第4届LHCb前沿物理研讨会

烟台，2024.07.27-31

Quarkonia production in heavy ion collisions

Jiaxing Zhao (赵佳星)

(HFHF/GSI/Goethe Uni.)

31/07/2024

HFHF Helmholtz Forschungsakademie Hessen für FAIR

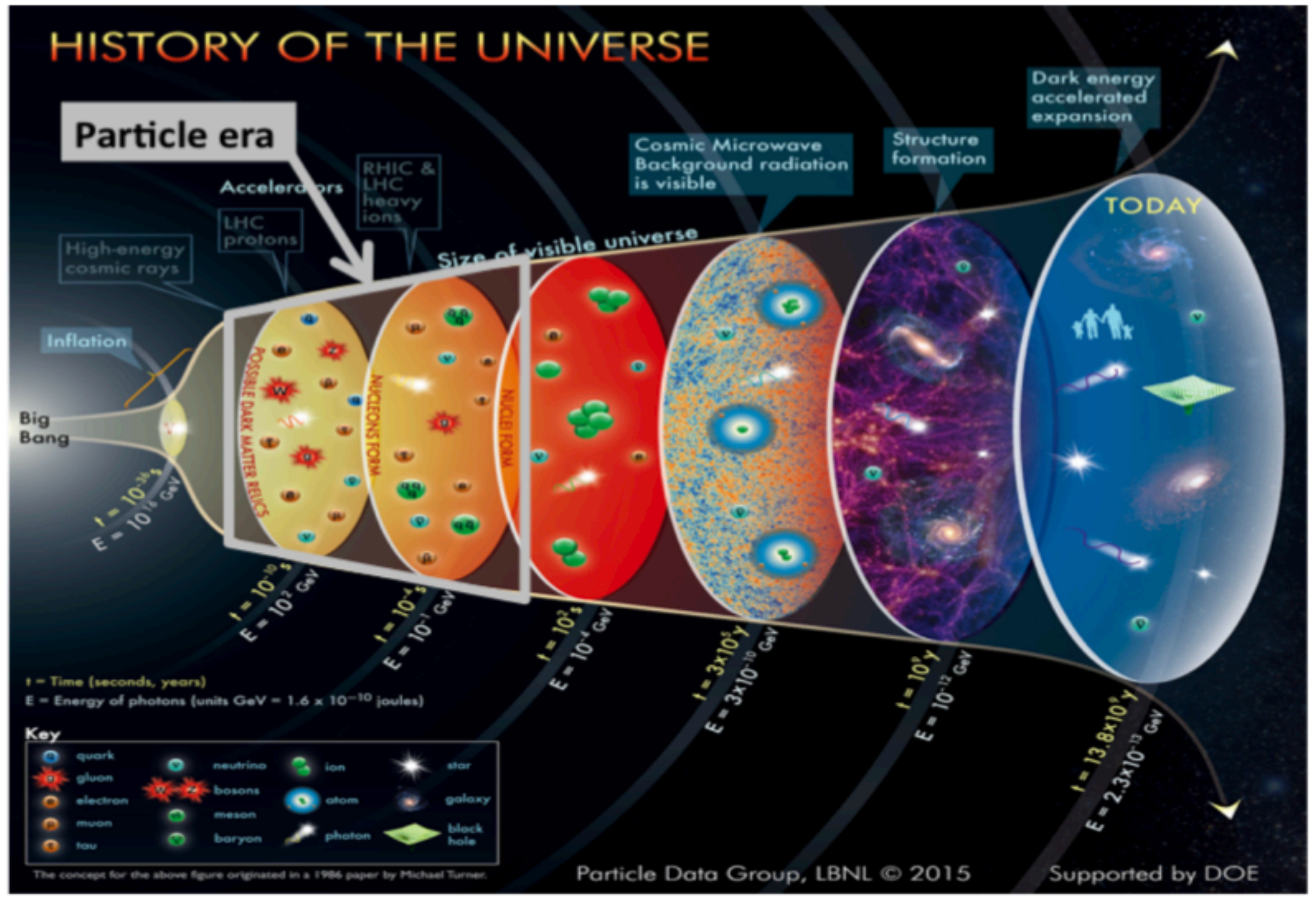
GSI

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

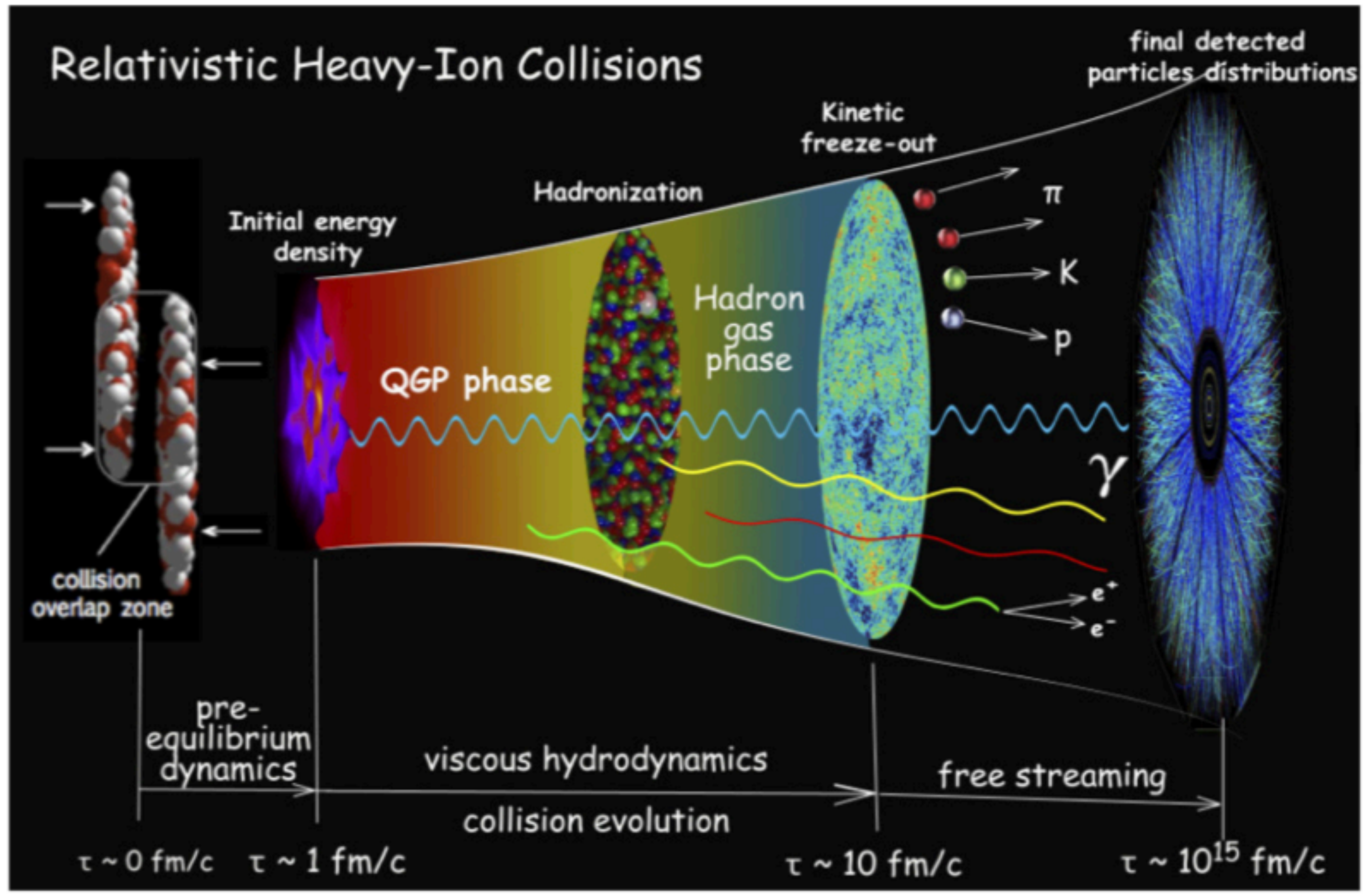
CRC-TR 211 Strong-interaction matter under extreme conditions

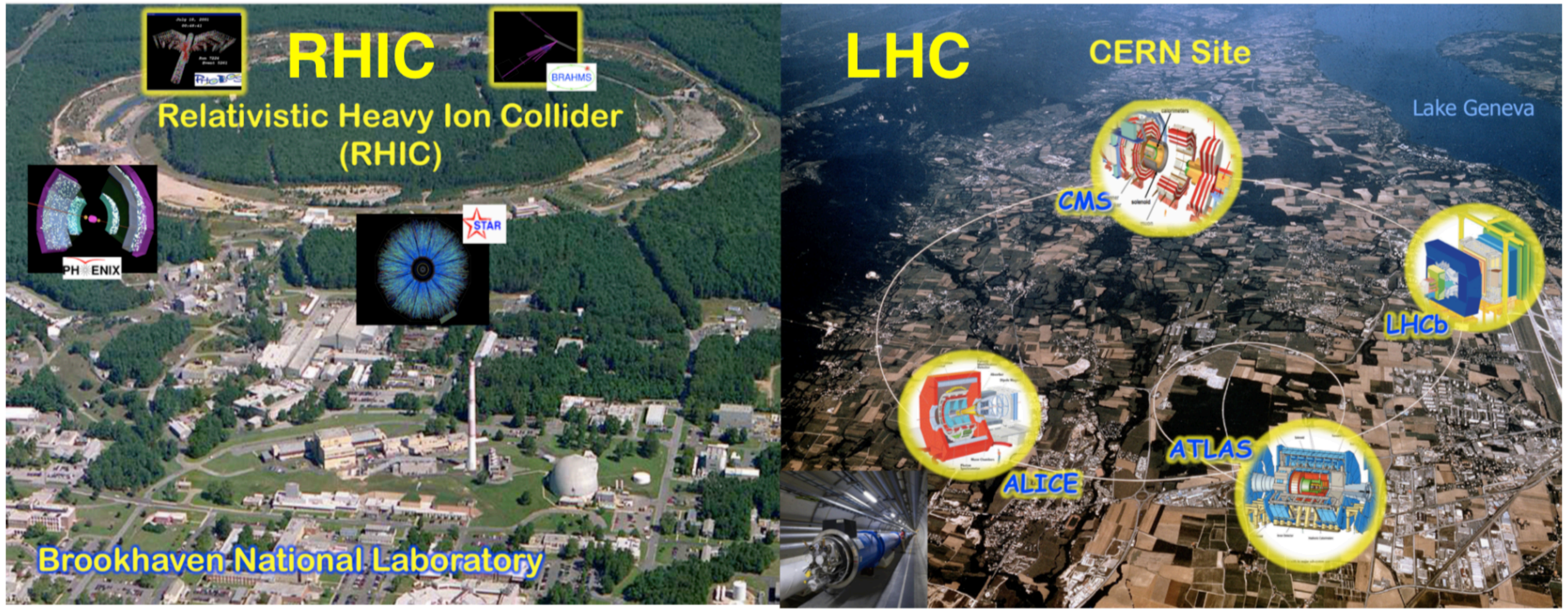
Relativistic heavy ion collisions

Big Bang



Little Bang





CBM / NICA / HAIF /…

PbPb, AuAu, OO, CuCu, ArAr, …pPb **Hengne Li's talk this morning.**

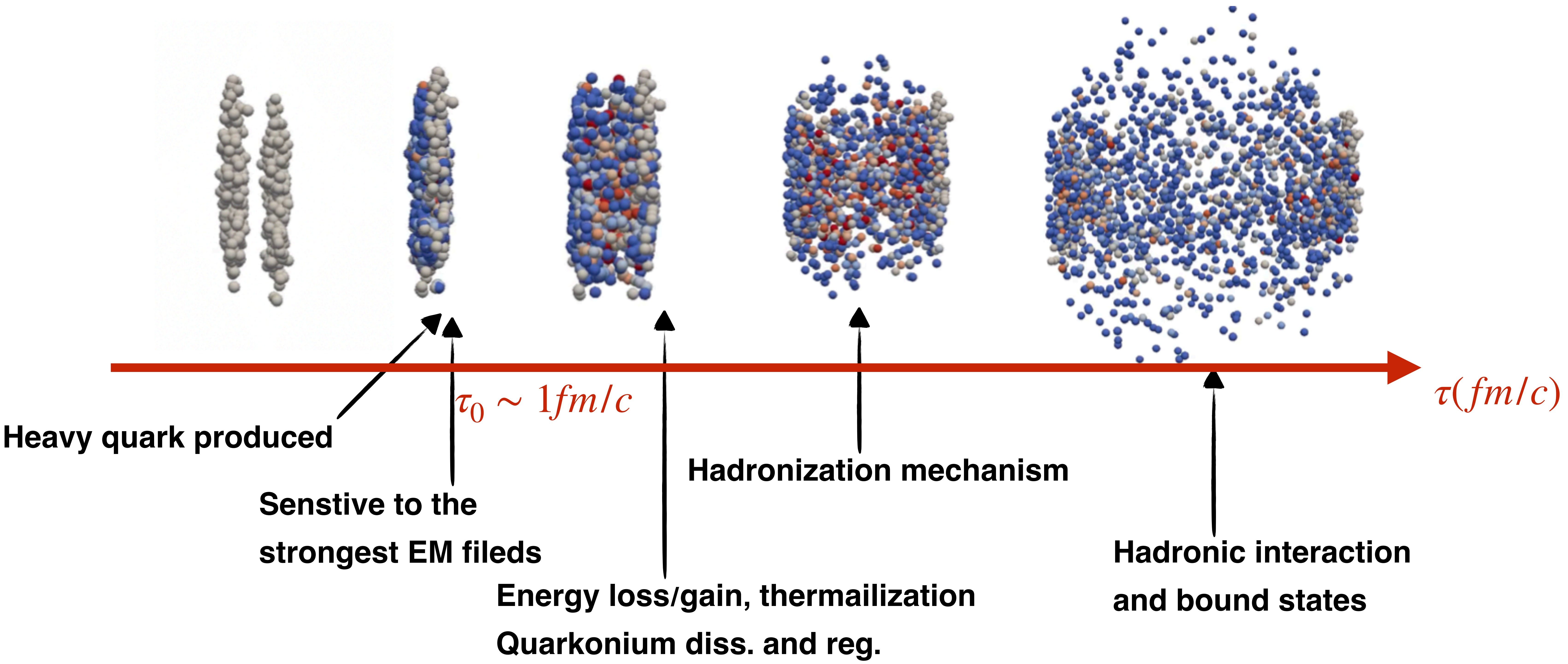
$\sqrt{s_{NN}}$ = 3GeV - 5.02TeV.

- ➡ QCD phase transition
- ➡ Properties of the quark-gluon plasma (QGP)
- ➡ Perturabitive and non-perturabitive QCD in thermal, high μ_B,,…
- ➡ Searching for new hadrons/exotic hadrons
- ➡ …

Heavy flavor probes

$m_c \sim 1.3\text{GeV}, \text{m}_\text{b} \sim 4.2\text{GeV}$

- $\tau_c \sim 1/m_c, \tau_b \sim 1/m_b < \tau_0 \sim 1 fm/c$, “see” full system evolution.
- $m_c, m_b \gg \Lambda_{QCD}$, produced by hard scattering, pQCD.
- $m_c, m_b \gg T$, number is conserved during the evolution (thermal production can be neglected).
- $m \gg T \sim q$, can be treated as a Brownian particle.

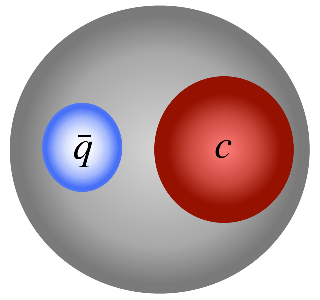


Heavy flavor is a nice probe of the hot QCD medium created !

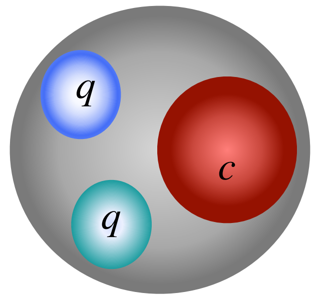
Open heavy flavor

$D^0, D^+, D_s^+, \Lambda_c, \Xi_c, \Omega_c, \ldots$

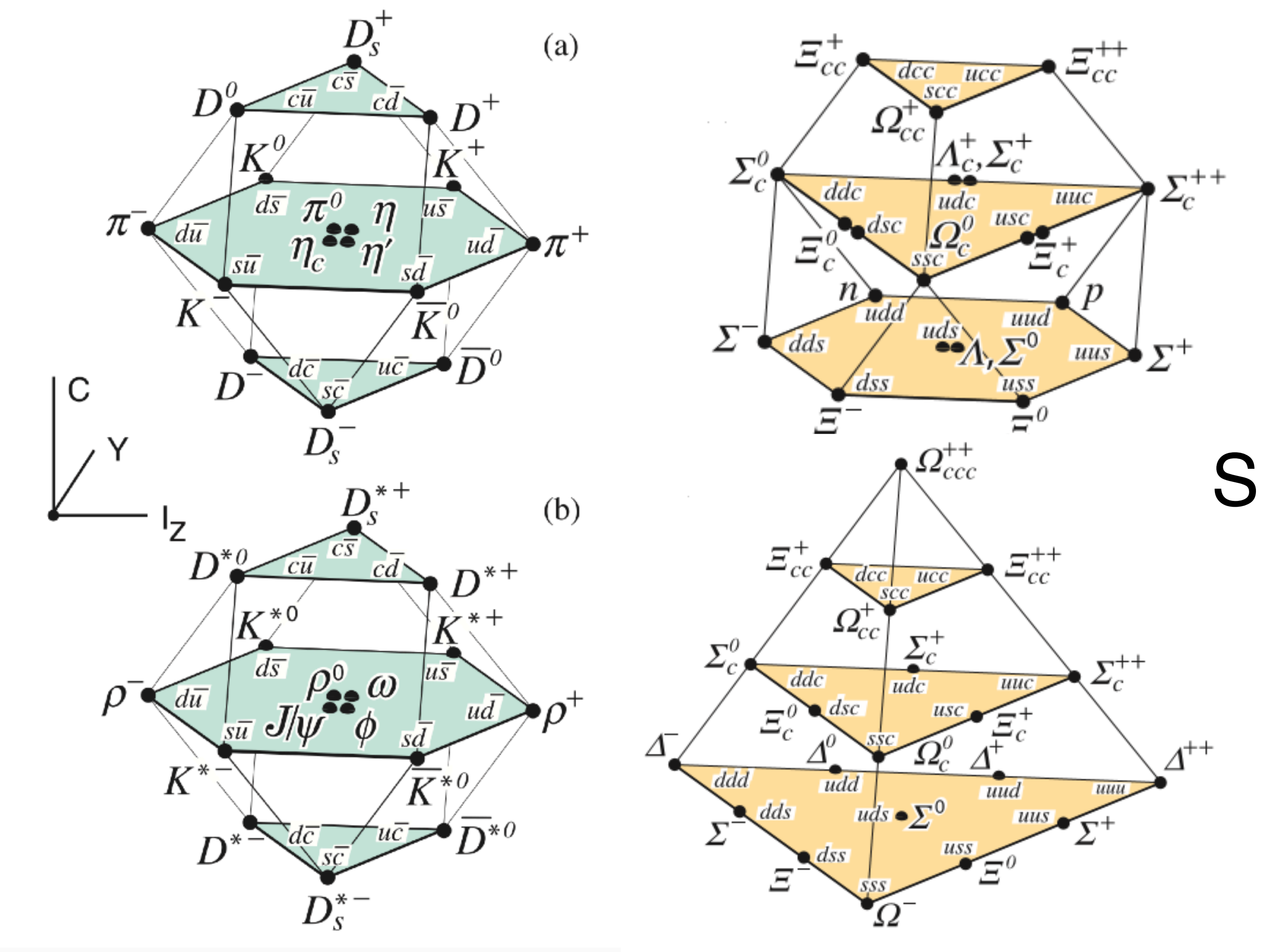
$B^0, B^-, B_s^0, \Lambda_b, \Xi_b, \Omega_b, \ldots$



Meson



Baryon



SU(4) quark model

Heavy quark energy loss can be used to study the medium properties and perturbabtive QCD !

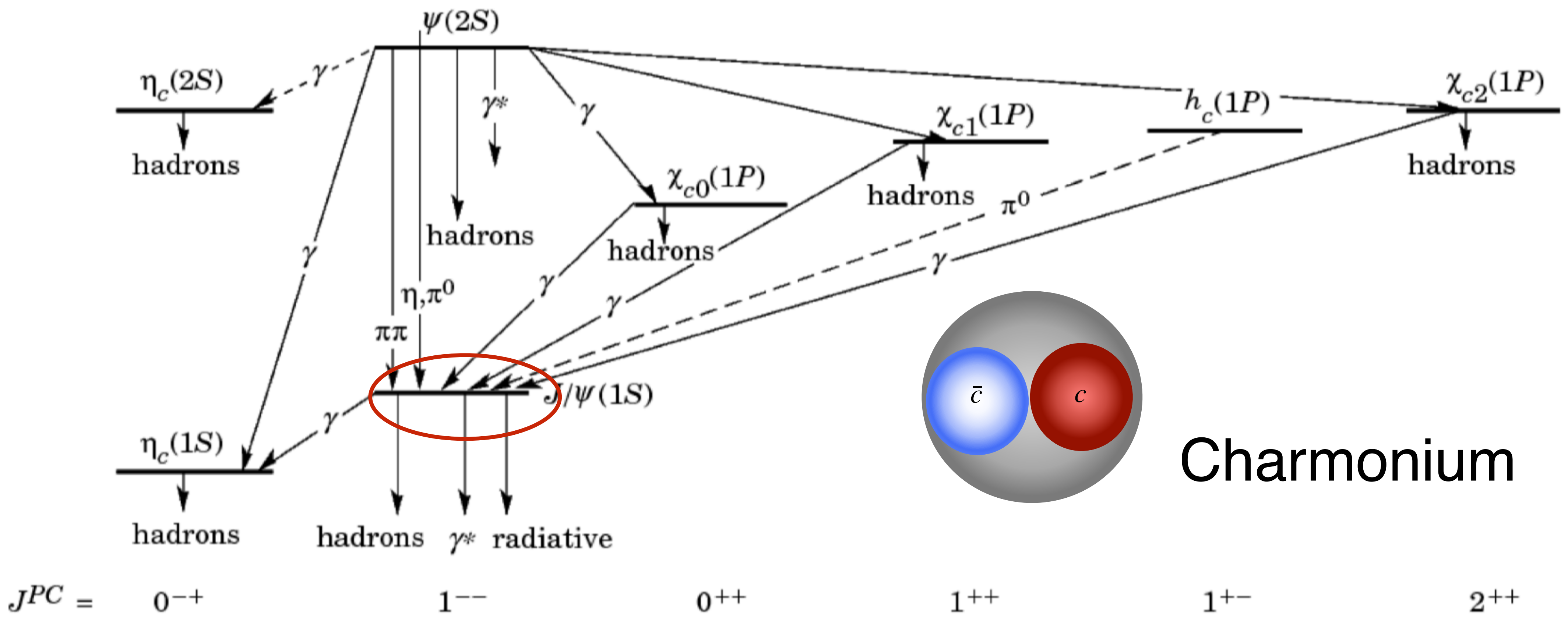
Hadronization mechanism in the thermal medium -> non-perturbabtive QCD !

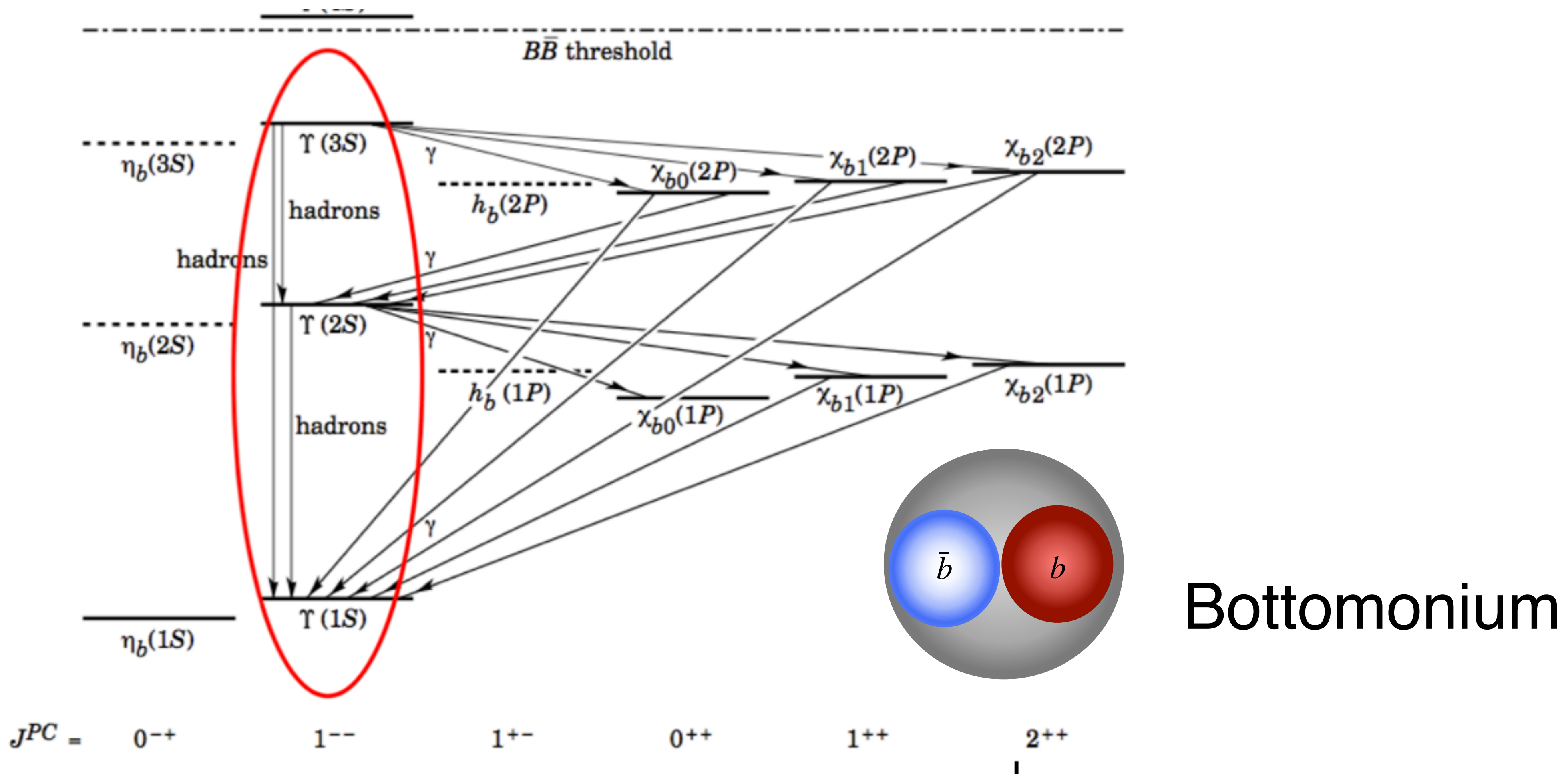
PHYSICAL REVIEW C **109**, 054912 (2024)

Hadronization of heavy quarks

Jiaxing Zhao,[1] Jörg Aichelin,[1] Pol Bernard Gossiaux,[1] Andrea Beraudo,[2] Shanshan Cao,[3] Wenkai Fan,[4] Min He,[5] Vincenzo Minissale,[6,7] Taesoo Song,[8] Ivan Vitev,[9] Ralf Rapp,[10] Steffen Bass,[4] Elena Bratkovskaya,[8,11,12] Vincenzo Greco,[6,7] and Salvatore Plumari[6,7]

Quarkonium

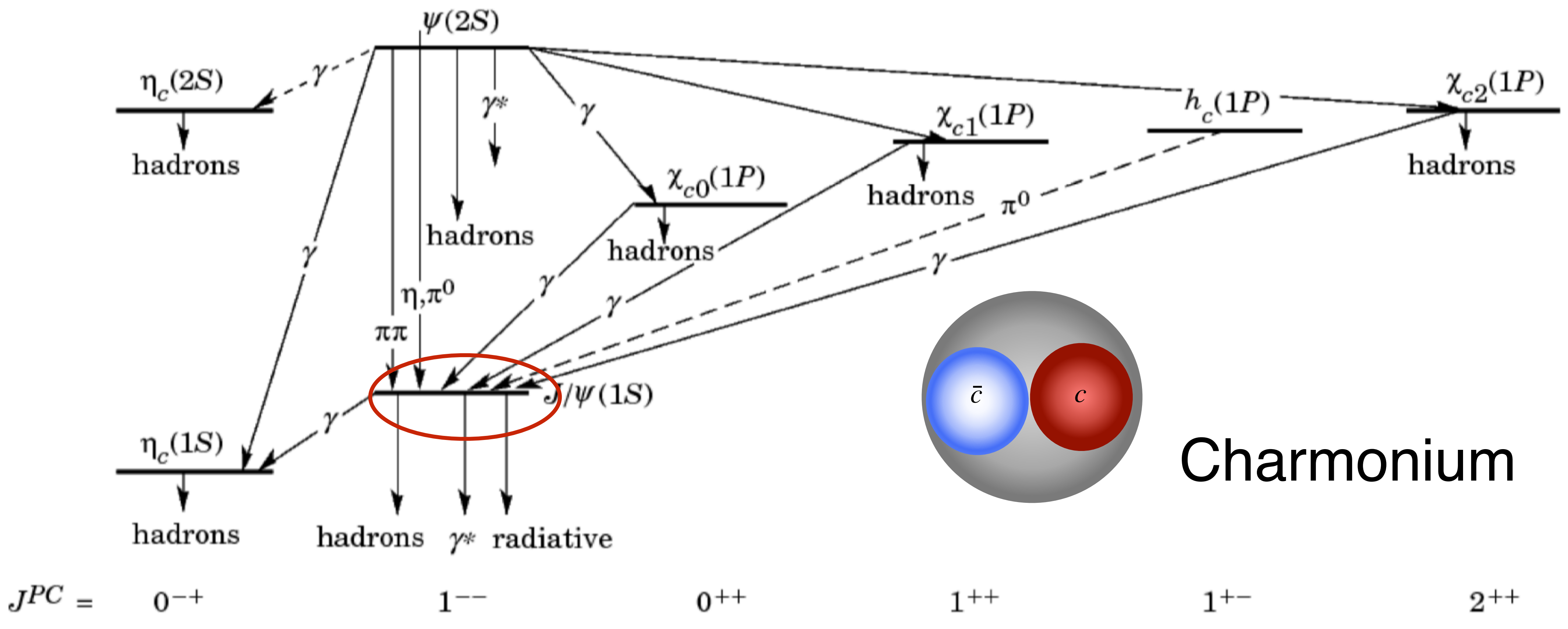


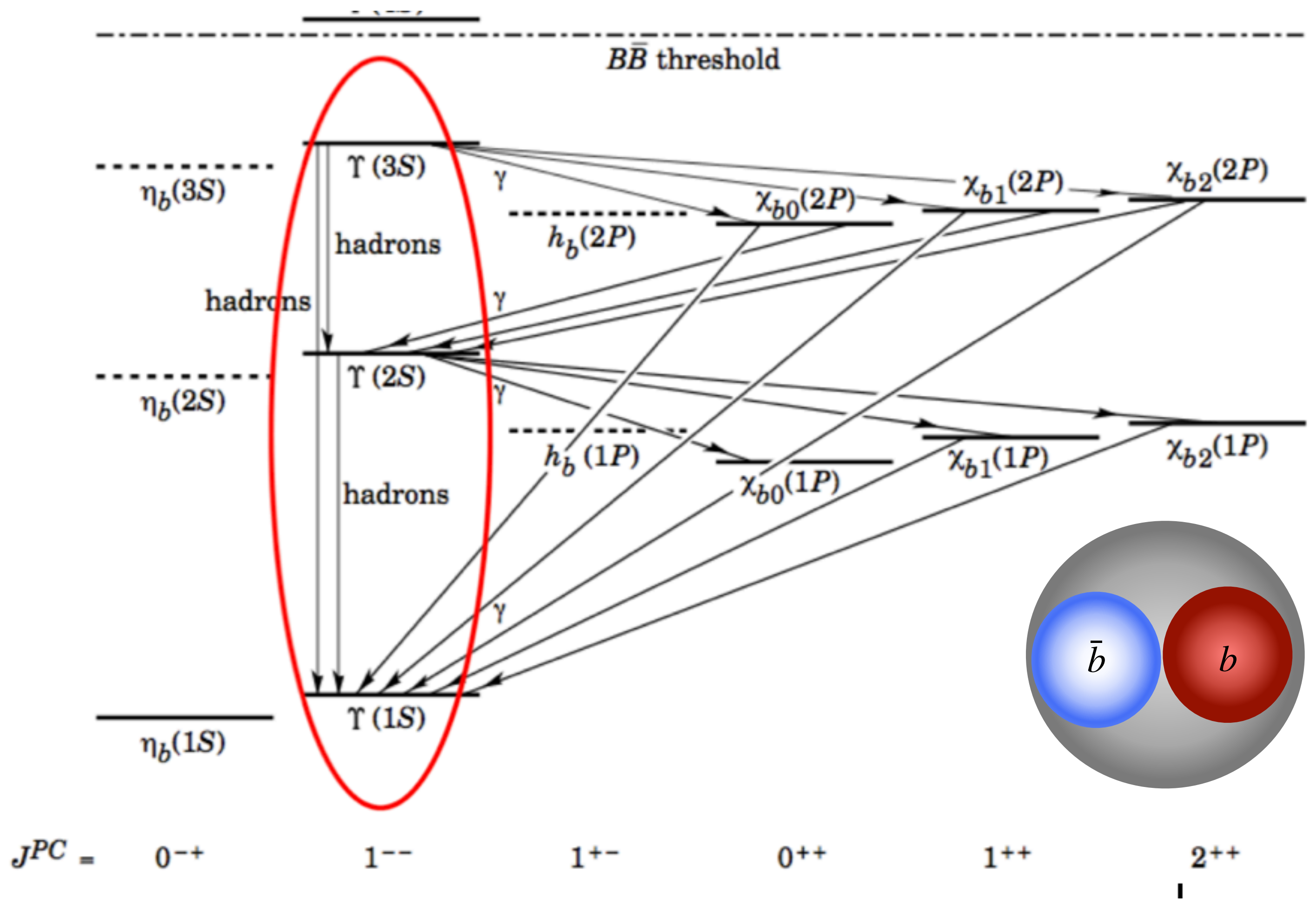


Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …)

From yield and distribution -> deduce in-medium properties and infer the fundamental interaction in QCD matter !

Quarkonium





Bottomonium

Quarkonia suppression has been considered as a smoking gun of the QGP (Matsui, Satz at 1986, …)
From yield and distribution -> deduce in-medium properties and infer the fundamental interaction in QCD matter !

Outline:

- **Vacuum and finite-temperature properties of quarkonium**

  Strong interaction in vacuum and under extreme conditions

- **Progress on quarkonium real-time evolution in HICs**

  Connect the experimental data to the first-principle QCD

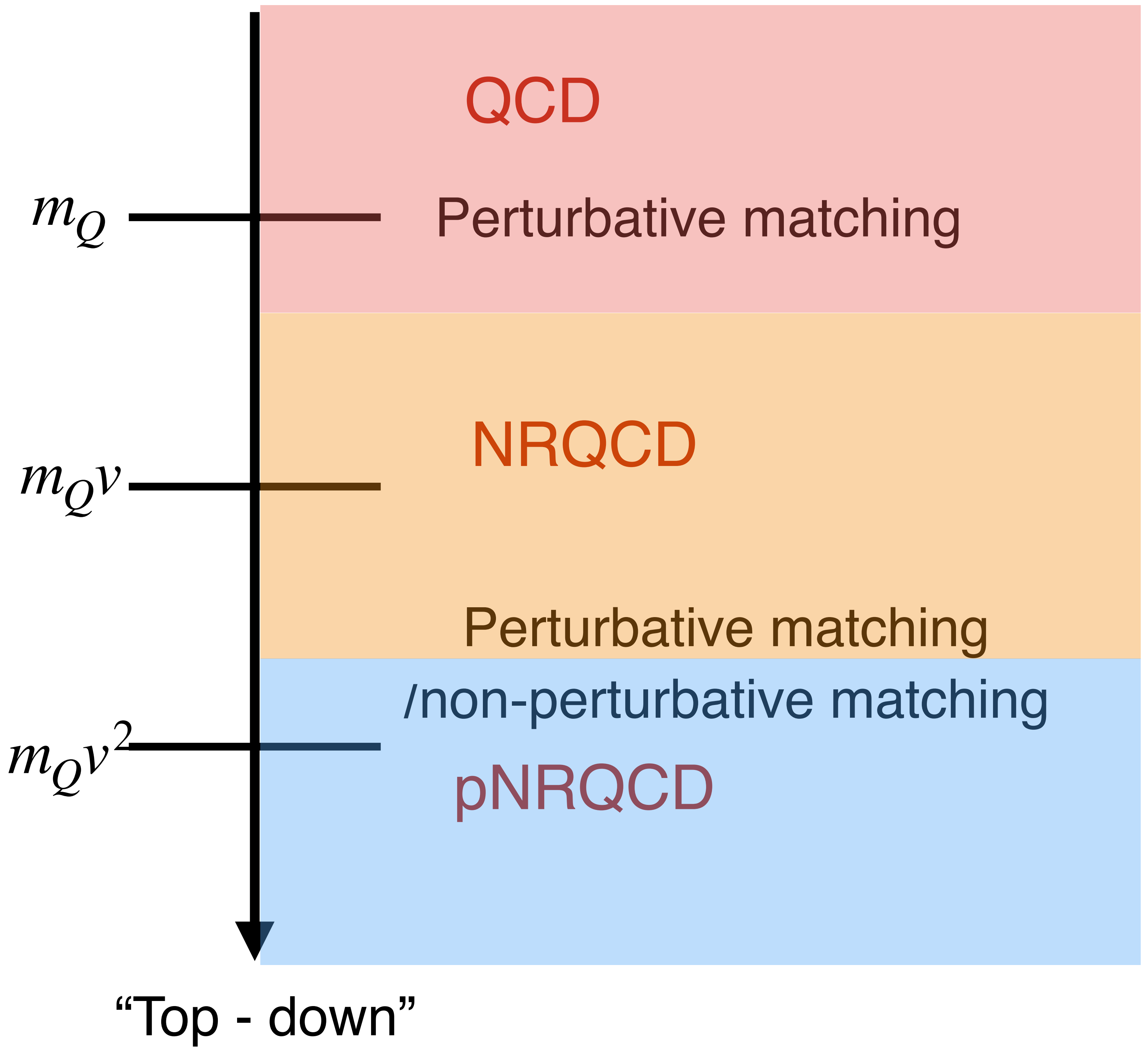
Vacuum and finite-temperature properties of quarkonium

Quarkonium static properties in a vacuum

From QCD to the potential model

$m_c \sim 1.3\ \text{GeV}, m_b \sim 4.2\ \text{GeV}$

Separation of scales: $m_Q \gg m_Q v \gg m_Q v^2$



$$
\begin{aligned}
\mathcal{L}_{pNRQCD} &= \int d^3r \mathrm{Tr}\left[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\right] \\
&+ V_A(r)\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}S + S^\dagger \mathbf{r}\cdot g\mathbf{E}O] \\
&+ \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_l
\end{aligned}
$$

Singlet field S; Octet field O.

$$
\begin{aligned}
H_S &= \{c_1^s(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^s(r)\frac{\mathbf{P}^2}{2M} + V_S^{(0)} + \frac{V_S^{(1)}}{m_Q} + \frac{V_S^{(2)}}{m_Q^2}, \\
H_O &= \{c_1^o(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^o(r)\frac{\mathbf{P}^2}{2M} + V_O^{(0)} + \frac{V_O^{(1)}}{m_Q} + \frac{V_O^{(2)}}{m_Q^2}.
\end{aligned}
$$

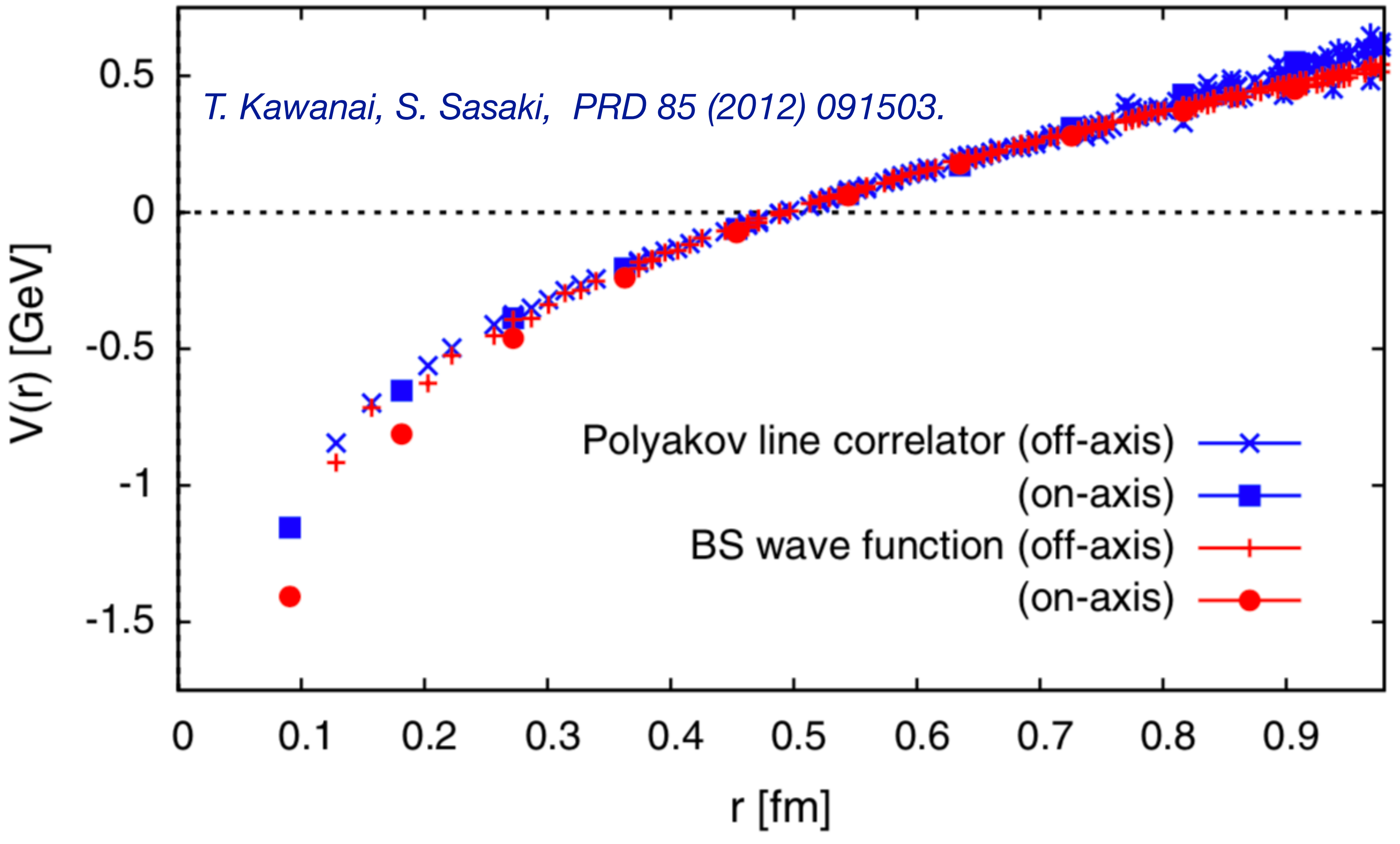
The potential model：two-body Schrödinger equation

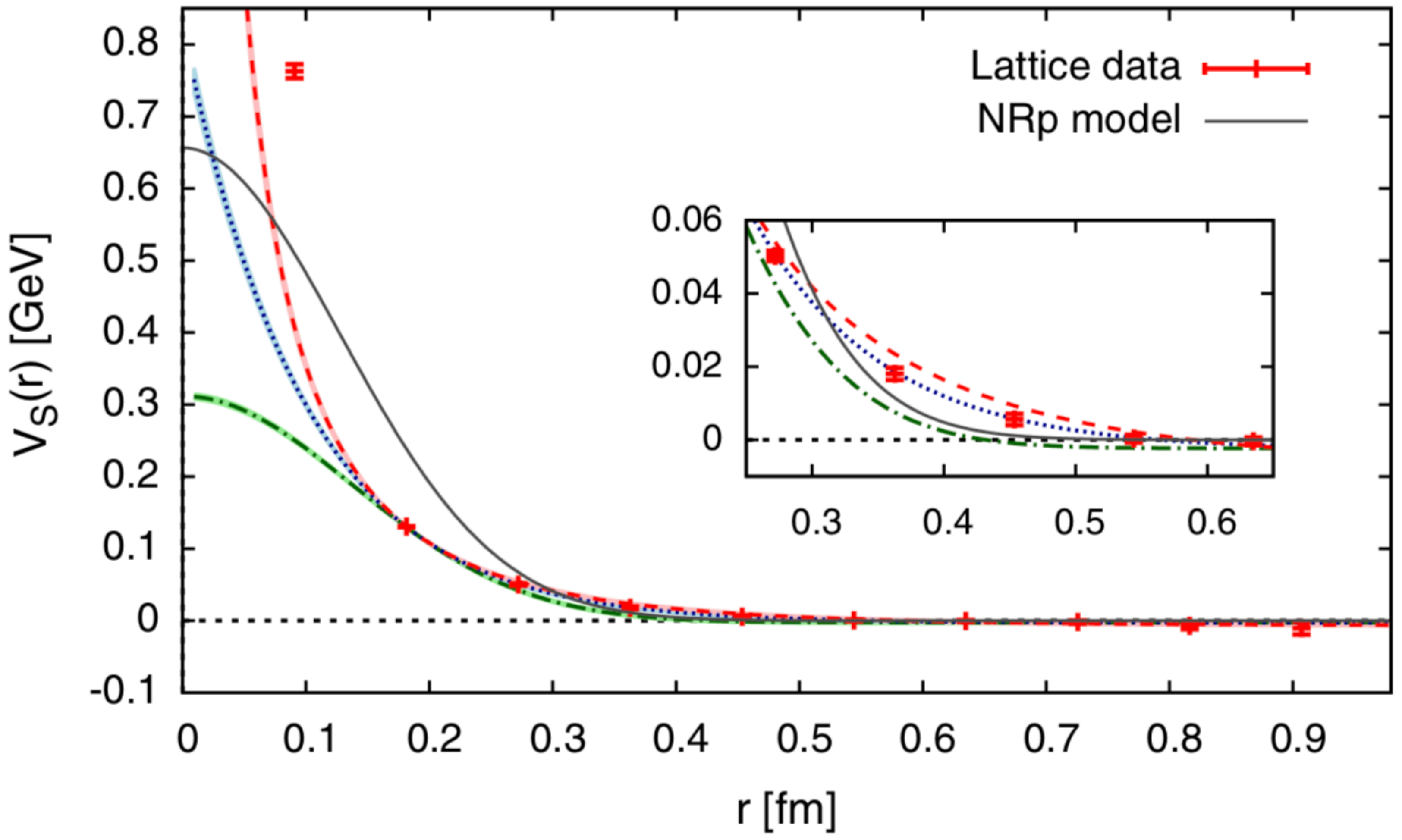
$$
\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_1, \mathbf{s}_2)\right]\psi = E\psi
$$

See for e.g.:
W. Caswell, G. Lepage, Phys. Lett. B 167 (1986) 437.
N. Brambilla, A. Pineda, J. Soto, A. Vairo, Nucl. Phys. B 566 (2000) 275.

Quarkonium static properties in a vacuum

Cornell potential + Spin-spin interaction

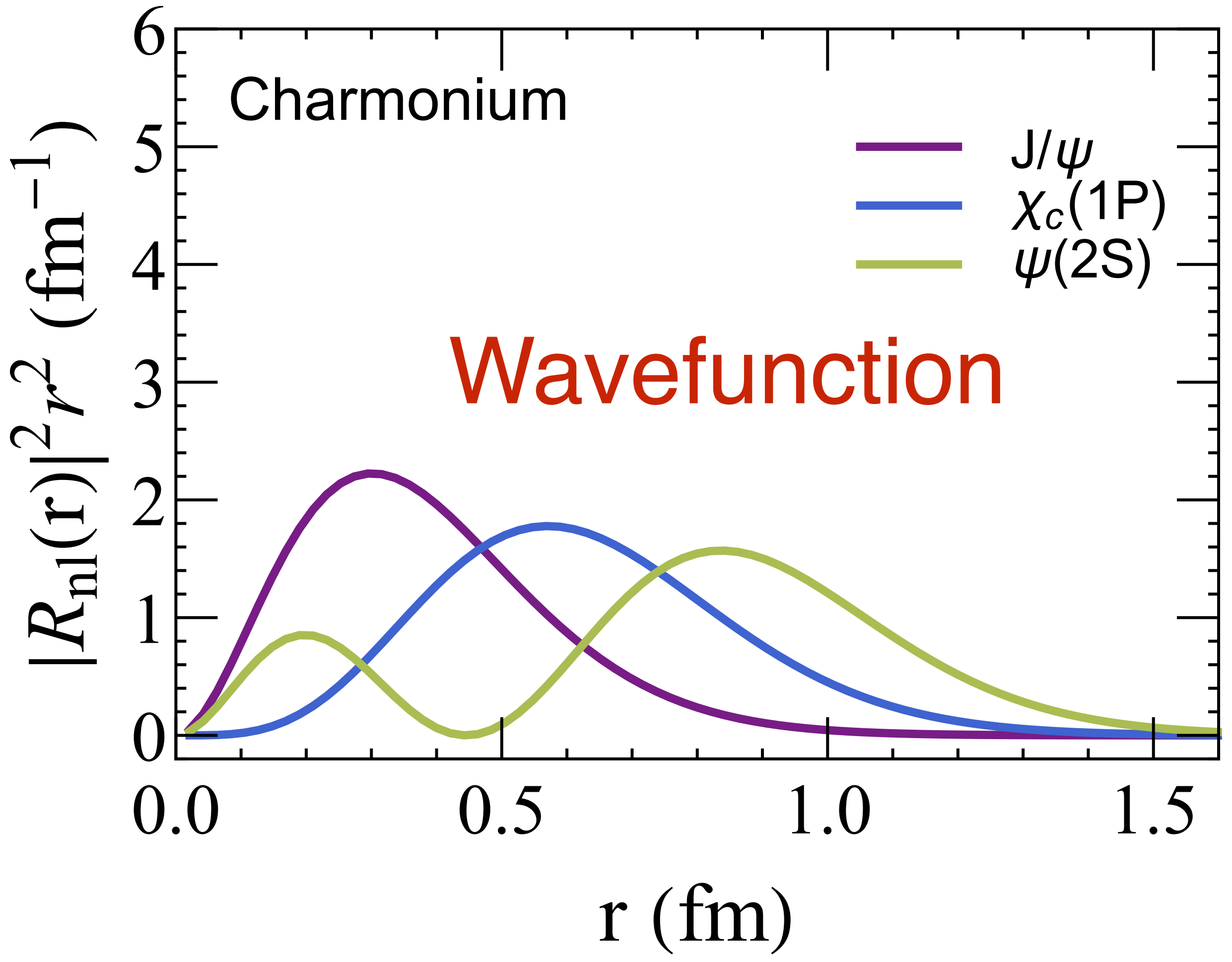


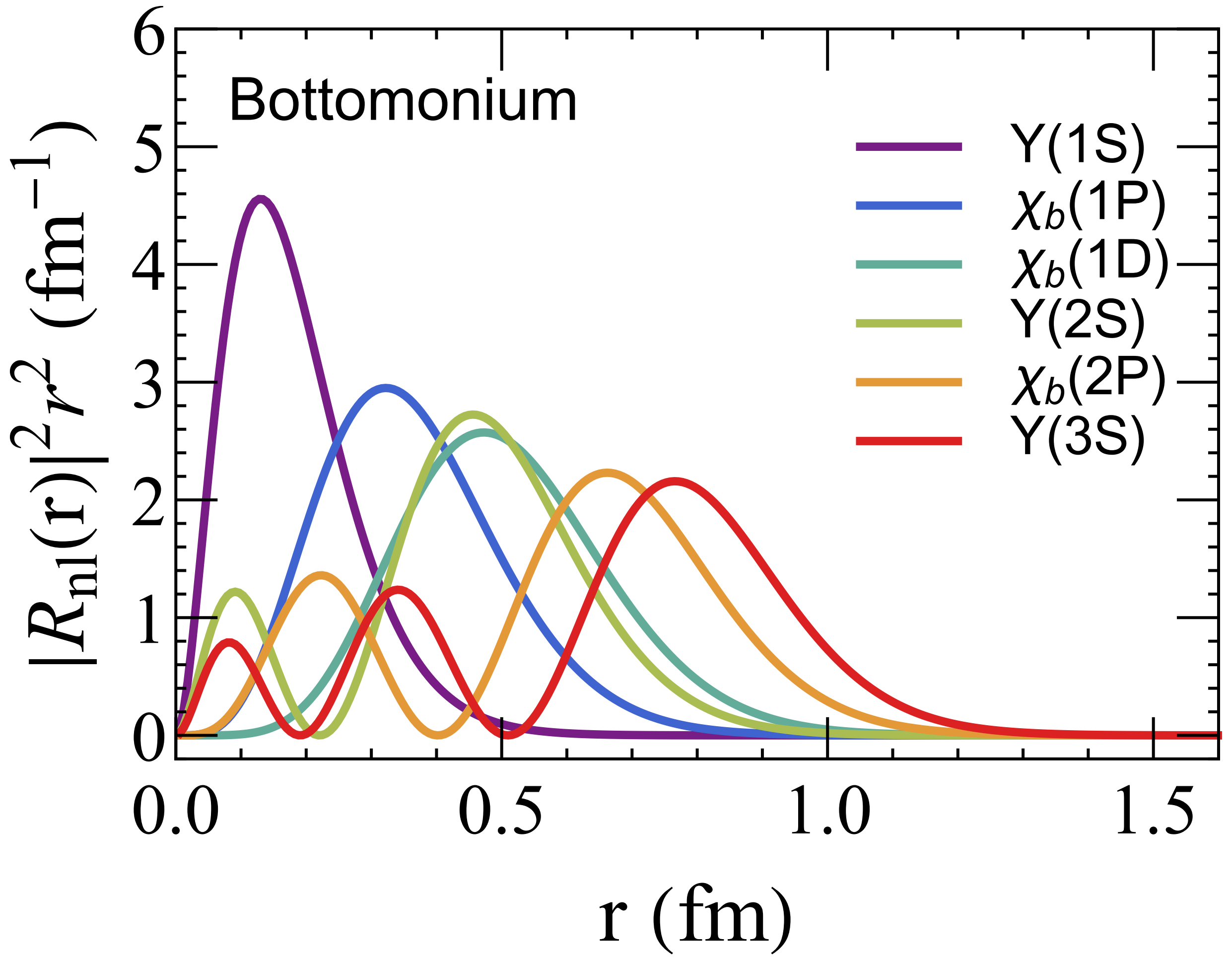


States	$\eta_c(1S)$	$J/\psi(1S)$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$h_c(2P)$	$\chi_c(2P)$
$M_{Exp.}$(GeV)	2.981	3.097	3.525	3.556	3.639	3.686	-	3.927
$M_{Th.}$(GeV)	2.967	3.102	3.480	3.500	3.654	3.720	3.990	4.000
$\langle r\rangle$(fm)	0.365	0.427	0.635	0.655	0.772	0.802	0.961	0.980
States	$\eta_b(1S)$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$M_{Exp.}$(GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269	10.355
$M_{Th.}$(GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221	10.325
$\langle r\rangle$(fm)	0.200	0.214	0.377	0.387	0.465	0.474	0.603	0.680

JZ, *K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

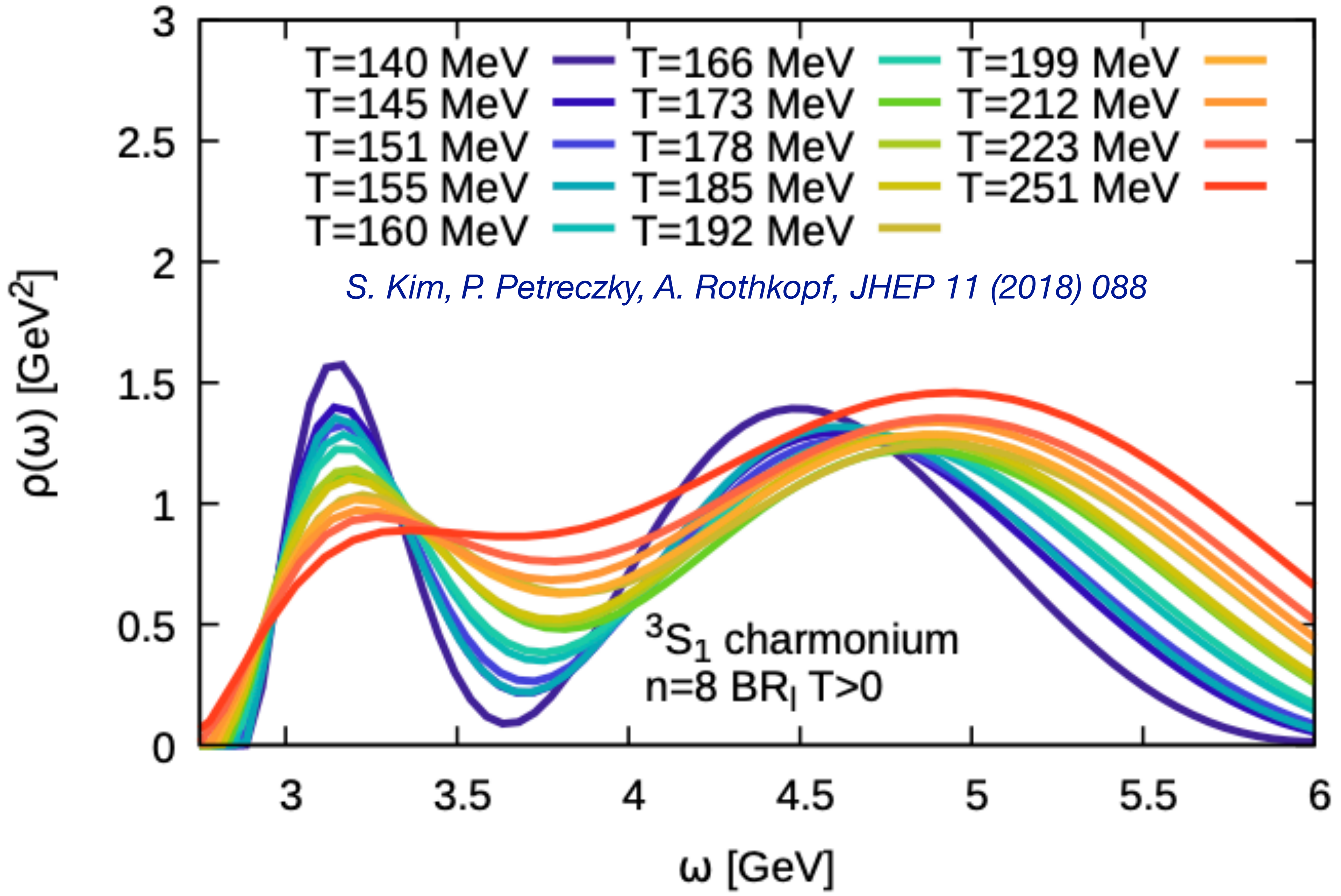
The mass spectra can be explained very well!





Quarkonium in the hot medium

All in-medium properties of quarkonium are encoded in their spectra function



peak position shifts and becomes broader as temperature increases.

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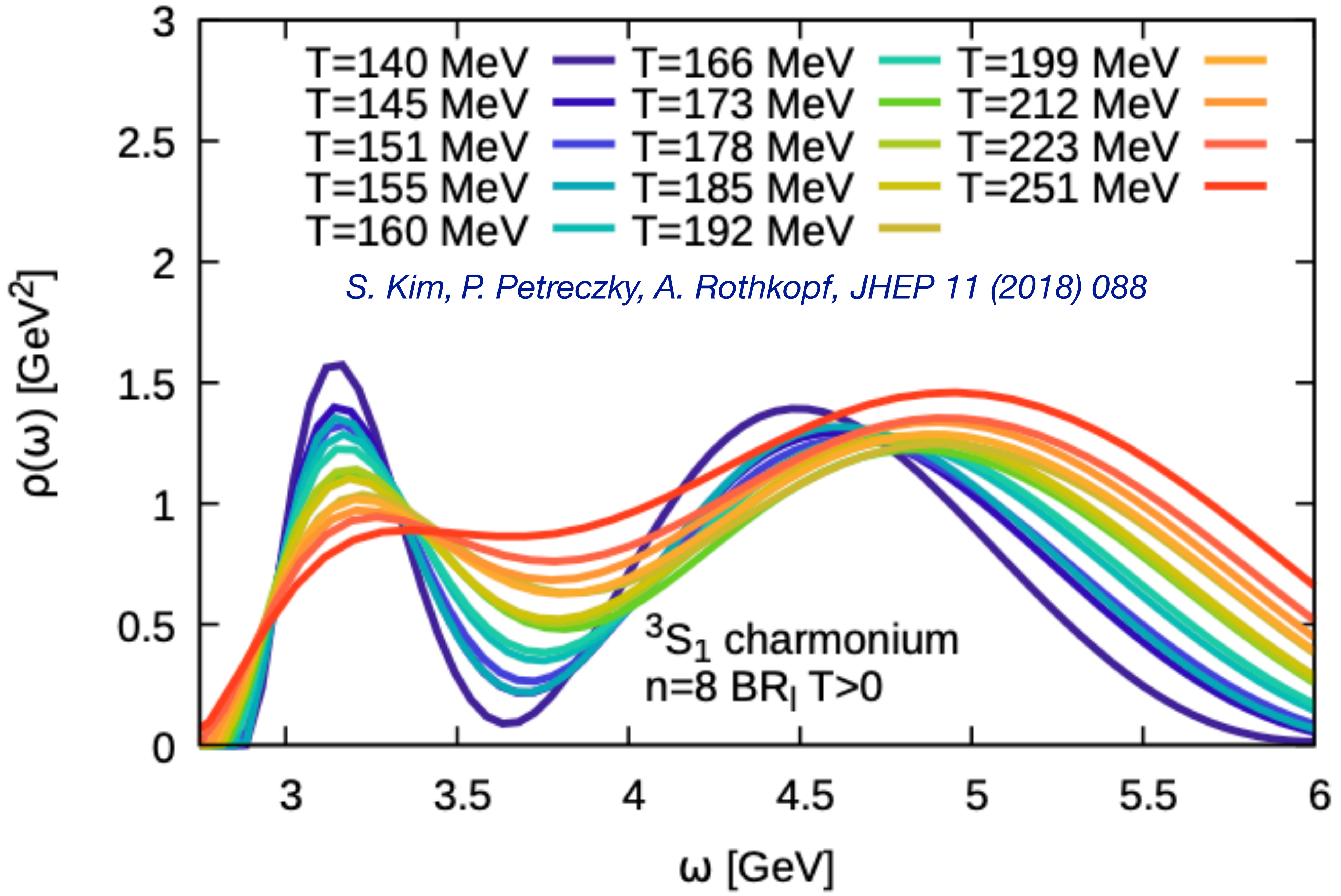


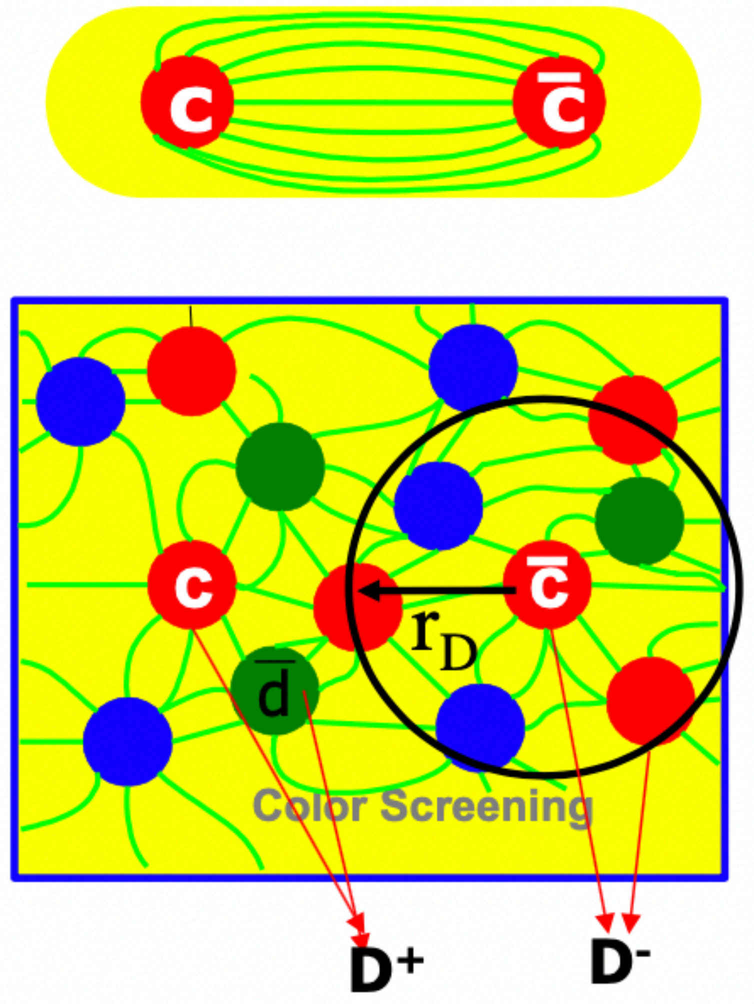
peak position shifts and becomes broader as temperature increases.

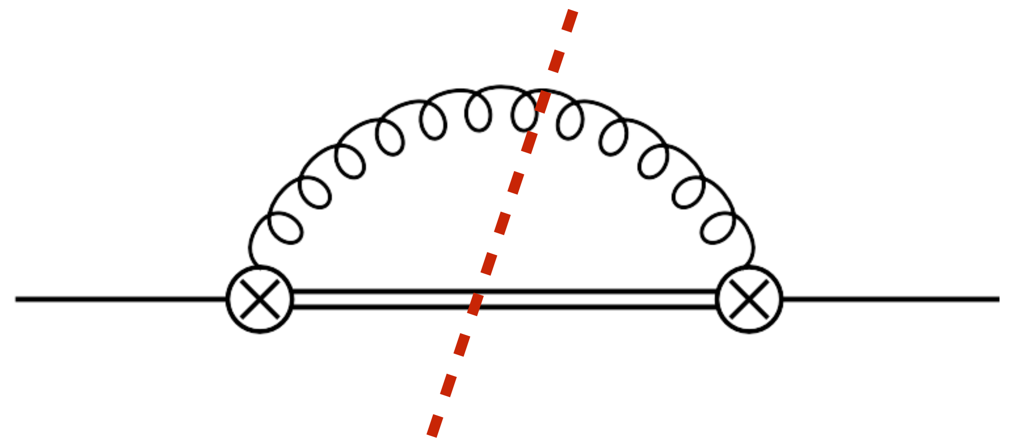
In the perturbative point of view:

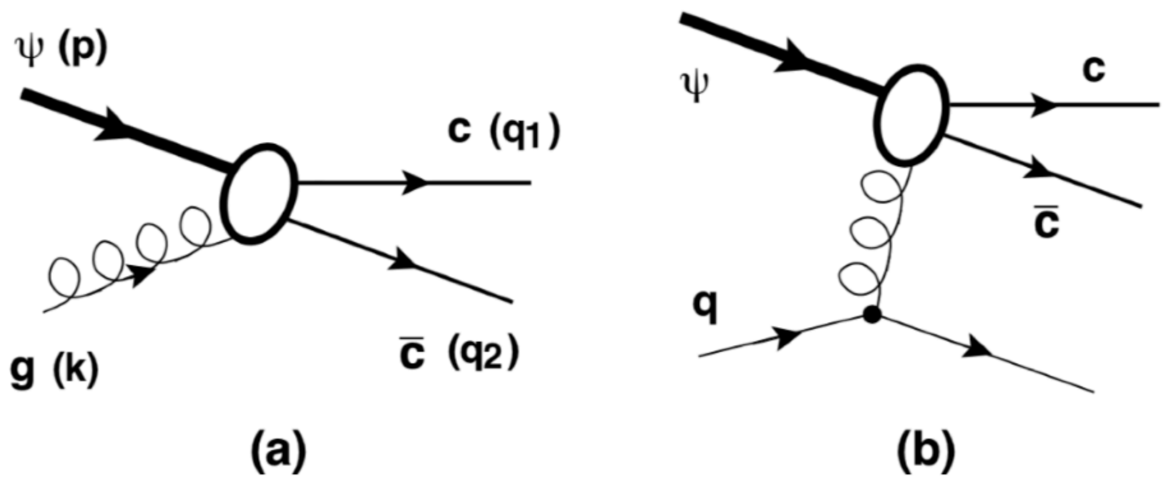
Mass shift -> static color screening

Singlet-octet thermal break up -> gluon-dissociation

Landau damping-> inelastic scattering (quasifree limit)







N. Brambilla, M. Escobedo, J. Ghiglieri, M. Laine, O. Philipsen, P. Romatschke, M. Tassler, P. Petreczky, et al, JHEP 03, 054 (2007). PRD 78, 014017 (2008). JHEP 09, 038 (2010). JHEP 1112 (2011) 116…

Heavy Quark Potential at finite temperature

In-medium properties can be absorbed in a temperature-dependent heavy quark potential.

If the heavy quarks interact with the medium for a very long time, the potential is equivalent to the free energy (given by LQCD).

How the heavy potential is modified at scales comparable to the internal time scale of quarkonium?

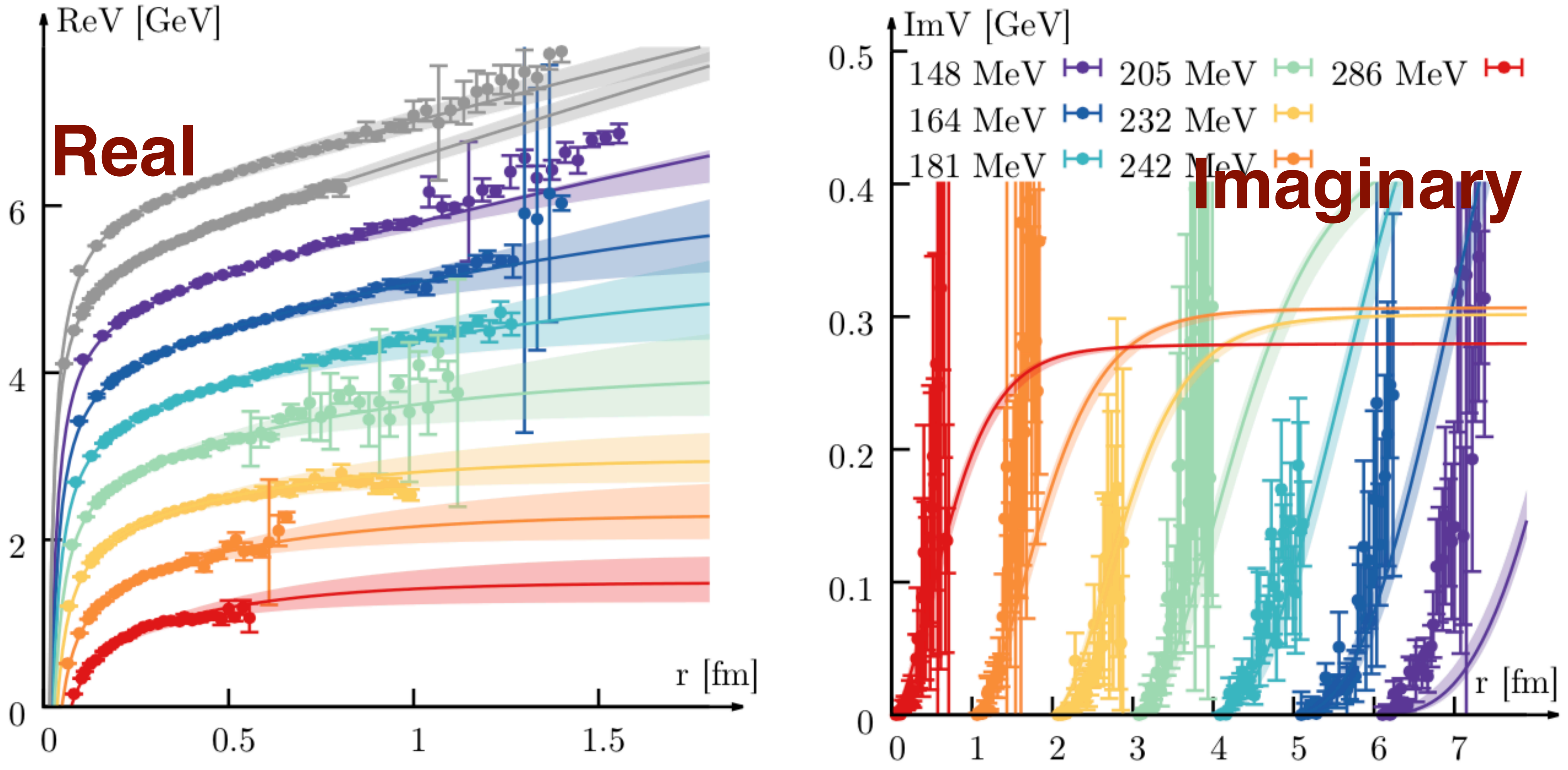
✤ In the weak-coupling regime (High temperature–> HTL,…)

M. Laine, O. Philipsen, P. Romatschke, M. Tassler, JHEP 03 (2007) 054

$$V(r,T) \;=\; -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_{\rm D} r) \qquad \phi(x) = 2\int_0^\infty \frac{{\rm d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

Real *Imaginary*

✤ In the strong-coupling regime (Lattice QCD,…)



D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .

Obvious screening for the real part potential, the imaginary part larger than HTL results.

Heavy Quark Potential at finite temperature

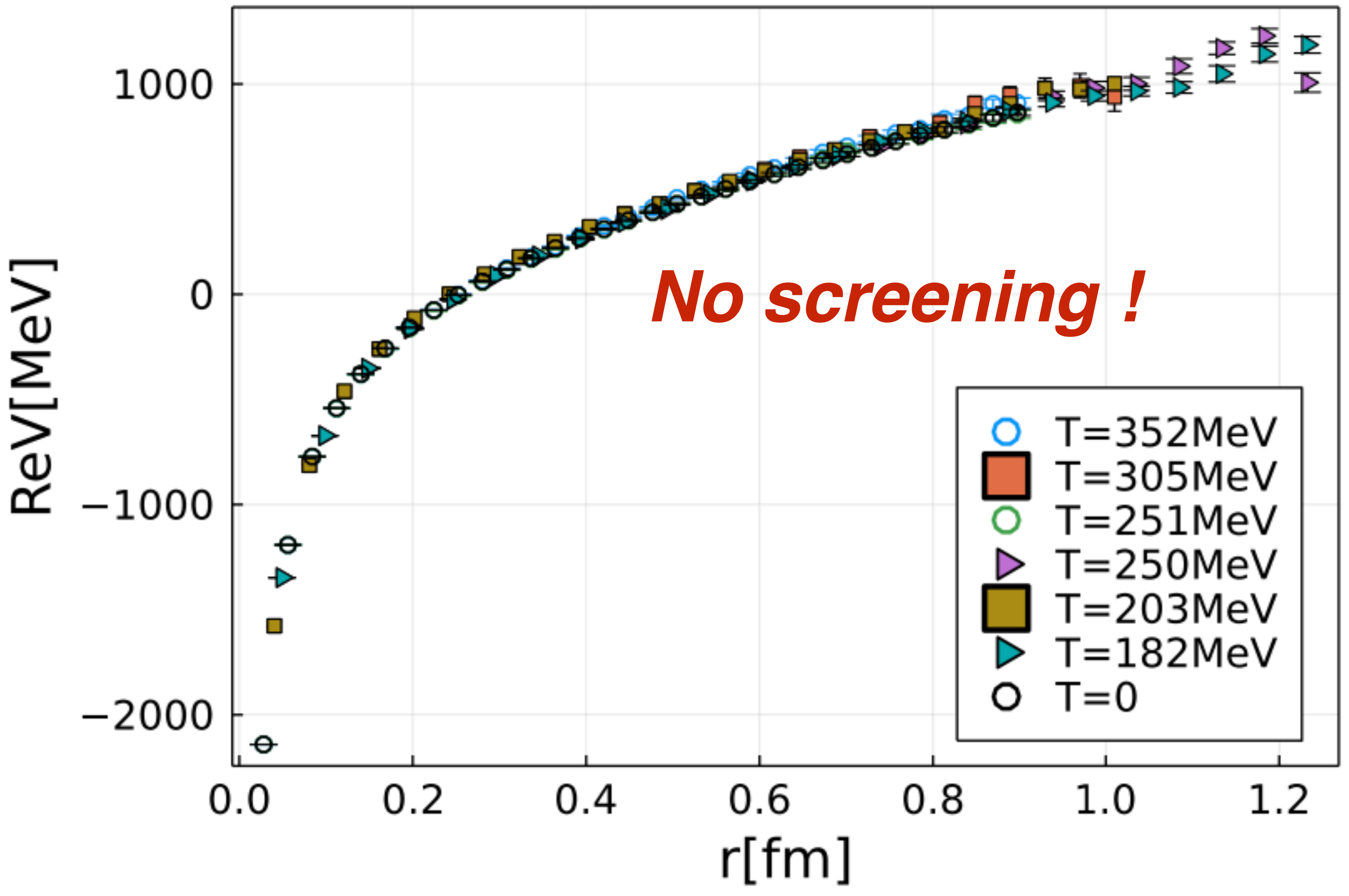
Reconstructing spectral functions through Euclidean correlation functions is an ill-posed inverse problem. Big difference caused by the extraction strategy !

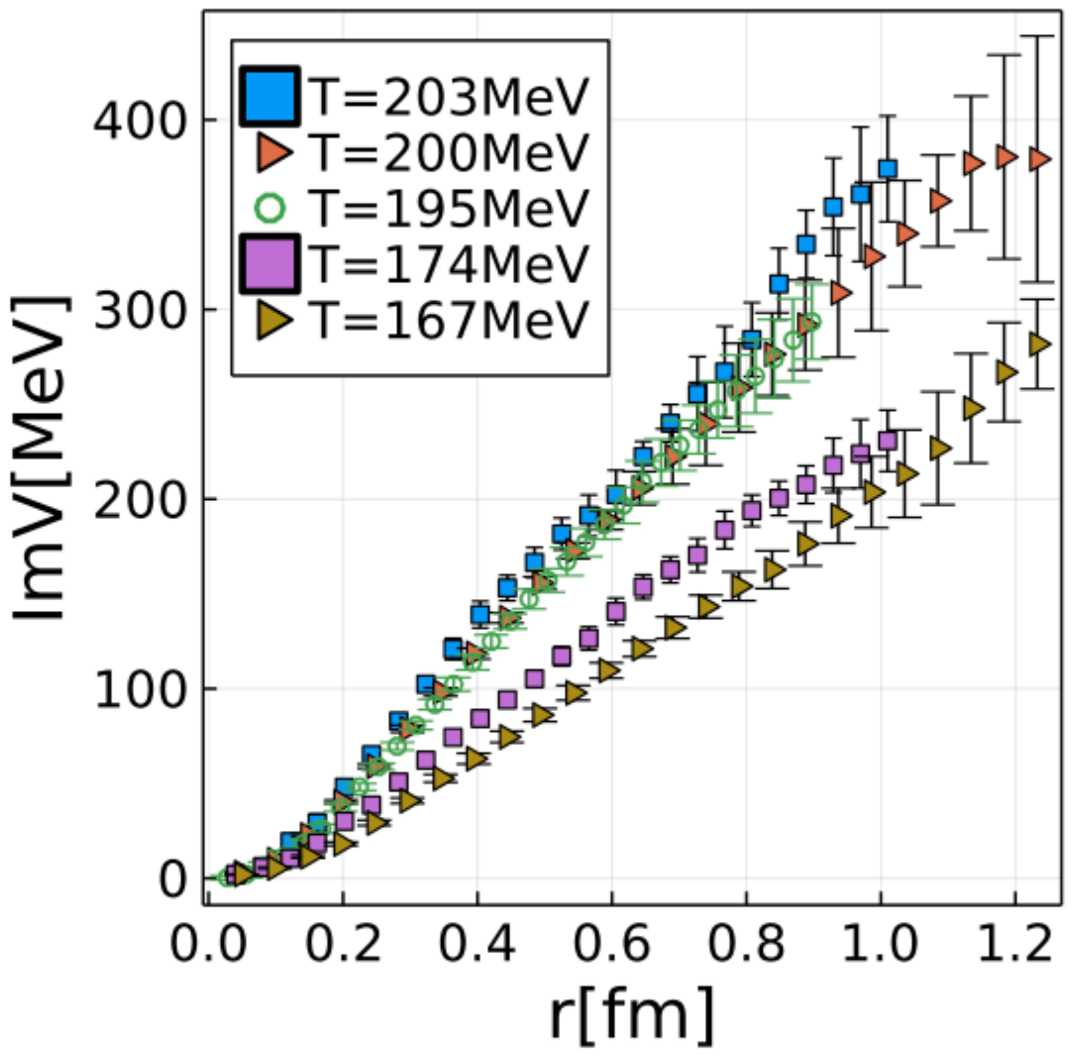
Extract the spectral functions from correlators with four different methods:

D. Bala et al, Phys.Rev.D 105, 054513 (2022).

1. Gaussian fit;
2. HTL inspired fit;
3. Pade fit;
4. Bayesian reconstruction (BR) method.

A physically appealing parametrization of spectrum -> Lorentzian form:





A.Bazavov, D. Hoying, O. Kaczmarek, R.N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf, J.H. Weber , Phys.Rev.D 109, 074504 (2024)

Lattice QCD with dynamical fermions indicates no screening in static quark-antiquark potential !

Heavy Quark Potential at finite temperature

Also supported by many recent studies:

1. Extraction of the HQ Potential from bottomonium mass and width (Lattice NRQCD)

   *S. Shi, K. Zhou, **JZ**, S. Mukherjee, and P. Zhuang. PRD 105 (2022) 1, 1.*

2. Extraction of the HQ Potential from Bottomonium Observables (R_{AA})

   X. Du, S. Liu, R. Rapp. Phys.Lett.B 796 (2019) 20-25.

3. Extraction of the potential by fitting the Wilson line correlators and EOS in T-matrix approach

   Z. Tang, S. Mukherjee, P. Petreczky, R. Rapp. Eur.Phys.J.A 60 (2024) 4, 92.

4. HQ potential with HTL resummed perturbation method within the Gribov-Zwanziger approach

   *W. Wu, G. Huang, **JZ**, P. ZHuang. PRD 107 (2023) 11, 114033*

   M.Debnath, R.Ghosh and N.Haque, Eur.Phys.J.C 84 (2024) 3, 313

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   M.Debnath, R.Ghosh and N.Haque, Eur.Phys.J.C 84 (2024) 3, 313

New heavy quark potential: No /a little color screening for the real part and a large imaginary part !

A different picture of quarkonium melting in the QGP -> dynamic dissociation plays an dominant role

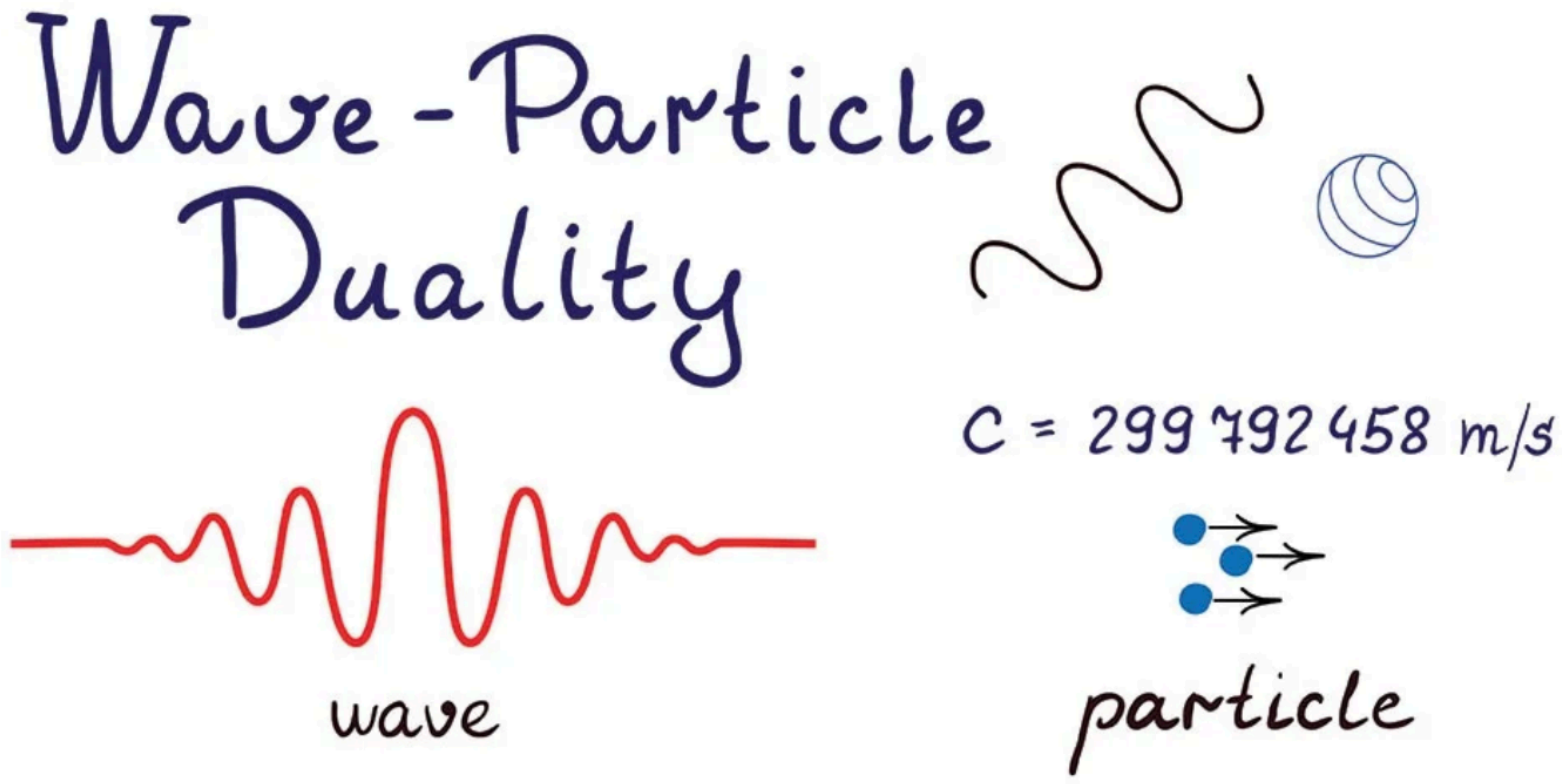
How to distinguish in the experiment? Which observable?

More phenomenological studies with quarkonium real-time evolution in the QGP are needed !

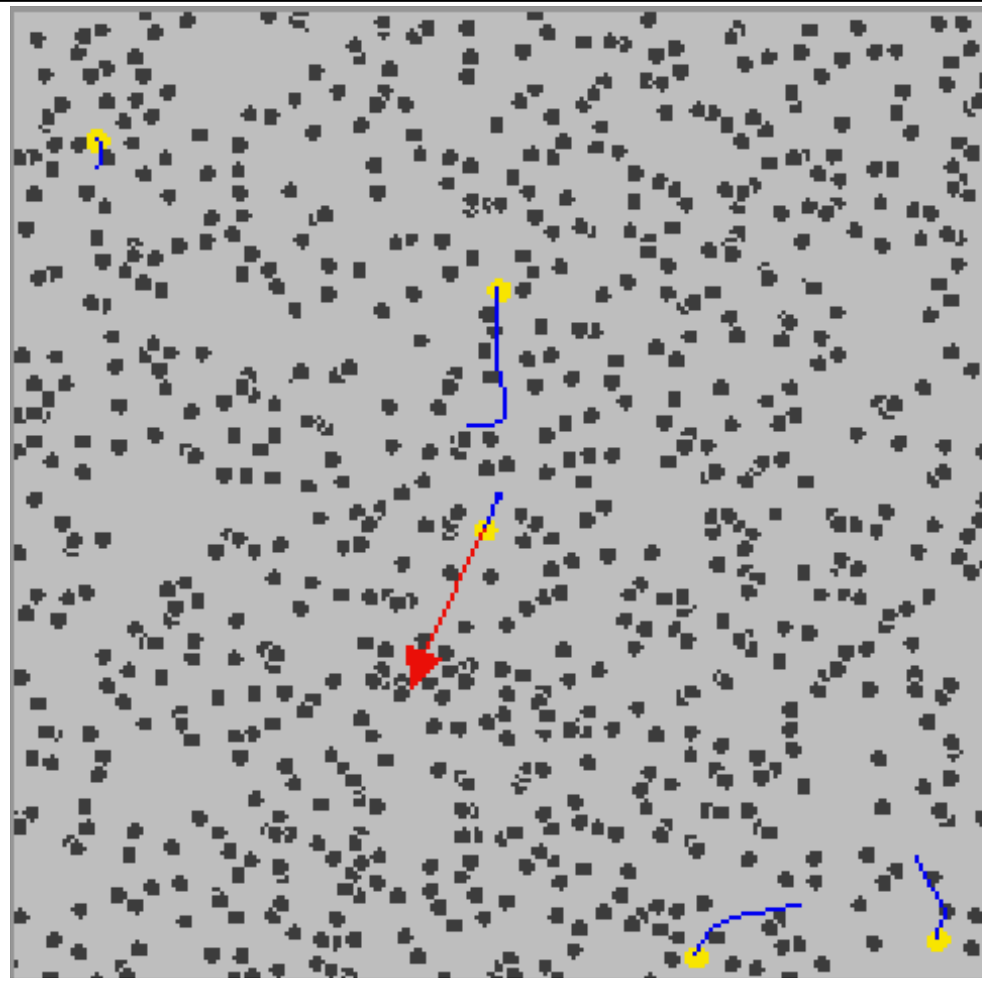
Quarkonium real-time evolution in HICs

Quarkonium real-time evolution in heavy-ion collisions

Is quarkonium a wave or a particle in heavy ion collisions ?



Quarkonium real-time evolution in heavy-ion collisions



Assume the quarkonium is a **classical particle**!

Charmonium are not fully dissociated. Dissociation and regeneration happen gradually in QGP.

➡Transport description (Boltzmann equation; Tsinghua model)

$$p^\mu \partial_\mu f_\psi = -\alpha E f_\psi + \beta E$$

$$\alpha = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g} W_{g\psi}^{c\bar{c}}(s) f_g(p_g, x)$$ Gluon-dissociation

$$\beta = \frac{1}{2E_T}\int \frac{d^3\mathbf{p}_g}{(2\pi)^3 2E_g}\frac{d^3\mathbf{p}_c}{(2\pi)^3 2E_c}\frac{d^3\mathbf{p}_{\bar{c}}}{(2\pi)^3 2E_{\bar{c}}} W_{c\bar{c}}^{g\psi}(s) f_c(p_c, x) f_{\bar{c}}(p_{\bar{c}}, x)(2\pi)^4 \delta^{(4)}(p + p_g - p_c - p_{\bar{c}})$$ Regeneration

Dissociation and regeneration are related to each other via the detailed balance.

➡Transport description (Rate equation; TAMU model)

$$\frac{dN_\psi(\tau)}{d\tau} = -\Gamma_\psi [N_\psi(\tau) - N_\psi^{\rm eq}(\tau)]$$

Dissociation rate

Include both gluon-dissociation and NLO (quasifree) process

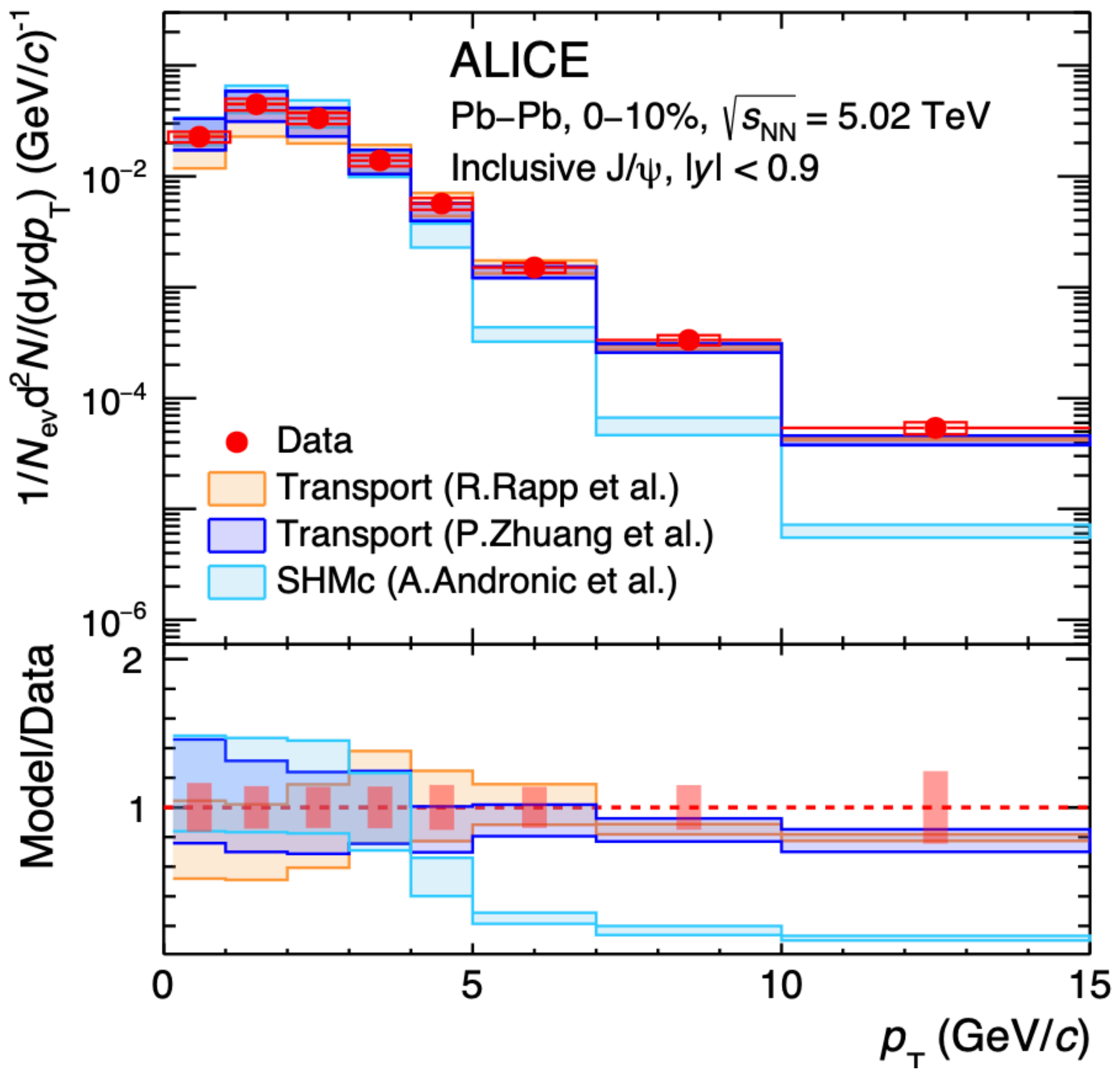
equilibrium limit of each state (Satisfied obviously.)

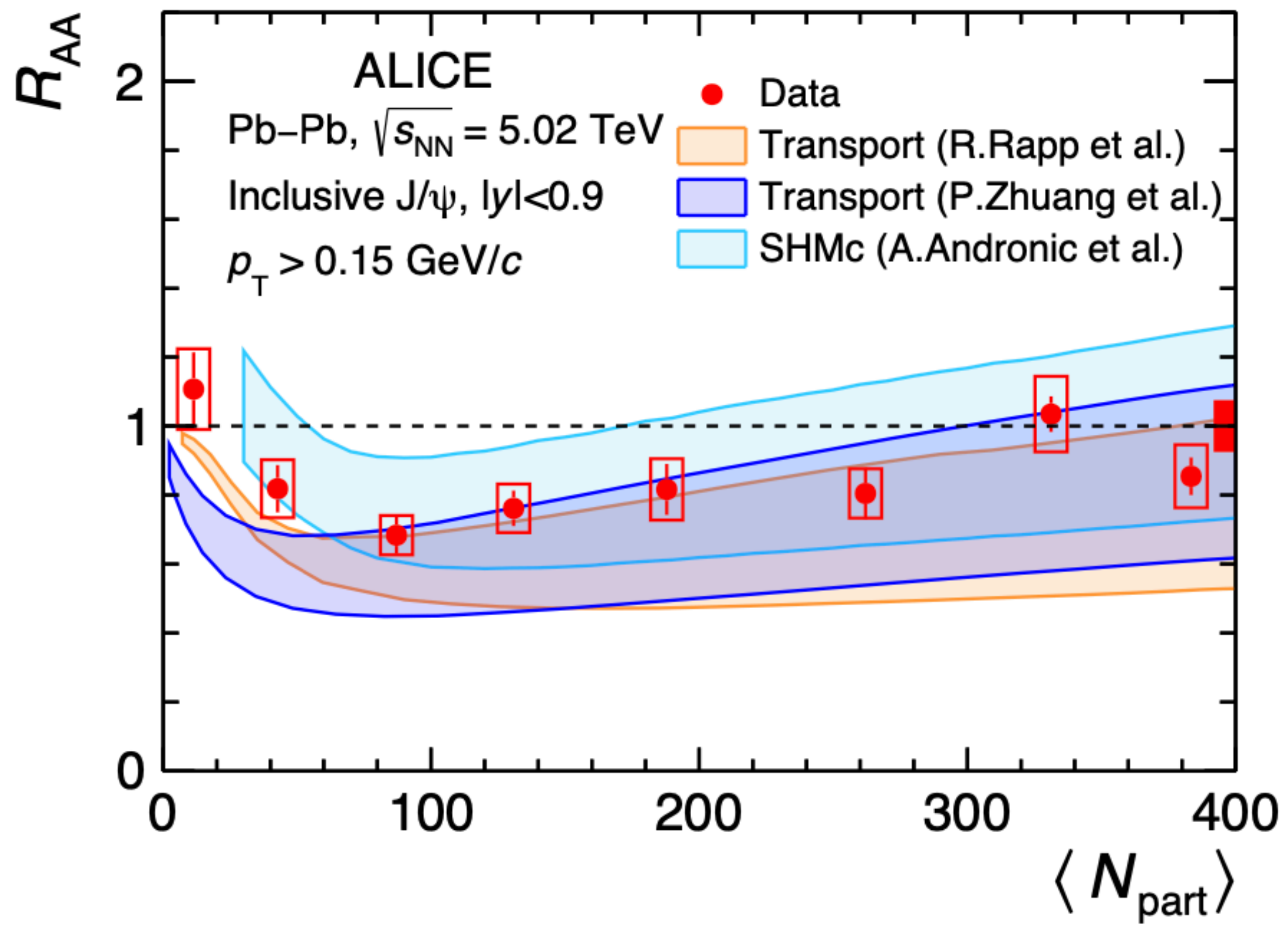
$$N_\psi^{\rm eq} = g_c^2 N^{\rm eq}$$

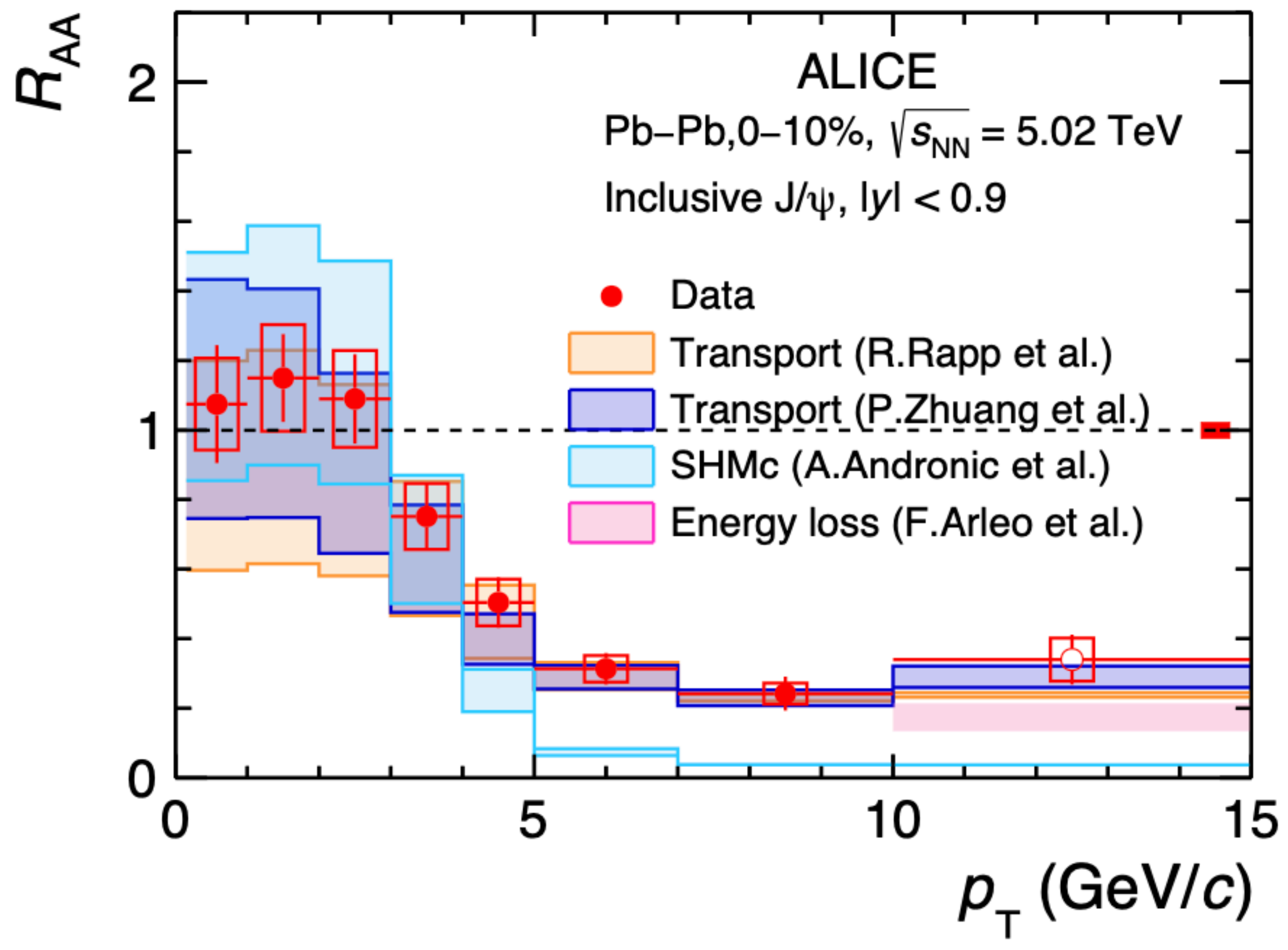
Quarkonium real-time evolution in heavy-ion collisions

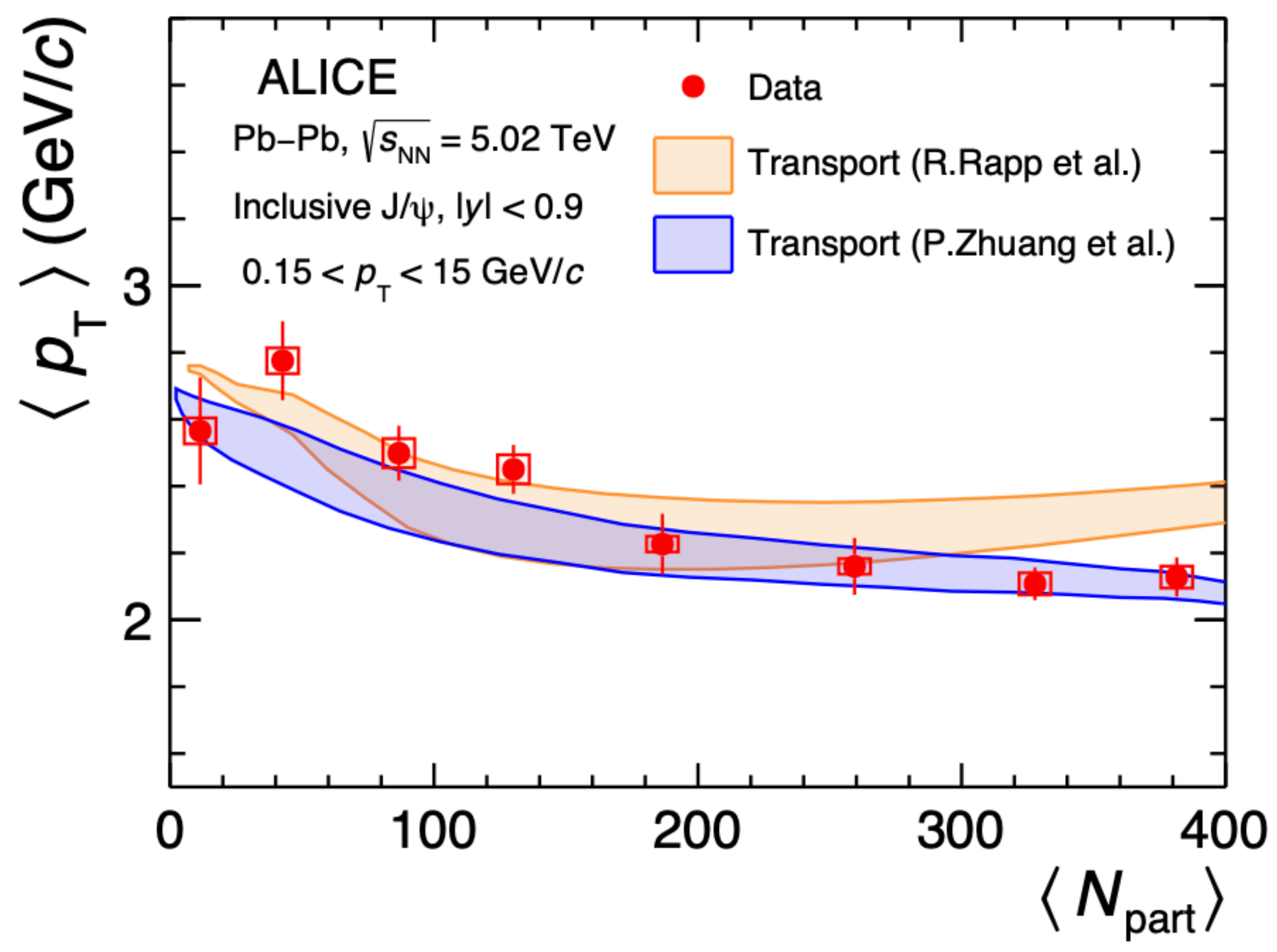
Assume the quarkonium is a **classical particle**!

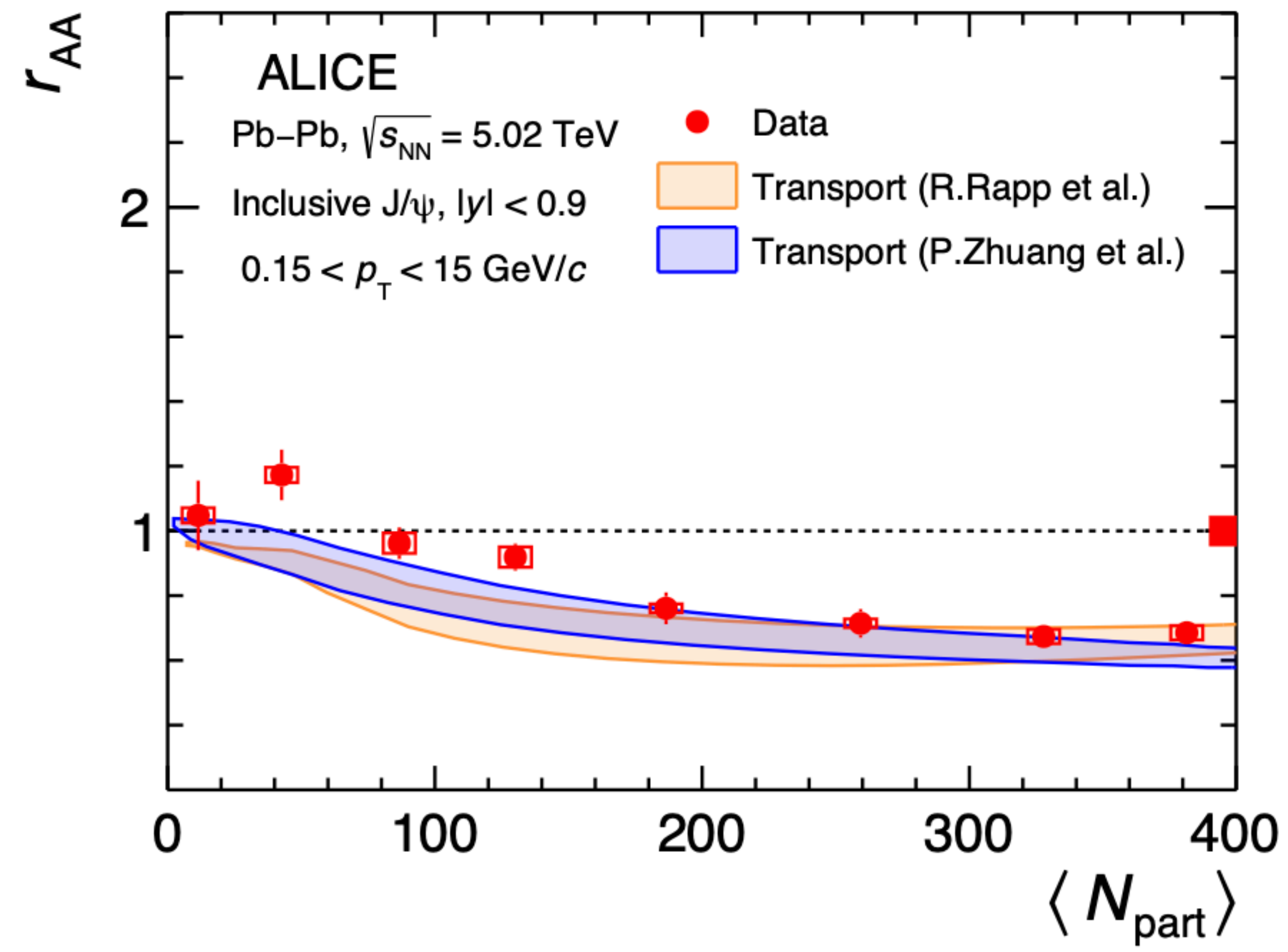
Models (such as TAMU, Tsinghua, Comover) can explain the experimental observables, like charmonium, spectra, R_{AA}, v_2, r_{AA}, $\langle p_T \rangle$ …**compared to the recent ALICE results.**











ALICE Collaboration, Phys.Lett.B 849 (2024) 138451

Quarkonium real-time evolution in heavy-ion collisions

Assume the quarkonium is a **classical particle**!

Models (such as TAMU, Tsinghua, Comover) can explain the experimental observables, like charmonium, spectra, R_{AA}, v_2, r_{AA}, $\langle p_T \rangle$ ….**new progress**

1. Directed flow v_1, elliptic flow v_2, and triangular flow v_3 of J/ψ.
2. Quarkonium polarization.
3. Probe the initial nuclear deformation.
4. Quarkonium with EM fields.
5. B_c, X(3872)

…

*B. Chen, M. Hu, H. Zhang, and **JZ**, PLB802 (2020) 135271; **JZ**, B. Chen, and P. Zhuang, PRC 105 (2022) 3, 034902*

D.Yang and X.Yao, arXiv:2405.20280; Y. Zhao, X. Sheng, S. Li and D.Hou, arXiv:2403.07468; JZ and B. Chen, arXiv:2312.01799

***JZ** and S. Shi, Eur.Phys.J.C 83 (2023) 6, 511.*

*See review: S. Iwasaki, M. Oka and K. Suzuki, Eur.Phys.J.A 57 (2021) 7, 222; **JZ**, K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

*B. Wu, Z. Tang, M. He, R. Rapp, Phys. Rev. C 109(1), 014906 (2024). B. Wu, X. Du, M. Sibila, R. Rapp, Eur. Phys. J. A 57(4), 122 (2021). **JZ** and P. Zhuang, arXiv: 2209.13475.*

A.Esposito, E. Ferreiro, A.Pilloni, A. Polosa and C. Salgado, Eur.Phys.J.C 81 (2021) 7, 669. Y. Guo, X. Guo, J. Liao, E. Wang and H. Xing, arXiv:2302.03828.

Quarkonium real-time evolution in heavy-ion collisions

Are quantum effects important? What are they?

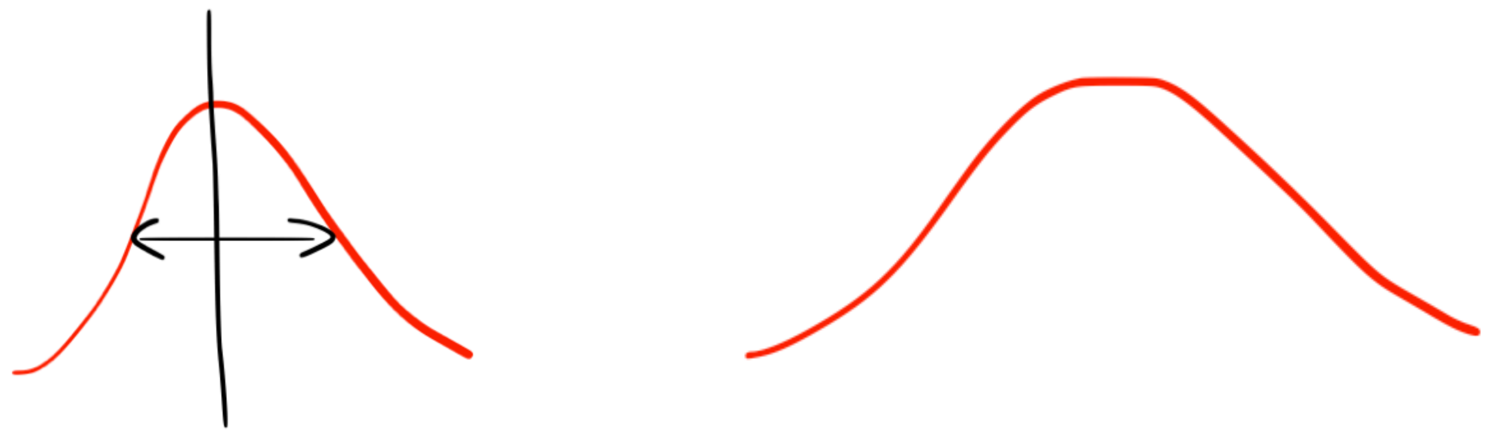
- Quantum coherence and decoherence

  Superposition state of various eigen states,…
  Usually absorbed into a phenomenological parameter "formation time" in the transport approach.

$$\Psi = \sum_i c_i \psi_{nl}$$

- Define and evolve a particle with a large width in the hot medium

$$\Gamma \gtrsim E$$



- Spin related physics, such as polarization

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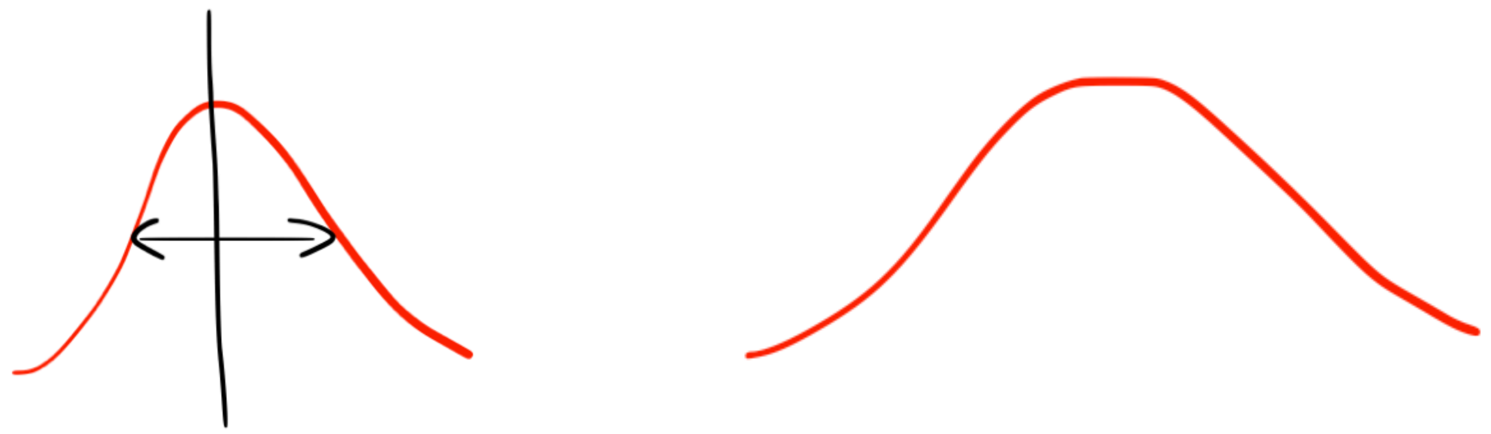
- Define and evolve a particle with a large width in the hot medium

$$\Gamma \gtrsim E$$



- Spin related physics, such as polarization

Assume the quarkonium is a **quantum wavefunction**!

Models (such as time-dependent Schrödinger equation + complex potential) have been used to describe the bottomonium evolution and production in heavy ion collisions.

no regeneration from uncrorrelated $b\bar{b}$ ($\leq$ 1 pair/event)

A. Islam and M. Strickland, JHEP 21, 235 (2020); Phys.Lett.B 811 (2020) 135949;
*L. Wen and B. Chen, Phys. Lett. B 839, 137774 (2023); G. Chen, B. Chen and **JZ**, arXiv:2402.11316;…*

Quarkonium real-time evolution in heavy-ion collisions

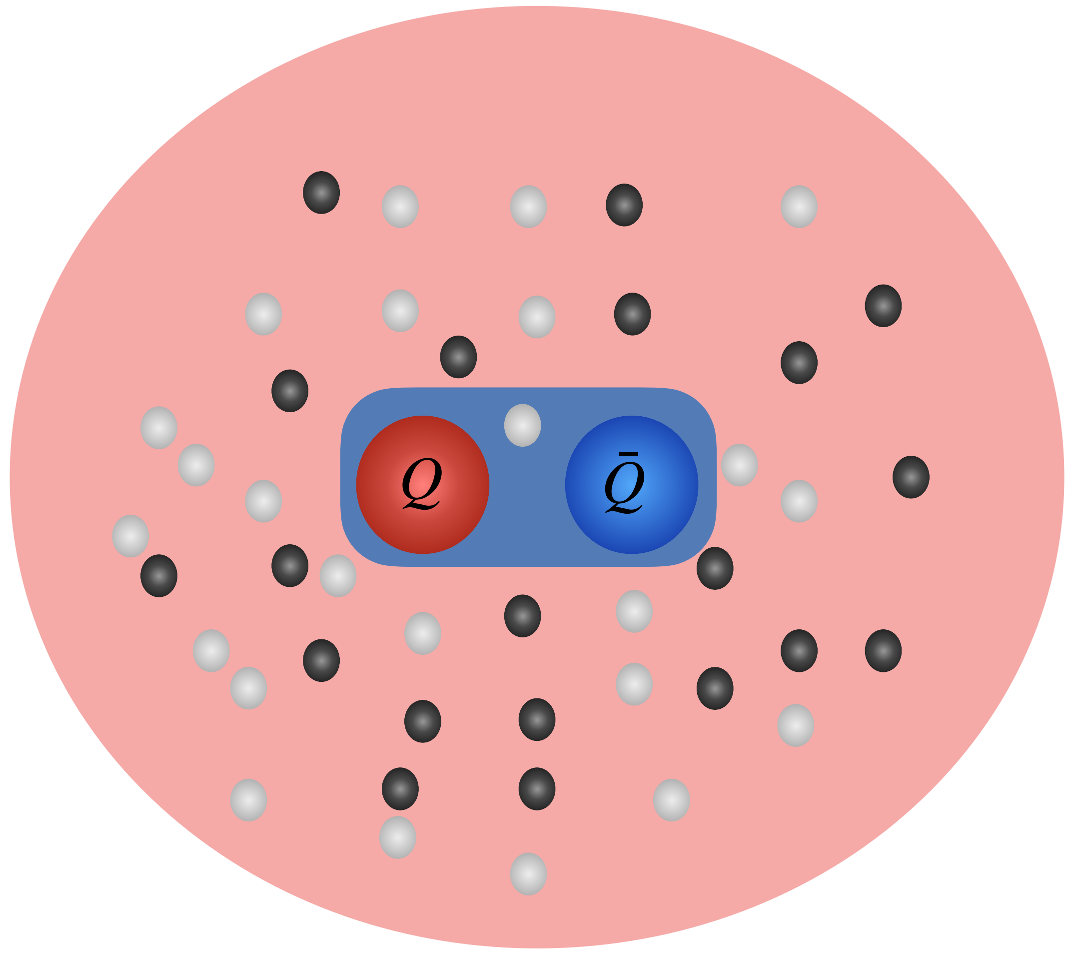
“pure” state (**wavefunction**) -> “mixed” state (**density operator**)

Open quantum system (OQS)

$$\hat{\rho}_{tot} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \quad \text{von Neumann equation:} \quad \frac{d\hat{\rho}_{tot}}{dt} = -i[\hat{H}_{tot}, \hat{\rho}_{tot}]$$

$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int},$$

Subsystem **Environment** **Interaction**



Trace over the environment degrees of freedom :

Quantum master equation

$$i\hbar\dot{\hat{\rho}}_s(t) = \mathrm{Tr}_e[\hat{H}_{tot}, \hat{\rho}_{tot}] = [\hat{H}_s, \hat{\rho}_s] + \mathrm{Tr}_e[I_s \otimes \hat{H}_e + \hat{H}_{int}, \hat{\rho}_{tot}]$$

➡ Separation of time-scales:

Environment relaxation time scale $\tau_e \sim \frac{1}{\pi T}.$

Intrinsic time scale of subsystem $\tau_s \sim \frac{1}{E_{bind}}.$

Subsystem relaxation time scale $\tau_r \sim \frac{1}{\eta} \approx \frac{M}{T^2}.$

Quarkonium real-time evolution in heavy-ion collisions

$\tau_e \sim 1/(\pi T)$, $\tau_s \sim 1/E_{bind}$, $\tau_r \sim M/T^2$

Non-Markovian

Markovian approximation: $\tau_e \ll \tau_r$ **, memory lose; reasonalbe for HICs**

Quantum master equation –> Lindblad equation

T

$\sim m_Q$

$\tau_e \ll \tau_s$

$(E_{bind} \ll m_D)$

$\tau_s \ll \tau_r$

$(E_{bind} \gg m_D)$

Quantum Brownian Motion

A broader peak (e.g. excited states)

Short quantum decoherence time

Classical limit:

Fokker-Planck equation

Munich-Kent

Nantes

Osaka

…

Quantum optical Limit

Well defined quarkonium state

Long quantum decoherence time

Classical limit:

Boltzmann equation/ Rate equation

TAMU

Tsinghua

Comover

Duke-MIT

…

Recent review papers: *R. Sharma, Eur.Phys.J.ST 230 (2021) 3, 697-718* *Y. Akamatsu, Prog.Part.Nucl.Phys. 123 (2022) 103932.* *X. Yao, Int.J.Mod.Phys.A 36 (2021) 20, 2130010.*

Duke-MIT Approach

- ✦ pNRQCD+OQS works in quantum optical limit $M \gg Mv \gg Mv^2 \gtrsim T \gtrsim m_D$
- ✦ A semi-classical (gradient) expansion and w/o quantum effect anymore
- ✦ Used for bottomonium.

X. Yao, T. Mehen, W. Ke, Y. Xu, S.Bass, B. Muller. Phys.Rev.D 99 (2019) 9, 096028; JHEP 01 (2021) 046.

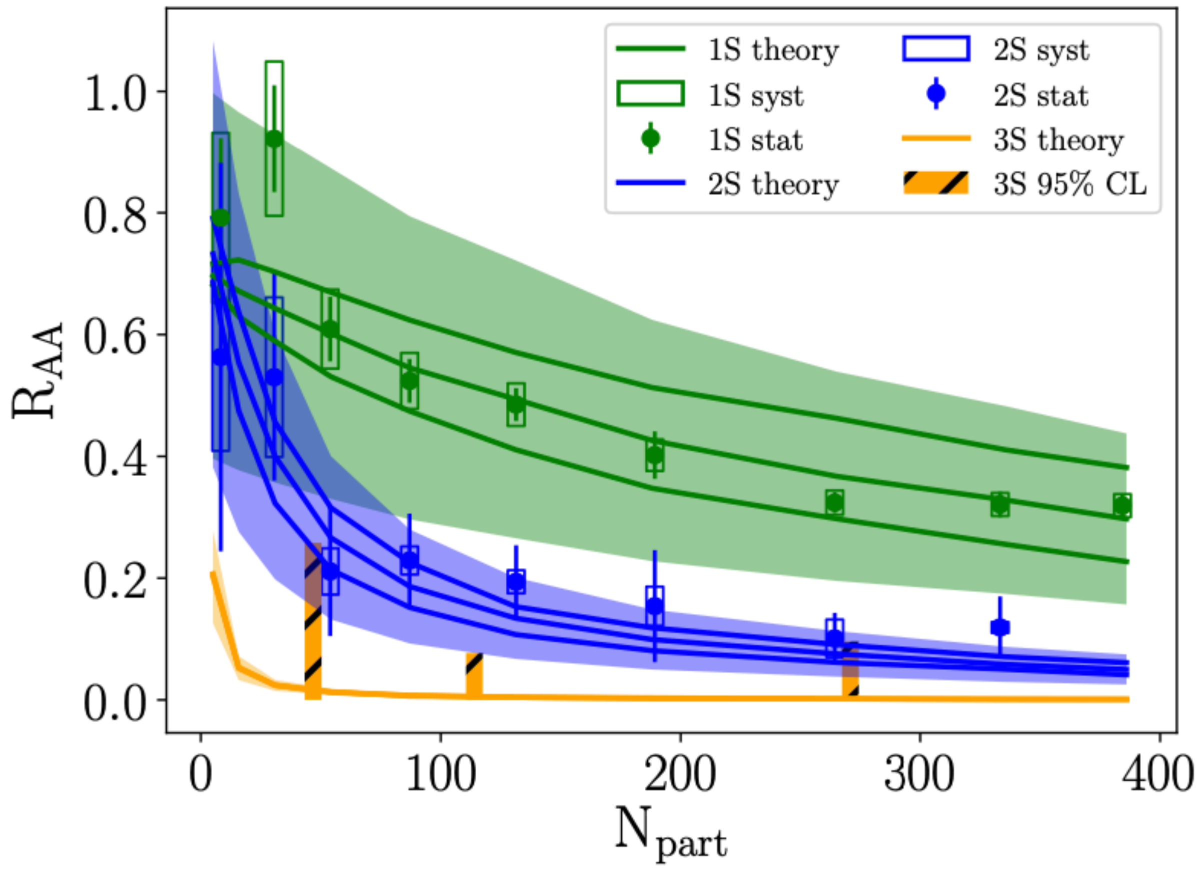
$$\rho_S(t) = \rho_S(0) + \sum_{a,b,c,d} \gamma_{ab,cd}(t)\left(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S(0)\}\right) - i\sum_{a,b}\sigma_{ab}(t)[L_{ab}, \rho_S(0)] + \mathcal{O}(H_I^3)\,.$$

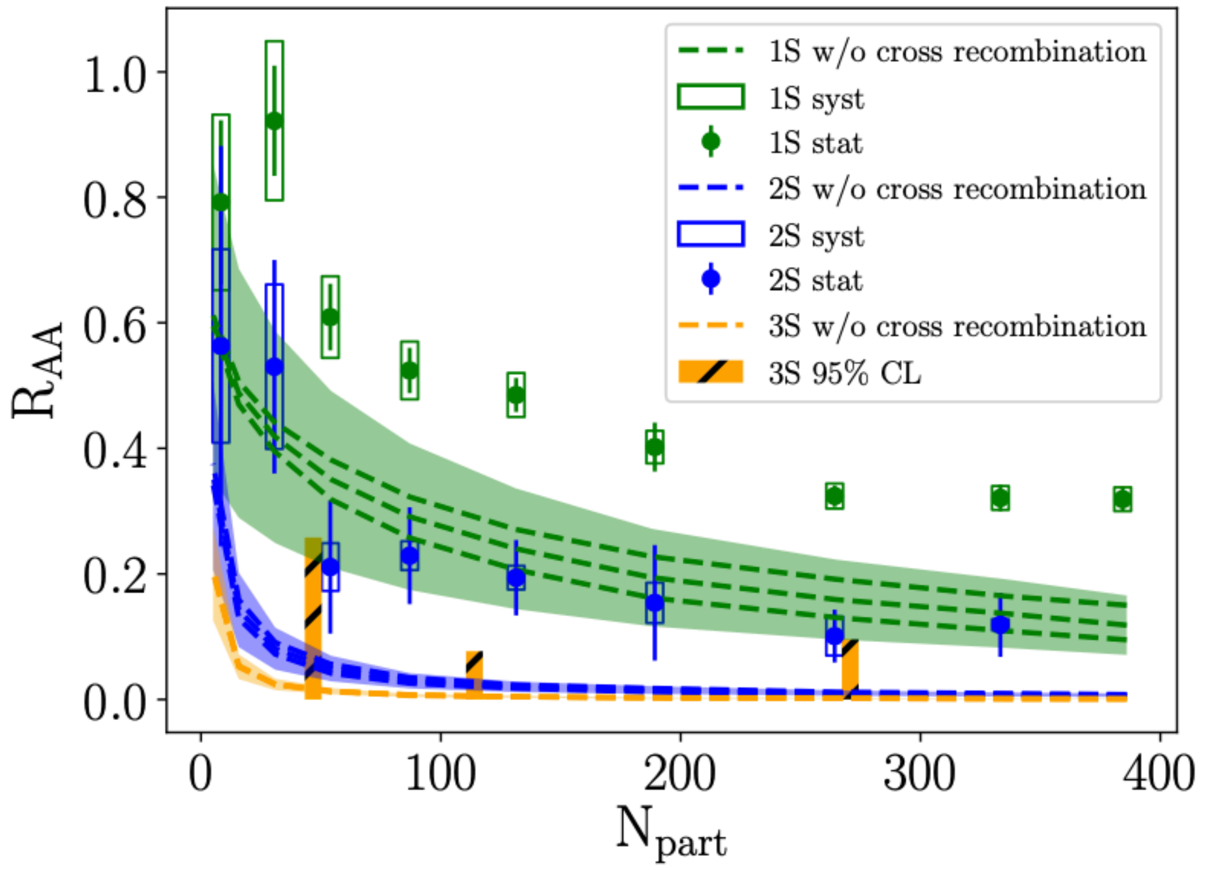
Lindblad equation

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

Similar to the TAMU and Tsinghua model





Importance of recombination from correlated $b\bar{b}$!

Gives a connection between the OQS and Boltzmann equation in the quantum optical limit!

Munich-Kent Approach

- **pNRQCD+OQS works in Quantum Brownian motion Regime** $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{bind}$
- **Expansion of E_{bind}/T from LO to NLO; the quantum jumps are now implemented.**
- **Used for bottomonium.**

N.Brambilla, M.Escobedo, M.Strickland, A.Vairo, J.Weber, Phys.Rev.D 104 (2021) 9, 094049; JHEP 05 (2021) 136;JHEP 08 (2022) 303; Phys.Rev.D 108 (2023) 1, L011502.

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] + \sum_n \left(C_n \rho(t) C_n^\dagger - \frac{1}{2}\left\{ C_n^\dagger C_n, \rho(t) \right\} \right)$$

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix},$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix},$$

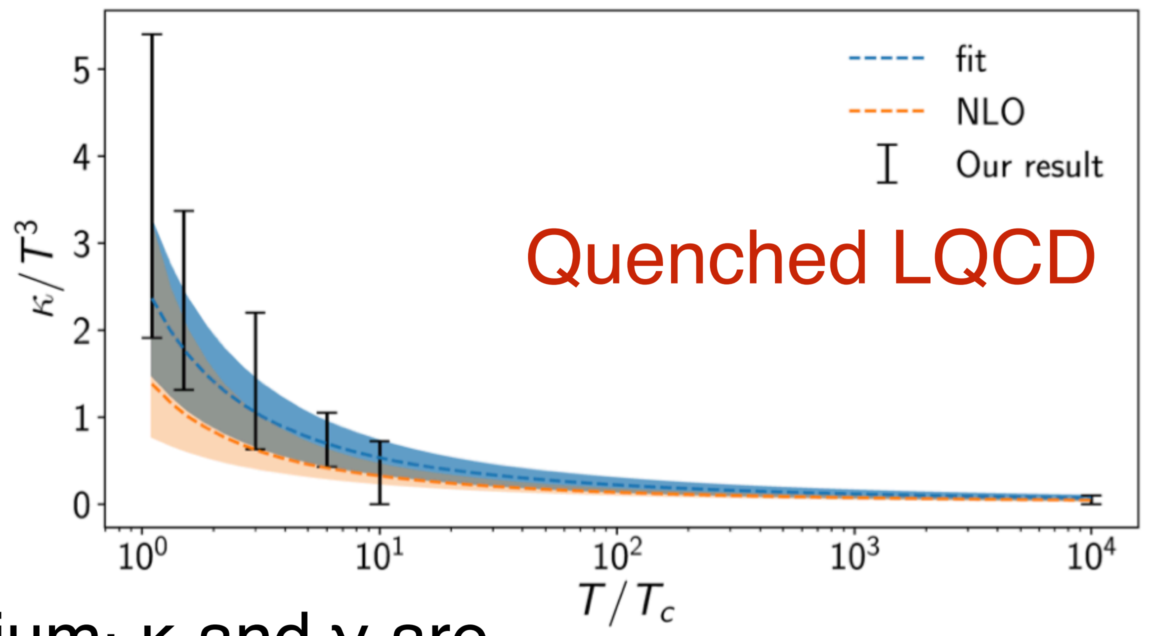
$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Quantum jump

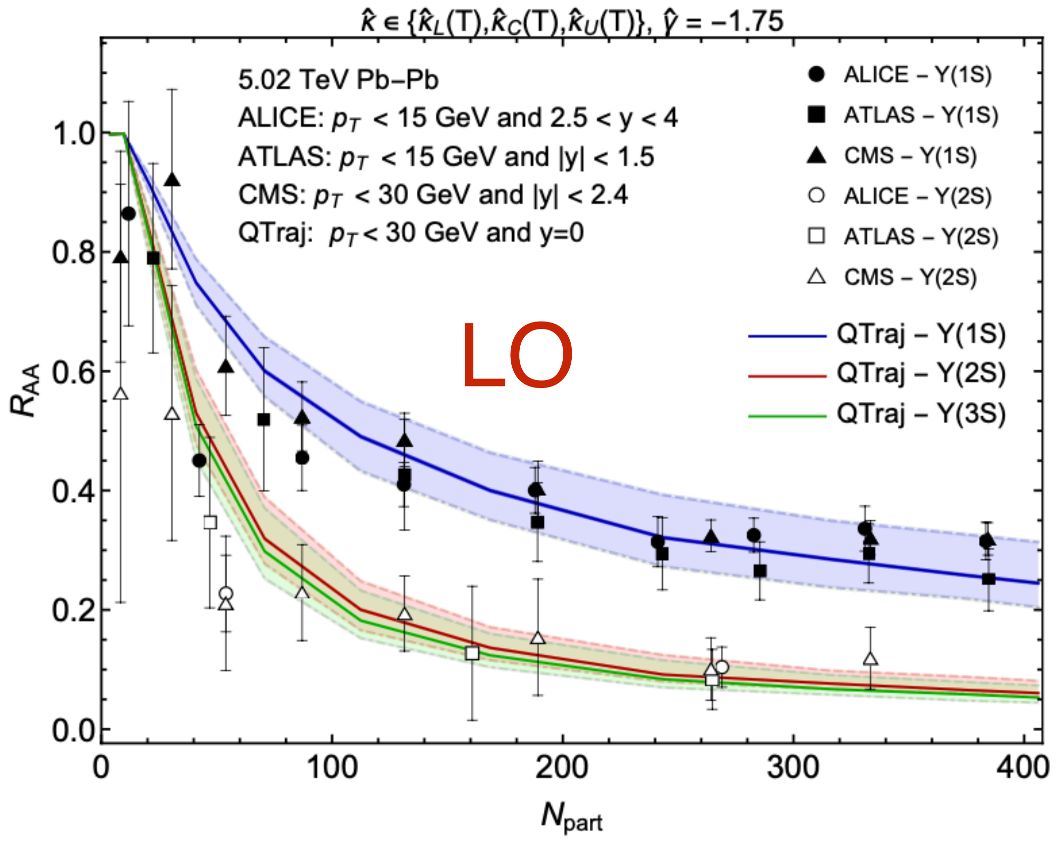
$$\kappa = \frac{g^2}{18}\int_0^\infty dt \left\langle \left\{ \tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right\} \right\rangle,$$

$$\gamma = -i\frac{g^2}{18}\int_0^\infty dt \left\langle \left[\tilde{E}^{a,i}(t,\mathbf{0}), \tilde{E}^{a,i}(0,\mathbf{0}) \right] \right\rangle$$

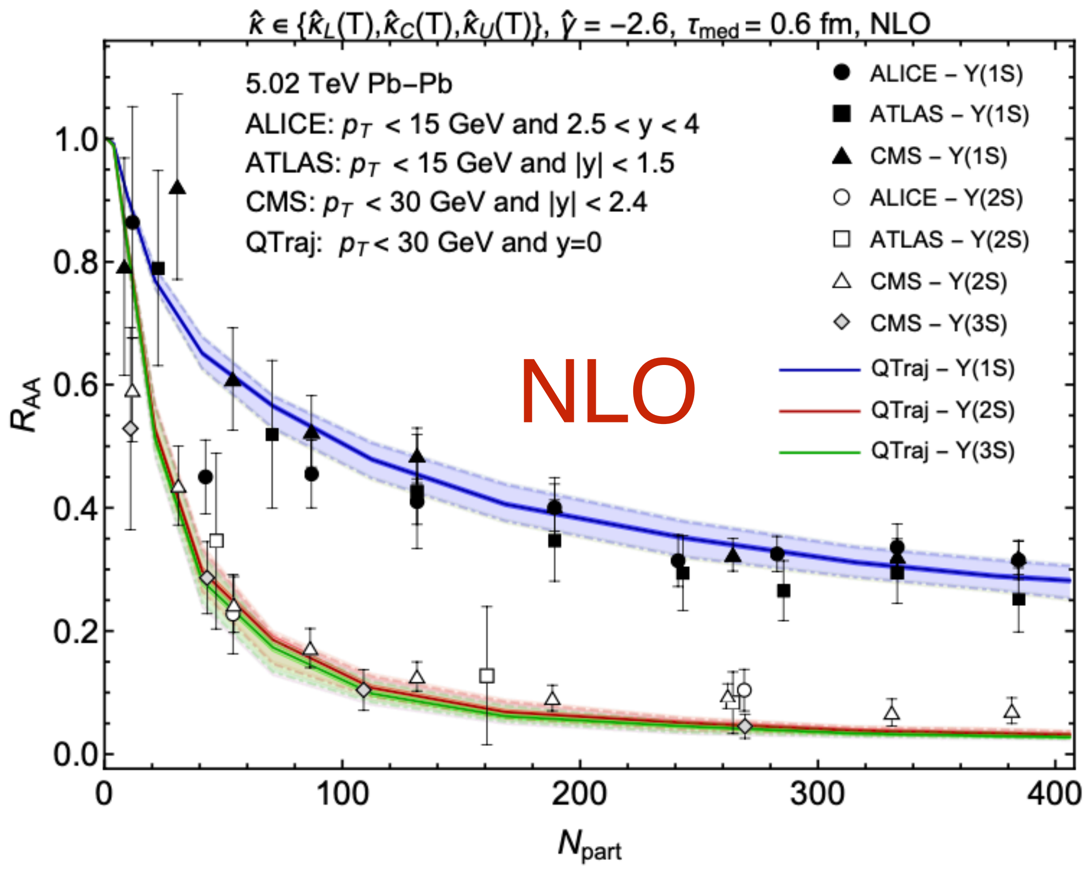


Strong coupling between heavy quark and medium; κ and γ are related to the thermal width Γ and mass shift δM of the bottomonium.

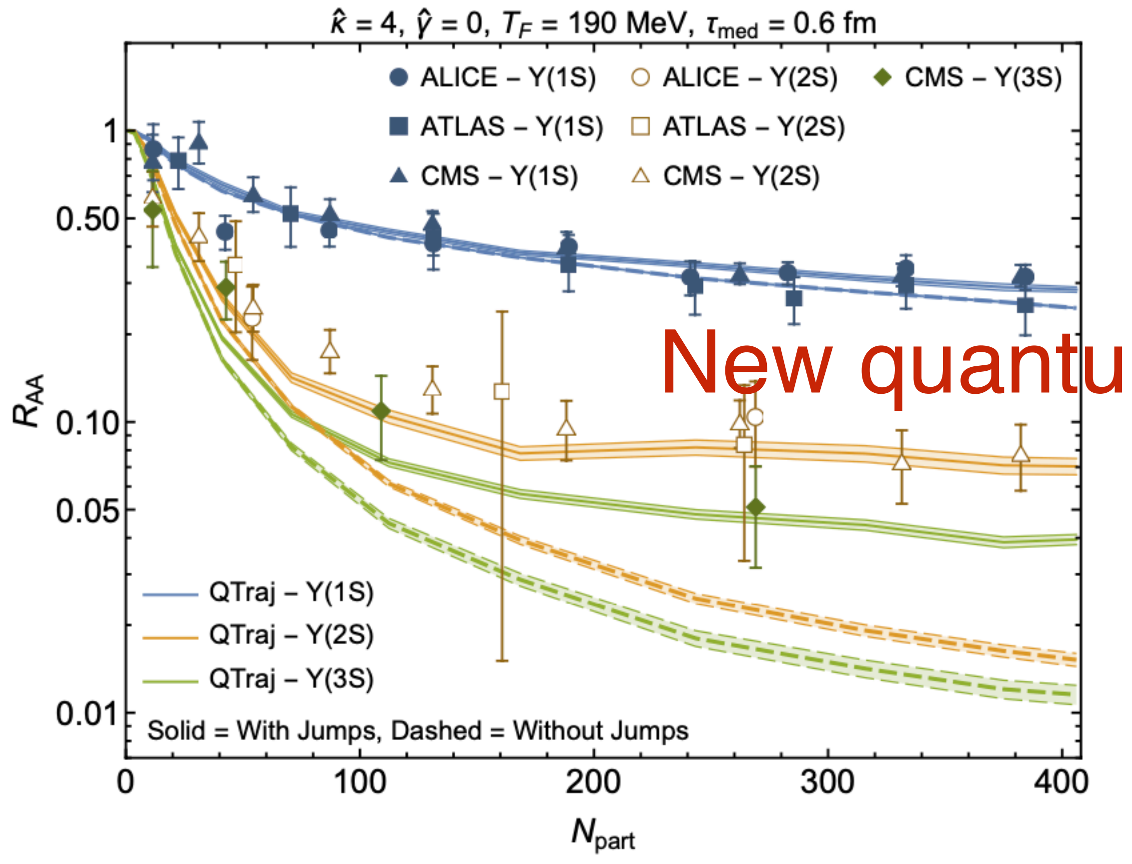
T_f = 250MeV



T_f = 190MeV



T_f = 190MeV, $\hat{\kappa} = 4, \hat{\gamma} = 0$ -> w/o screening



The new results with quantum jumps and w/o color screening agree well with the R_{AA} and double ratios!

Nantes Approach

- ✦ **NRQCD+OQS works in Quantum Brownian motion Regime** $M \gg T \sim m_D \gtrsim E_{bind}$
- ✦ **Expansion of τ_e/τ_s.**
- ✦ **Used for bottomonium and charmonium in 1D.**

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q, \qquad \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

J. Blaizot, M. Escobedo, JHEP **06**, 034 (2018).
S. Delorme, T. Gousset, R. Katz, P.B. Gossiaux, Acta Phys. Pol. B Proc. Suppl. **16**, 1–112 (2023) ;
Eur. Phys. J. A **58**(10), 198 (2022); arXiv:2402.04488.

Real part potential

$$\mathcal{L}_0\mathcal{D} = -i\,[H_Q, \mathcal{D}]$$

$$\mathcal{L}_1\mathcal{D} = -\frac{i}{2}\int_{xx'} V(x-x')\,[n^a_x n^a_{x'}, \mathcal{D}]$$

$$\mathcal{L}_2\mathcal{D} = \frac{1}{2}\int_{xx'} W(x-x')\,(\{n^a_x n^a_{x'}, \mathcal{D}\} - 2n^a_x \mathcal{D} n^a_{x'})$$

fluctuations

$$\mathcal{L}_3\mathcal{D} = -\frac{i}{4T}\int_{xx'} W(x-x')\left(\dot{n}^a_x \mathcal{D} n^a_{x'} - n^a_x \mathcal{D} \dot{n}^a_{x'} + \frac{1}{2}\{\mathcal{D}, [\dot{n}^a_x, n^a_{x'}]\}\right)$$

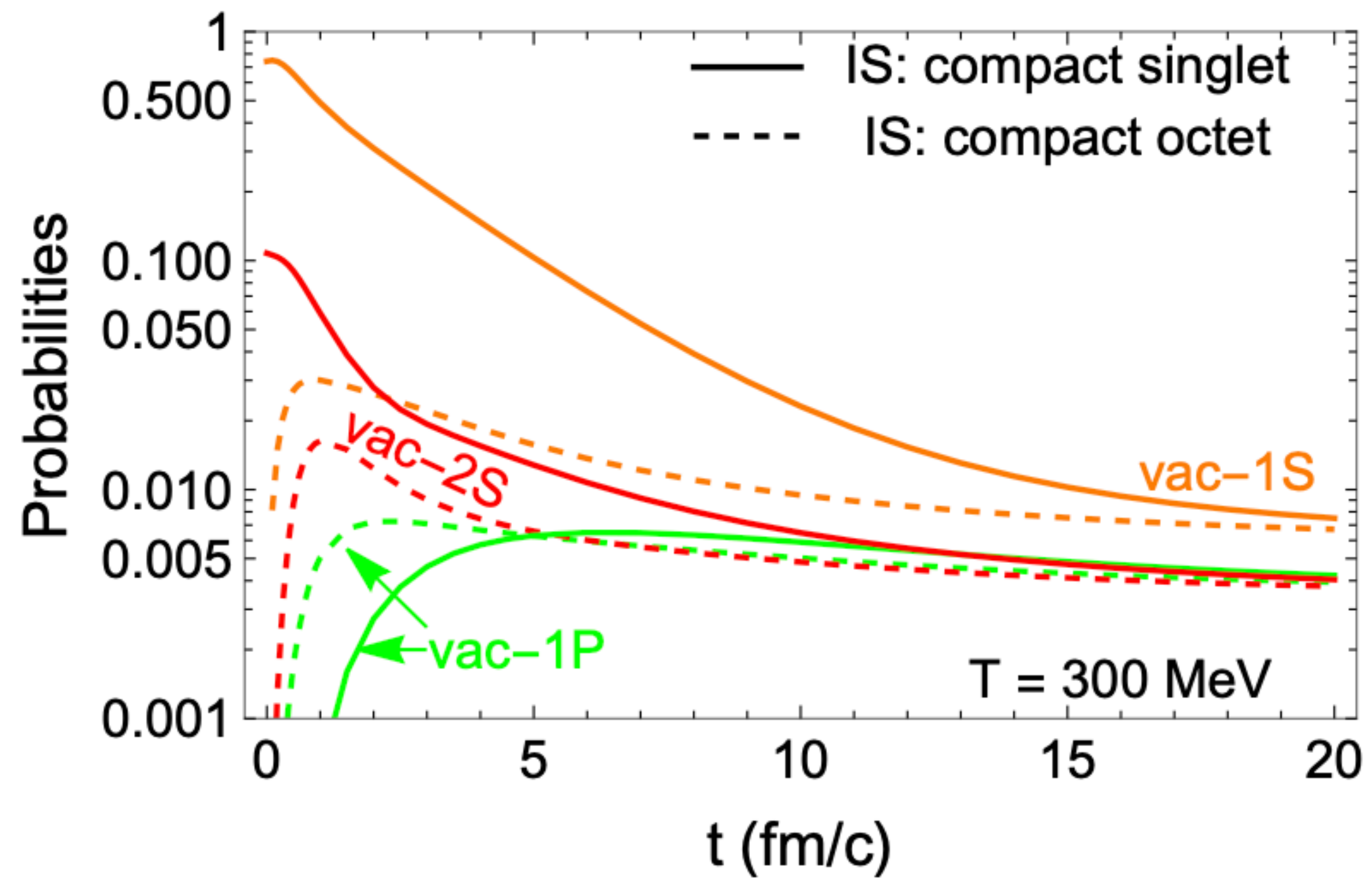
dissipation

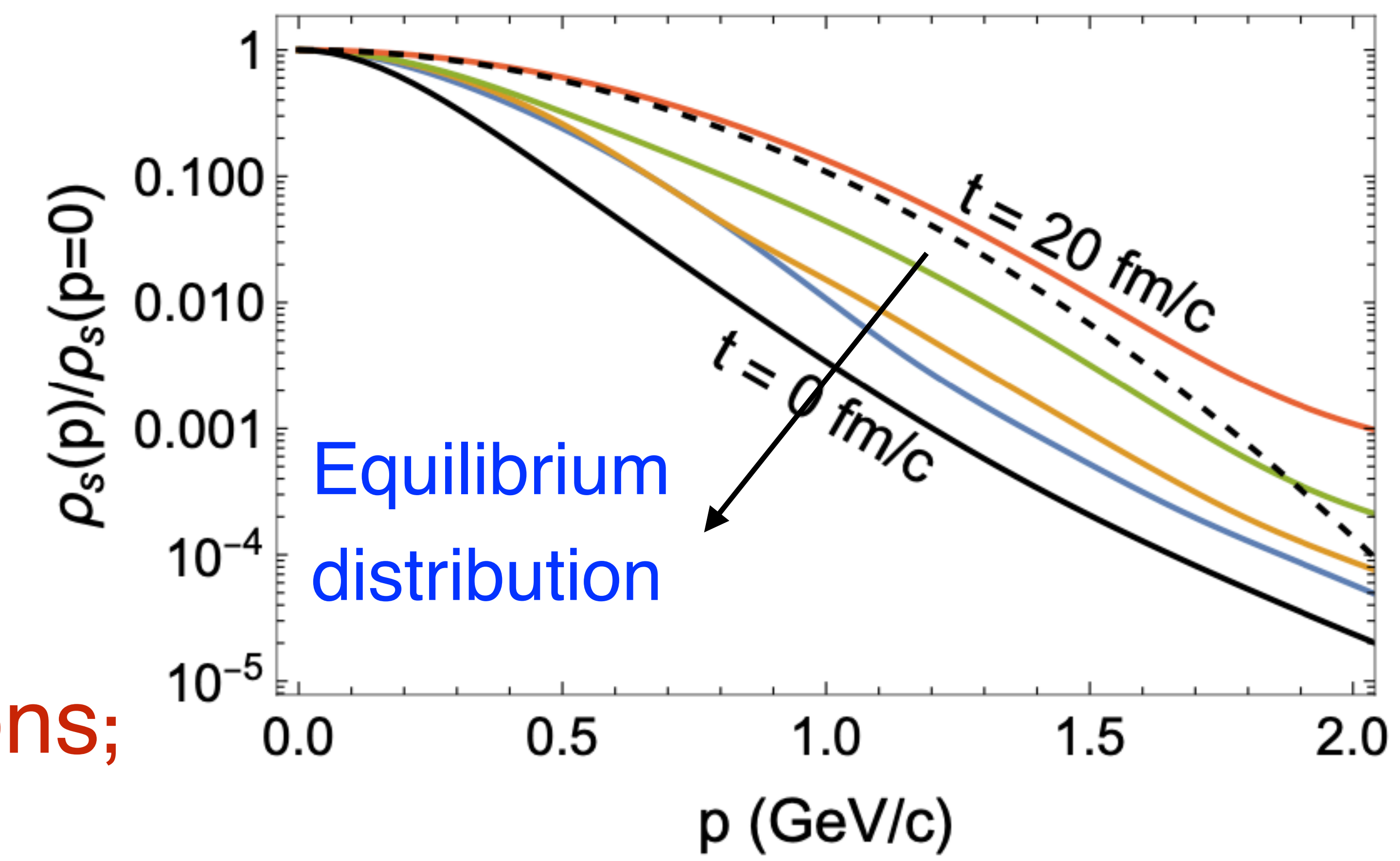
$$\mathcal{L}_4\mathcal{D} = \frac{1}{32T^2}\int_{xx'} W(x-x')\,(\{\dot{n}^a_x \dot{n}^a_{x'}, \mathcal{D}\} - 2\dot{n}^a_x \mathcal{D} \dot{n}^a_{x'})\,.$$

preservation of positivity

Imaginary part potential

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{s},\mathbf{s}',t)\\ \mathcal{D}_o(\mathbf{s},\mathbf{s}',t)\end{pmatrix}, \qquad \mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$





Beyond the dipole approximation;
The equations are solved with different initial states and medium configurations;
Equilibrium is checked.

Osaka Approach

- ✦ **NRQCD+OQS works in Quantum Brownian motion Regime** $M \gg T \sim m_D \gtrsim E_{bind}$
- ✦ **Weak coupling (strict) and go beyond the weak coupling (approximation)**
- ✦ **Used for bottomonium.**

T. Miura, Y. Akamatsu, M. Asakawa, et al, PRD 87 (2013) 045016; PRD 91 (2015) 5, 056002.; PRD97 (2018), 014003.; Phys.Rev.D 106 (2022) 7, 074001.

$$\frac{d}{dt}\rho_r(t) = -i[H_{\text{eff}}^{(r)}, \rho_r] + \sum_{\vec{k}a}\left(2L_{\vec{k}a}^{(r)}\rho_r L_{\vec{k}a}^{(r)\dagger} - L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\rho_r - \rho_r L_{\vec{k}a}^{(r)\dagger}L_{\vec{k}a}^{(r)}\right),$$

$$H_{\text{eff}}^{(r)} = \frac{\vec{p}^2}{M} + V(\vec{r})(t^a \otimes t^{a*}) - \frac{1}{4MT}\left\{\vec{p}, \vec{\nabla}D(\vec{r})\right\}(t^a \otimes t^{a*}),$$

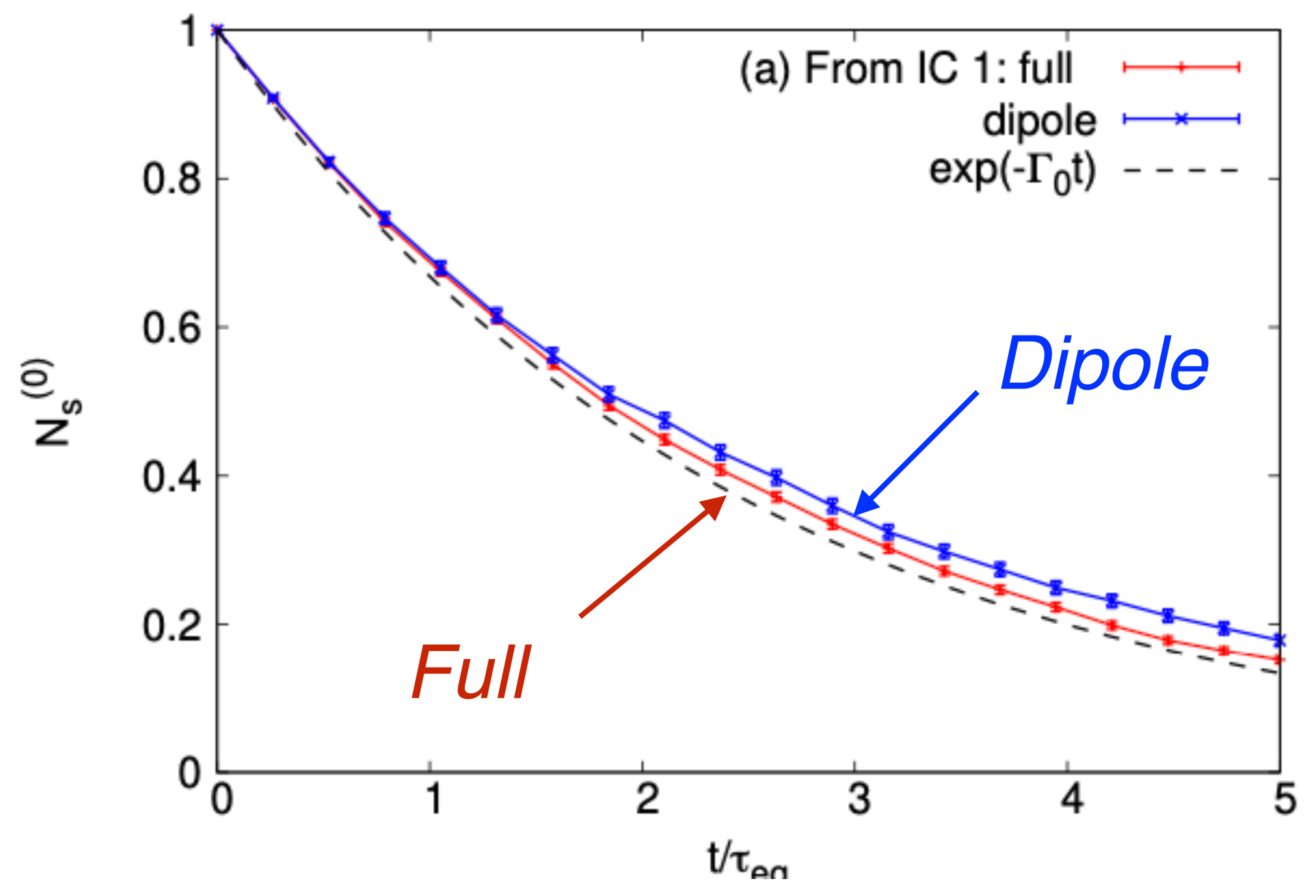
$$L_{\vec{k}a}^{(r)} = \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\text{CM}} + \vec{p}\right)\right]e^{\frac{i\vec{k}\cdot\vec{r}}{2}}(t^a \otimes 1)$$

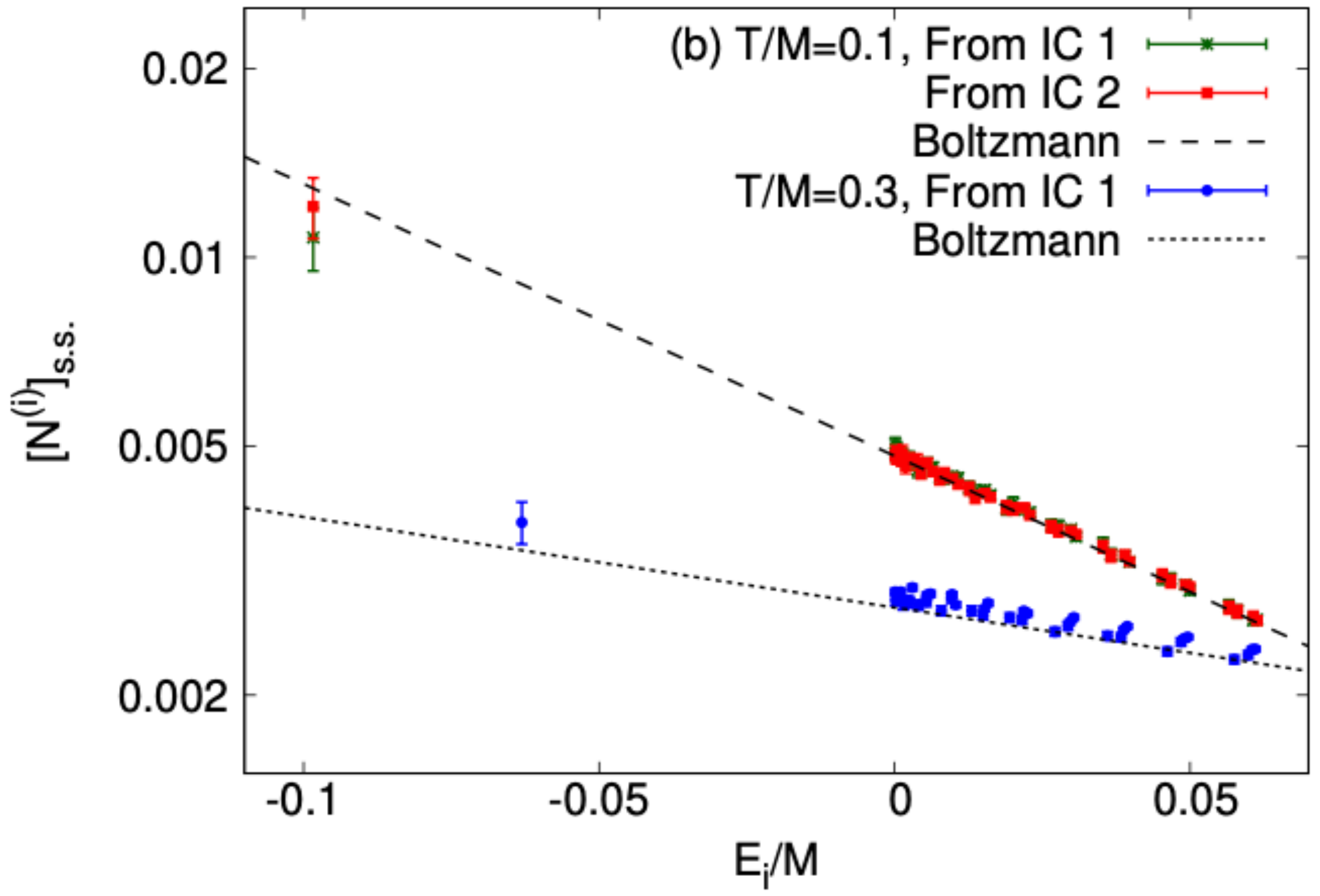
$$- \sqrt{\frac{\tilde{D}(\vec{k})}{2L^3}}\left[1 - \frac{\vec{k}}{4MT}\cdot\left(\frac{1}{2}\vec{P}_{\text{CM}} - \vec{p}\right)\right]e^{-\frac{i\vec{k}\cdot\vec{r}}{2}}(1 \otimes t^{a*}).$$

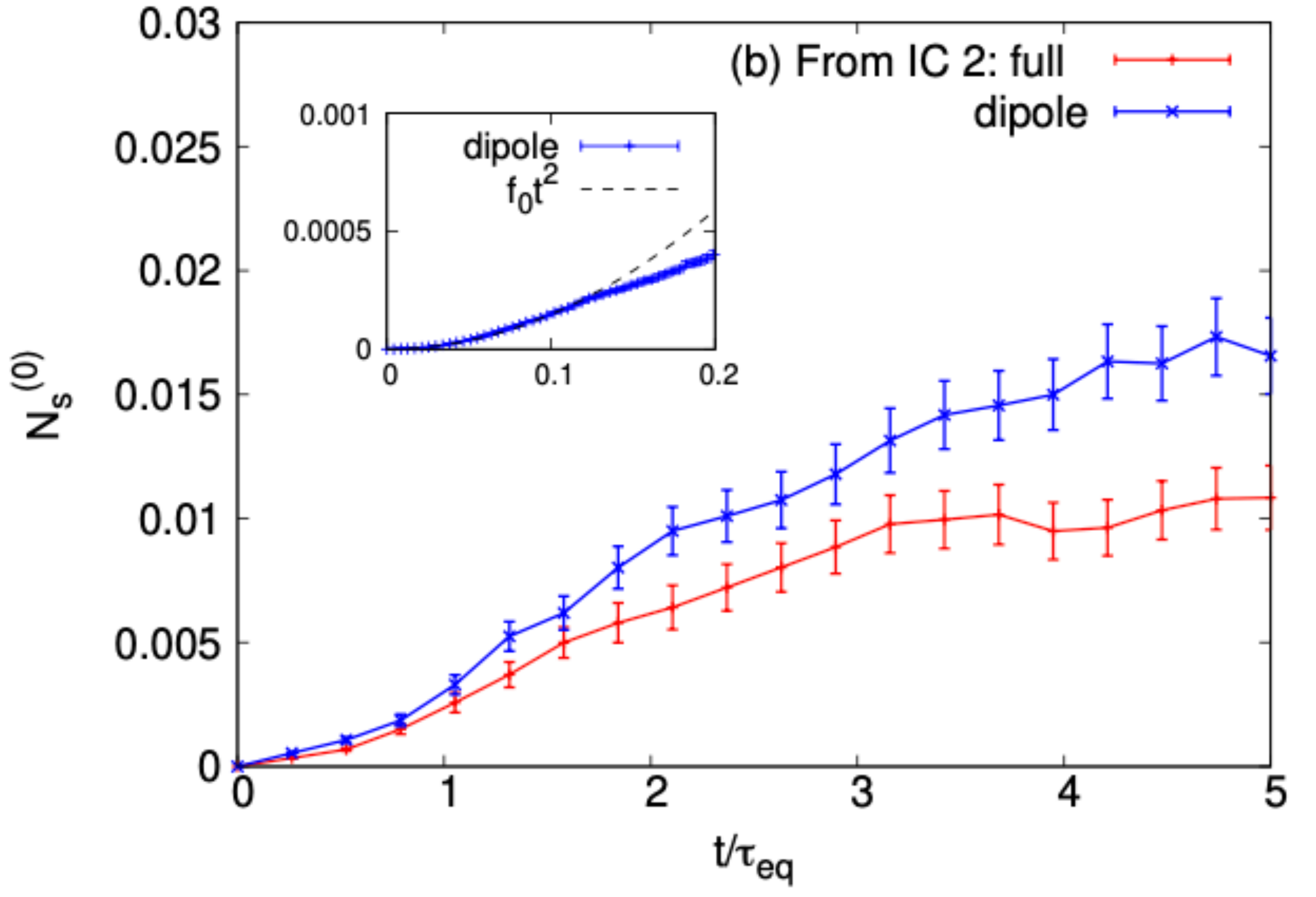
Beyond the weak coupling and assume the real and imaginary potential:

$$V(x) = -\frac{\alpha}{\sqrt{x^2 + x_c^2}}e^{-m_D|x|},$$

$$D(x) = \gamma\exp(-x^2/\ell_{\text{corr}}^2).$$







Beyond the dipole approximation;

The dipole approximation is an efficient alternative method, but it depends on the initial condition!

Equilibrium is satisfied.

Recent comparison paper

Eur. Phys. J. A (2024) 60:88
https://doi.org/10.1140/epja/s10050-024-01306-6

THE EUROPEAN PHYSICAL JOURNAL A

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Review

Comparative study of quarkonium transport in hot QCD matter

A. Andronic[1,a], P. B. Gossiaux[2,b], P. Petreczky[3,c], R. Rapp[4,d], M. Strickland[5,e], J. P. Blaizot[6], N. Brambilla[7], P. Braun-Munzinger[8,9], B. Chen[10], S. Delorme[11], X. Du[12], M. A. Escobedo[13,12], E. G. Ferreiro[12], A. Jaiswal[14], A. Rothkopf[15], T. Song[8], J. Stachel[9], P. Vander Griend[16], R. Vogt[17], B. Wu[4], J. Zhao[2], X. Yao[18]

Quarkonium real-time evolution in heavy-ion collisions

$\tau_e \sim 1/(\pi T),\ \tau_s \sim 1/E_{bind},\ \tau_r \sim M/T^2$

Non-Markovian

Markovian approximation: $\tau_e \ll \tau_r$ **, memory lose**

Quantum master equation –> Lindblad equation

T

$\sim m_Q$

$\tau_e \ll \tau_s$

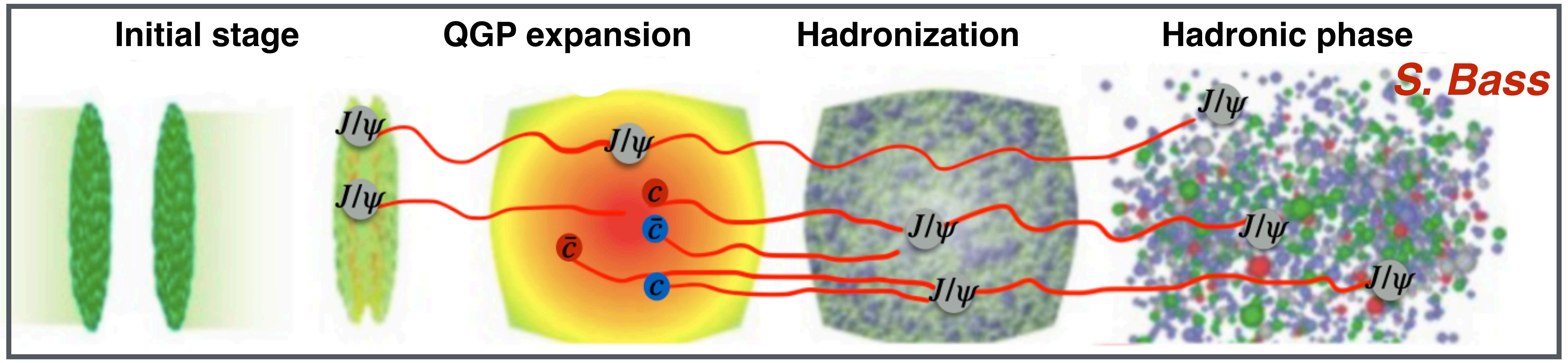
$(E_{bind} \ll m_D)$

$\tau_s \ll \tau_r$

$(E_{bind} \gg m_D)$

Quantum Brownian Motion

Quantum optical Limit



Further needs:

1. Connect the Quantum Brownian Motion at high temperture to the Quantum optical regime at low temperature.
2. One pair –> many pairs and regeneration from uncorrelated heavy quarks.

PHSD-Nantes Approach

- Start from the OQS and works from the QBM to QOL
- N pairs of $Q\bar{Q}$. Used for bottomonium and charmonium.
- N-body Wigner density is approximated as a classical phase space distribution

D. Villar, **JZ**, J. Aichelin, and P. Gossiaux, Phys.Rev.C 107 (2023) 5, 054913.
T. Song, J. Aichelin, and E. Bratkovskaya, Phys.Rev.C 107 (2023) 5, 054906.
T. Song, J. Aichelin, **JZ**, P. Gossiaux and E. Bratkovskaya,PRC108 (2023) 5, 054908.

Probability that at time t the state Φ is produced:

$$P^{\Phi}(t) = \mathrm{Tr}[\rho^{\Phi}\hat{\rho}_{tot}(t)] \qquad \rho^{\Phi} = |\Phi><\Phi|$$

So: $$P^{\Phi}(t) = P^{\Phi}(0) + \int_0^t \Gamma^{\Phi}(t)dt$$

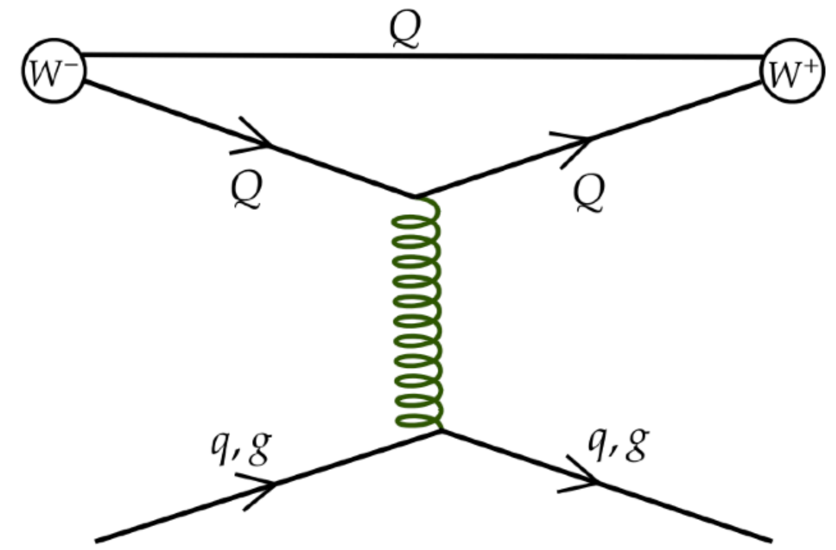
$$\Gamma^{\Phi} = \frac{dP^{\Phi}}{dt} = \frac{d}{dt}\mathrm{Tr}[\rho^{\Phi}\rho_{tot}] \approx \frac{d}{dt}\prod \frac{d^3r_id^3p_i}{(2\pi)^{3N}} W^{\Phi}(r,p)W_N(r_1,p_1,\ldots,r_N,p_N)$$

$$W_N \approx W_N^{\mathrm{C(classical)}} = \prod_{i=1}^{N}\delta(\mathbf{r}_i - \mathbf{r}_i^*(t))\delta(\mathbf{p}_i - \mathbf{p}_i^*(t))$$

Collisional rate

$$\Gamma_{\mathrm{coll}}(t) = \sum_k^2 \sum_{i\geq 3}^N \sum_n \delta(t - t_{ki}(n)) \int \prod_{j=1}^{N} d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2)\left[W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t+\epsilon) - W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t-\epsilon)\right]$$

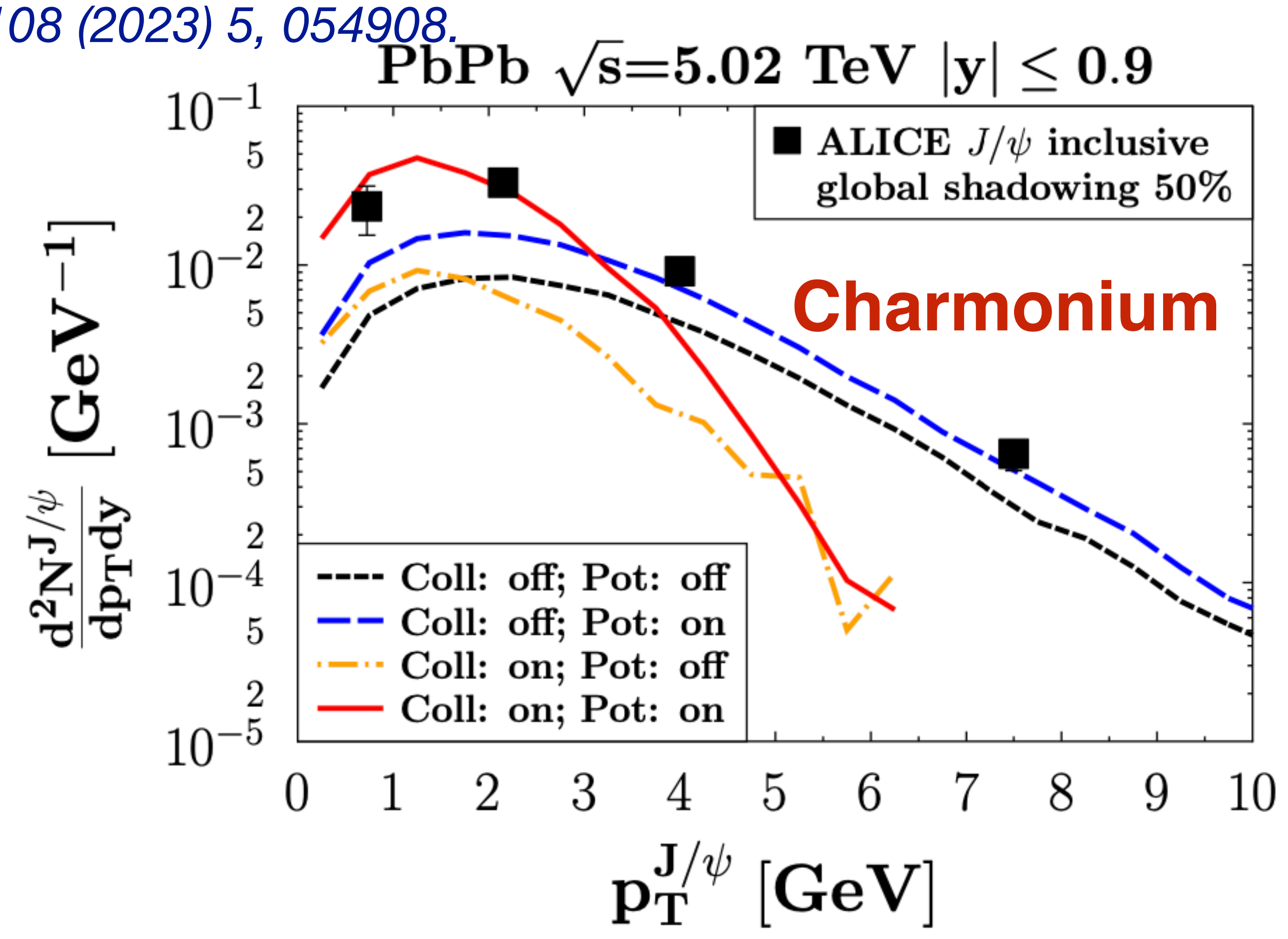
W^+ $\qquad$ W^-

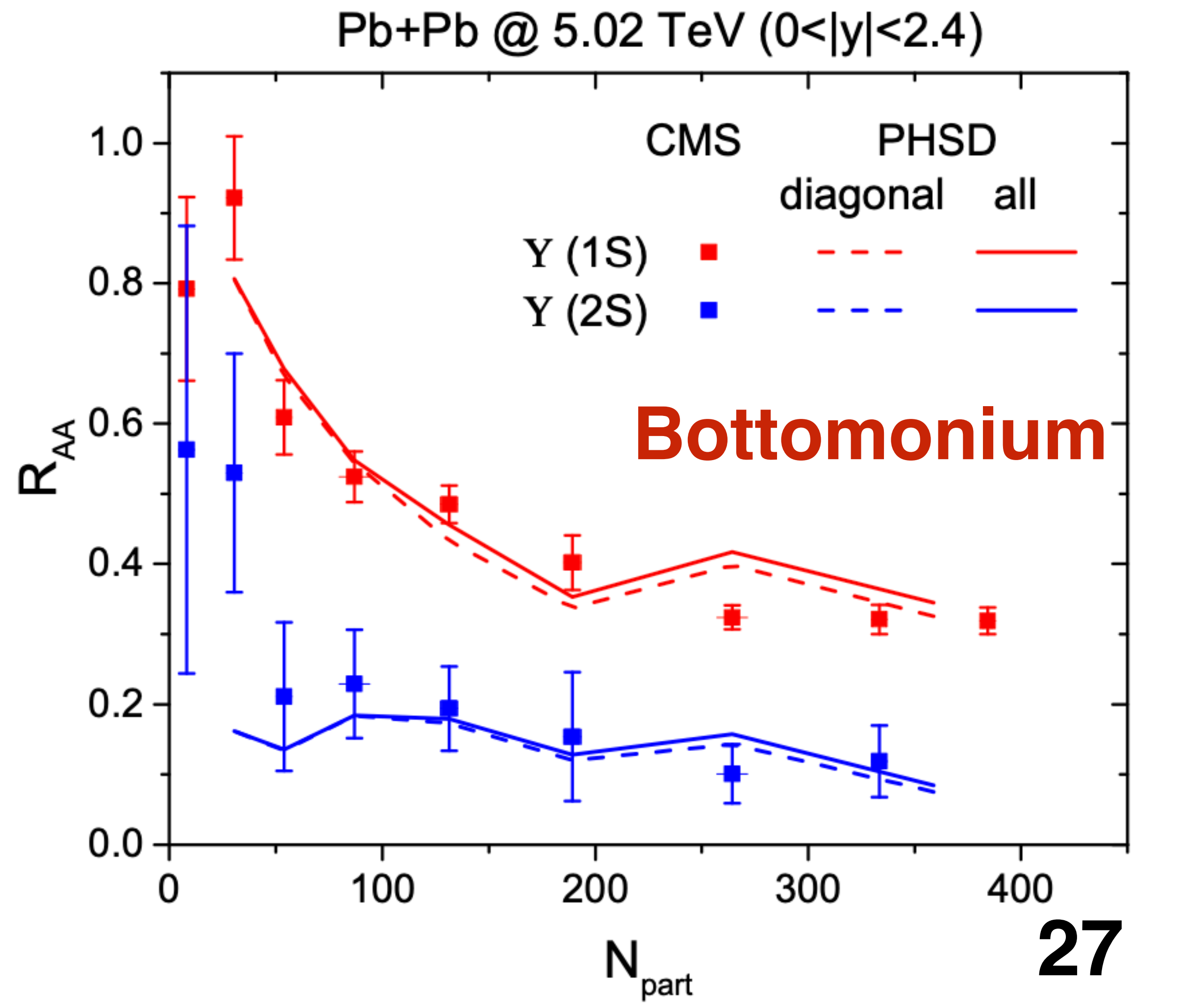


Wigner density of quarkonium states is temperature or time dependent->another term:

Local rate

$$\Gamma_{\mathrm{local}}(t) = \int \prod_{j=1}^{N} d^3\mathbf{r}_j d^3\mathbf{p}_j \dot{W}^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2,T(t))W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t)$$





Summary

- The vacuum properties are well described by the potential model. The in-medium properties can mostly be absorbed in the finite-temperature potential, which has both real and imaginary parts. Recent studies show: The HQ potential has no/a small color screening effect and a large imaginary part.

- With the assumption of a classical particle of quarkonium, the transport model can describe quite well the experimental data, which help us to understand the HQ in-medium potential, HQ energy loss, QGP properties,… Also extended to B_c and $X(3872)$.

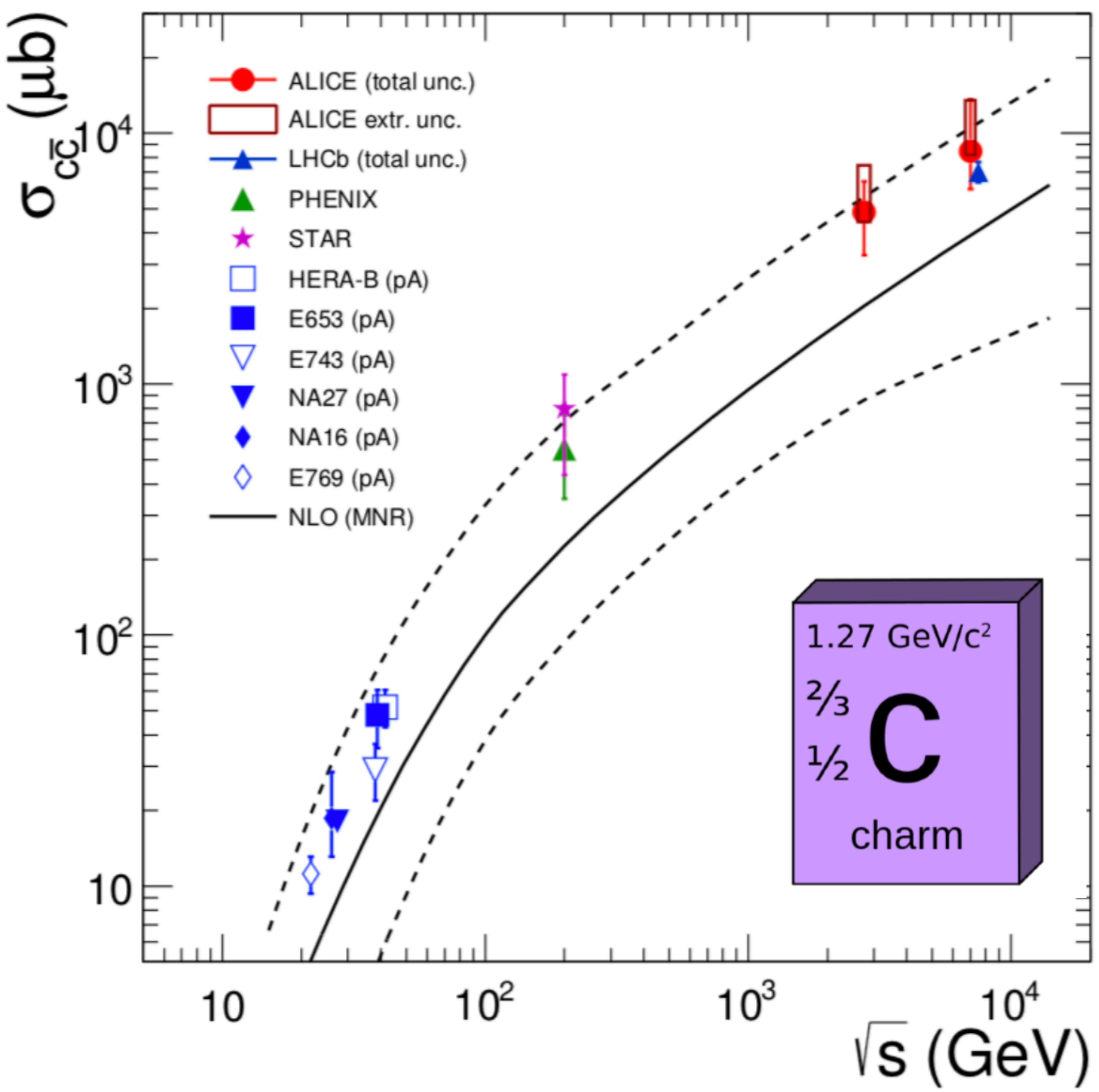
- Aiming to include the quantum effects and to build a genuine first principles based real time evolution framework, OQS is used and developed in different ways based on heavy quark effective theories. Much progress has been made in the Quantum Brownian Motion regime, where a bound state is difficult to define.

For LHCb:

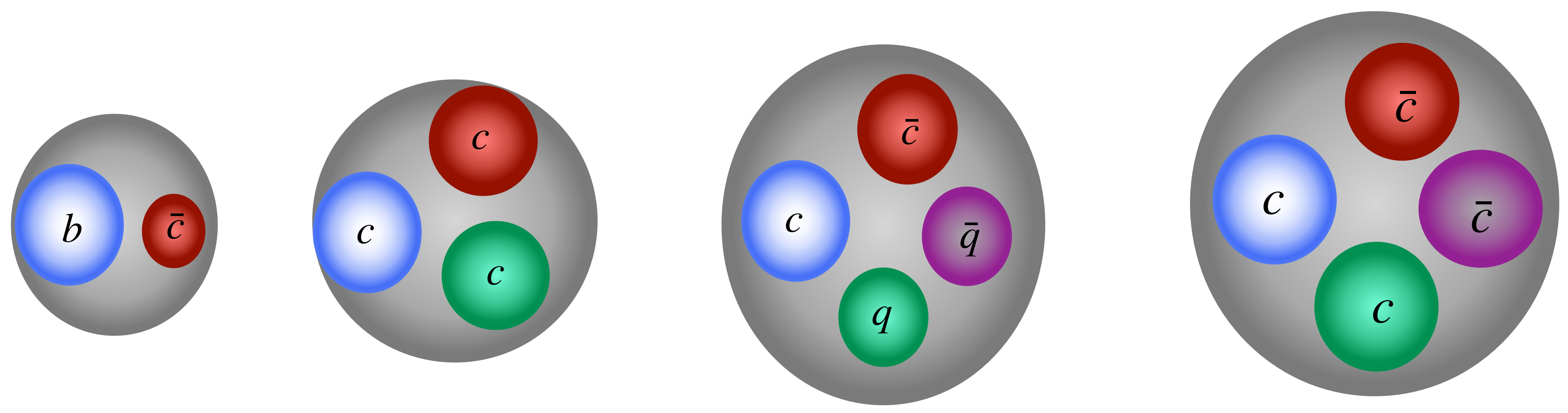
- Forward rapidity quarkonium production
- Direct quarkonium production, $J/\psi, \chi_c, \psi'$

Thanks for your attention

Exotic states



Andronic A, et al. Eur. Phys. J. C76(3):107 (2016).



pp, e+e-: 2 or 3 pairs of $Q\bar{Q}$

AA: a lot of uncorrelated $Q\bar{Q}$ pairs

N-body Schrödinger equation ->
Wigner function ->
production in quark-gluon plasma

__JZ__, H. He, Y. Liu, P. Zhuang. Phys. Lett. B 746(2015); Phys. Lett. B 771 (2017) 349-353; Few Body Syst. 58 (2017) 2, 100.

$N_{c\bar{c}} \sim T_A T_B\ \sigma_{c\bar{c}} \sim o(100)$ charm quarks in quark-gluon plasma at LHC!

a most "charming" system produced in relativistic HICs!

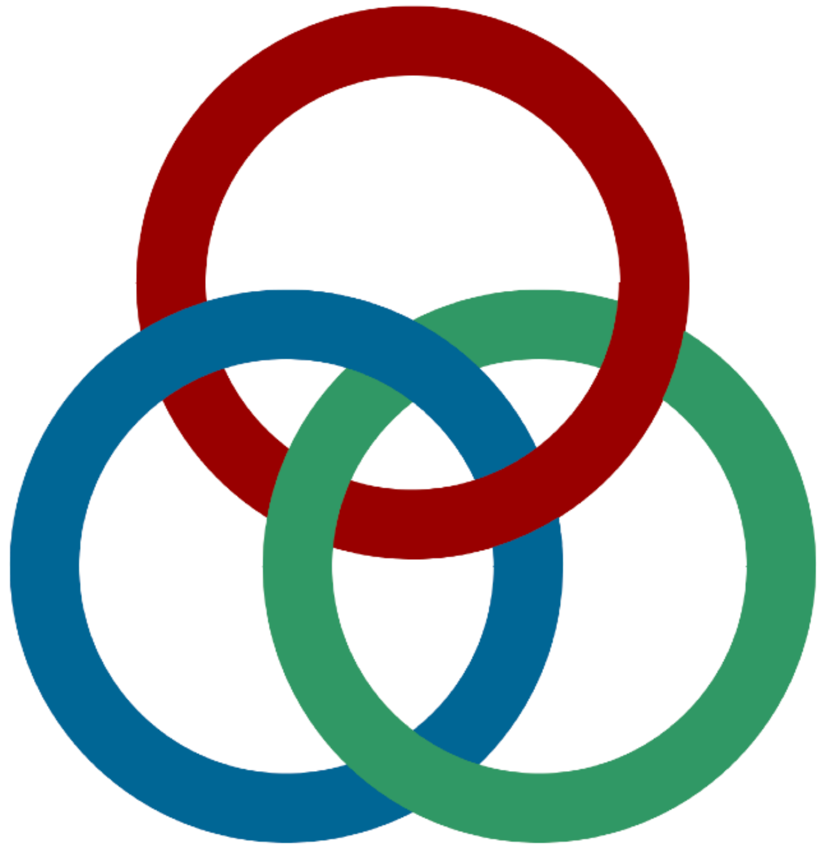
It is most probable to observe/discover B_c , Ξ_{cc} , Ω_{ccc} , and $X_{cc\bar{c}\bar{c}}$ in HICs!

How about pPb at LHCb ?

Bulid a unified framework

To learn more: To combine the light with heavy, open heavy flavor with quarkonium.

Light hadrons



Thermal medium properties:
EOS, lifetime,temperature,
velocity, shear viscosity…

- Catania model
- Duke model
- EPOS4 model
- LBT model
- PHSD model
- TAMU model
- Torino model
- …

Open heavy flavor

Quarkonium

- Comover model
- TAMU model
- Tsinghua model
- OQS based approaches:
- Munich-Kent
- Nantes
- Osaka
- PHSD-Nantes
- …

Heavy quark energy loss
Correlations
Quark number conservation
…

➡ **EPOS4 is now ready for light and open heavy flavors, and the quarkonium part is coming soon.**

K. Werner, PRC 109 (2024) 1, 014910
***JZ,** J.Aichelin, P.B. Gossiaux, V. Ozvenchuk, K.Werner, arXiv:2401.17096*
***JZ,** J.Aichelin, P.B. Gossiaux, K.Werner, Phys.Rev.D 109 (2024) 5, 054011*