

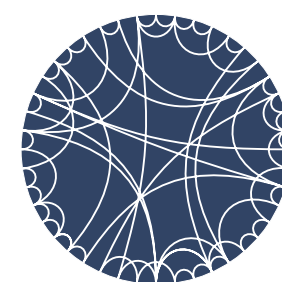
Locating the critical end point and the phase boundary of QCD from first principle QCD

Jan M. Pawłowski

Universität Heidelberg & ExtreMe Matter Institute

Wuhan, November 2nd 2024

for the fQCD collaboration



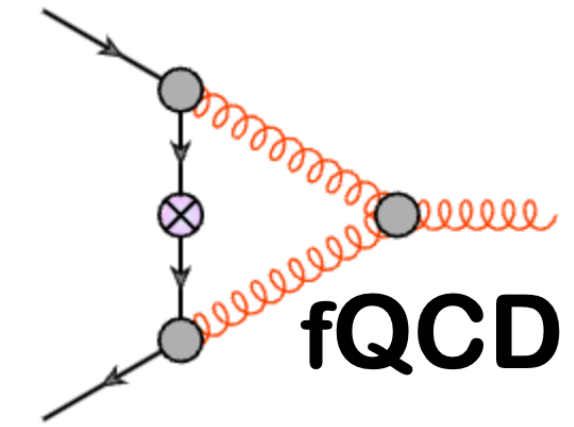
STRUCTURES
CLUSTER OF
EXCELLENCE



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HEIDELBERG
ZUKUNFT
SEIT 1386



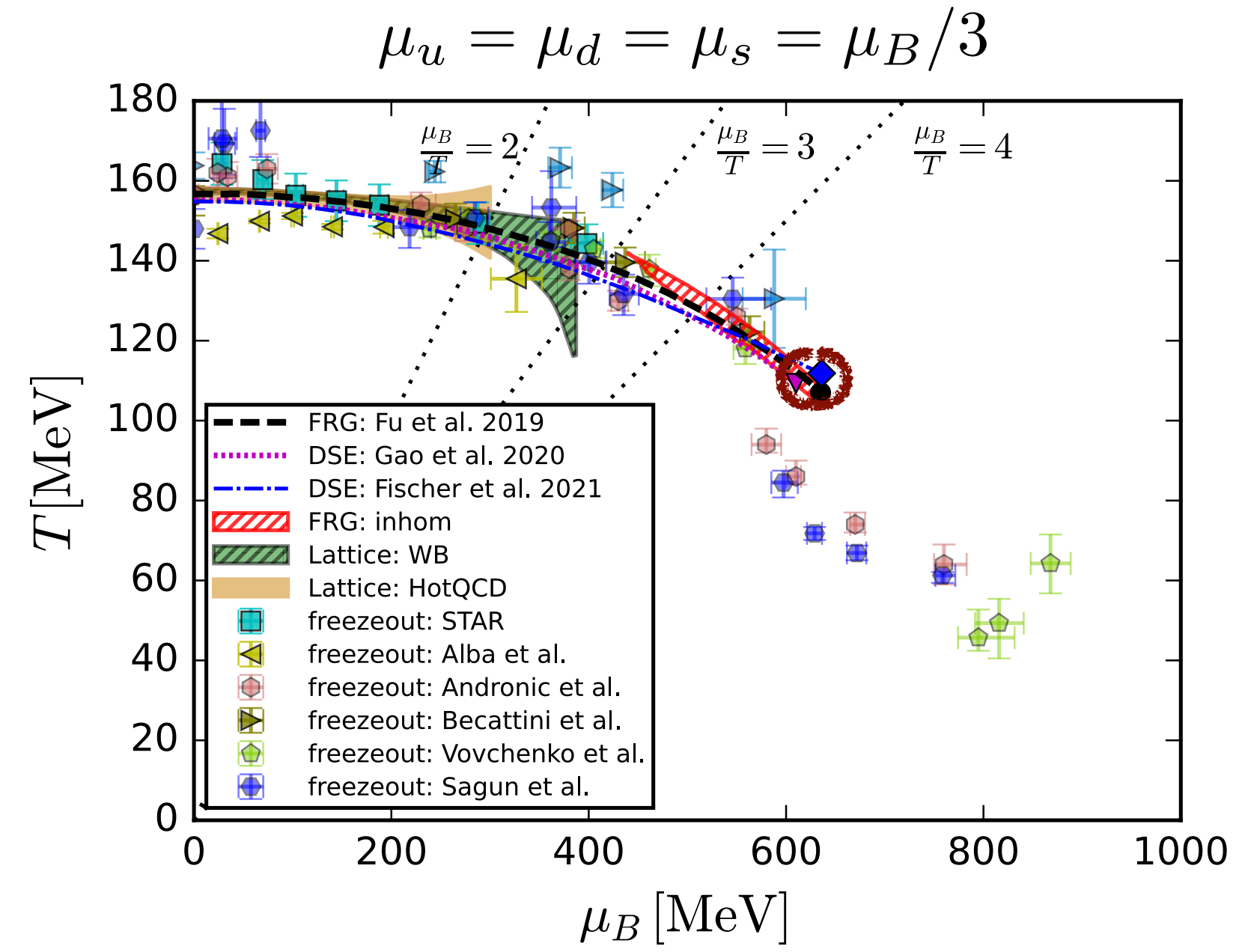
fQCD collaboration



Dalian, Beijing, Darmstadt, Heidelberg, Gießen

**Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler,
Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Yin, Zheng, Zorbach**

Phase structure of QCD and the CEP



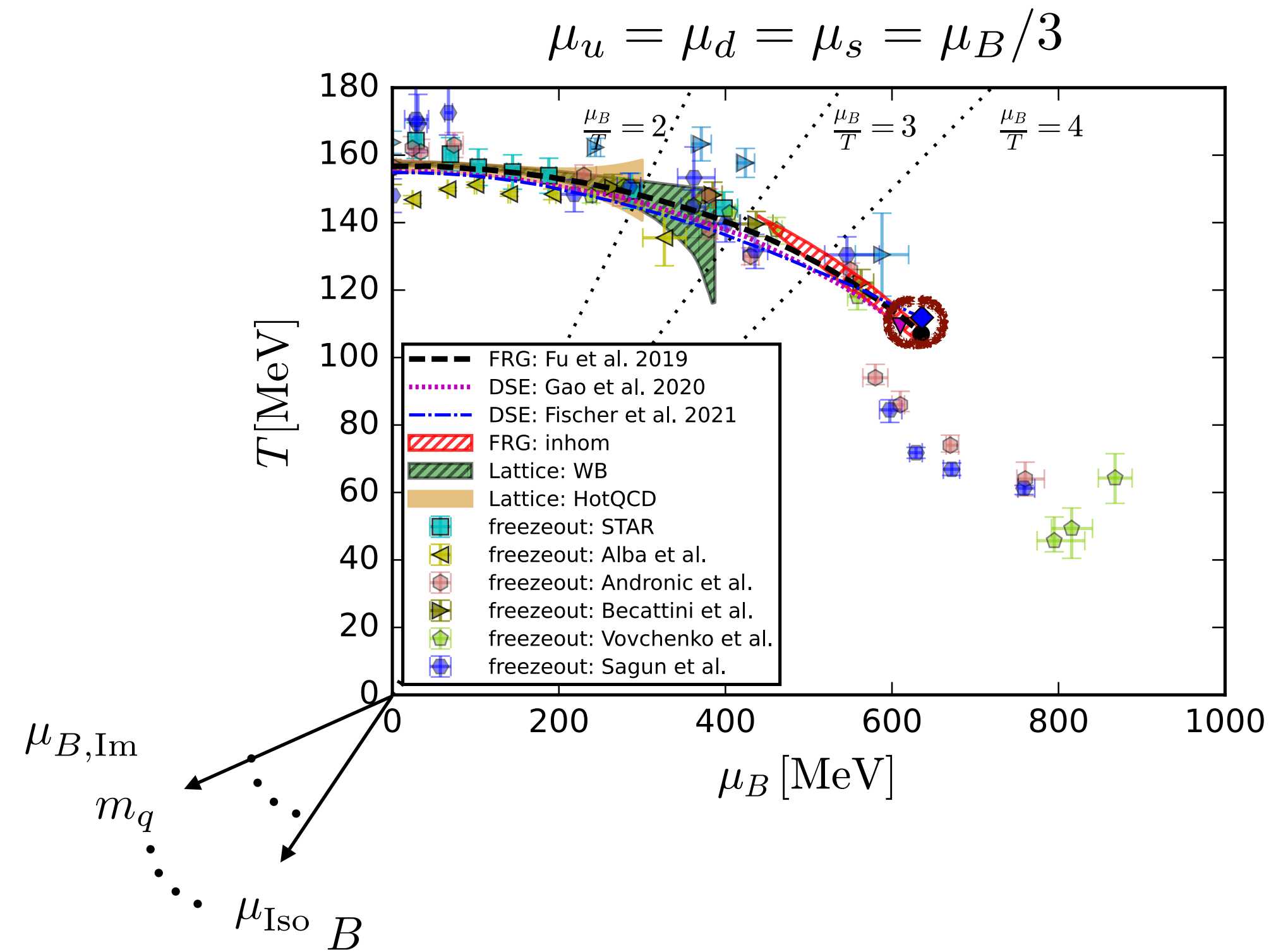
Functional QCD: CEP estimate

fRG: Fu, JMP, Rennecke, PRD 101 (2020) 054032

DSE: Gao, JMP, PLB 820 (2021) 136584
Gunkel, Fischer, PRD 104 (2021) 054022

$$(\mu_B, T)_{\text{CEP}} \sim (600 - 650, 105 - 115) \text{ MeV}$$

Phase structure of QCD and the CEP



Functional QCD: CEP estimate

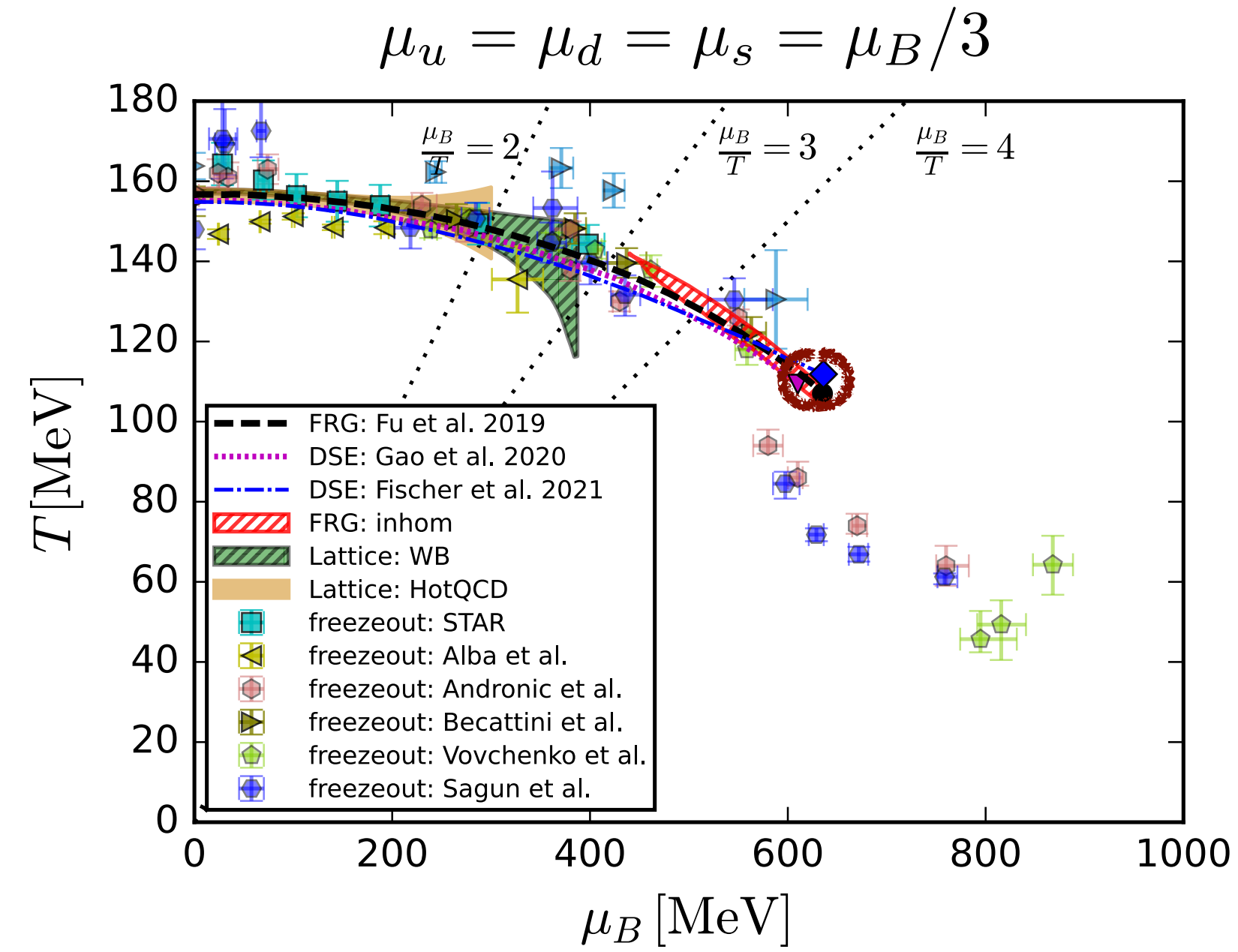
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**Collect all possible information/structure
for
physics understanding & extrapolations**

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Functional QCD: CEP estimate

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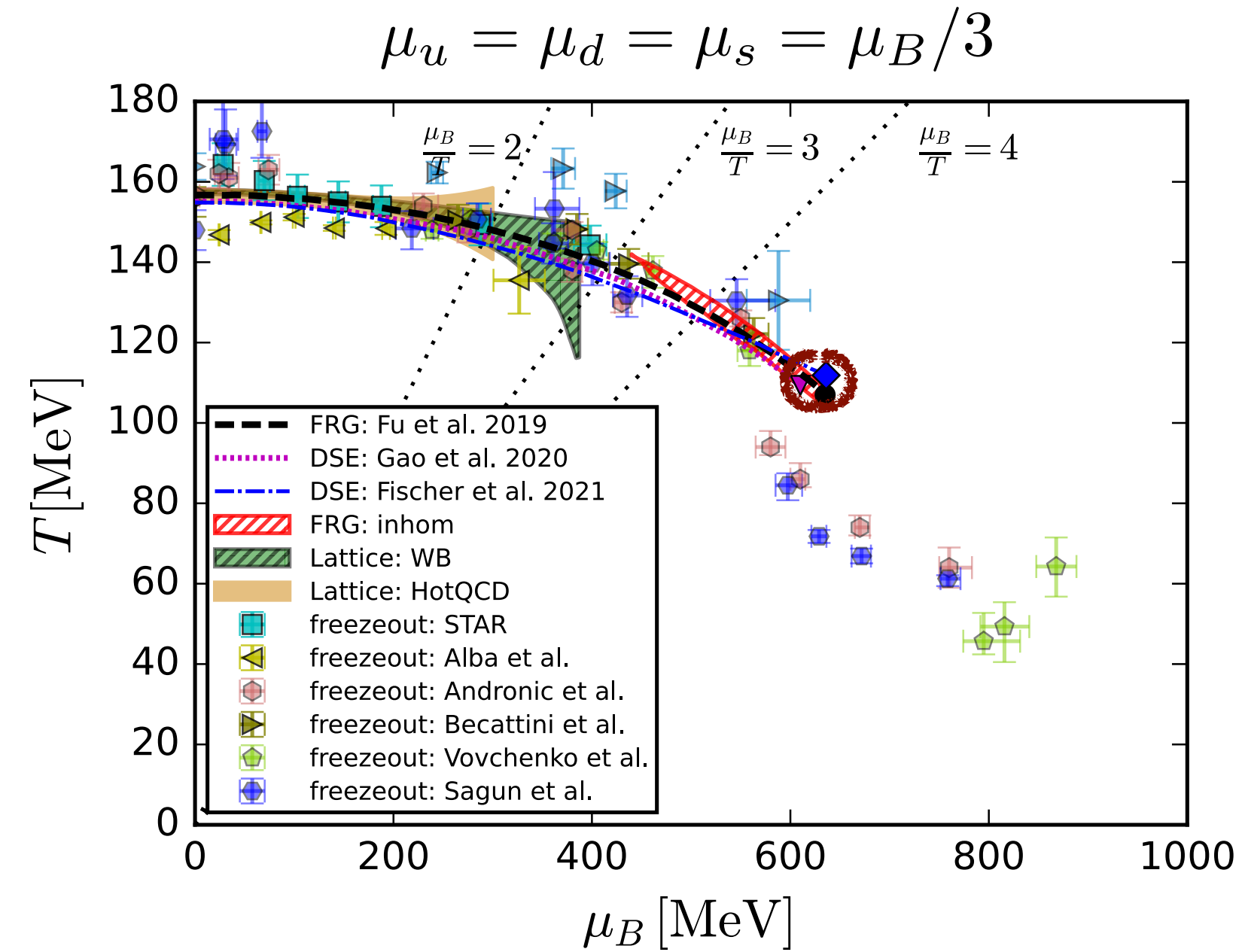
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Estimates & predictions

Requires computations in 1st principle QCD at

$$(\mu_B, T) \sim (\mu_B, T)_{\text{CEP}}$$

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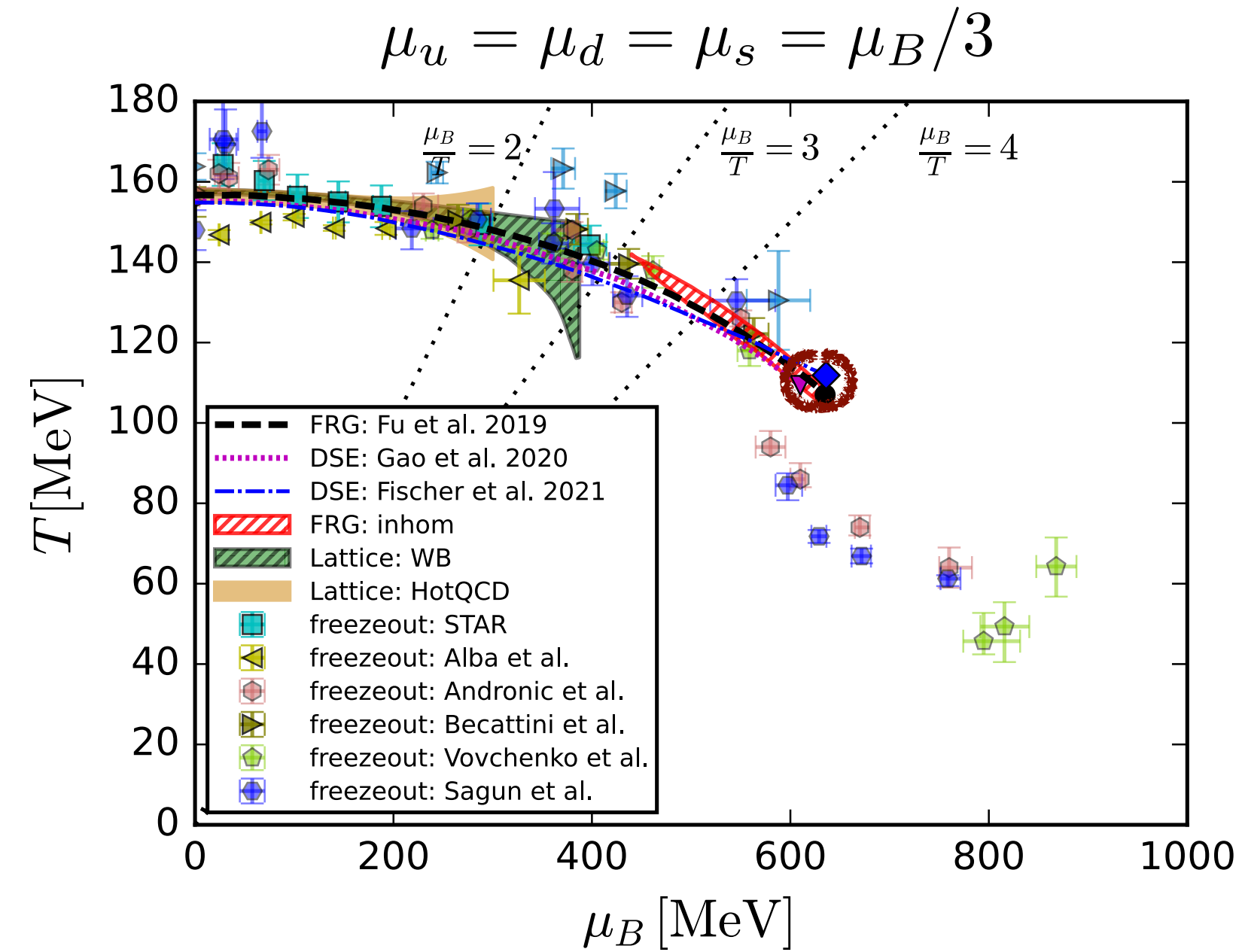
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Extrapolations for Pheno

Requires a discussion of the explicit & implicit assumptions

Phase structure of QCD and the CEP



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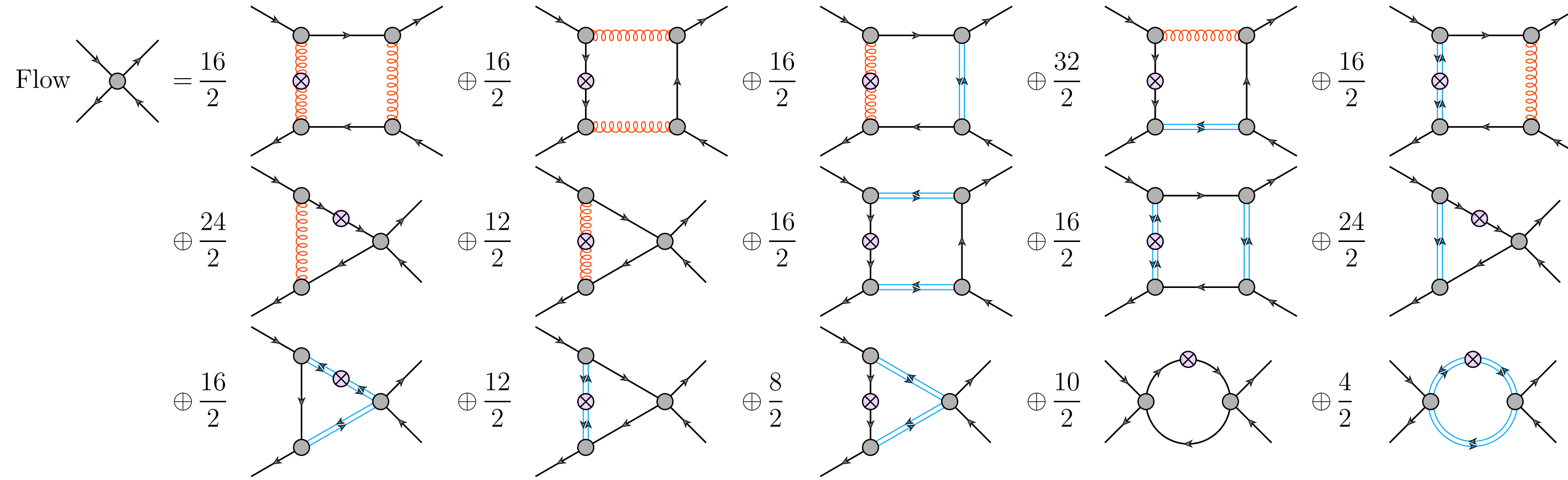
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Lattice extrapolations

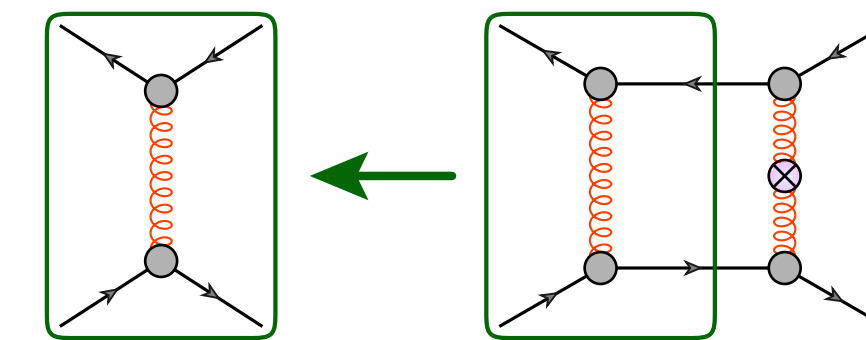
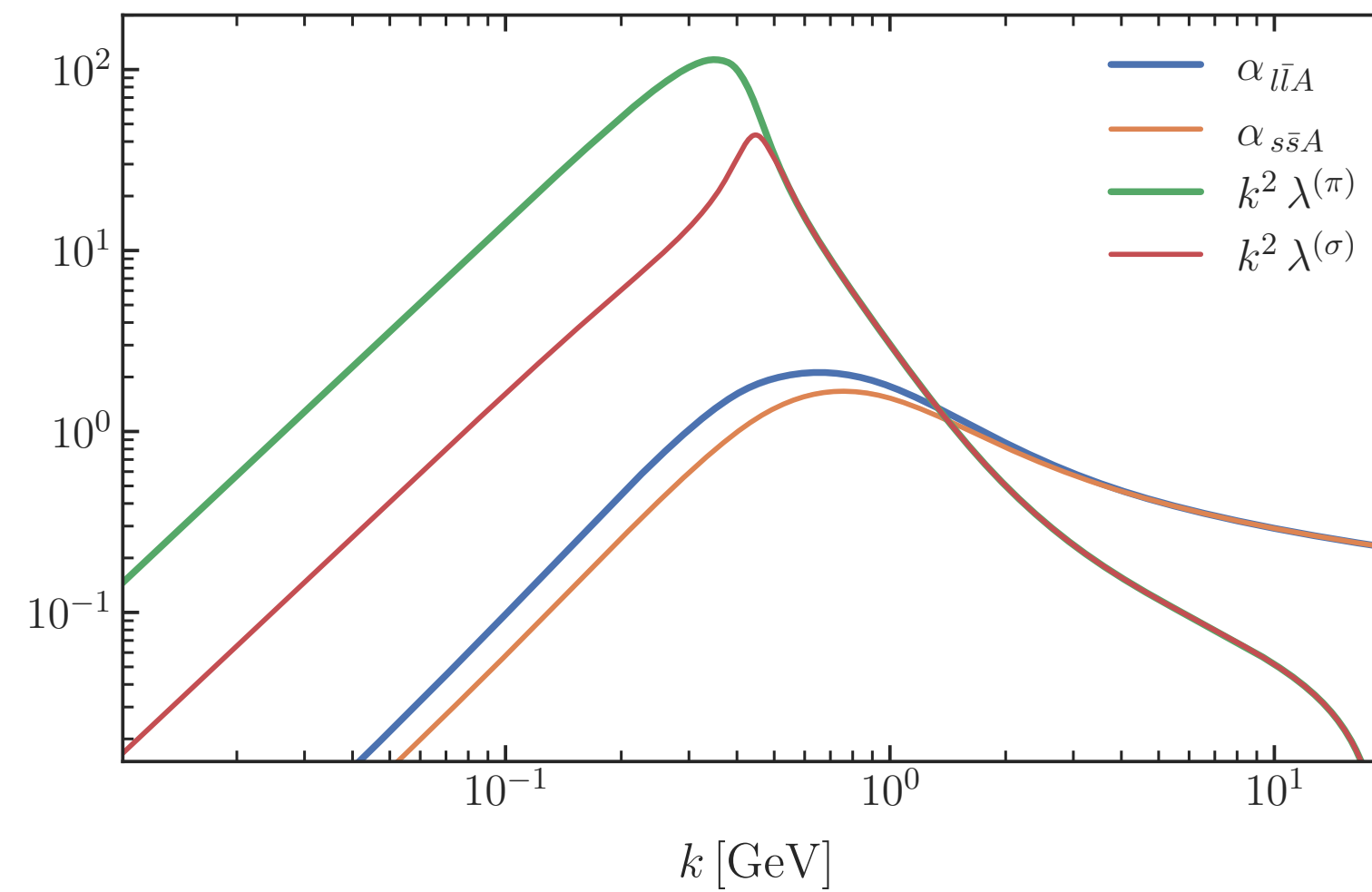
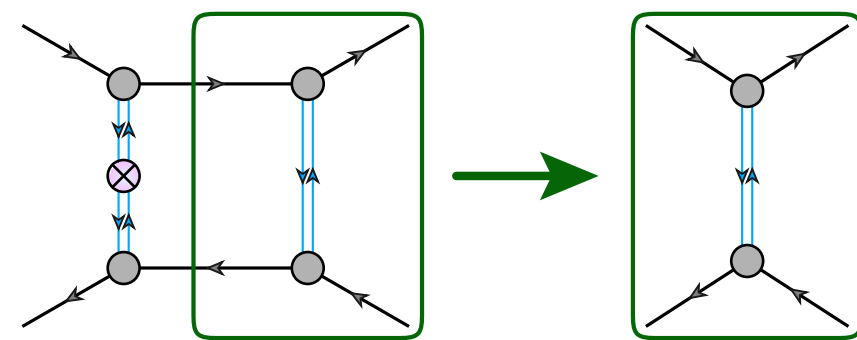
low energy effective theories:
QM, NJL, PQM, PNJL, ..., Holography

Phase structure from functional QCD: how to

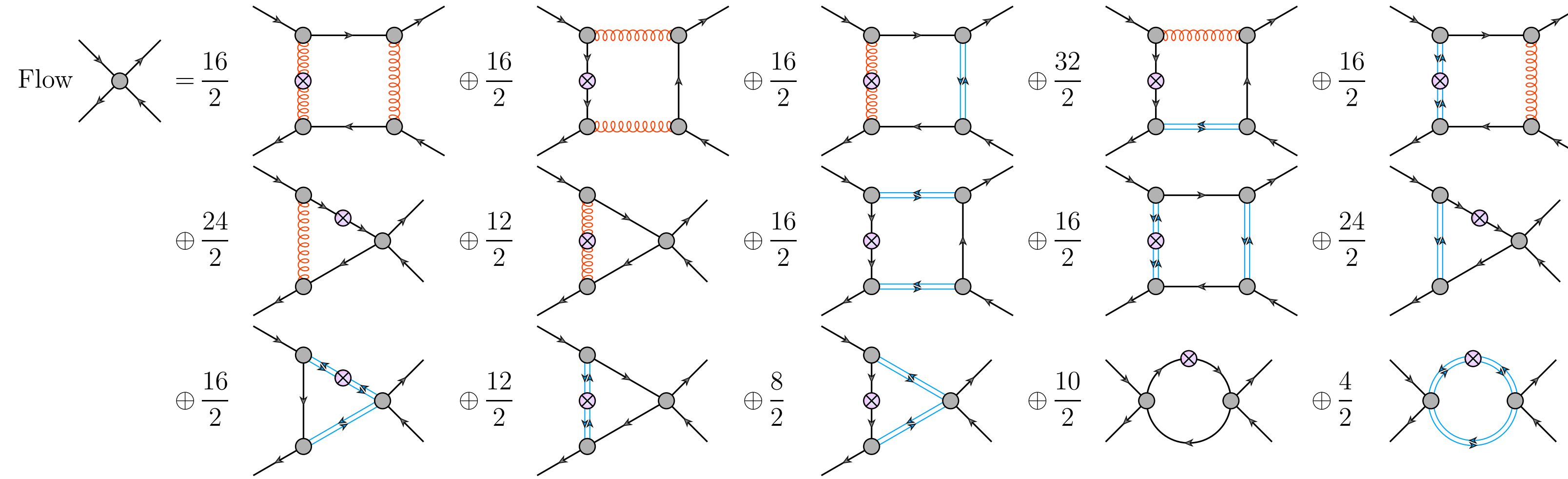
How to: systematic error estimates & the LEGO[®] principle



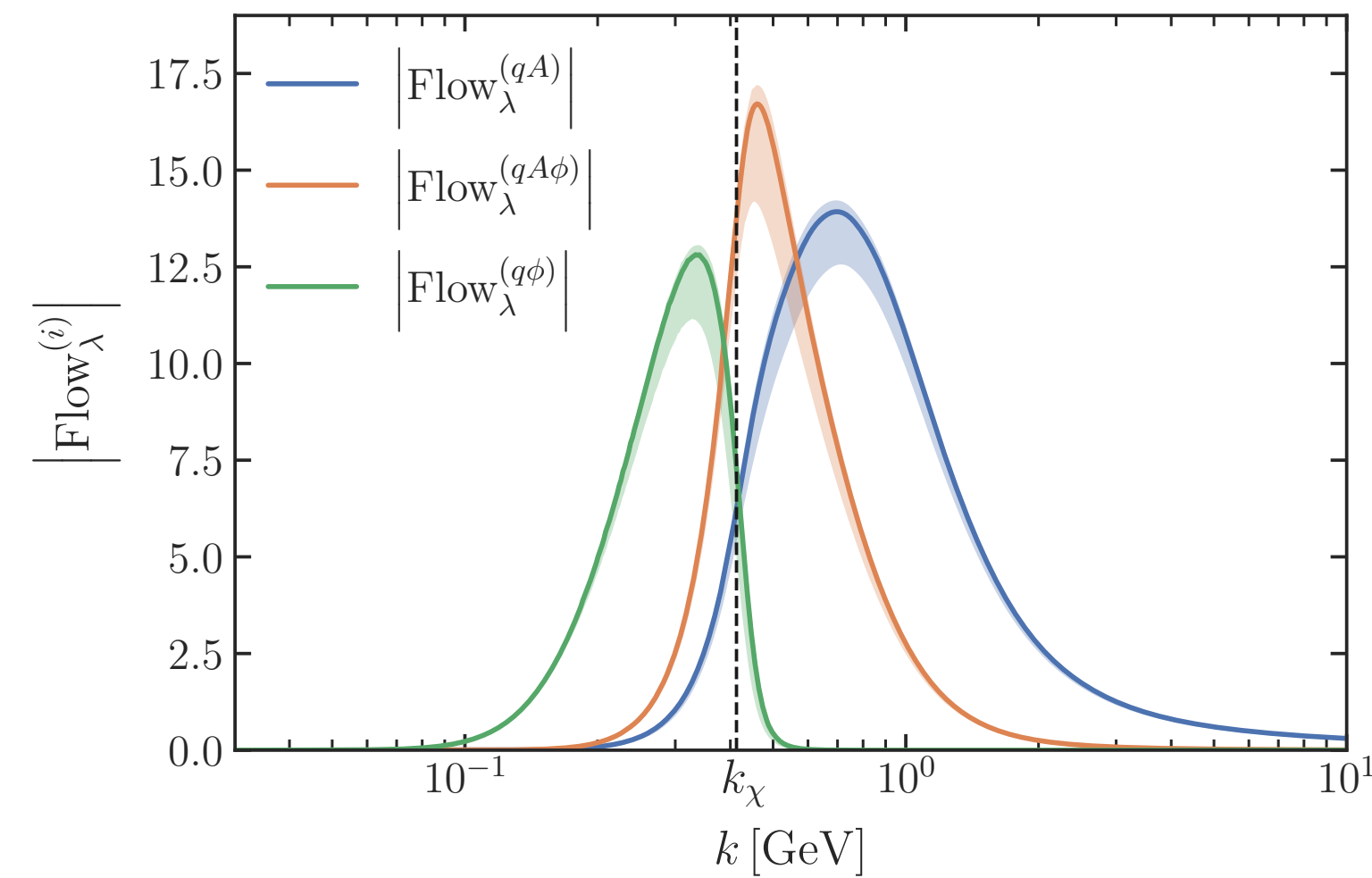
Example: 4-quark scattering vertex



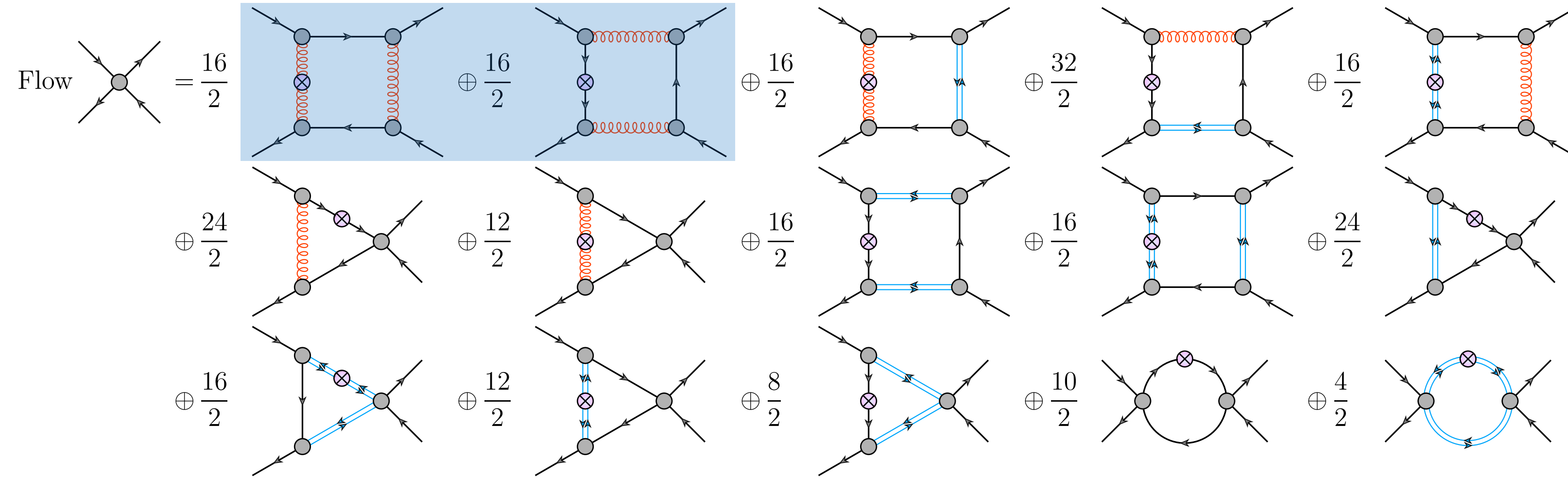
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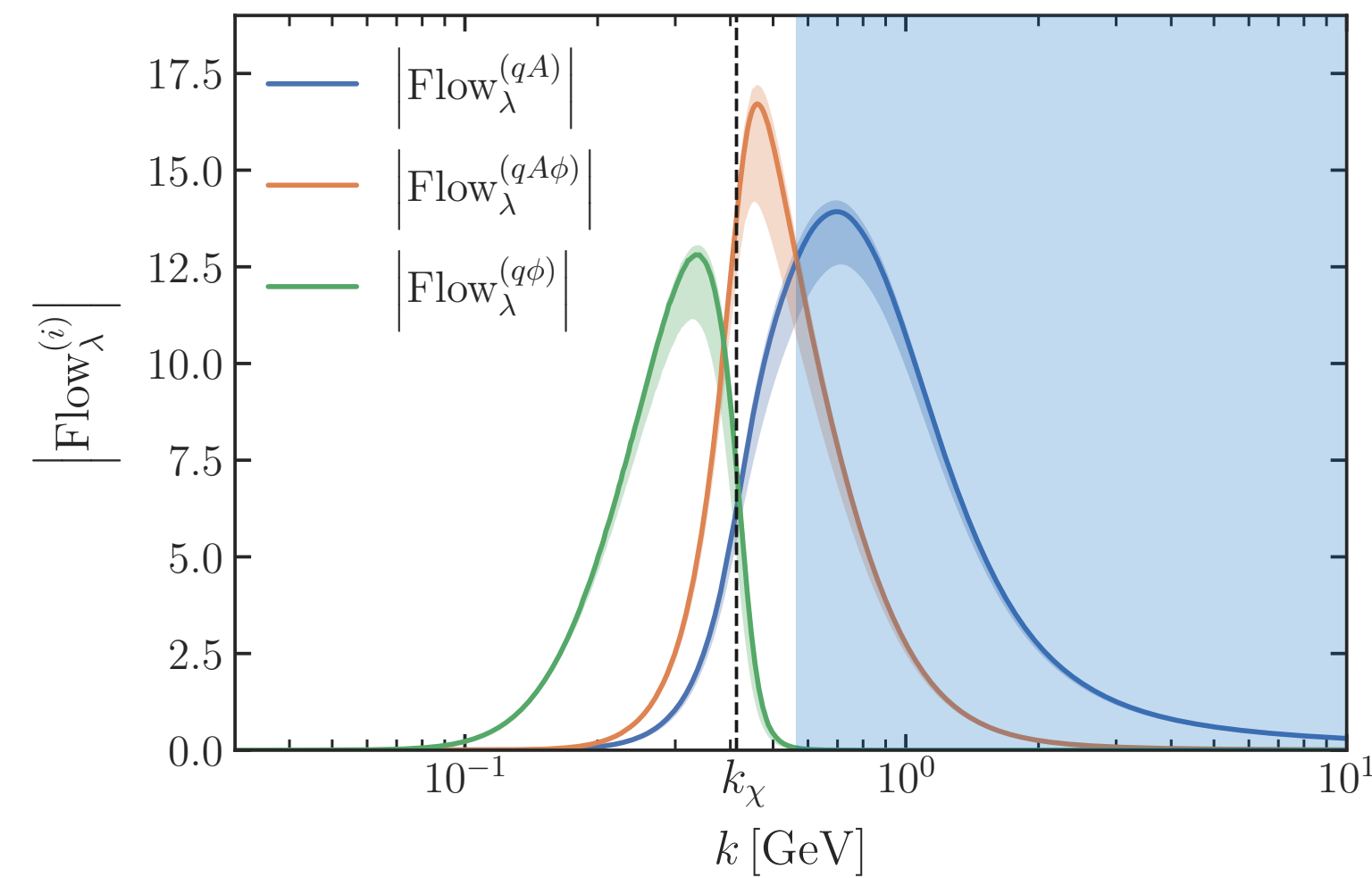
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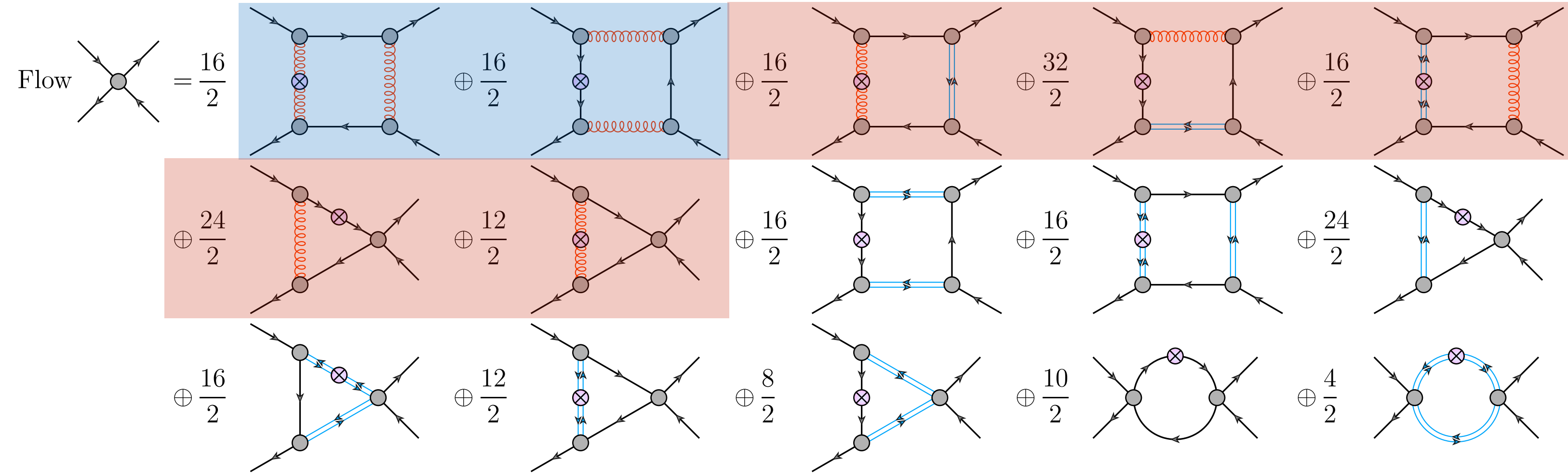
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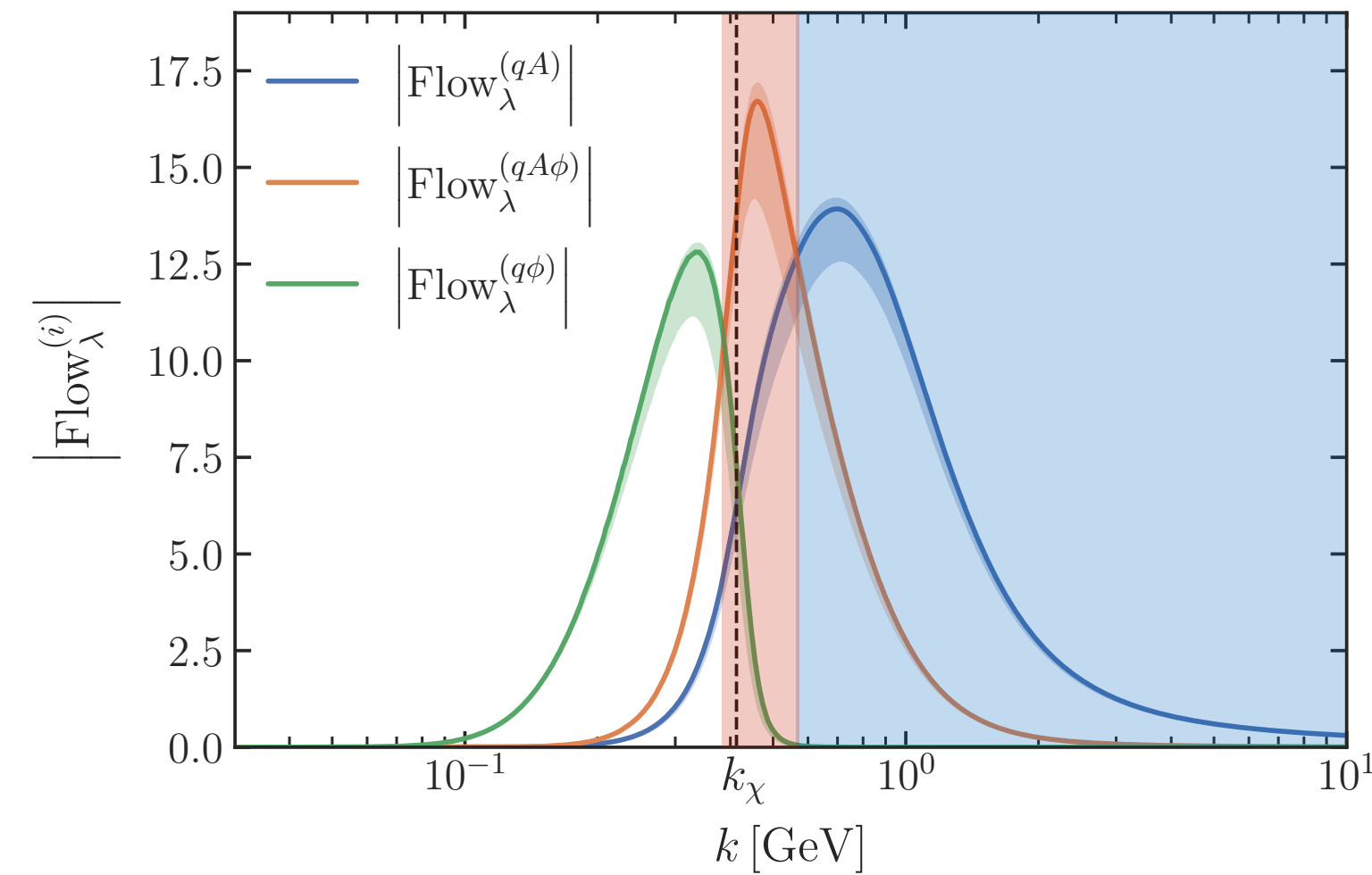
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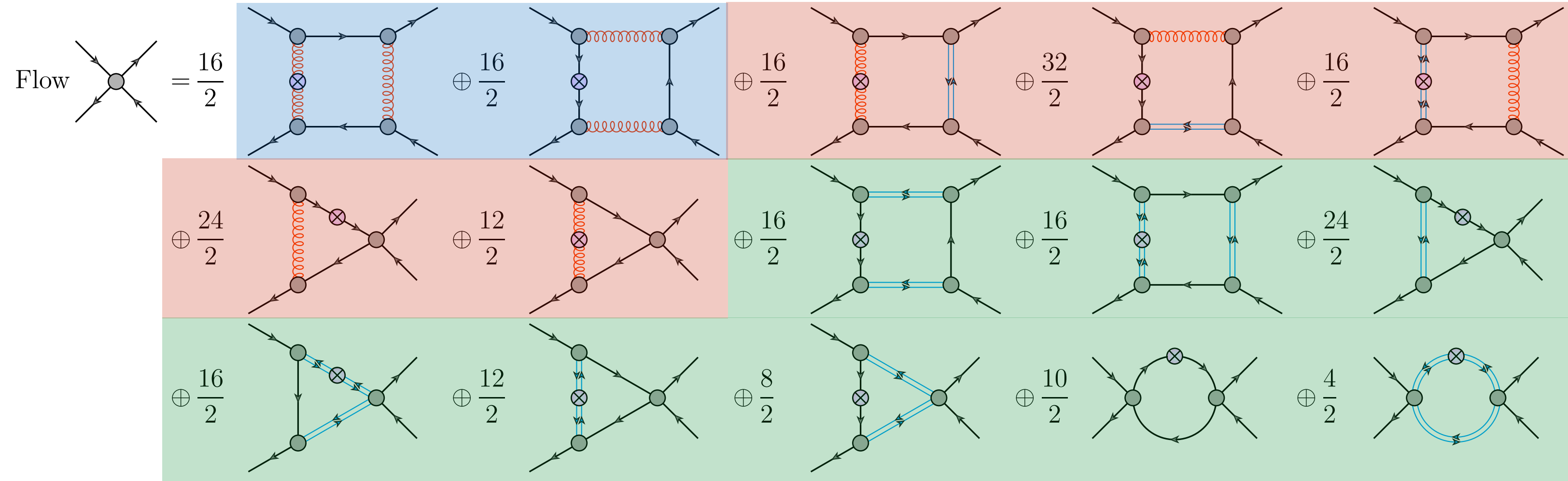
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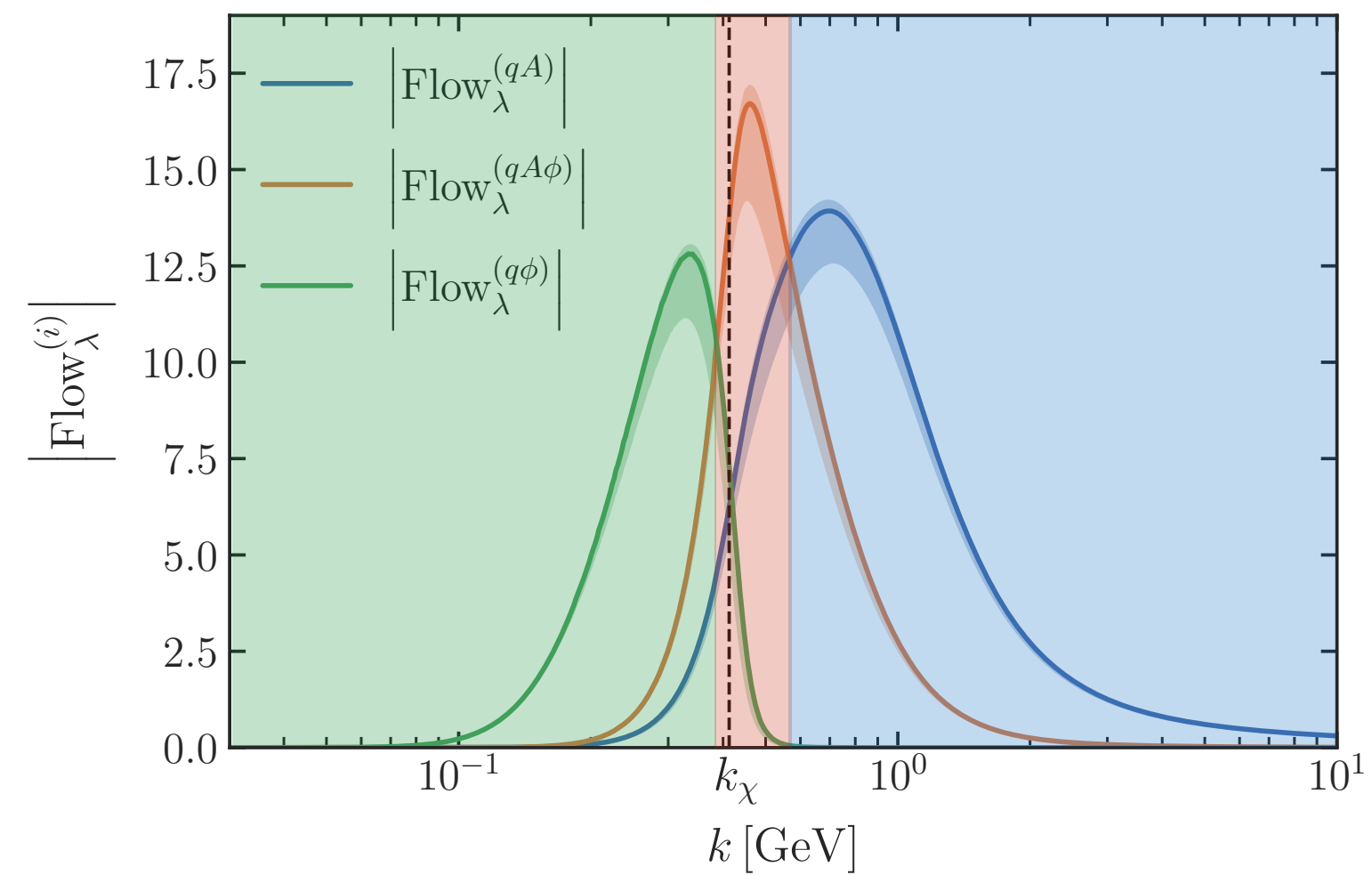
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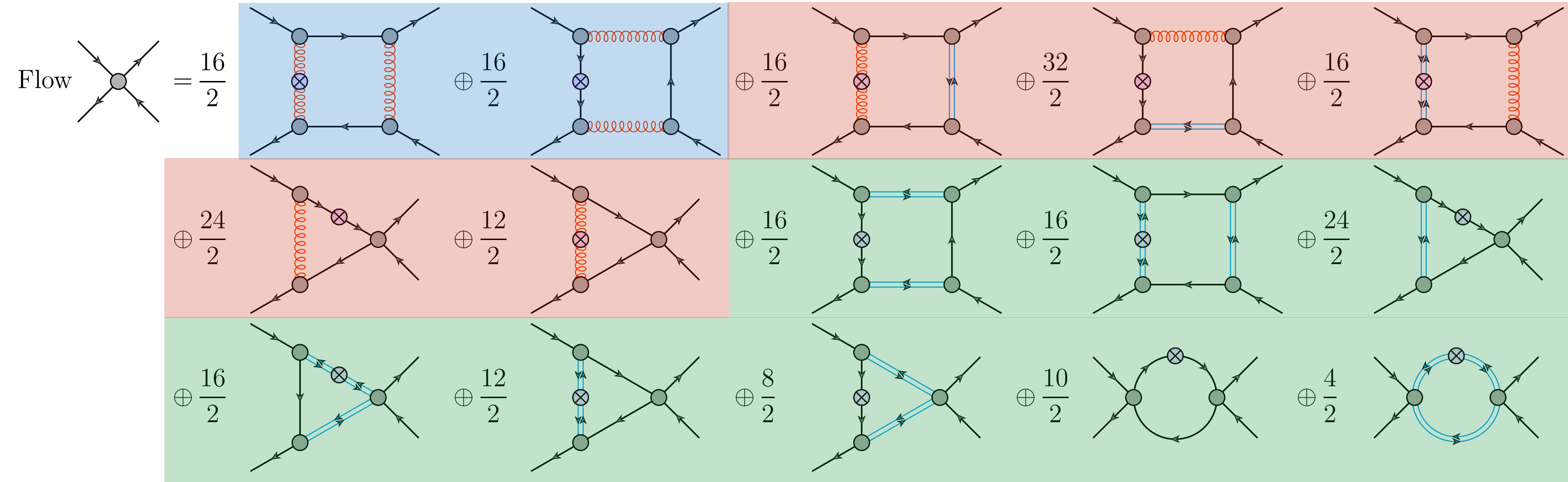
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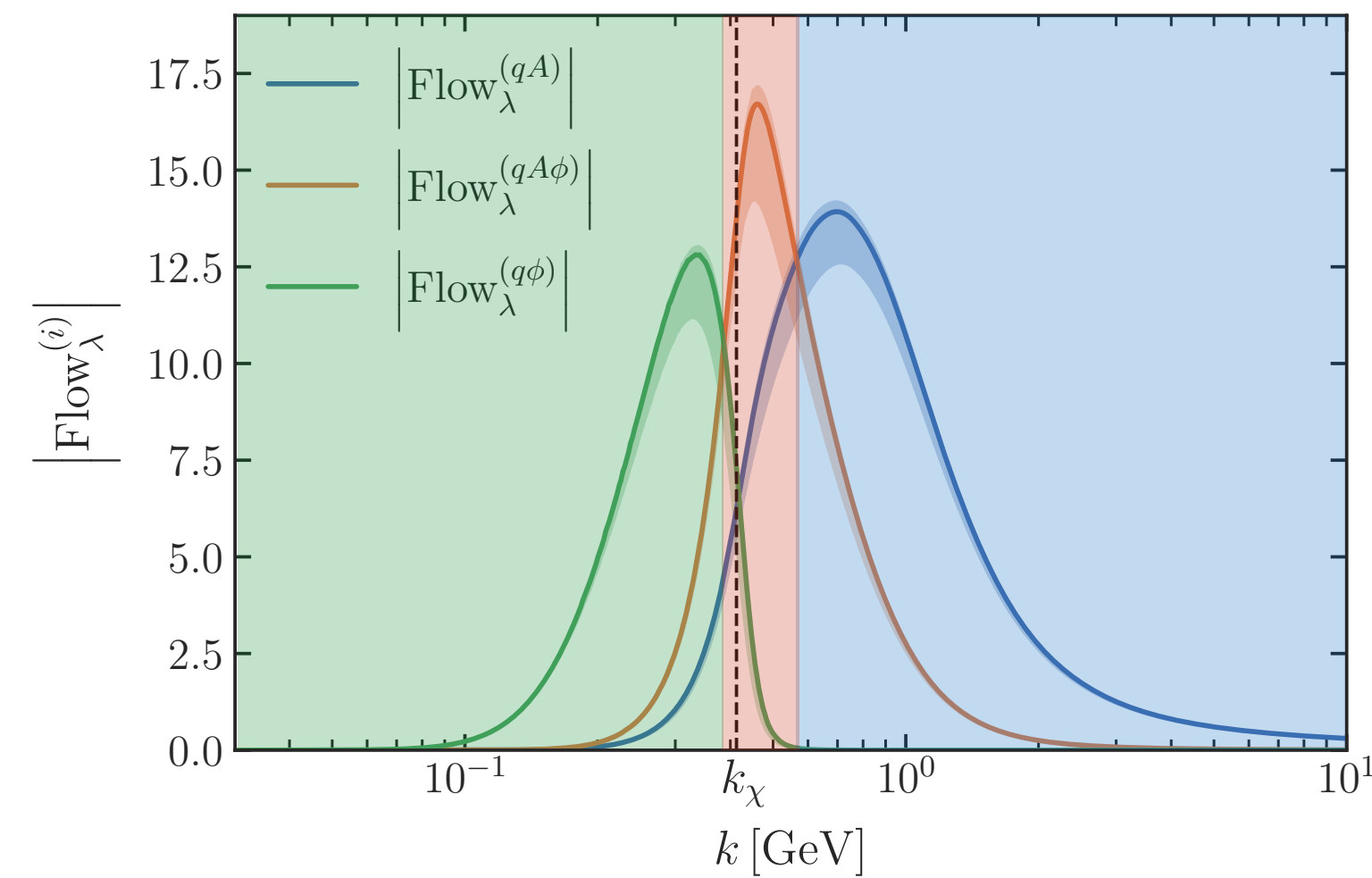
Example: 4-quark scattering vertex



How to: systematic error estimates & the LEGO[®] principle



The unreasonable effectiveness of low energy effective theories

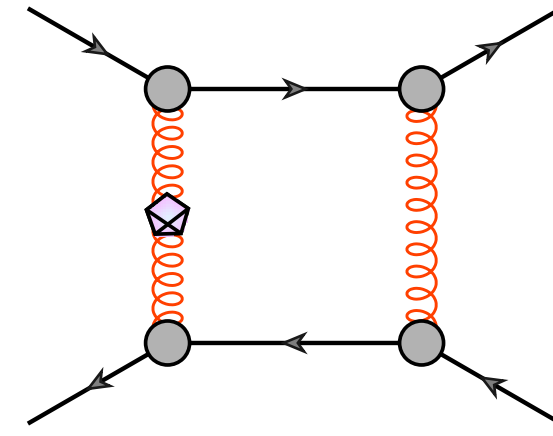


Access and combined use of error estimates from functional QCD & LEFTs

How to: systematic error estimates & the LEGO[®] principle



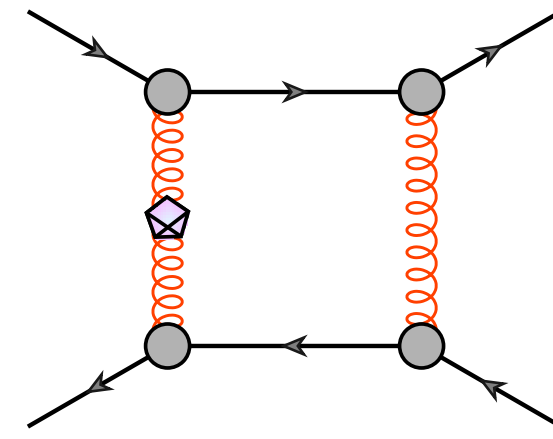
Example: Dissect quark-gluon box



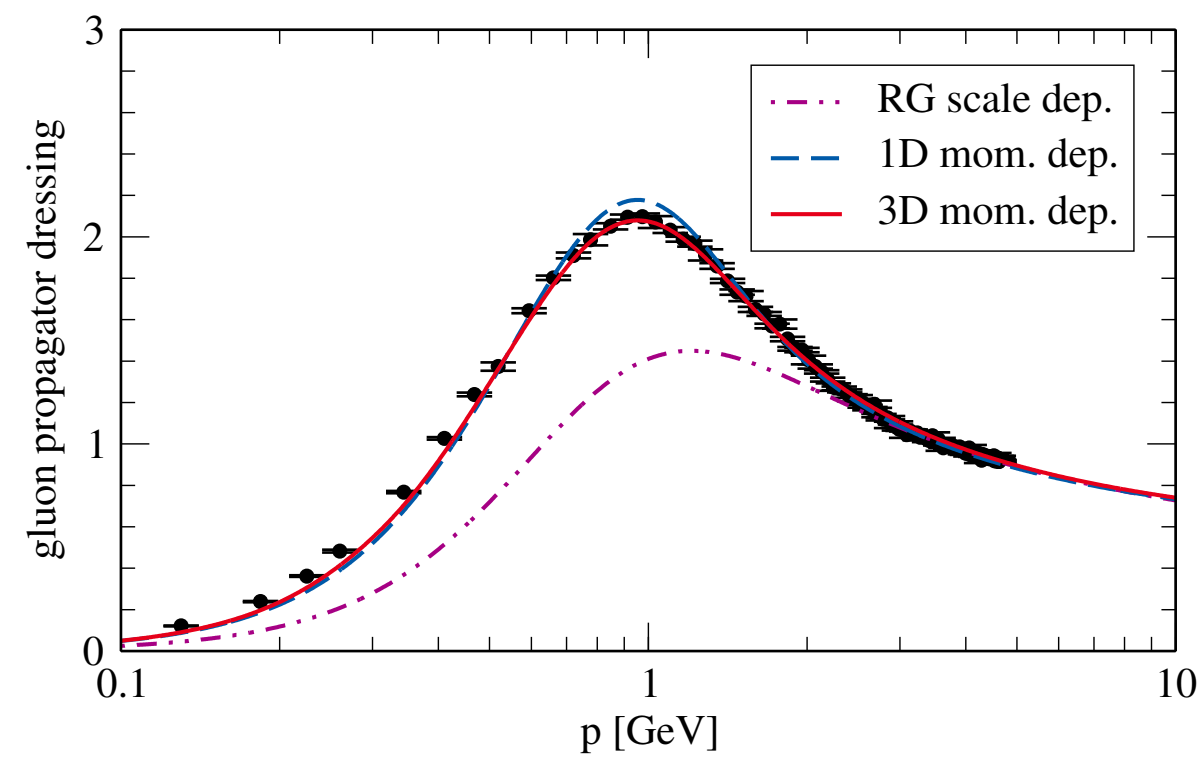
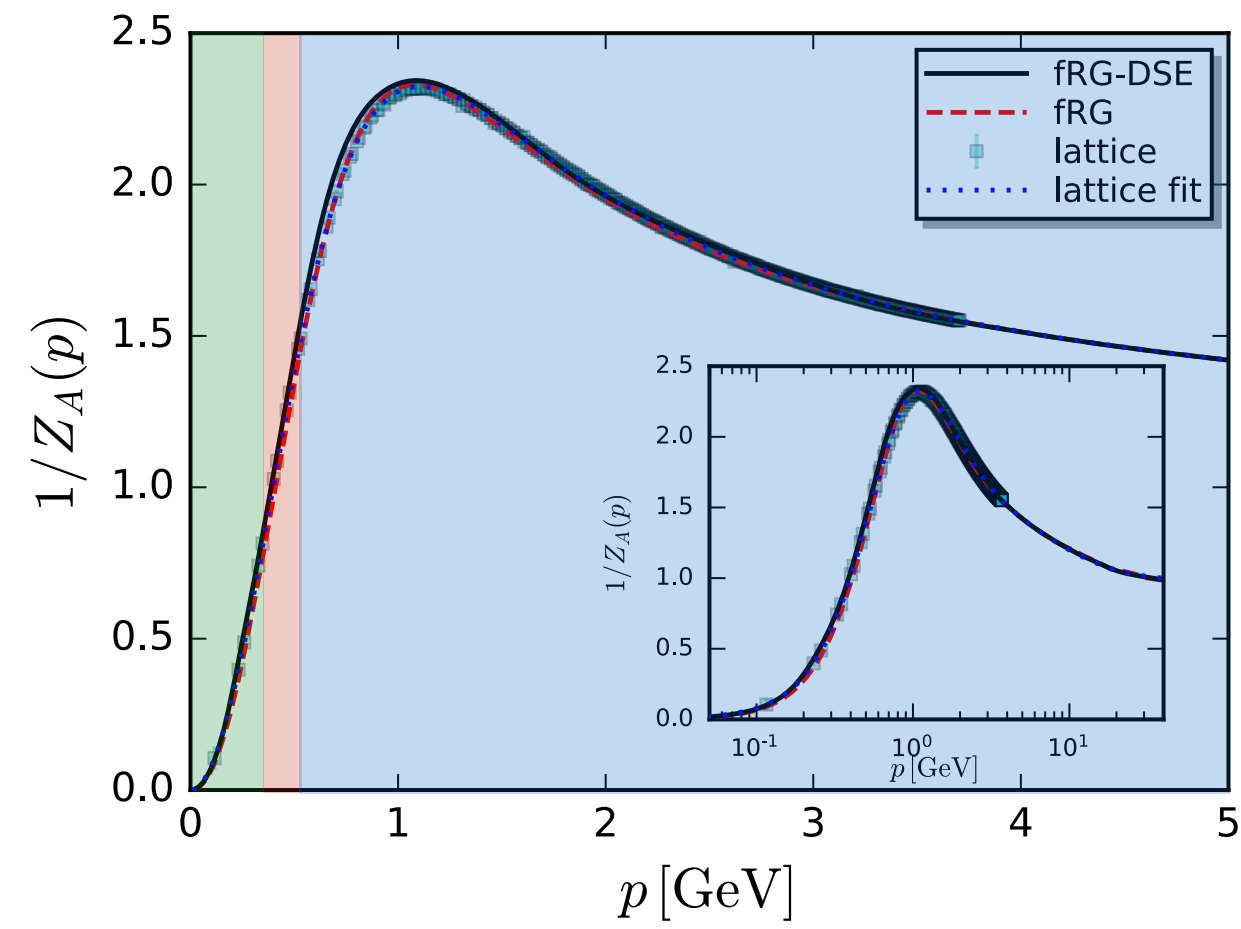
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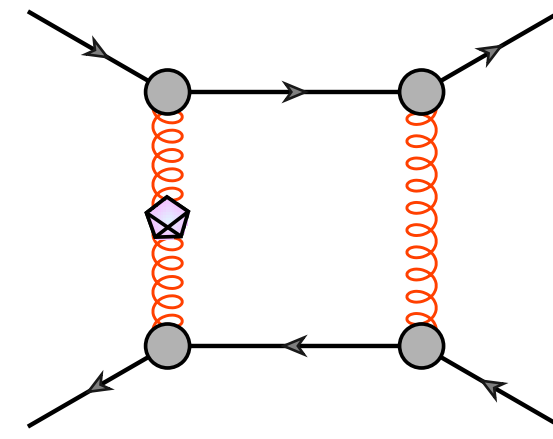
gluon two-point correlator 



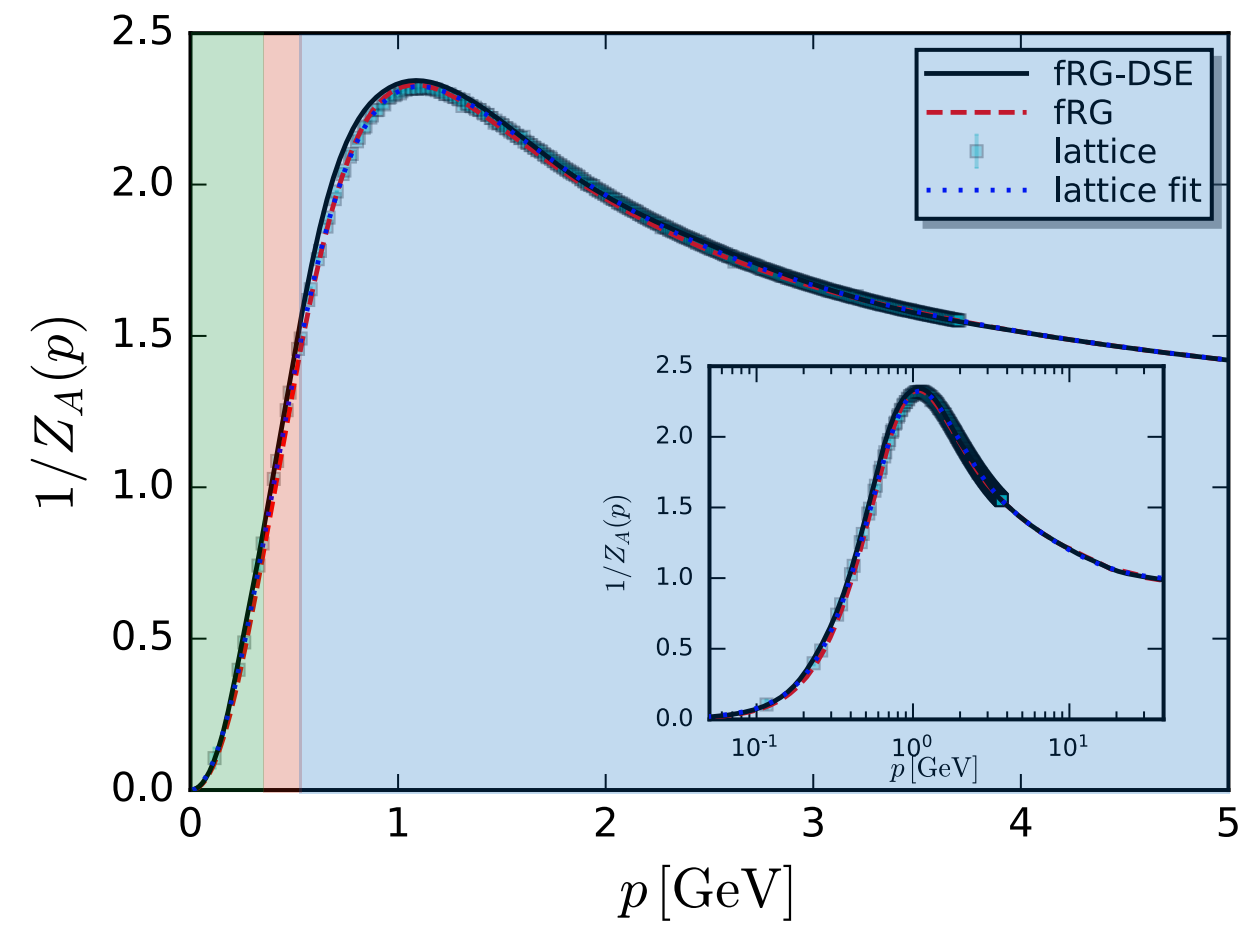
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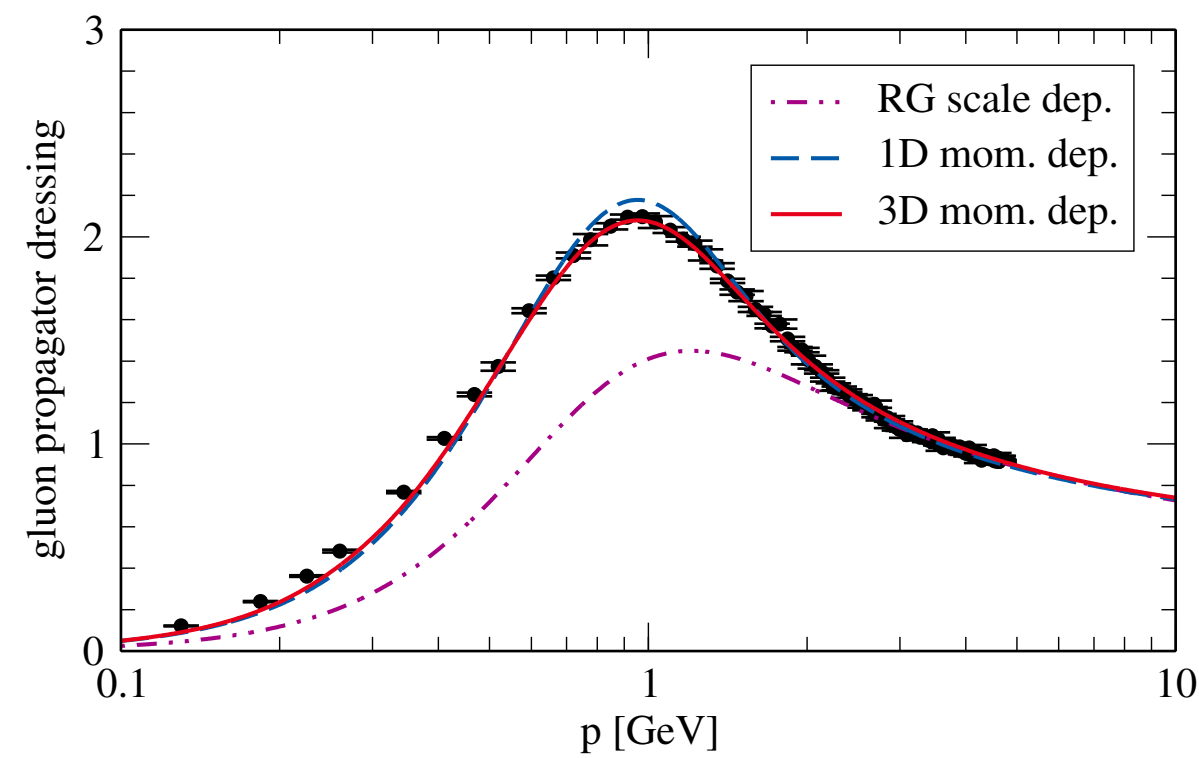
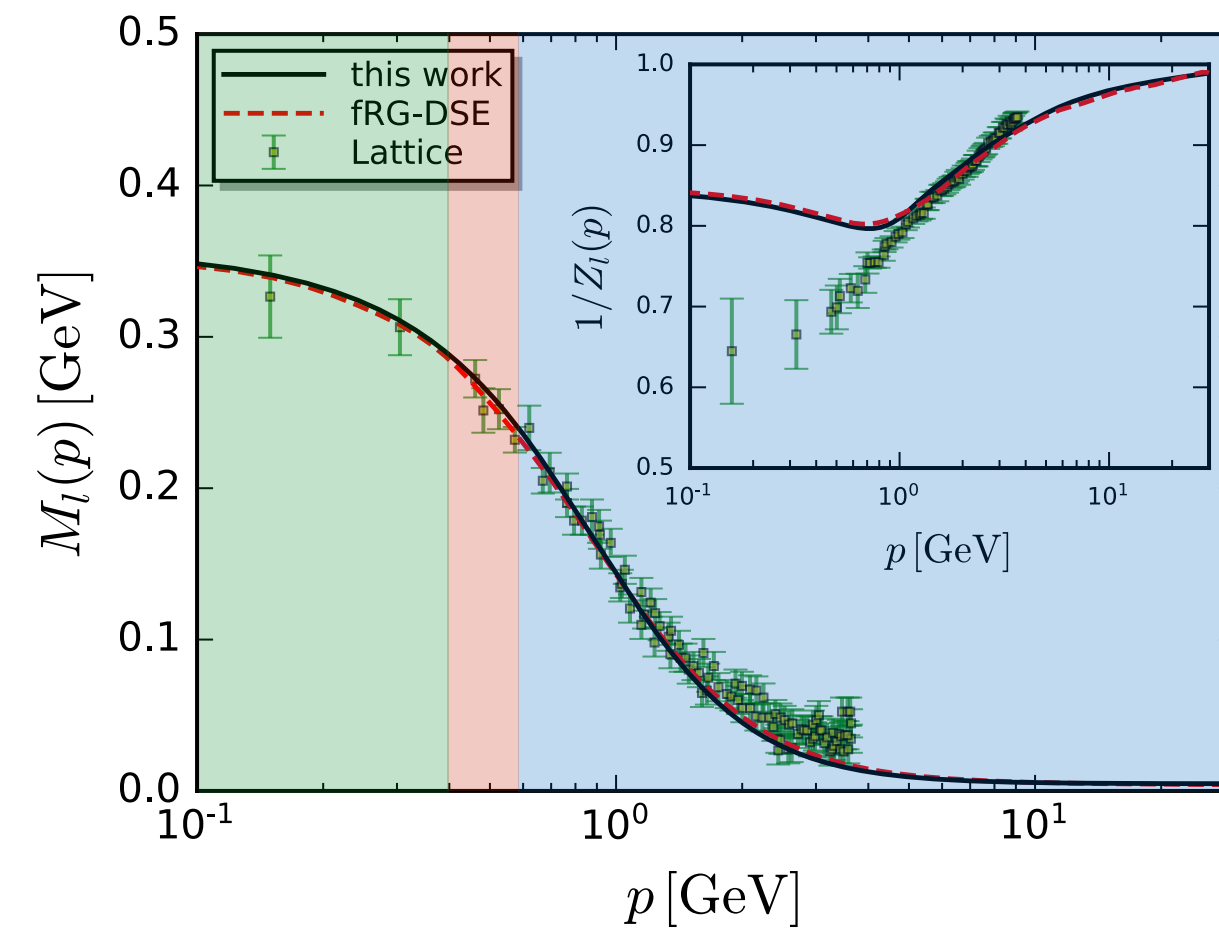
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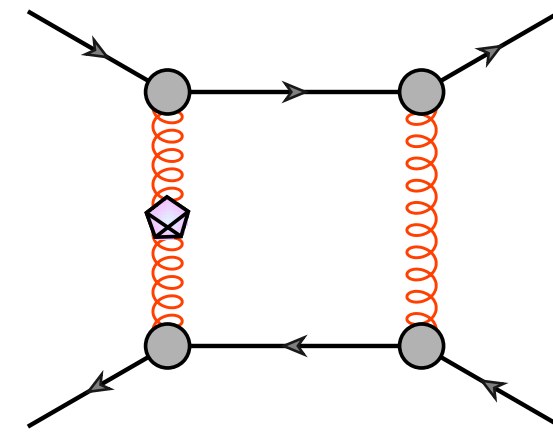
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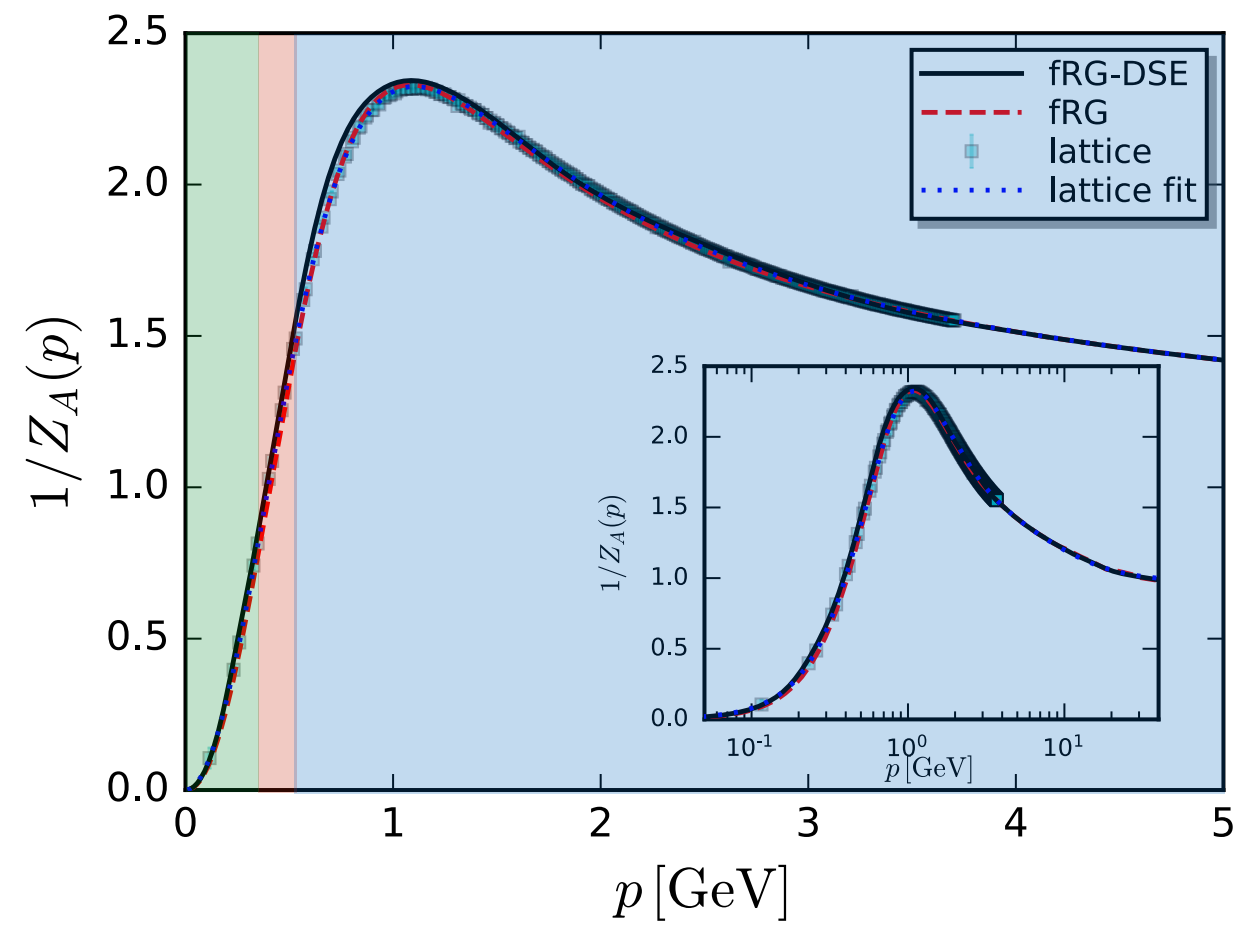


Example: Disect quark-gluon box

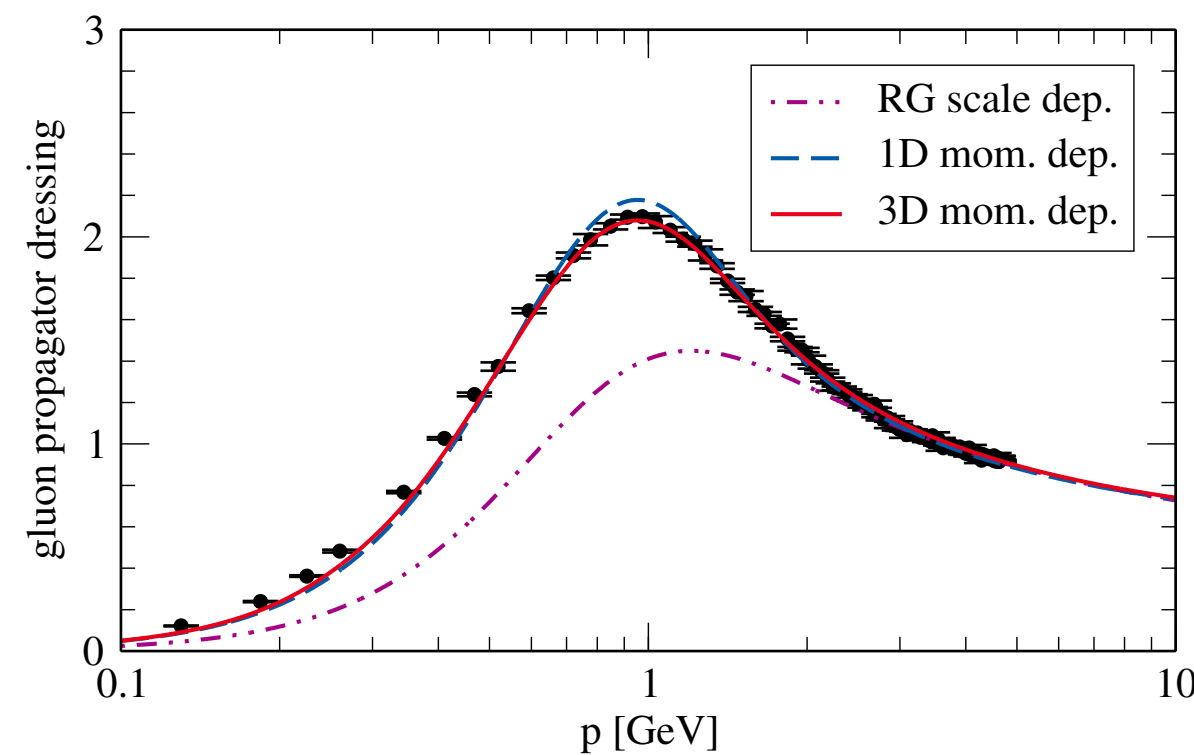
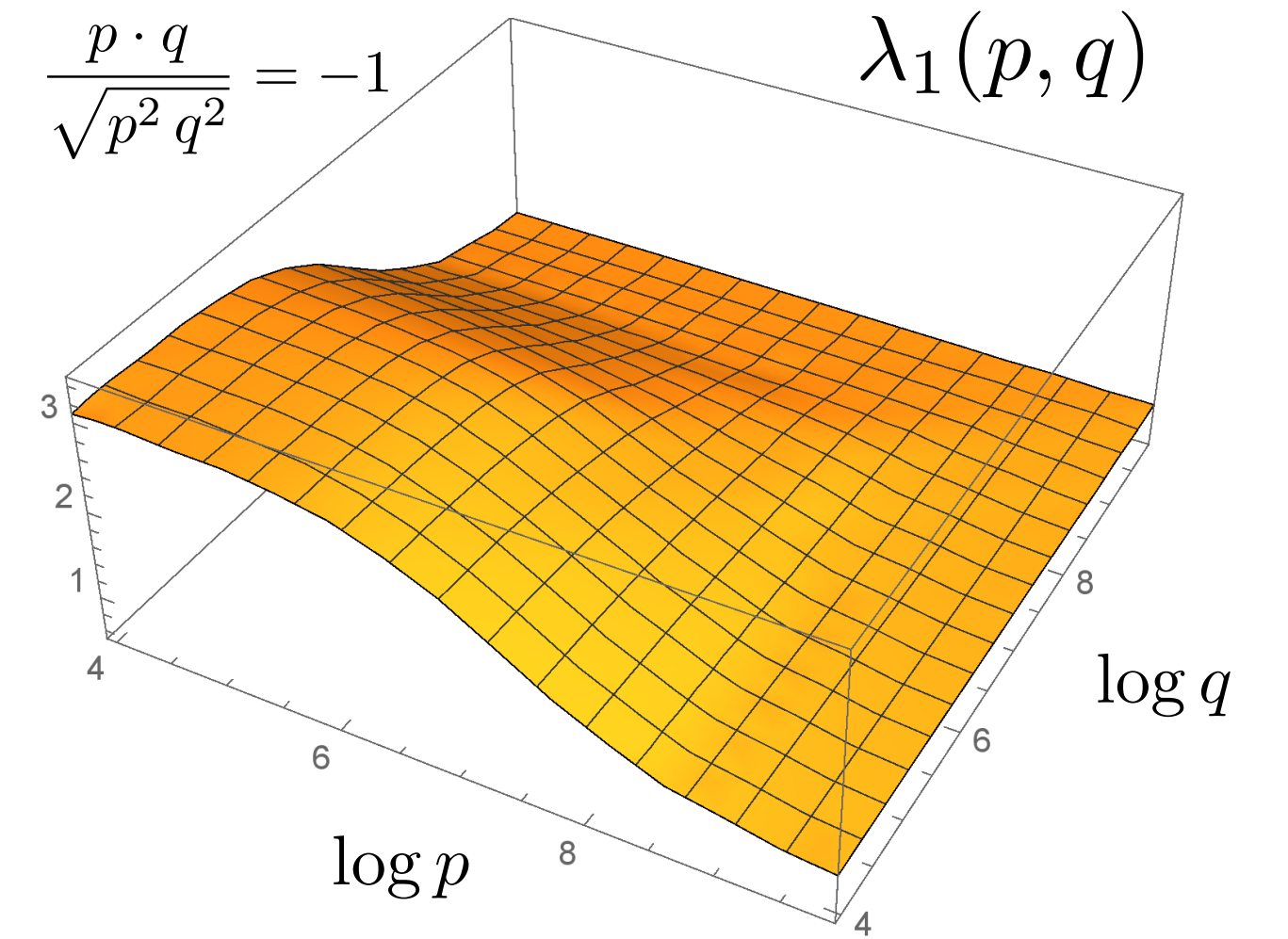
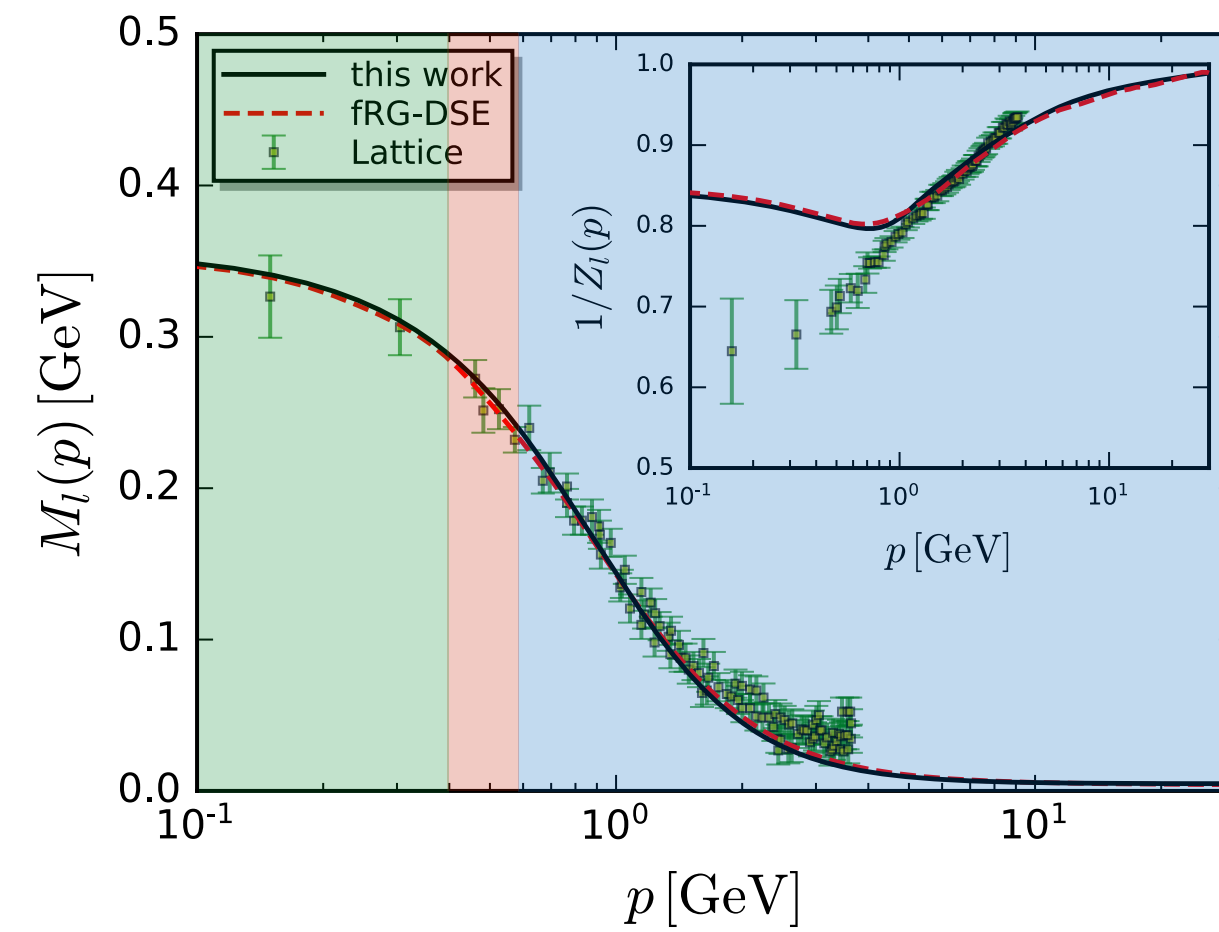


gluon two-point correlator 

quark-gluon scattering 



quark two-point correlator 



$\lambda_1(p, q), \dots, \lambda_8(p, q)$

Relevant: $\lambda_1, \lambda_4, \lambda_7$

How to: direct computations and the minimal point of view

Those are my interpretations,
and if you don't like them....
well, I have others

- Self-consistent truncations to functional relations define analytic functions in μ_B , eg:

$$\partial_t \langle q(x) \bar{q}(y) \rangle_{\mu_B} = \text{Loop} \left[\langle q(x) \bar{q}(y) \rangle_{\mu_B}, \langle q(x) A_\mu(y) \bar{q}(z) \rangle_{\mu_B}, \dots ; \mu_B \right]$$

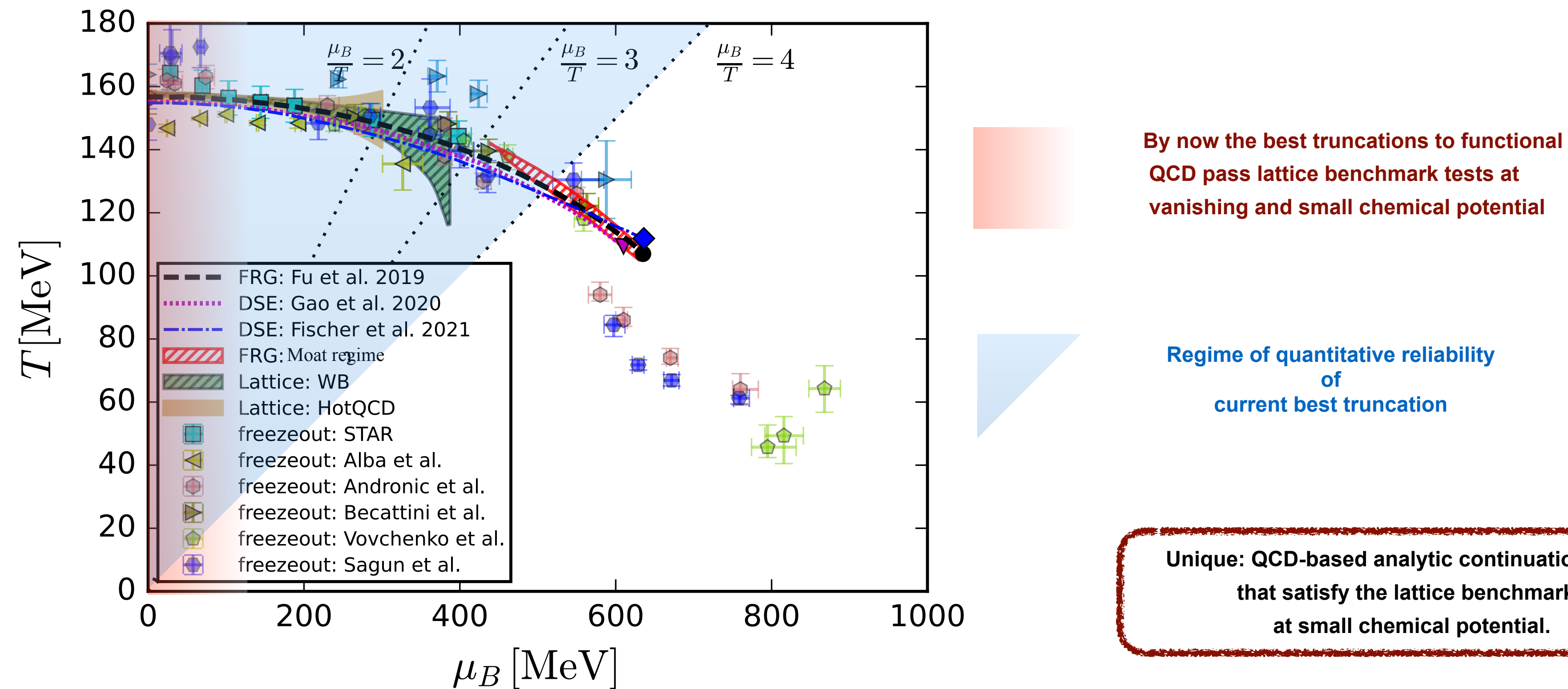
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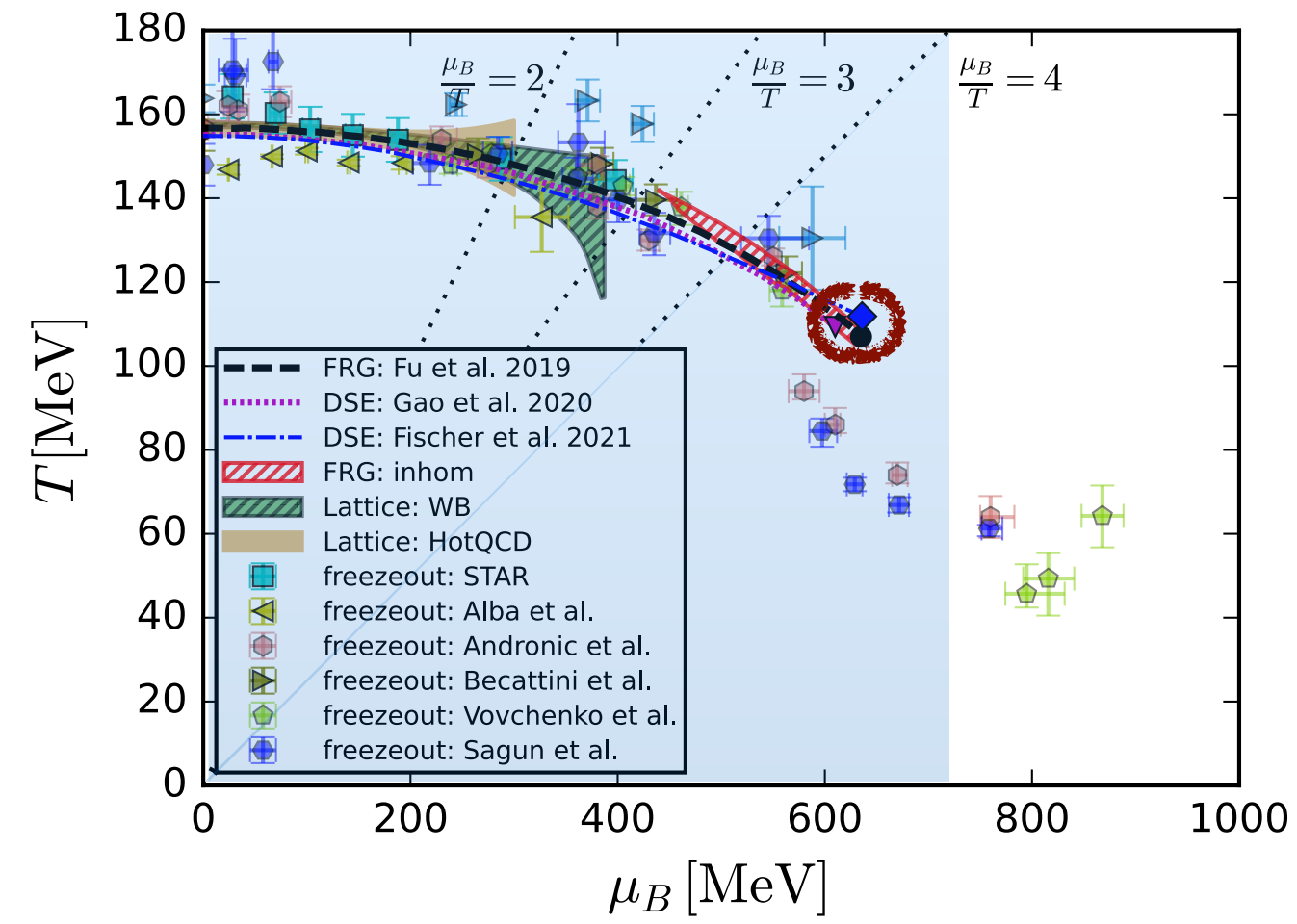
- Consequences for functional QCD predictions at finite density



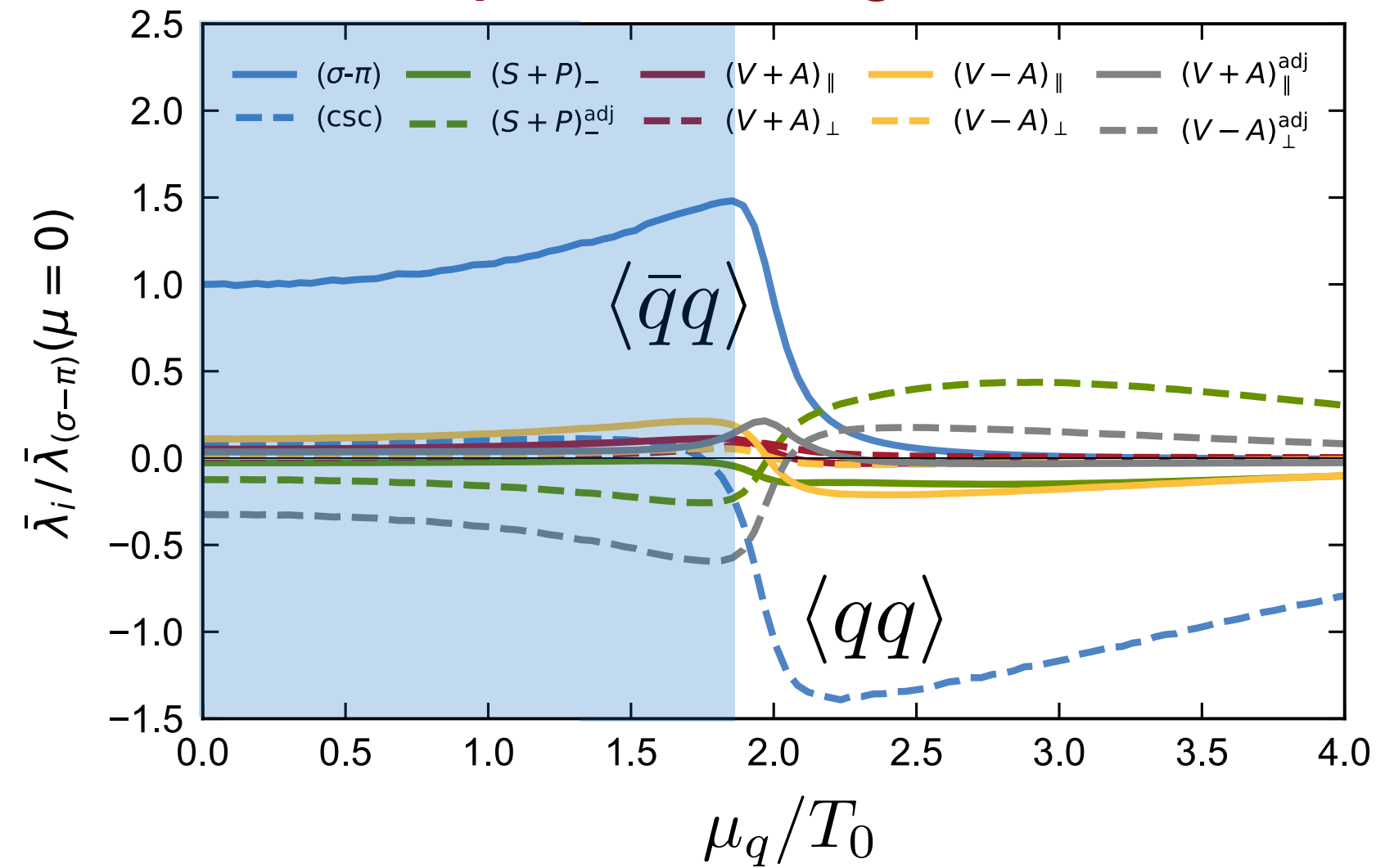
Great opportunity for a combined analysis of high density QCD (Exp. data + lattice QCD + functional QCD + LEFTs)

Phase structure from functional QCD: Predictions & estimates

Predictions & estimates



Four-quark scattering channels

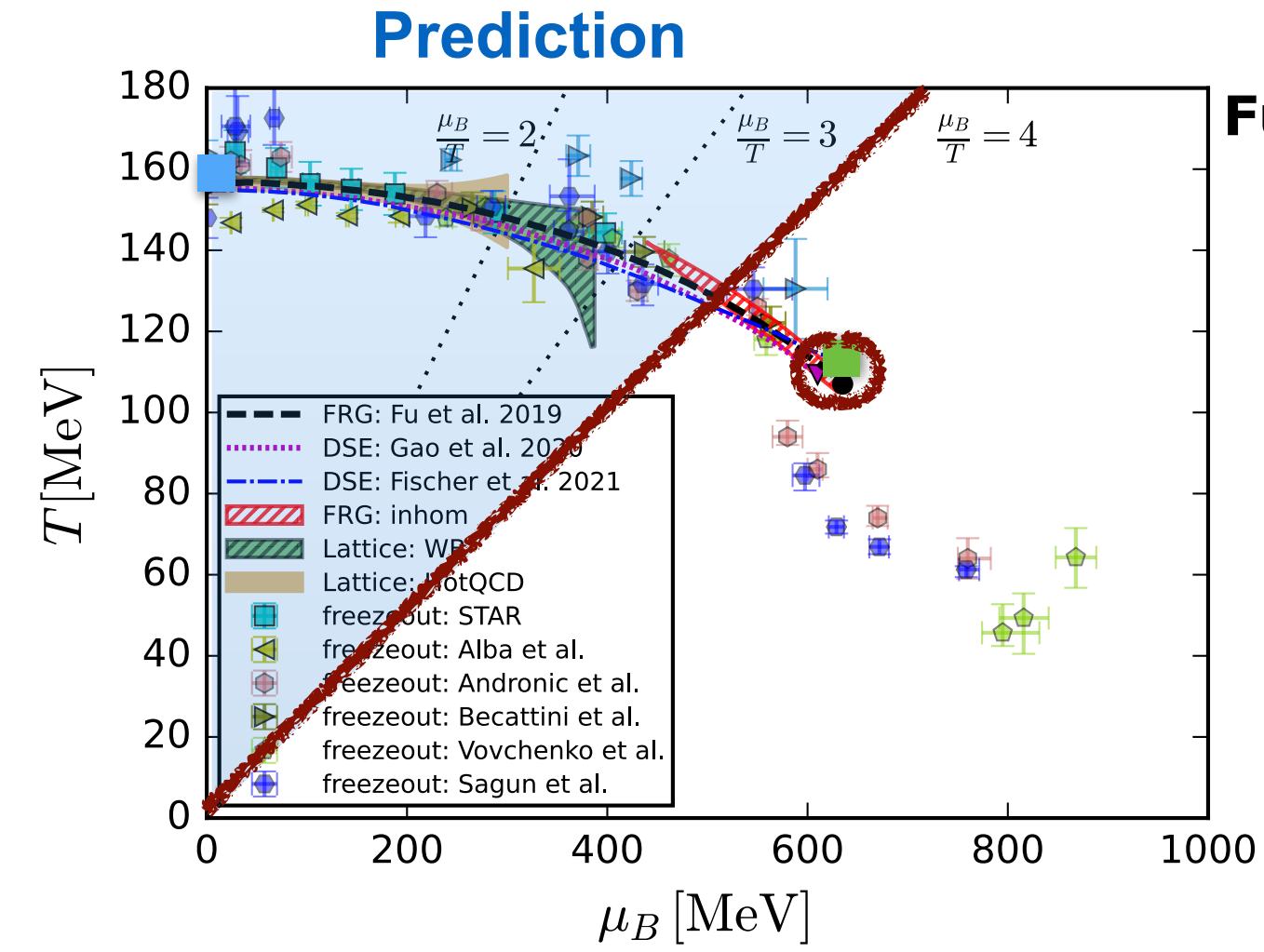


Dominance of scalar-pseudoscalar fluctuations
Pions & sigma mode

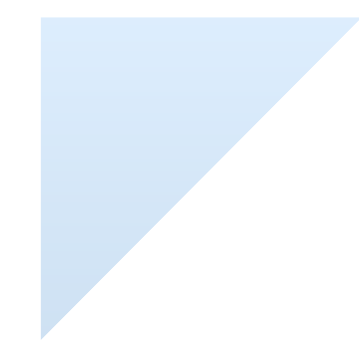
Predictions & estimates

Moat regime

Pisarski, Rennecke, PRL 127 (2021) 152302

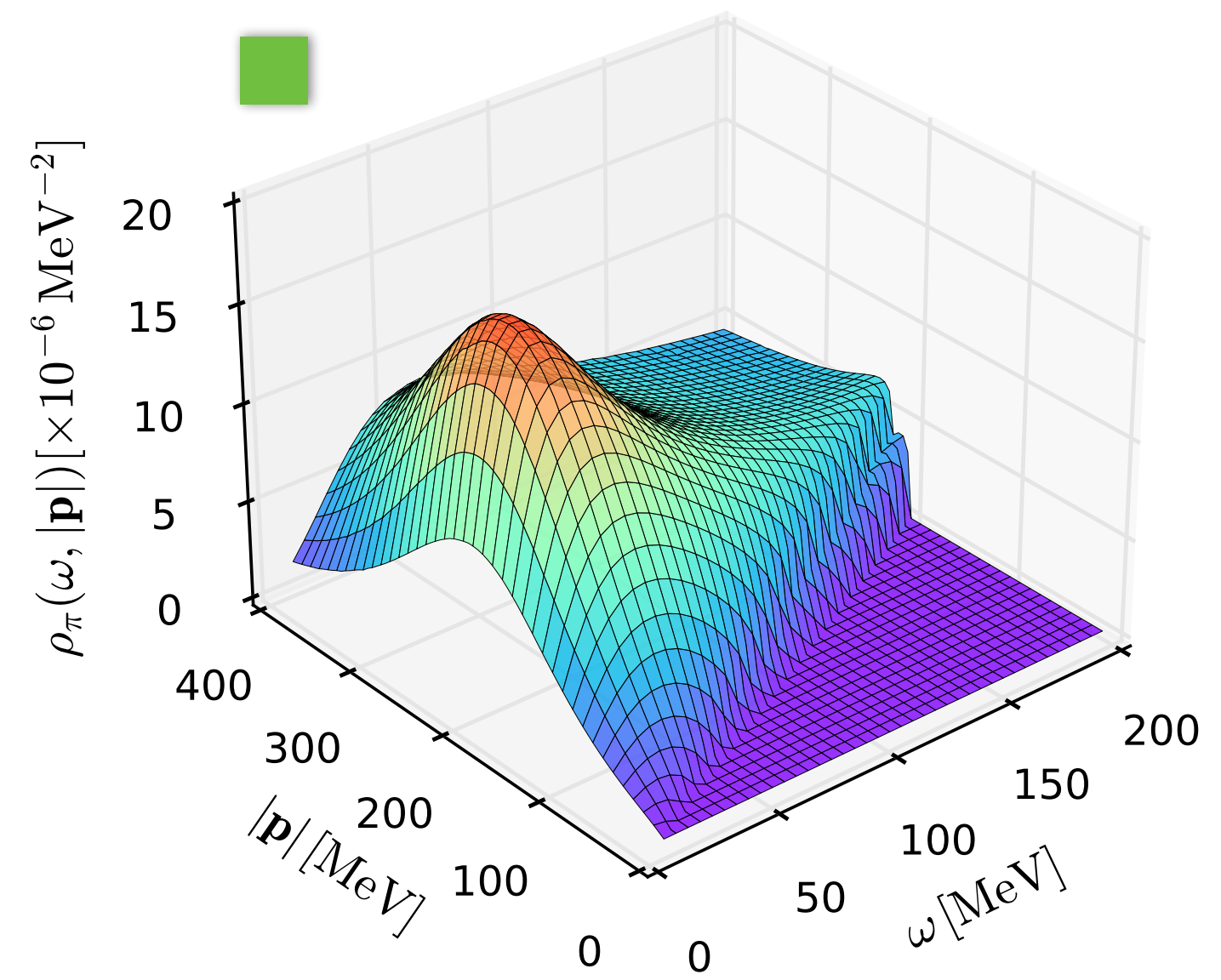


Fu, JMP, Rennecke, PRD 101 (2020) 054032

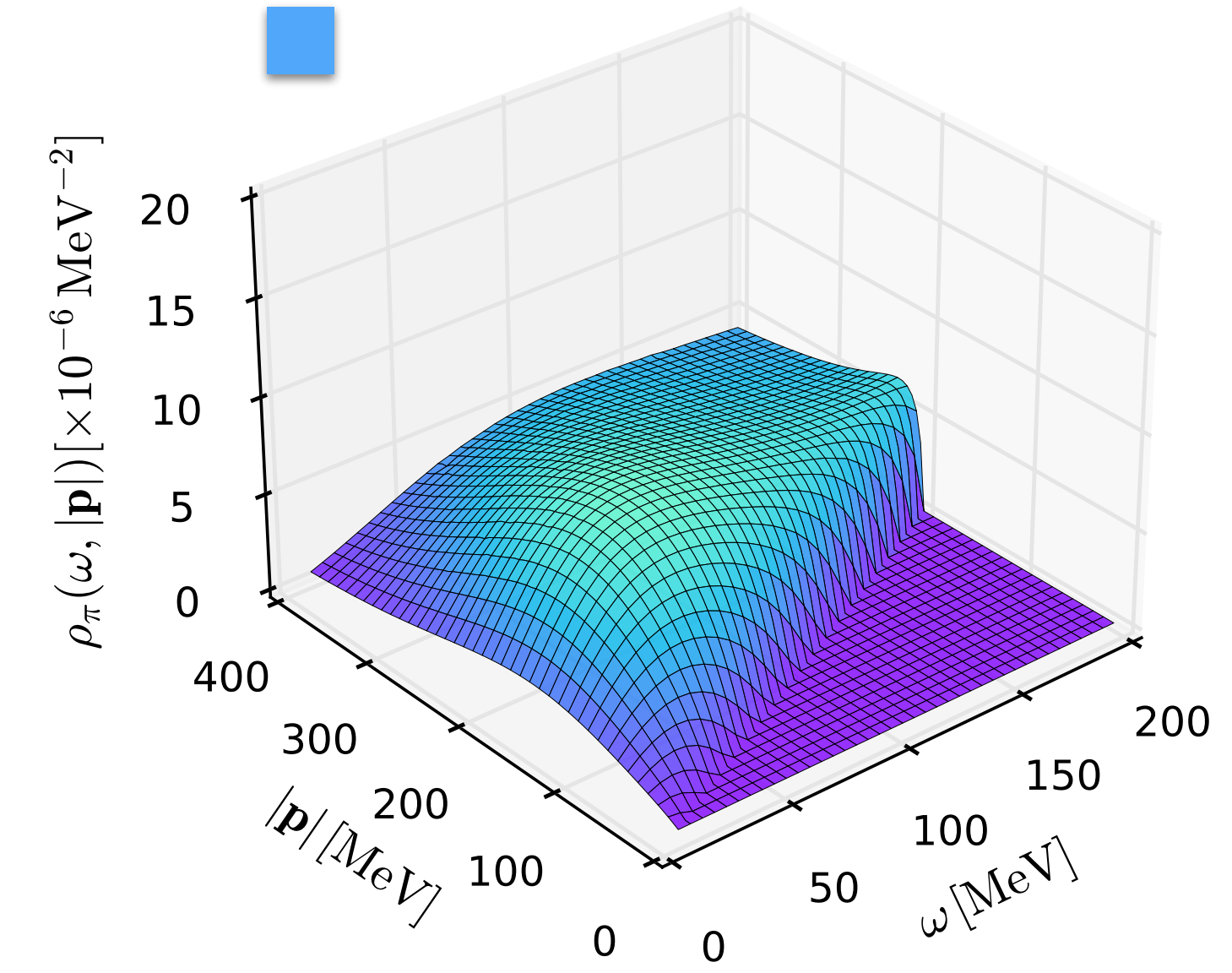


**Regime of quantitative reliability
of
current best truncation**

T=114 MeV & $\mu_B=630$ MeV



T=160 MeV & $\mu_B=0$ MeV



Pion spectral functions

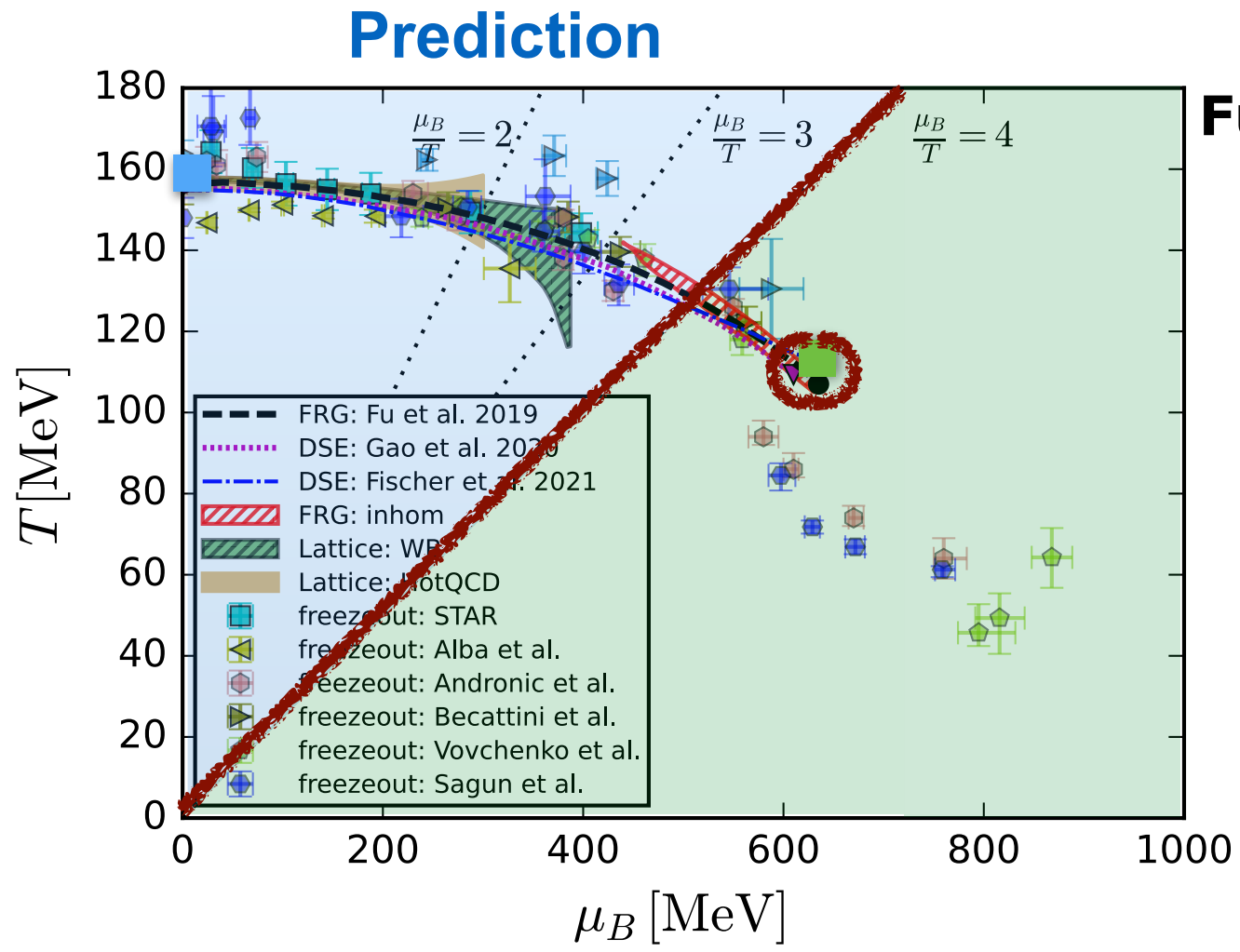
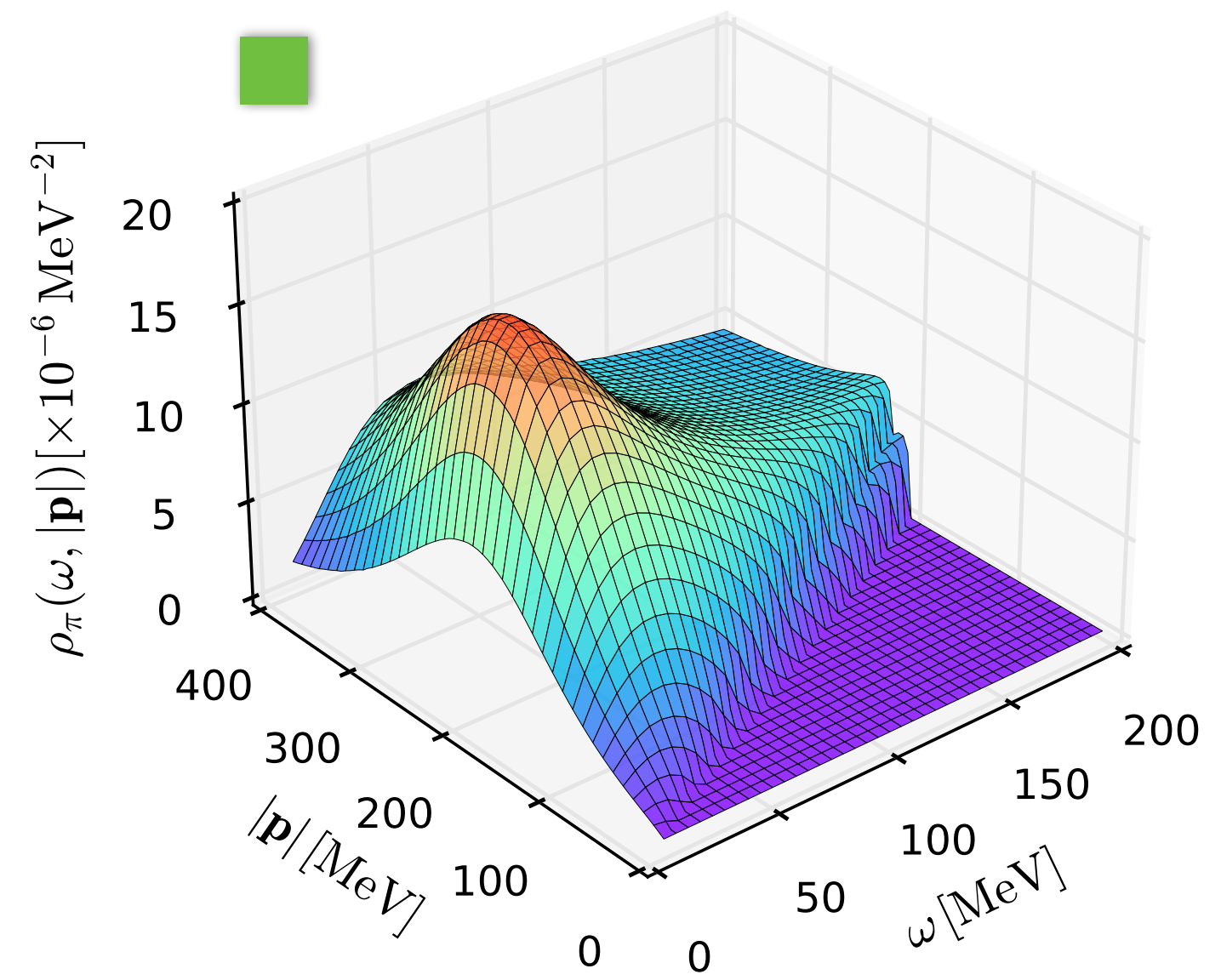
Fu, JMP, Pisarski, Rennecke, Wen, Yin, in prep

Predictions & estimates

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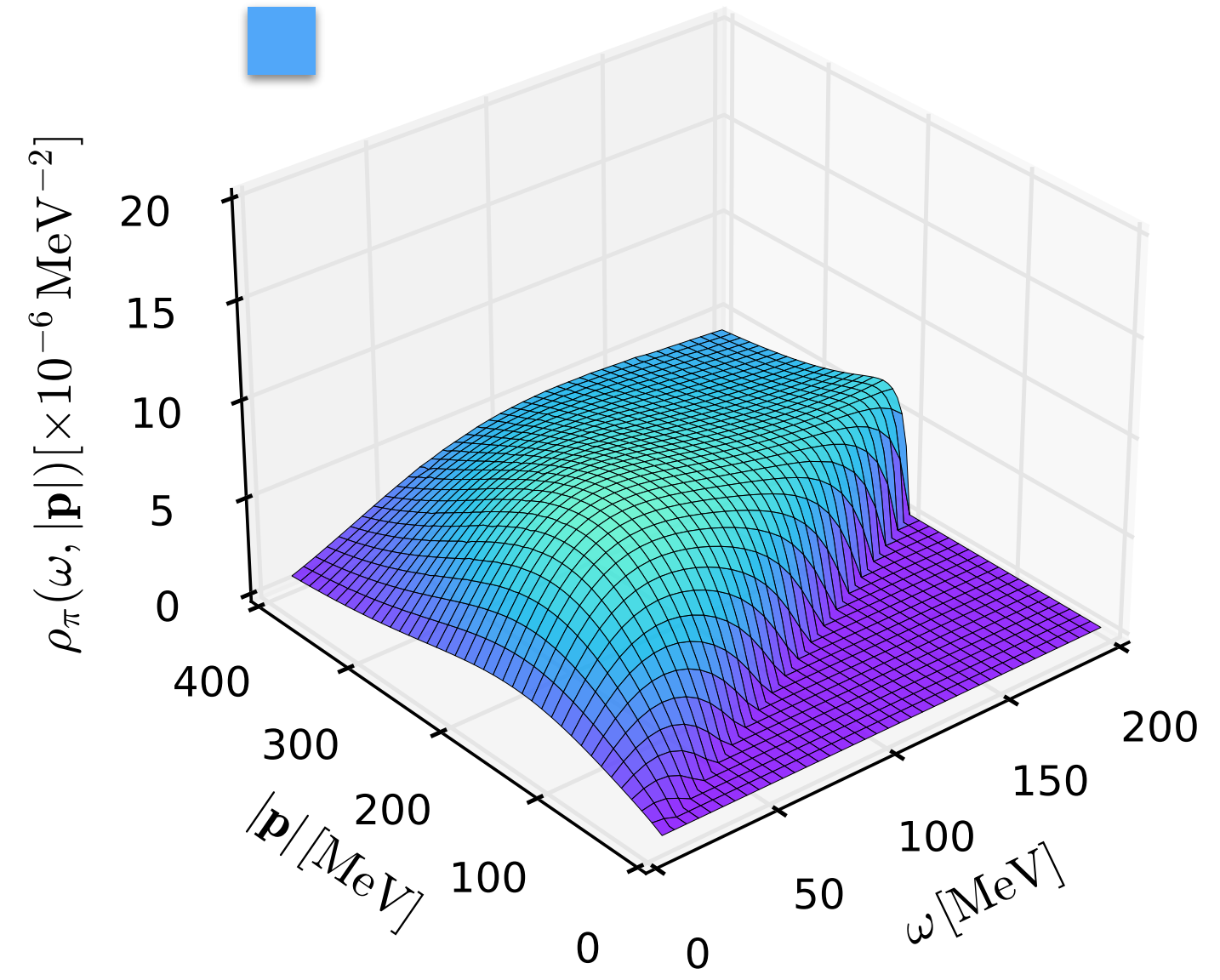
Estimate

Moat regime is not captured quantitatively

Pion spectral functions

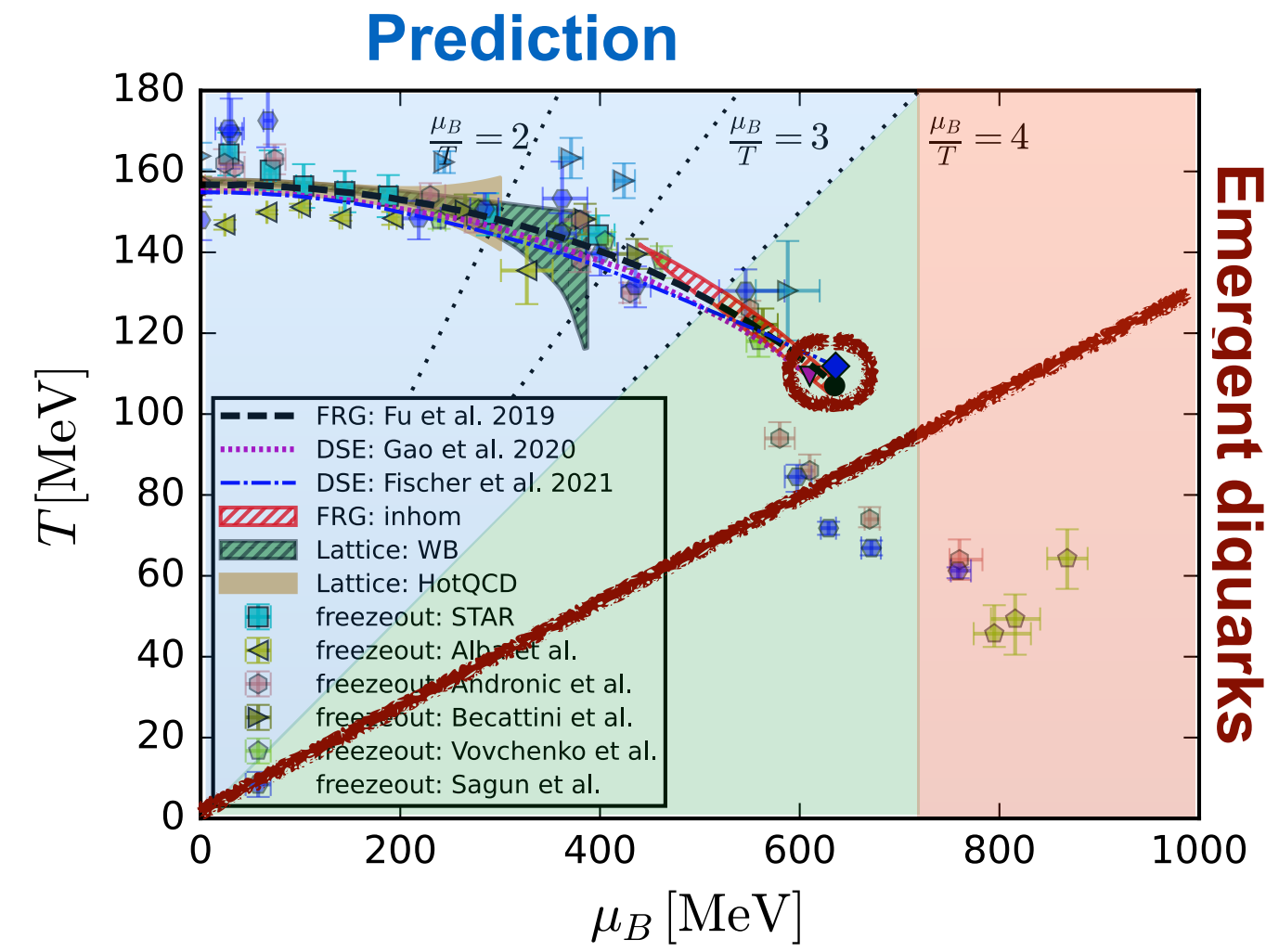
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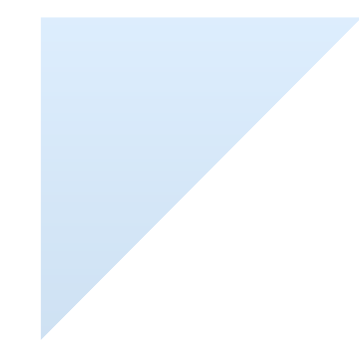


Predictions & estimates

Emergent diquarks

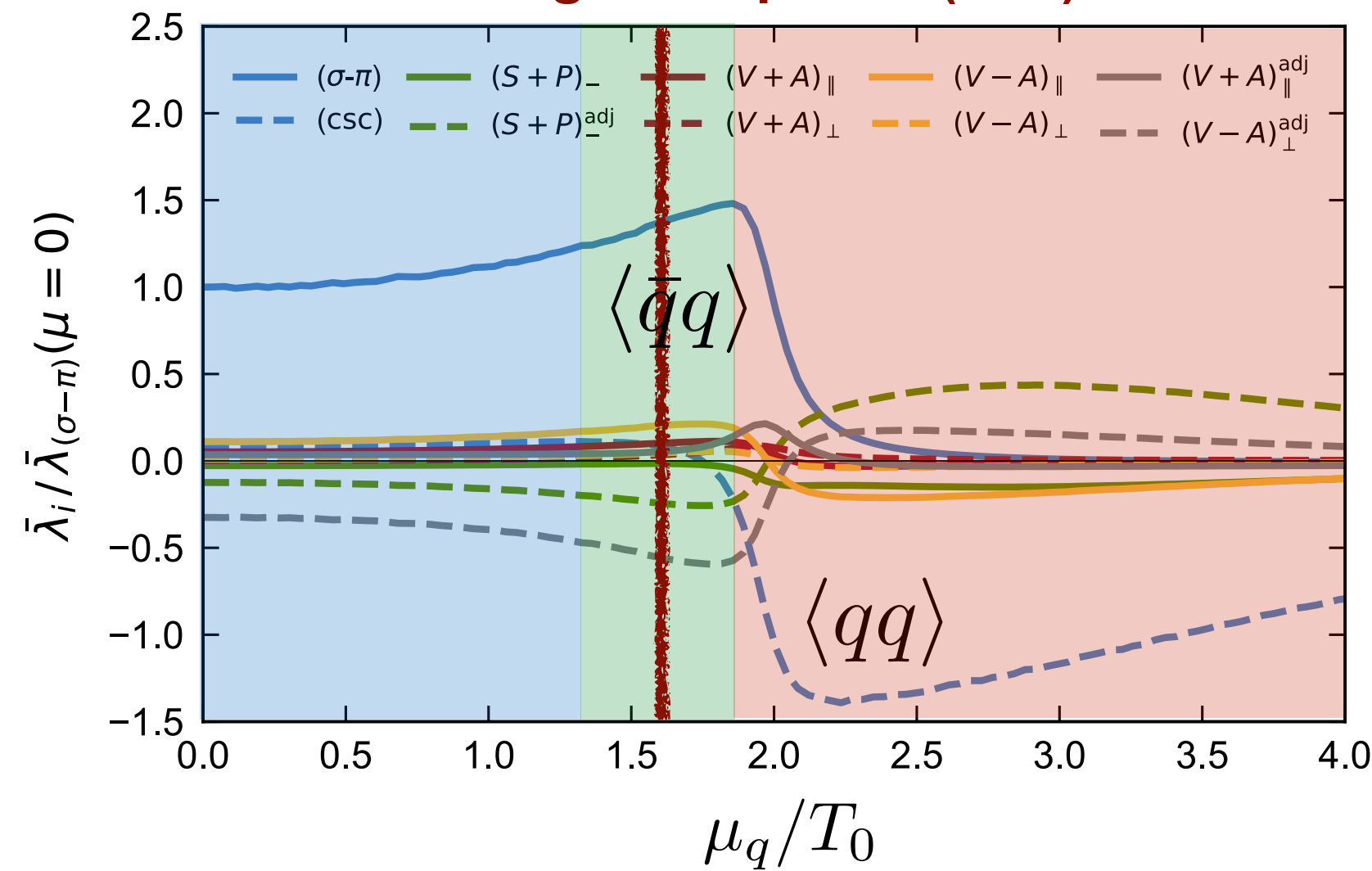


Estimate



**Regime of quantitative reliability
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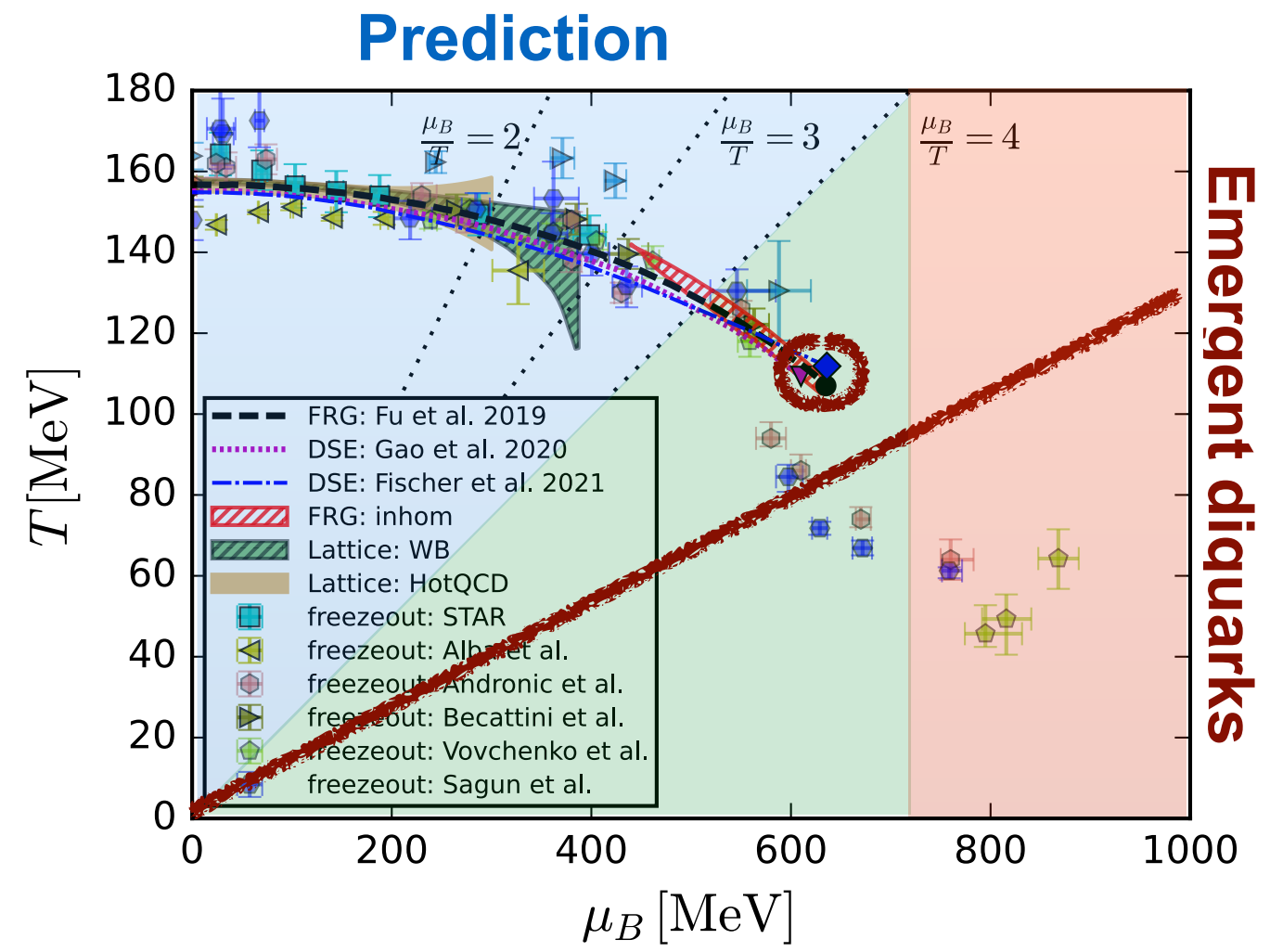
Emergent diquarks (fRG)



**Emergent diquarks are not captured
by extrapolations**

Predictions & estimates

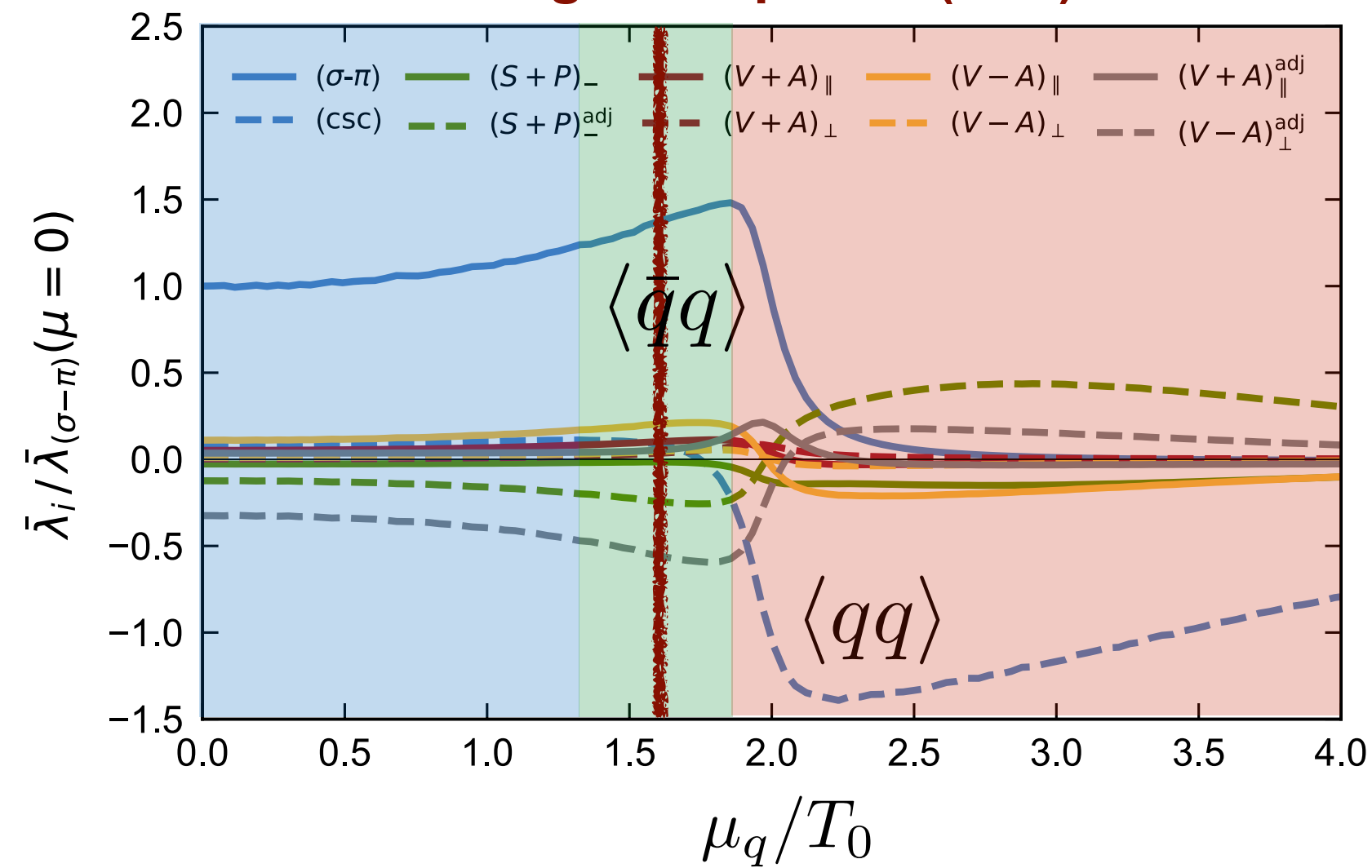
Emergent diquarks



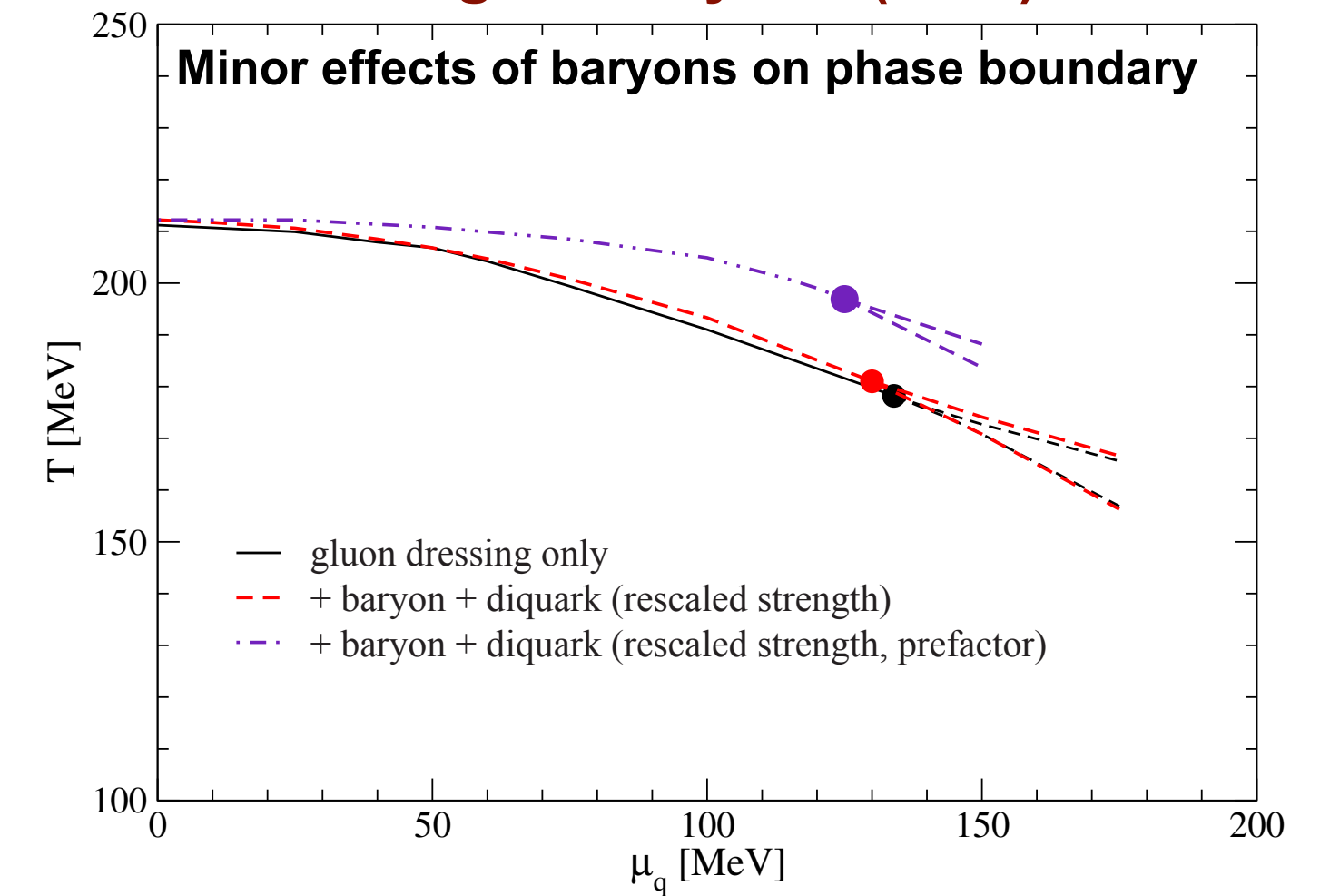
Regime of quantitative reliability of current best truncation

Estimate

Emergent diquarks (fRG)



Emergent baryons (DSE)

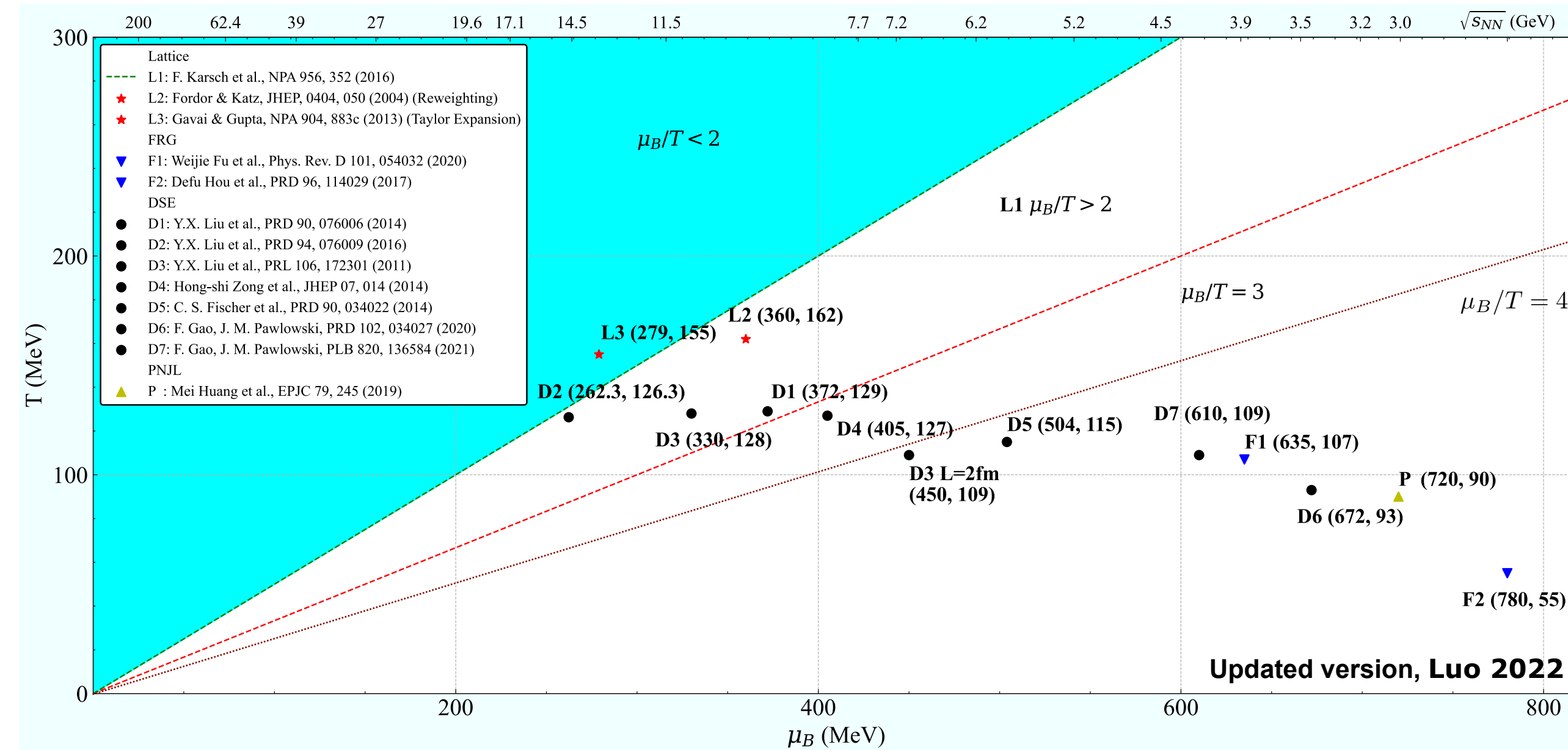


Predictions, estimates & extrapolations and how to judge them



Location of CP : Theoretical Prediction

Preliminary collection from Lattice, DSE, FRG and PNJL (2004-2020)



Large uncertainties for the estimation of CP location.

Disclaimer

Most functional computations (LEFT or QCD) have not been set-up for CEP-predictions!

Lack of predictive power for CEP-predictions is no quality measure!

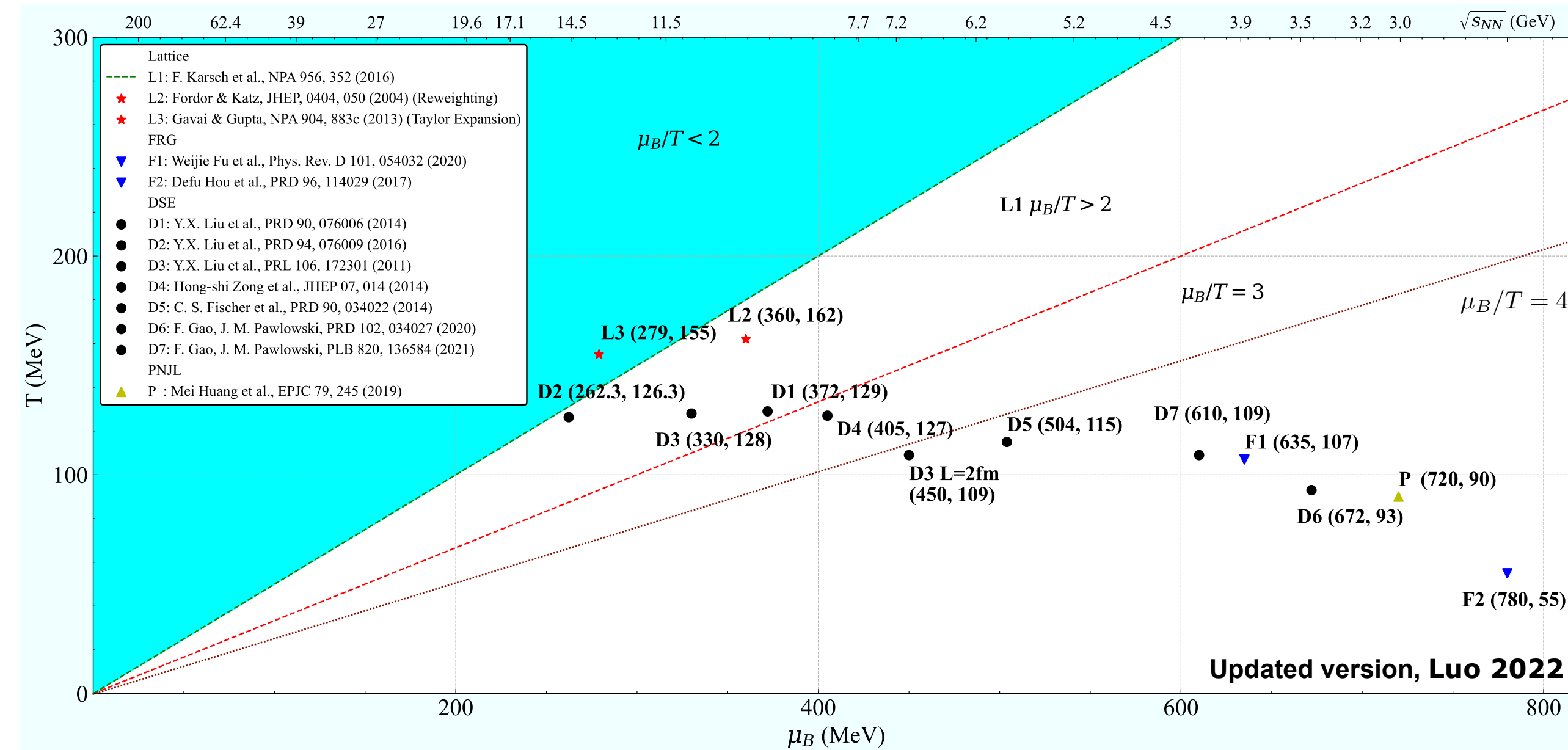
CEP is standing for 'regime with new physics'

Predictions, estimates & extrapolations and how to judge them



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Common folklore since ~2004



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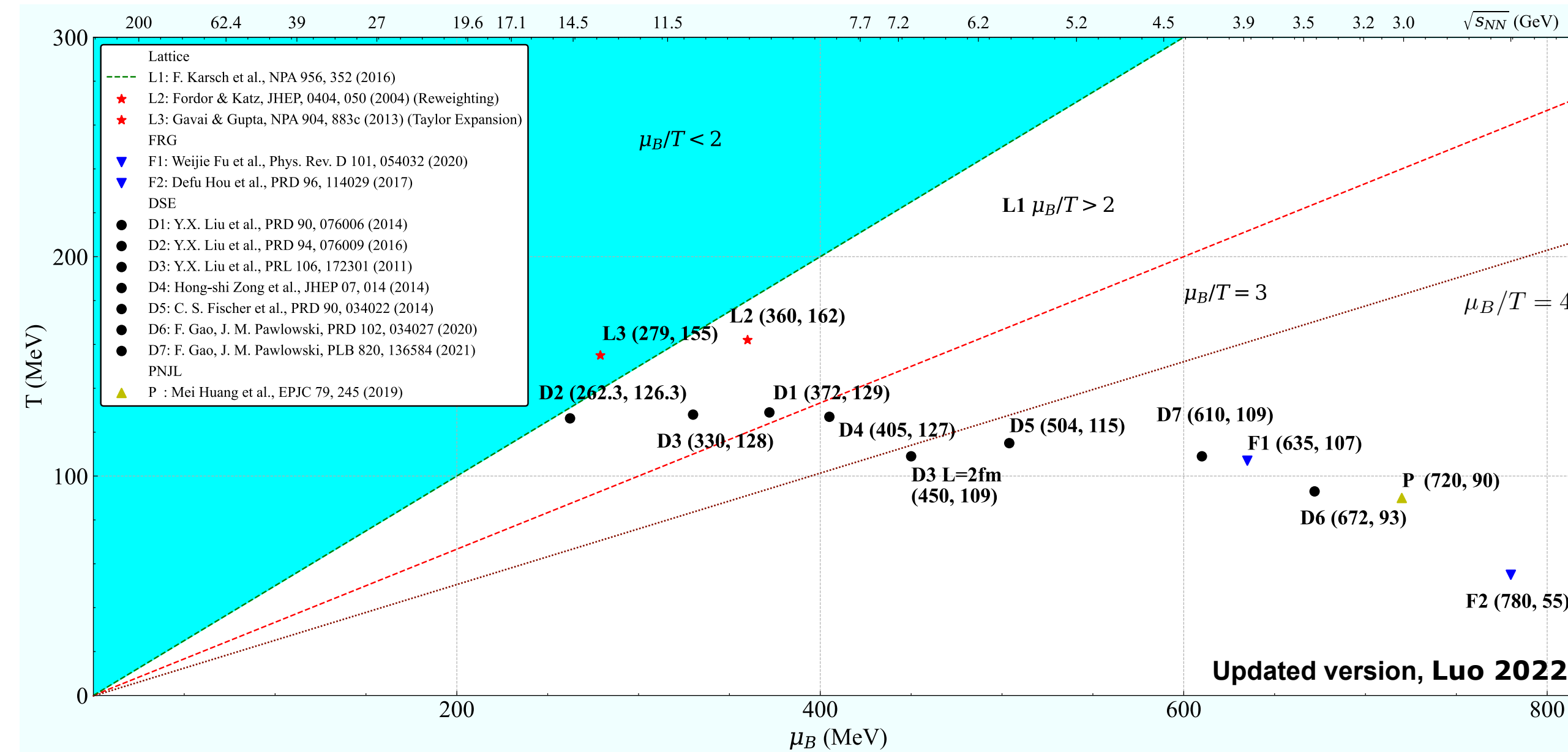
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Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

(i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)

(ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$

(iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

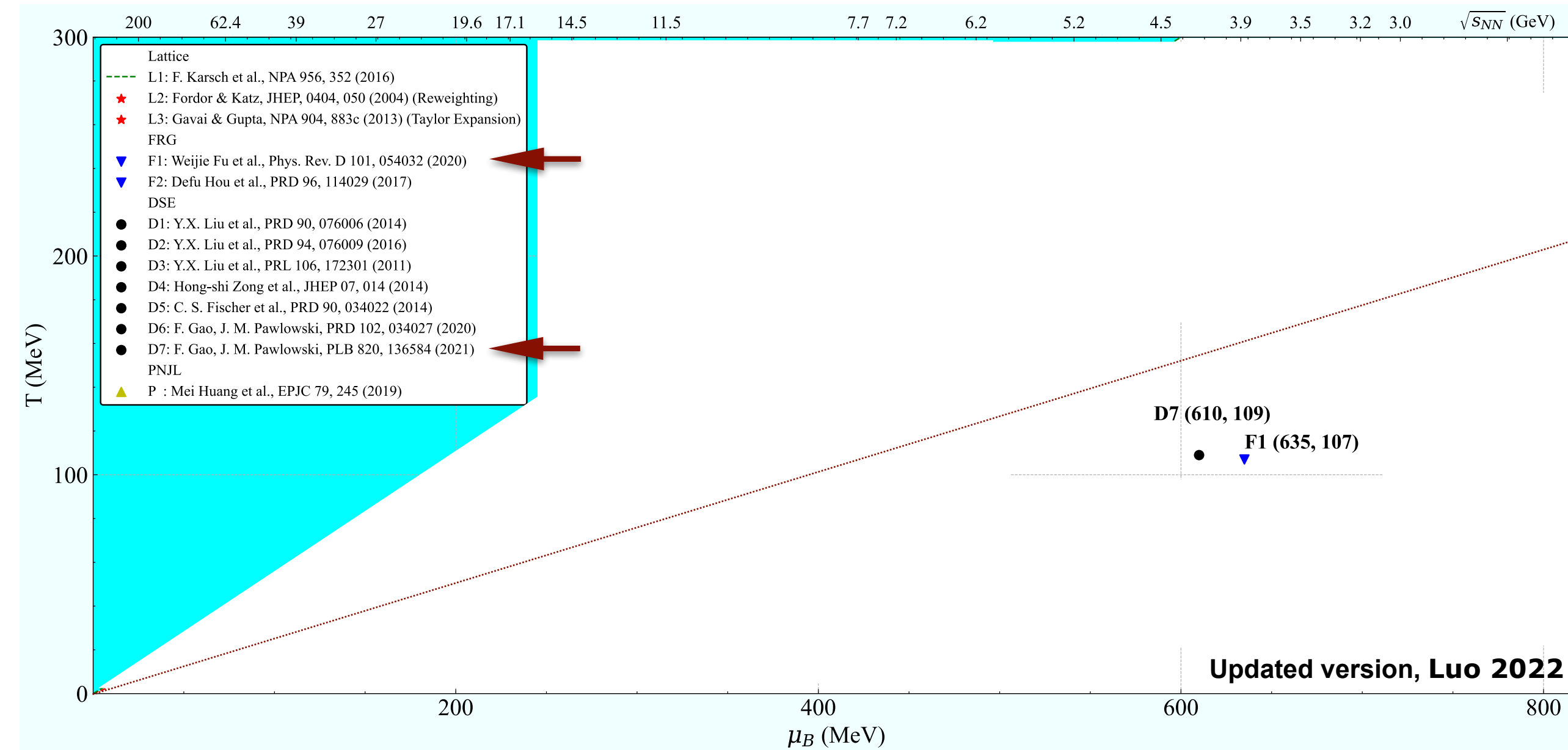
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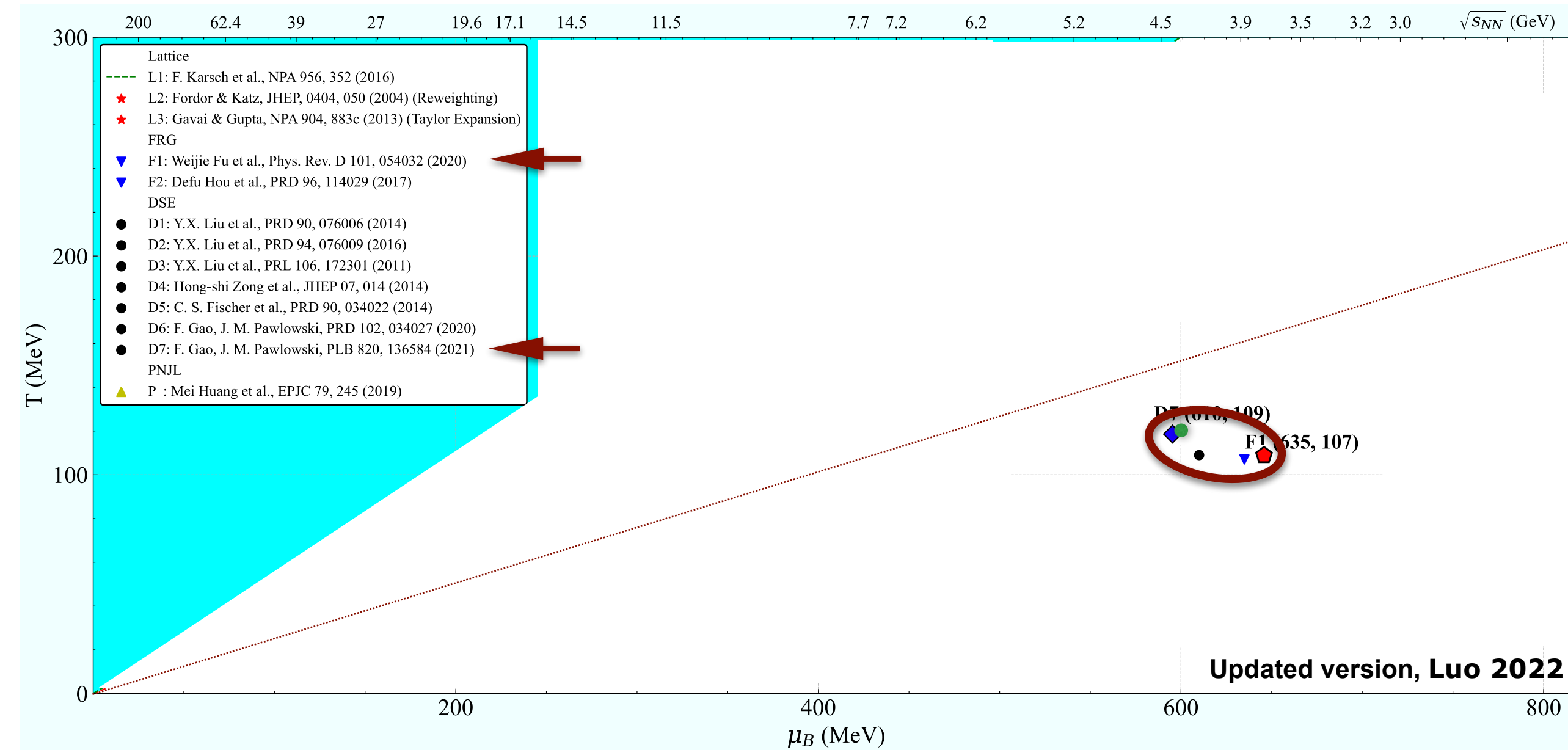


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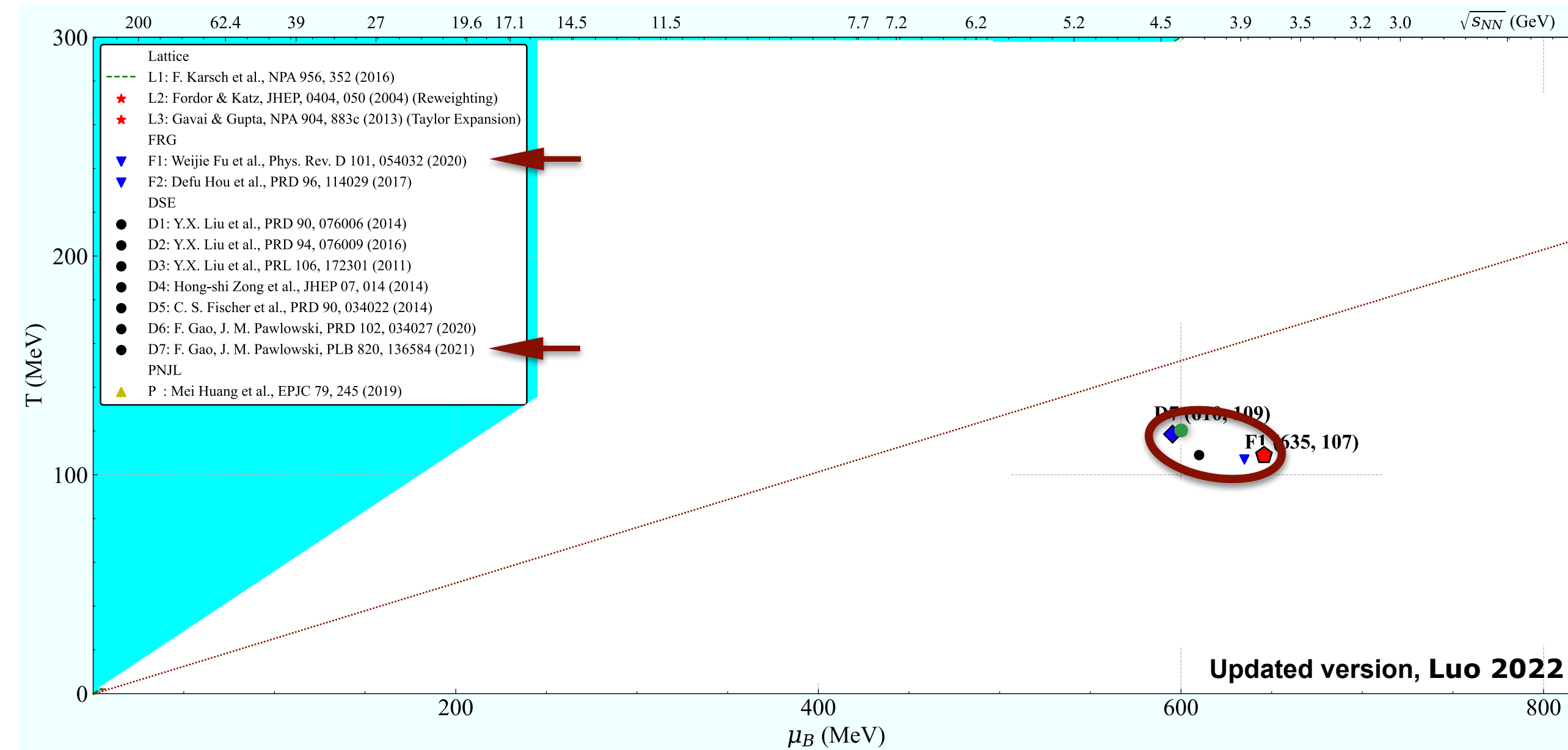


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Still small uncertainties for the estimation of **CP location / Onset of new phases**

Remove CEP-predictions

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Predictions, estimates & extrapolations and how to judge them



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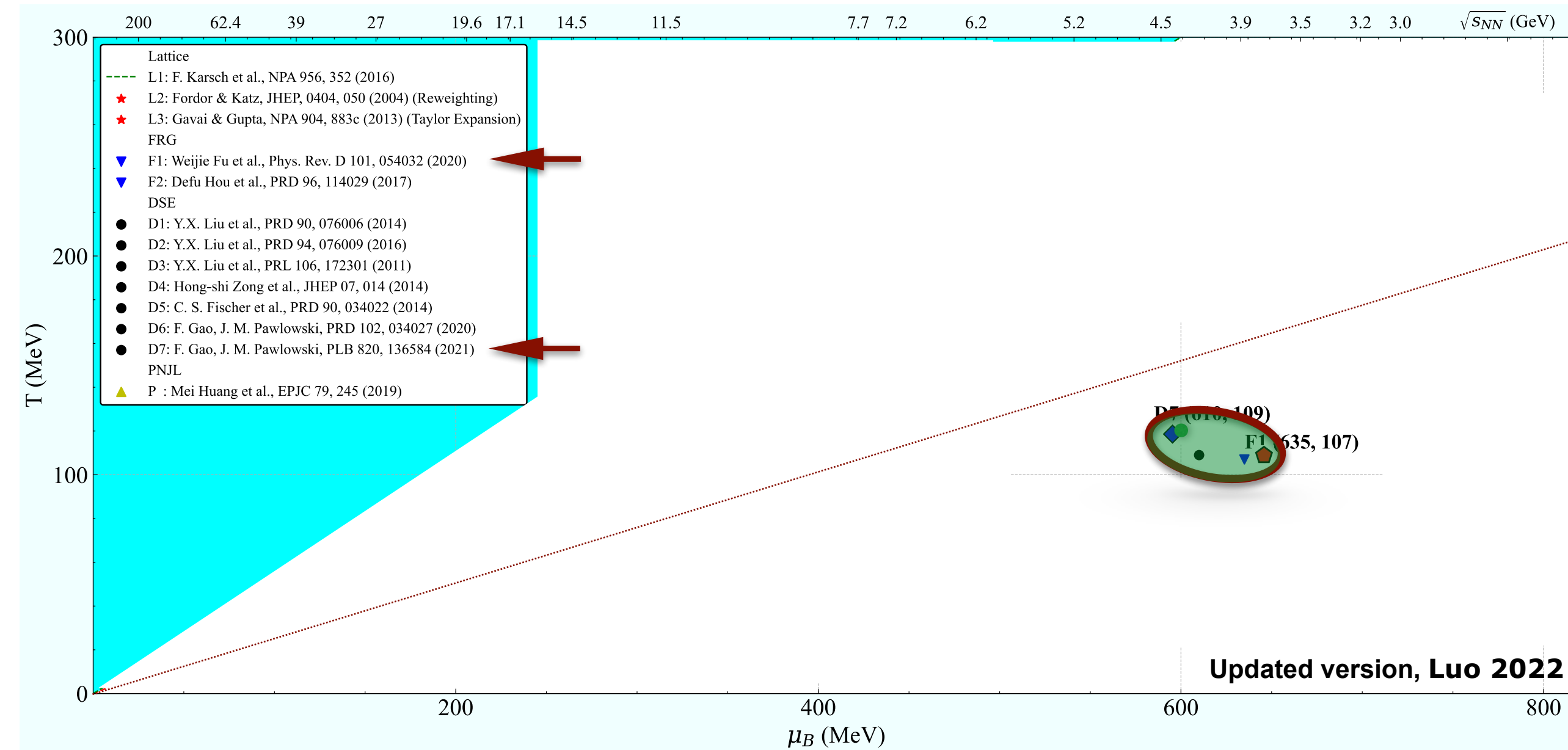
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Extrapolations

- Lattice extrapolations: Basar, PRC 110 (2024) 015203
 Bielefeld-Parma, arXiv:2405.10196
 ⋮
- Holographic models: Hippert, Grefa, Manning, Noronha,
 Noronha-Hostler, Portillo Vazquez, Ratti,
 Rougemont, Trujillo, arXiv: 2309.00579
 ⋮



Still small uncertainties for the estimation of **CP location / Onset of new phases**

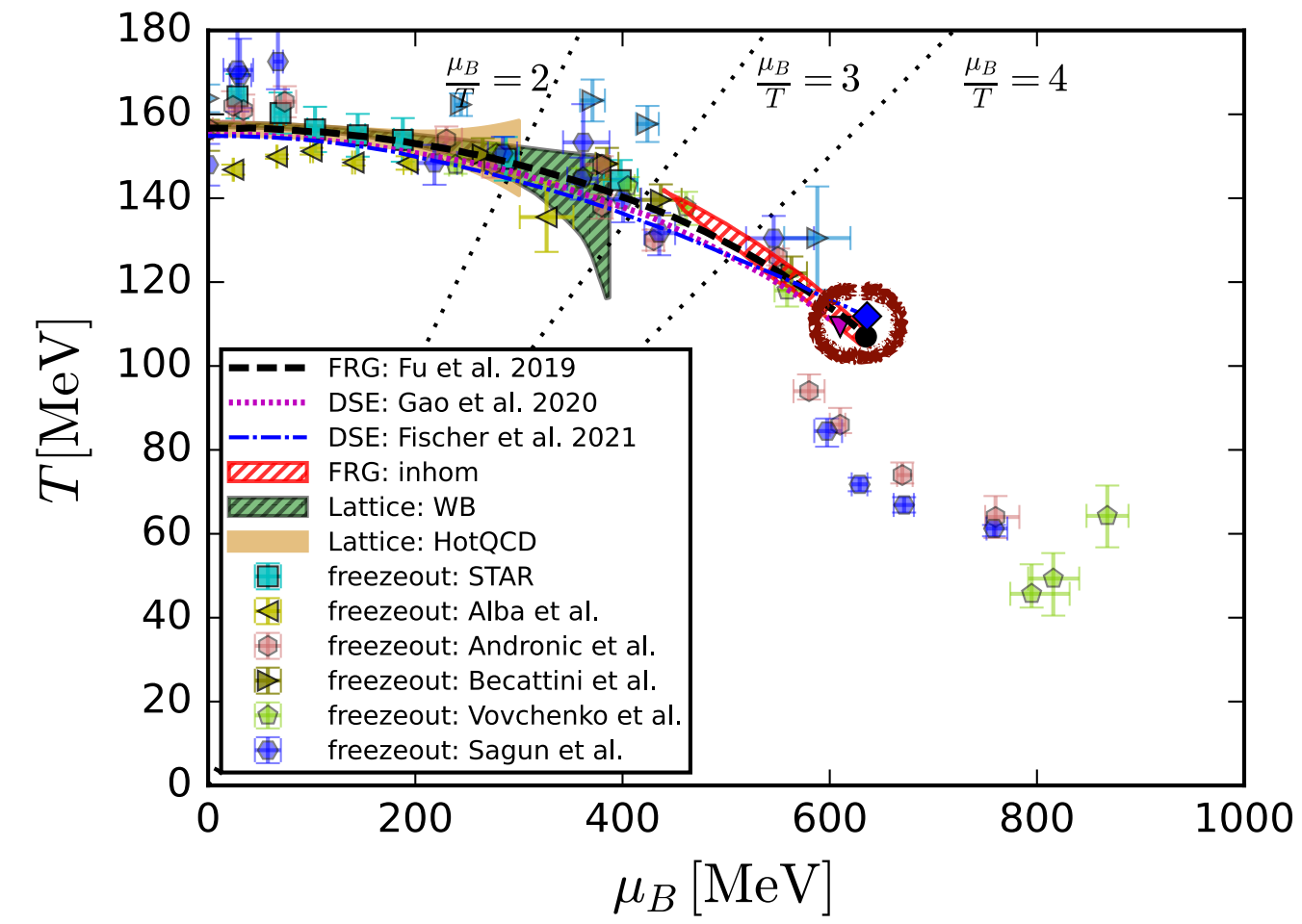
Remove CEP-predictions

RHIC-BES Seminar Oct. 6th 2020, Xiaofeng Luo

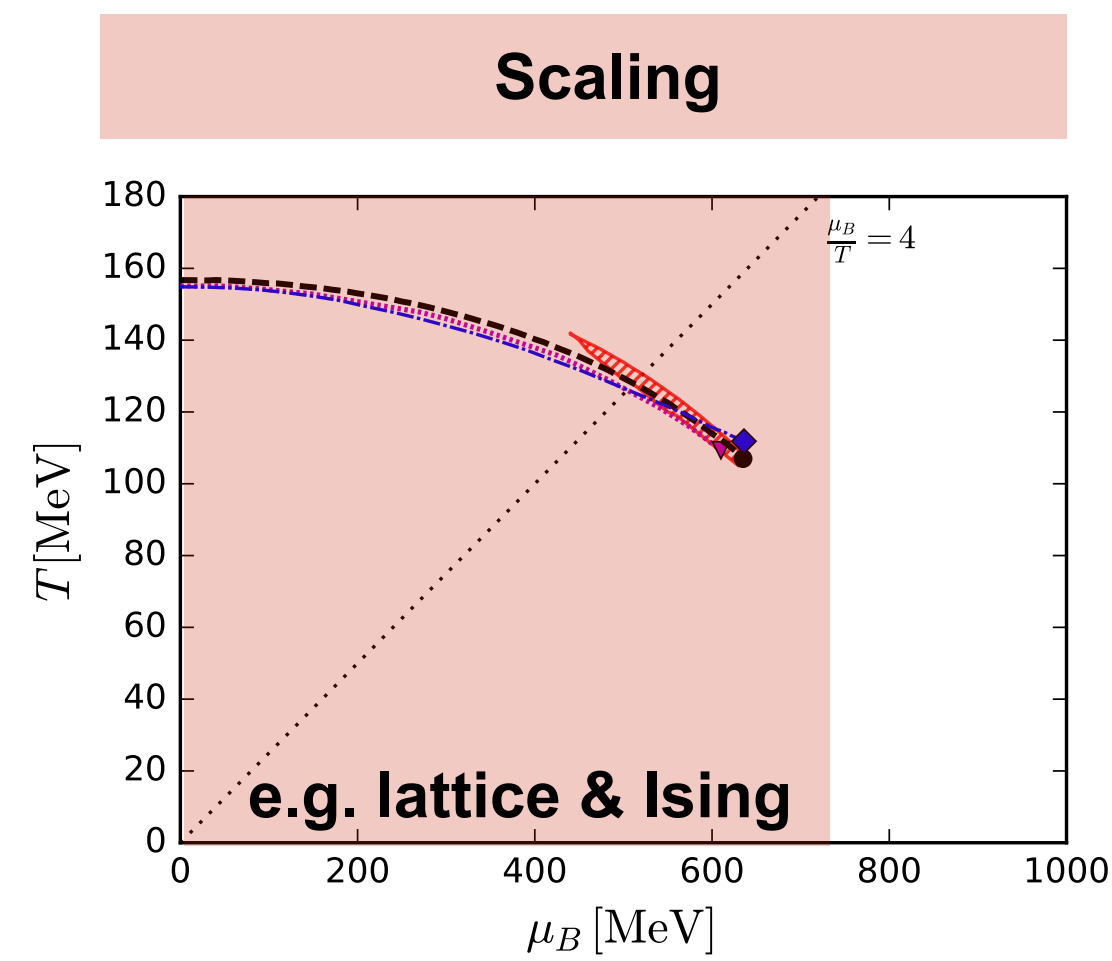
- (i) 'old' CEPs: lattice, Functional QCD approaches, LEFTS (updated computations available)
- (ii) LEFTs & Functional Results (qualitative approximations) that miss lattice benchmarks at $\mu_B = 0$
- (iii) LEFTs with CEPs at large density (missing quark-gluon back reaction)

Predictions, estimates & extrapolations and how to use them

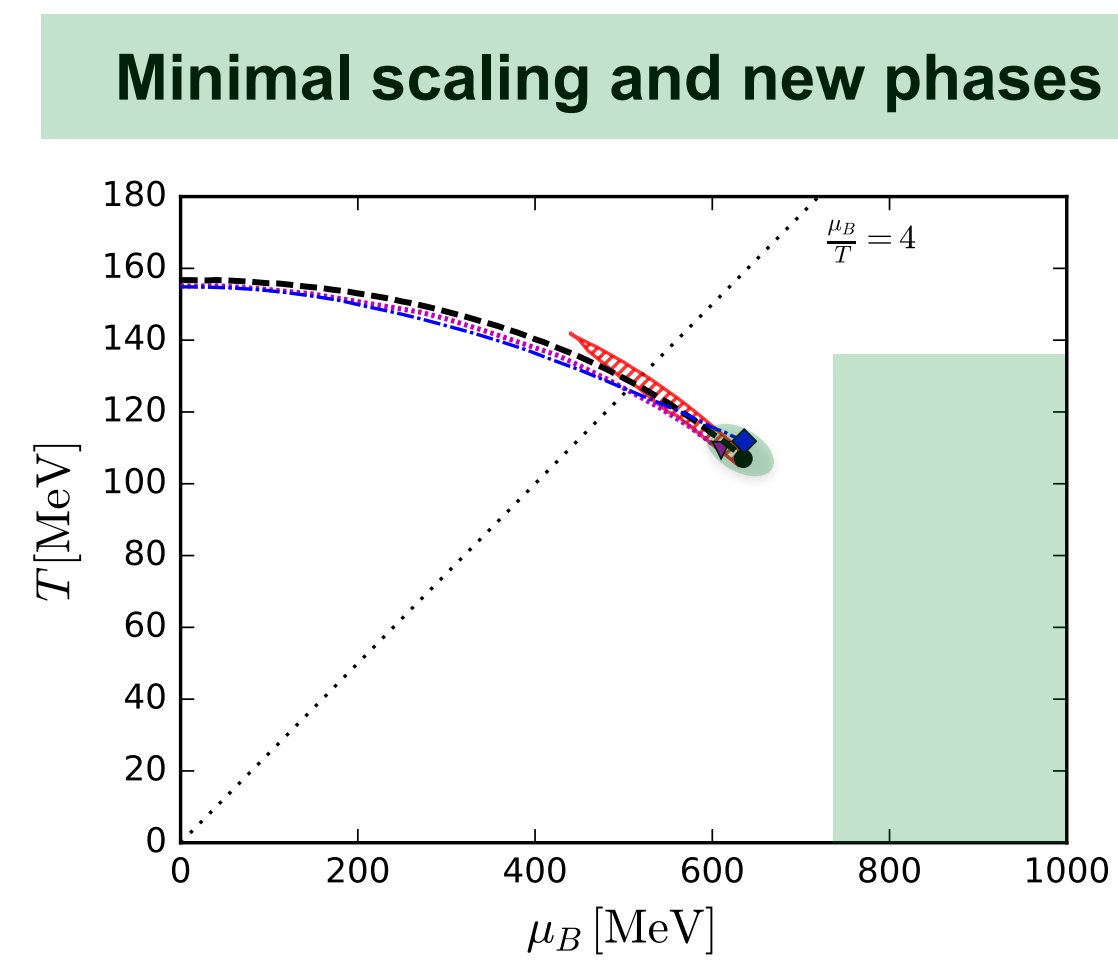
Predictions, estimates & extrapolations and how to use them



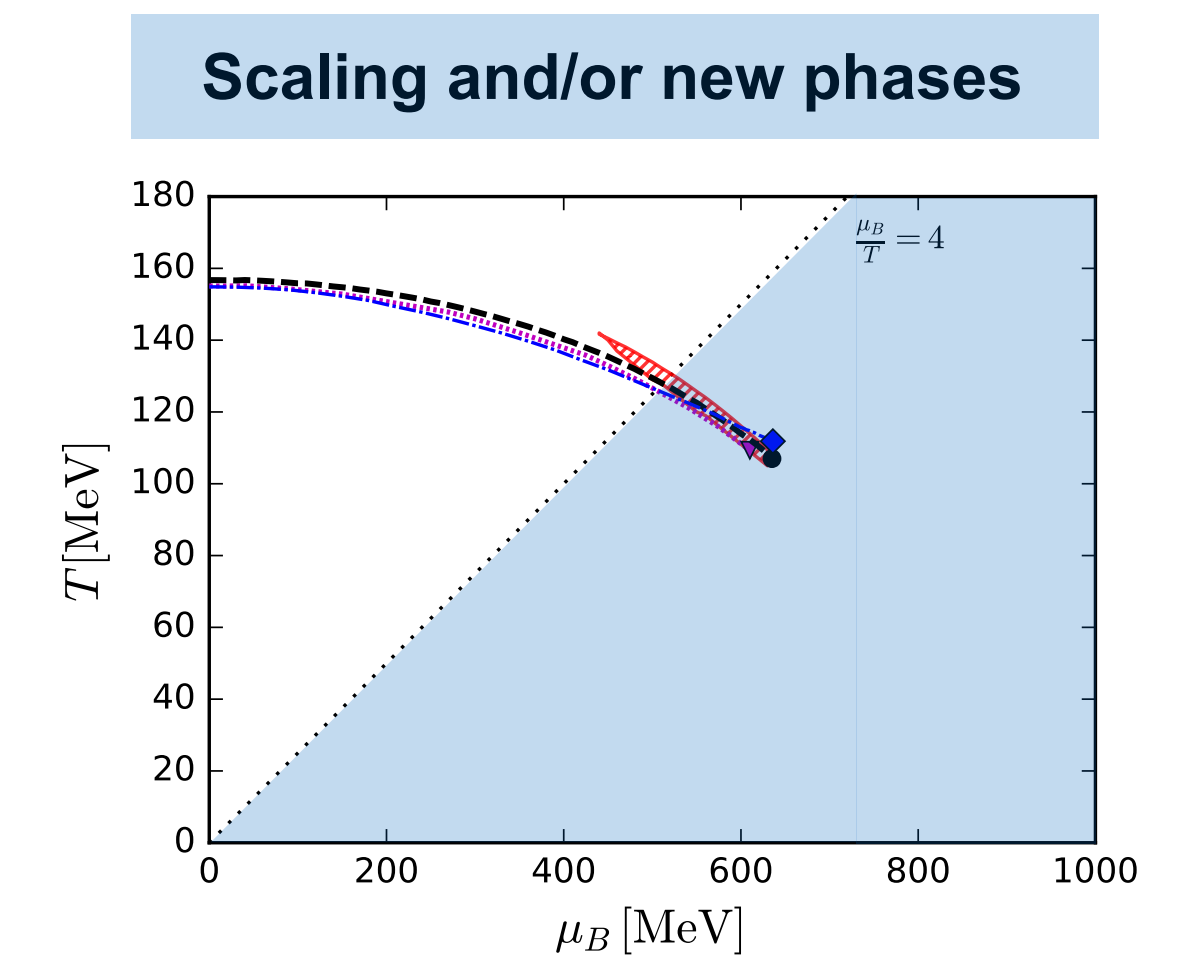
Scenario I



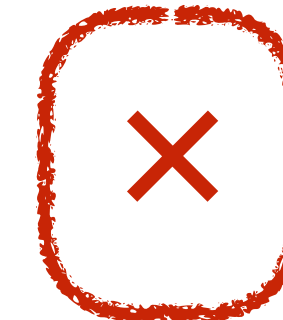
Scenario II



Scenario III

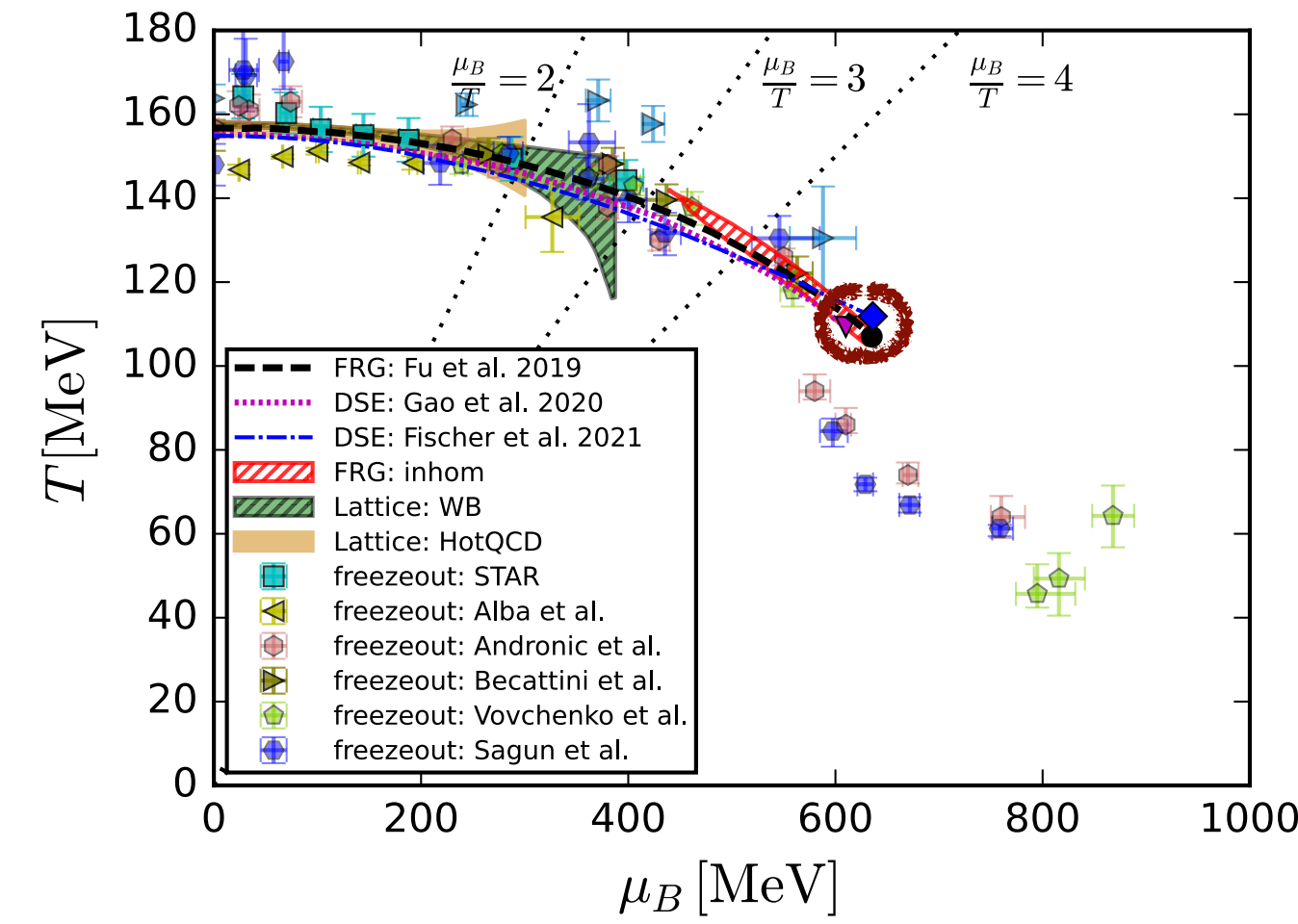


Extrapolations
for
Pheno



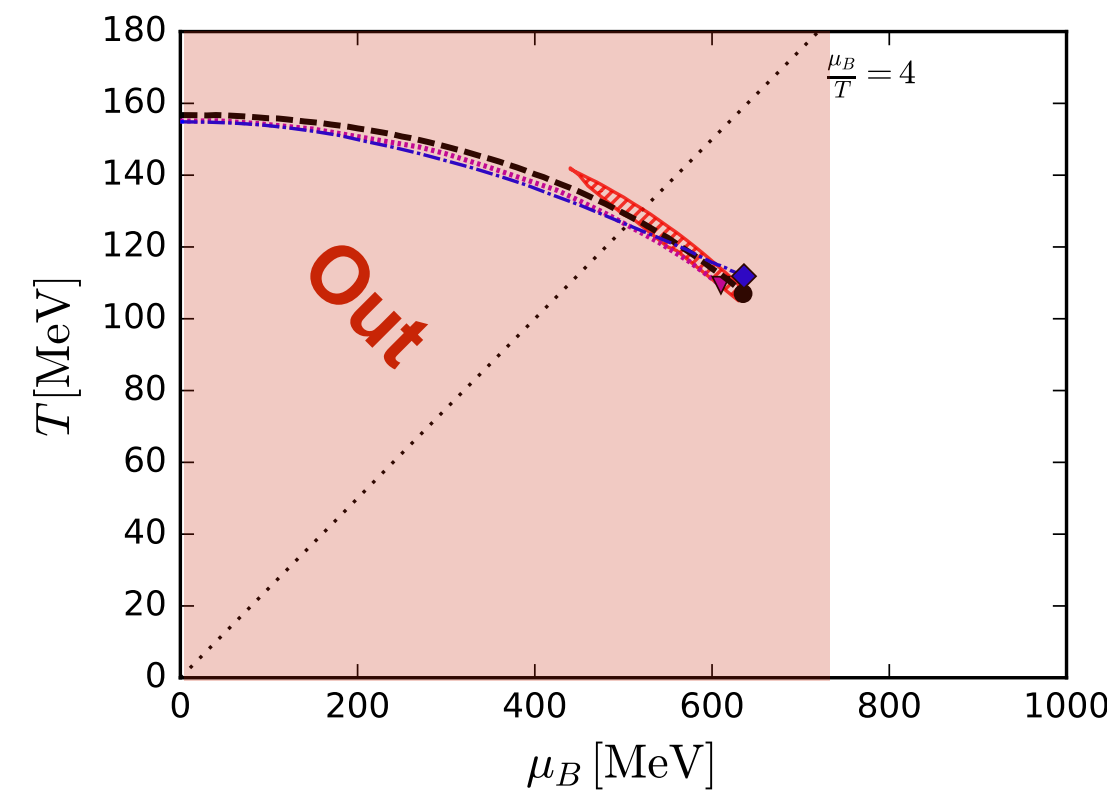
Predictions, estimates & extrapolations and how to use them

Out by the LEGO[®] principle
 Fu, JMP, Rennecke, PRD 101 (2020) 054032
 +
 Size of scaling regime in LEFTs
 Schaefer, Wambach, PRD 75 (2007) 085015
 Braun, Klein, Piasecki, EPJC 71 (2011) 1576
 ⋮



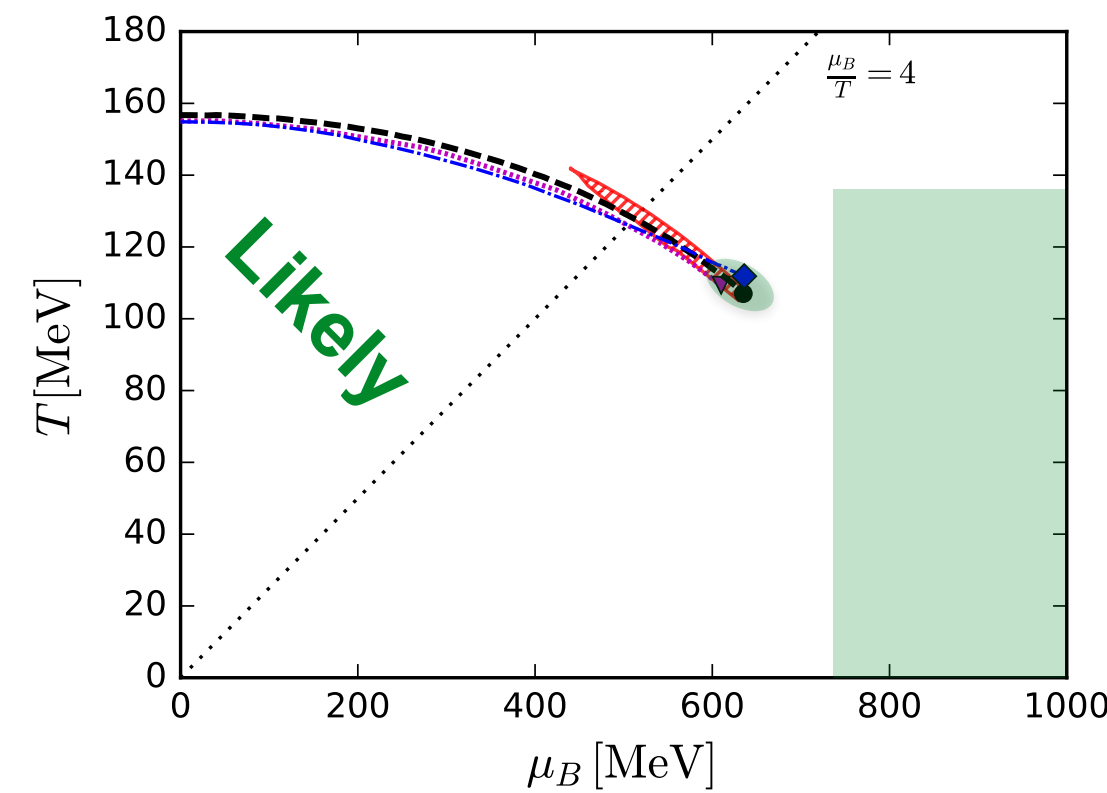
Scenario I

Scaling



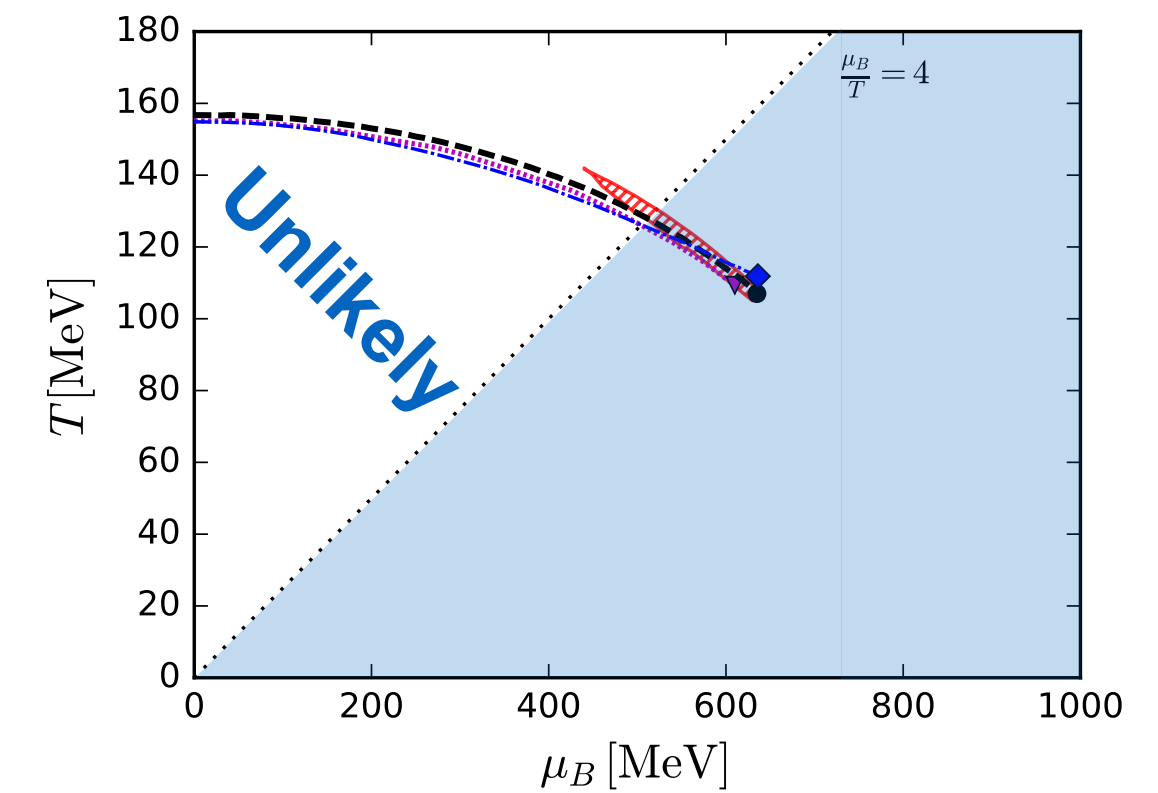
Scenario II

Minimal scaling and new phases



Scenario III

Scaling and/or new phases



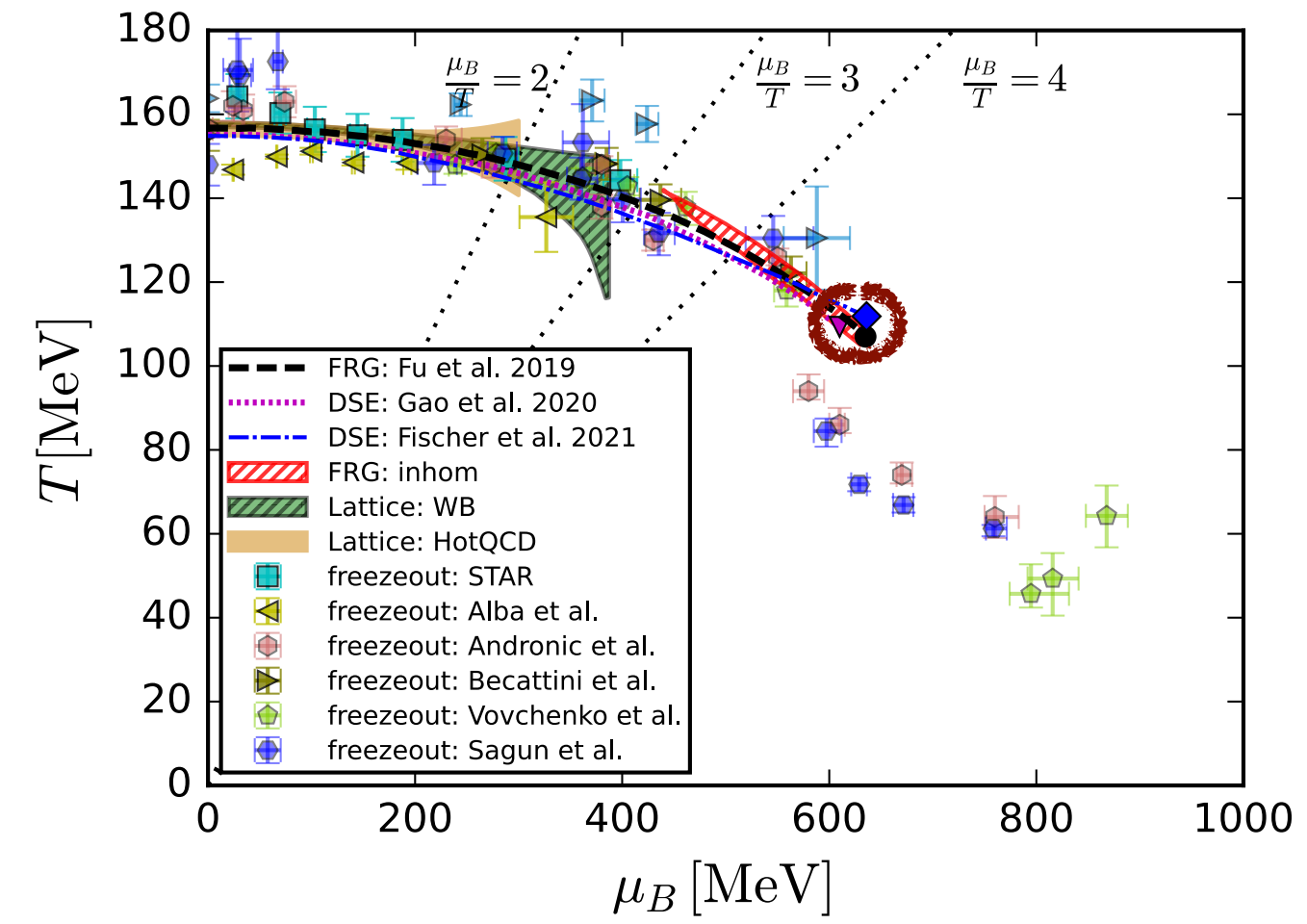
Braun, Fu, JMP, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

Gao, JMP, PRD 105 (2022) 094020

Soft modes in hot QCD matter: Braun, Chen, Fu, Gao, Huang, Ihssen, JMP, Rennecke, Sattler, Tan, Wen, Yin, arXiv:2310.19853

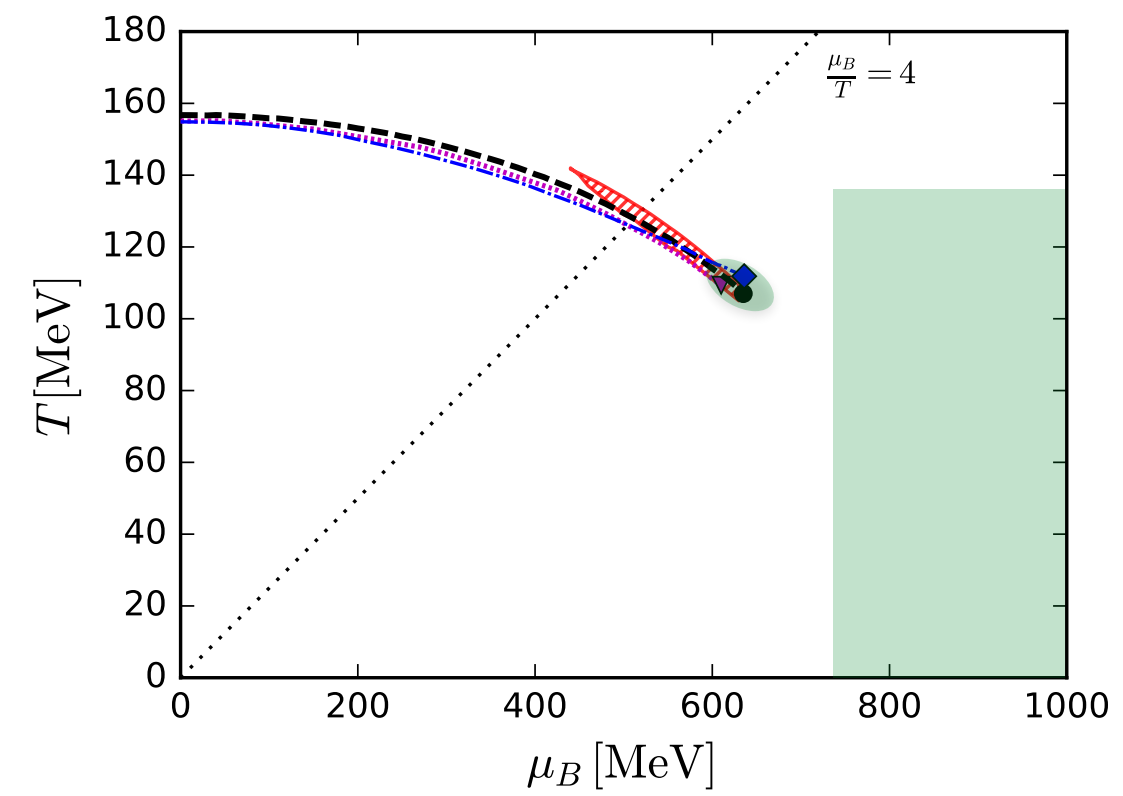
+ many results in dynamical low energy effective theories
LEGO[®] principle

Predictions, estimates & extrapolations and how to use them



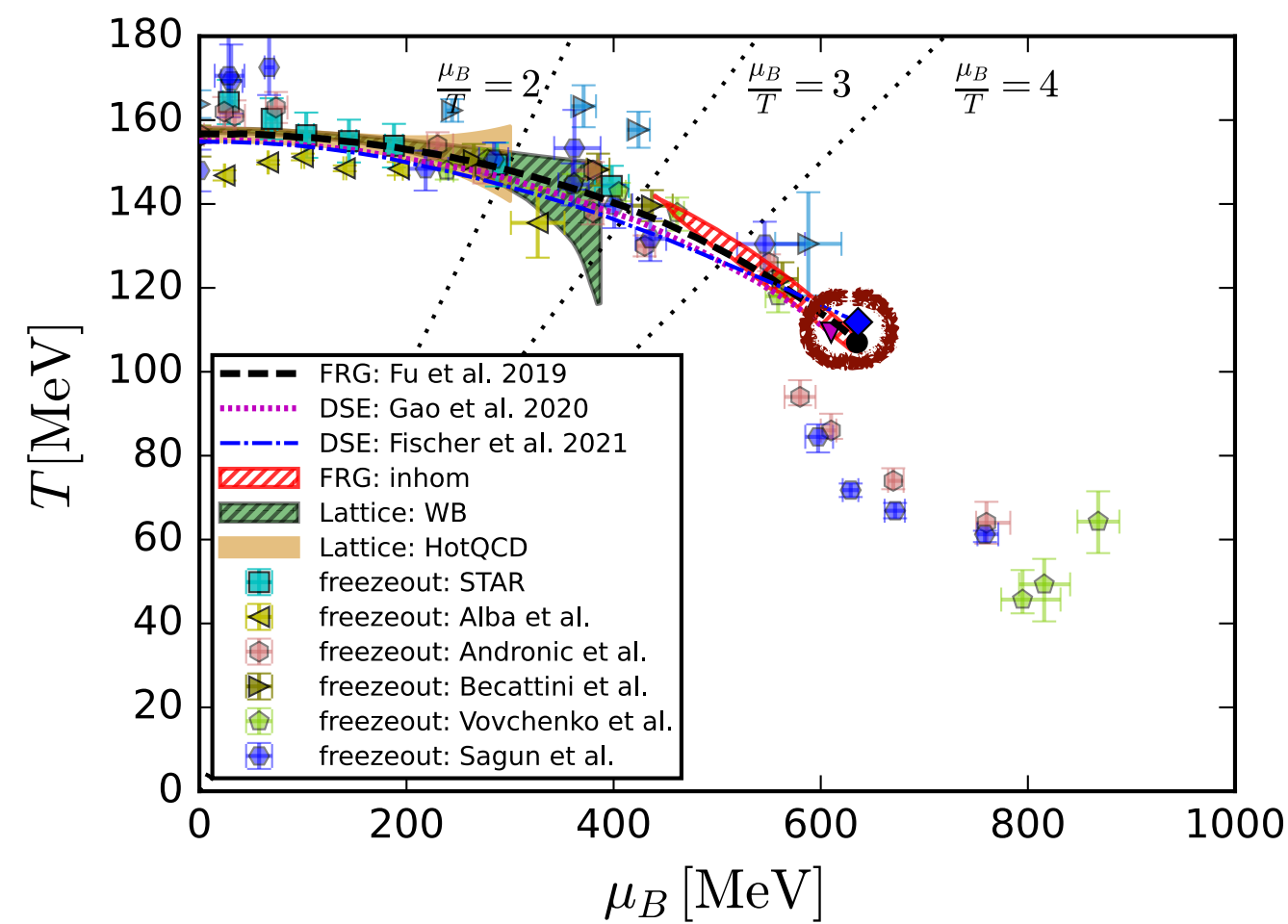
Scenario II

Minimal scaling and new phases



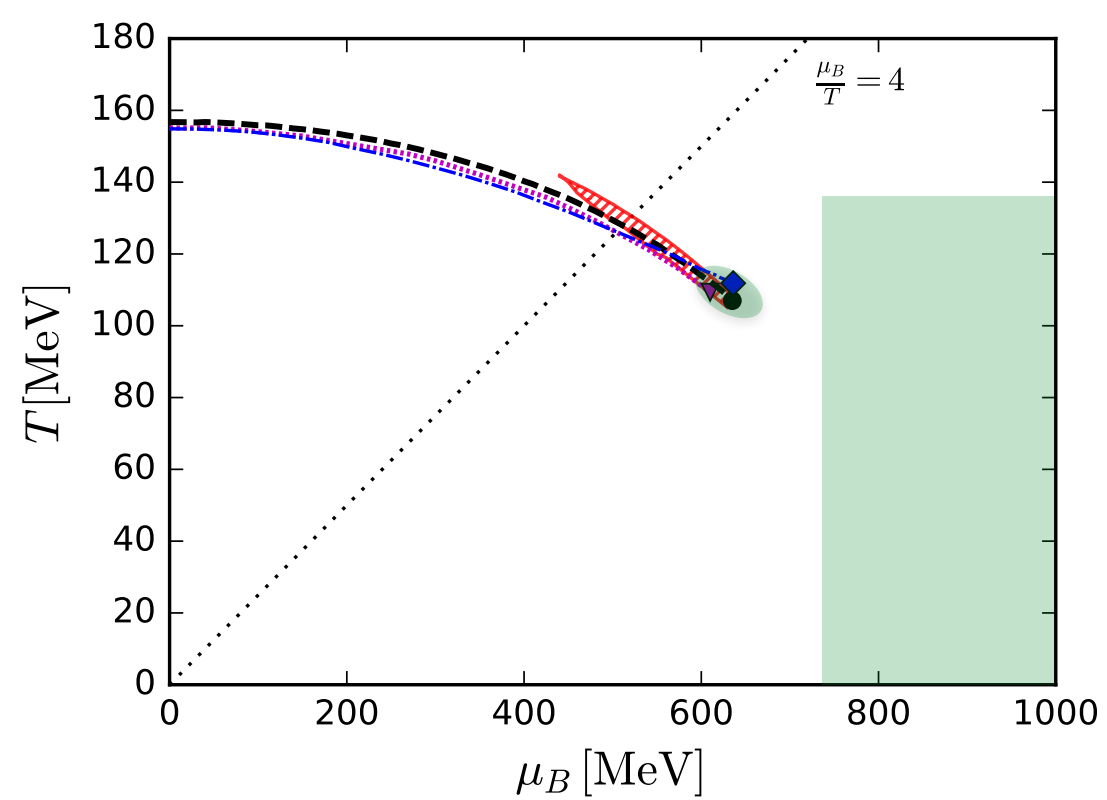
Extrapolations
for
Pheno

Predictions, estimates & extrapolations and how to use them



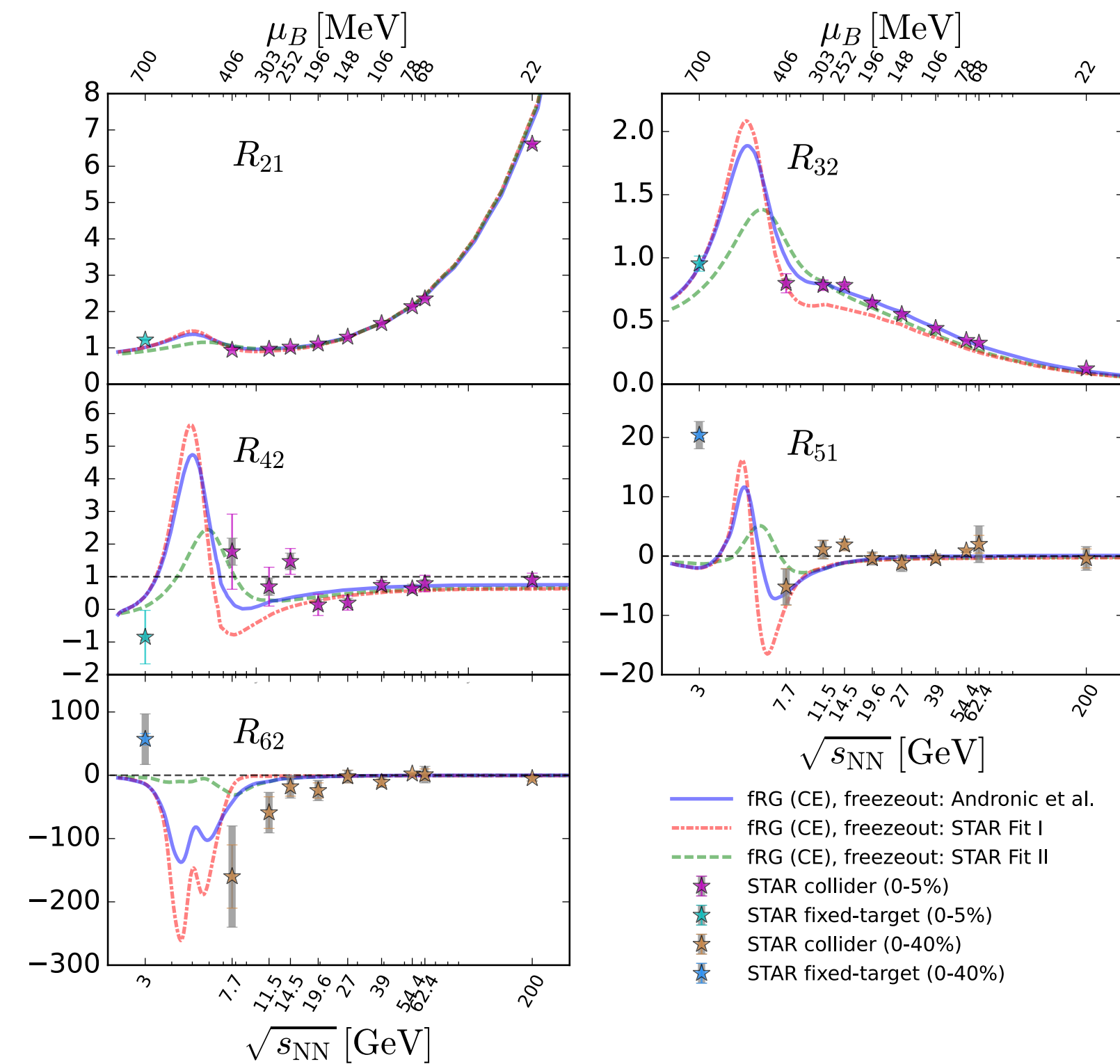
Scenario II

Minimal scaling and new phases



Ripples of the critical end point

baryon & proton number fluctuations



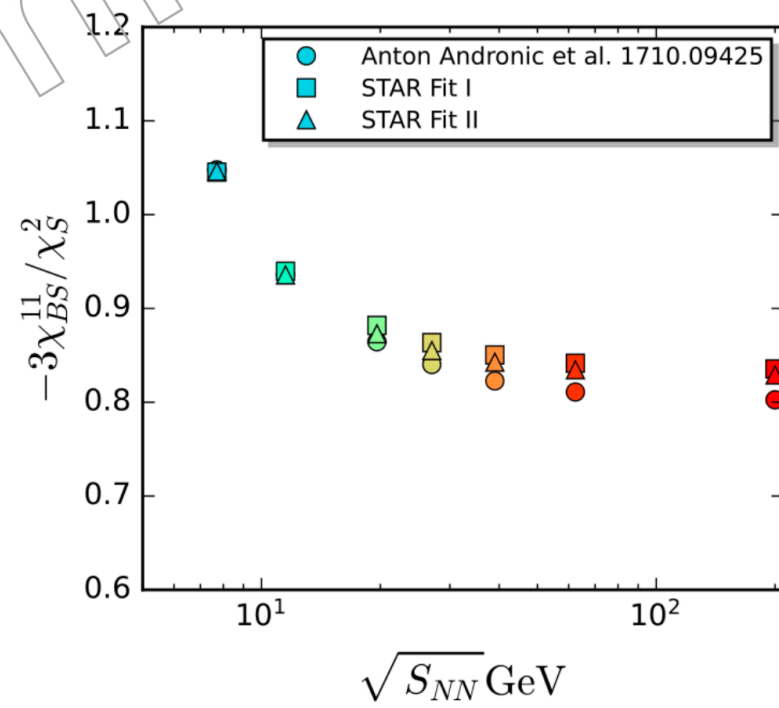
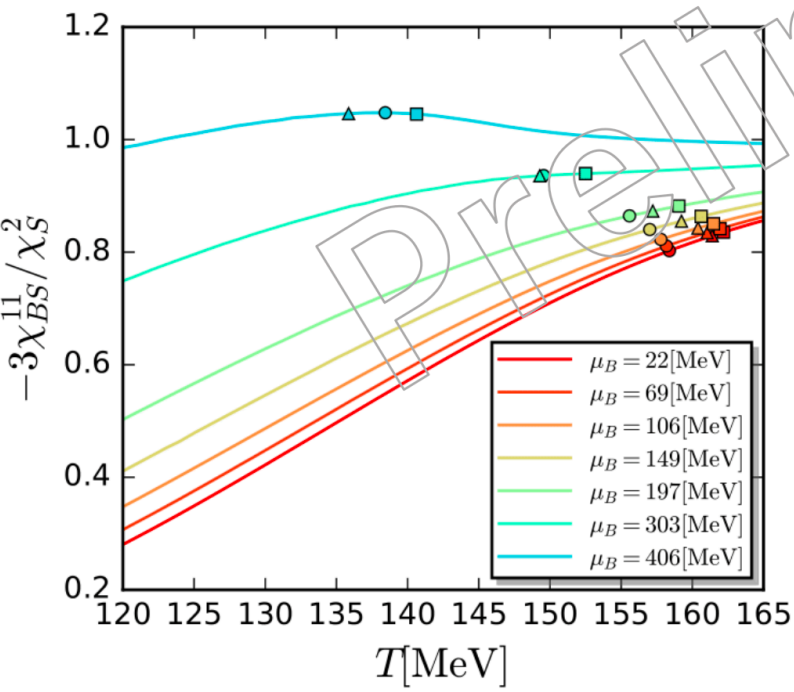
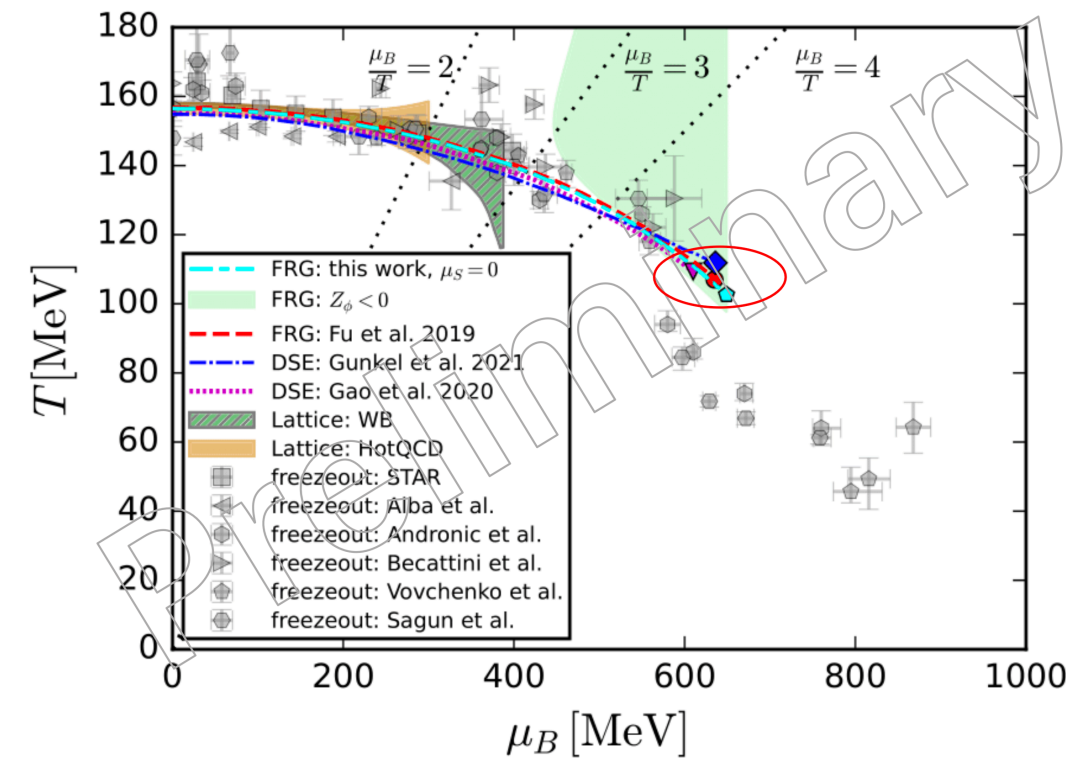
Extrapolations
for
Pheno



Predictions, estimates & extrapolations and how to use them

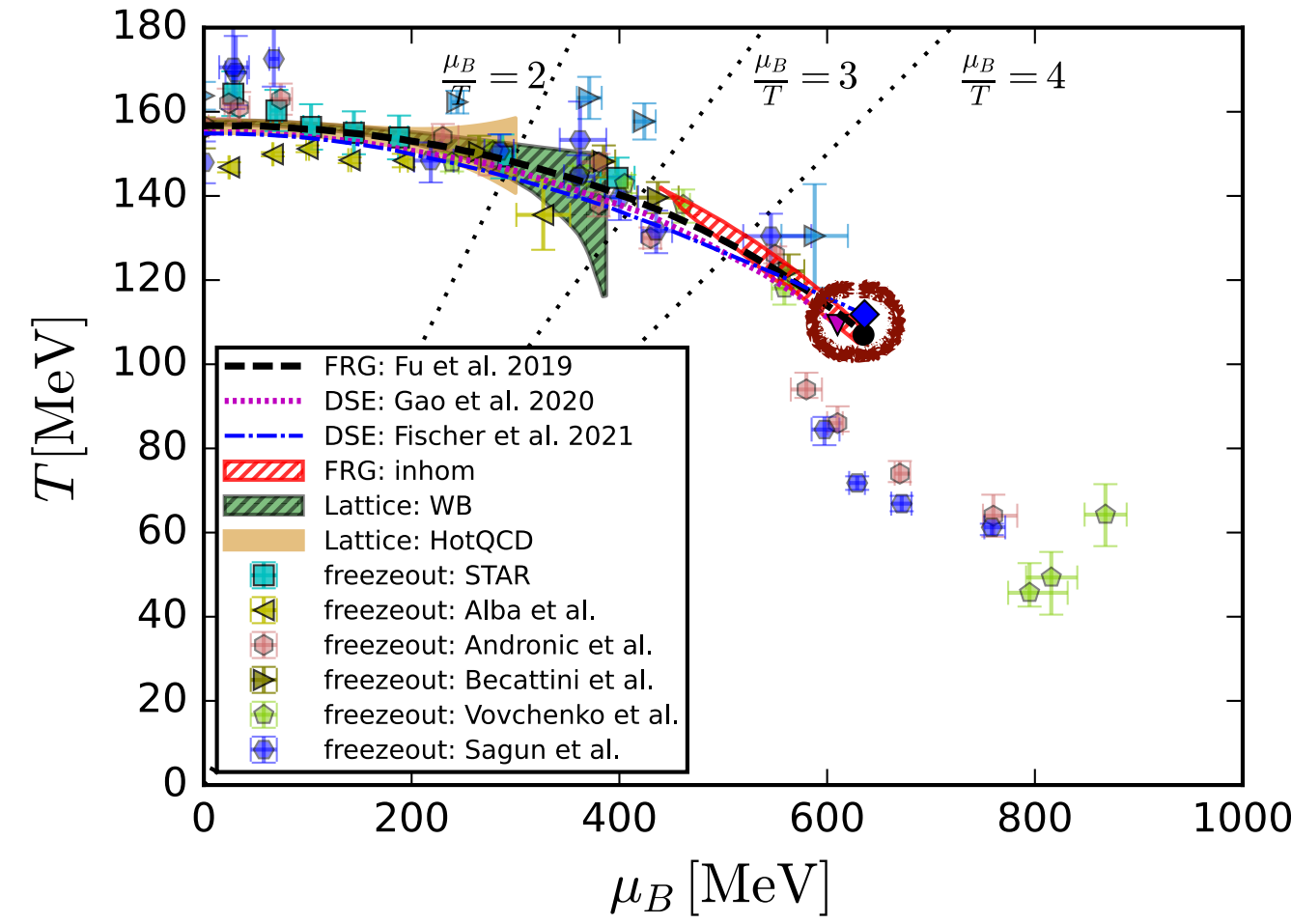
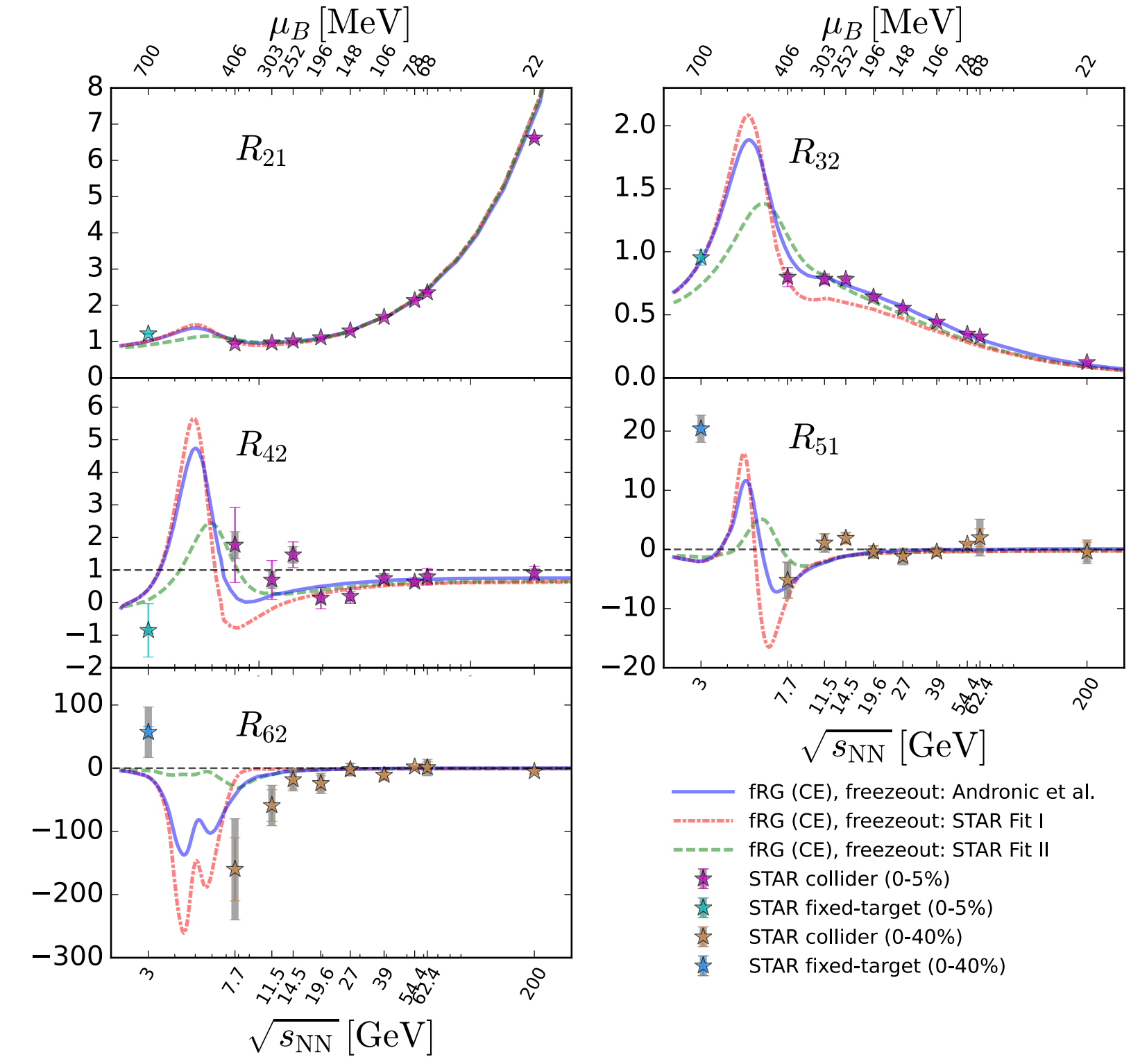
Strangeness neutrality

baryon & proton number fluctuations



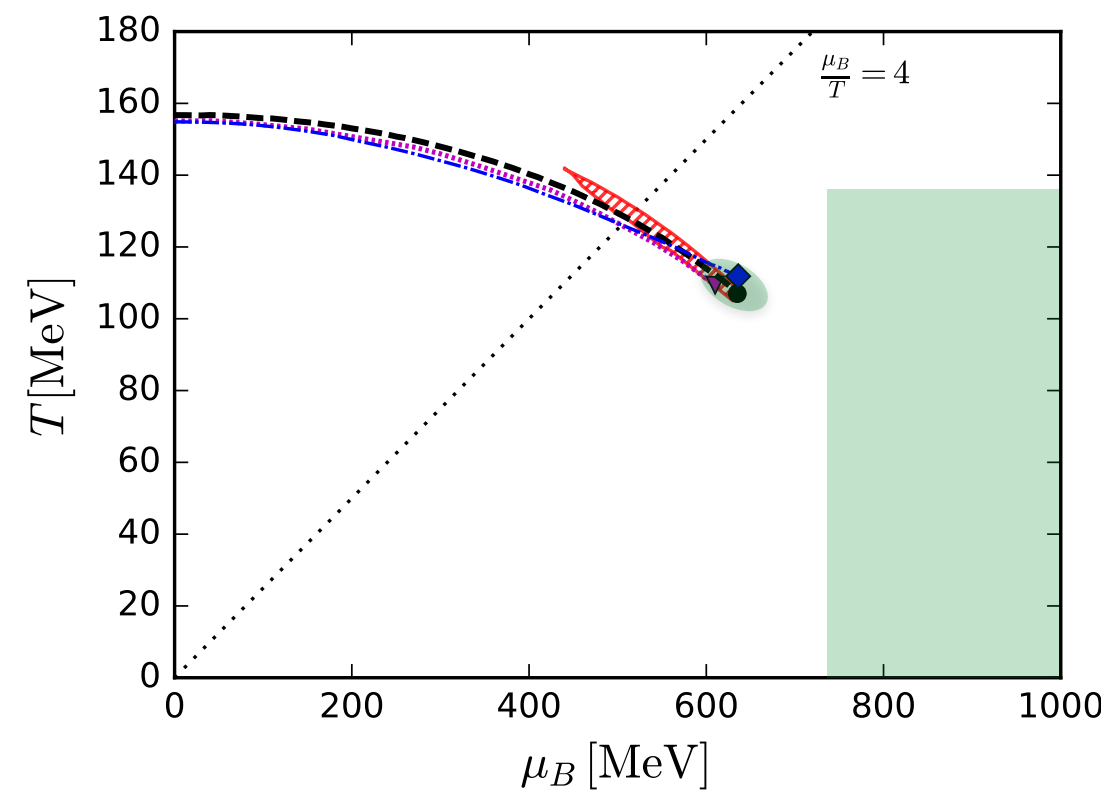
Ripples of the critical end point

baryon & proton number fluctuations



Scenario II

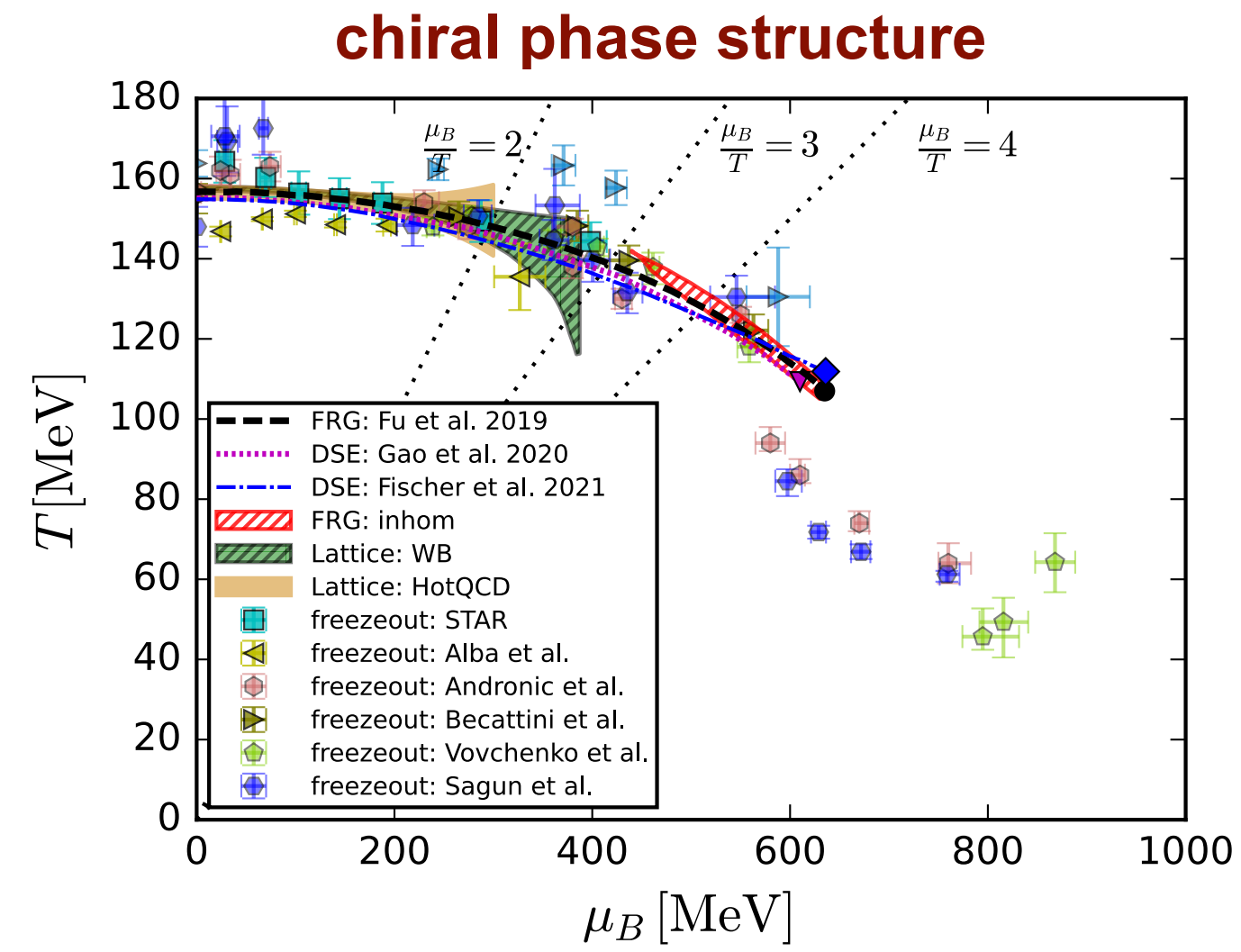
Minimal scaling and new phases



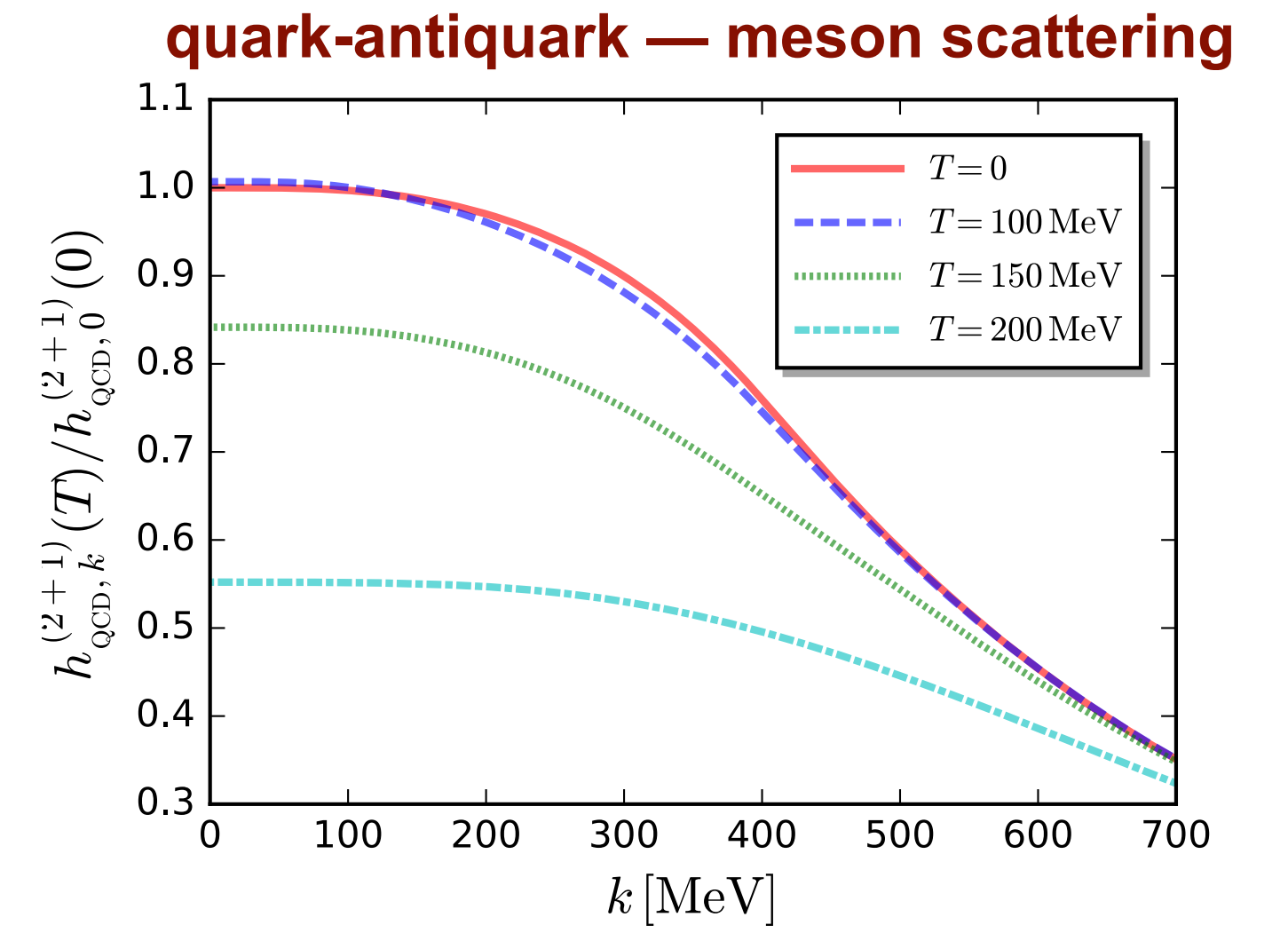
Ripples of the CEP

QCD-assisted low energy effective theory

Direct QCD input

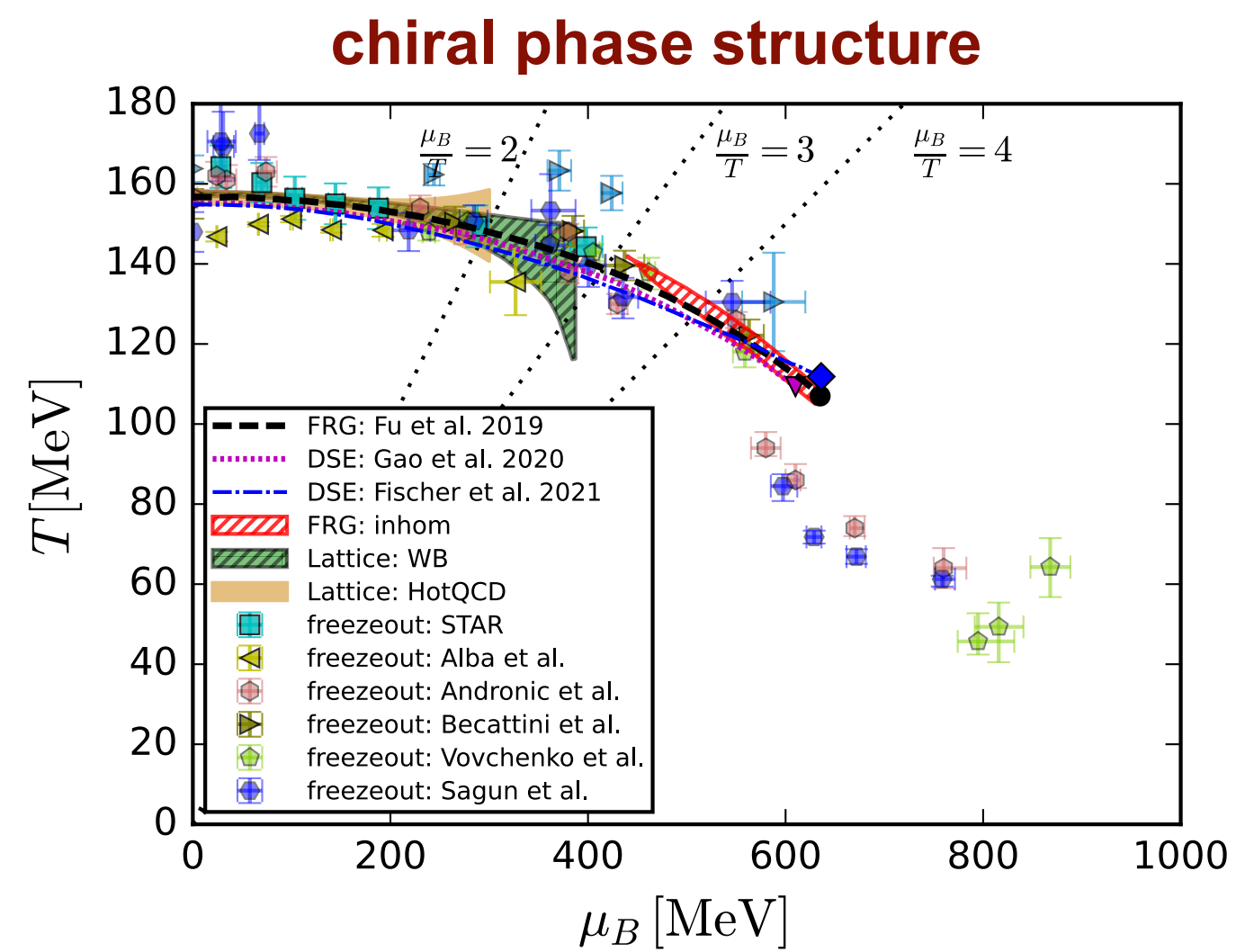


+

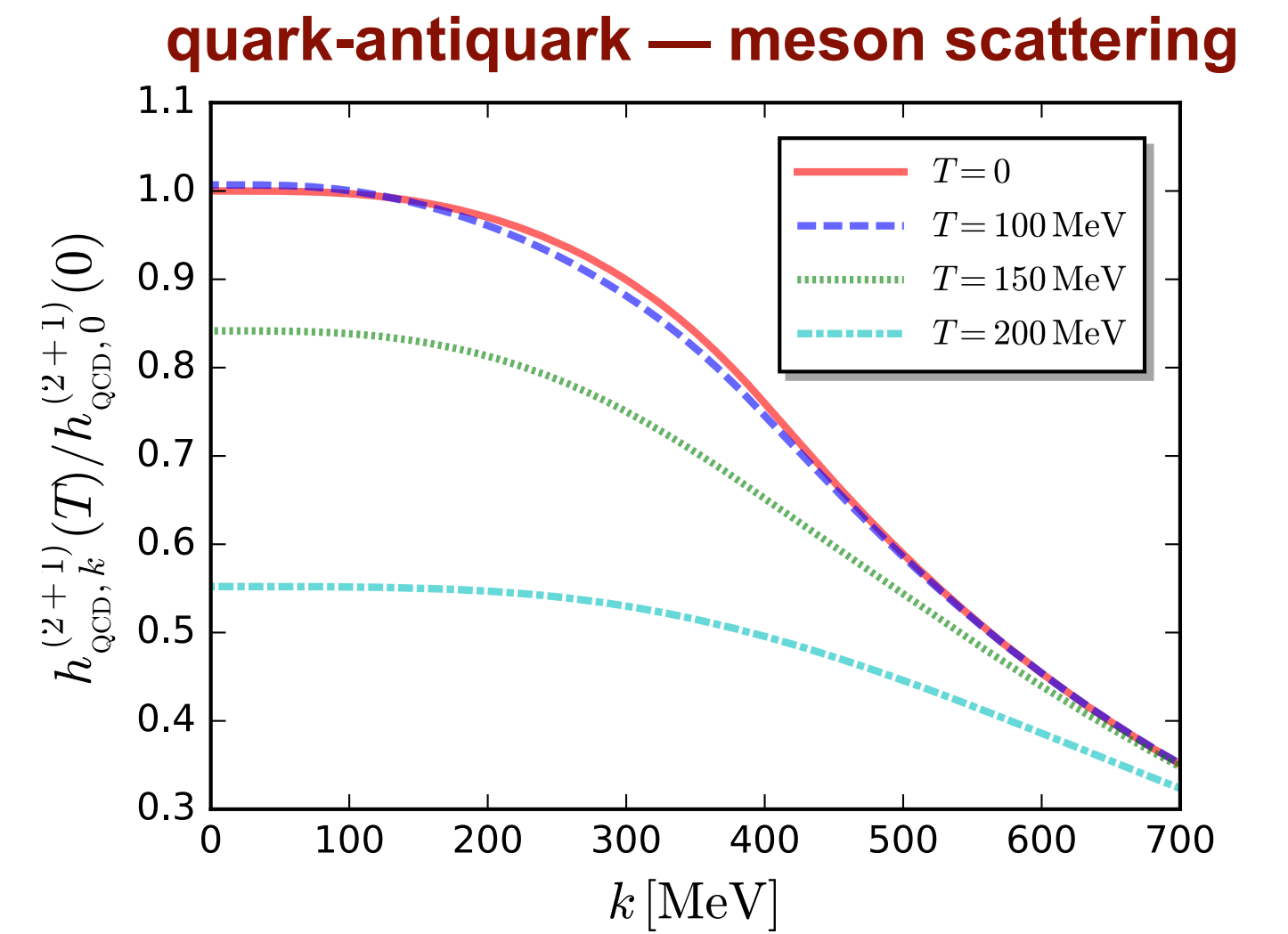


QCD-assisted low energy effective theory

Direct QCD input



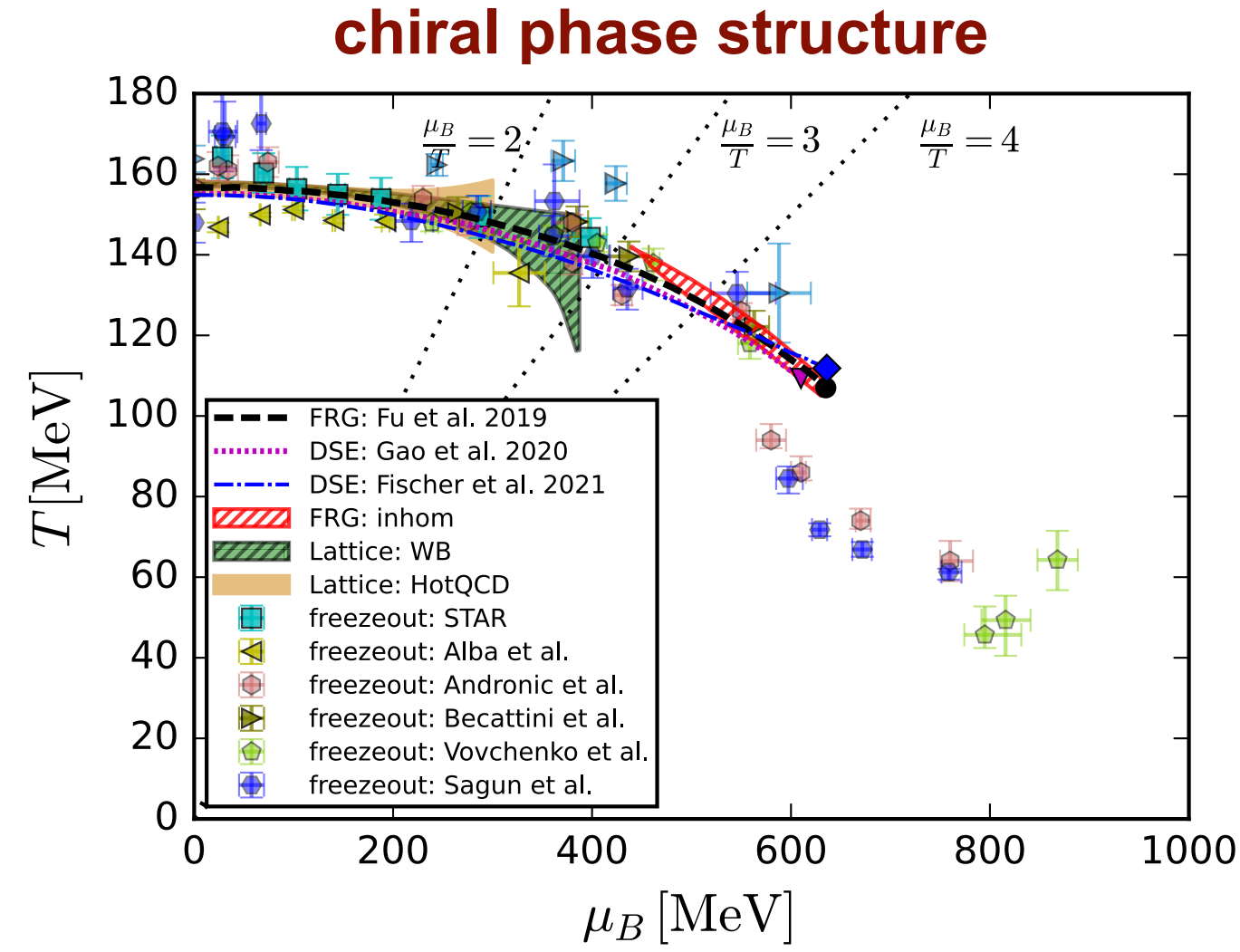
+



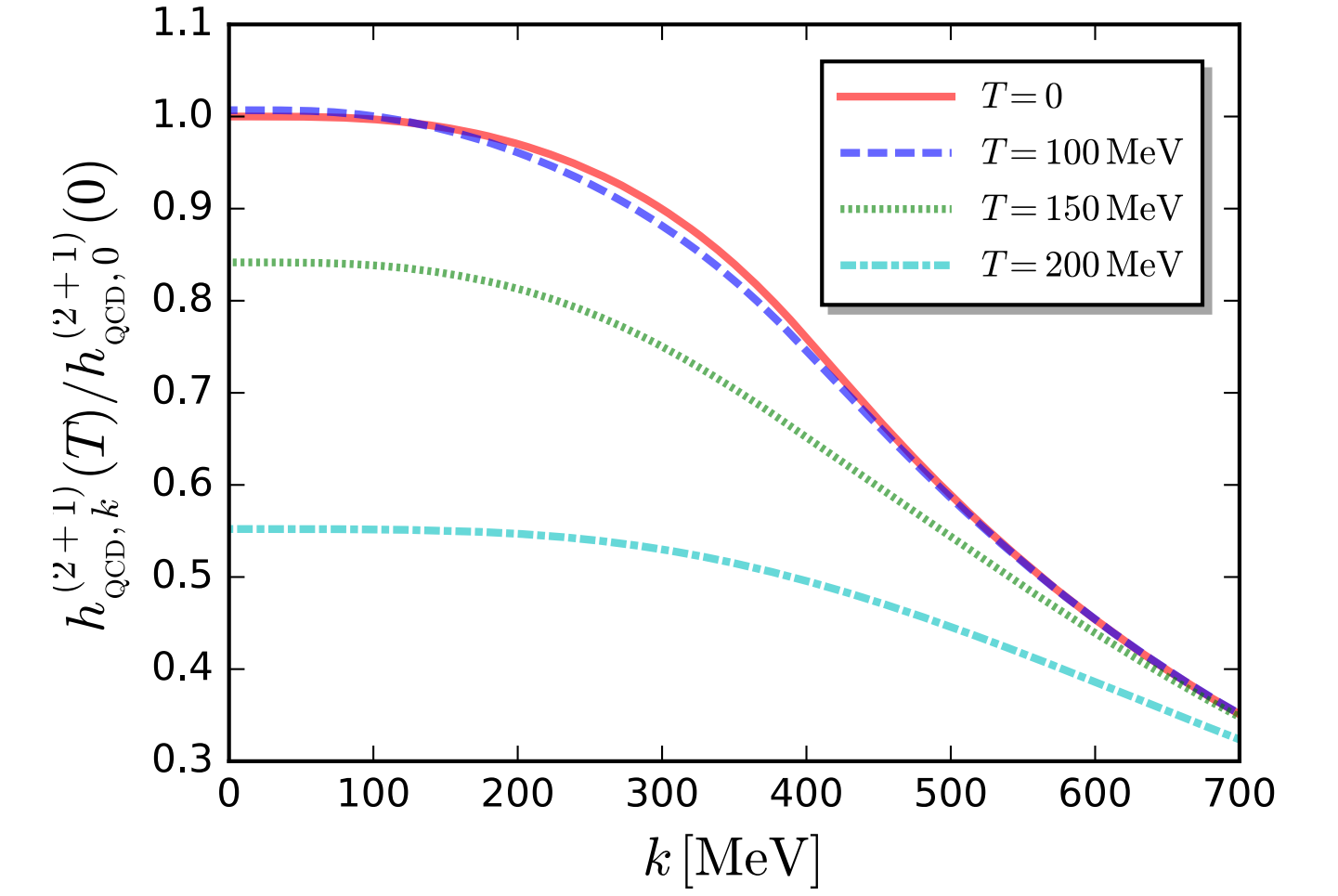
low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

QCD-assisted low energy effective theory

Direct QCD input



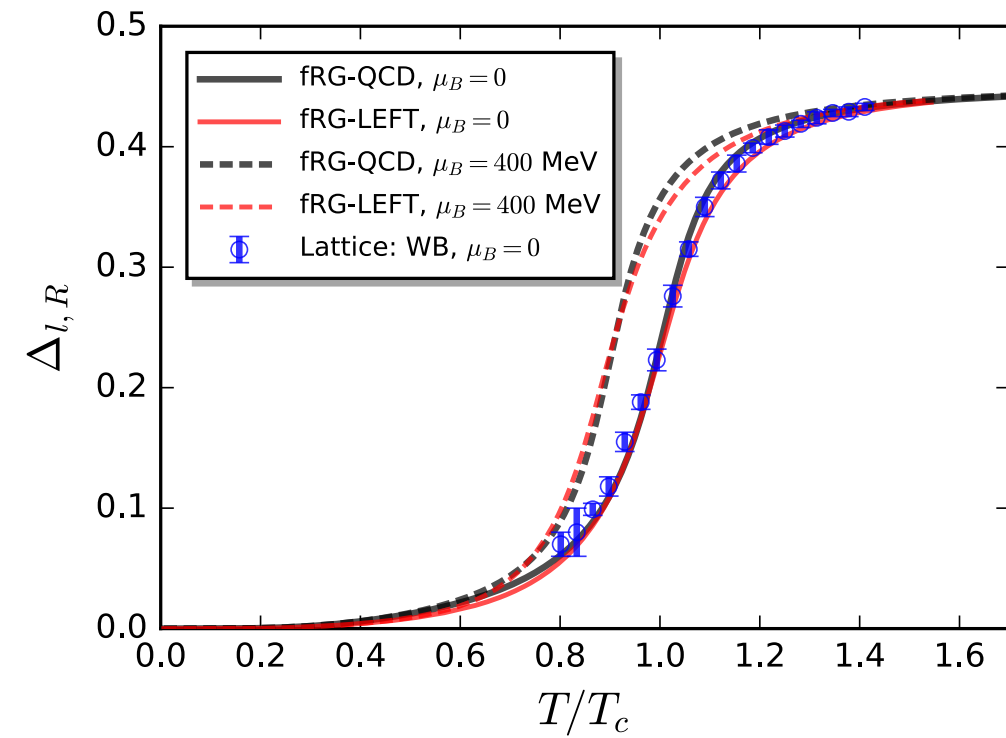
quark-antiquark — meson scattering



+

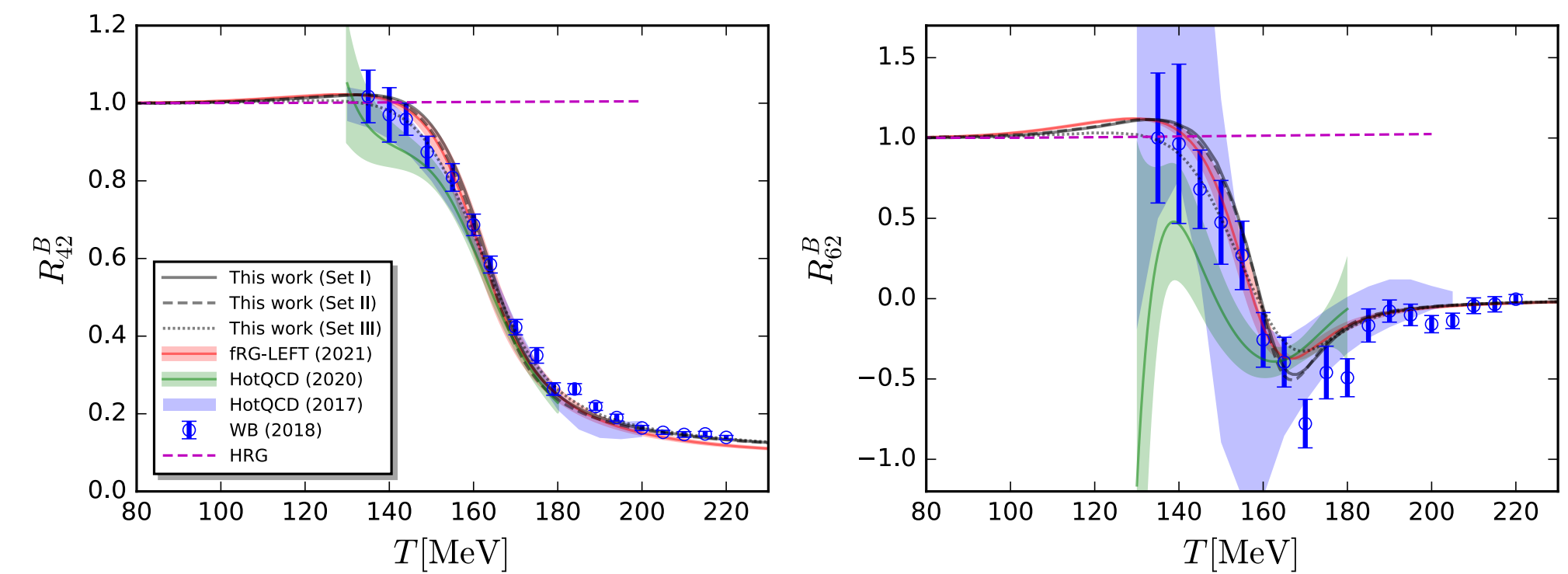
low energy quantum, thermal & density fluctuations via fRG (QCD-assisted PQM model)

renormalised chiral condensate



Benchmarks with lattice and fQCD
at
vanishing density and fQCD at finite density

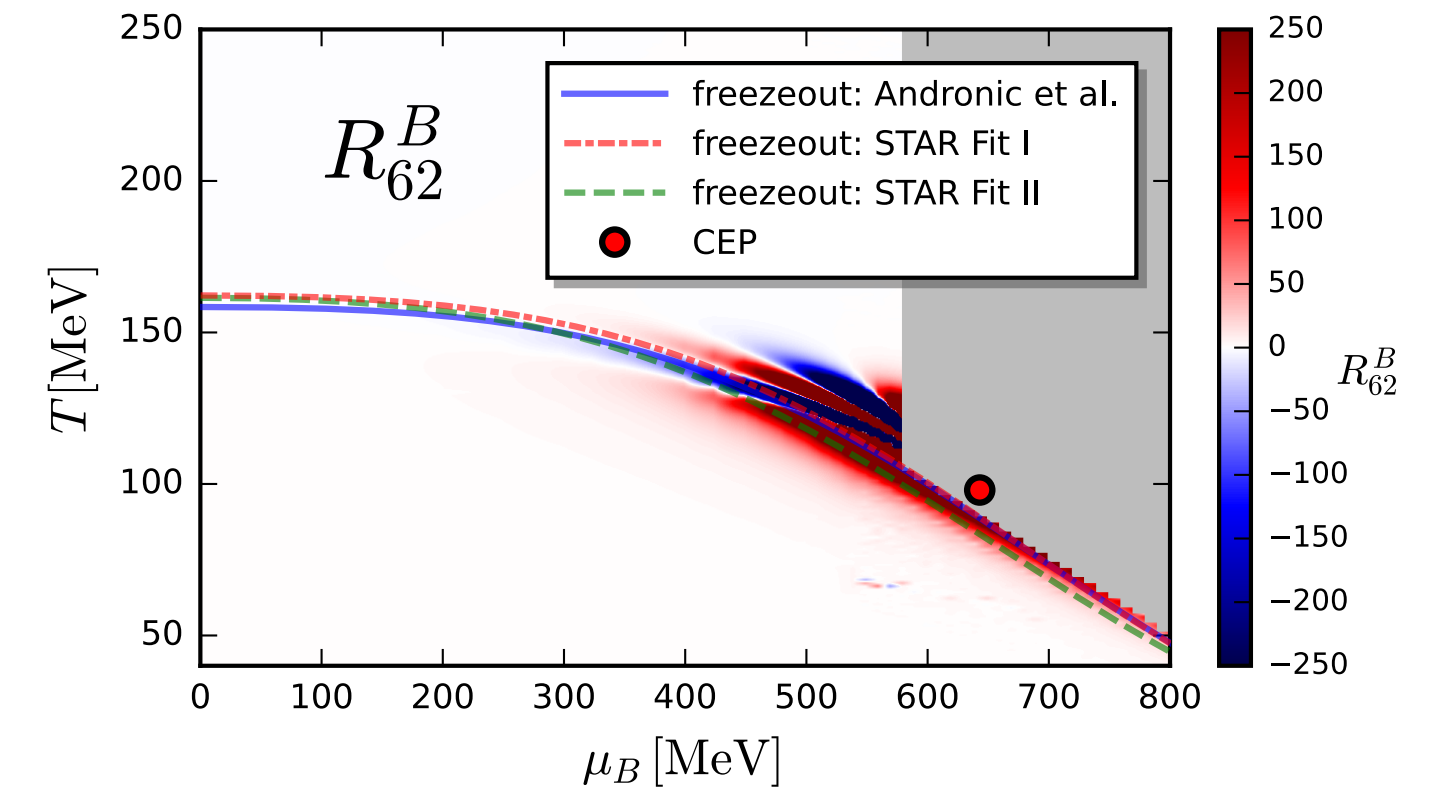
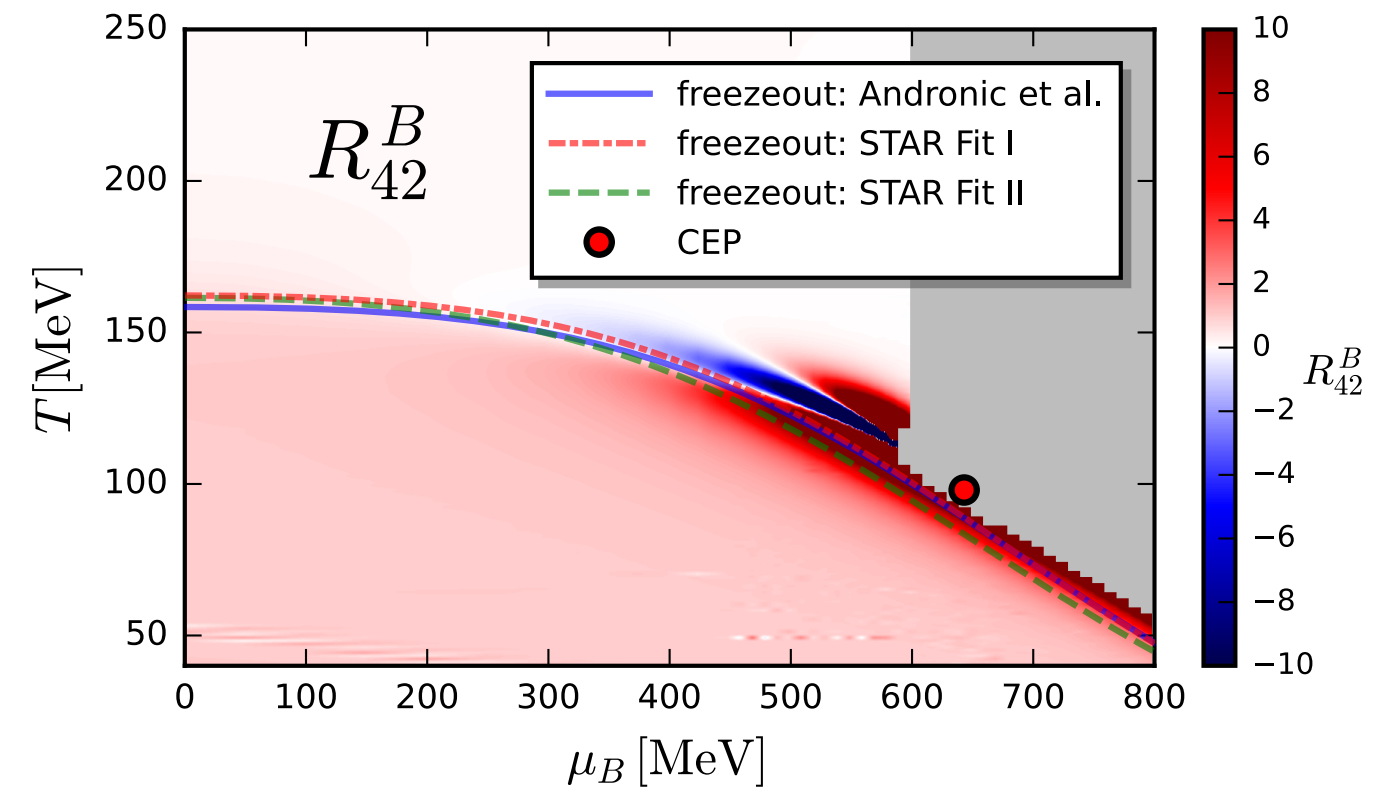
baryon number fluctuations



Ripples of the critical point

baryon number fluctuations in the phase structure

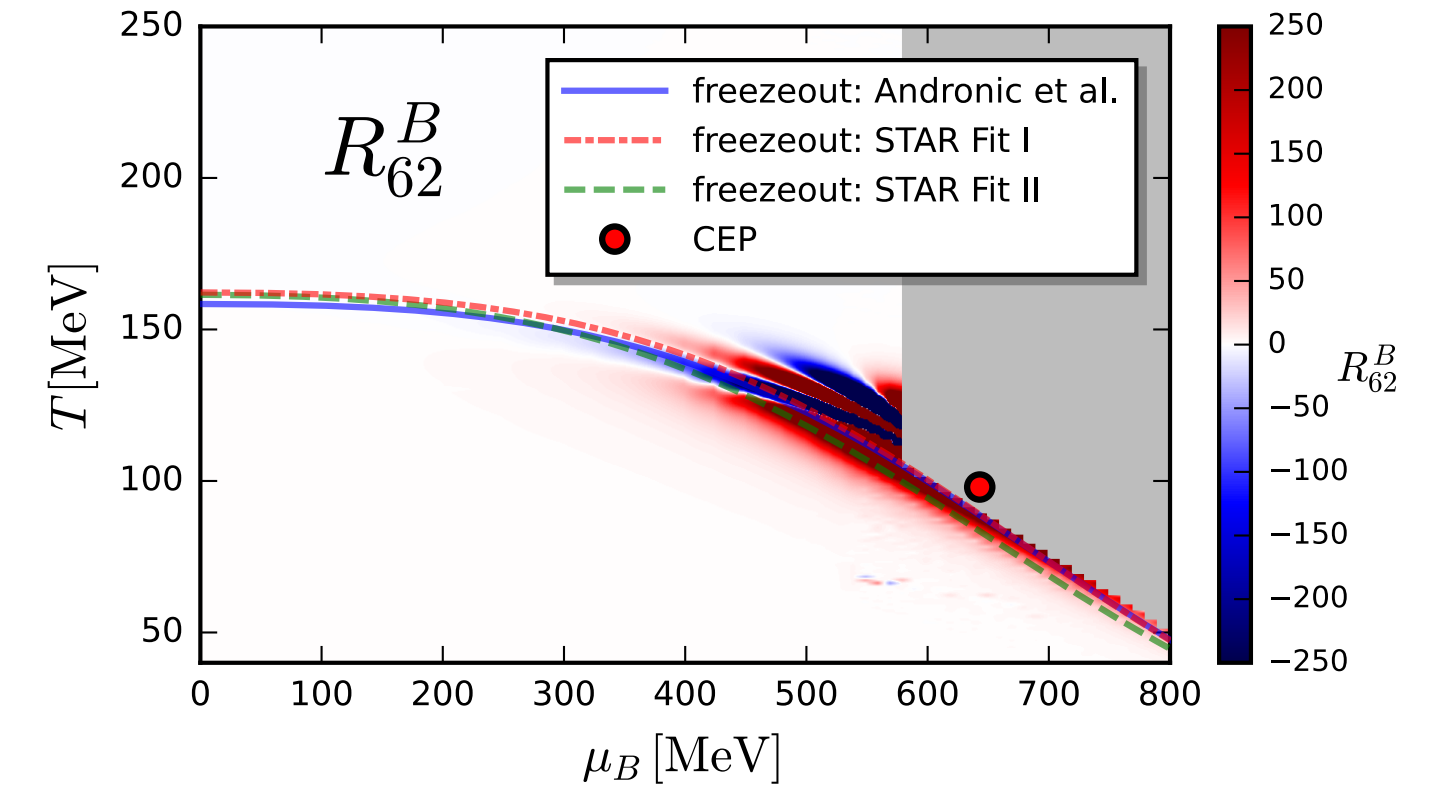
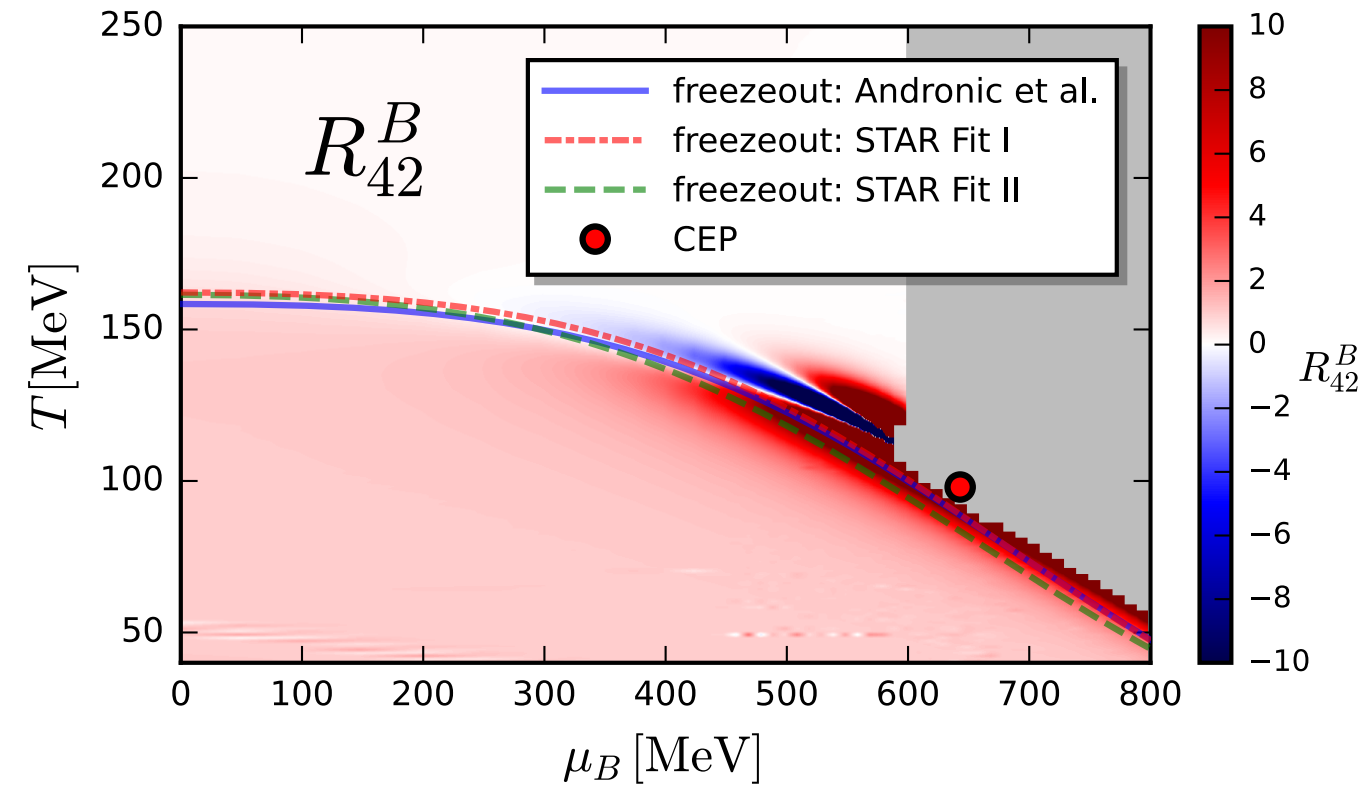
$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643) \text{ MeV}$$



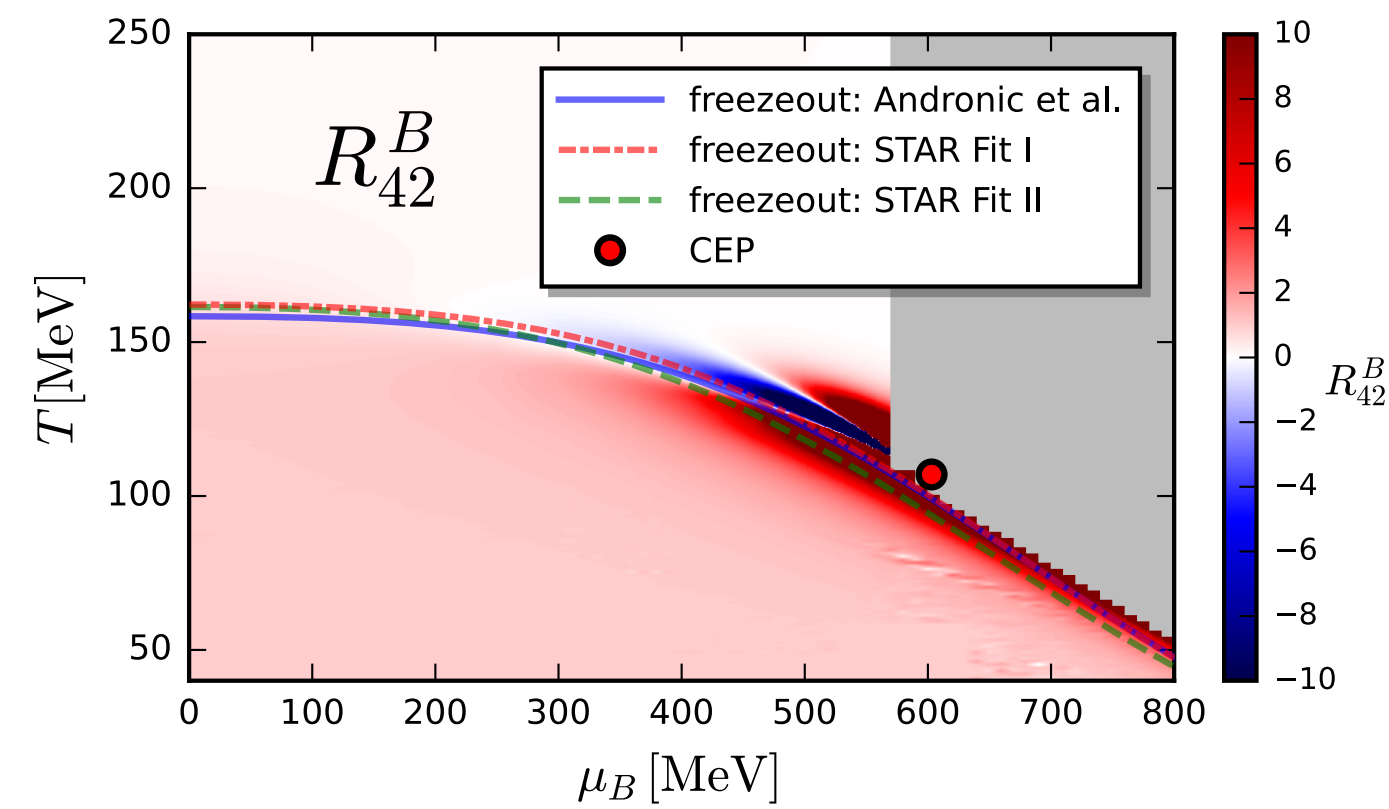
Ripples of the critical point

baryon number fluctuations in the phase structure

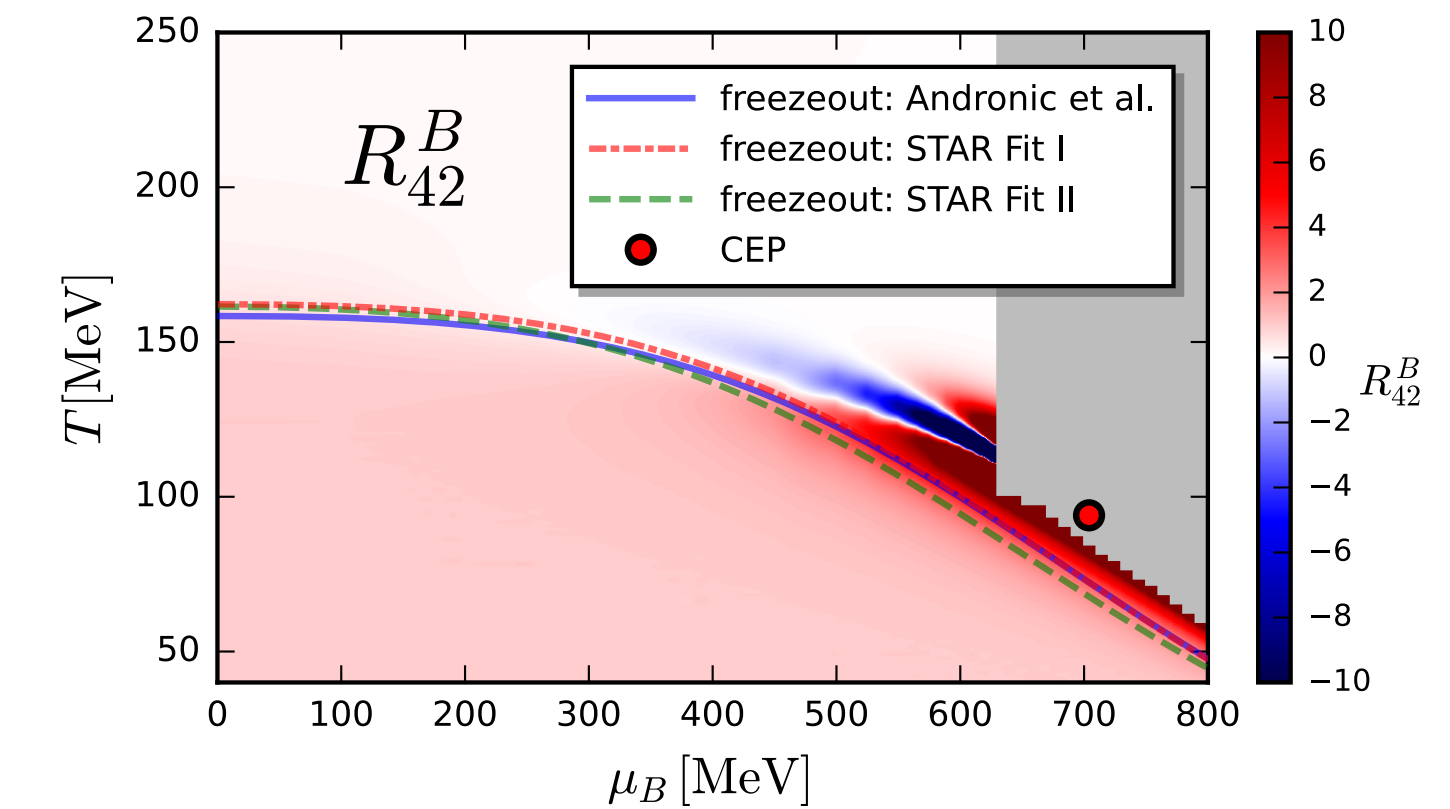
$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (98, 643) \text{ MeV}$$



Variations of the CEP in the allowed estimate regime



$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (108, 604) \text{ MeV}$$



$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (94, 704) \text{ MeV}$$

Ripples of the critical point

Canonical corrections via subensemble acceptance

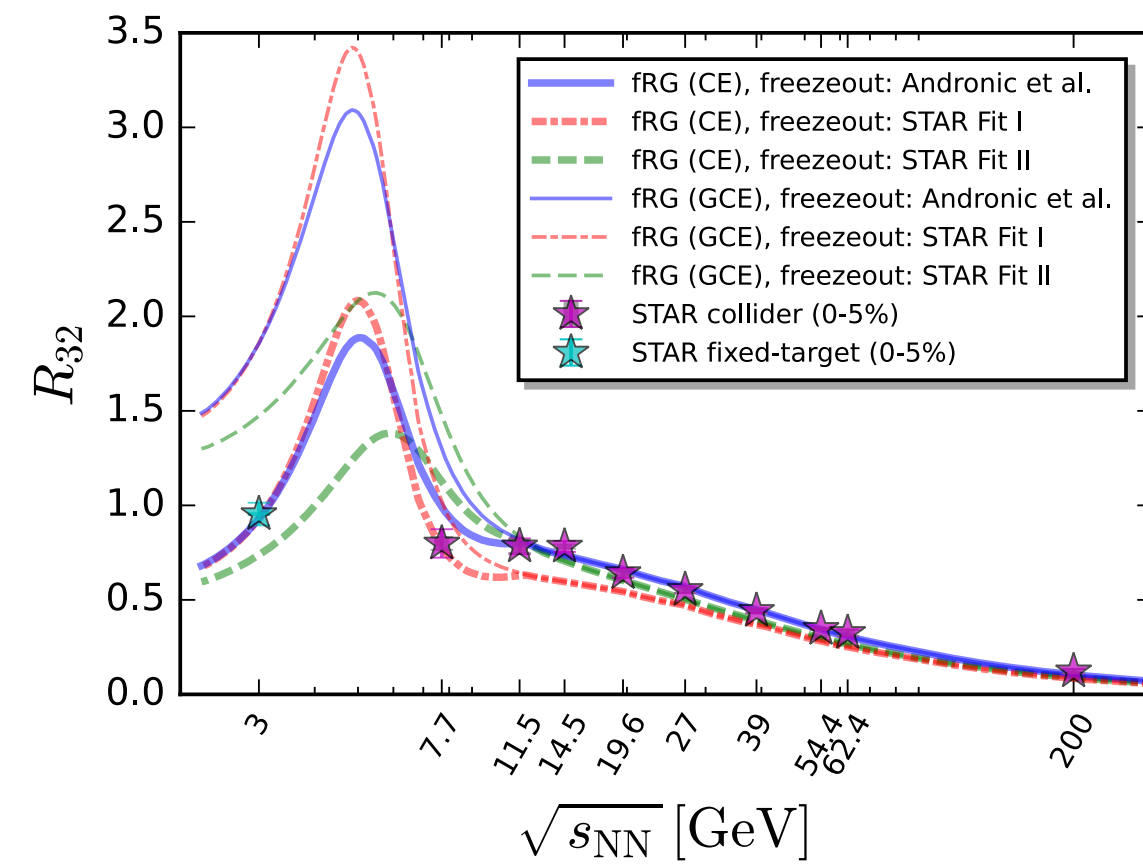
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha)R_{32}^B$$



Ripples of the critical point

Canonical corrections via subensemble acceptance

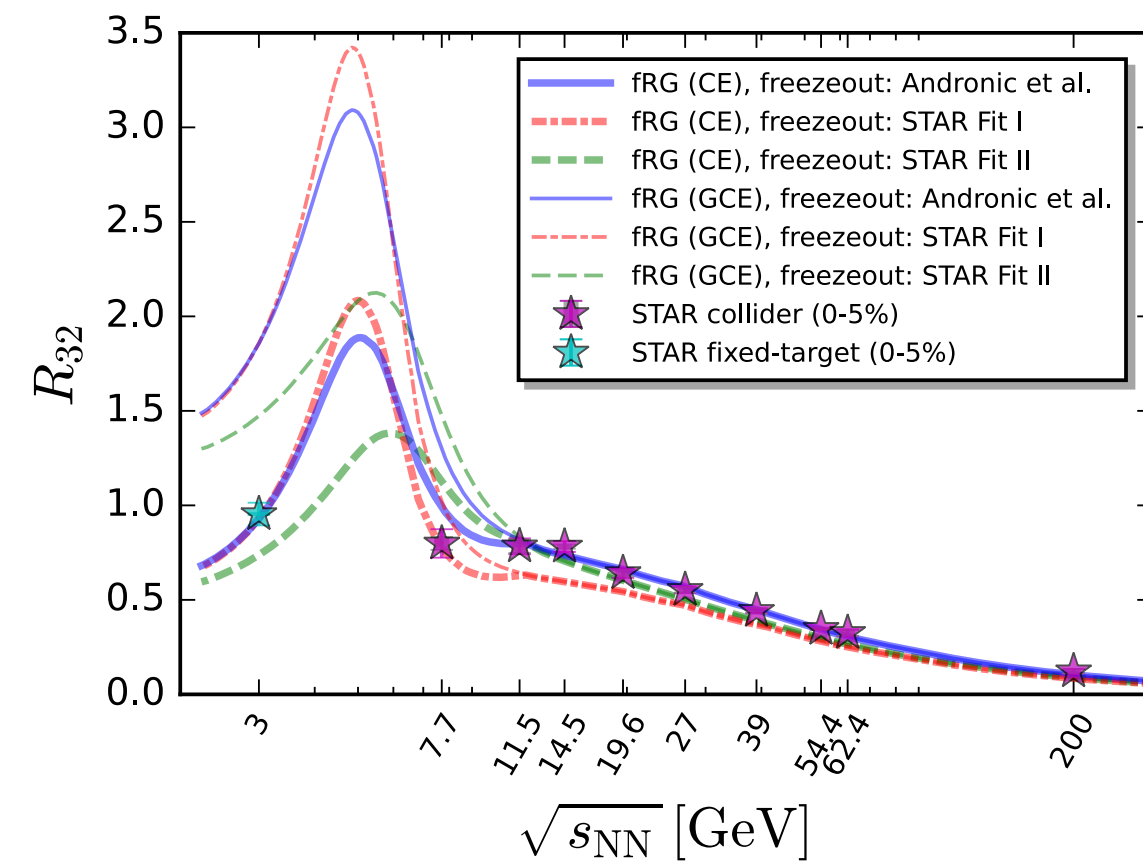
Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

subensemble volume system volume

$$V_1 = \alpha V$$

$$\bar{R}_{32}^B = (1 - 2\alpha)R_{32}^B$$



qualitative adjustment

$$\alpha(\bar{s}) = a \left(1 - \sqrt{\bar{s}}\right) \theta(1 - \bar{s})$$

$$a = 0.33 \quad \sqrt{\bar{s}} = \frac{\sqrt{s_{\text{NN}}}}{11.9 \text{ GeV}}$$

Ripples of the critical point

Canonical corrections via subensemble acceptance

Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

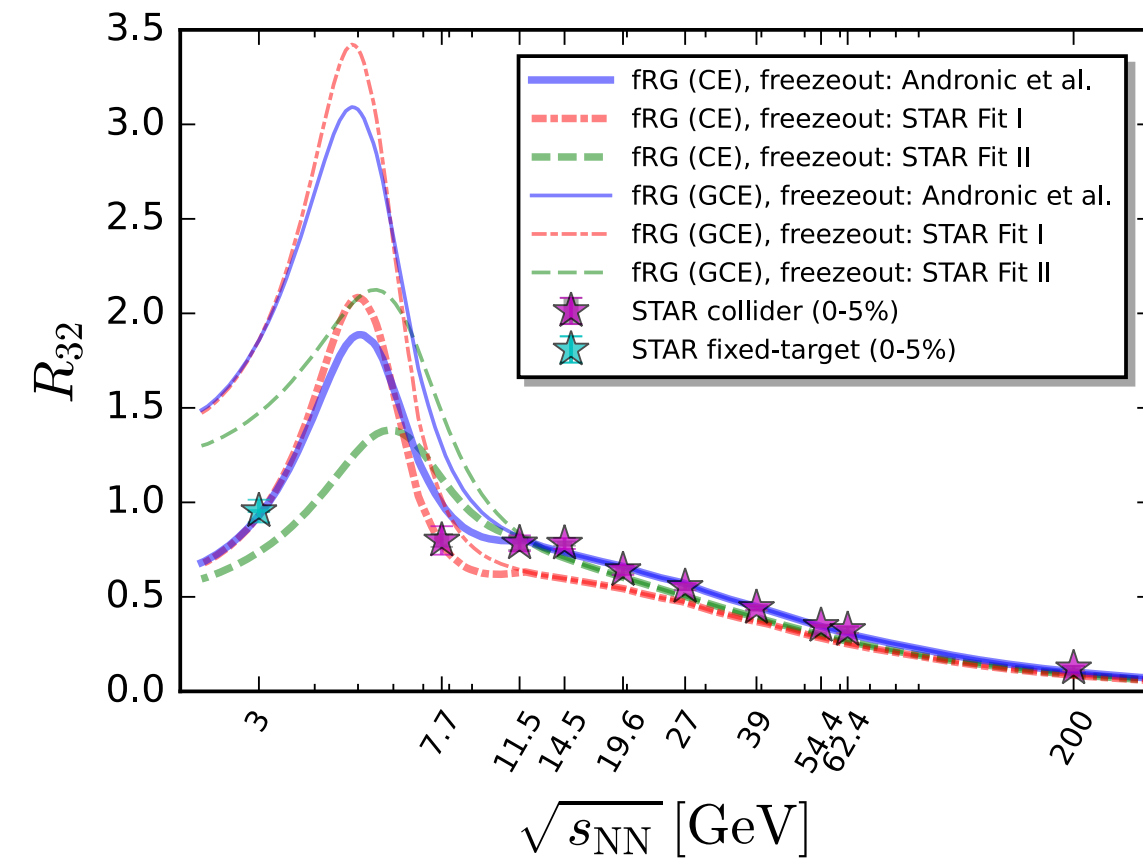
fixing the subensemble volume

baryon & proton number fluctuations

subensemble volume system volume

$$V_1 = \alpha V$$

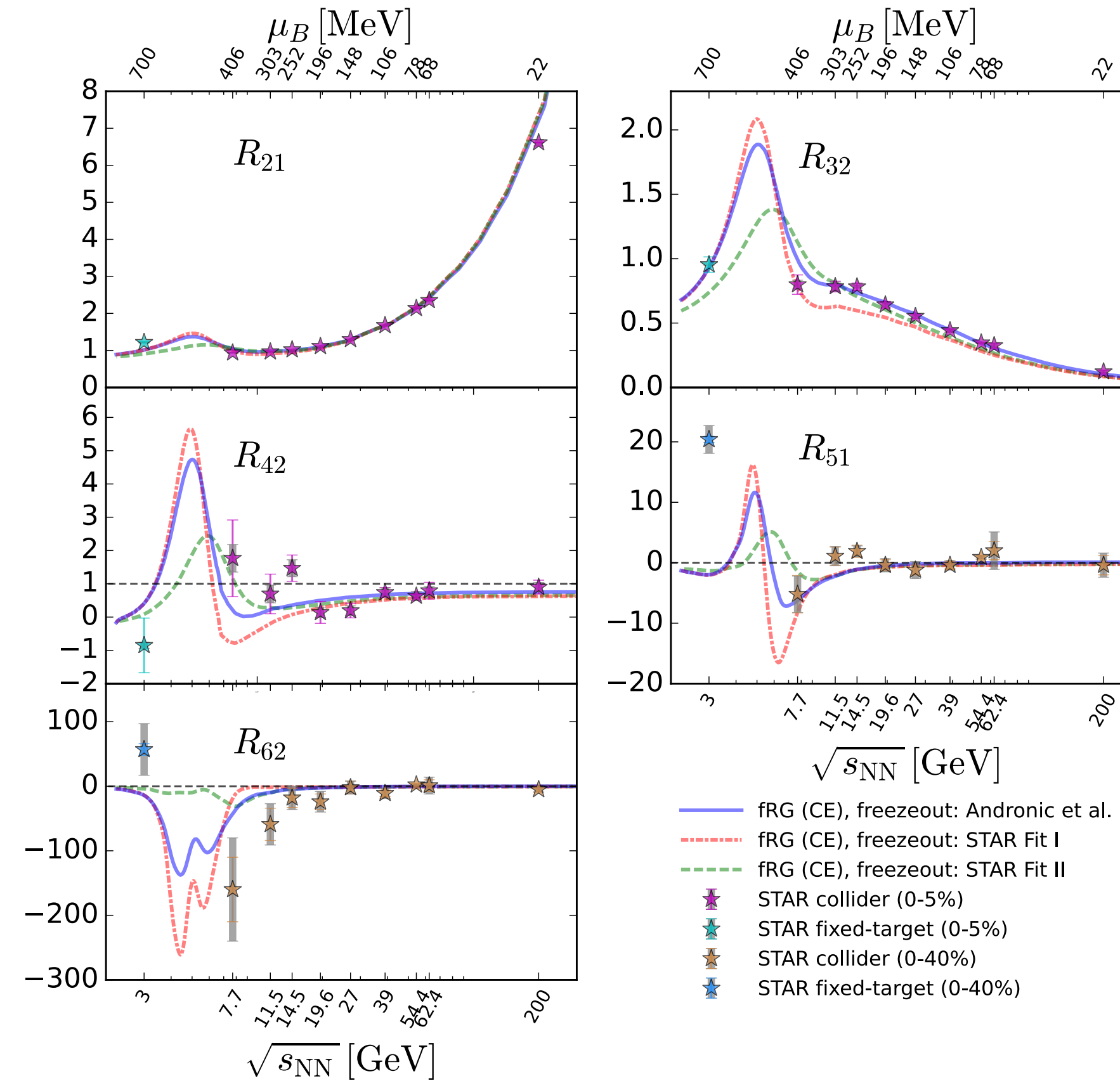
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Ripples of the critical point

Canonical corrections via subensemble acceptance

Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

fixing the subensemble volume

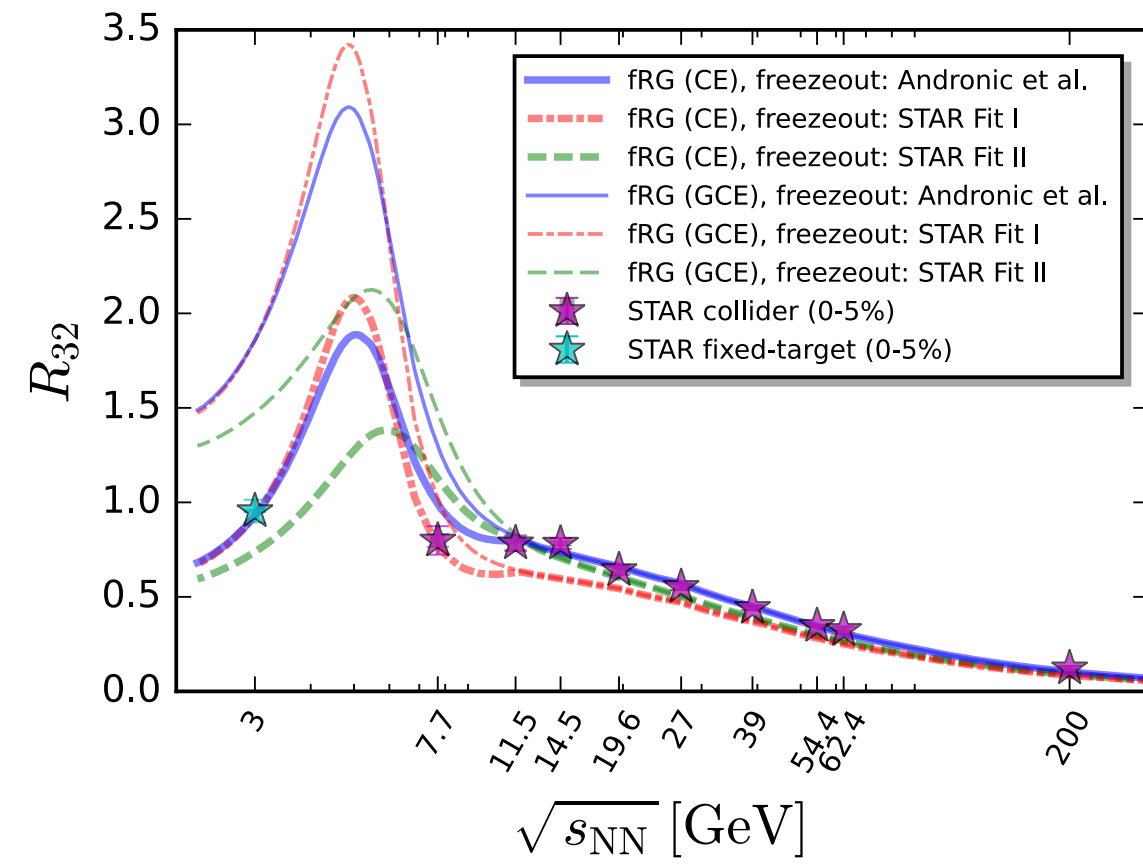
baryon & proton number fluctuations

Sneak preview

subensemble volume system volume

$$V_1 = \alpha V$$

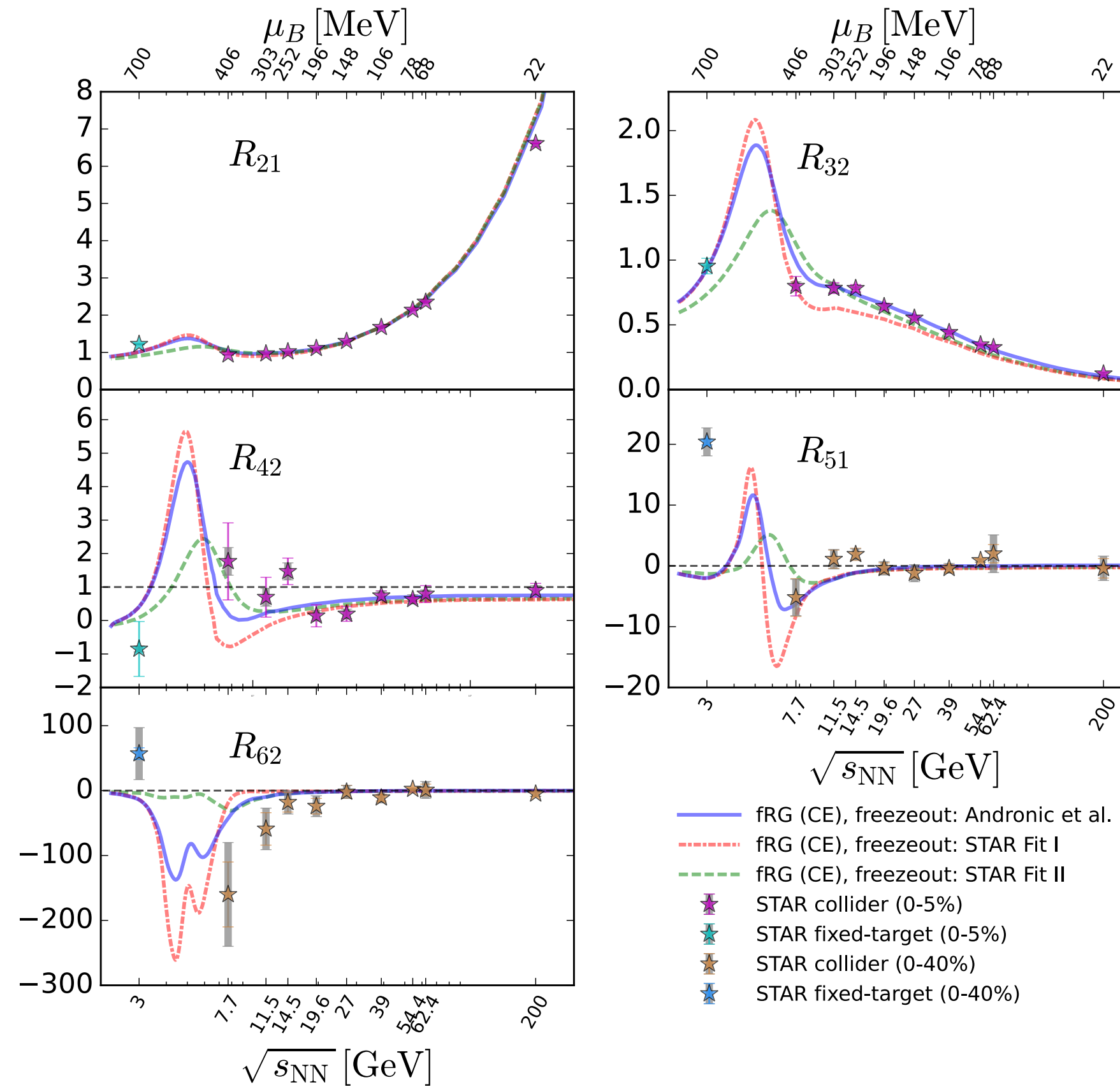
$$\bar{R}_{32}^B = (1 - 2\alpha)R_{32}^B$$



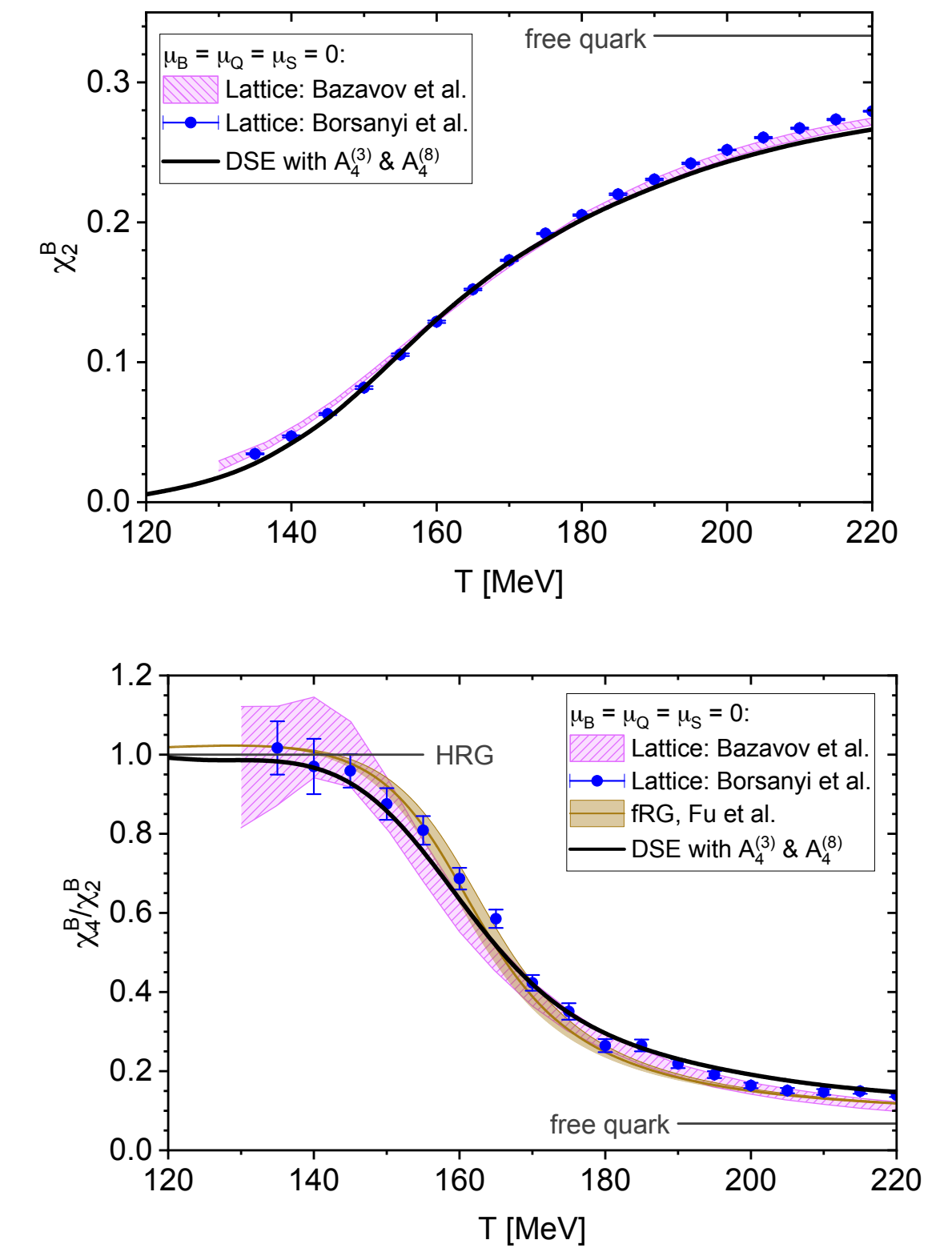
qualitative adjustment

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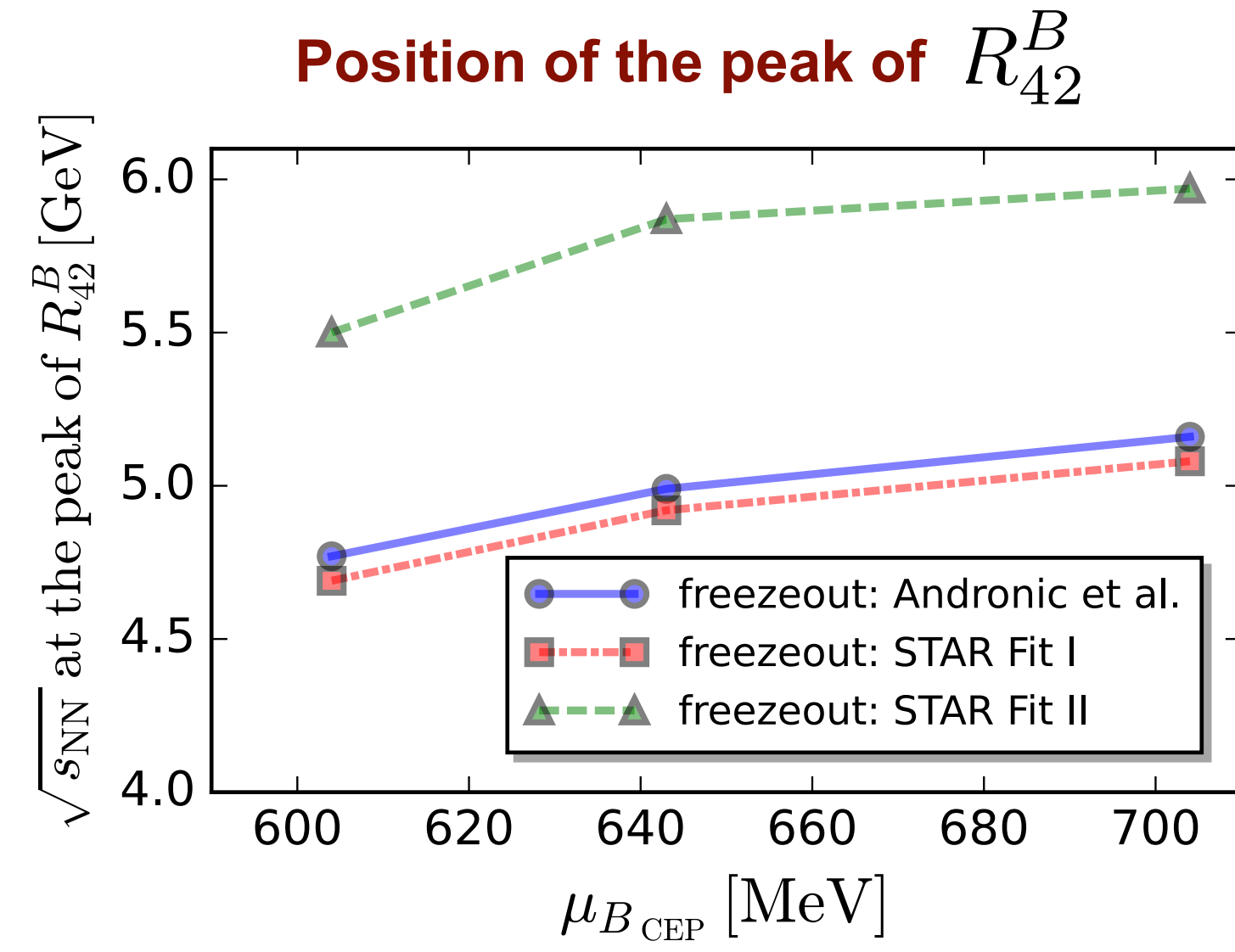


baryon fluctuations with functional QCD

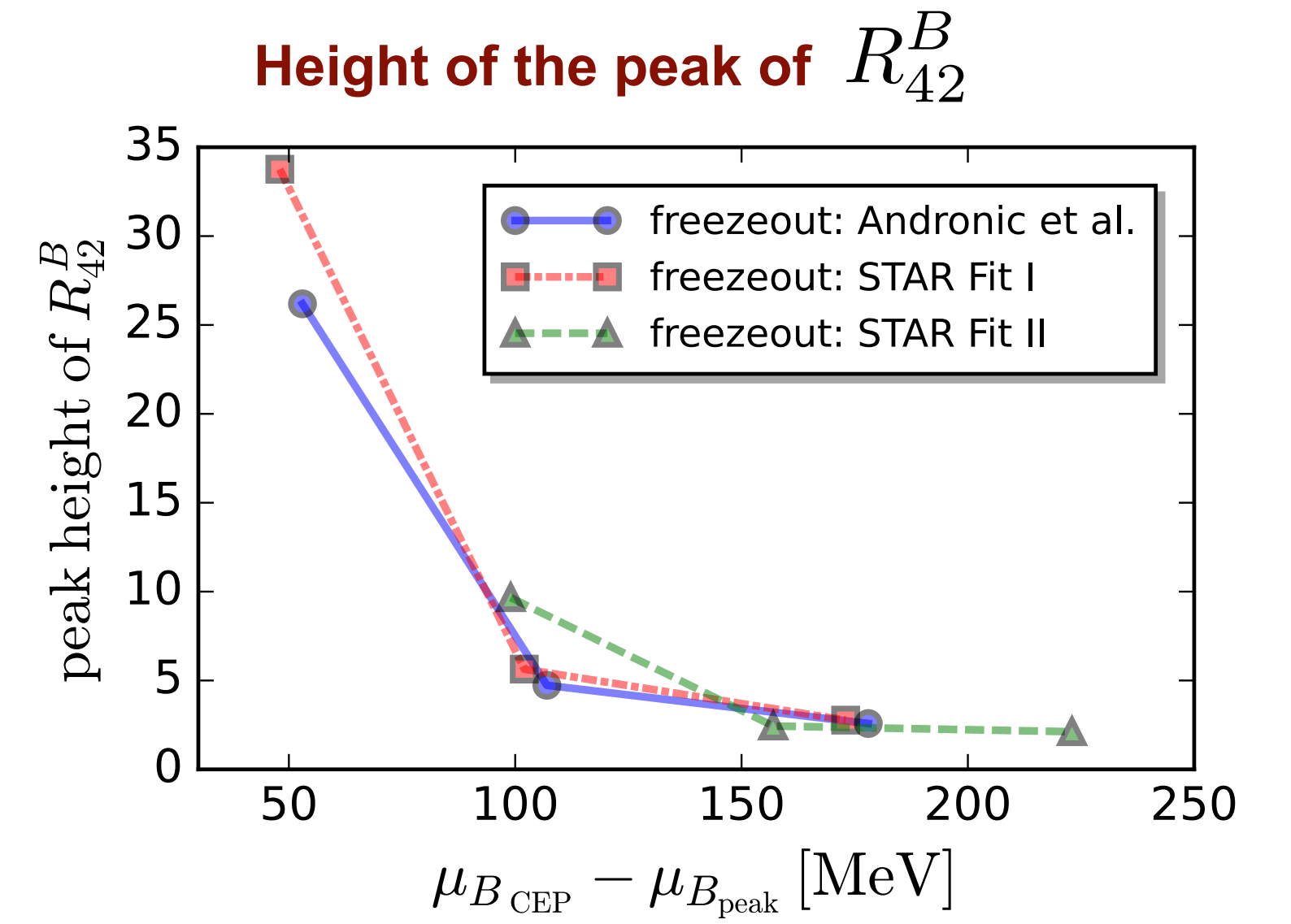


Gao, Lu, JMP, in preparation

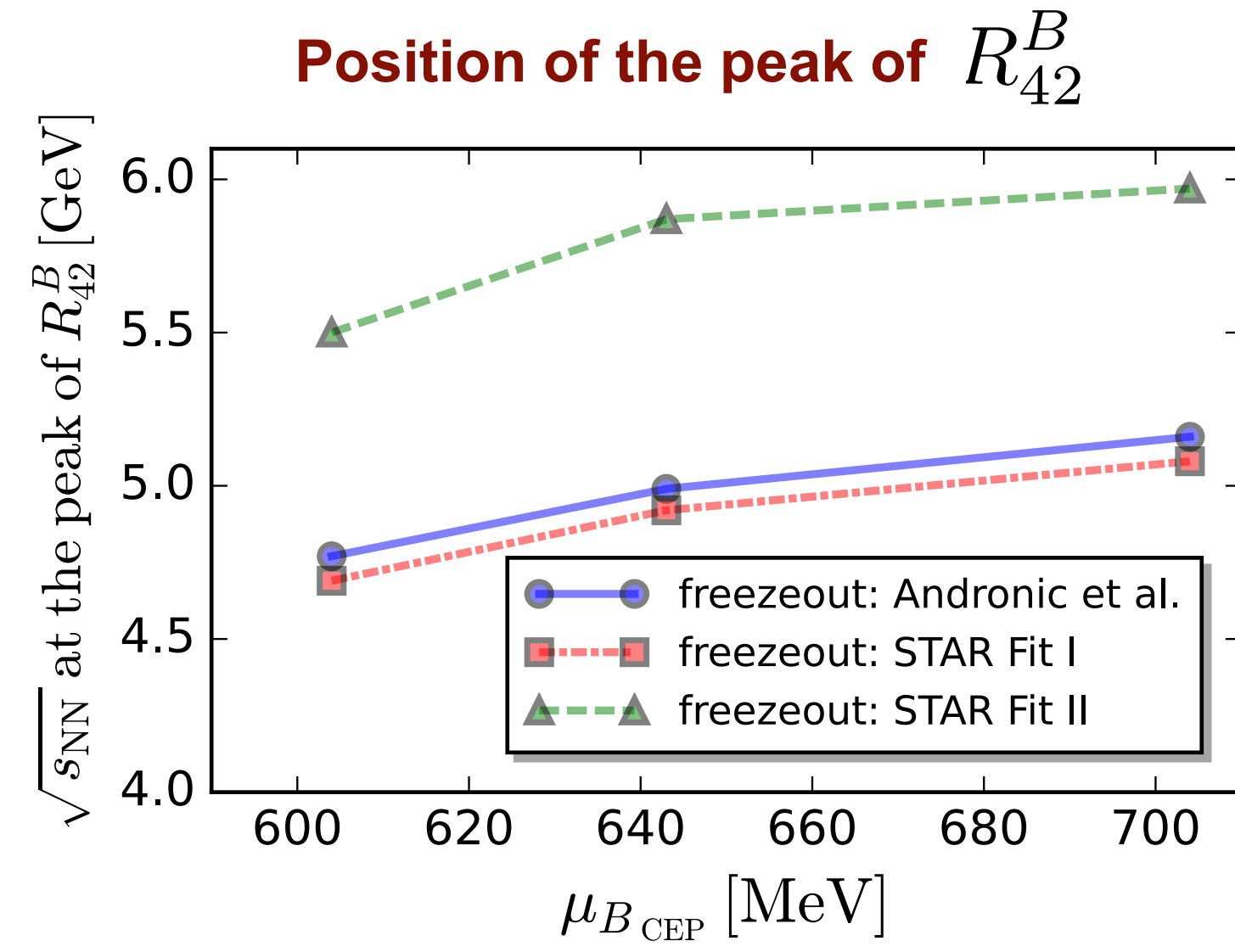
Ripples of the critical point



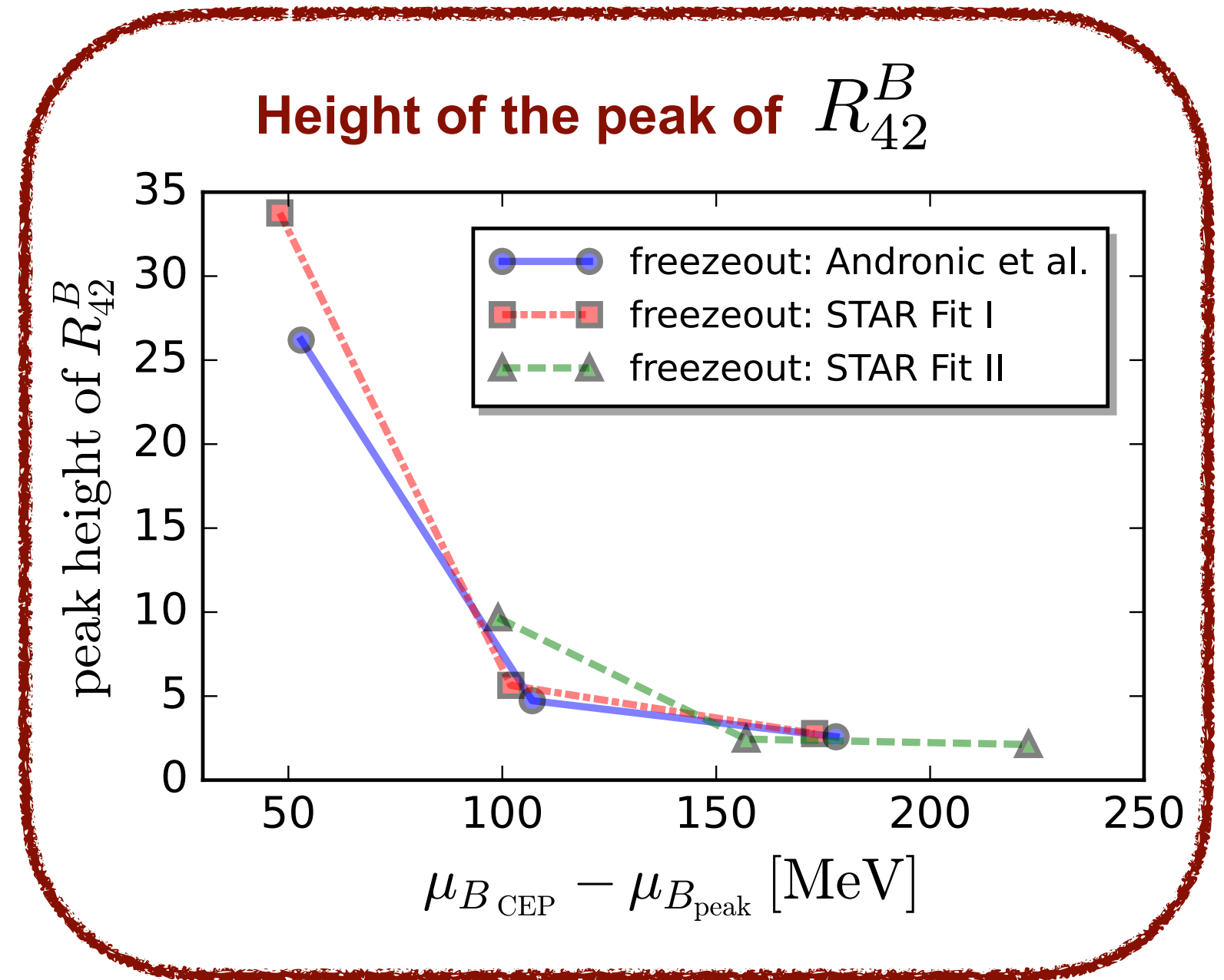
Reconstructing the CEP



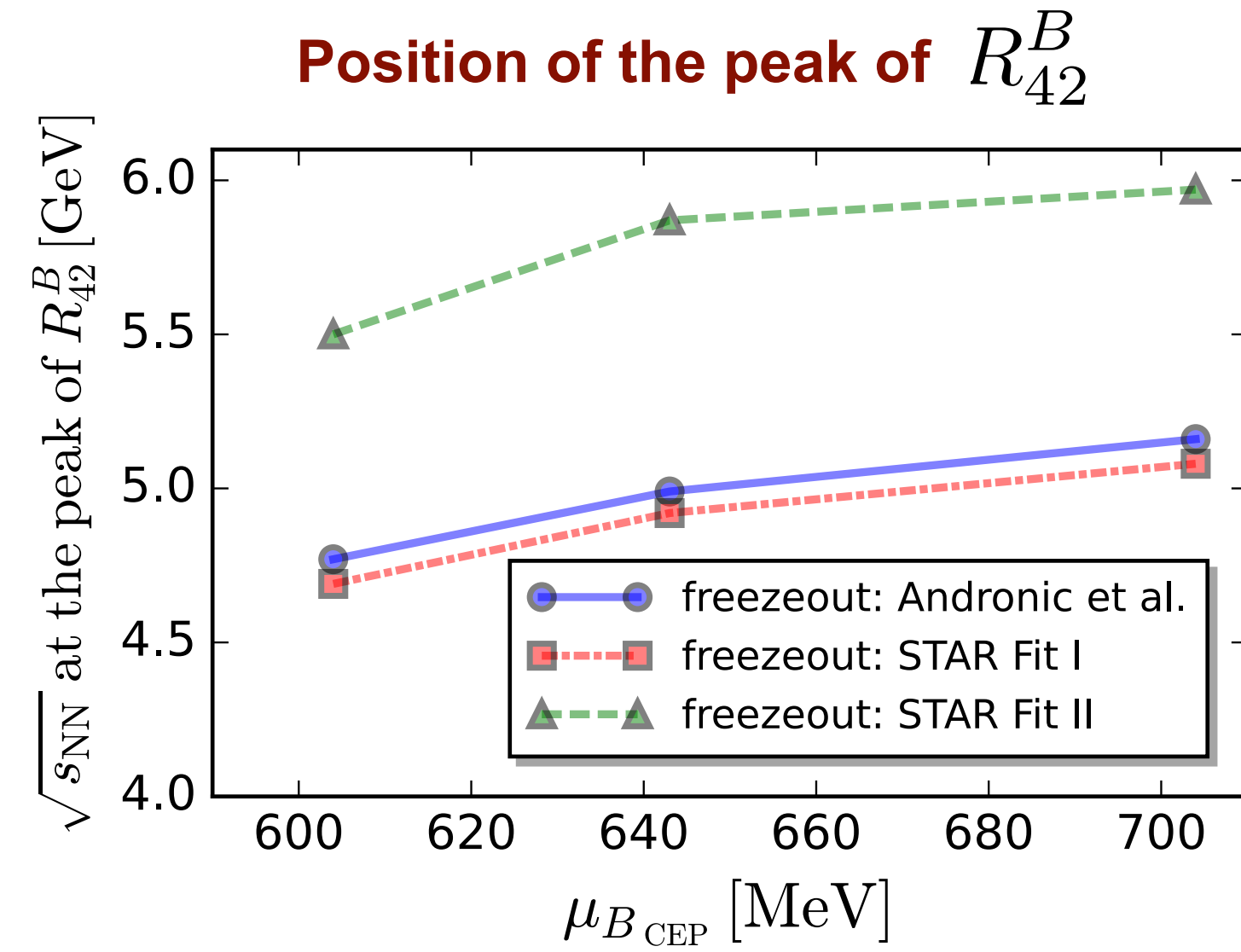
Ripples of the critical point



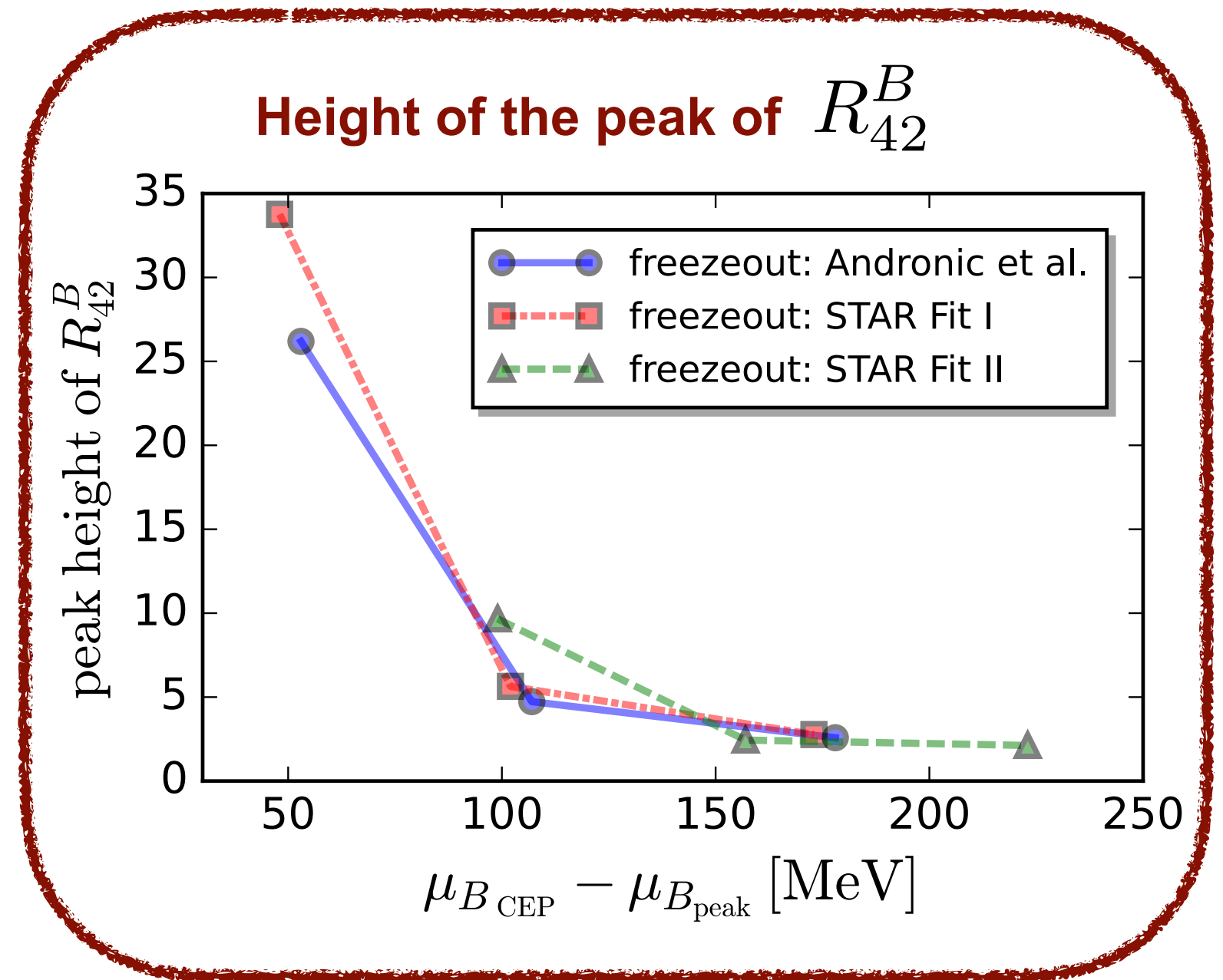
Reconstructing the CEP



Ripples of the critical point



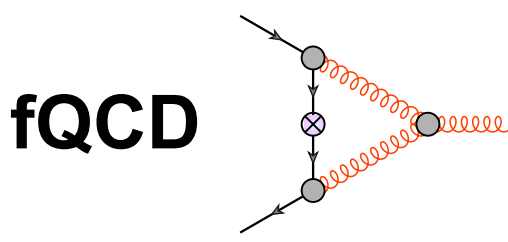
Reconstructing the CEP



Unfolding the high density regime with new phases & physics

Great opportunity for a combined high precision analysis of high density QCD (Exp. data + lattice QCD + functional QCD)

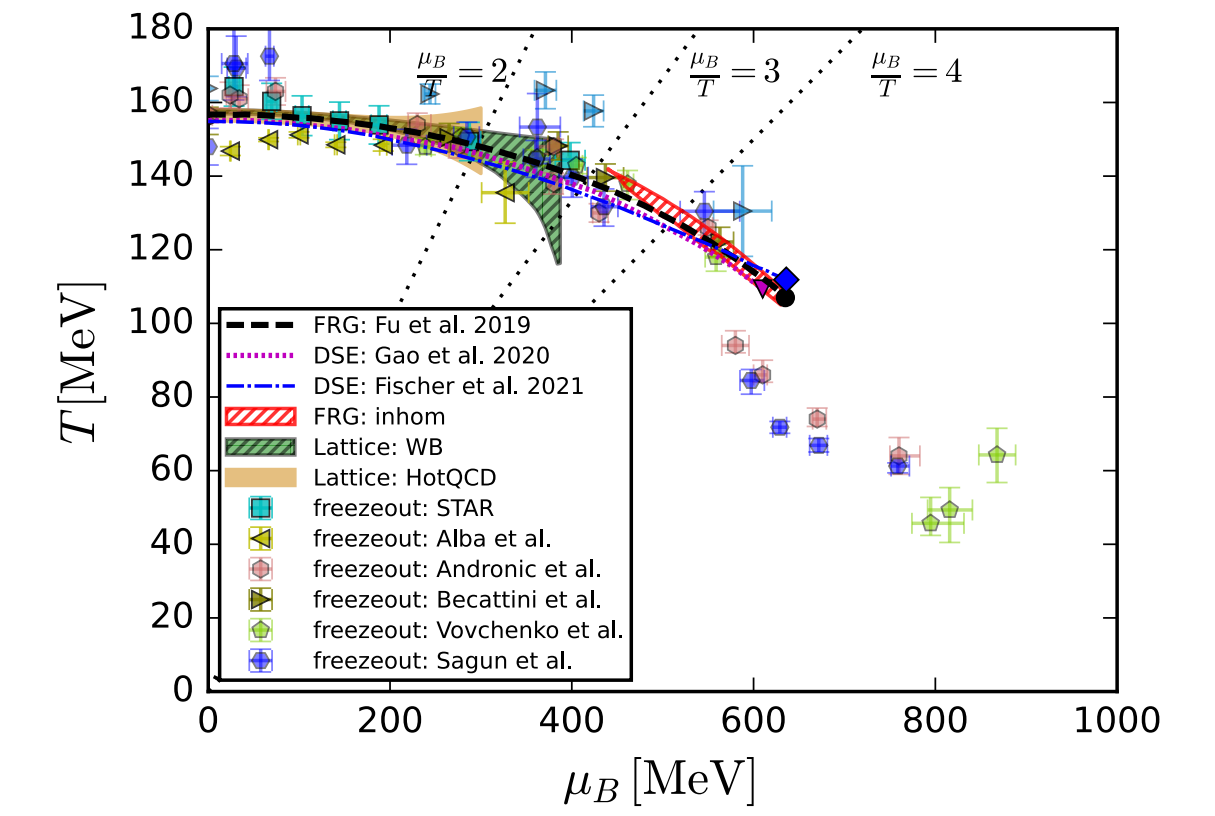
Summary



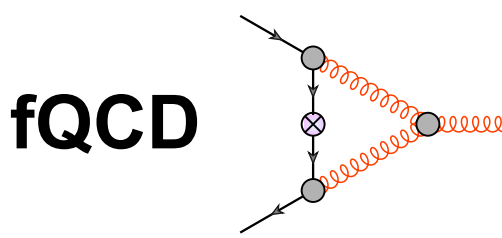
- Functional QCD provides direct 1st principle results for the phase structure at finite density

- Predictions: $\frac{\mu_B}{T} \lesssim 4$

- Estimates: $4 \lesssim \frac{\mu_B}{T} \lesssim 8$



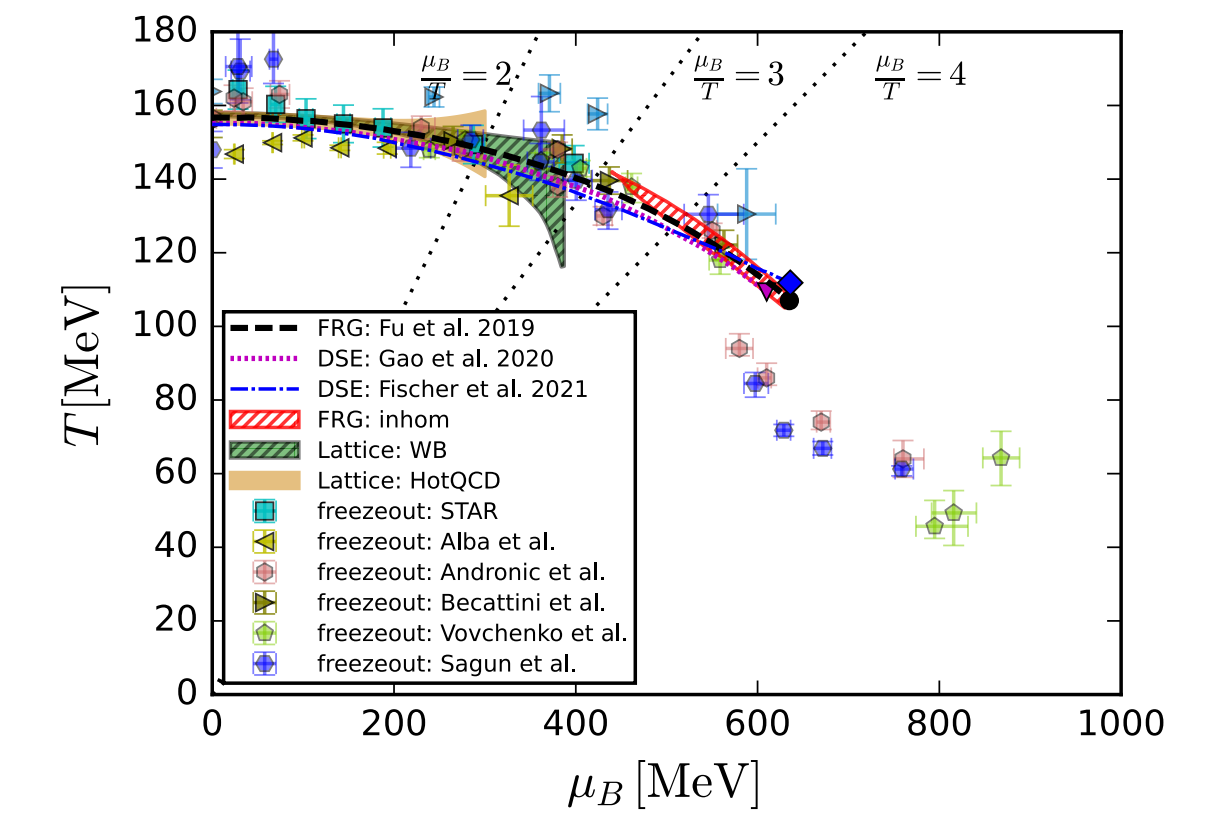
Summary



- Functional QCD provides direct 1st principle results for the phase structure at finite density

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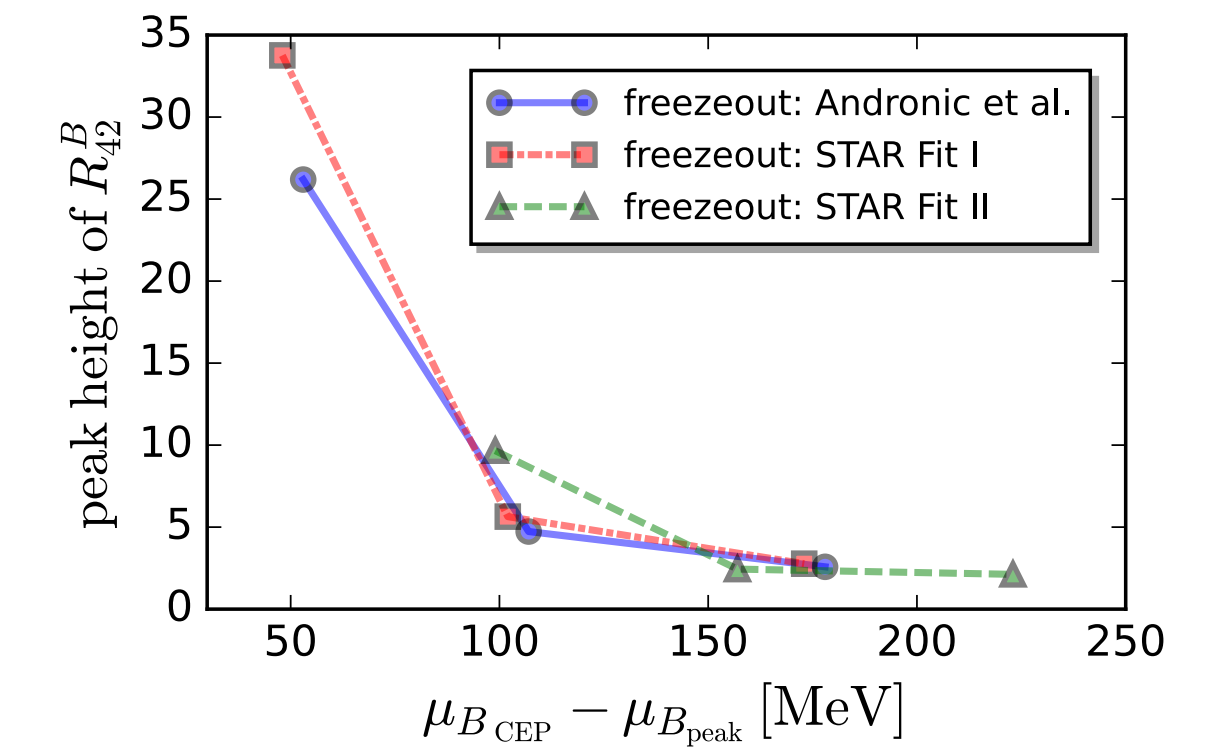


- Functional QCD results support the use of low energy effective theories for phenomenological applications

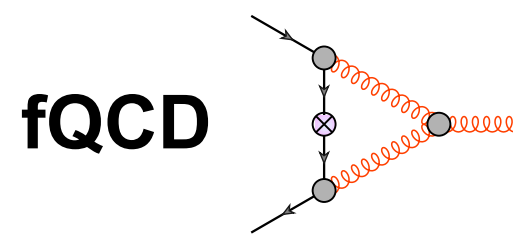
- Explanation for the convergence of CEP locations of extrapolation approaches

- Likely scenario: tiny critical regime

- Access to observables such as fluctuations of conserved charges



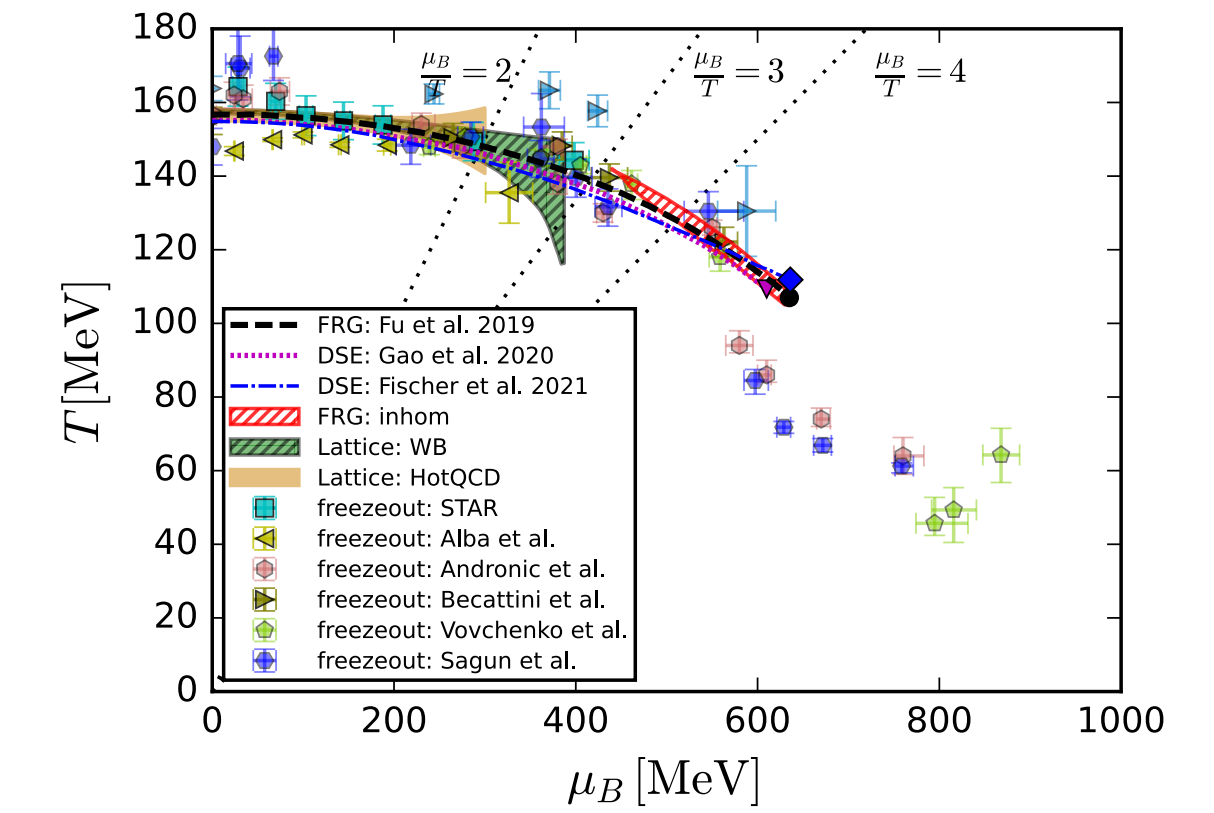
Summary



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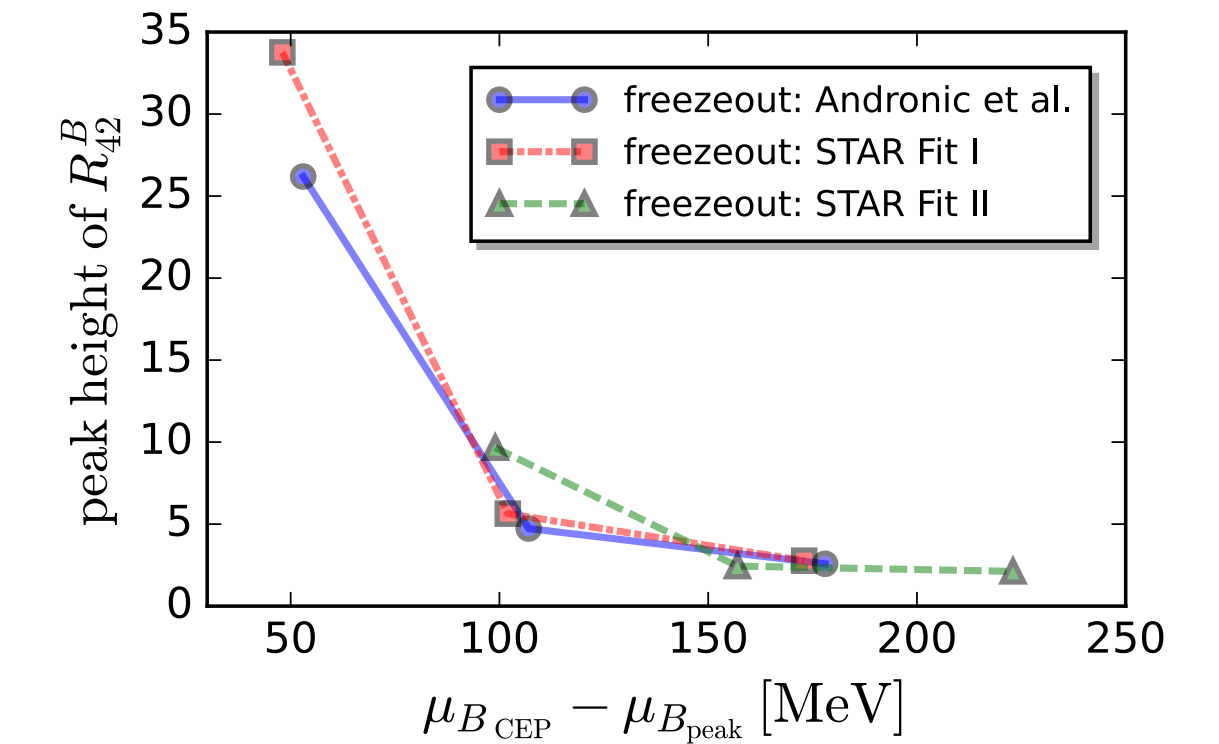
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- Functional QCD results support the use of low energy effective theories for phenomenological applications

- Explanation for the convergence of CEP locations of extrapolation approaches
- Likely scenario: tiny critical regime
- Access to observables such as fluctuations of conserved charges



- Systematic error estimates with the LEGO[®] principle

CEP Estimate \rightarrow CEP Prediction

Diquarks/baryons: ✓ Density channel/mode: (✓) Moat/inhomogeneous regime: ((✓))

Stay tuned