

Four-body baryonic $B \rightarrow B_1 \bar{B}'_1 B_2 \bar{B}'_2$ decays

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The4thLHCbFrontierPhysicsWorkshop

2024.07.29

PLB845, 138158 (2023)

Outline:

1. Introduction
2. Formalism
3. Some results
4. Summary

Introduction

Four-body baryonic $B \rightarrow B_1 \bar{B}'_1 B_2 \bar{B}'_2$ decays

- LHCb newly observed $\bar{B}^0 \rightarrow p\bar{p}p\bar{p}$ with

$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p}) = (2.2 \pm 0.4 \pm 0.1 \pm 0.1) \times 10^{-8}$$

[PRL131, 091901 (2023)].

- In comparison with

$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}) = (1.25 \pm 0.27 \pm 0.18) \times 10^{-8} \text{ (LHCb)},$$

$$\mathcal{B}(B^- \rightarrow p\bar{p}\pi^-) = (1.62 \pm 0.20) \times 10^{-6} \text{ (PDG)},$$

$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}\pi^0) = (5.0 \pm 1.9) \times 10^{-7} \text{ (PDG)},$$

$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}\pi^+\pi^-) = (3.0 \pm 0.2 \pm 0.2 \pm 0.1) \times 10^{-6} \text{ (PDG)},$$

$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p})$ is unexpectedly small.

Introduction

- $m_B > m_{\mathbf{B}} + m_{\bar{\mathbf{B}}'}$ for $\mathbf{B}\bar{\mathbf{B}}'$

1. $B \rightarrow \mathbf{B}\bar{\mathbf{B}}' M$

$\mathcal{B}(\bar{B}^0 \rightarrow n\bar{p}D^{*+}) \simeq 10^{-3}$ observed in 2001 (CLEO)

$\mathcal{B}(B^- \rightarrow p\bar{p}K^-) \simeq 10^{-6}$ observed in 2002 (BELLE)

2. $B \rightarrow \mathbf{B}\bar{\mathbf{B}}'$, LHCb

$\mathcal{B}(B^- \rightarrow \Lambda(1520)\bar{p}) = (3.15 \pm 0.48 \pm 0.27) \times 10^{-7}$ (2014)

$\mathcal{B}(B^- \rightarrow \Lambda\bar{p}) = (2.4^{+1.0}_{-0.8} \pm 0.3) \times 10^{-7}$ (2017)

$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}) = (1.25 \pm 0.27 \pm 0.18) \times 10^{-8}$ (2017)

Introduction

3. 1st observation of a baryonic \bar{B}_s^0 decay (LHCb, 2017)

$$\mathcal{B}(\bar{B}_s^0 \rightarrow \Lambda \bar{p} K^+ + \bar{\Lambda} p K^-)$$

$$= (5.46 \pm 0.61 \pm 0.57 \pm 0.50 \pm 0.32) \times 10^{-6}$$

4. $B_{(s)} \rightarrow \mathbf{B}\bar{\mathbf{B}}'MM'$

$$\mathcal{B}(B^- \rightarrow \Lambda \bar{p} \pi^+ \pi^-) = (5.92^{+0.88}_{-0.84} \pm 0.69) \times 10^{-6} \text{ (BELLE, 2009)}$$

$$\mathcal{B}(\bar{B}^0 \rightarrow p \bar{p} \pi^+ \pi^-) = (3.0 \pm 0.2 \pm 0.2 \pm 0.1) \times 10^{-6} \text{ (LHCb, 2017)}$$

$$\mathcal{B}(\bar{B}^0 \rightarrow p \bar{p} K^\mp \pi^\pm) = (6.6 \pm 0.3 \pm 0.3 \pm 0.3) \times 10^{-6} \text{ (LHCb, 2017)}$$

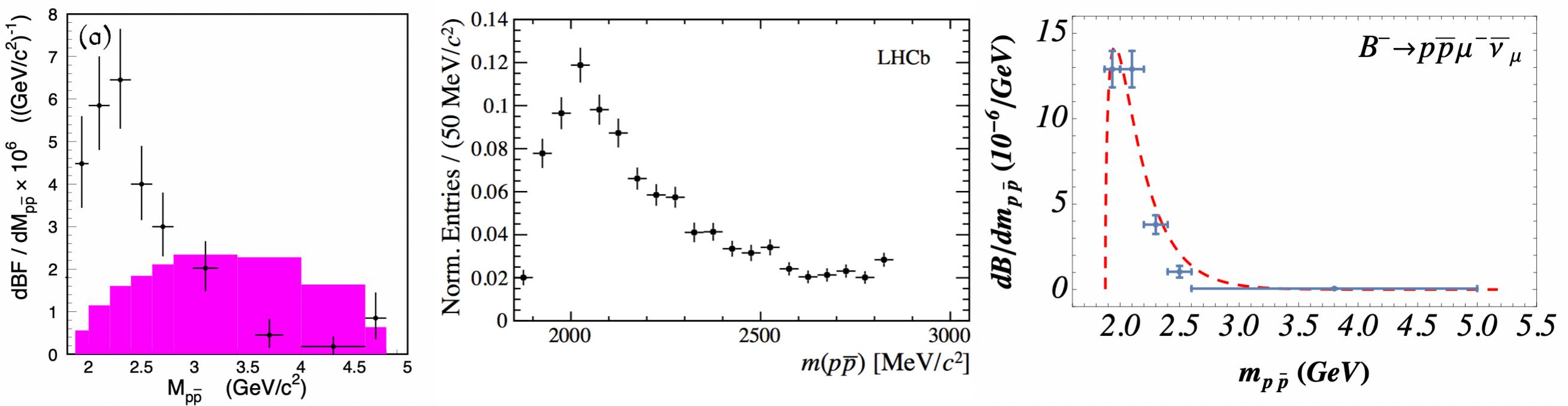
5. radiative and semileptonic

$$\mathcal{B}(B^- \rightarrow \Lambda \bar{p} \gamma) = (2.16^{+0.58}_{-0.53} \pm 0.20) \times 10^{-6} \text{ (BELLE, 2005)}$$

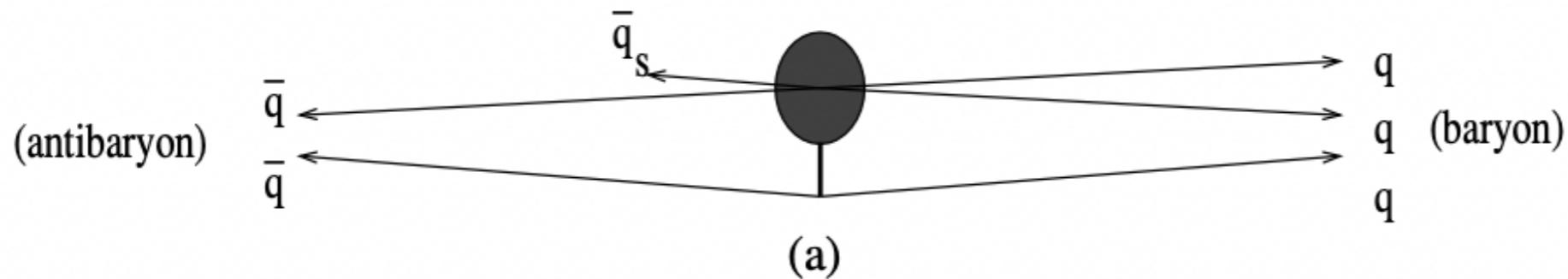
$$\mathcal{B}(B^- \rightarrow p \bar{p} \ell^- \bar{\nu}_\ell) = (5.8^{+2.4}_{-2.1} \pm 0.9) \times 10^{-6} \text{ (BELLE, 2014)}$$

Threshold effect

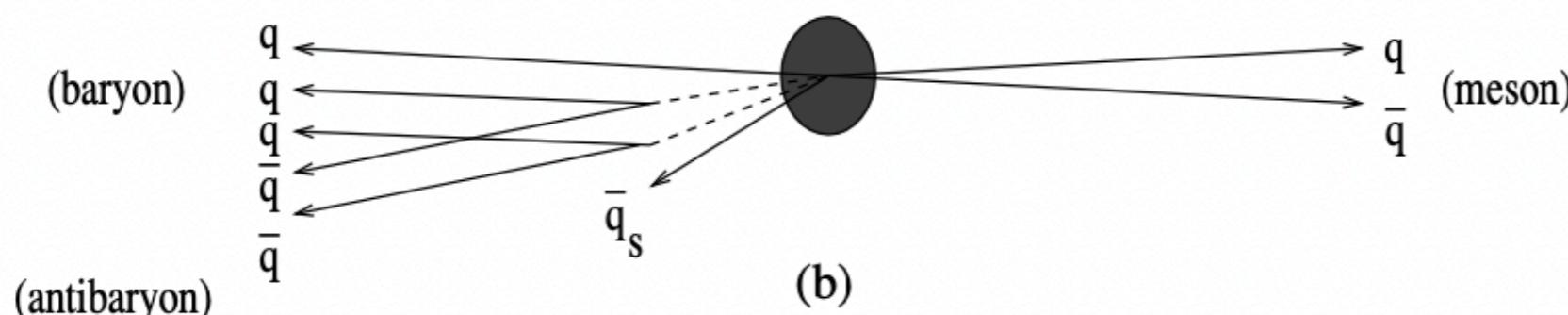
- From (non-)observation of $\bar{B}^0 \rightarrow n\bar{p}D^{*+}$ ($\bar{B}^0 \rightarrow p\bar{p}$),
conjecturing a preference of $B\bar{B}'$ formation [Hou, Soni, PRL (2001)]
- Threshold effect, an enhancing factor
A peak near the threshold area of $m_{B\bar{B}'} \simeq m_B + m_{\bar{B}'}$
Observed in $B \rightarrow B\bar{B}'M$, $B\bar{B}'MM'$, $B\bar{B}'\gamma$, $B\bar{B}'L\bar{L}'$



- The hard off-shell gluon suppresses $\mathcal{M}(B \rightarrow B\bar{B}')$ by the power of α_s/q^2 [Suzuki, JPG 34, 283 (2007)]



back-to-back $q\bar{q}$, energetic

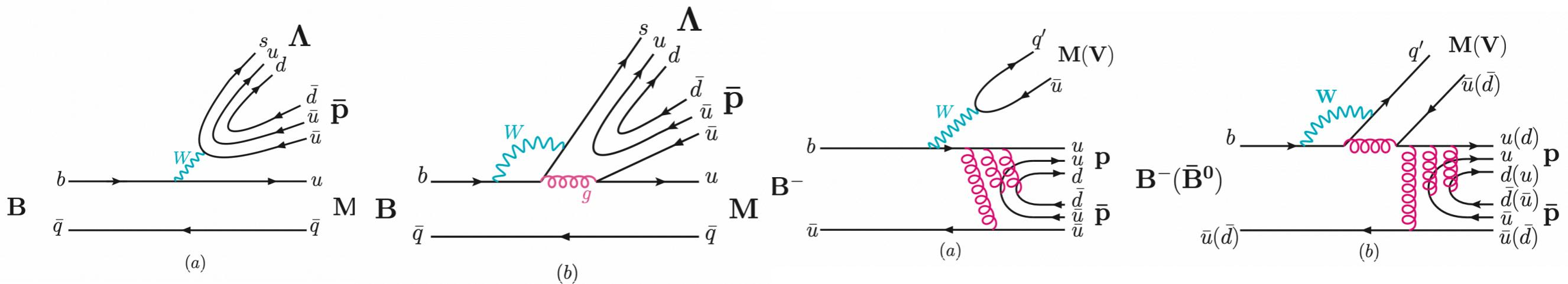


collinear $q\bar{q}$, not energetic

Factorization

$$\mathcal{M}_1(B \rightarrow \mathbf{B}\bar{\mathbf{B}}' M) \propto \langle \mathbf{B}\bar{\mathbf{B}}' |(\bar{q}q')|0\rangle \langle M|(\bar{q}b)|B\rangle$$

$$\mathcal{M}_2(B \rightarrow \mathbf{B}\bar{\mathbf{B}}' M) \propto \langle M|(\bar{q}q')|0\rangle \langle \mathbf{B}\bar{\mathbf{B}}' |(\bar{q}b)|B\rangle$$



pQCD counting rules

$$\langle \mathbf{B}\bar{\mathbf{B}}' |(\bar{q}q')|0\rangle, F_{\mathbf{B}\bar{\mathbf{B}}'} \propto 1/t^2 \text{ (Brodsky, Farrar, 1973)}$$

$$\langle \mathbf{B}\bar{\mathbf{B}}' |(\bar{q}b)|B\rangle, F'_{\mathbf{B}\bar{\mathbf{B}}'} \propto 1/t^3 \text{ (Chua, Hou, Tsai, 2002)}$$

$B \rightarrow \mathbf{B}_1 \bar{\mathbf{B}}'_1 \mathbf{B}_2 \bar{\mathbf{B}}'_2$

- $\mathcal{M} \propto \langle \mathbf{B}_1 \bar{\mathbf{B}}'_1 | (\bar{q}q') | 0 \rangle \langle \mathbf{B}_2 \bar{\mathbf{B}}'_2 | (\bar{q}b) | B \rangle$
- Two dibaryon formations, threshold effect exists.
- anticipated to be of order 10^{-6} . In fact,

$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p}) < 2.0 \times 10^{-7} \text{ (Babar, 2018)}$$

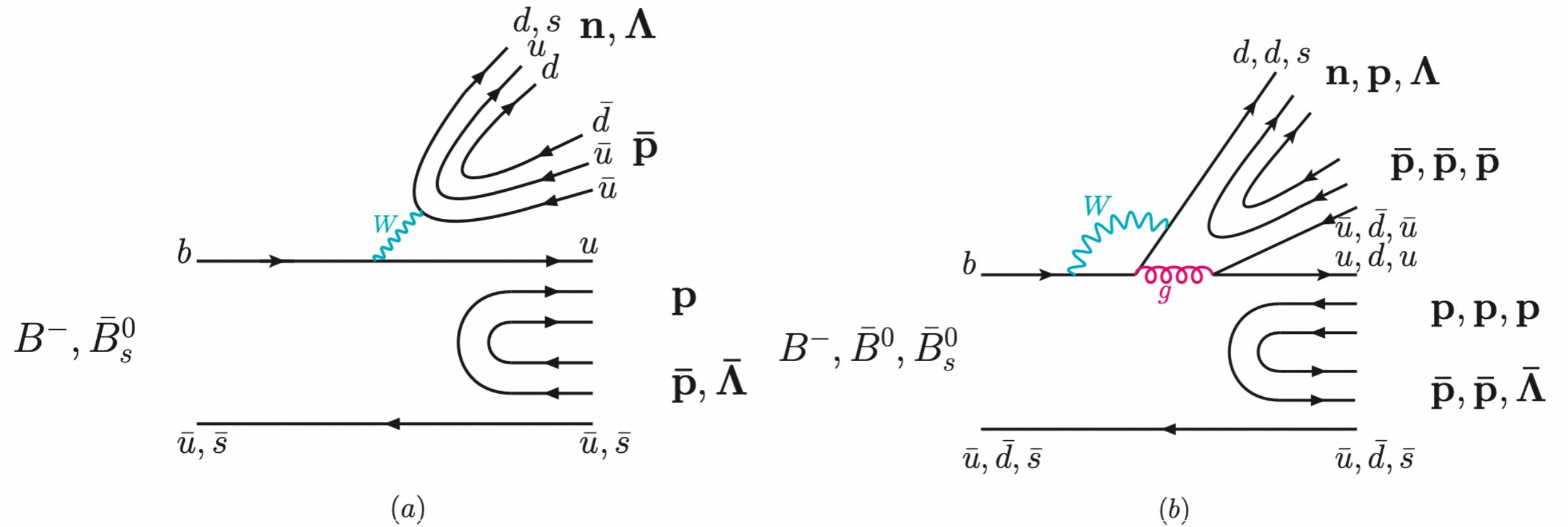
$$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p}) = (2.2 \pm 0.4 \pm 0.1 \pm 0.1) \times 10^{-8} \text{ (LHCb, 2022)}$$

as small as $\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p})$

- An interpretation is needed.
- Providing more $B \rightarrow \mathbf{B}_1 \bar{\mathbf{B}}'_1 \mathbf{B}_2 \bar{\mathbf{B}}'_2$ decay channels for future measurements.

Amplitudes

$$(B^-, \bar{B}_s^0) \rightarrow (n\bar{p}p\bar{p}, \Lambda\bar{p}p\bar{\Lambda})$$



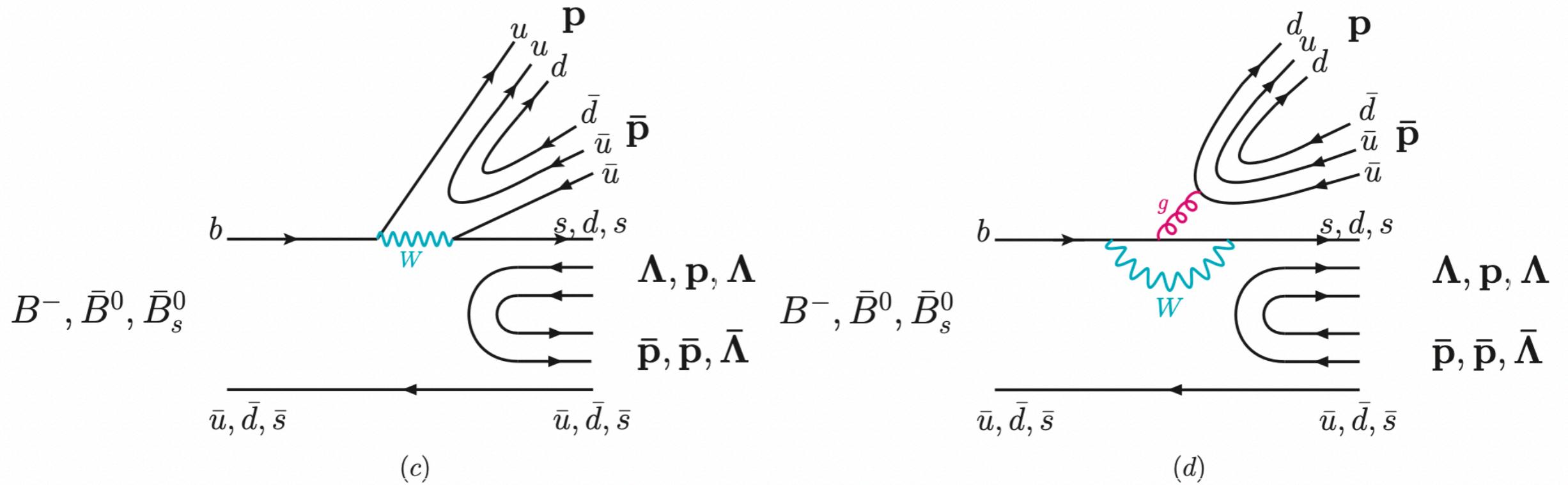
$$\bar{\mathcal{M}}(B^- \rightarrow n\bar{p}p\bar{p}) =$$

$$(\alpha_1^d + \alpha_4^d) \langle n\bar{p}|(\bar{d}u)_{V-A}|0\rangle \langle p\bar{p}|(\bar{u}b)_{V-A}|B^-\rangle$$

$$+ \alpha_6^d \langle n\bar{p}|(\bar{d}u)_{S+P}|0\rangle \langle p\bar{p}|(\bar{u}b)_{S-P}|B^-\rangle$$

Amplitudes

$$(B^-, \bar{B}^0, \bar{B}_s^0) \rightarrow p\bar{p}(\Lambda\bar{p}, p\bar{p}, \Lambda\bar{\Lambda})$$



$$\bar{\mathcal{M}}(\bar{B}^0 \rightarrow p\bar{p}pp\bar{p}) =$$

$$\begin{aligned}
 & [\langle p\bar{p} | \alpha_+^d (\bar{u}u)_V - \alpha_-^d (\bar{u}u)_A | 0 \rangle + \langle p\bar{p} | \beta_+^d (\bar{d}d)_V - \beta_-^d (\bar{d}d)_A | 0 \rangle \\
 & + (\alpha_4^d - \alpha_{10}^d/2) \langle p\bar{p} | (\bar{d}d)_{V-A} | 0 \rangle] \langle p\bar{p} | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \\
 & + \alpha_6^d \langle p\bar{p} | (\bar{d}d)_{S+P} | 0 \rangle \langle p\bar{p} | (\bar{d}b)_{S-P} | \bar{B}^0 \rangle
 \end{aligned}$$

Timelike baryonic form factors

$$\langle \mathbf{B}_1 \bar{\mathbf{B}}'_1 | (\bar{q}q')_V | 0 \rangle = \bar{u} \left[F_1 \gamma_\mu + \frac{F_2}{m_{\mathbf{B}_1} + m_{\bar{\mathbf{B}}'_1}} i \sigma_{\mu\nu} q^\nu \right] v$$

$$\langle \mathbf{B}_1 \bar{\mathbf{B}}'_1 | (\bar{q}q')_A | 0 \rangle = \bar{u} \left[g_A \gamma_\mu + \frac{h_A}{m_{\mathbf{B}_1} + m_{\bar{\mathbf{B}}'_1}} q_\mu \right] \gamma_5 v$$

$$\langle \mathbf{B}_1 \bar{\mathbf{B}}'_1 | (\bar{q}q')_S | 0 \rangle = f_S \bar{u} v$$

$$\langle \mathbf{B}_1 \bar{\mathbf{B}}'_1 | (\bar{q}q')_P | 0 \rangle = g_P \bar{u} \gamma_5 v$$

$$F_1 = \frac{\bar{C}_{F_1}}{s^2}, \quad g_A = \frac{\bar{C}_{g_A}}{s^2}, \quad f_S = \frac{\bar{C}_{f_S}}{s^2}, \quad g_P = \frac{\bar{C}_{g_P}}{s^2},$$

$$s = (p_1 + p'_1)^2$$

$$\bar{C}_i = C_i [\ln(t/\Lambda_0^2)]^{-\gamma} \text{ with } \gamma = 2.148 \text{ and } \Lambda_0 = 0.3 \text{ GeV.}$$

$$(C_{F_1}, C_{g_A}, C_{f_S}, C_{g_P}) = \sqrt{\frac{3}{2}} (C_{||}, C_{||}^*, -\bar{C}_{||}, -\bar{C}_{||}^*)$$

$$(\text{for } \langle \Lambda \bar{p} | (\bar{s}u)_{V,A,S,P} | 0 \rangle)$$

$$\text{with } C_{||(||)}^* \equiv C_{||(||)} + \delta C_{||(||)} \text{ and } \bar{C}_{||}^* \equiv \bar{C}_{||} + \delta \bar{C}_{||}.$$

$$C_{F_1} = \frac{5}{3}C_{||} + \frac{1}{3}C_{\overline{||}}, \quad C_{g_A} = \frac{5}{3}C_{||}^* - \frac{1}{3}C_{\overline{||}}^*, \quad (\text{for } \langle p\bar{p}|\bar{u}\gamma_\mu(\gamma_5)u|0\rangle)$$

$$C_{F_1} = \frac{1}{3}C_{||} + \frac{2}{3}C_{\overline{||}}, \quad C_{g_A} = \frac{1}{3}C_{||}^* - \frac{2}{3}C_{\overline{||}}^*, \quad (\text{for } \langle p\bar{p}|\bar{d}\gamma_\mu(\gamma_5)d|0\rangle)$$

$$C_{f_S} = \frac{1}{3}\bar{C}_{||}, \quad C_{g_P} = \frac{1}{3}\bar{C}_{||}^*, \quad (\text{for } \langle p\bar{p}|\bar{d}(\gamma_5)d|0\rangle)$$

$$C_{F_1} = \sqrt{\frac{3}{2}}C_{||}, \quad C_{g_A} = \sqrt{\frac{3}{2}}C_{||}^*, \quad (\text{for } \langle \Lambda\bar{p}|\bar{s}\gamma_\mu(\gamma_5)u|0\rangle)$$

$$C_{f_S} = -\sqrt{\frac{3}{2}}\bar{C}_{||}, \quad C_{g_P} = -\sqrt{\frac{3}{2}}\bar{C}_{||}^*, \quad (\text{for } \langle \Lambda\bar{p}|\bar{s}(\gamma_5)u|0\rangle)$$

$B \rightarrow \mathbf{B}_2 \bar{\mathbf{B}}'_2$ transition form factors

$$\langle \mathbf{B}_2 \bar{\mathbf{B}}'_2 | (\bar{s}b)_V | B \rangle = i\bar{u}[g_1\gamma_\mu + g_2i\sigma_{\mu\nu}p^\nu + g_3p_\mu + g_4q_\mu + g_5(p_{\bar{\mathbf{B}}'_2} - p_{\mathbf{B}_2})_\mu]\gamma_5 v$$

$$\langle \mathbf{B}_2 \bar{\mathbf{B}}'_2 | (\bar{s}b)_A | B \rangle = i\bar{u}[f_1\gamma_\mu + f_2i\sigma_{\mu\nu}p^\nu + f_3p_\mu + f_4q_\mu + f_5(p_{\bar{\mathbf{B}}'_2} - p_{\mathbf{B}_2})_\mu]v$$

$$\langle \mathbf{B}_2 \bar{\mathbf{B}}'_2 | (\bar{s}b)_S | B \rangle = i\bar{u}[\bar{g}_1\cancel{p} + \bar{g}_2(E_{\bar{\mathbf{B}}'_2} + E_{\mathbf{B}_1}) + \bar{g}_3(E_{\bar{\mathbf{B}}'_2} - E_{\mathbf{B}_1})]\gamma_5 v$$

$$\langle \mathbf{B}_2 \bar{\mathbf{B}}'_2 | (\bar{s}b)_P | B \rangle = i\bar{u}[\bar{f}_1\cancel{p} + \bar{f}_2(E_{\bar{\mathbf{B}}'_2} + E_{\mathbf{B}_2}) + \bar{f}_3(E_{\bar{\mathbf{B}}'_2} - E_{\mathbf{B}_2})]v$$

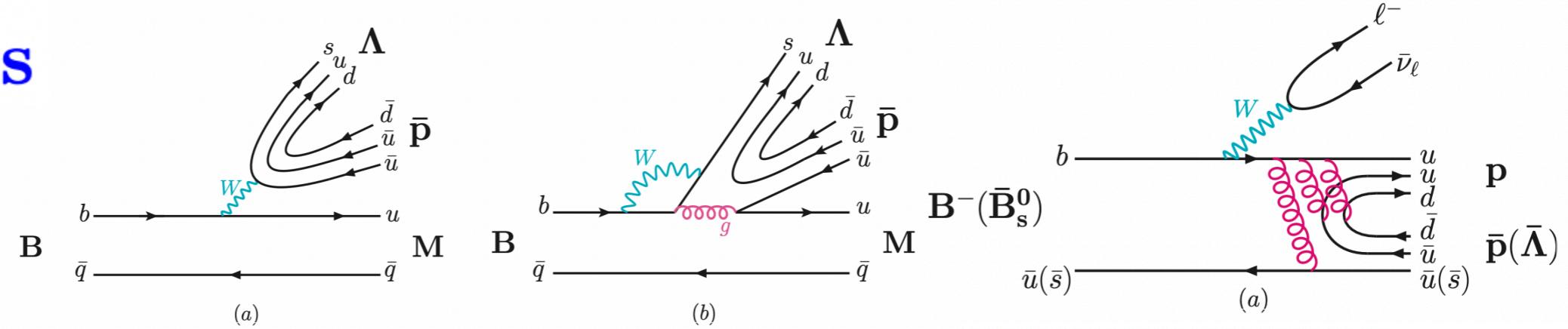
$$f_i = \frac{D_{f_i}}{t^3}, \quad g_i = \frac{D_{g_i}}{t^3}, \quad \bar{f}_i = \frac{D_{\bar{f}_i}}{t^3}, \quad \bar{g}_i = \frac{D_{\bar{g}_i}}{t^3},$$

$$t = (p_2 + p'_2)^2$$

$$D_{g_1} = D_{f_1} = \sqrt{\frac{3}{2}}D_{||}, \quad D_{g_{3,4,5}} = -D_{f_{3,4,5}} = -\sqrt{\frac{3}{2}}D_{||}^{3,4,5}$$

(for $\langle \Lambda \bar{p} | (\bar{s}b)_{V,A} | B^- \rangle$)

Extractions



$$(C_{||}, \delta C_{||}, C_{\overline{|}} , \delta C_{\overline{|}}) =$$

$$(150.8 \pm 5.7, 31.9 \pm 7.1, 27.4 \pm 27.3, -317.8 \pm 169.1) \text{ GeV}^4$$

X. Huang, Hsiao, J. Wang and L. Sun,

“Angular asymmetries in $B \rightarrow \Lambda \bar{p} M$ decays,” PRD105, 076016 (2022)

$$(D_{||}, D_{\overline{|}}) = (11.2 \pm 43.5, 332.3 \pm 17.2) \text{ GeV}^5$$

$$(D_{||}^2, D_{||}^3, D_{||}^4, D_{||}^5) =$$

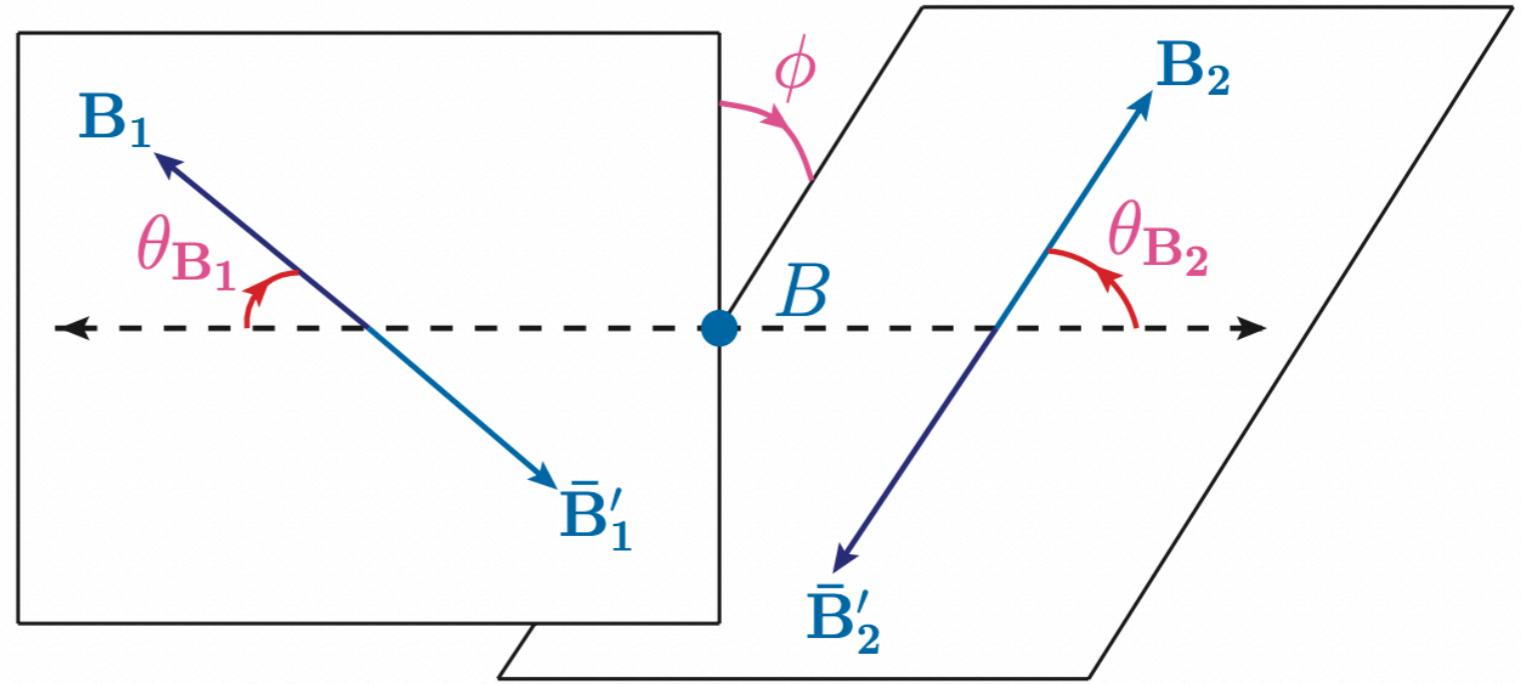
$$(47.7 \pm 10.1, 442.2 \pm 103.4, -38.7 \pm 9.6, 80.7 \pm 27.2) \text{ GeV}^4$$

$$(\bar{D}_{||}, \bar{D}_{\overline{|}}, \bar{D}_{||}^2, \bar{D}_{||}^3) =$$

$$(-59.9 \pm 12.9, 23.8 \pm 6.8, 90.9 \pm 11.1, 131.7 \pm 330.7) \text{ GeV}^4$$

Hsiao, “Semileptonic baryonic B decays,” EPJC83, 300 (2023).

Phase space



$$d\Gamma = \frac{|\mathcal{M}|^2}{4(4\pi)^6 m_B^3} X \alpha_{\mathbf{B}_1} \alpha_{\mathbf{B}_2} ds dt d \cos \theta_{\mathbf{B}_1} d \cos \theta_{\mathbf{B}_2} d\phi$$

$$X = [(m_B^2 - s - t)^2/4 - st]^{1/2}$$

$$\alpha_{\mathbf{B}_1} = \lambda^{1/2}(s, m_1^2, m_1'^2)/s, \alpha_{\mathbf{B}_2} = \lambda^{1/2}(t, m_2^2, m_2'^2)/t$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$$

Allowed ranges

$$(m_1 + m'_1)^2 \leq s \leq (m_B - \sqrt{t})^2, (m_2 + m'_2)^2 \leq t \leq (m_B - m_1 - m'_1)^2$$

$$0 \leq \theta_{\mathbf{B}_1, \mathbf{B}_2} \leq \pi, 0 \leq \phi \leq 2\pi$$

Identical particles, indistinguishable amplitudes

- In our case

$$(B^-, \bar{B}_s^0) \rightarrow (n\bar{p}p\bar{p}, \Lambda\bar{p}p\bar{\Lambda})$$

$$(B^-, \bar{B}^0, \bar{B}_s^0) \rightarrow p\bar{p}(\Lambda\bar{p}, p\bar{p}, \Lambda\bar{\Lambda})$$

- By following the studies of $(\pi^0, K_L, \bar{B}_s^0) \rightarrow e^+e^-e^+e^-$:

1. T. Miyazaki and E. Takasugi, PRD8, 2051 (1973),

“Internal conversion of pseudoscalar mesons into lepton pairs.”

2. L. Zhang and J. L. Goity, PRD57, 7031 (1998),

“The Decays $K_L \rightarrow \ell^+\ell^-\ell'^+\ell'^-$ revisited.”

3. Dincer, Sehgal, PLB556, 169 (2003),

“Electroweak effects in the double Dalitz decay $B_s \rightarrow \ell^+\ell^-\ell'^+\ell'^-$.”

4. Ivanov and Melikhov, PRD105, 094038 (2022),

“Theoretical analysis of the leptonic decays $B \rightarrow \ell\ell\ell\bar{\nu}_\ell$:

Identical leptons in the final state.”

- $\bar{B}^0 \rightarrow p\bar{p}p\bar{p}$ for illustration

$$\mathcal{M} = \mathcal{M}_{\text{dir}} + \mathcal{M}_{\text{ex}}$$

$$\mathcal{M}_{\text{dir}}[\bar{B}^0 \rightarrow p(p_1)\bar{p}(p'_1)p(p_2)\bar{p}(p'_2)]$$

$$\mathcal{M}_{\text{ex}}[\bar{B}^0 \rightarrow p(p_2)\bar{p}(p'_1)p(p_1)\bar{p}(p'2)]$$

$$\mathcal{B} = \mathcal{B}_{\text{dir}} + \mathcal{B}_{\text{ex}} + \mathcal{B}_{\text{dir}} + \mathcal{B}_{\text{dir} \times \text{ex}},$$

where $\mathcal{B}_{\text{dir} \times \text{ex}} \propto \mathcal{M}_{\text{dir}} \mathcal{M}_{\text{ex}}^* + \mathcal{M}_{\text{dir}}^* \mathcal{M}_{\text{ex}}$.

No doubt: $\mathcal{B}_{\text{dir}} = \mathcal{B}_{\text{ex}}$, but how to evaluate $\mathcal{B}_{\text{dir} \times \text{ex}}$?

$$\mathcal{M}_{\text{dir}}: F_{\mathbf{B}\bar{\mathbf{B}}'} \propto 1/s^2, \hat{F}_{\mathbf{B}\bar{\mathbf{B}}'} \propto 1/t^3.$$

$$\mathcal{M}_{\text{ex}}: F_{\mathbf{B}\bar{\mathbf{B}}'}^{\text{ex}} \propto 1/s'^2, \hat{F}_{\mathbf{B}\bar{\mathbf{B}}'}^{\text{ex}} \propto 1/t'^3,$$

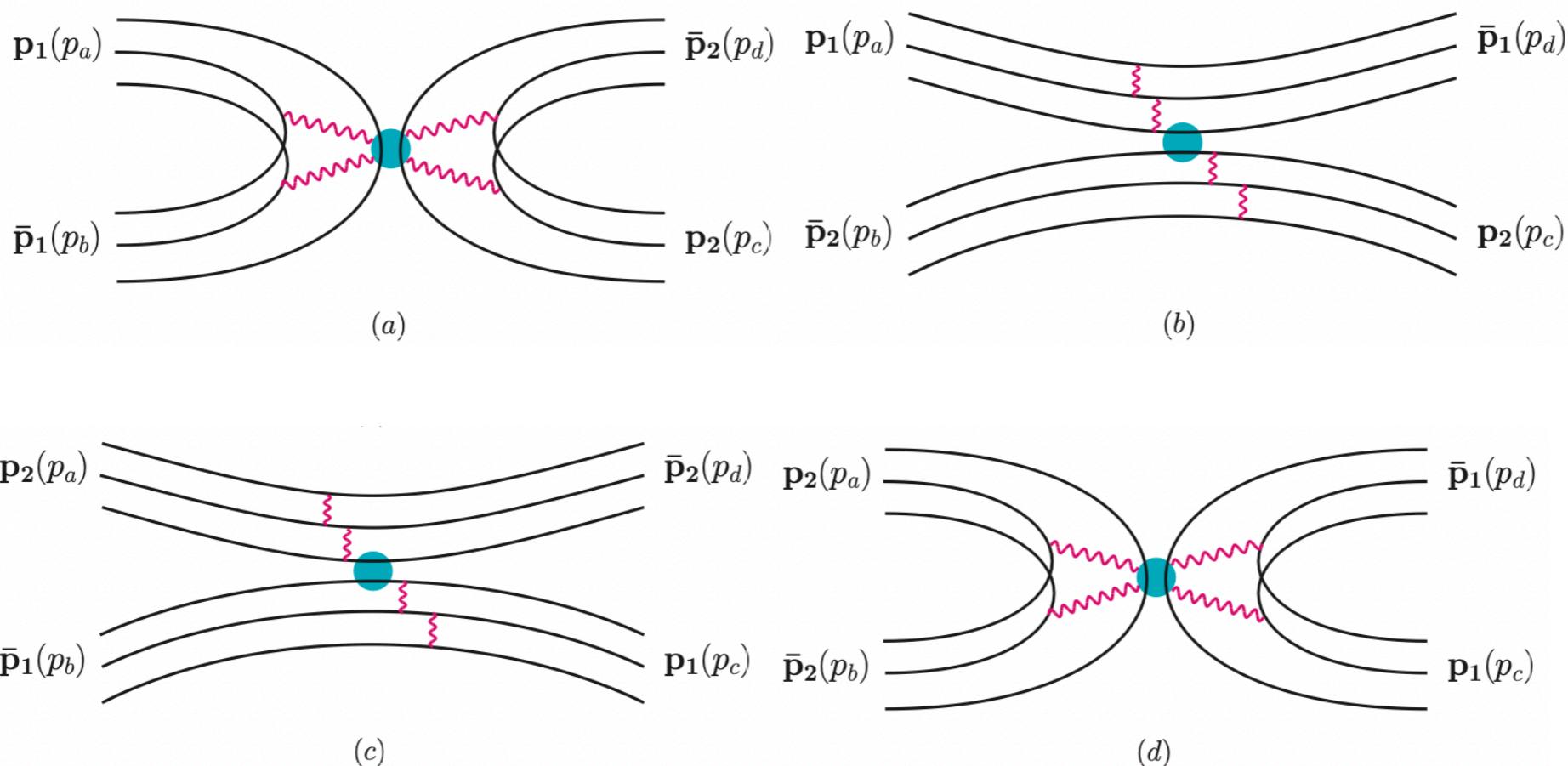
$$s' \equiv (p_2 + p'_1)^2 \text{ and } t' \equiv (p_1 + p'_2).$$

Approximation around the threshold

$\mathcal{B}_{\text{dir} \times \text{ex}} \sim 0.14 \times 10^{-8}$, negligible.

$\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}pp\bar{p}) \simeq (\mathcal{B}_{\text{dir}} + \mathcal{B}_{\text{ex}})/4$,

1/4 for the 4 indistinguishable amplitudes.



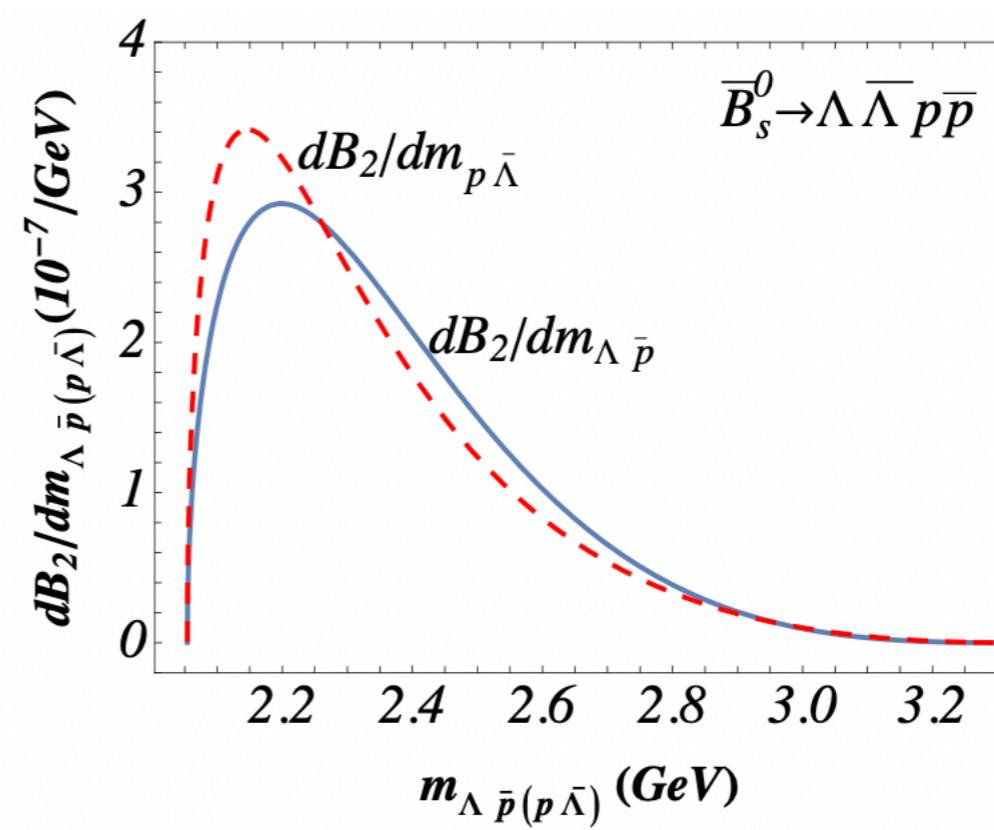
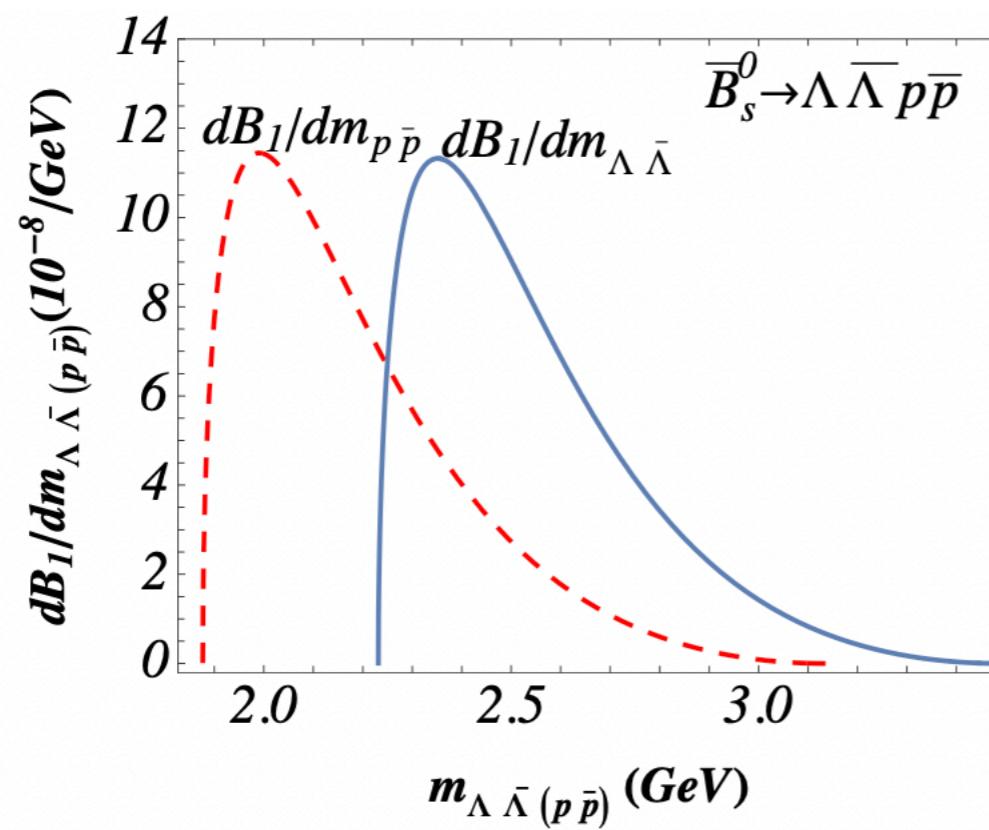
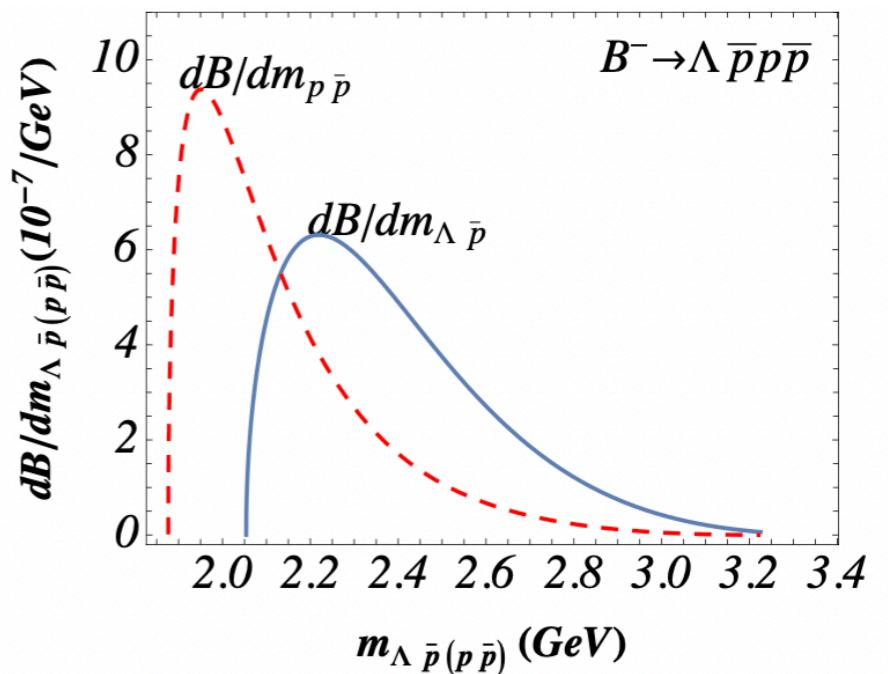
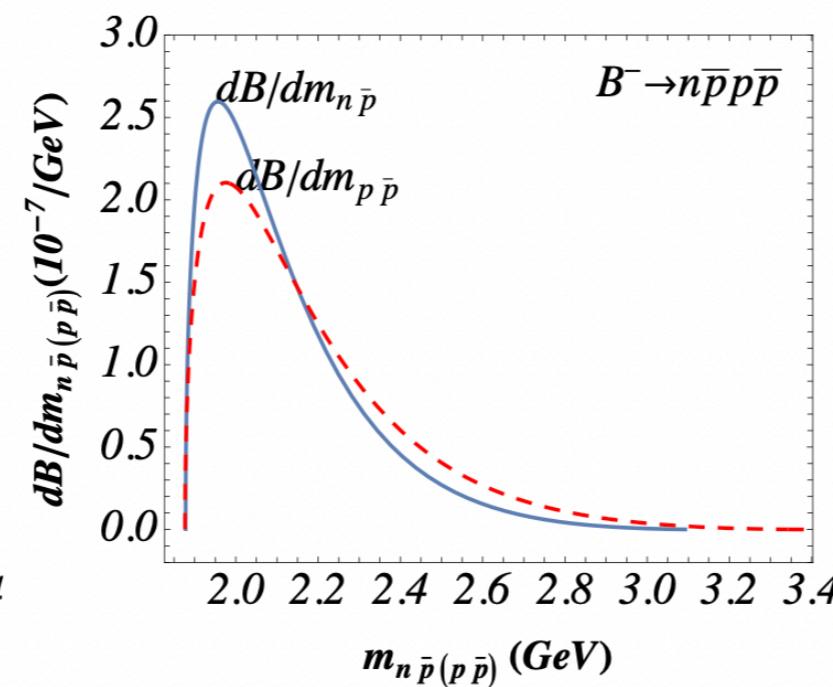
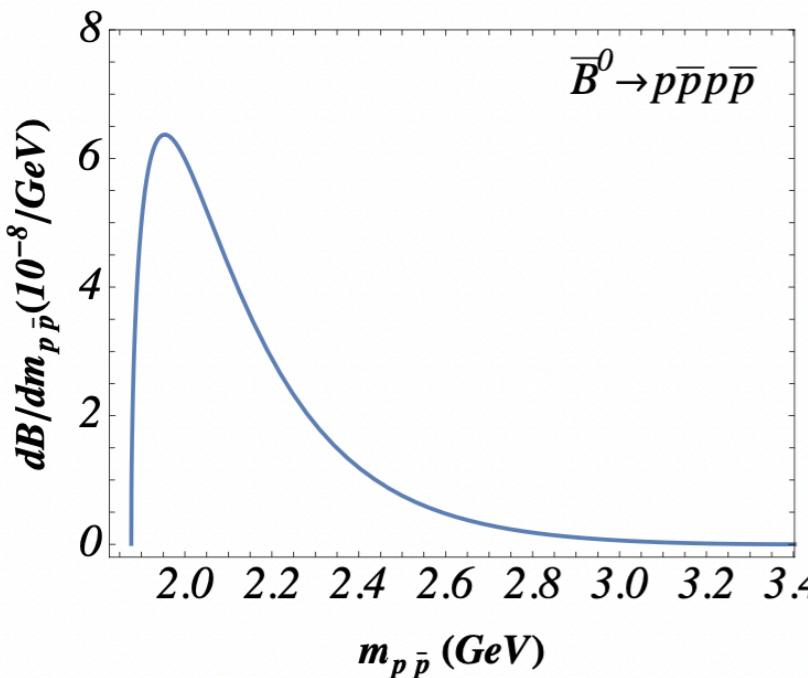
Numerical results

decay mode	our work	data
$10^8 \mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p})$	$2.2 \pm 0.4 \pm 0.1 \pm 0.4$	2.2 ± 0.4
$10^8 \mathcal{B}(B^- \rightarrow n\bar{p}p\bar{p})$	$8.4^{+2.1}_{-1.0} \pm 0.4^{+3.4}_{-1.9}$	—
$10^7 \mathcal{B}(B^- \rightarrow \Lambda\bar{p}p\bar{p})$	$3.7^{+0.3}_{-0.1} \pm 0.02^{+1.8}_{-1.3}$	—
$10^7 \mathcal{B}(\bar{B}_s^0 \rightarrow \Lambda\bar{\Lambda}p\bar{p})$	$1.9^{+0.3}_{-0.1} \pm 0.01^{+1.1}_{-0.6}$	—

Errors come from non-factorizable QCD corrections,
CKM matrix elements, and the form factors, in order.

- Except for $\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p})$, $\mathcal{B}(B \rightarrow \mathbf{B}_1 \bar{\mathbf{B}}'_1 \mathbf{B}_2 \bar{\mathbf{B}}'_2) \sim 10^{-7}$ not as large as $\mathcal{B}(B \rightarrow \mathbf{B} \bar{\mathbf{B}}' M) \sim 10^{-6}$.
- “Double” threshold effect:
a strong preference of the phase space,
an unexpected restriction.

Double threshold effect



Summary

- We explained $\mathcal{B}(\bar{B}^0 \rightarrow p\bar{p}p\bar{p})$ as small as 10^{-8} .
- We predicted $\mathcal{B}(B^- \rightarrow n\bar{p}p\bar{p})$, $\mathcal{B}(B^- \rightarrow \Lambda\bar{p}p\bar{p})$, and $\mathcal{B}(\bar{B}_s^0 \rightarrow \Lambda\bar{\Lambda}p\bar{p})$ of order 10^{-7} ,
accessible to the LHCb and Belle II experiments.
- Double threshold effect,
used to test the theoretical model.

Thank You