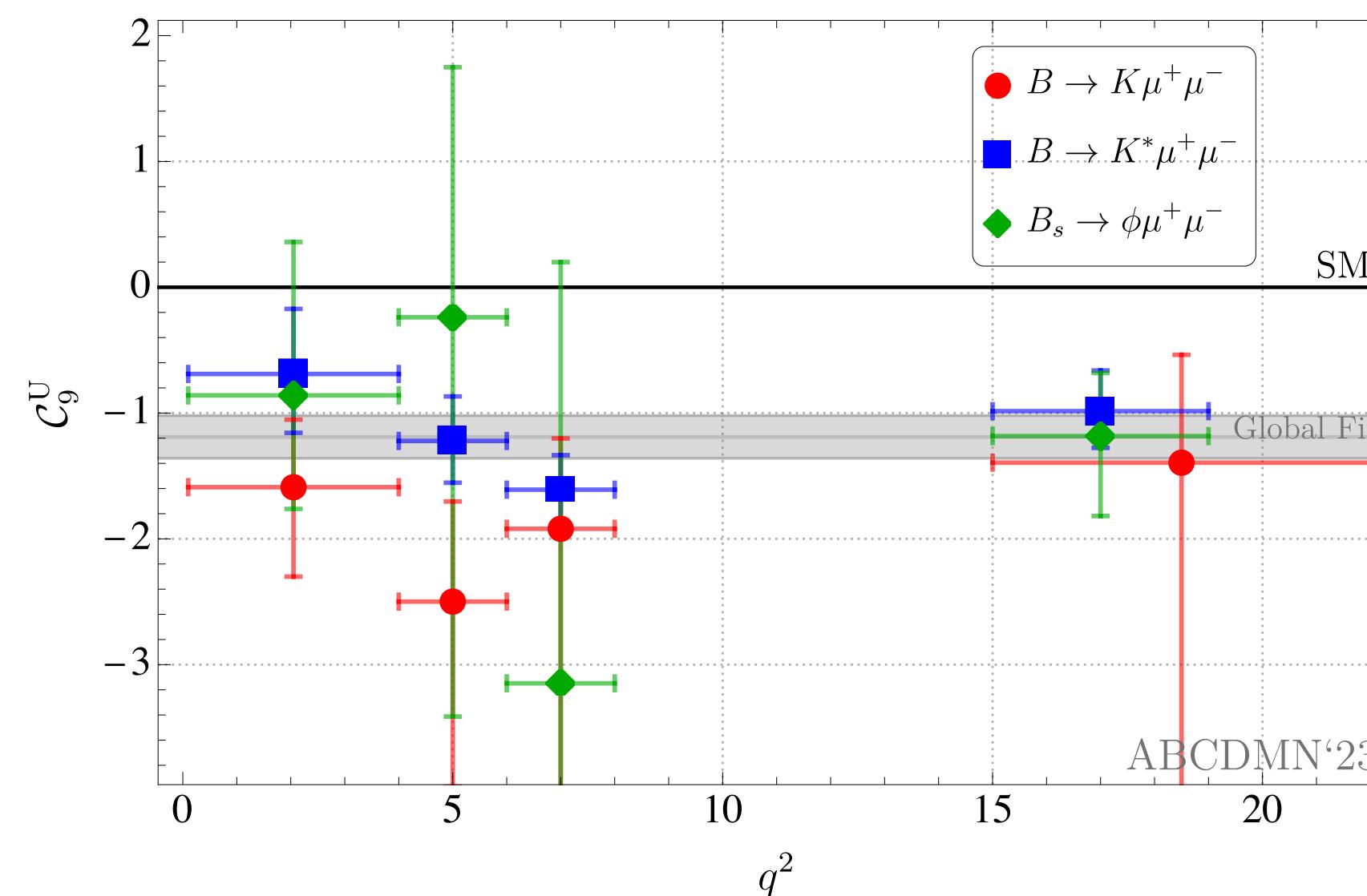
- Global fit prefer to  $C_{9e} = C_{9\mu} \neq C_9^{\text{SM}} \Leftarrow \mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}}$  is consistent with SM
- Charm-loop could mimic  $C_{9e} = C_{9\mu}$

$$C_{9e} = C_{9\mu} = C_9^{\text{SM}} + \Delta C_9^U, \text{charm loop} + \Delta C_9^U, \text{NP}$$

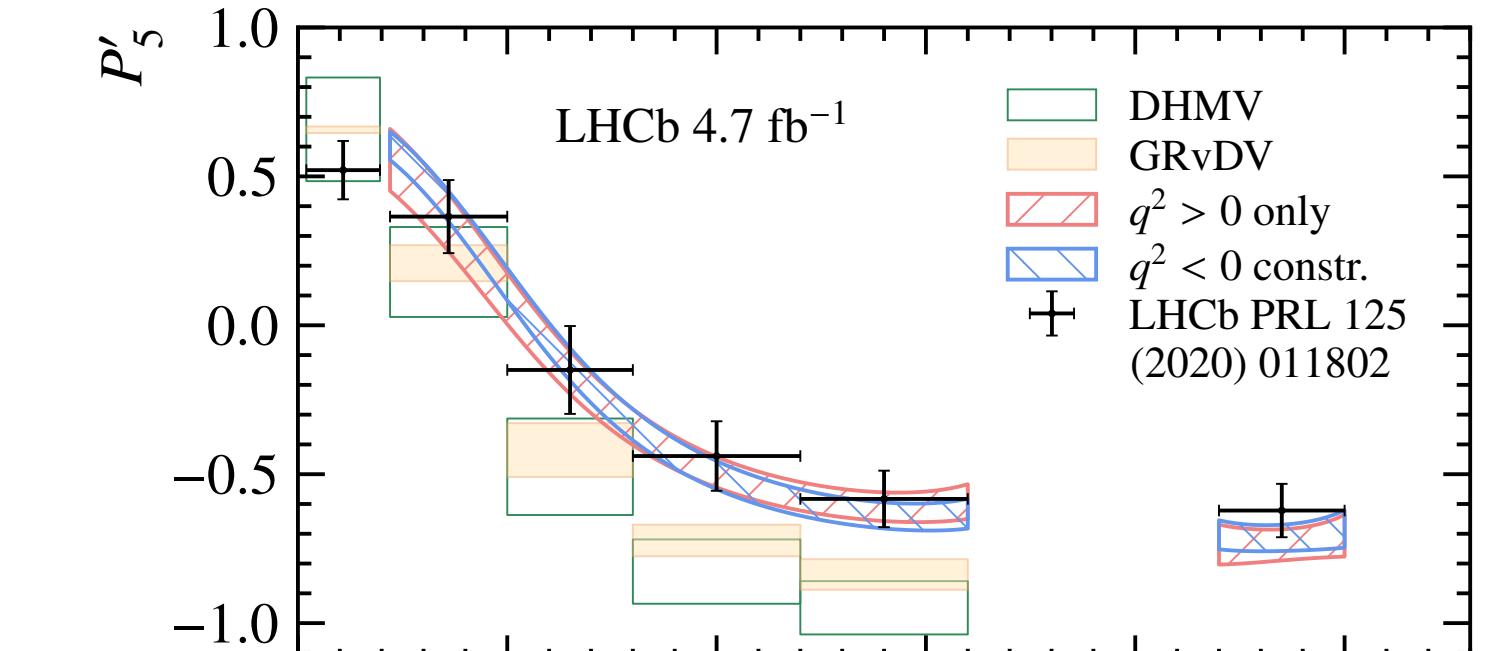
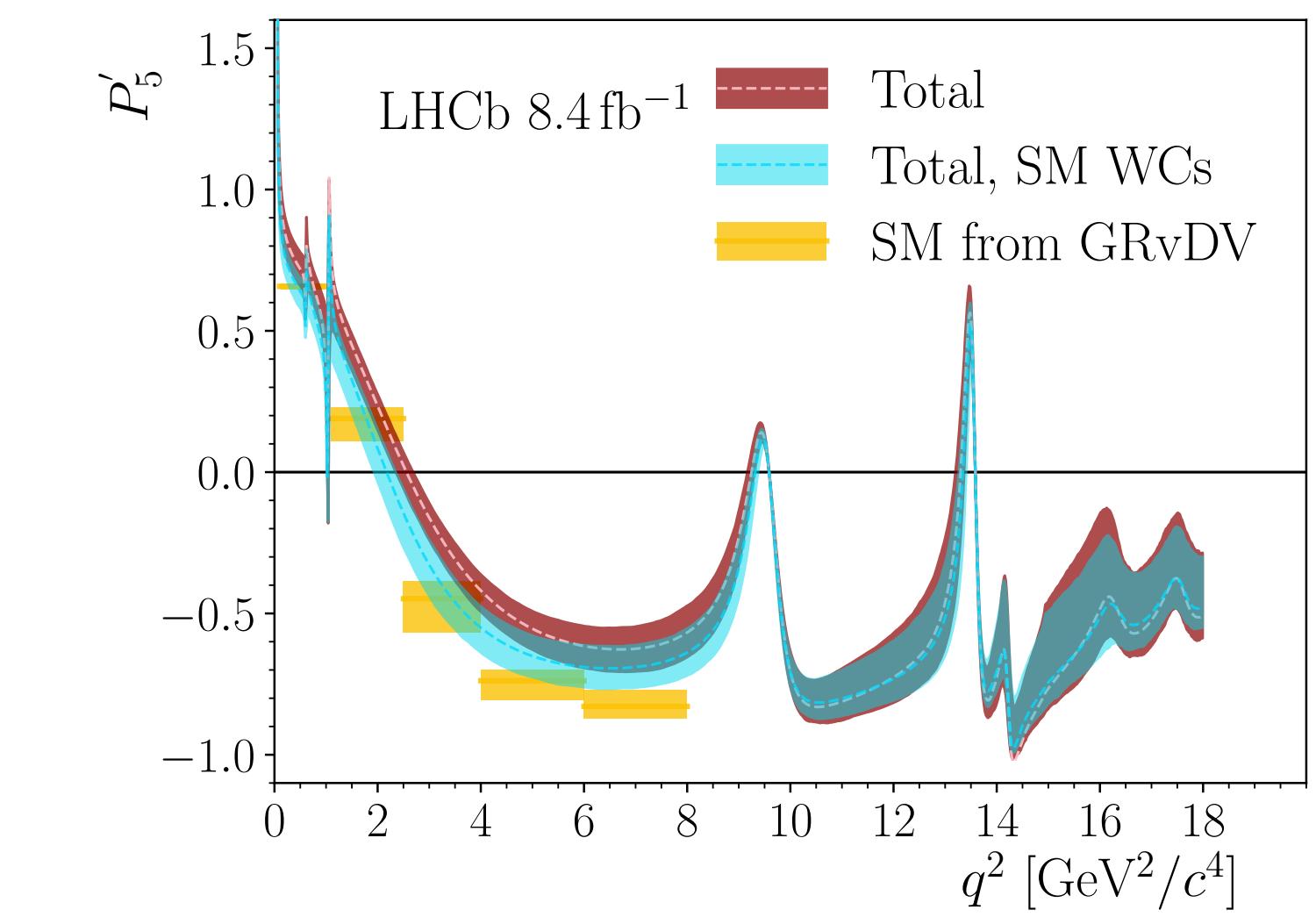
$$O_9 = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \ell)$$

$$O_{10} = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

- Charm-loop contribution is expected to be  $\Delta C_9^U(q^2)$ , but not  $\Delta C_9^U$



LHCb, PRL132(2024)131801  
 LHCb, PRD109(2024)052009  
 LHCb, 2405.17347 (charmonium region is open)



# Introduction

## Lepton Flavour Universality Test

$\pi, K, \tau$  system,  $W, Z$  boson,  $R_{D^{(*)}}$  and  $R_{K^{(*)}}$

## Implications for New Physics

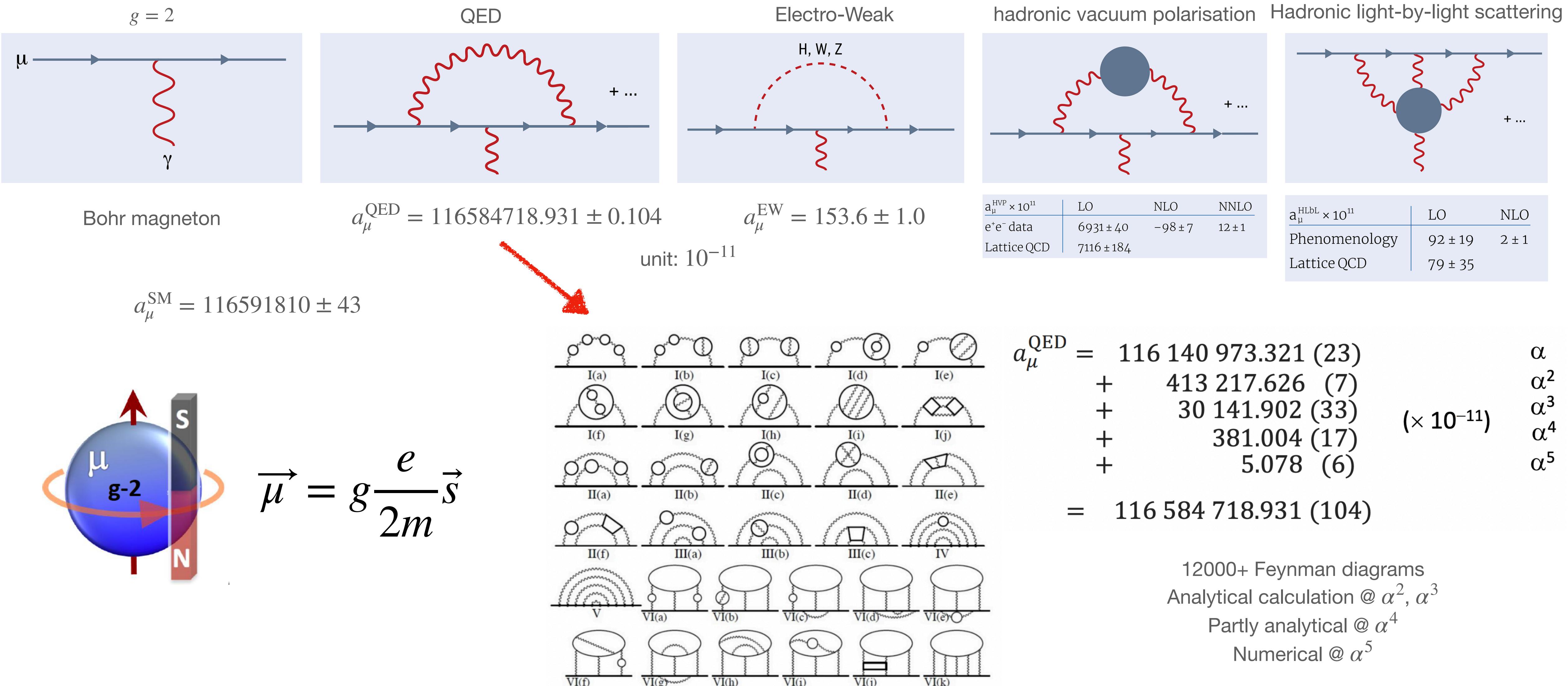
**Connections to other anomalies**  $\left\{ \begin{array}{l} (g-2)_\mu \text{ anomaly} \\ B^+ \rightarrow K^+ \nu \bar{\nu} \text{ excess at Belle II} \\ \text{Cabibbo angle anomaly} \end{array} \right.$

## Origin of LFU violation

## Summary

# $(g - 2)_\mu$

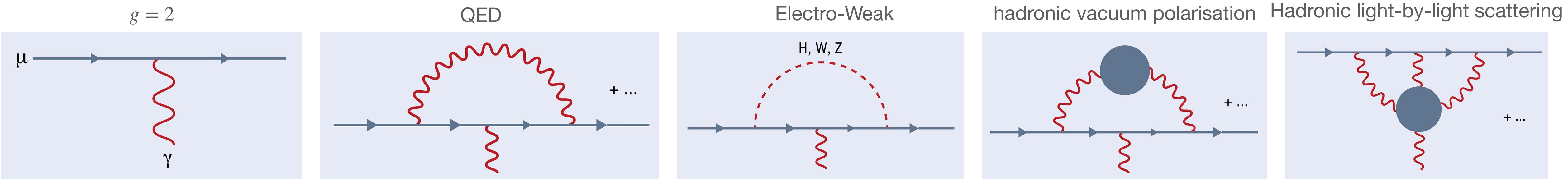
$$a_\mu = (g - 2)/2$$



Aoyama, Hayakawa, Kinoshita, Nio (2012-2019)

# $(g - 2)_\mu$

$$a_\mu = (g - 2)/2$$



Bohr magneton

$$a_\mu^{\text{SM}} = 116591810 \pm 43$$

$$a_\mu^{\text{QED}} = 116584718.931 \pm 0.104$$

$$a_\mu^{\text{exp}} = 116592061 \pm 41$$

unit:  $10^{-11}$

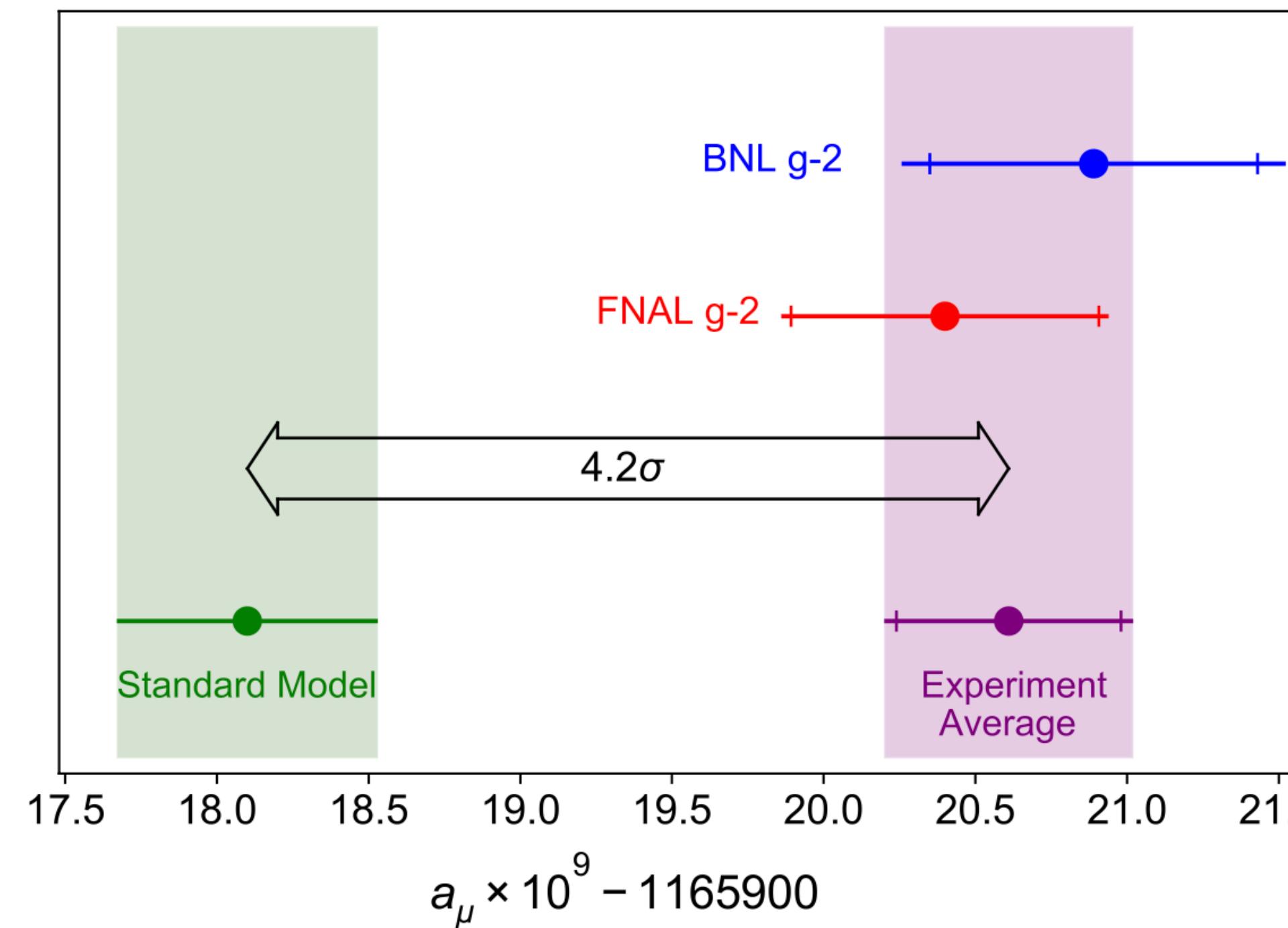
$$a_\mu^{\text{EW}} = 153.6 \pm 1.0$$

	LO	NLO	NNLO
$a_\mu^{\text{HVP}} \times 10^{11}$ e <sup>+</sup> e <sup>-</sup> data	$6931 \pm 40$	$-98 \pm 7$	$12 \pm 1$
Lattice QCD	$7116 \pm 184$		

	LO	NLO
$a_\mu^{\text{HLbL}} \times 10^{11}$ Phenomenology	$92 \pm 19$	$2 \pm 1$
Lattice QCD	$79 \pm 35$	

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251 \pm 59$$

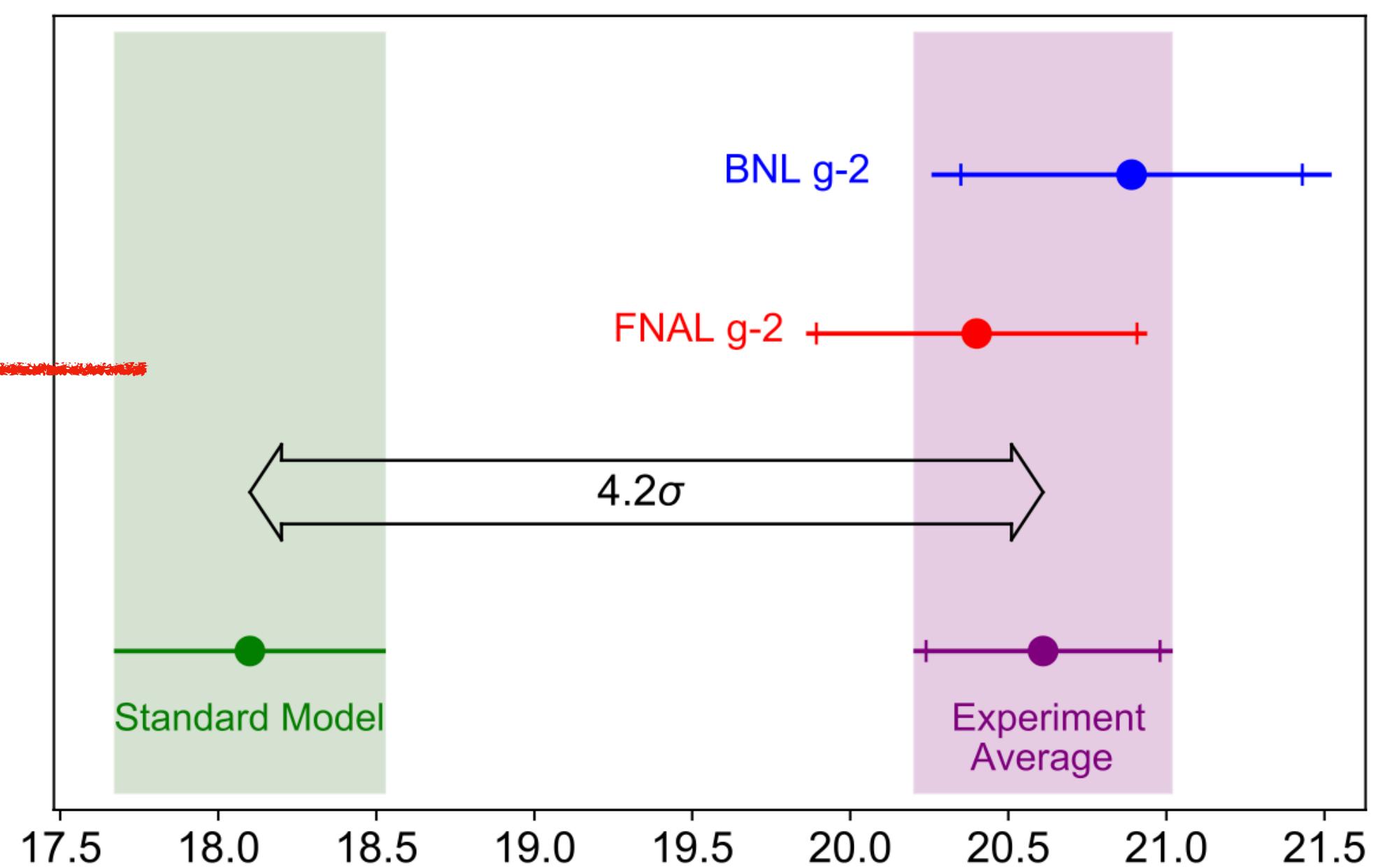
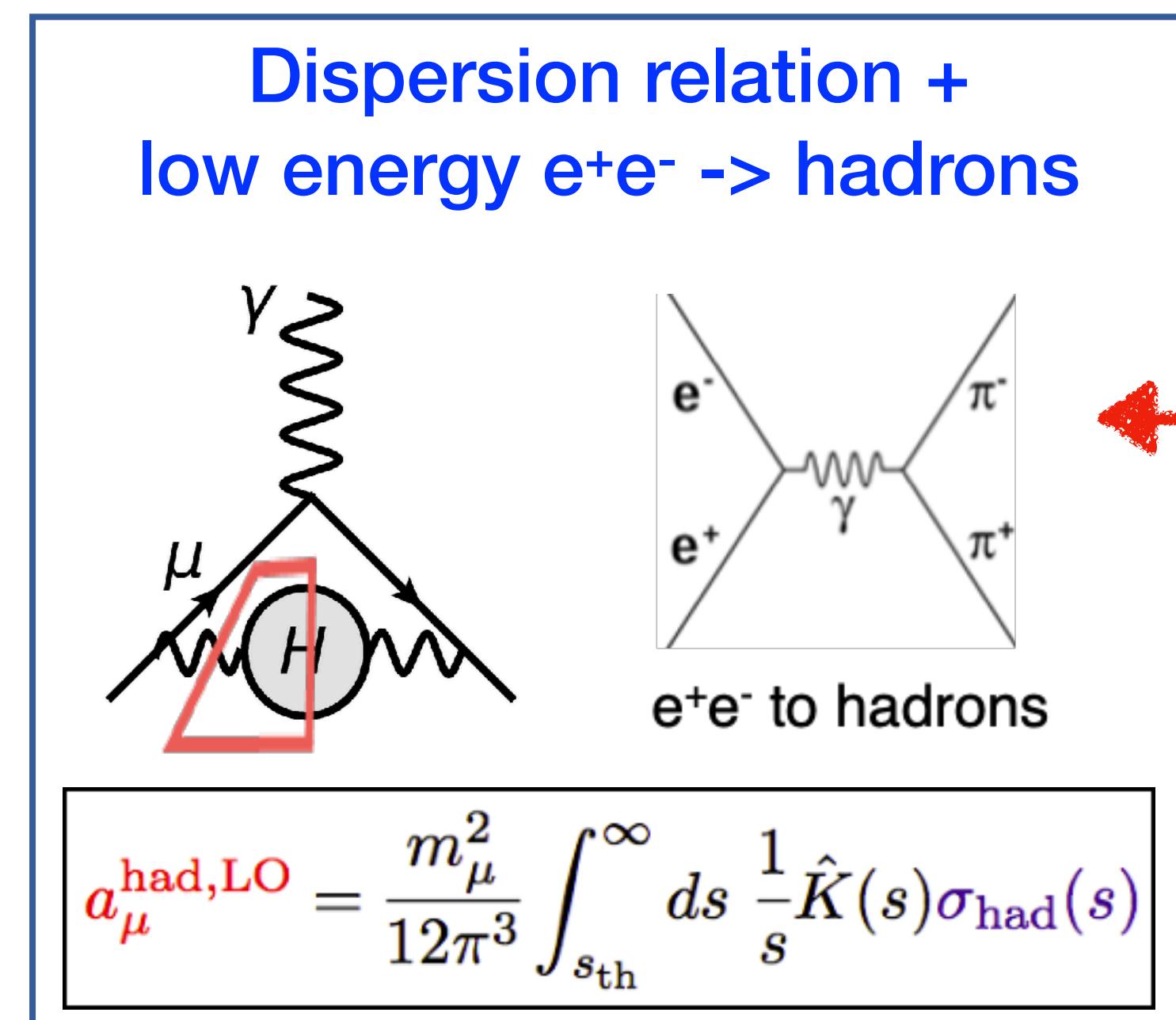
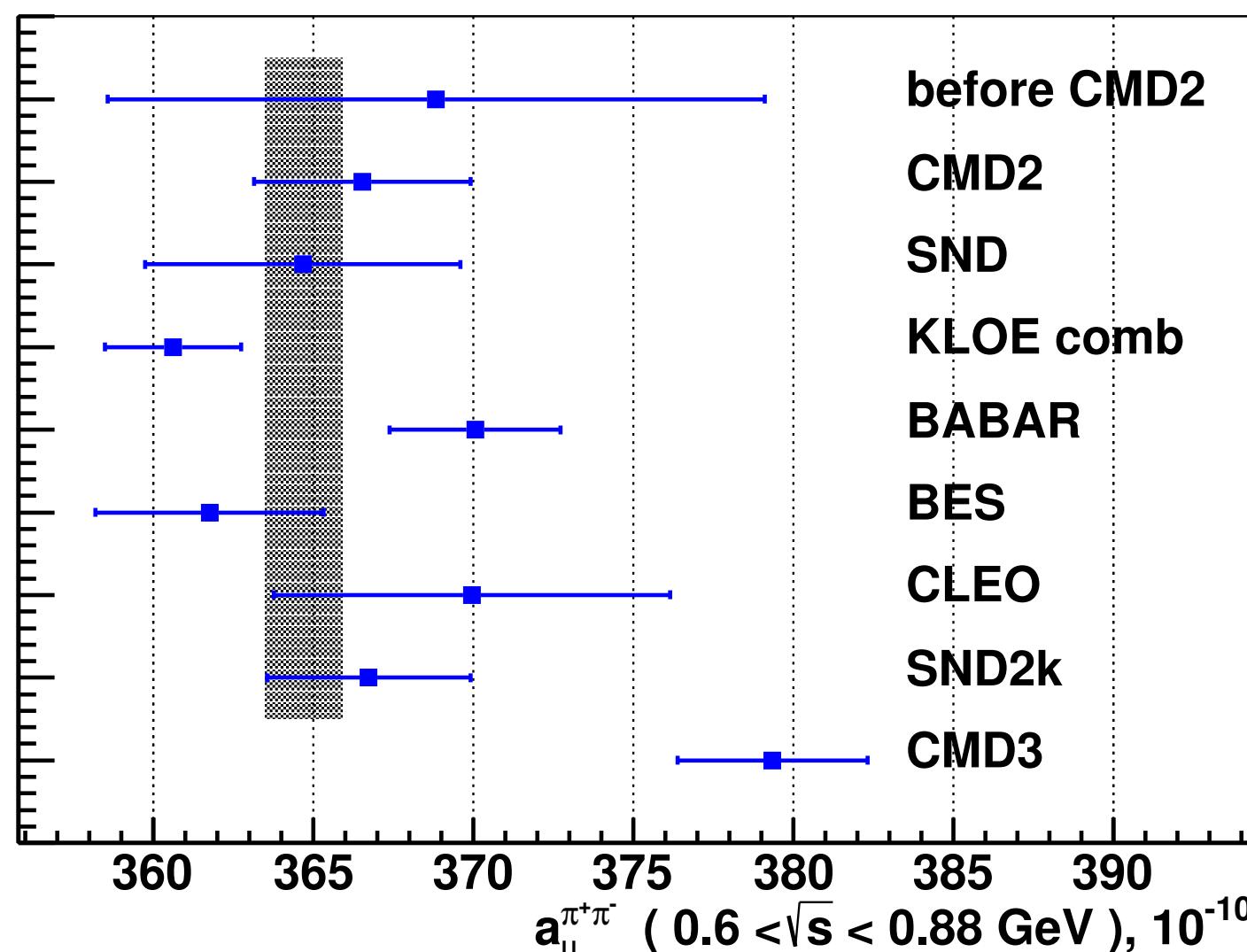
4.2 sigma difference !!!



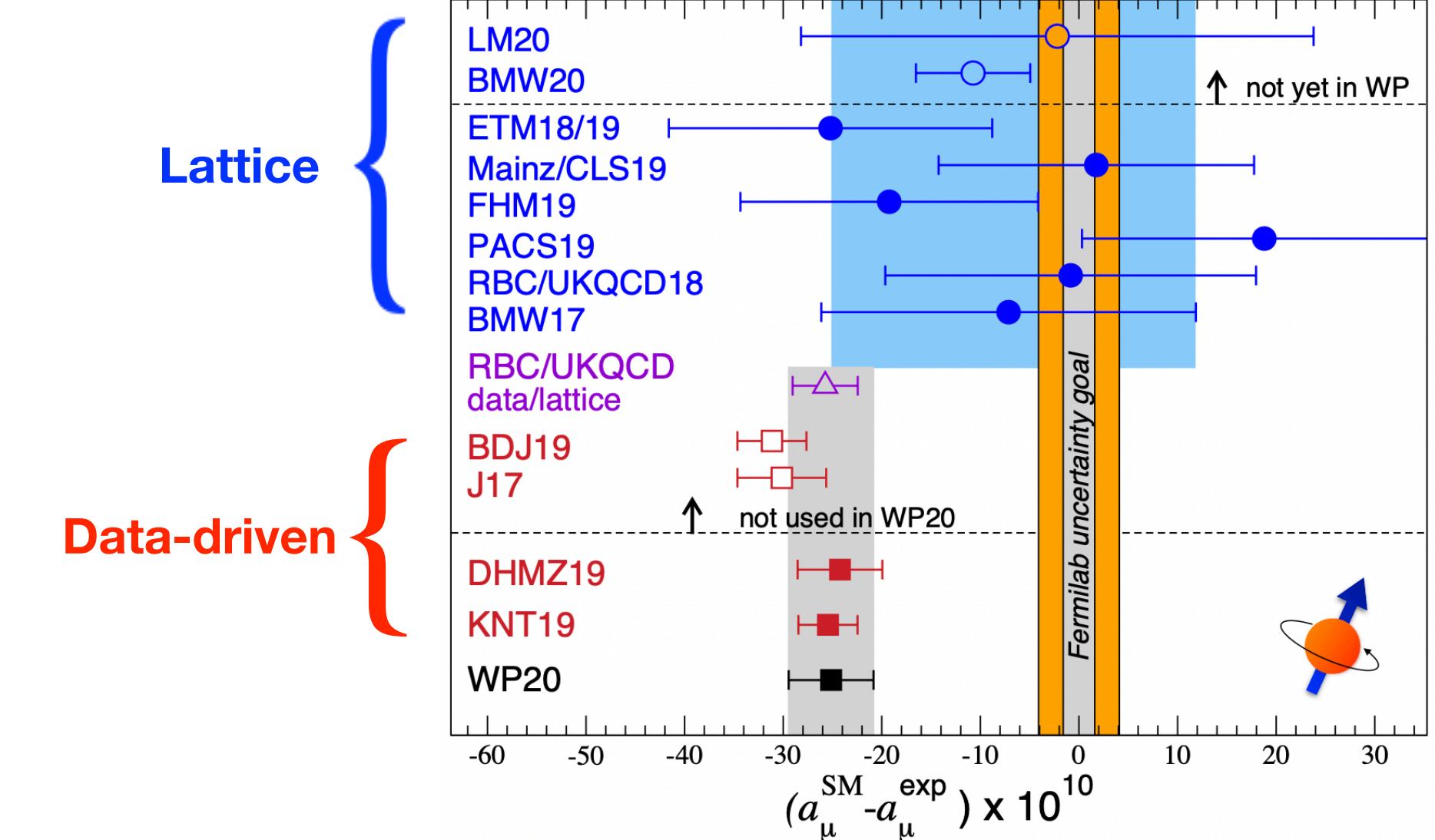
Experiment	Beam	Measurement	$\delta a_\mu/a_\mu$	Required th. terms
Columbia-Nevis (57)	$\mu^+$	$g=2.00 \pm 0.10$		$g=2$
Columbia-Nevis (59)	$\mu^+$	$0.001\ 13(+16)(-12)$	12.4%	$\alpha/\pi$
CERN 1 (61)	$\mu^+$	$0.001\ 145(22)$	1.9%	$\alpha/\pi$
CERN 1 (62)	$\mu^+$	$0.001\ 162(5)$	0.43%	$(\alpha/\pi)^2$
CERN 2 (68)	$\mu^+$	$0.001\ 166\ 16(31)$	265 ppm	$(\alpha/\pi)^3$
CERN 3 (75)	$\mu^\pm$	$0.001\ 165\ 895(27)$	23 ppm	$(\alpha/\pi)^3 + \text{had}$
CERN 3 (79)	$\mu^\pm$	$0.001\ 165\ 911(11)$	7.3 ppm	$(\alpha/\pi)^3 + \text{had}$
BNL E821 (00)	$\mu^+$	$0.001\ 165\ 919\ 1(59)$	5 ppm	$(\alpha/\pi)^3 + \text{had}$
BNL E821 (01)	$\mu^+$	$0.001\ 165\ 920\ 2(16)$	1.3 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak}$
BNL E821 (02)	$\mu^+$	$0.001\ 165\ 920\ 3(8)$	0.7 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$
BNL E821 (04)	$\mu^-$	$0.001\ 165\ 921\ 4(8)(3)$	0.7 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$
> FNAL Run1 (21)	$\mu^+$	$0.001\ 165\ 920\ 40(54)$	0.46 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$

60 years  
of g-2  
measurements

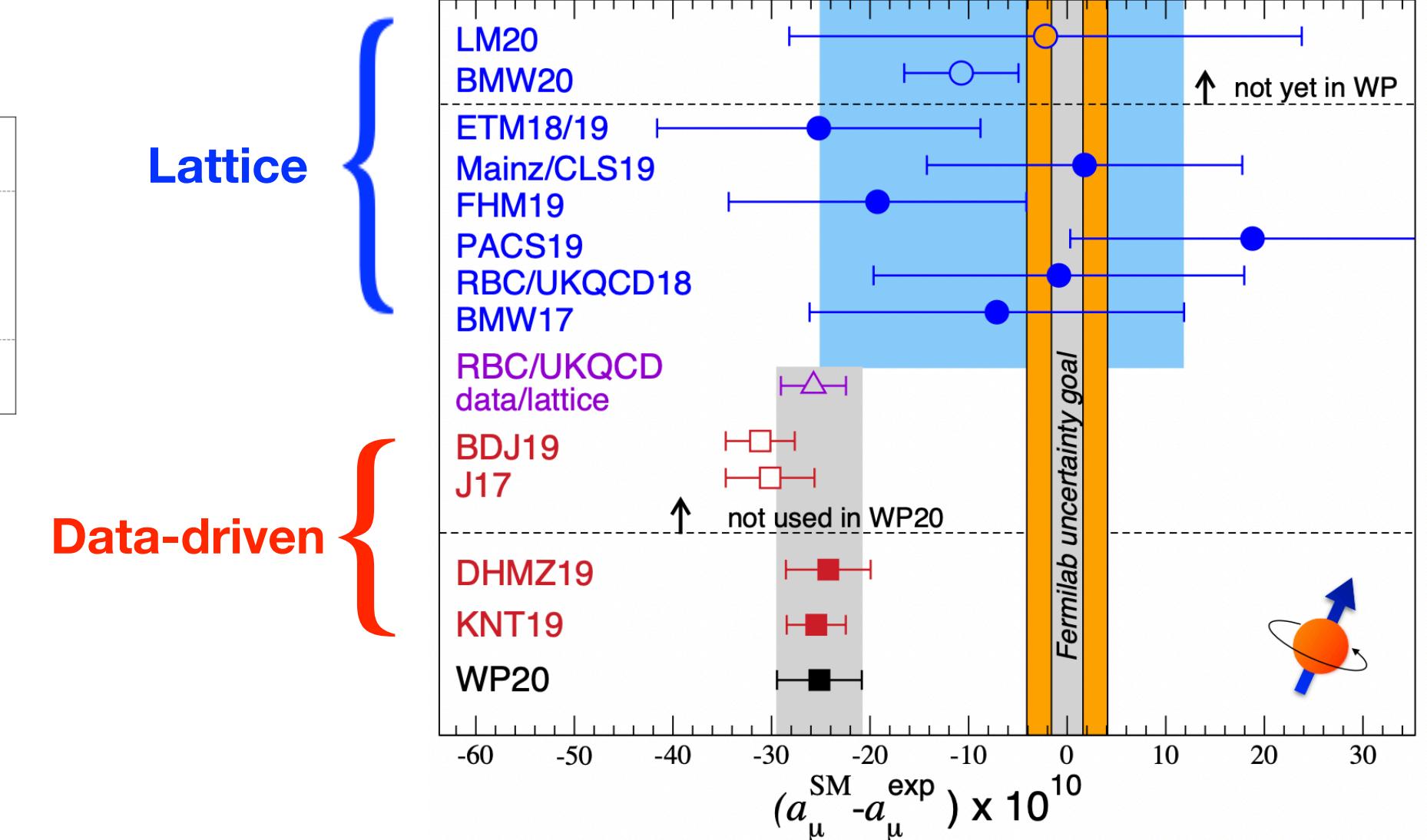
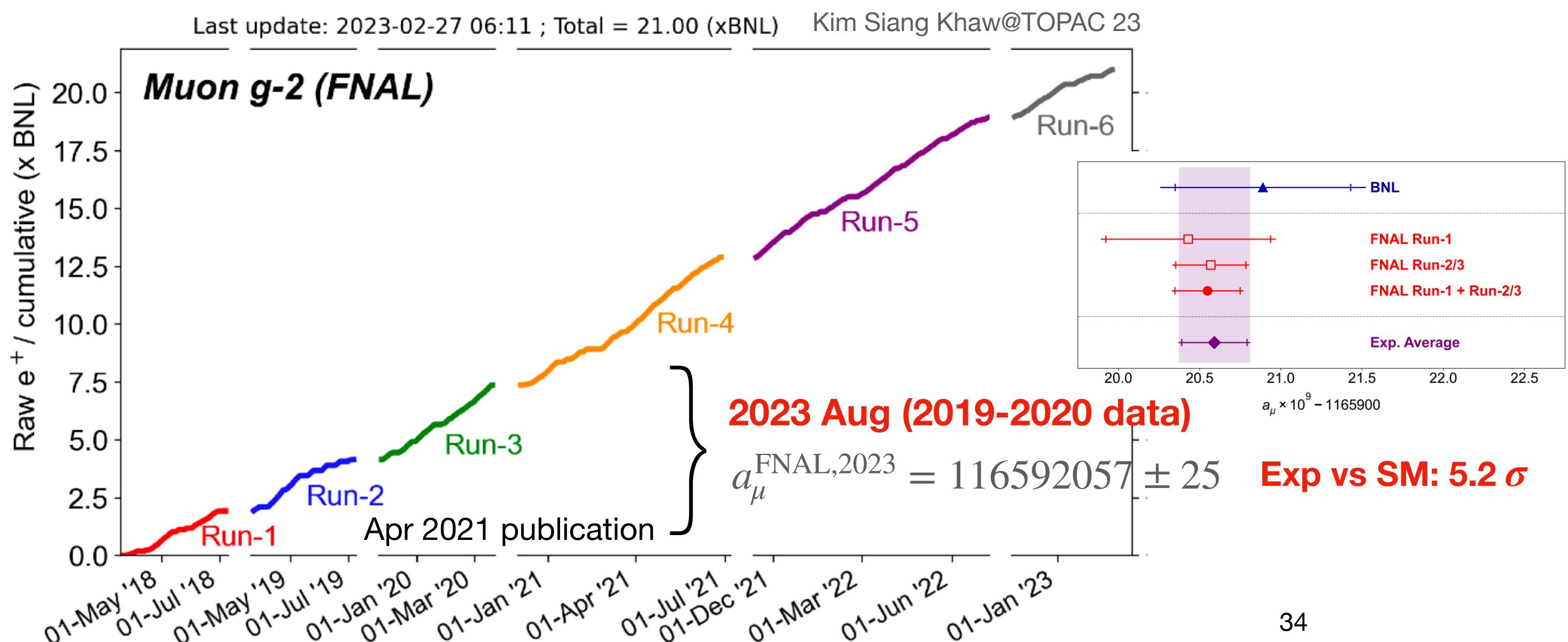
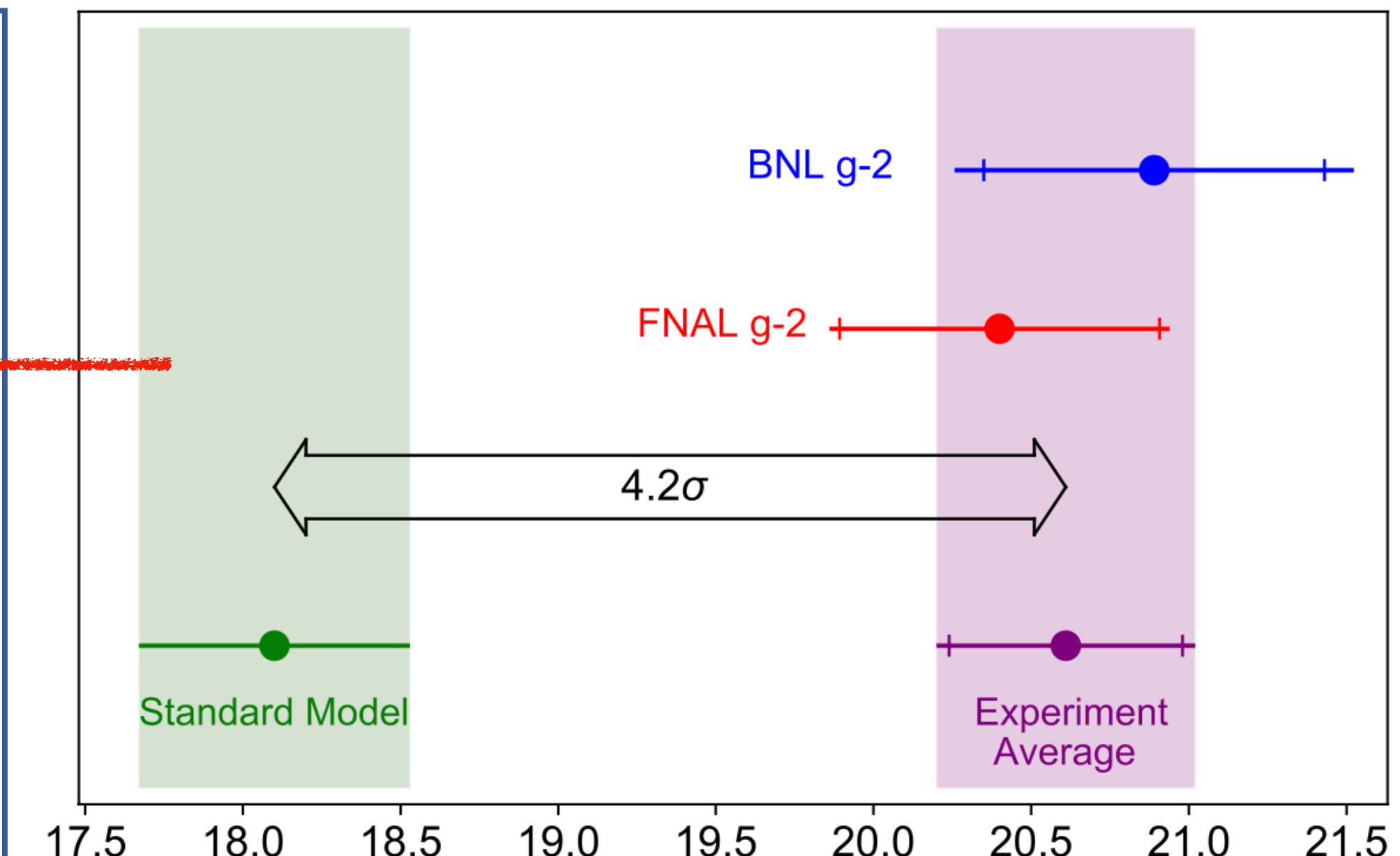
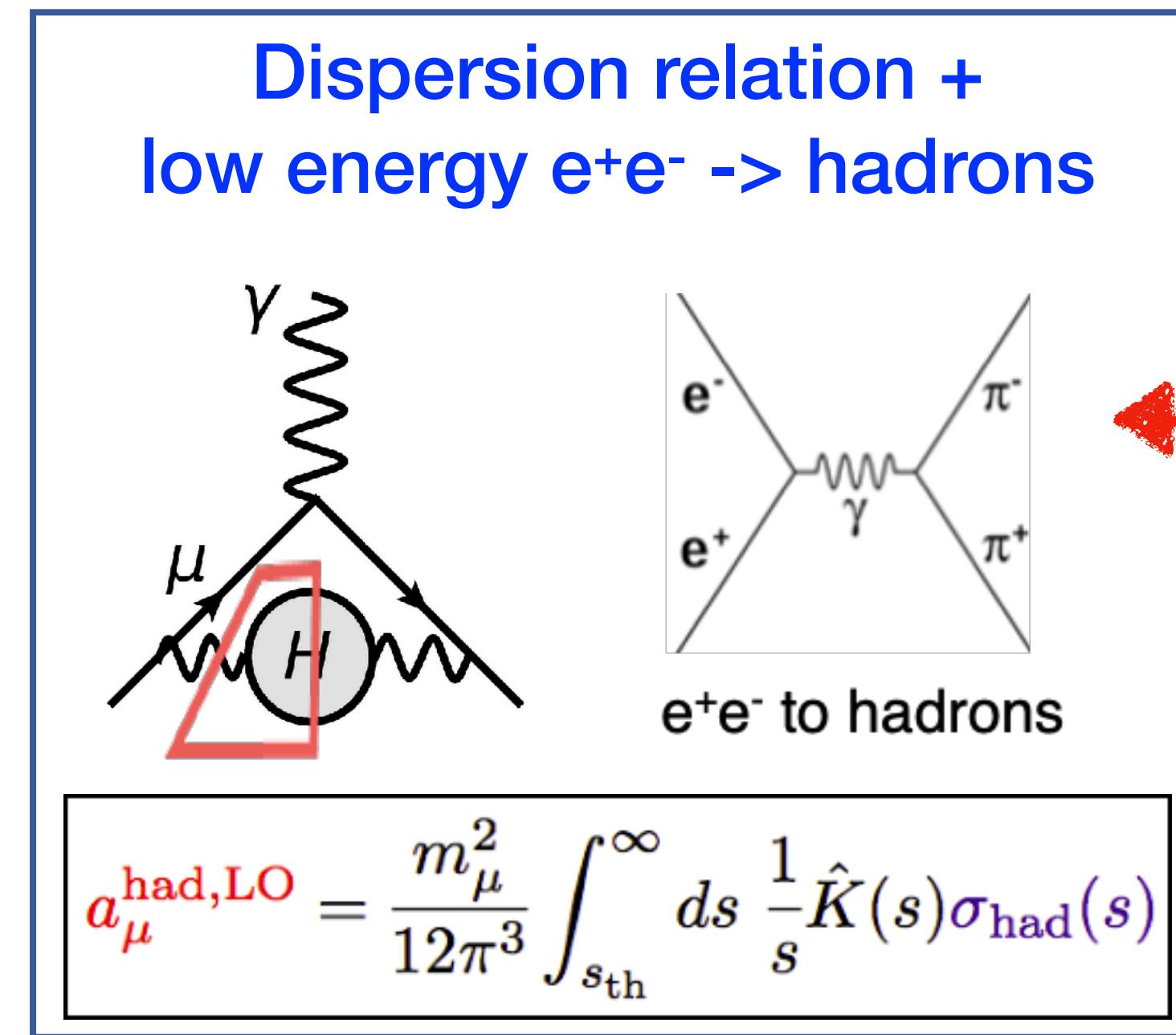
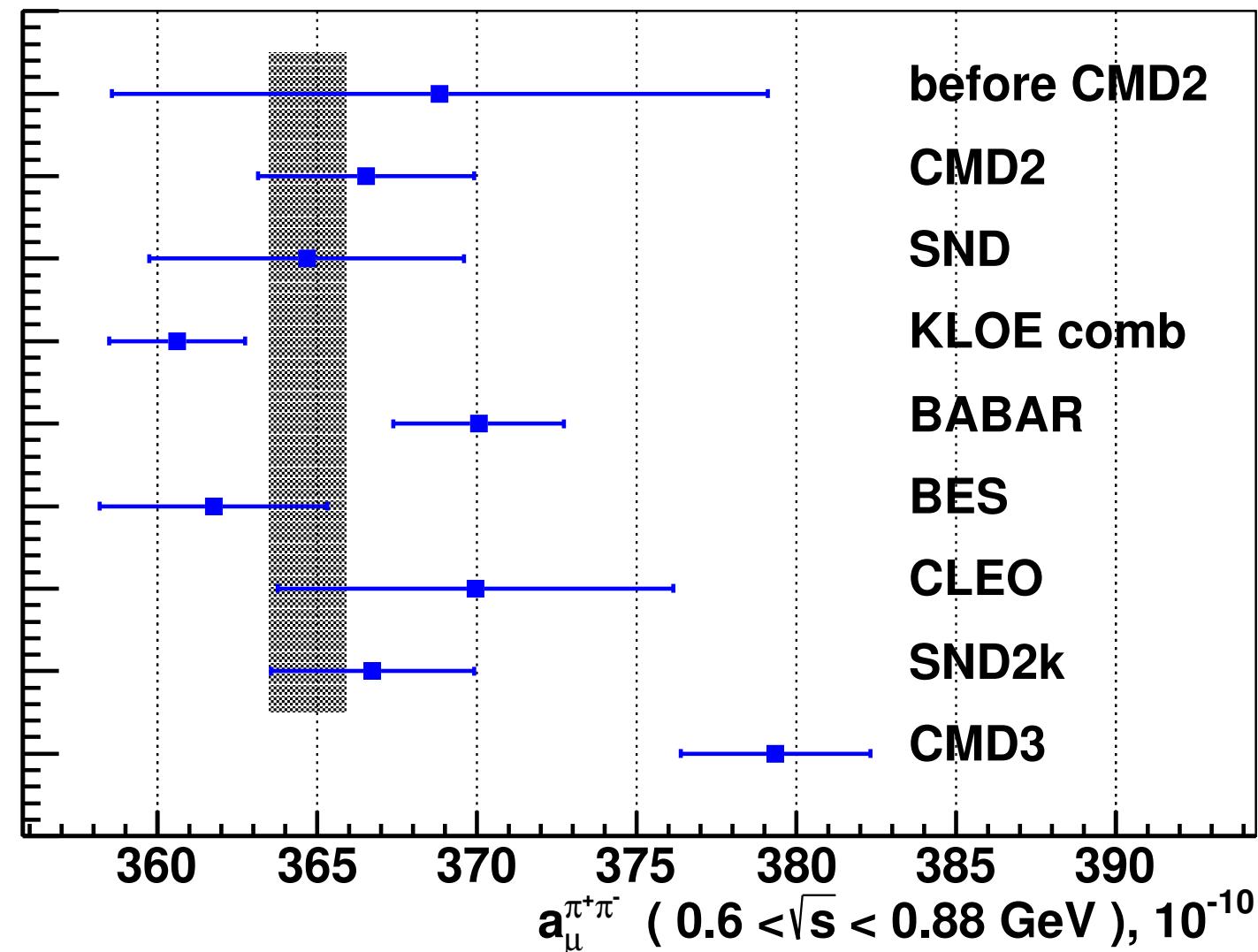
# $(g - 2)_\mu$



- CMD-3 different from CDM-2 and others
- New results from BES-III, Belle II, STCF expected !!!
- More time needed to resolve these puzzles
- Tension between data-driven and lattice**
- Uncertainty in BMW20 much smaller than others (huge computing power)
- Central value closer to experimental measurement
- Discrepancy between BMW20 and Data-driven
- Require more checks among the several Lattice groups
- Recent progress



# $(g - 2)_\mu$



# $(g - 2)_\mu$ and $b \rightarrow s\ell\ell$ anomaly

## Recent Global Fit

1D Hyp.	All			
	Best fit	$1\sigma/2\sigma$	$\text{Pull}_{\text{SM}}$	p-value
$C_{9\mu}^{\text{NP}}$	-0.67	$[-0.82, -0.52]$ $[-0.98, -0.37]$	4.5	20.2 %
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.19	$[-0.25, -0.13]$ $[-0.32, -0.07]$	3.1	9.9 %

2D Hyp.	All			
	Best fit	$\text{Pull}_{\text{SM}}$	p-value	
$(C_{9\mu}^{\text{NP}}, C_{10\mu}^{\text{NP}})$	$(-0.82, -0.17)$	4.4	21.9%	
$(C_{9\mu}^{\text{NP}}, C_{7'}^{\text{NP}})$	$(-0.68, +0.01)$	4.2	19.4%	
$(C_{9\mu}^{\text{NP}}, C_{9'\mu}^{\text{NP}})$	$(-0.78, +0.21)$	4.3	20.7%	
$(C_{9\mu}^{\text{NP}}, C_{10'\mu}^{\text{NP}})$	$(-0.76, -0.12)$	4.3	20.5%	
$(C_{9\mu}^{\text{NP}}, C_{9e}^{\text{NP}})$	$(-1.17, -0.97)$	5.6	40.3%	

Scenario	Best-fit point	$1\sigma$	$\text{Pull}_{\text{SM}}$	p-value
Scenario 0 $C_{9\mu}^{\text{NP}} = C_{9e}^{\text{NP}} = C_9^U$	-1.17	$[-1.33, -1.00]$	5.8	39.9 %
Scenario 5 $C_{9\mu}^V$	-1.02	$[-1.43, -0.61]$		
Scenario 5 $C_{10\mu}^V$	-0.35	$[-0.75, -0.00]$	4.1	21.0 %
Scenario 5 $C_9^U = C_{10}^U$	+0.19	$[-0.16, +0.58]$		
Scenario 6 $C_{9\mu}^V = -C_{10\mu}^V$	-0.27	$[-0.34, -0.20]$	4.0	18.0 %
Scenario 6 $C_9^U = C_{10}^U$	-0.41	$[-0.53, -0.29]$		
Scenario 7 $C_{9\mu}^V$	-0.21	$[-0.39, -0.02]$	5.6	40.3 %
Scenario 7 $C_9^U$	-0.97	$[-1.21, -0.72]$		
Scenario 8 $C_{9\mu}^V = -C_{10\mu}^V$	-0.08	$[-0.14, -0.02]$	5.6	41.1 %
Scenario 8 $C_9^U$	-1.10	$[-1.27, -0.91]$		

Ciuchini et al 2212.10516  
Alguero et al 2304.07330  
Qiaoyi Wen, Fanrong Xu 2305.19038

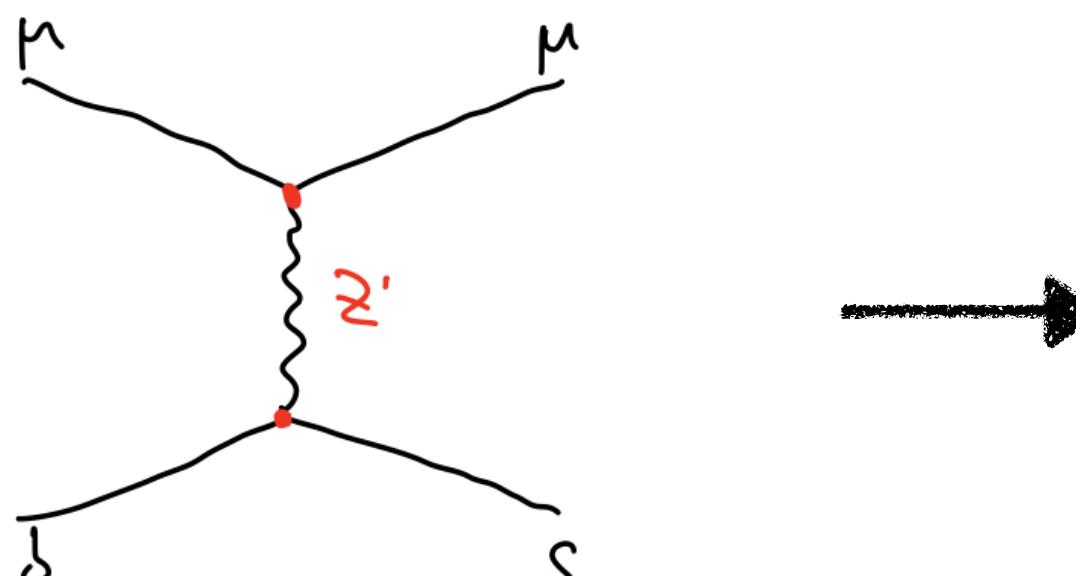
$$O_9 = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \ell)$$

$$O_{10} = (\bar{b}\gamma^\mu P_L s)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

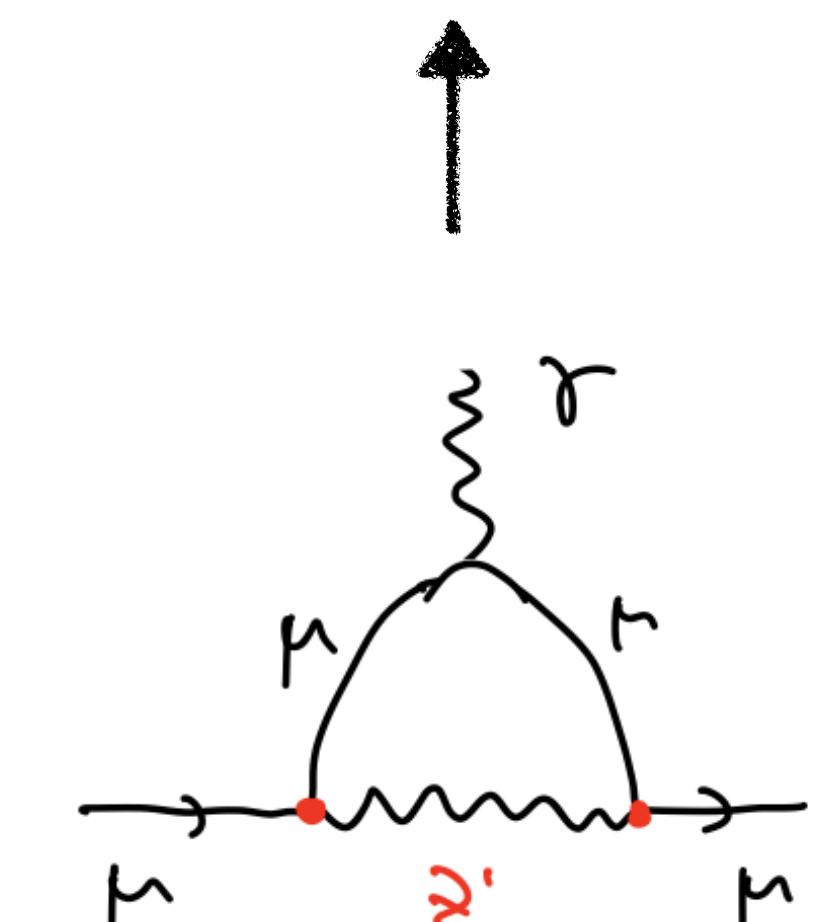
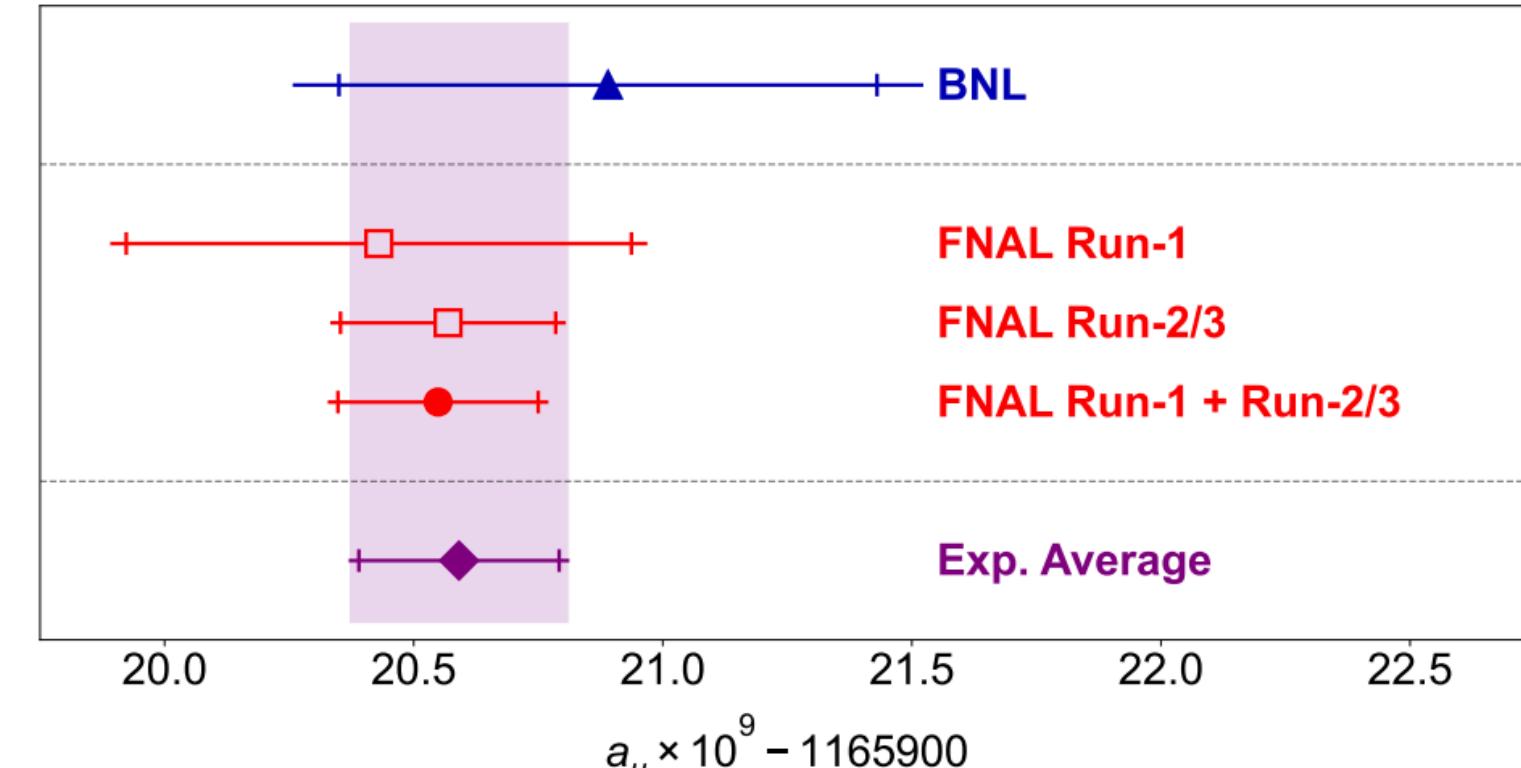
$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  consistent with SM  
( $C_{10}$  can't be too large)

Current global fit implies  
non-zero  $C_9^{\text{NP}}$

Z'  $\ell^+ \ell^-$  interaction should  
be almost vector-type

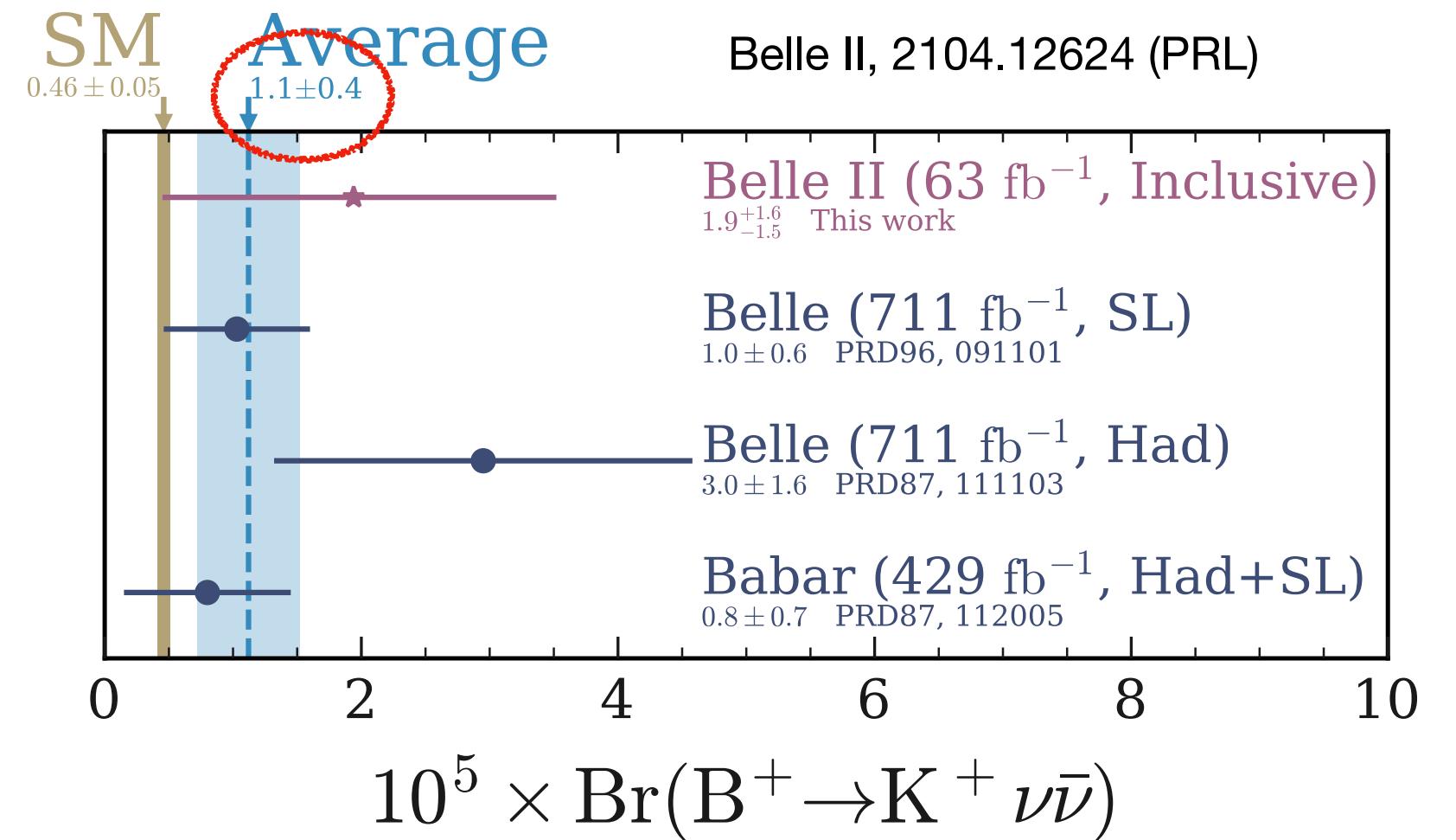


$$\Delta(g - 2)_\mu \propto -5g_A^2 + g_V^2$$



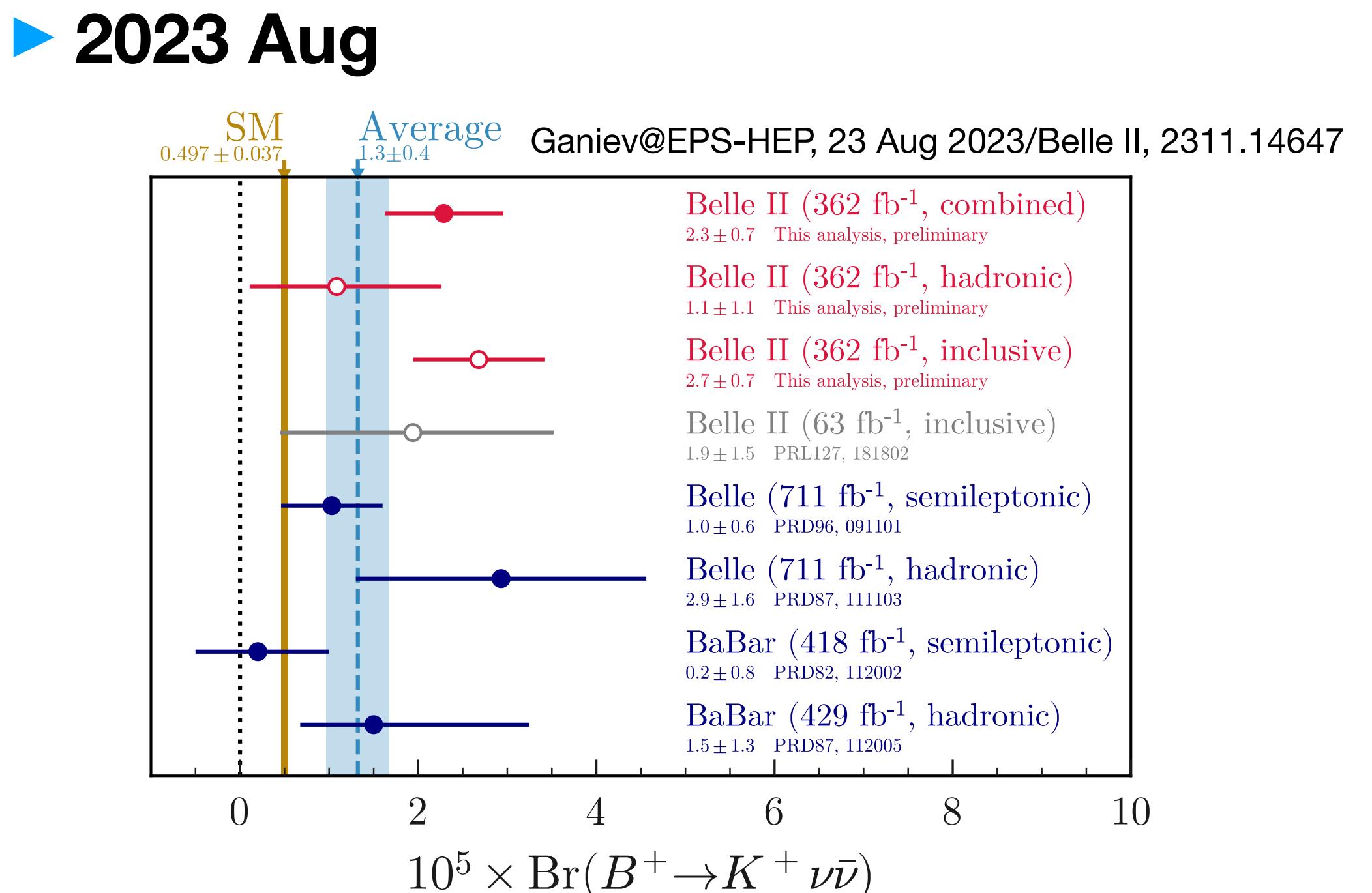
# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

► 2021 Apr



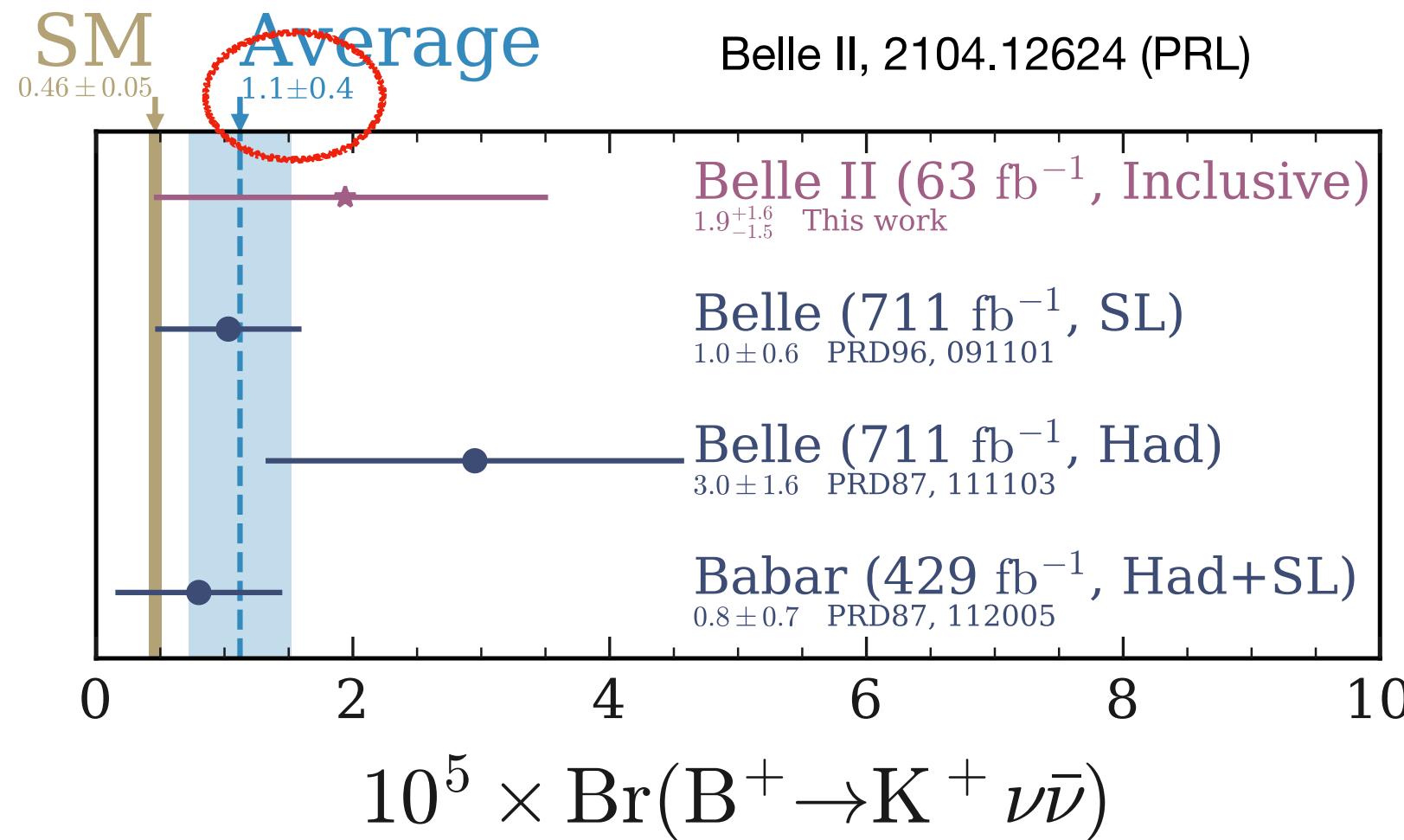
30+ theory papers !

- Impact of  $B \rightarrow K\nu\nu$  measurements on beyond the Standard Model theories #69  
Thomas E. Browder (Hawaii U.), Nilendra G. Deshpande (Oregon U.), Rusa Mandal (Siegen U.), Rahul Sinha (IMSc, Chennai and Bhubaneswar, Inst. Phys.) (Jul 2, 2021)  
Published in: *Phys.Rev.D* 104 (2021) 05, 053007 • e-Print: 2107.01080 [hep-ph]
- A tale of invisibility: constraints on new physics in  $b \rightarrow s\nu\bar{\nu}$  #65  
Tobias Felkl (New South Wales U.), Sze Lok Li (New South Wales U.), Michael A. Schmidt (New South Wales U.) (Nov 8, 2021)  
Published in: *JHEP* 12 (2021) 118 • e-Print: 2111.04327 [hep-ph]
- Explaining the  $B^+ \rightarrow K^+\nu\bar{\nu}$  excess via a massless dark photon #16  
E. Gabrielli, L. Marzola, K. Müürsepp, M. Raidal (Feb 8, 2024)  
e-Print: 2402.05901 [hep-ph]
- Phenomenological study of a gauged  $L_\mu - L_\tau$  model with a scalar leptoquark #42  
Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and NCTS, Taipei), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and NCTS, Taipei), Chun-Wei Su (Taiwan, Natl. Taiwan U.) (May 16, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 5, 5 • e-Print: 2305.09256 [hep-ph]
- Decoding the  $B \rightarrow K\nu\nu$  excess at Belle II: kinematics, operators, and masses #27  
Kåre Fridell, Mitrajyoti Ghosh, Takemichi Okui, Kohsaku Tobioka (Dec 19, 2023)  
e-Print: 2312.12507 [hep-ph]
- Higgs portal interpretation of the Belle II  $B^+ \rightarrow K^+\nu\nu$  measurement #29  
David McKeen (TRIUMF), John N. Ng (TRIUMF), Douglas Tuckler (TRIUMF and Simon Fraser U.) (Dec 1, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075006 • e-Print: 2312.00982 [hep-ph]
- Light new physics in  $B \rightarrow K^{(*)}\nu\nu$ ? #30  
Wolfgang Altmannshofer (UC, Santa Cruz, Inst. Part. Phys.), Andreas Crivellin (Zurich U.), Huw Haigh (Vienna, OAW), Gianluca Inguglia (Vienna, OAW), Jorge Martin Camalich (IAC, La Laguna) (Nov 24, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075008 • e-Print: 2311.14629 [hep-ph]
- $B \rightarrow K\nu\nu$ , MiniBooNE and muon g - 2 anomalies from a dark sector #31  
Alakabha Datta (Mississippi U. and SLAC and UC, Santa Cruz), Danny Marfatia (Hawaii U.), Lopamudra Mukherjee (Nankai U.) (Oct 23, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 3, L031701 • e-Print: 2310.15136 [hep-ph]
- Implications of an enhanced  $B \rightarrow K\nu\nu$  branching ratio #39  
Rigo Bause (Tech. U., Dortmund (main)), Hector Gisbert (INFN, Padua and Padua U.), Gudrun Hiller (Tech. U., Dortmund (main) and Sussex U.) (Aug 31, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 1, 015006 • e-Print: 2309.00075 [hep-ph]
- $B$  meson anomalies and large  $B^+ \rightarrow K^+\nu\bar{\nu}$  in non-universal  $U(1)'$  models #40  
Peter Athron (Nanjing Normal U.), R. Martínez (Colombia, U. Natl.), Cristian Sierra (Nanjing Normal U.) (Aug 25, 2023)  
Published in: *JHEP* 02 (2024) 121 • e-Print: 2308.13426 [hep-ph]
- $B \rightarrow K^*M_X$  vs  $B \rightarrow KM_X$  as a probe of a scalar-mediator dark matter scenario #33  
Alexander Berezhnoy (SINP, Moscow), Dmitri Melikhov (SINP, Moscow and Dubna, JINR and Vienna U.) (Sep 29, 2023)  
Published in: *EPL* 145 (2024) 1, 14001 • e-Print: 2309.17191 [hep-ph]
- Flavor anomalies in leptoquark model with gauged  $U(1)_{L_\mu - L_\tau}$  #34  
Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and Unlisted, TW), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and Unlisted, TW) (Sep 22, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075004 • e-Print: 2309.12904 [hep-ph]
- Revisiting models that enhance  $B^+ \rightarrow K^+\nu\nu$  in light of the new Belle II measurement #35  
Belle-II Collaboration • Xiao-Gang He (Tsung-Dao Lee Inst., Shanghai and Taiwan, Natl. Taiwan U.) et al. (Sep 22, 2023)  
Published in: *Phys.Rev.D* 109 (2024) 7, 075019 • e-Print: 2309.12741 [hep-ph]
- A new look at  $b \rightarrow s$  observables in 331 models #18  
Francesco Loparco (Jan 22, 2024)  
e-Print: 2401.11999 [hep-ph]
- Correlating  $B \rightarrow K^{(*)}\nu\bar{\nu}$  and flavor anomalies in SMEFT #19  
Feng-Zhi Chen, Qiaoyi Wen, Fanrong Xu (Jan 21, 2024)  
e-Print: 2401.11552 [hep-ph]
- Recent  $B^+ \rightarrow K^+\nu\bar{\nu}$  Excess and Muon g - 2 Illuminating Light Dark Sector with Higgs Portal #20  
Shu-Yu Ho, Jongkuk Kim, Pyungwon Ko (Jan 18, 2024)  
e-Print: 2401.10112 [hep-ph]
- SMEFT predictions for semileptonic processes #4  
Siddhartha Karmakar, Amol Dighe, Rick S. Gupta (Apr 15, 2024)  
e-Print: 2404.10061 [hep-ph]
- Implications of  $B \rightarrow K\nu\bar{\nu}$  under Rank-One Flavor Violation hypothesis #5  
David Marzocca, Marco Nardeccia, Alfredo Stanzione, Claudio Toni (Apr 9, 2024)  
e-Print: 2404.06533 [hep-ph]
- The quark flavor-violating ALPs in light of B mesons and hadron colliders #20  
Tong Li (Nankai U.), Zhiou Qian (Hangzhou Normal U.), Michael A. Schmidt (Sydney U. and New South Wales U.), Man Yuan (Nankai U.) (Feb 21, 2024)  
Published in: *JHEP* 05 (2024) 232 • e-Print: 2402.14232 [hep-ph]
- Scalar dark matter explanation of the excess in the Belle II  $B^+ \rightarrow K^+ + \text{invisible}$  measurement #9  
Xiao-Gang He, Xiao-Dong Ma, Michael A. Schmidt, German Valencia, Raymond R. Volkas (Mar 19, 2024)  
e-Print: 2403.12485 [hep-ph]
- Status and prospects of rare decays at Belle-II #10  
Elisa Manoni (Mar 12, 2024)  
Published in: *PoS WIFAI2023* (2024) 024 • Contribution to: *WIFAI 2023*, 024
- Rare  $B$  and  $K$  decays in a scotogenic model #11  
Chuan-Hung Chen, Cheng-Wei Chiang (Mar 5, 2024)  
e-Print: 2403.02897 [hep-ph]

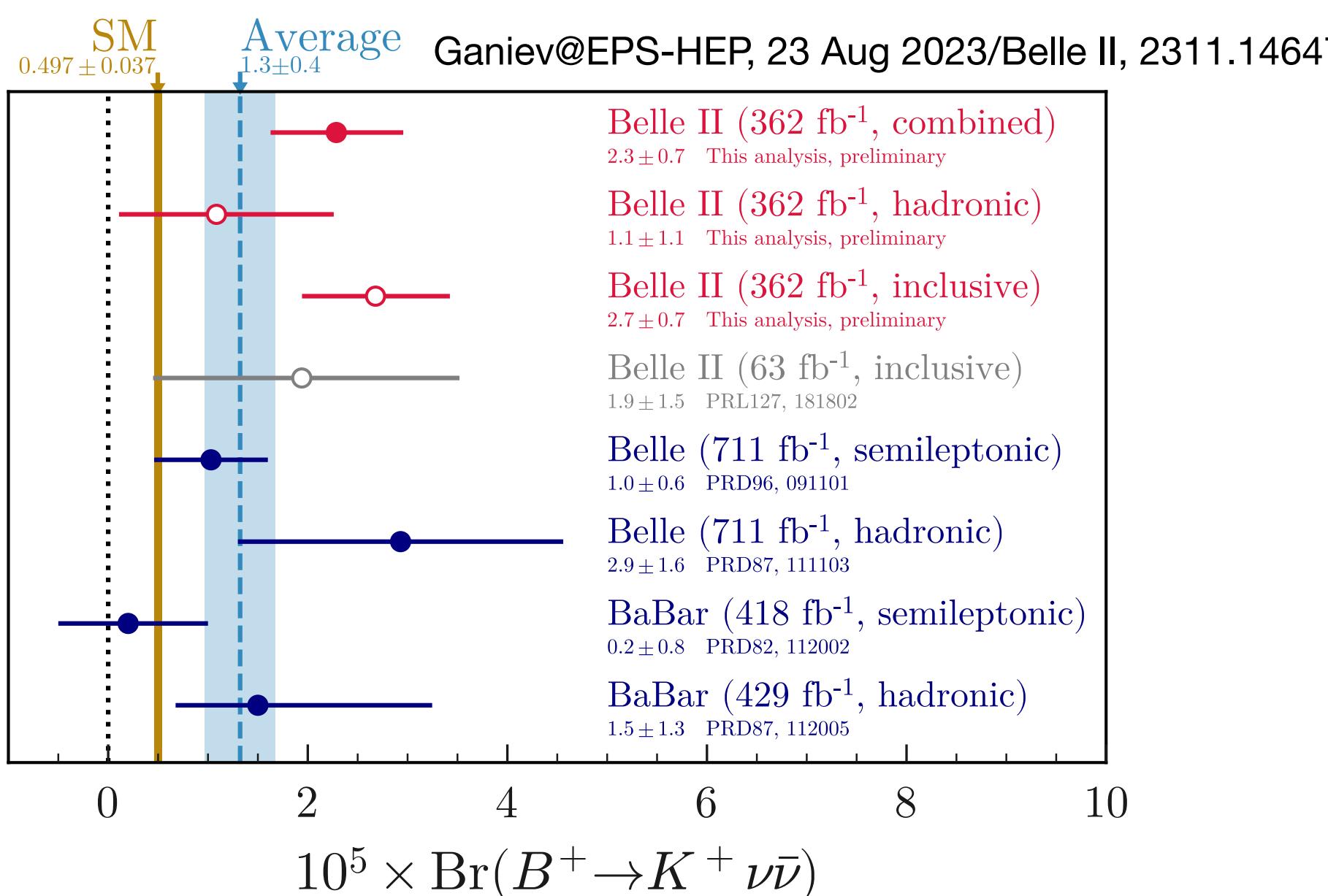


# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

► 2021 Apr



► 2023 Aug

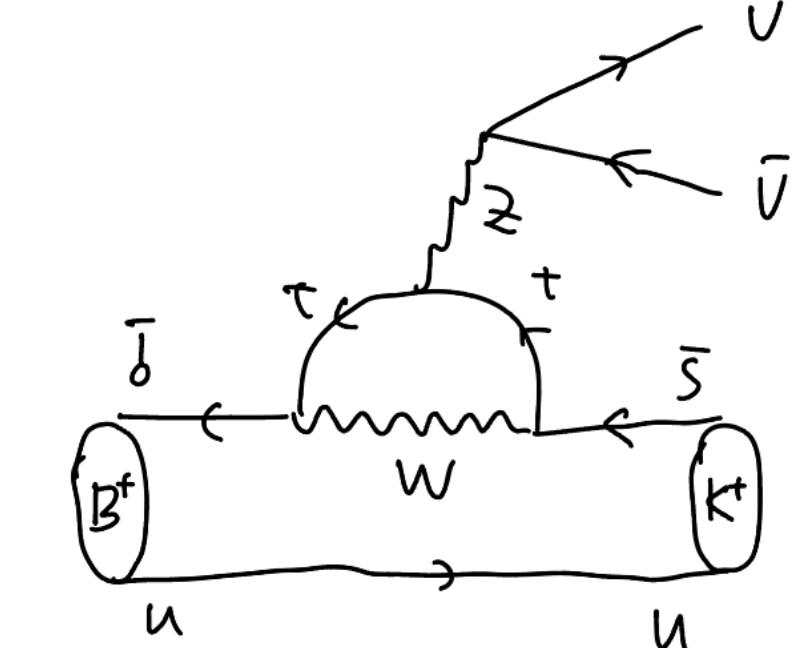


► Exp vs SM  $[10^{-6}]$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{SM}} = 4.16 \pm 0.57$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{exp}} = 23 \pm 7$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{exp}} \gtrsim 10 \text{ (2}\sigma \text{ lower bound)}$$



**2.7 $\sigma$  difference**  
**NP/SM  $\gtrsim 2$**

► Theoretical prediction

Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s} \gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu} \gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

theoretically, simple and clean  
one of the cleanest channels in  
flavour physics

$$\mathcal{O}_L = (\bar{s} \gamma_\mu P_L b)(\bar{\nu} \gamma^\mu P_L \nu) \text{ in the SM}$$

$$\mathcal{O}_R = (\bar{s} \gamma_\mu P_R b)(\bar{\nu} \gamma^\mu P_L \nu) \text{ possible in BSM}$$

**simple interaction but complicated flavour**

operator structure highly  
constrained by LH neutrino

$$\mathcal{O}_L = (\bar{s} P_L b)(\bar{\nu} P_L \nu) \times$$

$$\mathcal{O}_R = (\bar{s} P_R b)(\bar{\nu} P_R \nu) \times$$

$$\mathcal{O}_T = (\bar{s} \sigma_{\mu\nu} b)(\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

$$\mathcal{O}_{T5} = (\bar{s} \sigma_{\mu\nu} \gamma_5 b)(\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

$b \rightarrow s$

Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5^{+5}_{-4}$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^0\nu\bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
$\mathcal{B}(B^+ \rightarrow K^{*+}\nu\bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \phi\nu\bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$

$b \rightarrow d$

$\mathcal{B}(B^+ \rightarrow \pi^+\nu\bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \pi^0\nu\bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
$\mathcal{B}(B^+ \rightarrow \rho^+\nu\bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \rho^0\nu\bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$

$s \rightarrow d$

$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$	$8.42 \pm 0.61$	$10.6^{+4.0}_{-3.4} \pm 0.9$	$10^{-11}$
$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$	$3.41 \pm 0.45$	$< 300$	$10^{-11}$

$B^0 \rightarrow K^{*0}\nu\bar{\nu}$  can put strong constraints on related BSM effects.

## ► Exp vs SM [10<sup>-6</sup>]

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{SM}} = 4.16 \pm 0.57$$

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{exp}} = 23 \pm 7$$

$$\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{exp}} \gtrsim 10 \text{ (2}\sigma \text{ lower bound)}$$

2.7 $\sigma$  difference  
NP/SM  $\gtrsim 2$

## ► Theoretical prediction

### Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s}\gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu}\gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

theoretically, simple and clean  
one of the cleanest channels in  
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$$\mathcal{O}_L = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}\gamma^\mu P_L \nu) \text{ in the SM}$$

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simple interaction but complicated flavour

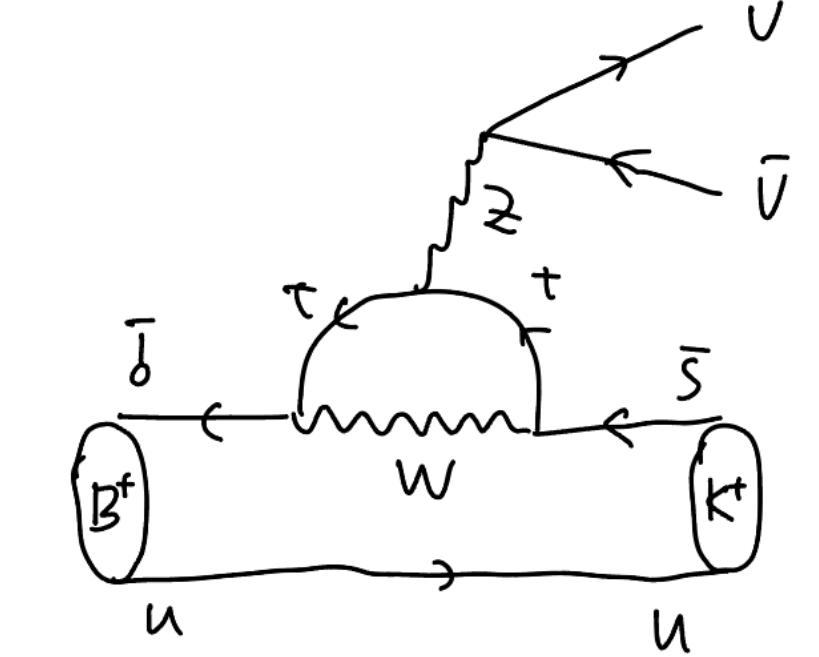
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$$\mathcal{O}_T = (\bar{s}\sigma_{\mu\nu} b)(\bar{\nu}\sigma^{\mu\nu} \nu) \times$$

$$\mathcal{O}_{T5} = (\bar{s}\sigma_{\mu\nu} \gamma_5 b)(\bar{\nu}\sigma^{\mu\nu} \nu) \times$$



# $b \rightarrow s\nu\bar{\nu}$ : SMEFT

B.F.Hou, X.Q.Li, M.Shen, Y.D.Yang, **XBY**, 2402.19208

## Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+\nu\bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

## prediction

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{SMEFT}} = (50^{+17}_{-16}) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0}\nu\bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

## Only $\mathcal{O}_{lq}^{(3)}$ is relevant with $R_{D^{(*)}}$

## $\mathcal{O}_{ld}$ can explain the $B^+ \rightarrow K^+\nu\bar{\nu}$ data

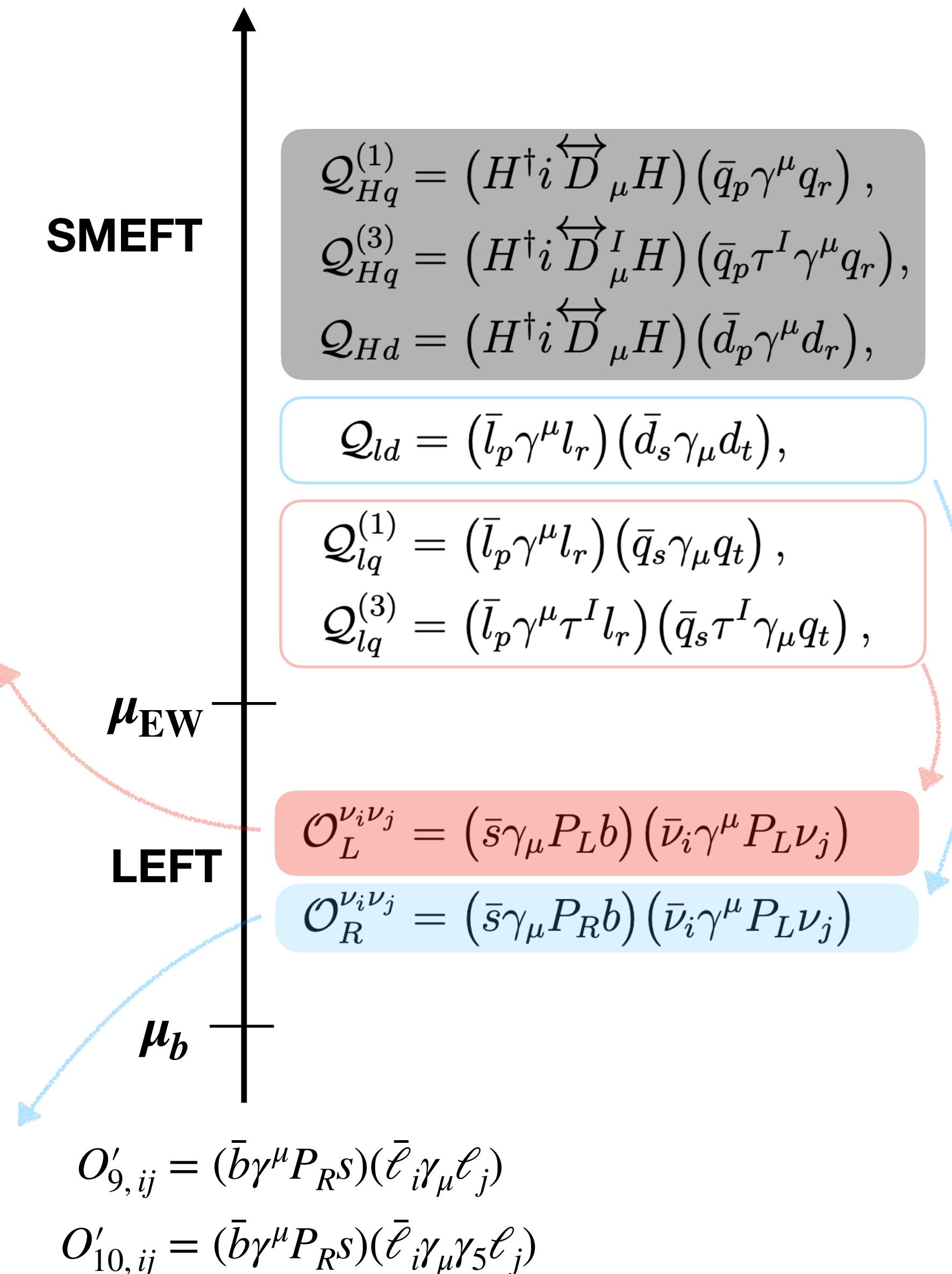
## $\mathcal{O}_{ld}$ also induce $O'_{9,ij}$ and $O'_{10,ij}$

## They can't improve the $b \rightarrow s\ell\ell$ fit

$O'_{9e}$  and  $O'_{10\mu}$  worsen the fit. **weird** (LFV,  $\tau\tau \gg ee, \mu\mu$ )

$O'_{9,ij}$  and  $O'_{10,ij}$  with  $i = j = \tau$  has no effect.

$O'_{9,ij}$  and  $O'_{10,ij}$  with  $i \neq j$  (i.e. LFV) has no effect.



SMEFT notation:  $l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ ,  $q = \begin{pmatrix} u \\ d \end{pmatrix}_L$ ,  $d = d_R$

induce  $\bar{s}bZ$  interaction,  
Thus, universally affect  
 $b \rightarrow se^+e^-, \mu^+\mu^-, \tau^+\tau^-$

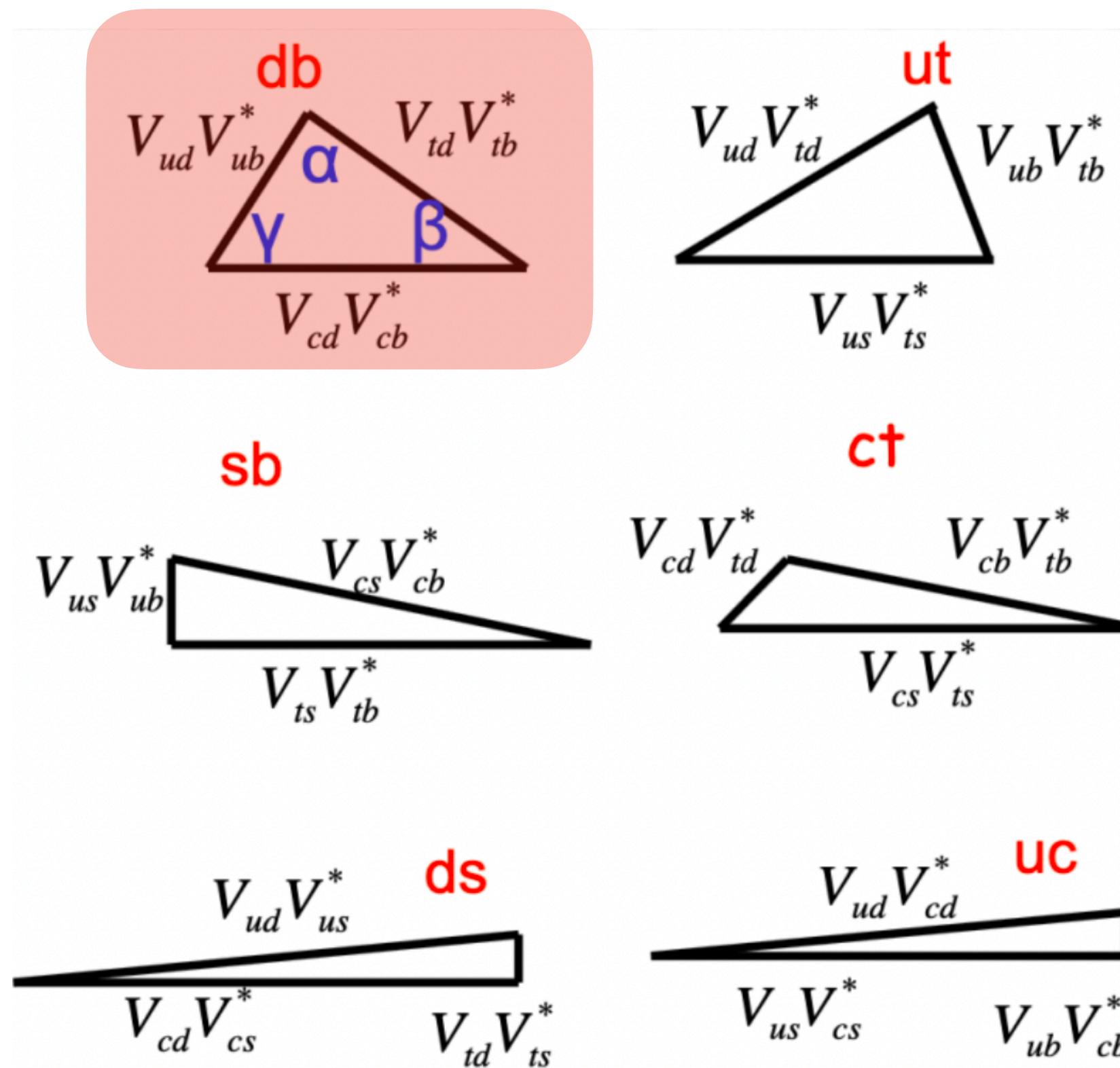
one LEFT operator!  
just the SM operator

# Cabibbo Angle Anomaly

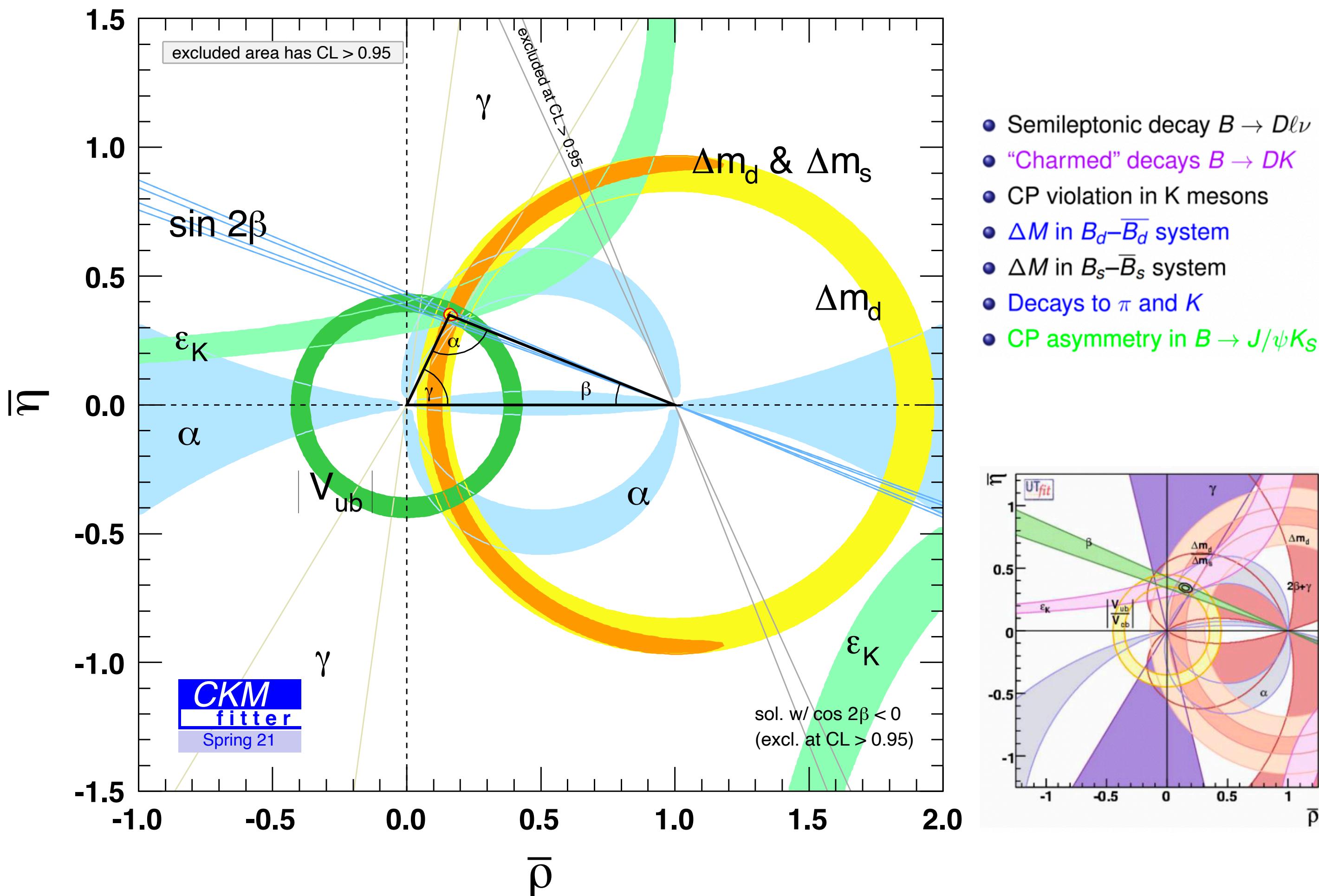
► unitarity of CKM:  $VV^\dagger = V^\dagger V = 1$

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \rightarrow \sum_i V_{ij} V_{ik}^* = \delta_{jk}$$

► unitarity triangle



All measurements agree with the CKM picture in the SM !!! However, ...



# Cabibbo Angle Anomaly

Belfatto, Beradze, Berezhiani 1906.02714

- From unitarity for the first row of CKM ( $|V_{ub}|^2 \sim 10^{-5}$ )

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Crivellin, Kirk, Kitahara, Mescia, 2212.06862

$$\text{► } ||V_{ud}|_{\text{global}}^2 + |V_{us}|_{\text{global}}^2 + |V_{ub}|^2 - 1 = -0.00151(53) \quad 2.8\sigma$$

$$|V_{ud}|_{\beta}^2 + |V_{us}|_{K_{\ell 3}}^2 + |V_{ub}|^2 - 1 = -0.00176(54), \quad 3.3\sigma$$

$$|V_{ud}|_{\beta}^2 + |V_{us}|_{K_{\mu 2}/\pi_{\mu 2}, \beta}^2 + |V_{ub}|^2 - 1 = -0.00098(56), \quad 1.8\sigma$$

$$|V_{ud}|_{K_{\mu 2}/\pi_{\mu 2}, K_{\ell 3}}^2 + |V_{us}|_{K_{\ell 3}}^2 + |V_{ub}|^2 - 1 = -0.0163(62), \quad 2.6\sigma$$

$|V_{ud}|$

$$K_{\mu 2}/\pi_{\mu 2} = \Gamma(K^+ \rightarrow \mu^+\nu)/\Gamma(\pi^+ \rightarrow \mu^+\nu)$$

$$\tau_K/\tau_\pi = \Gamma(\tau \rightarrow K^-\nu)/\Gamma(\tau \rightarrow \pi^-\nu)$$

$$K_{\ell 3}/\pi_{e 3} = \Gamma(K \rightarrow \pi\ell\nu)/\Gamma(\pi^+ \rightarrow \pi^0 e^+\nu)$$

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# Cabibbo Angle Anomaly

Belfatto, Beradze, Berezhiani 1906.02714

- From unitarity for the first row of CKM ( $|V_{ub}|^2 \sim 10^{-5}$ )

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

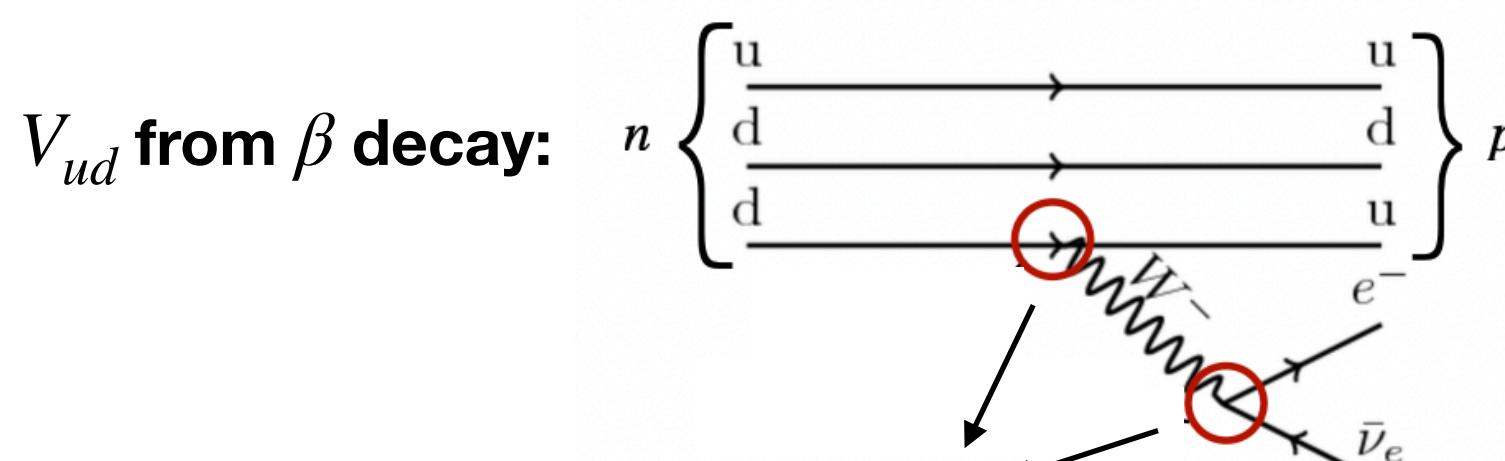
- From data

Crivellin, Kirk, Kitahara, Mescia, 2212.06862

$$|V_{ud}|_{\text{global}}^2 + |V_{us}|_{\text{global}}^2 + |V_{ub}|^2 - 1 = -0.00151(53) \quad 2.8\sigma$$

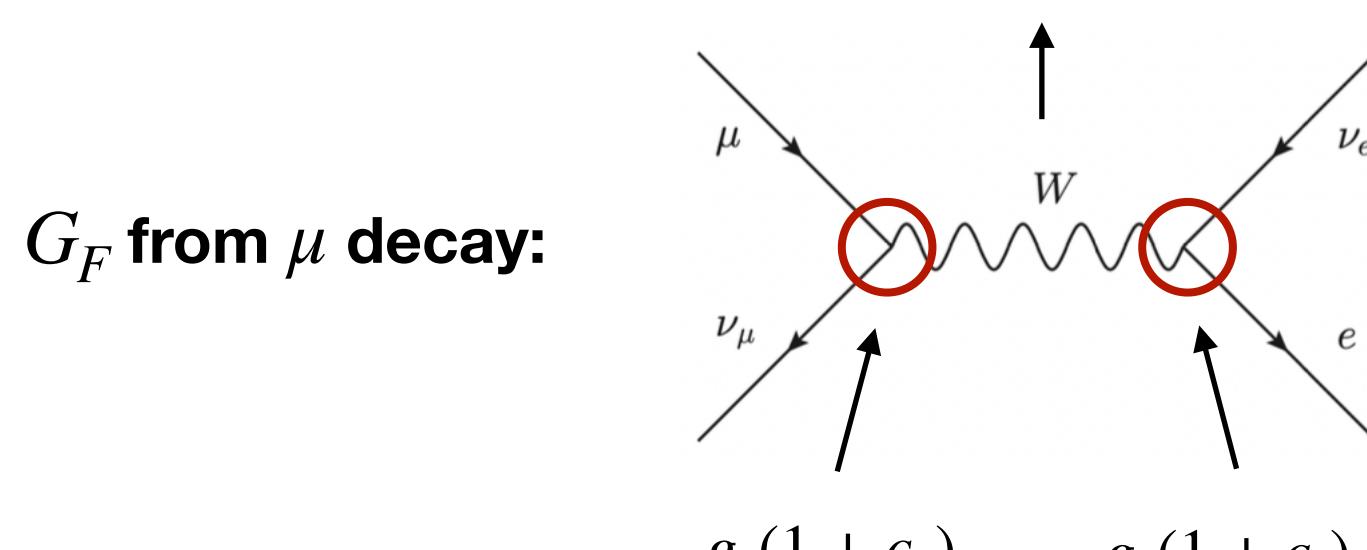
- Connection to LFUV

Crivellin, Hoferichter, 2002.07184



$$G_F^0 \cdot (1 + \epsilon_e) \cdot V_{ud}^0 = G_F \cdot (1 - \epsilon_\mu) \cdot V_{ud}^0 \quad \longrightarrow$$

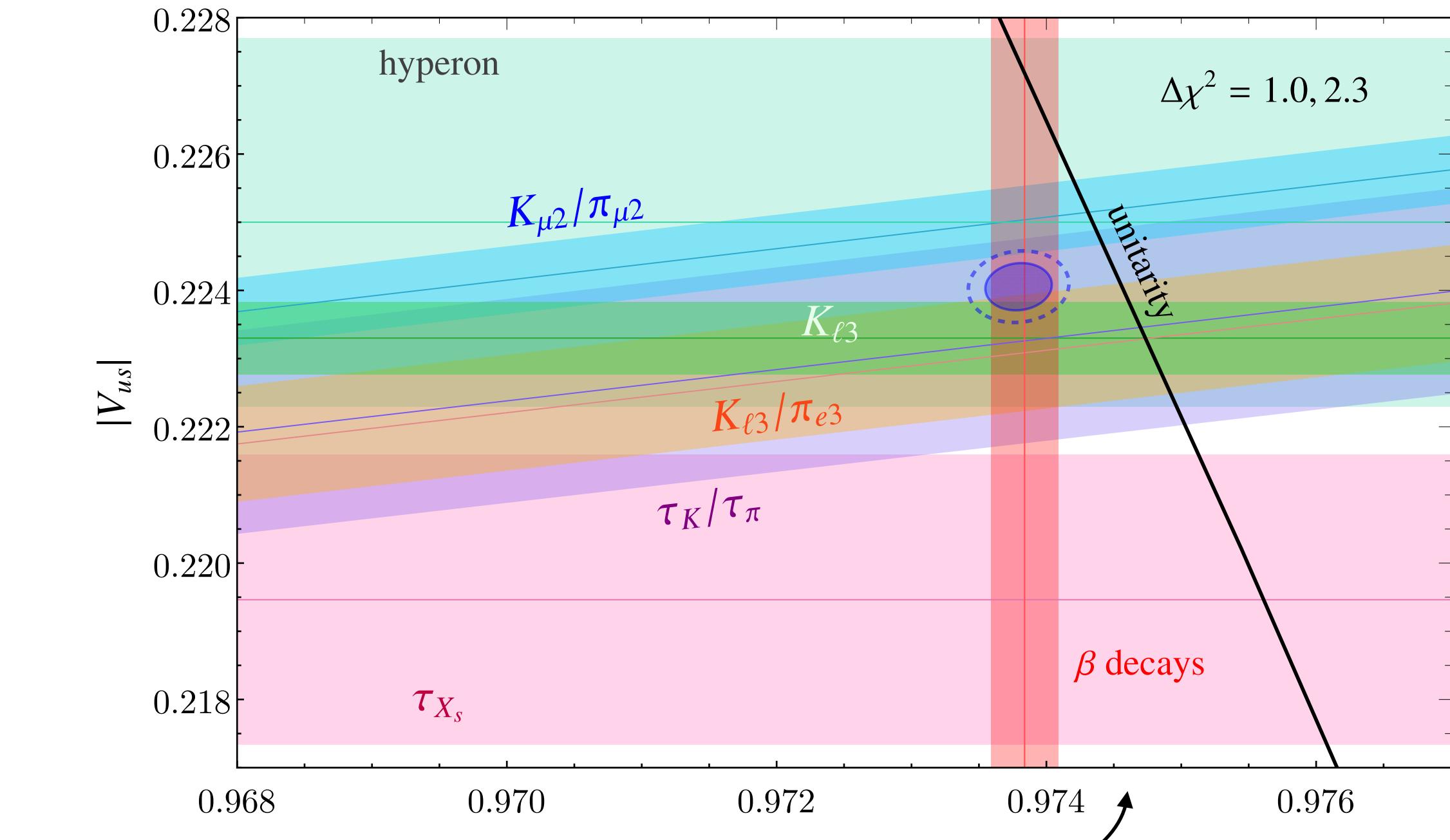
$$G_F = G_F^0(1 + \epsilon_e + \epsilon_\mu)$$



$$g_2(1 + \epsilon_\mu) \quad g_2(1 + \epsilon_e)$$

$$|V_{ud}|_{\text{global}} = 0.97379(25)$$

$$|V_{us}|_{\text{global}} = 0.22405(35)$$



$$V_{ud}^\beta = V_{ud}^0 \cdot (1 - \epsilon_\mu)$$

$V_{ud}^\beta$  can receive the contribution from  $\epsilon_\mu$ , but no  $\epsilon_e$

Observable	Measurement
$\frac{K \rightarrow \pi \mu \bar{\nu}}{K \rightarrow \pi e \bar{\nu}} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	1.0010(25) [77]
$\frac{K \rightarrow \mu \nu}{K \rightarrow e \nu} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	0.9978(18) [3, 78, 79]
$\frac{\pi \rightarrow \mu \nu}{\pi \rightarrow e \nu} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	1.0010(9) [3, 80–82]
$\frac{\tau \rightarrow \mu \nu \bar{\nu}}{\tau \rightarrow e \nu \bar{\nu}} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	1.0018(14) [3, 32]
$\frac{W \rightarrow \mu \bar{\nu}}{W \rightarrow e \bar{\nu}} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	0.9960(100) [83, 84]
$\frac{B \rightarrow D^{(*)} \mu \nu}{B \rightarrow D^{(*)} e \nu} \simeq 1 + \epsilon_{\mu\mu} - \epsilon_{ee}$	0.9890(120) [85]
$R(V_{us}) \simeq 1 - \left(\frac{V_{ud}}{V_{us}}\right)^2 \epsilon_{\mu\mu}$	0.9891(33) [11]
	0.9927(39) [14]

# Origin of LFU

## ► LFU in SM

$$\bar{\ell}'_R \gamma^\mu Z_\mu Y \ell'_R \propto [\bar{e}'_R \ \bar{\mu}'_R \ \bar{\tau}'_R] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{bmatrix} = [\bar{e}_R \ \bar{\mu}_R \ \bar{\tau}_R] U_R^\dagger \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} U_R \begin{bmatrix} e_R \\ \mu_R \\ \tau_R \end{bmatrix} = [\bar{e}_R \ \bar{\mu}_R \ \bar{\tau}_R] \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_R \\ \mu_R \\ \tau_R \end{bmatrix}$$

↑ mass eigenbasis      ↑ interaction eigenbasis

hypercharge of  $U(1)_Y$   
 $Y_{e_R} = Y_{\mu_R} = Y_{\tau_R} = -1$

$\ell'_R = U_R \ell_R$   
 $U_R$ : complex  $3 \times 3$  unitary matrix

flavour universal

## ► LFUV in BSM

$$\bar{\ell}'_R \gamma^\mu Z'_\mu Y' \ell'_R \propto [\bar{e}'_R \ \bar{\mu}'_R \ \bar{\tau}'_R] \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{bmatrix} = [\bar{e}_R \ \bar{\mu}_R \ \bar{\tau}_R] U_R^\dagger \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} U_R \begin{bmatrix} e_R \\ \mu_R \\ \tau_R \end{bmatrix} = [\bar{e}_R \ \bar{\mu}_R \ \bar{\tau}_R] \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \begin{bmatrix} e_R \\ \mu_R \\ \tau_R \end{bmatrix}$$

↑

hypercharge of  $U(1)'$   
 $Y'_{e_R} \neq Y'_{\mu_R} \neq Y'_{\tau_R}$

flavour non-universality is generated,  
but flavour violation usually can't be avoided.

## ► We learn that

$b \rightarrow s\mu\mu/b \rightarrow see \neq \text{SM} \implies \begin{cases} b \rightarrow s\text{e}\mu, b \rightarrow s\text{e}\tau, b \rightarrow s\mu\tau \\ b \rightarrow d\mu\mu/b \rightarrow dee, s \rightarrow d\mu\mu/s \rightarrow dee, b\bar{b} \rightarrow \mu\mu/b\bar{b} \rightarrow ee, \dots \end{cases}$

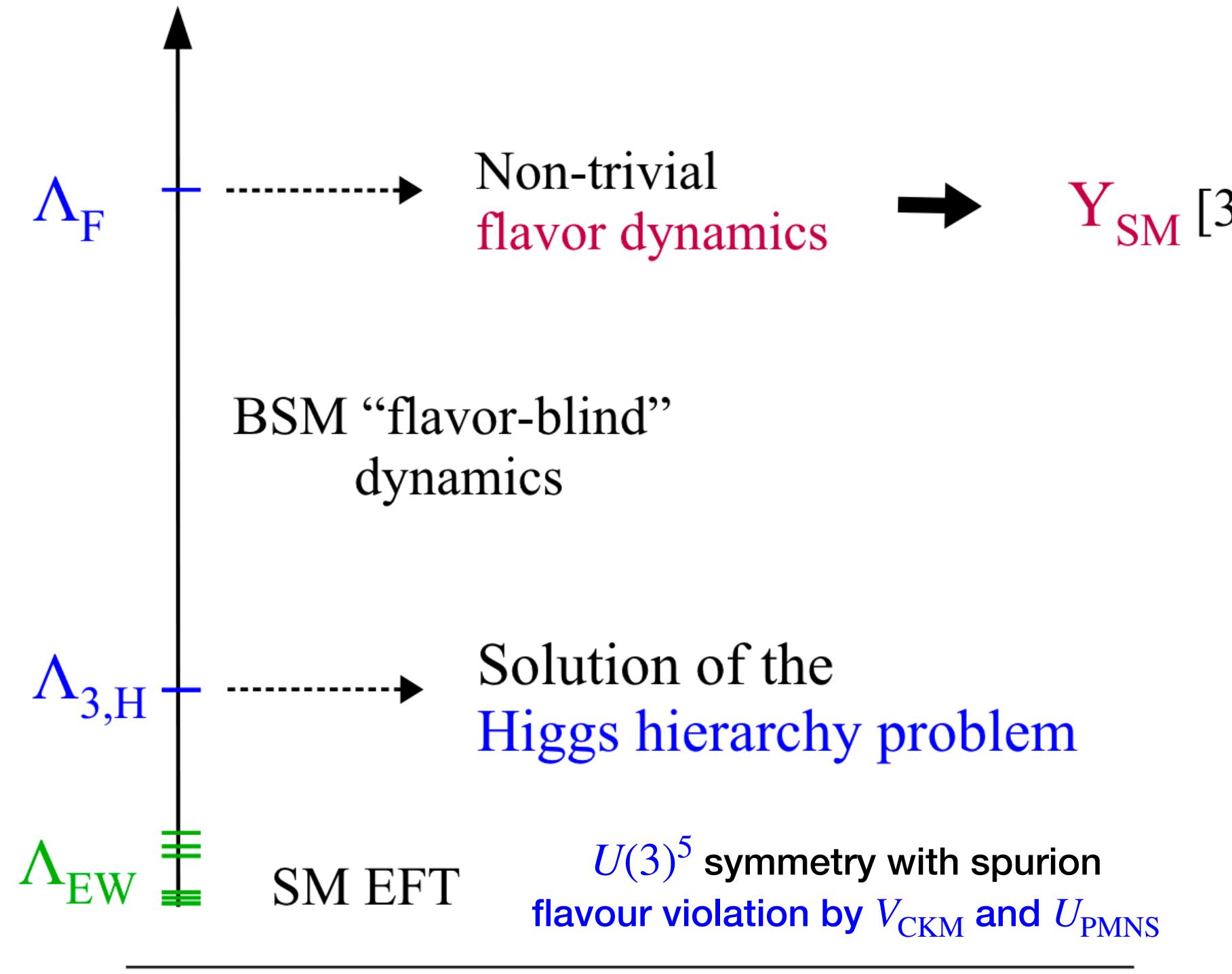
$b \rightarrow c\tau\nu/b \rightarrow c\mu\nu \neq \text{SM} \implies b \rightarrow u\tau\nu/b \rightarrow u\mu\nu, c \rightarrow s\tau\nu/c \rightarrow s\mu\nu, \dots$

What's the magnitude of NP effects in these related channels? Can they satisfy the current exp bound?

Flavour structure !

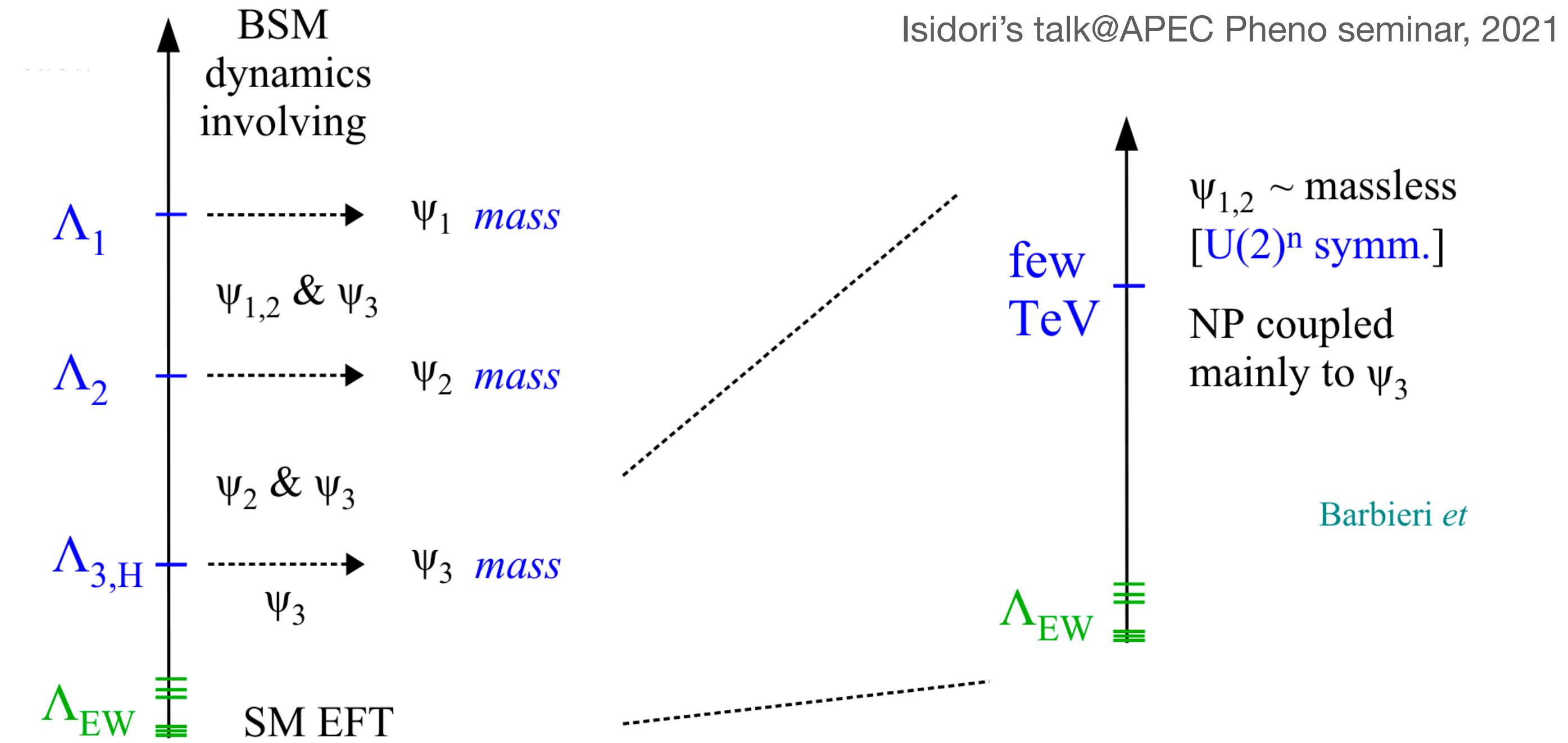
# Origin of LFU: connection to flavour structure

## ► Minimal Flavour Violation



Decay mode	Branching fractions		
	Measured upper limit at 90% CL [8,84]	Prediction maximum [or range]	
	NO	IO	
$B \rightarrow K e^\pm \mu^\mp$	$3.8 \times 10^{-8}$	$2.9 \times 10^{-9}$	$3.0 \times 10^{-9}$
$B \rightarrow K^* e^\pm \mu^\mp$	$5.1 \times 10^{-7}$	$7.8 \times 10^{-9}$	$7.8 \times 10^{-9}$
$B_s \rightarrow e^\pm \mu^\mp$	$1.1 \times 10^{-8}$	$8.6 \times 10^{-12}$	$9.0 \times 10^{-12}$
$B \rightarrow \pi e^\pm \mu^\mp$	$9.2 \times 10^{-8}$	$1.2 \times 10^{-10}$	$1.3 \times 10^{-10}$
$B \rightarrow \rho e^\pm \mu^\mp$	$3.2 \times 10^{-6}$	$3.1 \times 10^{-10}$	$3.2 \times 10^{-10}$
$B^0 \rightarrow e^\pm \mu^\mp$	$2.8 \times 10^{-9}$	$2.6 \times 10^{-13}$	$2.7 \times 10^{-13}$
....			

## ► Flavour deconstruction



- Flavour non-universal interactions already at the TeV scale.
- 1st and 2nd gen have small masses due to couple to NP at heavier scales.
- 3 gens are not identical copies up to high scales.

Covone, Davighi, Isidori, Pesut/2407.10950  
Fuentes-Martín, Lizana/2402.09507  
Davighi, Gosnay, Miller, Renner/2312.13346  
Barbieri, Isidori/2312.14004

Isidori/2308.11612  
Navarro, King/2305.07690  
Davighi, Stefanek/2305.16280  
Davighi, Isidori/2303.01520

C.W.Chiang, X.G.He, J.Tandean, **XBY**, 1706.02696  
C.S.Kim, **XBY**, Y.J.Zheng, 1602.08107

# Introduction

## Lepton Flavour Universality Test

$\pi, K, \tau$  system,  $W, Z$  boson,  $R_{D^{(*)}}$  and  $R_{K^{(*)}}$

## Implications for New Physics

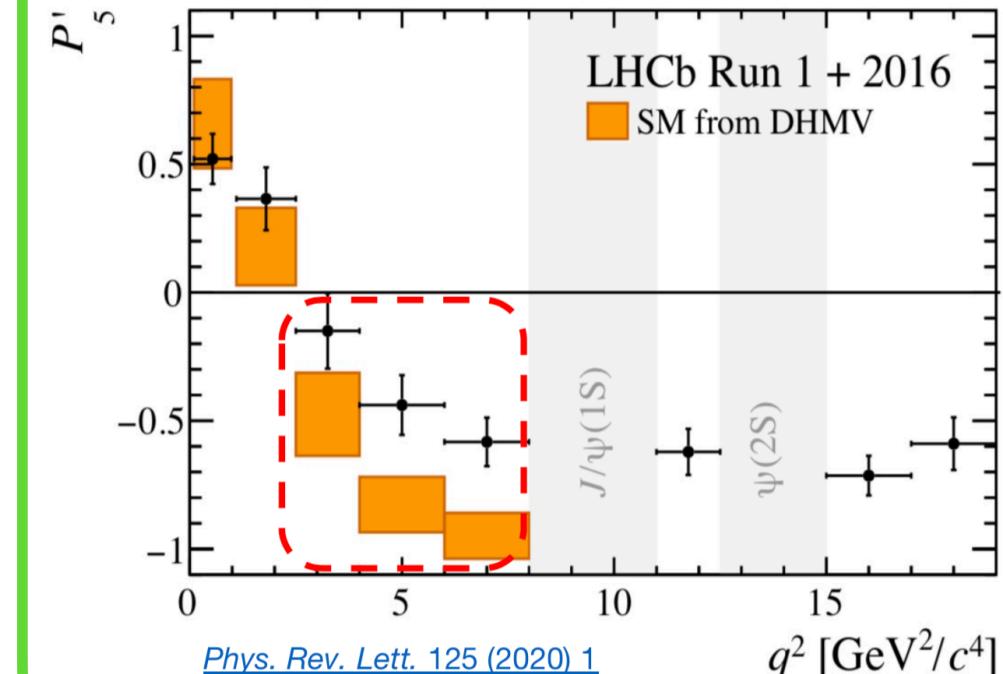
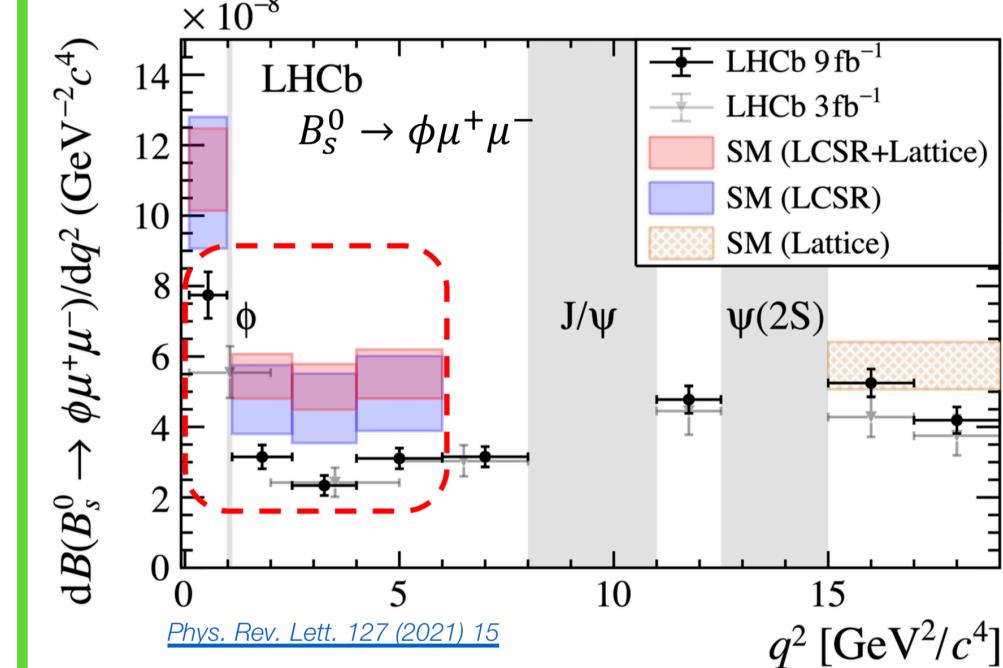
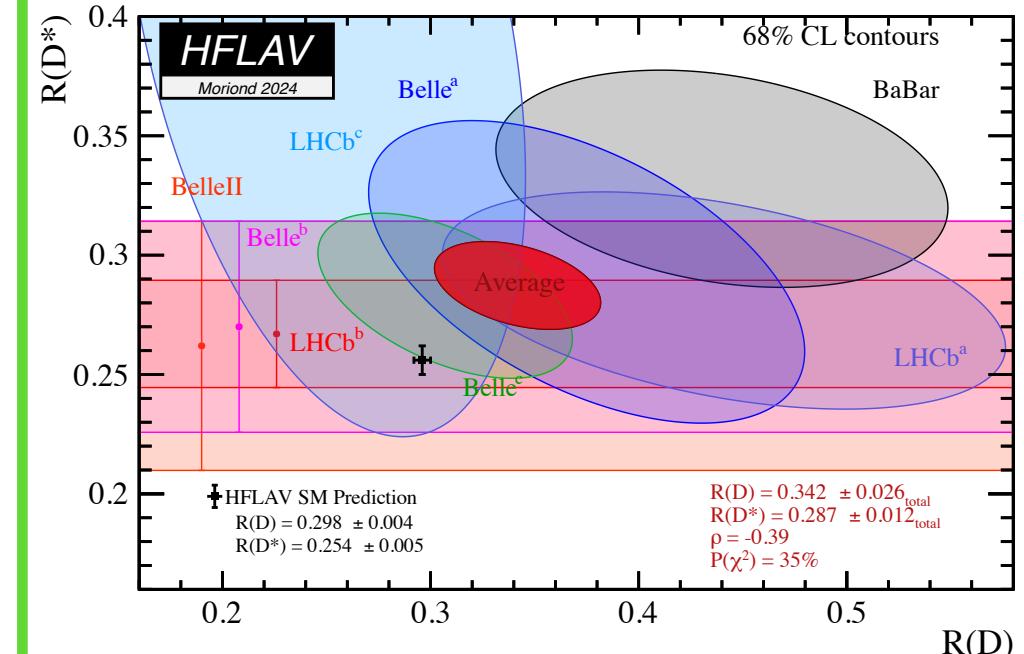
Connections to other anomalies  $\left\{ \begin{array}{l} (g-2)_\mu \text{ anomaly} \\ B^+ \rightarrow K^+ \nu \bar{\nu} \text{ excess at Belle II} \\ \text{Cabibbo angle anomaly} \end{array} \right.$

Origin of LFU violation

## Summary

# Summary

## exp measurement



SMEFT

inverse problem  
(e.g., Fermi theory to SM)

RGE  
(well understood ! )

LEFT/WET

QCD corrections  
form factor  
non-local matrix element

NP model

non-universal  
gauge interaction

leptoquark

big question

flavour structure in the SM

EW hierarchy problem

Grand Unified Theory

Dark matter

Strong CP

Neutrino mass

## Non-local matrix element

$$\begin{aligned} \mathcal{A}_\lambda^{L,R} = & \mathcal{N} \left\{ \left[ (\mathcal{C}_9 \pm \mathcal{C}'_9) \mp (\mathcal{C}_{10} \pm \mathcal{C}'_{10}) \right] \mathcal{F}_\lambda(q^2, k^2) \right. \\ & \left. + \frac{2m_b M_B}{q^2} \left[ (\mathcal{C}_7 \pm \mathcal{C}'_7) \mathcal{F}_\lambda^T(q^2, k^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2, k^2) \right] \right\} \end{aligned}$$

## Definition of Pull

To quantify the level of agreement between a given hypothesis and the data, we compute the corresponding  $p$ -value of *goodness-of-fit*:

$$p = \int_{\chi_{\min}^2}^{\infty} d\chi^2 f(\chi^2; n_{\text{dof}}), \quad (8)$$

where  $n_{\text{dof}} = N_{\text{obs}} - n$ . Finally, to compare the descriptions offered by two different nested hypotheses  $H_0$  and  $H_1$  (with  $n_{H_0}, n_{H_1}$  the respective number of degrees of freedom and  $n_{H_0} < n_{H_1}$ ), we compute their relative Pull, measured in units of Gaussian standard deviations ( $\sigma$ ):

$$\text{Pull}_{H_0 H_1} = \sqrt{2} \operatorname{Erf}^{-1} [F(\Delta\chi_{H_0 H_1}^2; n_{H_0 H_1})], \quad (9)$$

with  $\Delta\chi_{H_0 H_1}^2 = \chi_{H_0, \min}^2 - \chi_{H_1, \min}^2$ ,  $n_{H_0 H_1} = n_{H_1} - n_{H_0}$ ,  $F$  the  $\chi^2$  cumulative distribution function and  $\operatorname{Erf}^{-1}$  the inverse error function. Most of the time, we compare a given NP scenario with the SM case, denoting the result as  $\text{Pull}_{\text{SM}}$  unless there is a risk of ambiguity. Our statistical

### A. $(g - 2)_\mu$ and $(g - 2)_e$

According to the window observable theory [53, 54] and the SM prediction [55], the current disagreement concerning the  $\Delta a_\mu$  designations has emerged as follows [55–58]:

$$\Delta a_\mu^{\text{window}} = (1.81 \pm 0.47) \times 10^{-9}, \quad (17)$$

On the other hand, the electron magnetic dipole moment [59] with its experiment measurement through Rb in 2020 [60, 61], the discrepancy with SM prediction can be expressed as follows:

$$\Delta a_e^{\text{Rb}} = 34(16) \times 10^{-14}, \quad (18)$$

whereas the Cs atoms measurement method search has obtained a lower bound [62],

$$\Delta a_e^{\text{Cs}} = -102(26) \times 10^{-14}, \quad (19)$$

$$\text{Me/m_mu} = 0.5 \text{MeV}/105.6 \text{MeV} = 5 \times 10^{-3}$$