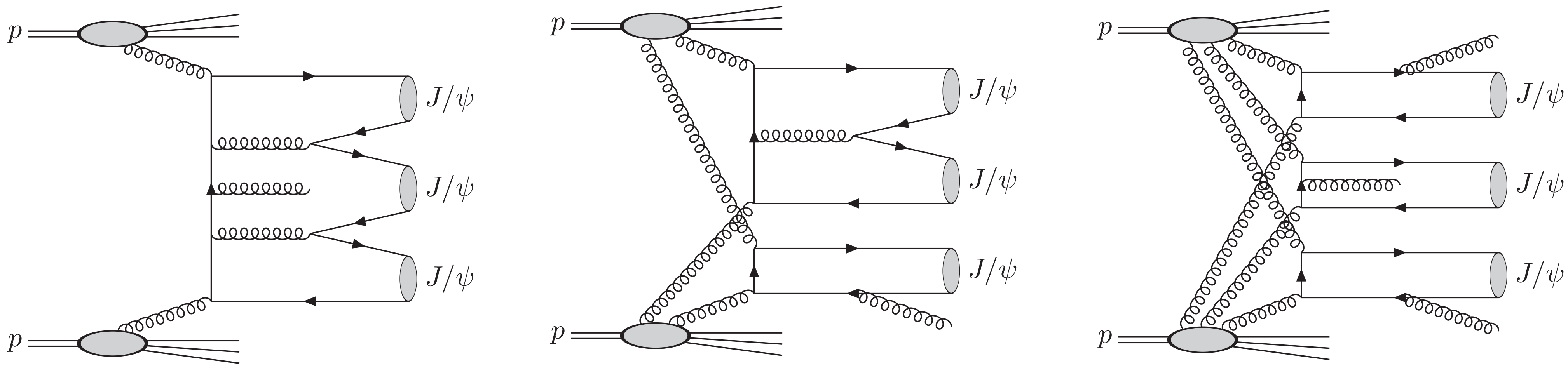
Quarkonium and multiple parton scattering



Hua-Sheng Shao

4th workshop on the frontiers of physics at the LHCb experiment

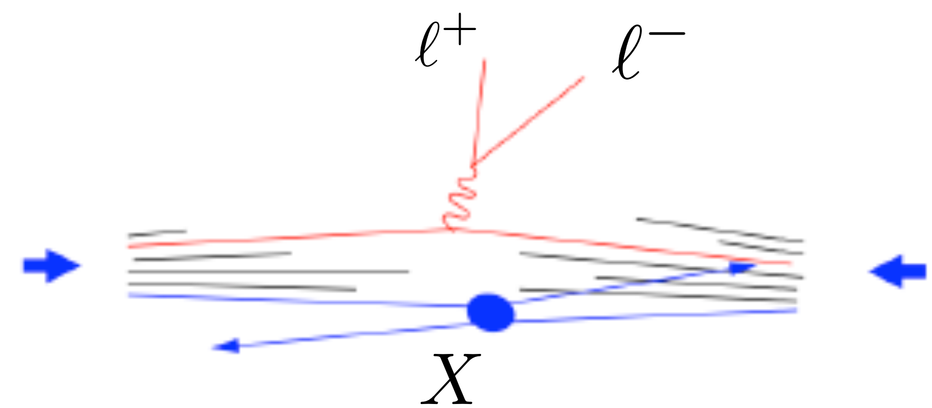
31 July 2024

A Brief Introduction

- **Cross section from factorization theorem (conjecture)**

$$\text{cross section} = \text{parton distribution} \times \text{partonic cross section}$$

- **Spectator-spectator interactions**
  - cancel in inclusive cross sections (unitarity)
  - affect final state X



- **Additional interaction (blue) will be sensitive if we probe X simultaneously**

- **If the second interaction is also hard** → **Double Parton Scattering**

  e.g. $pp \to Z + H + X \to l\bar{l} + b\bar{b} + X$

- **DPS contributes to signals and to backgrounds in many analyses at the LHC**

- **Inclusive cross section:**

$$\sigma_{\mathrm{SPS}} \sim \frac{1}{Q^2} \quad \text{vs} \quad \sigma_{\mathrm{DPS}} \sim \frac{\Lambda^2_{\mathrm{QCD}}}{Q^4}$$

- **Higher energy** ⇒ **Larger parton density** ⇒ **enhance DPS**

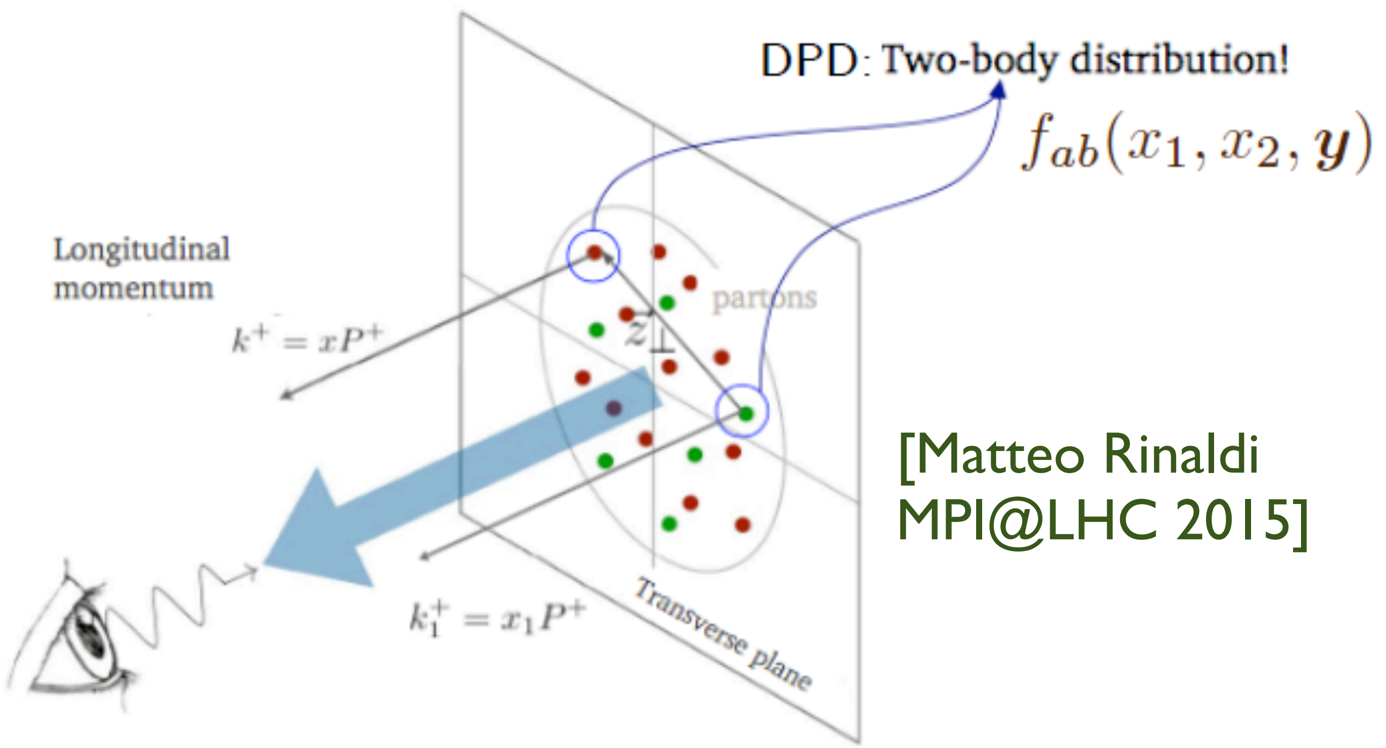
$$\sigma_{\mathrm{SPS}} \propto (\text{parton density})^2 \quad \text{vs} \quad \sigma_{\mathrm{DPS}} \propto (\text{parton density})^4$$

A DPS Theory Foundation

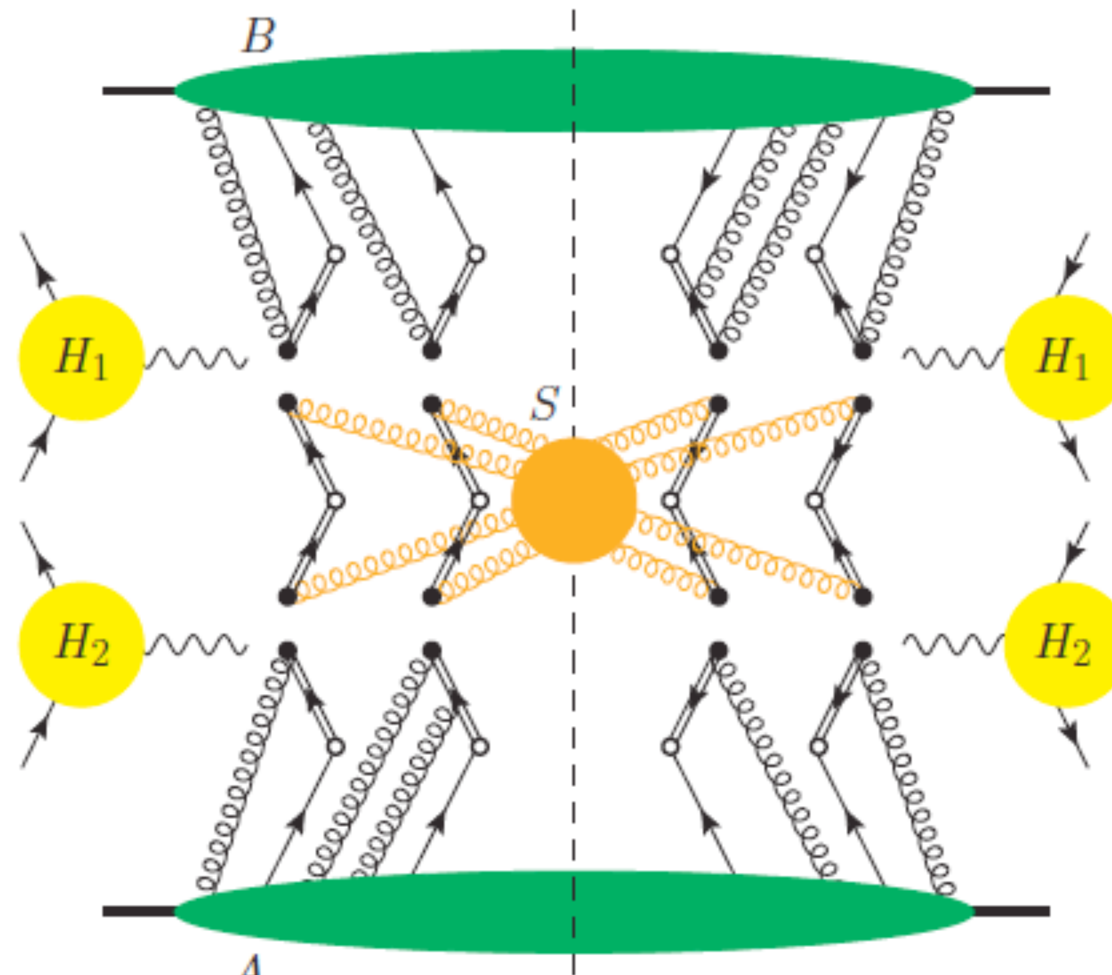
- **Like SPS, we now have a first proven factorisation theorem for DPS (double Drell-Yan)**

[Diehl, Gaunt, Ostermeier, Ploessl, Schafer (2015); Diehl, Nagar (2018)]

$$\sigma_{Q_1Q_2} = \frac{1}{1+\delta_{Q_1Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2\mathbf{b}_1 d^2\mathbf{b}_2 d^2\mathbf{b} \times \Gamma_{ij}(x_1,x_2,\mathbf{b}_1,\mathbf{b}_2)\, \hat{\sigma}^{Q_1}_{ik}(x_1,x'_1)\, \hat{\sigma}^{Q_2}_{jl}(x_2,x'_2)\, \Gamma_{kl}(x'_1,x'_2,\mathbf{b}_1-\mathbf{b},\mathbf{b}_2-\mathbf{b}),$$



[Matteo Rinaldi MPI@LHC 2015]



A new way to access the information of the nonperturbative structure of nucleons

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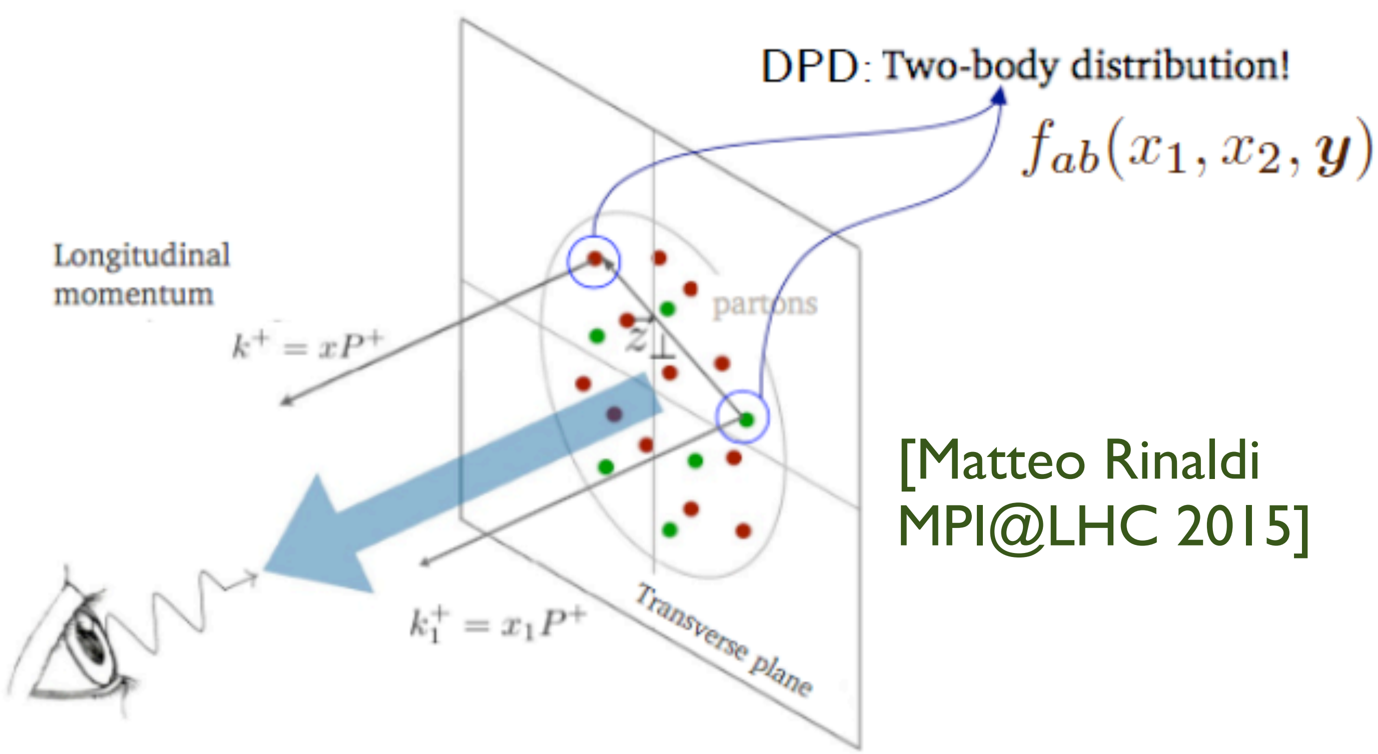
Generalised double parton distribution



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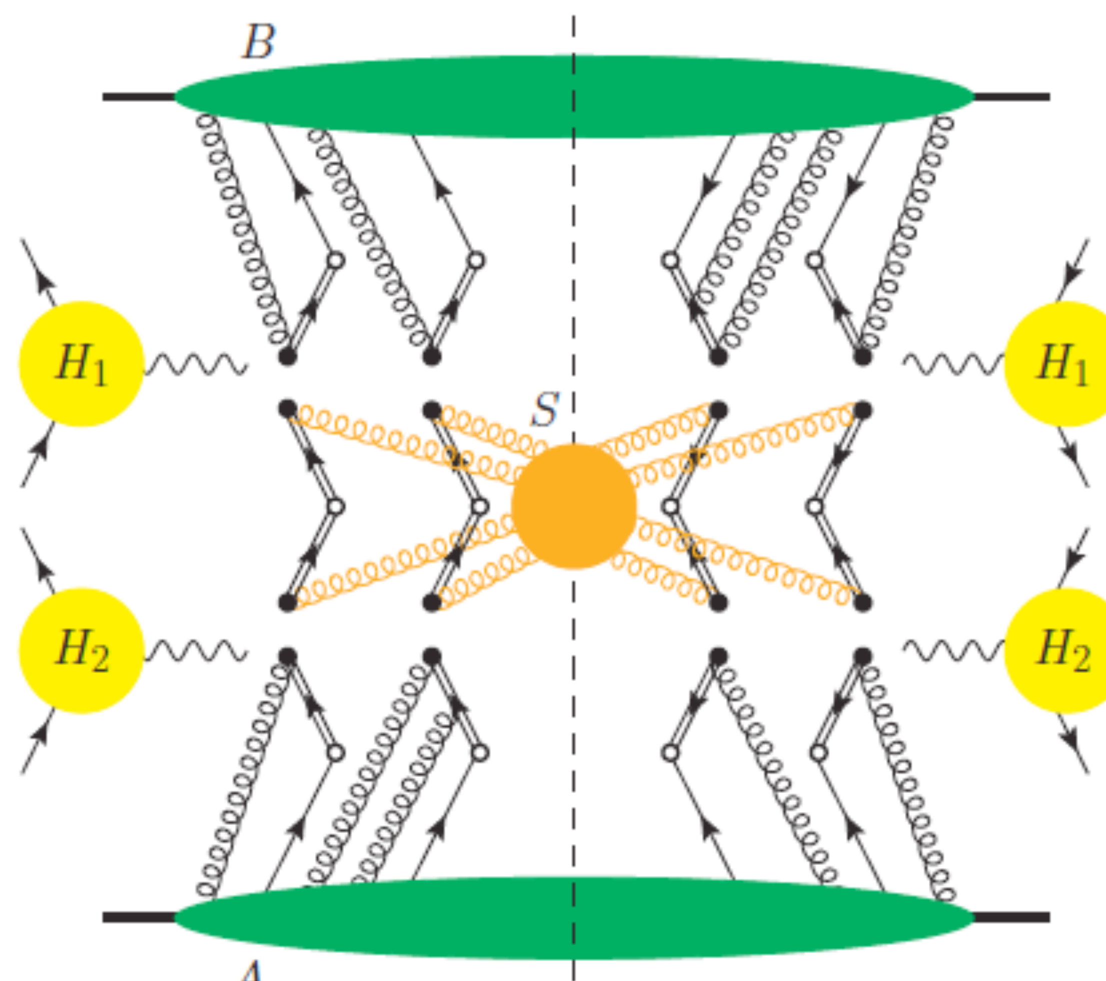

A NEW WAY TO ACCESS THE INFORMATION OF THE NONPERTURBATIVE STRUCTURE OF NUCLEONS

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LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

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- **Widespread simplifications (most phenomenology relies on. Go beyond ?)**
  - factorization I $\quad \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2) = D_{ij}(x_1, x_2) T_{ij}(\mathbf{b}_1, \mathbf{b}_2),$ dPDF
  - factorization II $\quad D_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2),$ PDF
    $T_{ij}(\mathbf{b}_1, \mathbf{b}_2) = T_i(\mathbf{b}_1) T_j(\mathbf{b}_2),$
  - assume flavor universality in T

$$\sigma_{\text{eff}} = \left[\int d^2\mathbf{b} F(\mathbf{b})^2\right]^{-1}.$$

$$\sigma_{Q_1Q_2} = \frac{1}{1+\delta_{Q_1Q_2}} \frac{\sigma_{Q_1}\sigma_{Q_2}}{\sigma_{\text{eff}}},$$

$$F(\mathbf{b}) = \int T(\mathbf{b}_i) T(\mathbf{b}_i - \mathbf{b}) d^2\mathbf{b}_i,$$

Pocket Formula

A DPS Theory Foundation

LPTHE LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

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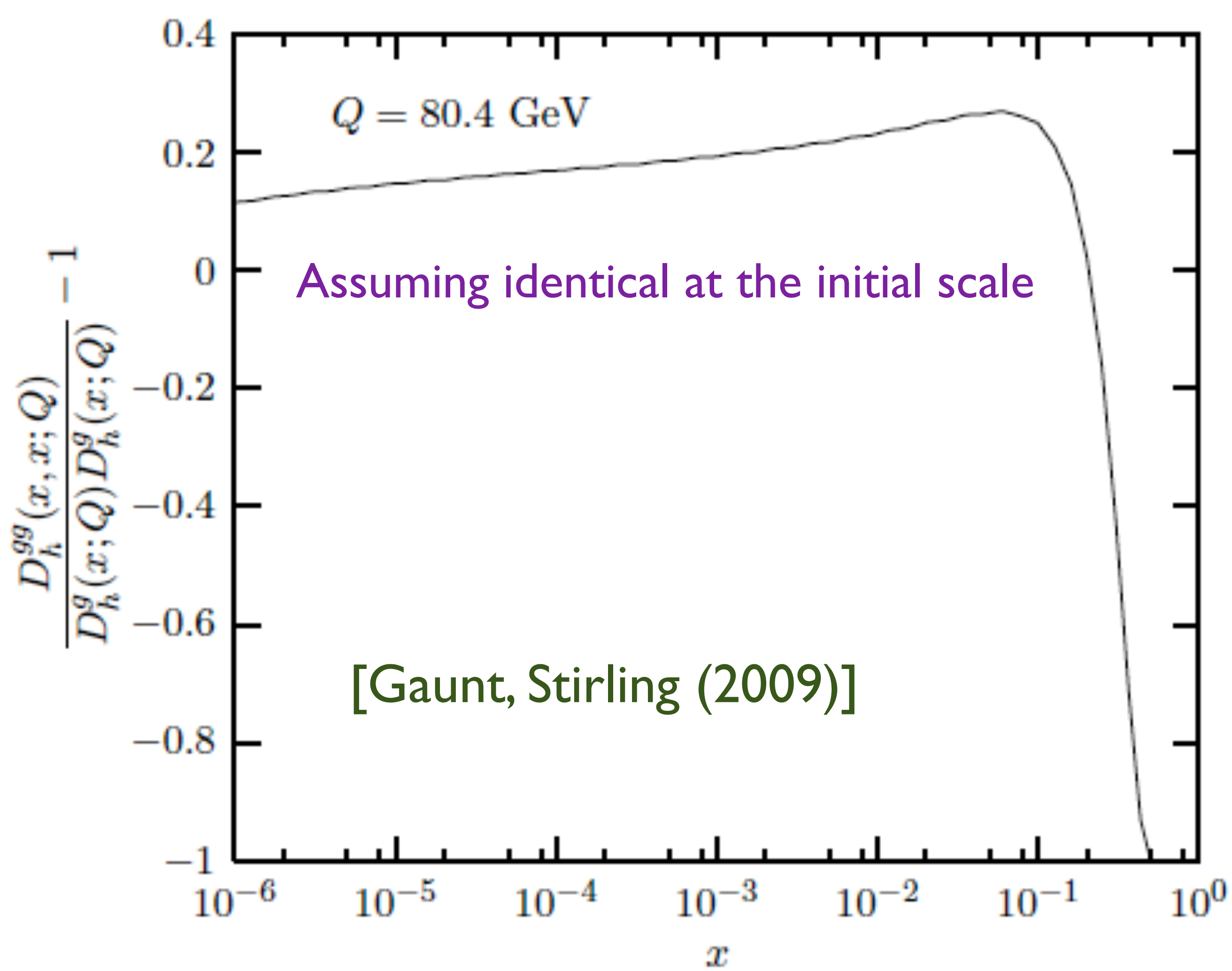
Pocket Formula

- Even these are complex objects to treat numerically

[Gaunt, Stirling; Elias, Golec-Biernat, Stasto; Diehl, Nagar, Tackmann]

DPS Theory Progress

- **Let us start with the pocket formula and take any deviation wrt experiment as an indication of calling for a more rigorous treatment.**
- **Possible deviations (a few examples):**
  - dDGLAP evolution (note high x !)



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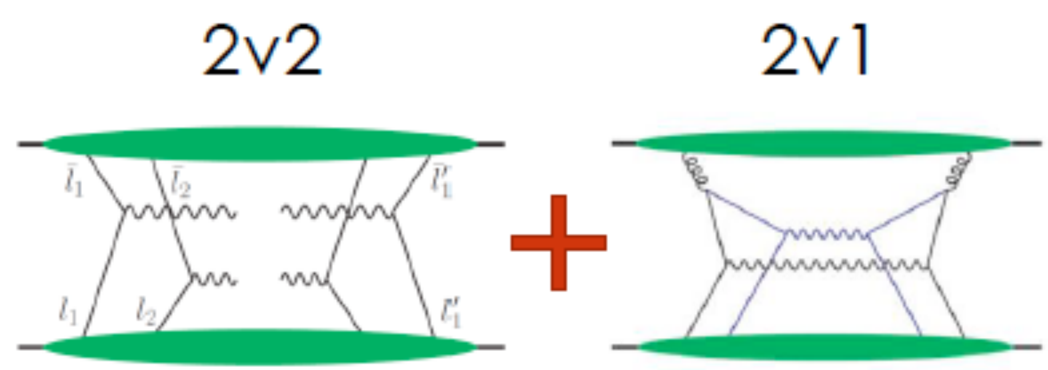
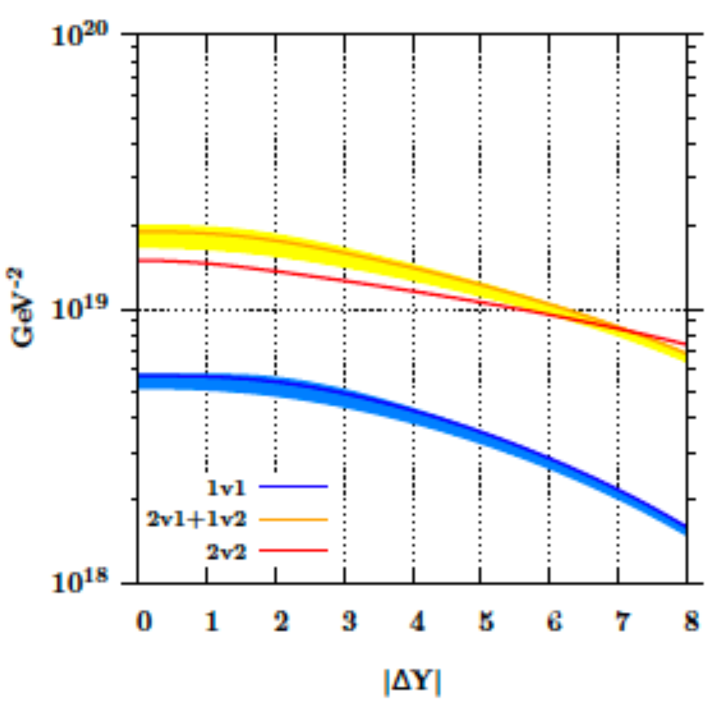


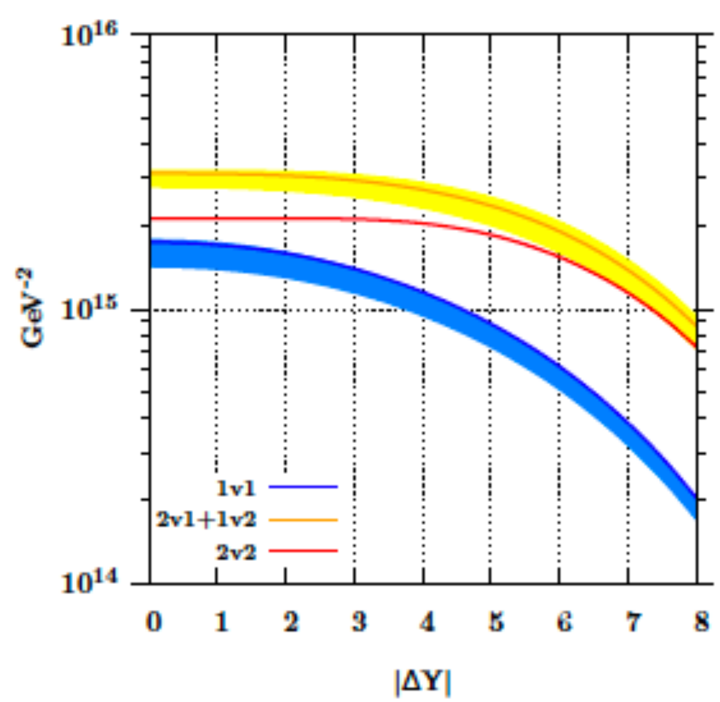
[Gaunt, Stirling (2011); Block et al. (2012); Manohar, Waalewijn (2012)]

parton luminosity is not suppressed !

[Riccardo Nagar @ Quarkonia As Tools 2020]



$\mathcal{L}_{gggg}\,(m_{J/\Psi}, m_{J/\Psi})$



$\mathcal{L}_{gggg}\,(m_W, m_{J/\Psi})$

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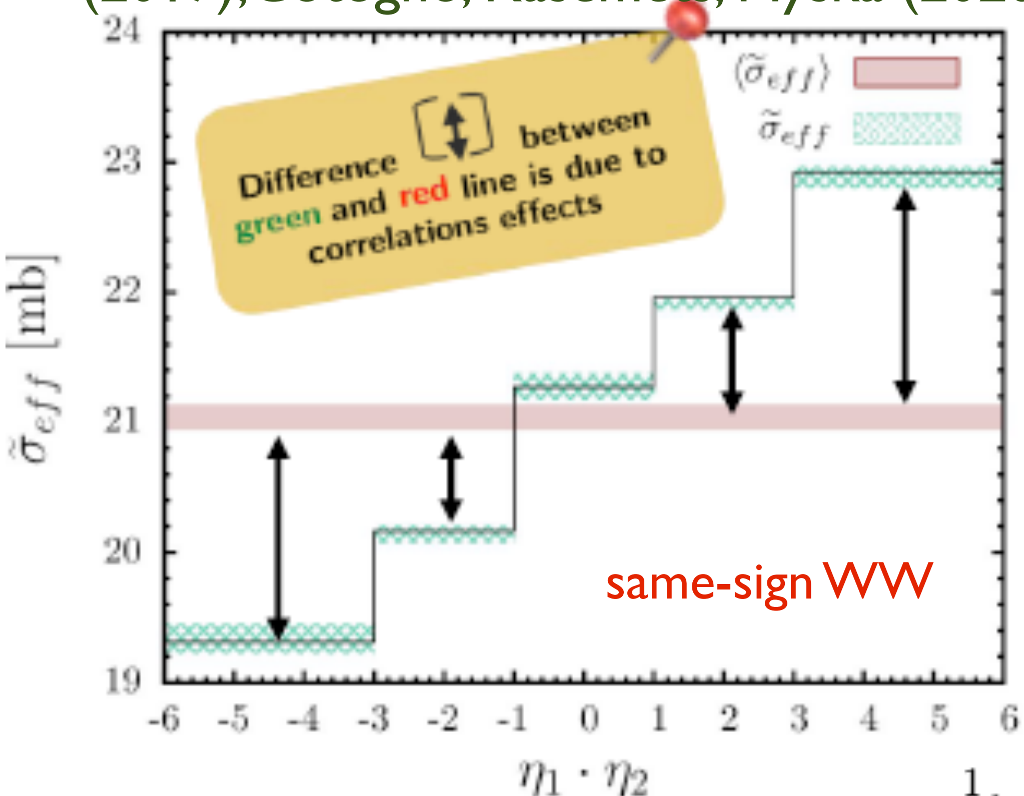
  - parton-parton correlations

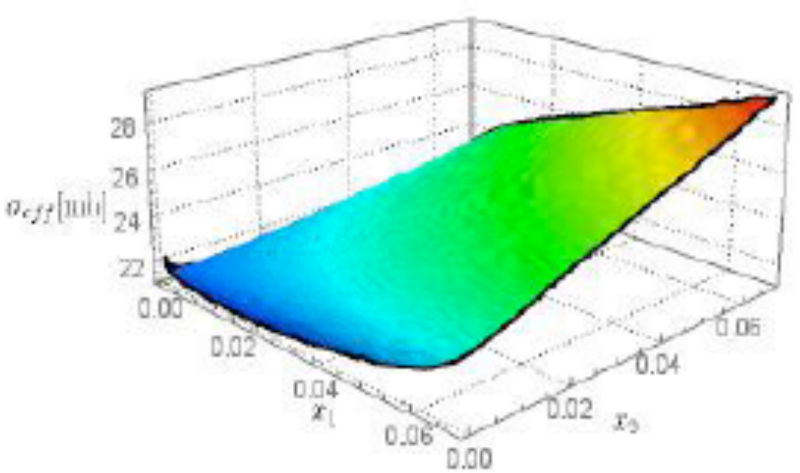
[Matteo Rinaldi @ Quarkonia As Tools 2020; Ceccopieri, Rinaldi, Scopetta (2017);Cotogno, Kasemets, Myska (2020)]



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} ln\frac{x_1}{x_3} ln\frac{x_2}{x_4}$$

the first and the last bins differ by 1 sigma,

$\mathcal{L} = 1000\ \text{fb}^{-1}$ is necessary to observe correlations



Gluons ⊗ Gluons

$$\sigma_{\text{eff}} \to \sigma_{\text{eff}}(x_1, x_2, \mu_F)$$

DPS Theory Progress

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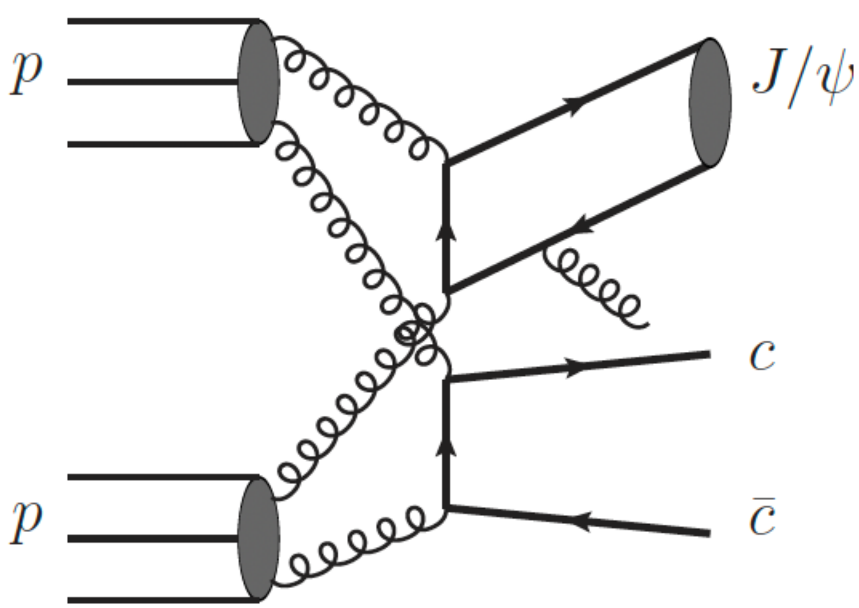
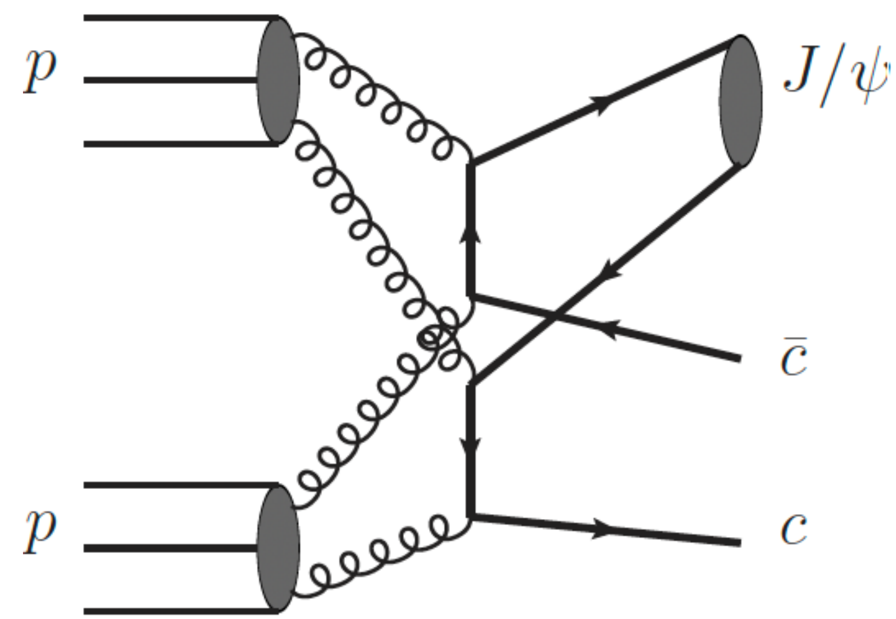
  - DPS coalescence [HSS PRD'20]



Traditional DPS

VS



Coalescence

DPS Theory Progress

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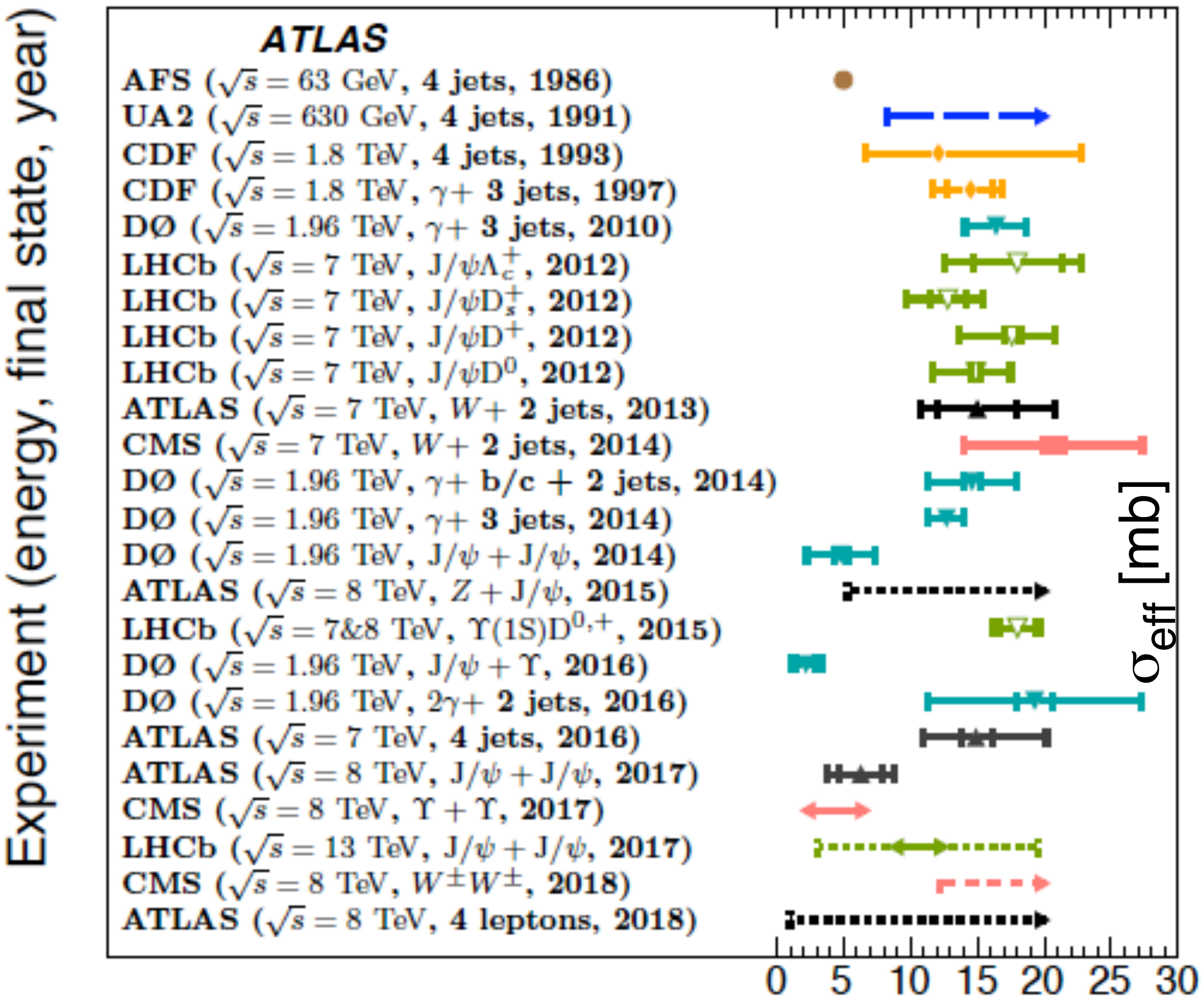
  - DPS coalescence [HSS PRD'20]
- **A few recent theoretical developments**
  - DPS shower dShower [Cabouat, Gaunt,Ostrolenk (2019); Cabouat, Gaunt (2020)]
  - dDGLAP evolution beyond LO ChiliPDF [Diehl et al. (2023)]
  - Double parton distributions from lattice QCD [Bali et al. (2021); Zhang (2023); Jaarsma et al. (2023)]

Also see the section 7 in arXiv:2012.14161

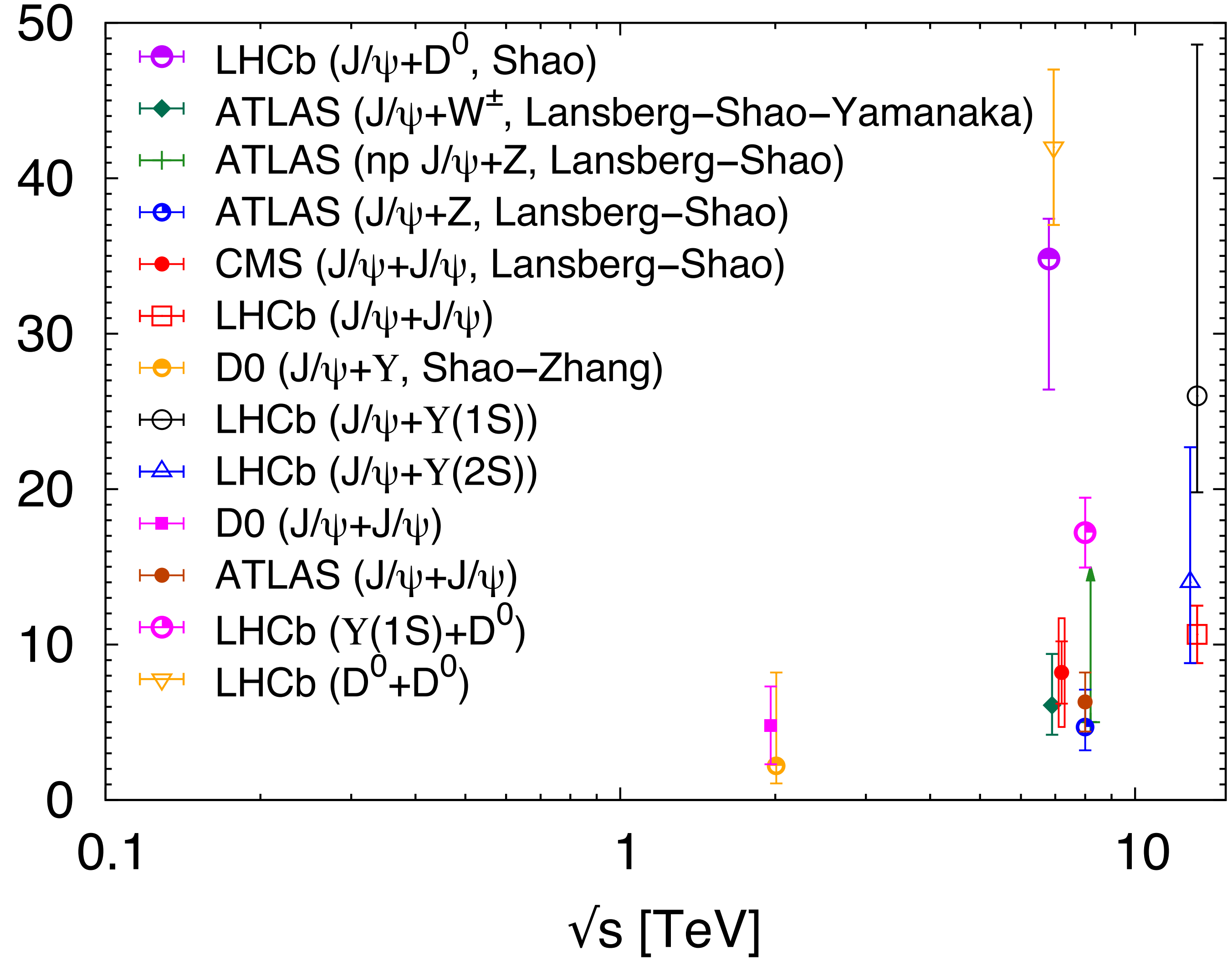
DPS Measurements

- **Many DPS measurements at the LHC (Tevatron) in pp (ppbar)**



[ATLAS (PLB'19)]

σ_{eff} [mb]



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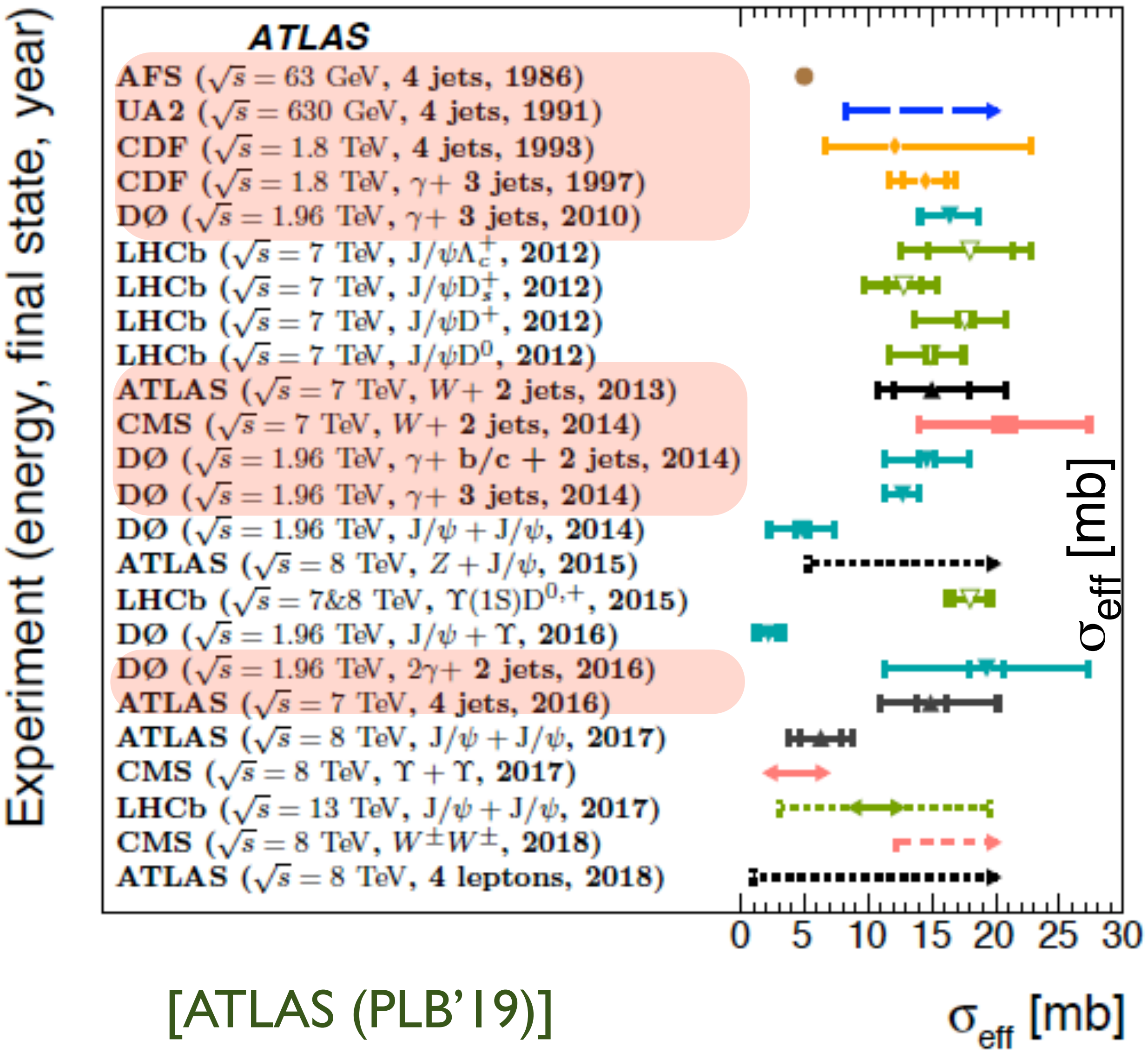

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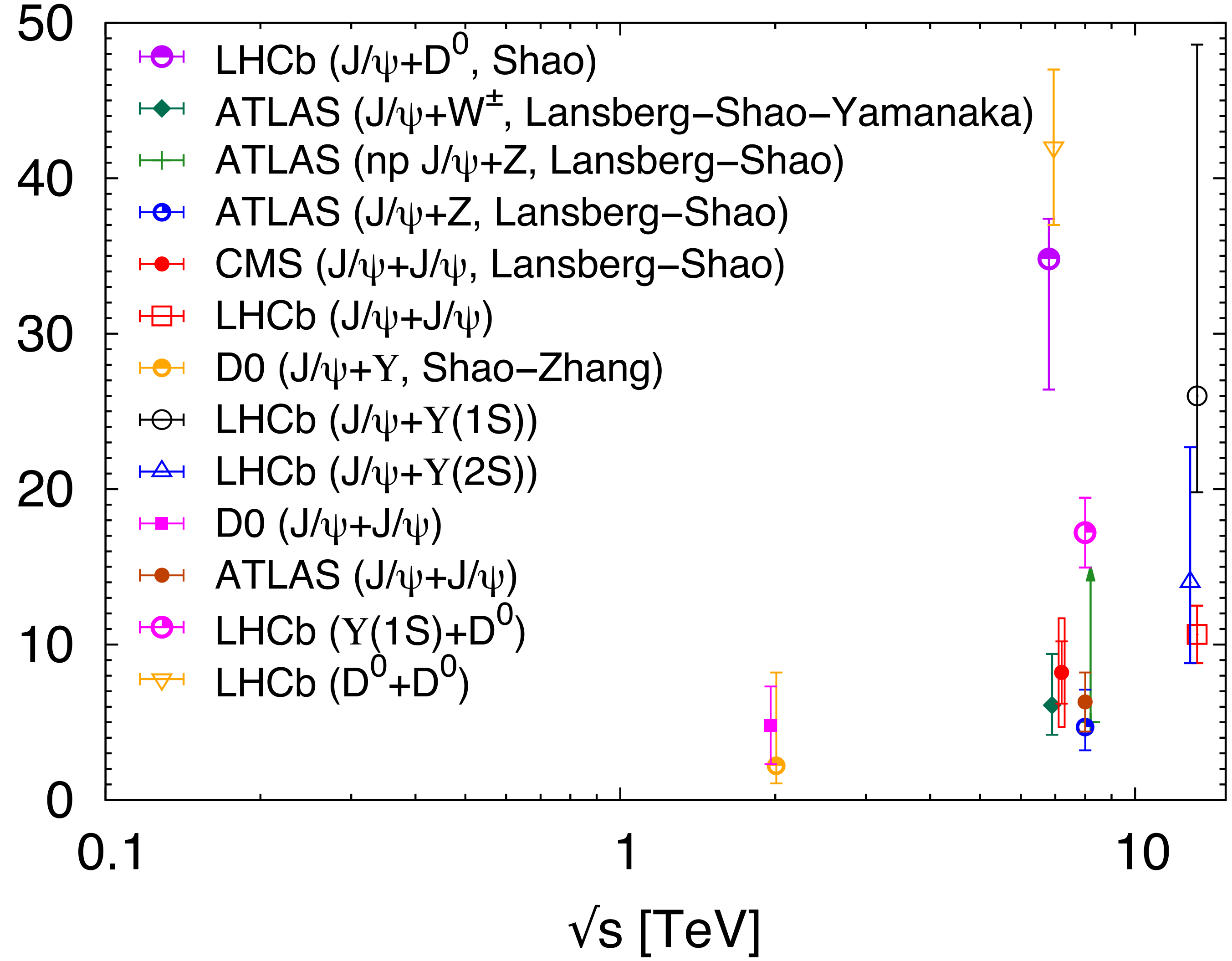

jets

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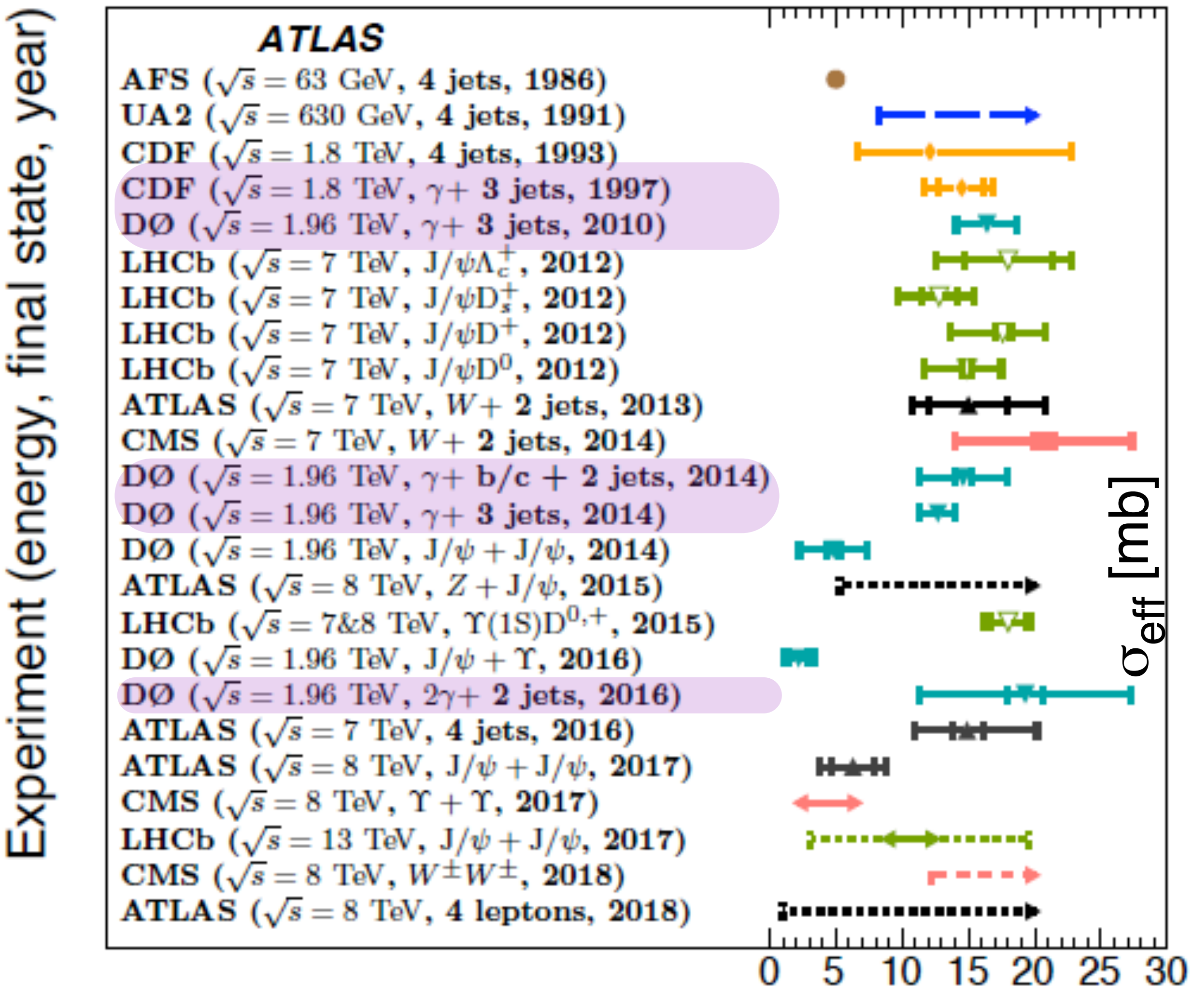

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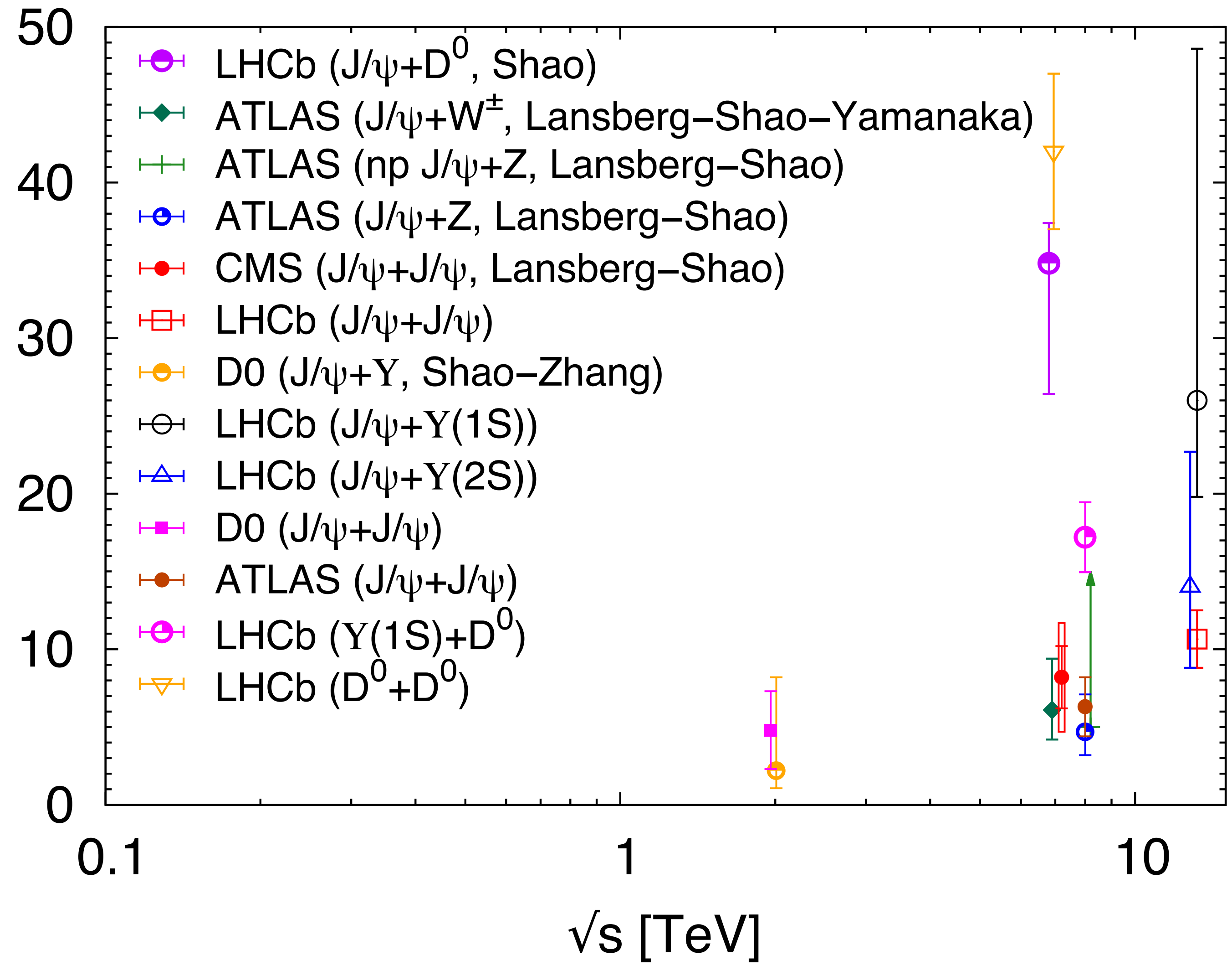

jets photons

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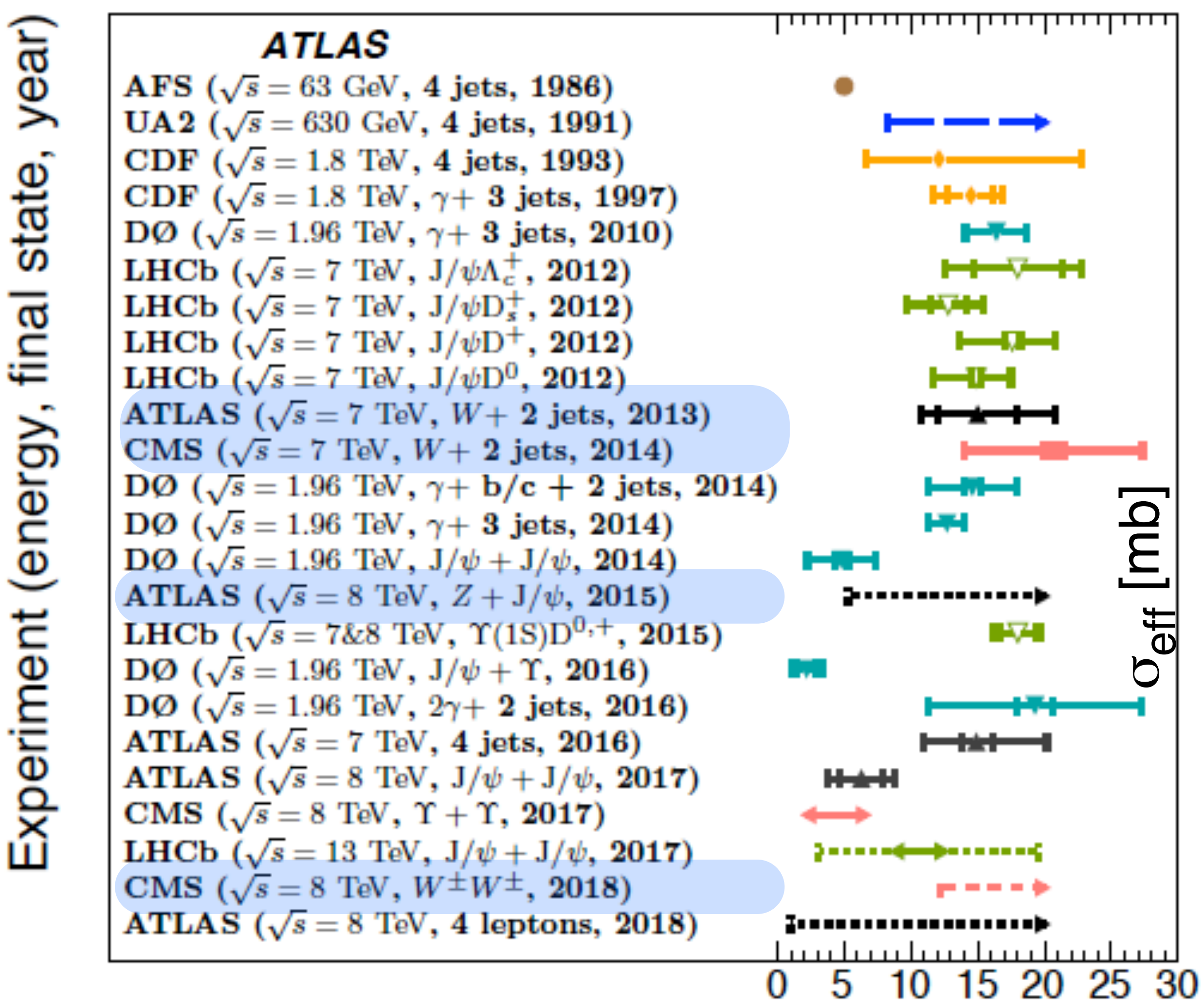

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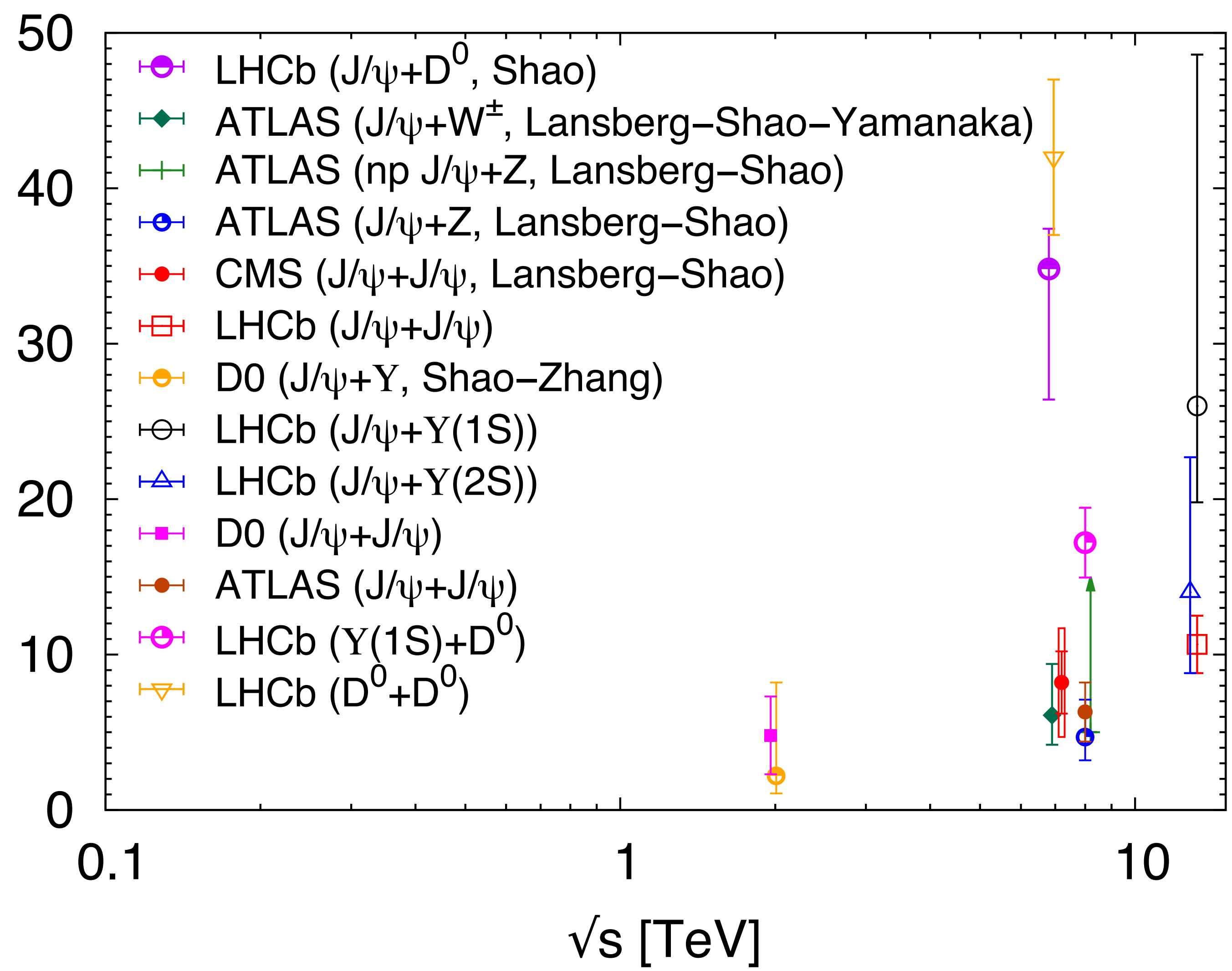

jets | photons | W & Z

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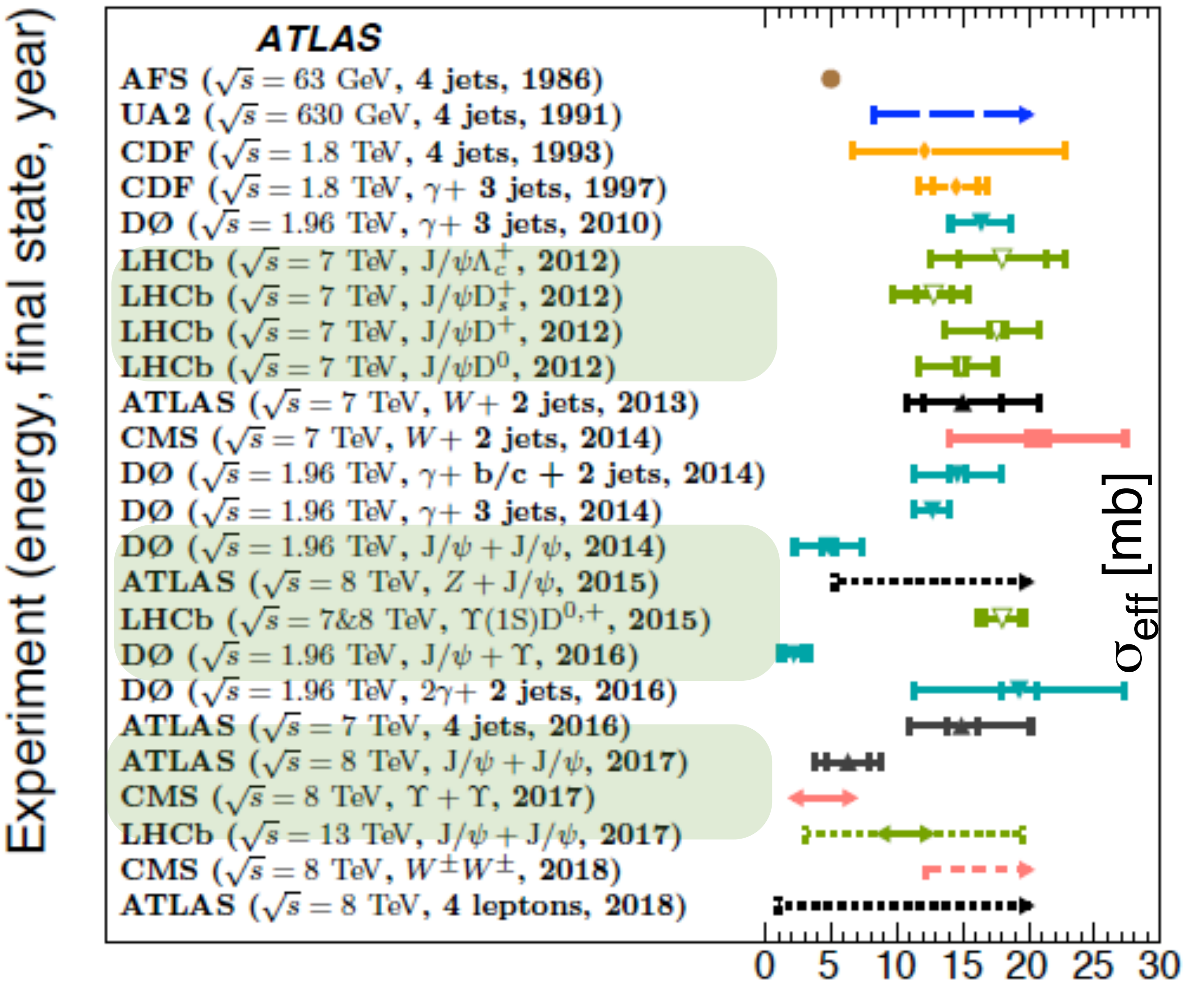

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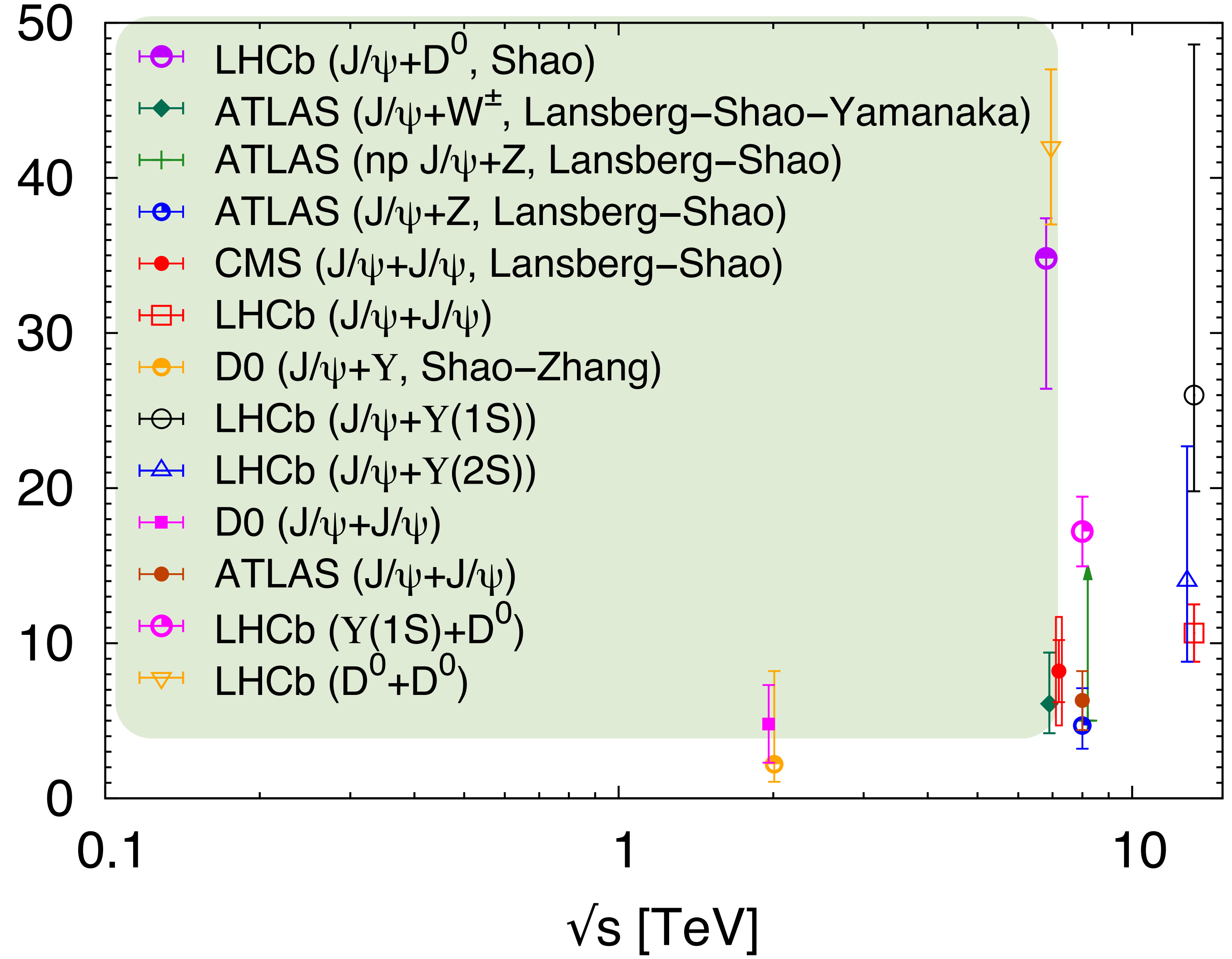

jets | photons | W & Z | heavy flav. & onia

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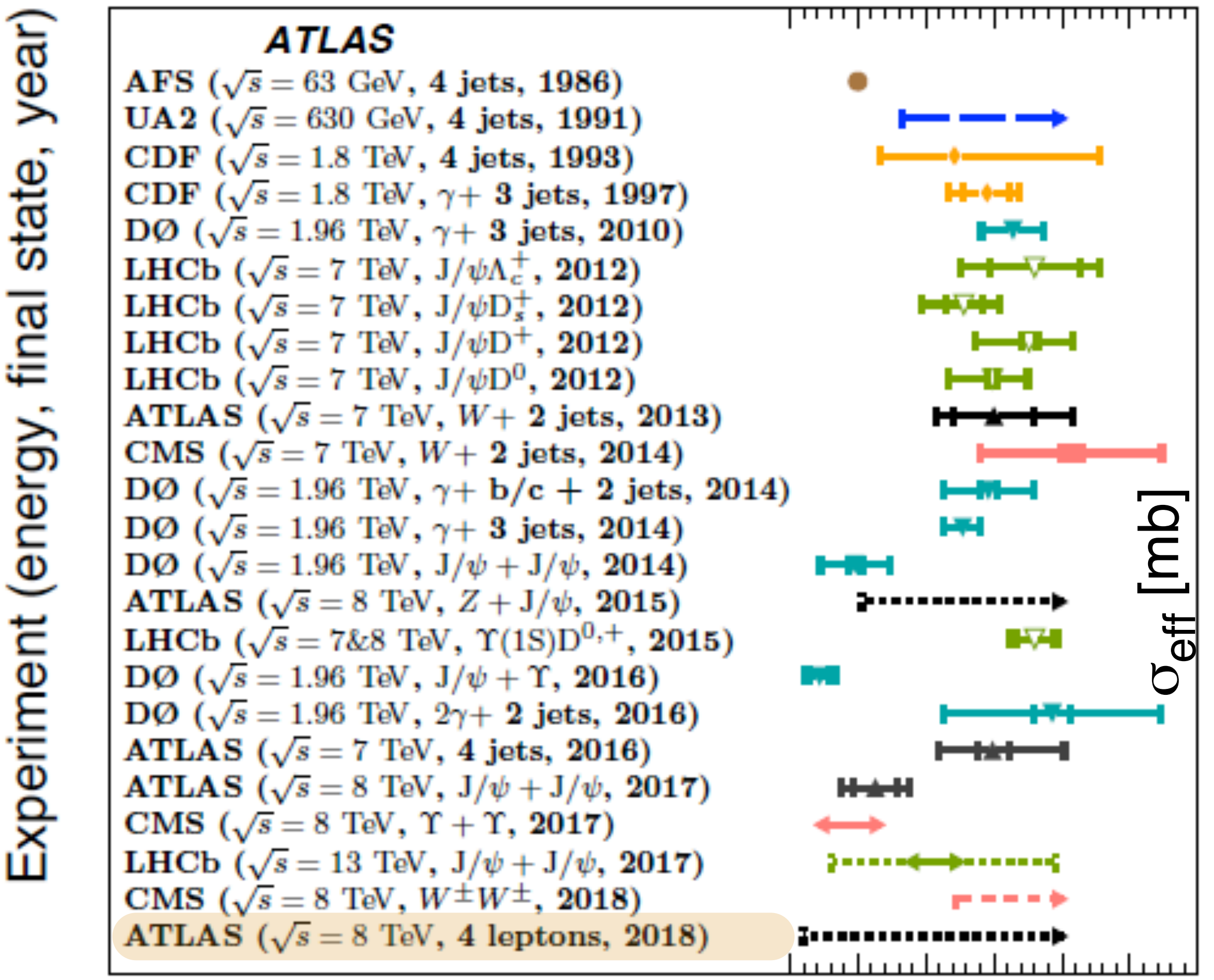

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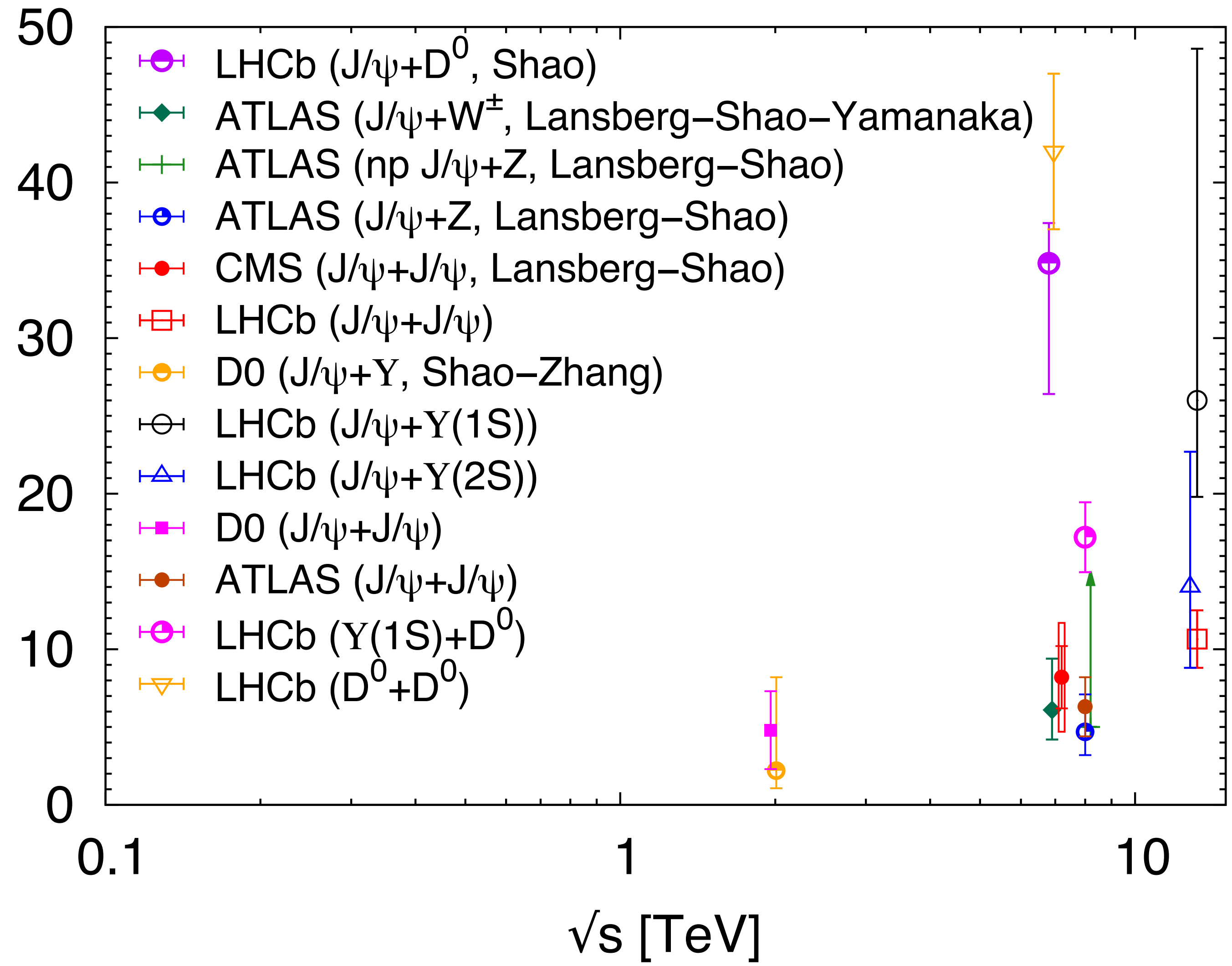

jets | photons | W & Z | heavy flav. & onia | double DY

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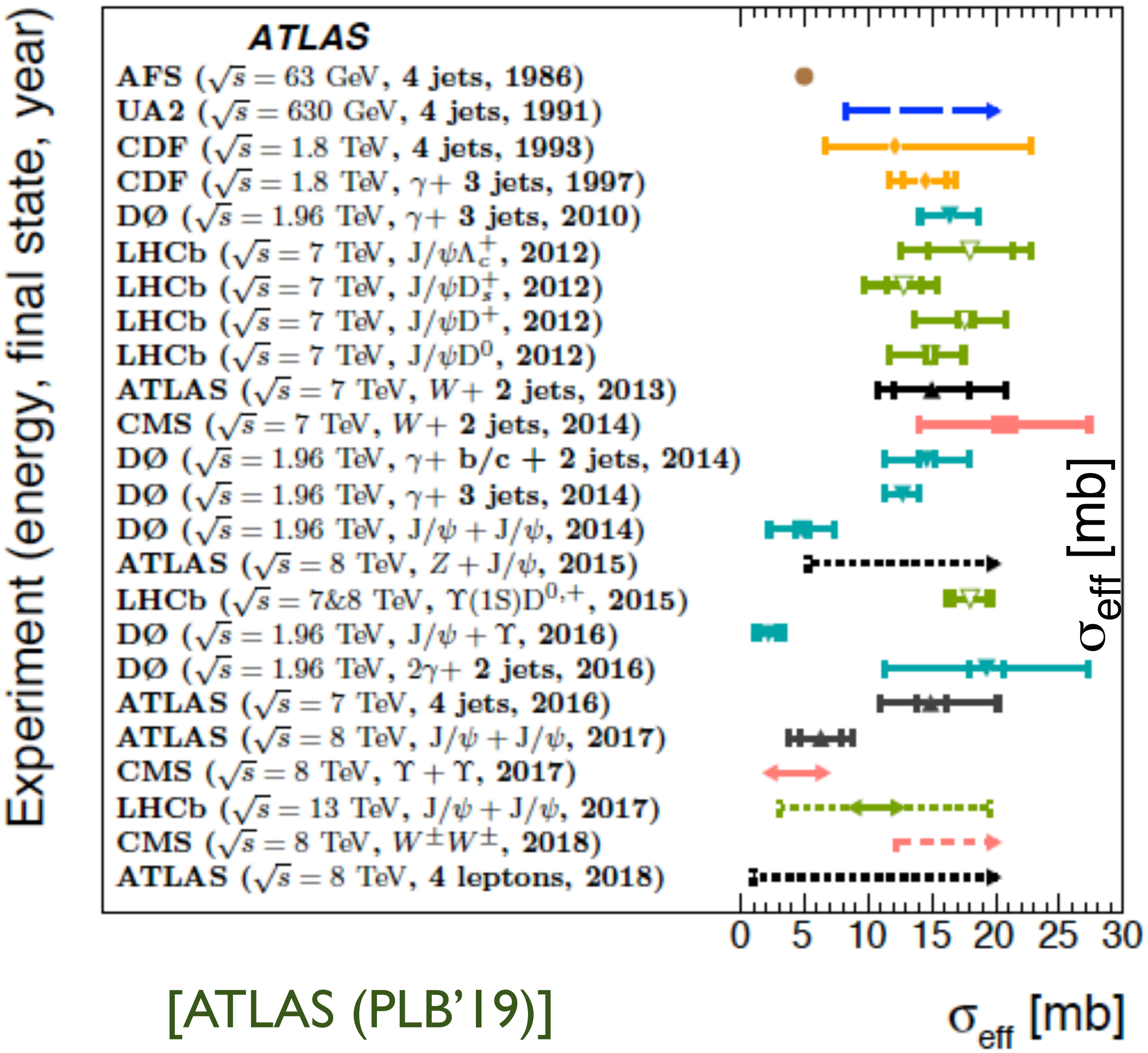
$\sigma_{\rm eff}$:
- flavour dependent ?
- energy dependent ?
- kinematic dependent ?



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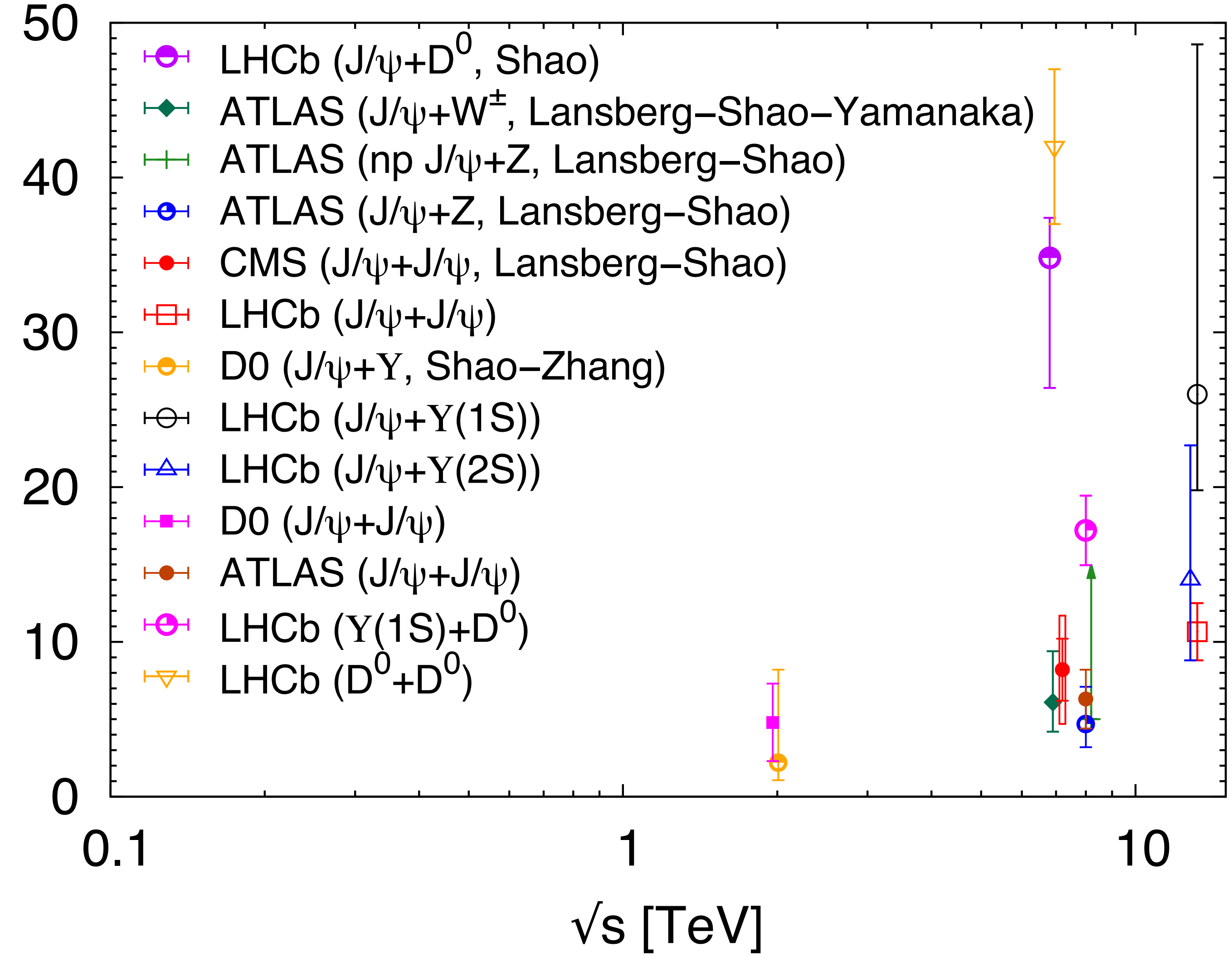

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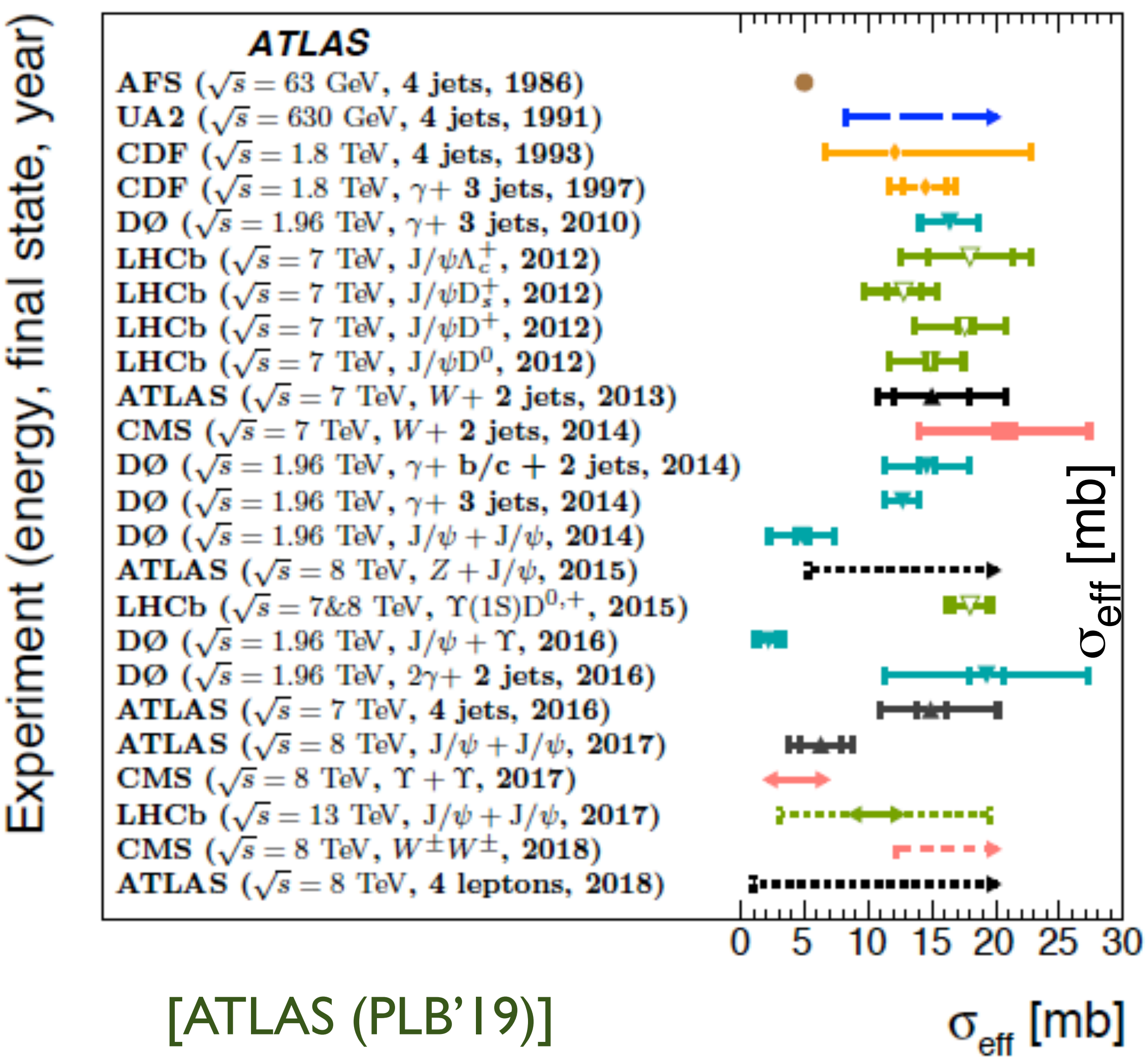

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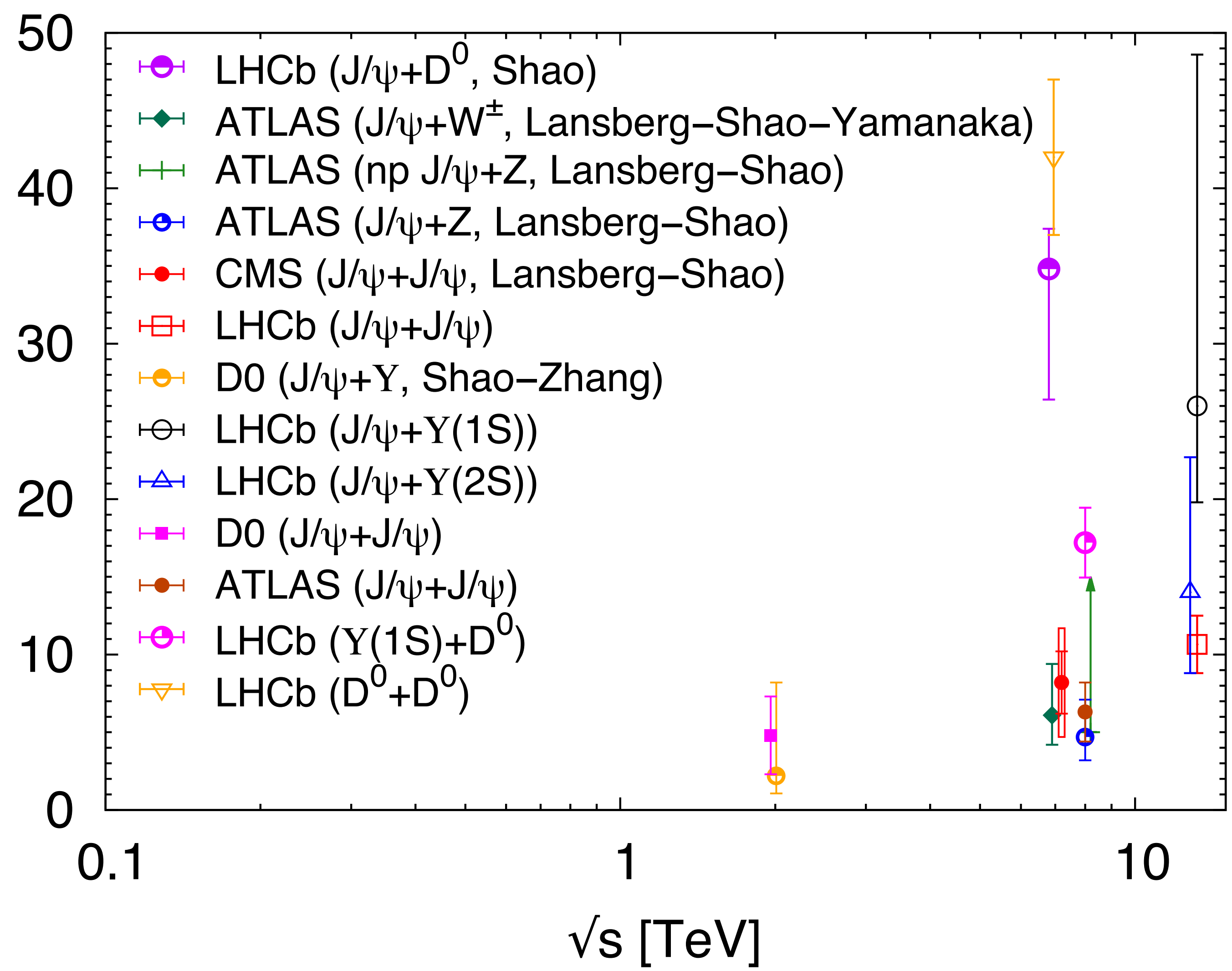

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Can LHCb measure kinematic dependence ?

DPS Measurements

- **Many DPS measurements at the LHC (Tevatron) in pp (ppbar)**
  - Caveats with different extractions (challenging in differ. SPS & DPS)
    - How good are we understanding/controlling SPS ?

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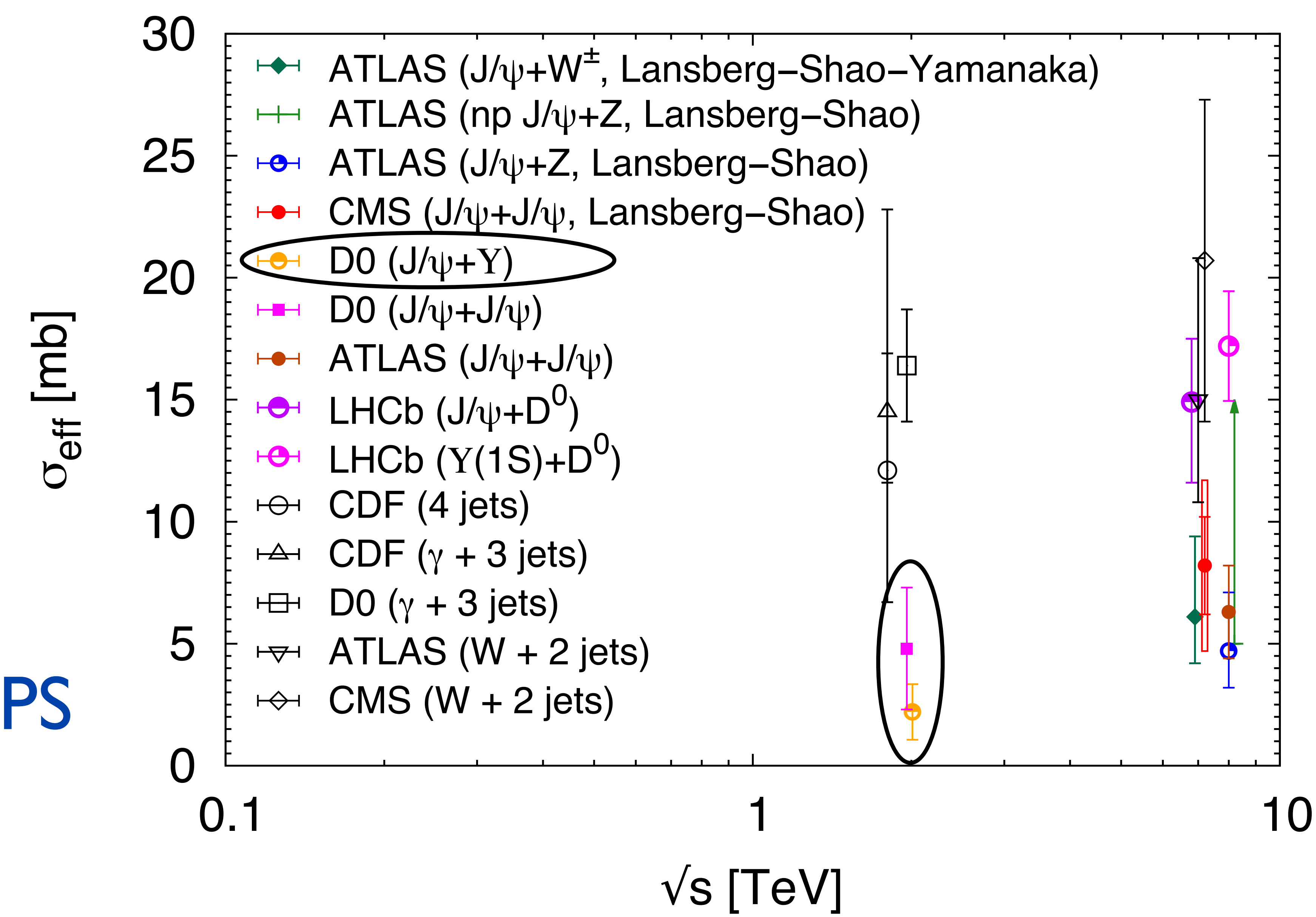

w/o SPS

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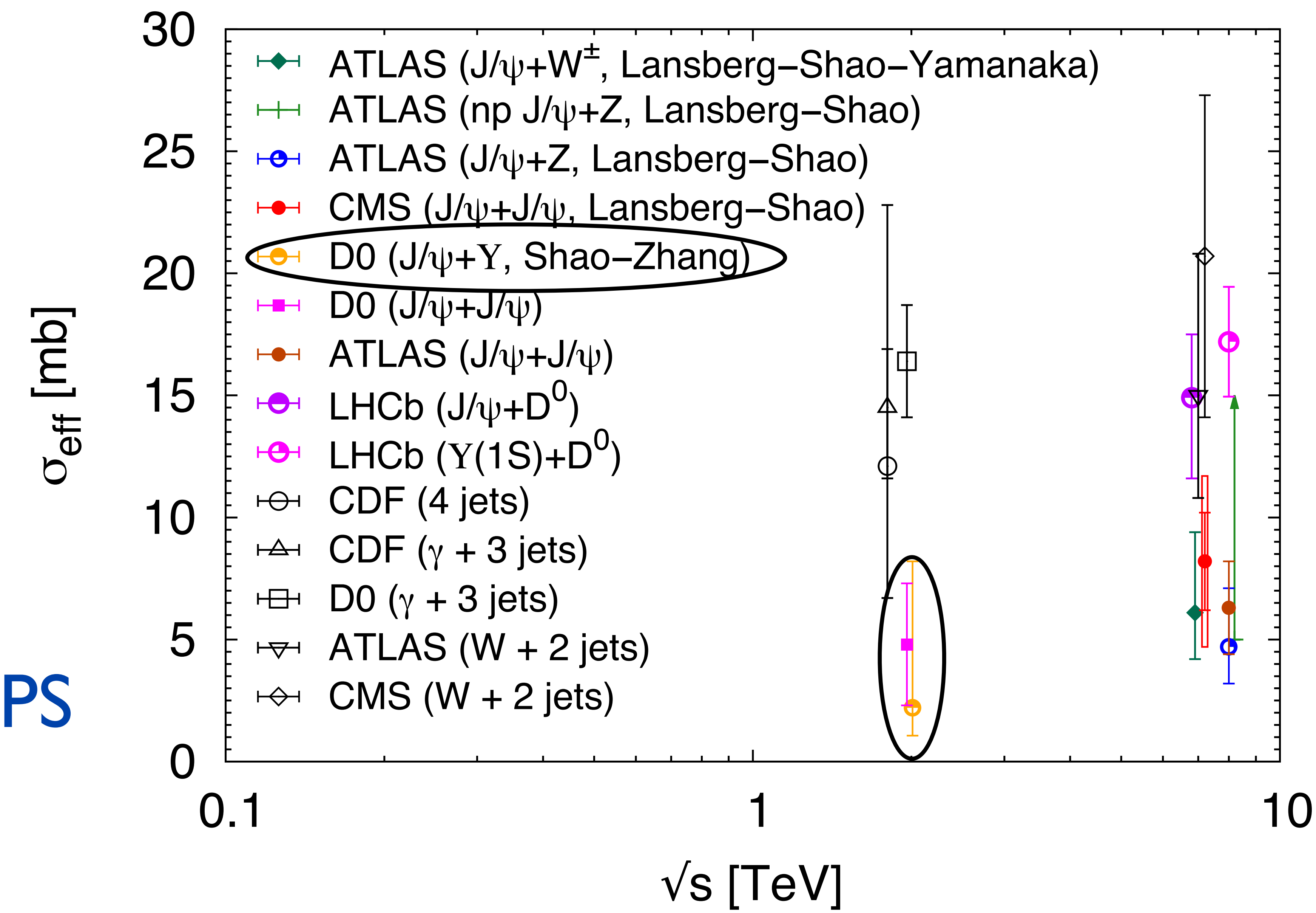
w SPS



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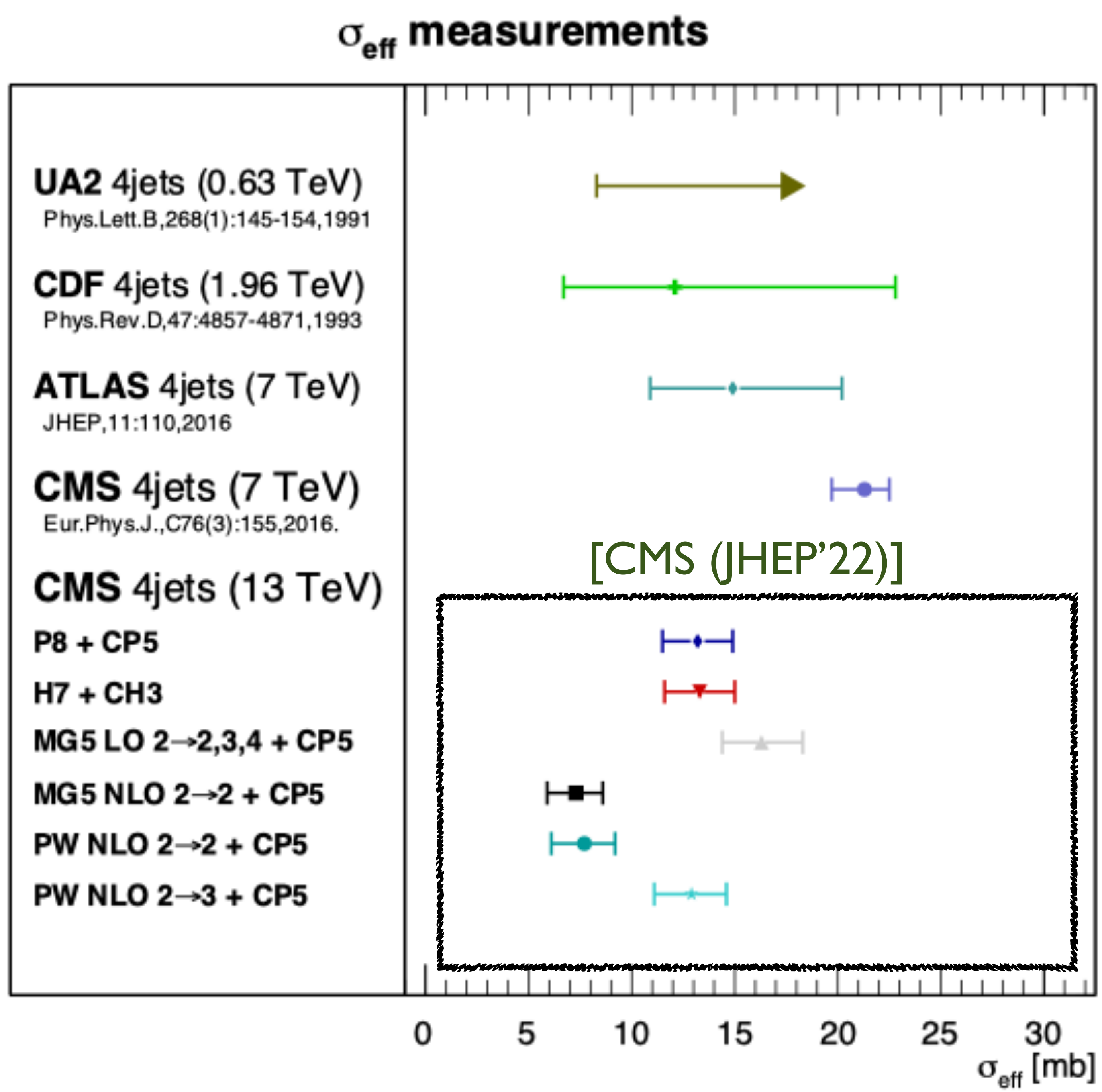


Same observable but different ME+MC

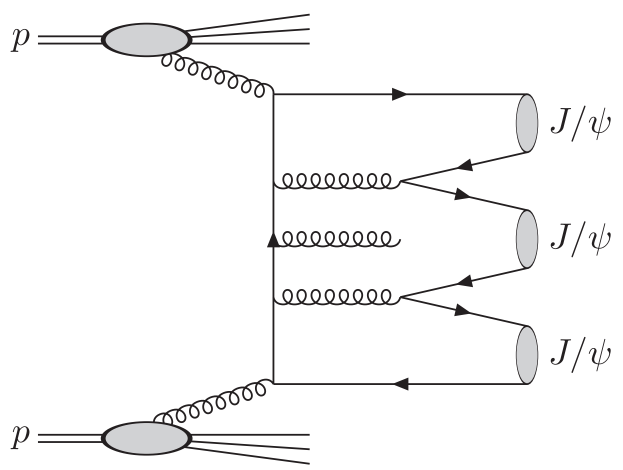
Two novel observables

- **In the rest of the talk, I will focus on two novel observables that have been firstly measured by CMS and LHCb respectively**

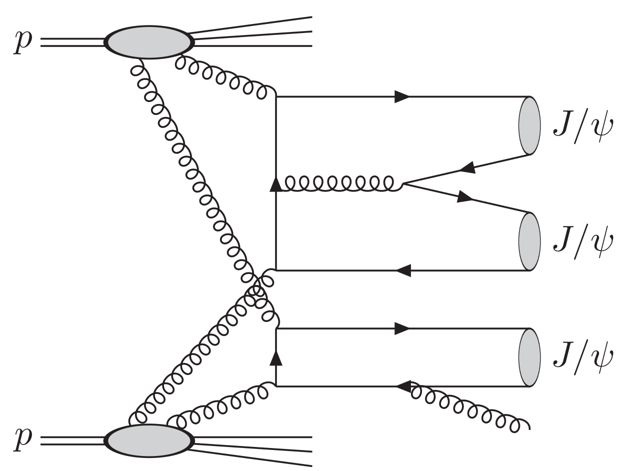
Triple Parton Scattering in pp

DPS in heavy-ion collisions

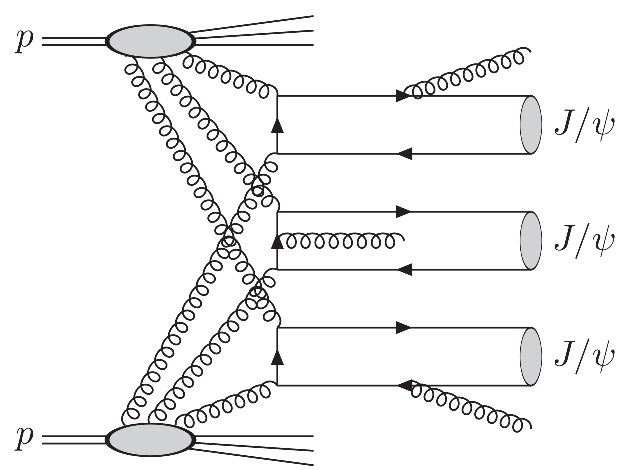
Triple Parton Scattering in pp



SPS



DPS



TPS

Triple Parton Scattering in pp

- **Analogously, ignoring the parton correlations, the NPS pocket formula:**

[D. d'Enterria, A. Snigirev (1708.07519)]

$$\sigma^{\mathrm{NPS}}_{f_1\cdots f_N} = \frac{m}{N!}\frac{\prod_{i=1}^{N}\sigma^{\mathrm{SPS}}_{f_i}}{\left(\sigma_{\mathrm{eff},N}\right)^{N-1}}$$

- **A pure geometric consideration leads to**

[D. d'Enterria, A. Snigirev (PRL'17)]

$$\sigma_{\mathrm{eff},3} = (0.82 \pm 0.11) \times \sigma_{\mathrm{eff},2}$$

- **In general, the inclusive cross sections scale as**

$$\sigma_{\mathrm{SPS}} \sim \frac{1}{Q^2} \quad \textbf{v.s.} \quad \sigma_{\mathrm{DPS}} \sim \frac{\Lambda^2_{\mathrm{QCD}}}{Q^4} \quad \textbf{v.s.} \quad \sigma_{\mathrm{TPS}} \sim \frac{\Lambda^4_{\mathrm{QCD}}}{Q^6}$$

Triple Parton Scattering in pp

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$$\sigma_{\rm SPS} \sim \frac{1}{Q^2} \quad \text{v.s.} \quad \sigma_{\rm DPS} \sim \frac{\Lambda^2_{\rm QCD}}{Q^4} \quad \text{v.s.} \quad \sigma_{\rm TPS} \sim \frac{\Lambda^4_{\rm QCD}}{Q^6}$$

- **Any chance to see TPS at the LHC ?**

Triple Parton Scattering in pp

- **Analogously, ignoring the parton correlations, the NPS pocket formula:**

[D. d'Enterria, A. Snigirev (1708.07519)]

$$\sigma^{\mathrm{NPS}}_{f_1\cdots f_N} = \frac{m}{N!}\frac{\prod_{i=1}^{N}\sigma^{\mathrm{SPS}}_{f_i}}{\left(\sigma_{\mathrm{eff},N}\right)^{N-1}}$$

- **A pure geometric consideration leads to**

[D. d'Enterria, A. Snigirev (PRL'17)]

$$\sigma_{\mathrm{eff},3} = (0.82 \pm 0.11) \times \sigma_{\mathrm{eff},2}$$

- **In general, the inclusive cross sections scale as**

$$\sigma_{\mathrm{SPS}} \sim \frac{1}{Q^2} \quad \text{v.s.} \quad \sigma_{\mathrm{DPS}} \sim \frac{\Lambda^2_{\mathrm{QCD}}}{Q^4} \quad \text{v.s.} \quad \sigma_{\mathrm{TPS}} \sim \frac{\Lambda^4_{\mathrm{QCD}}}{Q^6}$$

- **Any chance to see TPS at the LHC ?**

⇨ $J/\psi J/\psi J/\psi \to \mu^+\mu^-\mu^+\mu^-\mu^+\mu^-$

Triple Parton Scattering in pp

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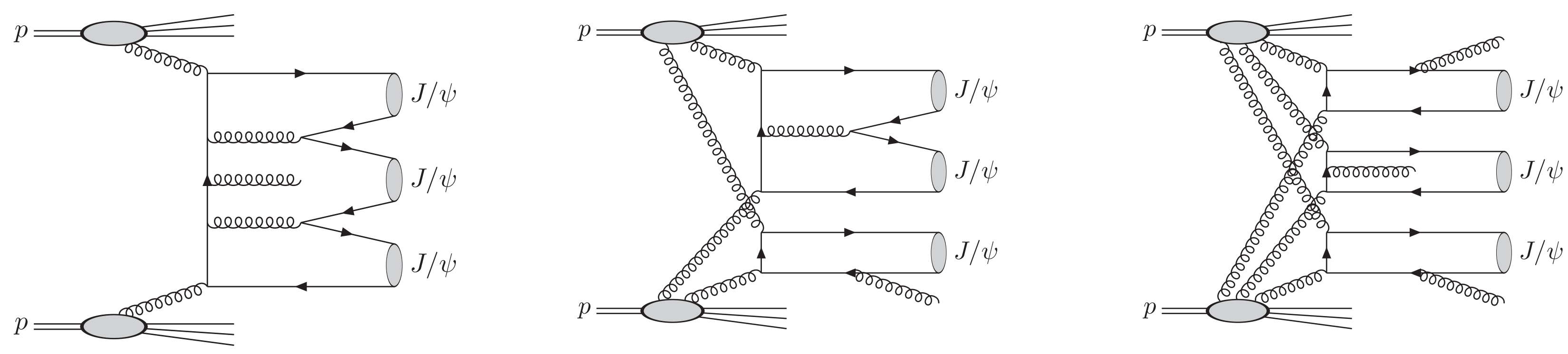
- **A first complete study of prompt triple J/psi as a probe of TPS**

[HSS, Zhang (PRL'19)]



SPS DPS TPS

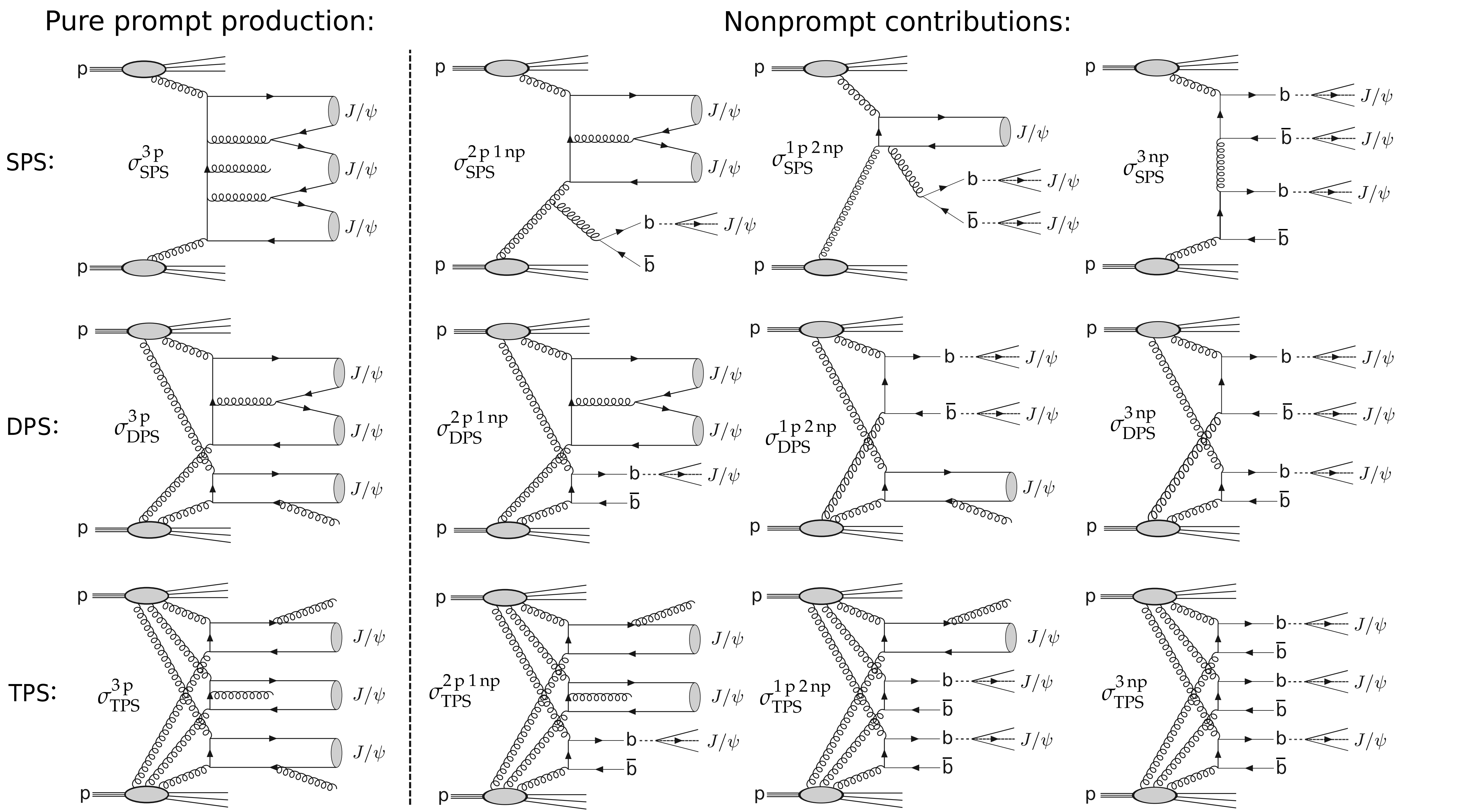
		inclusive	$2.0 < y_{J/\psi} < 4.5$	$\lvert y_{J/\psi}\rvert < 2.4$
13 TeV	SPS	$0.41^{+2.4}_{-0.34} \pm 0.0083$	$(1.8^{+11}_{-1.5} \pm 0.18) \times 10^{-2}$	$(8.7^{+56}_{-7.5} \pm 0.098) \times 10^{-2}$
	DPS	$(190^{+501}_{-140}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(7.0^{+18}_{-5.1}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(50^{+140}_{-37}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$130 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$1.3 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$18 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
27 TeV	SPS	$0.46^{+2.9}_{-0.39} \pm 0.022$	$(3.2^{+22}_{-2.8} \pm 0.21) \times 10^{-2}$	$(5.8^{+39}_{-5.1} \pm 0.29) \times 10^{-2}$
	DPS	$(560^{+2900}_{-480}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(19^{+97}_{-16}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(120^{+630}_{-100}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$570 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$5.0 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$57 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
75 TeV	SPS	$0.59^{+4.4}_{-0.52} \pm 0.016$	$(3.0^{+25}_{-2.7} \pm 0.23) \times 10^{-2}$	$(7.2^{+63}_{-6.5} \pm 0.38) \times 10^{-2}$
	DPS	$(1900^{+11000}_{-1600}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(57^{+340}_{-50}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(310^{+2000}_{-270}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$3900 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$27 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$260 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$
100 TeV	SPS	$1.1^{+8.4}_{-1.0} \pm 0.044$	$(4.5^{+33}_{-4.0} \pm 0.72) \times 10^{-2}$	$(36^{+290}_{-32} \pm 1.8) \times 10^{-2}$
	DPS	$(3400^{+19000}_{-2900}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(100^{+550}_{-86}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$	$(490^{+3000}_{-430}) \times \frac{10\text{ mb}}{\sigma_{\text{eff},2}}$
	TPS	$6500 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$45 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$	$380 \times \left(\frac{10\text{ mb}}{\sigma_{\text{eff},3}}\right)^2$

- With our knowledge of single J/psi and double J/psi, the process is predicted to be DPS and TPS dominant
- The number of events is large enough to be seen at the LHC unless $\sigma_{\text{eff},2}$ and $\sigma_{\text{eff},3}$ are significantly larger than 10 mb

Triple Parton Scattering in pp

- **First observation by CMS at 13 TeV in pp**

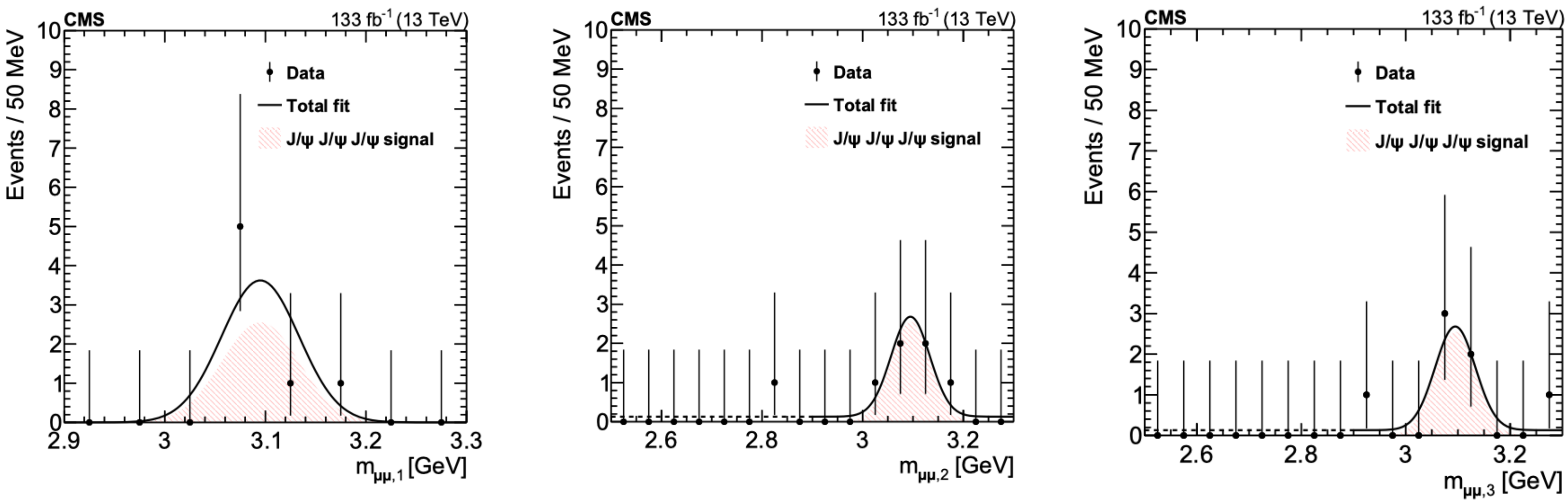
[CMS (Nature Physics'23)]



Triple Parton Scattering in pp

- **First observation by CMS at 13 TeV in pp**

[CMS (Nature Physics'23)]



- **Observation: 5 signal events + 1 background event**

- **The measurement of fiducial cross section**

$$\sigma(pp \to J/\psi J/\psi J/\psi X) = 272^{+141}_{-104}(\text{stat}) \pm 17(\text{syst})\ \text{fb}$$

Triple Parton Scattering in pp

- **Theoretical interpretation of the CMS measurement** [CMS (Nature Physics'23)]
  - Using the pocket formula, we need to know the following theoretical inputs

SPS single-J/ψ production		SPS double-J/ψ production			SPS triple-J/ψ production			
HO(DATA)	MG5NLO+PY8	HO(NLO*)	HO(LO)+PY8	MG5NLO+PY8	HO(LO)	HO(LO)+PY8	HO(LO)+PY8	MG5NLO+PY8
σ^{1p}_{SPS}	σ^{1np}_{SPS}	σ^{2p}_{SPS}	σ^{1p1np}_{SPS}	σ^{2np}_{SPS}	σ^{3p}_{SPS}	σ^{2p1np}_{SPS}	σ^{1p2np}_{SPS}	σ^{3np}_{SPS}
570 ± 57 nb	600^{+130}_{-220} nb	40^{+80}_{-26} pb	24^{+35}_{-16} fb	430^{+95}_{-130} pb	< 5 ab	$5.2^{+9.6}_{-3.3}$ fb	14^{+17}_{-8} ab	12 ± 4 fb

HO: *HELAC-Onia*

MG5NLO: *MadGraph5_aMC@NLO*

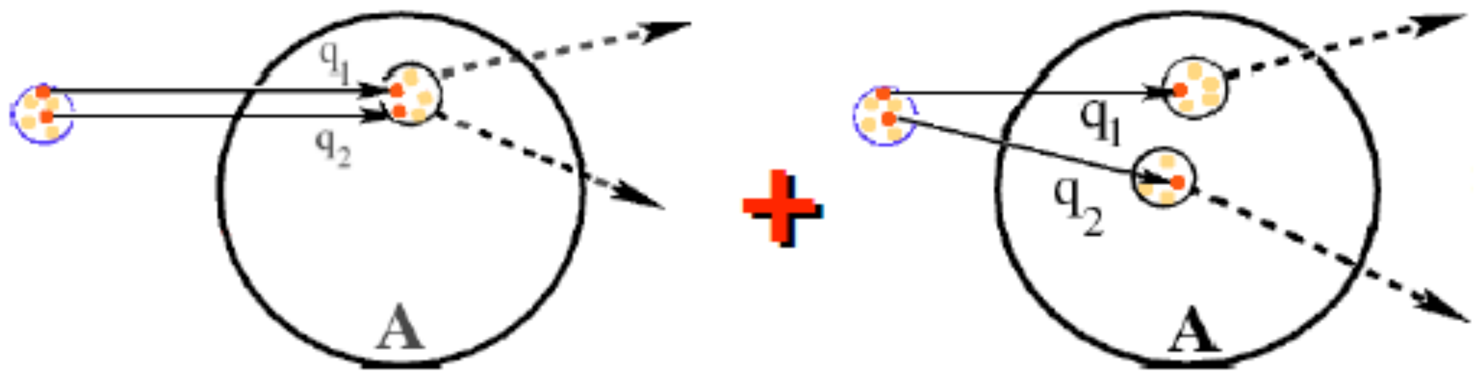
PY8: *Pythia8.2*

- Fixing $\sigma_{\mathrm{eff},3} = (0.82 \pm 0.11) \times \sigma_{\mathrm{eff},2}$ and fitting $\sigma_{\mathrm{eff},2}$

$$\sigma_{\mathrm{eff},2} = 2.7^{+1.4}_{-1.0}(\mathrm{exp})^{+1.5}_{-1.0}(\mathrm{theo})\ \mathrm{mb}$$

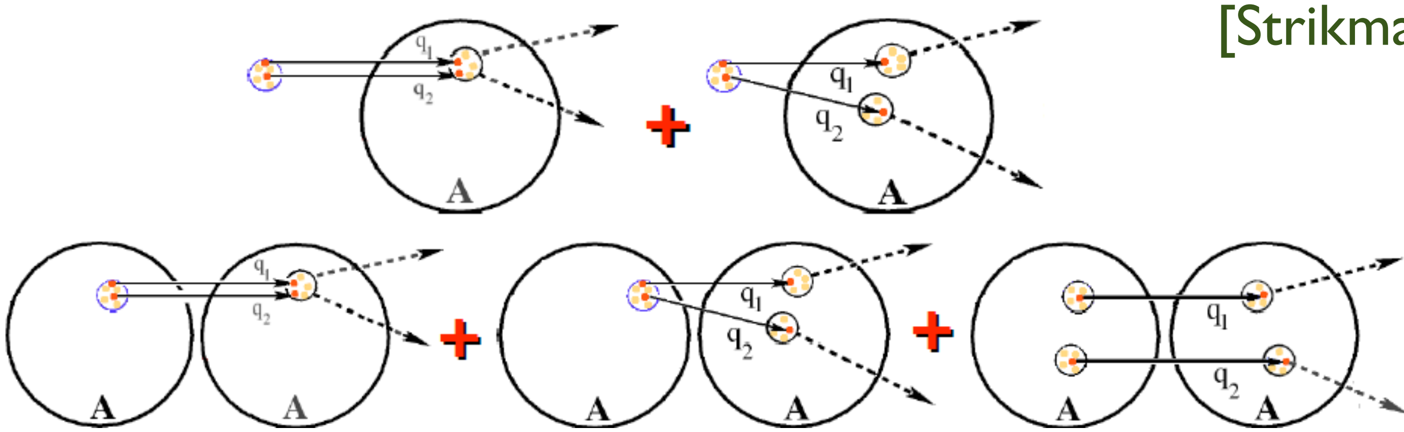
- Triple-J/psi fractions: ~6% SPS, ~74% DPS, ~20% TPS

DPS in heavy-ion collisions



DPS in Heavy-Ion Collisions

- **Geometrical enhancement because of several nucleons in a nucleus**



[Strikman, Treleani (2002); D. d'Enterria, A. Snigirev (2013, 2014)]

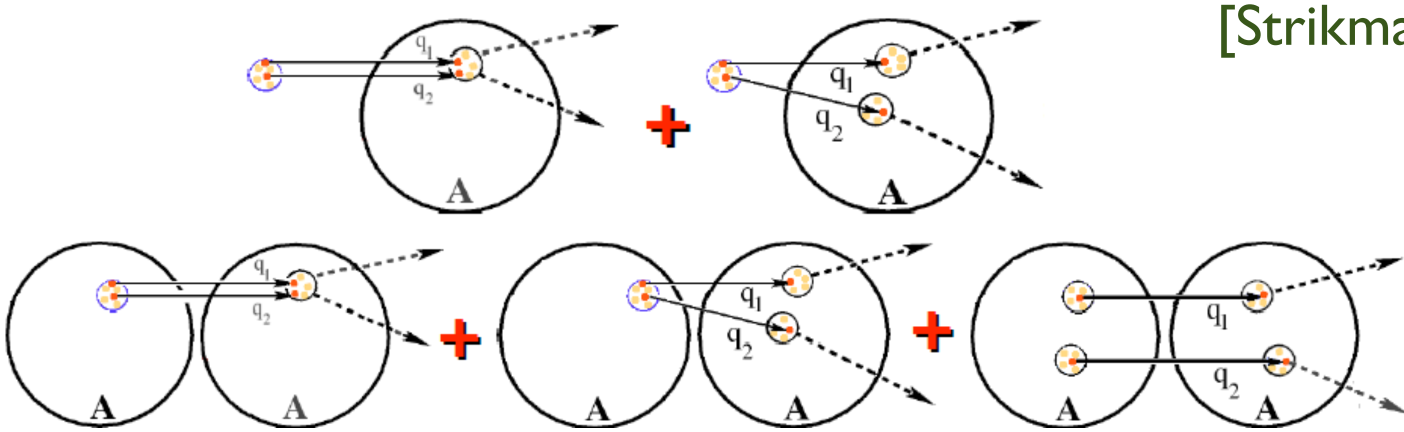
$$\sigma_{pA}^{\rm DPS} \approx 3A\sigma_{pp}^{\rm DPS}, \sigma_{pA}^{\rm SPS} \approx A\sigma_{pp}^{\rm SPS}$$

$$\sigma_{AA}^{\rm DPS} \approx \frac{A^{3.3}}{5}\sigma_{pp}^{\rm DPS}, \sigma_{AA}^{\rm SPS} \approx A^2\sigma_{pp}^{\rm SPS}$$

Assumptions: no nuclear modification and $\sigma_{\rm eff,\mathit{pp}} \simeq 15$ mb

DPS in Heavy-Ion Collisions

- **Geometrical enhancement because of several nucleons in a nucleus**

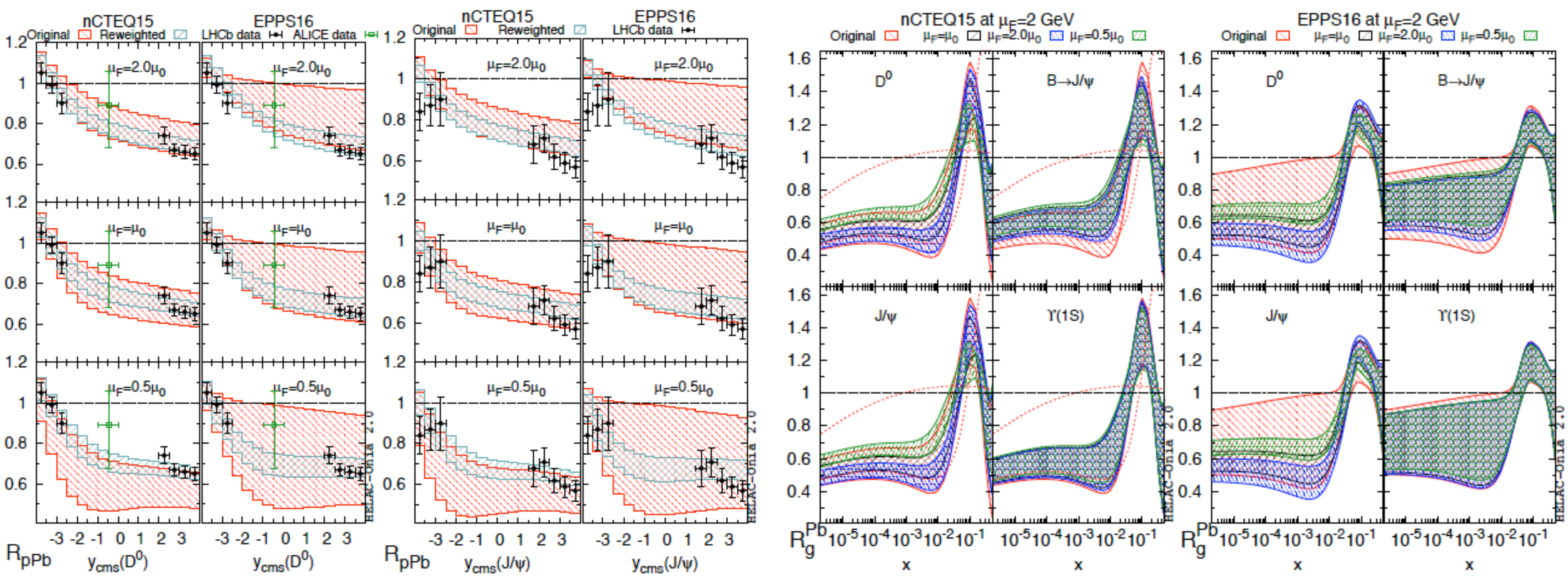


[Strikman, Treleani (2002); D. d'Enterria, A. Snigirev (2013, 2014)]

$$\sigma_{pA}^{\rm DPS} \approx 3A\sigma_{pp}^{\rm DPS}, \sigma_{pA}^{\rm SPS} \approx A\sigma_{pp}^{\rm SPS}$$

$$\sigma_{AA}^{\rm DPS} \approx \frac{A^{3.3}}{5}\sigma_{pp}^{\rm DPS}, \sigma_{AA}^{\rm SPS} \approx A^{2}\sigma_{pp}^{\rm SPS}$$

- **Of course, we know we cannot neglect the nuclear modifications …**
  - E.g. gluon (anti)shadowing for heavy flavour and quarkonia [Kusina, et al. (PRL'18)]

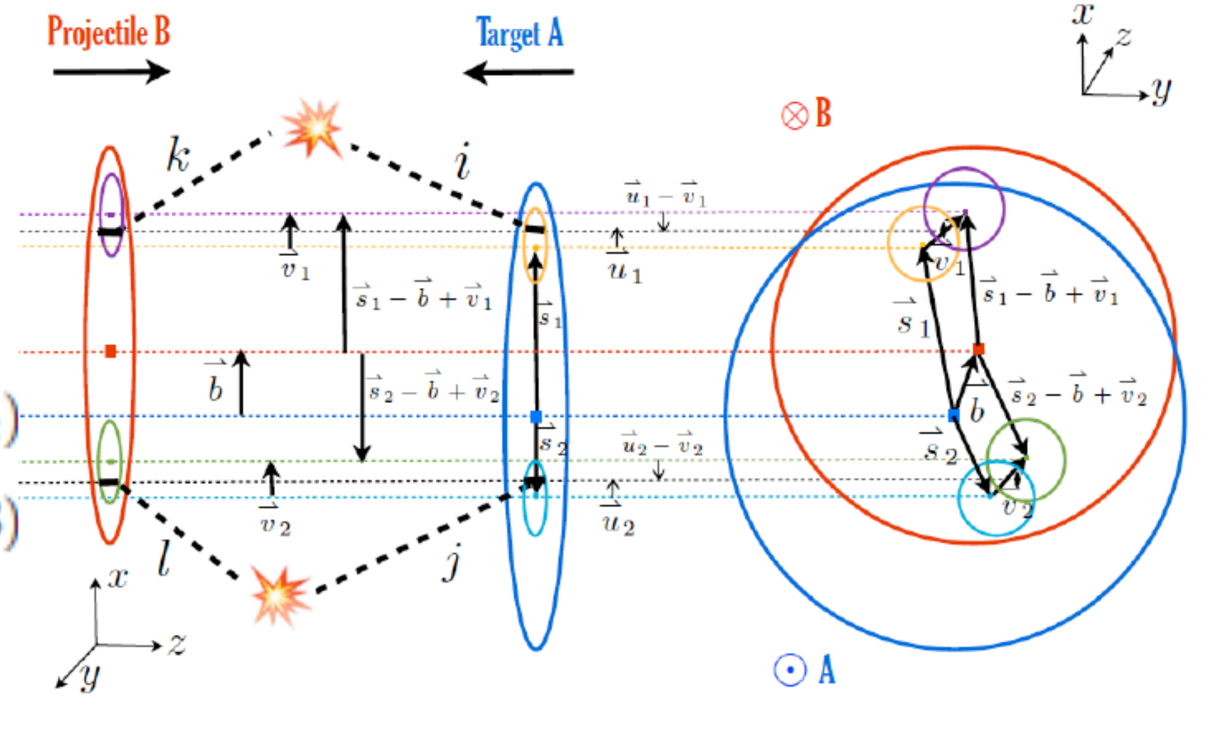


DPS in Heavy-Ion Collisions

- **Let us accommodate both nPDF and geometric effect** [HSS (PRD'20)]

$$\sigma_{Q_1Q_2} = \frac{1}{1+\delta_{Q_1Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx_1' dx_2' d^2\mathbf{b}_1 d^2\mathbf{b}_2 d^2\mathbf{b}$$
$$\times \Gamma_{ij}(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2)\, \hat{\sigma}^{Q_1}_{ik}(x_1, x_1')\, \hat{\sigma}^{Q_2}_{jl}(x_2, x_2')\, \Gamma_{kl}(x_1', x_2', \mathbf{b}_1 - \mathbf{b}, \mathbf{b}_2 - \mathbf{b}),$$

$$\sigma^{\rm DPS}_{AB\to f_1 f_2} = \frac{1}{1+\delta_{f_1 f_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx_1' dx_2'$$
$$\Gamma^{ij}_A(x_1, x_2, \vec{s}_1, \vec{s}_2, \vec{u}_1, \vec{u}_2)\hat{\sigma}^{f_1}_{ik}(x_1, x_1')\hat{\sigma}^{f_2}_{jl}(x_2, x_2') \times$$
$$\Gamma^{kl}_B(x_1', x_2', \vec{s}_1 - \vec{b} + \vec{v}_1, \vec{s}_2 - \vec{b} + \vec{v}_2, \vec{u}_1 - \vec{v}_1, \vec{u}_2 - \vec{v}_2)$$
$$d^2\vec{u}_1 d^2\vec{u}_2 d^2\vec{v}_1 d^2\vec{v}_2 d^2\vec{s}_1 d^2\vec{s}_2 d^2\vec{b}, \qquad (8)$$



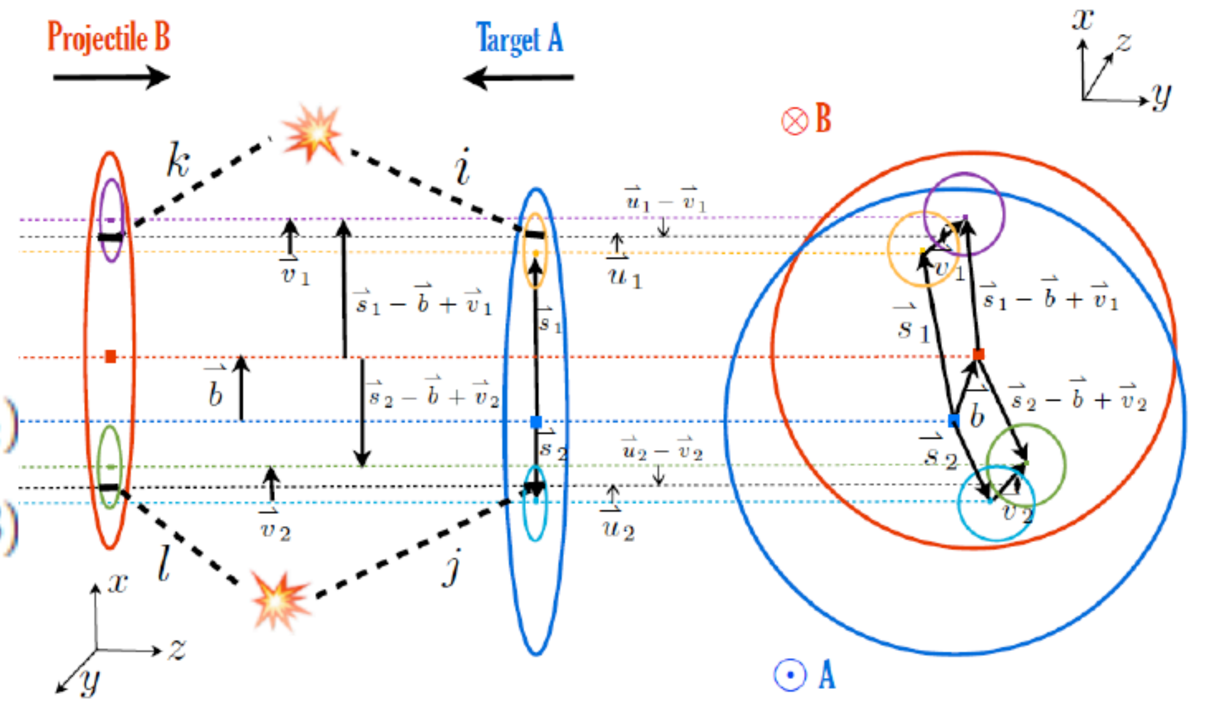
(a) Side view (b) Beam view

DPS in Heavy-Ion Collisions

- **Let us accommodate both nPDF and geometric effect** [HSS (PRD'20)]

$$\sigma_{Q_1Q_2} = \frac{1}{1+\delta_{Q_1Q_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2 d^2\mathbf{b}_1 d^2\mathbf{b}_2 d^2\mathbf{b} \times \Gamma_{ij}(x_1,x_2,\mathbf{b}_1,\mathbf{b}_2)\,\hat{\sigma}^{Q_1}_{ik}(x_1,x'_1)\,\hat{\sigma}^{Q_2}_{jl}(x_2,x'_2)\,\Gamma_{kl}(x'_1,x'_2,\mathbf{b}_1-\mathbf{b},\mathbf{b}_2-\mathbf{b}),$$

$$\sigma^{\rm DPS}_{AB\to f_1f_2} = \frac{1}{1+\delta_{f_1f_2}} \sum_{i,j,k,l} \int dx_1 dx_2 dx'_1 dx'_2\, \Gamma^{ij}_A(x_1,x_2,\vec{s}_1,\vec{s}_2,\vec{u}_1,\vec{u}_2)\hat{\sigma}^{f_1}_{ik}(x_1,x'_1)\hat{\sigma}^{f_2}_{jl}(x_2,x'_2)\times \Gamma^{kl}_B(x'_1,x'_2,\vec{s}_1-\vec{b}+\vec{v}_1,\vec{s}_2-\vec{b}+\vec{v}_2,\vec{u}_1-\vec{v}_1,\vec{u}_2-\vec{v}_2)\, d^2\vec{u}_1 d^2\vec{u}_2 d^2\vec{v}_1 d^2\vec{v}_2 d^2\vec{s}_1 d^2\vec{s}_2 d^2\vec{b}, \quad (8)$$



- **We also need the knowledge of nuclear modification at different positions**

$$R^A_k(x,\vec{b})-1 = \left(R^A_k(x)-1\right) G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \qquad k=g,q,\bar{q}$$

  - Ubiquitous in centrality-dependent observables
  - ...but mostly assume $G\left(\frac{T_A(\vec{s})}{T_A(\vec{0})}\right) = \frac{AT_A(\vec{s})}{T_{AA}(\vec{0})}$

DPS in Heavy-Ion Collisions

- **For example, considering** $p\mathrm{Pb} \to D^0 D^0 X$ [HSS (PRD'20)]

$$R^{\mathrm{DPS}}_{p\mathrm{Pb}\to D^0+D^0} = R^{D^0}_{p\mathrm{Pb}} R^{D^0}_{p\mathrm{Pb}} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right]$$

$$+\left(R^{D^0}_{p\mathrm{Pb}} + R^{D^0}_{p\mathrm{Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$

$$+\left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^a}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

$$R^f_{pA} \equiv \frac{\sigma_{pA\to f}}{A\sigma_{pp\to f}}$$

DPS in Heavy-Ion Collisions

- **For example, considering** $p\mathrm{Pb} \to D^0 D^0 X$ [HSS (PRD'20)]

$$R^{\mathrm{DPS}}_{p\mathrm{Pb}\to D^0+D^0} = R^{D^0}_{p\mathrm{Pb}} R^{D^0}_{p\mathrm{Pb}} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right]$$
$$+ \left(R^{D^0}_{p\mathrm{Pb}} + R^{D^0}_{p\mathrm{Pb}} \right) \left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^{a}}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} \right) \right]$$
$$+ \left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^{a}}{(a+4)} \right) \right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

Either calculable or fixable by other measurements

$$R^f_{pA} \equiv \frac{\sigma_{pA\to f}}{A\sigma_{pp\to f}}$$

DPS in Heavy-Ion Collisions

- **For example, considering** $p\mathrm{Pb} \to D^0 D^0 X$ [HSS (PRD'20)]

$$R^{\mathrm{DPS}}_{p\mathrm{Pb}\to D^0+D^0} = R^{D^0}_{p\mathrm{Pb}} R^{D^0}_{p\mathrm{Pb}} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right]$$

$$+ \left(R^{D^0}_{p\mathrm{Pb}} + R^{D^0}_{p\mathrm{Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$

$$+ \left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^a}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

$$\frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1) \simeq 5.23\left(\frac{\sigma_{\mathrm{eff},pp}}{34.8\ \mathrm{mb}}\right)$$

$$R^f_{pA} \equiv \frac{\sigma_{pA\to f}}{A\sigma_{pp\to f}}$$

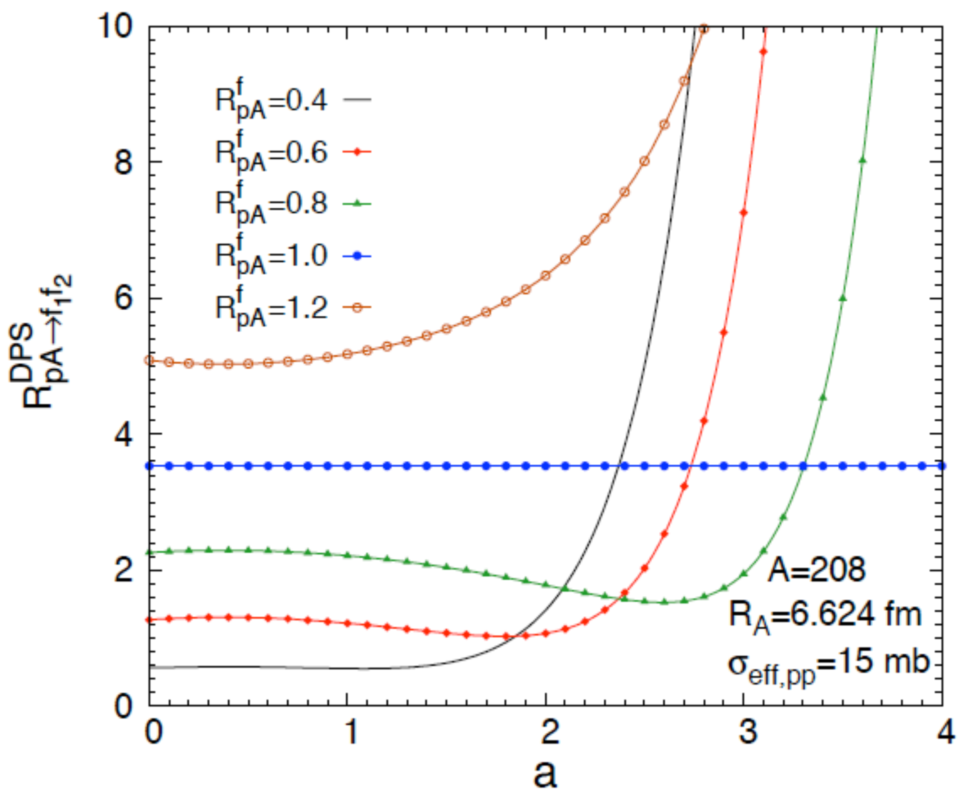
DPS in Heavy-Ion Collisions

- **For example, considering** $pPb \to D^0 D^0 X$ [HSS (PRD'20)]

$$R^{\rm DPS}_{p{\rm Pb}\to D^0+D^0} = R^{D^0}_{p{\rm Pb}} R^{D^0}_{p{\rm Pb}} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{{\rm eff},pp}}{\pi R_A^2}(A-1)\frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right]$$
$$+\left(R^{D^0}_{p{\rm Pb}} + R^{D^0}_{p{\rm Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{{\rm eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^{a}}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$
$$+\left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{{\rm eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^{a}}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

$$\frac{\sigma_{{\rm eff},pp}}{\pi R_A^2}(A-1) \simeq 5.23\left(\frac{\sigma_{{\rm eff},pp}}{34.8\ {\rm mb}}\right)$$



$$R^f_{pA} \equiv \frac{\sigma_{pA\to f}}{A\sigma_{pp\to f}}$$

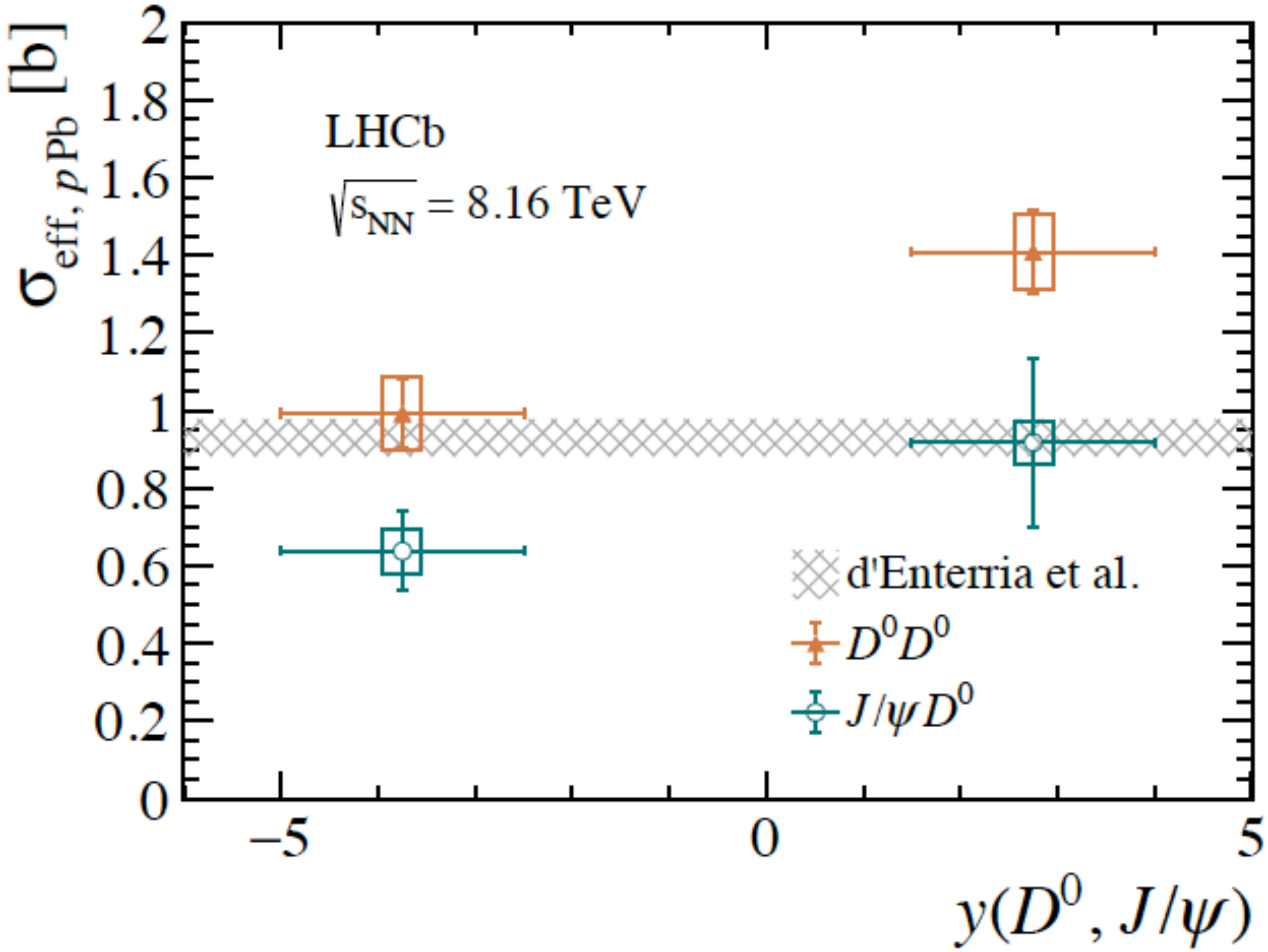
- DPS in heavy-ion potential to constrain G() !

DPS in Heavy-Ion Collisions

- **First DPS measurement in heavy-ion collisions by LHCb** [LHCb (PRL'20)]

$$\sigma_{\rm eff} = \frac{1}{1+\delta_{f_1 f_2}} \frac{\sigma_{p{\rm Pb}\to f_1}\sigma_{p{\rm Pb}\to f_2}}{\sigma_{p{\rm Pb}\to f_1 f_2}}$$

Caveats: $\sigma_{\rm eff} \equiv \sigma_{{\rm eff},p{\rm Pb}} \neq \sigma_{{\rm eff},pp}$

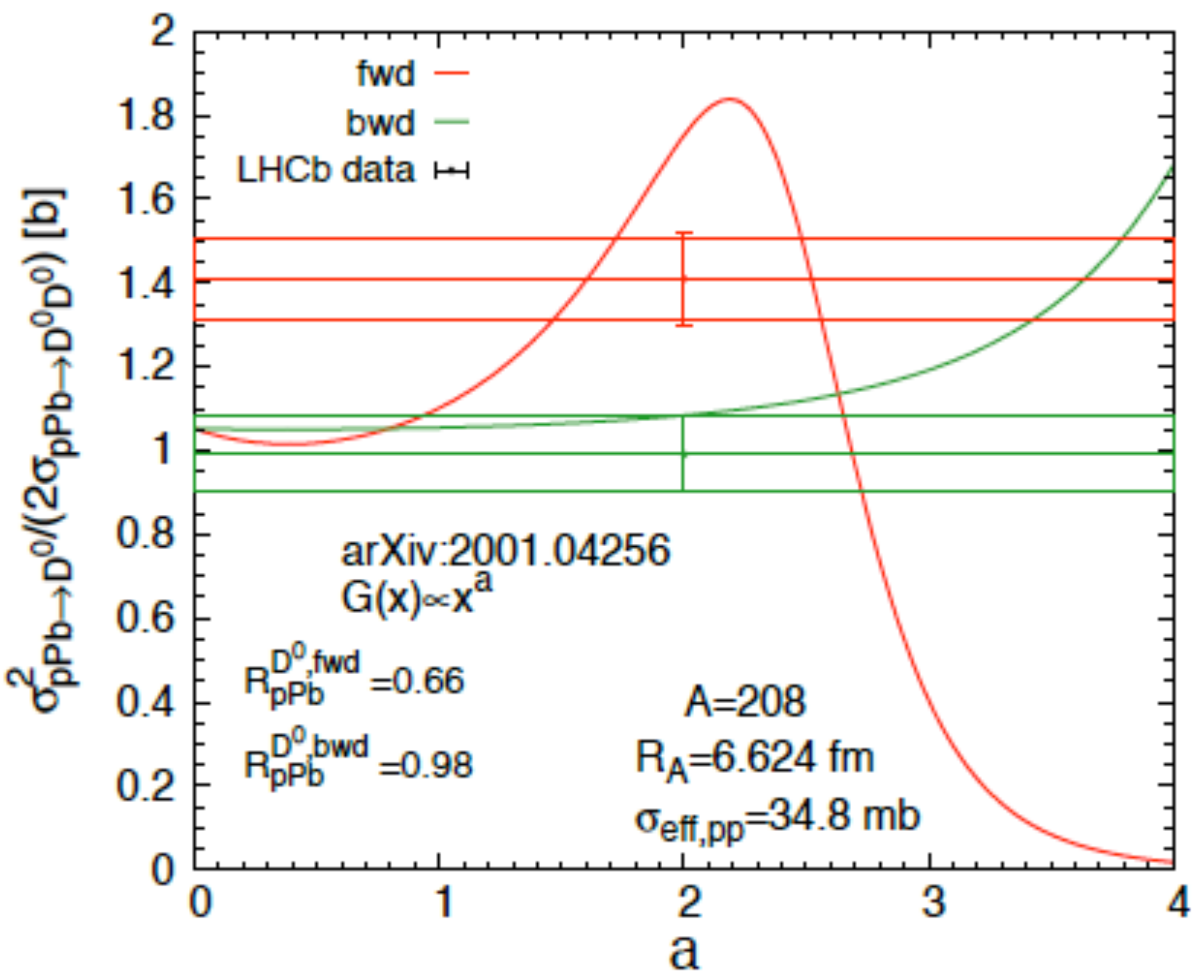


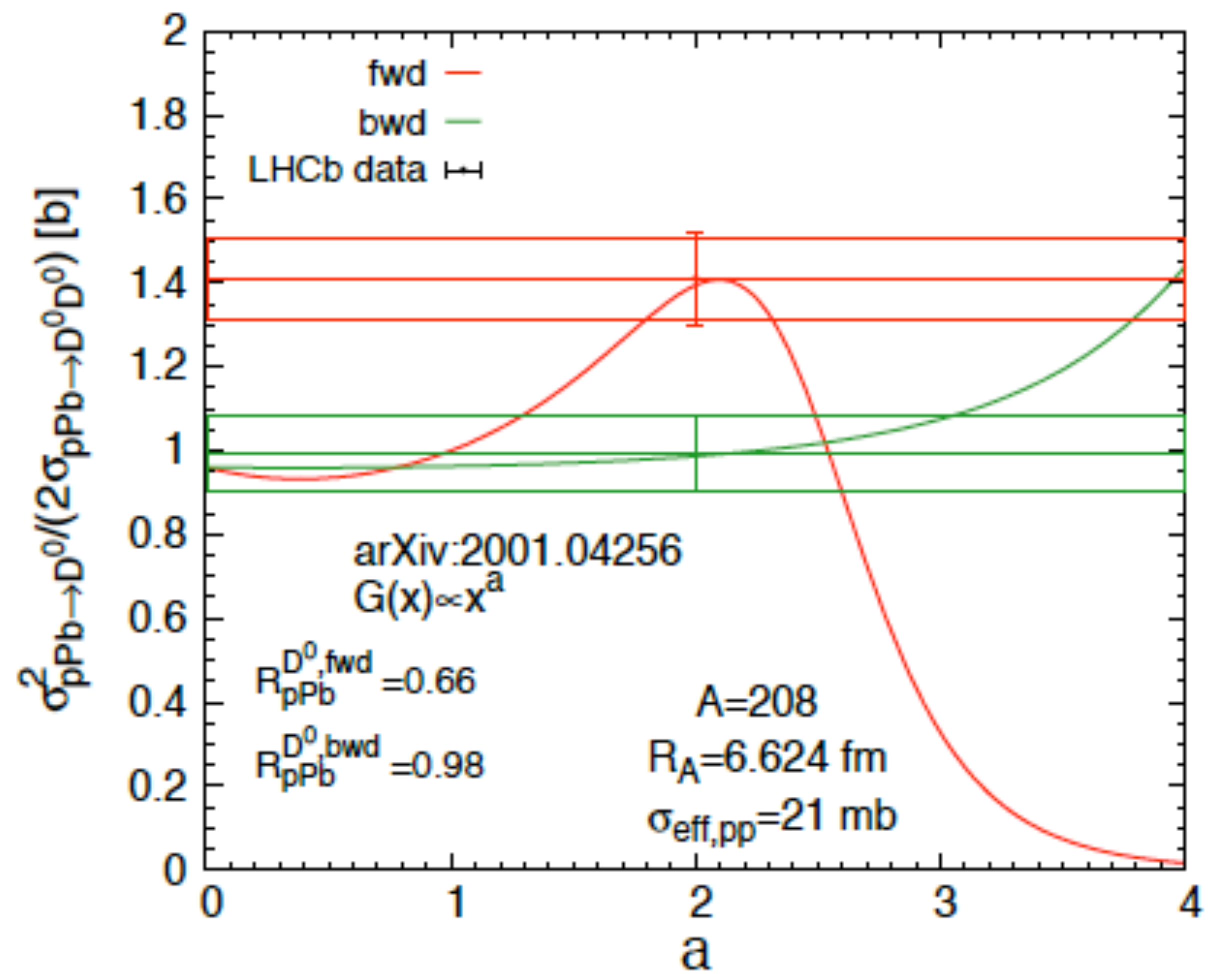
Only geometric effect

- Observe ~3A enhancement in DPS wrt ~A enhancement in SPS by comparing pA vs pp xs
- The pure geometric effect cannot explain the rapidity dependence

DPS in Heavy-Ion Collisions

- **Theoretical interpretation of the LHCb measurement**
  - J/psi+D^0 has the sizable SPS component [HSS (PRD'20)]
  - The SPS of D^0+D^0 is negligible in NLO pQCD caclulations [Helenius, Paukkunen (PLB'20)]
  - The b-dependent gluon shadowing can explain the rapidity dependence [HSS (PRD'20)]



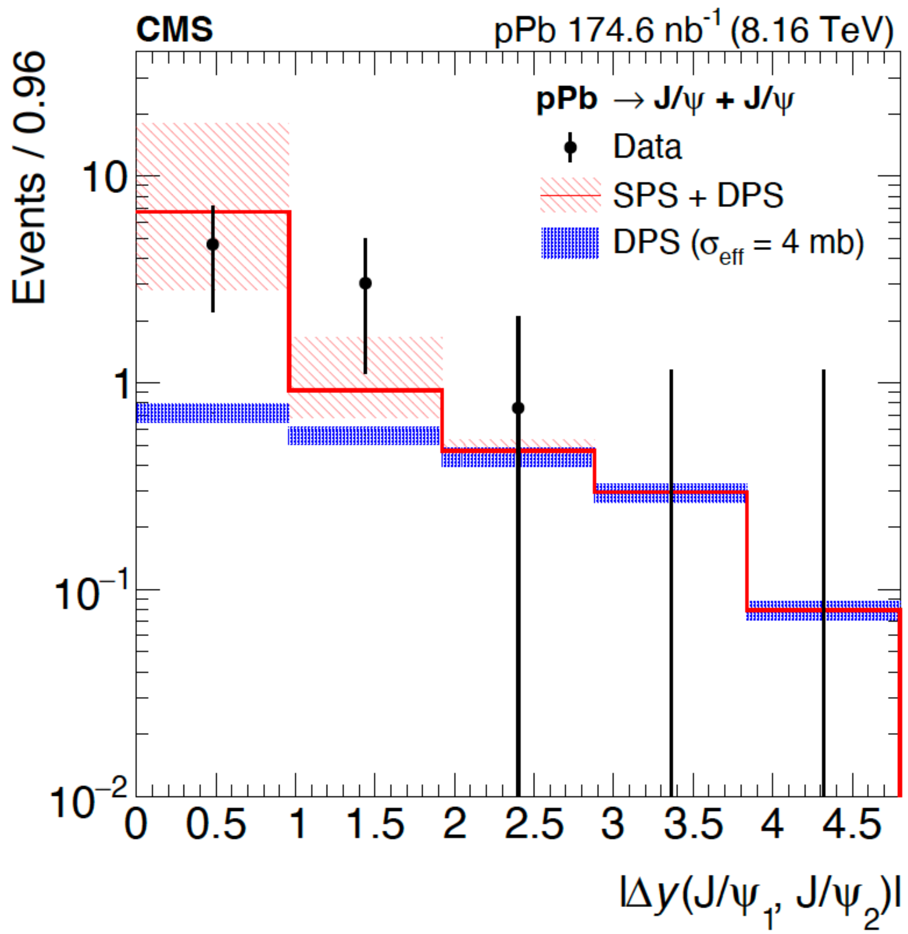


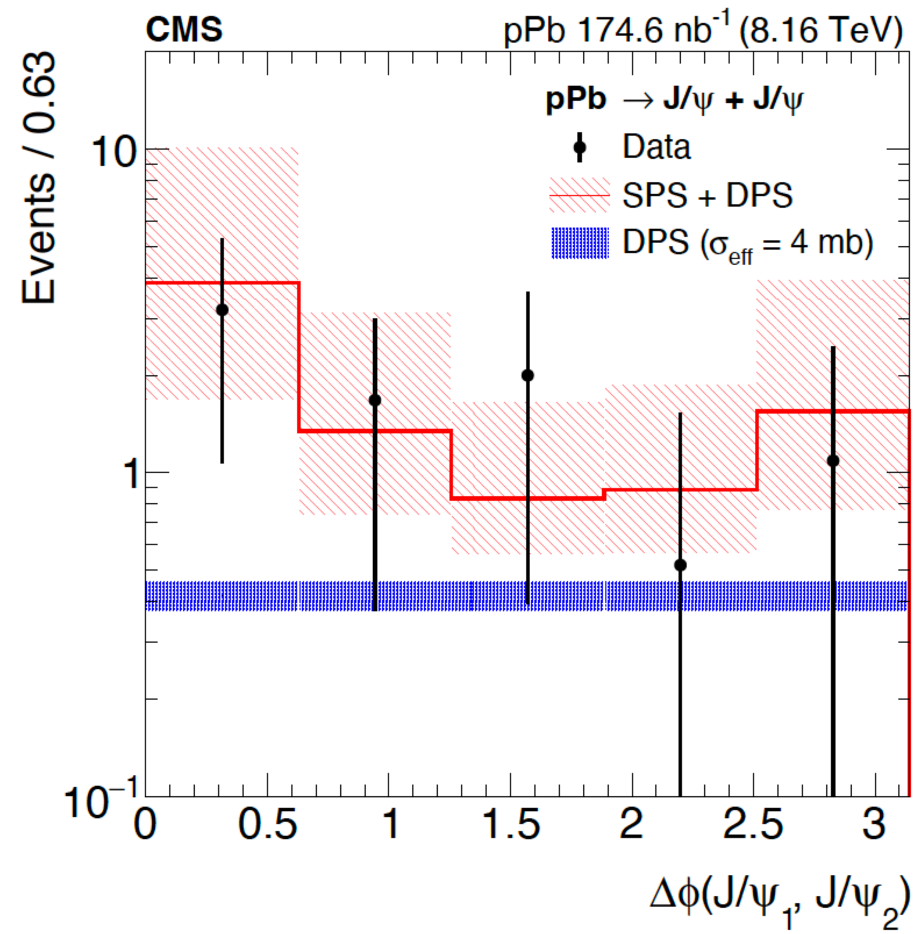
$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

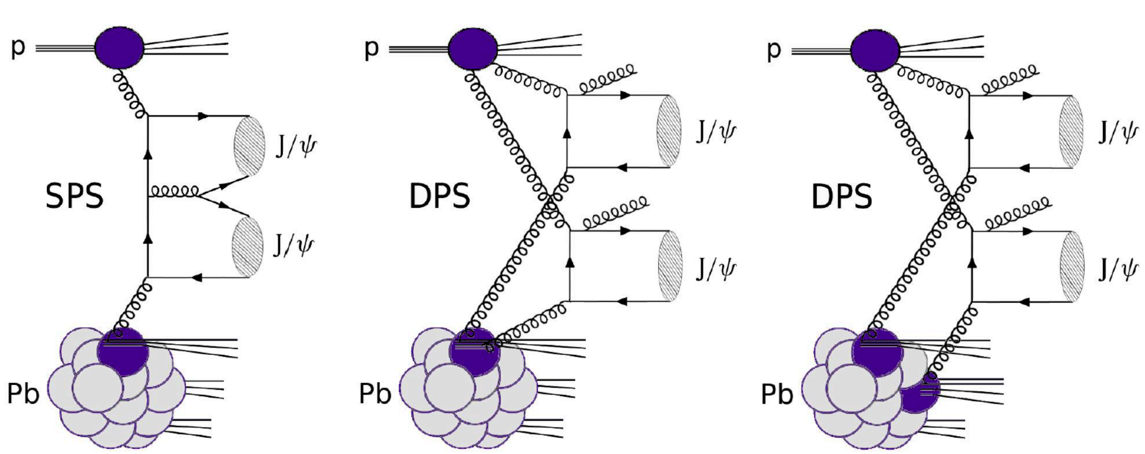
favoring a~2 and disfavoring a~1

DPS in Heavy-Ion Collisions

- **DPS measurement in heavy-ion collisions by CMS** [CMS (arXiv:2407.03223)]



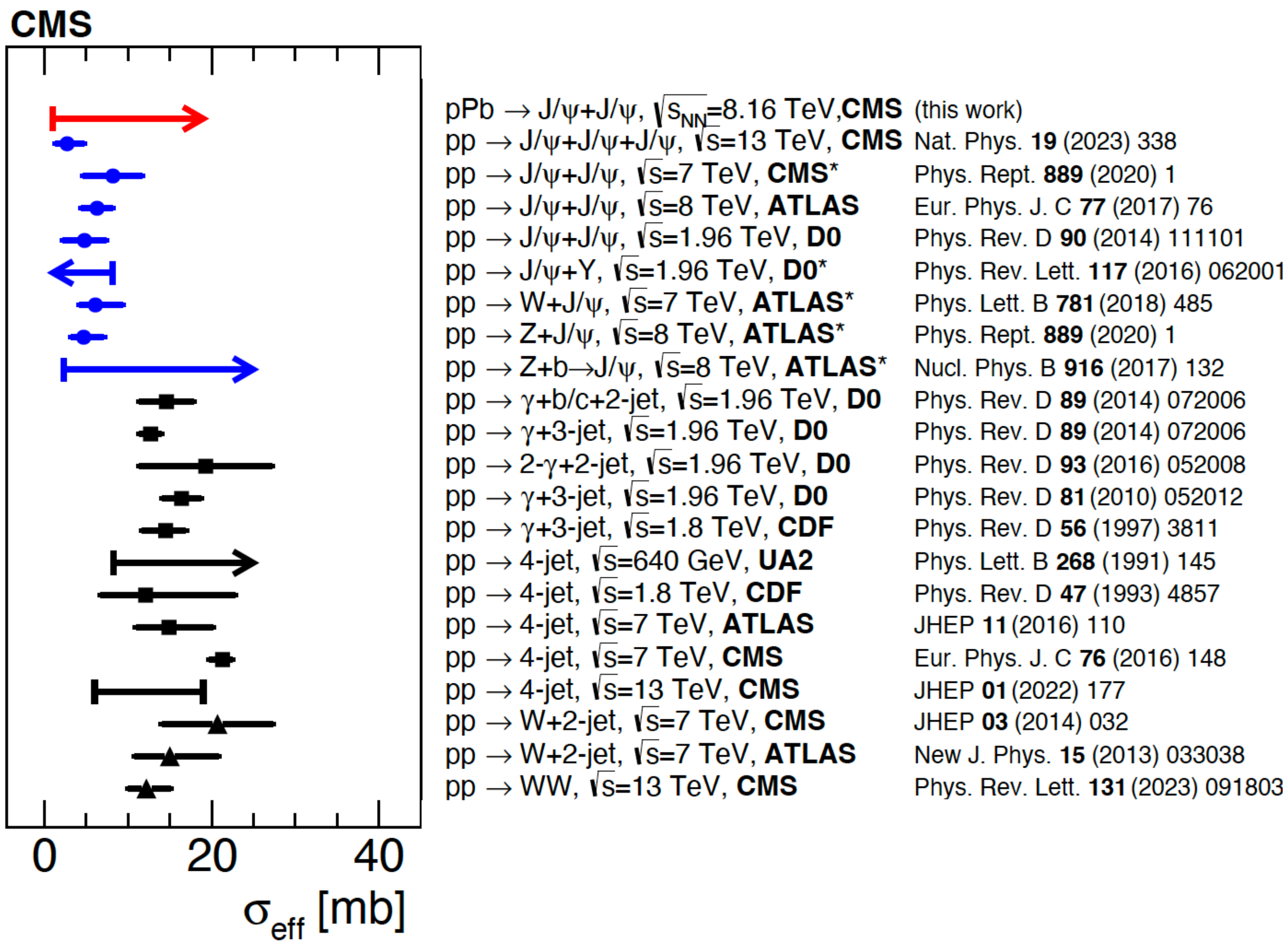


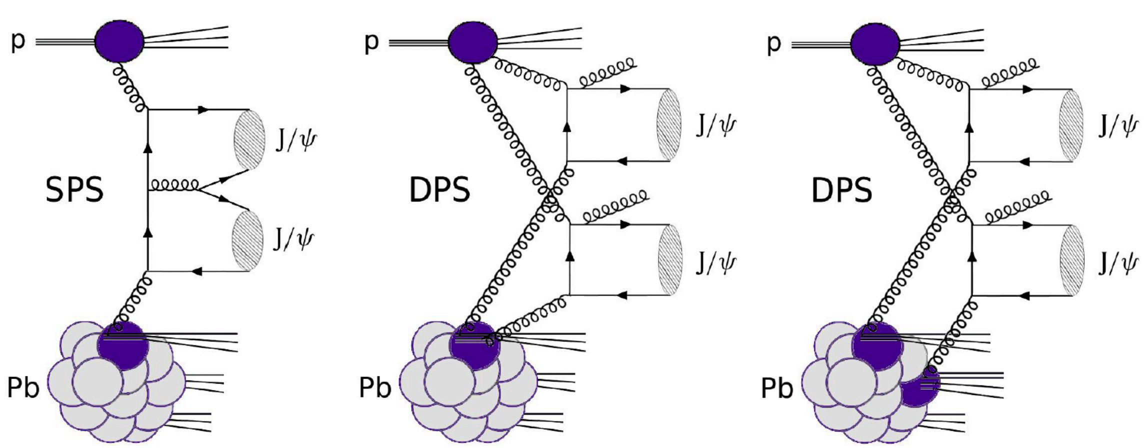


- **No enough statistics to extract DPS**

Particle	Fiducial requirement
Muons	$p_T > 3.4$ GeV for $\lvert\eta\rvert < 0.3$
	$p_T > 3.3$ GeV for $0.3 < \lvert\eta\rvert < 1.1$
	$p_T > 5.5–2.0\lvert\eta\rvert$ GeV for $1.1 < \lvert\eta\rvert < 2.1$
	$p_T > 1.3$ GeV for $2.1 < \lvert\eta\rvert < 2.4$
J/ψ mesons	$p_T > 6.5$ GeV and $\lvert y\rvert < 2.4$

- **DPS measurement in heavy-ion collisions by CMS** [CMS (arXiv:2407.03223)]





- **No enough statistics to extract DPS**

Particle	Fiducial requirement
Muons	$p_T > 3.4\,\text{GeV}$ for $\lvert\eta\rvert < 0.3$
	$p_T > 3.3\,\text{GeV}$ for $0.3 < \lvert\eta\rvert < 1.1$
	$p_T > 5.5–2.0\lvert\eta\rvert\,\text{GeV}$ for $1.1 < \lvert\eta\rvert < 2.1$
	$p_T > 1.3\,\text{GeV}$ for $2.1 < \lvert\eta\rvert < 2.4$
J/ψ mesons	$p_T > 6.5\,\text{GeV}$ and $\lvert y\rvert < 2.4$

DPS in Heavy-Ion Collisions

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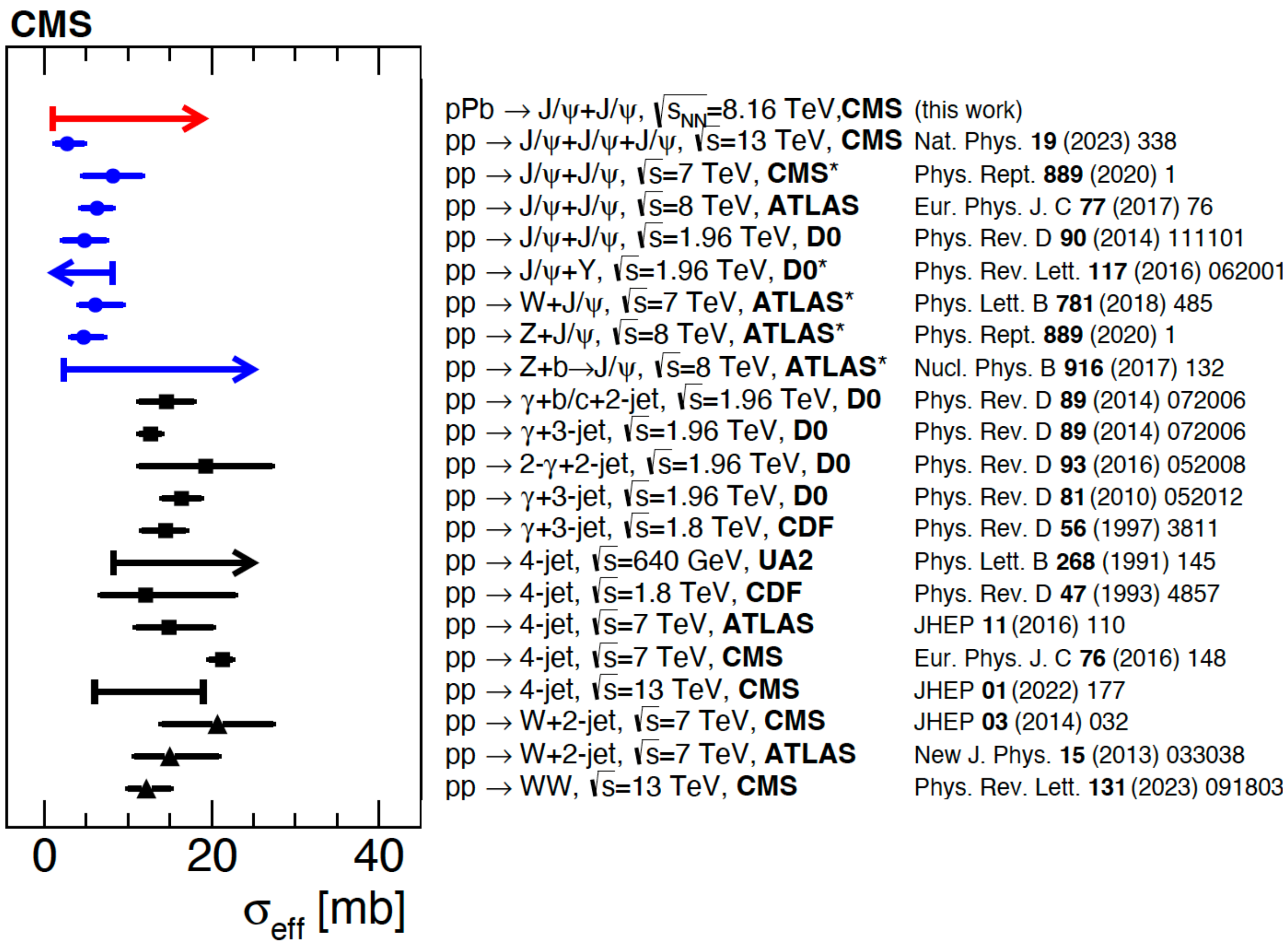
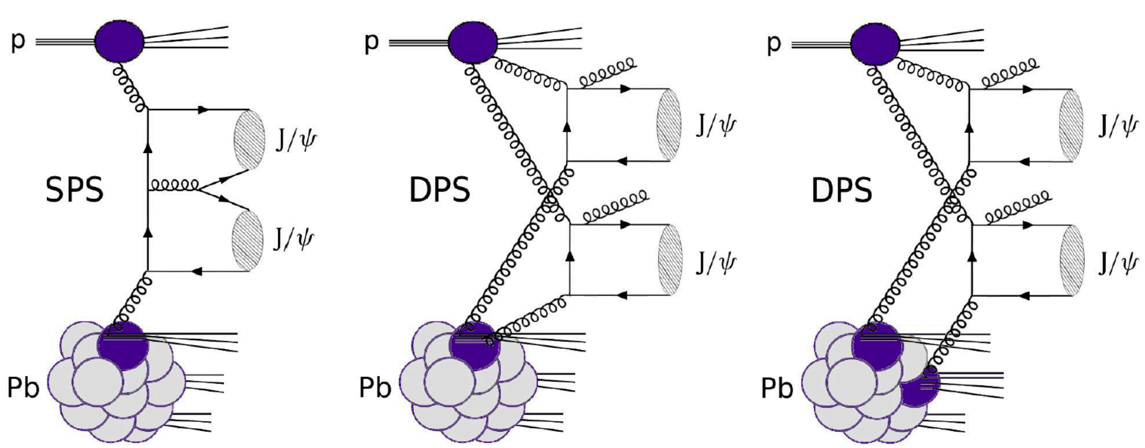


LHCb ?



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Particle	Fiducial requirement
Muons	$p_T > 3.4$ GeV for $\lvert\eta\rvert < 0.3$
	$p_T > 3.3$ GeV for $0.3 < \lvert\eta\rvert < 1.1$
	$p_T > 5.5$–$2.0\lvert\eta\rvert$ GeV for $1.1 < \lvert\eta\rvert < 2.1$
	$p_T > 1.3$ GeV for $2.1 < \lvert\eta\rvert < 2.4$
J/ψ mesons	$p_T > 6.5$ GeV and $\lvert y\rvert < 2.4$

Conclusion

- **LHC program offers an unprecedented avenue to study DPS & TPS.**
- **A lot of theoretical, phenomenological and experimental progress.**
- **MPS will reveal the first-ever multiple-body parton correlations in nucleon and nucleus** ⟶ such as $\sigma_{\rm eff}(p_T)$
- **Some novel observables can even tell us more (e.g. the impact parameter-dependent gluon shadowing)**
- **Don't be shy to attempt a 1st-ever measurement (e.g. TPS in pPb or DPS in PbPb ?)**

Conclusion

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Thank you for your attention !

Backup Slides

A Brief Introduction

- **How to probe DPS at the LHC ?**

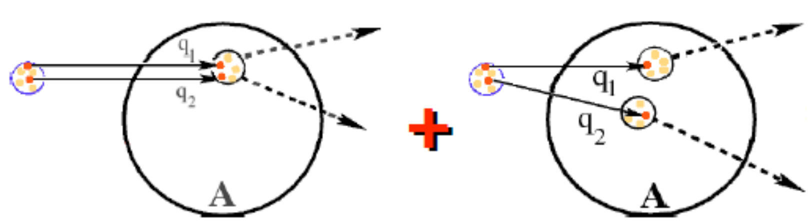
- **Processes of low hard scale Q (but still in the perturbative regime)**
  - multiply hadron production, e.g. $J/\psi + J/\psi$

- **Processes of large yields**
  - multi-jet production

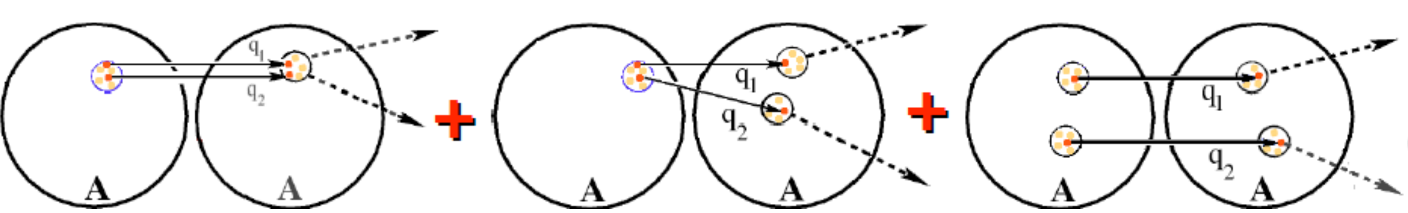
- **Processes of precision measurements**
  - multi-lepton production

- **Enhancement of parton luminosity**
  - higher energy [8 TeV to 14 TeV to 100 TeV (FCC)]
  - probe in proton-nucleus and nucleus-nucleus collisions

[Strikman, Treleani (2002); D. d'Enterria, A. Snigirev (2013, 2014)]



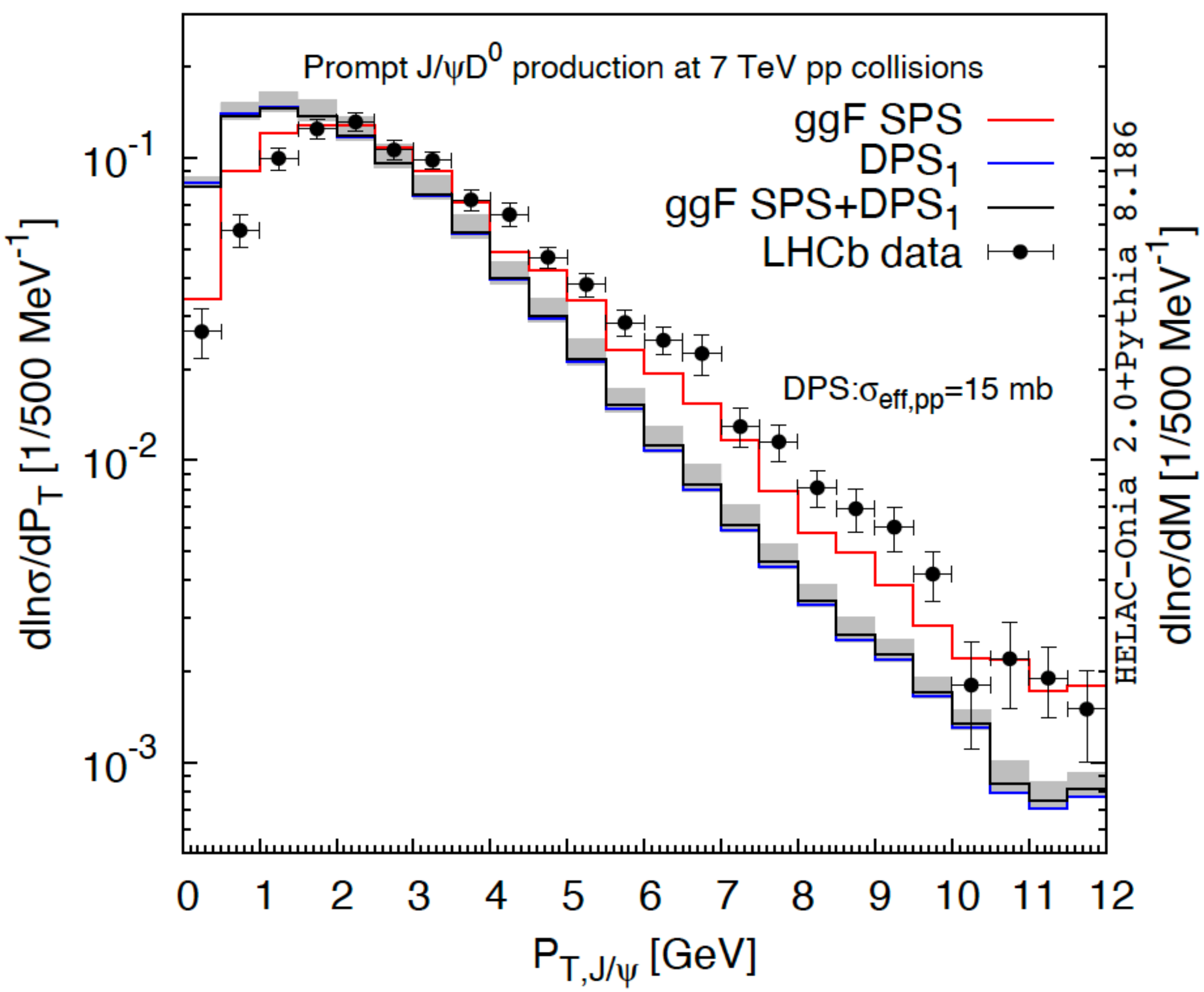
$$\sigma_{\mathrm{eff,pp}}/\sigma_{\mathrm{eff,pA}} \approx 3\,A \approx 600$$



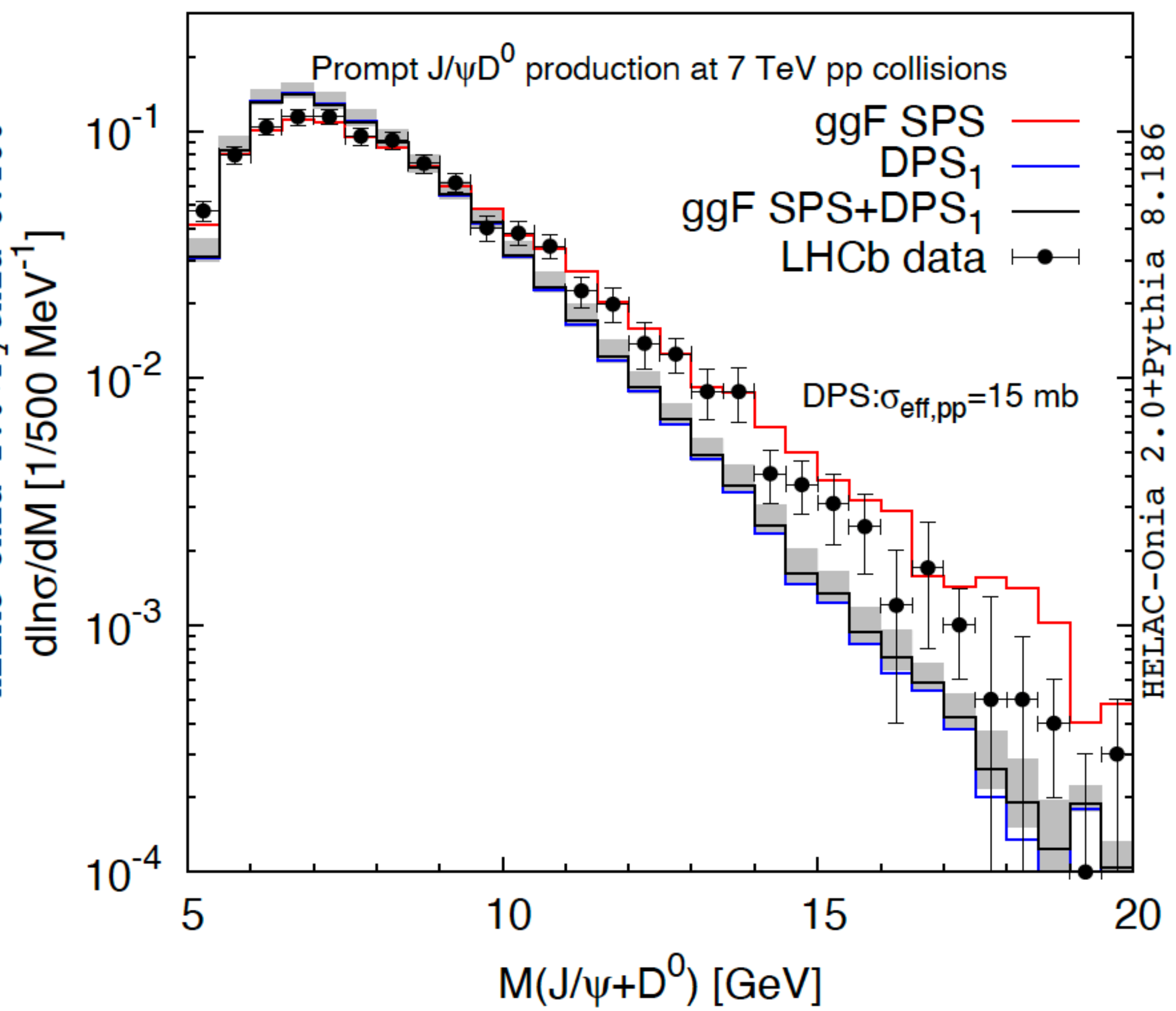
$$\sigma_{\mathrm{eff,pp}}/\sigma_{\mathrm{eff,AA}} \propto \mathrm{A}^{3.3}/5 \simeq 9 \cdot 10^6$$

J/psi+open charm

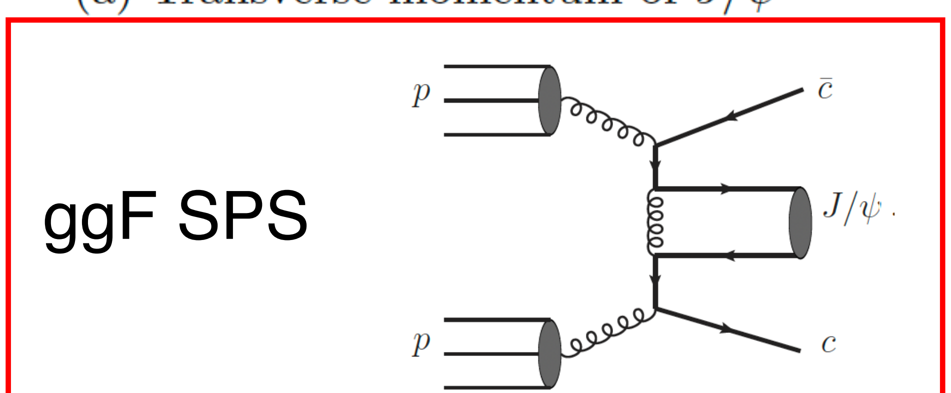
- **J/psi+open charm by LHCb cannot explain by DPS alone** [HSS (PRD'20)]

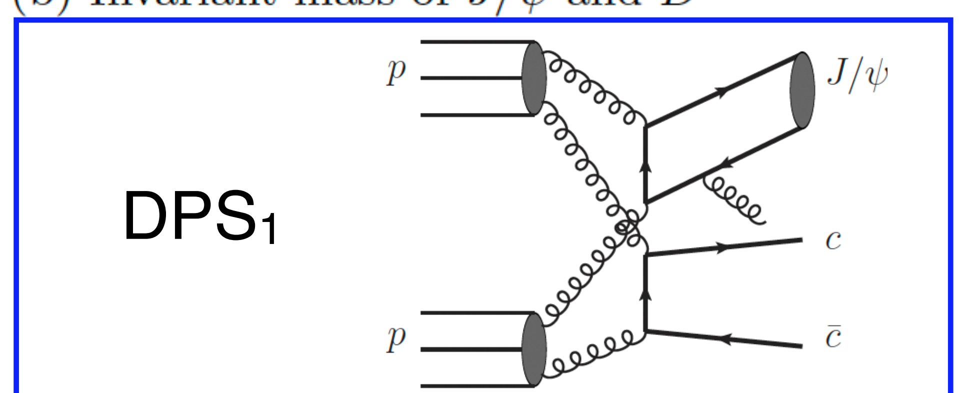


(a) Transverse momentum of J/ψ



(b) Invariant mass of J/ψ and D^0

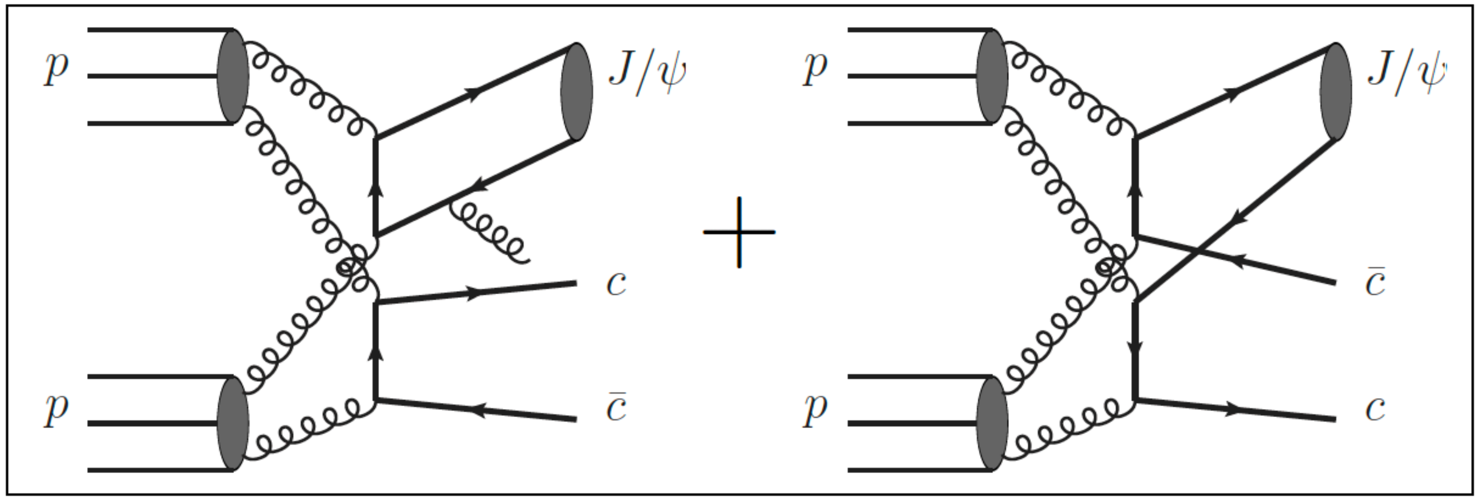




J/psi+open charm

DPS₁ DPS₂

[HSS (PRD'20)]



(a) DPS

Cross section \ Final state	$J/\psi + D^0$	$J/\psi + D^+$	$J/\psi + D_s^+$	$J/\psi + \Lambda_c^+$
σ^{DPS_1} $[\frac{15\ \mathrm{mb}}{\sigma_{\mathrm{eff},pp}} \cdot \mathrm{nb}]$	159.2	65.8	25.9	52.0
σ^{DPS_2} $[\frac{15\ \mathrm{mb}}{\sigma_{\mathrm{eff},pp}} \cdot \mathrm{nb}]$	$(3.4^{+12.7+1.1}_{-\ 2.8-1.0}) \cdot 10^{-1}$	$(1.5^{+5.5+0.5}_{-1.2-0.4}) \cdot 10^{-1}$	$(4.8^{+17.9+1.6}_{-\ 4.0-1.4}) \cdot 10^{-2}$	$(5.6^{+21.1+1.8}_{-\ 4.7-1.6}) \cdot 10^{-2}$

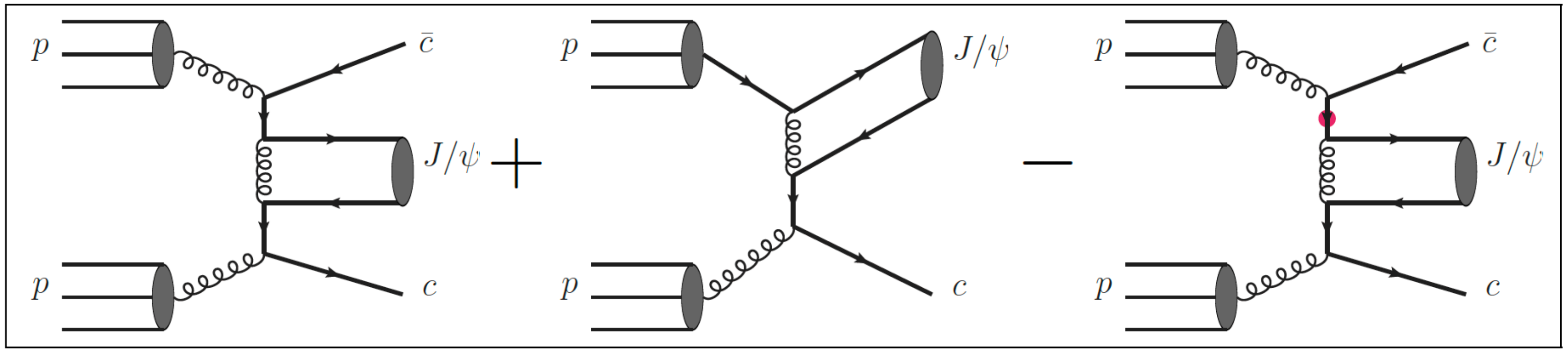
- **Colour octet and feed-down is not large enough**

Final state \ LDME Set	1	2	3	4	5	6	7	8	9
$J/\psi + D^0$	1.49	1.65	1.48	1.06	1.20	1.29	1.56	1.51	1.43
$J/\psi + D^+$	1.53	1.71	1.51	0.99	1.17	1.24	1.60	1.54	1.41
$J/\psi + D_s^+$	1.46	1.60	1.45	1.14	1.23	1.34	1.50	1.47	1.43
$J/\psi + \Lambda_c^+$	1.40	1.53	1.41	1.25	1.29	1.38	1.45	1.42	1.51

NRQCD/direct CSM

J/psi+open charm

[HSS (PRD'20)]



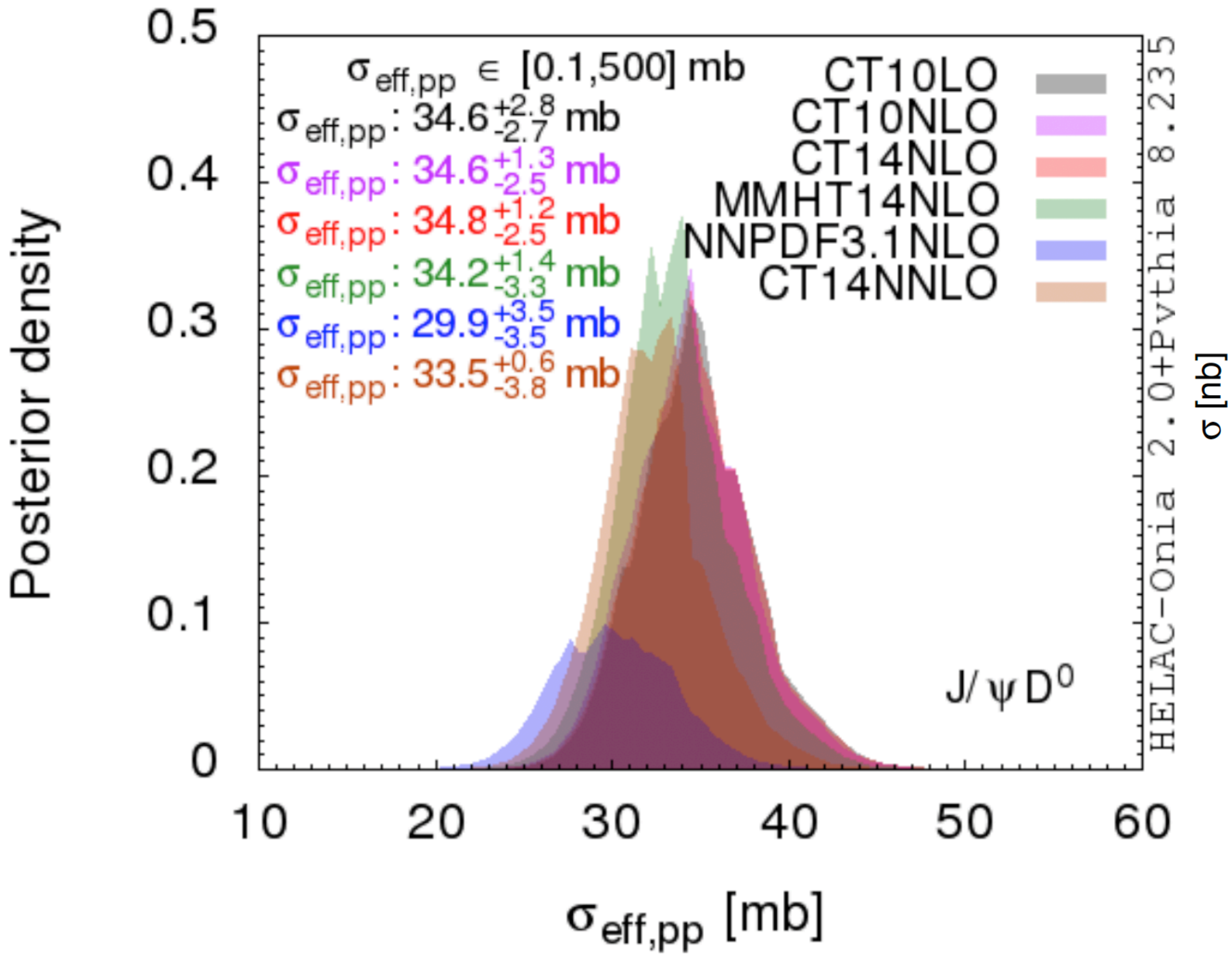
(b) SPS

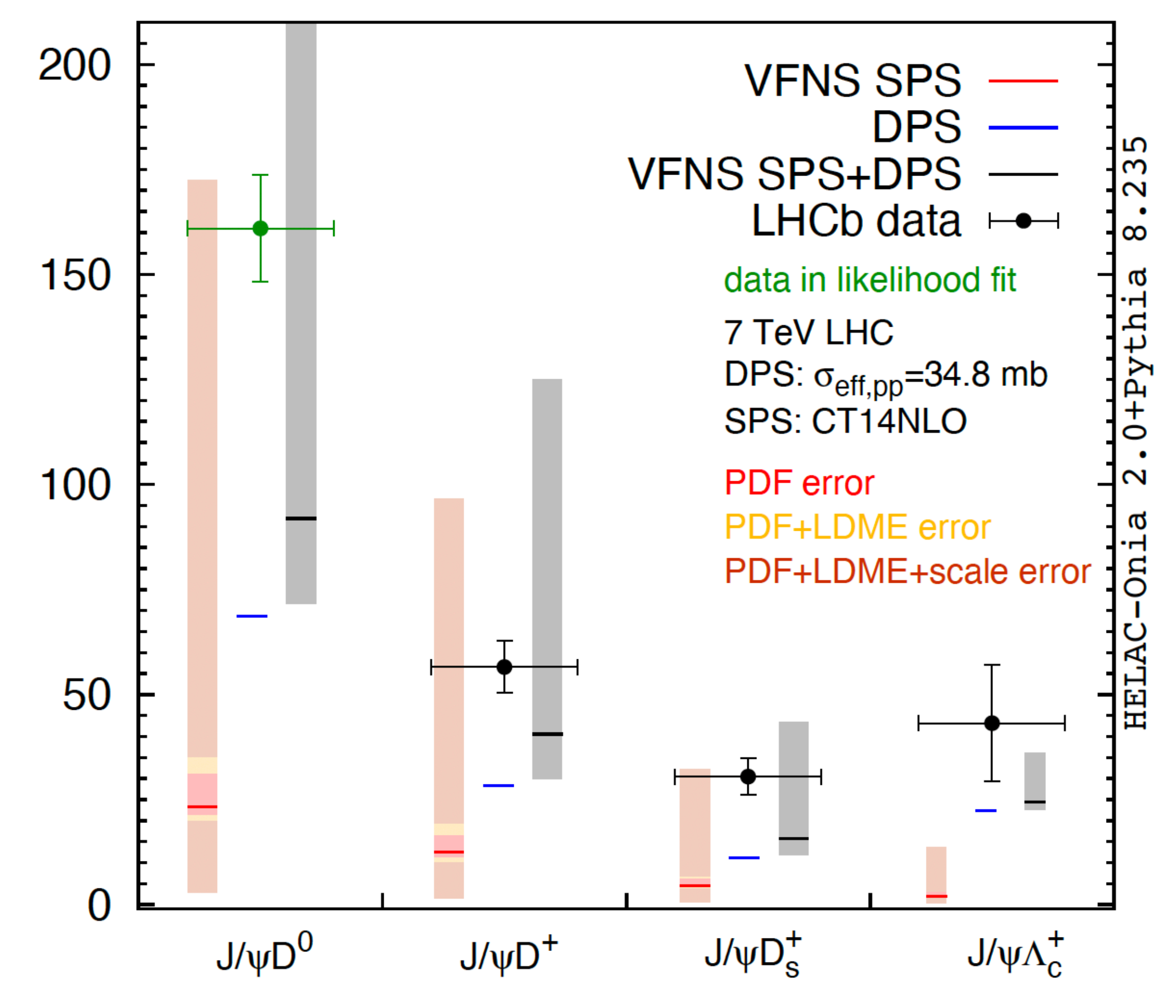
Final state	PDF	3FS (ggF)	4FS (cgF)	CT	VFNS
$J/\psi + D^0$	CT10LO	$6.4^{+19.5+0.9}_{-4.9-0.5}$	$17.4^{+105.3+5.7}_{-15.8-1.8}$	$4.9^{+32.5+1.2}_{-4.6-0.5}$	$18.8^{+92.4+5.4}_{-16.1-1.8}$
	CT10NLO	$6.5^{+18.6+0.9}_{-4.9-0.5}$	$17.2^{+92.0+5.6}_{-15.6-1.8}$	$4.9^{+28.3+1.2}_{-4.5-0.5}$	$18.8^{+82.3+5.3}_{-16.0-1.8}$
	CT14NLO	$6.6^{+18.7+1.0}_{-4.9-0.4}$	$17.9^{+95.1+6.9}_{-16.2-1.5}$	$5.0^{+28.8+1.4}_{-4.6-0.4}$	$19.4^{+84.9+6.5}_{-16.5-1.5}$
	CT14NNLO	$5.8^{+17.6+0.8}_{-4.5-0.3}$	$14.4^{+80.3+5.4}_{-13.2-1.0}$	$4.2^{+26.7+1.1}_{-3.9-0.3}$	$16.0^{+71.2+5.1}_{-13.8-1.0}$
	MMHT14NLO	$6.7^{+19.0+0.4}_{-5.0-0.3}$	$18.6^{+101.4+2.4}_{-17.1-1.9}$	$5.1^{+29.3+0.5}_{-4.7-0.4}$	$20.2^{+91.0+2.2}_{-17.3-1.8}$
	NNPDF3.1NLO	$5.9^{+17.4+0.5}_{-4.6-0.5}$	$12.1^{+72.2+2.0}_{-11.3-2.0}$	$4.3^{+25.9+0.6}_{-4.0-0.6}$	$13.7^{+63.8+1.9}_{-11.9-1.9}$
$J/\psi + D^+$	CT10LO	$3.5^{+10.7+0.5}_{-2.6-0.3}$	$9.5^{+57.6+3.1}_{-8.6-1.0}$	$2.7^{+17.9+0.7}_{-2.5-0.3}$	$10.2^{+50.3+2.9}_{-8.7-1.0}$
	CT10NLO	$3.4^{+9.9+0.5}_{-2.6-0.3}$	$9.4^{+50.1+3.0}_{-8.5-1.0}$	$2.6^{+15.5+0.7}_{-2.4-0.3}$	$10.2^{+44.5+2.9}_{-8.7-1.0}$
	CT14NLO	$3.5^{+10.1+0.5}_{-2.6-0.2}$	$9.7^{+51.6+3.8}_{-8.8-0.8}$	$2.7^{+15.7+0.8}_{-2.5-0.2}$	$10.5^{+46.0+3.5}_{-9.0-0.8}$
	CT14NNLO	$3.1^{+9.4+0.4}_{-2.4-0.2}$	$7.8^{+44.1+3.0}_{-7.2-0.6}$	$2.3^{+14.5+0.6}_{-2.1-0.1}$	$8.6^{+39.0+2.8}_{-7.5-0.6}$
	MMHT14NLO	$3.6^{+10.3+0.2}_{-2.7-0.2}$	$10.2^{+55.6+1.3}_{-9.4-1.1}$	$2.8^{+16.2+0.3}_{-2.6-0.2}$	$11.0^{+49.7+1.2}_{-9.4-1.0}$
	NNPDF3.1NLO	$3.2^{+9.6+0.3}_{-2.5-0.3}$	$6.6^{+39.4+1.1}_{-6.2-1.1}$	$2.3^{+13.9+0.3}_{-2.1-0.3}$	$7.5^{+35.1+1.0}_{-6.5-1.0}$
$J/\psi + D_s^+$	CT10LO	$1.2^{+3.7+0.2}_{-0.9-0.1}$	$3.3^{+20.2+1.1}_{-3.0-0.3}$	$0.96^{+6.41+0.25}_{-0.89-0.10}$	$3.5^{+17.5+1.0}_{-3.0-0.3}$
	CT10NLO	$1.2^{+3.4+0.2}_{-0.9-0.1}$	$3.3^{+17.8+1.1}_{-3.0-0.3}$	$0.91^{+5.49+0.16}_{-0.85-0.09}$	$3.5^{+15.7+1.0}_{-3.0-0.3}$
	CT14NLO	$1.2^{+3.5+0.2}_{-0.9-0.1}$	$3.4^{+18.5+1.4}_{-3.1-0.3}$	$0.94^{+5.57+0.27}_{-0.88-0.07}$	$3.7^{+16.5+1.3}_{-3.2-0.3}$
	CT14NNLO	$1.1^{+3.2+0.2}_{-0.8-0.1}$	$2.7^{+15.6+1.1}_{-2.5-0.2}$	$0.80^{+5.20+0.22}_{-0.75-0.05}$	$3.0^{+13.7+1.0}_{-2.6-0.2}$
	MMHT14NLO	$1.2^{+3.4+0.1}_{-0.9-0.1}$	$3.6^{+19.7+0.5}_{-3.3-0.4}$	$0.98^{+5.67+0.10}_{-0.91-0.08}$	$3.8^{+17.4+0.4}_{-3.3-0.3}$
	NNPDF3.1NLO	$1.1^{+3.3+0.1}_{-0.9-0.1}$	$2.3^{+14.0+0.4}_{-2.2-0.4}$	$0.80^{+4.92+0.12}_{-0.75-0.12}$	$2.6^{+12.4+0.4}_{-2.3-0.4}$
$J/\psi + \Lambda_c^+$	CT10LO	$0.46^{+1.42+0.07}_{-0.35-0.02}$	$1.2^{+7.0+0.5}_{-1.1-0.1}$	$0.35^{+2.33+0.10}_{-0.33-0.02}$	$1.3^{+6.1+0.4}_{-1.2-0.1}$
	CT10NLO	$0.52^{+1.51+0.07}_{-0.39-0.04}$	$1.5^{+8.1+0.5}_{-1.3-0.2}$	$0.41^{+2.48+0.11}_{-0.38-0.04}$	$1.6^{+7.2+0.5}_{-1.4-0.2}$
	CT14NLO	$0.50^{+1.48+0.08}_{-0.38-0.03}$	$1.5^{+8.3+0.6}_{-1.4-0.1}$	$0.42^{+2.53+0.10}_{-0.39-0.03}$	$1.6^{+7.3+0.6}_{-1.4-0.1}$
	CT14NNLO	$0.46^{+1.42+0.07}_{-0.35-0.02}$	$1.2^{+7.0+0.5}_{-1.1-0.1}$	$0.35^{+2.33+0.10}_{-0.33-0.02}$	$1.3^{+6.1+0.4}_{-1.2-0.1}$
	MMHT14NLO	$0.48^{+1.38+0.03}_{-0.36-0.02}$	$1.7^{+9.6+0.2}_{-1.5-0.2}$	$0.43^{+2.52+0.04}_{-0.40-0.04}$	$1.7^{+8.4+0.2}_{-1.5-0.2}$
	NNPDF3.1NLO	$0.50^{+1.50+0.04}_{-0.39-0.04}$	$1.0^{+6.3+0.2}_{-1.0-0.2}$	$0.36^{+2.25+0.05}_{-0.34-0.05}$	$1.2^{+5.6+0.2}_{-1.0-0.2}$

direct CSM

J/psi+open charm

[HSS (PRD'20)]





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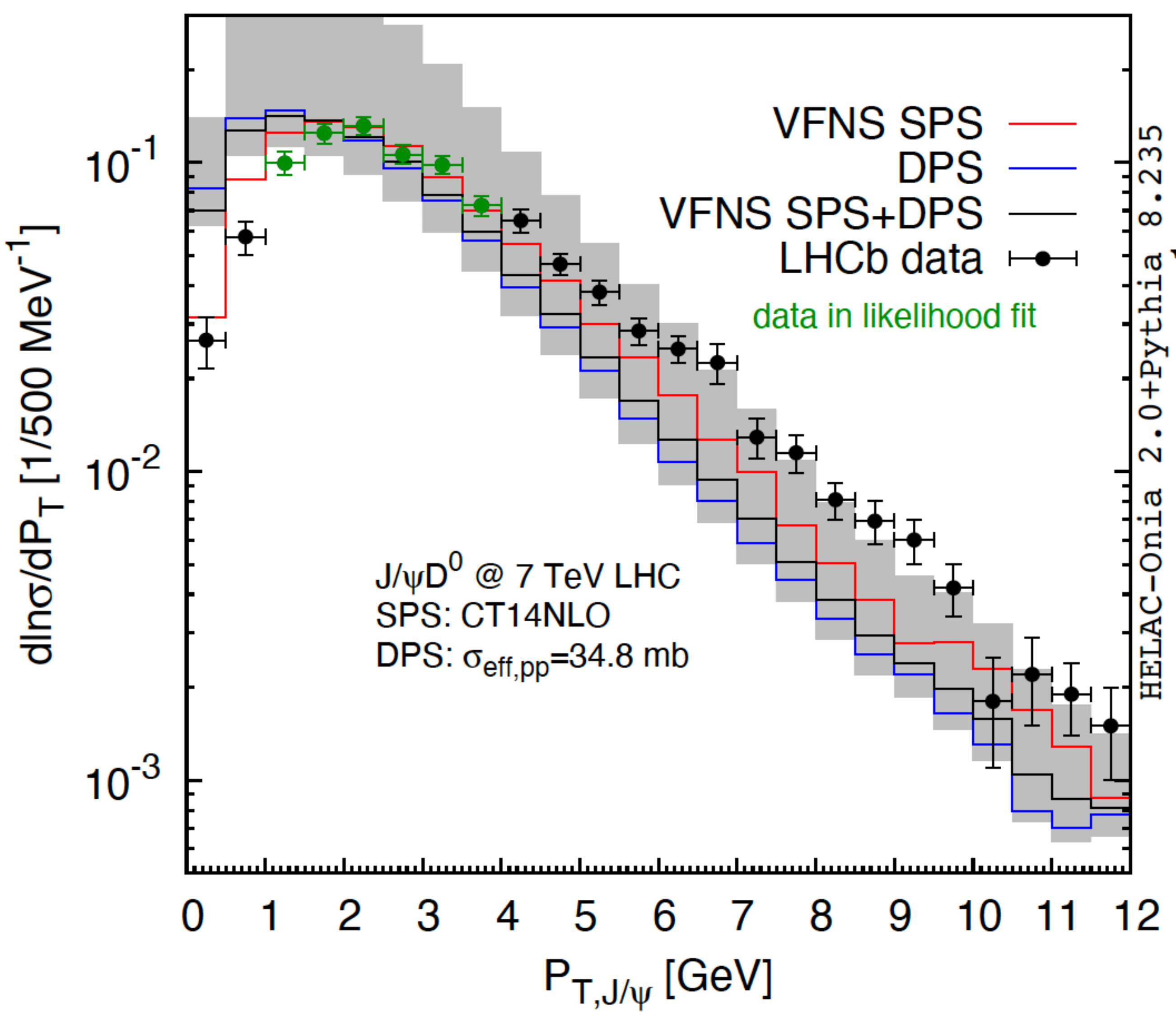

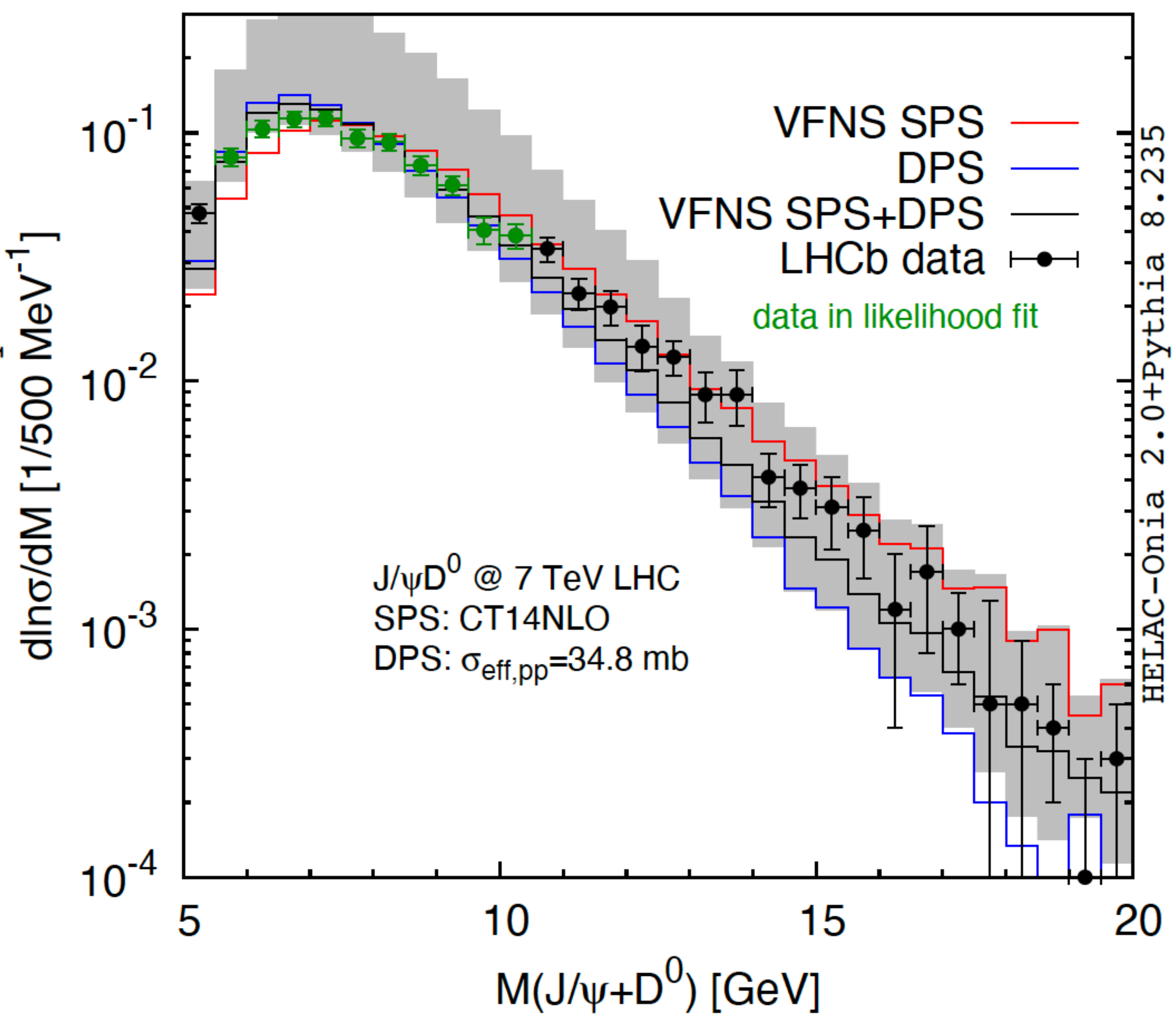

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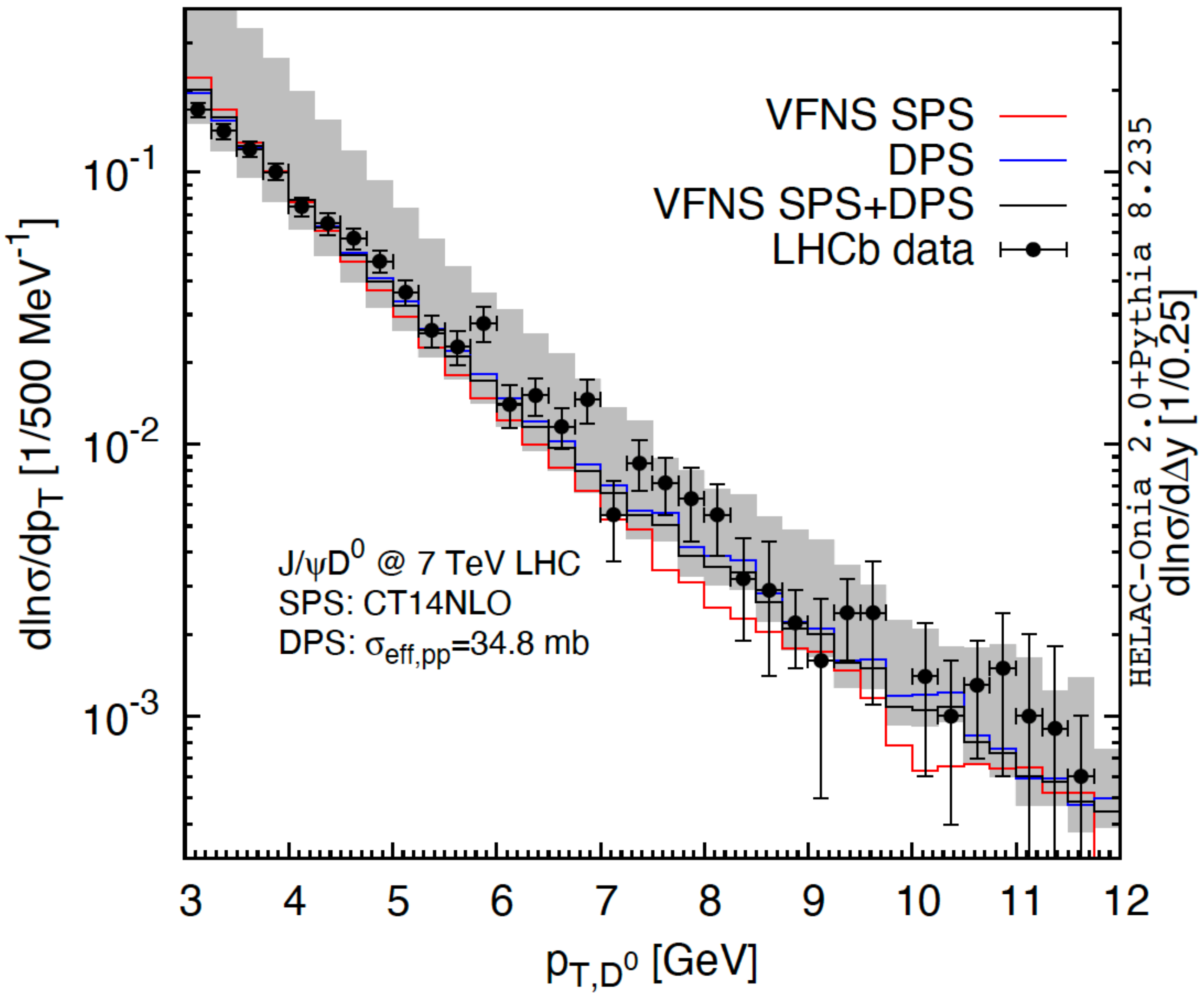

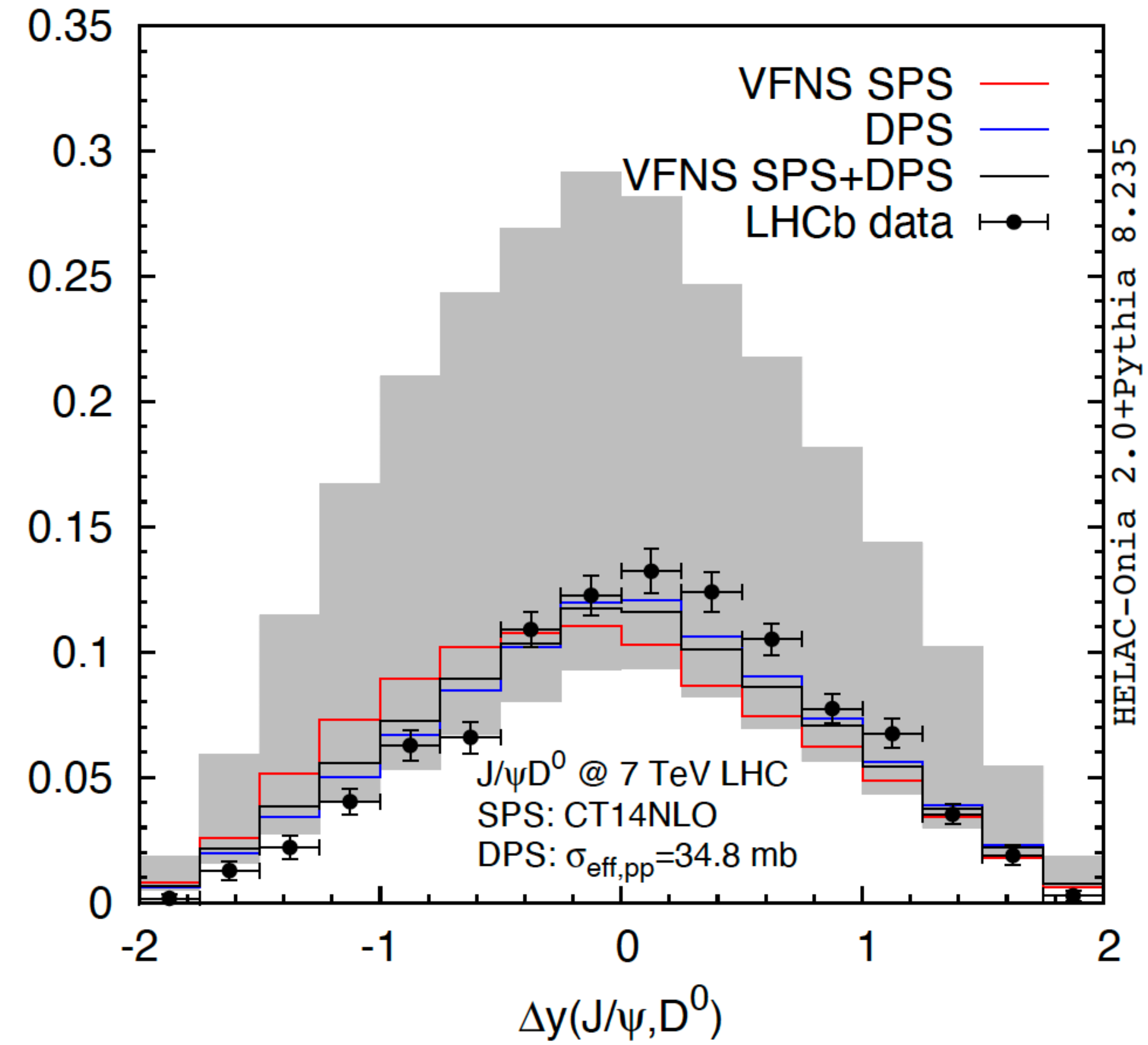

DPS in Heavy-Ion Collisions

- **As a concrete example, let us take** $p\mathrm{Pb} \to J/\psi + D^0$ [HSS (PRD'20)]

$$R^{\mathrm{DPS}}_{p\mathrm{Pb}\to J/\psi+D^0} = R^{J/\psi}_{p\mathrm{Pb}} R^{D^0}_{p\mathrm{Pb}} \left[\frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right]$$
$$+ \left(R^{J/\psi}_{p\mathrm{Pb}} + R^{D^0}_{p\mathrm{Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$
$$+ \left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^a}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

DPS in Heavy-Ion Collisions

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$$+\left(R^{J/\psi}_{p\text{Pb}} + R^{D^0}_{p\text{Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\text{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^a}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$

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$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

Either calculable or fixable by other measurements

DPS in Heavy-Ion Collisions

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$$+\left(R^{J/\psi}_{p\mathrm{Pb}} + R^{D^0}_{p\mathrm{Pb}}\right)\left[1 - \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{3^{2-a}(a+3)^{a}}{2(a+4)} - \frac{9^{1-a}(a+3)^{2a}}{4(a+2)}\right)\right]$$

$$+\left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^{a}}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

$$\boxed{\frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1) \simeq 5.23\left(\frac{\sigma_{\mathrm{eff},pp}}{34.8\ \mathrm{mb}}\right)}$$

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$$+\left[-1 + \frac{3^{1-2a}(a+3)^{2a}}{2a+3} + \frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1)\left(\frac{9}{8} + \frac{9^{1-a}(a+3)^{2a}}{4(a+2)} - \frac{3^{2-a}(a+3)^{a}}{(a+4)}\right)\right]$$

$$G\left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right) \propto \left(\frac{T_A(\vec{b})}{T_A(\vec{0})}\right)^a$$

$$\frac{\sigma_{\mathrm{eff},pp}}{\pi R_A^2}(A-1) \simeq 5.23\left(\frac{\sigma_{\mathrm{eff},pp}}{34.8\ \mathrm{mb}}\right)$$