Constraining the hidden-charm pentaquarks prediction and discriminating the spins of Pc(4440) and Pc(4457) through Effective Range Expansion

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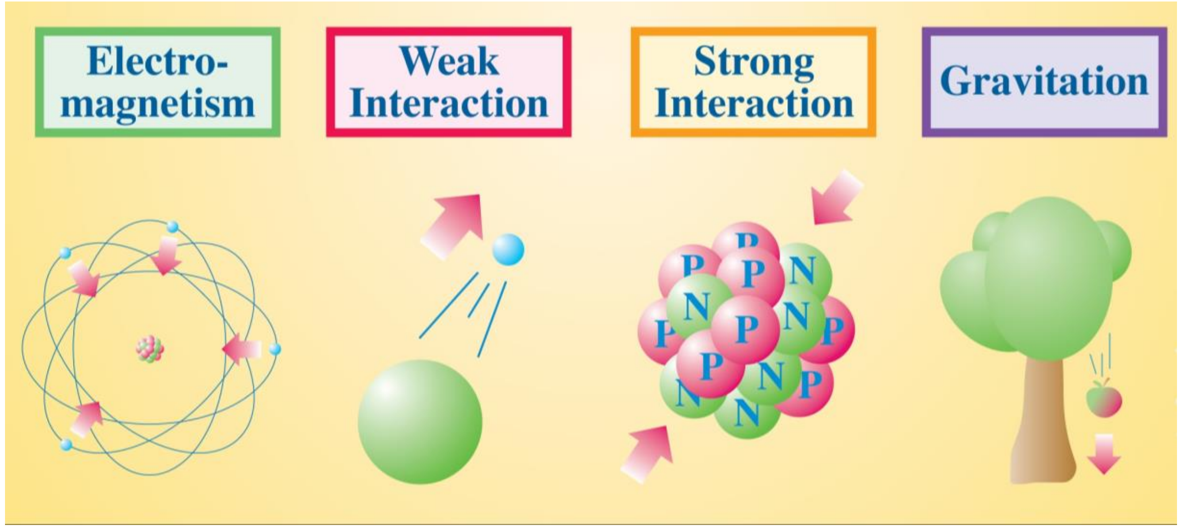
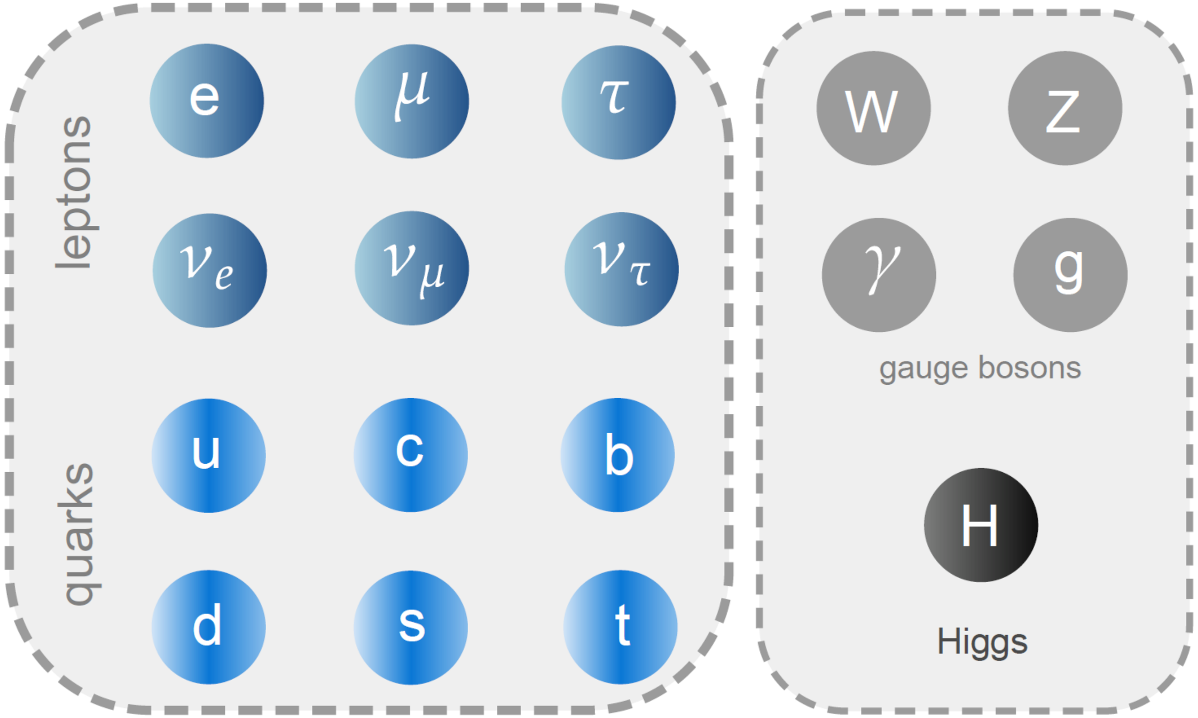
Outlines

1. Introduction of exotic hadrons and Pc states
2. Effective range expansion and compositeness
3. Molecule descriptions of Pc states with contact effective field theory
4. Matching effective range with NLO contact potential
5. Results and discussion

QCD and exotic hadrons

- SU(3)$_c$ gauge symmetry --- Quantum chromodynamics (QCD)





$$\mathcal{L} = \sum_{i,j=1}^{N_f} \bar{\psi}_i (i\gamma^\mu D_{\mu ij} - m_{ij})\psi_j - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

1. Asymptotic freedom

2. Quark confinement

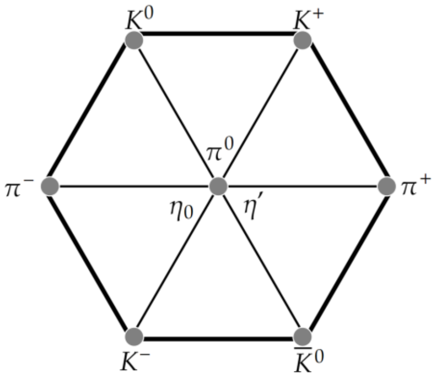
→ Matter made by quarks and gluons should be color singlet !

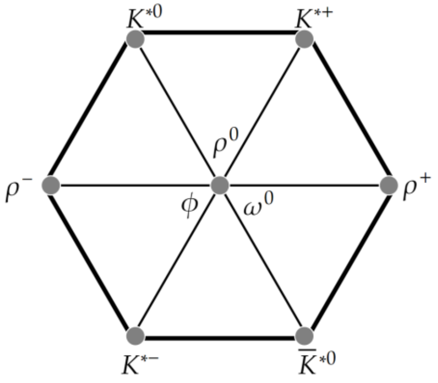
Hadrons classification

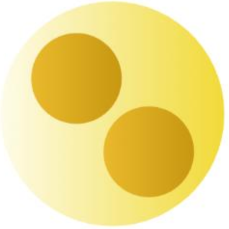
u d s **SU(3) flavor**

u d s c **SU(4) flavor**

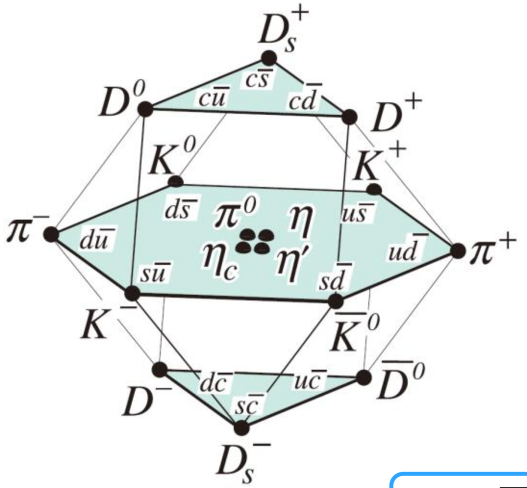
- pseudo-scalar mesons and vector octet mesons

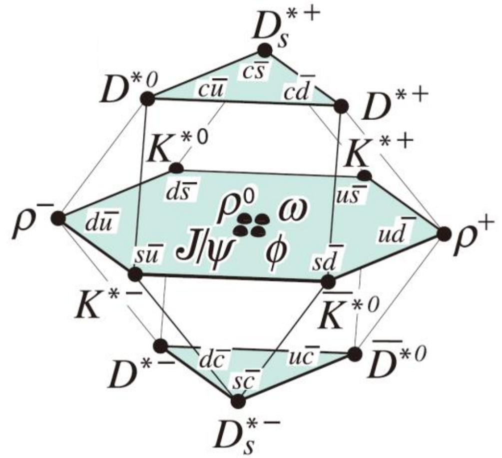






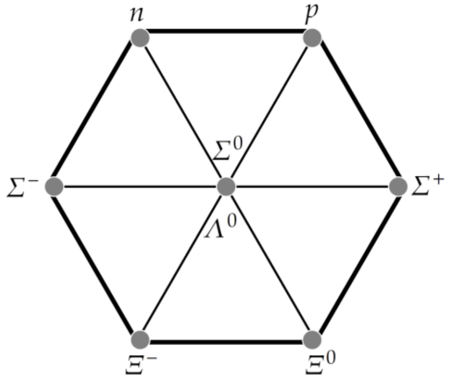
$$3 \otimes \bar{3} = 8 \oplus 1$$

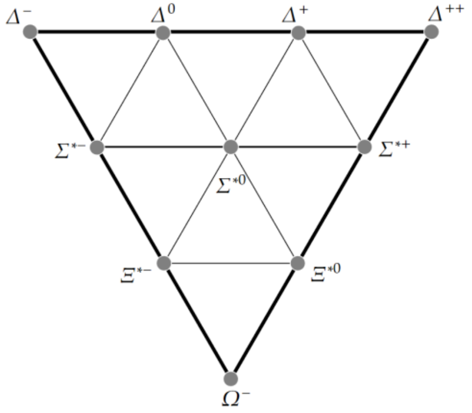


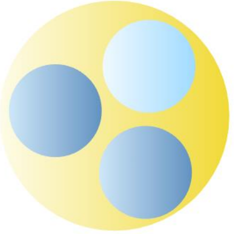


$$4 \otimes \bar{4} = 15 \oplus 1$$

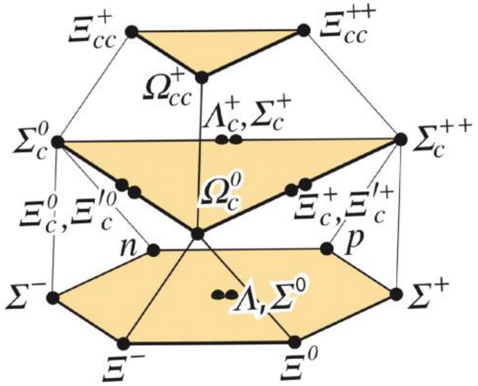
- $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons

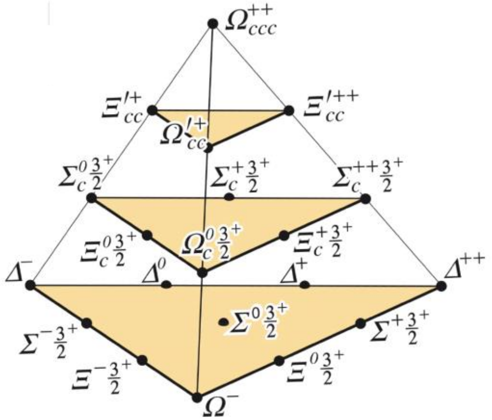






$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

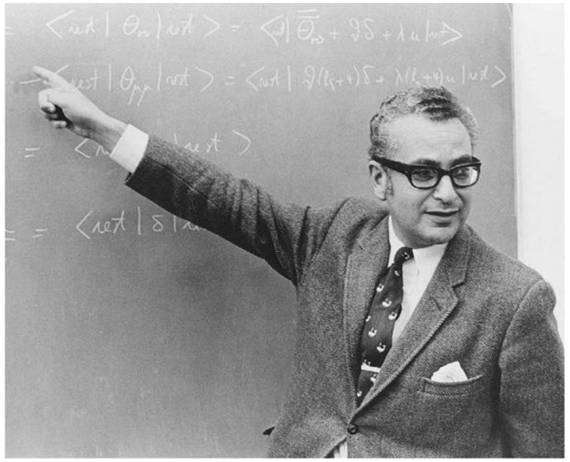




$$4 \otimes 4 \otimes 4 = \bar{4}_A \oplus 20_S \oplus 20_{MS} \oplus 20_{MA}$$

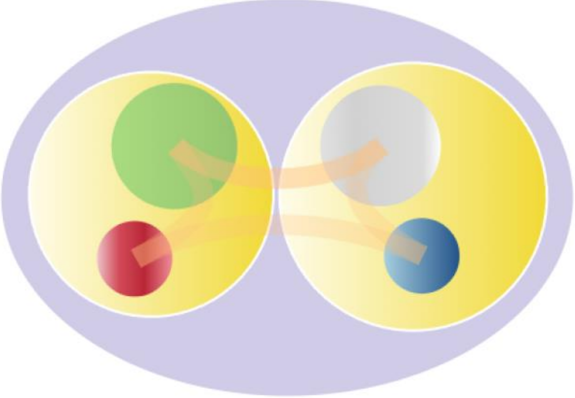
Exotic hadrons

- **Multiquark states**

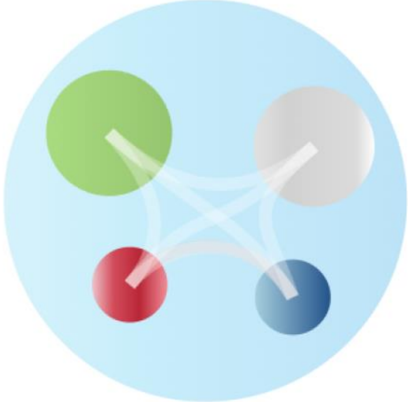


Color singlet

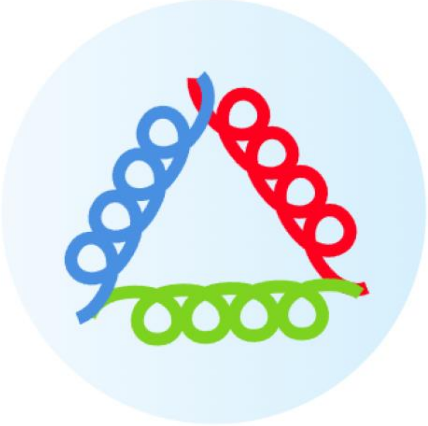
Tetraquarks	$q\bar{q}q\bar{q}$
Pentaquarks	$qqqq\bar{q}$
Hexaquarks	$q\bar{q}q\bar{q}q\bar{q}$ $qqqqqq$
…	



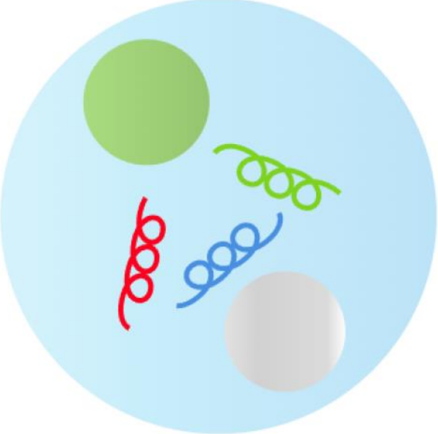
molecules



compact quarks



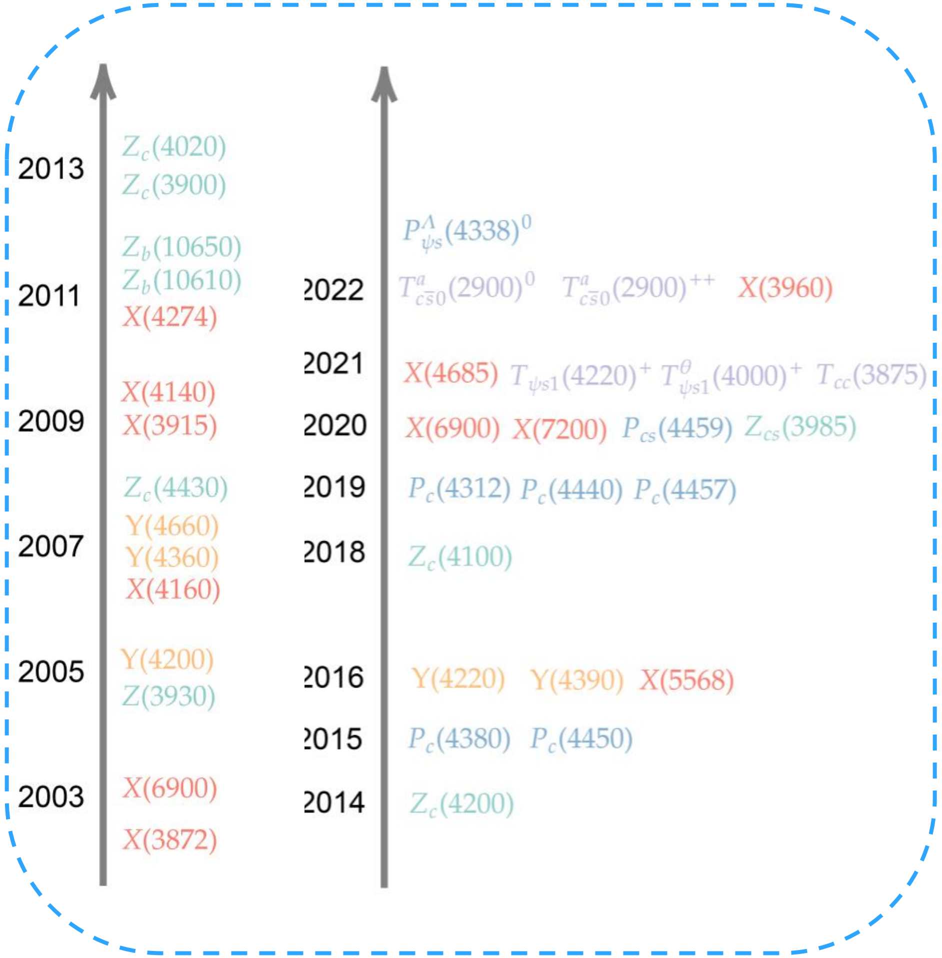
glueballs

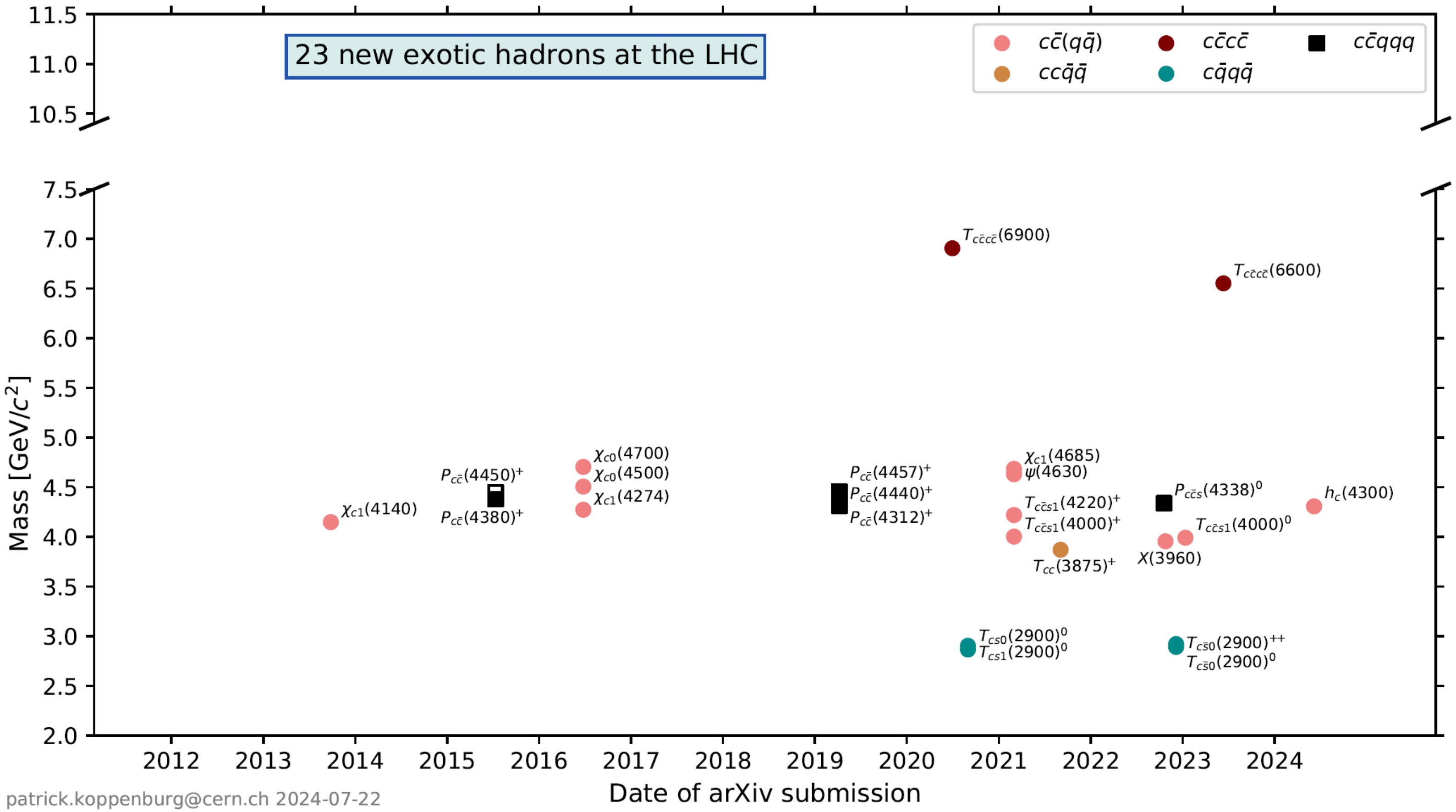


hibrids

Exotic hadrons

- **23 New exotic hadrons discovered by LHCb**



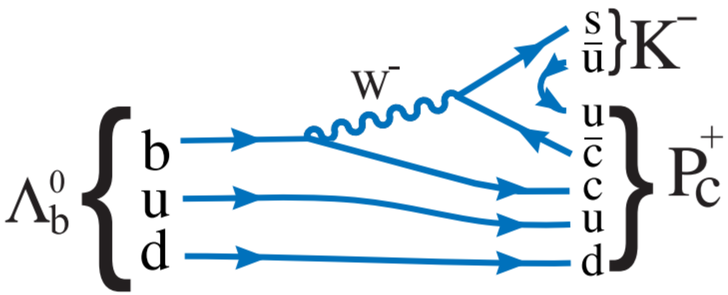


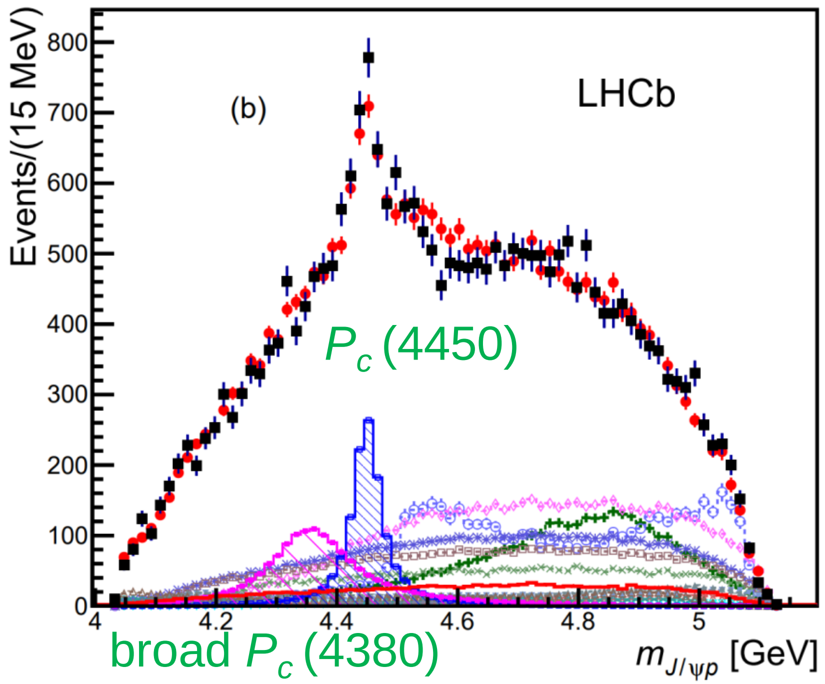
Pentaquarks and their HQSS partners

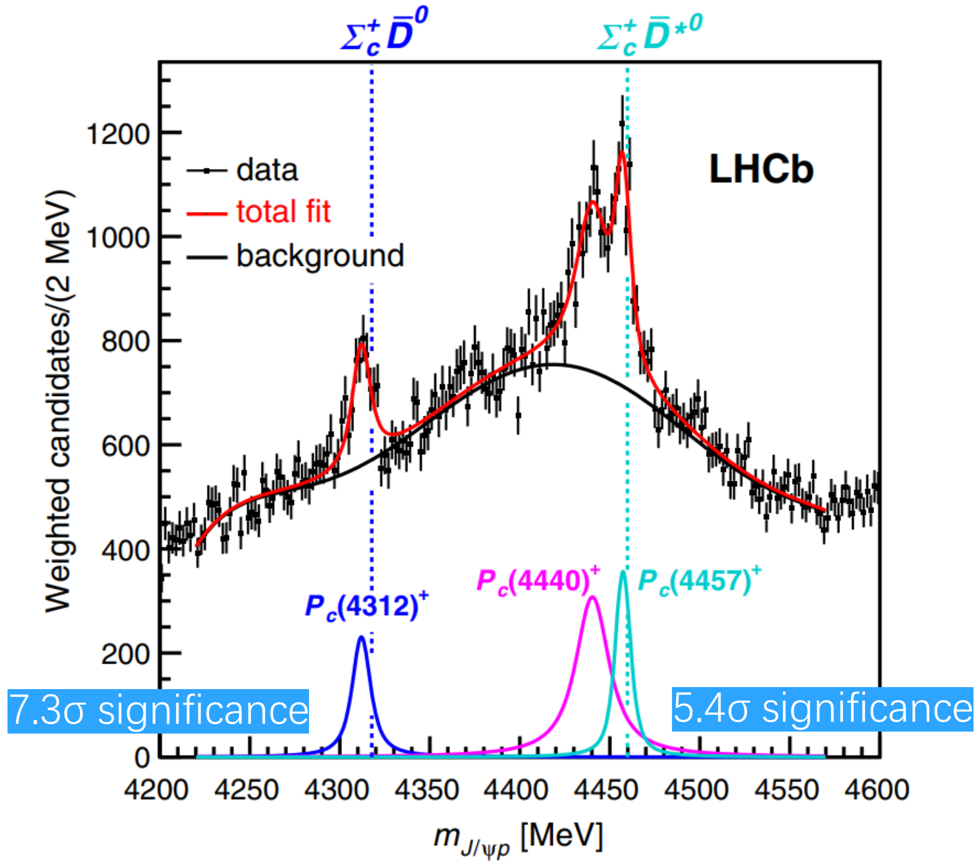
[PhysRevLett. 115,072001, LHCb Collaboration, 2019]

[PhysRevLett. 122,222001, LHCb Collaboration, 2019]

LHCb : $\Lambda_b^0 \to J/\Psi p K^-$







$$m_{P_{c1}} = 4311.9 \pm 0.7^{+6.8}_{-0.6}, \quad \Gamma_{P_{c1}} = 9.8 \pm 2.7^{+3.7}_{-4.5}$$

$$m_{P_{c2}} = 4440.3 \pm 1.3^{+4.1}_{-4.7}, \quad \Gamma_{P_{c2}} = 20.6 \pm 4.9^{+8.7}_{-10.1}$$

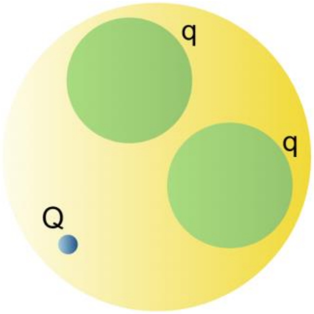
$$m_{P_{c3}} = 4457.3 \pm 0.6^{+4.1}_{-1.7}, \quad \Gamma_{P_{c3}} = 6.4 \pm 2.0^{+5.7}_{-1.9}$$

Pentaquarks and their HQSS partners

- **heavy quark spin symmetry (HQSS)**

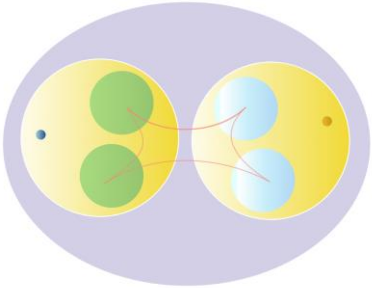
$$\Lambda_{QCD} << \Lambda << m_Q$$

$$p^\mu = m_Q v^\mu + q^\mu$$



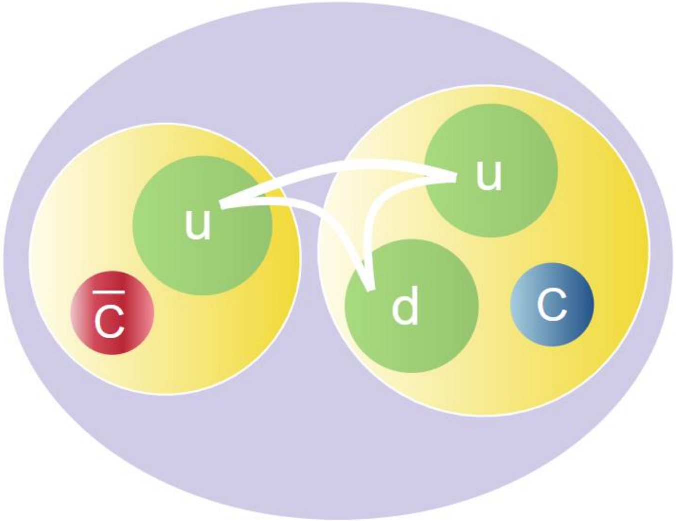
$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\not{D} - m_Q)\psi \\ &= (\bar{q}_v + \bar{Q}_v)(i\not{D} - m_Q + m_Q \not{v})(q_v + Q_v) \\ &= \bar{q}_v(iv\cdot D)q_v - \bar{Q}_v(iv\cdot D + 2m_Q)Q_v + \bar{q}_v(i\not{D}_\perp)q_v\end{aligned}$$

LO, no gamma term



$$\vec{S}_i^q \cdot \vec{S}_j^q$$

$\overline{D}^{(*)}\ \Sigma_c^{(*)}$



Heavy mesons | **Heavy baryons**

$$H_c = \frac{1}{\sqrt{2}}[D + \vec{D}^* \cdot \vec{\sigma}] \qquad \vec{S}_c = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*$$

$$\mathcal{L} = C_a \vec{S}_c^\dagger \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] + C_b \sum_{i=1}^{3} \vec{S}_c^\dagger \cdot (J_i \vec{S}_c)\mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c]$$

$$V = C_a + C_b\ \vec{\sigma}_L \cdot \vec{J}_L$$

Pentaquarks and their HQSS partners

Potentials

Molecule	J^P	V
$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	C_a
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$

$$T = V + VGT$$
$$\Rightarrow \quad T(1 - VG) = V$$
$$\Rightarrow \quad T = \frac{V}{1 - VG}$$

nonrelativistic propagator

$$G = \int \frac{d^3q}{(2\pi)^3} \frac{1}{B + \frac{q^2}{2\mu}} \exp[2(\frac{q}{\Lambda})^2]$$

$\Lambda \quad 0.5 - 1$ GeV

Scenario A

Pc (4440) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

Pc (4457) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

Scenario B

Pc (4440) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

Pc (4457) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

Results :

Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$(1/2)^-$	7.8–9.0	4311.8–4313.0
A	$\bar{D}\Sigma_c^*$	$(3/2)^-$	8.3–9.2	4376.1–4377.0
A	$\bar{D}^*\Sigma_c$	$(1/2)^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$(3/2)^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$(1/2)^-$	25.7–26.5	4500.2–4501.0
A	$\bar{D}^*\Sigma_c^*$	$(3/2)^-$	15.9–16.1	4510.6–4510.8
A	$\bar{D}^*\Sigma_c^*$	$(5/2)^-$	3.2–3.5	4523.3–4523.6
B	$\bar{D}\Sigma_c$	$(1/2)^-$	13.1–14.5	4306.3–4307.7
B	$\bar{D}\Sigma_c^*$	$(3/2)^-$	13.6–14.8	4370.5–4371.7
B	$\bar{D}^*\Sigma_c$	$(1/2)^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$(3/2)^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$(1/2)^-$	3.1–3.5	4523.2–4523.6
B	$\bar{D}^*\Sigma_c^*$	$(3/2)^-$	10.1–10.2	4516.5–4516.6
B	$\bar{D}^*\Sigma_c^*$	$(5/2)^-$	25.7–26.5	4500.2–4501.0

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Weinberg compositness criterion

- **Many works have been done to study the spin problem of Pc(4440) and Pc(4457)**

- **Mass spectrum and decays**
- **Hexaquark predictions**
- **One boson exchange model**
- **Machine learning on line shape**
- **femtoscopic correlation functions**
- **...**

[R. Chen, Z.F. Sun, X. Liu, and S.L. Zhu, Phys. Rev. D 100, 011502(R), 2019]
[M.Z. Liu, L.S. Geng, M.V. Valderrama, J.J. Xie, Phys. Rev. D 103, 054004, 2021]
[N. Yalikun, Y.H. Lin, F.K. Guo, Y. Kamiya, and B.S. Zou, Phys. Rev. D 104, 094039, 2021]
[Z.Y. Zhang, J.H. Liu, J.F. Hu, Q. Wang, U.-G. Meißner, j.scib.2023.04.018., 2023]
...

To determine which scenario of the Pc(4440) and Pc(4457) spins should be correct under two-body hadronic molecular $\bar{D}^{*}\Sigma_c$ description, it's helpful to reconsider the nature of "molecule" or "composite" picture.

$$|\Phi\rangle = \sqrt{Z}\,|\phi\rangle + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\lambda(\mathbf{k})\,|h_1 h_2(\mathbf{k})\rangle$$

$$Z = |\langle\Phi|\phi\rangle|^2$$

$$1 - Z = \int \frac{d^3\mathbf{k}}{(2\pi)^3}|\lambda(\mathbf{k})|^2$$

- **Weinberg compositness criterion was originally proposed on studying deuteron to discriminate between elementary particle and composite molecular state**

- **this criterion has analogously be extended to be applicable on exotic hadronic states**

Effective range expansion

[Y. Li, F.K. Guo, J.Y. Pang, and J.J. Wu, Phys.Rev.D 105. 7, L071502, 2022]
[Baru, and X.K. Dong, M.L. Du, Filin, F.K. Guo, Hanhart, Nefediev, Nieves and Q. Wang, j.physletb.2022.137290, 2022]

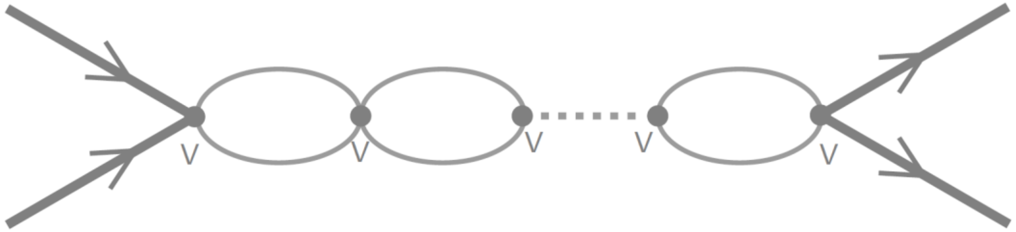
- **Low energy effective range expansion**

$$\mathcal{T} = \frac{2\pi}{\mu}\frac{1}{k\cot\delta_0 - ik} = \frac{2\pi}{\mu}\frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0k^2 + \cdots}$$

phase shift

$$k^{2l+1}\cot\delta_l = -\frac{1}{a_l} + \frac{1}{2}r_lk^2 + \cdots$$

- **Dispersion relation of amplitude T**



$$\mathcal{T} \sim \frac{g}{\sqrt{s} - E_R + i\frac{\Gamma}{2}}$$

$$1 - Z = \int \frac{d^3\vec{q}}{(2\pi)^3}\frac{g^2}{(E_B + \frac{\vec{q}^2}{(2\mu)})^2}(1 + \mathcal{O}(\frac{\vec{q}^2}{\beta^2})) = \frac{\mu^2 g^2}{2\pi\gamma}(1 + \mathcal{O}(\frac{\gamma}{\beta}))\,.$$

$$\mathcal{T} = \frac{4\pi g^2\mu\gamma}{2\pi\gamma(\gamma^2 + k^2) + g^2\mu^2\gamma^2 - g^2\mu^2k^2 + ig^2\mu^2\gamma k}$$

Effective range expansion

- **Weinberg compositness**

[Landau and Lifshits, Quantum Mechanics: Non-Relativistic Theory, Course of Theoretical Physics, Vol. v.3]
[A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, Phys. Rev. D 105, L031503, 2022]
[vanKolck, arxiv: 2209.08432, 2022]

$$a_0 = 2\frac{(1-Z)}{\gamma(2-Z)} + \mathcal{O}(\frac{1}{m_{ex}}),$$
$$r_0 = -\frac{Z}{\gamma(1-Z)} + \mathcal{O}(\frac{1}{m_{ex}}).$$

$$Z = 0$$

the effective range r_0 >0 is positive with the value around $O\left(\frac{1}{m_{ex}}\right)$, while $a_0 \sim \frac{1}{\gamma} + O\left(\frac{1}{m_{ex}}\right)$

- **2 assumptions**

[Matuschek, Baru, F.K. Guo, Hanhart, epja/s10050-021-00413-y, 2021]

- ☐ **bound states, not virtual states or resonances**

$$1 - Z = \sqrt{\frac{1}{1 + 2|\frac{r_0}{a_0}|}}$$

- ☐ **pure molecular states with $Z \sim 0$, which consistent well with the 3 Pc states**

e.g.

$$\beta < \frac{1-Z}{Z}\gamma$$

~~$Z \geq 0.2$~~

500 MeV ~ 100 MeV

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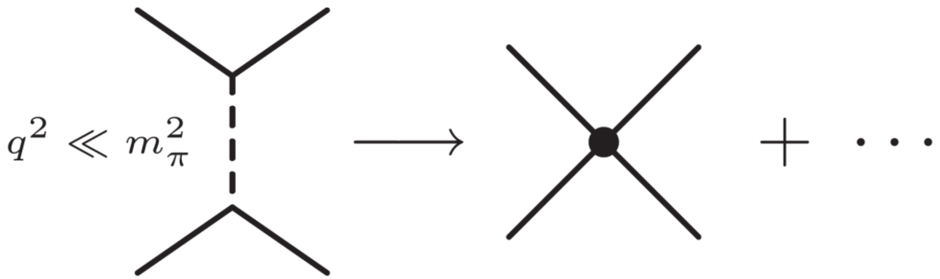
Contact effective field theory

[RevModPhys.92.025004, Hammer, Sebastian, van Kolck, 2020]



Nonrelativistic contact EFT

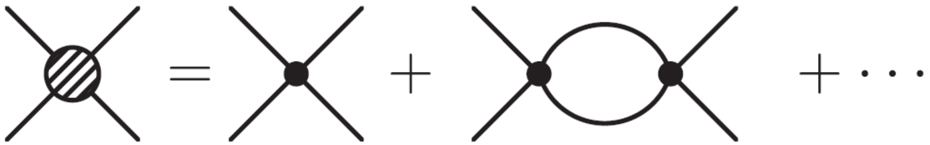
$O(P^0)$

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N} \right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}[(\psi\psi)^\dagger(\psi\overleftrightarrow{\boldsymbol{\nabla}}^2\psi) + \text{H.c.}] + \cdots$$

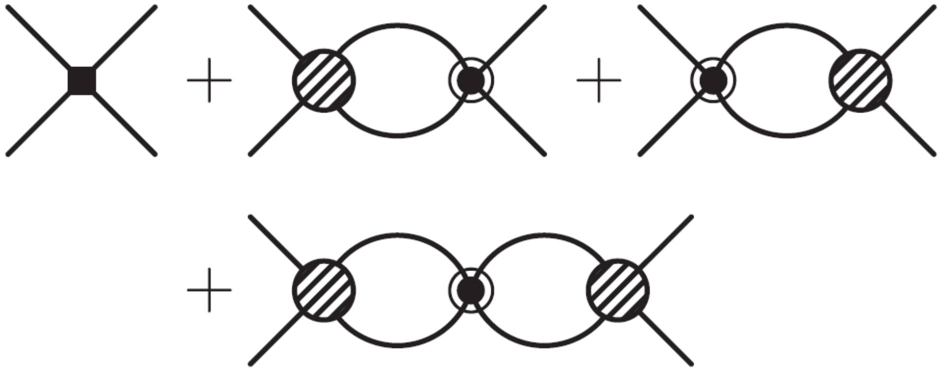
$O(P^2)$

$$V_{2N}(\mathbf{p}', \mathbf{p}) = C_0 + C_2(\mathbf{p}'^2 + \mathbf{p}^2) + \cdots$$

LO from the C_0 interaction



NLO from the C_2 interaction



Contact effective field theory up to NLO with spins

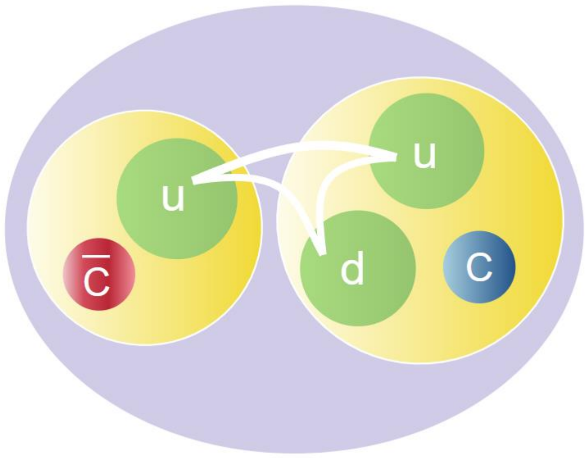
[E. Epelbaum, W. Glöckleb, U.-G. Meißner, Nuclear Physics A 747, 2005]

[J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, j.nuclphysa.2013.06.008, 2013]

$$\mathcal{L} = \psi^\dagger(i\partial_t + \frac{\nabla^2}{2m})\psi + C_0\psi^\dagger\psi\psi^\dagger\psi + C_2[(\psi^\dagger\psi^\dagger)(\psi\overleftrightarrow{\nabla}^2\psi) + H.c.] + \dots$$

$$\begin{aligned} V = {} & C_0 + C_2^1 q^2 + C_2^2 k^2 + (C_2^3 q^2 + C_2^4 k^2)\sigma_1\cdot\sigma_2 \\ & + \frac{i}{2}C_2^5(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(q\times k) + C_2^6(q\cdot\vec{\sigma}_1)(q\cdot\vec{\sigma}_2) \\ & + C_2^7(k\cdot\vec{\sigma}_1)(k\cdot\vec{\sigma}_2) + \frac{i}{2}C_2^8(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(q\times k) \\ & + \cdots \end{aligned}$$

$\overline{D}^{(*)}\ \Sigma_c^{(*)}$



S-wave Molecules

$$\begin{aligned} V(\bar{D}\Sigma_c) &= C_a + 2D_a p_{cm1}^2 \\ V(\bar{D}^*\Sigma_c, \frac{1}{2}^-) &= C_a - \frac{4}{3}C_b + (2D_a - 2D_b)p_{cm2}^2 \\ V(\bar{D}^*\Sigma_c, \frac{3}{2}^-) &= C_a + \frac{2}{3}C_b + (2D_a + D_b)p_{cm2}^2 \end{aligned}$$

Ca, Cb, Da, Db

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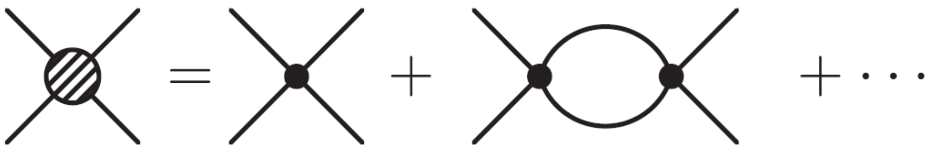
Matching effective range for Pc states

- **Solving T to get the couplings**

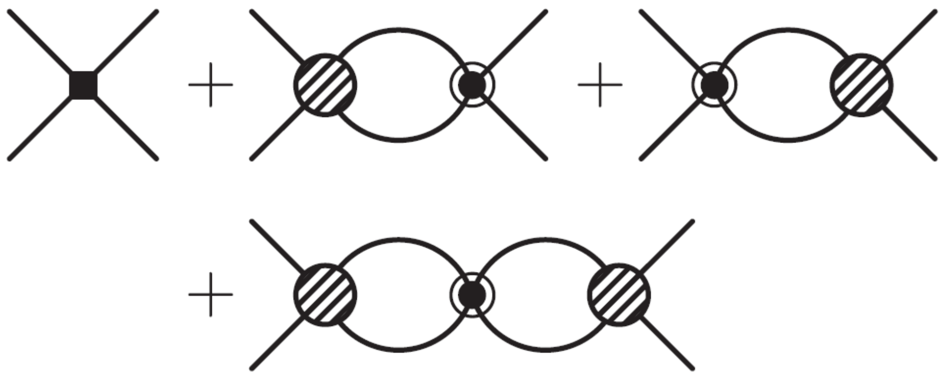
$$\mathcal{T} = V[1 + V\,G(q,\Lambda) + (V\,G(q,\Lambda))^2 + \cdots] = \frac{V}{1 + VG(q,\Lambda)}$$

$$G(E_B,\Lambda) = \frac{\mu}{\pi^2}\int_0^{+\infty} dq \frac{q^2}{2\mu\, E_B + q^2}\, e^{-\frac{q^2}{\Lambda^2}}$$

LO from the C_0 interaction



NLO from the C_2 interaction



- **Expanding T while $a_0 \sim \frac{1}{\gamma} > \frac{1}{\Lambda}$**

NLO

$$\mathcal{T} = -\frac{2\pi}{\mu}\frac{1}{\frac{1}{a_0} + ik}\left[1 + \frac{r_0}{2(\frac{1}{a_0} + ik)}k^2 + \frac{r_0^2}{4(\frac{1}{a_0} + ik)^2}k^4 + \cdots\right]$$

LO

Matching effective range for Pc states

[E. Epelbaum, arxiv: 1001.3229, 2010]

$$C_0 = \frac{2\pi}{\mu} \frac{1}{\frac{1}{a_0} - \frac{2\pi}{\mu} G(0,\Lambda)},$$

$$C_2 = \frac{2\pi}{\mu} \frac{1}{(\frac{1}{a_0} - \frac{2\pi}{\mu} G(0,\Lambda))^2} \frac{r_0}{2}.$$

$$r_0 = \frac{4\pi}{\mu} \frac{C_2}{C_0^2}$$

[D.B. Kaplan, J.M. Savage, and M.B. Wise, Nucl.Phys.B 534, 1998]

- **Notation about the regulator methods and power counting**

$$\mathcal{T} = -\frac{2\pi}{\mu} \frac{1}{\frac{1}{a_0} + ik} [1 + \frac{r_0}{2(\frac{1}{a_0} + ik)} k^2 + \frac{r_0^2}{4(\frac{1}{a_0} + ik)^2} k^4 + \cdots]$$

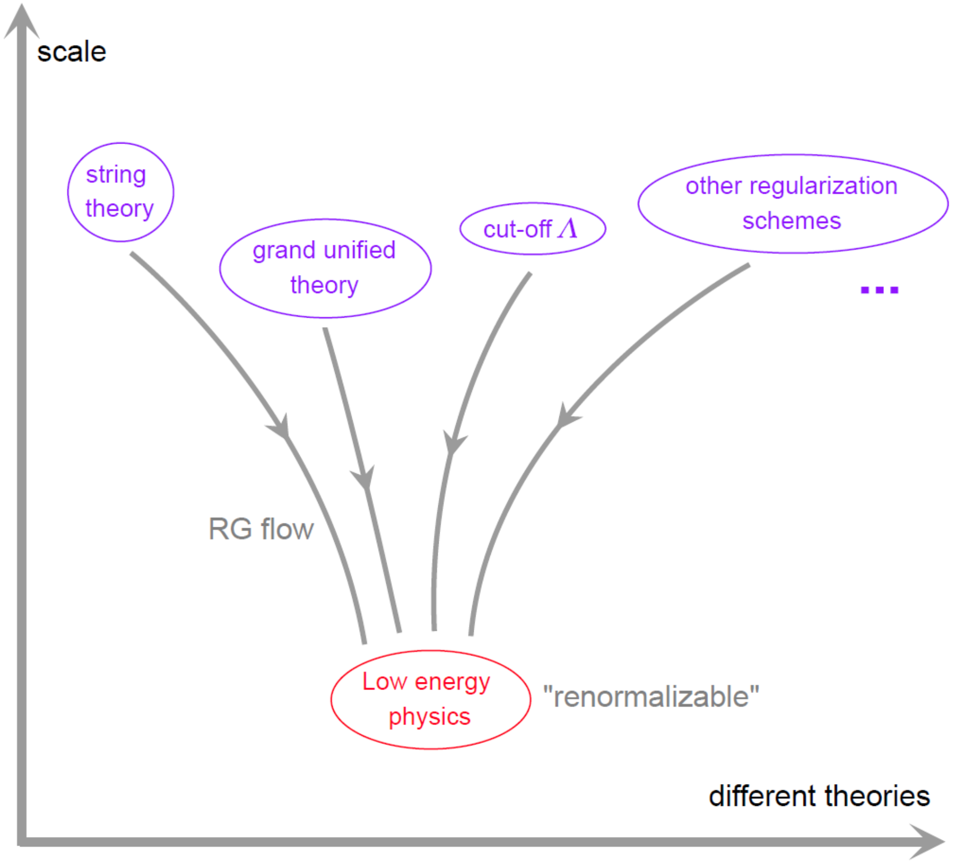
$O(k^{-1})$ $O(k^0)$

avoid the problem about the discussion from renormalization

How to deal with 4 couplings with 3 inputs from Pc(4312), Pc(4440) and Pc(4457) ?

- **Scheme A:** in this scheme, we just use the potential by neglecting the spin-spin interaction relevant term, namely, setting Db=0
- **Scheme B:** by bring in the Pc(4380) discovered by LHCb in 2015, we now have 4 mass inputs of the Pc(4312), Pc(4440), Pc(4457) and Pc(4380) states
- **Scheme C:** within this scheme, the dimensional analysis of the low energy couplings or Wilsonian coefficients in effective field theory can be used to determine the Db term

Effective field theory



$$\mathcal{Z} = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{iS(\phi_L,\phi_H)} = \int \mathcal{D}\phi_L e^{iS_\Lambda(\phi_L)}$$

$$\rightarrow \quad S_\Lambda(\phi_L) = \int d^d x \sum_n \lambda_n O_n$$

- **As the energy scale flow from ultraviolet (UV) to infrared (IR) zone, the contributions of non-renormalizable irrelevant operators will be suppressed while the finite marginal and relevant operators become more and more important, thus the low energy theory often shown to be renormalizable.**

$$\mathcal{L}_{eff} = \sum_n C_n O_n \quad \rightarrow \quad \frac{C_i}{C_j} = \frac{\Lambda^{d_j}}{\Lambda^{d_i}}$$

$$\begin{aligned} V(\bar{D}\Sigma_c) &= C_a + 2D_a p_{cm1}^2 \,, \\ V(\bar{D}^*\Sigma_c\,, \frac{1}{2}^-) &= C_a - \frac{4}{3}C_b + (2D_a - 2D_b)p_{cm2}^2 \,, \\ V(\bar{D}^*\Sigma_c\,, \frac{3}{2}^-) &= C_a + \frac{2}{3}C_b + (2D_a + D_b)p_{cm2}^2 \end{aligned}$$

$$\frac{C_2}{C_0} = \Lambda^{-2}$$

Outlines

1. Introduction of exotic hadrons and Pc states
2. Effective range expansion and compositeness
3. Molecule descriptions of Pc states with contact effective field theory
4. Matching effective range with NLO contact potential
5. Results and discussion

Results from Scheme A & B

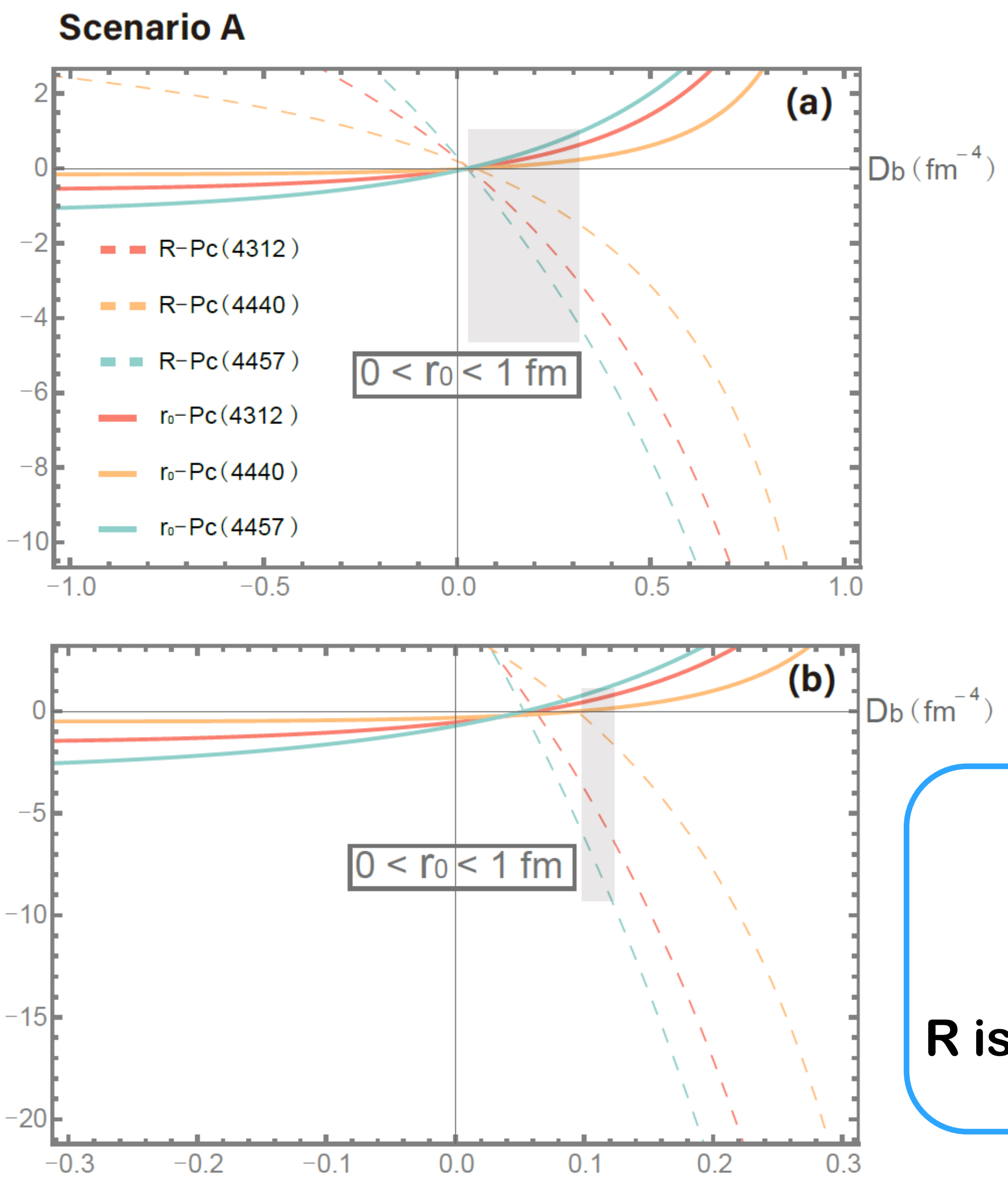
Scenario	Λ(GeV)	C_a(fm^{-2})	C_b(fm^{-2})	D_a(fm^{-4})	$r_{0P_{c1}}$(fm)	$r_{0P_{c2}}$(fm)	$r_{0P_{c3}}$(fm)	$R_{P_{c1}}$	$R_{P_{c2}}$	$R_{P_{c3}}$
A	0.5	-2.2	0.59	-0.09	-0.04	-0.02	-0.06	0.3	0.2	0.3
B	0.5	-1.77	-0.15	0.82	0.61	0.53	0.75	3.0	3.4	2.8
A	1	-0.89	0.20	-0.18	-0.54	-0.31	-0.72	5.3	4.1	6.2
B	1	-0.75	-0.05	0.12	0.50	0.44	0.58	4.1	4.5	4.0

TABLE I: The effective range of $P_c(4440)$ and $P_c(4457)$ derived from scheme (A), where D_b term has been neglected and two cutoffs of 0.5GeV and 1GeV are used for Λ.

Scenario	Λ(GeV)	C_a(fm^{-2})	C_b(fm^{-2})	D_a(fm^{-4})	D_b(fm^{-4})	$r_{0P_{c1}}$(fm)	$r_{0P_{c2}}$)(fm)	$r_{0P_{c3}}$(fm)	$r_{0P_{c4}}$(fm)	$R_{P_{c1}}$	$R_{P_{c2}}$	$R_{P_{c3}}$	$R_{P_{c4}}$
A	0.5	-1.44	0.19	1.50	0.54	1.68	0.75	2.28	1.66	6.7	3.6	8.6	6.7
B	0.5	-1.44	-0.22	1.50	-0.91	1.68	0.93	4.10	1.66	6.7	13.5	4.2	6.7
A	1	-0.64	0.07	0.33	0.17	1.87	0.65	2.62	1.85	13.2	5.5	17.9	13.2
B	1	-0.64	-0.07	0.33	-0.28	1.87	0.89	4.62	1.85	13.2	28.9	7.0	13.2

TABLE II: The effective range of $P_c(4380)$, $P_c(4440)$ and $P_c(4457)$ derived from scheme (B) with two cutoffs of 0.5GeV and 1GeV are used for Λ.

Results from Scheme C



$$N \sim 10$$

$$D_b \sim N \times \frac{1}{\Lambda^2} \times [-C, C]$$

$$\frac{D}{C} = \frac{1}{M_{Lo} M_{Hi}}$$

$$M_{Lo} \sim \gamma \sim 100 - 200\ MeV$$

$$M_{Hi} \lesssim \Lambda$$

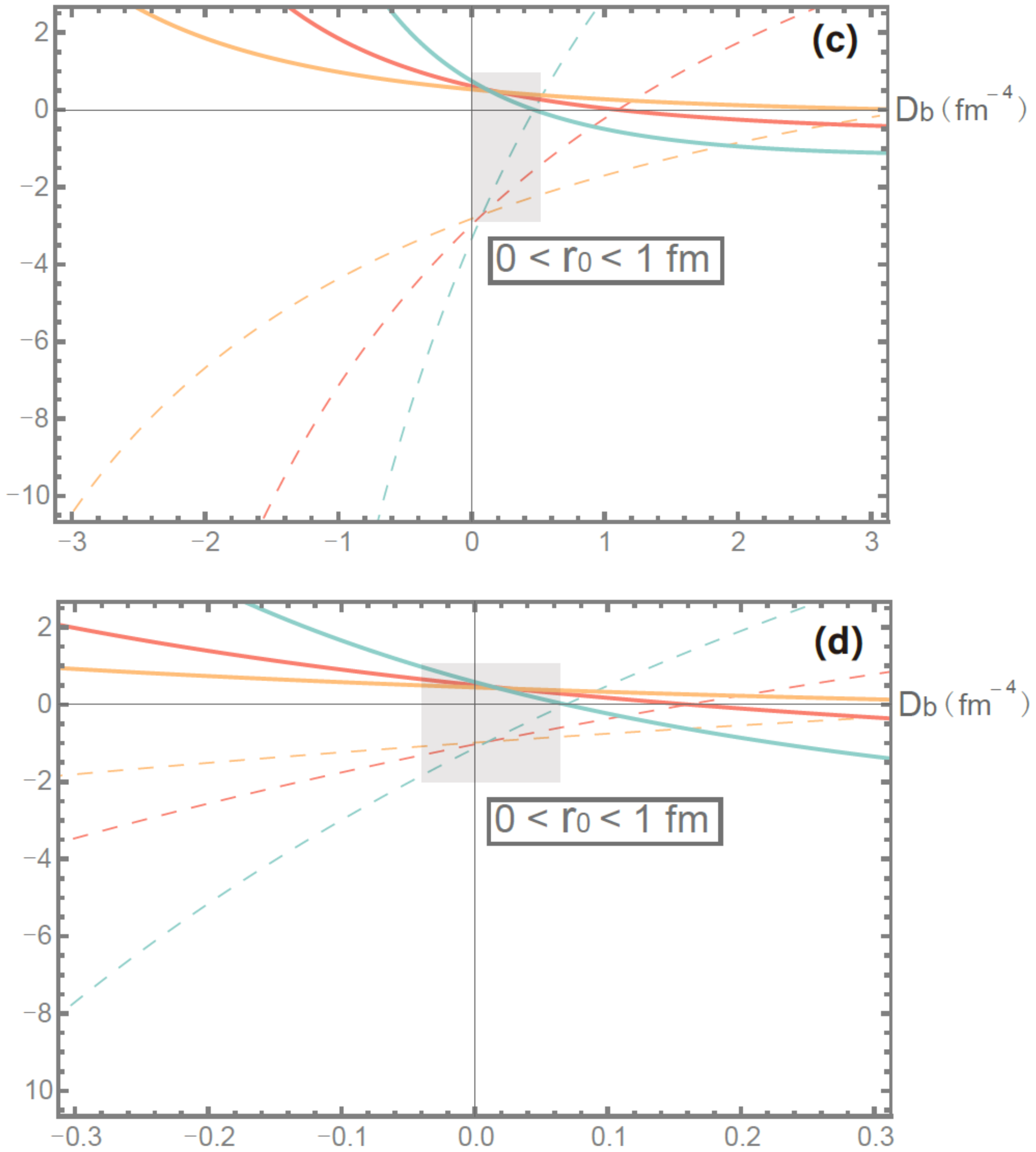
$$R := \frac{D}{C/\Lambda^2}$$

$$r_0 \sim O\left(\frac{1}{m_{ex}}\right)$$

R is reasonable to be O(1) when $r_0 \sim 0.5$ fm for Pc states

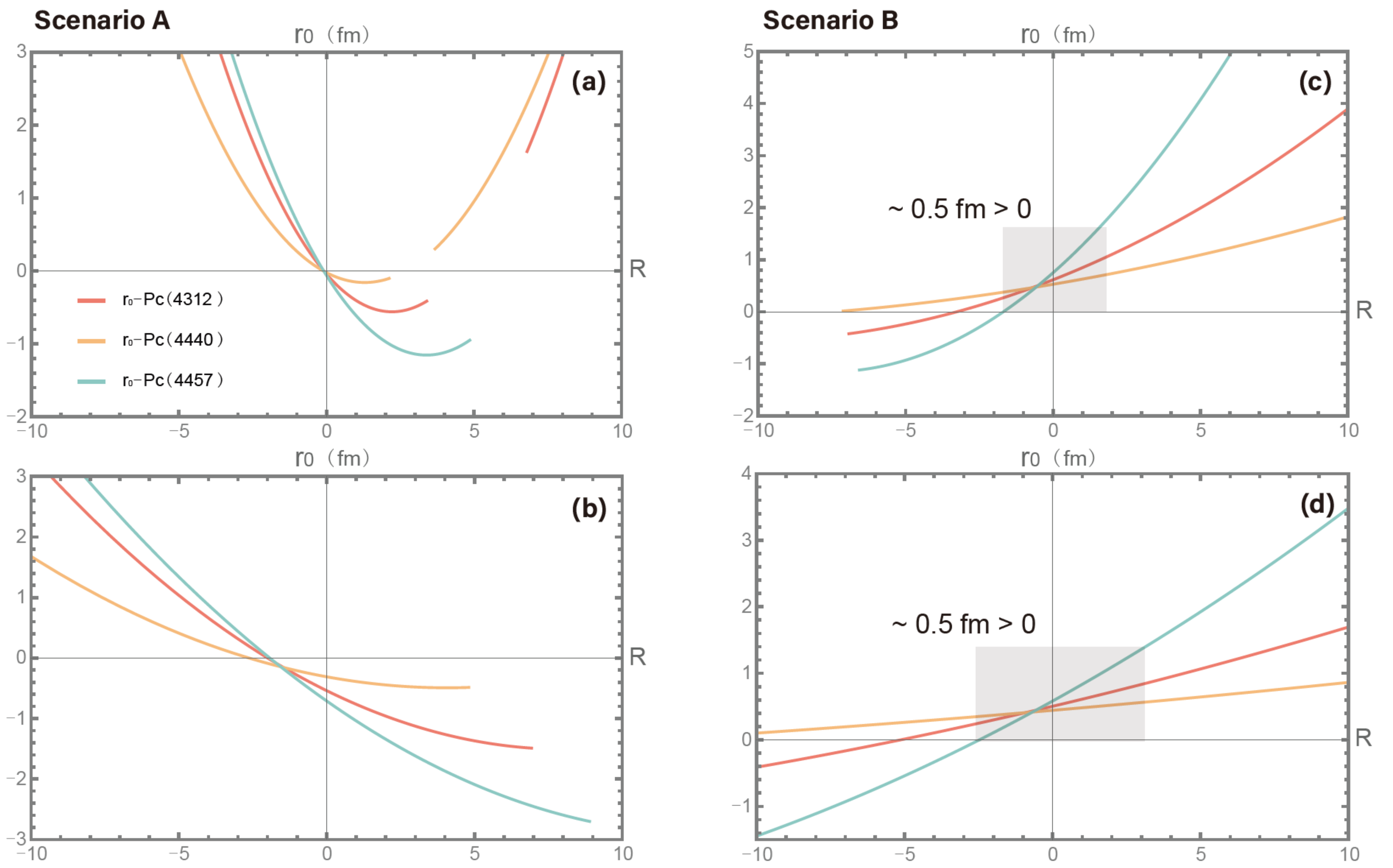
Results from Scheme C

Scenario B



- We get the reasonable R for the r_0 ~ 0.5 fm area
- All the 3 pentaquarks get a same r_0 ~ 0.5 fm with the same R

Results from Scheme C



Conclusions

- **To summarize, matching the effective range expansion from scattering amplitude, we conclude it is more natural to assign the quantum numbers $J^P = \frac{3}{2}^-$ to Pc(4440) and $J^P = \frac{1}{2}^-$ to Pc(4457) in molecular $\bar{D}^*\Sigma_c$ states within 3 schemes**

- **Besides, the results presented from scheme (B) suggest that the Pc(4380) may not be appropriately considered as part of the pure molecular system alongside Pc(4312), Pc(4440) and Pc(4457) states. This conclusion aligns with the observation that the broad Pc(4380) resonance, initially reported in 2015, has not been confirmed in later LHCb experiments conducted in 2019**

- **The above findings hold significant importance for future experimental investigations and further theoretical research on pentaquarks' nature**

THANK YOU !