Charming Opportunities in CP violation

NPG workshop, HIAS

PRD 109, L071302 (2024), arXiv:2404.19166
arXiv:24XX.XXXX

劉 佳 韋

TDLI

Aug. 28, 2024

Collaborators:耿朝强、何小刚

Histories of Charm Quark - November revolution

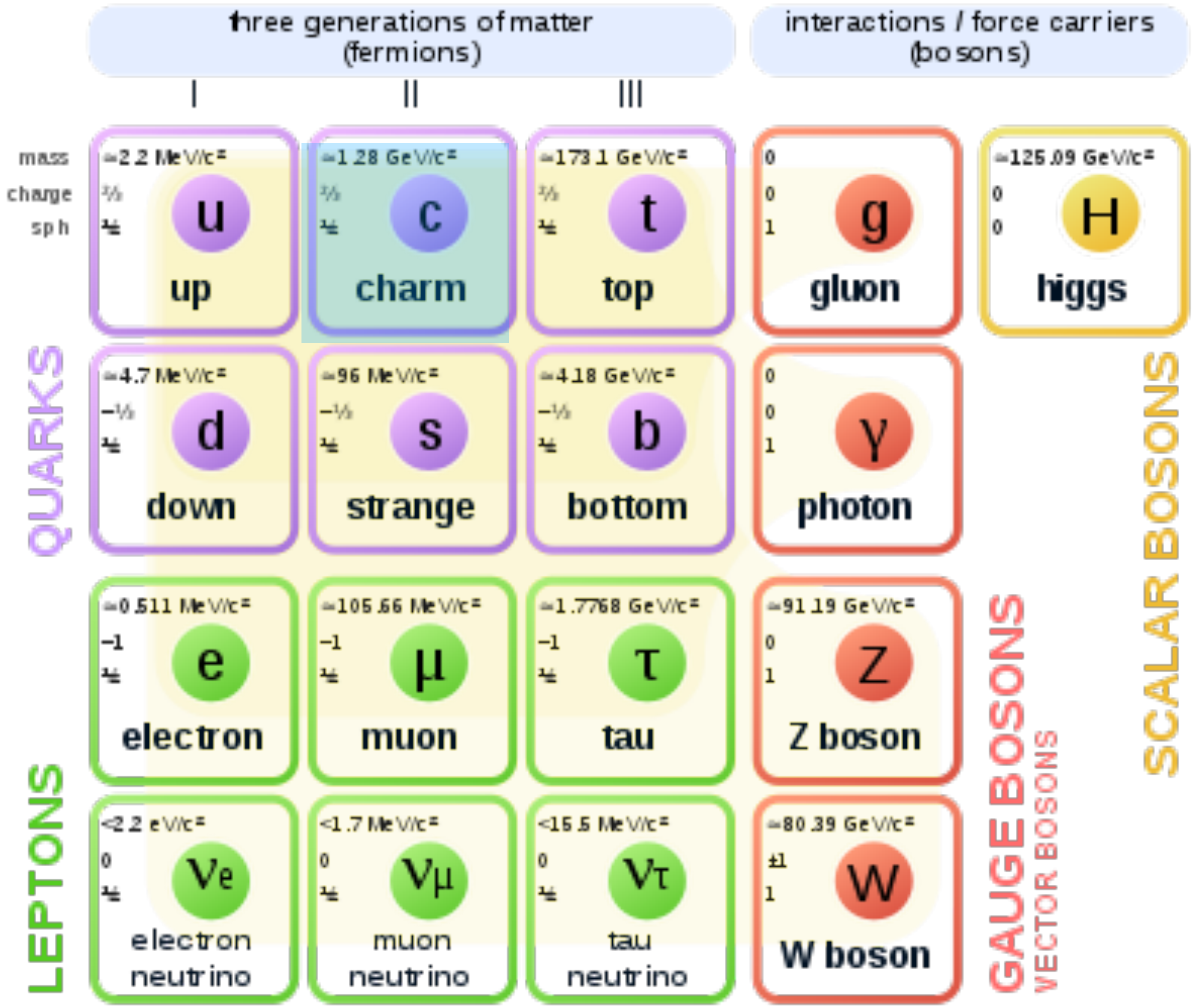
A discovery of extremely massive, narrow and high pyramid.

Charm

China element

中国元素

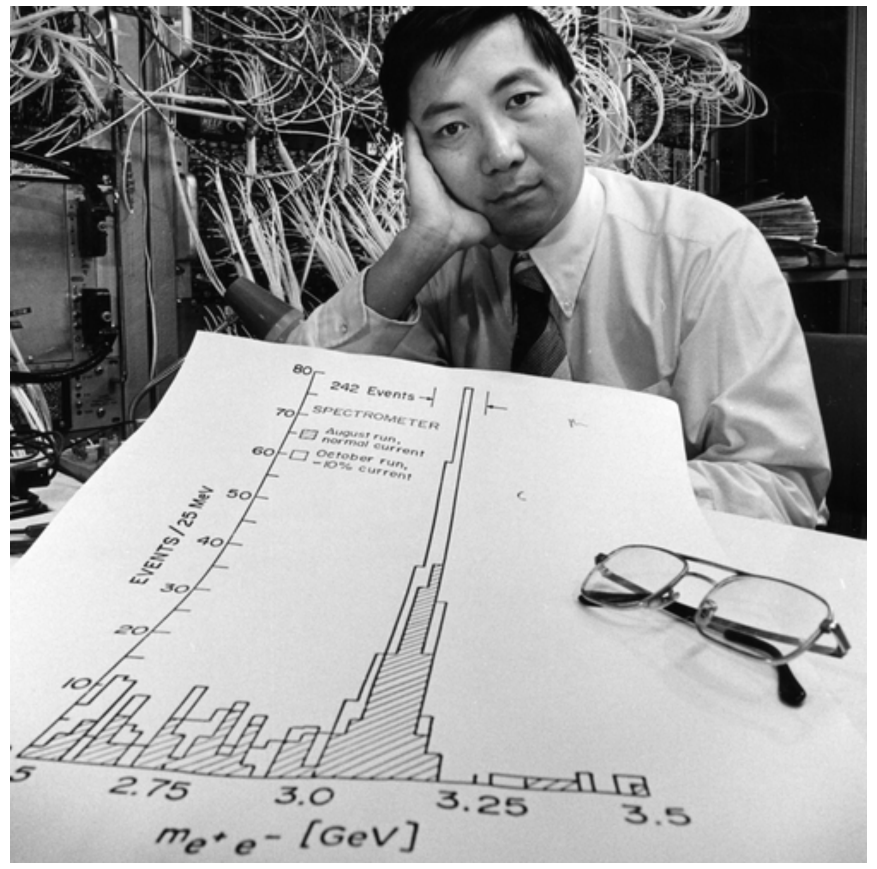
Standard Model of Elementary Particles



A. Khare, ``The November J/ψ revolution: Twenty five years later,''
Curr. Sci. 77, 1210 (1999)

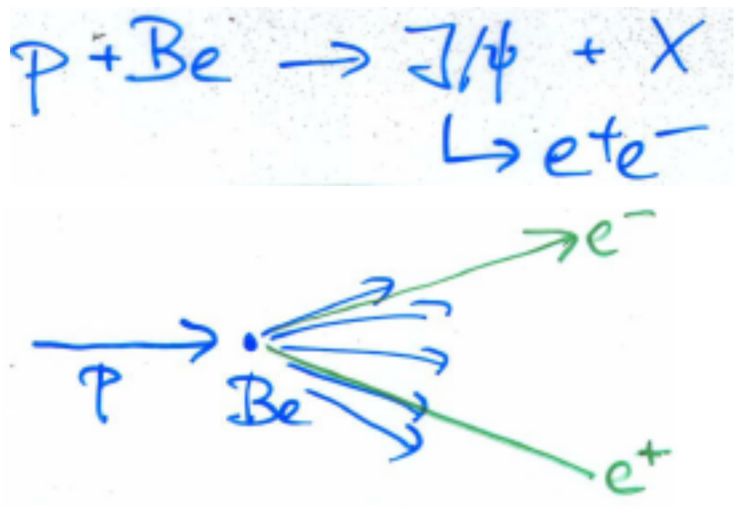
Scanning energies from 2~4 GeV for two weeks Aug. 22, 1974

At the East coast of US: Received by PRL on Nov. 12, 1974



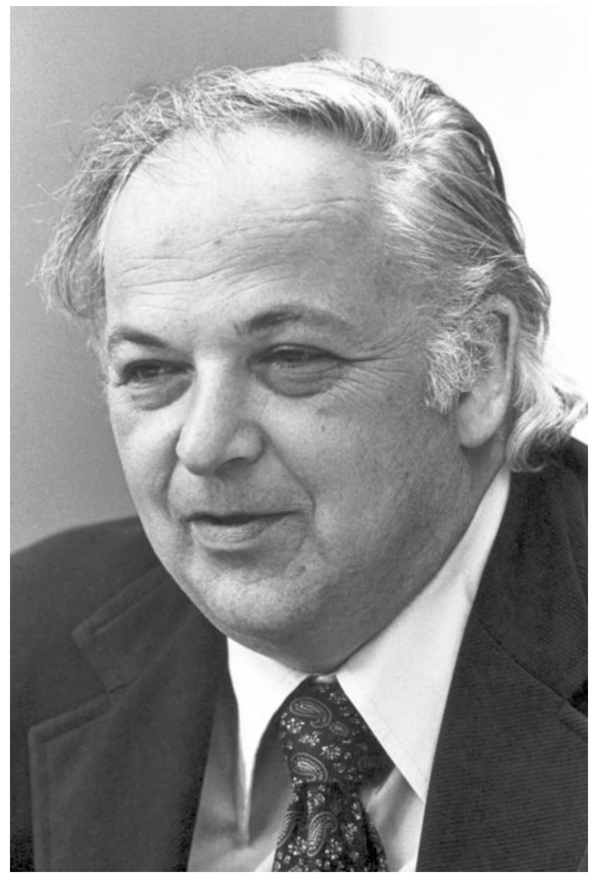
丁肇中

Brookhaven (Proton Synchrotron)



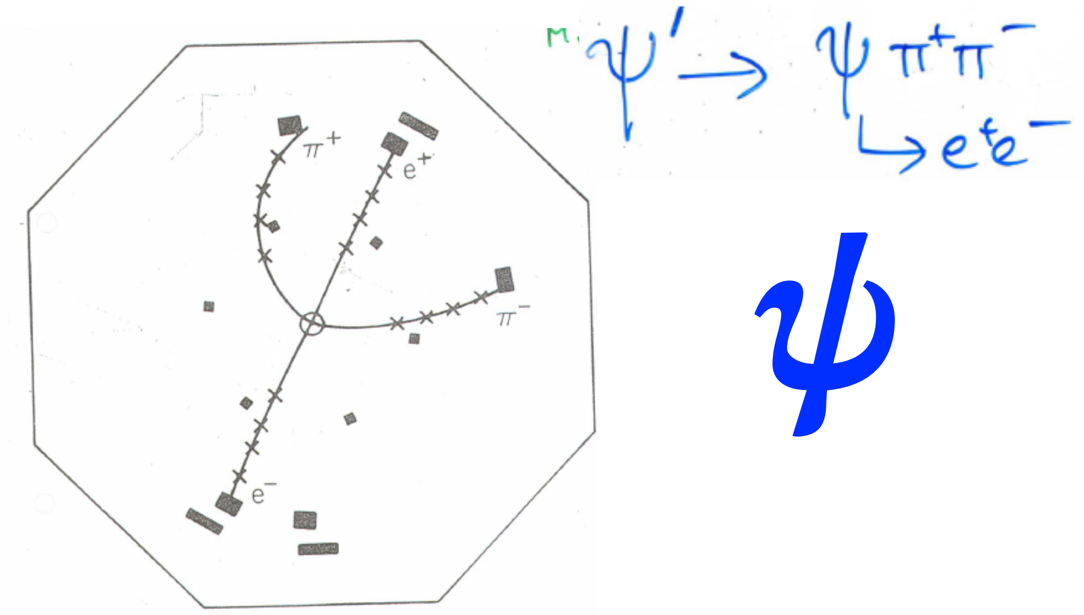
Adjusted the machine to 3.1 GeV Nov. 9, 1974

At the West coast of US: Received by PRL on Nov. 13, 1974



Burton Richter

SLAC (e^+e^- storage ring at 4.5-6 GeV)



Slide from Pro. Chaoqiang Geng

Why are there matters?

Where are antimatters?

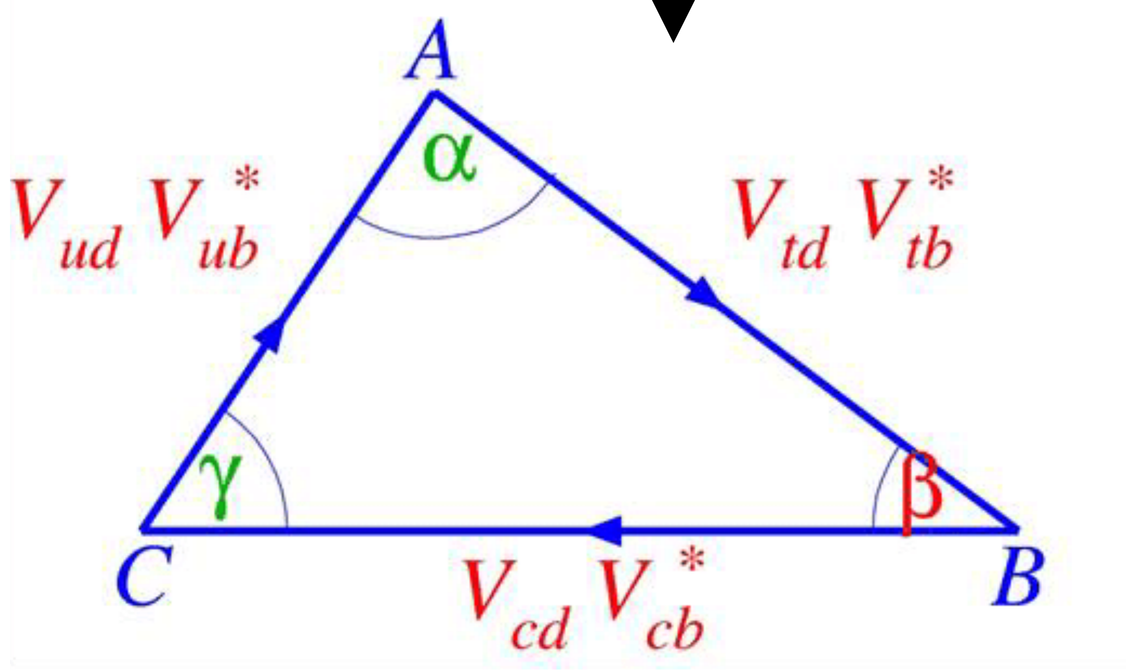
- **Charming physics - CP violation**

$$D^0 \to \pi^+\pi^-$$

$$a_{CP}^{\pi\pi} \sim 2\mathrm{Im}\left(\frac{V_{cs}V_{us}^*}{V_{cd}V_{ud}^*}\right)\left|\frac{\text{Penguin}}{\text{Tree}}\right| \sin\delta_{QCD}$$

CKM as input

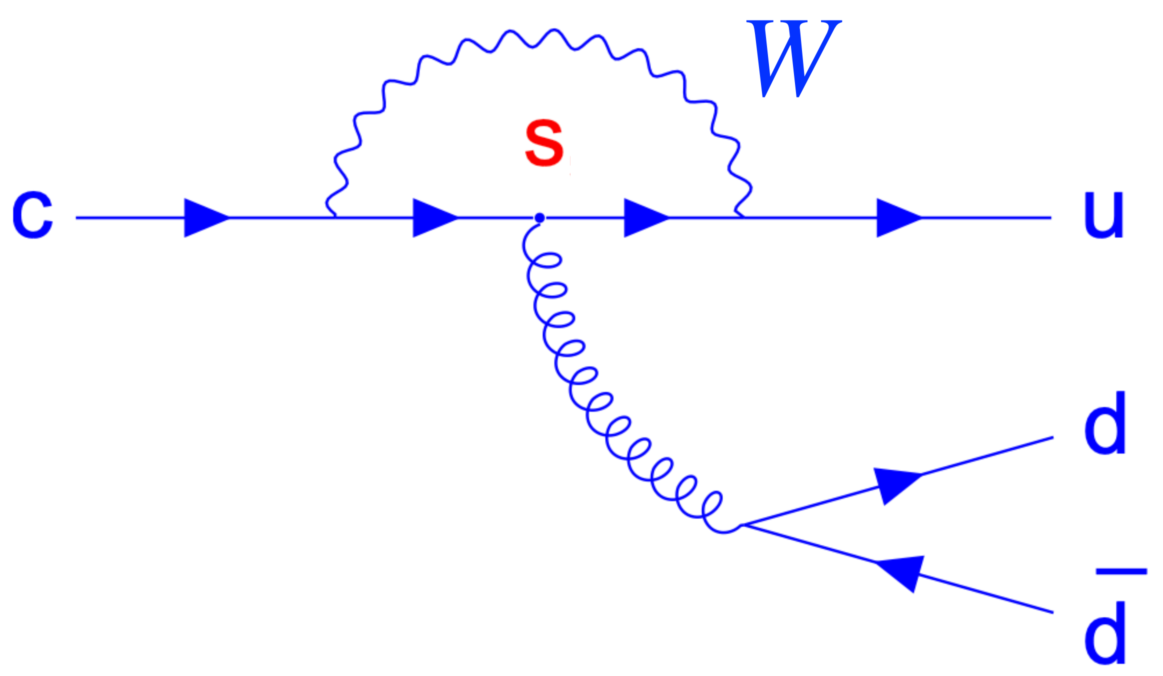
$\mathcal{O}(10^{-3})$



$c\bar{u} \to u\bar{d}, d\bar{u}$

Penguin to generate second weak phase !

$\mathcal{O}(10^{-1})$



- **Charming physics - CP violation**

$$D^0 \to \pi^+\pi^-$$

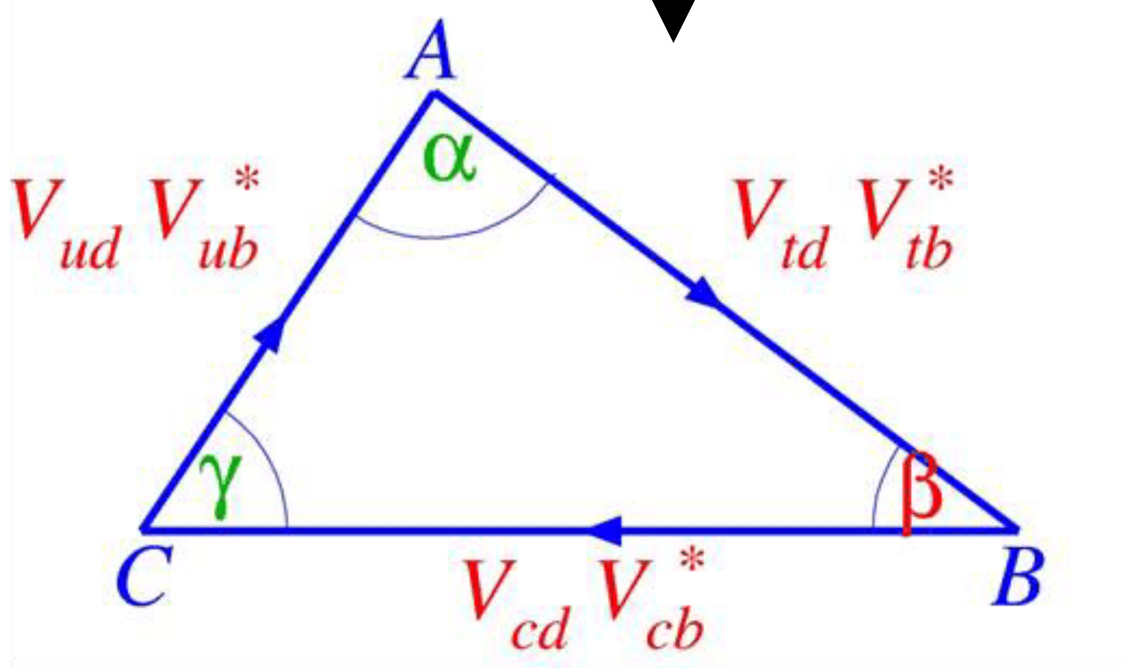
$$a_{CP}^{\pi\pi} \sim 2\mathrm{Im}\left(\frac{V_{cs}V_{us}^*}{V_{cd}V_{ud}^*}\right)\left|\frac{\text{Penguin}}{\text{Tree}}\right| \sin\delta_{QCD}$$

$$D^0 \to K^+K^-$$

$$a_{CP}^{KK} \sim 2\mathrm{Im}\left(\frac{V_{cd}V_{ud}^*}{V_{cs}V_{us}^*}\right)\left|\frac{\text{Penguin}}{\text{Tree}}\right| \sin\delta_{QCD}$$

CKM as input

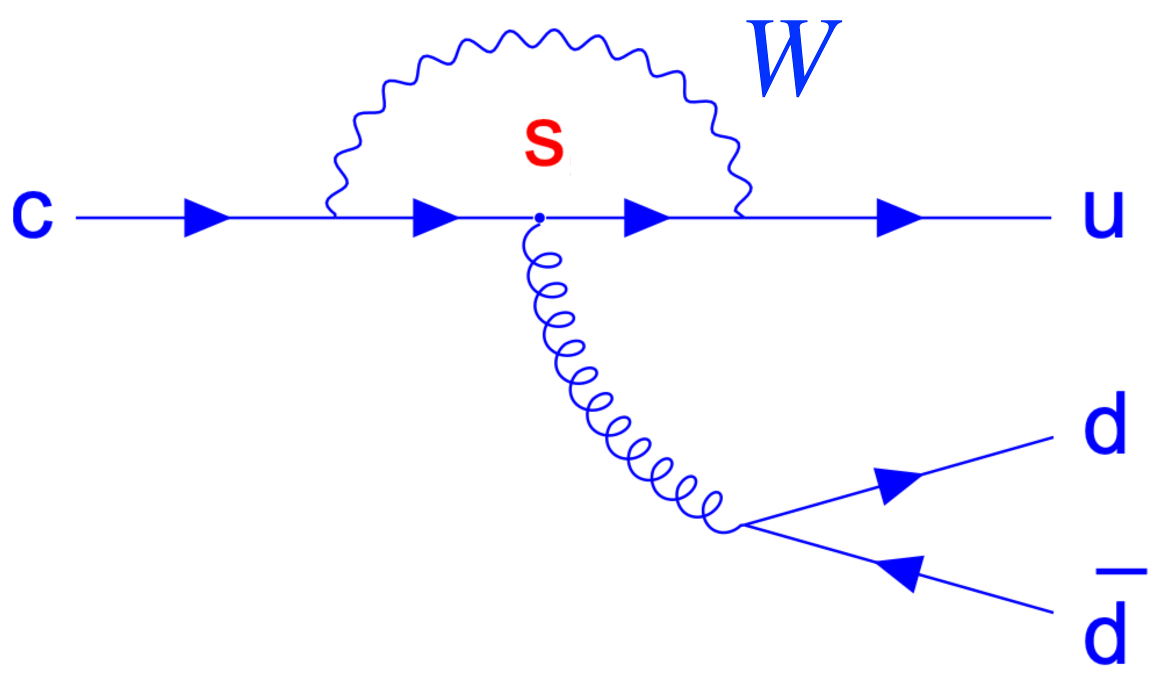
$\mathcal{O}(10^{-3})$



$$c\bar{u} \to u\bar{d}, d\bar{u}$$

Penguin to generate second weak phase !

$\mathcal{O}(10^{-1})$



Using $V_{cd}V_{ud}^* + V_{cs}V_{us}^* + V_{cb}V_{ub}^* = 0\,,\quad |V_{cs}V_{us}^*| \gg |V_{cb}V_{ub}^*|$ $\longrightarrow$ $a_{CP}^{\pi\pi} + a_{CP}^{KK} \approx 0$

$$a_{CP}^{\pi\pi} = (23.2 \pm 6.1) \times 10^{-4} \qquad a_{CP}^{KK} = (7.7 \pm 5.7) \times 10^{-4}$$

The sign was flipped !

PRL 131, 091802 (2023)

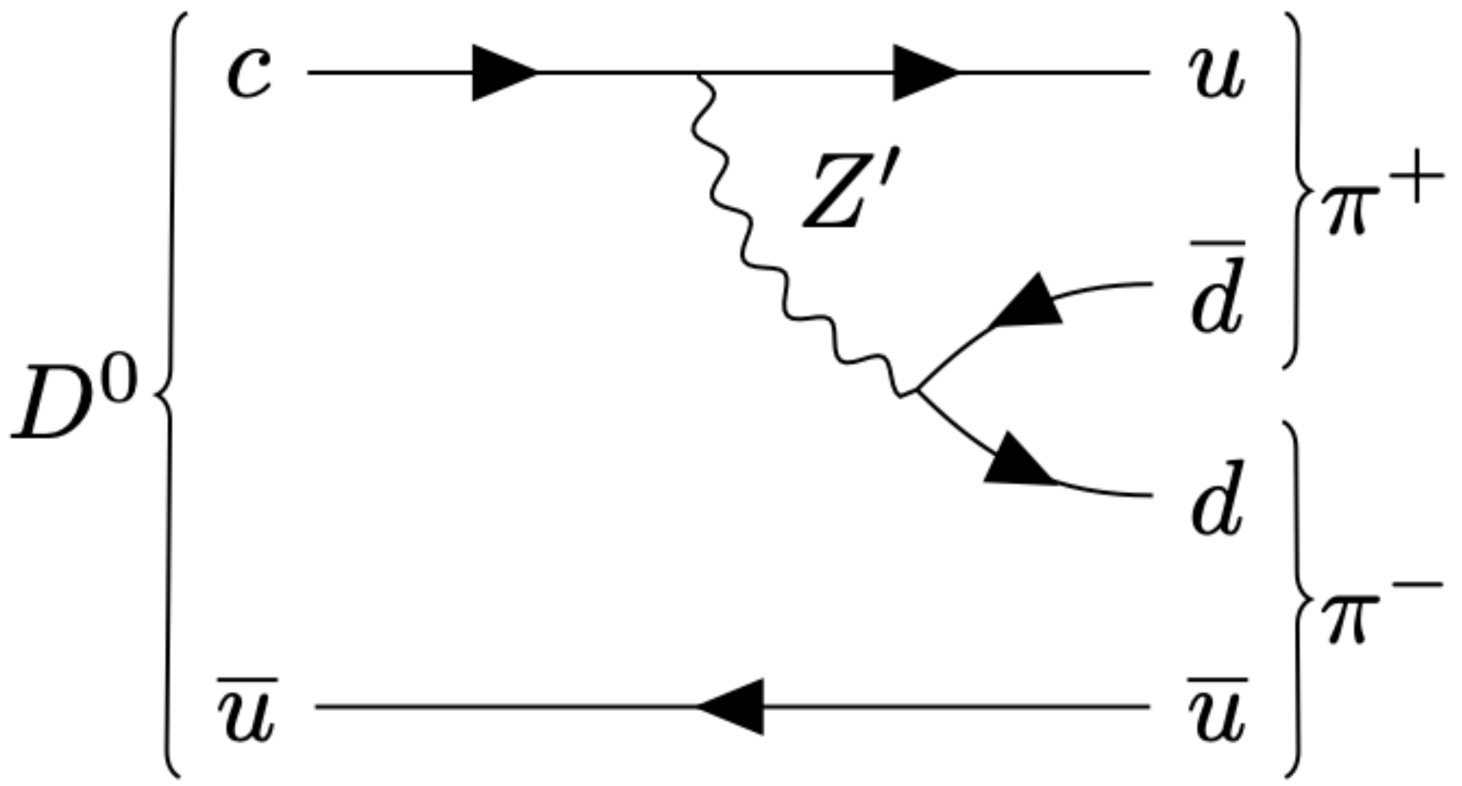
• Charming physics - CP violation

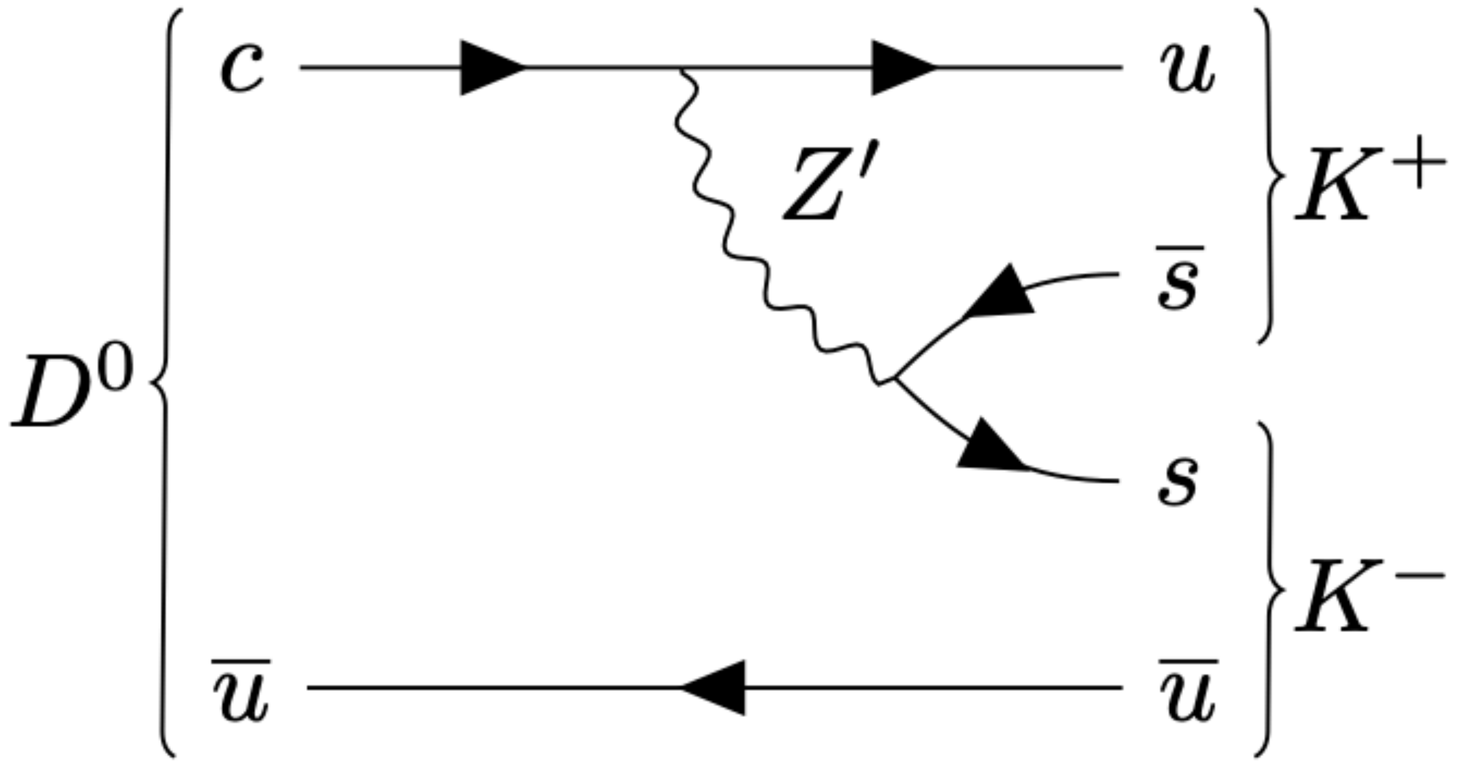
Why shall we work on charm CP violation?

- SM ***naively*** predicts tiny CP asymmetries but they were found to be an order larger!

- SM ***naively*** predicts $a_{CP}^{\pi\pi} = -\, a_{CP}^{KK}$ but two of them are found to be in the same sign !

- We know there must be additional CPV sources !





PRD **108**, 035005 (2023)

- **Charming physics - CP violation**

Reasons to go beyond charmed meson.

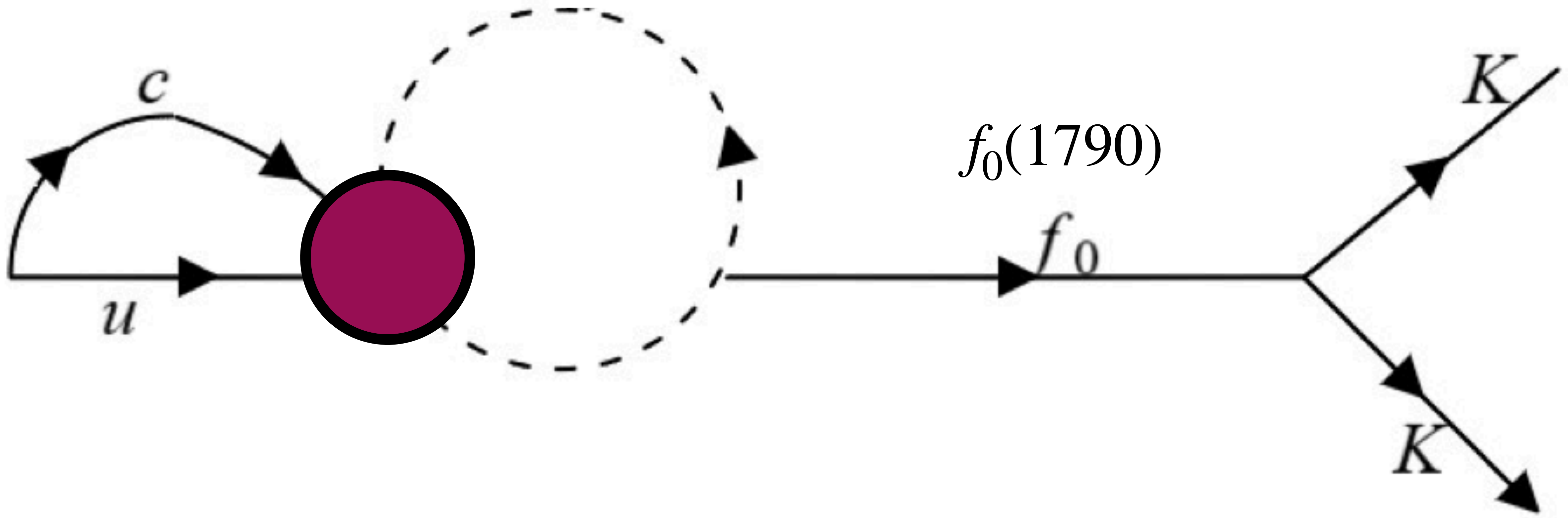
PHYSICAL REVIEW D **81,** 074021 (2010)

Two-body hadronic charmed meson decays

Hai-Yang Cheng[1,2] and Cheng-Wei Chiang[1,3]

Enhancement of charm CP violation due to nearby resonances

Stefan Schacht[a,*], Amarjit Soni[b]   PLB **825,** 136855 (2022)



$$a_{CP}^{\pi\pi} \sim 2\mathrm{Im}\left(\frac{V_{cs}V_{us}^*}{V_{cd}V_{ud}^*}\right)\left|\frac{\text{Penguin}}{\text{Tree}}\right| \sin\delta_{QCD}$$

1. f_0 might be a glueball which mainly decays to kaons. Amplitude $\propto m_q$.
2. Its mass is too close to D meson, enhancing SU(3) breaking effects from mass splitting.
3. Unlike $D^0 \to h^+h^-$, CP even phase shifts in baryon decays can be directly measured.

Experimental status of charmed hadron decays

2019: First evidence of CP violation in charm sector

PRL **122**, 211803 (2019)

$$A_{CP}^{dir}(D^0 \to K^+K^-) - A_{CP}^{dir}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

An order larger than theoretical expectations!

* First evidence of CP violation in charm hadron decays.

2022: The first measurement of CP violation in charmed baryon two-body decays

Sci. Bull. **68**, 583-592 (2023)

$$A_{CP}(\Lambda_c^+ \to \Lambda K^+) = 0.021 \pm 0.026$$

* The most precise CP violation measurement by far in charmed baryons.

2023: Measurements of strong phases in $\Lambda_c^+ \to \Xi^0 K^+$

PRL **132**, 031801 (2024)

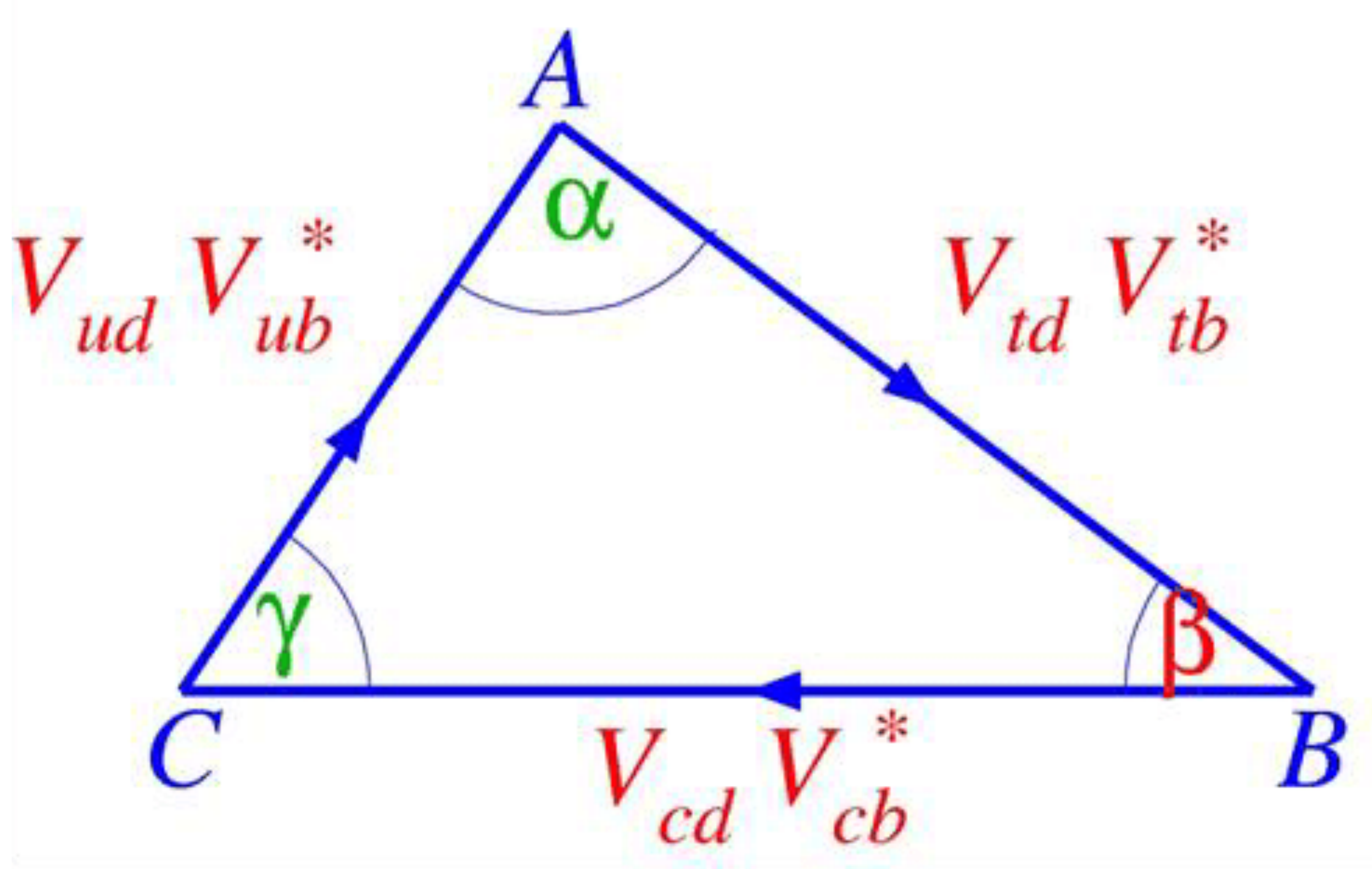
$$\delta_P - \delta_S = -1.55 \pm 0.27 (+\pi), \quad \alpha = 0.01 \pm 0.16$$

* CP even and Cabibbo-favored, but very important to studies of CP violation!

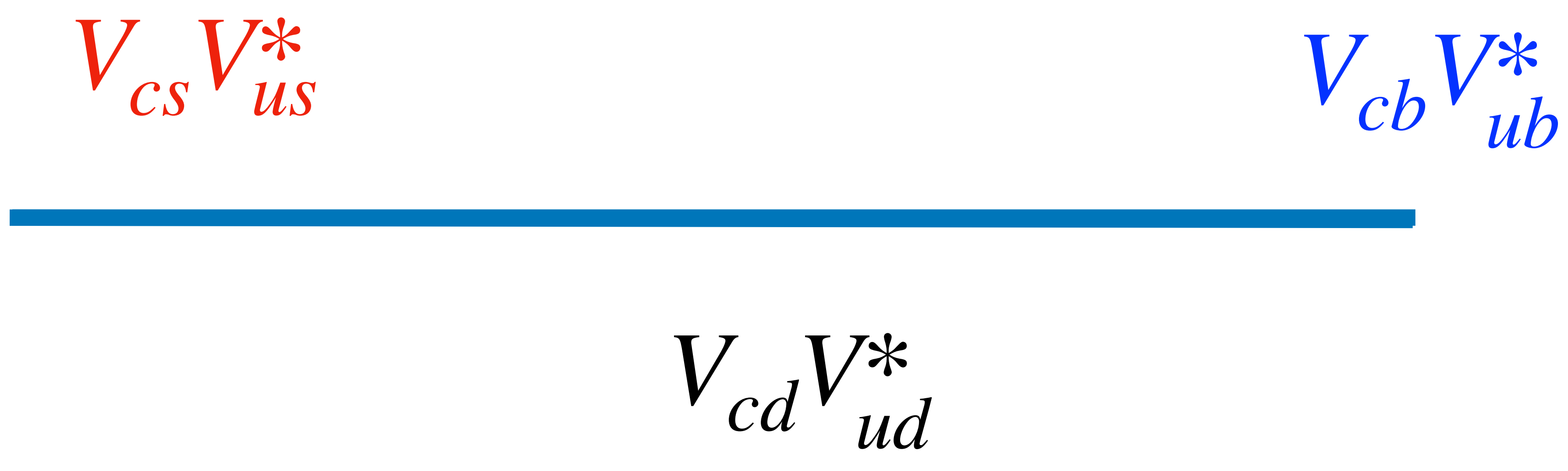
- **SU(3) flavor perspective of charmed baryon decays**

$$\text{Amplitudes}: V_{cs}V_{us}^* \overbrace{F^{s-d}}^{\text{5 parameters}} + V_{cb}V_{ub}^* \overbrace{F^{b}}^{\text{4 parameters}}$$

Do not need to consider F^b in studying CP-even quantities. ⟷ F^b cannot be determined with CP-even quantities.



CKM triangle for $b \to d$

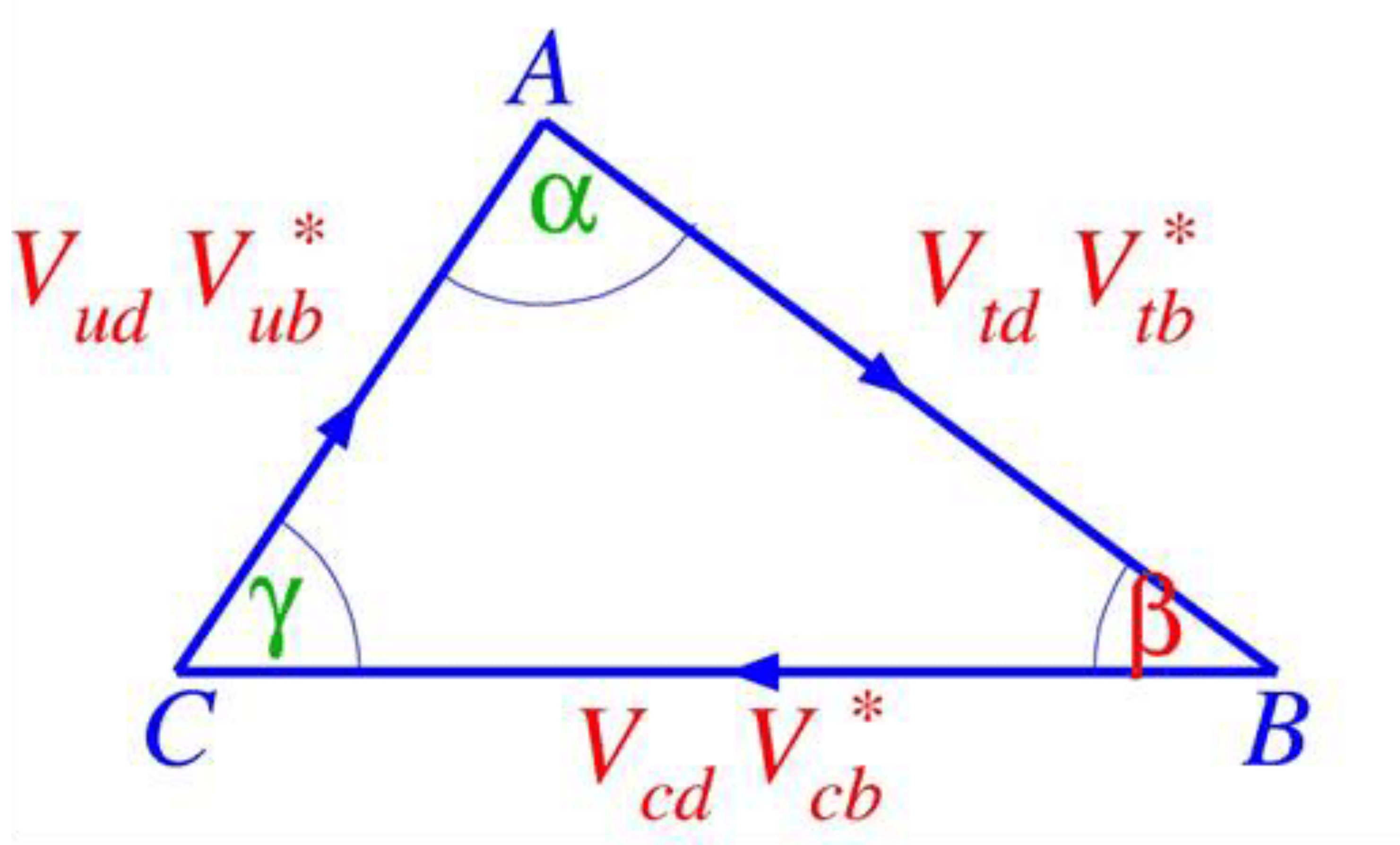


CKM triangle for $c \to u$

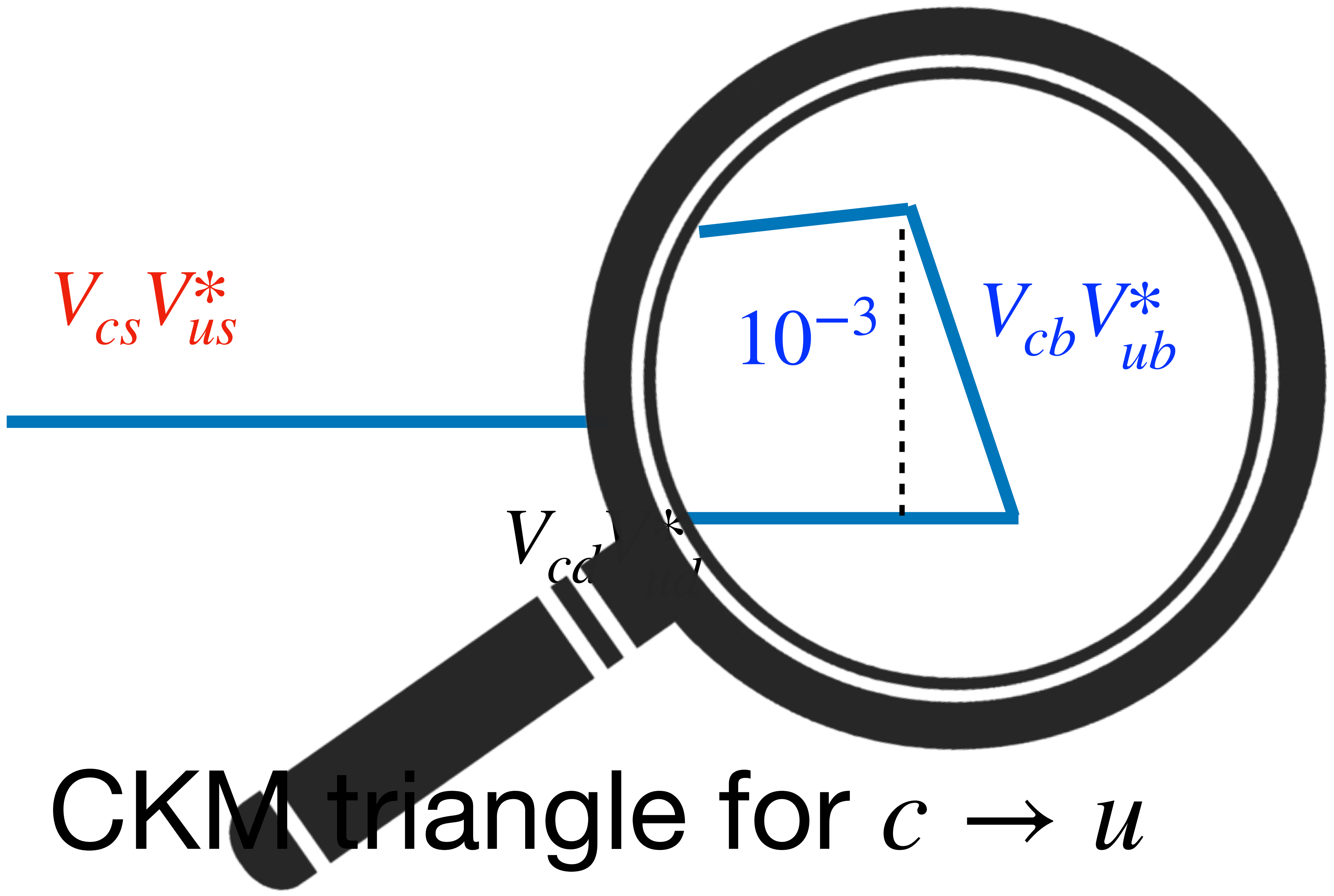
- **SU(3) flavor perspective of charmed baryon decays**

$$\text{Amplitudes}: V_{cs}V_{us}^* \overbrace{F^{s-d}}^{5\text{ parameters}} + V_{cb}V_{ub}^* \overbrace{F^{b}}^{4\text{ parameters}}$$

Do not need to consider F^b in studying CP-even quantities. ⟷ F^b cannot be determined with CP-even quantities.



CKM triangle for $b \to d$



CKM triangle for $c \to u$

SU(3) flavor analysis

$V_{cs}^* V_{us}$ Tree + ~~$V_{cb}^* V_{ub}$ Penguin~~

Insensitive to CP-even quantities & undetermined

Final State Rescattering

$V_{cs}^* V_{us}$ Tree + $V_{cb}^* V_{ub}$ Tree × (Penguin / Tree)

Determined by the rescattering

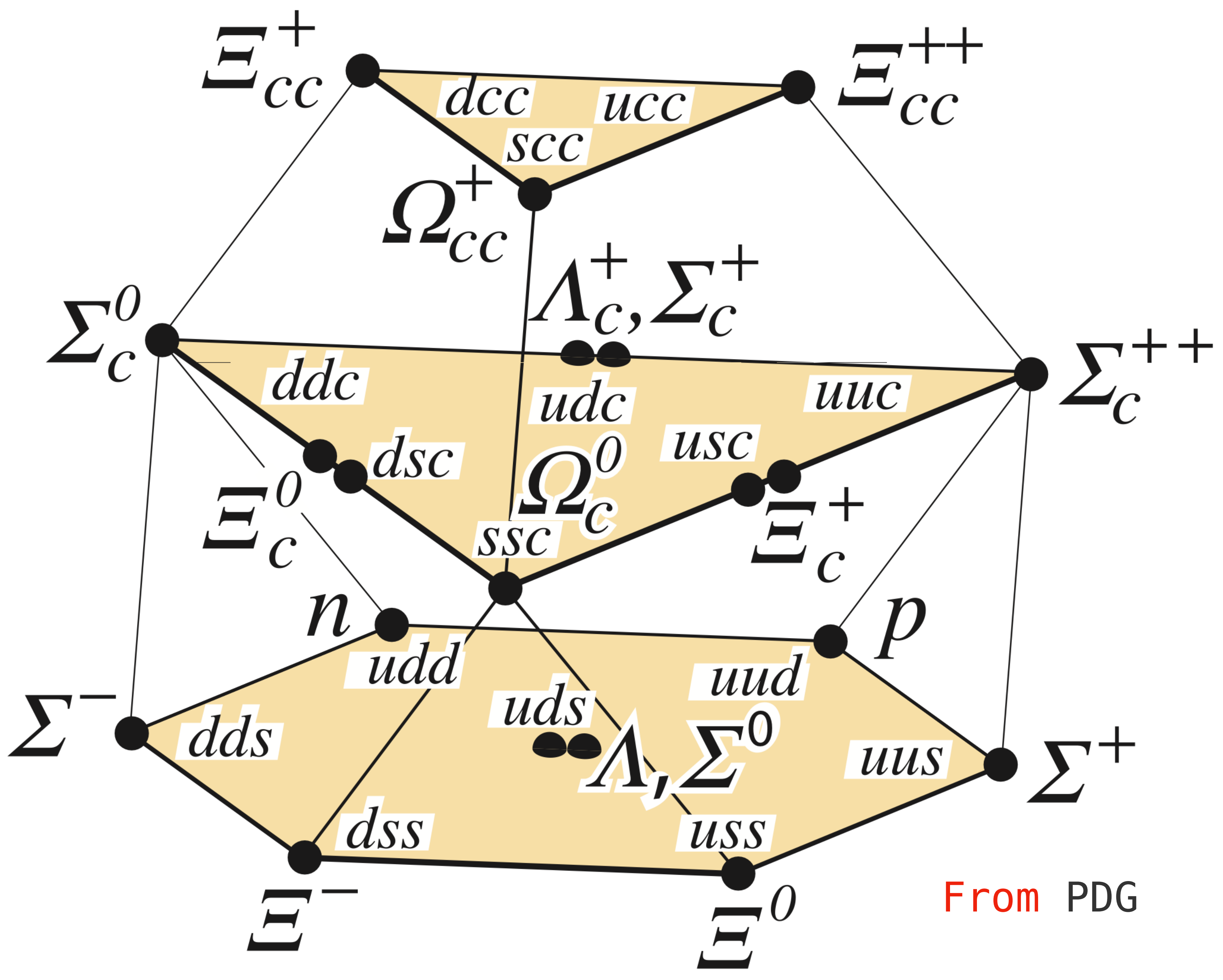
- **SU(3) flavor analysis — Tree**

SU(3) flavor representations :

$$\mathbf{B}_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)\,,$$

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix},$$

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c_\phi \eta + s_\phi \eta') & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\pi^0 + c_\phi \eta + s_\phi \eta') & K^0 \\ K^- & \bar{K}^0 & -s_\phi \eta + c_\phi \eta' \end{pmatrix},$$



From PDG

- **SU(3) flavor analysis — Tree**

*V – A dirac strucutre implied

$$\mathscr{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1(\bar{u}q)(\bar{q}c) + C_2(\bar{q}q)(\bar{u}c) \right) + \lambda_b \sum_{i=3\sim6} C_i Q_i \right] + \text{(H.c.)}$$

$\lambda_q = V^*_{cq} V_{uq}$ $\qquad \lambda_d + \lambda_s + \lambda_b = 0$

Cabibbo-suppressed decays $(c \to u)$

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

$$\begin{aligned}\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \Bigg\{ & \frac{\lambda_s - \lambda_d}{2} \Big[C_+ \big((\bar{u}s)(\bar{s}c) + (\bar{s}s)(\bar{u}c) - (\bar{d}d)(\bar{u}c) - (\bar{u}d)(\bar{d}c) \big)_{\mathbf{15}} \\ & + C_- \big((\bar{u}s)(\bar{s}c) - (\bar{s}s)(\bar{u}c) + (\bar{d}d)(\bar{u}c) - (\bar{u}d)(\bar{d}c) \big)_{\bar{\mathbf{6}}} \Big] \\ & - \frac{\lambda_b}{4} \Big[C_+ \big((\bar{u}d)(\bar{d}c) + (\bar{d}d)(\bar{u}c) + (\bar{s}s)(\bar{u}c) + (\bar{u}s)(\bar{s}c) - 2(\bar{u}u)(\bar{u}c) \big)_{\mathbf{15}} \\ & + C_+ \sum_{q=u,d,s} \big((\bar{u}q)(\bar{q}c) + (\bar{q}q)(\bar{u}c) \big)_{\mathbf{3}_+} + 2C_- \sum_{q=d,s} \big((\bar{u}q)(\bar{q}c) - (\bar{q}q)(\bar{u}c) \big)_{\mathbf{3}_-} \Big] \Bigg\}\end{aligned}$$

Leading terms

Provide CP phase

• SU(3) flavor analysis — Tree

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

Generalized Wigner-Eckart theorem

$\tilde{f}$:Free parameters

$$F^{s-d} = \tilde{f}^a (P^\dagger)^l_l \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^j_k + \tilde{f}^b \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^l_k (P^\dagger)^j_l + \underbrace{\tilde{f}^c \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (P^\dagger)^l_k (\mathbf{B}^\dagger)^j_l}_{SU(3)_F \text{ tensors}}$$
$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j \,,$$

$$F^b = \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j + \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j$$
$$+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i \,,$$

$$\mathcal{H}(\overline{\mathbf{6}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & V^*_{cs} V_{ud} & -\lambda_s - \frac{\lambda_b}{2} \\ 0 & -\lambda_s - \frac{\lambda_b}{2} & V^*_{cd} V_{us} \end{pmatrix} \qquad \mathcal{H}(\mathbf{15})^{ij}_k = \left(\begin{pmatrix} \frac{\lambda_b}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & -\lambda_s - \frac{3\lambda_b}{4} & V^*_{cs} V_{ud} \\ -\lambda_s - \frac{3\lambda_b}{4} & 0 & 0 \\ V^*_{cs} V_{ud} & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & V^*_{cd} V_{us} & \lambda_s + \frac{\lambda_b}{4} \\ V^*_{cd} V_{us} & 0 & 0 \\ \lambda_s + \frac{\lambda_b}{4} & 0 & 0 \end{pmatrix}_{ij} \right)_k$$

arXiv:2310.05491 [hep-ph]

• SU(3) flavor analysis — Tree

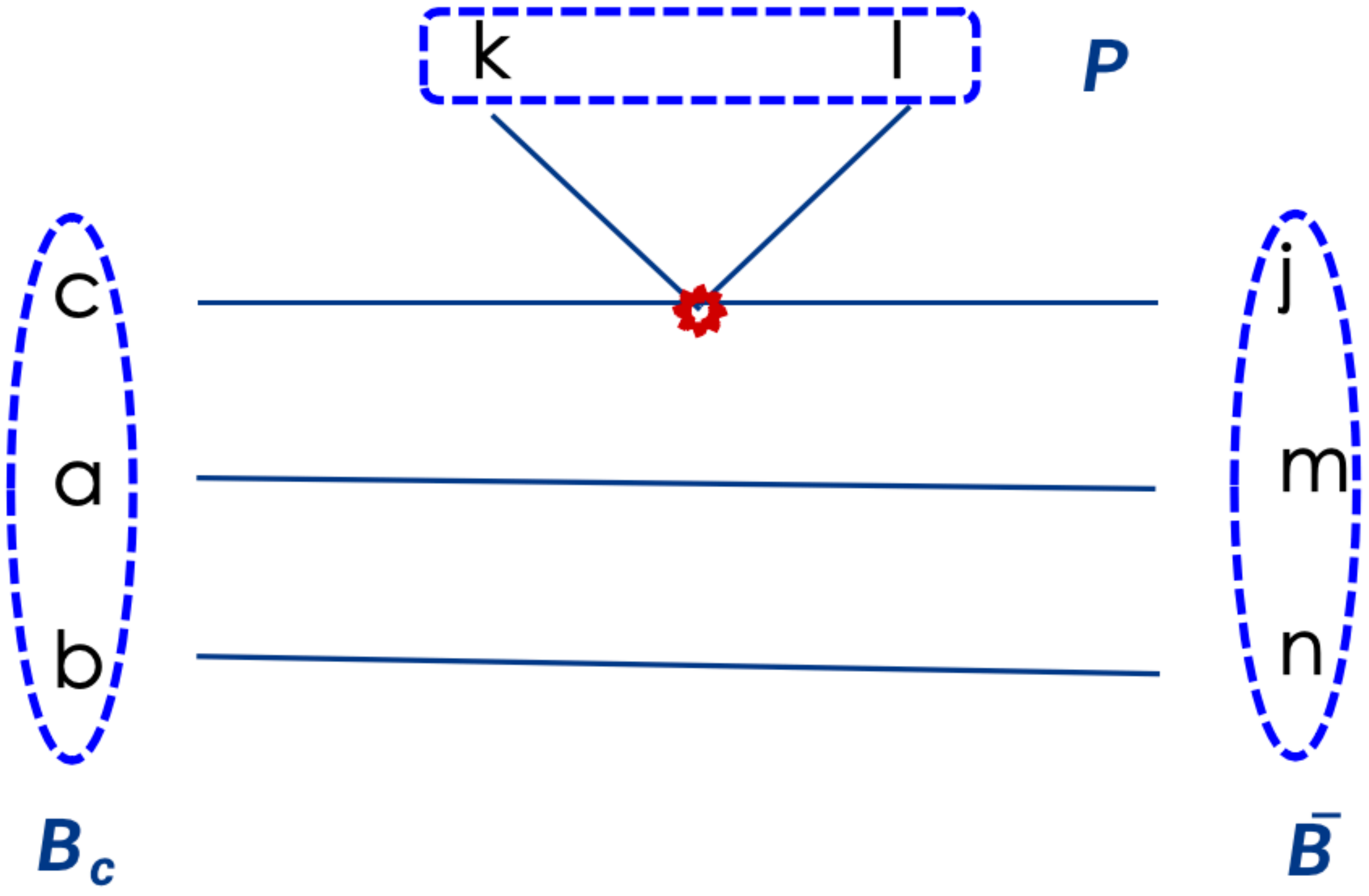
He, Shi, Wang   Zhong, Xu, Cheng   Wang, Luo

Equivalence to the quark diagrams analysis; see arXiv : 1811.03480, 2404.01350, 2406.14061

$$f^e (P)^l_k \bar{B}^i_j H(15)^{jk}_l (B_c)_i$$

↓

$$f^e (P)^l_k \varepsilon^{mni} (\bar{B})_{mnj} H(15)^{jk}_l \varepsilon_{abi} (B_c)^{ab}$$



arXiv:2310.05491 [hep-ph]

• SU(3) flavor analysis — Tree

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

Generalized Wigner-Eckart theorem

$\tilde{f}$:Free parameters

$$F^{s-d} = \tilde{f}^a (P^\dagger)^l_l \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^j_k + \tilde{f}^b \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^l_k (P^\dagger)^j_l + \tilde{f}^c \underbrace{\mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (P^\dagger)^l_k (\mathbf{B}^\dagger)^j_l}_{SU(3)_F \text{ tensors}}$$
$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j ,$$

$$F^b = \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j + \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j$$
$$+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i ,$$

To date, there are in total **30** data points but ~~9~~ **5** $\times$ 2(S & P waves) $\times$ 2(complex) $- 1 =$ ~~35~~ **19**

CP-even

$\tilde{f}^{a,b,c,d,e}$, $\tilde{f}^{a,b,c,d}_{\mathbf{3}}$

arXiv:2310.05491 [hep-ph]

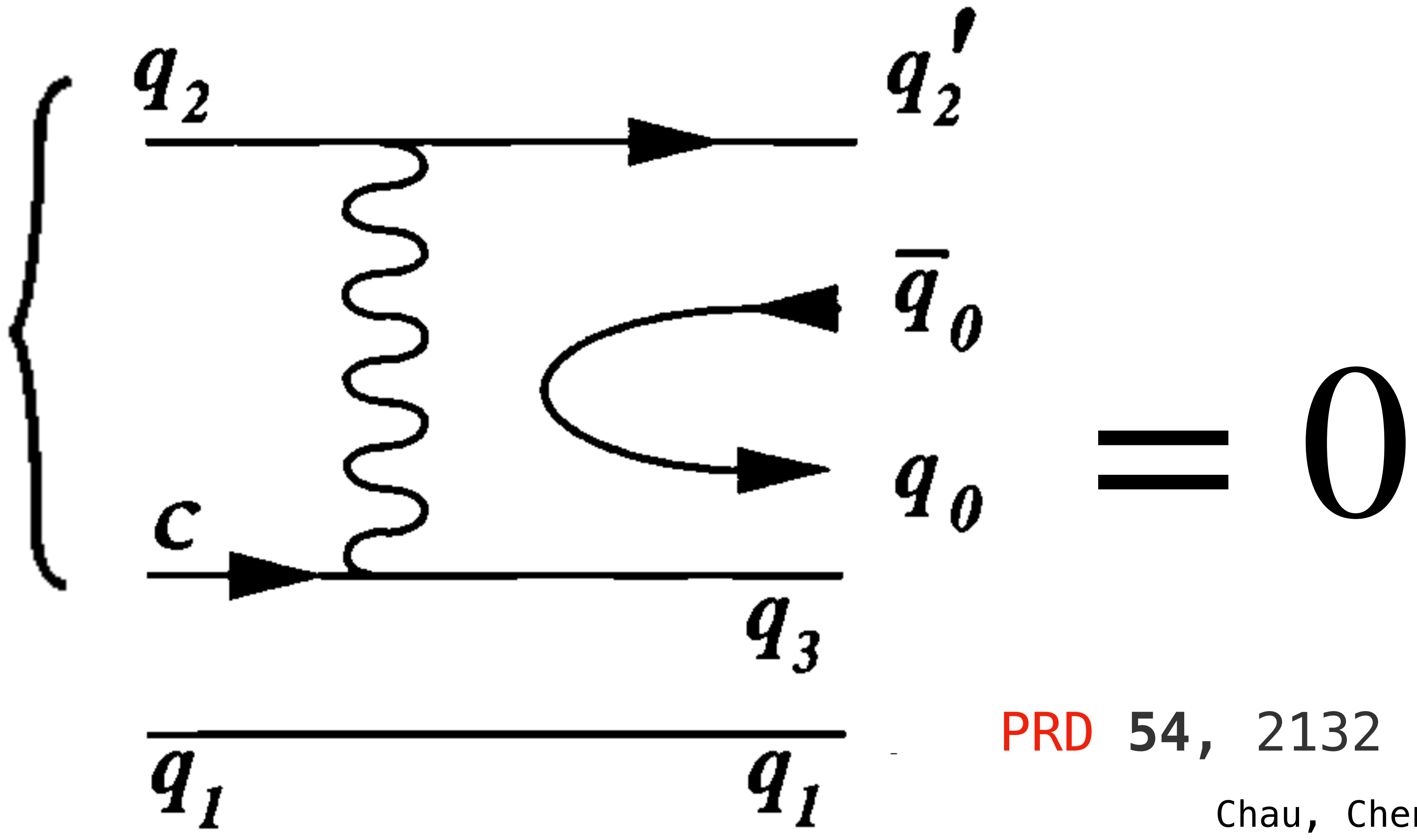
• SU(3) flavor analysis — Tree

New understanding of charming physics.

- **Körner-Pati-Woo theorem:**

$$\langle q_a q_b q_c | O_+^{qq'} | \mathbf{B}_i \rangle = 0$$

Color symmetric   Color singlet



PRD **54**, 2132 (1996)
Chau, Cheng, Tseng

- **Eliminate 4 redundancies in $\mathscr{H}(\mathbf{15})$**

- **Predict direct relations:**

$$\Gamma(\Lambda_c^+ \to \Sigma^+ K_S^0) = \Gamma(\Lambda_c^+ \to \Sigma^0 K^+) = s_c^2 \Gamma(\Xi_c^0 \to \Xi^0 \pi^0)$$

PLB **794**, 19(2019)

$\mathscr{B}(\Lambda_c^+ \to \Sigma^+ K_S^0, \Sigma^0 K^+)$ BESIII	$\mathscr{B}(\Xi_c^0 \to \Xi^0 \pi^0)$ BELLE
$(4.7 \pm 1.0) \times 10^{-4}$	$(7.1 \pm 0.4)_{th} \times 10^{-3}$
$\approx (4.8 \pm 1.4) \times 10^{-4}$	$(6.9 \pm 1.4)_{exp} \times 10^{-3}$
PRD **106**, 052003 (2022)	arXiv:2406.04642

- **Works without considering color-symmetry**

PRD **93**, 056008 (2016), PRD **97**, 073006 (2018)
Lü, Wang, Yu   Geng, Hsiao, Liu, Tsai

JHEP **09**, 035 (2022), JHEP **03**, 143 (2022), NPB **956**, 115048 (2020)
Hsiao, Wang, Zhao   Huang, Xing, He   Jia, Wang, Yu

Not able to determine both complex phases.

arXiv:2310.05491 [hep-ph]

- **SU(3) flavor analysis — Tree**

- **Sizable strong phases are found.**

$$\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.38 \pm 0.44)\,\%$$

LQCD, CPC **46**, 011002 (2022)

$$\times$$

$$\frac{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)}{\mathcal{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e)} = 1.37 \pm 0.08$$

BELLE PRL **127** 121803 (2021)

$$||$$

$$\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (3.26 \pm 0.63)\,\%$$

Values within parentheses represent the backward digit count of uncertainties, such as $1.59(8) = 1.59 \pm 0.08$.

Channels	$\mathcal{B}_{\rm exp}(\%)$	$\alpha_{\rm exp}$	$\mathcal{B}(\%)$	α	β
$\Lambda_c^+ \to pK_S$	$1.59(8)$	$^*0.18(45)$	$1.55(7)$	$-0.40(49)$	$0.32(29)$
$\Lambda_c^+ \to n\pi^+$	$0.066(13)$	BESIII	$0.067(8)$	$-0.78(12)$	$-0.63(15)$
$\Lambda_c^+ \to \Sigma^+ K_S$	$0.048(14)$		$0.036(2)$	$-0.52(10)$	$0.48(24)$
$\Lambda_c^+ \to p\pi^0$	< 0.008	BELLE	0.016(2)		$-0.82(32)$
$\Lambda_c^+ \to \Sigma^+\eta$	$0.32(4)$	$-0.99(6)$	$0.32(4)$	$-0.93(4)$	$-0.32(16)$
$\Lambda_c^+ \to p\eta$	$0.142(12)$		$0.145(26)$	$-0.42(61)$	$0.64(40)$
$\Lambda_c^+ \to \Sigma^+\eta'$	$0.437(84)$	$-0.46(7)$	$0.420(70)$	$-0.44(25)$	$0.86(6)$
$\Lambda_c^+ \to p\eta'$	$0.0484(91)$		$0.0520(114)$	$-0.59(9)$	$0.76(14)$
$\Xi_c^+ \to \Xi^0\pi^+$	$1.6(8)$	BELLE	$0.90(16)$	$-0.94(6)$	$0.32(21)$
$\Xi_c^0 \to \Xi^-\pi^+$	$^{****}1.43(32)$	$^* -0.64(5)$	$2.72(9)$	$-0.71(3)$	$0.36(20)$

29 data points with 10 complex parameters.

arXiv:2310.05491 [hep-ph]

SU(3) flavor analysis

$$V_{cs}^{*}V_{us}\ \text{Tree} + \underbrace{\cancel{V_{cb}^{*}V_{ub}\ \text{Penguin}}}$$

Insensitive to CP-even quantities & undetermined

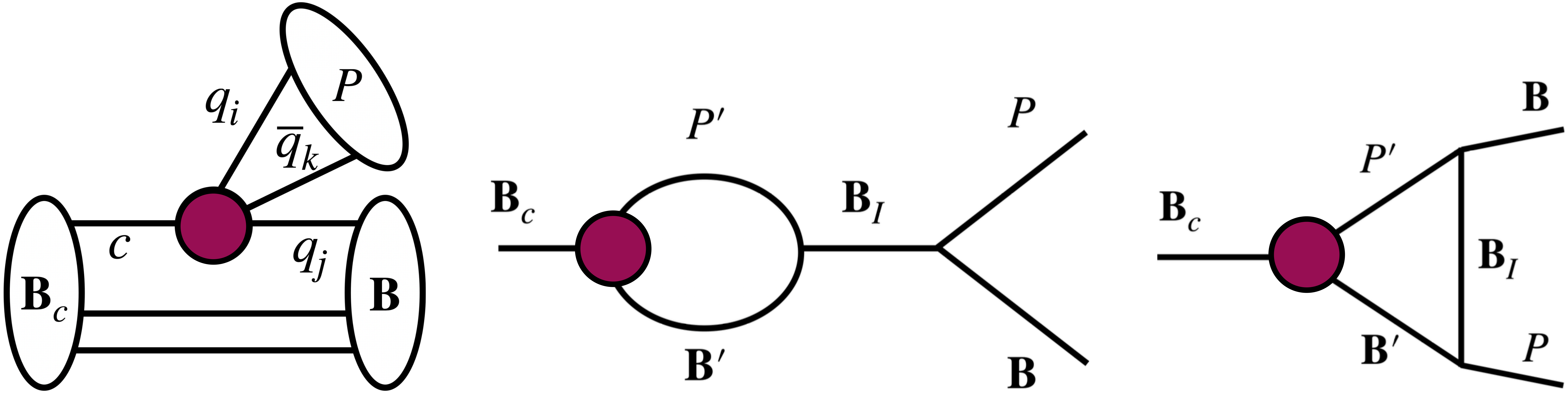
Final State Rescattering

$$V_{cs}^{*}V_{us}\ \text{Tree} + V_{cb}^{*}V_{ub}\ \text{Tree} \times \underbrace{(\ \text{Penguin} / \text{Tree}\ )}$$

Determined by the rescattering

- **Rescattering, solving penguin/tree**

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\rm Tree}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\rm FSR-s}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\rm FSR-t}_{\mathbf{B}_c\mathbf{B}P}$$

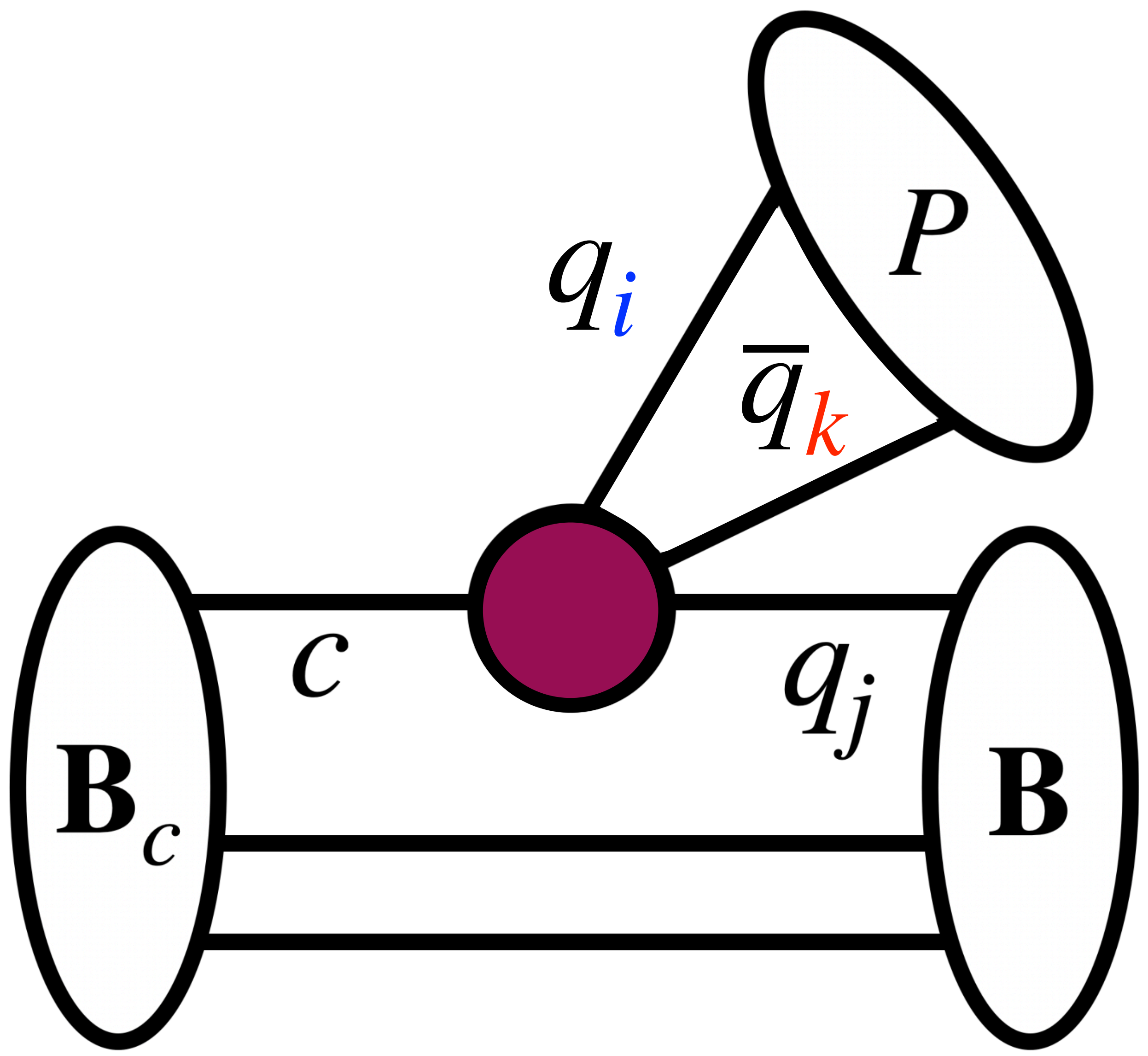


Assumptions:

1. Short distance transitions are dominated by the W-emission, including both color-enhanced and color-suppressed.
2. $\mathbf{B}_I \in$ lowest-lying baryons of both parities.
3. The re-scattering is closed, *i.e.* $\mathbf{B}'P'$ belong to the same $SU(3)_F$ group of $\mathbf{B}P$.

- **Rescattering, solving penguin/tree**

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\rm Tree}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\rm FSR-s}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\rm FSR-t}_{\mathbf{B}_c\mathbf{B}P}$$



From figure, we deduce:

$$\mathscr{L}^{\rm Tree}_{\mathbf{B}_c\mathbf{B}P} = \left(P^\dagger\right)^k_i (\overline{\mathbf{B}})^l_j \left(\tilde{F}^+_V \left(\mathscr{H}_+\right)^{ij}_k + \tilde{F}^-_V \left(\mathscr{H}_-\right)^{ij}_k \right) \left(\mathbf{B}_c\right)_l$$

where

$$(\mathscr{H}_+)^{ij}_k = \frac{\lambda_s - \lambda_d}{2} \mathscr{H}(\mathbf{15}^{s-d})^{ij}_k + \lambda_b \left(\mathscr{H}(\mathbf{15}^b)^{ij}_k + \mathscr{H}(\mathbf{3}_+)^i \delta^j_k + \mathscr{H}(\mathbf{3}_+)^j \delta^i_k \right)$$

$$(\mathscr{H}_-)^{ij}_k = \frac{\lambda_s - \lambda_d}{2} \mathscr{H}(\overline{\mathbf{6}})_{kl} \epsilon^{lij} + 2\lambda_b \left(\mathscr{H}(\mathbf{3}_-)^i \delta^j_k - \mathscr{H}(\mathbf{3}_-)^j \delta^i_k \right)$$

$$\mathcal{H}(\overline{\mathbf{6}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & V^*_{cs}V_{ud} & -\lambda_s - \frac{\lambda_b}{2} \\ 0 & -\lambda_s - \frac{\lambda_b}{2} & V^*_{cd}V_{us} \end{pmatrix} \quad \mathcal{H}(\mathbf{15})^{ij}_k = \left(\begin{pmatrix} \frac{\lambda_b}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & -\lambda_s - \frac{3\lambda_b}{4} & V^*_{cs}V_{ud} \\ -\lambda_s - \frac{3\lambda_b}{4} & 0 & 0 \\ V^*_{cs}V_{ud} & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & V^*_{cd}V_{us} & \lambda_s + \frac{\lambda_b}{4} \\ V^*_{cd}V_{us} & 0 & 0 \\ \lambda_s + \frac{\lambda_b}{4} & 0 & 0 \end{pmatrix}_{ij} \right)_k$$

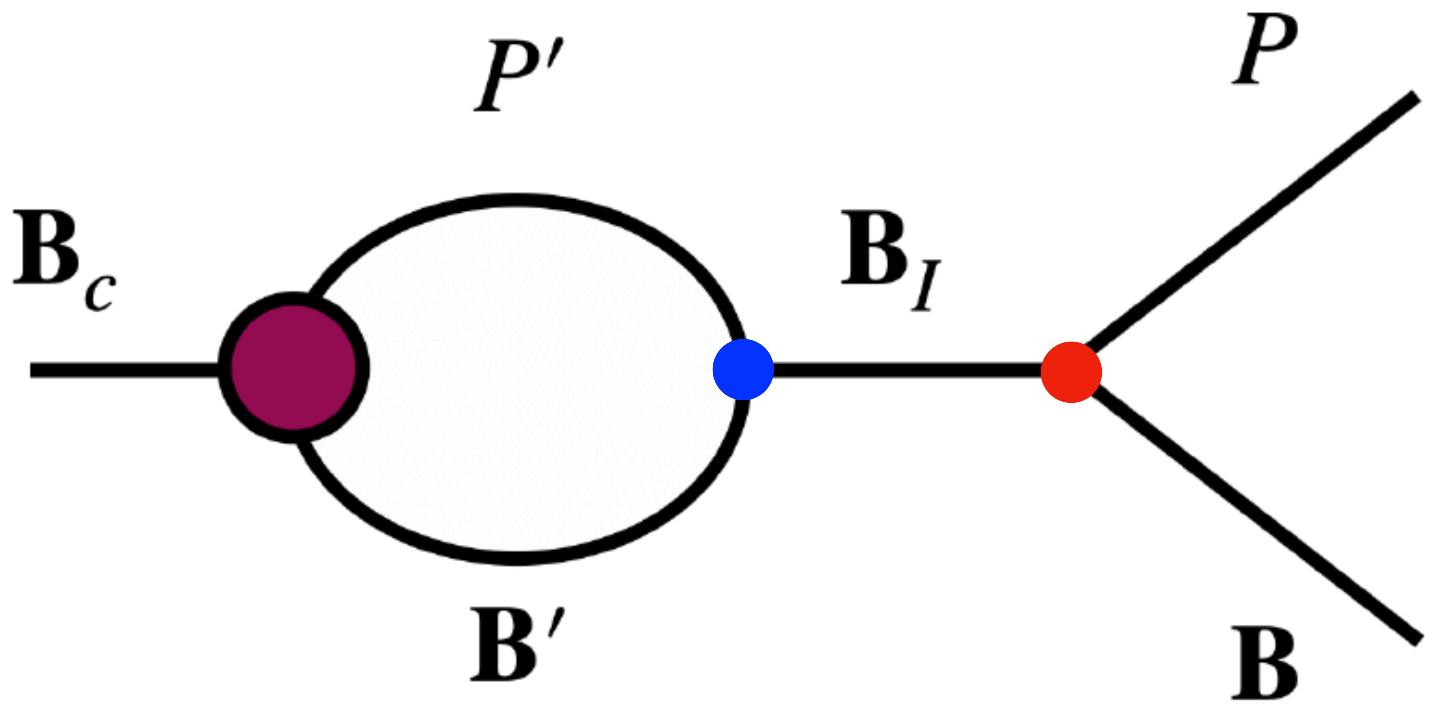
It is very important that $\mathbf{15}$, $\overline{\mathbf{6}}$ and $\mathbf{3}$ share two parameters $\tilde{F}^\pm_V$!

- **Rescattering, solving penguin/tree**

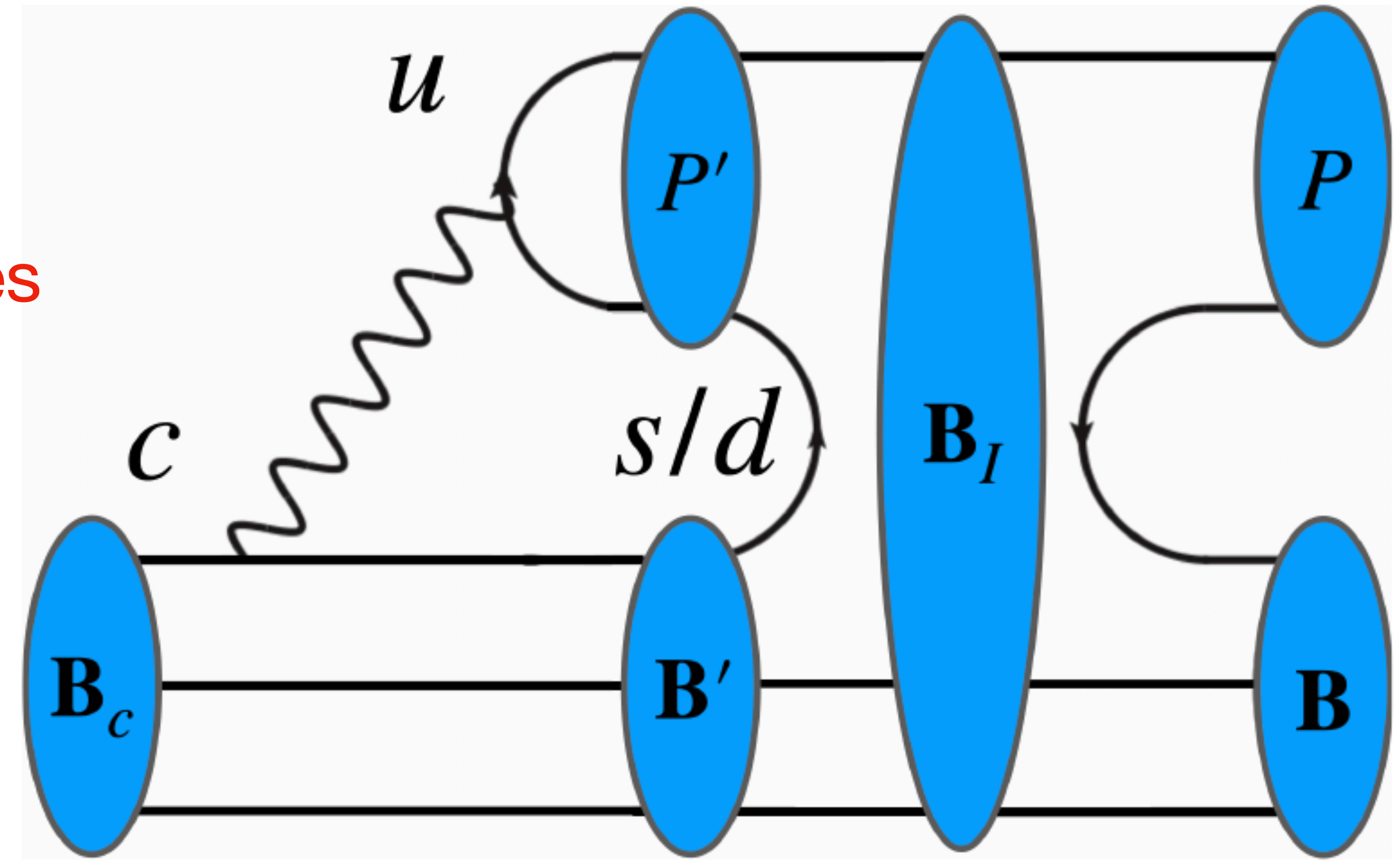
$$\langle \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} \rangle = \sum_{\mathbf{B}_I,\mathbf{B}',P'} \bar{u}_{\mathbf{B}} \left(\int \frac{d^4q}{(2\pi)^4} g_{\mathbf{B}_I\mathbf{B}P} \frac{p^{\mu}_{\mathbf{B}_c}\gamma_\mu + m_I}{p^2_{\mathbf{B}_c} - m_I^2} g_{\mathbf{B}_I\mathbf{B}'P'} \frac{q^\mu\gamma_\mu + m_{\mathbf{B}'}}{q^2 - m^2_{\mathbf{B}'}} \frac{1}{(q - p_{\mathbf{B}_c})^2 - m^2_{P'}} F^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}'P'} \right) u_{\mathbf{B}_c}$$

1. $F^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}'P'}$ and $g_{\mathbf{B}_I\mathbf{B}'P'}$ depend on q^2 otherwise a cut-off has to be introduced.

2. Sum over the intermediate hadrons $\mathbf{B}_I$, $\mathbf{B}'$ and P'.



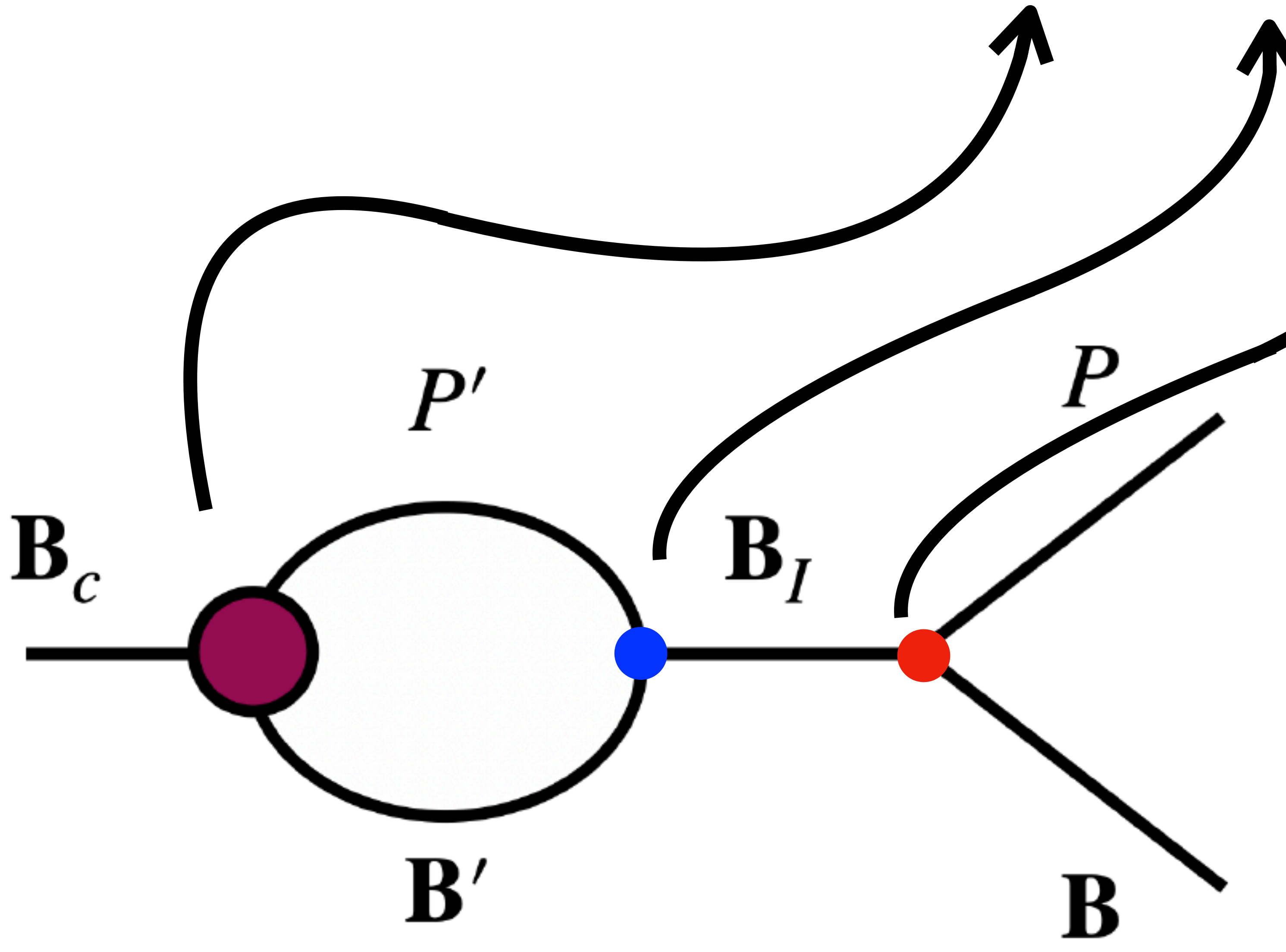
At quark level generates penguin topology



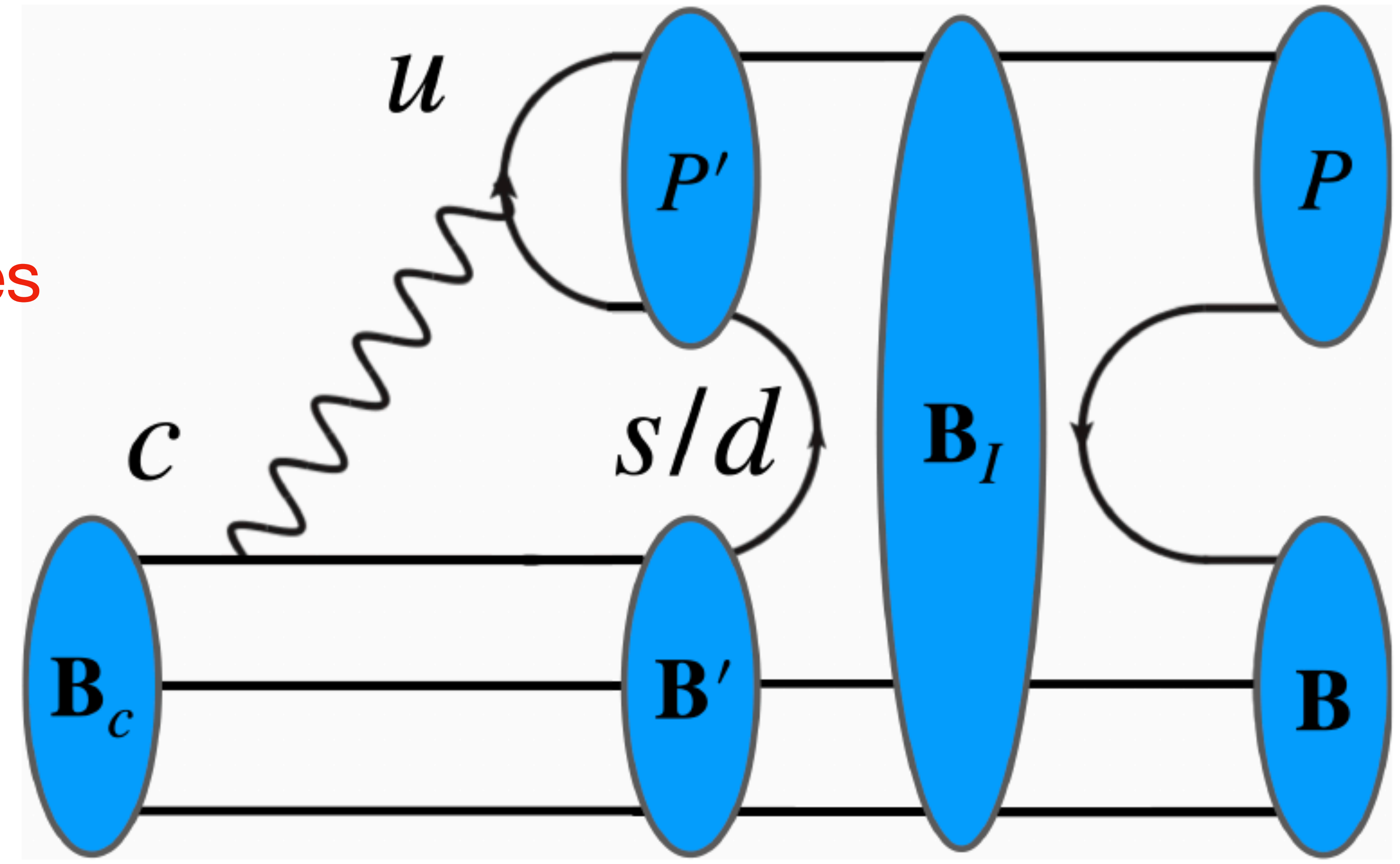
• Rescattering, solving penguin/tree

$$\langle \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} \rangle = \sum_{\mathbf{B}_I,\mathbf{B}',P'} \bar{u}_{\mathbf{B}} \left(\int \frac{d^4q}{(2\pi)^4} g_{\mathbf{B}_I\mathbf{B}P} \frac{p^\mu_{\mathbf{B}_c}\gamma_\mu + m_I}{p^2_{\mathbf{B}_c} - m_I^2} g_{\mathbf{B}_I\mathbf{B}'P'} \frac{q^\mu\gamma_\mu + m_{\mathbf{B}'}}{q^2 - m^2_{\mathbf{B}'}} \frac{1}{(q - p_{\mathbf{B}_c})^2 - m^2_{P'}} F^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}'P'} \right) u_{\mathbf{B}_c}$$

$$= \bar{u}_{\mathbf{B}} \left[\int \frac{d^4q}{(2\pi)^4} \left(\sum_{\mathbf{B}_I,\mathbf{B}',P'} F^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}'P'} g_{\mathbf{B}_I\mathbf{B}'P'} g_{\mathbf{B}_I\mathbf{B}P} \right) I(q^2) \right] u_{\mathbf{B}_c}$$



At quark level generates penguin topology



• Rescattering, solving penguin/tree

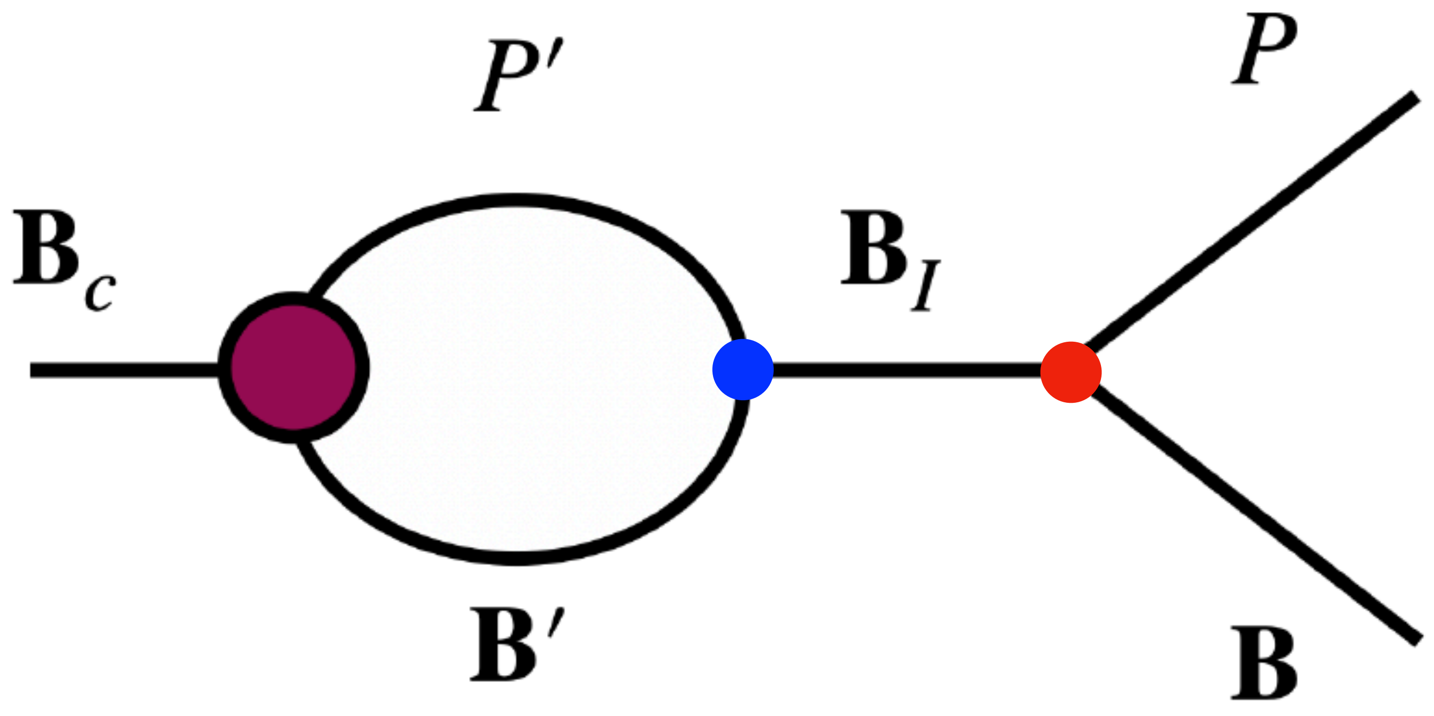
$$\sum_{\mathbf{B}_I,\mathbf{B}',P'} F^{\rm Tree}_{\mathbf{B}_c\mathbf{B}'P'}\, g_{\mathbf{B}_I\mathbf{B}'P'}\, g_{\mathbf{B}_I\mathbf{B}P}$$

Key of reduction rule: utilizing $\mathbf{B}_I$ belongs to $\mathbf{8}$.

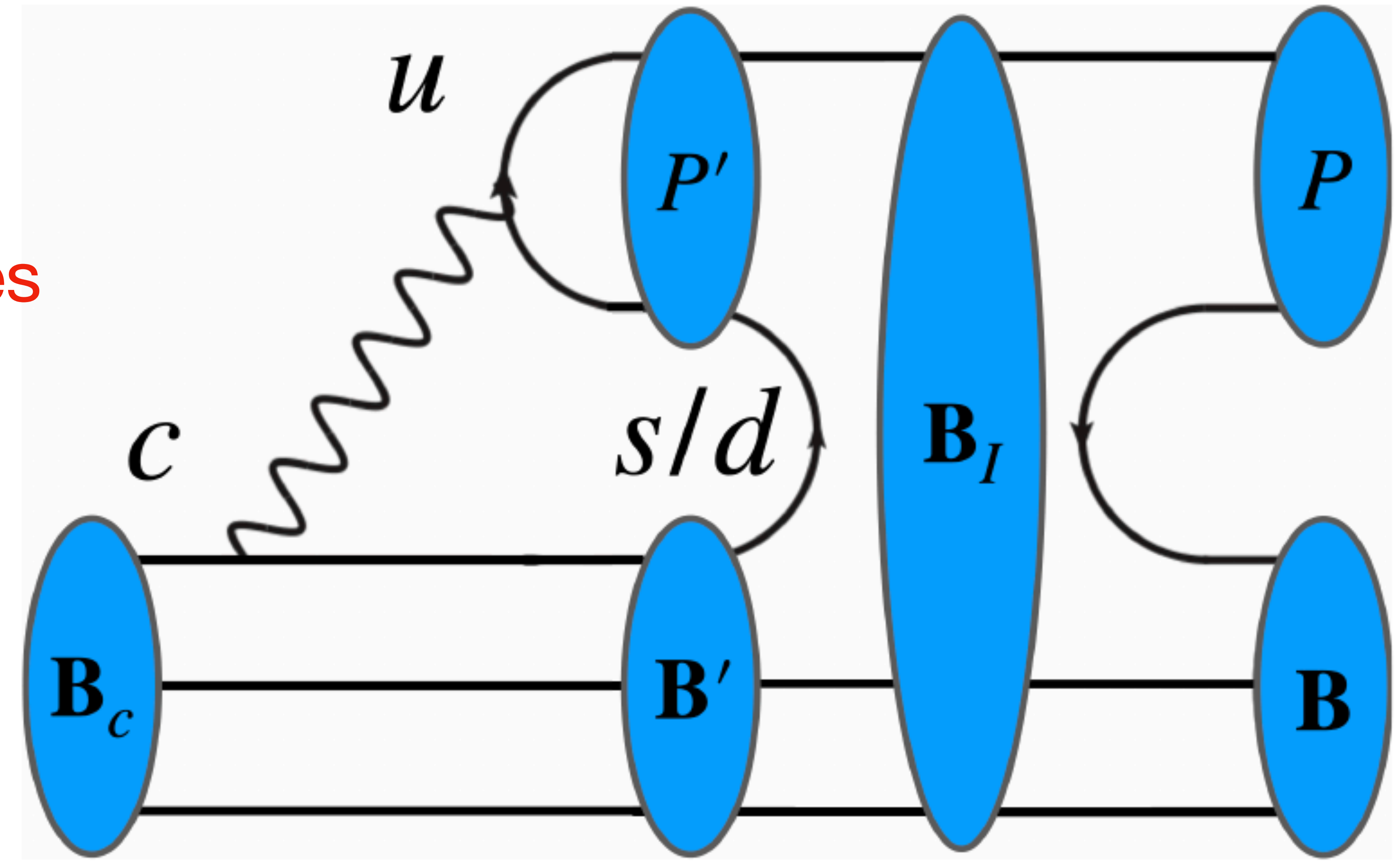
Substitute $\sum_{\mathbf{B}_I} \langle \overline{\mathbf{B}}_I\rangle^{k_1}_{i_1}\langle \mathbf{B}_I\rangle^{j_2}_{k_2}$ with $\frac{1}{2}\sum_{\lambda_a}(\lambda_a)^{k_1}_{i_1}(\lambda_a)^{j_2}_{k_2} = \delta^{j_2}_{i_1}\delta^{k_1}_{k_2} - \frac{1}{3}\delta^{k_1}_{i_1}\delta^{j_2}_{k_2}$

$$\propto \sum_{\mathbf{B}_I,\mathbf{B}',P'} \left(\langle P'^\dagger\rangle^k_i\langle\overline{\mathbf{B}}'\rangle^l_j(\mathscr{H}_-)^{ij}_k\langle\mathbf{B}_c\rangle_l\right)\left(\langle P'\rangle^{i_2}_{j_2}\langle\overline{\mathbf{B}}_I\rangle^{j_2}_{k_2}\langle\mathbf{B}'\rangle^{k_2}_{i_2} + r_-\langle P'\rangle^{j_2}_{k_2}\langle\overline{\mathbf{B}}_I\rangle^{i_2}_{j_2}\langle\mathbf{B}'\rangle^{k_2}_{i_2}\right)\left(\langle P^\dagger\rangle^{i_3}_{j_3}\langle\overline{\mathbf{B}}\rangle^{j_3}_{k_3}\langle\mathbf{B}_I\rangle^{k_3}_{i_3} + r_-\langle P^\dagger\rangle^{j_3}_{k_3}\langle\overline{\mathbf{B}}\rangle^{i_3}_{j_3}\langle\mathbf{B}_I\rangle^{k_3}_{i_3}\right)$$

$$\langle\mathscr{L}^{\rm FSR-s}_{\mathbf{B}_c\mathbf{B}P}\rangle = \tilde{S}^- \left(\langle P^\dagger\rangle^{i_1}_{j_1}\langle\overline{\mathbf{B}}\rangle^{j_1}_{k_1} + r_-\langle P^\dagger\rangle^{j_1}_{k_1}\langle\overline{\mathbf{B}}\rangle^{i_1}_{j_1}\right)\left(\delta^{k_1}_i\delta^k_{i_1} - \frac{1}{3}\delta^{k_1}_{i_1}\delta^k_i\right)\left((\mathscr{H}_-)^{ij}_k\langle\mathbf{B}_c\rangle_j + \frac{4r_-+1}{r_-+4}(\mathscr{H}_-)^{ji}_j\langle\mathbf{B}_c\rangle_k\right)$$

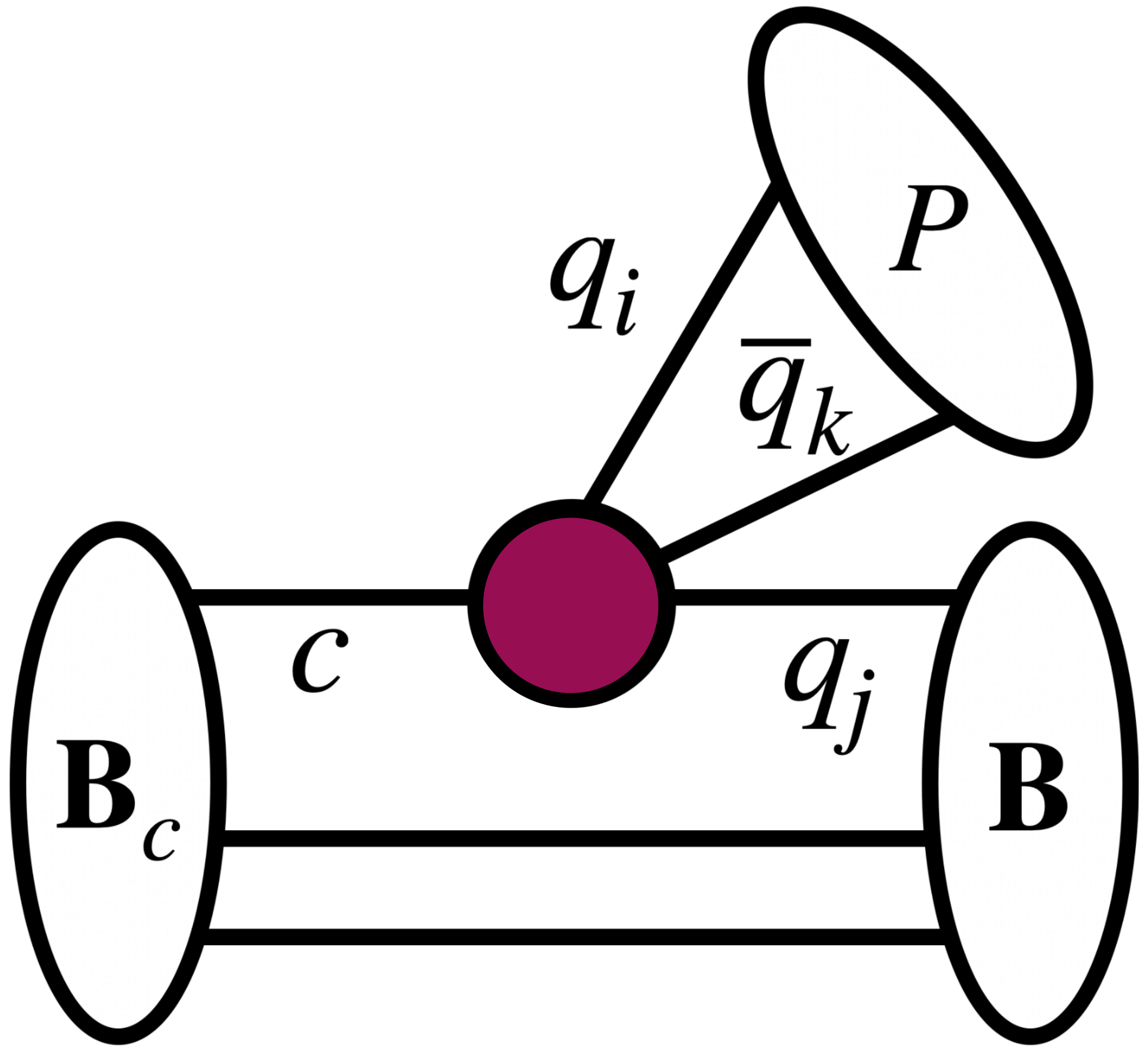


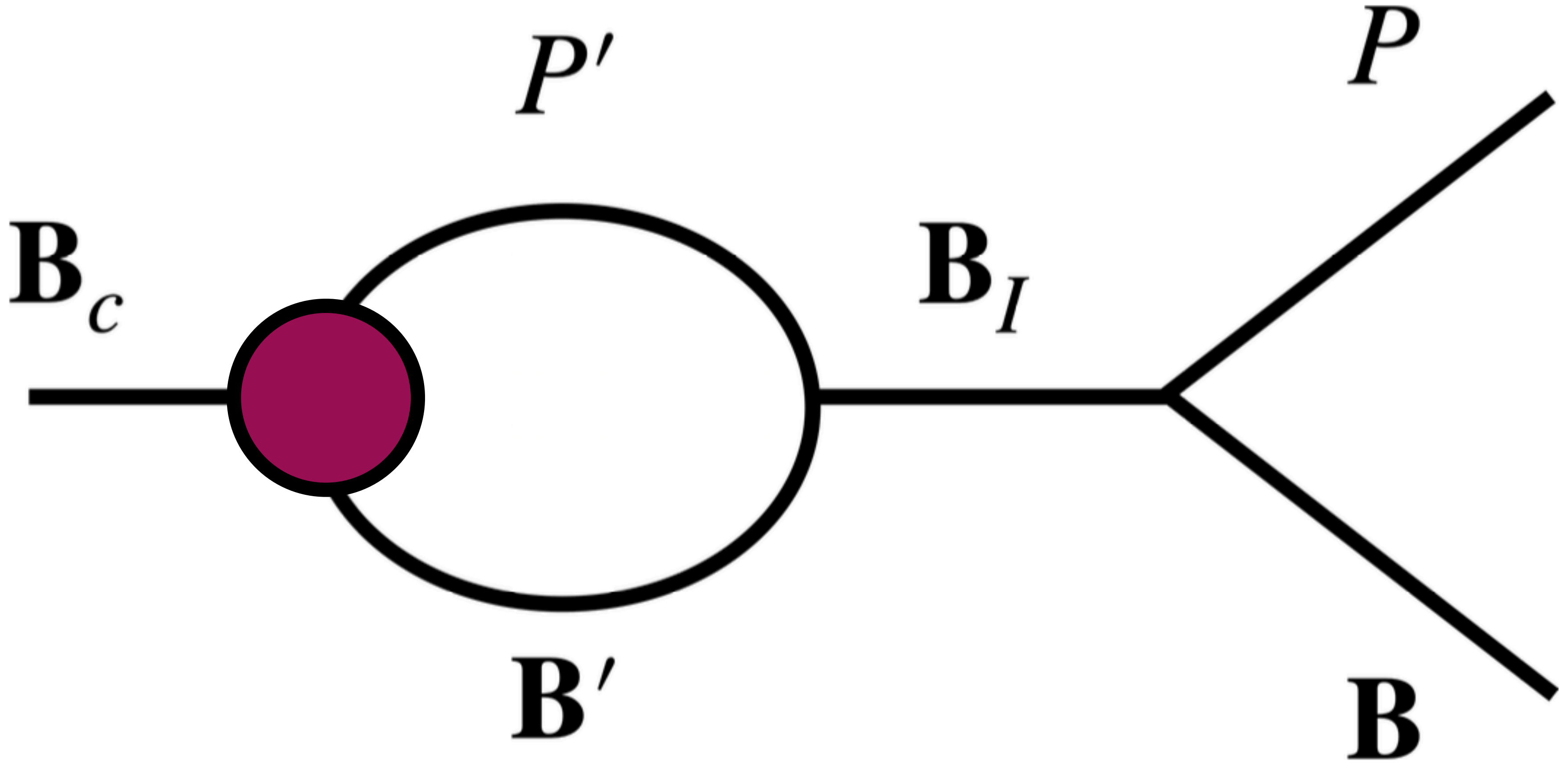
At quark level generates penguin topology

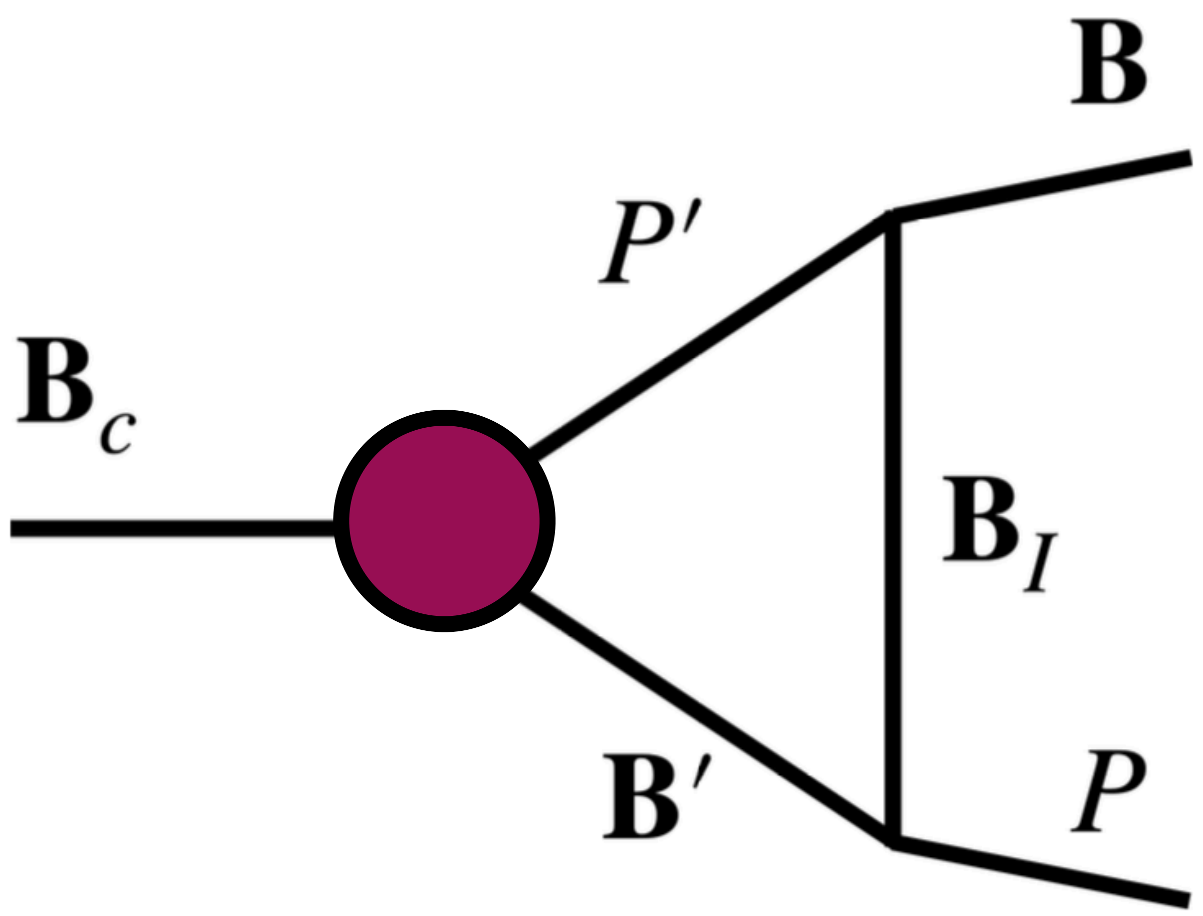


- ## Rescattering, solving penguin/tree

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-t}}_{\mathbf{B}_c\mathbf{B}P}$$







Induce two parameters:

$F_V^{\pm}$, including effective color number and form factors.

Induce one parameter:

$\tilde{S}^{-}$, containing the q^2 dependencies of couplings.

Induce one parameter:

$\tilde{T}^{-}$, containing the q^2 dependencies of couplings.

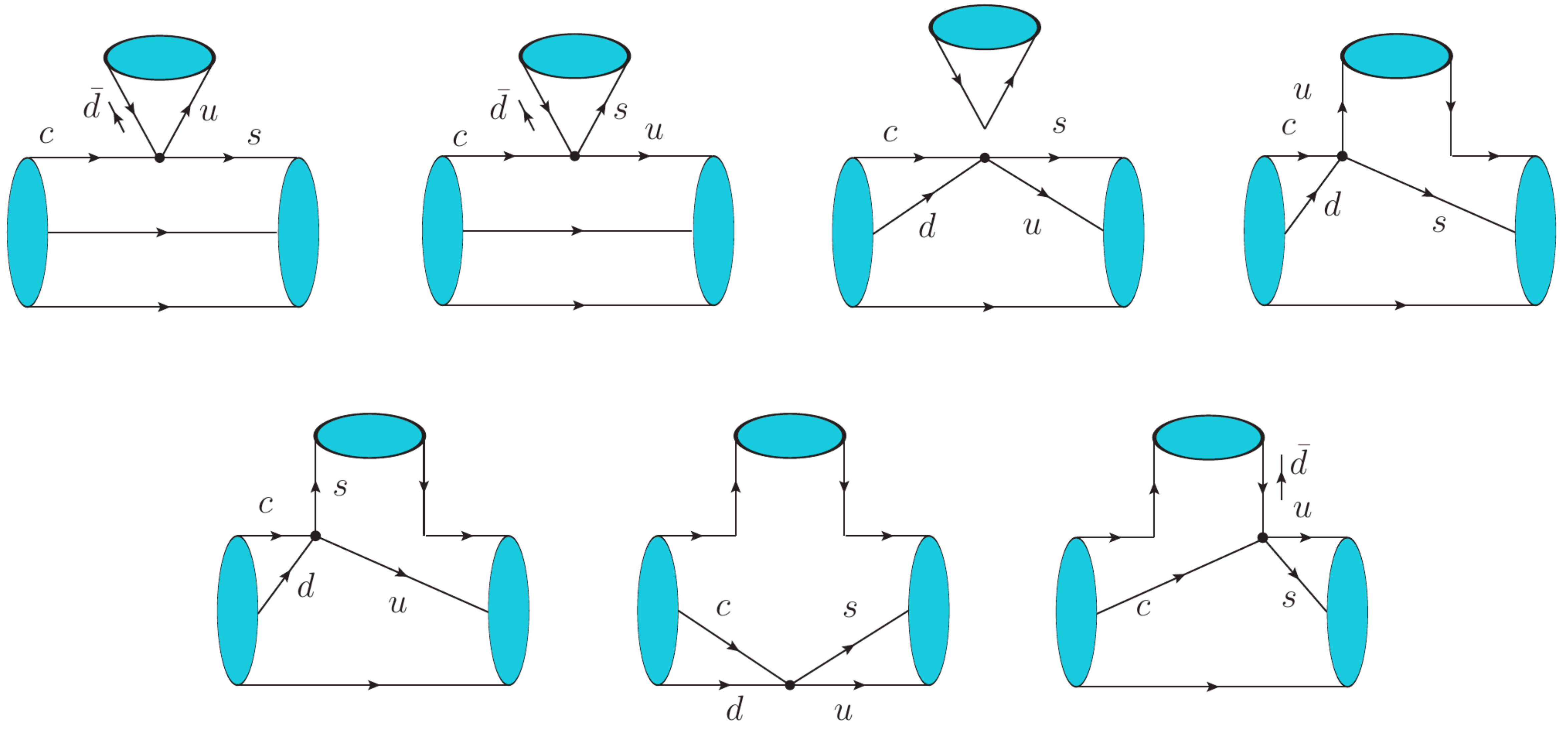
- **Rescattering, solving penguin/tree**

Amplitudes $\sim \dfrac{\lambda_s - \lambda_d}{2}\tilde{f} + \lambda_b \tilde{f}_{\mathbf{3}}$

$$\tilde{f}^b = \tilde{F}_V^- + \tilde{S}^- - \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}^c = r_-\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^- \,,$$

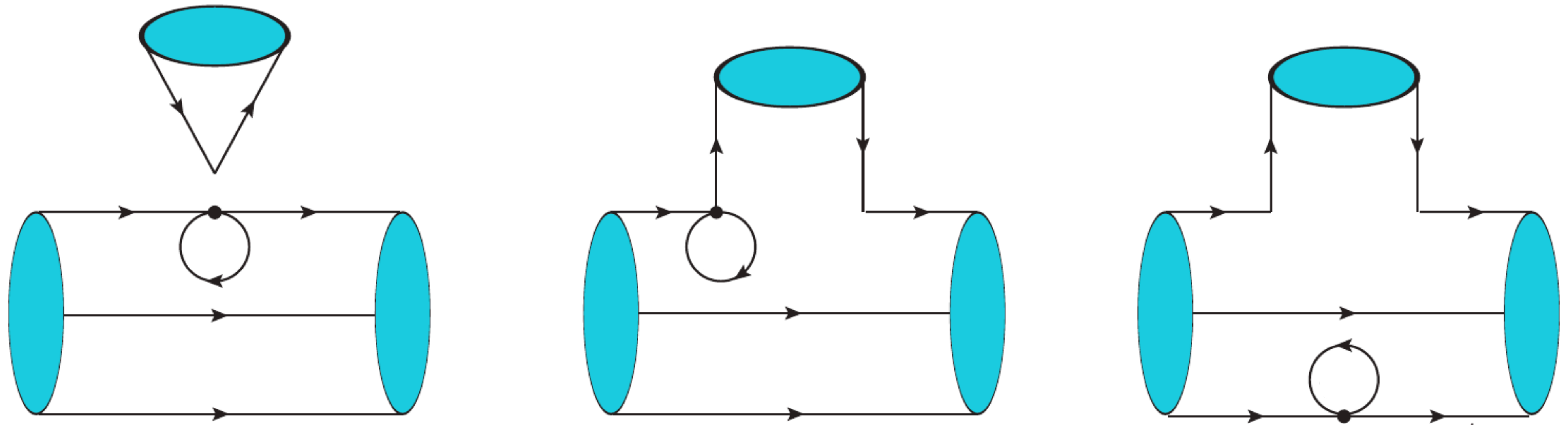
$$\tilde{f}^d = \tilde{F}_V^- - \sum_{\lambda=\pm}(2r_\lambda^2 - 2r_\lambda - 4)\tilde{T}_\lambda^- \,, \qquad \tilde{f}^e = \tilde{F}_V^+ \,,$$



$$\tilde{f}_{\mathbf{3}}^b = \frac{7r_- - 2}{8 + 2r_-}\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_{\mathbf{3}}^c = \frac{(r_- + 1)(2 - 7r_-)}{24 + 6r_-}\tilde{S}^- + \sum_{\lambda=\pm}\frac{1}{6}(r_\lambda^2 + 11r_\lambda + 1)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_{\mathbf{3}}^d = \frac{r_-(7r_- - 2)}{8 + 2r_-}\tilde{S}^- - \sum_{\lambda=\pm}\frac{1}{2}(r_\lambda + 1)^2\tilde{T}_\lambda^- - \frac{1}{4}\left(\tilde{F}_V^+ + 2\tilde{F}_V^-\right) \,,$$



$$\left(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e\right) \longleftrightarrow \left(\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-\right) \longrightarrow \left(\tilde{f}_{\mathbf{3}}^b, \tilde{f}_{\mathbf{3}}^c, \tilde{f}_{\mathbf{3}}^d\right)$$

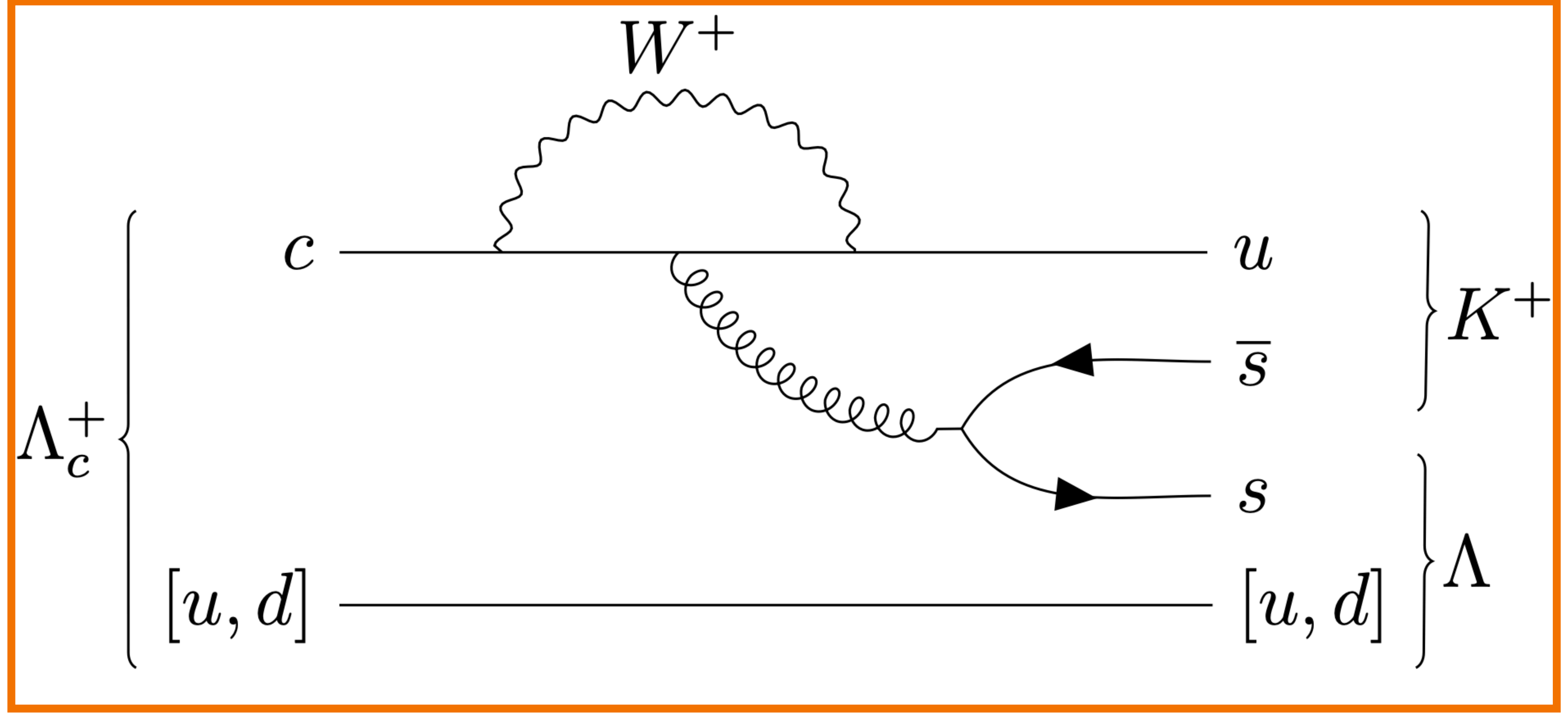
PRD **100**, 093002 (2019)

Much more complicated compared to $P^{LD} = E$ in *D* mesons !

- **Rescattering, solving penguin/tree**

Amplitudes $\sim \dfrac{\lambda_s - \lambda_d}{2}\tilde{f} + \lambda_b \tilde{f}_{\mathbf{3}}$

Corrections to A_{CP} are around 10%



$$
\begin{aligned}
\tilde{f}^b &= \tilde{F}_V^- + \tilde{S}^- - \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^- \,,\\
\tilde{f}^c &= r_-\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^- \,,\\
\tilde{f}^d &= \tilde{F}_V^- - \sum_{\lambda=\pm}(2r_\lambda^2 - 2r_\lambda - 4)\tilde{T}_\lambda^- \,, \qquad \tilde{f}^e = \tilde{F}_V^+ \,,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{f}^b_{\mathbf{3}} &= \frac{7r_- - 2}{8 + 2r_-}\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde{T}_\lambda^- \,,\\
\tilde{f}^c_{\mathbf{3}} &= \frac{(r_- + 1)(2 - 7r_-)}{24 + 6r_-}\tilde{S}^- + \sum_{\lambda=\pm}\frac{1}{6}(r_\lambda^2 + 11r_\lambda + 1)\tilde{T}_\lambda^- \,,\\
\tilde{f}^d_{\mathbf{3}} &= \frac{r_-(7r_- - 2)}{8 + 2r_-}\tilde{S}^- - \sum_{\lambda=\pm}\frac{1}{2}(r_\lambda + 1)^2\tilde{T}_\lambda^- - \frac{1}{4}\left(\tilde{F}_V^+ + 2\tilde{F}_V^-\right)
\end{aligned}
$$

$$
\left(1 + \frac{(3C_4 + C_3)\, m_c - \frac{2m_K^2}{m_s + m_u}(3C_6 + C_5)}{(C_+ + C_-)m_c}\right)
$$

$$
\left(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e\right) \longleftrightarrow \left(\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-\right) \longrightarrow \left(\tilde{f}^b_{\mathbf{3}}, \tilde{f}^c_{\mathbf{3}}, \tilde{f}^d_{\mathbf{3}}\right)
$$

PRD **100**, 093002 (2019)

Much more complicated compared to $P^{LD} = E$ in D mesons !

• Rescattering, numerical results

1. A_{CP} in the same size with the ones in D meson! If confirmed, it suggests the natural sizes of A_{CP} are around 10^{-3}.

2. In the U-spin limit, we have that

$$A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) = -A_{CP}\left(\Xi_c^0 \to pK^-\right) .$$

Hence it is reasonable to measure

$$\Delta A_{CP} = A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) - A_{CP}\left(\Xi_c^0 \to pK^-\right) .$$

3. The main uncertainties are from strong phases. Measurement on β can greatly improve!

Channels	$10^3\mathcal{B}$	$10^3 A_{CP}^{\alpha}$	$10^3 A_{CP}$
$\Xi_c^0 \to \Sigma^+\pi^-$	$0.21(2)$	0 $2.13(21)$	0 $-0.81(23)$
$\Xi_c^0 \to pK^-$	$0.20(2)$	0 $-2.51(33)$	0 $0.94(30)$
$\Lambda_c^+ \to p\pi^0$	$0.16(2)$	$-0.61(39)$ $-1.95(61)$	$0.42(1.15)$ $0.53(95)$
$\Lambda_c^+ \to n\pi^+$	$0.67(8)$	$0.12(20)$ $-0.68(69)$	$-0.15(42)$ $0.71(54)$
$\Lambda_c^+ \to \Lambda^0 K^+$	$0.63(2)$	$-0.03(10)$ $-0.49(12)$	$0.19(18)$ $0.02(21)$

29 data points with 10 complex parameters.

arXiv:2404.19166 [hep-ph]

Rescattering, numerical results

1. A_{CP} in the same size with the ones in D meson! If confirmed, it suggests the natural sizes of A_{CP} are around 10^{-3}.

2. In the U-spin limit, we have that

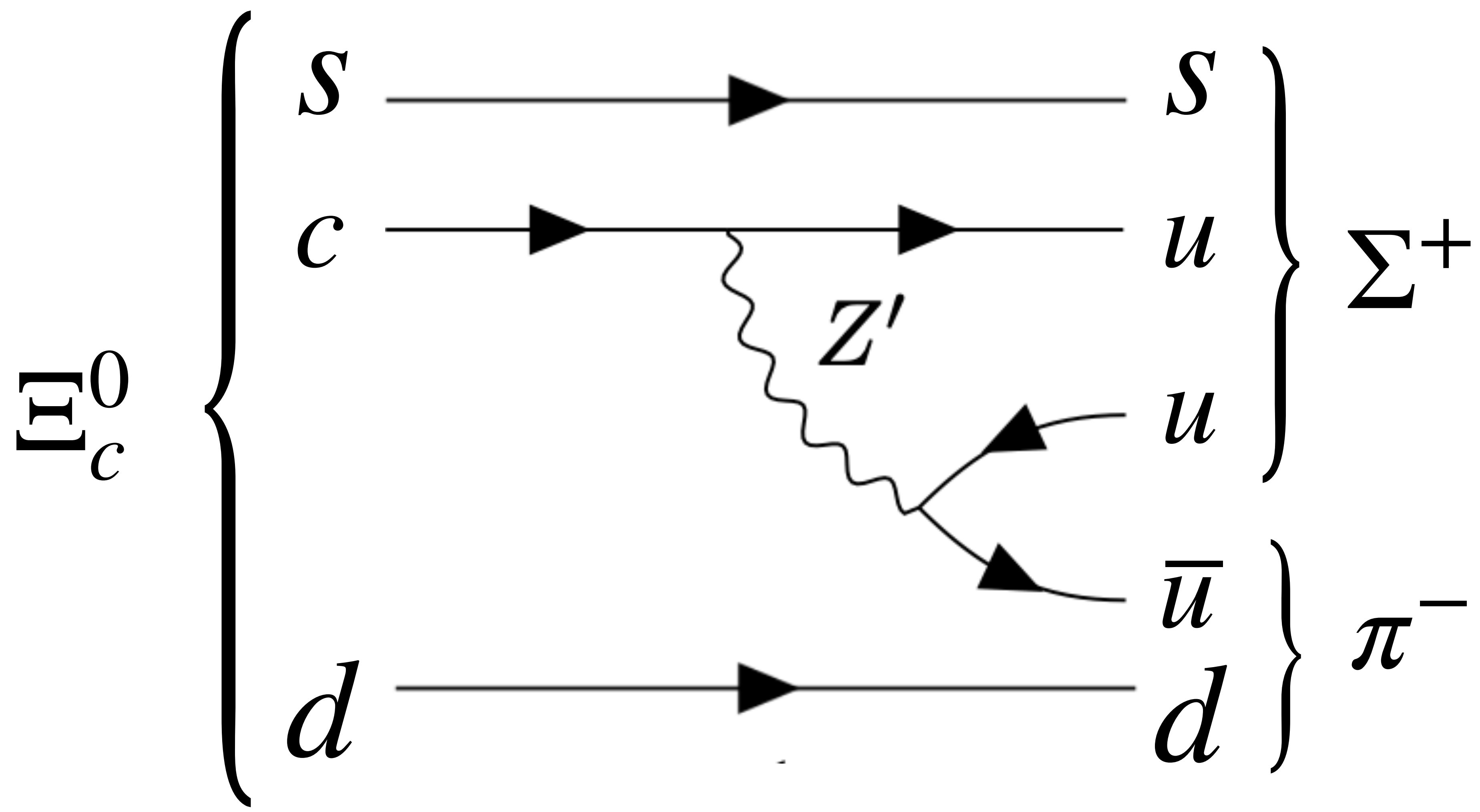
$$A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) = -A_{CP}\left(\Xi_c^0 \to pK^-\right) .$$

Hence it is reasonable to measure

$$\Delta A_{CP} = A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) - A_{CP}\left(\Xi_c^0 \to pK^-\right) .$$

3. The main uncertainties are from strong phases.
Measurement on β can greatly improve!

4. If there are indeed Z' contributions :



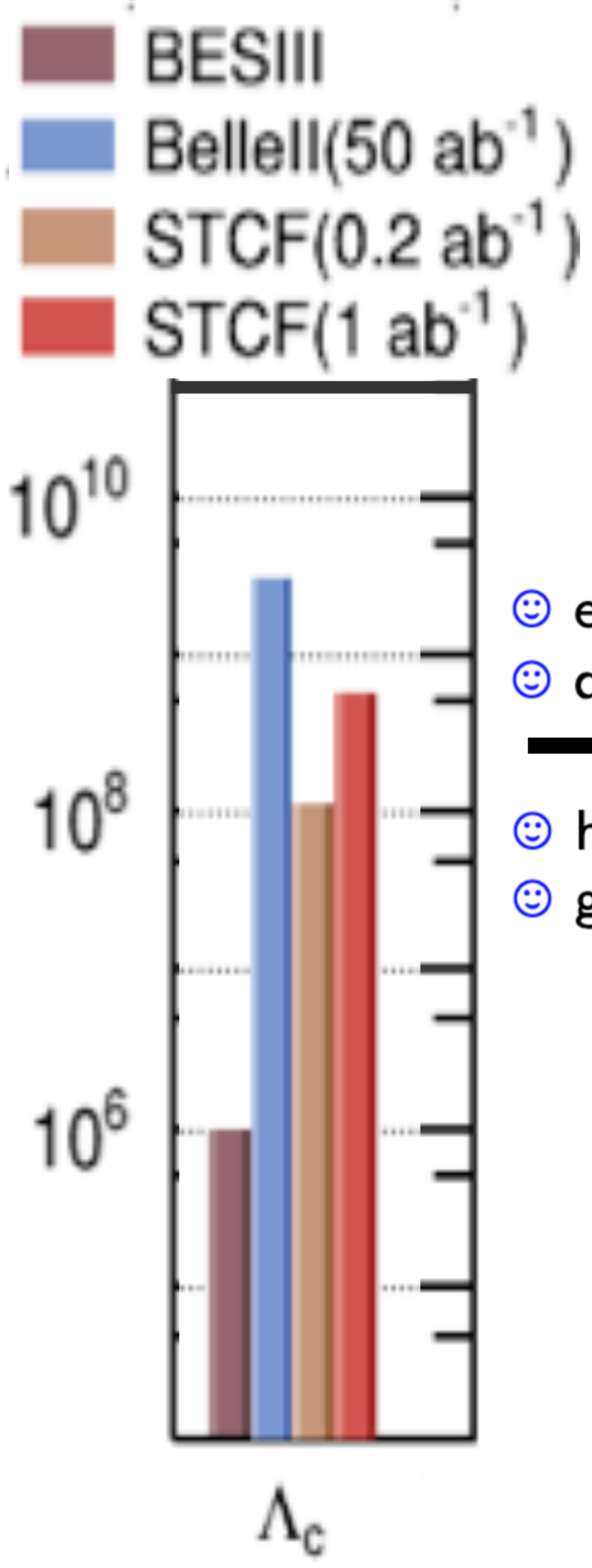
$$A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) \approx 2A_{CP}\left(\Xi_c^0 \to pK^-\right) .$$

(Preliminary result)

arXiv:2404.19166 [hep-ph]

Wish list on future experiments

*Rough estimate from statistics only



A_{CP} at $\mathcal{O}(10^{-3})$

☺ extremely clean environment
☺ quantum coherence

STCF

A_{CP} at $\mathcal{O}(10^{-3})$

↓ Upgrade

A_{CP} at $\mathcal{O}(10^{-4})$

LHCb

☺ very large production cross-section
☺ large boost, excellent time resolution

☺ high-efficiency detection of neutrals
☺ good trigger efficiency

BELLE

Belle : A_{CP} at $\mathcal{O}(10^{-2})$

↓ Upgrade

Belle II : A_{CP} at $\mathcal{O}(10^{-3})$

BESIII

☺ extremely clean environment
☺ quantum coherence

Measurements on β and γ extract important information of strong phases !

Diagram from 周小蓉

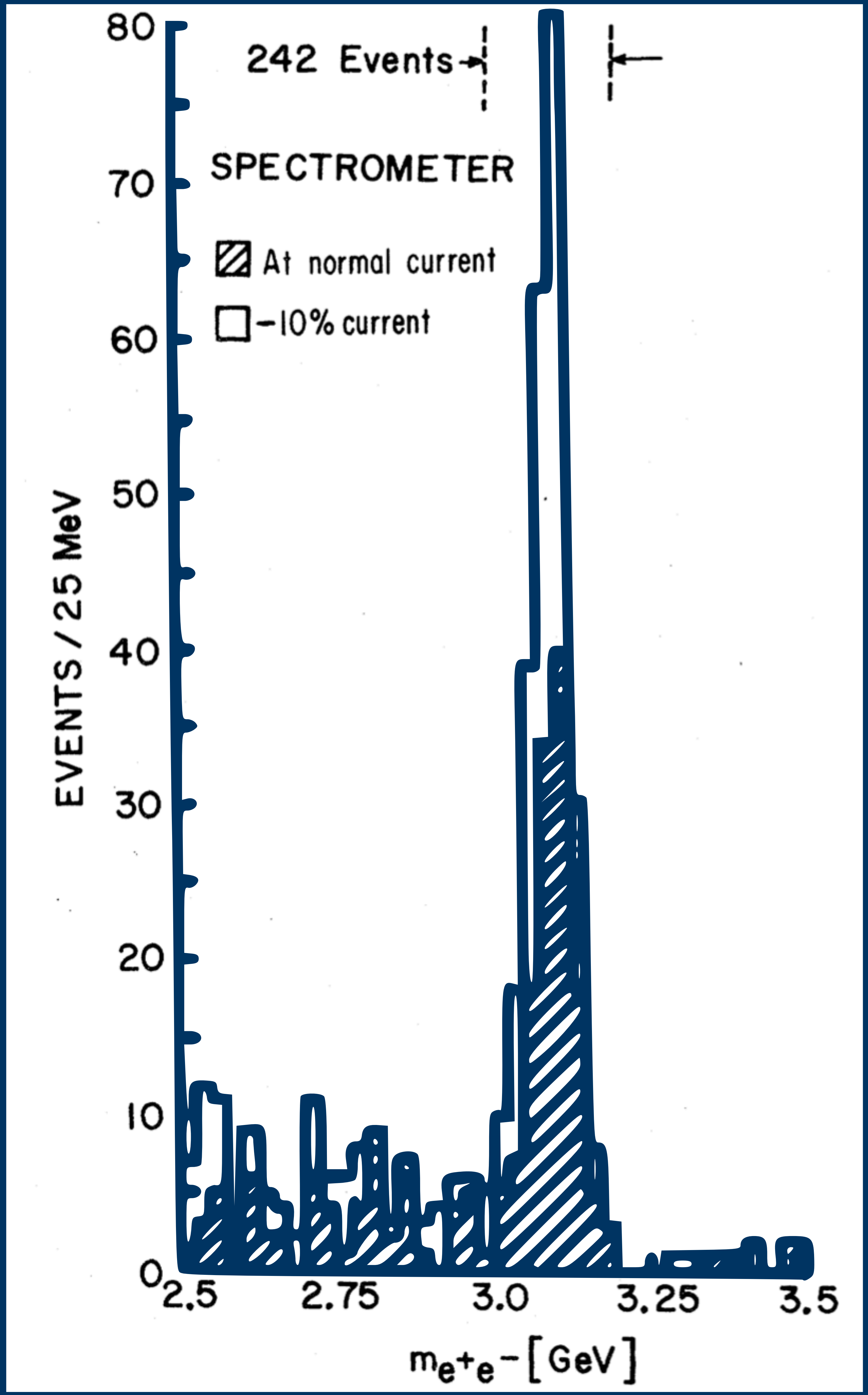
Special thanks to 李龙科

Conclusions

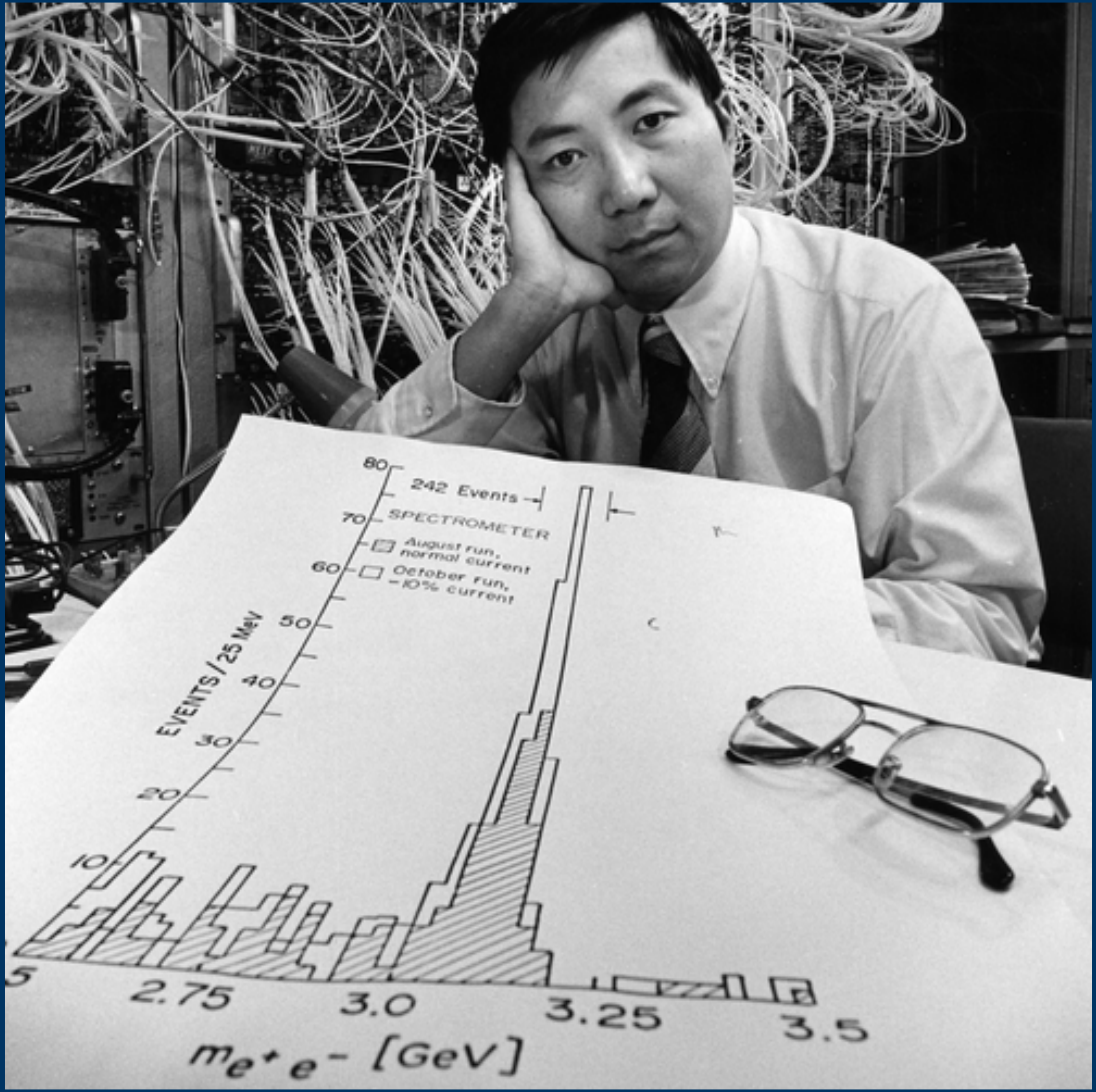
CP violation in charm is a powerful probe for NP!

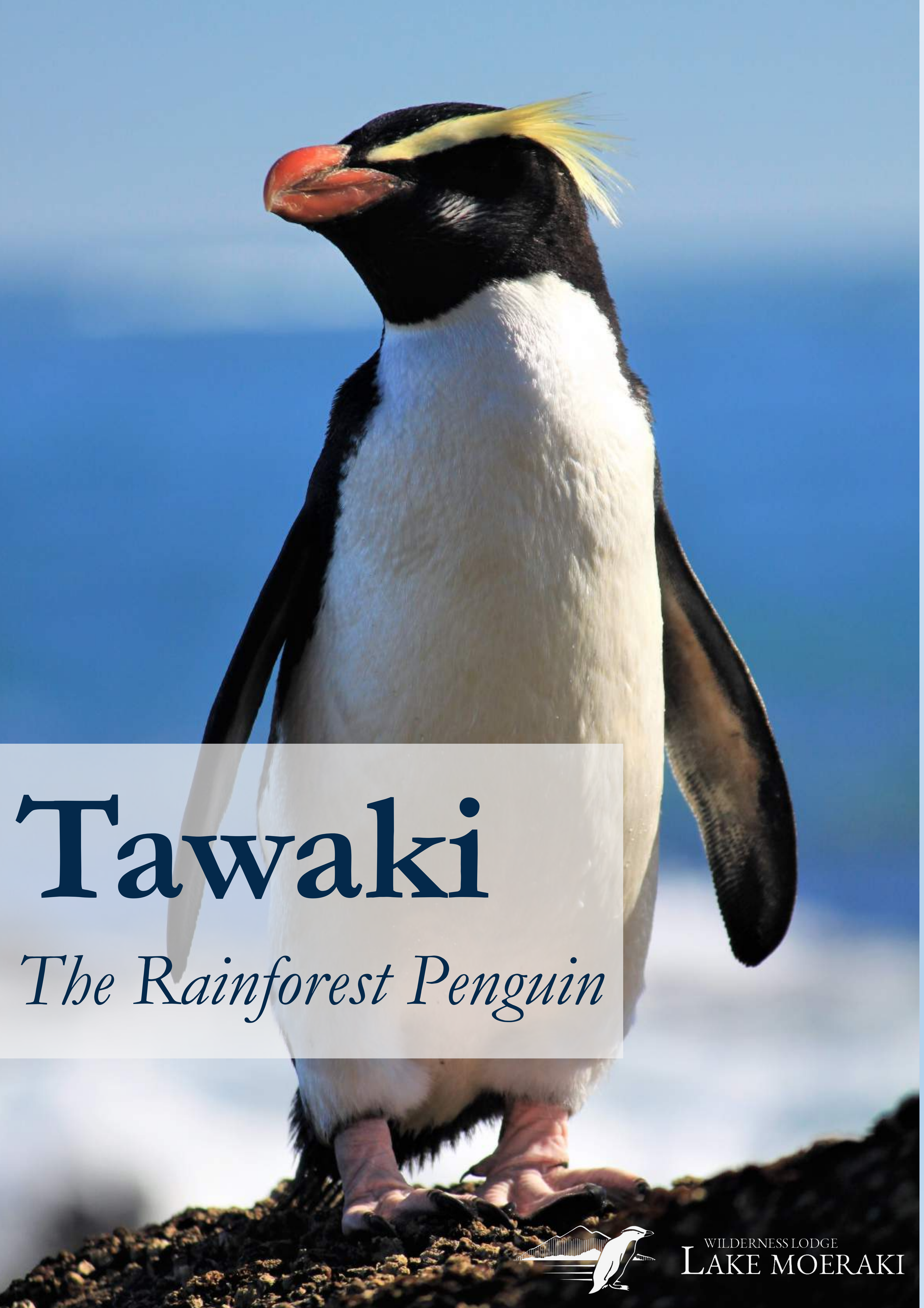
More measurements!

More theoretical studies!



Backup slides





WILDERNESS LODGE
LAKE MOERAKI

Tawaki: A Wildlife Treasure

Tawaki breed in jungle-like temperate rainforest along the rugged Lake Moeraki coastline. To see tawaki on wilderness beaches is one of New Zealand's great wildlife experiences.

The Rainforest Penguin

Tawaki, or the Fiordland Crested Penguin (*Eudyptes pachyrhynchus*), are unique among penguins.

They breed in temperate rainforest, only in the southwest corner of New Zealand. During the July to December breeding season they are most easily seen along the Lake Moeraki coastline.

Tawaki build their nests beneath logs and boulders. These will be deep in the forest, often hundreds of metres inland and up steep hillsides.

Adults must negotiate the pounding surf, wild beaches and dense undergrowth as they make their way between the Tasman Sea and their rainforest nests.

Guided Penguin Trips

Since 1989 Wilderness Lodge Lake Moeraki has taken guests to see tawaki under a special license from the Department of Conservation.

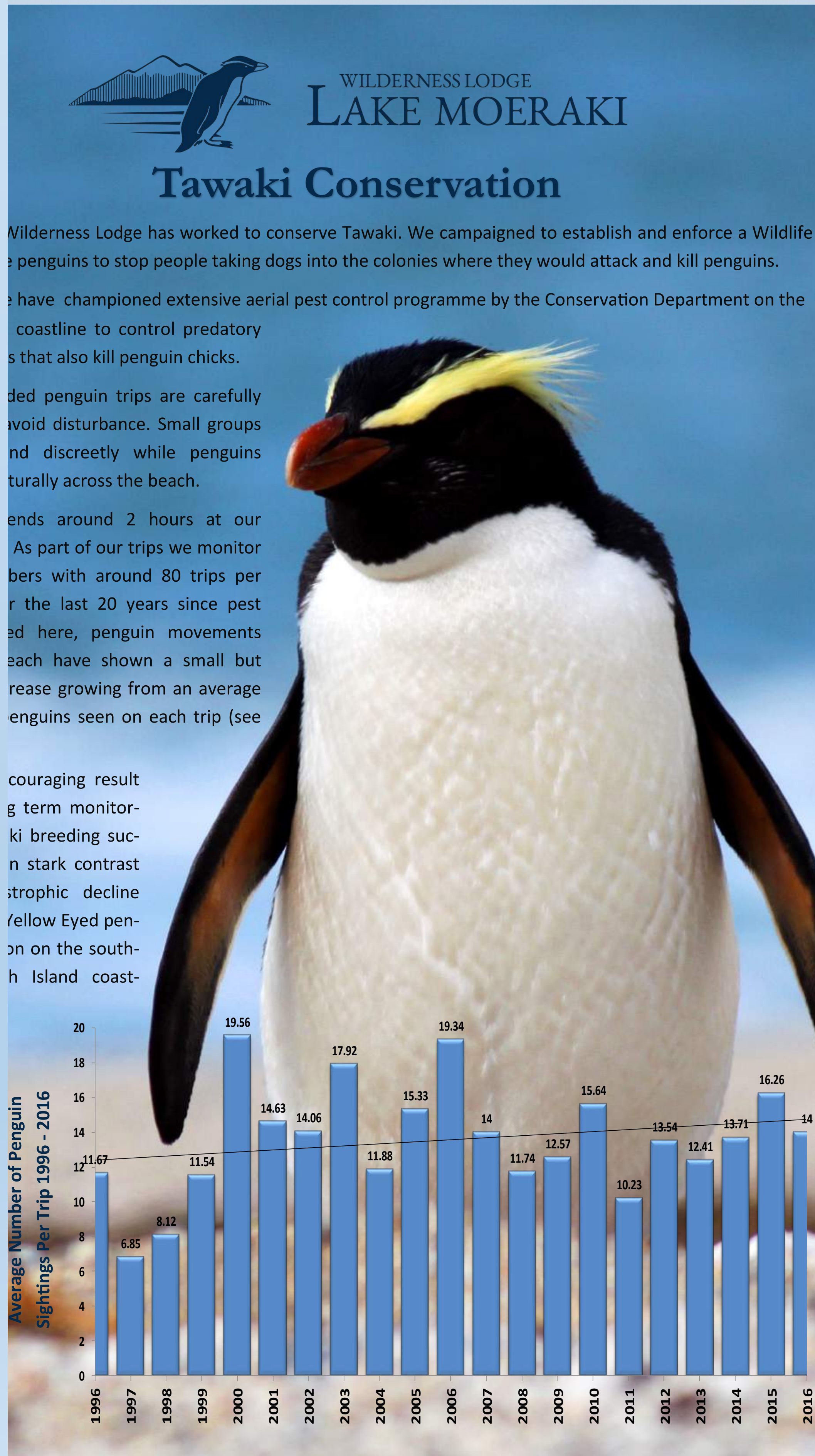
Our guides are experts in penguin ecology and delight in sharing this once in a lifetime experience with guests.

Hike through lush rainforest to a wilderness beach then sit quietly as penguins emerge from the surf and make their way across the beach and into the rainforest.

Guided penguin trips last about 3 hours, include light refreshments and require a low to moderate level of fitness. Group sizes are always kept small.

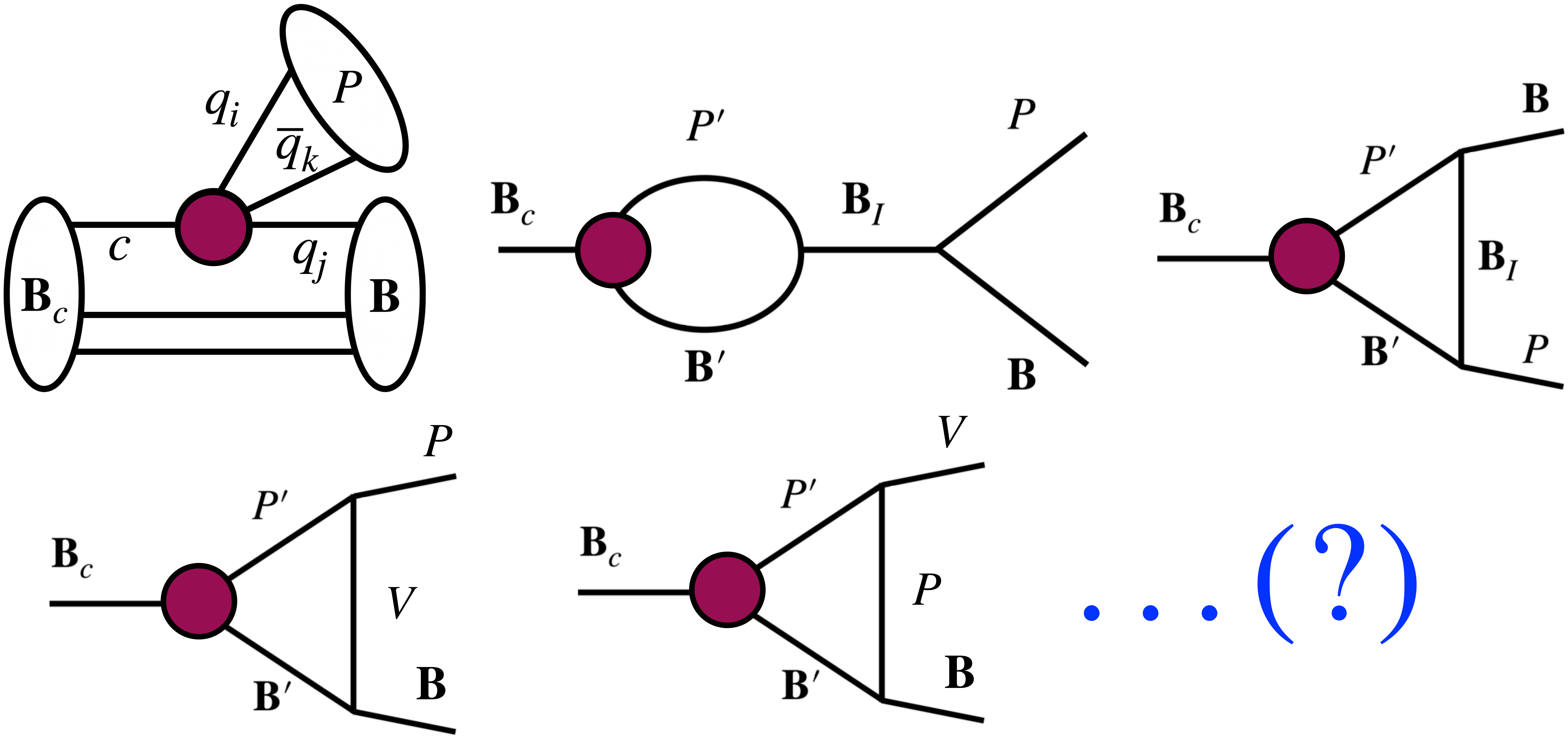
Tawaki Facts

- Tawaki are the world's only penguin to breed in temperate rainforest.
- They stand 60cm tall (2 ft) and weigh approx. 4kg.
- Females lay two eggs each year but only chick is ever feed. This chick grows quickly while the other generally won't survive more than a few days.
- The breeding season runs between July and early December. Outside of this period tawaki are at sea, fishing and sleeping on the surface of the ocean.
- The main threats to tawaki are domestic dogs, introduced stoats (weasel family) and disturbance.



- **Rescattering, solving penguin/tree**

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-t}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-u}}_{\mathbf{B}_c\mathbf{B}P} + \ldots (?)$$



• SU(3) flavor analysis — Tree

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

Generalized Wigner-Eckart theorem

$\tilde{f}$:Free parameters

$$F^{s-d} = \tilde{f}^a (P^\dagger)^l_l \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^j_k + \tilde{f}^b \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^l_k (P^\dagger)^j_l + \tilde{f}^c \underbrace{\mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (P^\dagger)^l_k (\mathbf{B}^\dagger)^j_l}_{SU(3)_F \text{ tensors}}$$

$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \boxed{\tilde{f}^e} (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j ,$$

$$F^b = \boxed{\tilde{f}^e} (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j + \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j$$

$$+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i ,$$

Naive assumption: $\tilde{f}^{a,b,c,d}_{\mathbf{3}} \to 0$

To date, there are in total **30** data points and $\mathbf{5} \times 2$(S & P waves) $\times 2$(complex) $- 1 = \mathbf{19}$

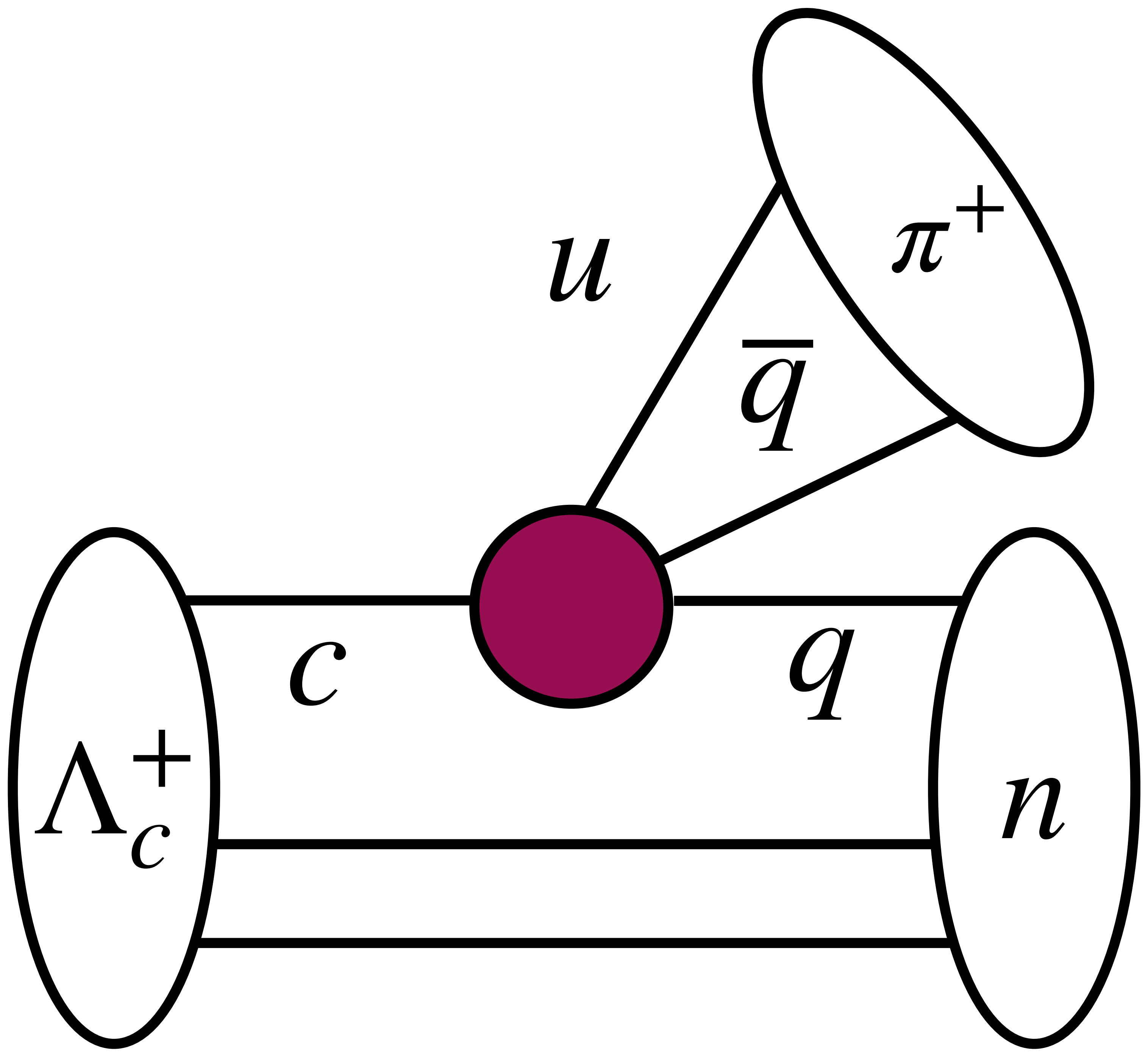
CP-even

arXiv:2310.05491 [hep-ph]

• SU(3) flavor analysis — Tree

$A_{CP}(\Lambda_c^+ \to n\pi^+) \neq 0$, **as parts of the tree interaction contain penguin topology.**

$$\mathcal{H}_{eff}^{\rm Tree} = \frac{G_F}{\sqrt{2}}\lambda_b \left(C_+ \sum_{q=u,d,s} ((\bar{u}q)(\bar{q}c) + (\bar{q}q)(\bar{u}c)) + 2C_- \sum_{q=d,s} ((\bar{u}q)(\bar{q}c) - (\bar{q}q)(\bar{u}c)) \right)_{\mathbf{3}} \cdots$$



Too small compared to *D* meson's:

$$A_{CP}^{dir}(D^0 \to K^+K^-) - A_{CP}^{dir}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

Channels	$\mathcal{B}(10^{-3})$	$A_{CP}^{\alpha}(10^{-3})$	$A_{CP}(10^{-3})$
$\Lambda_c^+ \to p\pi^0$	0.16(2)	−0.61(39)	0.42(1.15)
$\Lambda_c^+ \to p\eta$	1.45(25)	0.05(17)	−0.24(26)
$\Lambda_c^+ \to p\eta'$	0.52(11)	−0.02(7)	0.08(2)
$\Lambda_c^+ \to n\pi^+$	0.67(8)	0.12(20)	−0.15(42)
$\Lambda_c^+ \to \Lambda^0 K^+$	0.63(2)	−0.03(10)	0.19(18)

Experimental status of charmed hadron decays

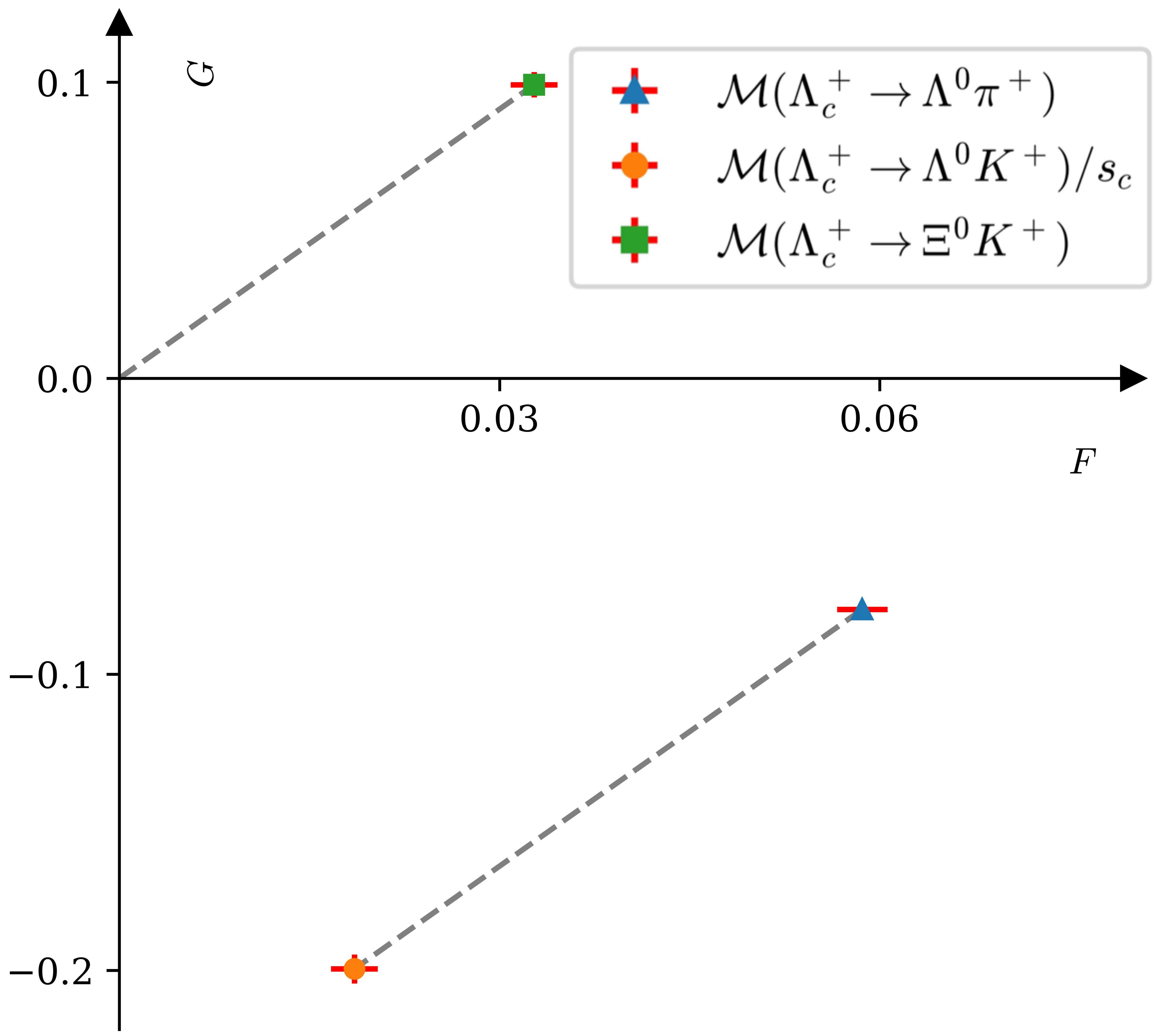
The SU(3) flavor relation:

$$\Gamma = \frac{p_f}{8\pi}\frac{\left(M_i + M_f\right)^2 - M_P^2}{M_i^2}\left(|F|^2 + \kappa^2 |G|^2\right), \quad \alpha = \frac{2\kappa \,\mathrm{Re}\,(F^* G)}{|F|^2 + \kappa^2 |G|^2}$$

$$F(\Lambda_c^+ \to \Xi^0 K^+) = \frac{2}{\sqrt{6}} F(\Lambda_c^+ \to \Lambda^0 \pi^+) - \frac{1}{s_c} F(\Lambda_c^+ \to \Lambda^0 K^+)$$

If F and G are real, they are solvable from experimental Γ and α!

→ Leads to $|\alpha(\Lambda_c^+ \to \Xi^0 K^+)| \approx 1$



2023: Measurements of strong phases in $\Lambda_c^+ \to \Xi^0 K^+$

PRL **132**, 031801 (2024)

$$\delta_P - \delta_S = -1.55 \pm 0.27(+\pi), \quad \alpha = 0.01 \pm 0.16$$

* CP even and Cabibbo-favored, but very important to studies of CP violation!