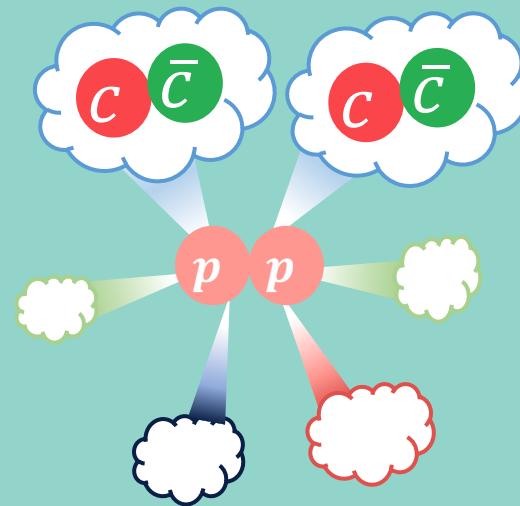


# Associated quarkonium production in $pp$ collisions at LHCb



安刘攀  
北京大学

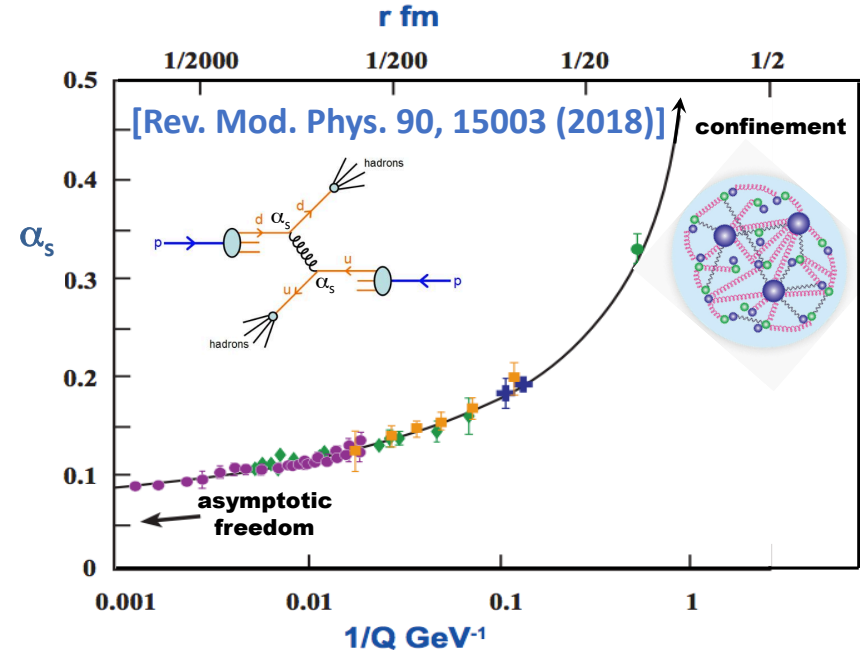


第四届LHCb前沿物理研讨会 @ 烟台, 2024年7月31日

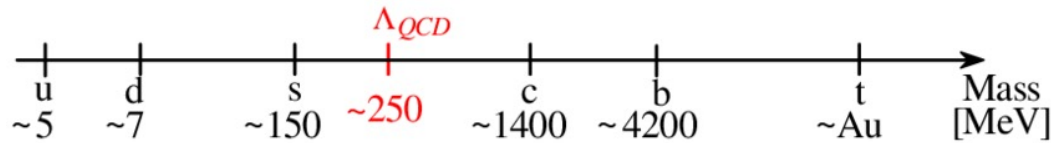
# Quarkonium for QCD

➤ **QCD dilemma:**  
understanding the non-perturbative property of QCD at low-energy scale

➤ **Confinement:**  
✓ no analytic proof exists  
⇒ what is the mechanism behind hadronization?



➤ **Heavy quarkonium:** ideal system to study hadronization mechanism



**Perturbative**

$$m_Q \gg$$

mass  
 $Q\bar{Q}$  creation

**Non-perturbative**

$$m_Q v \gg$$

momentum  
 $Q\bar{Q}$  expansion

$$m_Q v^2$$

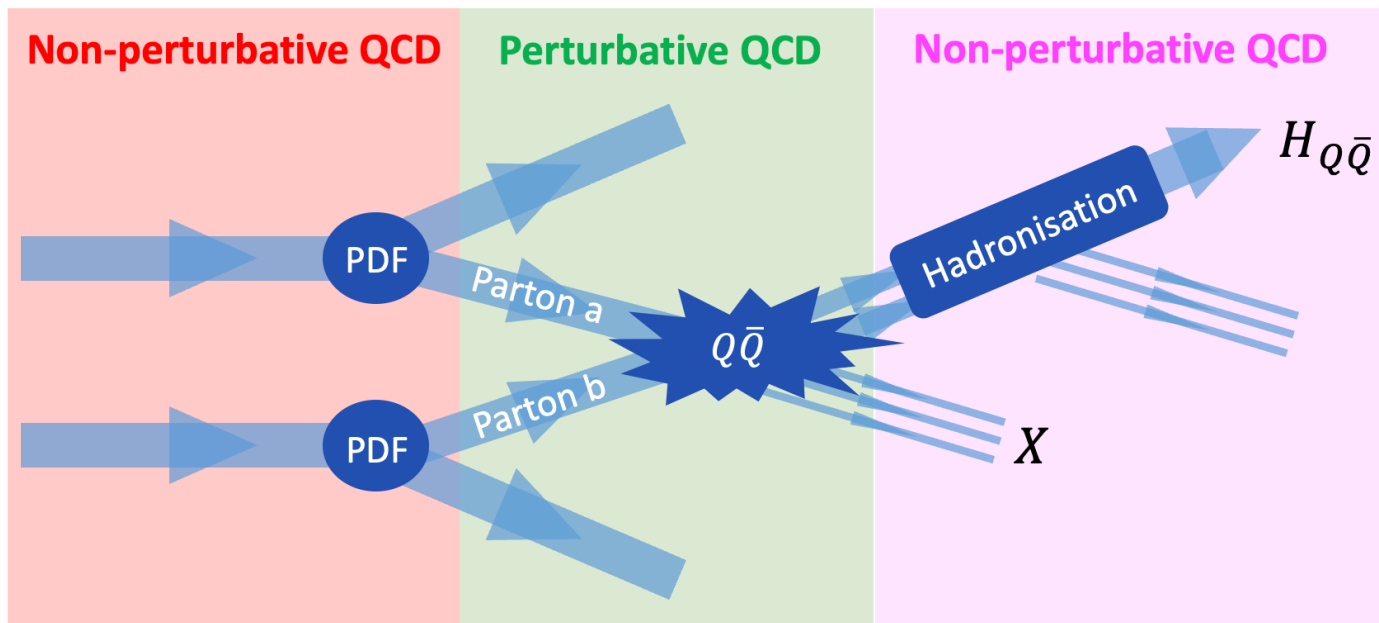
kinetic energy  
forming bound state

$$v^2 \sim 0.3 \text{ for } H_{c\bar{c}}$$

$$v^2 \sim 0.1 \text{ for } H_{b\bar{b}}$$

# Quarkonium production mechanism

- Non-relativistic QCD (NRQCD) provides the most successful description
- Yet not able to coherently describe prod.&pol. measurements in all collision systems



✓ excellent tool to study gluon PDF

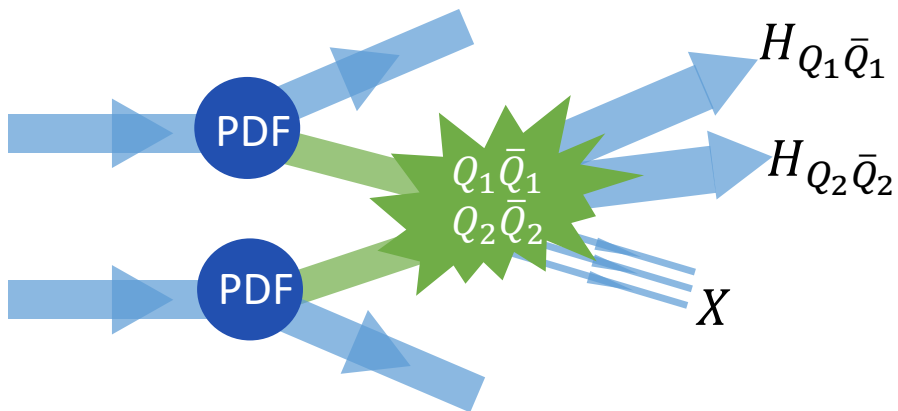
$$\sigma(H_{Q\bar{Q}}) = \sum_{a,b,n} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/p}(x_2) |\mathcal{A}(ab \rightarrow Q\bar{Q}[n] + X)|^2 \times \langle \mathcal{O}^H(n) \rangle$$

LDMEs: extracted from measurements & process independent

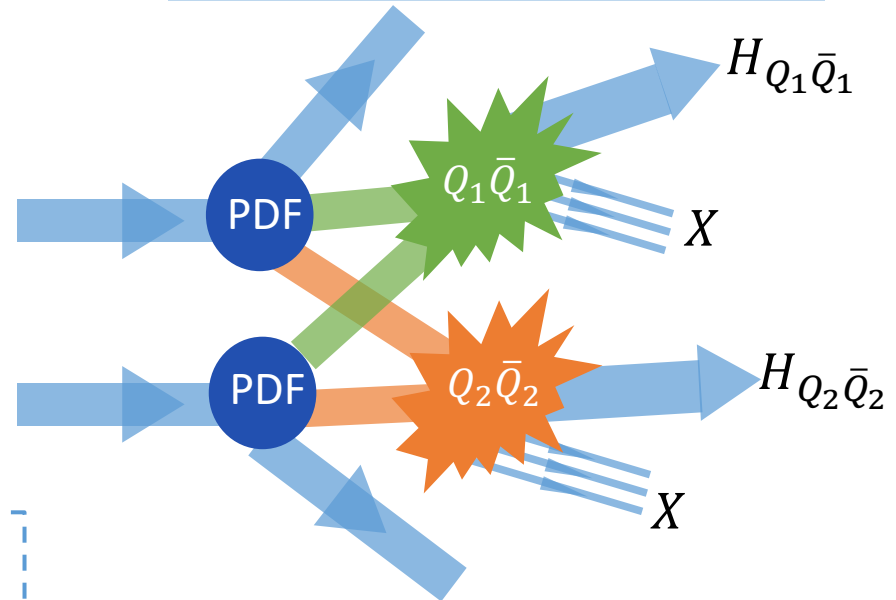
$$|J/\psi\rangle = \mathcal{O}(1) |c\bar{c} [{}^3S_1^{(1)}]\rangle + \mathcal{O}(v) |c\bar{c} [{}^3P_J^{(8)}] + g\rangle + \mathcal{O}(v^{3/2}) |c\bar{c} [{}^1S_0^{(8)}] + g\rangle + \dots$$

# Associated Quarkonium production

## Single-parton scattering (SPS)



## Double-parton scattering (DPS)



- ✓ To probe the quarkonium production mechanism puzzle
- ✓ Golden channel to probe gluon transverse momentum dependent (TMD) PDFs:
  - $h_1^{\perp g}(x, \mathbf{k}_T^2, \mu) \Rightarrow$  azimuthal asymmetry
  - $f_1^g(x, \mathbf{k}_T^2, \mu)$ : affect  $p_T$  spectrum
- [\[EPJC 80 \(2020\) 87\]](#)
- ✓ To search for fully heavy tetraquark states

- ✓ To provide information on parton transverse profile & correlations in colliding hadrons
- ✓ To understand multiparticle background ( $Z + b\bar{b}$ ,  $W^+W^+$  etc.) in both SM measurements and search for New Physics

# Double Parton Scattering

$$\sigma_{Q_1 Q_2}^{\text{DPS}} = \frac{1}{1 + \delta_{Q_1 Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2 \mathbf{b}_1 d^2 \mathbf{b}_2 d^2 \mathbf{b} \quad \text{Generalized double parton PDF}$$

$$\times \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) \times \hat{\sigma}_{ik}^{Q_1}(x_1, x'_1) \hat{\sigma}_{jl}^{Q_2}(x_2, x'_2) \times \Gamma_{kl}(x'_1, x'_2, \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b})$$

SPS parton-level cross-section

Assuming:

✓ factorization of trans. & long. components

$$\Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) = D_{ij}(x_1, x_2) T_{ij}(\mathbf{b}_1, \mathbf{b}_2)$$

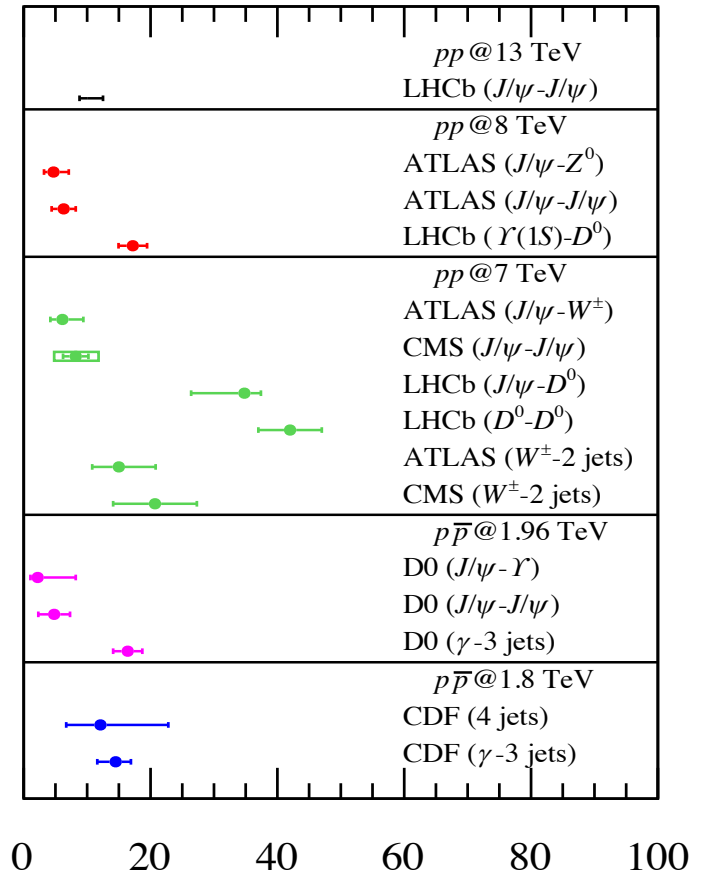
✓ no correlation between two sets of partons

$$D_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2), T_{ij}(\mathbf{b}_1, \mathbf{b}_2) = T_i(\mathbf{b}_1) T_j(\mathbf{b}_2)$$

$$\Rightarrow \sigma_{Q_1 Q_2} = \frac{1}{1 + \delta_{Q_1 Q_2}} \frac{\sigma_{Q_1} \sigma_{Q_2}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} = 1 / \left[ \int d^2 \mathbf{b} F(\mathbf{b})^2 \right], F(\mathbf{b}) = \int T(\mathbf{b}_i) T(\mathbf{b}_i - \mathbf{b}) d^2 \mathbf{b}_i$$

expected to be universal under the given assumptions



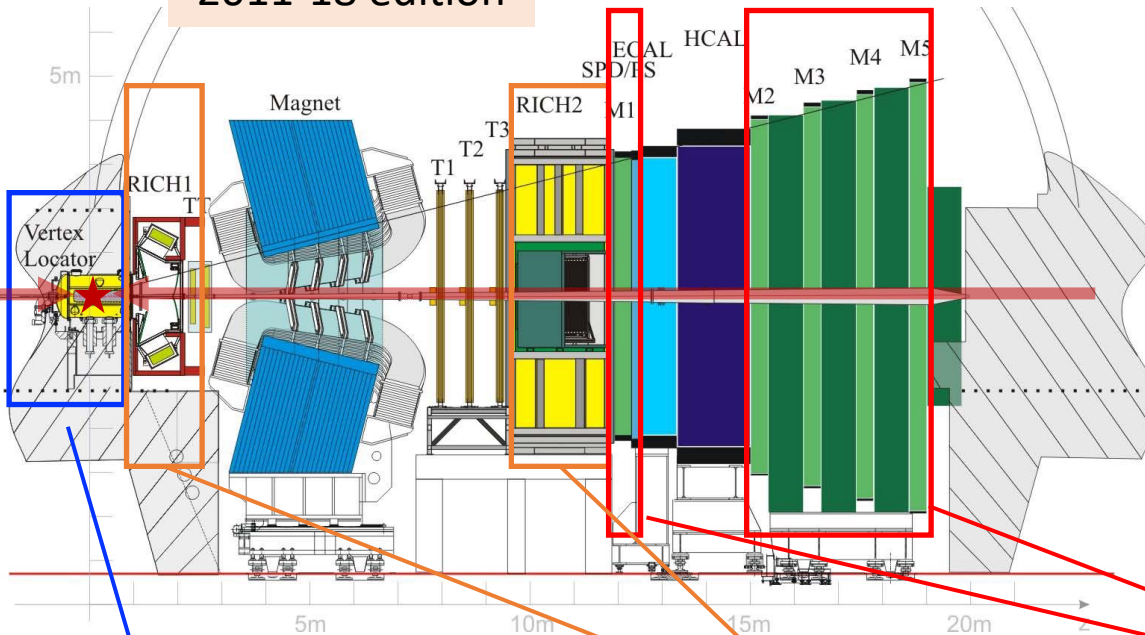
[PoS (LHCP2020) 172;  
arXiv: 2009.12555]

$\sigma_{\text{eff}}$  [mb]

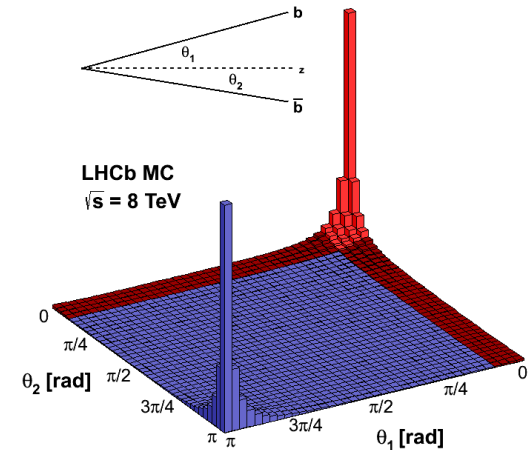
# The LHCb detector

➤ LHCb is a single-arm forward region spectrometer covering  $2 < \eta < 5$ , dedicated to heavy flavor physics at the Large Hadron Collider

2011-18 edition



2.4%  $4\pi$  angle  
 $\Rightarrow 25\% b\bar{b}$



**Vertex Locator:** high precision;  
 capable of separating  $b/c$   
 hadron production and decay  
 vertices

$$\sigma_{PV,x/y} \sim 10 \mu\text{m}, \sigma_{PV,z} \sim 60 \mu\text{m}$$

**RICHs:** efficient identification  
 of pions, kaons and protons

$$\varepsilon(K \rightarrow K) \sim 95\%$$

$$\text{@ misID rate } (\pi \rightarrow K) \sim 5\%$$

**Muon system (M1-M5):**  
 efficient muon  
 identification and trigger

$$\varepsilon(\mu \rightarrow \mu) \sim 97\%$$

$$\text{@ misID rate } (\pi \rightarrow \mu) \sim 1 - 3\%$$

# A glimpse of existing measurements

Observation of  $J/\psi$ -pair production in  $pp$  collisions at  $\sqrt{s} = 7$  TeV<sup>☆</sup>

LHCb Collaboration

Associated production of prompt  $J/\psi$  and  $\Upsilon$  mesons in  $pp$  collisions at  $\sqrt{s} = 13$  TeV



The LHCb collaboration

Measurement of the  $J/\psi$  pair production cross-section in  $pp$  collisions at  $\sqrt{s} = 13$  TeV



The LHCb collaboration

Measurement of  $J/\psi$ -pair production in  $pp$  collisions at  $\sqrt{s} = 13$  TeV and study of gluon transverse-momentum dependent PDFs



The LHCb collaboration

Measurement of associated  $J/\psi$ - $\psi(2S)$  production cross-section in  $pp$  collisions at  $\sqrt{s} = 13$  TeV



The LHCb collaboration

2012

2014

2016

2017

2023

2024

# A glimpse of existing measurements

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The LHCb collaboration

Measurement of associated  $J/\psi$ - $\psi(2S)$  production cross-section in  $pp$  collisions at  $\sqrt{s} = 13$  TeV



The LHCb collaboration

2012

2014

2016

2017

2023

2024

Observation of double charm production involving open charm in  $pp$  collisions at  $\sqrt{s} = 7$  TeV

The LHCb collaboration

Observation of associated production of a Z boson with a D meson in the forward region



The LHCb collaboration

Production of associated  $\Upsilon$  and open charm hadrons in  $pp$  collisions at  $\sqrt{s} = 7$  and 8 TeV via double parton scattering

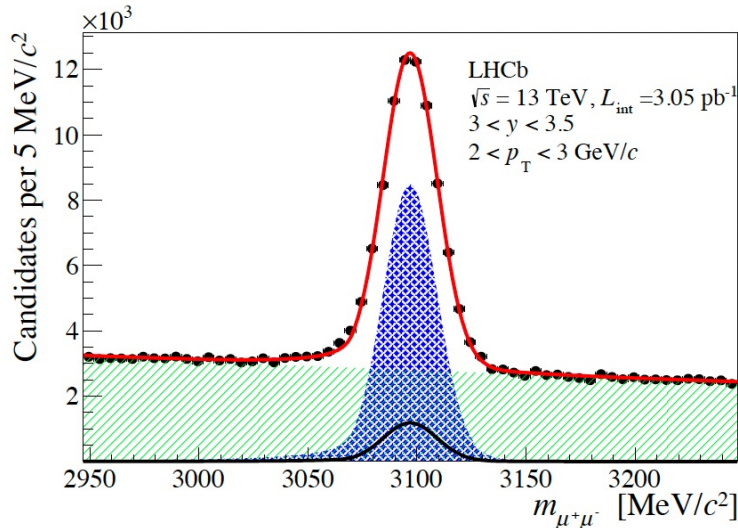
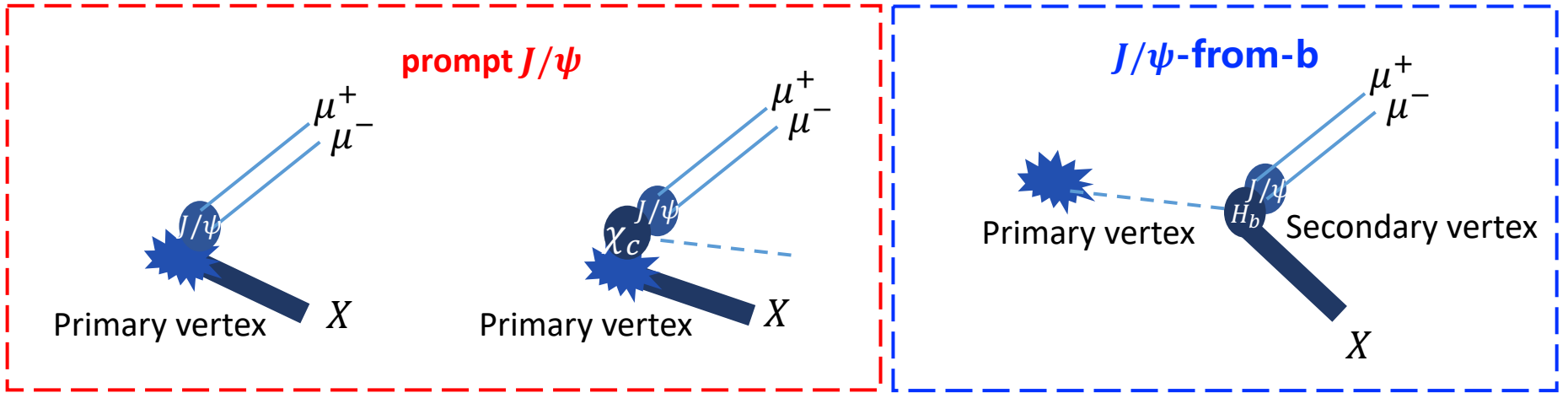


The LHCb collaboration

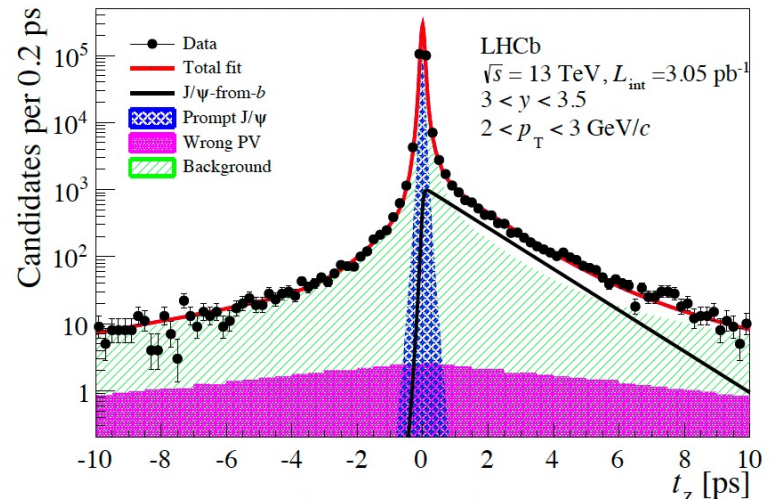


# Analysis strategy

$$\frac{d\sigma}{dv} = \frac{N}{\mathcal{L} \times \epsilon \times B^2(J/\psi \rightarrow \mu^+\mu^-) \times \Delta v}$$



[JHEP 10 (2015) 172]

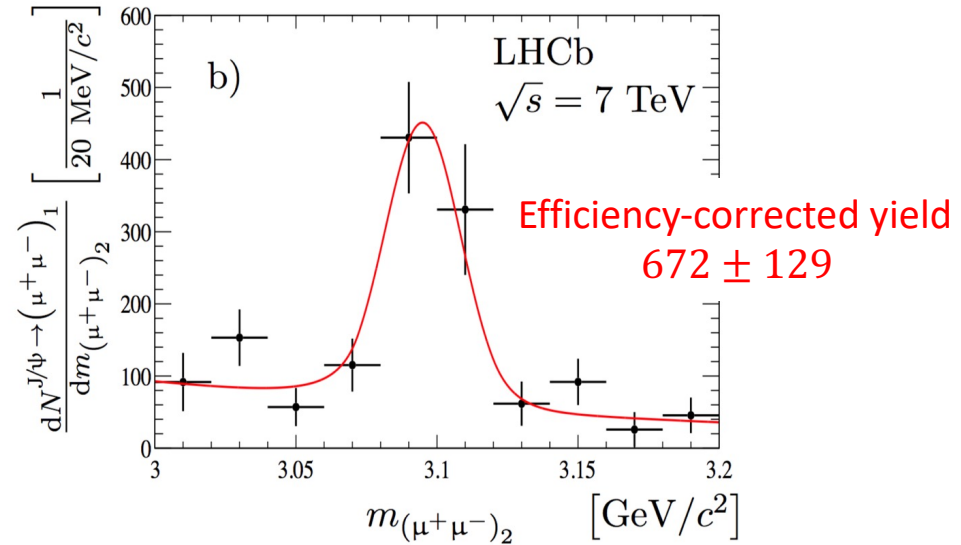
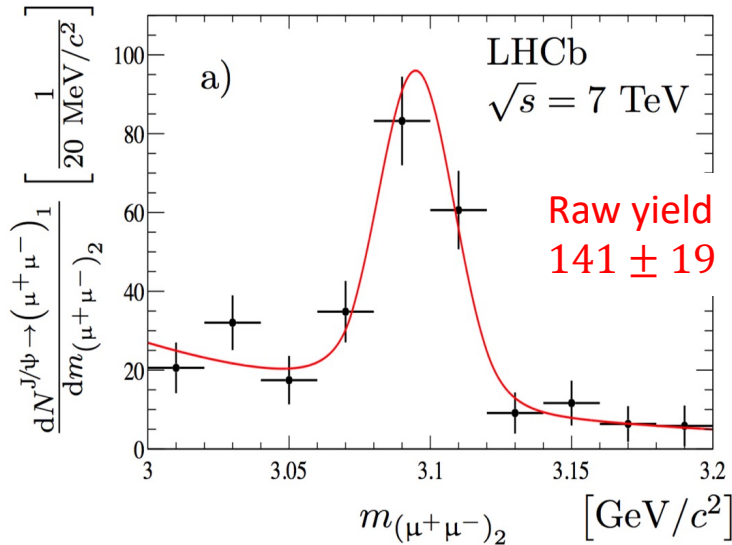


$$t_z = \frac{(z_{J/\psi} - z_{PV}) \times M_{J/\psi}}{p_z}$$

# di- $J/\psi$ @ 7 TeV

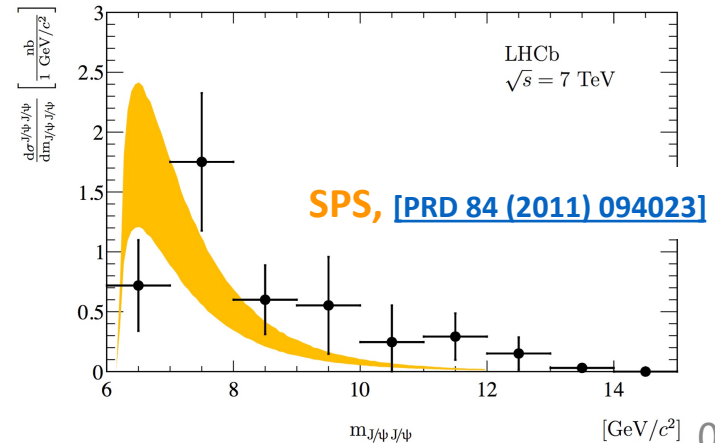
- Using  $37.5 \text{ pb}^{-1}$  data at  $\sqrt{s} = 7 \text{ TeV}$
- Fiducial region:  $2 < y^{J/\psi} < 4.5, p_T^{J/\psi} < 10 \text{ GeV}/c$
- Observed with significance  $> 6\sigma$  (from- $b$  contribution negligible)

[\[PLB 707 \(2012\) 52-59\]](#)



- $\sigma^{J/\psi J/\psi} = 5.1 \pm 1.0 \pm 1.1 \text{ nb}$ 
  - ✓  $\sigma_{\text{SPS}} = 4.1 \pm 1.2 \text{ nb}$
  - ✓  $\sigma_{\text{DPS}} \approx 2.5 \text{ nb}$

- Not enough events to disentangle SPS and DPS contributions



# di- $J/\psi$ @ 13 TeV

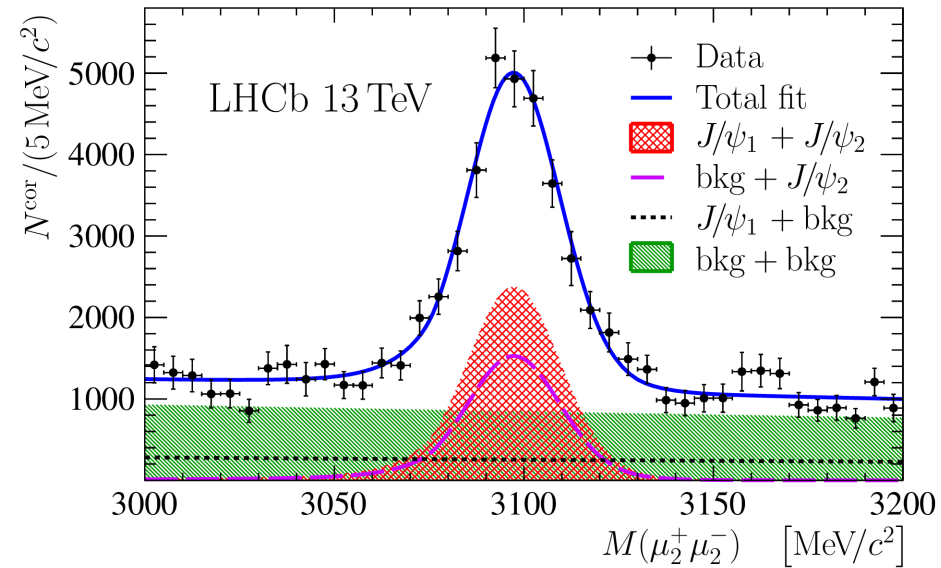
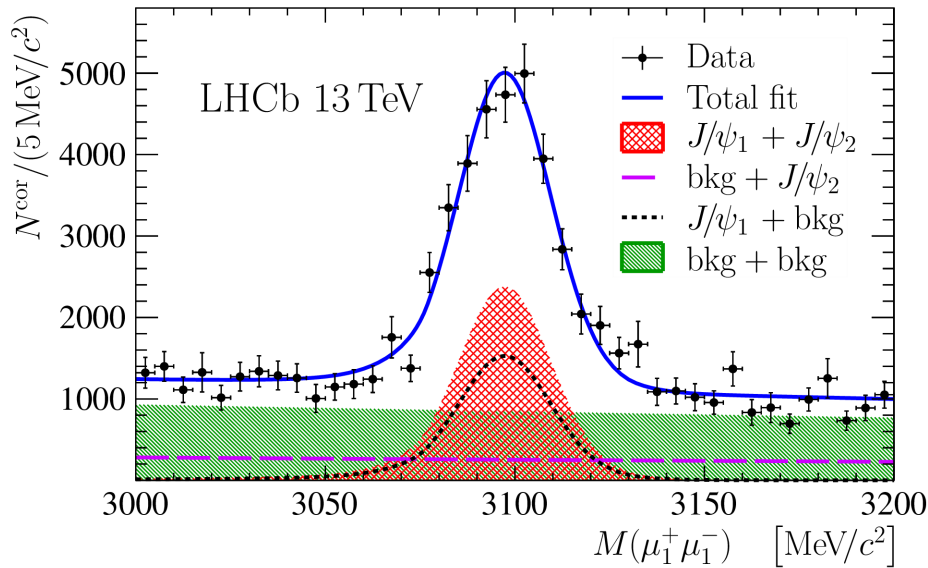
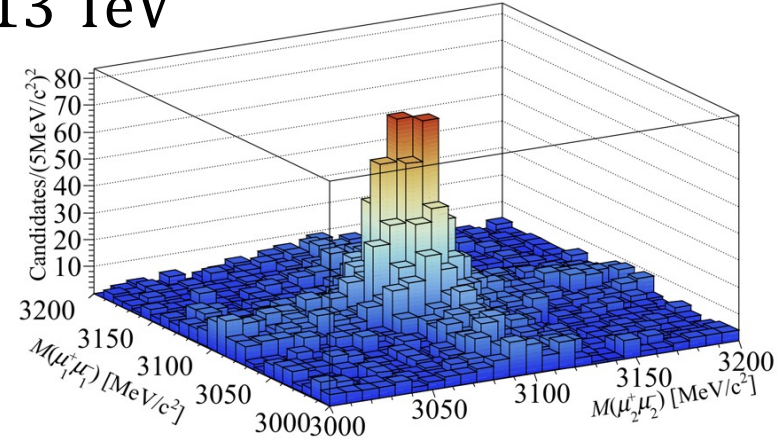
[JHEP 06 (2017) 047]

➤  $\mathcal{L} = 279 \text{ pb}^{-1}$ ,  $pp$  collisions @  $\sqrt{s} = 13 \text{ TeV}$

➤ Kinematic range of  $J/\psi$ :  
 $p_T < 10 \text{ GeV}/c$  for  $2.0 < y < 4.5$

➤ Signal yield determination

✓ Residual from- $b$  component determined using simulation together with  $\sigma(pp \rightarrow b\bar{b})$  and  $\sigma(\text{prompt } J/\psi)$



➤  $\sigma(J/\psi J/\psi) = 15.2 \pm 1.0(\text{stat}) \pm 0.9(\text{syst}) \text{ nb}$

# Comparison to theory

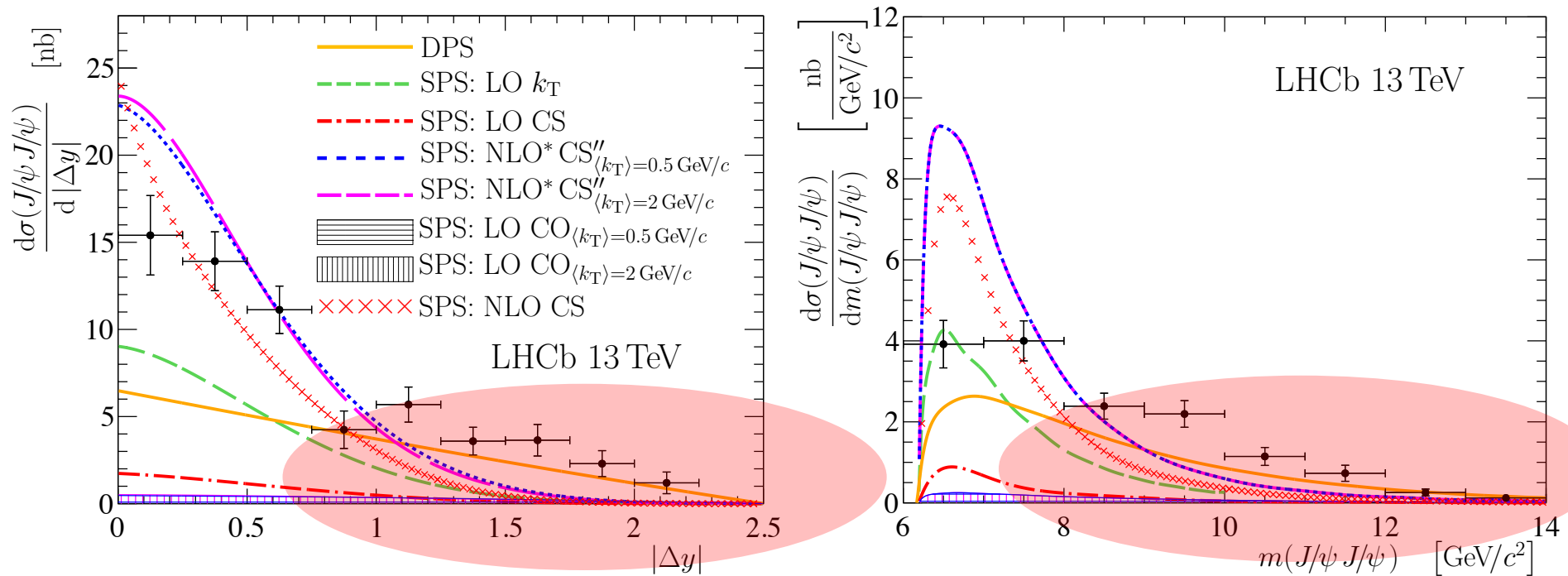
		$\sigma(J/\psi J/\psi)$ [nb]		
		<a href="#">[JHEP 06 (2017) 047]</a>		
		no $p_T$ cut	$p_T > 1 \text{ GeV}/c$	$p_T > 3 \text{ GeV}/c$
SPS	LO Colour-singlet	$1.3 \pm 0.1^{+3.2}_{-0.1}$	—	—
	LO Colour-octet	$0.45 \pm 0.09^{+1.42+0.25}_{-0.36-0.34}$	—	—
	LO $k_T$	$6.3^{+3.8+3.8}_{-1.6-2.6}$	$5.7^{+3.4+3.2}_{-1.5-2.1}$	$2.7^{+1.6+1.6}_{-0.7-1.0}$
	NLO* Colour-singlet'	—	$4.3 \pm 0.1^{+9.9}_{-0.9}$	$1.6 \pm 0.1^{+3.3}_{-0.3}$
	NLO* Colour-singlet''	$15.4 \pm 2.2^{+51}_{-12}$	$14.8 \pm 1.7^{+53}_{-12}$	$6.8 \pm 0.6^{+22}_{-5}$
	NLO Colour-singlet	$11.9^{+4.6}_{-3.2}$	—	—
	DPS	$8.1 \pm 0.9^{+1.6}_{-1.3}$	$7.5 \pm 0.8^{+1.5}_{-1.2}$	$4.9 \pm 0.5^{+1.0}_{-0.8}$
LHCb result		$15.2 \pm 1.0 \pm 0.9$	$13.5 \pm 0.9 \pm 0.9$	$8.3 \pm 0.6 \pm 0.5$

DPS: assuming  $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3}$  mb [\[PRD 56 \(1997\) 3811\]](#)

- LO Colour-octet : contribution very small
- LO Colour-singlet/ NLO\* Colour-singlet' and LO  $k_T$ : need DPS contribution
- NLO\* Colour-singlet'' and NLO Colour-singlet : consistent with our measurement by itself; overestimated if there is DPS contribution

# Differential cross-sections

- Differential cross-sections of different variables compared to theory predictions
  - ✓ Most significant indication of DPS comes from  $|\Delta y|$
  - ✓ DPS contribution essential for the region  $|\Delta y| > 1.5$
  - ✓ Also clear indication from  $m(J/\psi J/\psi)$

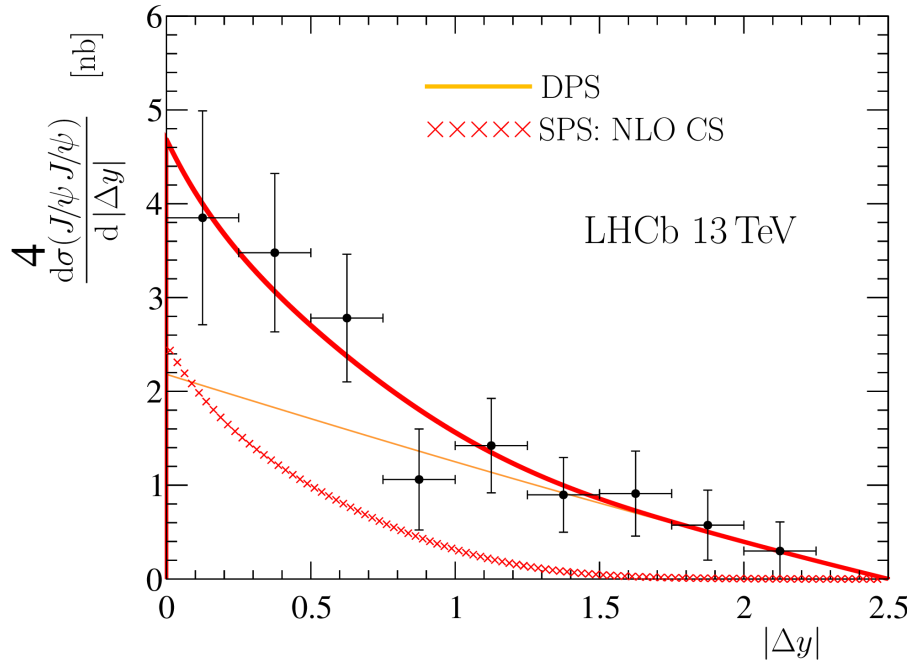


[JHEP 06 (2017) 047]

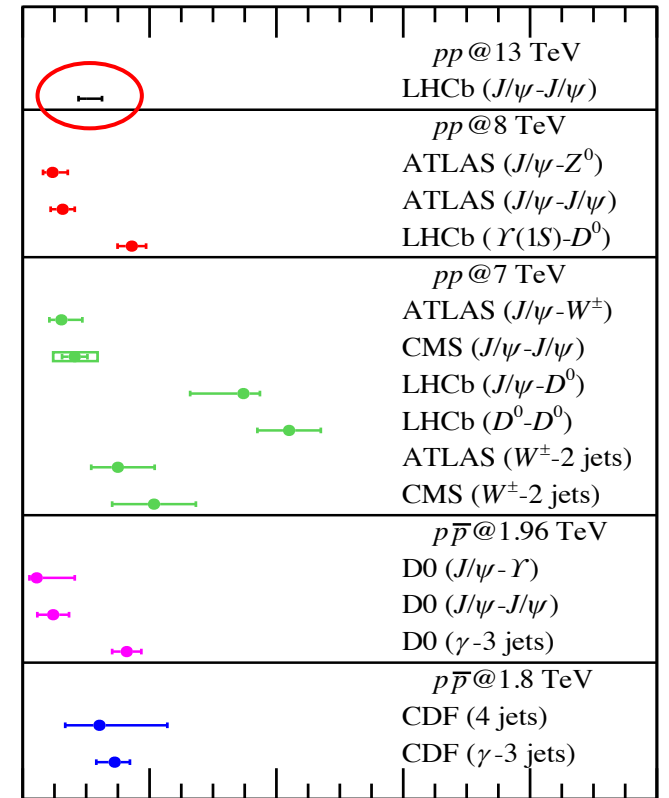
# DPS extraction

➤ Template DPS+SPS fits performed for different variables using various models

$$\frac{d\sigma}{dv} = \sigma_{\text{DPS}} F_{\text{DPS}}(v) + \sigma_{\text{SPS}} F_{\text{SPS}}(v)$$



$\sigma_{\text{eff}}: 8.8 \sim 12.5 \text{ mb}$



Variable	LO $k_T$	NLO* CS''		NLO CS
		$\langle k_T \rangle = 2 \text{ GeV}/c$	$\langle k_T \rangle = 0.5 \text{ GeV}/c$	
$p_T(J/\psi J/\psi)$	$9.7 \pm 0.5$	$8.8 \pm 5.6$	$9.3 \pm 1.0$	—
$y(J/\psi J/\psi)$	—	$11.9 \pm 7.5$	$10.0 \pm 5.0$	—
$m(J/\psi J/\psi)$	$10.6 \pm 1.1$	$10.2 \pm 1.0$	—	$10.4 \pm 1.0$
$ \Delta y $	$12.5 \pm 4.1$	$12.2 \pm 3.7$	$12.4 \pm 3.9$	$11.2 \pm 2.9$

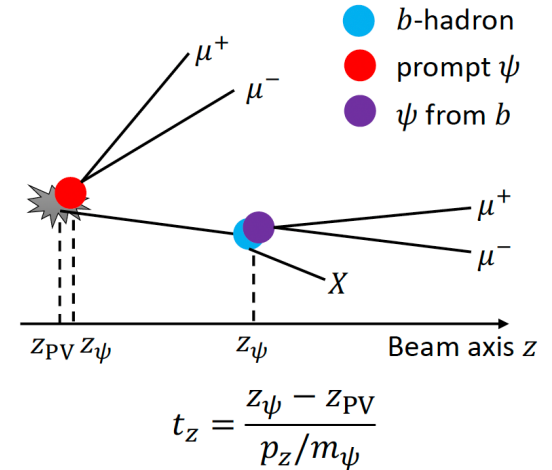
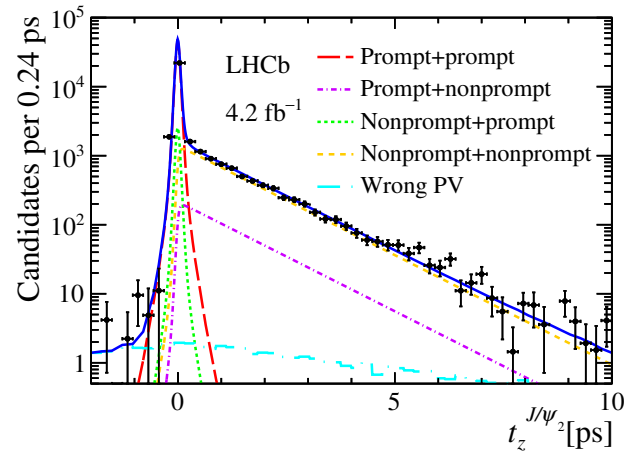
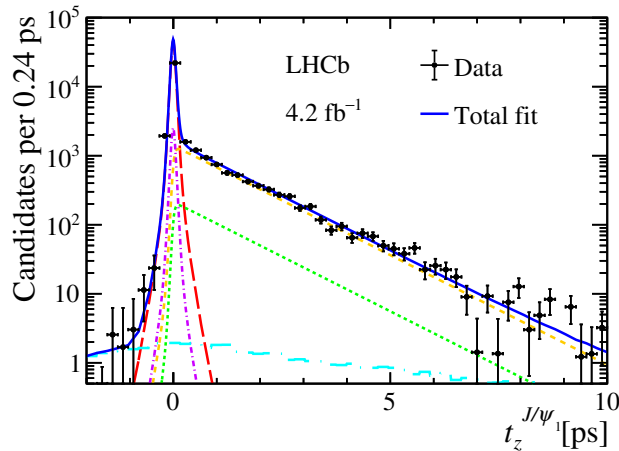
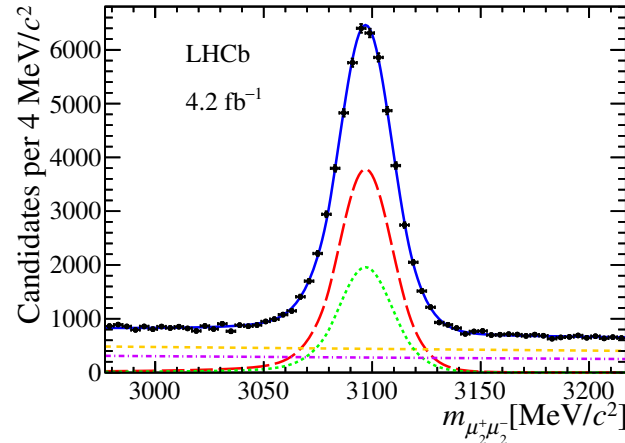
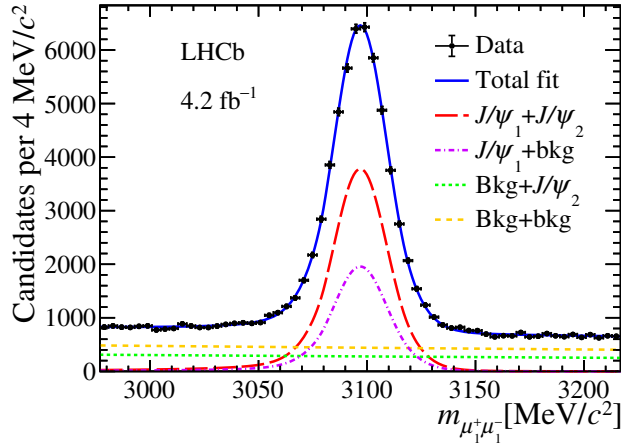
[PoS (LHCP2020) 172;  
arXiv: 2009.12555]

$\sigma_{\text{eff}} [\text{mb}]$

# di- $J/\psi$ @ 13 TeV update

[JHEP 03 (2024) 088]

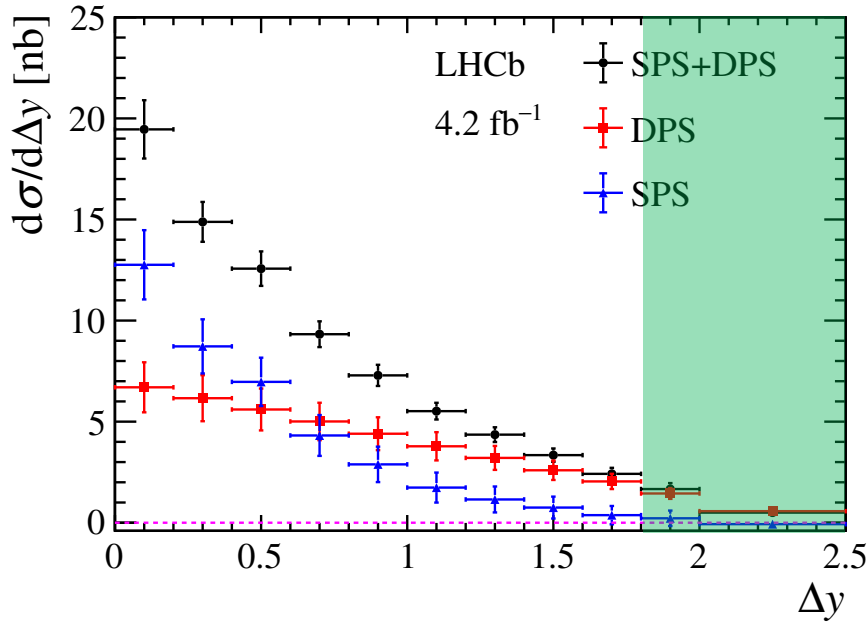
➤ Fiducial region:  $2 < y(J/\psi) < 4.5$ ,  $p_T(J/\psi) < 14$  GeV



$$N(J/\psi - J/\psi)_{\text{prompt}} = (2.187 \pm 0.020) \times 10^4$$

$$\sigma(J/\psi - J/\psi) = 16.36 \pm 0.28(\text{stat}) \pm 0.88(\text{syst}) \text{ nb}$$

# SPS and DPS separation

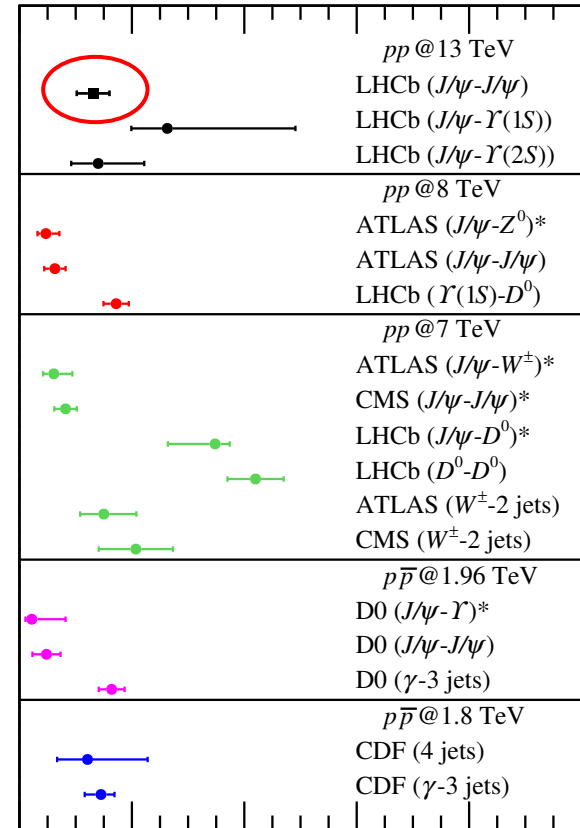


➤ SPS & DPS separated assuming negligible SPS contribution in  $1.8 < \Delta y < 2.5$  according to NRQCD predictions

$$\sigma(J/\psi - J/\psi)_{\text{DPS}} = 8.6 \pm 1.2(\text{stat}) \pm 1.0(\text{syst}) \text{ nb}$$

$$\sigma(J/\psi - J/\psi)_{\text{SPS}} = 7.9 \pm 1.2(\text{stat}) \pm 1.1(\text{syst}) \text{ nb}$$

$$\sigma_{\text{eff}} = 13.1 \pm 1.8(\text{stat}) \pm 2.3(\text{syst}) \text{ mb}$$

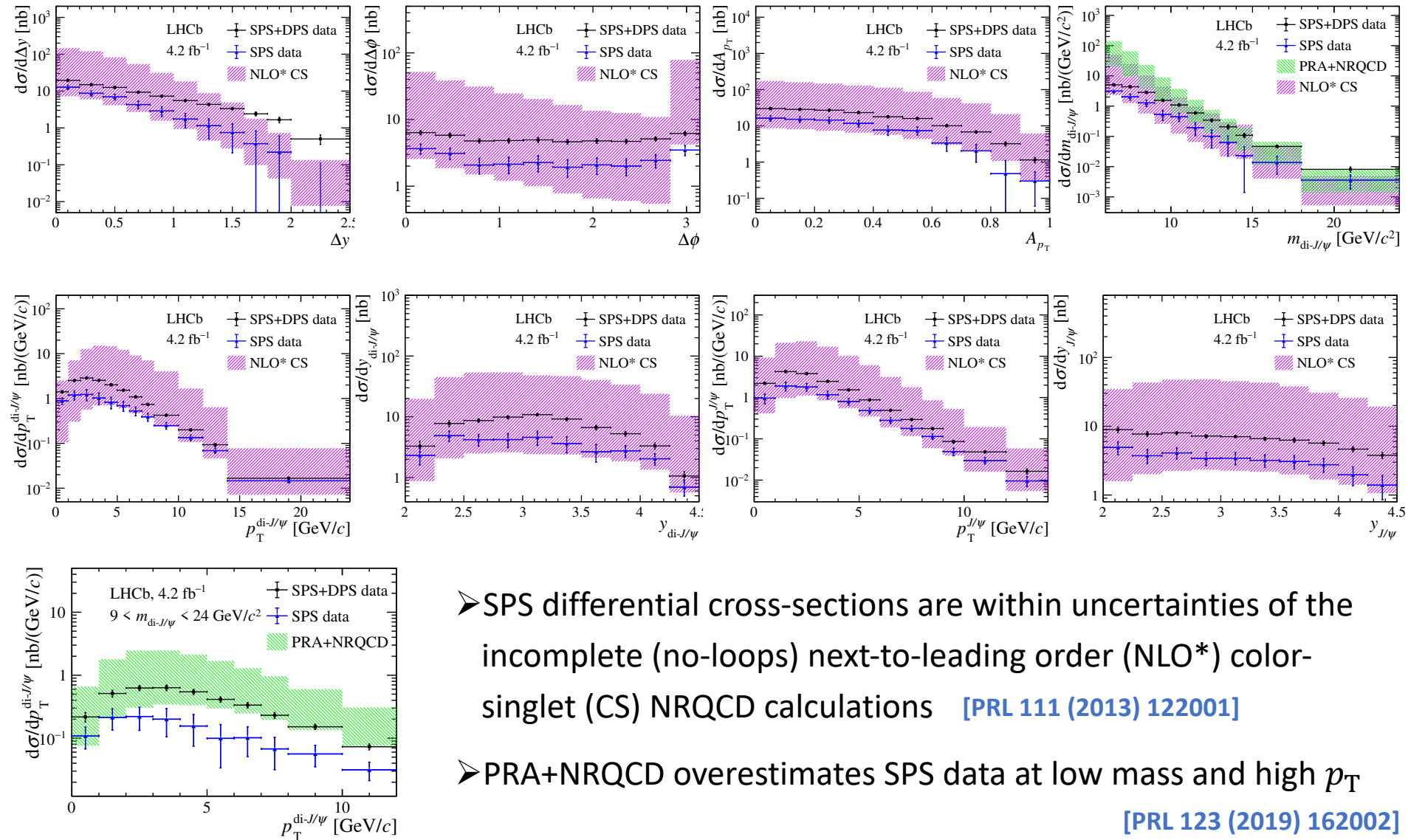


0 20 40 60 80 100



# Differential $J/\psi - J/\psi$ cross-section

[JHEP 03 (2024) 088]



➤ SPS differential cross-sections are within uncertainties of the incomplete (no-loops) next-to-leading order (NLO\*) color-singlet (CS) NRQCD calculations [PRL 111 (2013) 122001]

➤ PRA+NRQCD overestimates SPS data at low mass and high  $p_T$

[PRL 123 (2019) 162002]

[Comput. Phys. Commun. 184 (2013) 2562] [Comput. Phys. Commun. 198 (2016) 238]

# Gluon TMD PDFs study

[JHEP 03 (2024) 088]

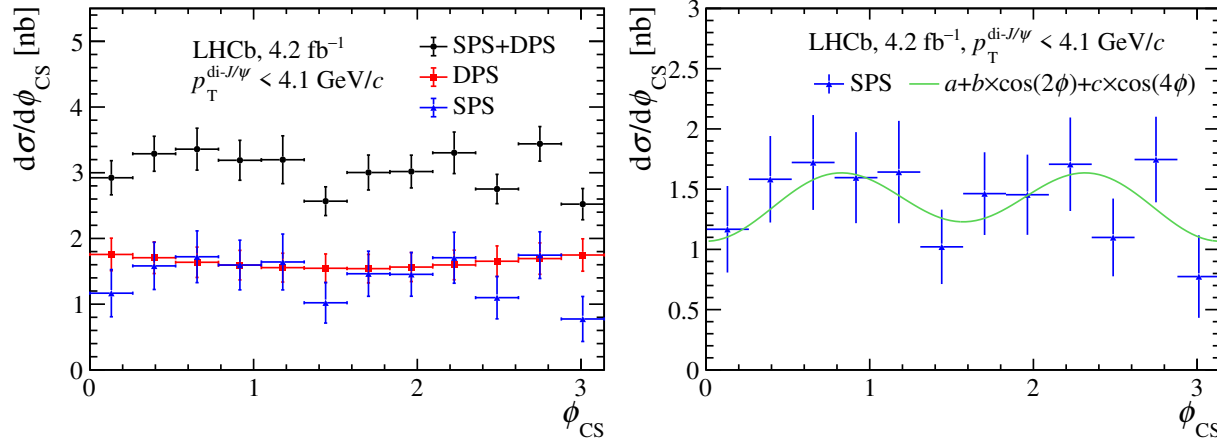
➤  $h_1^{\perp g}(x, \mathbf{k}_T^2, \mu) \Rightarrow$  azimuthal asymmetry

$$d\sigma/d\phi_{CS} = a + b \times \cos(2\phi_{CS}) + c \times \cos(4\phi_{CS})$$

$$a = F_1 C[f_1^g f_1^g] + F_2 C[w_2 h_1^{\perp g} h_1^{\perp g}],$$

$$b = F_3 C[w_3 f_1^g h_1^{\perp g}] + F_3' C[w_3' h_1^{\perp g} f_1^g],$$

$$c = F_4 C[w_4 h_1^{\perp g} h_1^{\perp g}],$$



$$\langle \cos(2\phi_{CS}) \rangle = b/2a$$

$$= -0.029 \pm 0.050 \pm 0.009$$

$$\langle \cos(4\phi_{CS}) \rangle = c/2a$$

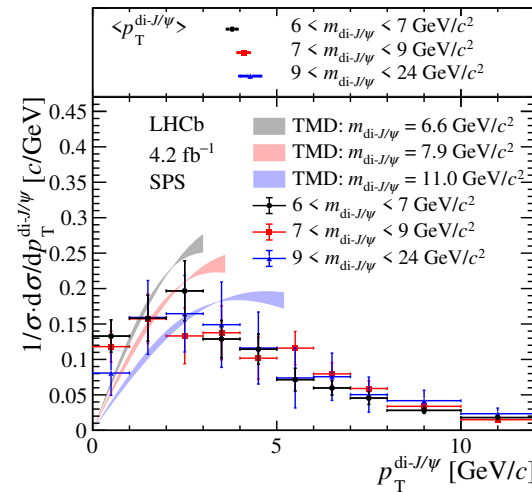
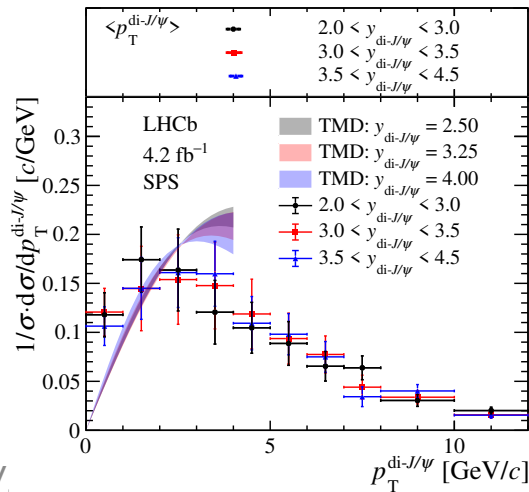
$$= -0.087 \pm 0.052 \pm 0.013$$

➤  $f_1^g(x, \mathbf{k}_T^2, \mu)$ : affect  $p_T$  spectrum

✓  $p_T$  shape shows no dependence on  $y$

[EPJC 80 (2020) 87]

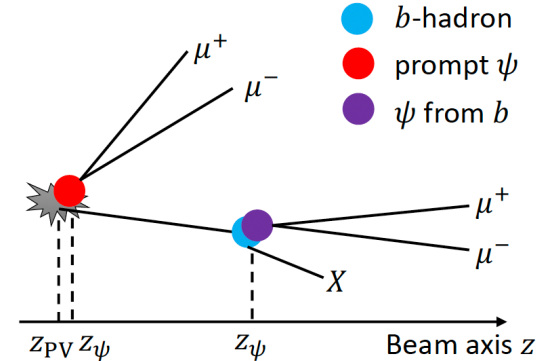
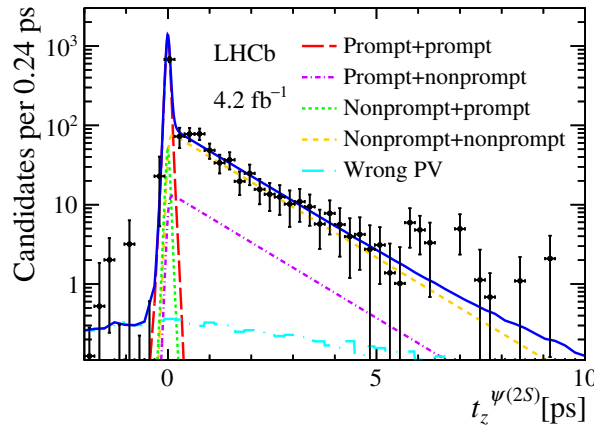
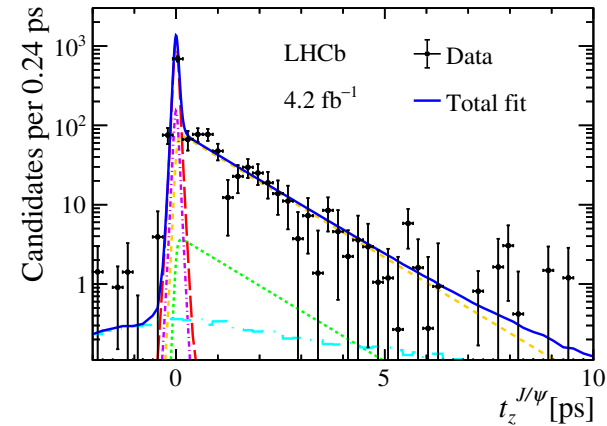
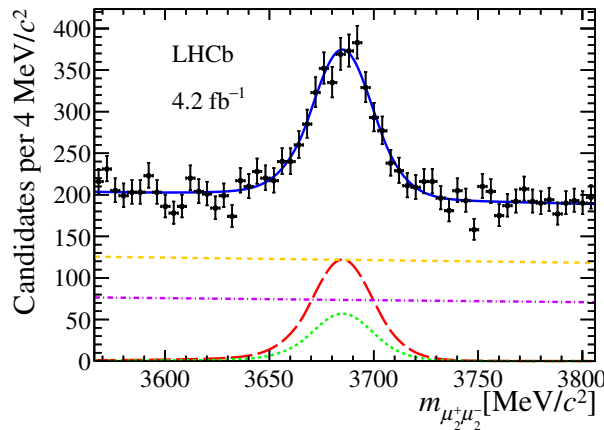
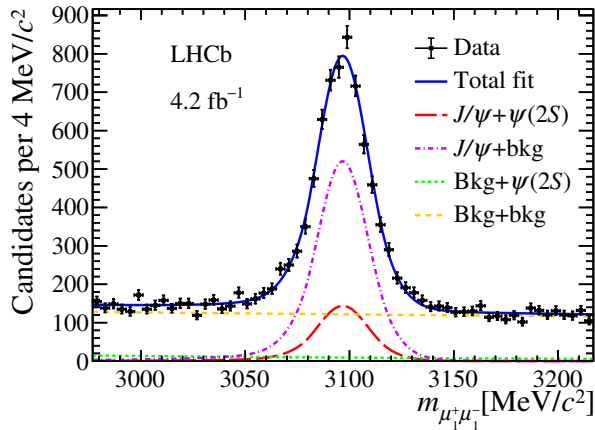
✓ No obvious broadening of  $p_T$  spectrum wrt increasing  $m$  given large uncertainties



# $J/\psi - \psi(2S)$ @ 13 TeV

[JHEP 05 (2024) 259]

➤ Fiducial region:  $2 < y(\psi) < 4.5$ ,  $p_T(\psi) < 14$  GeV



$$t_z = \frac{Z_\psi - Z_{PV}}{p_z/m_\psi}$$

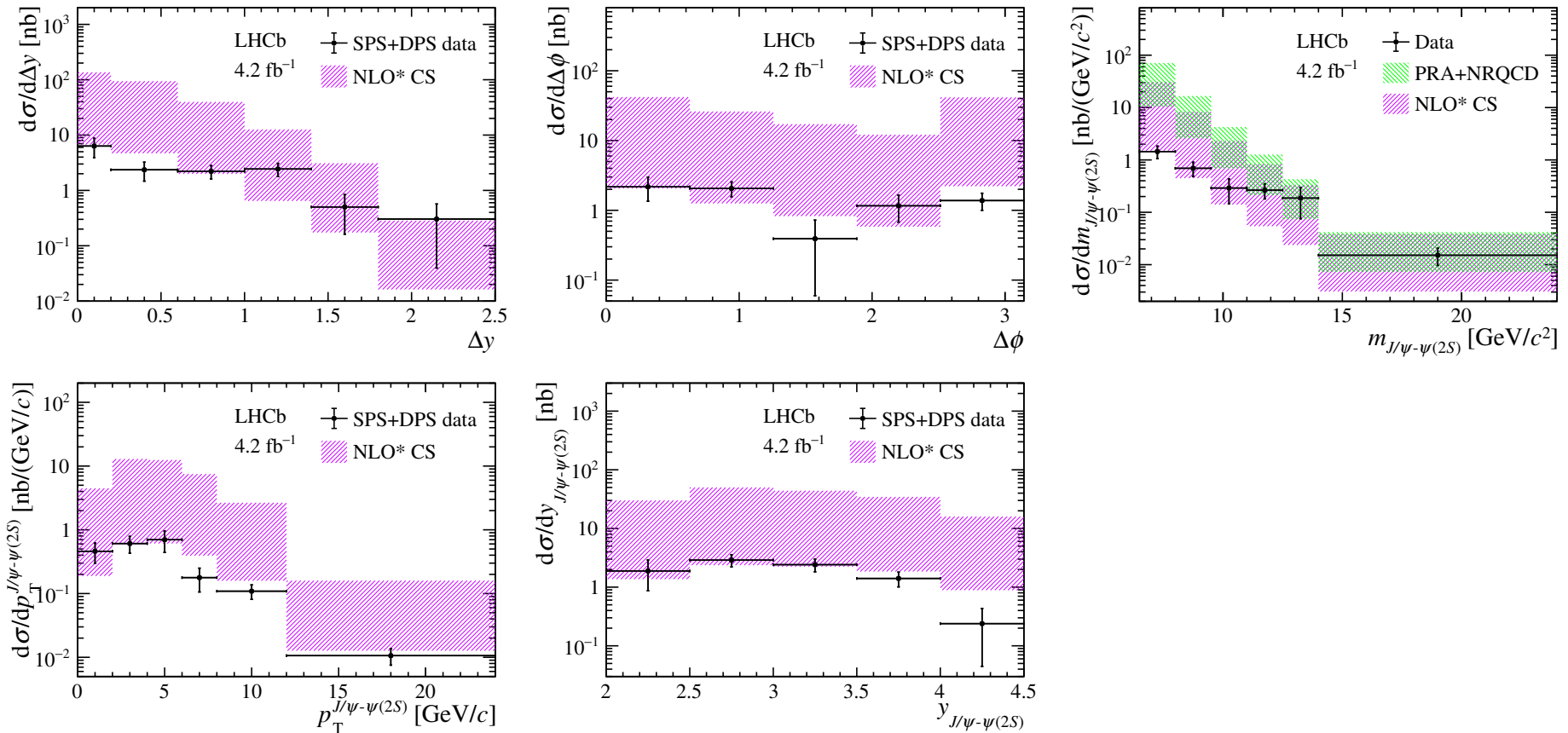
$$N(J/\psi - \psi(2S))_{\text{prompt}} = 629 \pm 50$$

$$\sigma(J/\psi - \psi(2S)) = 4.49 \pm 0.71(\text{stat}) \pm 0.26(\text{syst}) \text{ nb}$$

$$\sigma_{\text{eff}}(\text{lower limit}) = \frac{\sigma(J/\psi)\sigma(\psi(2S))}{\sigma(J/\psi - \psi(2S))} = 7.1 \pm 1.1(\text{stat}) \pm 0.8(\text{syst}) \text{ mb}$$

# Differential $J/\psi - \psi(2S)$ cross-section

[JHEP 05 (2024) 259]



➤ Results consistent with NLO\* CS NRQCD calculations albeit DPS is not subtracted

[PRL 111 (2013) 122001] [Comput. Phys. Commun. 184 (2013) 2562] [Comput. Phys. Commun. 198 (2016) 238]

➤ PRA+NRQCD overestimates SPS data at low mass

[PRL 123 (2019) 162002]

# $J/\psi - \psi(2S)$ vs. $J/\psi - J/\psi$

[JHEP 05 (2024) 259]

➤ Predictions on the ratio between  $\sigma(J/\psi - \psi(2S))$  and  $\sigma(J/\psi - J/\psi)$  give

✓ SPS:  $0.94 \pm 0.030$

[PLB 751 (2015) 479]

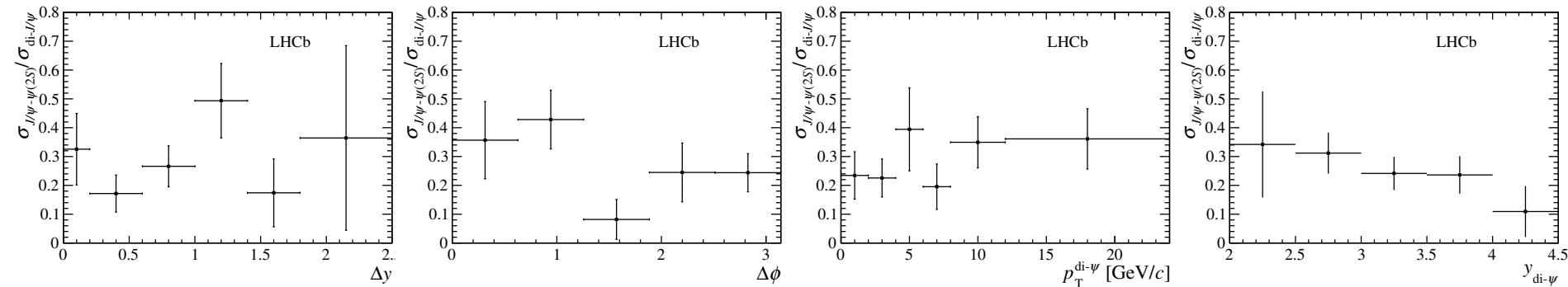
✓ DPS:  $0.282 \pm 0.027$

[JHEP 10 (2015) 172] [EPJC 80 (2020) 185]

$$\frac{\sigma(J/\psi - \psi(2S))}{\sigma(J/\psi - J/\psi)} = 0.274 \pm 0.044(\text{stat}) \pm 0.008(\text{syst})$$

⇒ it confirms a prominent DPS contribution to  $J/\psi - J/\psi$  production in a novel way, independent of the kinematic correlation of two  $J/\psi$  mesons

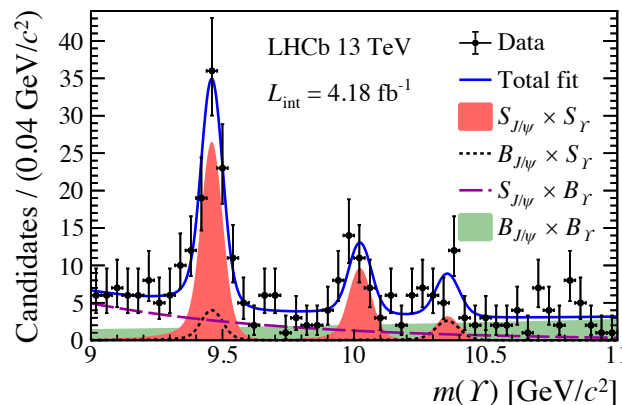
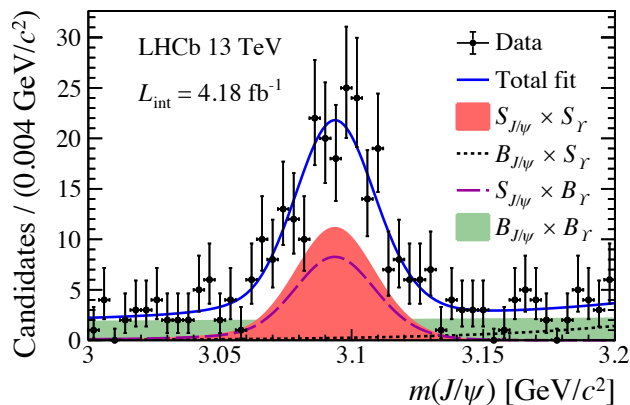
➤ Differential cross-section ratios are also measured, but more statistics needed



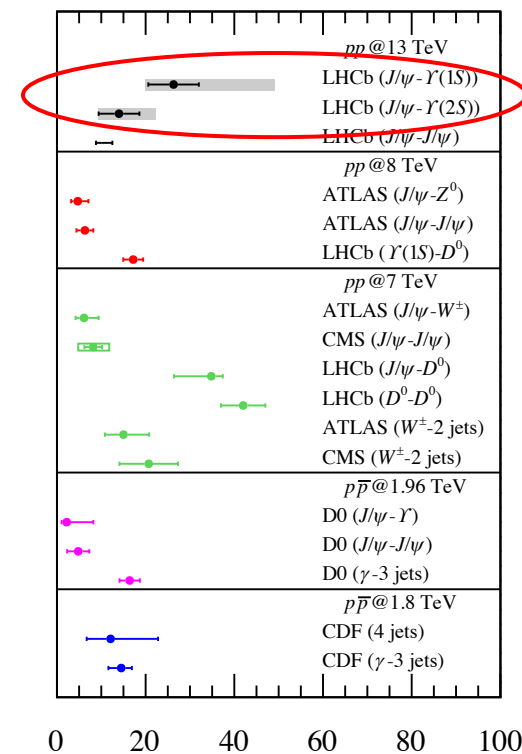
# $J/\psi - \Upsilon$ production

[JHEP 08 (2023) 093]

➤ Fiducial region:  $2 < y(J/\psi, \Upsilon) < 4.5$ ,  $p_T(J/\psi) < 10$  GeV,  $p_T(\Upsilon) < 30$  GeV



Signal	Raw yields	Significances
$J/\psi - \Upsilon(1S)$	$76 \pm 12$	$7.9 \sigma$
$J/\psi - \Upsilon(2S)$	$30 \pm 7$	$4.9 \sigma$
$J/\psi - \Upsilon(3S)$	$10 \pm 6$	$1.7 \sigma$



$$\sigma(J/\psi - \Upsilon(1S)) = 133 \pm 22(\text{stat}) \pm 7(\text{syst}) \pm 3(\mathcal{B}) \text{ pb}$$

$$\sigma(J/\psi - \Upsilon(2S)) = 76 \pm 21(\text{stat}) \pm 4(\text{syst}) \pm 7(\mathcal{B}) \text{ pb}$$

$$\sqrt{\sigma_{\text{eff}}(J/\psi - \Upsilon)} \equiv \frac{\sigma(J/\psi) \times \sigma(\Upsilon)}{\sigma_{\text{DPS}}(J/\psi - \Upsilon)} \text{ determined}$$

by subtracting SPS contribution

[PRL 117 (2016) 062001]

$$\sigma_{\text{SPS}}(J/\psi - \Upsilon(1S)) = 20_{-15}^{+52} \text{ pb}, \quad \sigma_{\text{SPS}}(J/\psi - \Upsilon(2S)) = 8_{-6}^{+22} \text{ pb}$$

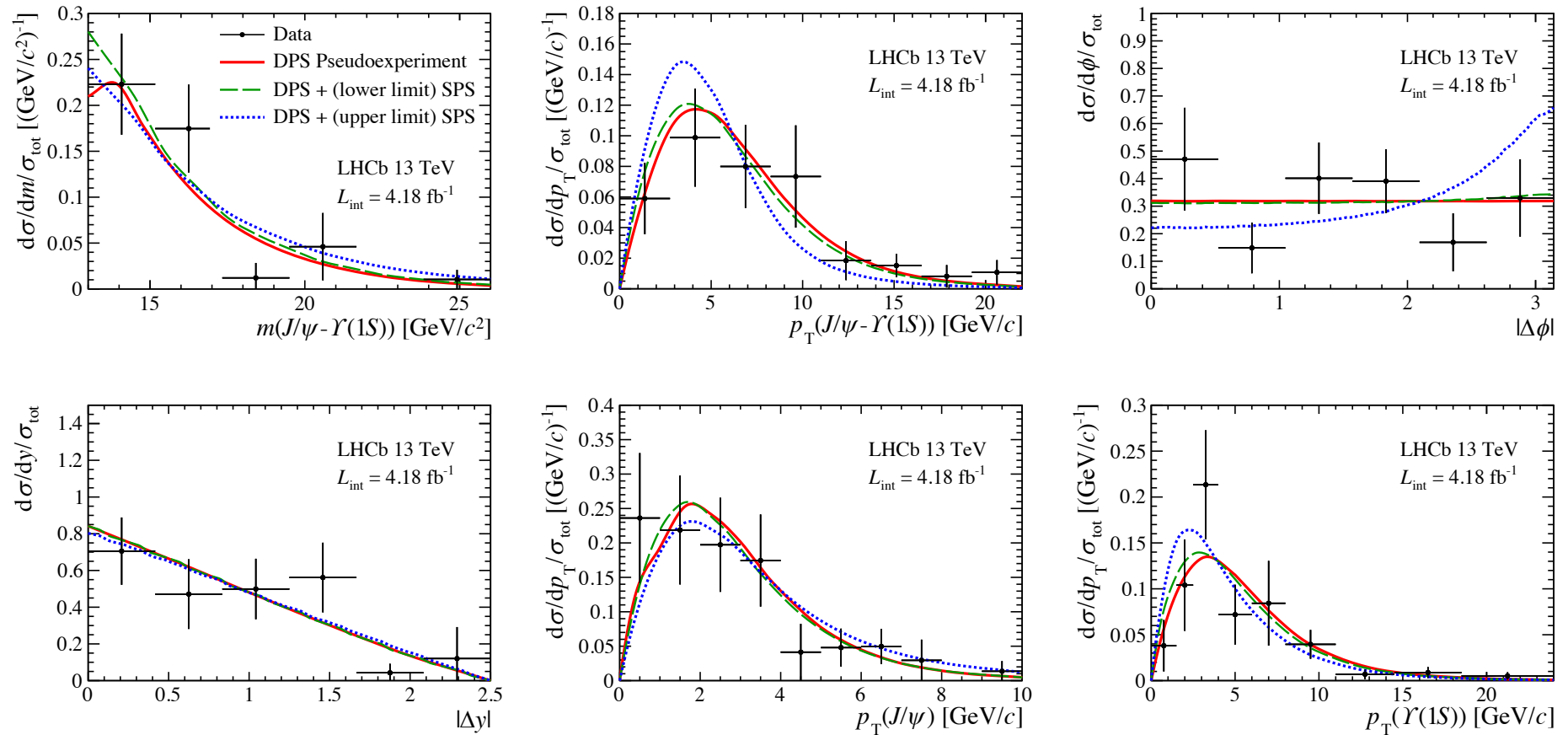
$$\sigma_{\text{eff}}(J/\psi - \Upsilon(1S)) = 26 \pm 5(\text{stat}) \pm 2(\text{syst}) \pm_{-3}^{+22}(\text{th}) \text{ mb}$$

$$\sigma_{\text{eff}}(J/\psi - \Upsilon(2S)) = 14 \pm 5(\text{stat}) \pm 1(\text{syst}) \pm_{-1}^{+7}(\text{th}) \text{ mb}$$

[PoS (LHCP2020) 172; arXiv: 2009.12555]  $\sigma_{\text{eff}}$  [mb]

# Differential $J/\psi - \Upsilon$ cross-sections

[JHEP 08 (2023) 093]



➤ Results consistent with both DPS-only and DPS+predicted SPS scenarios

# Where do we stand?

## ◆ Di-quarkonium production actively measured by the LHCb experiment

✓ di- $J/\psi$ : 7&13 TeV

✓  $J/\psi - \psi(2S)$ : 13 TeV

✓  $J/\psi - \Upsilon$ : 13 TeV

## ◆ What we have learned on DPS?

✓ Similar-level  $\sigma_{\text{eff}}$ : a good starting point

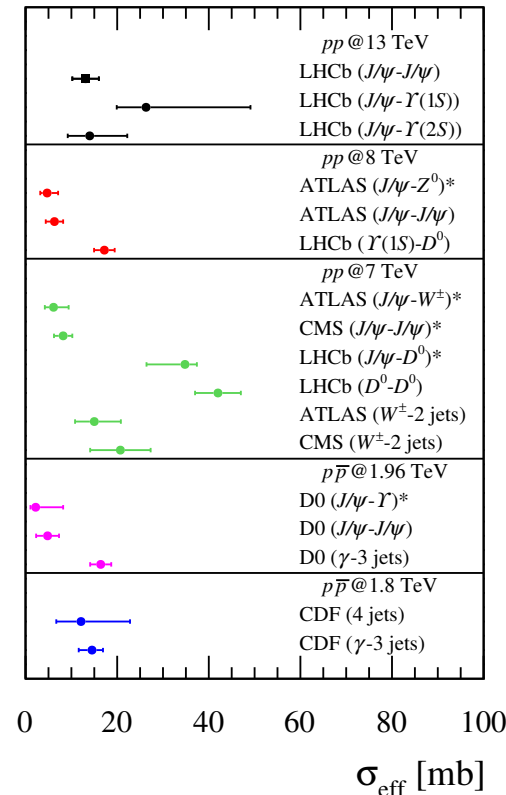
□ How to further investigate the discrepancies?

□ A unified way to separate SPS and DPS?

## ◆ What else we can learn from di-quarkonium prod.?

✓ gluon TMD

□ ...



[PoS (LHCP2020) 172;  
arXiv: 2009.12555]



# What we can further do?

## ◆ New di-quarkonium modes of interest?

✓  $J/\psi + \chi_c, J/\psi + \eta_c \dots$

## ◆ New observables?

✓ polarisation in associated production

## ◆ DPS $\rightarrow$ TPS (Triple-parton scattering)

✓ triple charm

✓  $J/\psi + \Upsilon + \phi$

## ◆ Quarkonium associated with others?

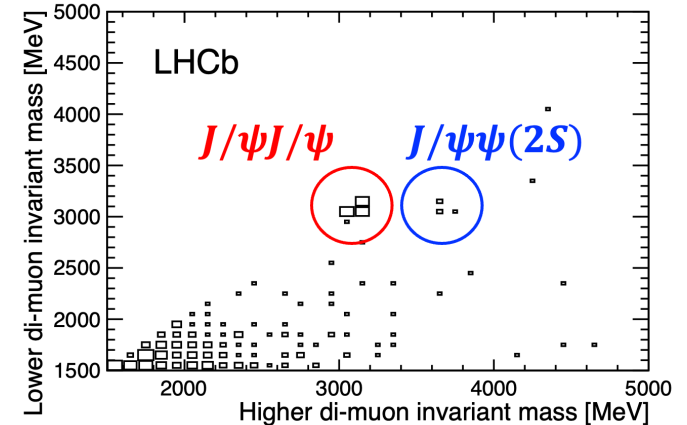
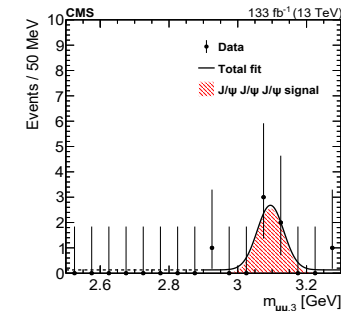
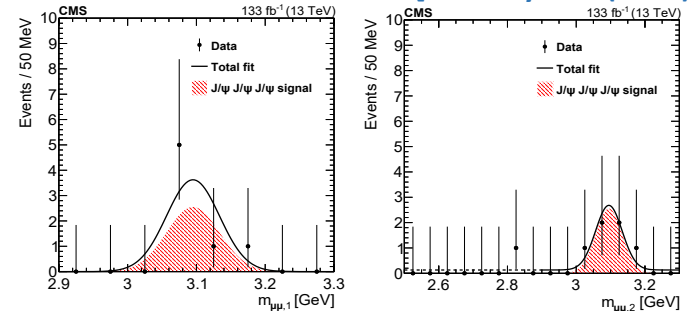
✓ quarkonium +  $W, Z$  (info beyond DPS?)

✓ quarkonium +  $\gamma$

## ◆ Di-quarkonium in exclusive production?

\*Tri- $J/\psi$  at CMS

[Nat. Phys. 19 (2023) 338]



[J. Phys. G: Nucl. Part. Phys. 41 (2014) 115002]

# Back up

# Leading twist TMD PDFs

[PR12-09-014]

Nucleon \ Quark	Unpol.	Long.	Trans.
Unpol.	$f_1 = \text{circle}$		$f_{1T}^\perp = \text{circle with up arrow} - \text{circle with down arrow}$
Long.		$g_{1L} = \text{circle with right arrow} - \text{circle with left arrow}$	$g_{1T} = \text{circle with right arrow and up arrow} - \text{circle with right arrow and down arrow}$
Trans.	$h_1^\perp = \text{circle with down arrow} - \text{circle with up arrow}$	$h_{1L}^\perp = \text{circle with right arrow and down arrow} - \text{circle with right arrow and up arrow}$	$h_{1T} = \text{circle with up arrow and down arrow} - \text{circle with up arrow and up arrow}$ $h_{1T}^\perp = \text{circle with up arrow and right arrow} - \text{circle with up arrow and left arrow}$

# Sketch of CS frame

